Priority Queue

What is that?
Implementation with linked list with O(n) behaviour
The Heap (O(log(n))
An implementation using an array
Your mission ...

- Store a collection of prioritized elements
 - Elements are comparable
- Allow insertion of an element
- Can only remove the element with highest priority
 - Comes first in order

Q.insert(e) e = Q.removeMin() Q.size()

Q.isEmpty()

Q.min()

We present 3 implementations

Priority queue

Example applications of a priority queue

- Dispatching processes in a computer
- Hospital waiting lists
- Standby passengers for a flight
- Queuing at call centres
- Internal data structure for another algorithm (graph algorithm)
- ...

- Store a collection of prioritized elements
 - Elements are comparable
- Allow insertion of an element
- Can only remove the element with highest priority
 - Comes first in order

Two examples on *comparing things*

This is a Vertex

```
nmport java.util.*;
public class Vertex implements Comparable<Vertex> {
   int index, degree, colour, saturation, nebDeg;
   boolean[] domain;

public Vertex (int index,int degree) {
    this.index = index;
    this.degree = degree;
    nebDeg = 0;
}
```

This is a Comparator for vertices

Using the comparator to sort an array of Vertex

```
boolean conflicts(int v,ArrayList<Integer> colourClass){
    for (int i=0;i<colourClass.size();i++){
        int w = colourClass.get(i);
        if (A[v][w] == 1) return true;
   return false;
}
void orderVertices(ArrayList<Integer> Colord){
    Vertex[] V = new Vertex[n];
   for (int i=0;i<n;i++) V[i] = new Vertex(i,degree[i]);</pre>
   for (int i=0; i<n; i++)
        for (int j=0; j<n; j++)
            if (A[i][i] == 1) V[i].nebDeq = V[i].nebDeq + degree[i];
    if (style == 1) Arrays.sort(V);
    if (style == 2) minWidthOrder(V);
    if (style == 3) Arrays.sort(V,new MCRComparator());
   for (Vertex v : V) Colord.add(v.index):
}
void minWidthOrder(Vertex[] V){
    ArrayList<Vertex> L = new ArrayList<Vertex>(n);
    Stack<Vertex> S = new Stack<Vertex>():
    for (Vertex v : V) L.add(v);
    while (!L.isEmpty()){
        Vertex v = L.get(0);
        for (Vertex u : L) if (u.degree < v.degree) v = u;
        5.push(v); L.remove(v);
        for (Vertex u : L) if (A[u.index][v.index] == 1) u.degree--;
    int k = 0;
    while (!s.isEmpty()) V[k++] = s.pop();
```

Another example: a Car

```
public class Car {
    String make, model;
    public Car(String s1,String s2){
        make = s1; model = s2;
    }
    public String make(){return make;}
    public String model(){return model;}
    public String toString(){return make +" "+ model;}
}
```

This is a CarComparator

```
import java.util.*;
public class CarComparator implements Comparator<Car> {
    public int compare(Car a,Car b){
        int c1 = a.make().compareTo(b.make());
        int c2 = a.model().compareTo(b.model());
        if (c1 == 0) return c2;
        return c1;
    }
    // make is most significant
}
```

Using the CarComparator

```
import java.util.*;
public class Test2 {
    public static void main(String args[]){
         TreeSet<Car> S = new TreeSet<Car>(new CarComparator());
         Car c1 = new Car("Citroen","C1");
         Car c2 = new Car("Ford", "Mustang");
Car c3 = new Car("Ferarri", "GTO");
Car c4 = new Car("Cadillac", "Elderado");
Car c5 = new Car("Ford", "Mustang");
         add(c1);
         5. add(c2);
         5. add(c3);
         5. add(c4);
         5. add(c5);
         System.out.println(5);
                 Z:\public_html\ads2\java\compare>javac Test2.java
                 Z:\public_html\ads2\java\compare>java Test2
                 [Cadillac Elderado, Citroen C1, Ferarri GT0, Ford Mustang]
                 Z:\public_html\ads2\java\compare>
```

We might use a linked list

- To insert we add to the front of the list
- To find the minimum we must iterate over entire the list
- To remove the minimum we must find the minimum and remove it
- Maintain a counter of number of elements in the list

Method	Time
size	O(1)
isEmpty	O(1)
insert	O(1)
removeMin	O(n)
min	O(n)

We might use a linked list

- The list is maintained in non-decreasing order
- To insert we scan to find position and splice in (see below)
- To find the minimum we deliver the first element in the list.
- To remove the minimum we return and remove the first element

```
public void insert(E s){
   if (head == null || head.getElement().compareTo(s) > 0)
      head = new Node<E>(s,head);
   else {
      Node<E> cursor = head;
      Node<E> next = cursor.getNext();
      while (next != null && next.getElement().compareTo(s) <= 0 ){
            cursor = next;
            next = next.getNext();
      }
      cursor.setNext(new Node<E>(s,next));
   }
   size++;
   Method
```

Method	Time
size	O(1)
isEmpty	O(1)
insert	O(n)
removeMin	O(1)
min	O(1)

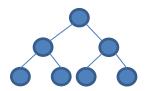
An alternative THE HEAP

• a heap H is a binary tree

heap

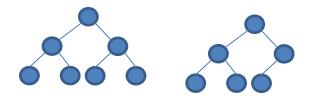
- a heap H is a binary tree
- H is a *complete* binary tree

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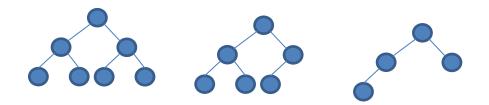
- each level but the last must be full
- in last level fill from left to right

- a heap H is a binary tree
- H is a *complete* binary tree



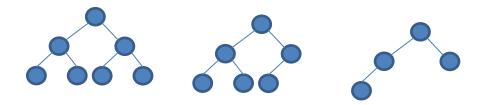
- each level but the last must be full
- in last level fill from left to right

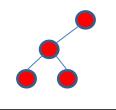
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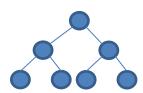


Not a heap!

- each level but the last must be full
- in last level fill from left to right

heap

- a heap H is a binary tree
- H is a *complete* binary tree
- *heap order property* is maintained



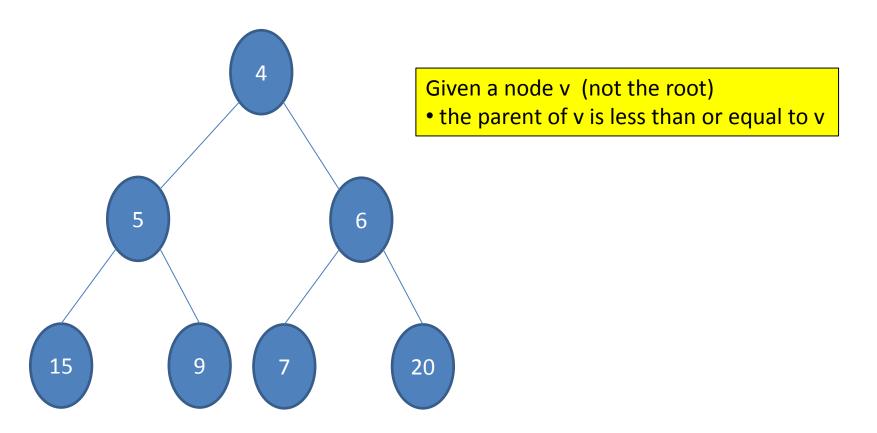
heap

- a heap H is a binary tree
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- heap order property is maintained

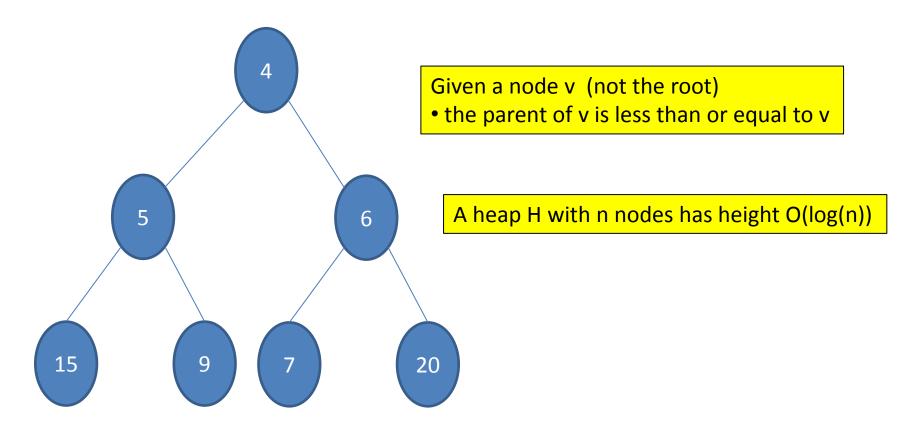
Given a node v (not the root)

• the parent of v is less than or equal to v

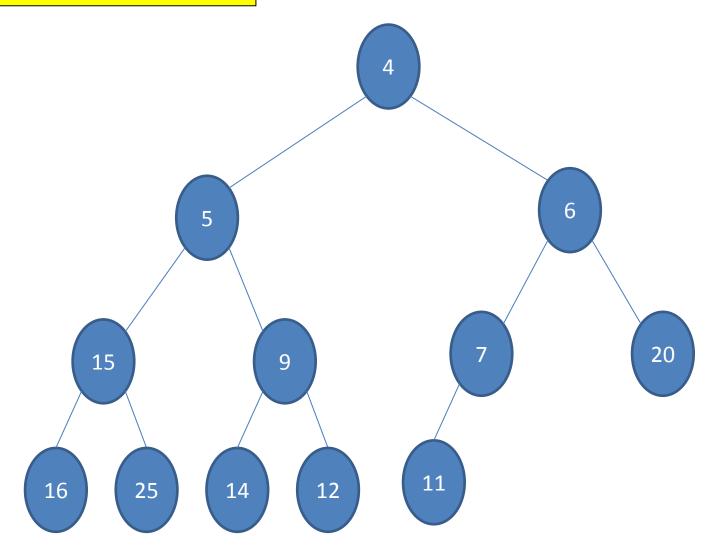
- a heap H is a binary tree
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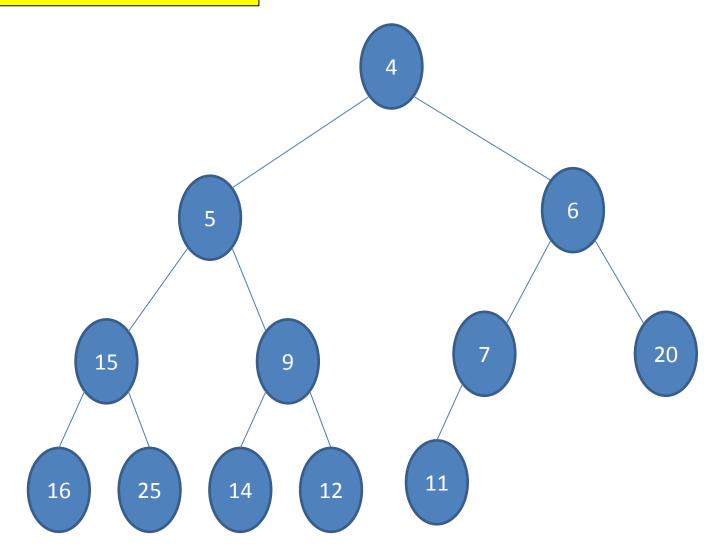


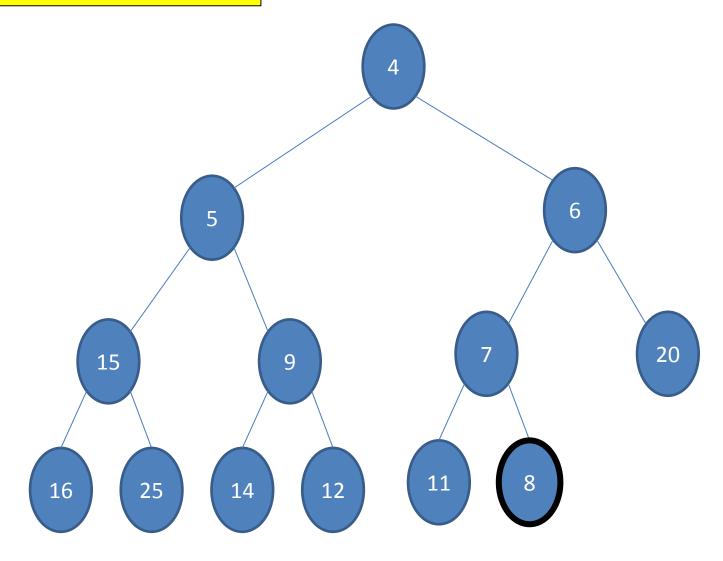
- a heap H is a binary tree
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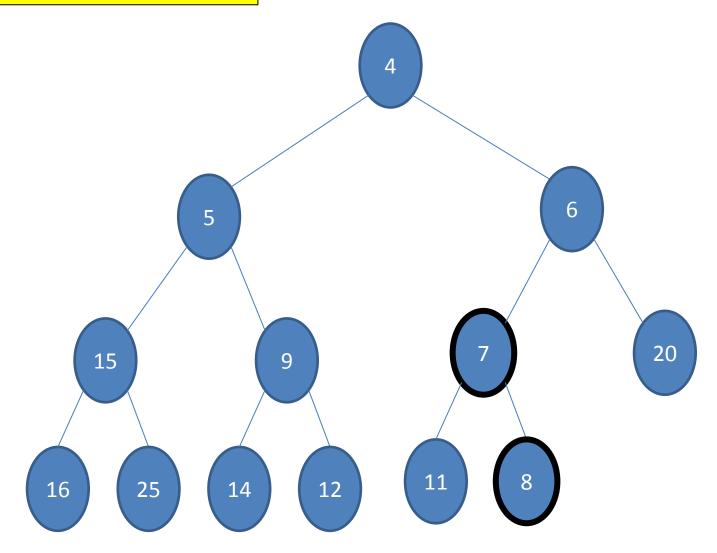


