zkML Course - Lesson 1

Today's topics

- Introduction to the course
- Fundamentals of ZKP and ML
  - Maths
  - Cryptography
  - Introduction to ZKP Theory
  - Computer architecture
    - Virtual machines
    - Opcodes

Introduction

Practical Details

The format will usually be 45 mins of theory followed by 45 mins practical

Homeworks

There will be a homework for each lesson, but you do not need to submit the homeworks.

About Extropy

Free Developer Workshops

- Basic
- Enterprise
- Advanced EVM
- Zero Knowledge Proofs

Business Workshops

Website : https://extropy.io

Email : info@extropy.io

Twitter : @extropy

Social Media :

- X : @Extropy

- Warpcast: @Extropy
- Discord Invite

For Q&A use Sli.do

Course Schedule

Week 1 - Introduction

Day	Lesson	Topic	Teacher
Monday	1	Fundamentals	Extropy
Tuesday	2	Introduction to ML	Extropy
Wednesday	3	Introduction to zkML / Use Cases	Extropy
Thursday	4	EZKL	EZKL

Week 2 - Deep Dive

Day	Lesson	Topic	Teacher
Monday	No Lesson		
Tuesday	5	zkML timeline / challenges	Extropy / Worldcoin
Wednesday	6	Giza	Giza / Extropy
Thursday	7	Modulus	Modulus

Week 3 - Advanced Topics

Day	Lesson	Topic	Teacher
Monday	8	Modulus	Extropy
Tuesday	9	Tensor Plonk / Zero Gravity	Extropy
Wednesday	10	Latest Research	Extropy
Thursday	11	Hardware / Games	Extropy

Week 4

Day	Lesson	Topic	Teacher
Monday	12	Privacy with FHE	Zama
Tuesday	13		
Wednesday	14		
Thursday	15		

Numbers and terminology

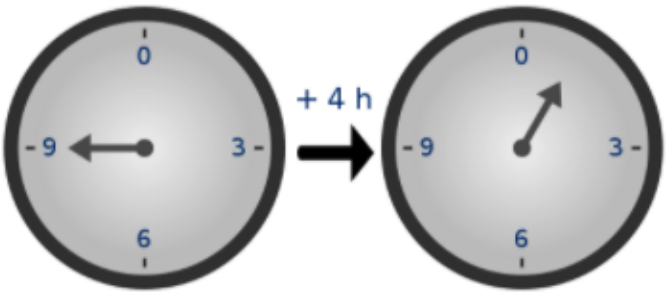
The set of Integers is denoted by $\mathbb{Z}$ e.g. $\{\cdots,-4,-3,-2,-1,0,1,2,3,4,\cdots\}$
The set of Rational Numbers is denoted by $\mathbb{Q}$ e.g. $\{\ldots 1, \frac{3}{2}, 2, \frac{22}{7} \ldots\}$
The set of Real Numbers is denoted by $\mathbb{R}$ e.g. $\{2, -4, 613, \pi, \sqrt{2}, \ldots\}$

Fields are denoted by $\mathbb{F}$, if they are a finite field or $\mathbb{K}$ for a field of real or complex numbers we also use $\mathbb{Z}_p^*$ to represent a finite field of integers mod prime p with multiplicative inverses.

We use finite fields for cryptography, because elements have “short”, exact representations and useful properties.

Modular Arithmetic

See this introduction



Because of how the numbers "wrap around", modular arithmetic is sometimes called "clock math"

When we write n mod k we mean simply the remainder when n is divided by k. Thus
25 mod 3 = 1
15 mod 4 = 3
The remainder should be positive.

Group Theory

Simply put a group is a set of elements {a,b,c,...} plus a binary operation, here we represent this as •

To be considered a group this combination needs to have certain properties

1. Closure
   For all a, b in G, the result of the operation, a • b, is also in G
2. Associativity
   For all a, b and c in G, (a • b) • c = a • (b • c)
3. Identity element
   There exists an element e in G such that, for every element a in G, the equation e • a = a • e = a holds. Such an element is unique and thus one speaks of the identity element.
4. Inverse element
   For each a in G, there exists an element b in G, commonly denoted a^{-1} (or −a, if the operation is denoted "+"), such that a • b = b • a = e, where e is the identity element.

Sub groups

If a subset of the elements in a group also satisfies the group properties, then that is a subgroup of the original group.

Cyclic groups and generators

A finite group can be cyclic. That means it has a generator element. If you start at any point and then apply the group operation with the generator as argument a certain number of times, you go around the whole group and end in the same place,

Fields

A field is a set of say Integers together with two operations called addition and multiplication.
One example of a field is the Real Numbers under addition and multiplication, another is a set of Integers mod a prime number with addition and multiplication.
The field operations are required to satisfy the following field axioms. In these axioms, a, b and c are arbitrary elements of the field $\mathbb{F}$.

1. Associativity of addition and multiplication: a + (b + c) = (a + b) + c and a · (b · c) = (a · b) · c.
2. Commutativity of addition and multiplication: a + b = b + a and a · b = b · a.
3. Additive and multiplicative identity: there exist two different elements 0 and 1 in $\mathbb{F}$ such that a + 0 = a and a · 1 = a.
4. Additive inverses: for every a in F, there exists an element in F, denoted −a, called the additive inverse of a, such that a + (−a) = 0.
5. Multiplicative inverses: for every a ≠ 0 in F, there exists an element in F, denoted by a^{-1}, called the multiplicative inverse of a, such that a $\cdot a^{-1}$ = 1.
6. Distributivity of multiplication over addition: a · (b + c) = (a · b) + (a · c).

Finite fields and generators

A finite field is a field with a finite set of elements, such as the set of integers mod p where p is a prime.

To try out operations on finite fields, see https://asecuritysite.com/encryption/finite

The **order** of the field is the number of elements in the field's set.

For a finite field the order must be either

- prime (a prime field)
  or
- the power of a prime (an extension field)

An element can be represented as an integer greater or equal than 0 and less than the field's order: {0, 1, ..., p-1} in a simple field.

Every finite field has a generator. A generator is capable of generating all of the elements in the set by exponentiating the generator .
So for generator g we can take g^0, g^1,g^2 and eventually this will give us all elements in the group

For example. taking the set of integers and prime p = 5, we get the group $\mathbb{Z}_5^*$ = {0,1,2,3,4}.
In the group $\mathbb{Z}_5^*$, operations are carried out modulo 5; hence, we don't have 3 × 4 = 12 but instead have 3 × 4 = 2, because 12 mod 5 = 2.

$\mathbb{Z}_5^*$ is cyclic and has two generators, 2 and 3, because $2^1 = 2, 2^2 = 4, 2^3 = 3, 2^4 = 1$, and $3^1 = 3, 3^2 = 4, 3^3 = 2, 3^4 = 1$

Fermat's little theorem

This is useful for finding a multiplicative inverse

$a^{-1} \equiv a^{p-2} (mod p)$
Let p = 7 and a = 2. We can compute the inverse of a as:
$a^{p-2} = 2^5 = 32 \equiv 4$ mod 7.

