In Exercises 11–16, compute and simplify.

11. $x^{1/2}(x^{2/3} - x^{4/3})$

12. $x^{1/2}(3x^{3/2} + 2x^{-1/2})$

13. $(x^{1/2} + y^{1/2})(x^{1/2} - y^{1/2})$

14. $(x^{1/3} + y^{1/2})(2x^{1/3} - y^{3/2})$

15. $(x + y)^{1/2}[(x + y)^{1/2} - (x + y)]$

16. $(x^{1/3} + y^{1/3})(x^{2/3} - x^{1/3}y^{1/3} + y^{2/3})$

In Exercises 17–22, factor the given expression. For example,

$$x - x^{1/2} - 2 = (x^{1/2} - 2)(x^{1/2} + 1).$$

17. $x^{2/3} + x^{1/3} - 6$

18. $x^{2/5} + 11x^{1/5} + 30$

19. $x + 4x^{1/2} + 3$

20. $x^{1/3} + 7x^{1/6} + 10$

21. $x^{4/5} - 81$

22. $x^{2/3} - 6x^{1/3} + 9$

In Exercises 23–28, write the given expression without using radicals.

23. $\sqrt[3]{a^2 + b^2}$

24. $\sqrt[4]{a^{-3} - b^3}$

25. $\sqrt[4]{\sqrt[4]{a^3}}$

26. $\sqrt{\sqrt[3]{a^3b^4}}$

27. $\sqrt[5]{t}\sqrt{16t^5}$

28. $\sqrt{x}(\sqrt[3]{x^2})(\sqrt[4]{x^3})$

In Exercises 29–42, simplify the expression without using a calculator.

29. $\sqrt{80}$

30. $\sqrt{96}$

31. $\sqrt{6}\sqrt{12}$

32. $\sqrt{8}\sqrt{96}$

33. $\dfrac{-6 + \sqrt{99}}{15}$

34. $\dfrac{5 - \sqrt{175}}{10}$

35. $\sqrt{50} - \sqrt{72}$

36. $\sqrt{75} + \sqrt{192}$

37. $5\sqrt{20} - \sqrt{45} + 2\sqrt{80}$

38. $\sqrt[3]{40} + 2 \cdot \sqrt[3]{135} - 5 \cdot \sqrt[3]{320}$

39. $\sqrt{16a^8b^{-2}}$

40. $\sqrt{24x^6y^{-4}}$

41. $\dfrac{\sqrt{c^2d^6}}{\sqrt{4c^3d^{-4}}}$

42. $\dfrac{\sqrt{a^{-10}b^{-12}}}{\sqrt{a^{14}d^{-4}}}$

In Exercises 43–48, rationalize the denominator and simplify your answer.

43. $\dfrac{3}{\sqrt{8}}$

44. $\dfrac{2}{\sqrt{6}}$

45. $\dfrac{3}{2 + \sqrt{12}}$

46. $\dfrac{1 + \sqrt{3}}{5 + \sqrt{10}}$

47. $\dfrac{2}{\sqrt{x} + 2}$

48. $\dfrac{\sqrt{x}}{\sqrt{x} - \sqrt{c}}$

In Exercises 49–52, find the difference quotient of the given function. Then rationalize its numerator and simplify.

49. $f(x) = \sqrt{x + 1}$

50. $g(x) = 2\sqrt{x + 3}$

51. $f(x) = \sqrt{x^2 + 1}$

52. $g(x) = \sqrt{x^2 - x}$

In Exercises 53–56, use the equation $y = 92.8935 \cdot x^{.6669}$ which gives the approximate distance y (in millions of miles) from the sun to a planet that takes x earth years to complete one orbit of the sun. Find the distance from the sun to the planet whose orbit time is given.

53. Mercury (.24 years)

54. Mars (1.88 years)

55. Saturn (29.46 years)

56. Pluto (247.69 years)

Between 1790 and 1860, the population y of the United States (in millions) in year x was given by $y = 3.9572(1.0299^x)$, where $x = 0$ corresponds to 1790. In Exercises 57–60, find the U.S. population in the given year.

57. 1800

58. 1817

59. 1845

60. 1859

61. Here are some of the reasons why restrictions are necessary when defining fractional powers of a negative number.
    (a) Explain why the equations $x^2 = -4$, $x^4 = -4$, $x^6 = -4$, etc., have no real solutions. Hence, we cannot define $c^{1/2}$, $c^{1/4}$, $c^{1/6}$ when $c = -4$.
    (b) Since 1/3 is the same as 2/6, it should be true that $c^{1/3} = c^{2/6}$, that is, that $\sqrt[3]{c} = \sqrt[6]{c^2}$. Show that this is false when $c = -8$.

62. Use a calculator to find a *six*-place decimal approximation of $(311)^{-4.2}$. Explain why your answer cannot possibly be the number $(311)^{-4.2}$.

63. (a) Graph $f(x) = x^5$ and explain why this function has an inverse function.
    (b) Show algebraically that the inverse function is $g(x) = x^{1/5}$.

64. If n is an odd positive integer, show that $f(x) = x^n$ has an inverse function and find the rule of the inverse function. [*Hint:* Exercise 63 is the case when $n = 5$.]

In Exercises 65–67, use the catalog of basic functions (page 214) and Section 3.4 to describe the graph of the given function.

65. $g(x) = \sqrt{x + 3}$

66. $h(x) = \sqrt{x} - 2$

67. $k(x) = \sqrt{x + 4} - 4$

68. (a) Suppose r is a solution of the equation $x^n = c$ and s is a solution of $x^n = d$. Verify that rs is a solution of $x^n = cd$.
    (b) Explain why part (a) shows that $\sqrt[n]{cd} = \sqrt[n]{c}\sqrt[n]{d}$.