SAT Math Equations

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Contents

1 Quadratics

A quadratic is a function in the form

$$f(x) = ax^2 + bx + c$$

Where $\{a, b, c\} \in \mathbb{R}$.

Or in the vertex form:

$$f(x) = a(x - h)^2 + k$$

Where $\{a, h, k\} \in \mathbb{R}$

Or in the roots form:

$$f(x) = a(x - r_1)(x - r_2)$$

Where $\{a, r_1, r_2\} \in \mathbb{R}$

1.1 FOIL

FOIL stands for First Outer Inner Last. Meaning, when you have

$$f(x) = (a_1x - r_1)(a_2x - r_2)$$

You get:

$$a_1a_2x^2 - a_1r_2x - a_2r_1x + r_1r_2 = a_1a_2x^2 - (a_1r_1 + a_2r_2)x + (r_1r_2)$$

1.2 Quadratic Formula

1.2.1 Derivation

$$ax^2 + bx + c = 0 \implies ax^2 + bx = -c \implies \frac{1}{a}ax^2 + \frac{b}{a}x = -\frac{c}{a} \implies x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Complete the square:

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} \implies (x + \frac{b}{2a})^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \implies (x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\implies \sqrt{(x + \frac{b}{2a})^2} = \frac{\pm\sqrt{b^2 - 4ac}}{2a} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1.2.2 Alternatives

In some scenarios it may be easier to use systems of equations.

Given a quadratic in the form:

$$ax^2 + bx + c = 0$$

We want to find r_1 and r_2 such that:

$$r_1 + r_2 = b \quad r_1r_2 = c$$

You can easily subsitute around until you get the answer. If $a \neq 1$ then it is difficult to use this formula.

1.3 Discriminant

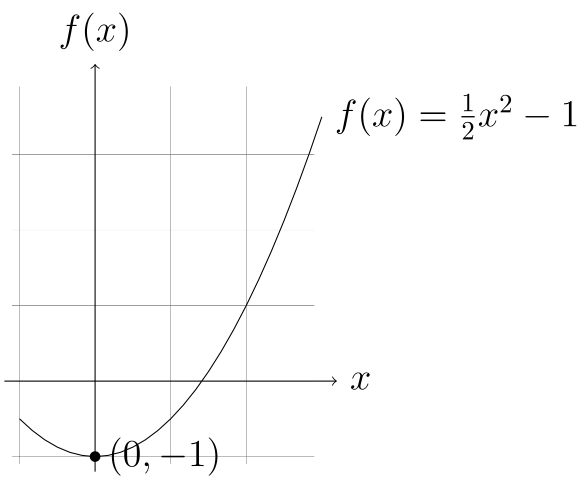
The discriminant tells you whether a quadratic is going to have real or imaginary roots. If we look at the quadratic formula for earlier:

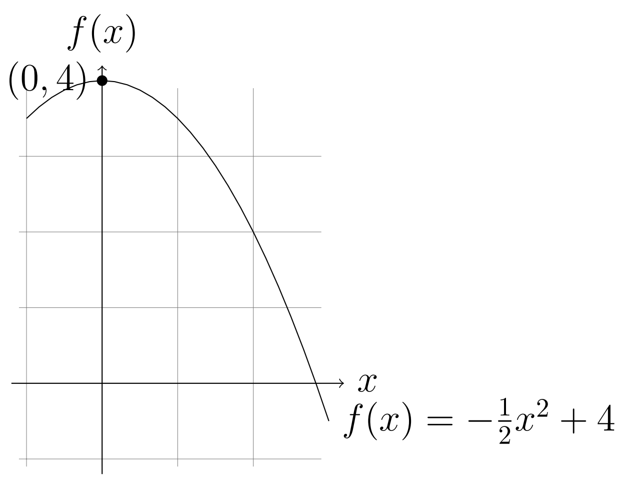
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The term $\pm\sqrt{b^2 - 4ac}$ denotes the possibility of two solutions or one solution or none. If $\sqrt{b^2 - 4ac} > 0$ then the $\pm$ would have an effect, making the two solutions $-\frac{-b \pm \sqrt{b^2-4ac}}{2a}$. If $\sqrt{b^2 - 4ac} = 0$ then the $\pm$ would have no effect, making the single solution $-\frac{-b}{2a}$. However, if $b^2 - 4ac < 0$ then the $\sqrt{}$ of it would be a complex number, meaning there would be **no real** solutions.

1.4 Vertex

The vertex is maximum (or minimum) of a quadratic in the form $(x, f(x))$





The vertex is always equidistant from the two roots. So if you find the average of the two roots, you can also find the vertex.

$$x = \frac{\frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}}{2} = \frac{\frac{-2b}{2a}}{2} = -\frac{b}{2a}$$

You can plug in this x value to find $f(x)$.

1.5 Factoring

Let's first assume a quadratic in the form:

$$x^2 + bx + c = 0$$

Where $a = 1$. We are solving for r_1 and r_2.

