

# SAT Math Equations

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## Contents

|          |                                   |          |
|----------|-----------------------------------|----------|
| <b>1</b> | <b>Quadratics</b>                 | <b>2</b> |
| 1.1      | FOIL . . . . .                    | 2        |
| 1.2      | Quadratic Formula . . . . .       | 2        |
| 1.2.1    | Derivation . . . . .              | 2        |
| 1.2.2    | Alternatives . . . . .            | 2        |
| 1.3      | Discriminant . . . . .            | 3        |
| 1.4      | Vertex . . . . .                  | 3        |
| 1.5      | Factoring . . . . .               | 3        |
| 1.6      | Aquarc Factoring Method . . . . . | 4        |
| 1.6.1    | Example . . . . .                 | 5        |
| <b>2</b> | <b>Sets</b>                       | <b>6</b> |
| 2.1      | Domain and Range . . . . .        | 6        |
| 2.2      | Standard Deviation . . . . .      | 6        |
| <b>3</b> | <b>Trigonometry</b>               | <b>6</b> |

# 1 Quadratics

A quadratic is a function in the form

$$f(x) = ax^2 + bx + c$$

Where  $\{a, b, c\} \in \mathbb{R}$ .

Or in the vertex form:

$$f(x) = a(x - h)^2 + k$$

Where  $\{a, h, k\} \in \mathbb{R}$

Or in the roots form:

$$f(x) = a(x - r_1)(x - r_2)$$

Where  $\{a, r_1, r_2\} \in \mathbb{R}$

## 1.1 FOIL

FOIL stands for First Outer Inner Last. Meaning, when you have

$$f(x) = (a_1x - r_1)(a_2x - r_2)$$

You get:

$$a_1a_2x^2 - a_1r_2x - a_2r_1x + r_1r_2 = a_1a_2x^2 - (a_1r_1 + a_2r_2)x + (r_1r_2)$$

## 1.2 Quadratic Formula

### 1.2.1 Derivation

$$ax^2 + bx + c = 0 \implies ax^2 + bx = -c \implies \frac{1}{a}ax^2 + \frac{b}{a}x = -\frac{c}{a} \implies x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Complete the square:

$$\begin{aligned} x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} &= -\frac{c}{a} + \frac{b^2}{4a^2} \implies (x + \frac{b}{2a})^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \implies (x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2} \\ &\implies \sqrt{(x + \frac{b}{2a})^2} = \frac{\pm\sqrt{b^2 - 4ac}}{2a} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

### 1.2.2 Alternatives

In some scenarios it may be easier to use systems of equations.

Given a quadratic in the form:

$$ax^2 + bx + c = 0$$

We want to find  $r_1$  and  $r_2$  such that:

$$r_1 + r_2 = b \quad r_1r_2 = c$$

You can easily substitute around until you get the answer. If  $a \neq 1$  then it is difficult to use this formula.

### 1.3 Discriminant

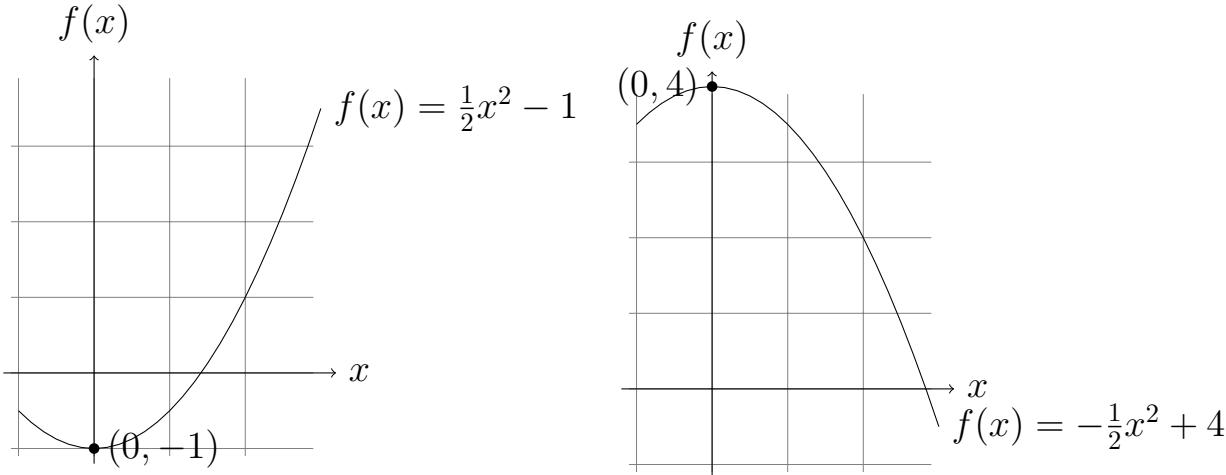
The discriminant tells you whether a quadratic is going to have real or imaginary roots. If we look at the quadratic formula for earlier:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The term  $\pm\sqrt{b^2 - 4ac}$  denotes the possibility of two solutions or one solution or none. If  $\sqrt{b^2 - 4ac} > 0$  then the  $\pm$  would have an effect, making the two solutions  $-\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . If  $\sqrt{b^2 - 4ac} = 0$  then the  $\pm$  would have no effect, making the single solution  $-\frac{-b}{2a}$ . However, if  $b^2 - 4ac < 0$  then the  $\sqrt{\phantom{x}}$  of it would be a complex number, meaning there would be **no real** solutions.