Example: adding to a heap

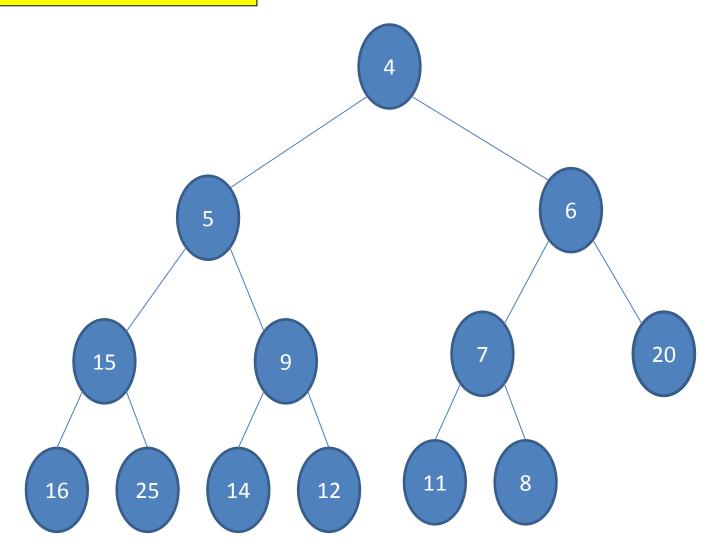


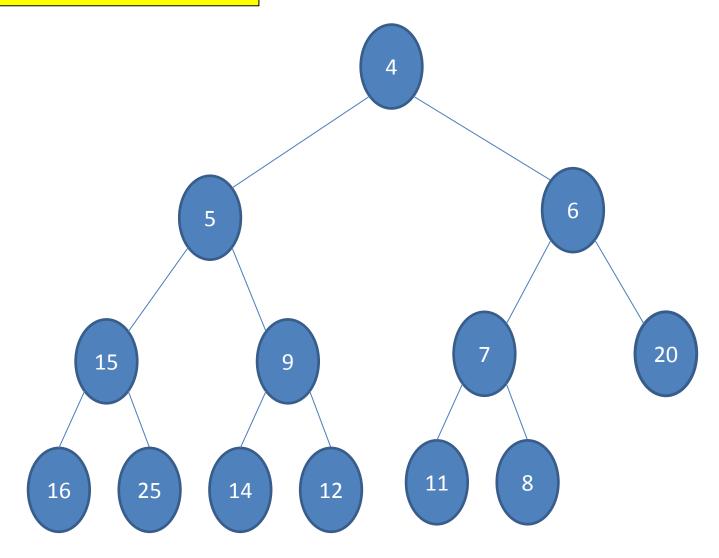


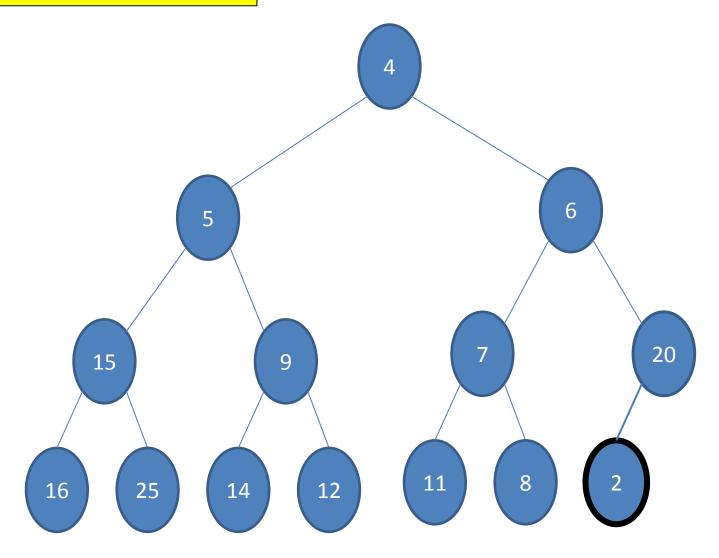


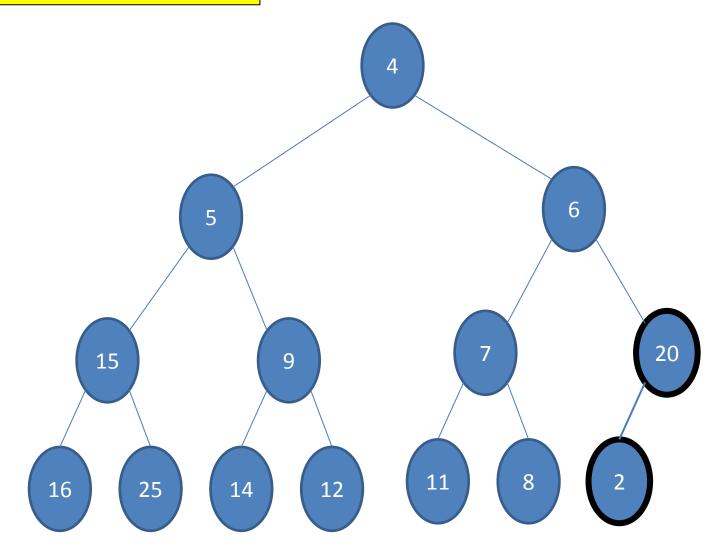


8 is greater than parent (7) ... done

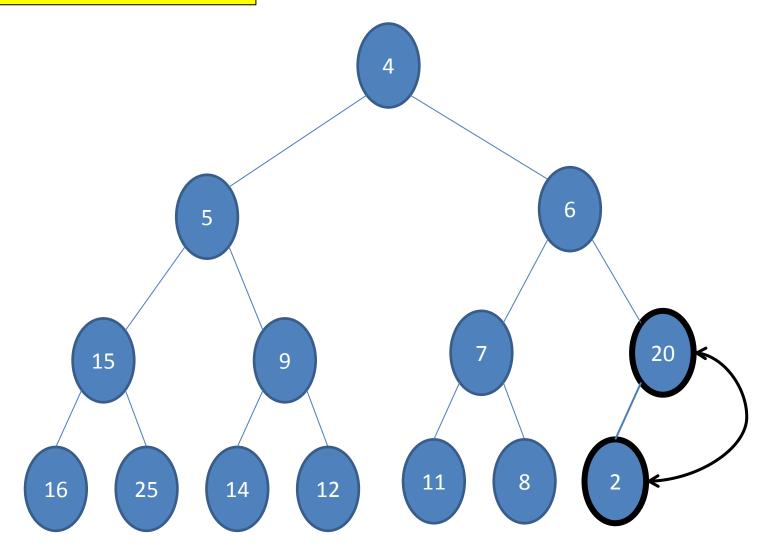


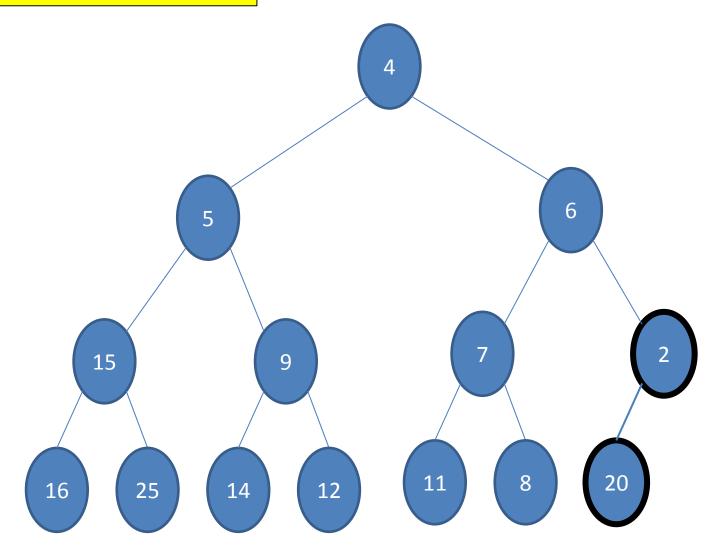


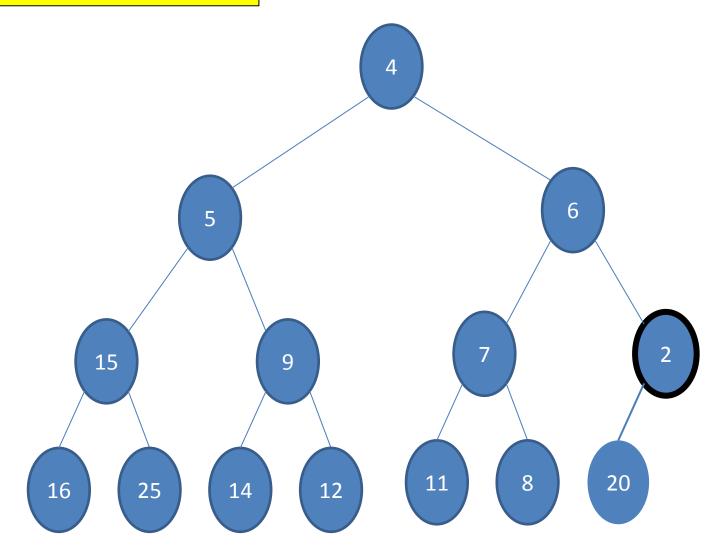


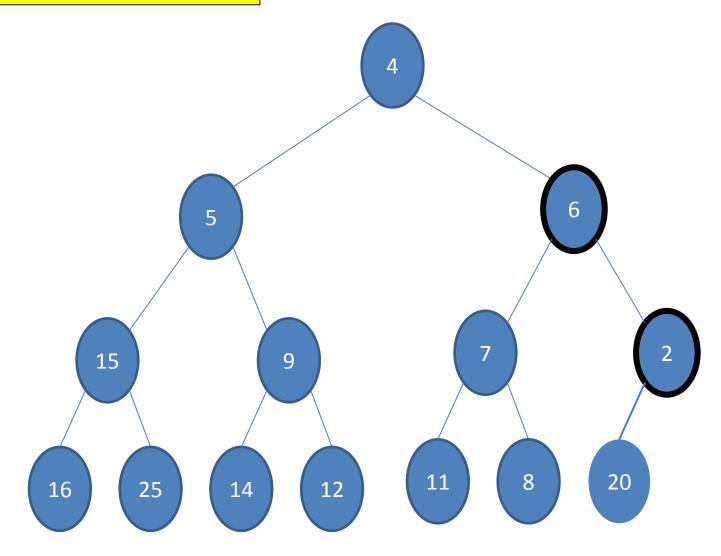


2 is less than parent 20

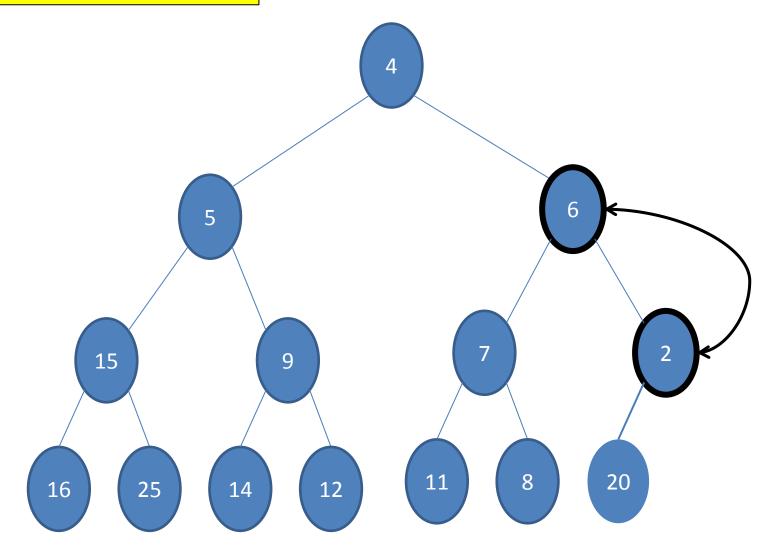




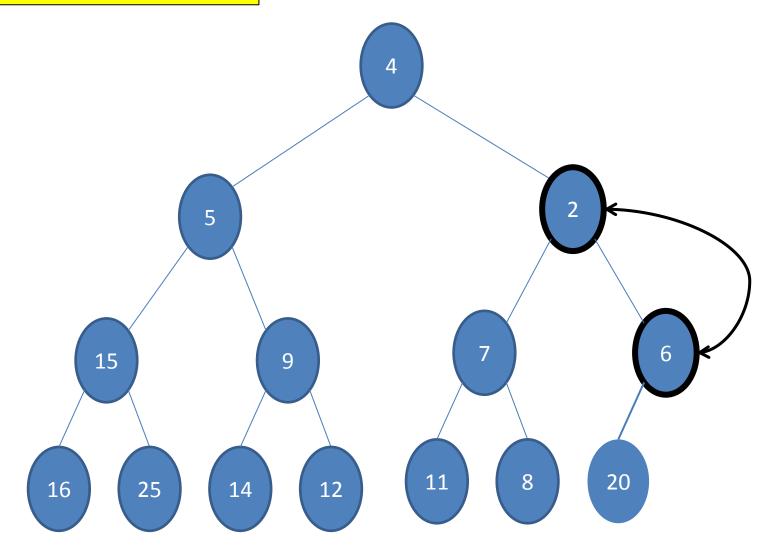




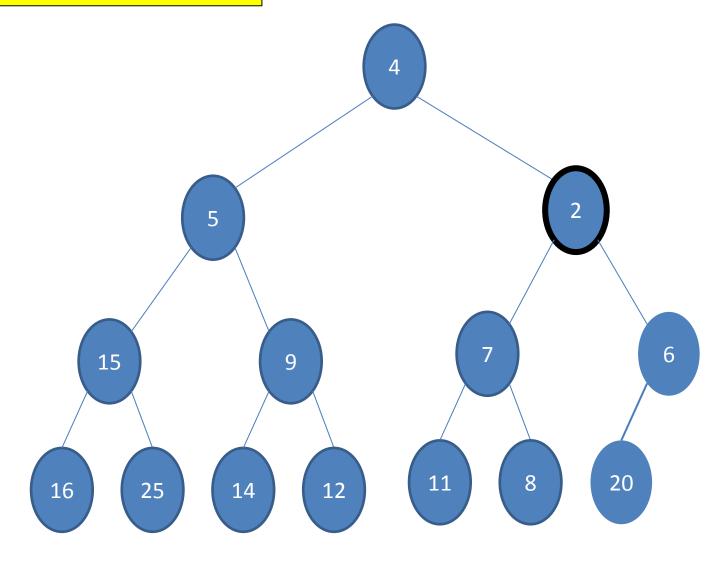
2 is less than parent 6

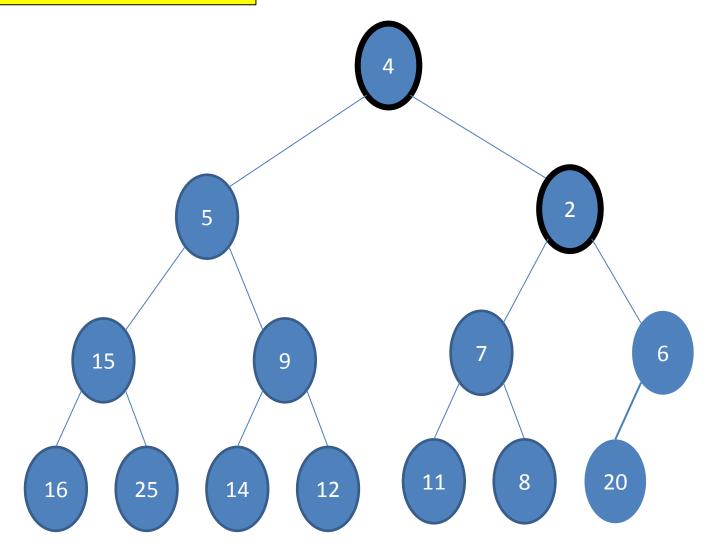


2 is less than parent 6 ... swap!

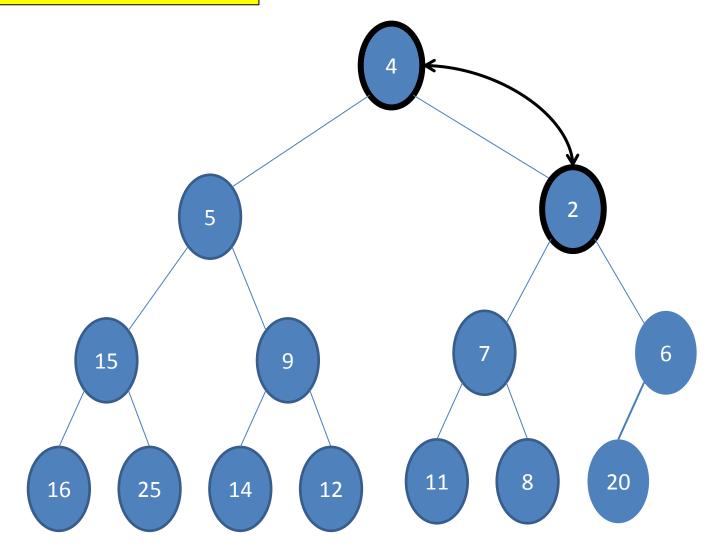


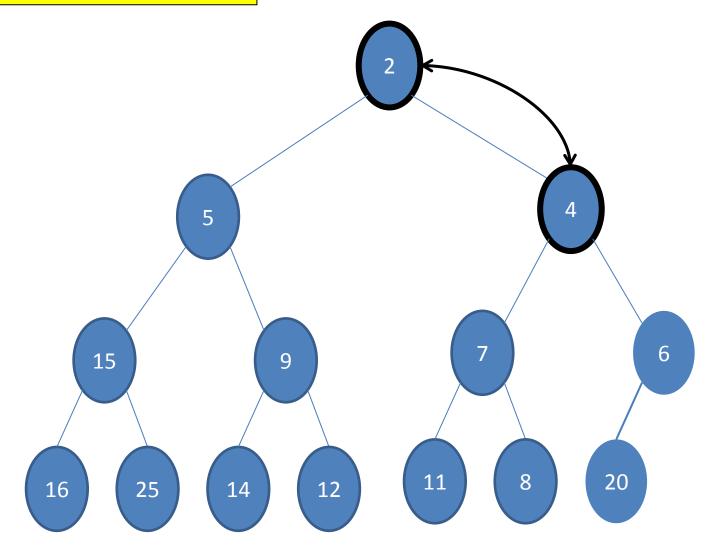
2 is less than parent 6 ... swap!



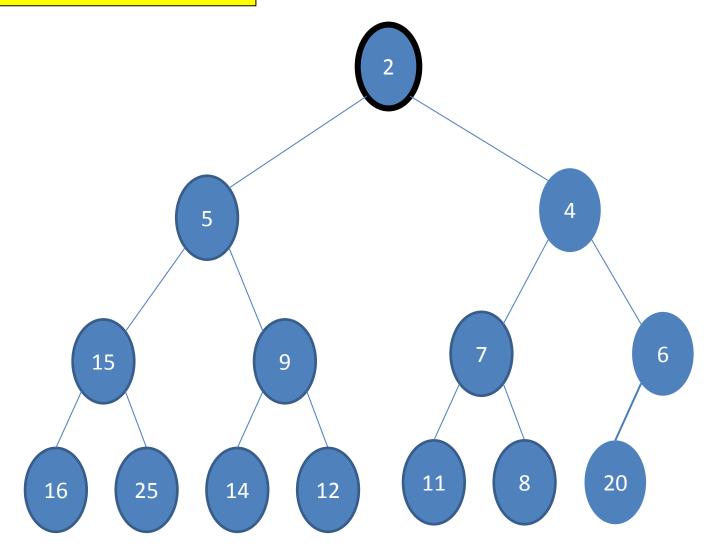


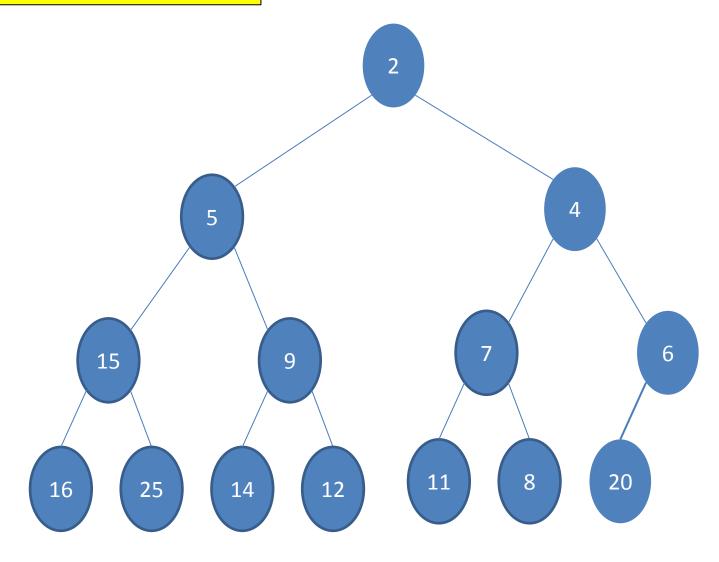
2 is less than parent 4



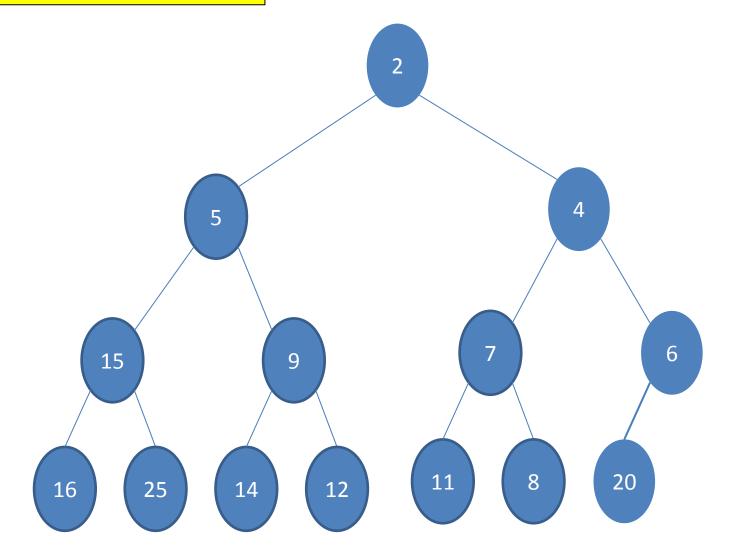


2 is less than parent 4 ... swap!



