4. Find a function whose *derivative* is

$$f'(x) = \frac{(3x^2-1)(x^2+3x-1)-(x^3-x+5)(2x+3)}{(x^2+3x-1)^2}.$$

5. Suppose $f(x)$ is a function with the following properties:

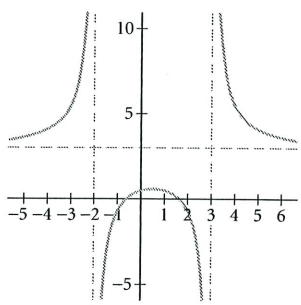
- $f(x)$ has domain $(-\infty, 4) \cup (4, \infty)$, and $f(-1) = 2$, $f(1) = 3$, $f(2) = 0$, $f(3) = -2$, and $f(6) = 3$.
- $f'(x)$ is zero only at $x = 1$ and $x = 3$, and $f'(x)$ does not exist at $x = 4$.
- $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = 1$.
- $\lim_{x\to4^+} f(x) = \infty$ and $\lim_{x\to4^-} f(x) = \infty$.

Given the information above, find the global extrema of $f(x)$ on each of the intervals below.

(a) $[-1, 4)$ **(b)** $[-1, 2]$ **(c)** $(-1, 2)$
(d) $[2, 6]$ **(e)** $(2, 6)$

6. Let f be the rational function graphed below. Determine graphically the global extrema of f (if any) on each of the following intervals:

(a) $[-1, 1]$ **(b)** $(-1, 1)$ **(c)** $[-1, 3)$
(d) $[-3, 1]$ **(e)** $(3, 5]$



Skills

■ **For each rational function f and value c below, use the definition of derivative (or the "alternative" definition) to calculate $f'(c)$.**

7. $f(x) = \dfrac{x-1}{x+3}$, $c = 2$

8. $f(x) = \dfrac{1}{x^2-1}$, $c = -2$

9. $f(x) = \dfrac{x^2-3x}{x^2-2x+1}$, $c = 0$

10. $f(x) = \dfrac{x-1}{(x+1)(x+2)}$, $c = 1$

■ **Calculate the derivatives of the rational functions below by using the definition of derivative (or the "alternative" definition of derivative).**

11. $f(x) = \dfrac{x-1}{x+3}$

12. $f(x) = \dfrac{1}{x^2-1}$

13. $f(x) = \dfrac{x^2-3x}{x^2-2x+1}$

14. $f(x) = \dfrac{x-1}{(x+1)(x+2)}$

15. $f(x) = \dfrac{x^3}{x+1}$

16. $f(x) = \dfrac{x^2-1}{x^2-x-2}$

■ **Use the quotient rule to calculate the derivatives of the following rational functions.**

17. $f(x) = \dfrac{2x-3}{5x+4}$

18. $f(x) = \dfrac{x^3}{x+1}$

19. $f(x) = \dfrac{x^2-3x}{x^2-2x+1}$

20. $f(x) = \dfrac{1}{x^3-2x^2+x-3}$

21. $f(x) = \dfrac{x^7-3x^5+4}{1-3x^4}$

22. $f(x) = \dfrac{x^2}{x^3+5x^2-3x}$

23. $f(x) = \dfrac{1}{(x+1)^3}$

24. $f(x) = \dfrac{x-1}{(x+1)(x+2)}$

25. $f(x) = \dfrac{(x-2)^2}{(x^2+1)(x-3)}$

■ **Find the critical points of each rational function f.**

26. $f(x) = \dfrac{1+x+x^2}{x^2+x-2}$

27. $f(x) = \dfrac{(x-1)^2}{x+2}$

28. $f(x) = \dfrac{x^3}{x^2-3x+2}$

29. $f(x) = \dfrac{x^2-2x+1}{x^2-1}$

30. $f(x) = \dfrac{x^2(x-1)}{(x-2)^2}$

31. $f(x) = \dfrac{1}{x^3-x}$

■ **Find the local extrema of the following rational functions. Do all work algebraically (by hand), and then check your answers with a graphing calculator. Notice that the first six problems involve the same functions you investigated in the block of problems above.**

32. $f(x) = \dfrac{1+x+x^2}{x^2+x-2}$

33. $f(x) = \dfrac{(x-1)^2}{x+2}$

34. $f(x) = \dfrac{x^3}{x^2-3x+2}$

35. $f(x) = \dfrac{x^2-2x+1}{x^2-1}$

36. $f(x) = \dfrac{x^2(x-1)}{(x-2)^2}$

37. $f(x) = \dfrac{1}{x^3-x}$

38. $f(x) = \dfrac{x^2-x-2}{x^3}$

39. $f(x) = \dfrac{1}{(x-2)^2}$

40. $f(x) = \dfrac{x^2-4x+4}{x-2}$