Finite Automata

204213 Theory of Computation

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Outline

An automatic door

- Recall our automatic door example from last time?

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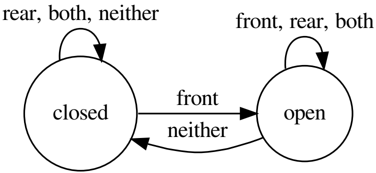
- Let's see a simulation.

What did you see?



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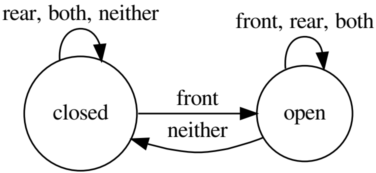

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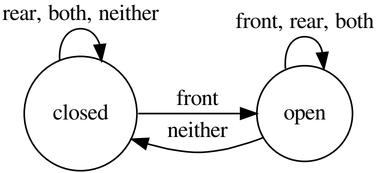

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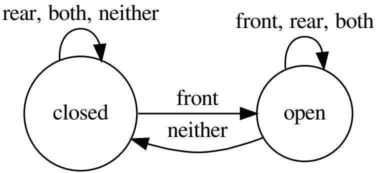

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- There are 4 possible inputs, and the state of the machine changes (or remains) after each input.

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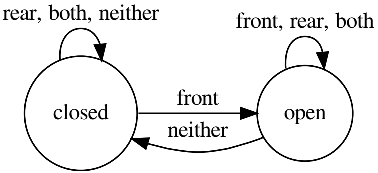
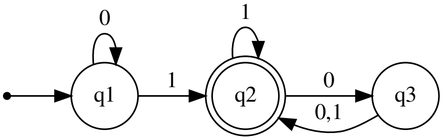


- There are two **states**: closed and open
- There are 4 possible inputs, and the state of the machine changes (or remains) after each input.
- See that in table form:

	neither	front	rear	both
closed	closed	open	closed	closed
open	closed	open	open	open

Another automaton: M_1



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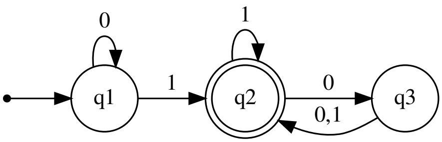

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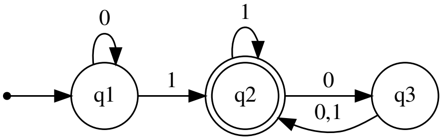

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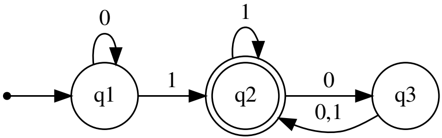

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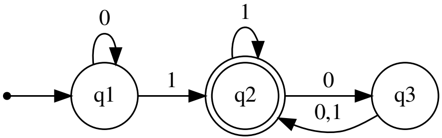

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- There are 3 states: q_1, q_2, q_3.
- q_1 is the **start state**. (see the arrow?)
- q_2 is the **accept state**. (see the double circle)
- Arrows are **transitions**.

Formal definition: why?

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Formal definition gives

- Precision

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- Notation

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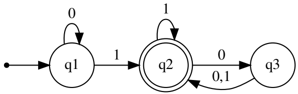
Definition [finite automaton]

A **finite automaton** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where

1. Q is a finite set called the *states*,
2. Σ is a finite set called the *alphabet*,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the *transition function*,
4. $q_0 \in Q$ is the *start state*, and
5. $F \subseteq Q$ is the *set of accept states*.

Formal definition of M_1



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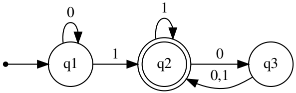


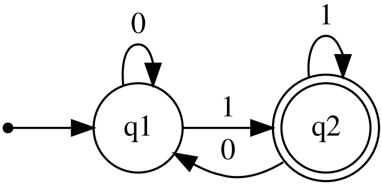
$M_1 = (Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0, 1\}$,
3. δ can be described as

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

4. q_1 is the start state, and
5. $F = \{q_2\}$.

Finite automaton M_2



Language of a machine

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- We write $L(M) = A$.
- We also say that **M recognizes A**.

