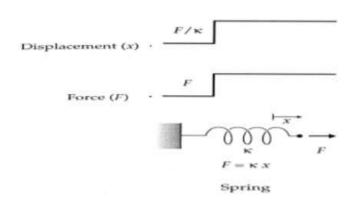


#### SPRING DASHSPOT SYSTEM

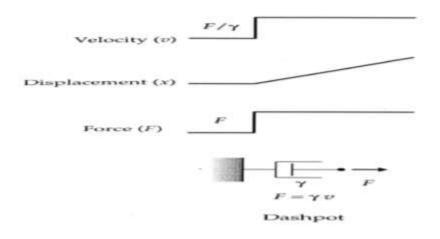
#### **SPRING**

Displacement directly proportional to the force applied



#### DASHSPOT

Velocity is directly proportional to the force applied







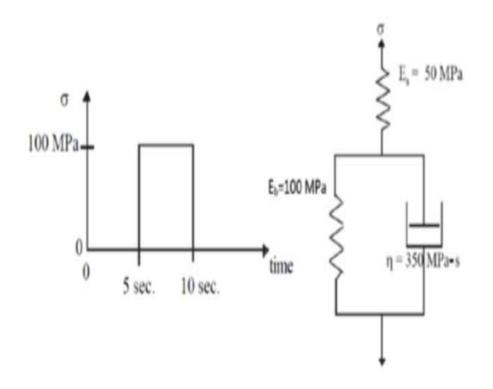


Quantity	In parallel	In series
Equivalent Spring Constant	Click to add text $k_2 = k_1 + k_2$	$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$
Equivalent Elongation	$x_{eq} = x_1 = x_2$	$x_{eq} = x_1 + x_2$
Equivalent Force	$F_{eq} = F_1 + F_2$	$F_{eq} = F_1 = F_2$
Equivalent Stored Energy	$E_{eq} = E_1 + E_2$	$E_{eq} = E_1 + E_2$

#### DASHSPOT SYSTEM

Quantity	In parallel	In series
Equivalent Drag Constant	$\gamma_{eq} = \gamma_1 + \gamma_2$	$\frac{1}{\gamma_{eq}} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2}$
Equivalent Elongation	$x_{eq} = x_1 = x_2$	$x_{eq} = x_1 + x_2$
Equivalent Force	$F_{eq} = F_1 + F_2$	$F_{eq} = F_1 = F_2$
Equivalent Stored Energy	$E_{eq} = E_1 + E_2$	$E_{eq} = E_1 + E_2$

## QUESTION



a)Plot strain as a function of time from t=0s to 20s

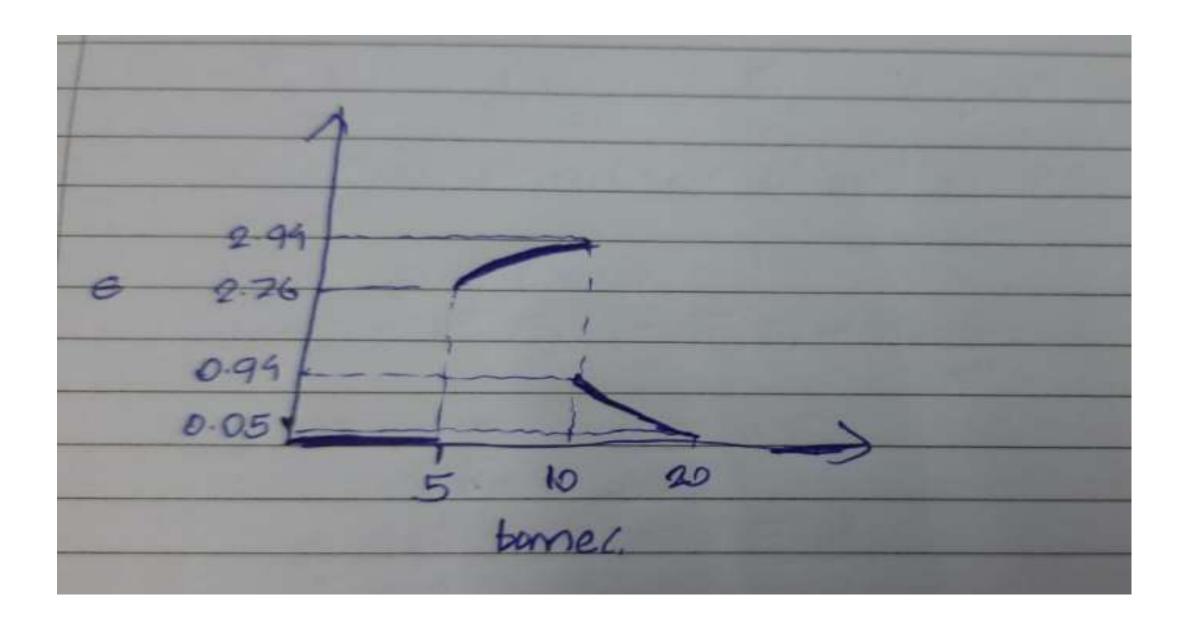
Assume: Uniform Area for all elements and all elements are at rest initially

