① $\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 32y = x$ $\quad y(0) = 1$ $\quad x(t) = 32u(t)$

$y'(0) = 2$

$$\frac{d^2y}{dt^2} \xrightarrow{\mathcal{L}} s^2Y - s\underset{1}{y(0)} - \underset{2}{y'(0)} + 12[sY - \underset{1}{y(0)}] + 32Y =$$

$$[s^2 + 12s + 32]Y - s - 14 = X$$

$$Y = \underbrace{\frac{X}{s^2+12s+32}}_{Y_{ZE}} + \underbrace{\frac{s+14}{s^2+12s+32}}_{Y_{ZI}} \rightarrow \text{Polos} \begin{cases} -4 \\ -8 \end{cases}$$

fracciones

$$Y_{ZE} = \frac{32}{s(s+8)(s+4)} = \frac{A}{s} + \frac{B}{s+8} + \frac{C}{s+4}$$

$$A = \left.\frac{32\,s}{s(s+8)(s+4)}\right|_{s=0} = 1$$

$$B = \left.\frac{32\,(s+8)}{s(s+8)(s+4)}\right|_{s=-8} = 1$$

$$C = \left.\frac{32\,(s+4)}{s(s+8)(s+4)}\right|_{s=-4} = -2$$

$$Y_{zr} = \frac{1}{s} + \frac{1}{s+8} - \frac{2}{s+4}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$\boxed{y_{zr}(t) = \left[1 + e^{-8t} - 2e^{-4t}\right]u(t)}$$

$$Y_{zi} = \frac{s+14}{(s+4)(s+8)} = \frac{D}{s+4} + \frac{E}{s+8}$$

$$D = \frac{(s+14)(s+4)}{(s+4)(s+8)}\Bigg|_{s=-4} = 2.5$$

$$E = \frac{(s+14)(s+8)}{(s+4)(s+8)}\Bigg|_{s=-8} = -1.5$$

$$Y_{zi} = \frac{2.5}{s+4} - \frac{1.5}{s+8}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$\boxed{y_{zi}(t) = \left[2.5e^{-4t} - 1.5e^{-8t}\right]u(t)}$$

$$y(t) = y_{zr}(t) + y_{zi}(t)$$

$$\boxed{y(t) = \left[1 + 0.5e^{-t} - 0.5e^{8t}\right]u(t)} \quad //$$

Residuos

$$y(t) = \sum \text{Res}\{Y(s)e^{st}\}\, u(t)$$

$$= \sum \text{Res}\left\{\frac{32e^{st}}{s(s+8)(s+4)} + \frac{(s+14)e^{st}}{(s+8)(s+4)}\right\} u(t)$$

$$= \sum \text{Res}\left\{\frac{s^2+14s+32}{s(s+8)(s+4)}e^{st}\right\} u(t)$$

$$a_{-1} = \lim_{s\to 0} sY(s)e^{st} = 1$$

$$b_{-1} = \lim_{s\to -8} (s+8)Y(s)e^{st} = -0.5e^{-8t}$$

$$c_{-1} = \lim_{s\to -4} (s+4)Y(s)e^{st} = 0.5e^{-4t}$$

$$\boxed{y(t) = [1 + 0.5e^{-4t} - 0.5e^{-8t}]\, u(t)} \quad //$$

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② $$F(s) = \frac{10}{s(s+2)(s+3)^2}$$

$$F(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3} + \frac{D}{(s+3)^2}$$

$$A = \left.\frac{10s}{s(s+2)(s+3)^2}\right|_{s=0} = \frac{10}{18} = \frac{5}{9}$$

$$B = \left.\frac{10(s+2)}{s(s+2)(s+3)^2}\right|_{s=-2} = \frac{10}{-2} = -5$$

$$D = \left.\frac{10(s+3)^2}{s(s+2)(s+3)^2}\right|_{s=-3} = \frac{10}{3}$$

Para C:

