Random Number Generation

Note Title 01-02-2021

- How to generate (pseudo) random numbers in C
- Dice example ✓
- Bernoulli & Binomial ✓
- Uniform RV ✓
- Exponential RV & Poisson process ✓
- General Continuous RVs ✓
- Gaussian RV — ? F() not known in closed form.

Know F() ✓

rand() and srand()

modulo

```
int dice = rand() %6 + 1 ;
```

{1, 2, 3, 4, 5, 6}
wp 1/6

Bernoulli $X = 1$ wp p; $= 0$ wp $1-p$

Uniform

```
float unif = rand / RAND_MAX ;
```

Uniform in [0,1] ✓

0 —— 1

```
float p = 0.75 ;
int Bern = 0 ;
if Unif <= p
    Bern = 1 ;
```

$$\text{Binomial}(n, p) = \sum_{i=1}^{n} \text{Bern}(p)$$

We know how to generate $U \sim \text{Uniform}[0,1]$

Random Variable $X \rightarrow$ Real valued

specified by a distribution function

$$F : \mathbb{R} \longrightarrow [0,1]$$

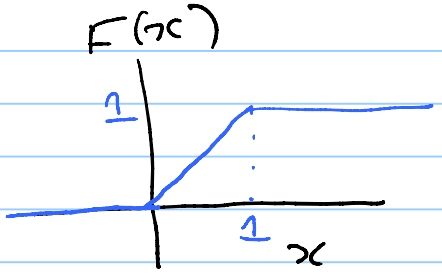
$F(\cdot)$

Non-decreasing →

$$F(x) \triangleq \mathbb{P}(X \leq x) \quad \text{CDF}$$

Eg $X \sim \text{Unif}[0,1]$

$$F(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$



Q How do I generate samples from an arbitrary distribution $F(\cdot)$?

(I know how to generate Uniform[0,1])

(Assume $F(\cdot)$ is strictly increasing)

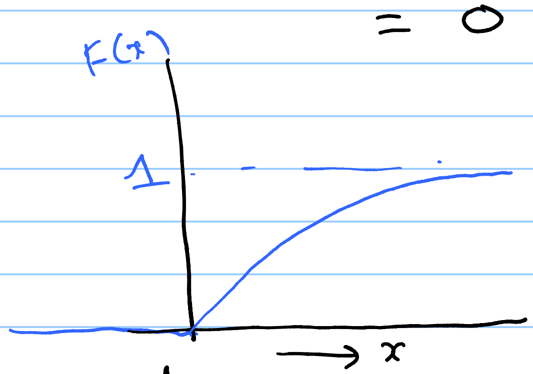
```
float Unif = rand() / RAND_MAX;
```

Ans $F^{-1}(\text{Unif})$ ← is distributed according to $F(\,)$.

why?

Generate an exponential RV whose CDF is

$$F(x) = 1 - e^{-\lambda x} \quad x \geq 0$$
$$= 0 \quad x < 0. \qquad \lambda > 0.$$

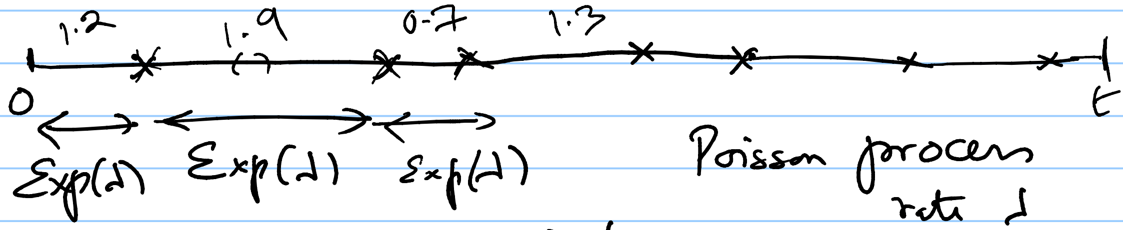


Unif [0,1]

$$F^{-1}(u) = -\frac{1}{\lambda} \ln(\underbrace{1-u}_{U}) \quad \text{for } \lambda > 0$$

<u>Exponential RV</u>

Models time interval between successive radioactive emissions, phone calls etc.



→ $t > 0$