SAT Math Equations

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Contents

1 Quadratics

A quadratic is a function in the form

$$f(x) = ax^2 + bx + c$$

Where $\{a, b, c\} \in \mathbb{R}$.
Or in the vertex form:

$$f(x) = a(x - h)^2 + k$$

Where $\{a, h, k\} \in \mathbb{R}$
Or in the roots form:

$$f(x) = a(x - r_1)(x - r_2)$$

Where $\{a, r_1, r_2\} \in \mathbb{R}$

1.1 FOIL

FOIL stands for First Outer Inner Last. Meaning, when you have

$$f(x) = (a_1 x - r_1)(a_2 x - r_2)$$

You get:

$$a_1 a_2 x^2 - a_1 r_2 x - a_2 r_1 x + r_1 r_2 = a_1 a_2 x^2 - (a_1 r_1 + a_2 r_2)x + (r_1 r_2)$$

1.2 Quadratic Formula

1.2.1 Derivation

$$ax^2 + bx + c = 0 \implies ax^2 + bx = -c \implies \frac{1}{a}ax^2 + \frac{b}{a}x = -\frac{c}{a} \implies x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Complete the square:

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} \implies (x + \frac{b}{2a})^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \implies (x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\implies \sqrt{(x + \frac{b}{2a})^2} = \frac{\pm\sqrt{b^2 - 4ac}}{2a} \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1.2.2 Alternatives

In some scenarios it may be easier to use systems of equations.
Given a quadratic in the form:

$$ax^2 + bx + c = 0$$

We want to find r_1 and r_2 such that:

$$r_1 + r_2 = b \quad r_1 r_2 = c$$

You can easily subsitute around until you get the answer. If $a \neq 1$ then it is difficult to use this formula.

You can use the two equations to do it

1.5.1   Aquarc Method

The fancy X thing

2   Sets

3   Word Problems

You can't really predict what these will be. For

4   Trigonometry