This is easy to verify: 2 x 4 ≡ 1 mod 7.

Equivalence classes

Since
6 mod 7 = 6
13 mod 7 = 6
20 mod 7 = 6
...
we can say that 6, 13, 20 ... form an equivalence class
more formally
modular arithmetic partitions the integers into N equivalence classes, each of the form $i + kN \mid k \in \mathbb{Z}$ for some i between 0 and $N - 1$.

Thus if we are trying to solve the equation
x mod 7 = 6
x could be 6, 13, 20 ...
This gives us the basis for a one way function.

Linear Algebra

See Introduction

Linear algebra is about linear combinations. That is, using arithmetic on columns of numbers called vectors and arrays of numbers called matrices, to create new columns and arrays of numbers.
For example we could have some equations in 2 unknowns, $x1, x2$

$y = 0.1 * x1 + 0.4 * x2$
$y = 0.3 * x1 + 0.9 * x2$
$y = 0.2 * x1 + 0.3 * x2$

The column of y values can be taken as a column vector of outputs from the equation.
The two columns of floating-point values are the data columns, say $a1$ and $a2$, and can be taken as a matrix A.
The two unknown values $x1$ and $x2$ can be taken as the coefficients of the equation and together form a vector of unknowns b to be solved.
We can write this compactly using linear algebra notation as:

$y = A.b$

Elliptic Curves

An elliptic curve is a set of points (x, y) that satisfy an equation such as
$y^2 = x^3 + ax + b$

where a and b are constants.
In addition to the points on the curve, there is also an "identity element" called the point at infinity.

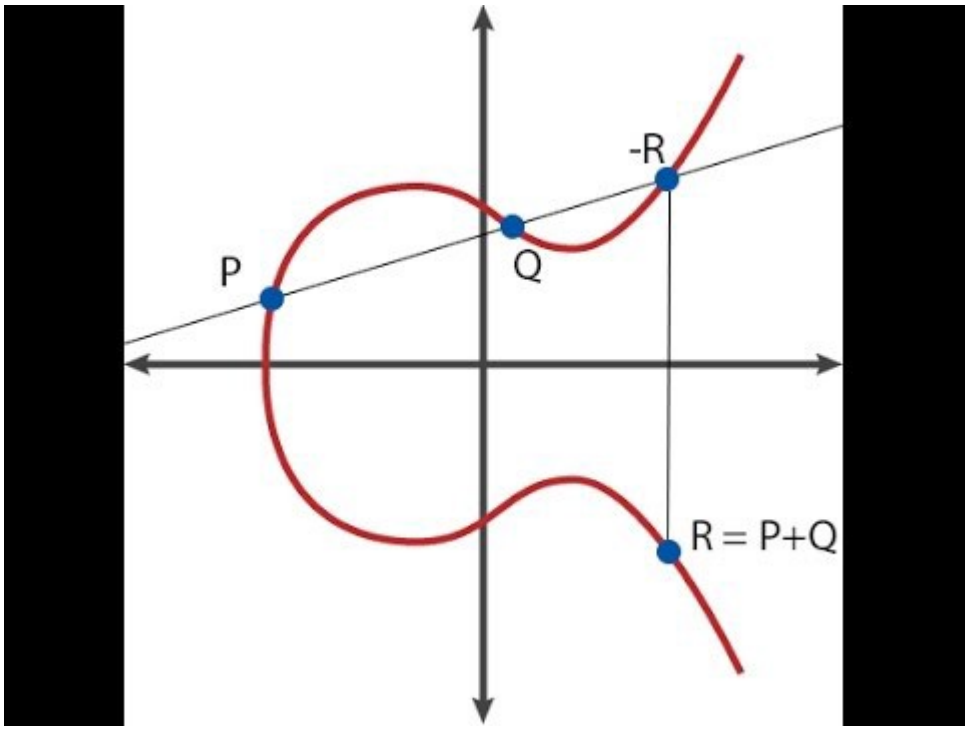
For certain equations they will satisfy the group axioms

- every two points can be added to give a third point (closure);
- it does not matter in what order the two points are added (commutatitivity);
- if you have more than two points to add, it does not matter which ones you add first either (associativity);
- there is an identity element.

We find that some curves do indeed form a group under the "point addition" operation.
Point addition combines two points on the curve to produce a third point on the curve.
The point addition operation has special rules based on the geometric properties of elliptic curves, which ensure that the result of the addition is also a point on the curve.
Point addition is geometrically motivated, and it allows us to define scalar multiplication, which is the basis for elliptic curve cryptography.

We often use 2 families of curves :

Montgomery Curves



For example curve 25519 with equation $y^2 = x^3 + 486662x^2 + x$

Curve 25519 gives 128 bits of security and is used in the Diffie–Hellman (ECDH) key agreement scheme
BN254 / BN_128 is the curve used in Ethereum for ZKSNARKS
BLS12-381 is the curve used by ZCash

Edwards Curves

The general equation is $ax^2 + y^2 = 1 + dx^2y^2$ with a = 1
for some scalar d which can be 0 or 1.

If a <> 1 they are called Twisted Edwards Curves
Every twisted Edwards curve is birationally equivalent to a Montgomery curve

Scalar multiplication

Scalar multiplication is an operation in which a point on an elliptic curve is added to itself a certain number of times.
The result of this operation is another point on the elliptic curve.

Given an elliptic curve E defined by the equation $y^2 = x^3 + ax + b$, where a and b are constants, and a point $P(x, y)$ on the curve E, scalar multiplication is defined as follows:

$kP = P + P + \ldots + P$ (k times)

where k is a scalar and the addition operation is the point addition operation defined on the elliptic curve.

