

**Exploration 8-2a: Sums of Squares of Residuals**

Date: \_\_\_\_\_

**Objective:** Find the sum of the squares of the residuals for a function found by linear regression.

Suppose that these data have been measured for the related variables  $x$  and  $y$ .

$x$	$y$
3	41
5	37
7	29
9	28
11	23
13	26
15	19
17	9
19	8
21	4

1. Show by linear regression that the best-fitting linear function is  $\hat{y} = -2x + 46.4$ . How do you interpret the fact that the correlation coefficient is negative?
2. Plot the given data on a scatter plot. On the same screen, plot the regression line. How well does the linear function fit the data?
3. On your grapher, make three lists, one showing  $\hat{y}$  for each point, one showing the residual  $y - \hat{y}$ , and a third showing the squares of the residuals,  $(y - \hat{y})^2$ . Copy the results into the table.
4. Calculate  $SS_{\text{res}}$  from the squares of the residuals. See if you can discover a time-efficient way to find the sum using built-in features on your grapher.

5. This table has the same data as before. Find  $SS_{\text{res}}$  again using  $y_2 = -2x + 46$ . How do you interpret the fact that the answer is *greater* than  $SS_{\text{res}}$  using the regression equation  $\hat{y} = -2x + 46.4$ ?

$x$	$y$
3	41
5	37
7	29
9	28
11	23
13	26
15	19
17	9
19	8
21	4

6. Calculate  $\bar{x}$  and  $\bar{y}$ , the averages of  $x$  and  $y$ . Show algebraically that the point  $(\bar{x}, \bar{y})$  is on the regression line  $\hat{y} = -2x + 46.4$ .
7. The line  $y_3 = -2.1x + 47.6$  also contains the “average-average” point  $(\bar{x}, \bar{y})$ , but it has a slope of  $-2.1$  instead of  $-2$ . Plot the line on the same screen as in Problem 2. Can you tell from the graphs which line fits the data better?
8. Find  $SS_{\text{res}}$  using the equation in Problem 7. Based on your answer, how can you tell that this line does *not* fit the data as well as does the regression line?
9. What did you learn as a result of doing this Exploration that you did not know before?