

Lecture 3: Projection of Planes

(Chapter 12, N.D. Bhatt)

ME 119

Time Table

- 12th Doubt Clearing session 2-3 PM
- 15th is holiday
- 19th Quiz+ sheet 3 submission
- 22nd Fourth sheet,
- 26th Sheet 4 submission and Quiz 4
- 3rd May is Mid Sem 2-5 PM (Safe app + Ms teams)

Motivation

- The surfaces of many 3D objects can be represented by planes

Typical Problems

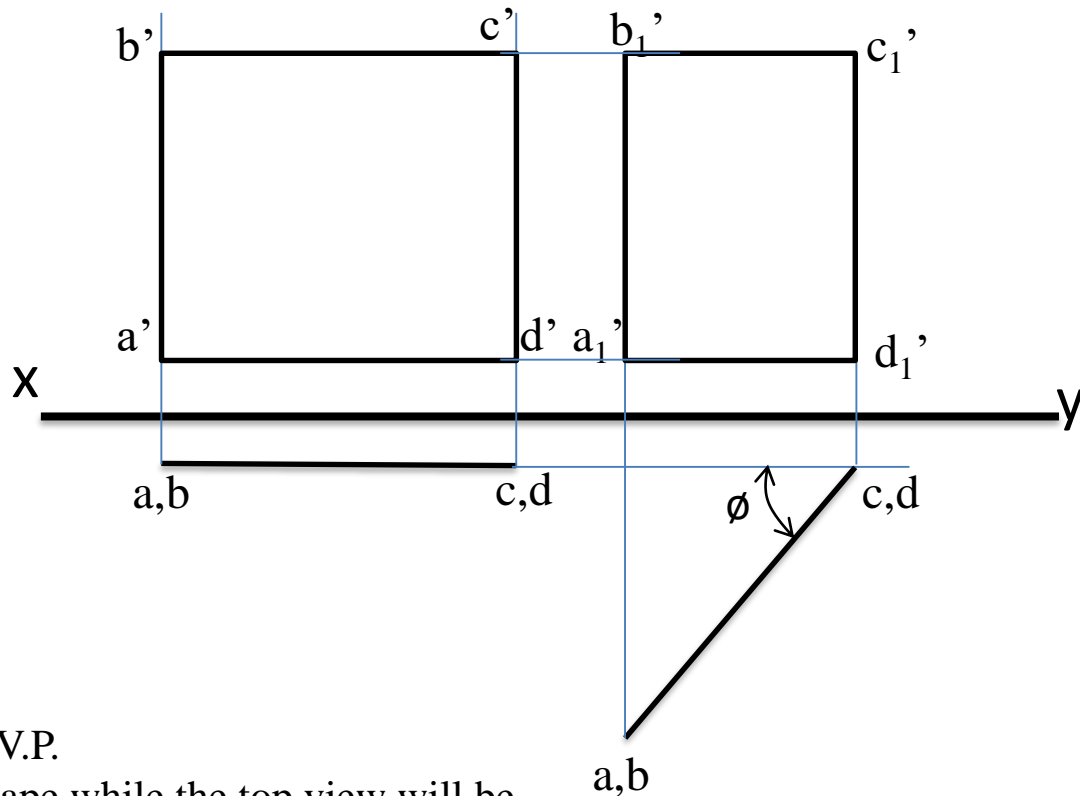
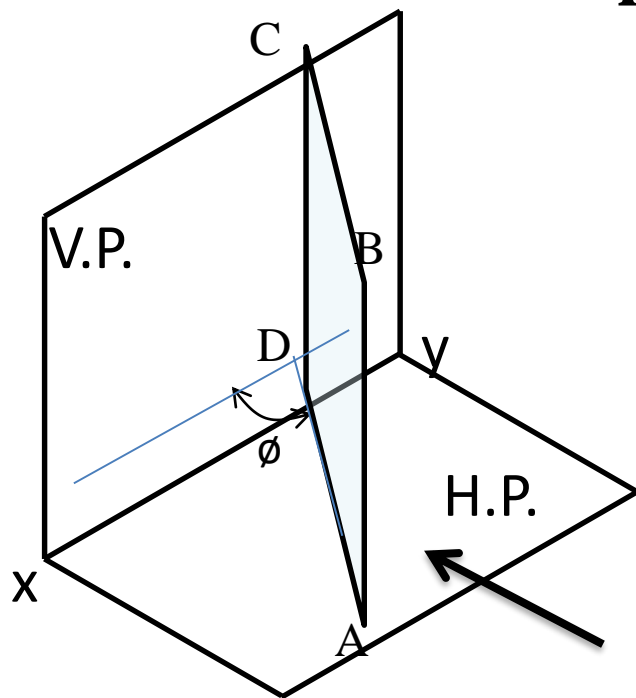
- To find the true shape of a plane given its principal views
- To find the angle of inclinations of the plane with the principal planes given its principal views
- To draw the principal views of a plane given its true shape and orientation