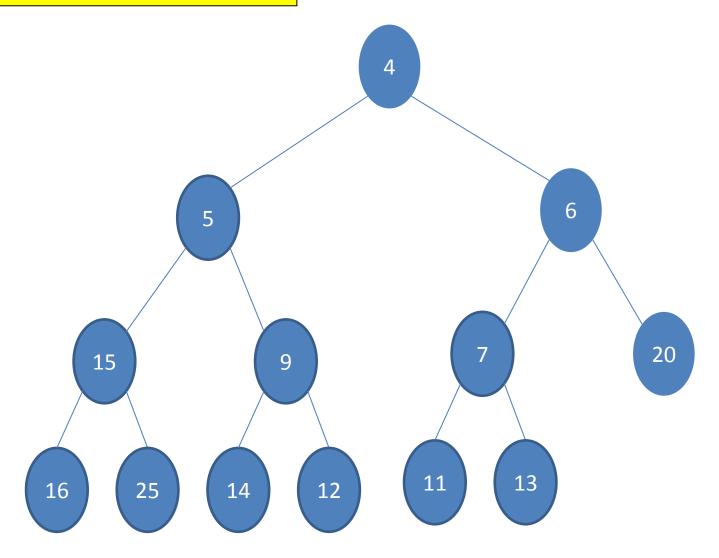


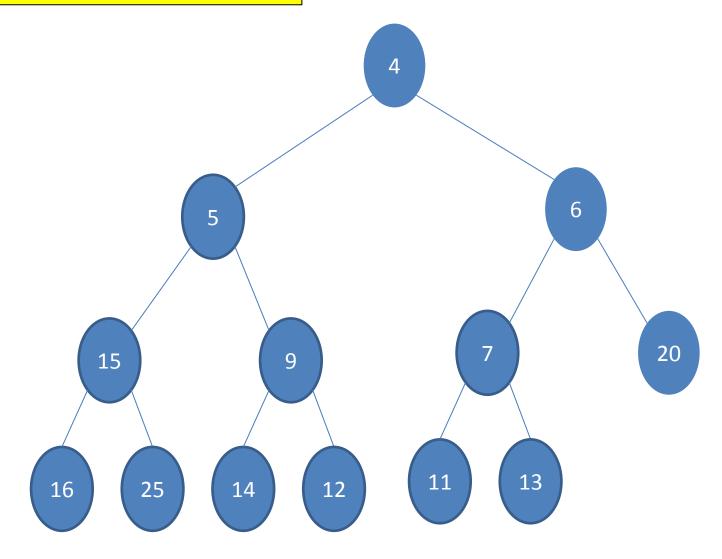
Done!



