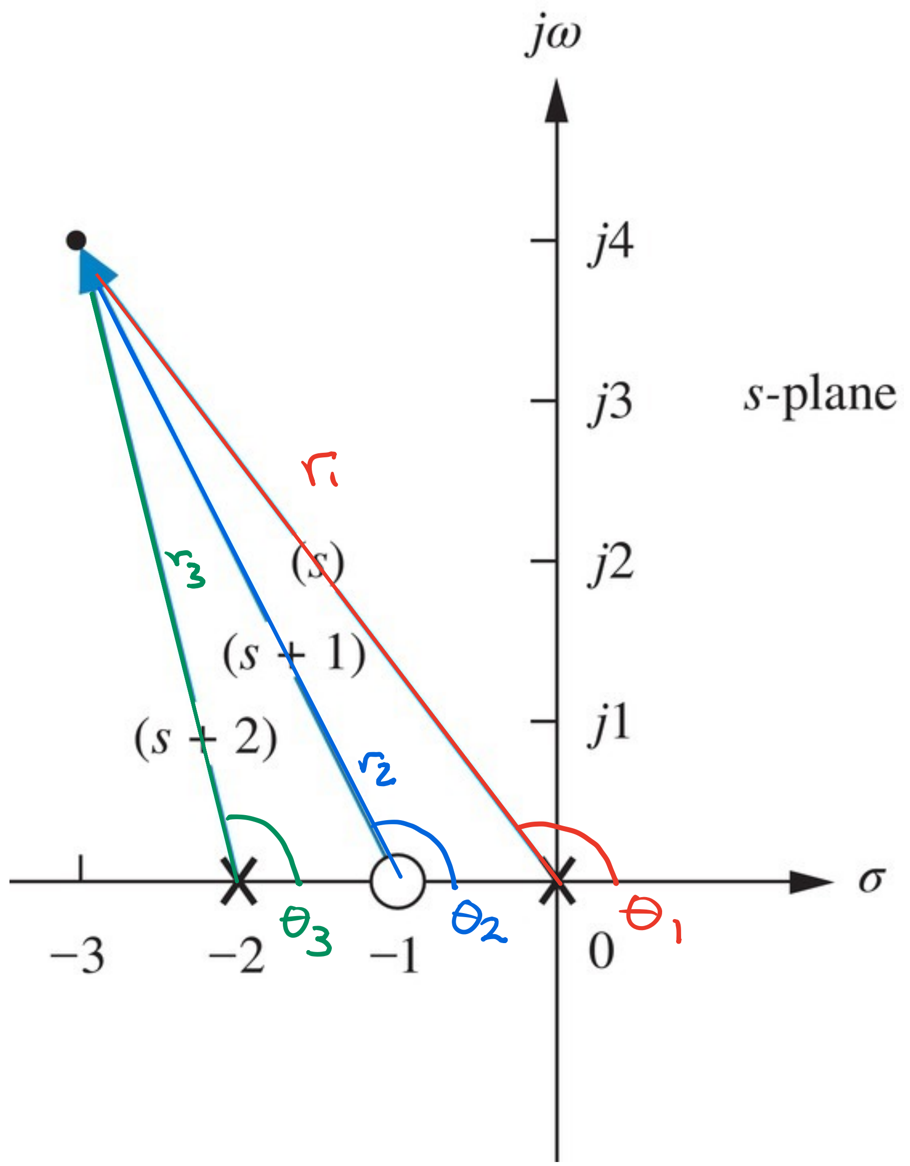
Ejemplo 1

$$F(s) = \frac{s+1}{s(s+2)}$$



$$-\theta_1 + \theta_2 - \theta_3 = -114^\circ$$

$$\theta_1 = \tan^{-1}\left(\frac{4}{-3}\right) \approx 127^\circ$$

$$\theta_2 = \tan^{-1}\left(\frac{4}{-2}\right) \approx 117^\circ$$

$$\theta_3 = \tan^{-1}\left(\frac{4}{-1}\right) \approx 104^\circ$$

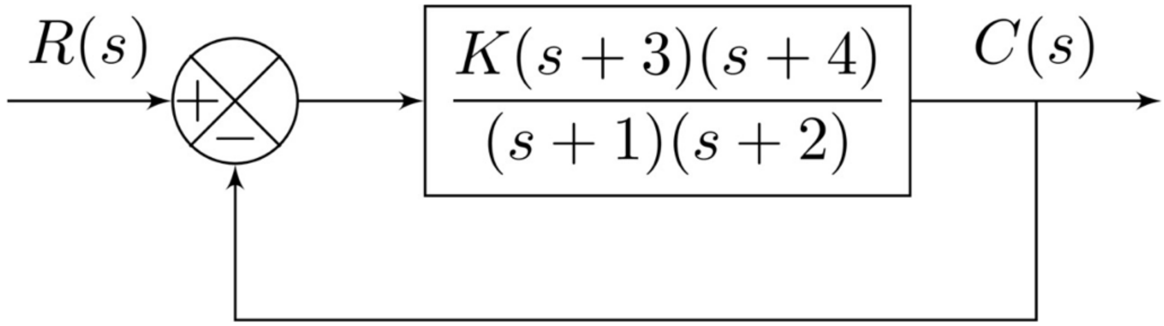
$$\frac{r_2}{r_1\, r_3} = \frac{2\sqrt{85}}{85} = 0.2169$$

$$r_1 = \sqrt{3^2 + 4^2} = 5$$

$$r_2 = \sqrt{2^2 + 4^2} = 2\sqrt{5}$$

$$r_3 = \sqrt{1 + 4^2} = \sqrt{17}$$

Ejemplo 2

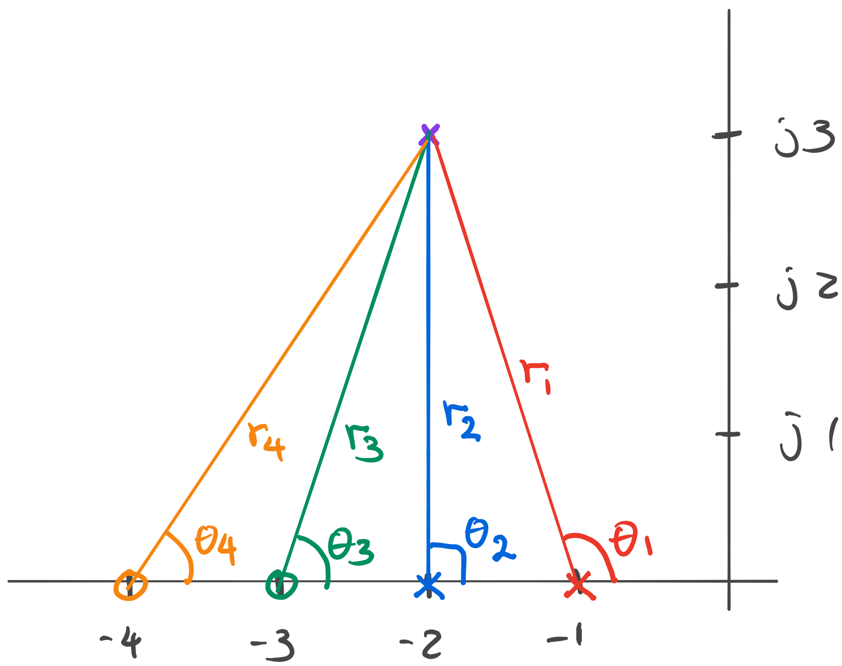


i) $\underbrace{|KG(s)H(s)| = 1}$, ii) $\underbrace{\measuredangle KG(s)H(s) = 180^\circ(2k+1)}$

No es muy relevante, dado que K se ajusta.

Se debe garantizar, ya que K, a lo sumo, aporta $\pm 180^\circ$

(a) $s_1 = -2 + j3$



$$-\theta_1 - \theta_2 + \theta_3 + \theta_4 = -70.560°$$ (No está sobre LGR)

$\theta_1 = \tan^{-1}\left(\frac{3}{-1}\right) = 108.435°$ $\theta_3 = \tan^{-1}\left(\frac{3}{1}\right) = 71.565°$

$\theta_2 = 90°$

$\theta_4 = \tan^{-1}\left(\frac{3}{2}\right) = 56.310°$

ⓑ $s_2 = -2 + j\frac{\sqrt{2}}{2}$

Los cálculos son similares, solo se cambia 3 por $\sqrt{2}/2$ en las $\tan^{-1}$

$\theta_1 = \tan^{-1}\left(\frac{\sqrt{2}}{-2}\right) = 144.736°$ $\theta_3 = \tan^{-1}\left(\frac{\sqrt{2}}{2}\right) = 35.264°$