Types of Planes

Depending on its orientation with respect to the reference planes, a plane can be classified as

- Perpendicular plane
 - Perpendicular to both the reference planes (profile plane)
 - Perpendicular to one and parallel to the other
 - Perpendicular to H.P. and parallel to V.P.
 - Perpendicular to V.P. and parallel to H.P.
 - Perpendicular to one and inclined to the other
 - Perpendicular to H.P. and inclined to V.P.
 - Perpendicular to V.P. and inclined to H.P.
- Oblique plane
 - Inclined to both the reference planes

Projection of a Plane Inclined to the V.P. and Perpendicular to the H.P.



- Assume the plane to be parallel to the V.P.
- The front view will show its true shape while the top view will be a line parallel to the xy line
- The plane is tilted so that it is inclined to the V.P. at the given angle
- The new top view will be inclined to the xy axis at the true inclination
- In the front view the corners of the plane will move parallel to the xy axis

The diagram illustrates the orthographic projection of a line inclined to the horizontal plane (H.P.). It is divided into two parts:

- Left Part (3D View):** Shows a line segment AB in the horizontal plane (H.P.) and its projection $A'B'$ in the vertical plane (V.P.). The line is inclined to the H.P. at an angle ϕ . The projection is labeled $V.P.$ and $H.P.$. The line is labeled AB and $A'B'$. The angle of inclination is labeled ϕ .
- Right Part (2D Projection):** Shows the orthographic projection of the line segment AB onto the H.P. and V.P. The line is represented by a horizontal line segment ab in the H.P. and a vertical line segment $a'b'$ in the V.P. The angle of inclination ϕ is shown between the horizontal line ab and the vertical line $a'b'$. The projection is labeled a, b, c, d and a', b', c', d' .

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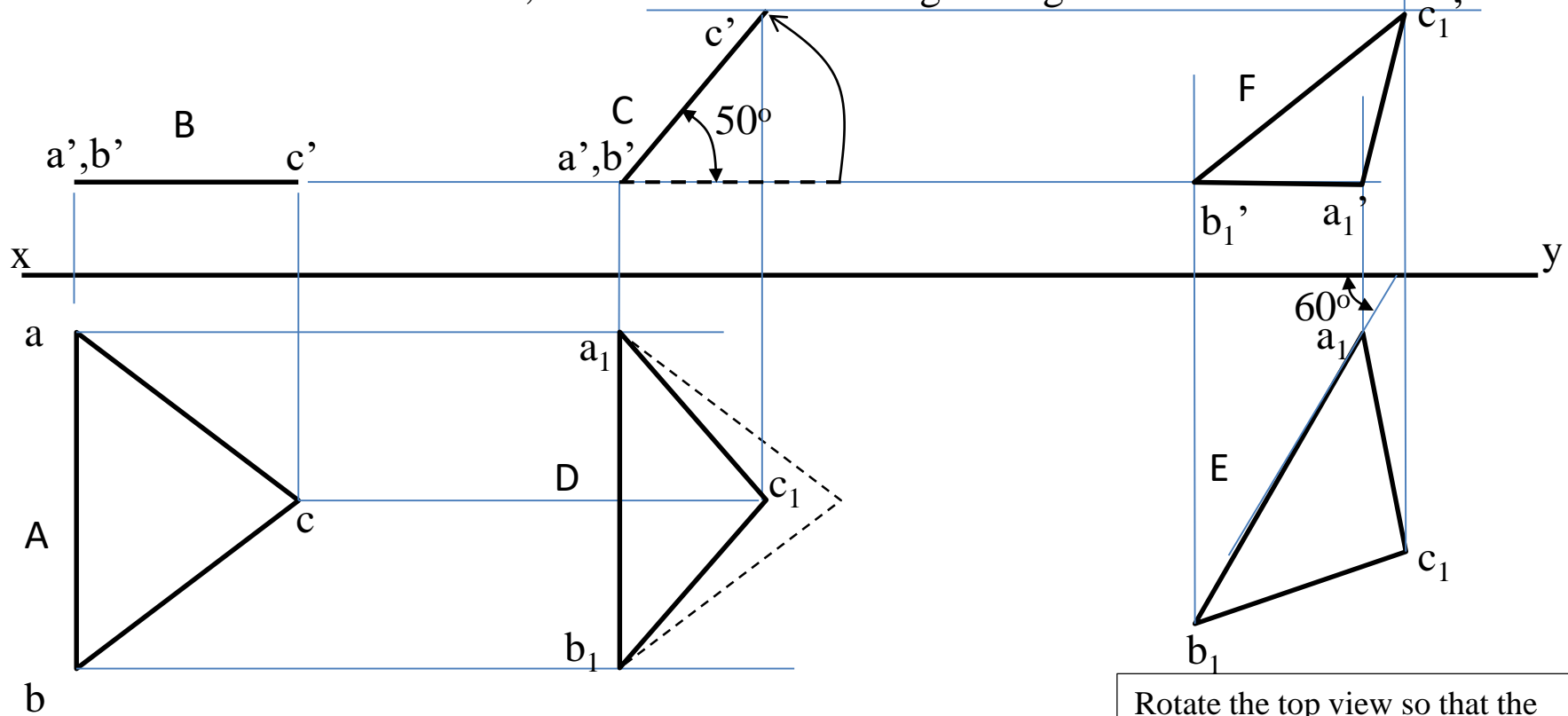
Summary: Projection of a plane inclined to one reference plane and perpendicular to the other reference plane