$$\left.\frac{10}{s(s+2)(s+3)^2}\right|_{s=1} = \left.\frac{5}{9s} - \frac{5}{s+2} + \frac{10}{3(s+3)^2} + \frac{C}{s+3}\right|_{s=1}$$

$$\frac{10}{48} = \frac{5}{9} - \frac{5}{3} + \frac{10}{48} + \frac{C}{4}$$

$$C = 4\left(\frac{10}{48} - \frac{5}{9} + \frac{5}{3} - \frac{10}{48}\right)$$

$$C = \frac{40}{9}$$

$$F(s) = \frac{5}{9s} - \frac{5}{s+2} + \frac{10}{3(s+3)^2} + \frac{40}{9(s+3)}$$

$$\mathcal{L}^{-1} \downarrow$$

$$\boxed{f(t) = \left[\frac{5}{9} - 5e^{-2t} + \frac{10}{3}te^{-3t} + \frac{40}{9}e^{-3t}\right]u(t)}$$

//

③ $$G(s) = \frac{s^3}{s^3+18s^2+104s+192}$$

$$G(s) = \frac{s^3+18s^2+104s+192-18s^2-104s-192}{s^3+18s^2+104s+192}$$

$$G(s) = 1 - \underbrace{\frac{18s^2+104s+192}{s^3+18s^2+104s+192}}_{G_1(s)}$$

$$G_1(s) = \frac{18s^2+104s+192}{s^3+18s^2+104s+192}$$

$$= \frac{18s^2+104s+192}{(s+4)(s+6)(s+8)}$$

$$= \frac{A}{s+4} + \frac{B}{s+6} + \frac{C}{s+8}.$$

$$A = \left.\frac{(18s^2+104s+192)(s+4)}{(s+4)(s+6)(s+8)}\right|_{s=-4} = 8$$

$$B = \left.\frac{(18s^2+40s^2+192)(s+6)}{(s+4)(s+6)(s+8)}\right|_{s=-6} = -54$$

$$C = \left.\frac{(18s^2+40s^2+192)(s+8)}{(s+4)(s+6)(s+8)}\right|_{s=-8} = 64$$

$$G_1(s) = \frac{8}{s+4} - \frac{54}{s+6} + \frac{64}{s+8}$$

$$G(s) = 1 - \frac{8}{s+4} + \frac{54}{s+6} - \frac{64}{s+8}$$

$$\mathcal{L}^{-1} \downarrow$$

$$\boxed{g(t) = \delta(t) + \left[54e^{-6t} - 8e^{-4t} - 64e^{-8t}\right]u(t)}$$

A

$$G(s) = \frac{s}{(s+4)(s+8)} = \frac{C(s)}{R(s)}$$

Si $R(s) = \frac{1}{s^2}$:

$$C(s) = \frac{s}{s^2(s+4)(s+8)} = \frac{1}{s(s+4)(s+8)}$$

$$C(s) = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+8}$$

$$A = \frac{s}{s(s+4)(s+8)}\Big|_{s=0} = \frac{1}{32}$$

$$B = \frac{s+4}{s(s+4)(s+8)}\Big|_{s=-4} = \frac{-1}{16}$$

$$C = \frac{s+8}{s(s+4)(s+8)}\Big|_{s=-8} = \frac{1}{32}$$

$$C(s) = \frac{1}{32s} - \frac{1}{16(s+4)} + \frac{1}{32(s+8)}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$\boxed{C_r(t) = \left[\frac{1}{32} - \frac{1}{16}e^{-4t} + \frac{1}{32}e^{-8t}\right]u(t)}$$

Para la respuesta al escalón, se deriva $C(t)$ ante la rampa.

$$C_u(t) = \frac{d}{dt} C_r(t)$$

$$\boxed{C_u(t) = \left[\frac{1}{4}e^{-4t} - \frac{1}{4}e^{-8t}\right]u(t)}$$

//