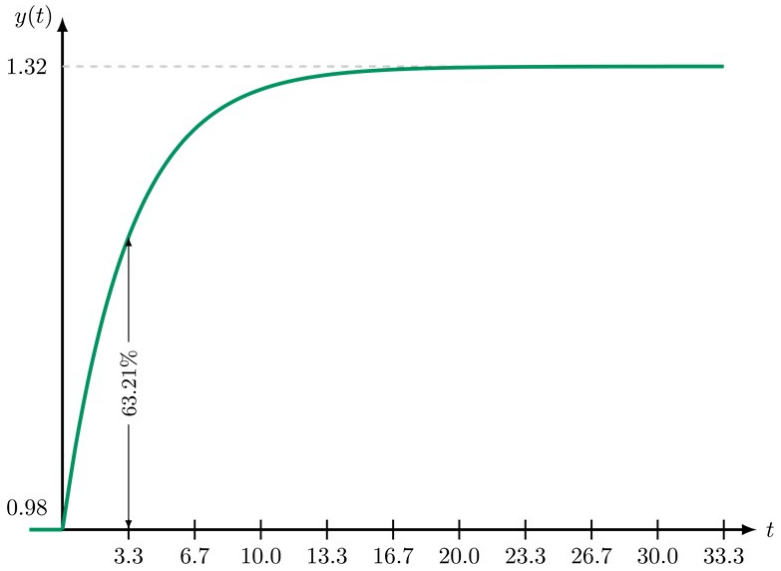
Ejemplo 1



$$\Delta y = 1.32 - 0.98$$

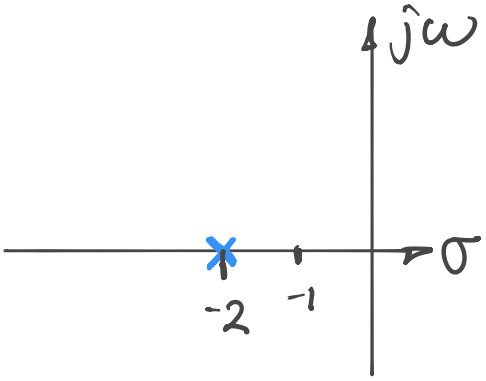
$$\Delta y = 0.34 = K$$

$$\frac{1}{a} = 3.3 \Rightarrow a = \frac{1}{3.3} = 0.3$$

$$H(s) = \frac{(0.34)(0.3)}{s+0.3} = \frac{0.102}{s+0.3}$$

Ejemplo 2

$$G_1(s) = \frac{2}{s+2}$$

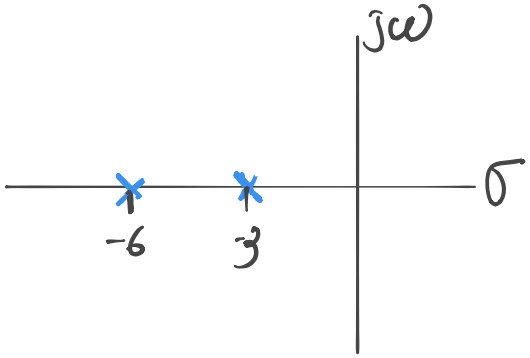


Polo $s = -2$

$$y_s(t) = A + Be^{-2t}$$

$$y_{ss} = 1$$

$$G_2(s) = \frac{5}{(s+3)(s+6)}$$

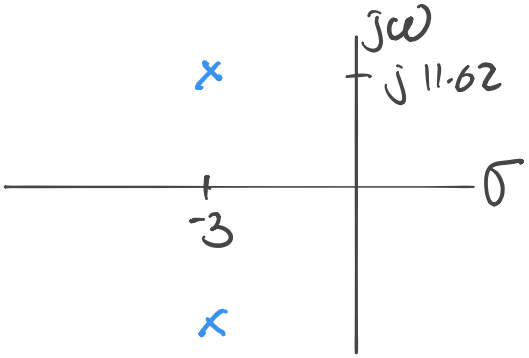


Polos, $s = -3, -6$

$$y_s(t) = A + Be^{-3t} + Ce^{-6t}$$

$$y_{ss} = \frac{5}{3 \times 6} = \frac{5}{18}$$

$$G_3(s) = \frac{20}{s^2+6s+144}$$

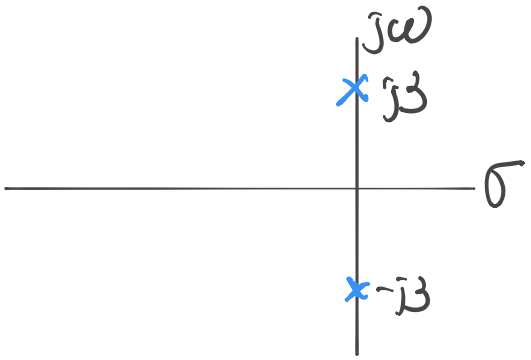


Polos: $-3 \pm j\,11.62$

$$y_s(t) = Ae^{-3t}\cos(11.62t + \phi)$$

$$y_{ss} \to 20/144$$

$$G_4(s) = \frac{s+2}{s^2+9}$$



poles : $\pm j3$

$y_s(t) = A\cos(3t+\phi)$

y_{ss} = no existe

Ejemplo 3

$$G(s) = \frac{100}{s^2 + 15s + 100}$$

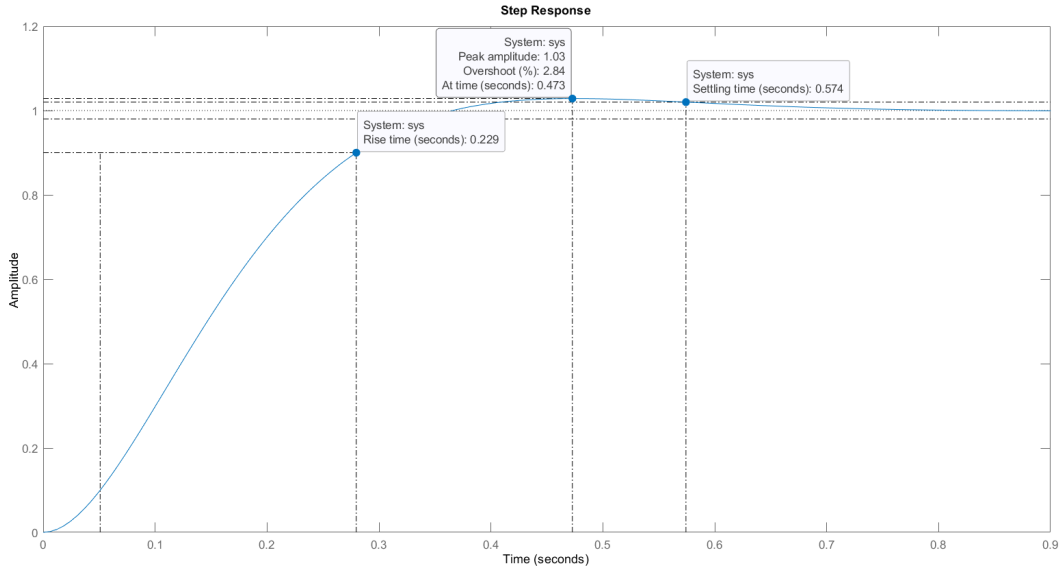
$$\omega_n = \sqrt{100} = 10$$

$$2\zeta\omega_n = 15 \Rightarrow \zeta = \frac{15}{20} = 0.75$$ ⇒ Subamortiguado ✓

