Name ______________________ Group ______________________ Date ______________

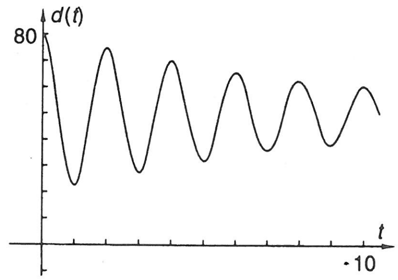
Exploration 9: Limits Involving Infinity

Objective: Discover what it means for a function to approach a limit as x approaches infinity.

A pendulum is pulled away from its rest position and let go. As it swings back and forth, its distance, $d(t)$ cm, from the wall is given by the equation

$$d(t) = 50 + 30(0.9)^t \cdot \cos \pi t,$$

where t is time in seconds since it was let go. The graph of function d shows that friction decreases the amplitude of the swings as time goes on.



1. Plot the graph of d using an x-window of about [20, 40] and the y-window shown in the graph. Sketch the result here.

2. What number does $d(t)$ stay close to as t becomes very large?

3. The number in Problem 2 is the **limit of $d(t)$ as t approaches infinity.** How large does t have to be in order for $d(t)$ to stay within $\varepsilon = 0.1$ unit of the limit for all larger values of t?

4. Tell the real-world meaning of the limit in Problem 2.

5. True or false: "The larger t gets, the closer $d(t)$ gets to the number in Problem 2." Explain how you arrived at your answer.

6. The definition of limit states that L is the limit of $f(x)$ as x approaches c if and only if for any $\varepsilon > 0$ there is a number $\delta > 0$ such that if x is within δ units of c $(x \neq c)$, then $f(x)$ is within ε units of L. But infinity is not a number; you can't keep x close to infinity. You can, however, keep x arbitrarily far away from zero. Using the symbol ∞ to stand for infinity, tell how the definition of limit can be modified to give meaning to

   $L = \lim_{x \to \infty} f(x).$

7. Why is the commonly used terminology ". . . $f(x)$ approaches L . . . " somewhat misleading? What words would better describe the relationship between $f(x)$ and L?

8. What did you learn as a result of doing this Exploration that you did not know before? (Over)