Name _____ Group _____ Date _____

Exploration 20: Derivative of a Product

<u>Objective</u>: Make a conjecture about an algebraic formula for the derivative of a product of two functions.

1. Let $g(x) = x^7$ and let $h(x) = x^{11}$.
    Let $f(x) = g(x) \cdot h(x)$.
    Find $g'(x)$ and $h'(x)$.

2. Write an equation for $f(x)$ as a single power of x.
    Then find an equation for $f'(x)$.

3. Show that $f'(x)$ does *not* equal $g'(x) \cdot h'(x)$.

4. It is possible to get the correct answer for $f'(x)$ by a clever combination of the equations for $g(x)$, $h(x)$, $g'(x)$, and $h'(x)$. For instance, you might notice that the 18 in $18x^{17}$ is the *sum* of the 7 and 11 in $7x^6$ and $11x^{10}$. Figure out what this combination is.

5. Make a conjecture about what $f'(x)$ equals in terms of $g(x)$, $h(x)$, $g'(x)$, and $h'(x)$.

6. Assume that your conjecture in Problem 5 is true for any product of two functions. If $f(x) = x^2 \sin x$, what would $f'(x)$ equal?

7. Plot on the same screen the graphs of $f(x)$, the numerical derivative of $f(x)$, and the equation for $f'(x)$ that you wrote in Problem 6. If the graphs refute your conjecture in Problem 5, change your conjecture and try again.

8. What did you learn as a result of doing this Exploration that you did not know before?