### 1.4 Vertex

The vertex is maximum (or minimum) of a quadratic in the form  $(x, f(x))$



The vertex is always equidistant from the two roots. So if you find the average of the two roots, you can also find the vertex.

$$x = \frac{\frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}}{2} = \frac{\frac{-2b}{2a}}{2} = -\frac{b}{2a}$$

You can plug in this  $x$  value to find  $f(x)$ .

### 1.5 Factoring

Let's first assume a quadratic in the form:

$$x^2 + bx + c = 0$$

Where  $a = 1$ . We are solving for  $r_1$  and  $r_2$ .

In this scenario, we know by FOIL that:

$$x^2 + (r_1 + r_2)x + (r_1 r_2) = 0$$

Therefore:

$$r_1 + r_2 = b \quad r_1 r_2 = c$$

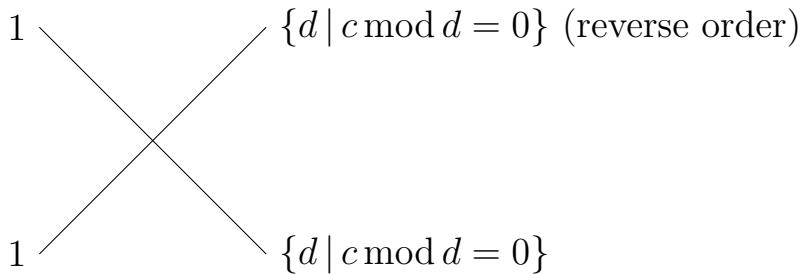
However if  $a \neq 1$  then it is difficult to use this formula. For that reason, we propose usage of the Aquarc Method.

## 1.6 Aquarc Factoring Method

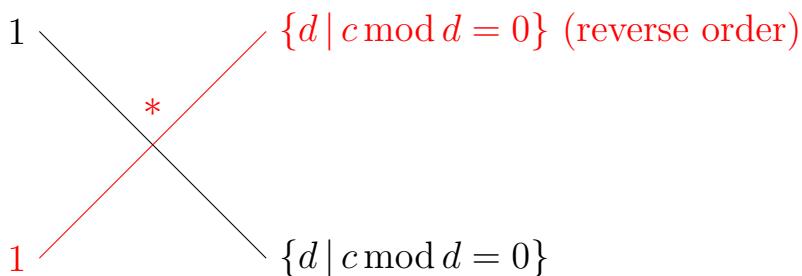
Let's take the same function  $f(x) = x^2 + bx + c$  again.

First, we identify all the factors of  $a$  and  $c$ . Since  $a = 1$ , we only need to find all the factors of  $c$ .

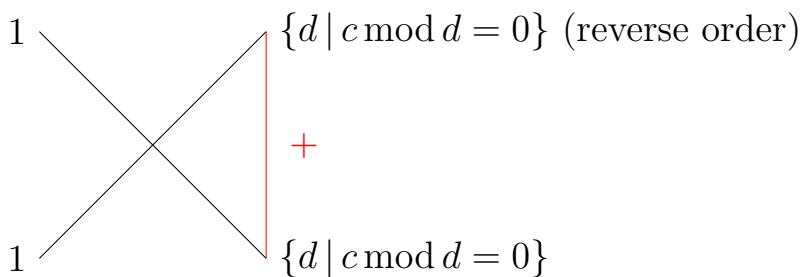
Let's represent all factors of  $c$  with  $\{d \mid c \bmod d = 0\}$ . Let  $d_r$  represent a random factor. And let's draw it out on a giant X.



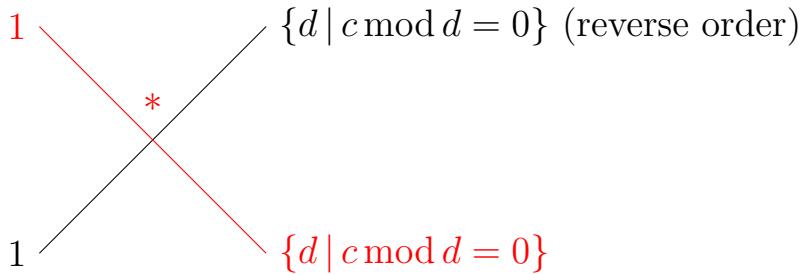
Multiply this first diagonal like  $1 * d_r$



Add the second diagonal by the vertical:



