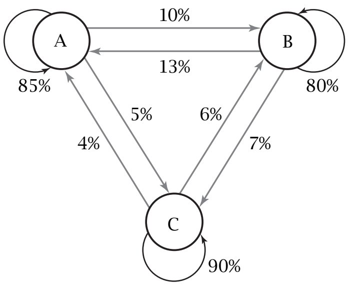
Name: ______________________ Group Members: ______________________

Exploration 11-4d: Markov Chain Problem

Date: __________

Objective: Use iterative multiplication of matrices in a nongeometrical real-world problem.

Television Network Loyalty Problem: In a poll of television viewers, a research company found the percentages of viewers who change from the evening news on one network to that on another network one month later.



For instance, for network A, 85% stayed with A, 10% switched to network B, and 5% switched to network C. The company uses these numbers as probabilities of what will happen in subsequent months. They arrange the numbers in **transition matrix** $[T]$.

$$[T] = \begin{array}{c} \begin{bmatrix} 0.85 & 0.10 & 0.05 \\ 0.13 & 0.80 & 0.07 \\ 0.04 & 0.06 & 0.90 \end{bmatrix} \begin{array}{l} A \\ B \\ C \end{array} \\ \begin{array}{ccc} A & \quad B & \quad C \end{array} \end{array}$$

Each element represents the probability that a viewer who watches the network in the *row* one month will be watching the network in the *column* the next month.

1. At the beginning of January, A has 35 million viewers, B has 20 million, and C has 43 million. These numbers are recorded in the "viewers" matrix, $[V_0]$.

   $$[V_0] = \begin{array}{c} [35 \quad 20 \quad 43] \\ \begin{array}{ccc} A & B & C \end{array} \end{array}$$

   Explain why the number of viewers at the beginning of February is given by

   $[V_1] = [V_0][T]$

2. Show that the number of viewers for A in $[V_1]$ is equal to the number who stayed with A plus the number that transferred from B and from C to A.

3. Show that the viewers matrix $[V_2]$ at the beginning of March can be found either as

   $[V_2] = [V_1][T]$ or $[V_2] = [V_0][T]^2$

4. In the most time-efficient way, find the number of viewers predicted for each network the following January, one full year later. Assume that the probabilities remain constant.

5. Assuming that the probabilities remain constant, the number of viewers for each network approach a fixed limit as the number of months becomes very large. Find approximations for these limits numerically.

6. The matrices $[V_0]$,$[V_1]$,$[V_2]$, . . . form a **Markov chain.** On the Internet or in some other reference source, find out who Markov is or was. Write a paragraph summarizing your findings.

7. What did you learn as a result of doing this Exploration that you did not know before?