$$\ddot{y} + 3\dot{y} + 2y = 2\dot{x} + x \quad , x(t) = 4e^{-3t}, \; y(0) = 2, \; \dot{y}(0) = 4$$

$$\ddot{y} \xrightarrow{\mathcal{L}} s^2 Y(s) - s y(0) - \dot{y}(0) = s^2 Y(s) - 2s - 4$$

$$\dot{y} \xrightarrow{\mathcal{L}} s Y(s) - y(0) = sY(s) - 2$$

$$y \xrightarrow{\mathcal{L}} Y(s)$$

$$\dot{x} \xrightarrow{\mathcal{L}} sX(s)$$

$$x \xrightarrow{\mathcal{L}} X(s)$$

$$\mathcal{L} \quad s^2 Y(s) - 2s - 4 + 3(sY(s) - 2) + 2Y(s) = 2sX(s) + X(s)$$

$$(s^2 + 3s + 2)Y(s) - (2s + 4 + 6) = (2s+1)X(s)$$

$$(s^2 + 3s + 2)Y(s) = (2s+1)X(s) + (2s+4+6)$$

$$Y(s) = \frac{2s+1}{s^2+3s+2} X(s) + \frac{2s+10}{s^2+3s+2}$$

$$Y(s) = \underbrace{\frac{2s+1}{(s+1)(s+2)} X(s)}_{Y_{zs}(s)} + \underbrace{\frac{2s+10}{(s+1)(s+2)}}_{Y_{zi}(s)}$$

Método de residuos

Estado cero:

como $X(s) = \dfrac{4}{s+3}$

$$Y_{zs}(s) = \frac{4(2s+1)}{(s+1)(s+2)(s+3)}$$

Como hay 3 singularidades $(-1, -2, -3)$, hay 3 residuos:

Para el polo en $s=-1$:

$$F(s) = \frac{4(2s+1)}{(s+1)(s+2)(s+3)}$$ (la expresión en t en $\mathcal{L}\{G(s)\}$)

$$\operatorname{Res}(F(s)e^{st}, -1) = a_{-1} = \lim_{s\to -1} \frac{4(2s+1)e^{st}}{(s+2)(s+3)}$$

$$= \frac{4(2(-1)+1)e^{-t}}{(-1+2)(-1+3)}$$

$$\boxed{a_{-1} = -2e^{-t}}$$

$$\operatorname{Res}(F(s)e^{st}, -2) = b_{-1} = \lim_{s\to -2} \frac{4(2s+1)e^{st}}{(s+1)(s+3)}$$

$$\boxed{b_{-1} = 12e^{-2t}}$$

$$\operatorname{Res}(F(s)e^{st}, -3) = c_{-1} = \lim_{s\to -3} \frac{4(2s+1)e^{st}}{(s+1)(s+2)}$$

$$\boxed{c_{-1} = -10e^{-3t}}$$

$$y_{zs}(t) = \sum \operatorname{Res}\{F(s)e^{st}\}$$

$$= a_{-1} + b_{-1} + c_{-1}$$

$$\boxed{y_{zs}(t) = -2e^{-t} + 12e^{-2t} - 10e^{-3t}}$$

Entrada Cero

$$y_{zi}(s) = \frac{2s+10}{(s+1)(s+2)}$$

$$\operatorname{Res}(F(s)e^{st}, -1) = a_{-1} = \lim_{s\to -1} \frac{2s+10}{s+2} e^{st}$$

$$\boxed{a_{-1} = 8e^{-t}}$$

$$\operatorname{Res}(F(s)e^{st}, -2) = b_{-1} = \lim_{s\to -2} \frac{2s+10}{s+2} e^{st}$$

$$\boxed{b_{-1} = -6e^{-2t}}$$

$$y_{zi}(t) = \sum \text{Res}\{F(s)e^{st}\}$$

$$= a_{-1} + b_{-1}$$

$$\boxed{y_{zi}(t) = 8e^{-t} - 6e^{-2t}}$$

Para la respuesta ante el impulso, se considera que el sistema está en reposo ($CI = 0$, y_{zs}) y $X(s) = 1$ (más detalle sobre esto la prox. clase)

$$H(s) = \frac{2s+1}{(s+2)(s+1)}$$

$$a_{-1} = \lim_{s \to -1} \frac{2s+1}{s+2} e^{st} = -e^{-t}$$

$$b_{-1} = \lim_{s \to -2} \frac{2s+1}{s+1} e^{st} = 3e^{-2t}$$

$$\boxed{h(t) = a_{-1} + b_{-1} = -e^{-t} + 3e^{-2t}}$$

Método de Fracciones Parciales

Estado cero:

$$Y_{zs}(s) = \frac{4(2s+1)}{(s+1)(s+2)(s+3)}$$

$$= \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

Para hallar A, B y C hay dos métodos:

I) Independencia lineal de coeficientes

II) Encubrimiento de Heaviside

Por método I

$$4(2s+1) = A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)$$

$$8s+4 = A(s^2+5s+6) + B(s^2+4s+3) + C(s^2+3s+2)$$

$$8s+4 = s^2(A+B+C) + s(5A+4B+3C) + (6A+3B+2C)$$

$$\left.\begin{aligned} A+B+C &= 0 \\ 5A+4B+3C &= 8 \\ 6A+3B+2C &= 4 \end{aligned}\right\} \quad \begin{aligned} A &= -2 \\ B &= 12 \\ C &= -10 \end{aligned}$$

Por método II

$$A = \lim_{s \to -1} \frac{4(2s+1)\cancel{(s+1)}}{\cancel{(s+1)}(s+2)(s+3)}$$ (por esto se llama encubrimiento)

$$A = -2$$

$$B = \lim_{s \to -2} \frac{4(2s+1)\cancel{(s+2)}}{(s+1)\cancel{(s+2)}(s+3)}$$

$$B = 12$$

$$C = \lim_{s \to -3} \frac{4(2s+1)\cancel{(s+3)}}{(s+1)(s+2)\cancel{(s+3)}}$$

$$C = -10$$

$$Y_{zs}(s) = \frac{-2}{s+1} + \frac{12}{s+2} - \frac{10}{s+3}$$

Cada fracción se busca en una tabla:

$$\frac{K}{s+a} \xrightarrow{\mathcal{L}^{-1}} Ke^{-at}$$

$$\boxed{y_{zs}(t) = -2e^{-t} + 12e^{-2t} - 10e^{-3t}}$$

Entrada Cero:

$$Y_{zi}(s) = \frac{2s+10}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \lim_{s \to -1} \frac{2s+10}{s+2} = 8$$

$$B = \lim_{s \to -2} \frac{2s+10}{s+1} = -6$$

$$Y_{zi}(s) = \frac{8}{s+1} - \frac{6}{s+2}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$\boxed{y_{zi}(t) = 8e^{-t} - 6e^{-2t}}$$

Respuesta al Impulso

$$H(s) = \frac{2s+1}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$A = \lim_{s\to -2} \frac{2s+1}{s+1} = 3$$

$$B = \lim_{s\to -1} \frac{2s+1}{s+2} = -1$$

$$H(s) = \frac{3}{s+2} - \frac{1}{s+1}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$\boxed{h(t) = 3e^{-2t} - e^{-t}}$$