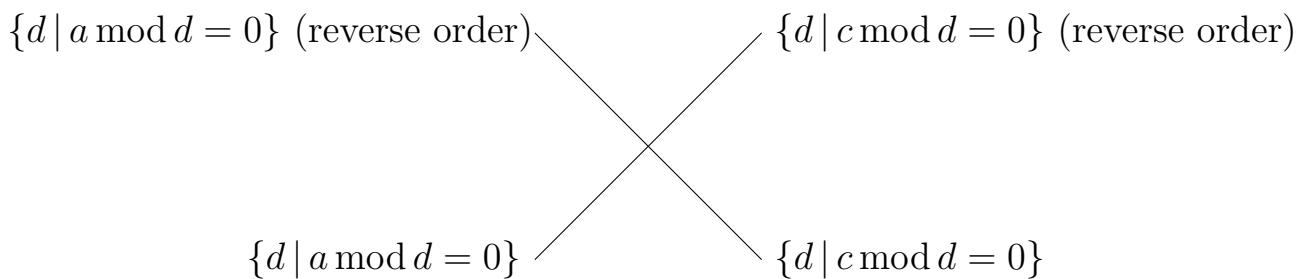
And multiply again with  $1 * \frac{c}{d_r}$



At the end you get:

$$1 * d_r + 1 * \frac{c}{d_r}$$

If  $a \neq 1$  then simply change the left hand side to include  $\{d | a \text{ mod } d = 0\}$ .



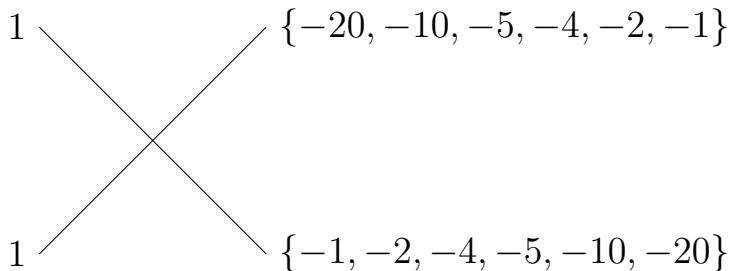
Note that:

- if  $c > 0$  then dependent on whether  $b > 0$  or  $b < 0$  pick the positive or negative factors in  $d$ .
- otherwise, one of the factors in  $d$  will be negative and the other chosen factor should be positive.

#### 1.6.1 Example

$$f(x) = 3x^2 - 27x + 60 \implies f(x) = 3(x^2 - 9x + 20)$$

Since  $c > 0$  and  $b < 0$ , then both sets have to be negative for the equation to be true.



$$1 * -4 + 1 * -5 = -9$$

Therefore:

$$f(x) = 3(x - 4)(x - 5)$$

Since  $a$  doesn't have to be 1, this method scales well with more difficult quadratics.

## 2 Sets

On the SAT, you may be given a set visually or in numbers. For this guide, we will use sets in this format:

$$\{a_1, a_2, \dots, a_n\}$$

### 2.1 Domain and Range

Domain is the total distance of the inputs. For the above set, the domain could be denoted as  $n - 1$  or  $[1, n]$  because the first element is  $a_1$  and the last is  $a_n$ .

The range is the total distance, which would be  $\max(\{a_1, a_2, \dots, a_n\}) - \min(\{a_1, a_2, \dots, a_n\})$ .

### 2.2 Standard Deviation

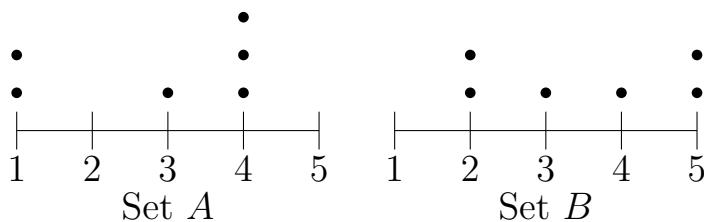
Standard deviation measures the total difference between the mean and each value in the set.

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2}$$

Where  $n$  is the length,  $x_i$  is each item in the set, and  $\mu$  is the mean.

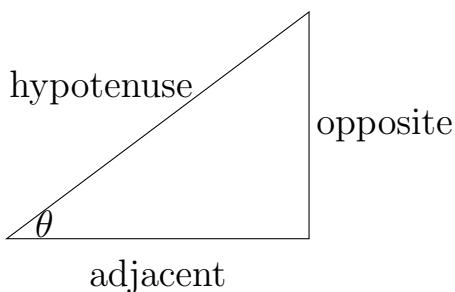
The SAT tests whether you can conceptually understand the deviation or not. So if your data typically is further from the mean, it has a higher standard deviation. Note that spread is unrelated to the range. Don't let outliers in the range confuse you.

Typically, on the SAT they will look like:



Set B has the smaller standard deviation.

## 3 Trigonometry



$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$