

SAT Math Equations

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1 Quadratics

A quadratic is a function in the form

$$f(x) = ax^2 + bx + c$$

Where $\{a, b, c\} \in \mathbb{R}$.

Or in the vertex form:

$$f(x) = a(x - h)^2 + k$$

Where $\{a, h, k\} \in \mathbb{R}$

Or in the roots form:

$$f(x) = a(x - r_1)(x - r_2)$$

Where $\{a, r_1, r_2\} \in \mathbb{R}$

1.1 FOIL

FOIL stands for First Outer Inner Last. Meaning, when you have

$$f(x) = (a_1x - r_1)(a_2x - r_2)$$

You get:

$$a_1a_2x^2 - a_1r_2x - a_2r_1x + r_1r_2 = a_1a_2x^2 - (a_1r_1 + a_2r_2)x + (r_1r_2)$$

1.2 Quadratic Formula

1.2.1 Derivation

$$ax^2 + bx + c = 0 \implies ax^2 + bx = -c \implies \frac{1}{a}ax^2 + \frac{b}{a}x = -\frac{c}{a} \implies x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Complete the square:

$$\begin{aligned} x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} &= -\frac{c}{a} + \frac{b^2}{4a^2} \implies \left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \implies \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \\ \implies \sqrt{\left(x + \frac{b}{2a}\right)^2} &= \frac{\pm\sqrt{b^2 - 4ac}}{2a} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

1.2.2 Alternatives

In some scenarios it may be easier to use systems of equations.

Given a quadratic in the form:

$$ax^2 + bx + c = 0$$

We want to find r_1 and r_2 such that:

$$r_1 + r_2 = b \quad r_1r_2 = c$$

You can easily substitute around until you get the answer. If $a \neq 1$ then it is difficult to use this formula.

1.3 Discriminant

The discriminant tells you whether a quadratic is going to have real or imaginary roots. If we look at the quadratic formula for earlier:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The term $\pm\sqrt{b^2 - 4ac}$ denotes the possibility of two solutions or one solution or none. If $\sqrt{b^2 - 4ac} > 0$ then the \pm would have an effect, making the two solutions $-\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. If $\sqrt{b^2 - 4ac} = 0$ then the \pm would have no effect, making the single solution $-\frac{-b}{2a}$. However, if $b^2 - 4ac < 0$ then the \sqrt of it would be a complex number, meaning there would be **no real** solutions.

1.4 Vertex

1.5 Factoring

You can use the two equations to do it

1.5.1 Aquarc Method

The fancy X thing

2 Sets

2.1 Domain and Range

2.2 Standard Deviation

3 Word Problems

You can't really predict what these will be. For

4 Trigonometry