50. The Gateway Arch (Figure 5–13) is 630 feet high and 630 feet wide at ground level. Suppose it were placed on a coordinate plane with the x-axis at ground level and the y-axis going through the center of the arch. Find a catenary function $g(x) = A(e^{kx} + e^{-kx})$ and a constant C such that the graph of the function $f(x) = g(x) + C$ provides a model of the arch. [*Hint*: Experiment with various values of A, k, C as in the Graphing Exploration on page 350. Many correct answers are possible.]

51. (a) A genetic engineer is growing cells in a fermenter. The cells multiply by splitting in half every 15 minutes. The new cells have the same DNA as the original ones. Complete the following table.

Time (hours)	Number of Cells
0	1
.25	2
.5	4
.75	
1	

(b) Write the rule of the function that gives the number of C cells at time t hours.

52. Do Exercise 51, using the following table.

Time (hours)	Number of Cells
0	300
.25	600
.5	1200
.75	
1	

53. A weekly census of the tree-frog population in Frog Hollow State Park produces the following results.

Week	1	2	3	4	5	6
Population	18	54	162	486	1458	4374

(a) Find a function of the form $f(x) = Pa^x$ that describes the frog population at time x weeks.
(b) What is the growth factor in this situation (that is, by what number must this week's population be multiplied to obtain next week's population)?
(c) Each tree frog requires 10 square feet of space and the park has an area of 6.2 square miles. Will the space required by the frog population exceed the size of the park in 12 weeks? In 14 weeks? [Remember: 1 square mile $= 5280^2$ square feet.]

54. An eccentric billionaire offers you a job for the month of September. She says that she will pay you 2¢ on the first day, 4¢ on the second day, 8¢ on the third day, and so on, doubling your pay on each successive day.
(a) Let $P(x)$ denote your salary in *dollars* on day x. Find the rule of the function P.
(b) Would you be better off financially if instead you were paid $10,000 per day? [*Hint:* Consider $P(30)$.]

55. Take an ordinary piece of typing paper and fold it in half; then the folded sheet is twice as thick as the single sheet was. Fold it in half again so that it is twice as thick as before. Keep folding it in half as long as you can. Soon the folded paper will be so thick and small that you will be unable to continue, but suppose you could keep folding the paper as many times as you wanted. Assume that the paper is .002 inches thick.
(a) Make a table showing the thickness of the folded paper for the first four folds (with fold 0 being the thickness of the original unfolded paper).
(b) Find a function of the form $f(x) = Pa^x$ that describes the thickness of the folded paper after x folds.
(c) How thick would the paper be after 20 folds?
(d) How many folds would it take to reach the moon (which is 243,000 miles from the earth)? [*Hint:* One mile is 5280 feet.]

*Based on data from *The Cable TV Financial Datebook* and *The Pay TV Newsletter.*