$$\phi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)$$

$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$

$$\Delta P = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$$

$$\langle x^2\rangle = \int_0^L \phi_n^* \, x^2 \, \phi_n \, dx$$

$$= \frac{2}{L}\int_0^L x^2 \sin^2\left(\frac{n\pi}{2}x\right) dx$$

$$\Rightarrow L^2\left(\frac{1}{3} - \frac{1}{2n^2\pi^2}\right)$$

$$\langle x\rangle = \frac{2}{L}\int_0^L x \sin^2\left(\frac{n\pi}{2}x\right) dx$$

$$= L/2$$

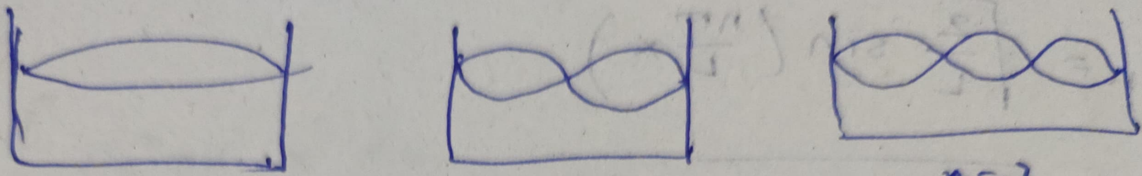
$$\Delta x = \sqrt{L^2\left(\frac{1}{3} - \frac{1}{2n^2\pi^2}\right) - \frac{L^2}{4}} = \sqrt{\frac{L^2}{4n^2\pi^2}\left(\frac{n^2\pi^2}{3} - 2\right)}$$

$$\langle p^2\rangle = \hbar^2 \left(\frac{n\pi}{L}\right)^2 \left(\frac{2}{L}\right) \int_0^L \sin^2 \frac{n\pi x}{L} \, dx$$

$$\Rightarrow \hbar^2\left(\frac{n\pi}{L}\right)^2$$

$$\Delta P = \sqrt{\hbar^2\left(\frac{n\pi}{L}\right)^2 - 0^2} = \hbar\frac{n\pi}{L}$$

$$\boxed{\therefore \Delta x \cdot \Delta P = \hbar\sqrt{\frac{n^2\pi^2 - 6}{12}}}$$



$n=1$ $n=2$ $n=3$

$$\lambda = \frac{2L}{n}$$

$$\frac{h}{p} = \frac{2L}{n}$$

$$\frac{p^2}{2m} = \frac{n^2 h^2}{4L^2 \cdot 2m}$$

$$\boxed{E = \frac{n^2 \hbar^2 \pi^2}{2mL^2}}$$

$$\boxed{\frac{\quad}{2mL^2}}$$

- There exists a non-zero zero point energy because energy can't be zero because then $P = 0$. as p is zero but $\Delta x = \frac{L}{2}$, this would break the uncertainity principle. ~~Hen~~