$$\ddot{y} + 3\dot{y} + 2y = 2\dot{x} + x \quad , \; x(t) = 4e^{-3t}, \; y(0) = 2, \; \dot{y}(0) = 4$$

Entrada cero:

$$\ddot{y} + 3\dot{y} + 2y = 0$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$(\lambda + 2)(\lambda + 1) = 0$$

$$y_{zi}(t) = c_1 e^{-2t} + c_2 e^{-t}$$

$$\dot{y}_{zi}(t) = -2c_1 e^{-2t} - c_2 e^{-t}$$

en $t = 0$

$$\left.\begin{aligned} c_1 + c_2 &= 2 \\ -2c_1 - c_2 &= 4 \end{aligned}\right\} \quad \begin{aligned} c_1 &= -6 \\ c_2 &= 8 \end{aligned}$$

$$\boxed{y_{zi}(t) = -6e^{-2t} + 8e^{-t}}$$

Estado cero:

$$\ddot{y} + 3\dot{y} + 2y = x$$

Dado que $x(t) = 4e^{-3t}$, se supone $y_p = Ke^{-3t}$

$$\dot{y}_f = -3Ke^{-3t}$$

$$\ddot{y}_f = 9Ke^{-3t}$$

$$\cancel{9Ke^{-3t}} \cancel{+3(-3)Ke^{-3t}} + 2K\cancel{e^{-3t}} = 4\cancel{e^{-3t}}$$

$$2K = 4$$

$$\boxed{K = 2}$$

$$y_o(t) = C_1 e^{-2t} + C_2 e^{-t} + 2e^{-3t}$$

$$\dot{y}_o(t) = -2C_1 e^{-2t} - C_2 e^{-t} - 6e^{-3t}$$

en $t=0$, $y_o(t) = \dot{y}_o(t) = 0$

$$\left.\begin{aligned} C_1 + C_2 &= -2 \\ -2C_1 - C_2 &= 6 \end{aligned}\right\} \quad \begin{aligned} C_1 &= -4 \\ C_2 &= 2 \end{aligned}$$

$$y_o(t) = -4e^{-2t} + 2e^{-t} + 2e^{-3t}$$

$$\dot{y}_o(t) = 8e^{-2t} - 2e^{-t} - 6e^{-3t}$$

$$\boxed{y_{zs}(t) = 2\dot{y}_o(t) + y_o(t) = 12e^{-2t} - 2e^{-t} - 10e^{-3t}}$$

respuesta al impulso

$$\ddot{y} + 3\dot{y} + 2y = 0 \qquad y(0) = 0,\ \dot{y}(0) = 1$$

$$h_0(t) = c_1 e^{-t} + c_2 e^{-2t}$$

$$\dot{h}_0(t) = -c_1 e^{-t} - 2c_2 e^{-2t}$$

en $t = 0$

$$\left.\begin{aligned} c_1 + c_2 &= 0 \\ -c_1 - 2c_2 &= 1 \end{aligned}\right\rangle \quad \begin{aligned} c_1 &= 1 \\ c_2 &= -1 \end{aligned}$$

$$h_0(t) = e^{-t} - e^{-2t}$$

$$\dot{h}_0(t) = -e^{-t} + 2e^{-2t}$$

$$h(t) = 2\dot{h}_0(t) + h_0(t)$$

$$\boxed{h(t) = -e^{-t} + 3e^{-2t}}$$