(1) $z_1 = \sqrt{2} + j\sqrt{2}$, $z_2 = 8e^{j\pi/3}$

$$r_1 = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2$$

$$\theta_1 = \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \pi/4 \quad (45°)$$

$$z_1 = 2e^{j\pi/4}$$

$$z_2 = 8\left(\cos(\pi/3) + j\,\mathrm{sen}(\pi/3)\right)$$

$$= 8\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$z_2 = 4 + j4\sqrt{3}$$

(a)
$$2z_1 - z_2 = 2(\sqrt{2} + j\sqrt{2}) - (4 + j4\sqrt{3})$$

$$= 2\sqrt{2} + j2\sqrt{2} - 4 - j4\sqrt{3}$$

$$= (2\sqrt{2} - 4) + j(2\sqrt{2} - 4\sqrt{3})$$

$$= -1.17 - j\,4.1$$

(b)
$$z_1^{-1} = (2e^{j\pi/4})^{-1} = \frac{1}{2}e^{-j\pi/4}$$

$$= \frac{1}{2}\left(\cos(\pi/4) - j\,\mathrm{sen}(\pi/4)\right)$$

$$= \frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4}$$

(c)
$$\frac{z_1}{z_2^2} = \frac{2e^{j\pi/4}}{(8e^{j\pi/3})^2} = \frac{2e^{j\pi/4}}{64e^{j2\pi/3}} = \frac{1}{32}e^{j(\pi/4 - 2\pi/3)}$$

$$= \frac{1}{32}e^{-j5\pi/12}$$

(d) $\sqrt[3]{z_1 z_2} = \sqrt[3]{(2e^{j\pi/4})(8e^{j\pi/3})}$

$= \sqrt[3]{16\, e^{j(\pi/4+\pi/3)}}$

$= \sqrt[3]{16\, e^{j7\pi/12}}$

$= \sqrt[3]{16}\, e^{j(7\pi/12 + 2\pi k)/3}$ $\quad k = 0, 1, 2$

$z_0 = \sqrt[3]{16}\, e^{j7\pi/36}$, $z_1 = \sqrt[3]{16}\, e^{j31\pi/36}$, $z_2 = \sqrt[3]{10}\, e^{j55\pi/36}$

(2) $X(\omega) = \dfrac{2+j\omega}{3+j4\omega}$

forma rectangular:

$X(\omega) = \dfrac{(2+j\omega)(3-j4\omega)}{(3+j4\omega)(3-j4\omega)}$

$= \dfrac{(6+4\omega^2)+j(-8\omega+3\omega)}{(9+16\omega^2)\cancel{+j(-12\omega+12\omega)}}$

$$X(\omega) = \frac{6-4\omega^2}{9+16\omega^2} - j\frac{5\omega}{9+16\omega^2}$$

forma polar:

$$|X(\omega)| = \frac{|2+j\omega|}{|3+j4\omega|} = \frac{\sqrt{4+\omega^2}}{\sqrt{9+16\omega^2}} = \sqrt{\frac{4+\omega^2}{9+16\omega^2}}$$

$$\measuredangle X(\omega) = \measuredangle(2+j\omega) - \measuredangle(3+j4\omega)$$

$$= \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{4\omega}{3}\right)$$

$$X(\omega) = \sqrt{\frac{4+\omega^2}{9+16\omega^2}}\, e^{j\left(\tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{4\omega}{3}\right)\right)}$$

3) $f(t) = \cos(\omega_0 t) - \sqrt{3}\,\operatorname{sen}(\omega_0 t)$

$$\cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$\sqrt{3}\,\operatorname{sen}(\omega_0 t) = \frac{\sqrt{3}}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)$$

$$f(t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} - \frac{\sqrt{3}}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)$$

$$= \frac{e^{j\omega_0 t}}{2} + \frac{e^{-j\omega_0 t}}{2} + j\frac{\sqrt{3}}{2}e^{j\omega_0 t} - j\frac{\sqrt{3}}{2}e^{-j\omega_0 t}$$

$$= \frac{e^{j\omega_0 t}}{2}\underbrace{\left(1 + j\sqrt{3}\right)}_{z_1} + \frac{e^{-j\omega_0 t}}{2}\underbrace{\left(1 - j\sqrt{3}\right)}_{z_1^*}$$

$$r_1 = \sqrt{1 + (\sqrt{3})^2} = \sqrt{4} = 2$$

$$\theta_1 = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}, \quad \theta_1^* = -\frac{\pi}{3}$$

$$f(t) = \frac{e^{j\omega_0 t}}{2}\left(2e^{j\pi/3}\right) + \frac{e^{-j\omega_0 t}}{2}\left(2e^{-j\pi/3}\right)$$

$$= 2\left(\frac{e^{j(\omega_0 t + \pi/3)} + e^{-j(\omega_0 t + \pi/3)}}{2}\right)$$

$$= 2\cos\left(\omega_0 t + \pi/3\right)$$

$$g(t) = \underbrace{-3\cos\left(\omega_0 t + \frac{\pi}{4}\right)}_{a} + \underbrace{4\sin\left(\omega_0 t - \frac{\pi}{6}\right)}_{b}$$

$$a = -3\left(\frac{e^{j(\omega_0 t + \frac{\pi}{4})} + e^{-j(\omega_0 t + \pi/4)}}{2}\right)$$

$$= -\frac{3}{2} e^{j\frac{\pi}{4}} e^{j\omega_0 t} - \frac{3}{2} e^{-j\pi/4} e^{-j\omega_0 t}$$

$$b = 4\left(\frac{e^{j(\omega_0 t - \pi/6)} - e^{-j(\omega_0 t - \pi/6)}}{2j}\right)$$

$$= -j\frac{4}{2} e^{j\omega_0 t} e^{-j\pi/6} + j\frac{4}{2} e^{-j\omega_0 t} e^{j\pi/6}$$

$$= 2 e^{j\omega_0 t} e^{-j(\pi/6 + \pi/2)} + 2 e^{-j\omega_0 t} e^{j(\pi/6 + \pi/2)}$$

$$= 2 e^{-j2\pi/3} e^{j\omega_0 t} + 2 e^{j2\pi/3} e^{-j\omega_0 t}$$

$$a + b = e^{j\omega_0 t}\left(2e^{-j2\pi/3} - \frac{3}{2} e^{j\pi/4}\right) + e^{-j\omega_0 t}\left(2e^{j2\pi/3} - \frac{3}{2} e^{-j\pi/4}\right)$$

$$\approx e^{j\omega_0 t}\left(3.4707 e^{-j2.2065}\right) + e^{-j\omega_0 t}\left(3.4707 e^{j2.2065}\right)$$

$$\approx 6.9414\left(\frac{e^{j(\omega_0 t - 2.2065)} + e^{-j(\omega_0 t - 2.2065)}}{2}\right)$$

$$g(t) \approx 6.9414\cos(\omega_0 t - 2.2065)$$