Options Arbitrage Technique

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Integrating:

$$\int_0^T dy(t) = \int_0^T \frac{\partial y}{\partial x} dx(t) \tag{1}$$

$$y(t)\Big|_0^T \approx \sum_{i=1}^{n} \frac{\partial y(t_i)}{\partial x} \Delta x_{t_i} \tag{2}$$

$$y(T) - y(0) \approx \sum_{i=1}^{n} q_i \left(x_i - x_{i-1}\right) \tag{3}$$

How arbitrage works? Any price not equal to fair option price $y(0)$ can be arbitraged by synthesizing the opposite position through rebalanced stock.

Say option trades at a discount M, buy option at $y(0) - M$, PNL is:

$$\begin{aligned} P_o &= y(T) - (y(0) - M) \\ &= (y(T) - y(0)) + M \end{aligned} \tag{4}$$

Sell delta-hedged stock, PNL is:

$$\begin{aligned} P_s &= -\left(\sum_{i=1}^{n} q_i (x_i - x_{i-1})\right) \\ &= -\left(y(T) - y(0)\right) \end{aligned} \tag{5}$$

Total PNL then:

$$P_o + P_s = M \tag{6}$$