# WRITE BASIC EQUATIONS FOR DASHSPOT AND SPRINGS

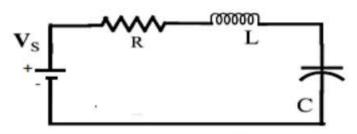
- $\bullet \sigma_{spring} = E \epsilon$
- $\bullet \sigma_{dashspot} = \eta \frac{d\epsilon}{dt}$
- •Here ,  $\sigma = {}^F/_A$  ,  $E = {}^k/_A$  ,  $\eta = {}^\gamma/_A$

Ep = 100MPx when \$510-50 A & b< 5 · 0 = Fa = Fb =, + 2 de 90, 62 - 0= Ea -FoG: = ndG: = 0 ; Gaz = 0 Since all element are at pest inhally When 5 < 6 < 10 6 = F662 = F66, + nd6, 

Case 3' 10 26 5205 0 = Each = Ebe, + 1 de, 6,00 6 = 6, + 62 - 0.94e-42 (b-10)



# COMPARISON WITH LCR CIRCUIT



Translational Mechanical System	Electrical System	
Force(F)	Voltage(V)	
Mass(M)	Inductance(L)	
Frictional Coefficient(B)	Resistance(R)	
Spring Constant(K)	Reciprocal of Capacitance $(\frac{1}{c})$	
Displacement(x)	Charge(q)	
Velocity(v)	Current(i)	

# COMPARISON WITH LCR CIRCUIT

#### MECHANICAL SYSTEM EQUATION

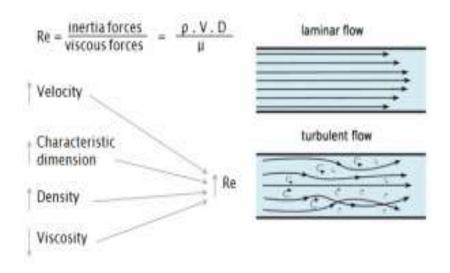
LCR SYSTEM EQUATION

$$L\frac{di}{dt} + \frac{q}{C} + Ri = \xi$$

#### REYNOLD'S NUMBER

It is a ratio of viscous forces to inertial forces

$$Re = \frac{Inertial\ force}{Viscous\ force} = \frac{\rho Rv}{\eta}$$



### QUESTION

Calculate Reynolds number for a spherical organism of radius 100 micro-m swimming at speed 10 micro-m/s in a medium whose viscosity is hundred times that of water and density is ten times that of water. The density of the organism is thousand times that of water

#### MODEL ANSWER

$$Re = \frac{9RV}{7} = \frac{10 \times 10^{+3} \times 100 \times 10^{-6} \times 10 \times 10^{-6}}{100 \times 10^{-3}}$$

$$= \frac{10^{7-12}}{10^{-1}} = 10^{7-12+1} = 10^{8-12} = 10^{-4}$$

$$\therefore Re = 10^{-4}$$

Note: You will take medium's density and not body's

#### STOKE'S LAW

- $\square$  For a spherical object ,  $F = 6\pi \eta r v$
- ☐ F-Viscous Drag Force
- $\square \eta$  –Viscosity of user
- ☐r-Radius
- □v- Velocity of the bodu

### QUESTION

#### Click to add text

Consider the sedimentation of a spherical bio-molecule of radius 1 nm, initially right below the surface, in an Eppendorf tube of length 1 centimeters filled with water. Suppose that density of this biomolecule is 7.5 times that of water and this bio-molecule sediments under the effect of gravity. Further assume that this bio-molecule attains a constant velocity as soon as it starts to descend in the Eppendorf tube. How much time (in seconds) this bio-molecule would take to descend 1mm in Eppendorf tube (Density water=1000 Kg m-3 and g=10m/s2)?