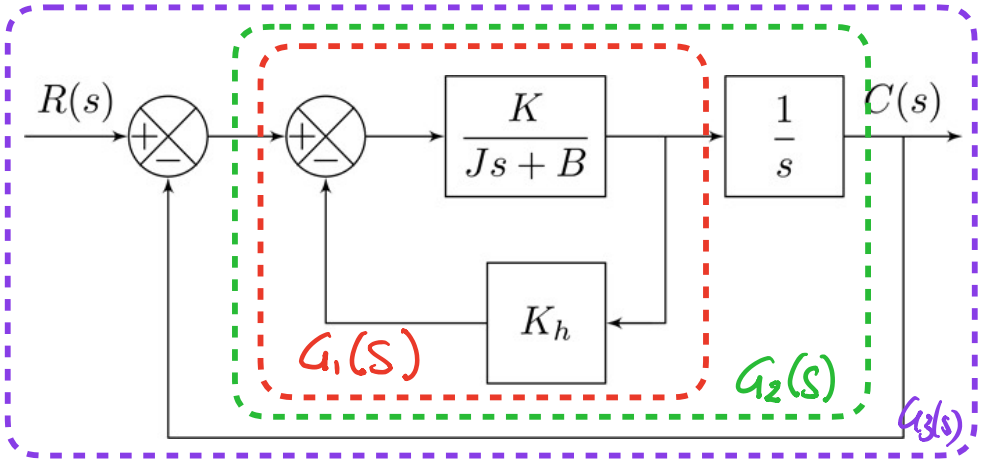
$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.48$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{7.5} = 0.53 \qquad T_r \approx 0.23$$

$$\%OS = 100\, e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = 2.84$$



Ejemplo 4



$$G_1(S) = \frac{\frac{K}{JS+B}}{1+\frac{KK_h}{JS+B}} = \frac{K}{JS+B+KK_h}, \quad G_2 = \frac{G_1}{S}$$

$$G_3(S) = \frac{\frac{K}{S(JS+B+KK_h)}}{1+\frac{K}{S(JS+B+KK_h)}}$$

$$G_3(S) = \frac{K}{JS^2+(B+KK_h)S+K}$$

Con $J=1$, $B=1$

$$G_3(S) = \frac{K}{S^2+(1+KK_h)S+K}$$

$K \rightarrow \omega_n^2$

$1+KK_h \rightarrow 2\zeta\omega_n$

$\%OS = 20\%$, $T_p = 1\,s$

$$\%OS = 100\, e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \Rightarrow \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2+\ln^2(\%OS/100)}}$$

$$\zeta = \frac{-\ln(0.2)}{\sqrt{\pi^2+\ln^2(0.2)}} = 0.46$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \Rightarrow \omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}} = \frac{\pi}{\sqrt{1-0.46^2}}$$

$$\omega_n = 3.54$$

$$s^2 + (1 + K K_n)s + K = s^2 + 2(0.46)(3.54)s + 3.54^2$$

$$K = \omega_n^2 = 12.53$$

$$1 + K K_n = 3.26$$

$$K_n = \frac{3.26 - 1}{12.53}$$

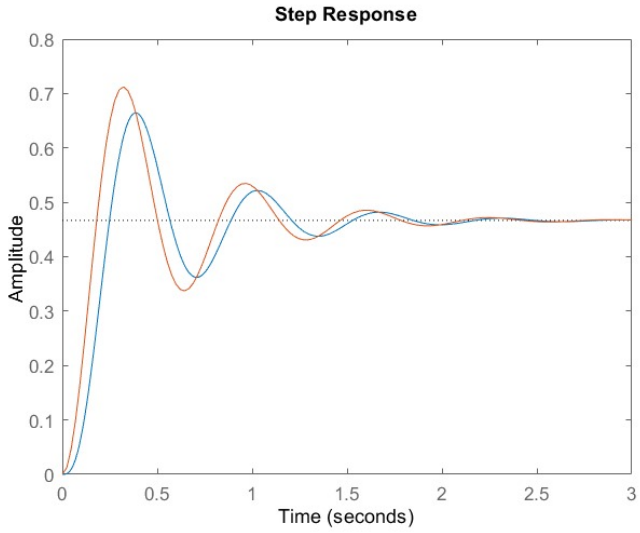
$$K_n = 0.18$$

Ejemplo 5

$$G_1(s) = \frac{700}{(s+15)(s^2+4s+100)} = \frac{(7/15)(15)(100)}{(s+15)(s^2+4s+100)} \approx \frac{700/15}{s^2+4s+10}$$

Polos: $-2 \pm j\omega_d$

Polo: $-15 \angle 20$



RMSE: 0.0299

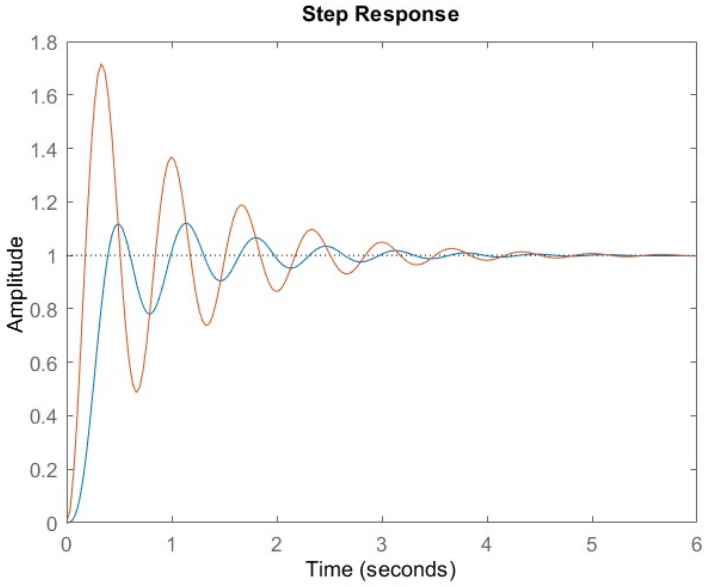
R^2: 0.7618

Aproximación Regular

$$G_2(s) = \frac{360}{(s+4)(s^2+2s+90)} = \frac{(4)(90)}{(s+4)(s^2+2s+90)} \approx \frac{90}{s^2+2s+90}$$