Roots of unity

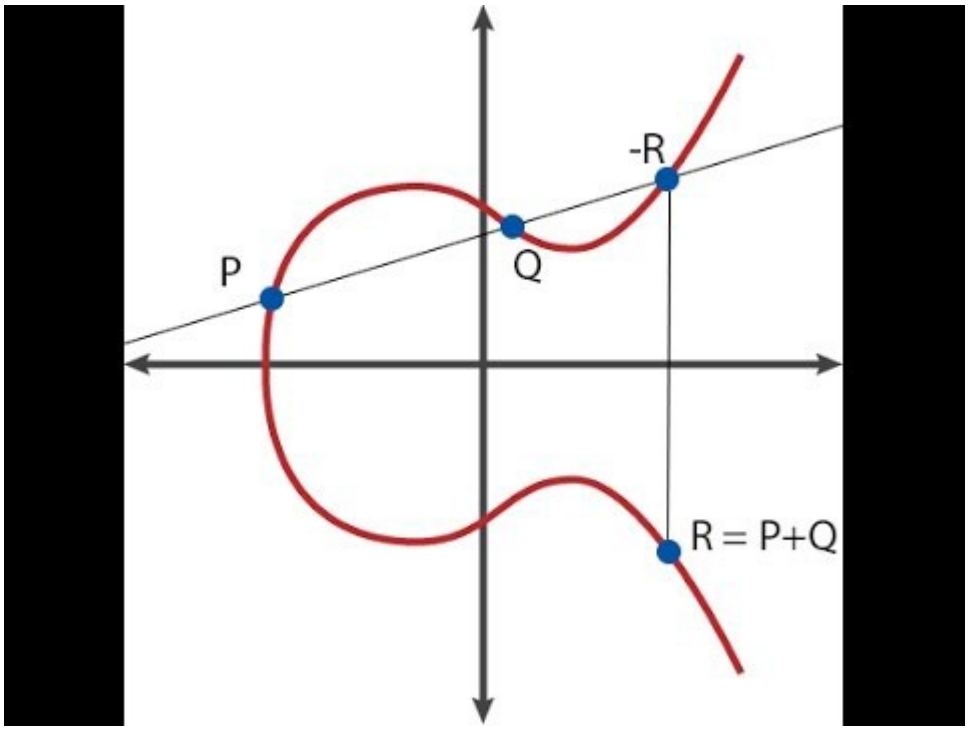
Recall, over a finite field $\mathbb{F}_p$, where p is a prime number, an elliptic curve has a finite number of points (including a special "point at infinity" denoted as O).
The set of points on an elliptic curve forms a group under point addition

Roots of unity are points on the curve that, when added to themselves a certain number of times using point addition, result in the identity element O.
In other words, for a point P on the elliptic curve, there exists a positive integer n such that:

$nP = P + P + \ldots + P$ (n times) = 1

Here, n is called the order of the point P.
The smallest positive integer n for which this condition holds is the order of the point P.

Visualisations



From
Serious Cryptography Jean-Philippe Aumasson

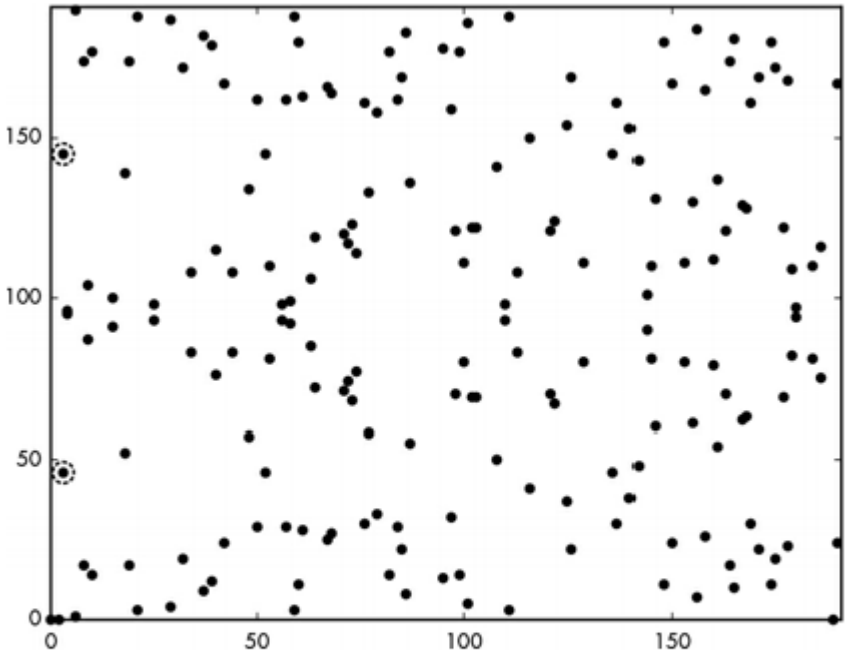


Figure 12-2: The elliptic curve with the equation $y^2 = x^3 - 4x$ *over* $\mathbb{Z}_{191}$*, the set of integers modulo 191*

This video gives further examples of elliptic curve arithmetic.

Polynomial Introduction

A polynomial is an expression that can be built from constants and variables by means of addition, multiplication and exponentiation to a non-negative integer power.

e.g. $3x^2 + 4x + 3$

Quote from Vitalik Buterin
"There are many things that are fascinating about polynomials. But here we are going to zoom in on a particular one: **polynomials are a single mathematical object that can contain an unbounded amount of information** (think of them as a list of integers and this is obvious)."
Furthermore, **a single equation between polynomials can represent an unbounded number of equations between numbers**.
For example, consider the equation A(x)+B(x)=C(x). If this equation is true, then it's also true that:

- A(0)+B(0)=C(0)
- A(1)+B(1)=C(1)
- A(2)+B(2)=C(2)
- A(3)+B(3)=C(3)

Adding, multiplying and dividing polynomials

We can add, multiply and divide polynomials, for examples
see https://en.wikipedia.org/wiki/Polynomial_arithmetic

Roots

For a polynomial P of a single variable x in a field K and with coefficients in that field, the root r of P is an element of K such that $P(r) = 0$

B is said to divide another polynomial A when the latter can be written as

$A = BC$

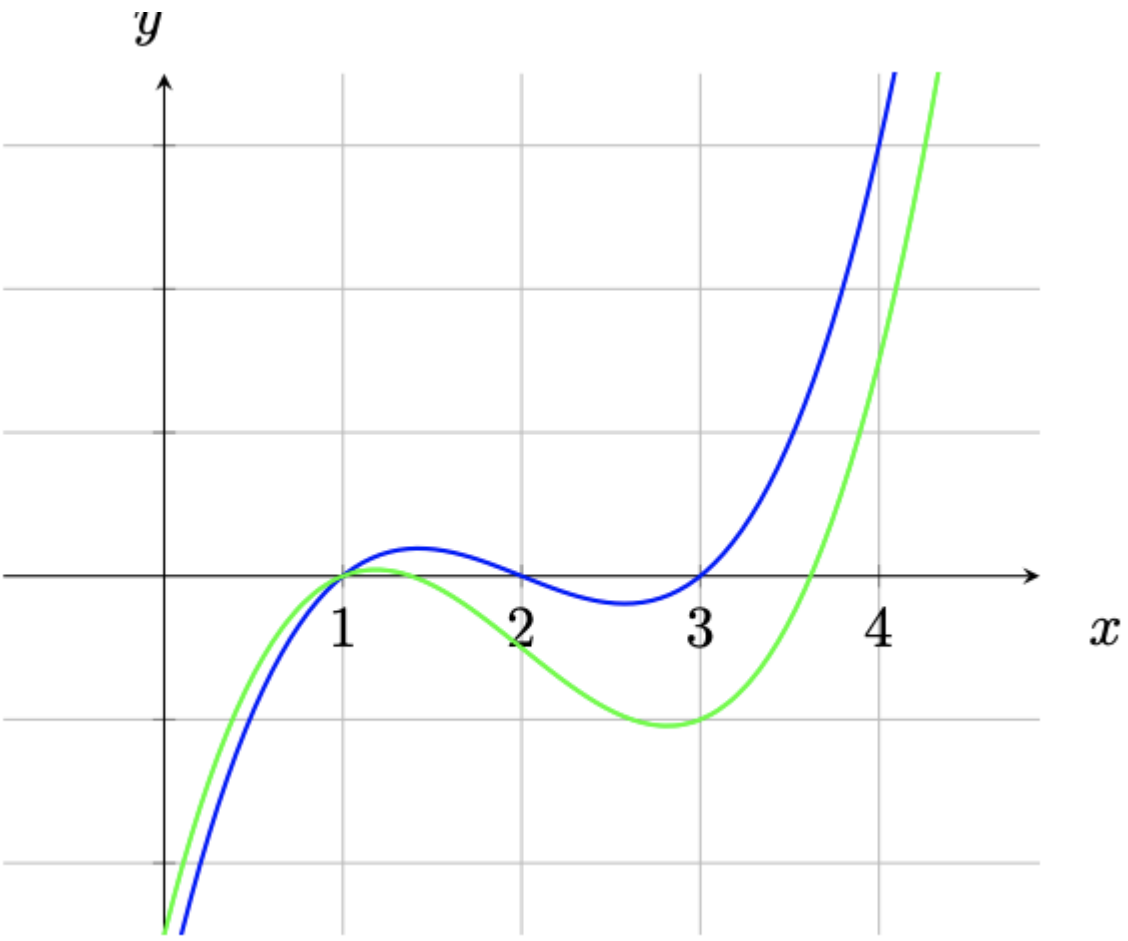
with C also a polynomial,the fact that B divides A is denoted $B|A$

