

Exploration 12-2a: Parametric Equations of Conic Sections

Date: _____

Objective: Plot graphs of conic sections in parametric form, and relate the result to the Cartesian equation.

1. Put your grapher in parametric mode and radian mode. Set a t -range of $0-2\pi$ and a window, with equal scales on the two axes, that has an x -range of $[-10, 10]$. Plot these **parametric equations** and sketch the result.

$$x = \cos t$$

$$y = \sin t$$

2. Square both sides of both equations in Problem 1. Then add the two equations, left side to left side and right side to right side. Use the Pythagorean property of circular functions to show that the result is equivalent to the **unit circle** $x^2 + y^2 = 1$.

3. Plot these parametric equations.

$$x = 5 \cos t$$

$$y = 3 \sin t$$

Describe verbally how the resulting **ellipse** is related to the unit circle in Problem 1.

4. Plot these parametric equations.

$$x = 2 + 5 \cos t$$

$$y = -1 + 3 \sin t$$

How is the graph related to the one in Problem 3?

5. Plot the **unit hyperbola** with these parametric equations and sketch the result.

$$x = \sec t$$

$$y = \tan t$$

6. Square both sides of both equations in Problem 5, and then combine the two equations in a way that shows that $x^2 - y^2 = 1$.

7. Plot this unit hyperbola and explain how it is related to the one in Problem 5.

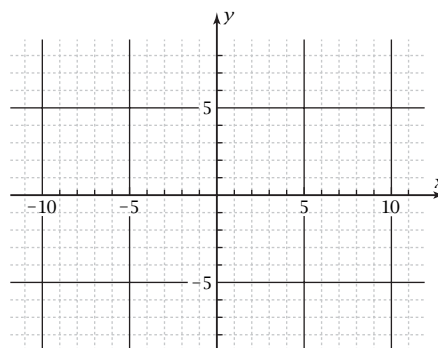
$$x = \tan t$$

$$y = \sec t$$

8. Plot this hyperbola. Sketch on the given axes.

$$x = -4 + 3 \sec t$$

$$y = 1 + 2 \tan t$$



9. What did you learn as a result of doing this Exploration that you did not know before?