Polos: $-1 \pm j\omega_d$

Polo: -4



RMSE: 0.1720

$R^2 = -0.4044$

Muy mala aproximación

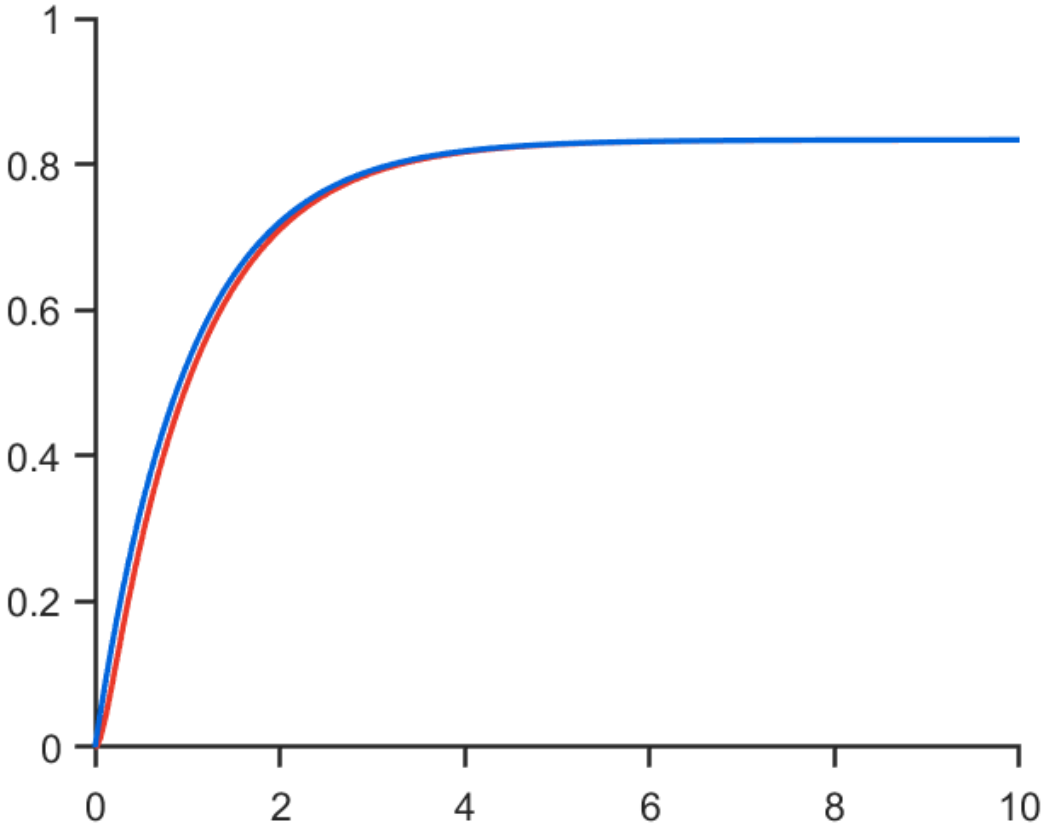
Ejemplo 6

$$G_1(s) = \frac{10}{s^2 + 13s + 12}$$

Polos: $s = -1, -12$ → Dominante

$$K = \frac{10}{12} = \frac{5}{6}$$

$$\tilde{G}_1(s) = \frac{(5/6)(1)}{s+1} = \frac{5}{6(s+1)}$$



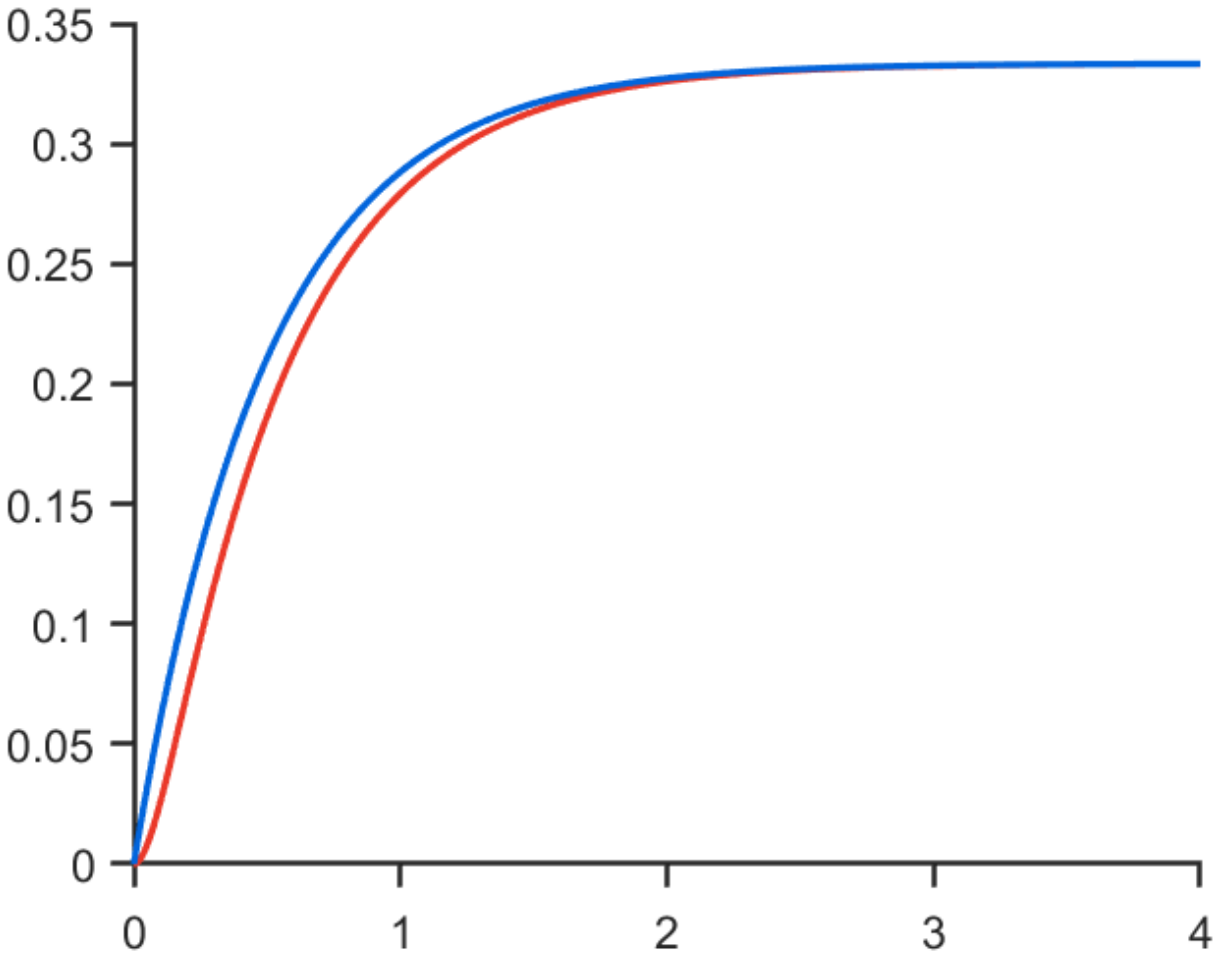
$R^2 = 0.9931$, RMSE = 0.0149

$$G_2(s) = \frac{8}{s^2+14s+24}$$

Polos: $\begin{cases} -2 \\ -12 \end{cases}$ → Dominante

$$K = \frac{8}{24} = \frac{1}{3}$$

