

## Exploration 45: Euler's Method

**Objective:** Given a differential equation, find an approximation to a particular solution by a numerical method.

1. For the differential equation

$$\frac{dy}{dx} = -\frac{x}{2y},$$

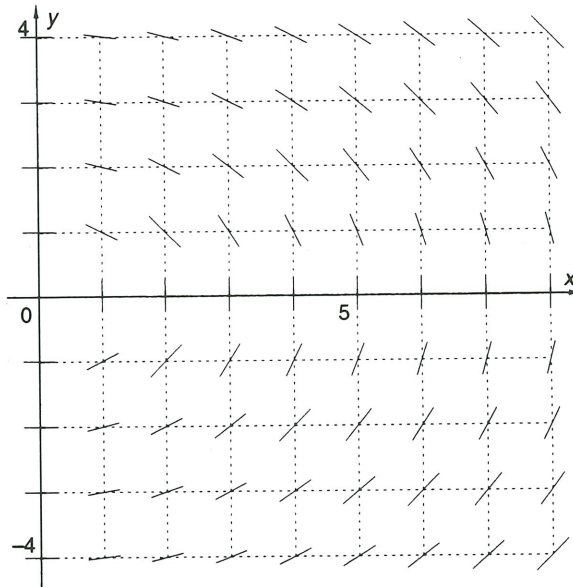
calculate the slope at the point (0, 3). Then calculate  $dy$  if  $dx = 0.5$ . Use the result to estimate the value of  $y$  at  $x = 0.5$ .

2. Calculate the slope at (0.5,  $y$ ) from Problem 1. Then calculate  $dy$  if  $dx = 0.5$ . Use the result to estimate the value of  $y$  at  $x = 1$ .

3. Repeat the computations in Problems 1 and 2 for values of  $x$  from 1.5 through 7. Record the values in the table. This technique is called **Euler's method** for solving differential equations.

$x$	$y$	Slope	$dy$
0	3	0	0
0.5	3	-0.0833...	-0.0416...
1	2.9583...		
1.5			
2			
2.5			
3			
3.5			
4			
4.5			
5			
5.5			
6			
6.5			
7			

4. The graph below shows the slope field for this differential equation from  $x = 0$  through  $x = 7$ . Plot the  $y$ -values from Problems 1 through 3 on this graph paper. For which values of  $x$  does the numerical solution by Euler's method seem to follow the slope field? For which values of  $x$  is the numerical solution clearly wrong?



5. Solve the differential equation in Problem 1 algebraically. Plot the particular solution that contains (0, 3). Explain why Euler's method gives meaningless answers for larger values of  $x$ .
6. What did you learn as a result of doing this Exploration that you did not know before?