## Step 1: Write Equation of Motion

## Step 2: Find Velocity

$$\Rightarrow V = \frac{6.5 \times 9 \times 4 \times 7^{3} g}{3 \times 6 \times 10^{2} \times 10^{2}}$$

$$= \frac{26 P Y^{2} g}{18 \eta} = \frac{26 \times 10^{3} \times 10^{-9} \times 10^{-9} \times 10^{-9}}{18 \times 10^{-3}}$$

$$= \frac{26}{18} \times 10^{-11} \text{ m/s}$$

## Step 3: Find Time

#### THERMAL ENERGY

- □ Proportional to the temperature of the molecule/body
- □Since thermal forces are **randomly directed**, the resulting thermal motion is characterized by **frequent changes in direction** and is called diffusion
- □ Diffusion of a free particle or object is called Brownian motion
- □Thermal Energy is given by ,  $E=k_BT$ ,

where  $k_B$ =Boltzmann constant = 1.38064852 × 10<sup>-23</sup> m<sup>2</sup> kg s<sup>-2</sup> K<sup>-1</sup>

### QUESTION

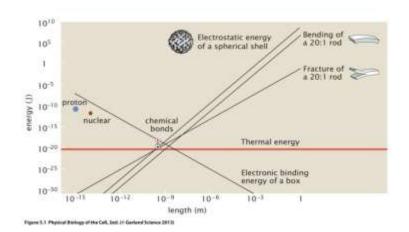
Hydrogen bonds are common in biological macromolecules such as proteins and DNA. Suppose that energy associated with hydrogen bond is 2 KJ/mol. Can thermal energy available at room temperature i.e. T=300 K can break a hydrogen bond?

#### ANSWER

```
KBT= 411 ×10-21 pNm at T= 300K
      = 411×10-21 J
E_{\text{H-Band}} = \frac{2 \text{ KJ}}{\text{mol}} = \frac{2000 \text{ } \times 10^{-23} \text{ J}}{6.023} = 332.06 \times 10^{-23} \text{ J}
         = 3.32×10-21 J
  " EH-Bond < KBT
  Hence, thermal energy can break this hydrogen bond.
```

# IMPORTANCE OF THERMAL ENERGY

- ☐ Thermal Energy is **independent of the size/scale** of the object
- ☐ In macroscopic scale, the deterministic forces heavily dominate
- □In microscopic scale (scale of molecule and chemical bonds), the thermal and deterministic energy are comparable and both play a significant role in determining the state of molecule



#### **BOLTZMAN LAW**

Probability of finding a molecule in a certain state depends on the energy of that state and surrounding temperature

$$p_i = \frac{1}{Z} e^{-\frac{E_i}{k_B T}}$$

Where  $z = \sum_{i} e^{-\frac{E_i}{k_B T}}$  is called partition function

The exponential term is called Boltzmann factor

### NERST EQUATION

$$\frac{p_1}{p_2} = \frac{C_1}{C_2} = \frac{e^{-\frac{zeV_1}{k_B T}}}{e^{-\frac{zeV_2}{k_B T}}}$$

$$V_2 - V_1 = \frac{k_B T}{ze} ln \frac{C_1}{C_2}$$

z is valancy of ion and  $e = 1.6 \times 10^{-19} C$ z = +1 for Na<sup>+</sup> and z = -1 for Cl<sup>-</sup>

# MICROSTATES AND FREE ENERGIES

The probability associated with any microstate is given by:

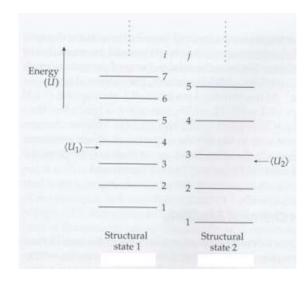
$$p_i = \frac{1}{Z}e^{-\frac{U_i}{k_B T}}$$