$$\hat{G}_2(s) = \frac{(1/3)(2)}{s+2} = \frac{2/3}{s+2}$$



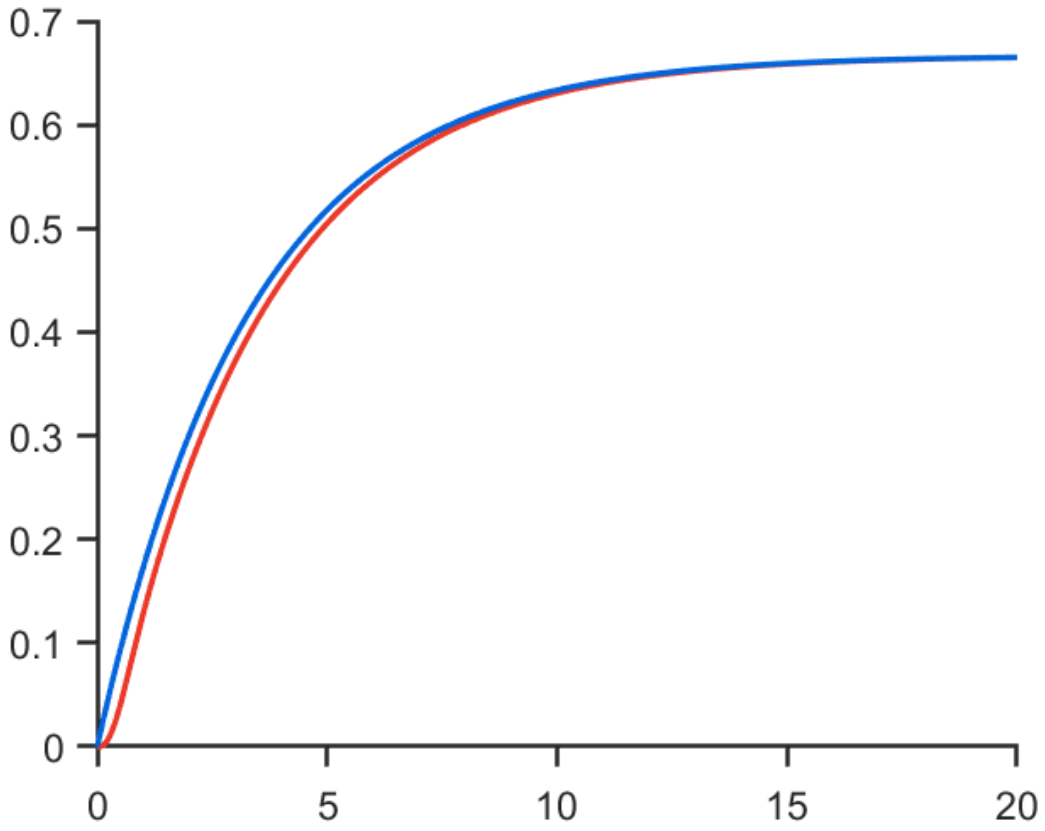
$R^2 = 0.9755$, RMSE = 0.0129

$$G_3(s) = \frac{5}{s^3 + 7.3s^2 + 27.1s + 7.5} = \frac{5}{(s+0.3)(s^2+7s+25)}$$

$$Polos = \begin{cases} -0.3 \longrightarrow \text{Dominante} \\ -3.5 \pm j\,\frac{\sqrt{51}}{2} \end{cases}$$

$$K = \frac{5}{7.5} = \frac{2}{3}$$

$$\tilde{G}_3(s) \approx \frac{(2/3)(0.3)}{s+0.3} = \frac{(2/3)\left(\frac{3}{10}\right)}{s+\frac{3}{10}} = \frac{2}{10s+3}$$



