Ejemplo 1

$D_1(s) = s^5 + 2s^4 + 4s^3 + 6s^2 + 8s + 12$

s^5	1	4	8
s^4	2	6	12
s^3	1	2	0
s^2	2	12	0
s^1	-4	0	0
s^0	12	0	0

- No estable
- 2 Cambios de signo → 2 polos RHP

//

$D_2(s) = s^3 + 14s^2 + 55s + 42$

s^3	1	55
s^2	14	42
s^1	52	0
s^0	42	0

Estable

$$D_3(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$$

$$\begin{array}{c|ccc}
s^5 & 1 & 3 & 5 \\
s^4 & 2 & 6 & 3 \\
s^3 & \boxed{0} & 7/2 & 0 \\
s^2 & & & \\
s^1 & & & \\
s^0 & & &
\end{array}$$

Caso especial I

a) ε

b) reciproco

a)

$$\begin{array}{c|ccc}
s^5 & 1 & 3 & 5 \\
s^4 & 2 & 6 & 3 \\
s^3 & \varepsilon & 7/2 & 0 \\
s^2 & \frac{6\varepsilon - 7}{\varepsilon} & 3 & 0 \\
s^1 & a & 0 & 0 \\
s^0 & 3 & 0 & 0
\end{array}$$

$$a = \frac{\left(\frac{6\varepsilon-7}{\varepsilon}\right)\left(\frac{7}{2}\right) - 3\varepsilon}{\frac{6\varepsilon-7}{\varepsilon}}$$

$$= \frac{42\varepsilon - 49 - 6\varepsilon^2}{12\varepsilon - 14}$$

$$\lim_{\varepsilon \to 0^-} \varepsilon \Rightarrow - \; \Big| \; \lim_{\varepsilon \to 0^+} (+)$$

$$\lim_{\varepsilon \to 0^-} \frac{6\varepsilon - 7}{\varepsilon} + \; \Big| \; \lim_{\varepsilon \to 0^+} (-)$$

$$\lim_{\varepsilon \to 0} a = \frac{-49}{-14} \Rightarrow +$$

Inestable, 2 polos RHP

b)

s^5	3	6	2
s^4	5	3	1
s^3	21/5	7/5	0
s^2	4/3	1	0
s^1	-7/4	0	0
s^0	1	0	0

⇒ Inestable
2 polos RHP

//

$D_4(s) = s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56$

s^5	1	6	8
s^4	7	42	56
s^3	0	0	0
s^2			
s^1			
s^0			

→ Caso especial II

-polinomio auxiliar

$7s^4 + 42s^2 + 56$

↓ d/ds

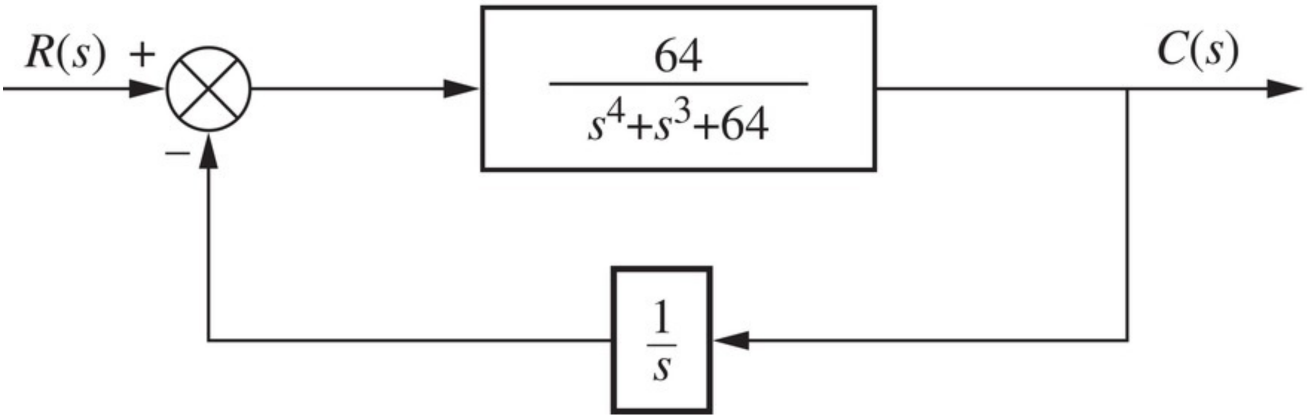
$28s^3 + 84s$

s^5	1	6	8
s^4	7	42	56
s^3	28	84	0
s^2	21	56	0
s^1	28/3	0	0
s^0	56	0	0

→ Marginalmente estable

(no hay cambios de signo de s^4 a s^0)

