1

$$2\ddot{z} + 0.8z - 0.4w + 0.2\dot{z}w = 0$$

$$4\dot{w} + 3w + 0.1w^3 - 6z = 8V$$

$x_1 = z$, $x_2 = \dot{z}$, $x_3 = w$, $u = V$

$\frac{d}{dt}$

→ 3 estados, 1 entrada

$$\dot{x}_1 = \dot{z} = \underbrace{x_2}_{f_1}$$

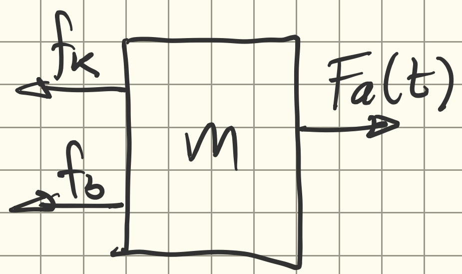
$$\dot{x}_2 = \ddot{z} = \frac{1}{2}\left(-0.8\underset{x_1}{z} + 0.4\underset{x_3}{w} - 0.2\underset{x_2}{\dot{z}}\underset{x_3}{w}\right) = \underbrace{-0.4x_1 + 0.2x_3 - 0.1x_2x_3}_{f_2}$$

$$\dot{x}_3 = \dot{w} = \frac{1}{4}\left(8\underset{u}{V} - 3\underset{x_3}{w} - 0.1\underset{x_3^3}{w^3} + 6\underset{x_1}{z}\right) = \underbrace{2u - 0.75x_3 - 0.025x_3^3 + 1.5x_1}_{f_3}$$

$$\boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.4x_1 + 0.2x_3 - 0.1x_2x_3 \\ \dot{x}_3 &= 1.5x_1 - 0.75x_3 - 0.025x_3^3 + 2u \end{aligned}}$$

//

②



$$Fa(t) - f_k - f_b = ma$$

$$Fa(t) - (k_1 z + k_3 z^3) - b\dot{z} = m\ddot{z}$$

$$m\ddot{z} + b\dot{z} + k_1 z + k_3 z^3 = Fa(t)$$

$x_1 = z, \; x_2 = \dot{z}, \; u = Fa$ → 2 estados, 1 entrada

$\frac{d}{dt} \downarrow$

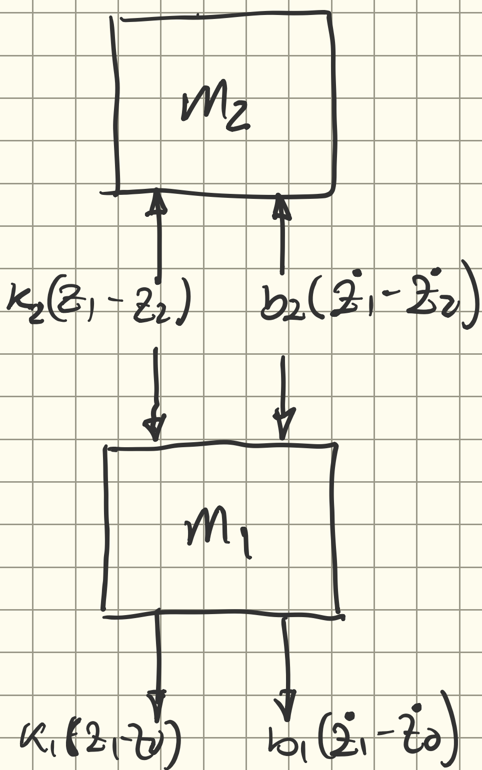
$$\dot{x}_1 = \dot{z} = x_2$$

$$\dot{x}_2 = \ddot{z} = \frac{1}{m}\Big(\underbrace{Fa(t)}_{u} - b\underbrace{\dot{z}}_{x_2} - k_1\underbrace{z}_{x_1} - k_3\underbrace{z^3}_{x_1^3}\Big)$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{k_1}{m}x_1 - \frac{k_3}{m}x_1^3 - \frac{b}{m}x_2 + \frac{1}{m}u$$