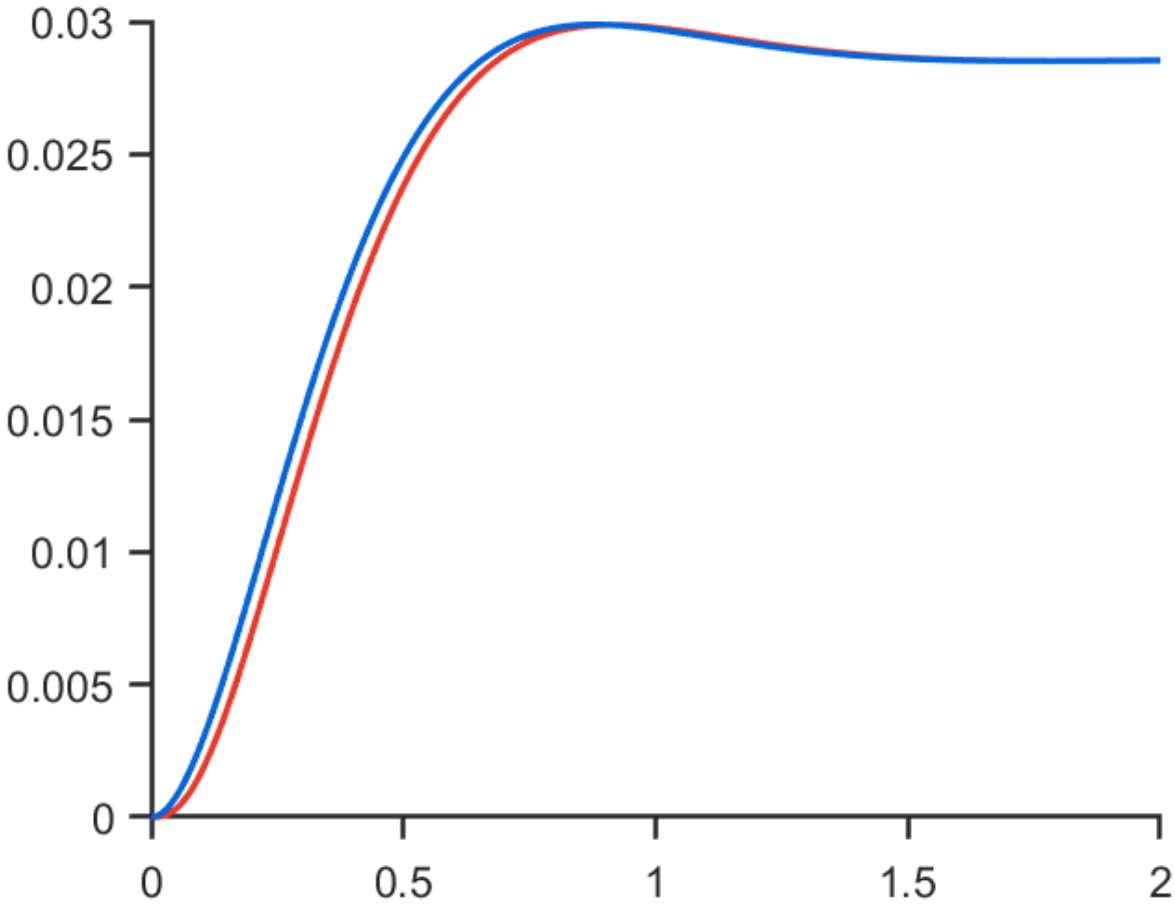
$R^2 = 0.9912$ $\quad$ $RMSE = 0.0160$

$$G_4(s) = \frac{25}{s^3 + 42s^2 + 270s + 875} = \frac{25}{(s+35)(s^2+7s+25)}$$

$$\text{Polos} = \begin{cases} -35 \\ -3.5 \pm j\frac{\sqrt{51}}{2} \end{cases} \longrightarrow \text{Dominante}$$

$$K = \frac{1}{35}$$

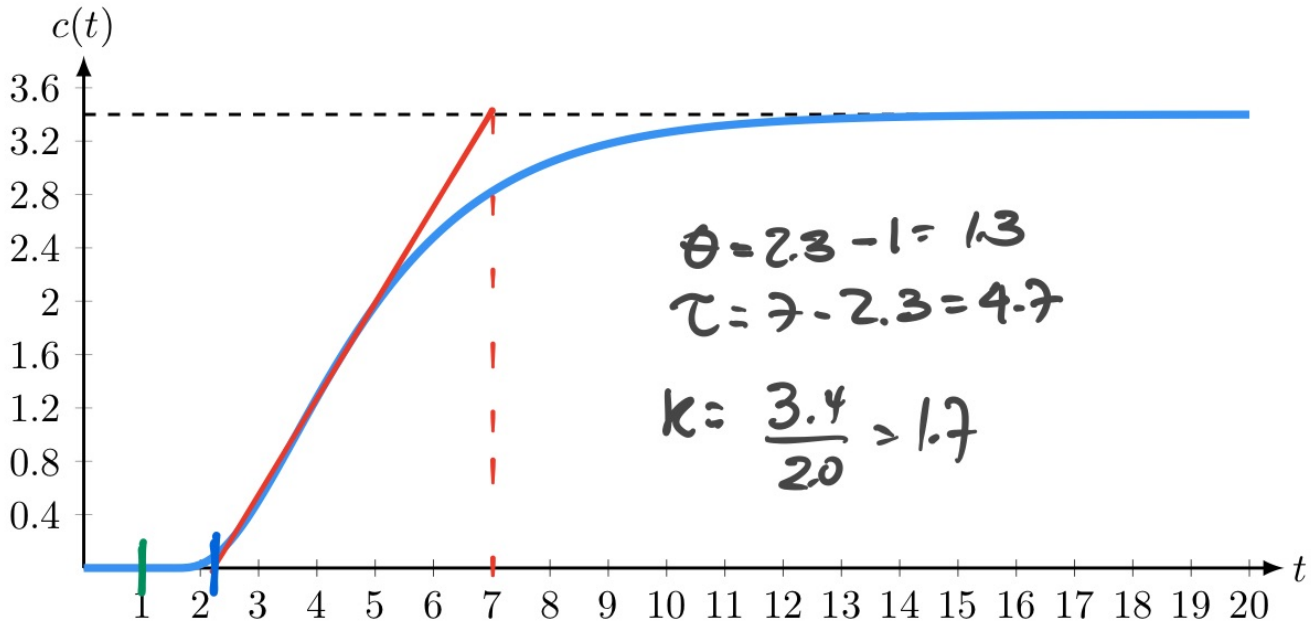
$$\widetilde{G}_4(s) = \frac{\left(\frac{1}{35}\right)(25)}{s^2+7s+25} = \frac{5/7}{s^2+7s+25}$$



$R^2 = 0.9925$, $RMSE = 7.64 \times 10^{-4}$

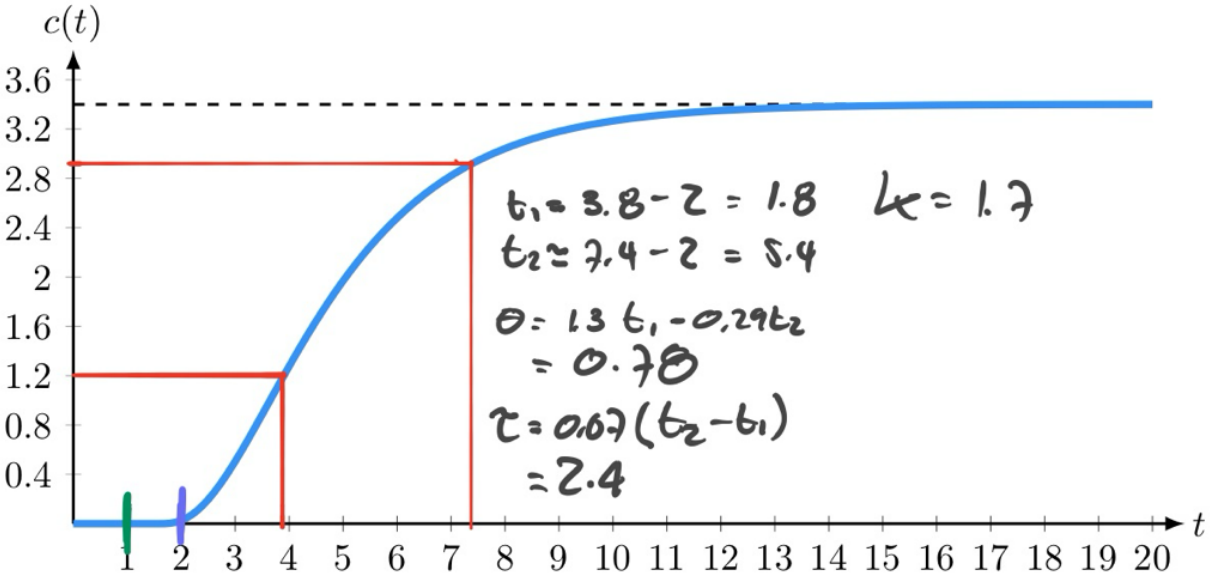
Ejemplo 7

FOPDT (Método I)