Ejemplo 2



$$T(s) = \frac{\frac{64}{s^4+s^3+64}}{1+\frac{1}{s}\left(\frac{64}{s^4+s^3+64}\right)} = \frac{64s}{s^5+s^4+64s+64}$$

$$\begin{array}{c|ccc}
s^5 & 1 & 0 & 64 \\
s^4 & 1 & 0 & 64 \\
s^3 & 4 & 0 & 0 \\
s^2 & \varepsilon & 64 & 0 \\
s^1 & -\frac{256}{\varepsilon} & 0 & 0 \\
s^0 & 64 & 0 & 0
\end{array}$$

s^4 row $\rightarrow$ $s^4+64 \Rightarrow d/ds \Rightarrow 4s^3$

s^2 row $\rightarrow$

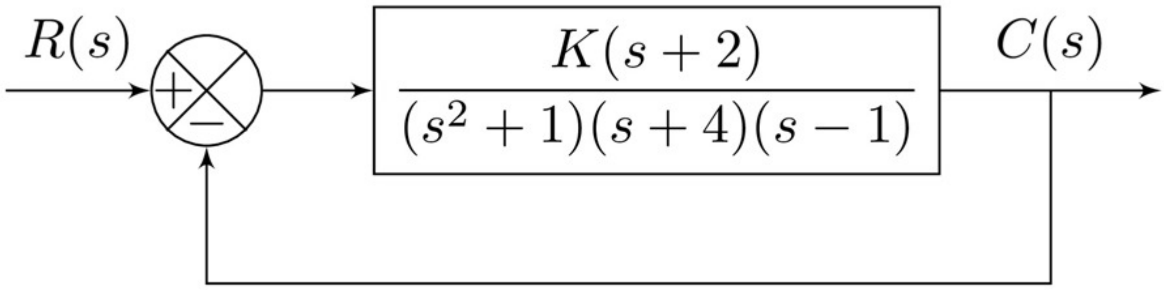
(+)	(+)
$\lim_{\varepsilon\to0^-} \varepsilon$ (−)	$\lim_{\varepsilon\to0^+} \varepsilon$ (−)
$\lim_{\varepsilon\to0^-} -\frac{256}{\varepsilon}$ (+)	$\lim_{\varepsilon\to0^+} -\frac{256}{\varepsilon}$ (−)
(+)	(+)

$\Rightarrow$ 2 polos RHP

3 polos LHP

0 polos $j\omega$

Ejemplo 3



$$T(s) \; \frac{K(s+2)}{(s^2+1)(s+4)(s-1)+K(s+2)} = \frac{K(s+2)}{s^4+3s^3-3s^2+(k+3)s+2k-4}$$

s^4	1	-3	$2k-4$
s^3	3	$k+3$	0
s^2	$\boxed{-\frac{k}{3}-4}$	$2k-4$	0
s^1	$\boxed{\frac{k(k+33)}{k+12}}$	0	0
s^0	$\boxed{2k-4}$	0	0