Helmholtz free energy

$$A = U - TS$$

Gibbs free energy

$$G = H - TS$$



# ENTROPY AND ITS RELATION TO MICROSTATES

Entropy is a measure of the microscopic degeneracy of a macroscopic state ("macrostate"

In other words, entropy can be computed by counting the number of possible microscopic arrangements/states ("micro-states") for a given macroscopic state ("macro-state")

$$S = k_B \ln W$$

W is the number of possible microscopic arrangements (or "micro-states")

# PROBABILITY ASSOCIATED WITH A MACROSTATE

$$p(X) = \frac{1}{Z}e^{-\frac{G(X)}{k_BT}}$$

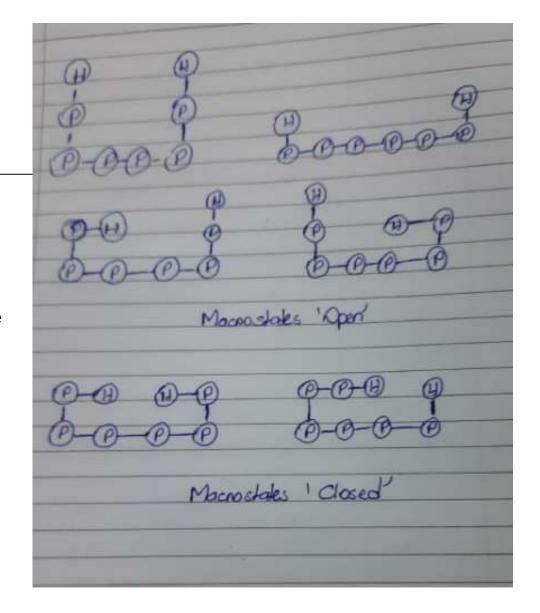
Note: Here Z will be summation over all macrostates

### QUESTION

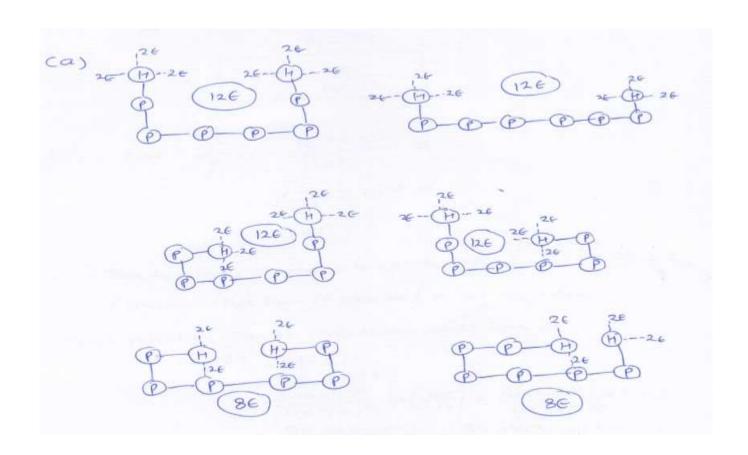
Consider a protein consisting of eight amino acids at temperature T. The sequence of the amino acids is HPPPPPH. H and P denote the Hydrophobic and Polar amino acids respectively. Bond length between two amino acids is b. Suppose that this protein can be found in two different macrostates-"Open" or "Closed". The microstates corresponding to these macrostates are shown in the figure beside.

Suppose that this protein is kept in a solvent (water) such that energy increases by  $\epsilon$  for every contact of H with either a solvent molecule or a P molecule within distance b (consider the initial energy of the protein to be zero when it is not in solvent)

- (i) What are the energies of the protein in different microstates?
- (ii) What are the free energies of "Open" and "Closed" macrostates?
- (iii) What is the critical temperature above which opened microstate will be favoured?

