Problem Set 3-2

Quick Review

Q1. Quick! What does derivative mean?

Q2. Simplify: $(x^2 - 81)/(x - 9)$

Q3. Find: $\lim_{x \to 9}(x^2 - 81)/(x - 9)$

Q4. Sketch the graph of $y = 2^x$.

Q5. Expand: $(3x - 7)^2$

Q6. Fill in the blank with the appropriate operation: $\log(x/y) = \log x$ —?— $\log y$.

Q7. Sketch a graph with a step discontinuity at $x = 3$.

Q8. Sketch a graph with a cusp at the point $(5, 2)$.

Q9. Who is credited with inventing calculus?

Q10. $\lim_{x \to c} kf(x) = k \lim_{x \to c} f(x)$ is a statement of which limit property?

A. Limit of a sum

B. Limit of a product

C. Limit of a constant

D. Limit of a constant times a function

E. Limit of the identity function

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1. Write the definition of derivative.

2. What are the physical and the graphical meanings of the derivative of a function?

For Problems 3 and 4,

a. Use the definition of derivative to calculate the value of $f'(c)$ exactly.

b. Plot the difference quotient in a window that includes c and sketch the result.

c. Plot the graph of the function in a window that includes c.

d. Plot a line through $(c, f(c))$ with slope $f'(c)$. Sketch the graph and the line.

3. $f(x) = 0.6x^2,\ c = 3$

4. $f(x) = -0.2x^2,\ c = 6$

For Problems 5–12, use the definition of derivative to calculate $f'(c)$ exactly.

5. $f(x) = x^2 + 5x + 1,\ c = -2$

6. $f(x) = x^2 + 6x - 2,\ c = -4$

7. $f(x) = x^3 - 4x^2 + x + 8,\ c = 1$

8. $f(x) = x^3 - x^2 - 4x + 6,\ c = -1$

9. $f(x) = -0.7x + 2,\ c = 3$

10. $f(x) = 1.3x - 3,\ c = 4$

11. $f(x) = 5,\ c = -1$

12. $f(x) = -2,\ c = 3$

13. From the results of Problems 9 and 10, what can you conclude about the derivative of a linear function? How does this conclusion relate to derivatives and tangent lines?

14. From the results of Problems 11 and 12, what can you conclude about the derivative of a constant function? How does this conclusion relate to derivatives and tangent lines?

15. *Local Linearity Problem:* Figure 3-2d shows the graph of $f(x) = x^2$, and a line of slope $f'(1)$ passing through the point on the graph of f where $x = 1$.

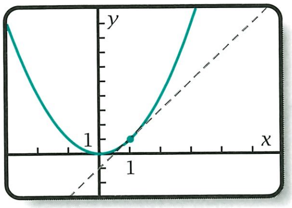

Figure 3-2d

a. Reproduce this graph on your grapher. Use a friendly window that includes the point $(1, 1)$. Describe how you plotted the tangent line.

b. Zoom in on the point $(1, 1)$. What do you notice about the line and the curve?

c. Zoom in several more times. How do the line and the curve seem to be related now?

d. The graph of f possesses a property called **local linearity** at $x = 1$. Why do you suppose these words are used to describe this property?

e. Explain why you could say that the value of the derivative at a point equals the "slope of the graph" at that point if the graph has local linearity.