(5)

$$A=\begin{bmatrix}-4 & -6\\ 1 & 0\end{bmatrix},\quad B=\begin{bmatrix}1\\ 0\end{bmatrix},\quad C=\begin{bmatrix}3 & 5\end{bmatrix},\quad D=0.$$

$$\frac{Y}{u}=C(sI-A)^{-1}B$$

$$sI-A=\begin{bmatrix}s+4 & +6\\ -1 & s\end{bmatrix}$$

$$(sI-A)^{-1}=\frac{1}{\det(sI-A)}\,\mathrm{adj}(sI-A)$$

$$\det(sI-A)=s(s+4)+6$$

$$\mathrm{adj}(sI-A)=\begin{bmatrix}s & -6\\ 1 & s+4\end{bmatrix}$$

$$\frac{Y}{u}=\frac{1}{s^2+4s+6}\begin{bmatrix}3 & 5\end{bmatrix}\begin{bmatrix}s & -6\\ 1 & s+4\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$\boxed{\frac{Y}{u}=\frac{3s+5}{s^2+4s+6}}$$

$$A=\begin{bmatrix}-4 & -1.5\\ 4 & 0\end{bmatrix},\ B=\begin{bmatrix}2\\ 0\end{bmatrix},\ C=\begin{bmatrix}1.5 & 0.625\end{bmatrix},\ D=0$$

$$sI-A=\begin{bmatrix}s+4 & 1.5\\ -4 & s\end{bmatrix}$$

$$\det(sI-A)=s(s+4)+\underbrace{(4)(1.5)}_{6}$$

$$\operatorname{adj}(sI-A)=\begin{bmatrix}s & -1.5\\ 4 & s+4\end{bmatrix}$$

$$\frac{Y}{U}=\frac{1}{s(s+4)+6}\begin{bmatrix}1.5 & 0.625\end{bmatrix}\begin{bmatrix}s & -1.5\\ 4 & s+4\end{bmatrix}\begin{bmatrix}2\\ 0\end{bmatrix}$$

$$\frac{Y}{U}=\frac{(1.5)(s)(2)+(0.625)(4)(2)}{s^2+4s+6}$$

$$\boxed{\frac{Y}{U}=\frac{3s+5}{s^2+4s+6}}$$

(b) $$G(s) = \frac{Y(s)}{U(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24}$$

$$(s^3 + 9s^2 + 26s + 24)Y(s) = 24U(s)$$

$x_1 = y$ $\xrightarrow{d/dt}$ $\dot{x}_1 = x_2$

$x_2 = \dot{y}$ $\xrightarrow{d/dt}$ $\dot{x}_2 = x_3$

$x_3 = \ddot{y}$ $\xrightarrow{d/dt}$ $\dot{x}_3 = -9x_3 - 26x_2 - 24x_1 + 24u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u$$

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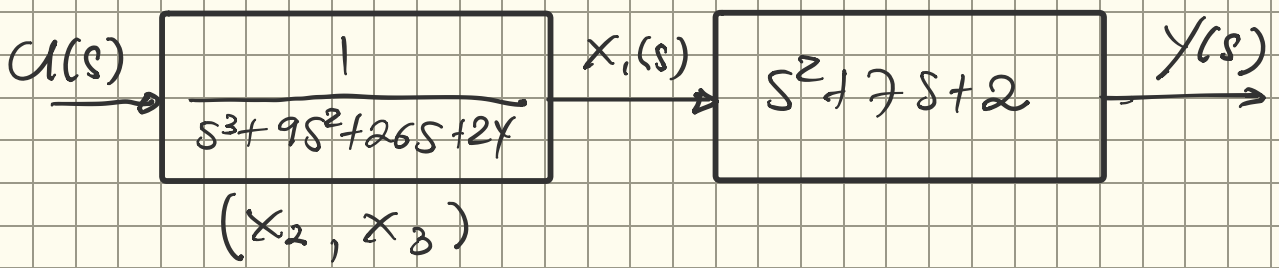
$$G(s) = \frac{s^2+7s+2}{s^3+9s^2+26s+24}$$



$$(s^3+9s^2+26s+24)X_1 = U$$

$$\underbrace{s^3X_1}_{\dot{x}_3} = -24\underbrace{x_1}_{x_1} - 26\underbrace{sx_1}_{x_2} - 9\underbrace{s^2x_1}_{x_3} + u$$

$$x_1 = x_1$$

$$x_2 = \dot{x}_1$$

$$x_3 = \ddot{x}_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$Y(s) = \underbrace{s^2x_1}_{x_3} + 7\underbrace{sx_1}_{x_2} + 2\underbrace{x_1}_{x_1}$$

$$y = \begin{bmatrix} 2 & 7 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u$$

⑧ $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix}$

$$\det(A-\lambda I) = \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ -24 & -26 & -9-\lambda \end{vmatrix}$$

$$= -\lambda\left(\lambda(9+\lambda)+26\right) + (-1)(24)$$

$$= -\lambda(9\lambda+\lambda^2+26) - 24$$

$$= -\lambda^3 - 9\lambda^2 - 26\lambda - 24$$

$$\boxed{\lambda^3 + 9\lambda^2 + 26\lambda + 24 = 0}$$