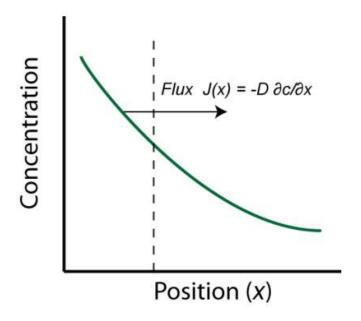


### Answer: Part a

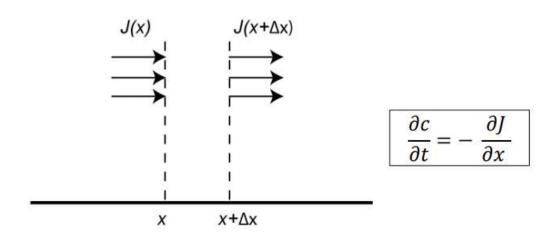


#### Answer: Part b and c

### FICK'S LAW



## CONTINUITY EQUATION



### DIFFUSION EQUATION

#### Combine Fick's Law and Continuity Equation

$$\frac{\partial c(x,t)}{\partial t} = D \frac{\partial^2 c(x,t)}{\partial x^2}$$

### QUESTION

Which one(s) of the following functions is/are solutions of diffusion equation in one dimension

(i) 
$$C(x,t) = \exp(-\alpha t^2) \sin(-\sqrt{\frac{\alpha}{D}}x)$$

(ii) 
$$C(x,t) = \exp(-\alpha t) \sin(\sqrt{\frac{\alpha}{D}}x^2)$$

(i) 
$$C(x,t) = \exp(-\alpha t^2) \sin(-\sqrt{\frac{\alpha}{D}}x)$$
 (ii)  $C(x,t) = \exp(-\alpha t) \sin(\sqrt{\frac{\alpha}{D}}x^2)$  (iii)  $C(x,t) = \exp(-\alpha t) \cos(\sqrt{\frac{\alpha}{D}}x^2)$  (iv)  $C(x,t) = \exp(-\alpha t) \sin(\sqrt{\frac{\alpha}{D}}x)$ 

(iv) 
$$C(x,t) = \exp(-\alpha t) \sin(\sqrt{\frac{\alpha}{D}}x)$$

[2 Marks]

### ANSWERS

(i) 
$$\frac{\partial C}{\partial t} = 2ate^{-att} \sin(\sqrt{g}n), \frac{\partial^{2}C}{\partial n} = \sqrt{g}e^{-att} \cos(\sqrt{g}n)$$
 $\frac{\partial^{2}C}{\partial n} = \frac{d}{dn}e^{-att} \sin(\sqrt{g}n)$ 

"!  $\frac{\partial C}{\partial t} \neq D\frac{\partial r}{\partial n} \Rightarrow \text{It is not a solution}$ 

(ii)  $\frac{\partial C}{\partial t} = -de^{-att} \sin(\sqrt{g}n^{2}), \frac{\partial C}{\partial n} = 2n\sqrt{g}e^{-att} \cos(\sqrt{g}n^{2})$ 
 $\frac{\partial r}{\partial n} = \frac{1}{D}\left[2e^{-att}\left\{\sqrt{g}\cos(n^{2}\sqrt{g}) - 2dn^{2}\sin^{2}(\sqrt{g}n^{2})\right\}\right]$ 

"!  $\frac{\partial C}{\partial t} \neq D\frac{\partial^{2}C}{\partial n} \Rightarrow \text{It is not a solution}$ 

"!  $\frac{\partial C}{\partial t} \neq D\frac{\partial^{2}C}{\partial n} \Rightarrow \text{It is not a solution}$ 

(iii) 
$$\frac{\partial C}{\partial t} = -\Delta e^{-\Delta t} \cos\left(n^{2} \int_{B}^{\infty}\right), \frac{\partial C}{\partial n} = -2n \int_{B}^{\infty} e^{-\Delta t} \sin\left(\frac{1}{B}n^{2}\right)$$

$$\frac{\partial^{2}C}{\partial n} = -\frac{1}{D}\left(2e^{-\Delta t}\left[t\right) \underbrace{\beta}_{B} \sin\left(n^{2} \int_{B}^{\infty}\right) + 2\alpha n^{2} \cos\left(\int_{B}^{\infty}n^{2}\right)^{2}\right)$$

$$\therefore \frac{\partial C}{\partial t} \neq D \underbrace{\frac{\partial^{2}C}{\partial n^{2}}} \Rightarrow \text{ It is not a solution}$$

$$\therefore \frac{\partial C}{\partial t} \neq D \underbrace{\frac{\partial^{2}C}{\partial n^{2}}} \Rightarrow \text{ It is not a solution}$$

$$\frac{\partial C}{\partial t} = -\Delta e^{-\Delta t} \sin\left(\int_{B}^{\infty}n\right), \frac{\partial C}{\partial n} = \underbrace{\int_{B}^{\infty}} e^{-\Delta t} \cos\left(\int_{B}^{\infty}n\right)$$

$$\frac{\partial^{2}C}{\partial n^{2}} = -\frac{\Delta}{D} e^{-\Delta t} \sin\left(\int_{B}^{\infty}n\right)$$

$$\frac{\partial^{2}C}{\partial n^{2}} = -\frac{\Delta}{D} e^{-\Delta t} \cos\left(\int_{B}^{\infty}n\right)$$