If one root r of a polynomial $P(x)$ of degree n is known then polynomial long division can be used to factor $P(x)$ into the form
$(x - r)(Q(x))$
where
$Q(x)$ is a polynomial of degree $n - 1$.
$Q(x)$ is simply the quotient obtained from the division process; since r is known to be a root of $P(x)$, it is known that the remainder must be zero.

Schwartz-Zippel Lemma

“different polynomials are different at most points”.

Polynomials have an advantageous property, namely, if we have two non-equal polynomials of degree at most d, they can intersect at no more than d points.



if f and g are polynomials and are equal, then
$f(x) = g(x)$ for all x

if f and g are polynomials and are NOT equal, then
$f(x) \neq g(x)$ for all pretty much any x

What does it mean to say 2 polynomials are equal ?

1. They evaluate to the same value or all points
2. They have the same coefficients

If we are working with real numbers, these 2 points would go together, however that is not the case when we are working with finite fields.

For example all elements of a field of size q satisfy the identity
$x^q = x$

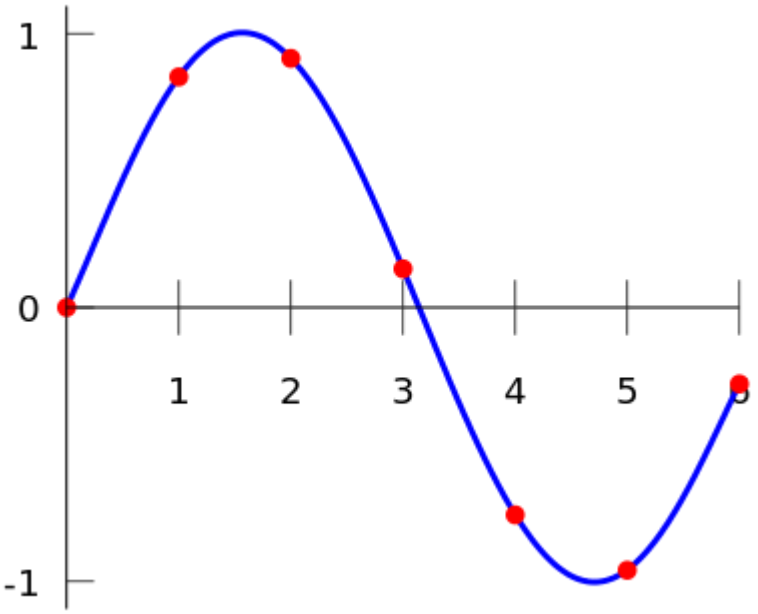
The polynomials X^q and X take the same values at all points, but do not have the same coefficients.

Lagrange Interpolation

If you have a set of points then doing a Lagrange interpolation on those points gives you a polynomial that passes through all of those points.

If you have two points on a plane, you can define a single straight line that passes through both, for 3 points, a single 2nd-degree curve (e.g. $5x^2 + 2x + 1$) will go through them etc.
For n points, you can create a n-1 degree polynomial that will go through all of the points.

(We can use this in all sorts of interesting schemes as well as zkps)



Representations

We effectively have 2 ways to represent polynomials

1. Coefficient form
   $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \dots$
2. Point value form
   $(x_1, y_1), (x_2, y_2), \dots$

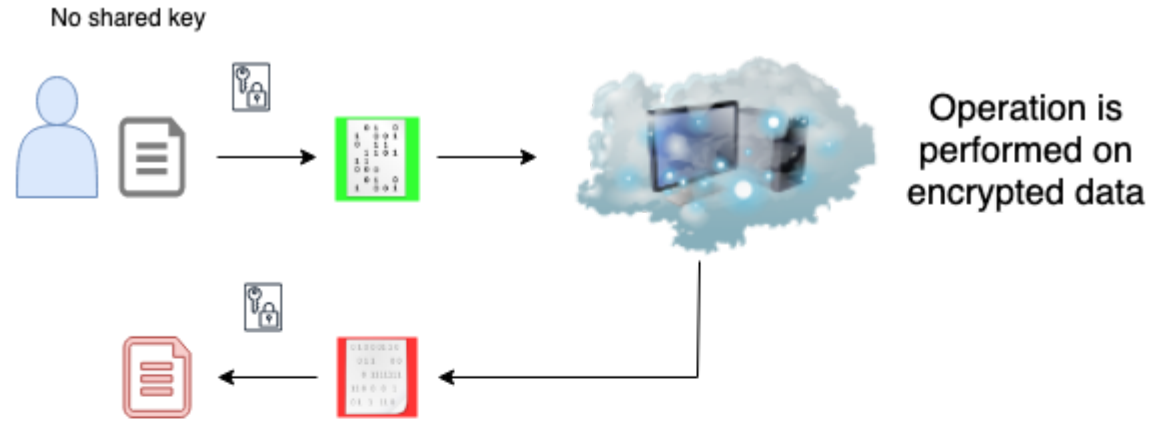
We can switch between the two forms by evaluation or interpolation.