In this scenario, we know by FOIL that:

$$x^2 + (r_1 + r_2)x + (r_1 r_2) = 0$$

Therefore:

$$r_1 + r_2 = b \quad r_1 r_2 = c$$

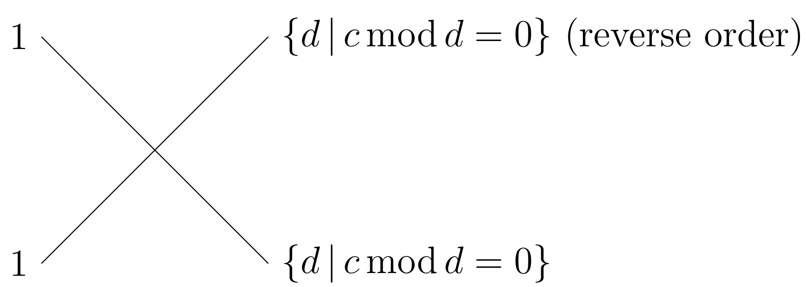
However if $a \neq 1$ then it is difficult to use this formula. For that reaosn, we propose usage of the Aquarc Method.

1.6 Aquarc Factoring Method

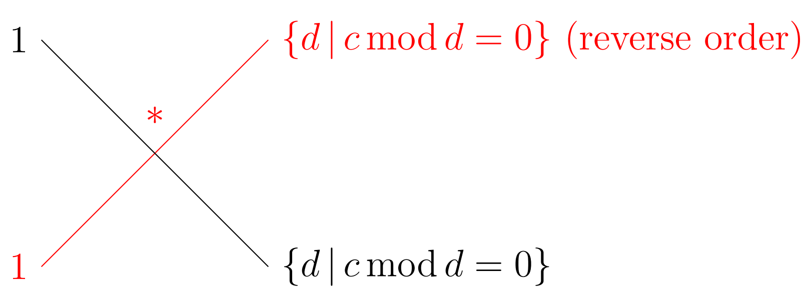
Let's take the same function $f(x) = x^2 + bx + c$ again.

First, we identify all the factors of a and c. Since $a = 1$, we only need to find all the factors of c.

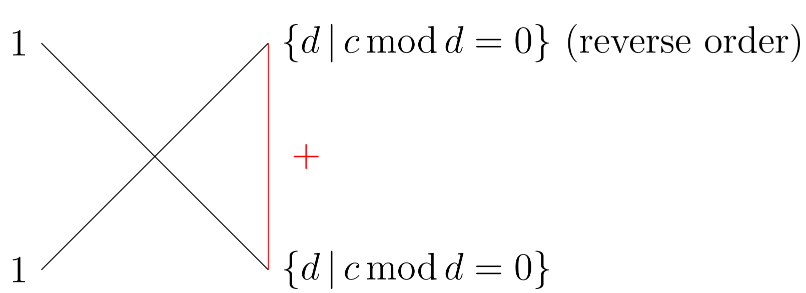
Let's represent all factors of c with $\{d \,|\, c \bmod d = 0\}$. Let d_r represent a random factor. And let's draw it out on a giant X.



Multiply this first diagonal like $1 * d_r$



Add the second diagonal by the vertical:



And multiply again with $1 * \frac{c}{d_r}$

1 $\qquad$ $\{d \,|\, c \bmod d = 0\}$ (reverse order)

$*$

1 $\qquad$ $\{d \,|\, c \bmod d = 0\}$

At the end you get:

$$1 * d_r + 1 * \frac{c}{d_r}$$

If $a \neq 1$ then simply change the left hand side to include $\{d \,|\, a \bmod d = 0\}$.

$\{d \,|\, a \bmod d = 0\}$ (reverse order) $\qquad$ $\{d \,|\, c \bmod d = 0\}$ (reverse order)

$\{d \,|\, a \bmod d = 0\}$ $\qquad$ $\{d \,|\, c \bmod d = 0\}$

Note that:

- if $c > 0$ then dependent on whether $b > 0$ or $b < 0$ pick the positive or negative factors in d.
- otherwise, one of the factors in d will be negative and the other chosen factor should be positive.

1.6.1 Example

$$f(x) = 3x^2 - 27x + 60 \Longrightarrow f(x) = 3(x^2 - 9x + 20)$$

Since $c > 0$ and $b < 0$, then both sets have to be negative for the equation to be true.

1 $\qquad$ $\{-20, -10, -5, -4, -2, -1\}$

1 $\qquad$ $\{-1, -2, -4, -5, -10, -20\}$

$$1 * \mathbf{-4} + 1 * \mathbf{-5} = -9$$

Therefore:

$$f(x) = 3(x - 4)(x - 5)$$

Since a doesn't have to be 1, this method scales well with more difficult quadratics.

2 Sets

2.1 Domain and Range

2.2 Standard Deviation

3 Word Problems

You can't really predict what these will be. For

4 Trigonometry