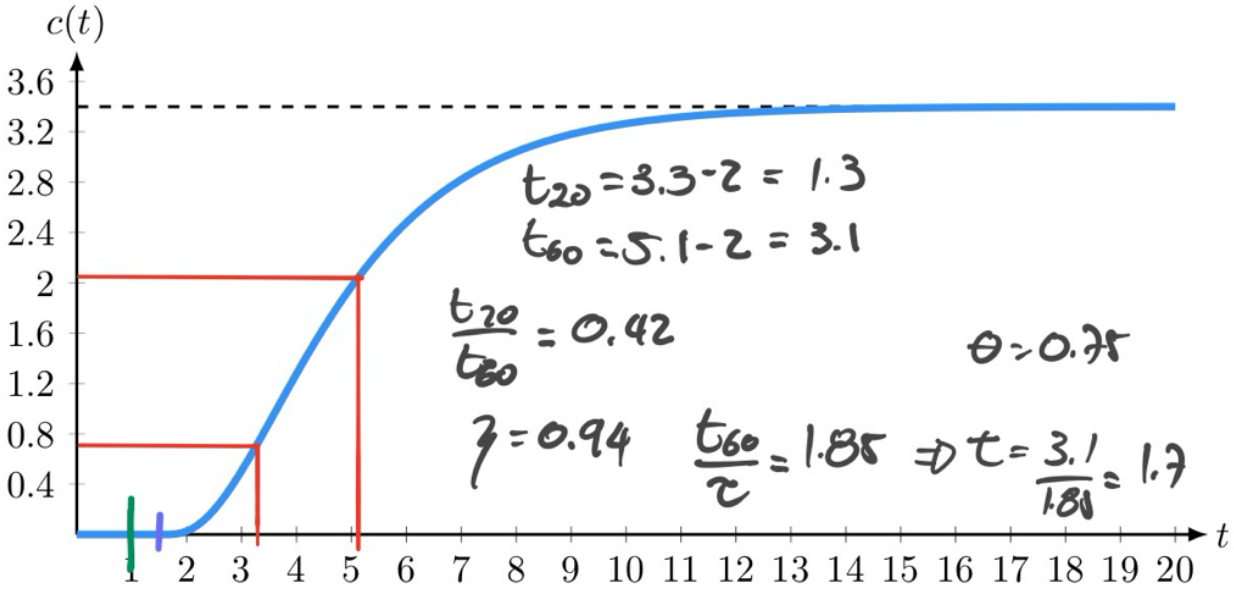
$$\tilde{G}_1(s) = \frac{1.7\,e^{-1.3s}}{4.7s+1}$$

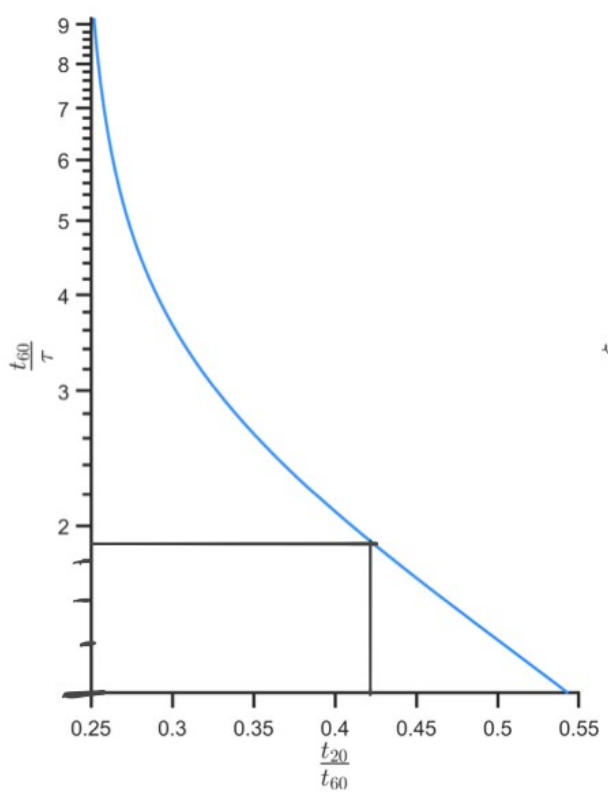
FOPDT (Método II)

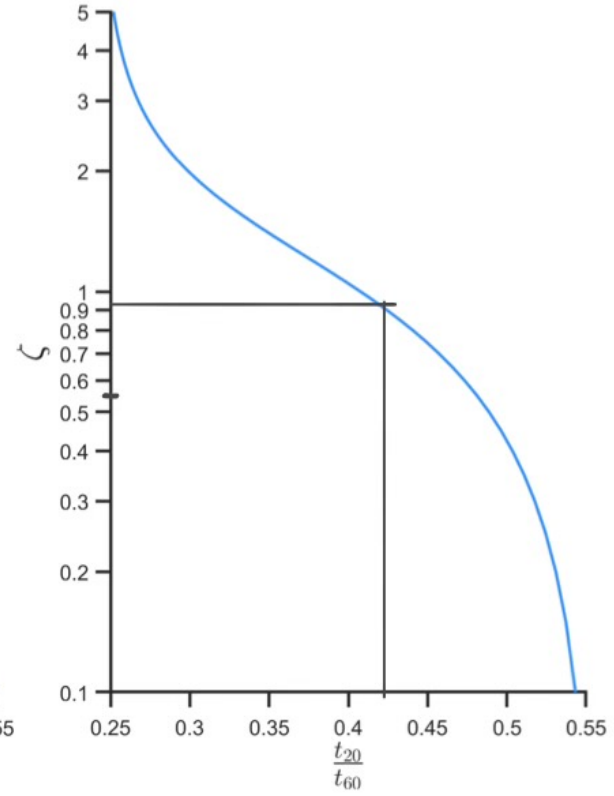


$$\tilde{G}_2(s) = \frac{1.7\,e^{-0.78s}}{2.4s+1}$$

SOPDT (Smith)