$\theta_2 = 90°$

$\theta_4 = \tan^{-1}\left(\frac{\sqrt{2}}{4}\right) = 19.471°$

$$-\theta_1 - \theta_2 + \theta_3 + \theta_4 = -180°$$ cumple para algún valor de K

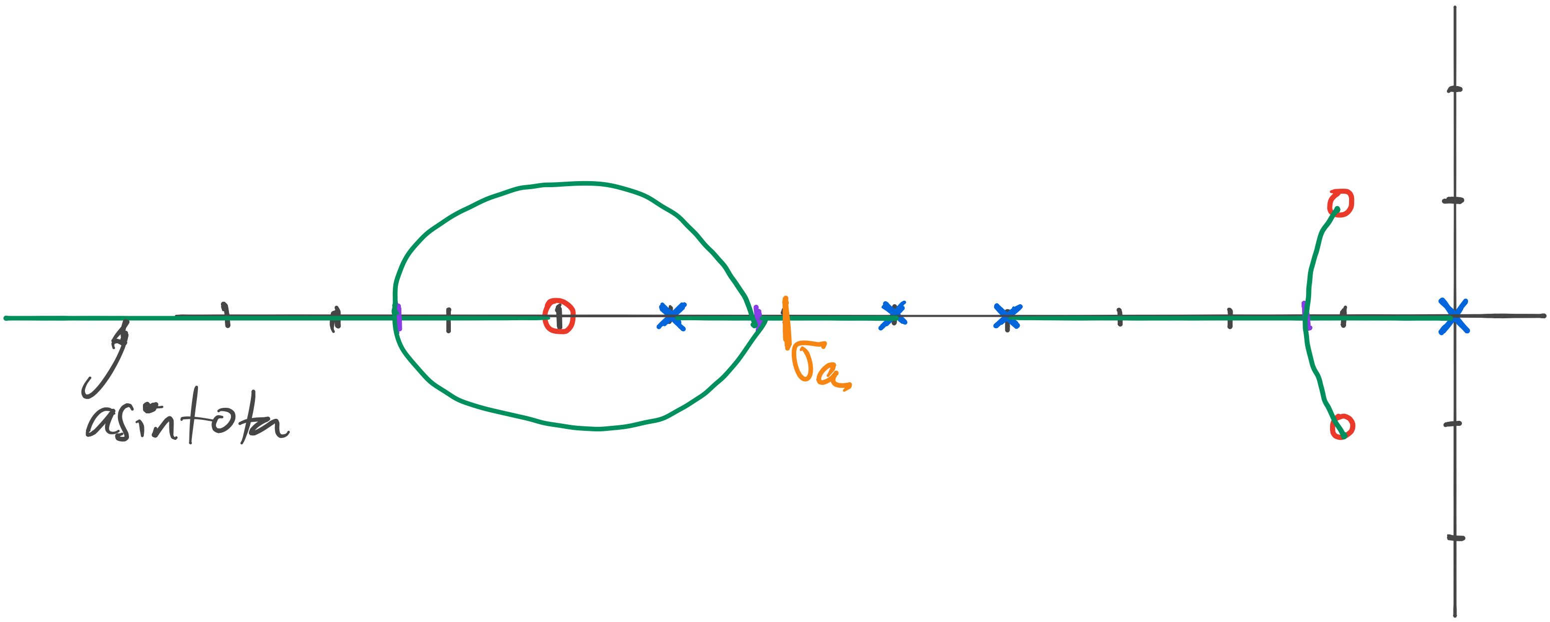
$$\frac{K\, r_3\, r_4}{r_1\, r_2} = 1 \Rightarrow K = \frac{r_1 r_2}{r_3 r_4} = \frac{\left(\frac{\sqrt{6}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{6}}{2}\right)\left(\frac{3\sqrt{2}}{2}\right)} = \frac{1}{3}$$

$r_1 = \sqrt{1 + \frac{2}{4}} = \frac{\sqrt{6}}{2}$ $r_3 = \sqrt{1 + \frac{2}{4}} = \frac{\sqrt{6}}{2}$