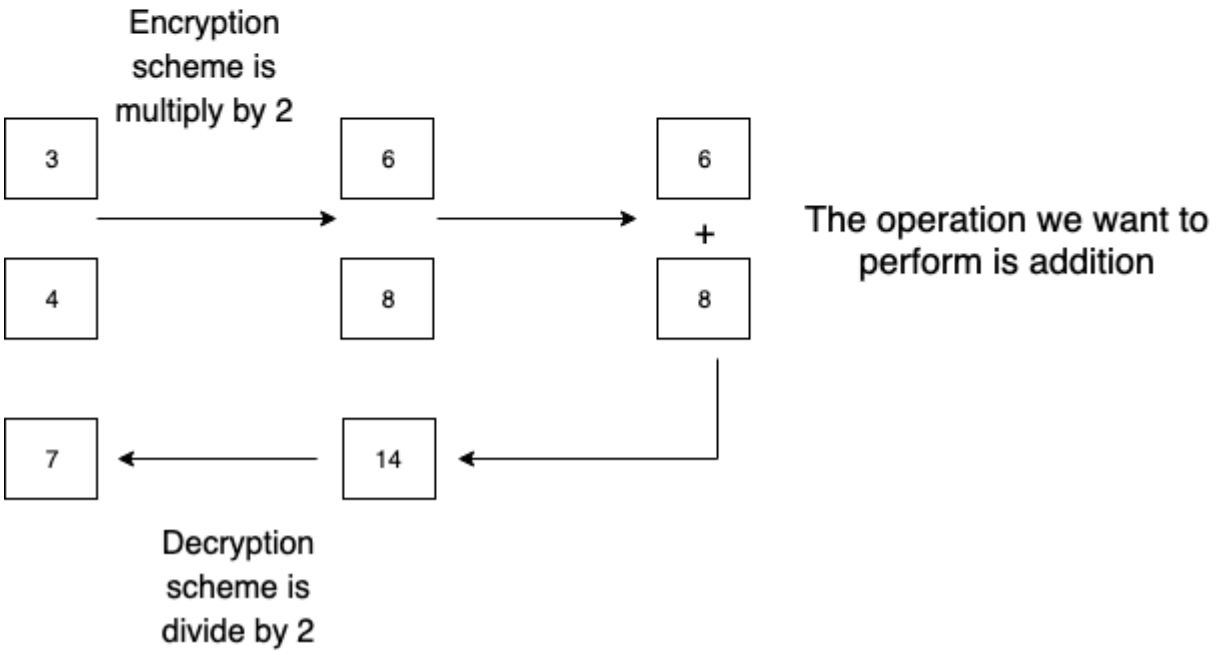
FHE

Background

Fully Homomorphic Encryption , the ‘holy grail’ of cryptography, is a form of encryption that allows arbitrary computations on encrypted data.

Homomorphic encryption is a form of encryption with an additional evaluation capability for computing over encrypted data without access to the secret key. The result of such a computation remains encrypted. Homomorphic encryption can be viewed as an extension of either symmetric-key or public-key cryptography. Homomorphic refers to homomorphism in algebra: the encryption and decryption functions can be thought as homomorphisms between plaintext and ciphertext spaces.





Alice, the data owner, encrypts data with her key and sends it to an outsourced machine for storage and processing.
The outsourced machine performs arbitrary computations on the encrypted data without learning anything about it.
Alice decrypts the results of those computations using her original key, retaining full confidentiality, ownership, and control.

Example :

Medical Data using FHE

Bitcoin split-key vanity mining

Bitcoin addresses are hashes of public keys from ECDSA key pairs. A vanity address is an address generated from parameters such that the resultant hash contains a human-readable string (e.g., 1BoatSLRHtKNngkdXEeobR76b53LETtpyT).
Given that ECDSA key pairs have homomorphic properties for addition and multiplication, one can outsource the generation of a vanity address without having the generator know the full private key for this address.

For example,
Alice generates a private key (a) and public key (A) pair, and publicly posts A.
Bob generates a key pair (b, B) such that hash(A + B) results in a desired vanity address. He sells b and B to Alice.
A, B, and b are publicly known, so one can verify that the address = hash(A + B) as desired.
Alice computes the combined private key (a + b) and uses it as the private key for the public key (A + B).
Similarly, multiplication could be used instead of addition.

Complexity Theory

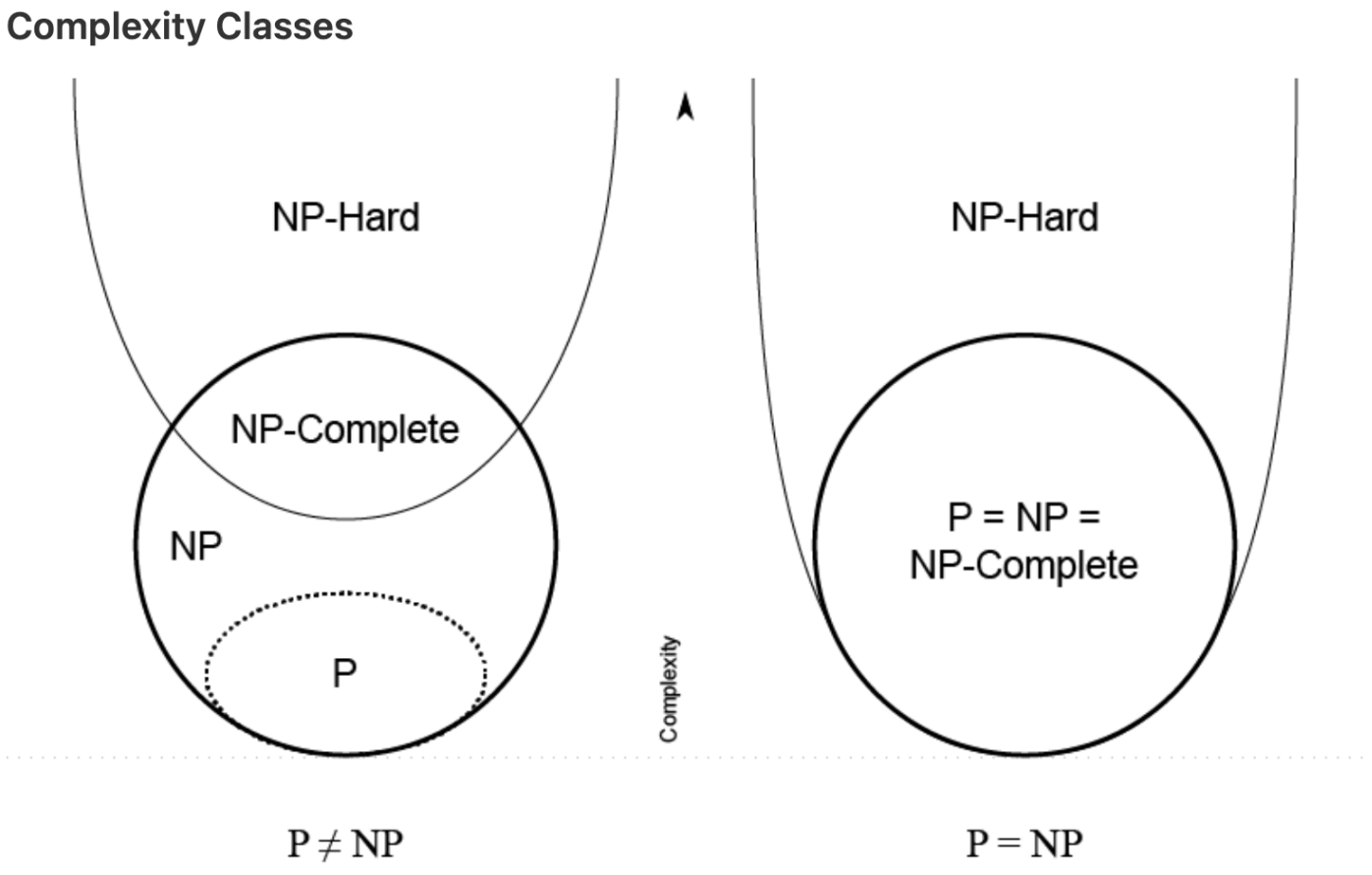
Complexity theory looks at the time or space requirements to solve a problem, particularly in terms of the size of the input.
We can classify problems according to the time required to find a solution, for some problems there may exist an algorithm to find a solution in a reasonable time, whereas for other problems we may not know of such an algorithm, and may have to 'brute force' a solution, trying out all potential solutions until one is found that works.

