Min/Max and Point of Inflection Cheatsheet

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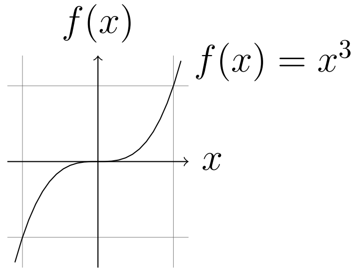
1 Min/Max

To check for a minimum or maximum, you set $f'(x) = 0$ to obtain the **critical points**. Not all of them are going to be maxima or minima, and sometimes neither.



1. If $f''(x) < 0$ and $f'(x) = 0$, then $(x, f(x))$ is a maximum
2. If $f''(x) > 0$ and $f'(x) = 0$, then $(x, f(x))$ is a minimum
3. If $f''(x) = 0$ and $f'(x) = 0$, then $(x, f(x))$ is indeterminate; check if signs change or use a Taylor approximation instead (not practical).

1.1 Taylor Approximation

A Taylor Series is used to approximate a point x from $f(x)$ using a series around c. It looks like:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n \tag{1}$$

Because no sole derivative will give you enough information from the function at one point (illustrated by $f(x) = x^3$, x^4, x^5...) you need to know all the derivatives. And if the function isn't smooth (differentiable for each x where $x \in \mathbb{R}$, this method can't be used analytically either.

2 Point of Inflection

Points of Inflection **may** exist when $f''(x) = 0$ or are undefined. You have to use Taylor approximations to figure out if it actually is a point of inflection. It is preferable to check if the signs change.

2.1 Examples

These problems are taken from *Calculus for the AP Course* by Michael Sullivan.

1. The function f has a second derivative given by $f''(x) = x^2(x-1)\sqrt{x+1}$. At what values of x does $f(x)$ have a point of inflection?

   At a quick glance, potential points of inflection are $x = -1, 0, 1$. However, we need to plug in values between each of these potential inflection points to find where $f''(x)$ switches from negative to positive or vice versa. You can also figure out that x^2 and $\sqrt{x+1}$ will always return a positive value, so the only solution is $x = 1$.

2. Let g be the function given by $g(x) = \int_0^{x^2} e^{-t^2} dt$. The function g has a point of inflection at $x =$

Let $f(x) = e^{-x^2}$. Let $F(x)$ represent its antiderivative.

$$\begin{aligned} g(x) &= F(x^2) - F(0) \\ g'(x) &= 2xF'(x^2) \\ \implies g'(x) &= 2xe^{-x^4} \\ g''(x) = 2e^{-x^4} &+ 2xe^{-x^4} * (-4x^3) \\ 0 &= 2e^{-x^4}(1 - 4x^4) \\ x^4 &= \frac{1}{4} x = \pm\frac{\sqrt{2}}{2} \end{aligned}$$

It is good practice to test that these inflection points are actually inflection points, but if you are short on time you can just pick the MCQ that includes eiither or both $\pm\frac{\sqrt{2}}{2}$.