● $-\frac{k}{3} - 4 > 0$

$-\frac{k}{3} > 4$

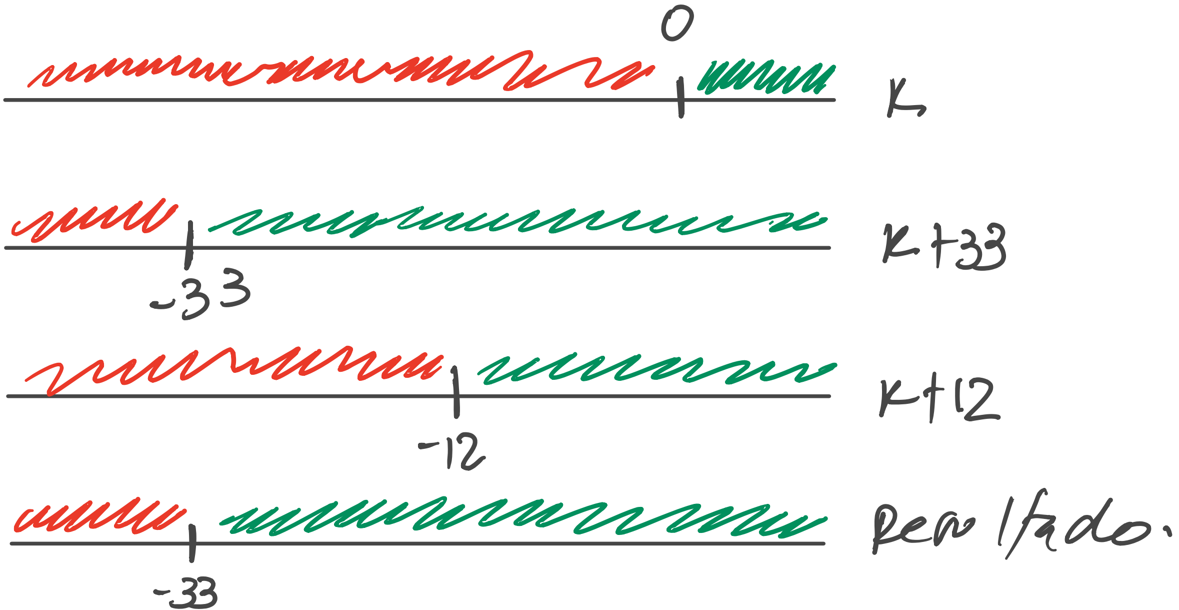
$\boxed{K < -12}$

● $2k - 4 > 0$

$2k > 4$

$\boxed{k > 2}$

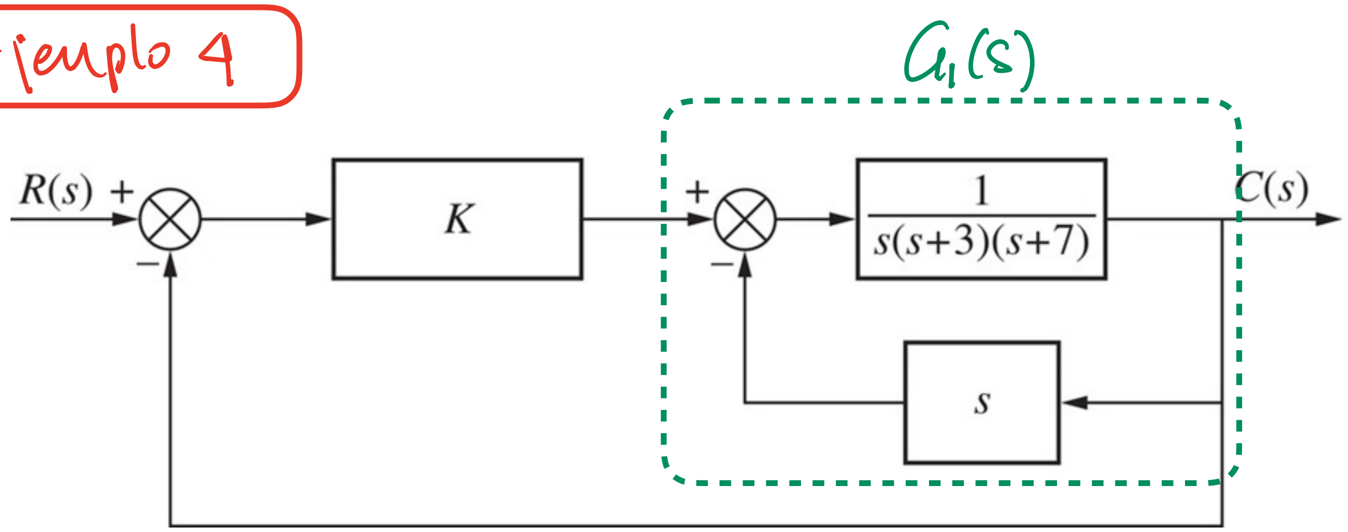
● $\frac{k(k+33)}{k+12} > 0$



$\boxed{K > -33}$

$\Rightarrow$ El sistema no es estable para ningún valor de K

Ejemplo 4



$$G_1(s) = \frac{\frac{1}{s(s+3)(s+7)}}{1 + (s)\left(\frac{1}{s(s+3)(s+7)}\right)}$$

$$= \frac{1}{s\left[(s+3)(s+7) + 1\right]}$$

$$= \frac{1}{s^3 + 10s^2 + 22s}$$

$$T(s) = \frac{\frac{K}{s^3+10s^2+22s}}{1 + \frac{K}{s^3+10s^2+22s}}$$

$$= \frac{K}{s^3+10s^2+22s+K}$$

s^3	1	22
s^2	10	K
s^1	$\frac{220-K}{10}$	0
s^0	K	0

ⓐ para estabilidad:

$K>0$, $220-K>0 \Rightarrow K<220$.