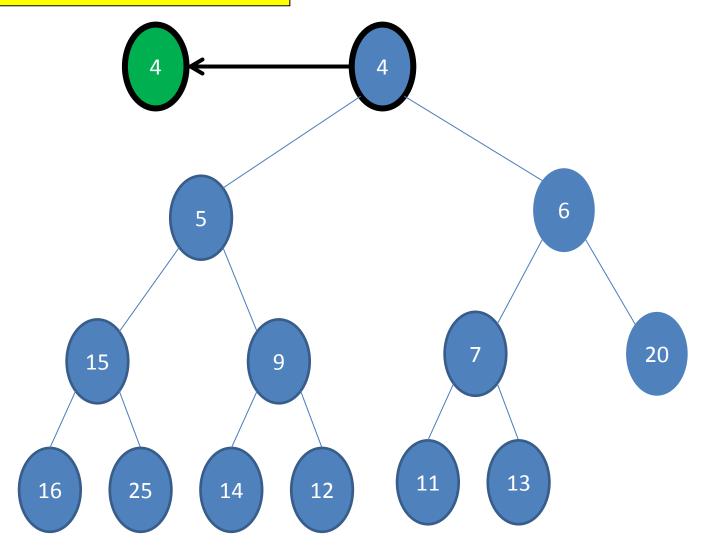
Example: removal from a heap

NOTE: it is a heap in a different state

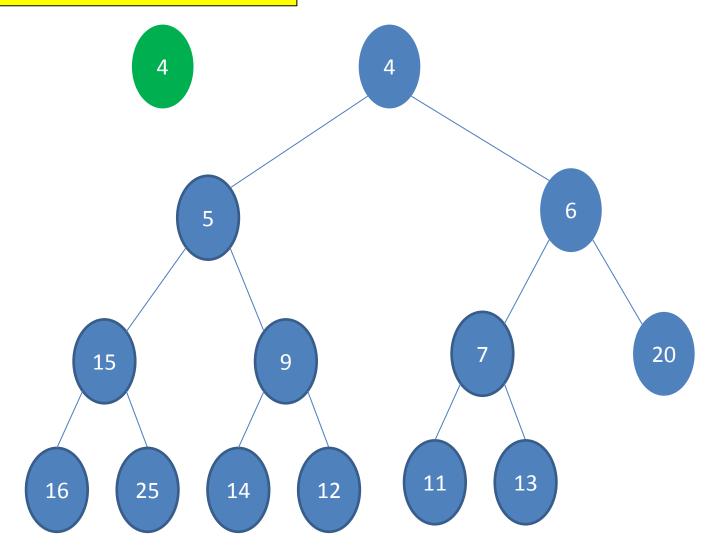


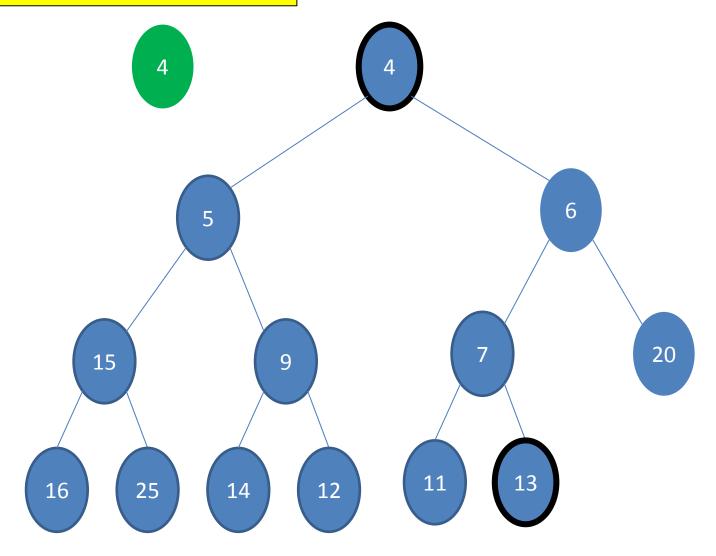


Save off top of heap



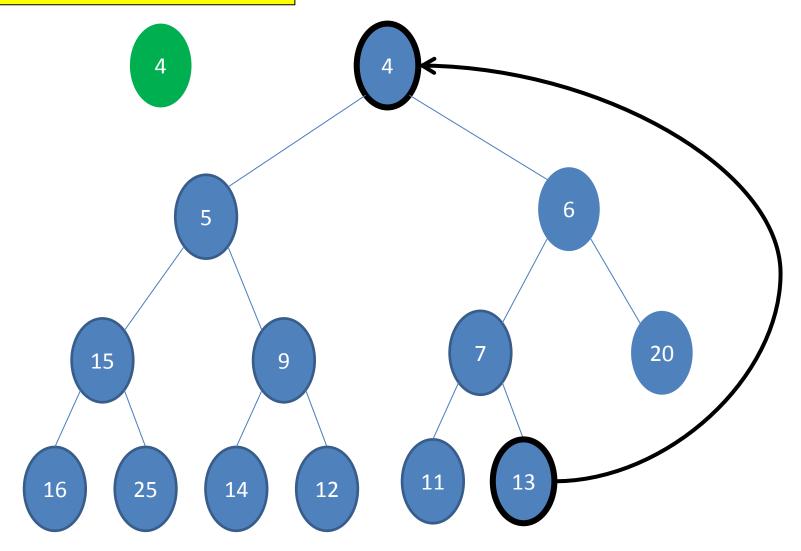
Save off top of heap





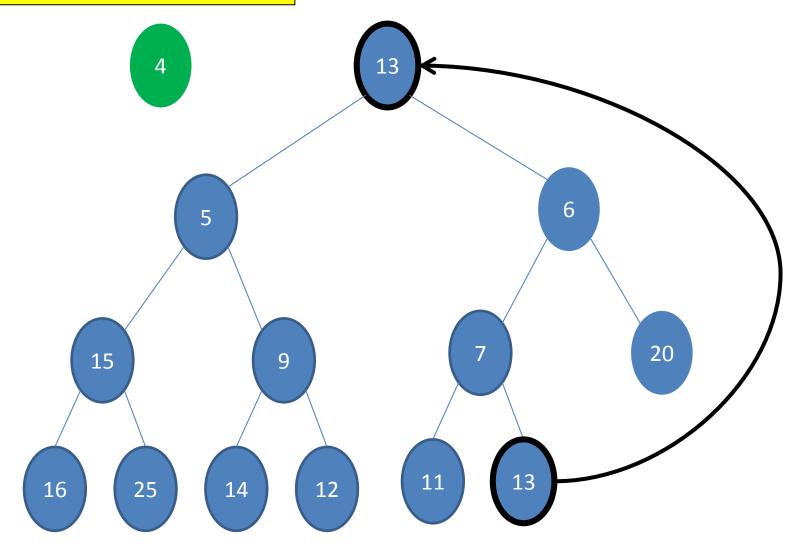


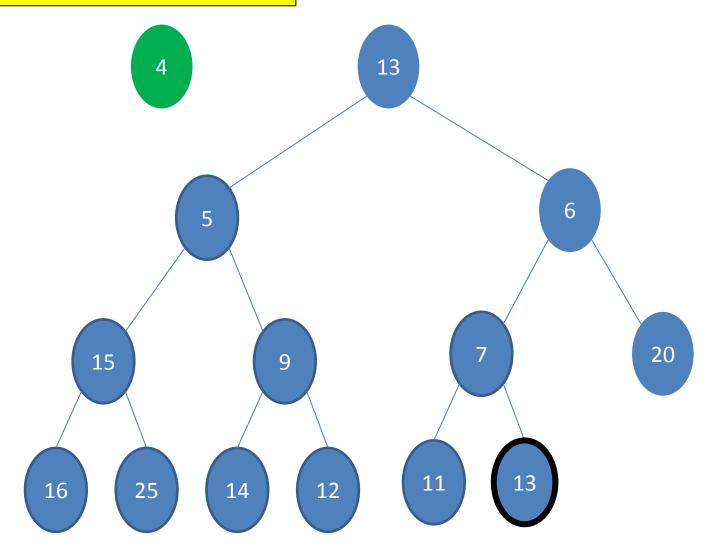


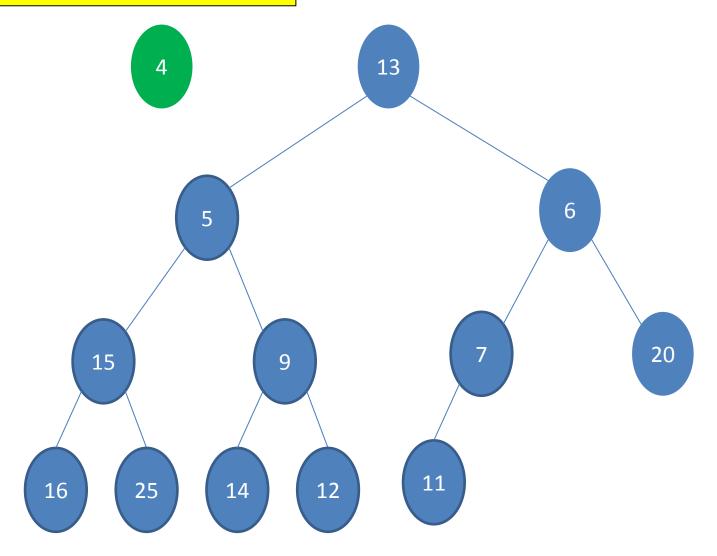


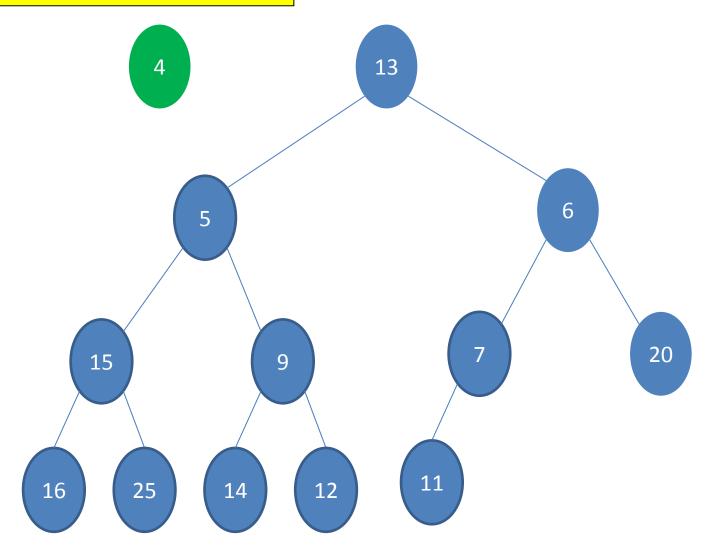




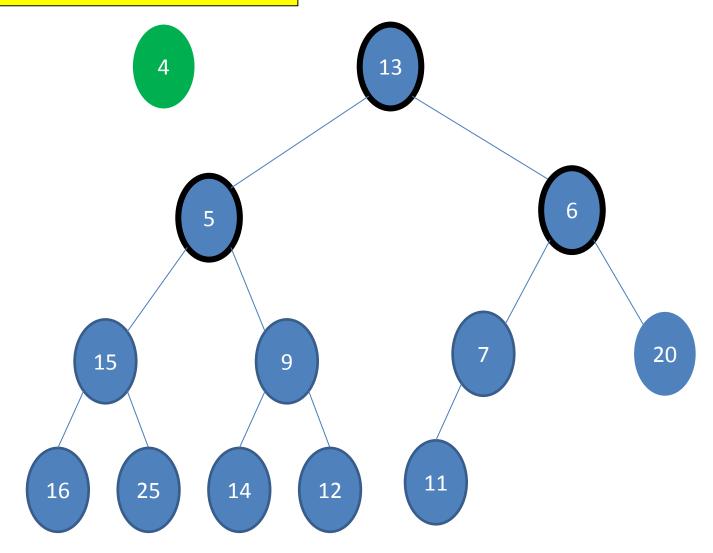








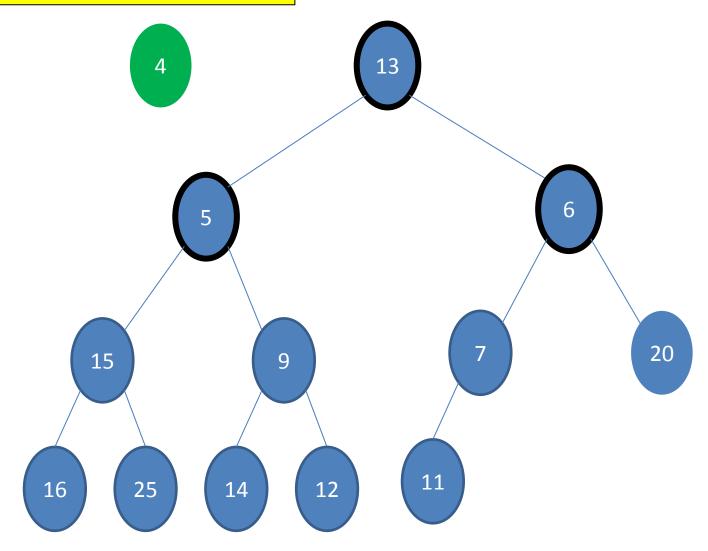
Compare current node with its children

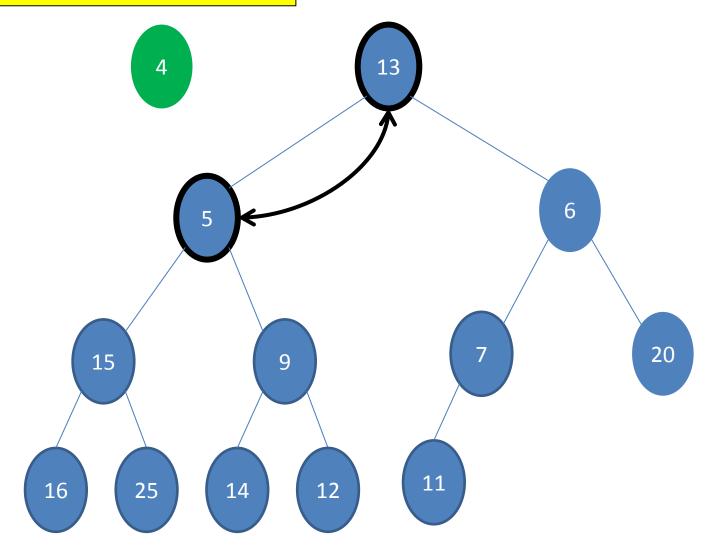


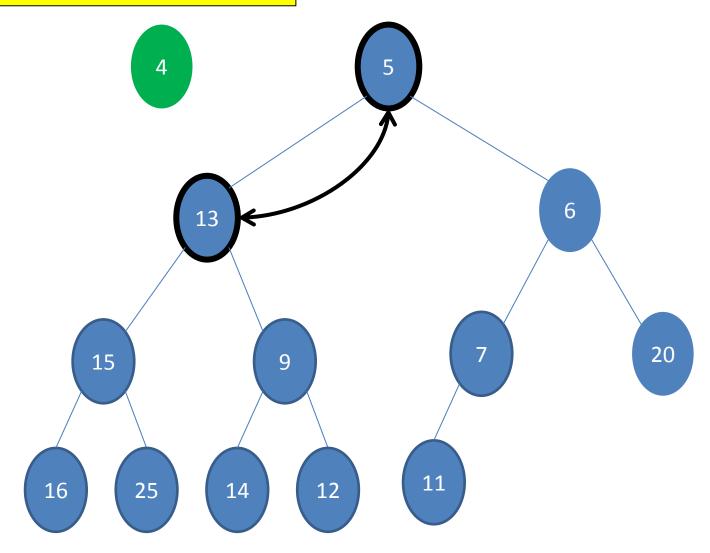
Compare current node with its children

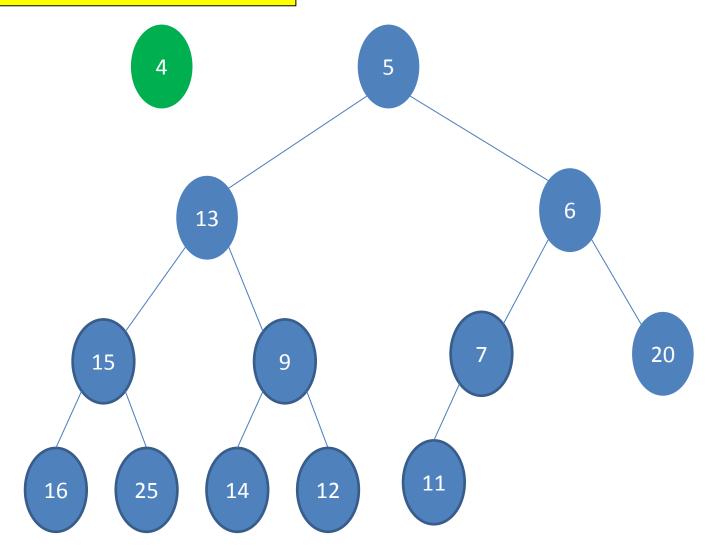


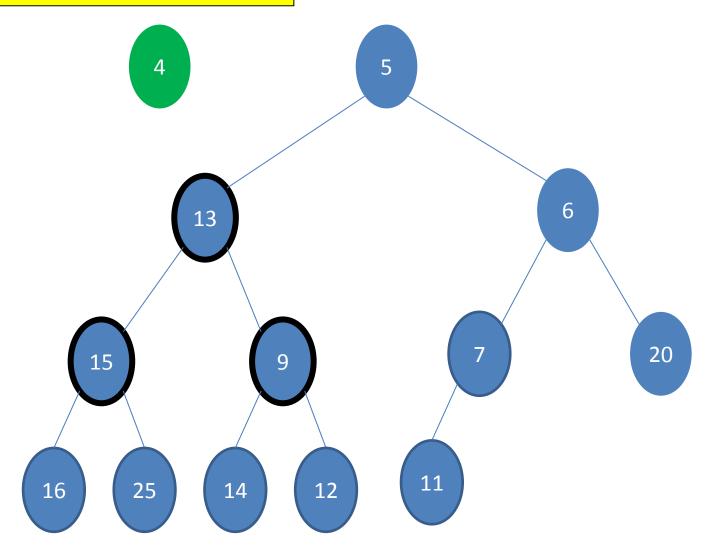




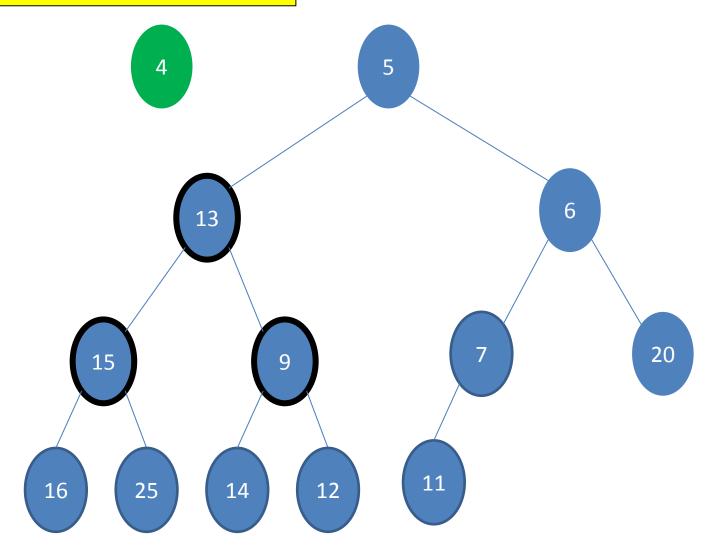


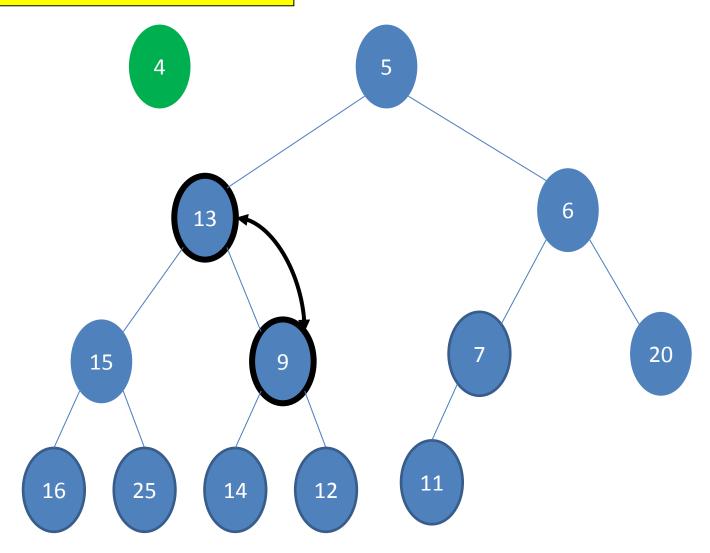


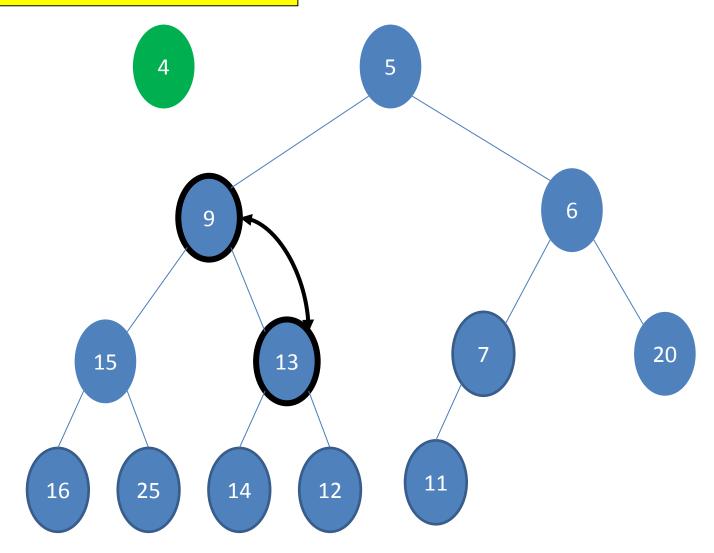


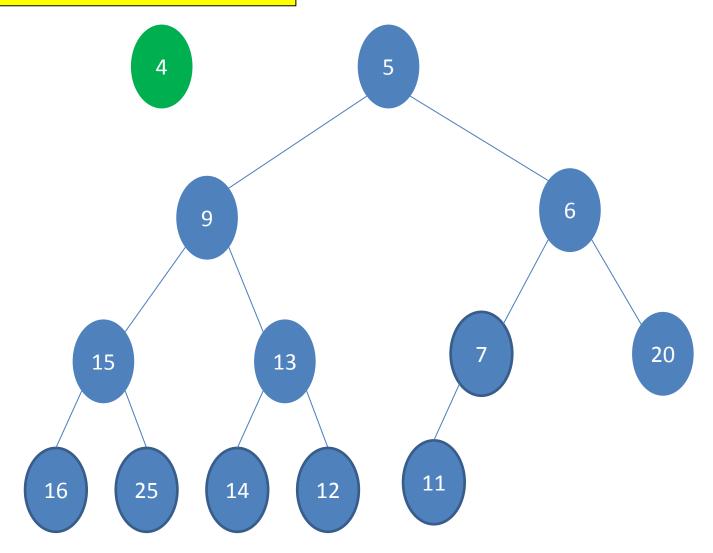


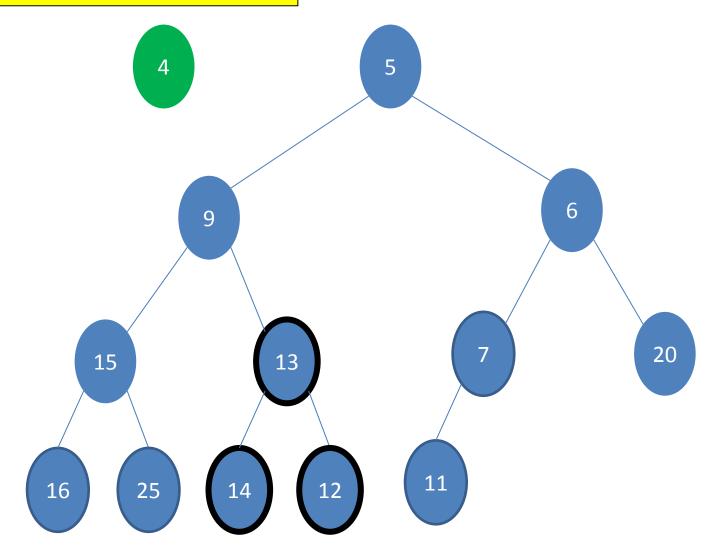
Compare current node with its children



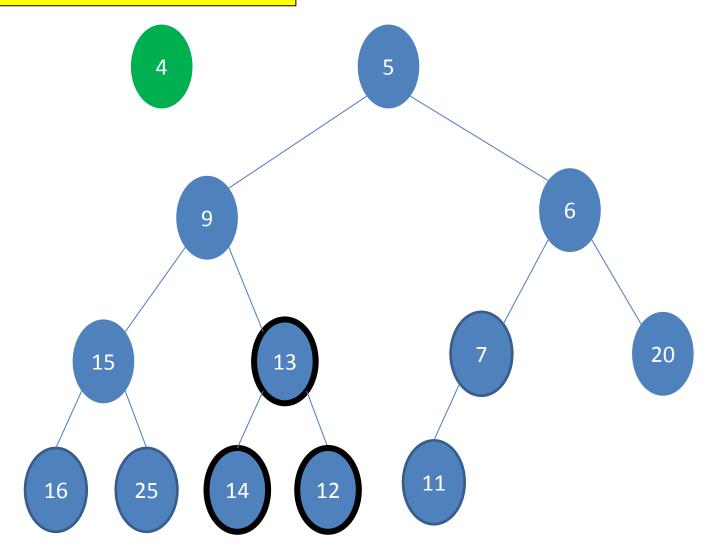


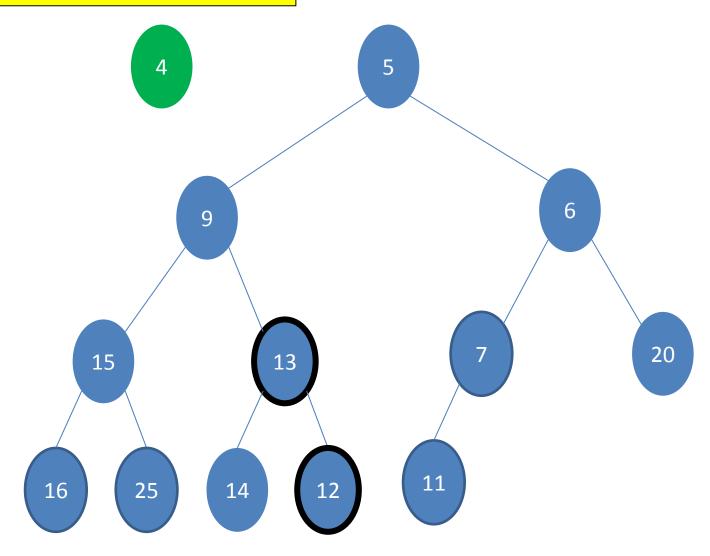


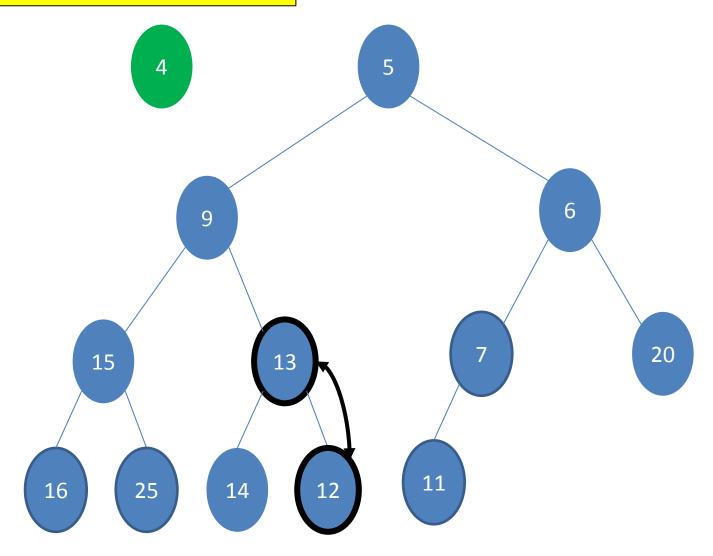


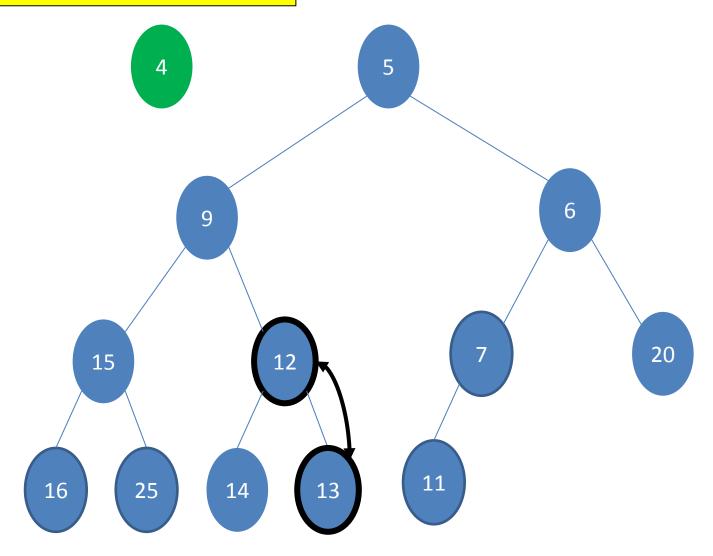


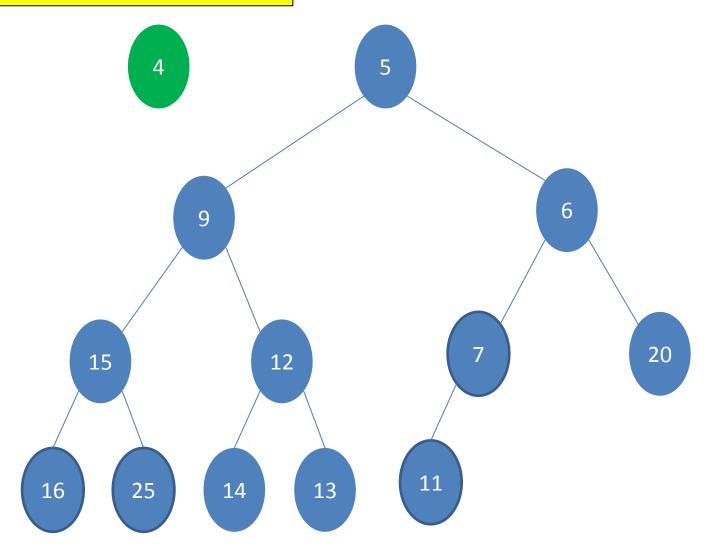