$r_2 = \sqrt{0 + \frac{2}{4}} = \frac{\sqrt{2}}{2}$ $r_4 = \sqrt{4 + \frac{2}{4}} = \frac{3\sqrt{2}}{2}$

Ejemplo 3

$$1 + \frac{K(s+8)(s^2+2s+2)}{s(s+4)(s+5)(s+7)} = 0$$



$$\sigma_a = \frac{\sum P_i - \sum Z_i}{\#P_i - \#Z_i} = \frac{(0-4-5-7) - (-8-1+\cancel{j}-1-\cancel{j})}{4-3}$$

$$\sigma_a = -6.$$

$$\theta_a = \frac{180^\circ(2k+1)}{\#P_i - \#Z_i} = 180^\circ(2k+1) = 180^\circ$$

Puntos de ruptura:

$$\frac{1}{\sigma+z_1} + \frac{1}{\sigma+z_2} + \frac{1}{\sigma+z_3} = \frac{1}{\sigma+p_1} + \frac{1}{\sigma+p_2} + \frac{1}{\sigma+p_3} + \frac{1}{\sigma+p_4}$$

$$\frac{1}{\sigma+8}+\frac{1}{\sigma+1-j}+\frac{1}{\sigma+1+j}=\frac{1}{\sigma}+\frac{1}{\sigma+4}+\frac{1}{\sigma+5}+\frac{1}{\sigma+7}$$

$$\frac{1}{\sigma+8}+\frac{\sigma+1+\cancel{j}+\sigma+1-\cancel{j}}{\sigma^2+2\sigma+2}=\frac{1}{\sigma}+\frac{1}{\sigma+4}+\frac{1}{\sigma+5}+\frac{1}{\sigma+7}$$

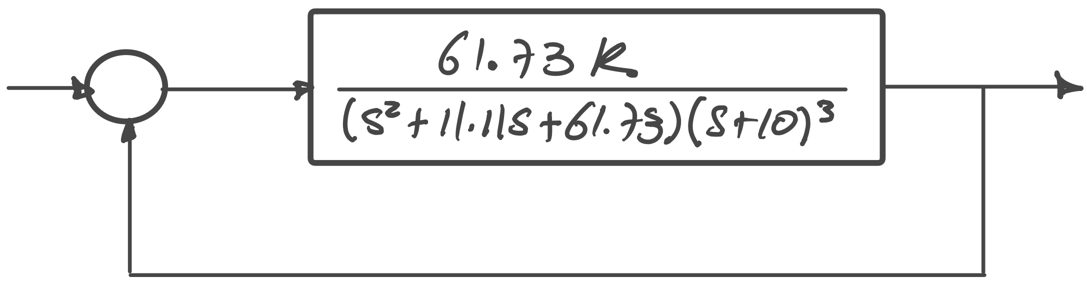
$$\frac{1}{\sigma+8}+\frac{2\sigma+2}{\sigma^2+2\sigma+2}-\frac{1}{\sigma}-\frac{1}{\sigma+4}-\frac{1}{\sigma+5}-\frac{1}{\sigma+7}=0$$

$$\sigma^6+20\sigma^5+131\sigma^4+360\sigma^3+862\sigma^2+2656\sigma+2240=0$$

$$\sigma=\begin{cases}-9.45\\-6.36\\-1.12\end{cases}$$

las demas raices no forman parte del LGR

Ejemplo 4



ⓐ el sistema oscila cuando el LGR se intersecta con el eje $j\omega$:

$$G(s) = \frac{61.73}{(s+10)^3(s^2+11.11s+61.73)}$$

$$= \frac{61.73}{s^5+41.11s^4+695.03s^3+6184.9s^2+29629s+61730}$$

En $j\omega$:

$$G(j\omega) = \frac{61.73}{j\omega^5+41.11\omega^4-j695.03\omega^3-6184.9\omega^2+j29629\omega+61730}$$

$$-\tan^{-1}\left(\frac{\omega^5-695.03\omega^3+29629\omega}{41.11\omega^4-6184.9\omega^2+61730}\right) = -180^\circ$$

$$\omega^5-695.03\omega^3+29629\omega = 0$$

$$\omega = \begin{cases} 0 \\ \pm 25.4834 \\ \pm 6.7546 \rightarrow \omega_1 \end{cases}$$

$$K = \left| \frac{1}{G(j\omega_1)} \right| = 2185$$