Projection of the plane is performed in 2 stages

1. Assume the plane parallel to that reference plane to which it is to be made inclined
2. Tilt the plane to the required inclination

Projection of Oblique Planes

Problem: Draw projections of an isosceles triangle ABC with side AB (base) parallel to the H.P. and inclined at 60° to the V.P., and the surface making an angle of 50° with the H.P.,



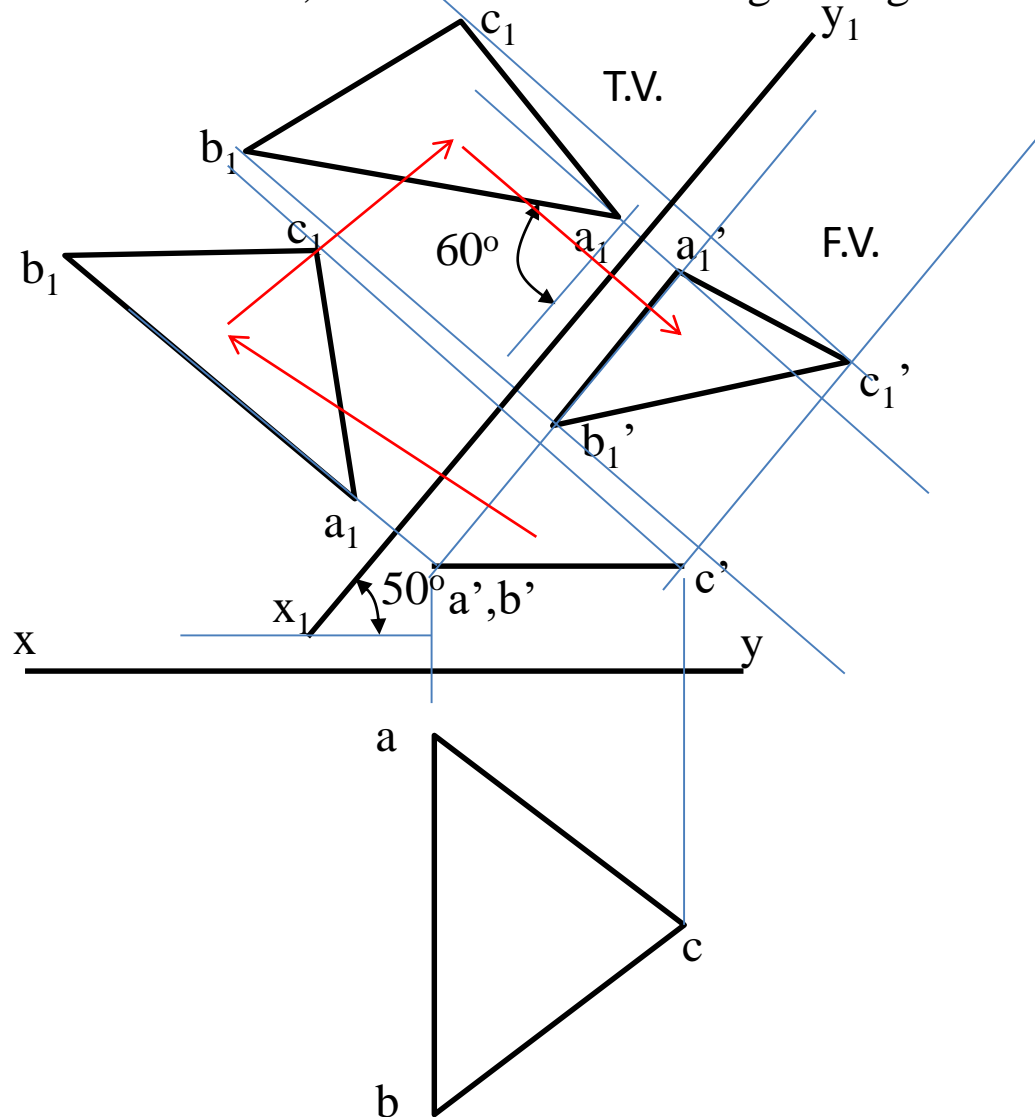
Assume that the triangle is parallel to the H.P. with edge AB perpendicular to the xy axis. The top view shows its true shape and size and the front view is a line parallel to the xy axis