Compare current node with its children





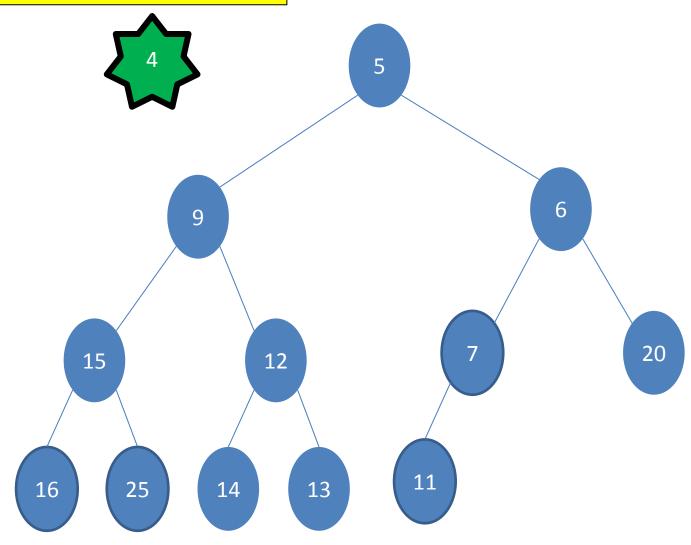




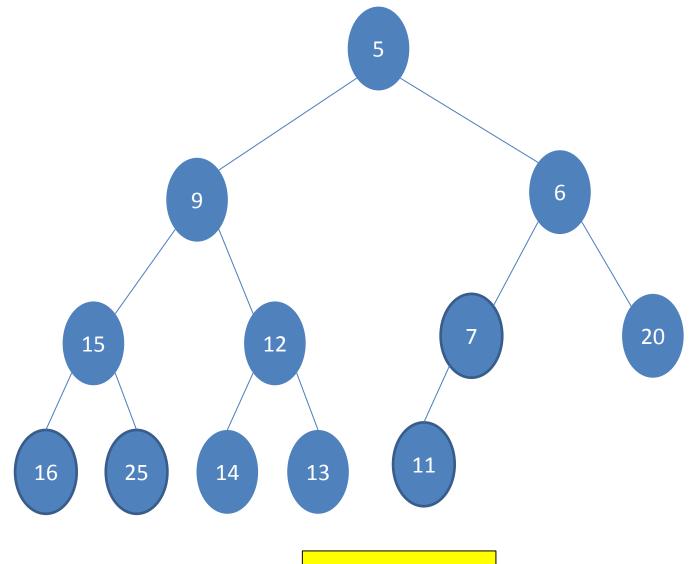








Return result



Done ©

What we just saw

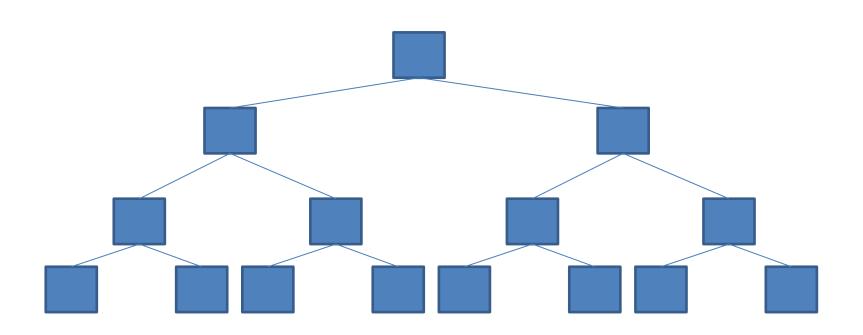
upheap bubbling: when we add to the heap

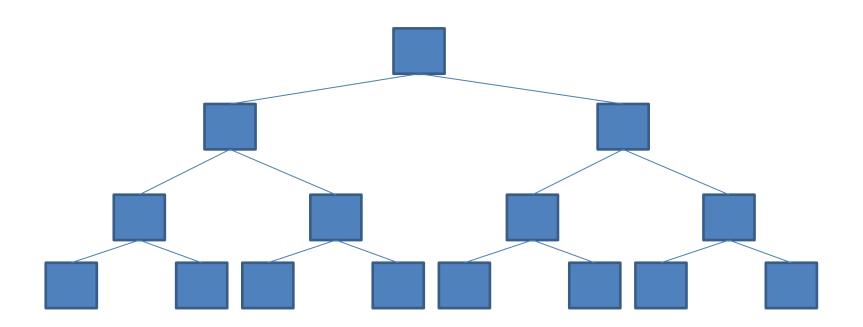
downheap bubbling: when we remove from the heap

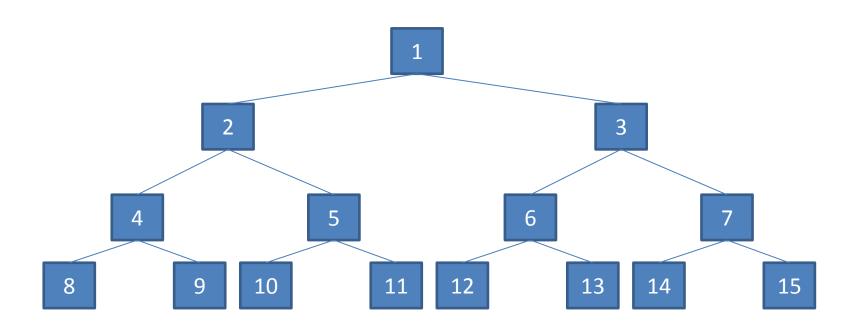


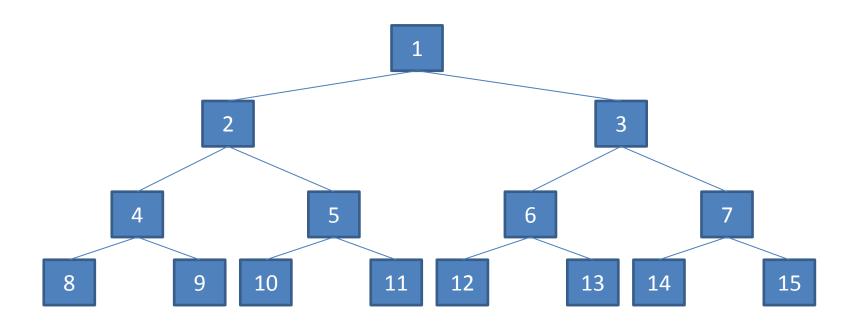
An implementation of a Heap data structure

An implementation of a Heap data structure

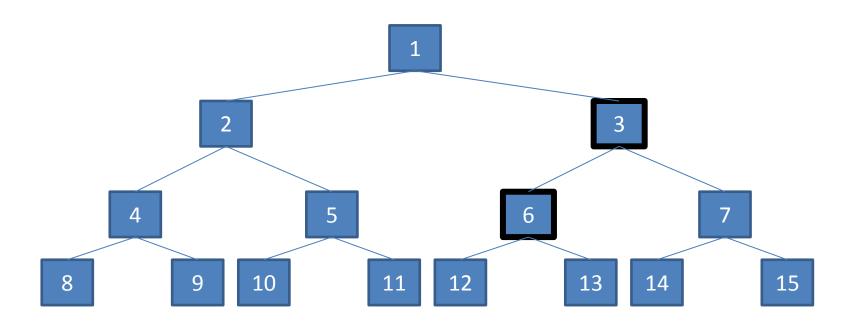




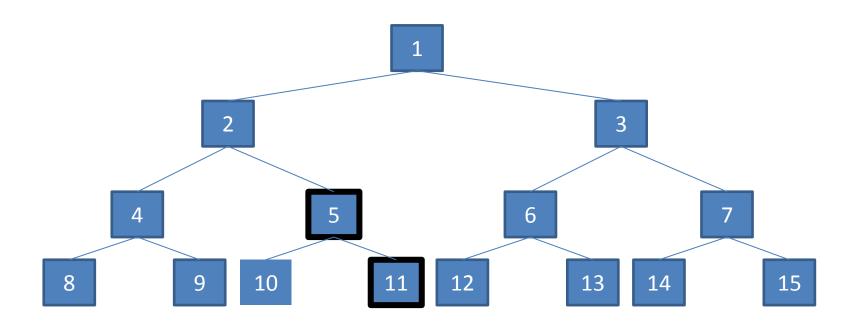




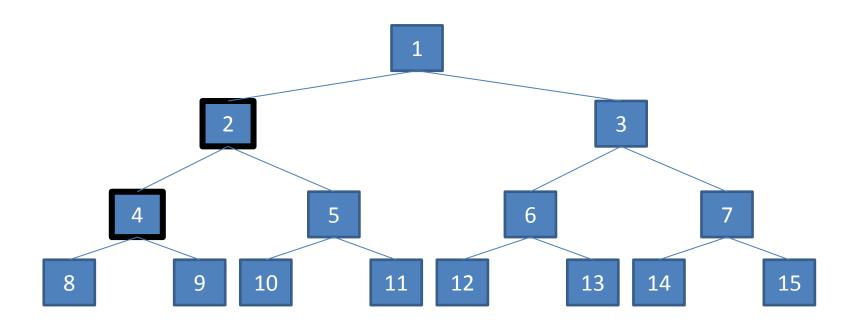
Note: *parent* of node i is i/2



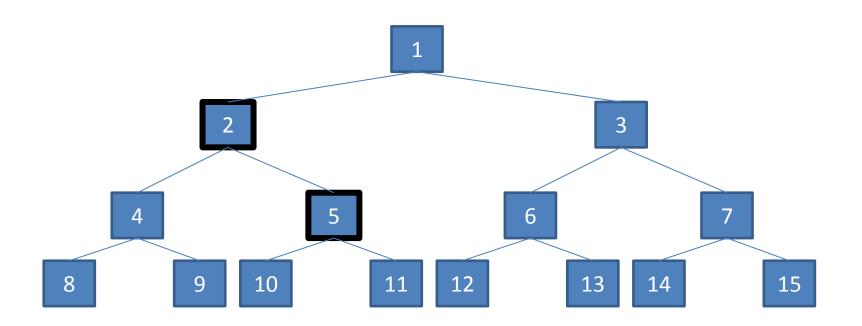
Note: *parent* of node i is i/2



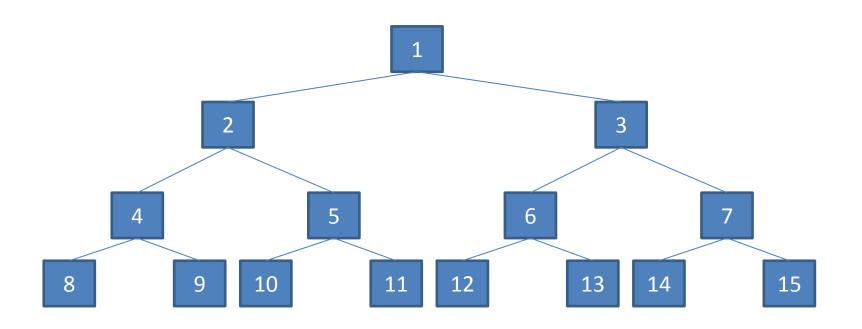
Note: *parent* of node i is i/2



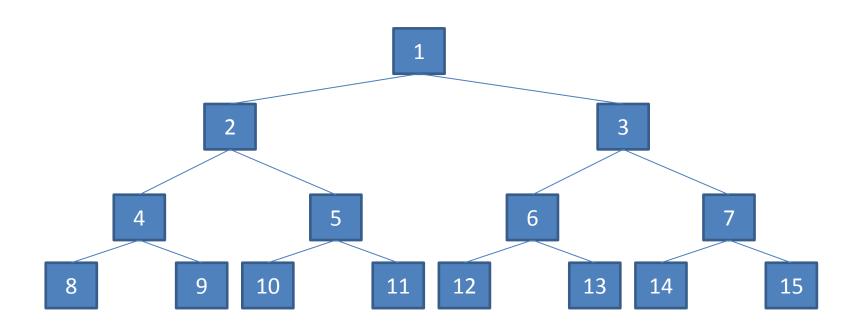
Note: *left child* of i is i×2

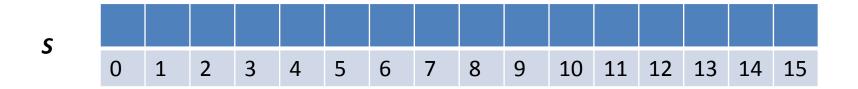


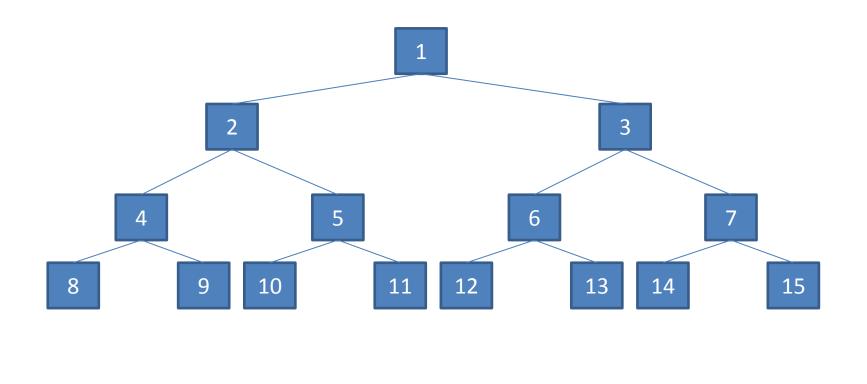
Note: *right child* of i is (i×2) +1

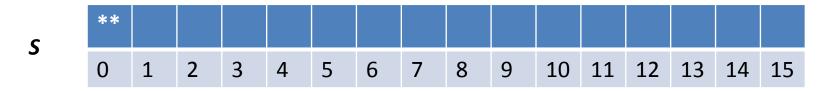


Represent as a one *dimensional* array

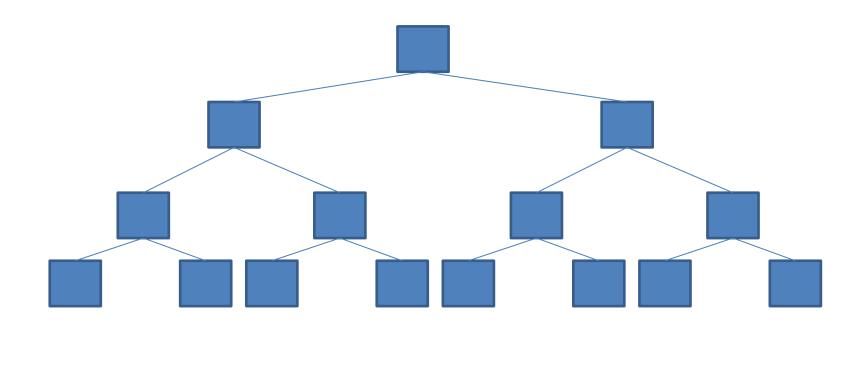


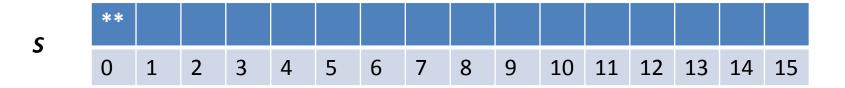




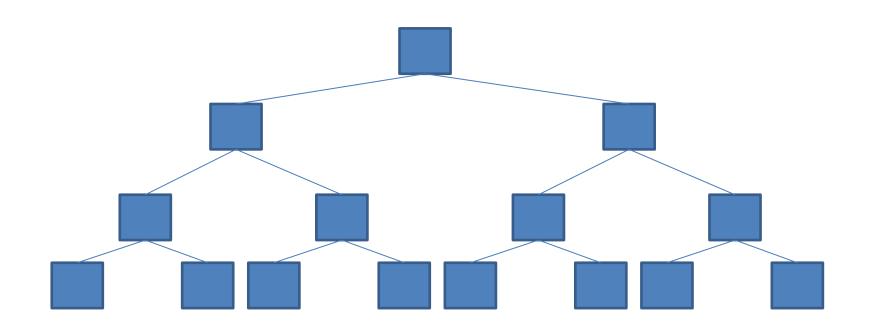


To simplify implementation we **do not** use S[0]