③



$$\sum f_{z_2} - \sum f_{z_1} = 0$$

$$m_2 \ddot{z}_2 + b_2 \dot{z}_2 + k_2 z_2 - b_2 \dot{z}_1 - k_2 z_1 = 0$$

$$\sum f_{z_1} - \sum f_{z_2} = \sum f_{z_0}$$

$$m_1 \ddot{z}_1 + (b_1 + b_2)\dot{z}_1 + (k_1 + k_2) z_1 - b_2 \dot{z}_2 - k_2 z_2 = b_1 \dot{z}_0 + k_1 z_0$$

$$x_1 = z_1, \quad x_2 = \dot{z}_1, \quad x_3 = z_2, \quad x_4 = \dot{z}_2, \quad u_1 = z_0, \quad u_2 = \dot{z}_0$$

$$y_1 = z_1, \quad y_2 = z_2$$

$\frac{d}{dt} \downarrow$

$$\dot{x}_1 = \dot{z}_1 = x_2$$

$$\dot{x}_2 = \ddot{z}_1 = \frac{1}{m_1}\left(-(b_1+b_2)\underset{x_2}{\underset{\downarrow}{\dot{z}_1}} - (k_1+k_2)\underset{x_1}{\underset{\downarrow}{z_1}} + b_2 \underset{x_4}{\underset{\downarrow}{\dot{z}_2}} + k_2 \underset{x_3}{\underset{\downarrow}{z_2}} + b_1 \underset{u_2}{\underset{\downarrow}{\dot{z}_0}} + k_1 \underset{u_1}{\underset{\downarrow}{z_0}}\right)$$

$$\dot{x}_3 = \dot{z}_2 = x_4$$

$$\dot{x}_4 = \ddot{z}_2 = \frac{1}{m_2}\left(-b_2 \underset{x_4}{\underset{\downarrow}{\dot{z}_2}} - k_2 \underset{x_3}{\underset{\downarrow}{z_2}} + b_2 \underset{x_2}{\underset{\downarrow}{\dot{z}_1}} + k_2 \underset{x_1}{\underset{\downarrow}{z_1}}\right)$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{k_1+k_2}{m_1}x_1 - \frac{b_1+b_2}{m_1}x_2 + \frac{k_2}{m_1}x_3 + \frac{b_2}{m_1}x_4 + \frac{k_1 u_1}{m_1} + \frac{b_1 u_2}{m_1}$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{k_2}{m_2}x_1 + \frac{b_2}{m_2}x_2 - \frac{k_2}{m_2}x_3 - \frac{b_2}{m_2}x_4$$

$$y_1 = x_1$$

$$y_2 = x_3$$

SSR:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & -\frac{b_1+b_2}{m_1} & \frac{k_2}{m_1} & \frac{b_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{b_2}{m_2} & -\frac{k_2}{m_2} & -\frac{b_2}{m_1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{k_1}{m_1} & \frac{b_1}{m_1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

(4) Como el sistema no es lineal, se debe linealizar, como son 3 ecuaciones diferenciales, usamos linealización Jacobiana.

$$\frac{dS}{dt} = \mu N - \beta \frac{SI}{N} - \mu S = f_1(S,I,R)$$

$$\frac{dI}{dt} = \beta \frac{SI}{N} - (\gamma+\mu) I = f_2(S,I,R)$$

$$\frac{dR}{dt} = \gamma I - \mu R = f_3(S,I,R)$$

definimos la matriz Jacobiana como:

$$J_F = \begin{bmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial I} & \frac{\partial f_3}{\partial R} \end{bmatrix}$$

$$\frac{\partial f_1}{\partial S} = -\frac{\beta I}{N} - \mu, \quad \frac{\partial f_1}{\partial I} = -\frac{\beta S}{N}, \quad \frac{\partial f_1}{\partial R} = 0$$

$$\frac{\partial f_2}{\partial S} = \frac{\beta I}{N}, \quad \frac{\partial f_2}{\partial I} = \frac{\beta S}{N} - (\gamma+\mu), \quad \frac{\partial f_2}{\partial R} = 0$$

$$\frac{\partial f_3}{\partial S} = 0, \quad \frac{\partial f_3}{\partial I} = \gamma, \quad \frac{\partial f_3}{\partial R} = -\mu$$

Si suponemos un punto de equilibrio $(\bar{S},\bar{I},\bar{R})$

$$\frac{d}{dt}\begin{bmatrix}\delta S\\ \delta I\\ \delta R\end{bmatrix} = Jf\Big|_{(\bar{S},\bar{I},\bar{R})} = \begin{bmatrix} -\frac{\beta\bar{I}}{N}-\mu & -\frac{\beta\bar{S}}{N} & 0\\ \frac{\beta\bar{I}}{N} & \frac{\beta\bar{S}}{N}-\gamma-\mu & 0\\ 0 & \gamma & -\mu\end{bmatrix}\begin{bmatrix}\delta S\\ \delta I\\ \delta R\end{bmatrix}$$

Es un sistema autónomo (sin entradas)