Tilt the triangle so that it makes the required angle with the H.P. Draw the new front view of triangle. It is inclined at the true angle with the H.P. Draw the new top view by moving points parallel to the xy axis

Rotate the top view so that the edge AB makes the required angle with the V.P. The shape and size of the top view does not change. In the front view the distances of the corners from xy remain the same as the second front view

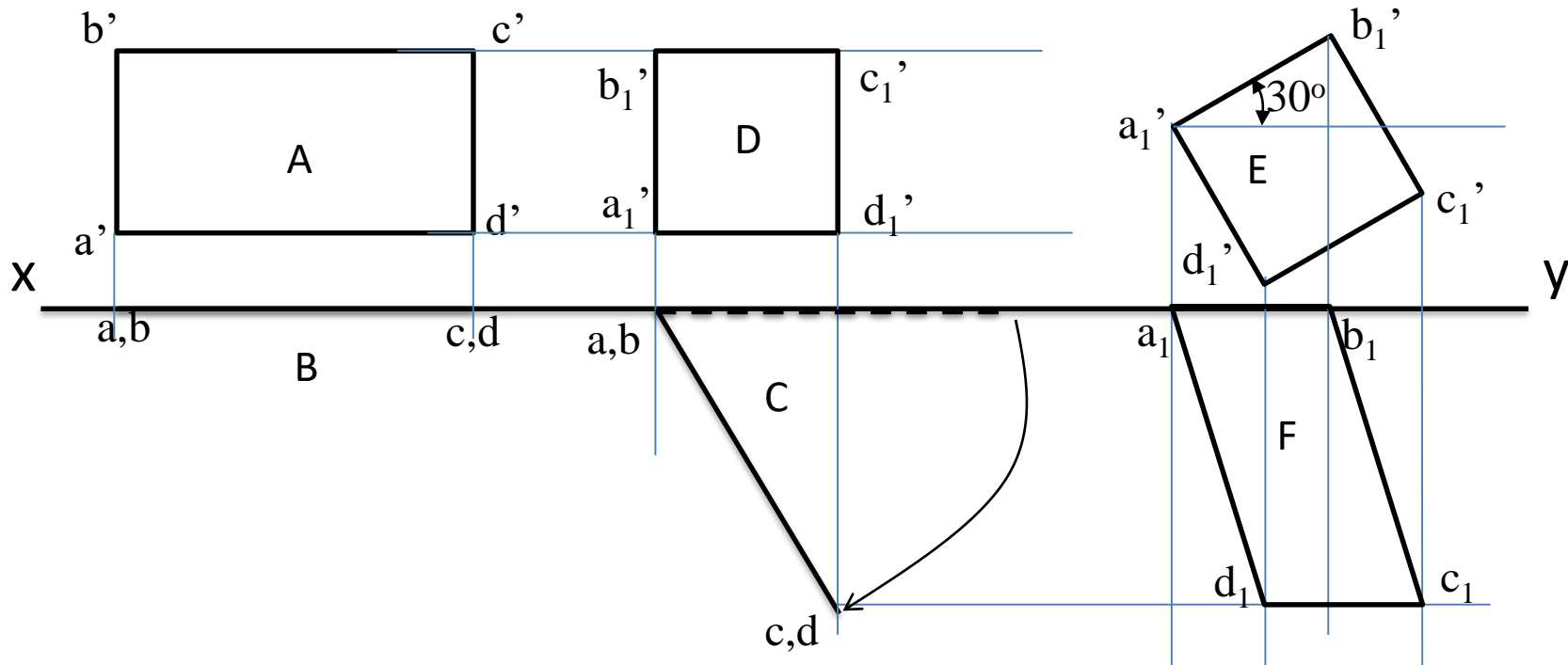
Projection of Oblique Planes Using Auxiliary Planes