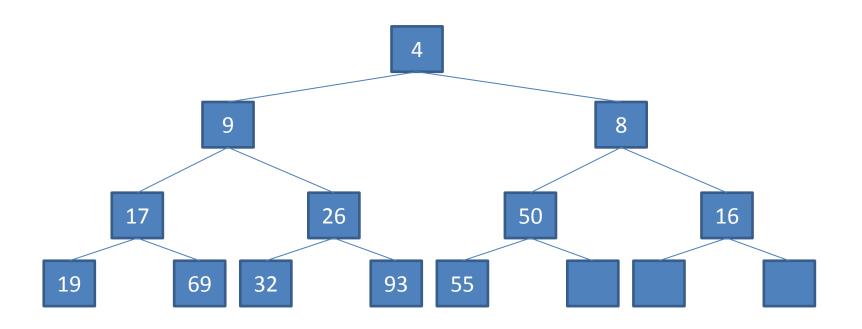
Require two integer variables, *last* and *capacity* where last is initially 0 In our example *capacity* is 15



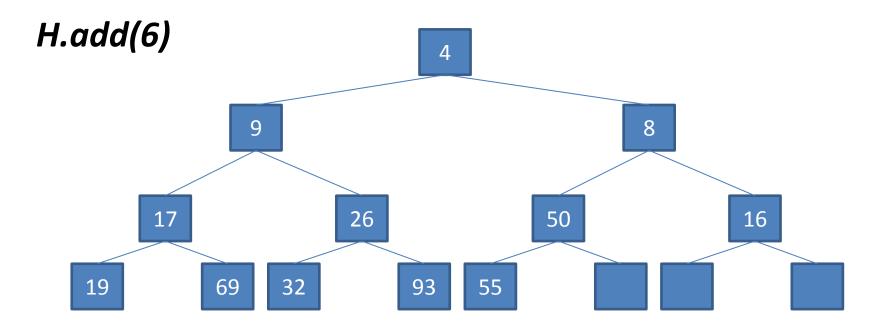
last: 0

An implementation of a Heap data structure

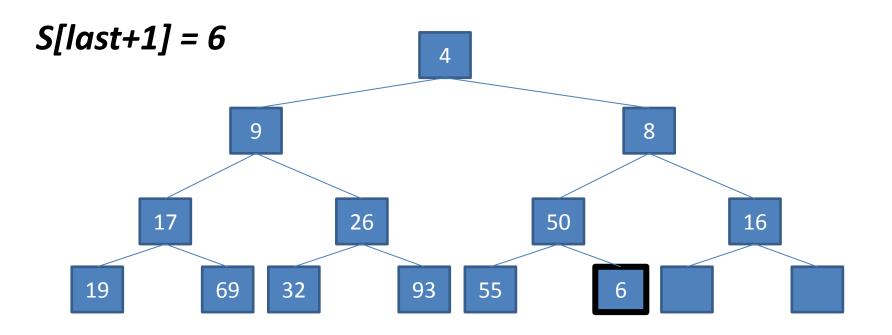
Consider the following heap H



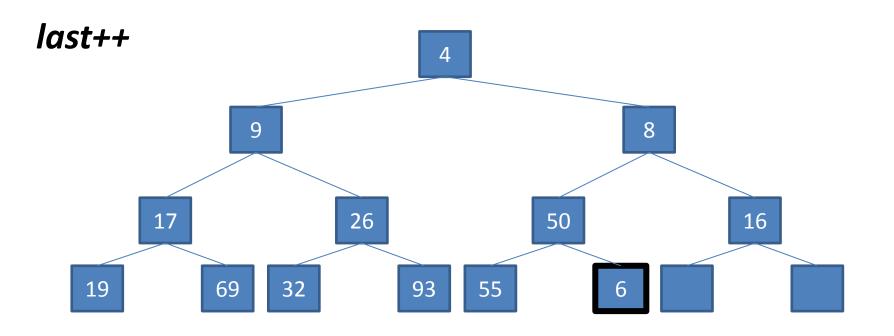
c	**	4	9	8	17	26	50	16	19	69	32	93	55			
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15



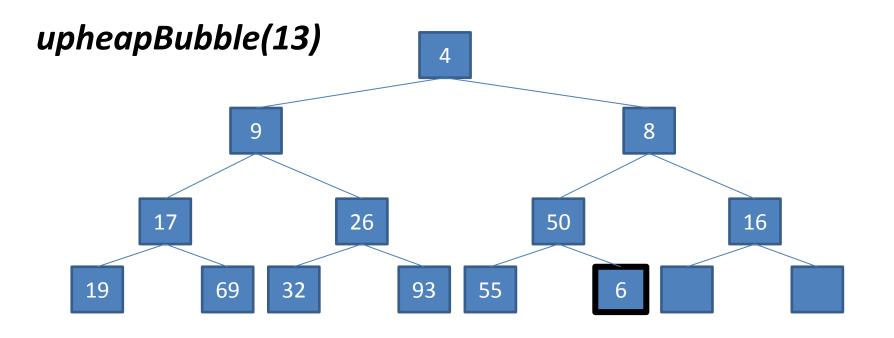
c	**	4	9	8	17	26	50	16	19	69	32	93	55			
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15



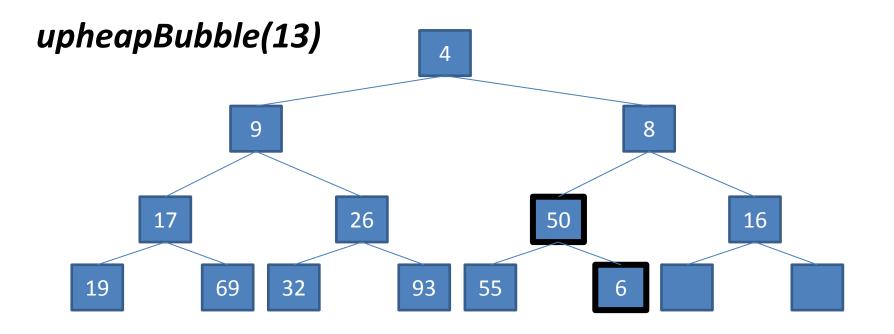
c	**	4	9	8	17	26	50	16	19	69	32	93	55	6		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15



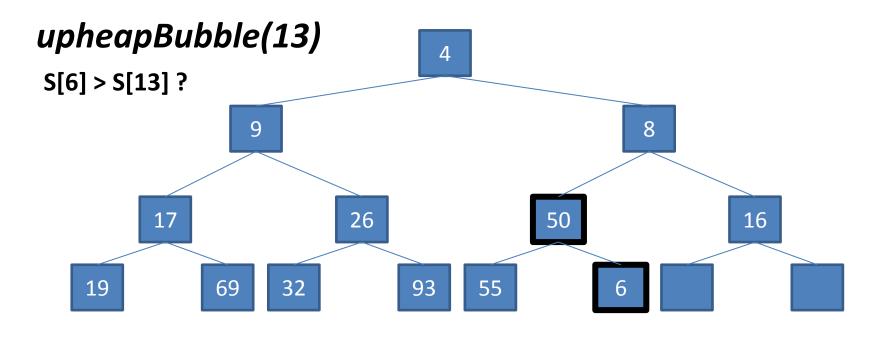
c	**	4	9	8	17	26	50	16	19	69	32	93	55	6		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

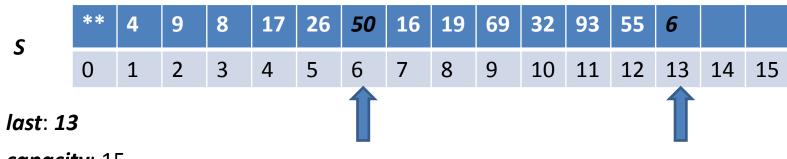


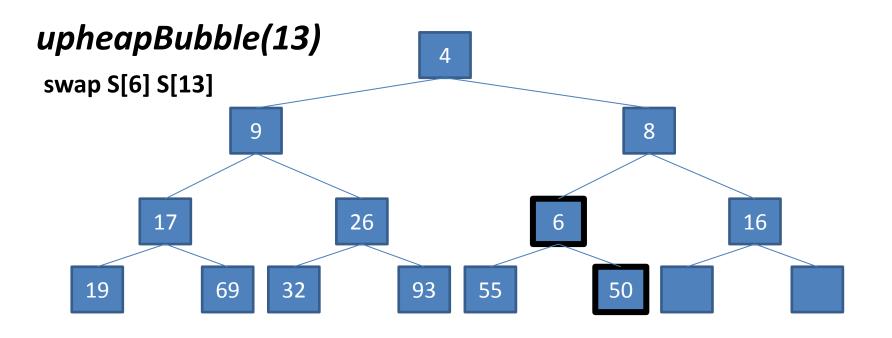
c	**	4	9	8	17	26	50	16	19	69	32	93	55	6		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

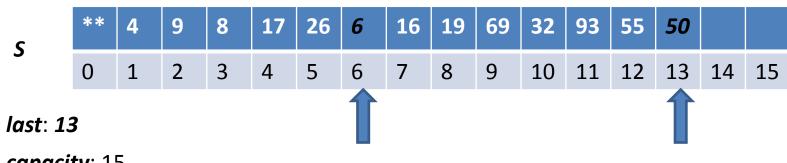


c	**	4	9	8	17	26	<i>50</i>	16	19	69	32	93	55	6		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

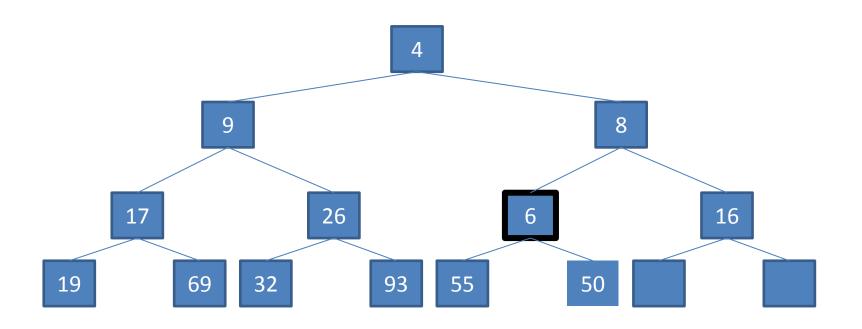






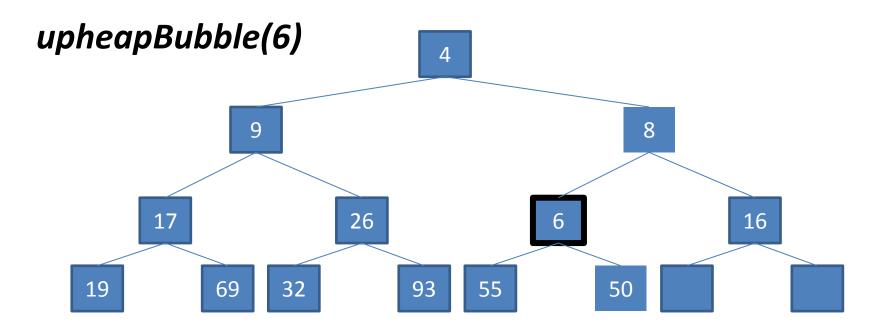


An implementation of a Heap data structure

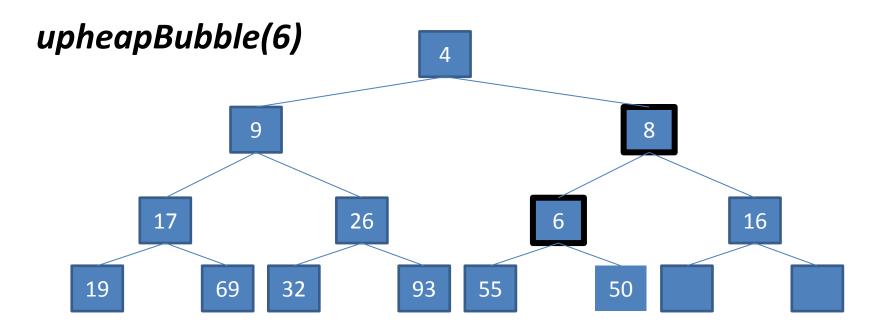


c	**	4	9	8	17	26	6	16	19	69	32	93	55	50		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

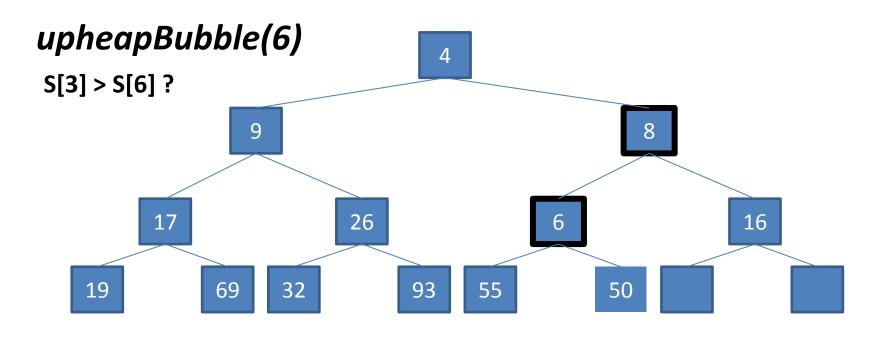
last: 13

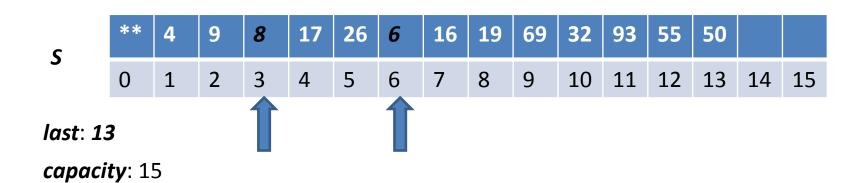


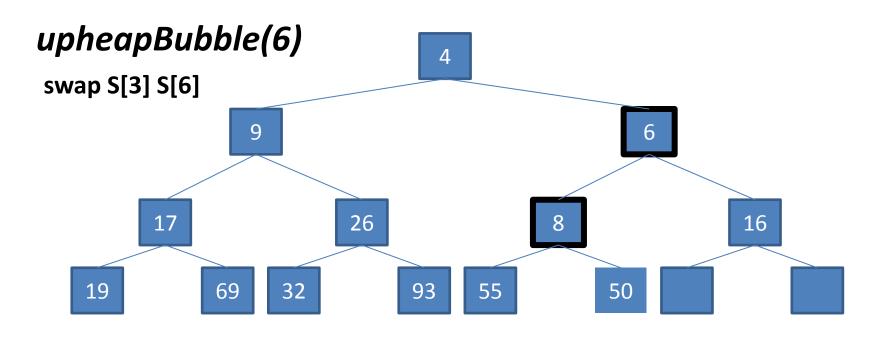
c	**	4	9	8	17	26	6	16	19	69	32	93	55	50		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

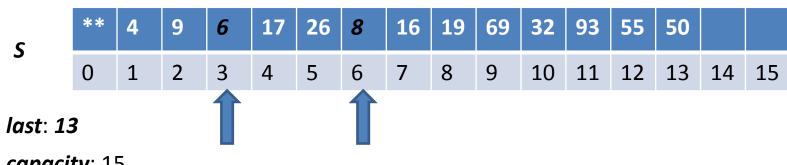


c	**	4	9	8	17	26	6	16	19	69	32	93	55	50		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

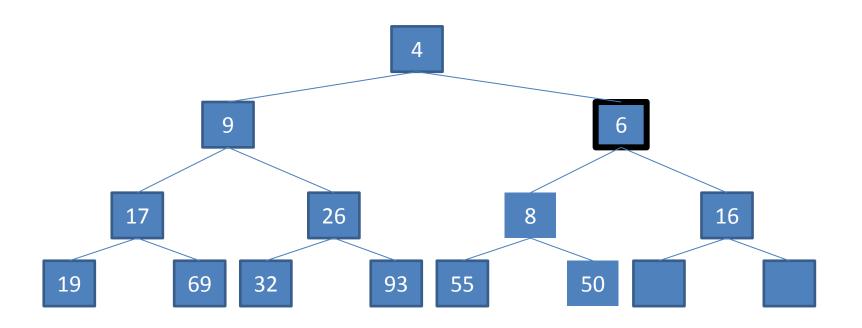






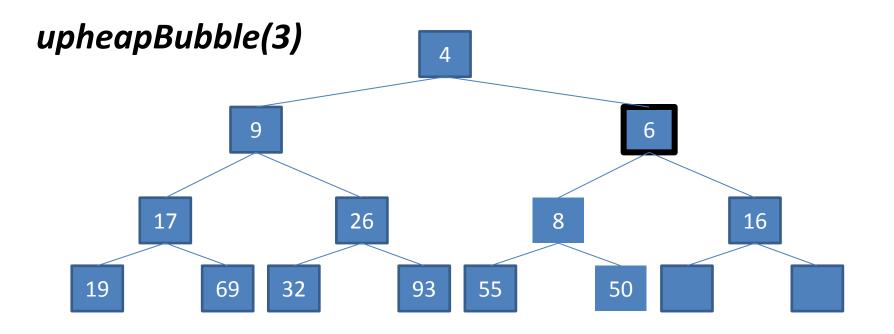


An implementation of a Heap data structure

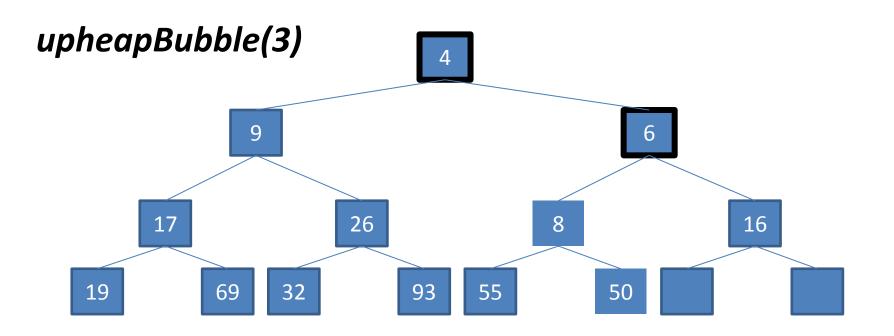


c	**	4	9	6	17	26	8	16	19	69	32	93	55	50		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

