6. *Rabbit Overpopulation Problem:* Figure 7-6e shows two logistic functions

$$y = \frac{1000}{1 + ae^{-x}}$$

Both represent the population of rabbits in a particular woods as a function of time x in years. The value of the constant a is to be determined under two different initial conditions.

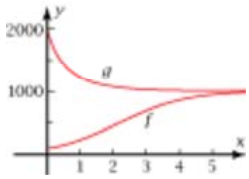

Figure 7-6e

a. For $y = f(x)$ in Figure 7-6e, 100 rabbits were introduced into the woods at time $x = 0$. Find the value of the constant a under this condition. Show that your answer is correct by plotting the graph of f on your grapher.

b. How do you interpret this mathematical model with regard to what happens to the rabbit population under the conditions in part a?

c. For $y = g(x)$ in Figure 7-6e, 2000 rabbits were introduced into the woods at time $x = 0$. Find the value of a under this condition. Show that the graph agrees with Figure 7-6e.

d. How do you interpret the mathematical model under the condition of part c? What seems to be the implication of trying to stock a region with a greater number of a particular species than the region can support?

7. Given the logistic function

$$f(x) = \frac{c}{1 + ae^{-0.4x}}$$

a. Let $a = 2$. Plot on the same screen the graphs of f for $c = 1$, 2, and 3. Use a domain of $x \in [-10, 10]$. Sketch the results. True or false? "c is a vertical dilation factor."

b. Figure 7-6f shows the graph of f with $c = 2$ and with $a = 0.2$, 1, and 5. Which graph is which? What transformation does a do on the graph?

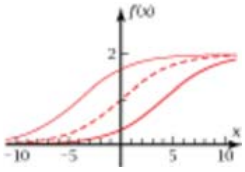

Figure 7-6f

c. Let $g(x) = \frac{c}{1 + ae^{-0.4(x-3)}}$. What transformation of f does this represent? Confirm that your answer is correct by plotting f and g on the same screen using $c = 2$ and $a = 1$.

d. What value of a in the equation for $f(x)$ would produce the same transformation as in part c?