Problem: Draw projections of an isosceles triangle ABC with side AB (base) parallel to the H.P. and inclined at 60° to the V.P., and the surface making an angle of 50° with the H.P.



Projection of Oblique Planes

Problem: A thin rectangular plane of sides 100 mm x 50 mm has its shorter side in the V.P. and Inclined at 30° to the H.P. Project its top view if the front view is a square of 50 mm long sides



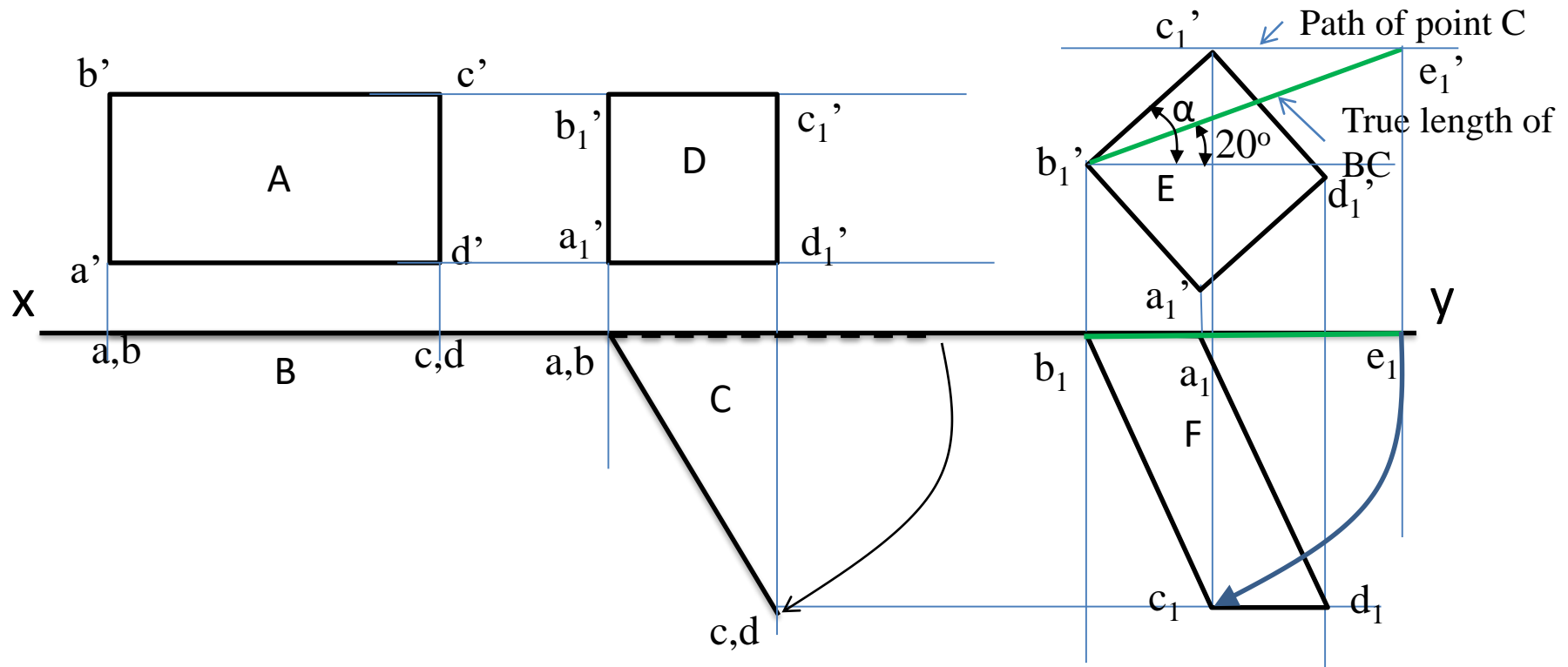
Assume that the rectangle is in the V.P. with edge AB perpendicular to the xy axis. The front view shows its true shape and size and the top view lies in the xy axis

