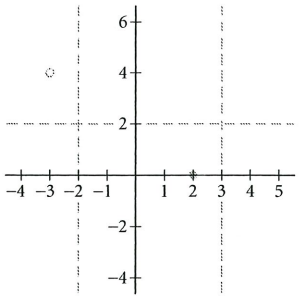
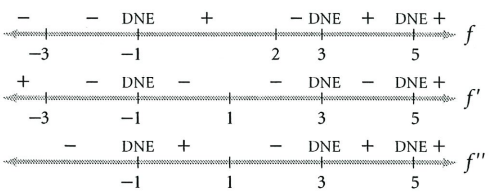


9. Given the number lines below, sketch a possible graph of f. Make sure that your graph reflects all of the information given in the number lines. Identify all roots, holes, and asymptotes of your graph. What additional information would you need in order to sketch a completely accurate graph of f?



10. Find an equation for a rational function that would have the number lines from Problem 9. You may have to do some educated guessing (and use a graphing calculator to check and refine your guesses).

11. Many of the rational functions in the graphing exercises are chosen precisely because they (and/or their derivatives) can be factored using the techniques in this course. In general, however, rational functions are not always this nice to work with! Think of a "random" rational function that isn't too simple or too complicated. Can you factor its numerator and denominator easily? What about the numerators and denominators of its first and second derivatives?

12. If you can factor the denominator of a rational function $f(x) = \frac{p(x)}{q(x)}$, can you also factor the denominators of f' and f''? Why or why not? What if you can factor the numerator of f; could you then factor the numerators of f' and f''?

Skills

Make rough sketches of the following rational functions without using any derivative information. Be as accurate as you can. Begin by using the factors and degrees of the numerator and denominator, and then, as necessary, plot points of the function by hand and/or do a sign analysis of the function. Find the equations of any horizontal, vertical, slant, or curve asymptotes.

13. $f(x) = \dfrac{(x-1)(x+2)}{(x+1)(x-1)}$

14. $f(x) = \dfrac{x^2+x-6}{x-2}$

15. $f(x) = \dfrac{x^2+1}{x-1}$

16. $f(x) = \dfrac{1}{16-x^4}$

17. $f(x) = \dfrac{3x(x+1)}{(x-5)^2(x+2)}$

18. $f(x) = \dfrac{2x^3+4x^2-6x}{x^2-4}$

19. $f(x) = \dfrac{2x^3+3x^2-2x-3}{x^2-2x-3}$

20. $f(x) = \dfrac{(x^2-1)^2}{2x^2-3x-2}$

21. $f(x) = \dfrac{(x^2-4)^2}{2x^2-3x-2}$

Sketch accurate graphs of the following rational functions. Use first and second derivative information as well as root, hole, and asymptote information. If possible, label the coordinates of any roots, extrema, or inflection points. Give the equations of any asymptotes (vertical, horizontal, slant, or curve).

22. $f(x) = \dfrac{x^2-1}{x^2-x-2}$

23. $f(x) = \dfrac{x-2}{(x-1)^2}$

24. $f(x) = \dfrac{x^2}{x-1}$

25. $f(x) = \dfrac{3x^2-2x+4}{x^2-4x+4}$

26. $f(x) = \dfrac{x(x+1)(x-1)}{(x-2)(x+1)}$

27. $f(x) = \dfrac{x^3-5x}{x-1}$

28. $f(x) = \dfrac{(x^2-1)^2}{3x^2-x-2}$

29. $f(x) = \dfrac{x^4}{x-2}$

30. $f(x) = \dfrac{x^3-3x^2-x+3}{x^3-6x^2+12x-8}$

For each of the graphs given below, find a rational function f that has the properties shown in the graph. Explain