68. (a) The half-life of radium is 1620 years. If you start with 100 milligrams of radium, what is the rule of the function that gives the amount remaining after t years?
(b) How much radium is left after 800 years? After 1600 years? After 3200 years?

Thinkers

69. Find a function $f(x)$ with the property $f(r+s) = f(r)f(s)$ for all real numbers r and s. [*Hint:* Think exponential.]

70. Find a function $g(x)$ with the property $g(2x) = (g(x))^2$ for every real number x.

71. (a) Using the viewing window with $-4 \le x \le 4$ and $-1 \le y \le 8$, graph $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = 2^x$ on the same screen. If you think of the y-axis as a mirror, how would you describe the relationship between the two graphs?
(b) Without graphing, explain how the graphs of $g(x) = 2^x$ and $k(x) = 2^{-x}$ are related.

72. Look back at Section 4.4, where the basic properties of graphs of polynomial functions were discussed. Then review the basic properties of the graph of $f(x) = a^x$ discussed in this section. Using these various properties, give an argument to show that for any fixed positive number $a(\neq 1)$, it is *not* possible to find a polynomial function $g(x) = c_n x^n + \cdots + c_1 x + c_0$ such that $a^x = g(x)$ for *all* numbers x. In other words, *no exponential function is a polynomial function.* However, see Exercise 73.

73. Approximating exponential functions by polynomials. For each positive integer n, let f_n be the polynomial function whose rule is

$$f_n(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!},$$

where $k!$ is the product $1 \cdot 2 \cdot 3 \cdots k$.

(a) Using the viewing window with $-4 \le x \le 4$ and $-5 \le y \le 55$, graph $g(x) = e^x$ and $f_4(x)$ on the same screen. Do the graphs appear to coincide?
(b) Replace the graph of $f_4(x)$ by that of $f_5(x)$, then by $f_6(x), f_7(x)$, and so on until you find a polynomial $f_n(x)$ whose graph appears to coincide with the graph of $g(x) = e^x$ in this viewing window. Use the trace feature to move from graph to graph at the same value of x to see how accurate this approximation is.
(c) Change the viewing window so that $-6 \le x \le 6$ and $-10 \le y \le 400$. Is the polynomial you found in part (b) a good approximation for $g(x)$ in this viewing window? What polynomial is?

74. This exercise provides a justification for the claim that the function $M(x) = c(.5)^{x/h}$ gives the mass after x years of a radioactive element with half-life h years. Suppose we have c grams of an element that has a half-life of 50 years. Then after 50 years, we would have $c\left(\frac{1}{2}\right)$ grams. After another 50 years, we would have half of that, namely, $c\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = c\left(\frac{1}{2}\right)^2$.
(a) How much remains after a third 50-year period? After a fourth 50-year period?
(b) How much remains after t 50-year periods?
(c) If x is the number of years, then $x/50$ is the number of 50-year periods. By replacing the number of periods t in part (b) by $x/50$, you obtain the amount remaining after x years. This gives the function $M(x)$ when $h = 50$. The same argument works in the general case (just replace 50 by h).