In Exercises 1–4:

a) Find the inverse f^{-1} of the function f, expressed as a function of x.

b) Graph f and f^{-1} together.

c) Verify Eq. (1) by evaluating df/dx at $x = a$ and df^{-1}/dx at $x = f(a)$.

1. $f(x) = 2x + 3, \quad a = -1$

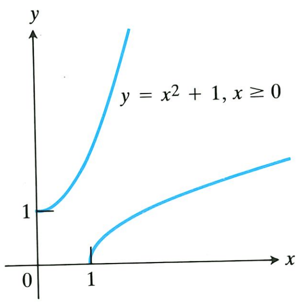
2. $f(x) = 5 - 4x, \quad a = 1/2$

3. $f(x) = (1/5)x + 7, \quad a = -1$

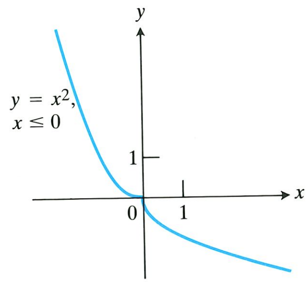
4. $f(x) = 2x^2, \ x \geq 0, \quad a = 5$

In Exercises 5–8, find $f^{-1}(x)$.

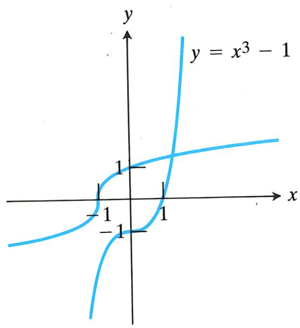
5. $f(x) = x^2 + 1, \quad x \geq 0$



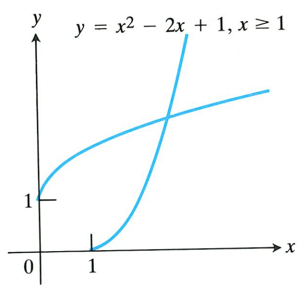
6. $f(x) = x^2, \quad x \leq 0$



7. $f(x) = x^3 - 1$



8. $f(x) = x^2 - 2x + 1, \quad x \geq 1$



In Exercises 9–16, find $f^{-1}(x)$ and show that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

9. $f(x) = x^5$

10. $f(x) = x^4, \quad x \geq 0$

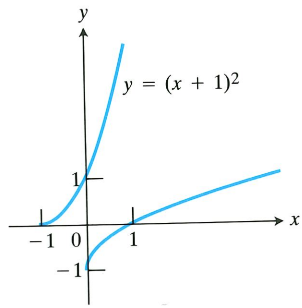
11. $f(x) = x^3 + 1$

12. $f(x) = (1/2)x - 7/2$

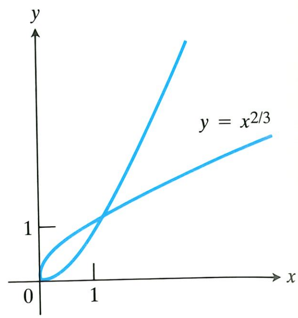
13. $f(x) = 1/x^2, \quad x > 0$

14. $f(x) = 1/x^3, \quad x \neq 0$

15. $f(x) = (x + 1)^2, \quad x \geq 1$



16. $f(x) = x^{2/3}, \quad x \geq 0$



17. Graph $y = x^3$ and $y = x^{1/3}$ together over the interval $-2 \leq x \leq 2$ and sketch the tangents at $(1, 1)$ and $(-1, -1)$. What lines are tangent to the curves at $x = 0$?

18. Graph the curve $y = 1/x, x > 0$ and notice its symmetry about the line $y = x$. Find the inverse of the function $f(x) = 1/x$.

19. One of the virtues of Eq. (1) is that it enables us to find values of df^{-1}/dx even when we do not have an explicit formula for the derivative. As a case in point, let $f(x) = x^2 - 4x - 3, \ x > 2$ and find the value of df^{-1}/dx at the point $x = -3 = f(4)$.

20. *Increasing functions and decreasing functions.* As we saw in Section 4.2, a function $f(x)$ increases on its domain if for any two points x_1 and x_2 in the domain,

$$x_2 > x_1 \quad \Rightarrow \quad f(x_2) > f(x_1).$$

Similarly, a function decreases on its domain if for any two points x_1 and x_2 in the domain,

$$x_2 > x_1 \quad \Rightarrow \quad f(x_2) < f(x_1).$$

Show that increasing functions and decreasing functions are one-to-one. That is, show that $x_2 \neq x_1$ always implies $f(x_2) \neq f(x_1)$.

TOOLKIT PROGRAM

Picard's Fixed Point Method

In addition to being a powerful equation solver, this program enables you to toggle back and forth between the graphs of a function and its inverse.