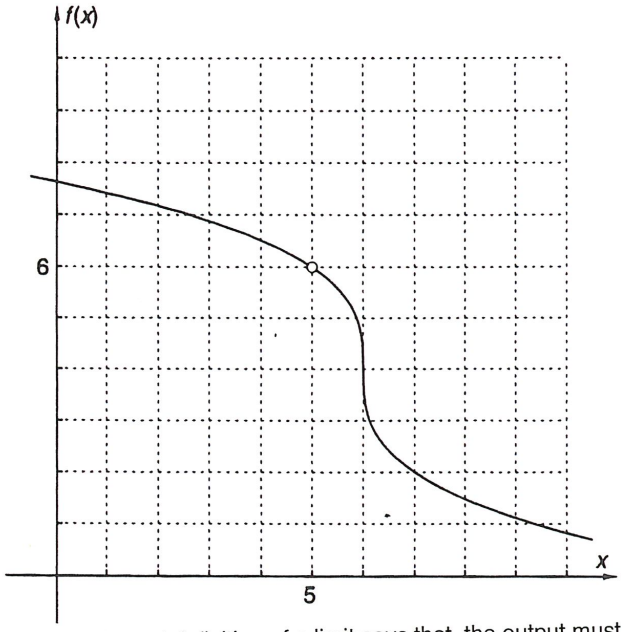
Name _____ Group _____ Date _____

The Definition of Limit

Objective: Interpret graphically and algebraically the definition of limit.

Let f be the function whose graph is shown here.



1. The formal definition of a limit says that, the output must approach within some amount "e" of the limiting value, if the input varies by some about "d". Rewrite this definition with input being 5 and output being 6 for the function f(x).

2. Let $e = 1$. From the graph, estimate how close to 5 on the left side x must be kept in order for $f(x)$ to be within e units of 6.

3. From the graph, estimate how close to 5 on the right side x must be kept in order for $f(x)$ to be within $e = 1$ unit of 6.

4. For $e = 1$, approximately what must d equal in the definition of limit in order for $f(x)$ to be within e units of 6 whenever x is within d units of 5 (but not equal to 5)?

5. The equation of the function graphed is $f(x) = 4 - 2(x - 6)^{1/3}$, for $x \neq 5$.
   Calculate precisely the value of d from Problem 4.

6. If $e = 0.01$, calculate precisely what d must equal in order for $f(x)$ to be within e units of 6 whenever x is within d units of 5 (but not equal to 5).

7. If $e = 0.0001$, calculate precisely what d must equal in order for $f(x)$ to be within e units of 6 whenever x is within d units of 5 ($x \neq 5$).

8. Does it appear that there is a positive value of d for *any* positive number e, no matter how small? See if you can find d algebraically in terms of e.

9. What did you learn as a result of doing this Exploration that you did not know before?