For example the travelling salesman problem tries to find the shortest route for a salesman required to travel between a number of cities, visiting every city exactly once. For a small number of cities, say 3, we can quickly try all alternatives to find the shortest route, however as the number of cities grows, this quickly becomes unfeasible.

Based on the size of the input n , we classify problems according to how the time required to find a solution grows with n.
If the time taken in the worst case grows as a polynomial of n, that is roughly proportional to n^k for some value k, we put these problems in class P for polynomial. These problems are seen as tractable.

We are also interested in knowing how long it takes to verify a potential solution once it has been found.

Decision Problem: A problem with a yes or no answer



P

P is a complexity class that represents the set of all decision problems that can be solved in polynomial time. That is, given an instance of the problem, the answer yes or no can be decided in polynomial time.

NP

NP is a complexity class that represents the set of all decision problems for which the instances where the answer is “yes” have proofs that can be verified in polynomial time, even though the solution may be hard to find.
This means that if someone gives us an instance of the problem and a witness to the answer being yes, we can check that it is correct in polynomial time, that is you can run some polynomial-time algorithm that will verify whether you’ve found an actual solution.
For example, the problem of recovering a secret key with a known plaintext is in NP, because you can check that a candidate key is the correct key by verifying that encrypting the plaintext with that key and showing that it equals the supplied cypher text.
The process of finding a potential key (the solution) can’t be done in polynomial time, but checking whether the key is correct is done using a polynomial-time algorithm.

NP-Complete

NP-Complete is a complexity class which represents the set of all problems X in NP for which it is possible to reduce any other NP problem Y to X in polynomial time.
Intuitively this means that we can solve Y quickly if we know how to solve X quickly.
Precisely, Y is reducible to X, if there is a polynomial time algorithm f to transform instances y of Y to instances
x = f(y) of X in polynomial time, with the property that the answer to y is yes, if and only if the answer to f(y) is yes

NP-hard

Intuitively, these are the problems that are at least as hard as the NP-complete problems. Note that NP-hard problems do not have to be in NP, and they do not have to be decision problems.
The precise definition here is that a problem X is NP-hard, if there is an NP-complete problem Y, such that Y is reducible to X in polynomial time.
But since any NP-complete problem can be reduced to any other NP-complete problem in polynomial time, all NP-complete problems can be reduced to any NP-hard problem in polynomial time. Then, if there is a solution to one NP-hard problem in polynomial time, there is a solution to all NP problems in polynomial time.

Even the task of winning in certain video games can sometimes be proven to be NP-complete (for famous games including Tetris, Super Mario Bros., Pokémon, and Candy Crush Saga). For example, the article “Classic Nintendo Games Are (Computationally) Hard” (https://arxiv.org/abs/1203.1895) considers “the decision problem of reachability” to determine the possibility of reaching the goal point from a particular starting point.

Some of these video game problems are actually even harder than NP-complete and are called NP-hard.

From "Everything provable is provable in zero knowledge"
https://dl.acm.org/doi/pdf/10.5555/88314.88333

"Assuming the existence of a secure probabilistic encryption scheme, we show that every language that admits an interactive proof admits a (computational) zero-knowledge interactive proof. This result extends the result of Goldreich, MiCali and Wigderson, that, under the same assumption, all of NP admits zero-knowledge interactive proofs."

