

4.2 Exercises

In Exercises 1 – 16, sketch the graph of a continuous function that satisfies all the given conditions.

1. Given conditions:

- a. $f(-1) = 2$.
- b. $f'(-1) = 0$.
- c. $f'(x) < 0$ if $x < -1$.
- d. $f'(x) > 0$ if $x > -1$.
- e. $f''(x) > 0$ for all x .

2. Given conditions:

- a. $f(3) = 4$.
- b. $f'(3) = 0$.
- c. $f'(x) < 0$ if $x > 3$.
- d. $f'(x) > 0$ if $x < 3$.
- e. $f''(x) < 0$ for all x .

3. Given conditions:

- a. $f(-2) = 4$, $f(-1) = 1$, $f(1) = -1$.
- b. $f'(-2) = 0$, $f'(1) = 0$.
- c. $f'(x) < 0$ if $-2 < x < 1$.
- d. $f'(x) > 0$ if $x < -2$ or $x > 1$.
- e. $f''(-1) = 0$.
- f. $f''(x) < 0$ if $x < -1$.
- g. $f''(x) > 0$ if $x > -1$.

4. Given conditions:

- a. $f(0) = -2$, $f(2) = 0$, $f(3) = 3$.
- b. $f'(0) = 0$, $f'(3) = 0$.
- c. $f'(x) < 0$ if $x < 0$ or $x > 3$.
- d. $f'(x) > 0$ if $0 < x < 3$.
- e. $f''(2) = 0$.
- f. $f''(x) < 0$ if $x > 2$.
- g. $f''(x) > 0$ if $x < 2$.

5. Given conditions:

- a. $f(-3) = 5$, $f(-1) = 2$, $f(0) = -1$.
- b. $f'(-3) = 0$, $f'(0) = 0$.
- c. $f'(x) < 0$ if $x < 0$ and $x \neq -3$.
- d. $f'(x) > 0$ if $x > 0$.
- e. $f''(-3) = 0$, $f''(-1) = 0$.
- f. $f''(x) < 0$ if $-3 < x < -1$.
- g. $f''(x) > 0$ if $x < -3$ or $x > -1$.

6. Given conditions:

- a. $f(1) = 2$, $f(2) = 3$, $f(4) = 4$,
 $f(6) = 2$.
- b. $f'(1) = 0$, $f'(4) = 0$.
- c. $f'(x) < 0$ if $x > 4$, $x < 1$.
- d. $f'(x) > 0$ if $1 < x < 4$.

7. Given conditions:

- a. $f(x) = ax^3 + bx + c$.
- b. $f(0) = 0$.
- c. $f(1) = 15$.
- d. $f'(-1) = 0$ and $x = -1$ is a local max.

8. Given conditions:

- a. $f(10) = 5$.
- b. $f'(5) = 0$.
- c. $f''(5) = 10$.
- d. $f''(x) < 0$ if $x > 10$.
- e. $f''(x) > 0$ if $x < 10$.

9. Given conditions:

- a. $f''(x) > 0$ if $x < 5$.
- b. $f''(5) = 0$.
- c. $f''(x) < 0$ if $x > 5$.
- d. $f'(x) > 0$ for all x .

10. Given conditions:

- a. $f(4) = 8$.
- b. $f'(4) = 0$.
- c. $f''(4) = 8$.