

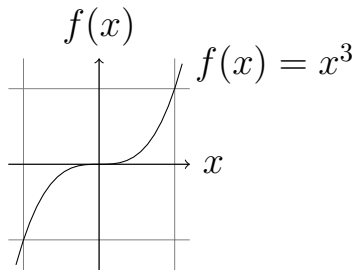
# Min/Max and Point of Inflection Cheatsheet

## Contents

<b>1</b>	<b>Min/Max</b>	<b>2</b>
1.1	Taylor Approximation . . . . .	2
<b>2</b>	<b>Point of Inflection</b>	<b>2</b>
2.1	Examples . . . . .	2

## 1 Min/Max

To check for a minimum or maximum, you set  $f'(x) = 0$  to obtain the **critical points**. Not all of them are going to be maxima or minima, and sometimes neither.



1. If  $f''(x) < 0$  and  $f'(x) = 0$ , then  $(x, f(x))$  is a maximum
2. If  $f''(x) > 0$  and  $f'(x) = 0$ , then  $(x, f(x))$  is a minimum
3. If  $f''(x) = 0$  and  $f'(x) = 0$ , then  $(x, f(x))$  is indeterminate; check if signs change or use a Taylor approximation instead (not practical).

### 1.1 Taylor Approximation

A Taylor Series is used to approximate a point  $x$  from  $f(x)$  using a series around  $c$ . It looks like:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} (x - c)^n \quad (1)$$

Because no sole derivative will give you enough information from the function at one point (illustrated by  $f(x) = x^3, x^4, x^5 \dots$ ) you need to know all the derivatives. And if the function isn't smooth (differentiable for each  $x$  where  $x \in \mathbb{R}$ , this method can't be used analytically either.

## 2 Point of Inflection

Points of Inflection **may** exist when  $f''(x) = 0$  or are undefined. You have to use Taylor approximations to figure out if it actually is a point of inflection. It is preferable to check if the signs change.

### 2.1 Examples

These problems are taken from *Calculus for the AP Course* by Michael Sullivan.

1. The function  $f$  has a second derivative given by  $f''(x) = x^2(x - 1)\sqrt{x + 1}$ . At what values of  $x$  does  $f(x)$  have a point of inflection?

At a quick glance, potential points of inflection are  $x = -1, 0, 1$ . However, we need to plug in values between each of these potential inflection points to find where  $f''(x)$  switches from negative to positive or vice versa. You can also figure out that  $x^2$  and  $\sqrt{x + 1}$  will always return a positive value, so the only solution is  $x = 1$ .

2. Let  $g$  be the function given by  $g(x) = \int_0^{x^2} e^{-t^2} dt$ . The function  $g$  has a point of inflection at  $x =$   
 Let  $f(x) = e^{-x^2}$ . Let  $F(x)$  represent its antiderivative.

$$\begin{aligned} g(x) &= F(x^2) - F(0) \\ g'(x) &= 2xF'(x^2) \\ \implies g'(x) &= 2xe^{-x^4} \\ g''(x) &= 2e^{-x^4} + 2xe^{-x^4} * (-4x^3) \\ 0 &= 2e^{-x^4}(1 - 4x^4) \\ x^4 &= \frac{1}{4}x = \pm \frac{\sqrt{2}}{2} \end{aligned}$$

It is good practice to test that these inflection points are actually inflection points, but if you are short on time you can just pick the MCQ that includes either or both  $\pm \frac{\sqrt{2}}{2}$ .