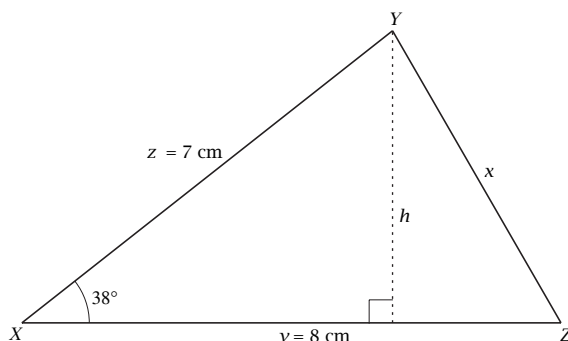


## Exploration 6-3a: Area of a Triangle and Hero's Formula

Date: \_\_\_\_\_

**Objective:** Derive a *quick* method to calculate the area of a triangle from two sides and the included angle.

For Problems 1-3,  $\triangle XYZ$  has sides  $y = 8$  cm,  $z = 7$  cm, and included angle  $X = 38^\circ$ .



1. Do you agree with the given measurements?

$y$  \_\_\_\_\_  $z$  \_\_\_\_\_  $\angle X$  \_\_\_\_\_

2. Use the given measurements to calculate altitude  $h$ . Measure  $h$ . Does it agree with the calculation?

3. You recall from geometry that the area of a triangle is  $\frac{1}{2}(\text{base})(\text{altitude})$ . Find the area of  $\triangle XYZ$ .

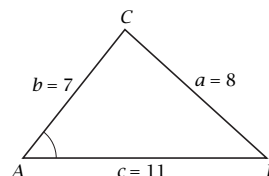
4. By substituting  $z \sin X$  in Problem 3 you get

$$\text{Area} = \frac{1}{2}yz \sin X, \text{ or, in general,}$$

$$\text{Area} = \frac{1}{2}(\text{side})(\text{side})(\text{sine of included angle}).$$

Sketch a triangle with sides 43 m and 51 m, and included angle  $143^\circ$ . Use this area formula to find the area of this triangle.

For Problems 5-8,  $\triangle ABC$  has sides  $a = 8$ ,  $b = 7$ , and  $c = 11$ .



5. Find the measure of angle  $A$  using the law of cosines. Store the answer without rounding.

6. Use the unrounded value of  $A$  and the area formula of Problem 3 to find the area of  $\triangle ABC$ .

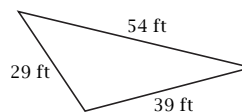
7. Calculate the **semiperimeter** (half the perimeter) of the triangle,  $s = \frac{1}{2}(a + b + c)$ .

8. Evaluate the quantity  $\sqrt{s(s-a)(s-b)(s-c)}$ . What interesting thing do you notice about the answer?

9. Use **Hero's formula**, namely,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

to find the area of this triangle.



10. What did you learn as a result of doing this Exploration that you did not know before?