Suppose that intra-cellular and extra-cellular concentrations of an ion Y across a cell membrane are  $9\times10^{-2}$  molecules/  $nm^3$  and  $3\times10^{-2}$  molecules/  $nm^3$  respectively. The diffusion constant for ion Y is  $2\times10^9$   $nm^2$  /s. Suppose that cell membrane has an ion channel of length 6 nm and area 0.1 nm2. Calculate the number of Y ions passing through this ion channel in 10 seconds?

## STEP 1; FIND THE FLUX(J)

The magnitude of Hux (ie total no. of ions crossing

the channel per unit area per unit time is given by

$$J = D \frac{\partial C}{\partial n}$$

$$= 2 \times 10^{9} \left( \frac{69 \times 10^{-2} - 3 \times 10^{-3}}{6 \times 10^{-2}} \right)$$

$$= 2 \times 10^{9} \times \frac{6 \times 10^{-2}}{6 \times 10^{-2}}$$

$$= 2 \times 10^{9} \times 10^{-2}$$

## STEP 2: MULTIPLY THE FLUX WITH AREA AND THE WITH TIME

```
Total no. of ions passing through the channel per secondis

given by JX0.1 nm = 2×10°×0.1

= 2×106 ions

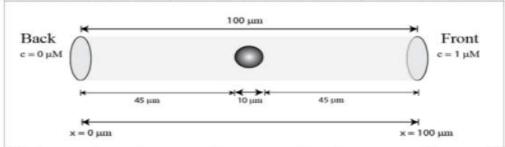
Therefore, no. of ions passing through this channel

in 10 seconds = 2×106×10

= 2×10° ions
```

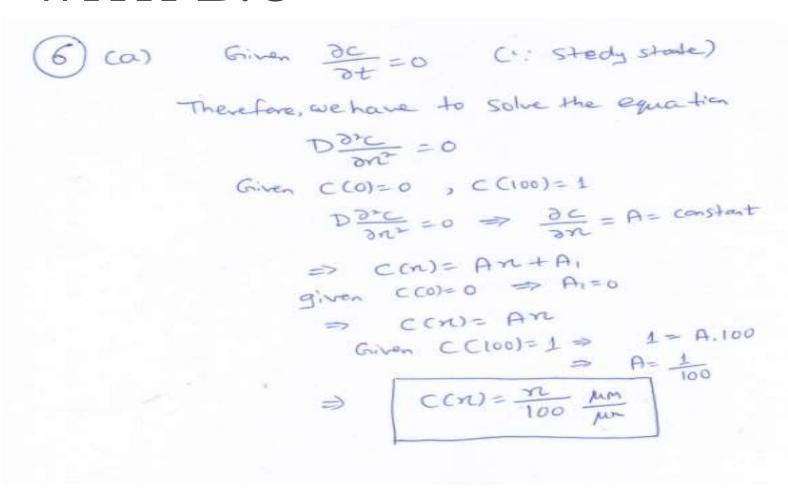
NOTE: FLUX= NO OF MOLECULES PASSING PER UNIT AREA PER UNIT TIME

Some cells have the ability to undergo chemotaxis i.e. the cell can sense a gradient of a chemical attractant from its front to its back, and move in the up-gradient direction. Figure below shows a cell that is initially stationary in the middle of a tube. The tube is  $100\mu m$  long and the cell can be approximated as sphere  $10\mu m$  in diameter. Independent flows can be used to control the concentration c of an attractant molecule A at each end of the tube. The concentration at the front end of the tube is maintained at  $1\mu M$ , and the concentration at the back end is maintained at  $0\mu M$ . Assume that a steady-state concentration profile has been reached inside the tube at T=300 K and the viscosity of water is 1 m Pa s.



