27. Line: $x = x_1 + t(x_2 - x_1), \quad y = y_1 + t(y_2 - y_1)$

28. Circle: $x = h + r\cos\theta, \quad y = k + r\sin\theta$

29. Ellipse: $x = h + a\cos\theta, \quad y = k + b\sin\theta$

30. Hyperbola: $x = h + a\sec\theta, \quad y = k + b\tan\theta$

In Exercises 31–38, use the results of Exercises 27–30 to find a set of parametric equations for the line or conic.

31. Line: Passes through $(0, 0)$ and $(6, -3)$

32. Line: Passes through $(2, 3)$ and $(6, -3)$

33. Circle: Center: $(3, 2)$; Radius: 4

34. Circle: Center: $(-3, 2)$; Radius: 5

35. Ellipse: Vertices: $(\pm 4, 0)$; Co-vertices: $(0, \pm 5)$

36. Ellipse: Vertices: $(4, 7), (4, -3)$;
Co-vertices: $(8,2), (0,2)$

In Exercises 39–46, find a set of parametric equations for the given rectangular equation using (a) $t = x$ and (b) $t = 2 - x$.

39. $y = 3x - 2$

40. $x = 3y - 2$

41. $y = x^2$

42. $y = x^3$

43. $y = x^2 + 1$

44. $y = 2 - x$

45. $y = \dfrac{1}{x}$

46. $y = \dfrac{1}{2x}$

47. The graph of the parametric equations $x = 2\sec t$ and $y = 3\tan t$ is given in the figure. Would the graph change for the equations $x = 2\sec(-t)$ and $y = 3\tan(-t)$? If so, how would it change?

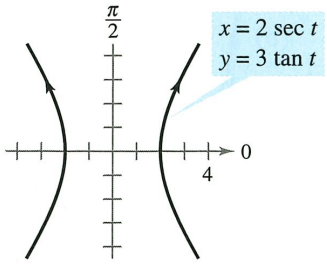


Figure for 47

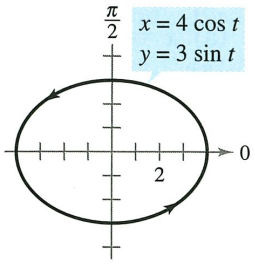


Figure for 48

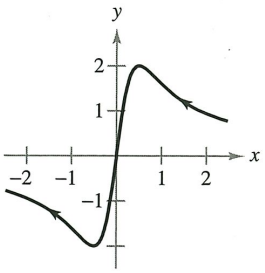
48. A moving object is modeled by the parametric equations $x = 4\cos t$ and $y = 3\sin t$, where t is time (see figure). How would the orbit change for the following?

(a) $x = 4\cos 2t, \quad y = 3\sin 2t$

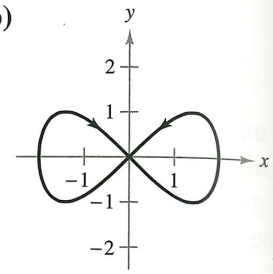
(b) $x = 5\cos t, \quad y = 3\sin t$

In Exercises 49–52, match the parametric equations with the correct graph and describe the domain and range. [The graphs are labeled (a) through (d).]

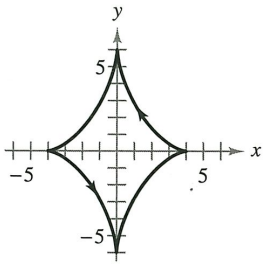
(a)



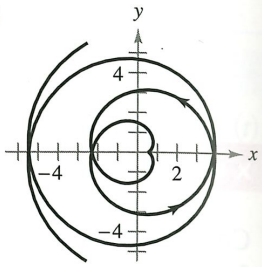
(b)



(c)



(d)



49. Lissajous curve: $x = 2\cos\theta$
$y = \sin 2\theta$

50. Evolute of ellipse: $x = 4\cos^3\theta$
$y = 6\sin^3\theta$

51. Involute of circle: $x = \frac{1}{2}(\cos\theta + \theta\sin\theta)$
$y = \frac{1}{2}(\sin\theta - \theta\cos\theta)$

52. Serpentine curve: $x = \frac{1}{2}\cot\theta$
$y = 4\sin\theta\cos\theta$

In Exercises 53–60, use a graphing utility to obtain a graph of the curve represented by the parametric equations.

53. Cycloid: $x = 4(\theta - \sin\theta)$
$y = 4(1 - \cos\theta)$

54. Cycloid: $x = \theta + \sin\theta$
$y = 1 - \cos\theta$

55. Prolate cycloid: $x = \theta - \frac{3}{2}\sin\theta$
$y = 1 - \frac{3}{2}\cos\theta$

56. Prolate cycloid: $x = 2\theta - 4\sin\theta$
$y = 2 - 4\cos\theta$

57. Hypocycloid: $x = 3\cos^3\theta$
$y = 3\sin^3\theta$

58. Curtate cycloid: $x = 8\theta - 4\sin\theta$
$y = 8 - 4\cos\theta$

59. Witch of Agnesi: $x = 2\cot\theta$
$y = 2\sin^2\theta$