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- Let $A = \{w | w$ contains at least one 1 and an even number of 0's follow the last 1$\}$.
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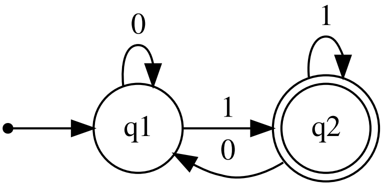
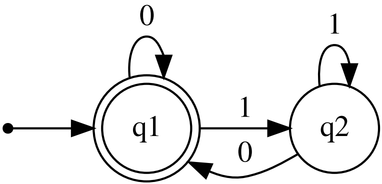
Finite automaton M_2



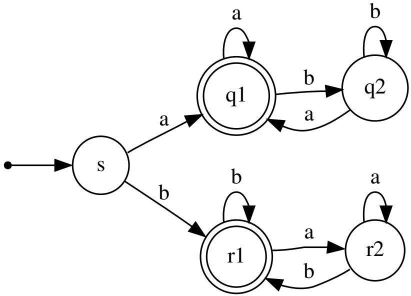
What is the language that M_2 recognizes?

Finite automaton M_3



What is the language that M_3 recognizes?

Finite automaton M_4



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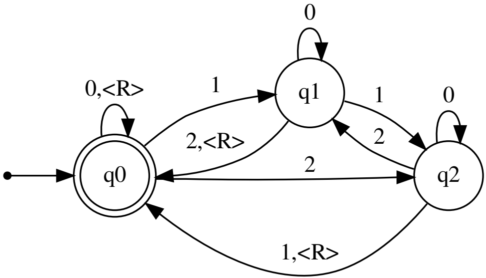
Finite automaton M_5



What is the language that M_5 recognizes?

Formal definition of computation

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton and let $w = w_1 w_2 \cdots w_n$ be a string over alphabet Σ. M **accepts** w if

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1. $r_0 = q_0$,
2. $\delta(r_i, w_{i+1}) = r_{i+1}$ for $i = 0, \ldots, n-1$, and
3. $r_n \in F$.

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- A language is called a **regular language** if some finite automaton recongnizes it.

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- Think about what you have to remember to make decision correctly. (That would be a set of states.)

Practice

Language consisting of all strings with an odd number of 1's.

Building more complex finite automata

- Let $\Sigma = \{0, 1, 2\}$.
- Can you build a finite automaton M_3 that accepts all strings whose sums are divisible by 3?

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Building more complex finite automata

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- Can you build a finite automaton M_5 that accepts all strings whose sums are divisible by 5?
- Can you build a finite automaton M_5 that accepts all strings whose sums are divisible by 3 or 5?

Construction from smaller building boxes

This is one of important ideas in computer science.

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  - some fun (?).

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- A collection of objects is **closed under some operation** if applying that operation to objects in that set only result in object in that set.
- E.g., a set of natural number $\mathcal{N}$ is closed under multiplication.

Definition [regular operations]

For a language A and B, the regular operations **union**, **concatenation**, and **star** can be defined as follows.

- Union: $A \cup B = \{x | x \in A \ \text{ or } \ x \in B\}$
- Concatenation: $A \circ B = \{xy | x \in A \ \text{ and } \ y \in B\}$
- Star: $A^* = \{x_1 x_2 \cdots x_k | k \geq 0 \ \text{ and each } \ x_i \in A\}$

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- Go back to the definition of regular languages.
  - **Goal:** to show that there exists a finite automaton recognizing $A_1 \cup A_2$,
  - **Given that:** there are finite automata M_1 and M_2 such that M_1 recognizes A_1 and M_2 that recognizes A_2.

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Approach for proving it.

- Proof by construction.
- We know that there are finite automata M_1 that recognizes A_1 and M_2 that recognizes A_2.
- We shall construct M that recognizes $A_1 \cup A_2$.

How can we show that?

- Again, pretend that you are a finite automaton trying to recognizes $A_1 \cup A_2$.

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  - Can we use them to recognize $A_1 \cup A_2$?

Given

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- Machine $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognizing A_1
- Machine $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognizing A_2

Machine M recognizing $A_1 \cup A_2$

Machine $M = (Q, \Sigma, \delta, q_0, F)$, such that

- $Q = Q_1 \times Q_2$,

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- $F = \{(r_1, r_2) | \; r_1 \in F_1 \text{ or } r_2 \in F_2\}$

Other regular operations

- Can we use the same technique to prove that $A_1 \circ A_2$ is regular?