The plane is inclined to the V.P. as the front view is not of true shape and size. Tilt the rectangle about the edge AB so that the new front view appears as a square. It is inclined at the true angle with the V.P. Draw the new front view by moving points parallel to the xy axis

Rotate the front view so that the edge AB makes the required angle with the H.P. The shape and size of the front view does not change. In the top view the distances of the corners from xy remain the same as the second top view

Projection of Oblique Planes

Problem: A thin rectangular plane of sides 100 mm x 50 mm has its shorter side in the V.P. The longer side is inclined to 20° to the H.P. Project its top view if the front view is a square of 50 mm long sides



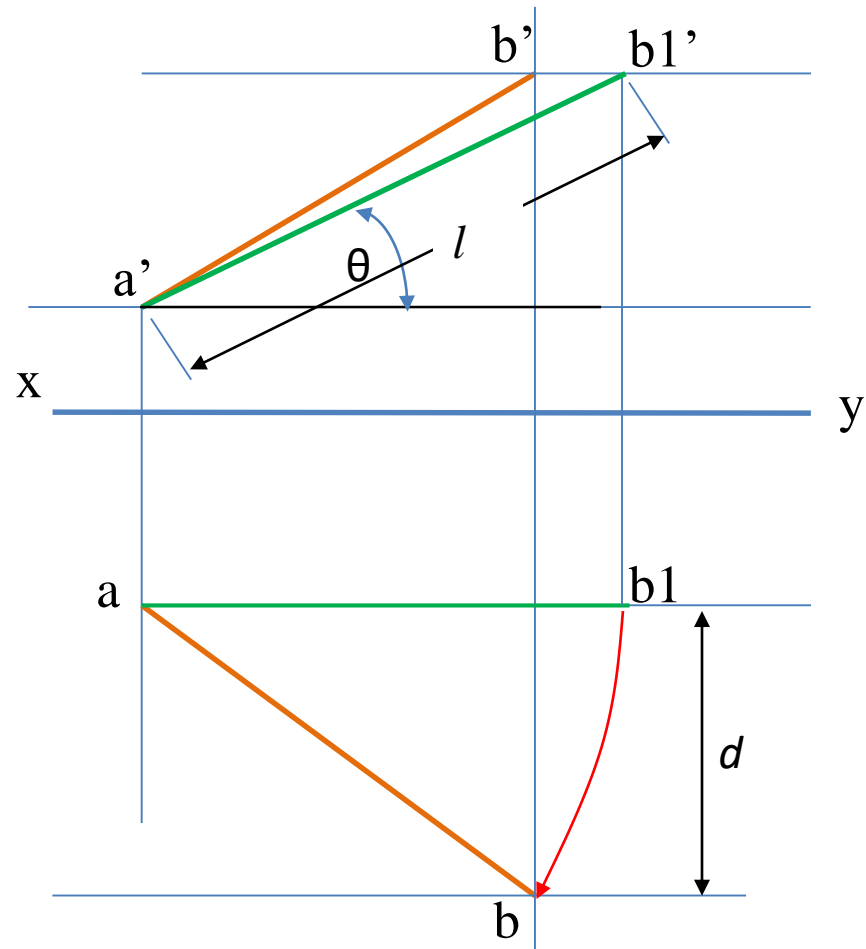
First two steps are the same as the previous problem

Rotate the front view so that the edge BC makes the required angle with the H.P. Length $b_1' c_1'$ does not represent the true length of side BC. Therefore the apparent of BC with xy axis (α) needs to be found out.