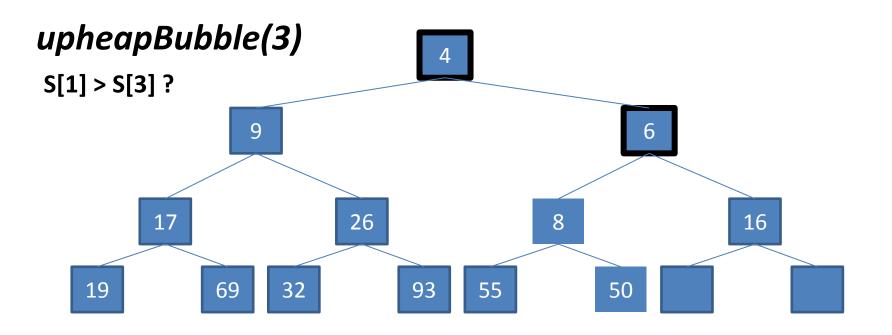
last: 13

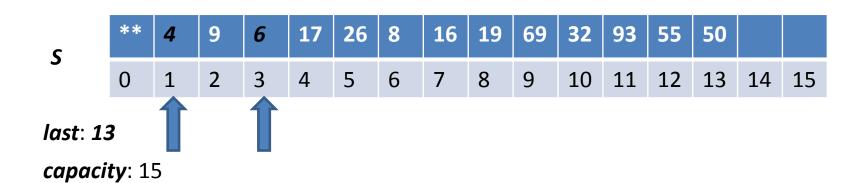


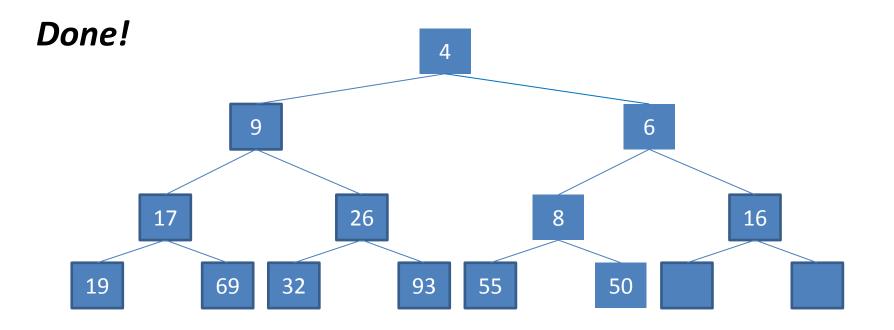
c	**	4	9	6	17	26	8	16	19	69	32	93	55	50		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15



c	**	4	9	6	17	26	8	16	19	69	32	93	55	50		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

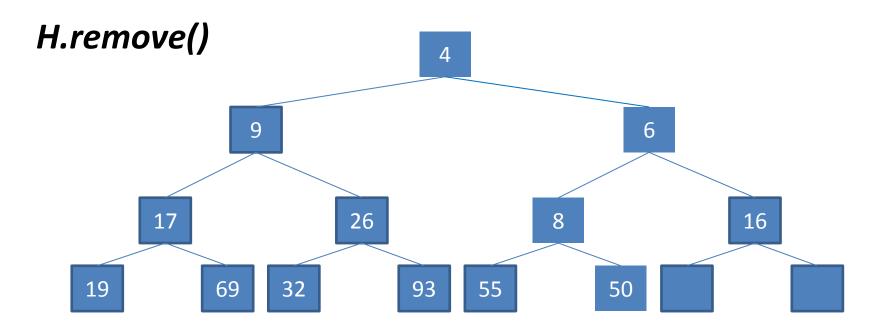






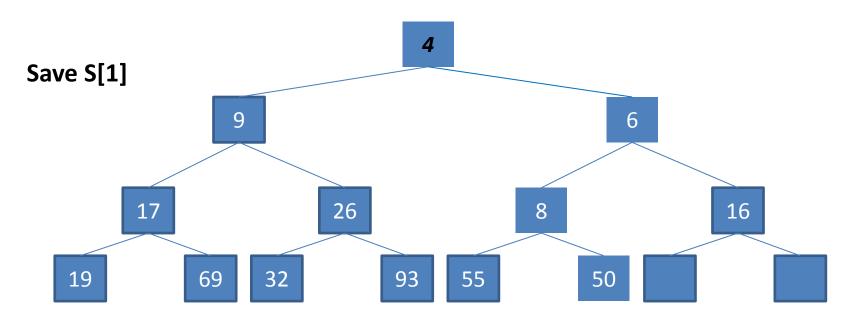
S	**	4	9	6	17	26	8	16	19	69	32	93	55	50		
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Removal from the heap H



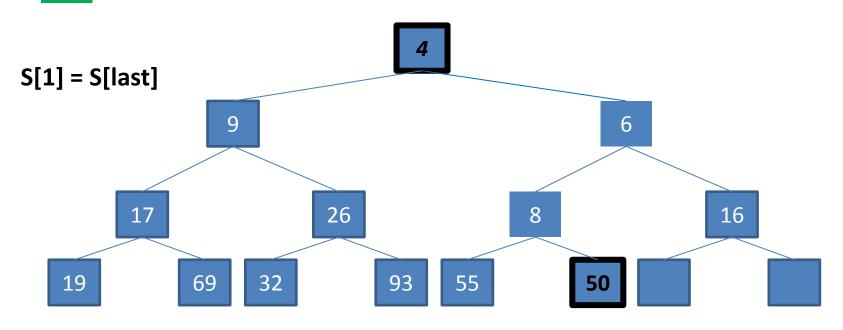
c	**	4	9	6	17	26	8	16	19	69	32	93	55	50		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15





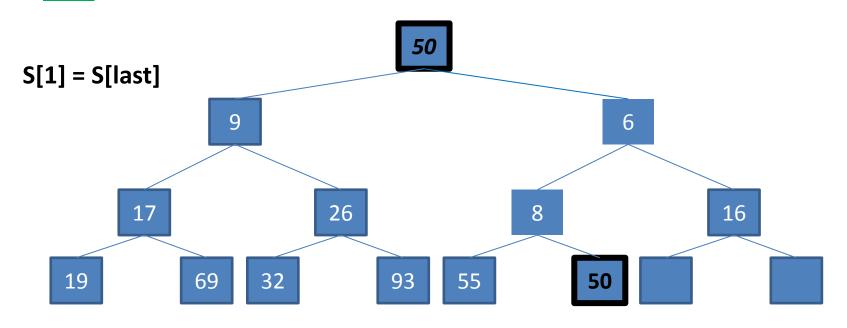
ς	**	4	9	6	17	26	8	16	19	69	32	93	55	50		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15





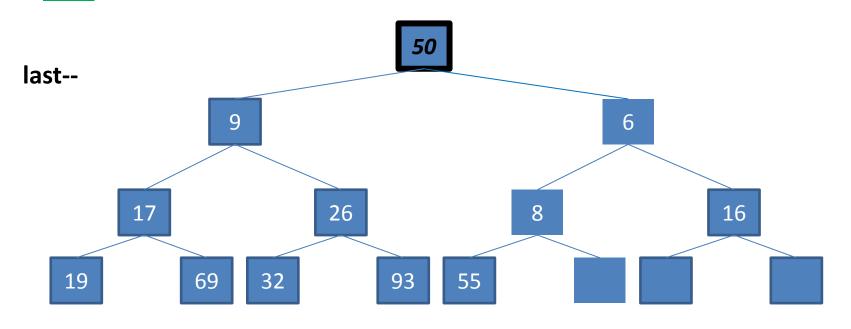
c	**	4	9	6	17	26	8	16	19	69	32	93	55	<i>50</i>		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15





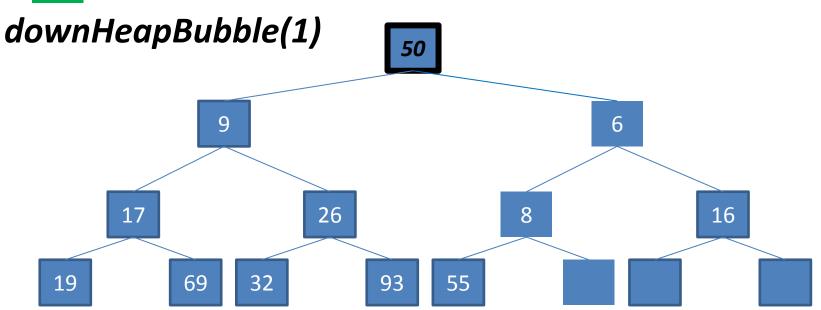
S	**	50	9	6	17	26	8	16	19	69	32	93	55	<i>50</i>		
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15





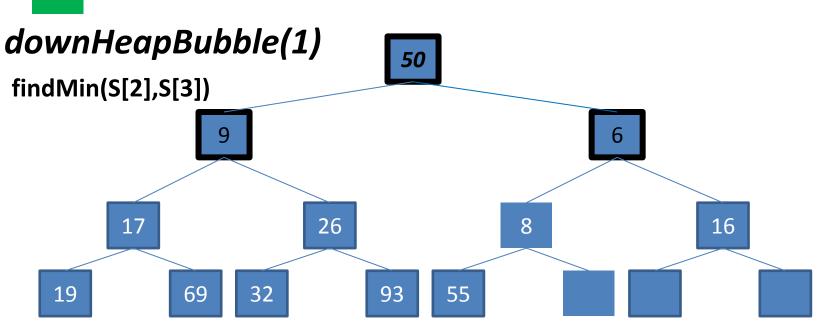
c	**	<i>50</i>	9	6	17	26	8	16	19	69	32	93	55			
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15





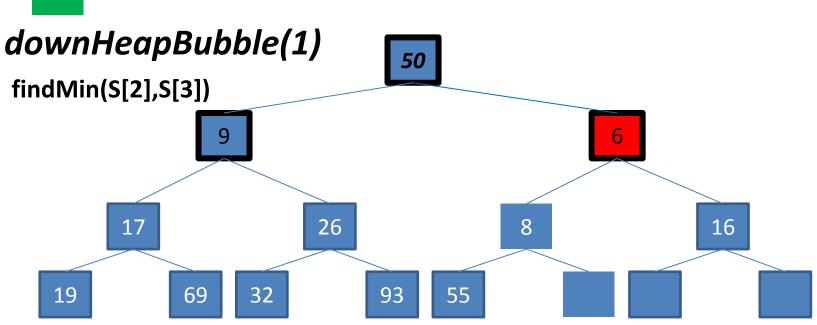
c	**	<i>50</i>	9	6	17	26	8	16	19	69	32	93	55			
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

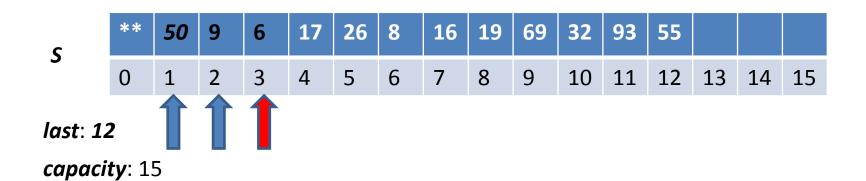




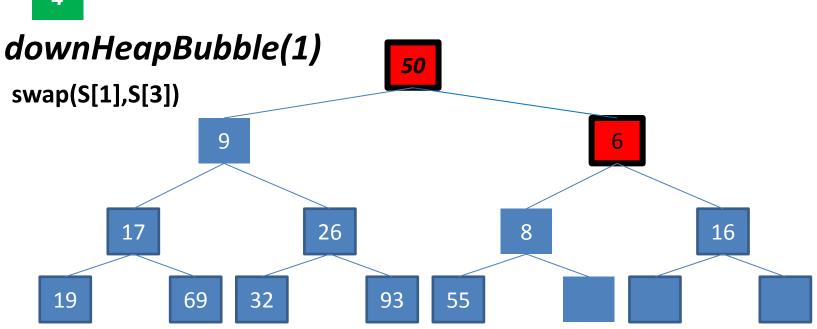


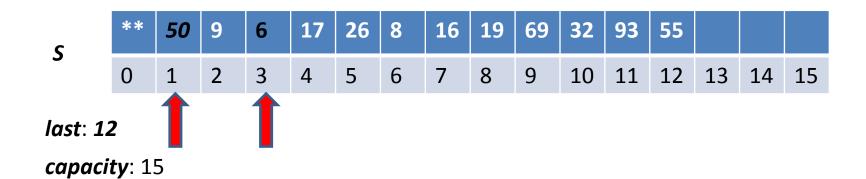




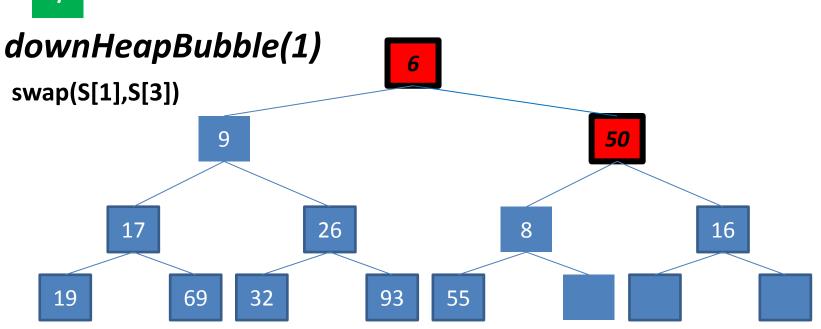


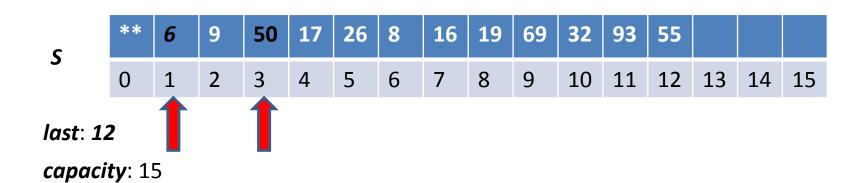




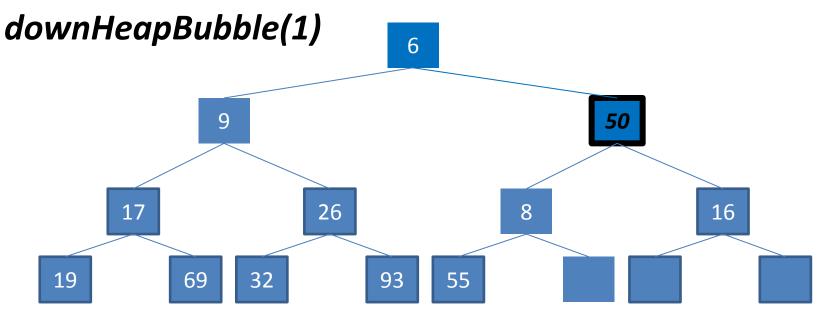






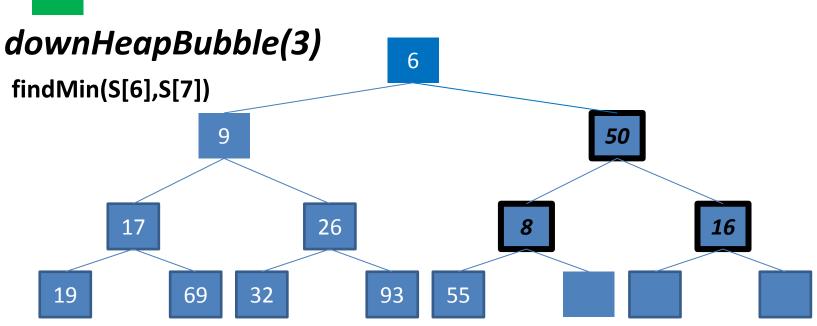


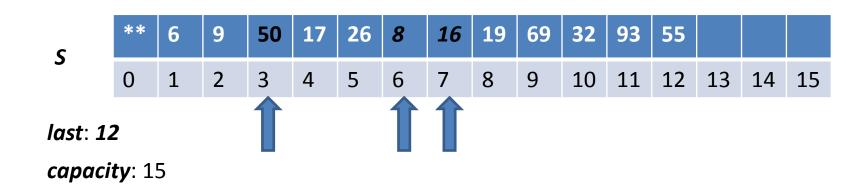




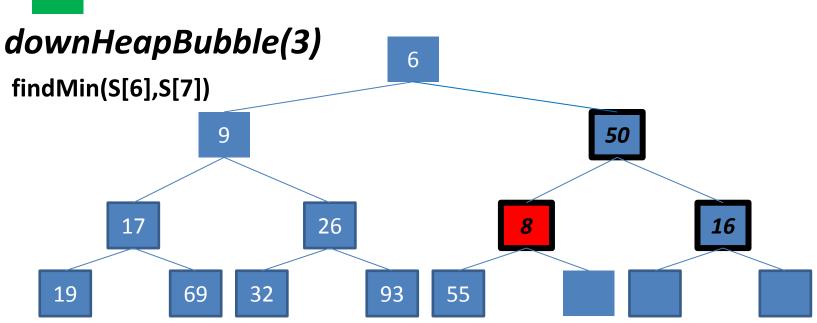
c	**	6	9	50	17	26	8	16	19	69	32	93	55			
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

