c. Pick another value of a and find b. Show that f is continuous for these values of a and b.

42. *Two Values of Constant Problem:* For function f, use one-sided limits in an appropriate way to find the *two* values of k that make f continuous at $x = 2$.

$$f(x) = \begin{cases} k^2 - x^2, & \text{if } x < 2 \\ 1.5kx, & \text{if } x \geq 2 \end{cases}$$

43. *River Crossing Problem:* Calvin stands at the beginning of a bridge that is perpendicular to the banks of a 120-ft-wide river (Figure 2-4n). He can walk across the bridge at 5 ft/s, or take a scenic trip in a rowboat at 3 ft/s, making an angle θ, in degrees, with the riverbank. The time he takes to get to the other side of the river is a piecewise function of θ. Write an equation for this function. Plot the graph in a suitable domain and sketch the result.

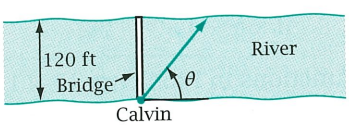

Figure 2-4n

44. *Surprise Function Problem!* Let

$$f(x) = x + 3 + \frac{10^{-20}}{x - 1}$$

a. Plot the graph on your grapher.

b. What appears to be the limit of $f(x)$ as x approaches 1?

c. Show that $f(x)$ is very close to the number in part b when $x = 1.0000001$.

d. Function f is not continuous at $x = 1$ because there is no value for $f(1)$. What type of discontinuity occurs at $x = 1$? (Be careful!)

45. *Continuity of Polynomial Functions:* The general polynomial function of degree n has an equation of the form

$$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n$$

Based on the closure axioms for real numbers and the properties of limits you have learned, explain why any polynomial function is continuous for all real values of x.

46. *The Signum Function:* Figure 2-4o shows the graph of the signum function, $f(x) = \text{sgn } x$. The value of the function is 1 when x is positive, -1 when x is negative, and 0 when x is zero. This function is useful in computing for testing a value of x to see what sign it has (hence the name *signum*). Here is the formal definition:

$$\text{sgn } x = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

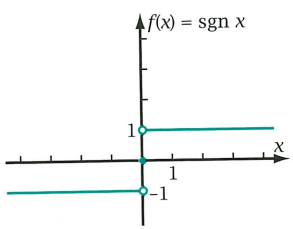

Figure 2-4o

In this problem you will explore various compositions of the signum function.

a. Does $r(x) = |\text{sgn } x|$ have a limit as x approaches 0? Does it have a function value at $x = 0$? Is it continuous at $x = 0$?

b. Sketch the graph of $g(x) = 3 \text{ sgn}(x - 2)$.

c. Sketch the graph of $h(x) = x^2 - \text{sgn } x$.

d. Show that the function $a(x) = |x|/x$ is equal to sgn x for all x except zero.

e. Sketch the graph of $f(x) = \cos x + \text{sgn } x$.