- (a) Solve the diffusion equation and compute the concentration of attractant, c(x), across the tube.
- (b) What is the difference in concentration felt by the front of the cell versus the back of the cell?
- (c) Upon binding to a receptor on the cell surface, attractant A triggers phosphorylation of another intra-cellular protein, B, at the front-end of the cell. Assume that B can diffuse in the cytoplasm only along the diameter of cell, and does so with a diffusion constant  $D = 10^{-10} \, \text{m}^2 \, \text{s}^{-1}$ . How long does it take for the phosphorylated protein B to diffuse to the back-end of the cell? Assume cell membrane has zero thickness for this part of question.
- (d) Now, instead assume that B is a protein that can only diffuse on outer cell membrane, and it does so with a diffusion constant  $D = 10^{-11} \text{ m}^2 \text{ s}^{-1}$ . How long does it take now for B to diffuse from front-end to the back-end of the cell?

# ANSWER- PART 1: USE CONTINUITY EUATION AND FICK'S LAWS ALONG WITH B.C



# PART 2: SUBSITUTE THE VALUES IN EXPRESSION OF C AND FIND THE DIFFERENCE

(b) front of cell is located at 
$$n = 55 \mu m$$

-:  $C(55) = \frac{55}{100} \mu m = 0.55 \mu m$ 

Back of the cell is located of  $n = 45 \mu m$ 

-:  $C(45) = \frac{45}{100} \mu m = 0.45 \mu m$ 

-: Difference in concentration =  $(0.55 - 0.45) \mu m$ 

=: 0.1  $\mu m$ 

# PART 3: Use relation for One Dimensional Equation for x

(c) Time taken can be colculated by using relation 
$$x1^2 = 2Dt$$

$$\Rightarrow t = \frac{x^2}{2D} = \frac{(10\mu\text{m})^2}{2 \times 10^{-10} \text{m}^2 \text{s}^{-1}}$$

$$= \frac{10 \times 10^{-6} \times 10 \times 10^{-6} \text{ m}^2}{2 \times 10^{-10} \text{m}^2 \text{s}^{-1}}$$

$$= \frac{10^{-10} \text{m}^2}{2 \times 10^{-10} \text{m}^2 \text{s}^{-1}}$$

$$= \frac{1}{2} \text{S} = 0.5 \text{ second}$$

# PART 4: Use relation for two Dimensional Equation for x

(d) Time taken can be colculated using rebtion
$$\chi^2 = 4Dt$$

$$= \frac{\chi^2}{4D} = \frac{(r \cdot R)^2}{4D}$$

$$= \frac{r^2 R^2}{4D} = \frac{(3^{11}4)^2 (5 \times 10^{-6})^2 m^2}{4 \times 10^{-10} m^2 S^{-1}}$$

$$= \frac{246.5 \times 10^{-12}}{4 \times 10^{-10}} = \frac{2.465 \times 10^{-10} \text{ S}}{4 \times 10^{-10}}$$

$$= 0.616 \text{ Second}$$

$$= 0.62 \text{ Second}$$

- 12. This problem explores how the diffusion of oxygen limits the size of organisms without a circulatory system. Assume a spherical organism of radius R, living in an aqueous medium. The organism is entirely made of tissues. Its tissues need to consume oxygen at a rate  $M = 0.6\mu L/(s.cm^3)$  for its survival. The concentration of oxygen in the surrounding water is  $C_s = 10 \mu L/cm^3$ . Oxygen diffuses through the tissue with a diffusion coefficient of  $D = 10 \times 10^{-5} cm^2/s$
- (i) Consider a spherical tissue of radius r < R inside the organism located such that center of tissue coincides with the center of organism. What is the required total oxygen consumption per unit time inside the spherical tissue? Calculate required flux of oxygen across the surface of the spherical tissue,  $J_r(r)$ ?. Give your results in terms of the variables r and M for all of the above.
- (ii) Using the results of (i) and the fact that  $C(R) = C_s$ , derive the form of the steady-state oxygen concentration C(r) inside the organism. Give your results in terms of r, R, M,  $C_s$  and D.
- (iii) From a simple constraint that organism would die if oxygen does not reach its center, derive the maximal possible radius of the organism.