Rotate the top view such that $b_1' c_1'$ makes an angle α with the xy axis. The shape and size of the front view does not change. In the top view the distances of the corners from xy remain the same as the second top view

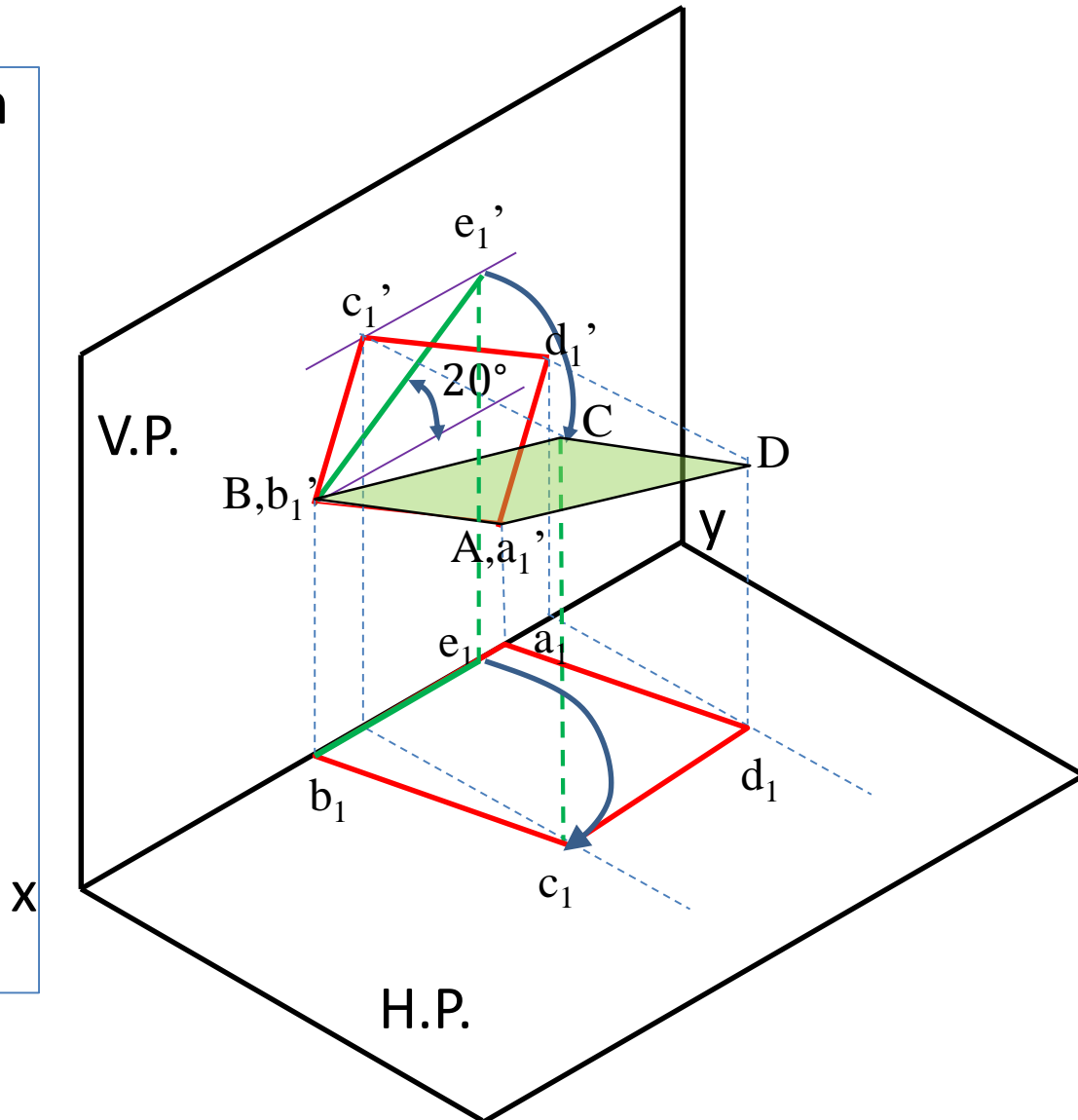
Explanation of Solution

- To understand the solution, recall from Sheet 3
 - Given true length of a line (l), true angle θ w.r.t. H.P., and distance d between horizontal projectors on T.V., draw projections of line
 - Main concept: Once the true line is rotated by true angle θ w.r.t. H.P., the point B has to be rotated around A keeping height of B from H.P. a constant
- In this example problem, true length of BC (100 mm) is given, it's angle w.r.t. H.P. (20°) is given, and distance of point C from V.P. is given (via T.V. of stage ii)



3D View of The Problem

- The 3D plane is shown in **green shade**
- The true length line is shown in **green**
- When $b'_1e'_1$ is rotated to b'_1C , the height of e'_1 and C from H.P. is kept the same.
- The T.V. and F.V. projections are shown in **red lines**
- e'_1C is a circular arc parallel to H.P.

