Find the global extrema of each of the following rational functions on the given intervals. Do all work algebraically (by hand), and then check your answers with a graphing calculator. Note that all of the functions in this block are also in the previous block of problems.

41. $f(x) = \dfrac{1 + x + x^2}{x^2 + x - 2}, \quad (-1, 1)$

42. $f(x) = \dfrac{1 + x + x^2}{x^2 + x - 2}, \quad (1, 5]$

43. $f(x) = \dfrac{(x-1)^2}{x+2}, \quad [-1, 3]$

44. $f(x) = \dfrac{(x-1)^2}{x+2}, \quad (-4, -2)$

45. $f(x) = \dfrac{1}{(x-2)^2}, \quad [0, 2)$

46. $f(x) = \dfrac{1}{(x-2)^2}, \quad [1, 4]$

47. $f(x) = \dfrac{x^2 - 2x + 1}{x^2 - 1}, \quad [-2, 1]$

48. $f(x) = \dfrac{x^2 - 2x + 1}{x^2 - 1}, \quad (-1, 2)$

Proofs

49. Use the quotient rule to prove that a rational function f is differentiable everywhere on its domain.

50. Prove that if f is a rational function with a horizontal asymptote, then its derivative f' also has a horizontal asymptote.

51. Prove that if f is a rational function with a slant asymptote, then its derivative f' has a horizontal asymptote.

52. Use the definition of derivative to prove the quotient rule. Justify each step in your calculation.