Name _____ Group _____ Date _____

Exploration 8: Continuous and Discontinuous Functions

Objective: Given a function specified by two different rules, make the function continuous at the boundary between the two branches.

Let f be the function defined by

$$f(x) = \begin{cases} x + 1, \text{ if } x < 2 \\ k(x - 5)^2, \text{ if } x \geq 2 \end{cases}$$

where k stands for a constant.

1. Plot the graph of f for $k = 1$. Sketch the result.

2. Function f is **discontinuous** at $x = 2$. Tell what it means for a function to be discontinuous.

3. Find $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$. (The second limit will be in terms of k.) What must be true of these two limits for f to be **continuous** at $x = 2$?

4. Find the value of k that makes f continuous at $x = 2$. Sketch the graph of f for this value of k.

5. The graph in Problem 4 has a **cusp** at $x = 2$. What is the origin of the word *cusp*, and why is it appropriate to use in this context?

6. Suppose someone asks, "Is $f(x)$ increasing or decreasing at $x = 2$ with k as in Problem 4?" How would you have to answer that question? What, then, can you conclude about the derivative of a function at a point where the graph has a cusp?

7. What did you learn as a result of doing this Exploration that you did not know before?