numbers. What does it mean to say that the limit of $f(x)$ is 2 as x approaches infinity? How is the line $y = 2$ related to the graph?

8. Let $g(x) = \sec x$.
   a. Sketch the graph of g.
   b. Find $\lim_{x\to\pi/2^-} g(x)$ and $\lim_{x\to\pi/2^+} g(x)$. Explain why, even though $\lim_{x\to\pi/2} g(x)$ is infinite, it is *not* correct to say that $\lim_{x\to\pi/2} g(x) = \infty$. What feature does the graph of g have at $x = \pi/2$?
   c. Find a value of x close to $\pi/2$ on the positive side for which $g(x) = -1000$. Choose several values of x closer to $\pi/2$ than this, and show numerically that $g(x) < -1000$ for all of these values. What does it mean to say that the limit of $g(x)$ is negative infinity as x approaches $\pi/2$ from the positive side? How is the line $x = \pi/2$ related to the graph of g?

9. Let $r(x) = 2 + \dfrac{\sin x}{x}$.
   a. Plot the graph of r. Use a friendly window with an x-range of about -20 to 20 for which $x = 0$ is a grid point. Sketch the result.
   b. Find the limit, L, of $r(x)$ as x approaches infinity.
   c. Show that $r(28)$ is within 0.01 unit of 2, but that there are values of $x > 28$ for which $r(x)$ is *more than* 0.01 unit away from 2. Use a suitable window to show this graphically, and sketch the result. Find a value $x = D$ large enough so that $r(x)$ is within 0.01 unit of 2 for *all* $x > D$.
   d. In part b, if you draw a horizontal line at $y = L$, will it be an asymptote? Explain.
   e. Make a conjecture about the limit of $r(x)$ as x approaches zero. Give evidence to support your conjecture.

10. Let $h(x) = \left(1 + \dfrac{1}{x}\right)^x$.
   a. Plot the graph of h. Use a friendly window with an x-range of 0 to about 100. You will have to explore to find a suitable y-range. Sketch the result.
   b. As x becomes large, $1/x$ approaches zero, so $h(x)$ takes on the form 1^∞. You realize that 1 to any power is 1, but the base is always greater than 1, and a number greater than 1 raised to a large positive power becomes infinite. Which phenomenon "wins" as x approaches infinity: 1, infinity, or some "compromise" number in between?

11. Figure 2-5h shows the graph of

$$y = \log x$$

Does the graph level off and approach a finite limit as x approaches infinity, or is the limit infinite? Justify your answer. The definition of logarithm is helpful here ($y = \log x$ if and only if $10^y = x$).

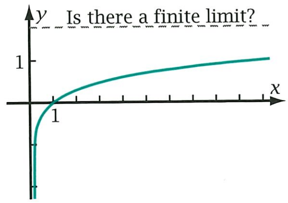

Figure 2-5h

12. Wanda Wye wonders why the form $1/0$ is infinite and why the form $1/\infty$ is zero. Explain to her what happens to the size of fractions such as $1/0.1$, $1/0.0001$, and so on, as the denominator gets close to zero. Explain what happens as the denominator becomes very large.

13. *Limits Applied to Integrals Problem:* Rhoda starts riding down the driveway on her tricycle. Being quite precocious, she figures her velocity, v, in ft/s is

$$v = \sqrt{t}$$

where t is time, in seconds, since she started. Figure 2-5i shows v as a function of t.

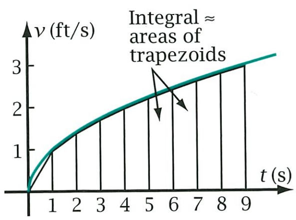

Figure 2-5i