Name: ______________________ Group Members: ______________________

Exploration 14-3e: Arithmetic and Geometric Series Problems

Objective: Find terms and partial sums of arithmetic or geometric series numerically or algebraically.

Pushups Problem: Emma Strong starts an exercise program. On the first workout, she does five pushups. The next workout, she does eight pushups. She decides to let the number of pushups in each workout be a term in an arithmetic series.

1. Find algebraically the number of pushups she does on the tenth workout.

2. Find algebraically the total number of pushups she has done after ten workouts.

3. Write the numbers of pushups Emma does on workouts 1, 2, 3, . . . , 10. Confirm that your answer to Problem 2 is correct by actually adding these terms.

4. If Emma were able to keep up the arithmetic series of pushups, one of the terms in the series would be 101. Calculate the term number algebraically.

Medication Problem: N. Hale takes 50 mg of allergy medicine each day. By the next day, some of the medicine has decomposed, but the rest is still active in his body. Mr. Hale finds that the amount still in his body after n days is given by this partial sum.

$$S_n = \sum_{k=1}^{n} 50(0.8^{k-1})$$

5. Demonstrate that you know what sigma notation means by writing out the first three terms of this series.

6. Find S_3 for the series in Problem 5.

7. Run your SERIES program to find S_{40}. Write the answer here, and check with your instructor. ________

8. After how many days does the amount in his body first exceed 200 mg?

9. Does the amount in Mr. Hale's body seem to be converging to a certain number, or does it just keep getting bigger without limit? How can you tell?

(Over)

Exploration 14-3e: Arithmetic and Geometric Series Problems continued

Loan Problem: Len de Monet borrows $200 from his parents to buy a new calculator. They require him to pay back 10% of his unpaid balance at the end of each month.

10. How much does Len pay at the end of the first month? What, then, is his unpaid balance after that payment?

11. Calculate Len's unpaid balances after 2 months and after 3 months.

12. Is the sequence of unpaid balances in Problems 10 and 11 geometric, arithmetic, or neither? How can you tell?

13. Calculate algebraically the unpaid balance at the end of 1 full year.

14. Len must pay off the rest of the loan when his unpaid balance has dropped below $5. After how many months will this have happened? Describe the method you use. (It is not enough to say, "I used my calculator" or "Guess and check.")

15. From the results of Problems 10 and 11, write the amounts of the payments Len makes at the end of 1, 2, and 3 months. The total Len has paid after n months is the partial sum of the series of payments. Find the third partial sum algebraically. Confirm that the answer is correct by adding the first three terms.

16. If Len were not required to pay off the loan when the balance dropped below $5, his payments would go on infinitely! Show algebraically that the total he would have paid after a long time converges to the original $200 he borrowed.

17. What did you learn as a result of doing this Exploration that you did not know before?