Hint: For a spherically symmetric diffusion problem in 3 dimensions, flux and concentration depend only on radius r, and are related by  $J_r(r) = -D \frac{d}{dr}C(r)$  where C(r) is the concentration of particles and  $J_r(r)$  is the radial flux of particles at distance r from the center of the sphere.

Total organ regund - M. 4 1103 Jan = Dane 2 Ode Now, Va mor - Magsins = JON - Me No. + Odgo = Mg  $= C - C_{5} = -M (R^{2} - R^{2})$   $= C(R) = -M (R^{2} - R^{2}) + C_{5}$ 11 CCOD-O R2 = 6 CED

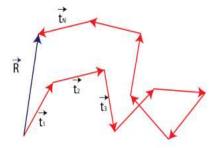
### Conformation of Polymer as Random Walk

Regard polymer as a string of N units, where each unit is joint to the next by a perfectly flexible joint. In the thermal equilibrium, joints will be at random angles. Freely Jointed Chain (FJC)

$$\langle R^2 \rangle = NL^2$$

A polymers can be treated as consisting of a number of linear segments

Persistance length is what determines the whether the polymer is bent or straight



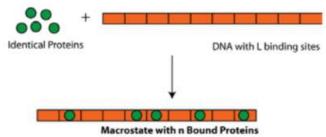
$$\langle R^2 \rangle = NL^2$$

### PROTEIN DNA BINDING

- ☐ Proteins and DNA being oppositely charged favours binding
- □ If there are m proteins and B binding sites , no of microstates W

$$W = \frac{N!}{m! (N-m)!}$$

10. Consider the Protein-DNA system discussed in lecture where identical protein molecules can bind to a DNA with total L binding sites as shown in figure below.



Suppose different possible macrostates of this protein-DNA system are characterized by the parameter  $\rho$  which measures the density of protein molecules bound to DNA.  $\rho$ =n/L where n is the number of protein molecules bound to DNA and L=10. Consider the three macrostates A, B and C of this protein-DNA system with densities  $\rho$ =0.2,  $\rho$ =0.5 and 0.8 respectively.

- (i) Calculate the number of microstates corresponding to macrostates A, B and C [1.5 Marks]
- (ii) Calculate the free energies of the macrostates A, B and C if energy (internal energy) of this system is zero and binding of protein does not increase/decrease energy. [1.5 Marks]

[1 Marks]

(iii) Which macrostate has lowest free energy at temperature T= 300 K?

(i) Ho. of microstates in macrostate 
$$A = \frac{10!}{2!(10-2)!}$$
  
=  $\frac{10!}{2!8!} = \frac{16 \times 9 \times 9!}{2! \times 9!}$ 

```
cii) free energy of macrostate A= 0-1kgln45 = -kgTln45

" B = 0-Tkgln252 = -kgTln252

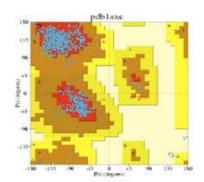
" C = 0-Tkgln45 = -kgTln45

" C = 0-Tkgln45 = -kgTln45

Ciii) Macrostate Buill have lowest free energy at T=300k
```

#### RAMACHANDRAN PLOT

- □ It is the set of allowed angles (phi and psi) for a proteins and helps in determining the stability of the protein structure
- ☐ The range of phi and psi is -180 to 180 degrees
- ☐ There are three kind of regions: i) Favoured ii) Allowed iii) Generously allowed



The red, brown, and yellow regions represent the favored, allowed, and "generously allowed" regions

#### ACTIN AND MYOSIN

- □Actin and myosin are both proteins that are found in every type of muscle tissue.
- ☐ Thick myosin filaments and thin actin filaments work together to generate muscle contractions and movement.
- □ Myosin is a type of molecular motor and converts chemical energy released from ATP into mechanical energy. This mechanical energy is then used to pull the actin filaments along, causing muscle fibers to contract and, thus, generating movement.

#### ACTIN AND MYOSIN

- □ A *hydrolysis reaction* releases energy from ATP, and the myosin works like a motor to convert this chemical energy into mechanical energy.
- ☐ The myosin uses that mechanical energy to move its head groups towards the middle of the sarcomere.
- □This movement pulls the actin filaments towards the center of the sarcomere, causing the sarcomere to shorten and contract.
- ☐ The contraction of the sarcomere causes the muscle fiber to contract and generates muscle movement