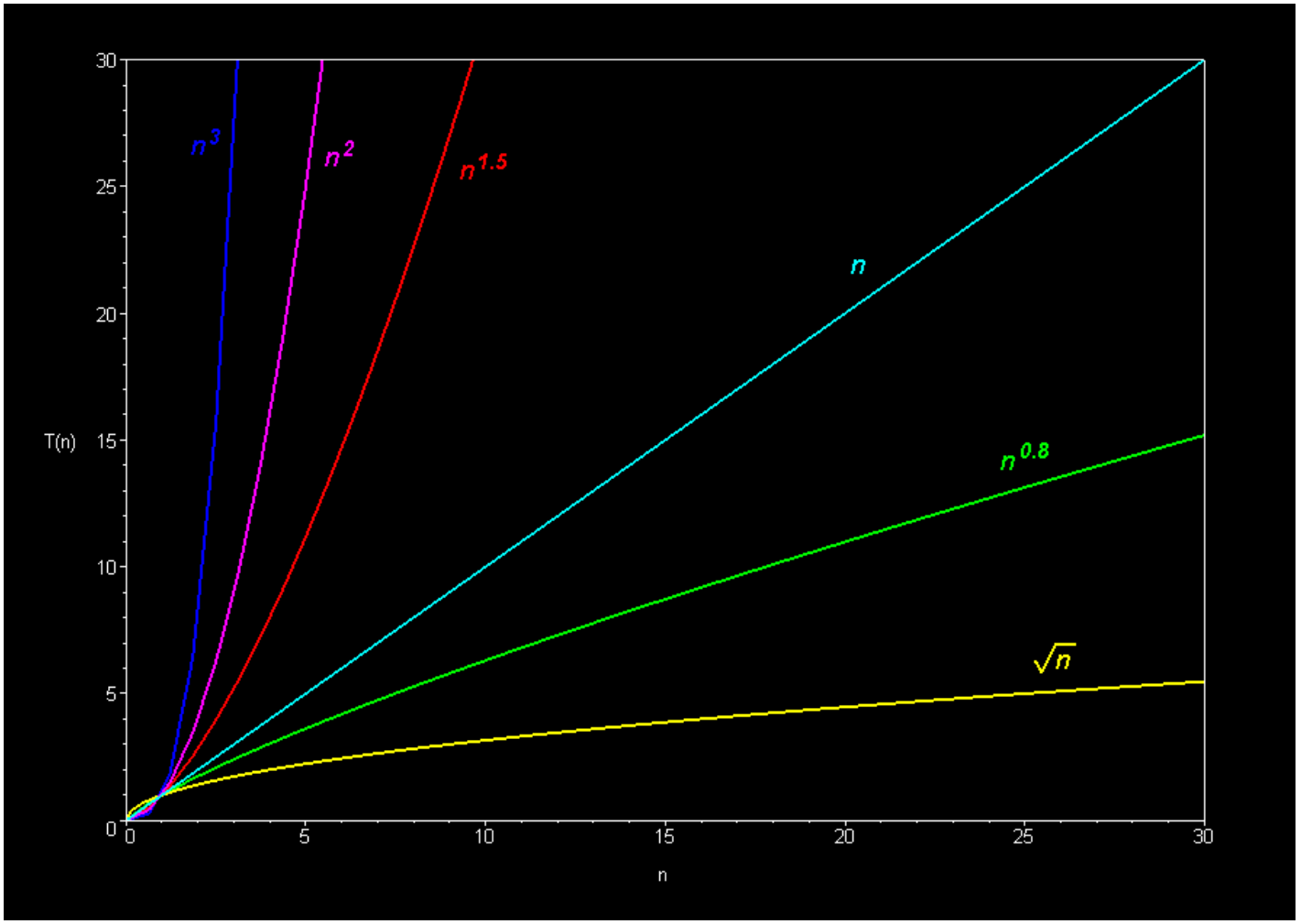
IP

A class that lies at the heart of zkps is the Interactive Proof class.
In complexity theory they are related to the other complexity classes, and we shall see how the idea of interactive proofs, as opposed to traditional static proofs is widely used in zk proving systems.
Interactive proof videos from Alessandro Chiesa

Big O notation

In plain words, Big O notation describes the **complexity** of your code using algebraic terms.
It describes the time or space required to solve a problem in the worse case in terms of the size of the input.

For example if we say for input size n
$O(n^2)$
we are saying that as n increases, the time taken to solve the problem goes up to proportional to the square of n.



We use this notation when comparing ZKP systems

	SNARKs	STARKs	Bulletproofs
Algorithmic complexity: prover	O(N * log(N))	O(N * poly-log(N))	O(N * log(N))
Algorithmic complexity: verifier	~O(1)	O(poly-log(N))	O(N)
Communication complexity (proof size)	~O(1)	O(poly-log(N))	O(log(N))
- size estimate for 1 TX	Tx: 200 bytes, Key: 50 MB	45 kB	1.5 kb
- size estimate for 10.000 TX	Tx: 200 bytes, Key: 500 GB	135 kb	2.5 kb
Ethereum/EVM verification gas cost	~600k (Groth16)	~2.5M (estimate, no impl.)	N/A
Trusted setup required?	YES 😒	NO 😄	NO 😄
Post-quantum secure	NO 😒	YES 😄	NO 😒
Crypto assumptions	Strong 😒	Collision resistant hashes 😄	Discrete log 😏

Zero Knowledge Proof Introduction

Intuitive grasp of Zero Knowledge Proofs

It is difficult to find zero knowledge resources that avoid the extremes of either over simplifying the subject, or presenting so much mathematical detail that the reader gets bogged down and loses interest.
In this course I hope to find an accessible but informative middle ground.

We start with some examples to show how zero knowledge proofs can proceed, and the situations where they could be used.

What is a zero knowledge proof

A loose definition
It is a proof that there exists or that we know something, plus a zero knowledge aspect, that is the person verifying the proof only gains one piece of information - that the proof is valid or invalid.

Actors in a Zero Knowledge Proof System

- Creator - optional, maybe combined with the prover
- Prover - I will call her Peggy
- Verifier - I will call him Victor

Examples to give an Intuitive grasp of zero-knowledge proofs

1. Colour blind verifier
   This is an interactive proof showing that the prover can distinguish between a red and a green billiard ball, whereas the verifier cannot distinguish them.

• The prover wants to show the verifier that they have different colours but does not want him to learn which is red and which is green.

• Step 1: The verifier takes the balls, each one in each hand, holds them in front of the prover and then hides them behind his back. Then, with probability 1/2 either swaps them (at most once) or keeps them as they are. Finally, he brings them out in front.

• Step 2: The prover has to say the verifier switched them or not.

• Step 3: Since they have different colours, the prover can always say whether they were switched or not.
But, if they were identical (the verifier is inclined to believe that), the prover would be wrong with probability 1/2.

• Finally, to convince the verifier with very high probability, the prover could repeat Step 1 to Step 3 k times to reduce the probability of the prover being successful by chance to a extremely small amount.

2. Wheres Wally
   Based on the pictures of crowds where Wally is distinctivly dressed, the aim being to find him within a sea of similar people.
   The proof procedes as follows :
   Imagine the Peggy has found Wally in the picture and wants to prove this fact to Victor, however if she just shows him, Victor is liable to cheat and claim he also found Wally.
   In order to prove to Victor that she has indeed found Wally, without giving away his location in the picture
3. Peggy cuts a hole in a (very) large sheet of paper, the hole should be the exact shape of Wally in the underlying picture.
4. Peggy places the paper sheet over the original picture, so that the location of the picture beneath the paper is obscured.
5. Victor can then see throught he hole that Wally has indeed been found, but since the alignment with the underlying picture cannot be seen, he doesn't gain any information about the location of Wally.

3. Sudoku
   An interactive proof can be created to prove the knowledge of a solution to a sudoku puzzle by placing cards in the sudoku grid. The process is descrived here Sudoku Proof

Quote from Vitalik Buterin
"You can make a proof for the statement "I know a secret number such that if you take the word ‘cow', add the number to the end, and SHA256 hash it 100 million times, the output starts with

`0x57d00485aa`". The verifier can verify the proof far more quickly than it would take for them to run 100 million hashes themselves, and the proof would also not reveal what the secret number is."

Zero Knowledge Proof Timeline

Changes have occurred because of

- Improvements to the cryptographic primitives (improved curves or hash functions for example)
- A fundamental change to the approach to zero knowledge
  See the excellent blog post from Starkware :
  The Cambrian Explosion

1984 : Goldwasser, Micali and Rackoff - Probabilistic Encryption.
1989 : Goldwasser, Micali and Rackoff - The Knowledge Complexity of Interactive Proof Systems
1991 O Goldreich, S. Micali and A. Wigderson. Proofs that Yield Nothing but their Validity or All Languages in NP Have Zero-Knowledge Proof Systems. Preliminary version in 1986. (Graph colouring problem)

....

2006 Groth, Ostrovsky and Sahai introduced pairing-based NIZK proofs, yielding the first linear size proofs based on standard assumptions.

