Name: _____ Group Members: _____

Exploration 14-4a: A Power Series for a Familiar Function

Date: _____

Objective: Learn what a power series is and how it can fit closely a particular function.

Let $P(x)$ be defined by

$$P(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n + \cdots$$

The letter P is used because the right side of the equation is a **power series.** It is also appropriate because the expression looks like a **polynomial,** except that it has an infinite number of terms. In this Exploration, you will calculate and plot values of $P(x)$ and try to figure out which familiar function P represents.

1. Calculate $P(0.6)$ three times, using 3, 4, and 5 terms of the series (term index $n = 2$, 3, and 4).

2. The values of $P(0.6)$ in Problem 1 are **partial sums** of the series. Use the SUM and SEQUENCE commands on your grapher to enter an equation into y_1 that will calculate $P(0.6)$ for $n = x$ terms. Then make a table of values of $P(0.6)$ using $n = 5$, 6, 7, 8, 9, and 10. Calculate $e^{0.6}$. What do you notice about the partial sums?

3. Change the equation in y_1 so that it calculates the 11th partial sum ($n = 10$) of the series for $P(x)$. Then plot the graph using a window with an x-range of $[-5, 5]$ and a y-range of $[-1, 10]$. Sketch the result.

4. On the same screen as in Problem 3, plot the graph of $y = e^x$. Does the graph support the conjecture that $P(x) = e^x$?

5. Find the 11th partial sum ($n = 10$) for $P(1)$. How close is the answer to e^1?

6. Show that the 11th partial sum for $P(10)$ is *not* close to e^{10}.

7. What did you learn as a result of doing this Exploration that you did not know before?