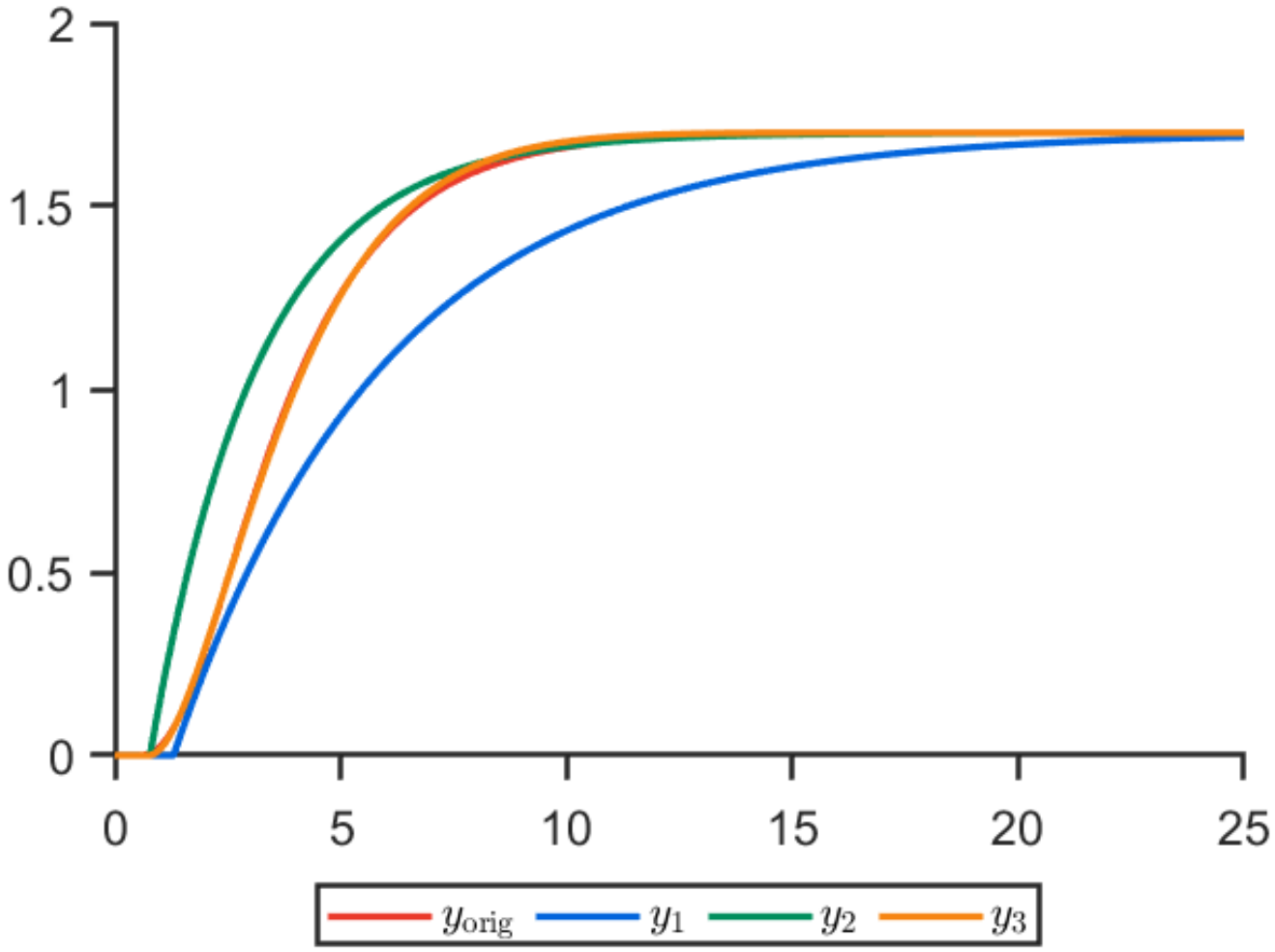






$$\tilde{G}_3(s) = \frac{1.7e^{-0.75s}}{(1.7)^2 s^2 + 2(1.7)(0.94)s + 1}$$

$$\tilde{G}_3(s) = \frac{1.7e^{-0.75s}}{2.89s^2 + 3.196s + 1}$$

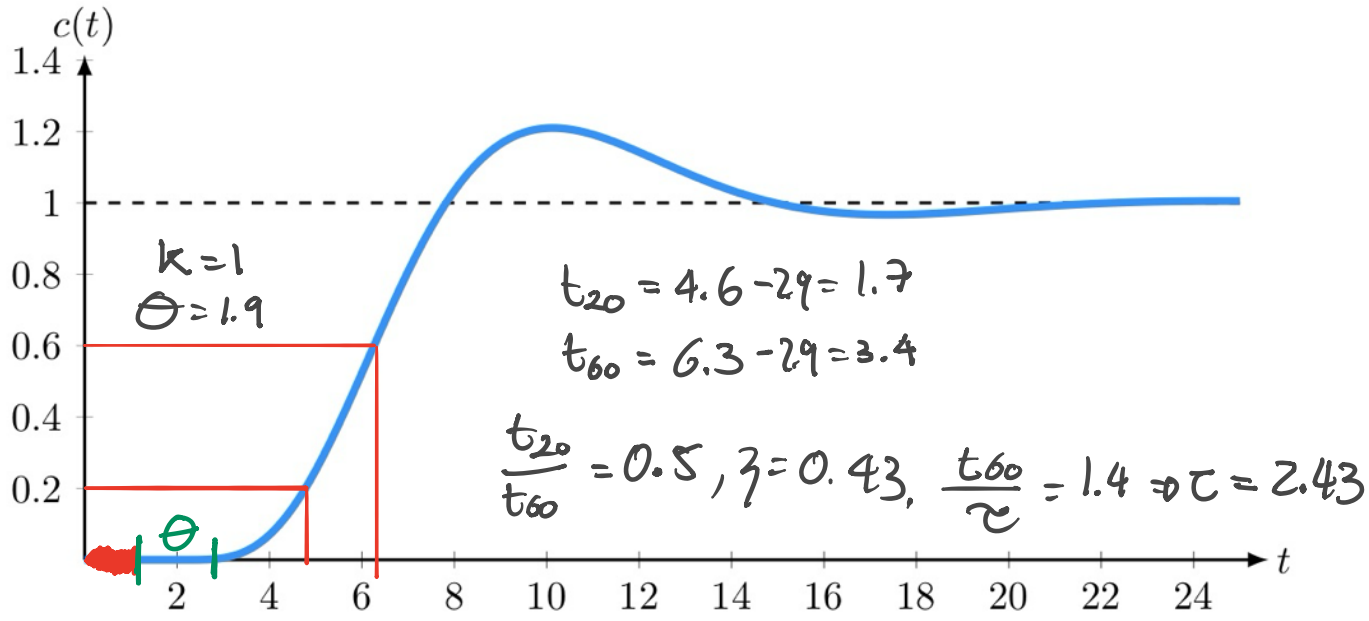


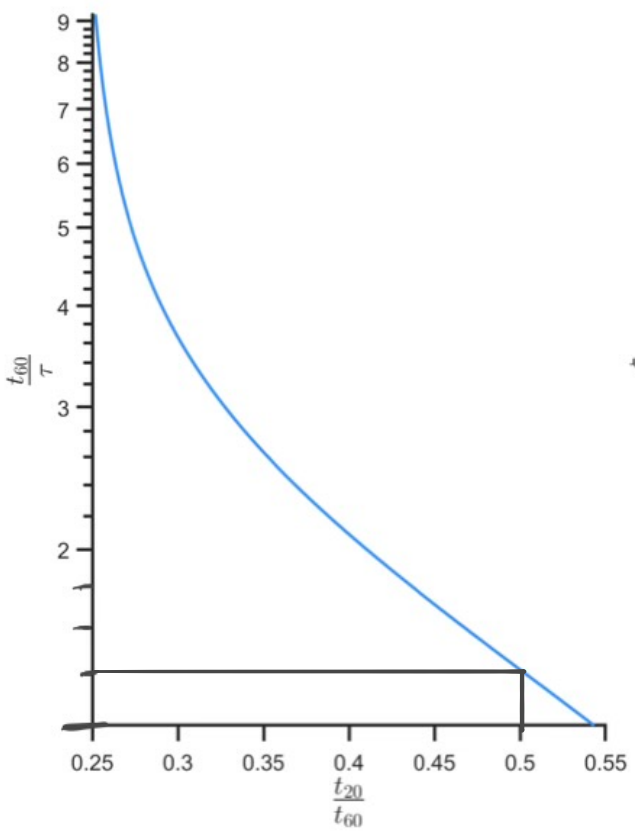
① $R^2 = 0.8654$
RMSE = 0.1971

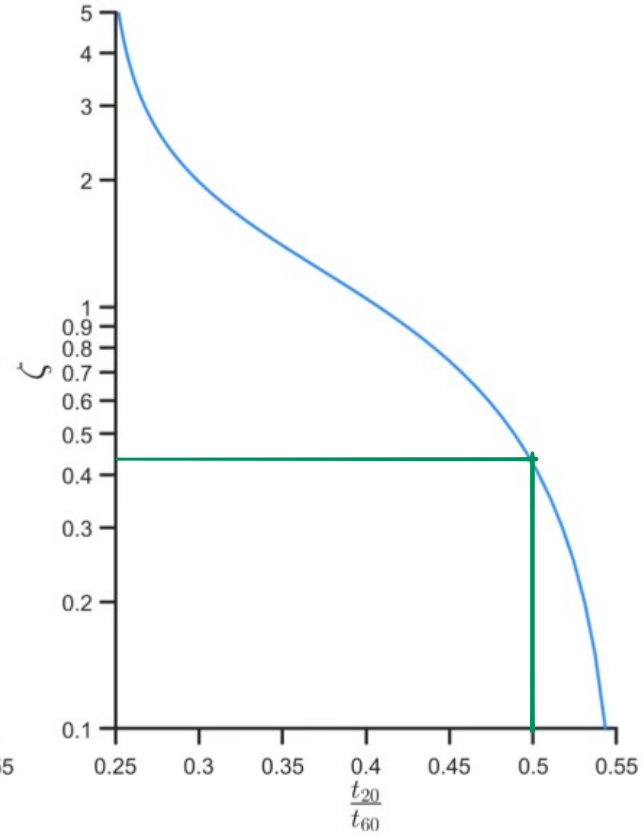
② $R^2 = 0.9334$
RMSE = 0.1387

③ $R^2 = 0.9998$
RMSE = 0.0077

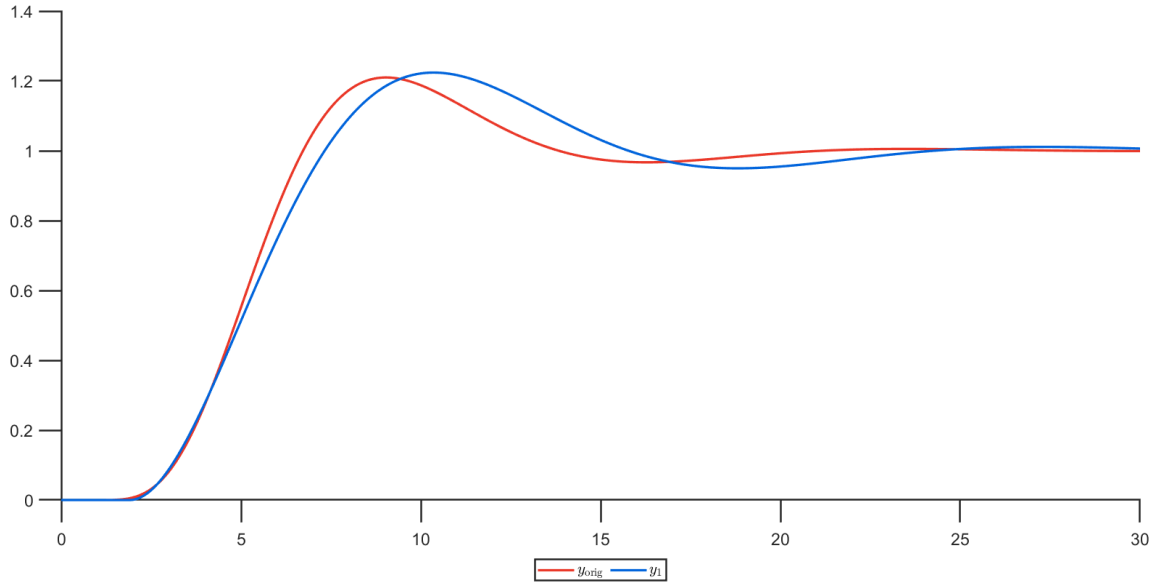
Ejemplo 8


c(t)
1.4
1.2
1
0.8
0.6
0.4
0.2
t
2
4
6
8
10
12
14
16
18
20
22
24
K=1
Θ=1.9
Θ
$t_{20} = 4.6 - 2.9 = 1.7$
$t_{60} = 6.3 - 2.9 = 3.4$
$\frac{t_{20}}{t_{60}} = 0.5, \zeta = 0.43, \frac{t_{60}}{\tau} = 1.4 \Rightarrow \tau = 2.43$


$\frac{t_{60}}{\tau}$
9
8
7
6
5
4
3
2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
$\frac{t_{20}}{t_{60}}$


ζ
5
4
3
2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.25
0.3
0.35
0.4
0.45
0.5
0.55
$\frac{t_{20}}{t_{60}}$

$$\tilde{G}_1(s) = \frac{e^{-1.9s}}{5.908s^2 + 2.098s + 1}$$



$R^2 = 0.9803$, $RMSE = 0.0493$