PH108 : Electricity & Magnetism : Problem Set 5

Only * problems are to be solved in the tut session

1.* A metal sphere with radius R_1 has charge Q. A second metal sphere with radius R_2 has zero charge. Now connect the spheres together using a fine conducting wire. Assume that the spheres are separated by a distance R which is large enough that the charge distribution on each ball remains uniform. Derive an expression for the final charge on the sphere with radius R_1.

2. (a) Show that the capacitance, C, of a conducting sphere of radius a is given by $C = 4\pi\epsilon_0 a$.

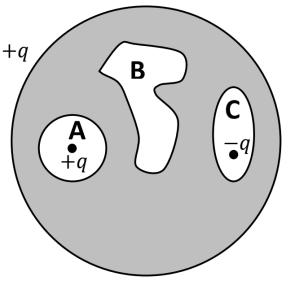
   (b) Two isolated conducting spheres, both of radius a, initially carry charges of q_1 and q_2 and are held far apart. The spheres are connected together by a conducting wire until equilibrium is reached, whereupon the wire is removed. Show that the total electrostatic energy stored in the spheres decreases by an amount U , given by

$$\Delta U = \frac{1}{16\pi\epsilon_0 Q}(q_1 - q_2)^2$$

   What happens to this energy?

3. A metal sphere of radius R carries a total charge Q. What is the force of repulsion between the "northern" hemisphere and the "southern" hemisphere?

4. A solid spherical conductor encloses 3 cavities, a cross-section of which are as shown in the figure. A net charge $+q$ resides on the outer surface of the conductor. Cavities A and C contain point charges $+q$ and $-q$, respectively. What are the net charges on the surfaces of the cavities?



5. A metal sphere of radius R, carrying charge q, is surrounded by a thick concentric metal shell (inner radius a, outer radius b). The shell carries no net charge.

   (a) Find the surface charge density σ at R, at a, and at b.

   (b) Find the potential at the center, using infinity as the reference point.

   (c) Now the outer surface is touched to a grounding wire, which drains off charge and lowers its potential to zero (same as at infinity). How do your answers to (a) and (b) change?

6.* A point charge q of mass m is released from rest at a distance d from an infinite grounded conducting plane. How long will it take for the charge to hit the plane?

7. Two infinite parallel grounded conducting planes are being held at distance a apart. A point charge q is placed in between them at a distance x from one plate. Find the force on q. Check your answer is correct for the special cases as $a \to \infty$ and $x = a/2$.

8. A conducting sphere (or a shell) of radius R has a charge Q.

   (a) Find the force of repulsion between the two hemispheres

   (b) Now suppose one has a solid sphere of radius R with charge Q distributed uniformly over its volume. What will be the force of repulsion between the two hemispheres?

   (c) Which case (a) vs (b) has the larger force of repulsion?

9.* (a) Find the average potential over a spherical surface of radius R due to a point charge q located inside. Show that, in general,

$$V_{ave} = V_{center} + \frac{Q_{enc}}{4\pi\epsilon_0 R}$$

   where V_{center} is the potential at the center due to all the external charges and Q_{enc} is the total enclosed charge.
   (b) Find the general solution to Laplace's equation in spherical coordinates for the case where V depends only on r. Do the same for cylindrical coordinates assuming V depends only on s.

10.* A point charge $+q$ is placed at a distance d from the centre of a conducting sphere of radius R ($d > R$). Show that if the sphere is grounded, the ratio of the charge on the part of the sphere visible from $+q$ to that on the rest is $\sqrt{\frac{d+R}{d-R}}$.

11.* Two infinite conducting plates (both grounded and perpendicular to the $x-y$ plane) meet at an angle of $60°$. A point charge $+q$ in the $x-y$ plane has plane polar coordinates $(a, 20°)$. Find all the image charges and their positions in polar coordinates.

12.* A rectangular pipe running parallel to the z-axis (from $-\infty$ to $+\infty$) has three grounded metal sides at $y = 0, y = a$ and $x = 0$. The fourth side at $x = b$ is maintained at a specified potential $V_0(y)$.
(a) Develop a general formula for the potential within the pipe.
(b) Find the potential explicitly, for the case $V_0(y) = V_0$ (a constant).