Name: _____ Group Members: _____

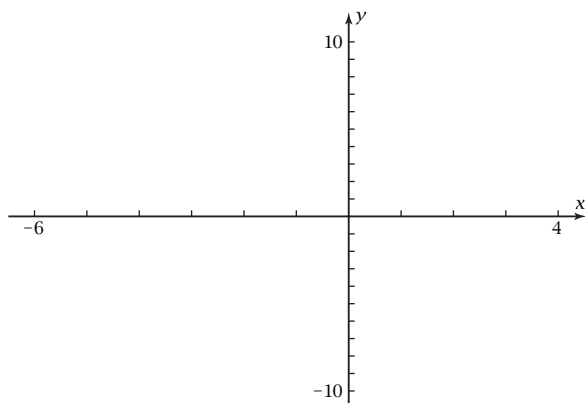# Exploration 15-4b: Limits and Curved Asymptotes

Date: _____

Objective: Analyze graphs of rational algebraic functions with the help of synthetic substitution and factoring.

1. Let

$$f(x) = \frac{x^3 + 2x^2 - 8x + 5}{x - 1}$$

   Plot the graph of f as y_1. Use a friendly window with an x-range of about $x = -6$ to $x = 4$ that includes $x = 1$ as a grid point. Have the grid off. Sketch the graph.

2. $f(1)$ is undefined because of division by zero. Trace to $x = 1$ on your graph. What feature does the graph have at this point? By tracing closer and closer to $x = 1$, find the **limit** $f(x)$ seems to be approaching as x approaches 1. Is the limit as x approaches 1 from the left side the same as when x approaches 1 from the right side?

3. Remove the discontinuity at $x = 1$ algebraically by factoring the numerator and reducing the fraction. Evaluate the resulting quotient polynomial at $x = 1$. Is the answer equal to the limit you found in Problem 2?

4. How do you read $\lim_{x \to 1} f(x) = -1$? What does this equation mean?

5. Let

$$g(x) = \frac{x^3 + 2x^2 - 8x + 6}{x - 1}$$

   Plot the graph of g as y_2 using "thick" style. What feature does the graph of g have at $x = 1$? Sketch the graphs of $f(x)$ and $g(x)$ here, showing their relationship to each other.
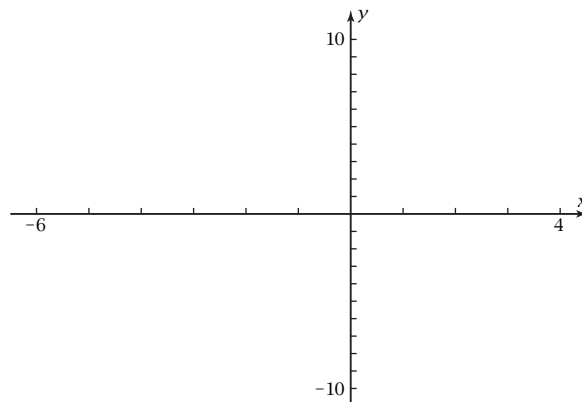
6. Try to find $\lim_{x \to 1} g(x)$ by tracing to x-values closer and closer to 1. Try x-values on both sides of 1. What happens to the quotient as x approaches 1?

7. To understand why the graph of $g(x)$ resembles the graph of $f(x)$, simplify the equation for $g(x)$ by synthetic substitution. Write the equation in "mixed-number" form as

$$g(x) = (\text{polynomial}) + \frac{\text{remainder}}{x - 1}$$

   What relationship do you notice between the equations for $f(x)$ and $g(x)$?

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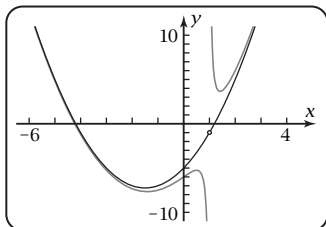
8. Let

$$h(x) = \frac{x^3 + 2x^2 - 8x + 4}{x - 1}$$

Plot the graph of $h(x)$ as y_3.

9. The graphs of f and g from Problem 5 should look like this. On this figure, sketch the graph of h.

10. Simplify the equation for $h(x)$ by long division or synthetic substitution. Write the result in mixed-number form as

$$h(x) = (\text{polynomial}) + \frac{\text{remainder}}{x - 1}$$

11. How is the polynomial part of $h(x)$ in Problem 10 related to the graph of $h(x)$?

12. Why is it important for your work to be 100% correct in problems like these that are sequential?

13. Write a paragraph summarizing the things you have learned about rational functions, removable discontinuities, asymptotes, etc. as a result of doing this Exploration.