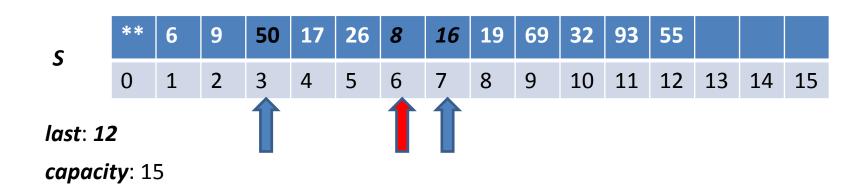




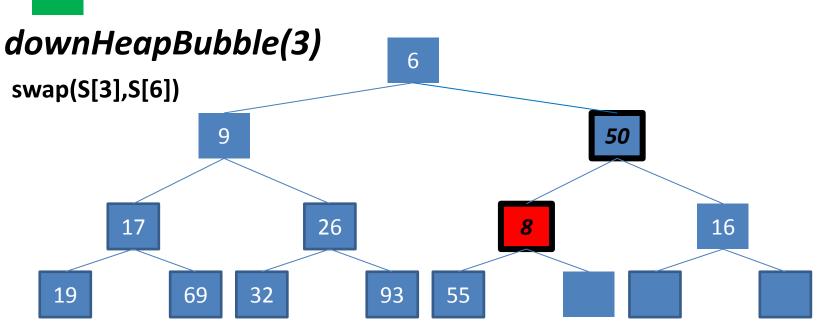


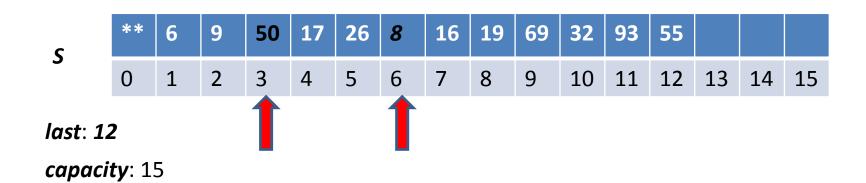




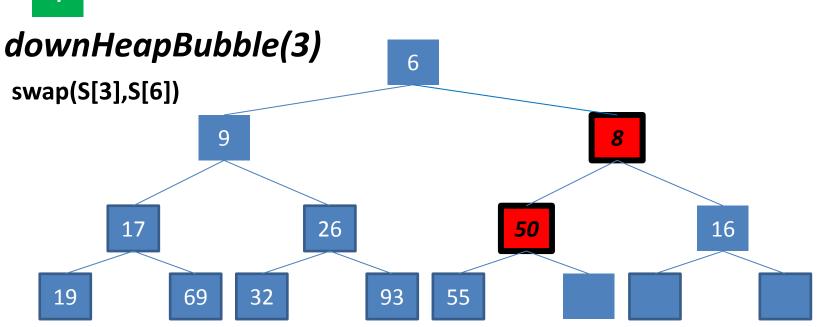


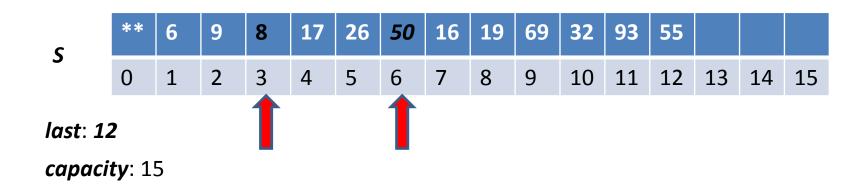




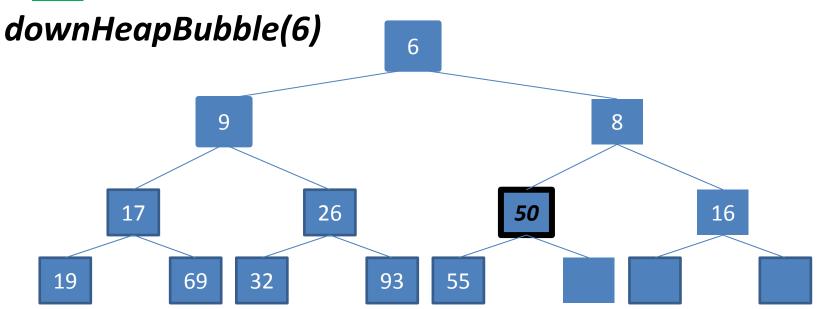






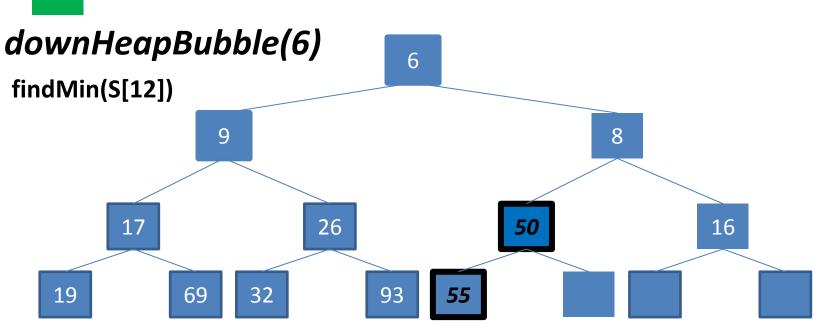


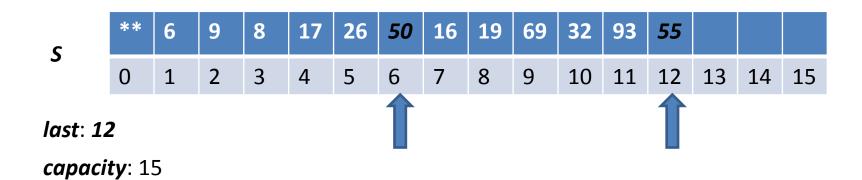




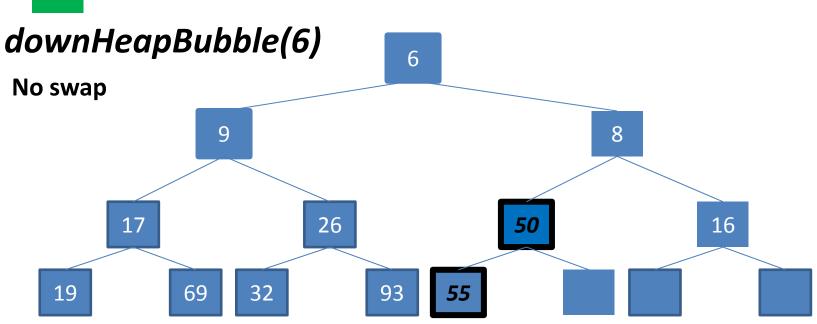
c	**	6	9	8	17	26	50	16	19	69	32	93	55			
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

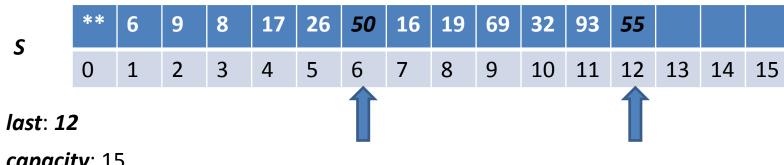




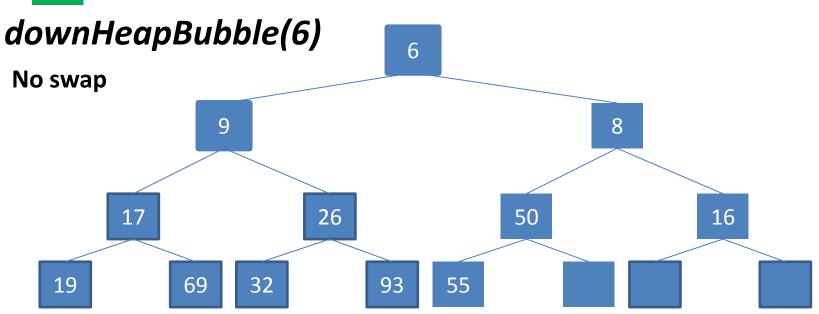






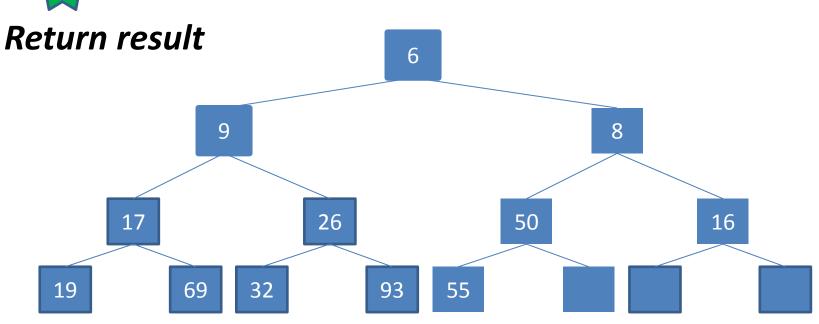






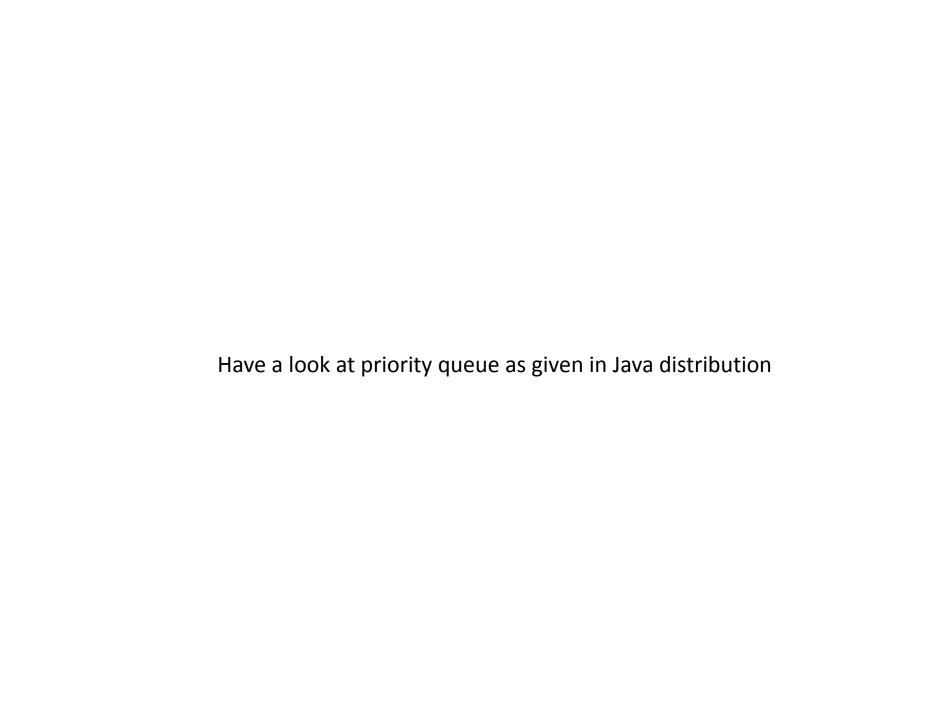
c	**	6	9	8	17	26	50	16	19	69	32	93	55			
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

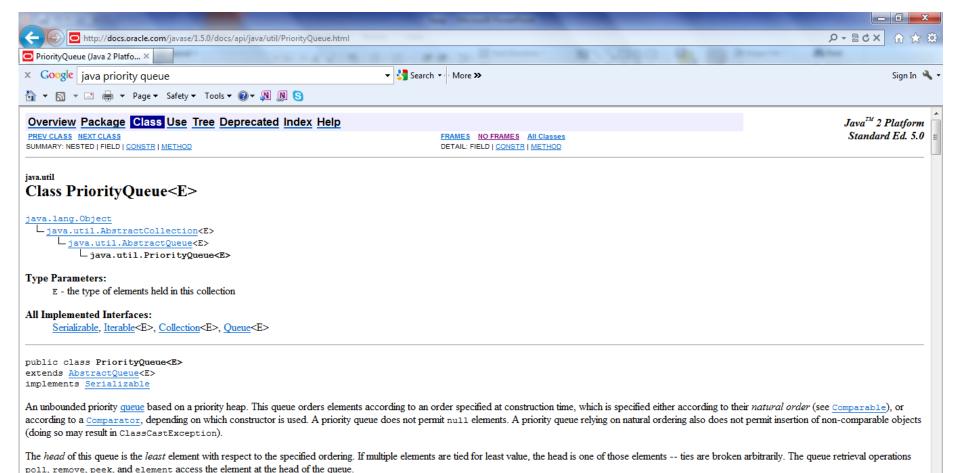




c	**	6	9	8	17	26	50	16	19	69	32	93	55			
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Method	Time
size	O(1)
isEmpty	O(1)
insert	O(log(n))
removeMin	O(log(n))
min	O(1)



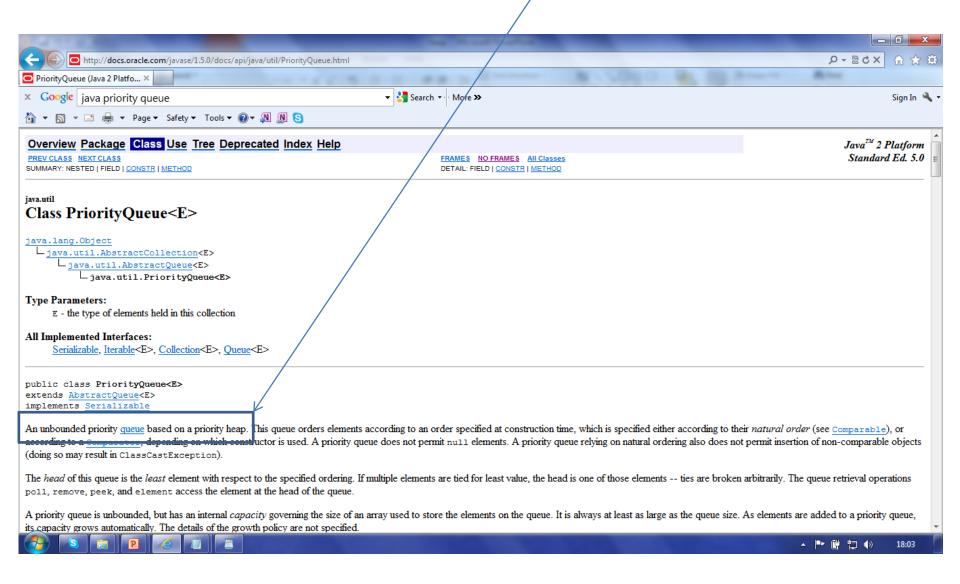


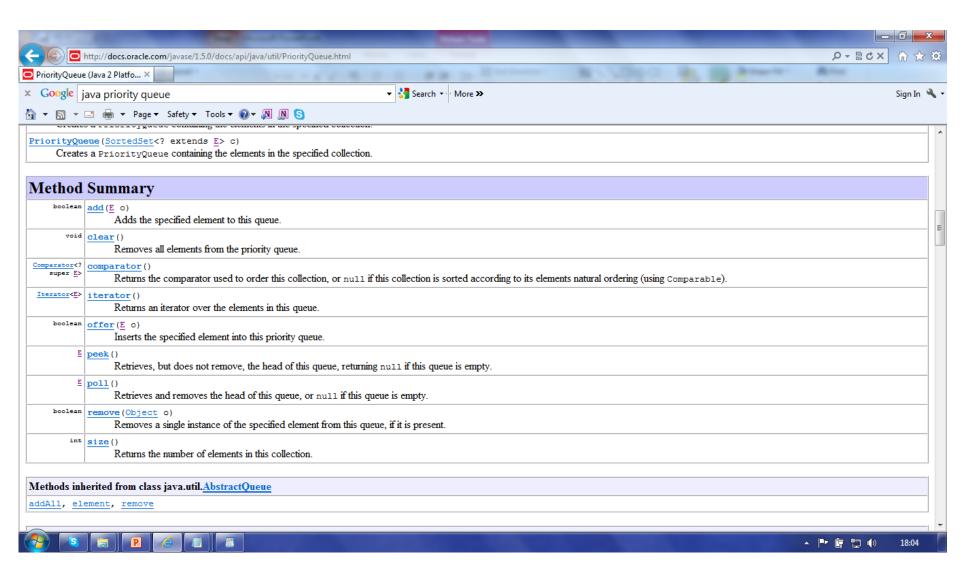
A priority queue is unbounded, but has an internal capacity governing the size of an array used to store the elements on the queue. It is always at least as large as the queue size. As elements are added to a priority queue,

18:03

its capacity grows automatically. The details of the growth policy are not specified.

... based on a priority heap





Different method names ... add rather than insert



Your mission, should you choose to accept it ...

Exercise 4 (assessed)

- Implement the Heap
- Use it for sorting



##