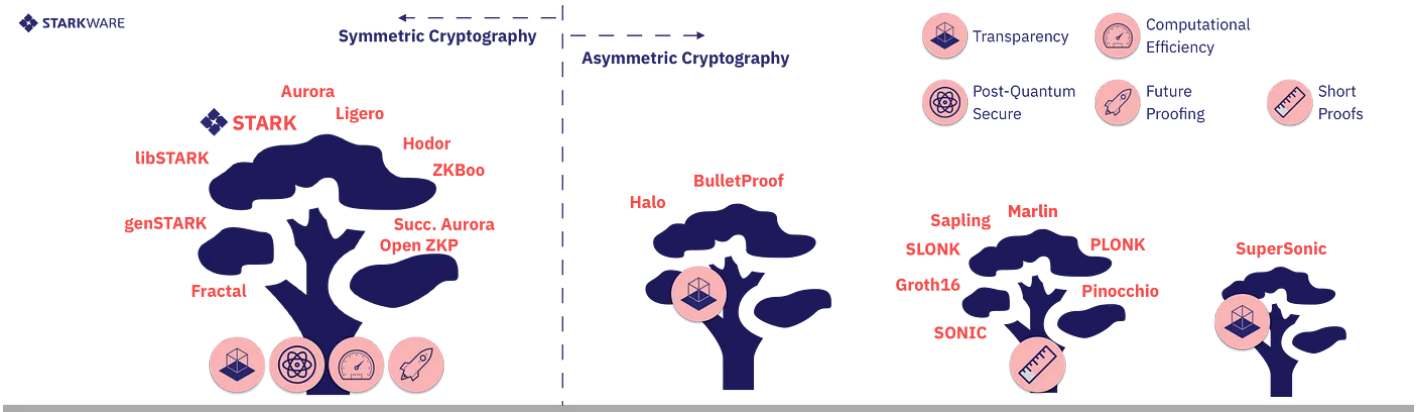
2010 Groth combined these techniques with ideas from interactive zero-knowledge arguments to give the first constant size NIZK arguments.

2016 : Jens Groth - On the Size of Pairing-based Non-interactive Arguments

From Matthew Green

Prior to Goldwasser et al., most work in this area focused the soundness of the proof system. That is, it considered the case where a malicious Prover attempts to ‘trick’ a Verifier into believing a false statement. What Goldwasser, Micali and Rackoff did was to turn this problem on its head. Instead of worrying only about the Prover, they asked: what happens if you don’t trust the Verifier?

ZKP Ecosystem



Cryptographic Assumptions	Collision-Resistant Hash	Elliptic Curve DLP	Knowledge of Exponent	Groups of Unknown Order
YEAR	1976	1980s-2000s	2000s-2017	2017-2019

from

[The Cambrian Explosion]

Proving Systems

A statement is a proposition we want to prove. It depends on:

- Instance variables, which are public.
- Witness variables, which are private.

Given the instance variables, we can find a short proof that we know witness variables that make the statement true (possibly without revealing any other information).

What do we require of a proof ?

- Completeness: there exists an honest prover P that can convince the honest verifier V of any correct statement with very high probability.
- Soundness: even a dishonest prover P running in super-polynomial time cannot convince an honest verifier V of an incorrect statement. Note: P does not necessarily have to run in polynomial time, but V does.

To make our proof zero knowledge we also need 'zero knowledginess'

To oversimplify: represented on a computer, a ZKP is nothing more than a sequence of numbers, carefully computed by Peggy, together with a bunch of boolean checks that Victor can run in order to verify the proof of correctness for the computation.

A zero-knowledge protocol is thus the mechanism used for deriving these numbers and defining the verification checks.

Interactive v Non Interactive Proofs

Non-interactivity is only useful if we want to allow multiple independent verifiers to verify a given proof without each one having to individually query the prover.

In contrast, in non-interactive zero knowledge protocols there is no repeated communication between the prover and the verifier. Instead, there is only a single "round", which can be carried out asynchronously.
Using publicly available data, Peggy generates a proof, which she publishes in a place accessible to Victor (e.g. on a distributed ledger).
Following this, Victor can verify the proof at any point in time to complete the “round”. Note that even though Peggy produces only a single proof, as opposed to multiple ones in the interactive version, the verifier can still be certain that except for negligible probability, she does indeed know the secret she is claiming.

Succint v Non Succint

Succinctness is necessary only if the medium used for storing the proofs is very expensive and/or if we need very short verification times.

Proof v Proof of Knowledge

A proof of knowledge is stronger and more useful than just proving the statement is true. For instance, it allows me to prove that I know a secret key, rather than just that it exists.

Argument v Proof

In a proof, the soundness holds against a computationally unbounded prover and in an argument, the soundness only holds against a polynomially bounded prover.
Arguments are thus often called "computationally sound proofs".

The Prover and the Verifier have to agree on what they’re proving. This means that both know the statement that is to be proven and what the inputs to this statement represent.

Virtual Machines

Introduction

See article
Virtual Machines (VMs) are designed to provide the functionality a computer, usually wholly in software (there is a subtle difference between virtualisation and emulation).

Virtual machine is an entity that emulates a guest interface on top of a host machine

- Language view:
  - Virtual machine = Entity that emulates an API (e.g., JAVA) on top of another
  - Virtualising software = compiler/interpreter –
- Process view:
  - Machine = Entity that emulates an ABI on top of another
    - Virtualising software = runtime –
- Operating system view:
  - Machine = Entity that emulates an ISA
  - Virtualising software = virtual machine monitor (VMM)

Virtual Machines vs Containers

A virtual machine is a complete operating system, that is walled off from the rest of the system.
A container (such as Docker) is less featured, it only has as much of the operating system as is needed for the app that it is running.

Virtual Machine interfaces

- Application programmer interface (API):
  - – High-level language library such as clib
- Application binary interface (ABI):
  – User instructions (User ISA)
  – System calls
- Hardware-software interface:
  - – Instruction set architecture (ISA)

The Instruction Set Architecture (ISA) is part of the abstract model of a computer that defines how the CPU is controlled by the software.

There are 3 main types of VMs,
- register based,
- stack based and
- accumulator based,

most VMs are of the first 2 types.
The Ethereum Virtual Machine is stack based.

zkVMs

A zkVM is a zero-knowledge proof-based virtual machine that combines a zk proof and a Virtual Machine.
The zkVM typically consists of two important components: a compiler that can compile high-level languages such as C++ and Rust into intermediate (IR) expressions for the ZK system to perform; the other is the ISA (Instruction Architecture) instruction set framework, which mainly executes instructions about CPU operations and is a series of instructions used to instruct the CPU to perform operations.

Opcodes

An opcode specifies the instructions that can executed by a CPU/processor in machine code. They are associated with data to be processed in the form of 'operands'
From [evm codes]

OPCODE	NAME	MINIMUM GAS	STACK INPUT	STACK OUTPUT	DESCRIPTION
00	STOP	0			Halts execution
01	ADD	3	`a` `b`	`a + b`	Addition operation
02	MUL	5	`a` `b`	`a * b`	Multiplication operation
03	SUB	3	`a` `b`	`a - b`	Subtraction operation
04	DIV	5	`a` `b`	`a // b`	Integer division operation