Problem Set 7-6

Do These Quickly

Q1. An exponential function has the —?— – —?— property.

Q2. A power function has the —?— – —?— property.

Q3. The equation $y = 3 + 5 \ln x$ defines a —?— function.

Q4. The function in Q3 has the —?— – —?— property.

Q5. The expression $\ln x$ is a logarithm with the number —?— as its base.

Q6. Write in exponential form: $h = \log_p m$

Q7. Write in logarithmic form: $c = 5^j$

Q8. If an object rotates at 100 revolutions per minute, how many radians per minute is this?

Q9. Write the general equation for a quadratic function.

Q10. $\cos \pi$ is

A. –1 B. 0 C. 1 D. $\frac{1}{2}$ E. Undefined

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1. Given the exponential function $f(x) = 1.2^x$ and the logistic function $g(x) = \dfrac{1.2^x}{1.2^x + 1}$,

   a. Plot both graphs on the same screen. Use a domain of $x \in [-10, 10]$. Sketch the result.

   b. How do the two graphs compare for large positive values of x? How do they compare for large negative values of x?

   c. Find the approximate x-value of the point of inflection for function g. For what values of x is the graph of function g concave up? Concave down?

   d. Explain algebraically why the logistic function has a horizontal asymptote at $y = 1$.

   e. Transform the equation of the logistic function so that an exponential term appears only *once.* Show numerically that the resulting equation is equivalent to $g(x)$ as given.

2. Figure 7-6d shows the logistic function

$$f(x) = \frac{3e^{0.2x}}{e^{0.2x} + 4}$$

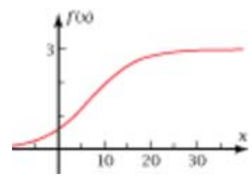

Figure 7-6d

   a. Explain algebraically why the graph has a horizontal asymptote at $y = 3$.

   b. Read the point of inflection from the graph. Find the x-coordinate algebraically.

   c. For what values of x is the graph concave up? Concave down?

   d. Transform the equation so that there is only *one* exponential term. Confirm by graphing that the resulting equation is equivalent to $f(x)$ as given.

3. *Spreading the News Problem:* You arrive at school and meet your mathematics teacher, who tells you today's test has been cancelled! You and your friend spread the good news. The table shows the number of students, y, who have heard the news after x minutes have passed since you and your friend heard the news.

x (min)	y (students)
0	2
10	5
20	13
30	35
40	90

   a. Plot the points. Imagine a function fit to the points. Is the graph of this function concave up or concave down or both?

   b. There are 1220 students in the school. Use the numbers of students at 0 minutes and at 40 minutes to find the equation of the logistic function that meets these constraints.

c. Plot the graph of the logistic function for the first three hours.

d. Based on the logistic model, how many students have heard the news at 9:00 a.m. if you heard it at 8:00 a.m.? How long will it be until all but 10 students have heard the news?

4. *Spreading the News Simulation Experiment:* In this experiment you will simulate the spread of the news in Problem 3. Number each student in your class starting at 1. Person 1 stands up and then selects two people at random to "tell" the news to. Do this by selecting two random integers between 1 and the number of students in your class, inclusive. (It is not actually necessary to tell any news!) The random number generator on one student's calculator will help make the random selection. The two people with the chosen numbers stand. Thus after the first iteration there will probably be three students standing (unless a duplicate random number came up). Each of these (three) people selects two more people to "tell" the news to by selecting a total of 6 (or 4?) more random integers. Do this for a total of 10 iterations or until the entire class is standing. At each iteration, record the number of iterations and the total number of people who have heard the news. Describe the results of the experiment. Include such things as

- The plot of the data points.
- A function that fits the data, and a graph of this function on the plot. Explain why you chose the function you did.
- A statement of how well the logistic model fits the data.
- The iteration number at which the good news was spreading most rapidly.

5. *Ebola Outbreak Epidemic Problem:* In the fall of 2000, an epidemic of the ebola virus broke out in the Gulu district of Uganda. The table shows the total number of people infected from the day the cases were diagnosed as ebola virus infections. The final number of people who got infected during this epidemic is 396. (Ebola is a virus that causes internal bleeding and is fatal in most cases.)

x **(days)**	***y*** **(total infections)**
1	71
10	182
15	239
21	281
30	321
50	370
74	394

a. Make a plot of the data points. Imagine a function that fits the data. Is the graph of this function concave upward or downward?

b. Use the second and last points to find the particular equation of a logistic function that fits the data.

c. Plot on the same screen as the plot in part a the logistic function from part b. Sketch the results.

d. Where does the point of inflection occur in the logistic model? What is the real-world meaning of this point?

e. Based on the logistic model, how many people were infected after 40 days?

f. Consult a reference on the Internet or elsewhere to find data about other epidemics. Try to model the spread of the epidemic for which you found data.

A Red Cross medical officer instructs villagers about the ebola virus in Kabede Opong, Uganda.

6. *Rabbit Overpopulation Problem:* Figure 7-6e shows two logistic functions

$$y = \frac{1000}{1 + ae^{-x}}$$

Both represent the population of rabbits in a particular woods as a function of time x in years. The value of the constant a is to be determined under two different initial conditions.

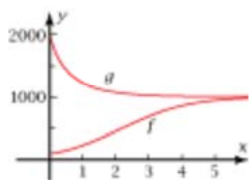

Figure 7-6e

a. For $y = f(x)$ in Figure 7-6e, 100 rabbits were introduced into the woods at time $x = 0$. Find the value of the constant a under this condition. Show that your answer is correct by plotting the graph of f on your grapher.

b. How do you interpret this mathematical model with regard to what happens to the rabbit population under the conditions in part a?

c. For $y = g(x)$ in Figure 7-6e, 2000 rabbits were introduced into the woods at time $x = 0$. Find the value of a under this condition. Show that the graph agrees with Figure 7-6e.

d. How do you interpret the mathematical model under the condition of part c? What seems to be the implication of trying to stock a region with a greater number of a particular species than the region can support?

7. Given the logistic function

$$f(x) = \frac{c}{1 + ae^{-0.4x}}$$

a. Let $a = 2$. Plot on the same screen the graphs of f for $c = 1, 2,$ and 3. Use a domain of $x \in [-10, 10]$. Sketch the results. True or false? "c is a vertical dilation factor."

b. Figure 7-6f shows the graph of f with $c = 2$ and with $a = 0.2, 1,$ and 5. Which graph is which? What transformation does a do on the graph?

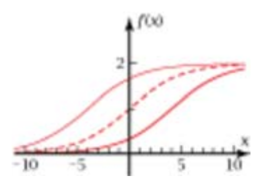

Figure 7-6f

c. Let $g(x) = \dfrac{c}{1 + ae^{-0.4(x-3)}}$.
What transformation of f does this represent? Confirm that your answer is correct by plotting f and g on the same screen using $c = 2$ and $a = 1$.

d. What value of a in the equation for $f(x)$ would produce the same transformation as in part c?