$$\boxed{0<K<220}$$

ⓑ Para oscilación

$$\boxed{K=220}$$

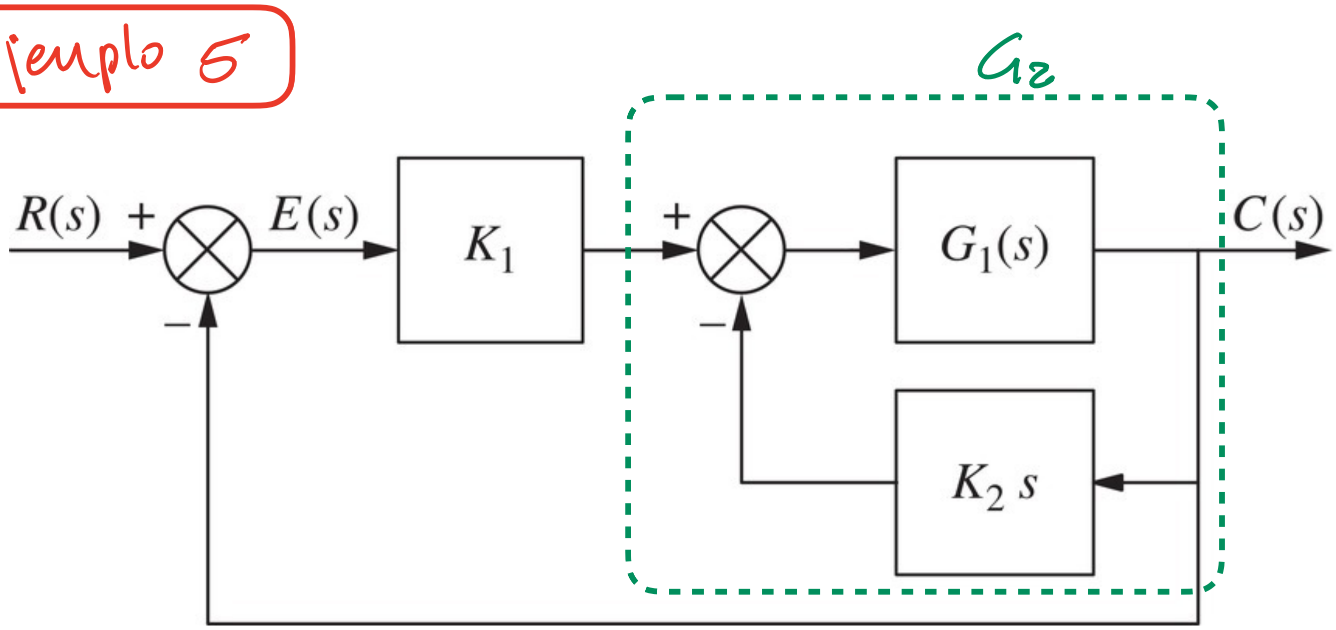
Con este valor

s^2: $10s^2+220=0$

$$s^2+22=0$$

$$\boxed{s=\pm j\sqrt{22} \rightarrow \omega_0}$$

Ejemplo 5



$$G_1(s) = \frac{1}{s(s+2)(s+4)} = \frac{1}{s^3+6s^2+8s}$$

ⓐ

$$G_2(s) = \frac{\frac{1}{s^3+6\cdot s^2+8s}}{1+(K_2 s)\left(\frac{1}{s^3+6\cdot s^2+8s}\right)}$$

$$= \frac{1}{s^3+6s^2+(8+K_2)s}$$

$$= \frac{1}{s(\underbrace{s^2+6s^2+8+K_2})}$$

para que sea trinomio cuadrado perfect
se requiere

$K_2 = 1$

$$T(s) = \frac{\dfrac{K_1}{s^3+6s^2+9s}}{1+\dfrac{K_1}{s^3+6s^2+9s}}$$

$$= \frac{K_1}{s^3+6s^2+9s+K_1}$$

$$\begin{array}{c|cc}
s^3 & 1 & 9 \\
s^2 & 6 & K_1 \\
s^1 & \frac{54-K_1}{6} & 0 \\
s^0 & K_1 & 0
\end{array}$$

$54-K_1>0$

$K_1<54$

$K_1>0$

$$\boxed{0<K_1<54}$$

ⓑ Para que haya un polo de lazo cerrado en $s=-5$

$$(s+5)(s^2+bs+c) = s^3+6s^2+9s+K_1$$

$$s^3+(b+5)s^2+(c+5b)s+5c = s^3+6s^2+9s+K_1$$

$b+5=6 \longrightarrow b=1$

$c+5b=9 \longrightarrow c=9-5(1)=4$

$5c=K_1 \longrightarrow \boxed{K_1=20}$

$$(s+5)(\underbrace{s^2+s+4}_{\text{Parte real: } -0.5})=0$$

$$\downarrow$$

$-5 \longrightarrow$ 10 veces ✓ OK

Sí se puede hacer la aproximación

$$\hat{T}(s) = \frac{4}{s^2+8+4}$$