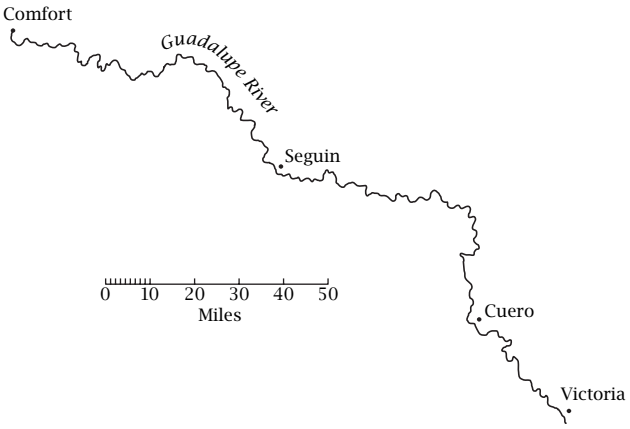
Name: _____ Group Members: _____

Exploration 11-6c: Fractal Dimension of a River

Date: _____

Objective: Apply Hausdorff's definition of *dimension* to the Guadalupe River.



The figure shows the Guadalupe River between the towns of Comfort and Victoria. As the crow flies, the two towns are 150 miles apart. But the river is longer because it meanders. In this Exploration, you will find out something surprising about the actual length of the river.

1. Draw the bee-line distance from Comfort to Victoria. Do you agree that this distance is 150 miles?

   _____

2. If you use a ruler 150 miles long, the river would measure $N = 1$ ruler length. If you use a ruler only 50 miles long, then measuring the distance between points on the river, you will get *more* than three ruler lengths. Starting at Comfort, draw a line segment to a point on the river a direct distance of 50 miles from Comfort. Draw other 50-mile segments connecting points on the river until you reach Victoria. (The last segment will be a fraction of 50 miles.) How long does the river appear to be, using a 50-mile ruler? To two decimal places, how many ruler lengths is this?

3. This table shows the number of ruler lengths, N, as a function of the length of the ruler. Do you agree with N for a 20-mile ruler? _____

Ruler (mi)	N Pieces	Ratio, r	$\frac{1}{r}$
150	1.0		
50	3.16		
20	8.2		
10	17.2		
5	35.0		
2	90.5		

4. In the table, fill in the columns for the ratio

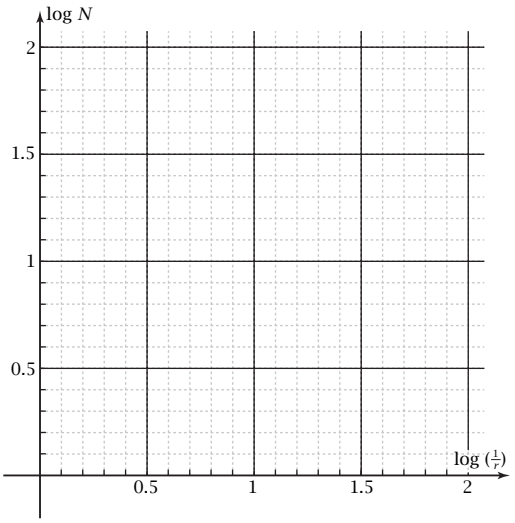
   $$r = \frac{\text{ruler length}}{150}$$

   and for the reciprocal, $\frac{1}{r}$.

5. Enter lists for $\frac{1}{r}$ and N on your grapher. Use these to calculate other lists containing values of $\log \frac{1}{r}$ and $\log N$. Write the values here, rounded to two decimal places.

$\log \frac{1}{r}$	$\log N$

6. Plot $\log N$ versus $\log \frac{1}{r}$ on this graph paper. If any points do not lie on a straight line, go back and check your work.



7. Perform linear regression to find the best-fitting linear function for $\log N$ as a function of $\log \frac{1}{r}$. Write the particular equation here.

(Over)

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Exploration 11-6c: Fractal Dimension of a River continued

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8. Use the equation in Problem 7 to calculate N if $\frac{1}{r} = 15$ (10-mile ruler). How close does the answer come to 17.2 in the table?

9. The graph in Problem 6 and the equation in Problem 7 show that the line goes almost through the origin. Assuming that it does, the equation is

   $$\log N = m \log \frac{1}{r}$$

   By dividing both sides by $\log \frac{1}{r}$, show that m satisfies Hausdorff's definition of *dimension*. What is the dimension of the Guadalupe River?

10. Based on your equation in Problem 7, what would N equal if you used a 1-inch-long ruler? How many miles long would the river be? Surprising?

11. What did you learn as a result of doing this Exploration that you did not know before?