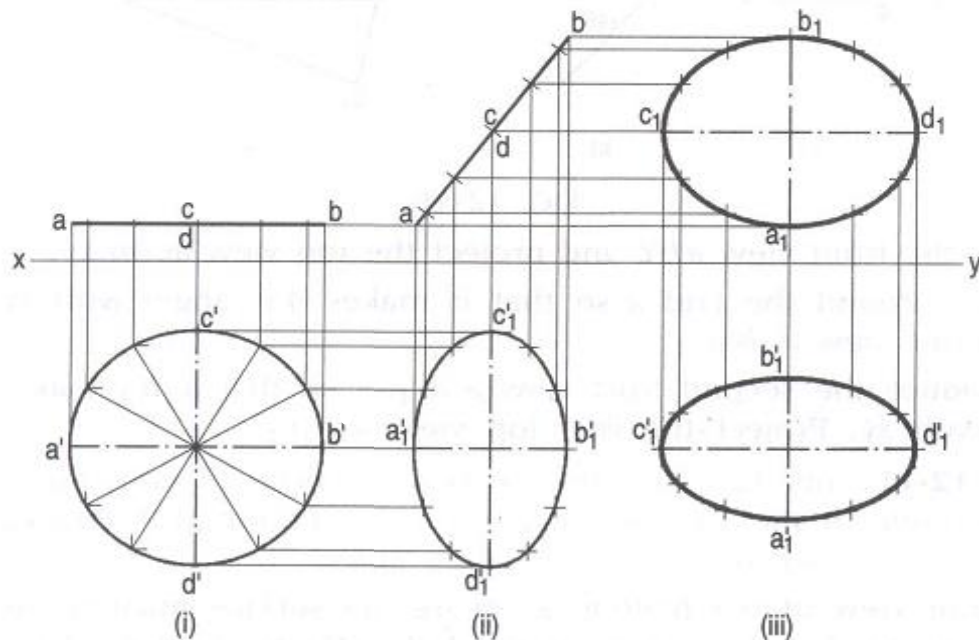


Problem: A thin circular plate of 50 mm diameter appears as an ellipse with major axis 50 mm and minor axis 30 mm in the top view. Draw the projections when the major axis appears horizontal

Stage (i): Draw true shape of the circle in T.V. and draw corresponding edge ab in F.V.

Stage (ii): Mark $a_1'b_1'=30$ mm, and tilt ab so on F.V. so that a and b lie on vertical projectors from a_1' and b_1' . Mark the projection of other points on the circle on the F.V. using the F.V. of stage (i)

Stage (iii): Turn T.V. of stage (ii) by 90° to get new T.V. Now use projectors from F.V. of stage (ii) and T.V. of stage (iii) to get new F.V.



Summary: Projection of an Oblique Plane

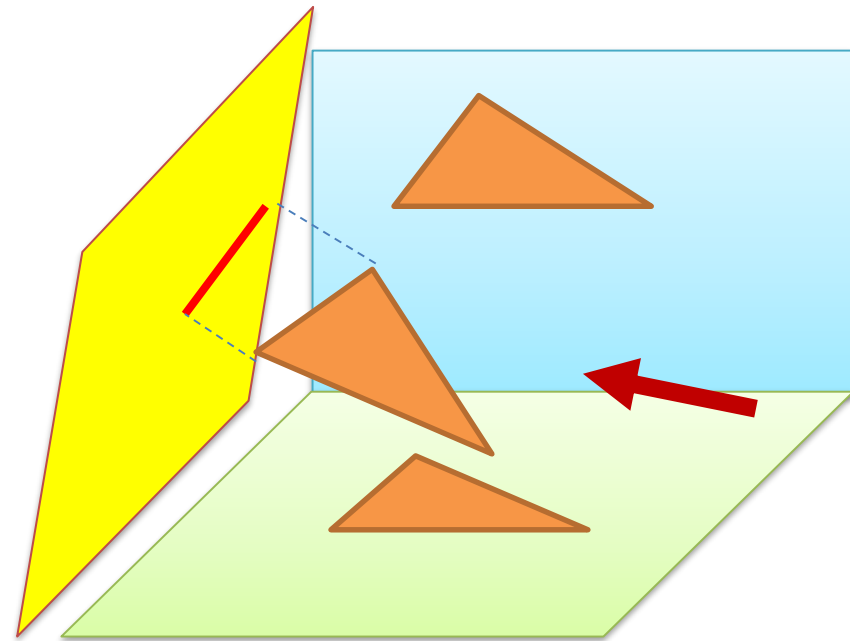
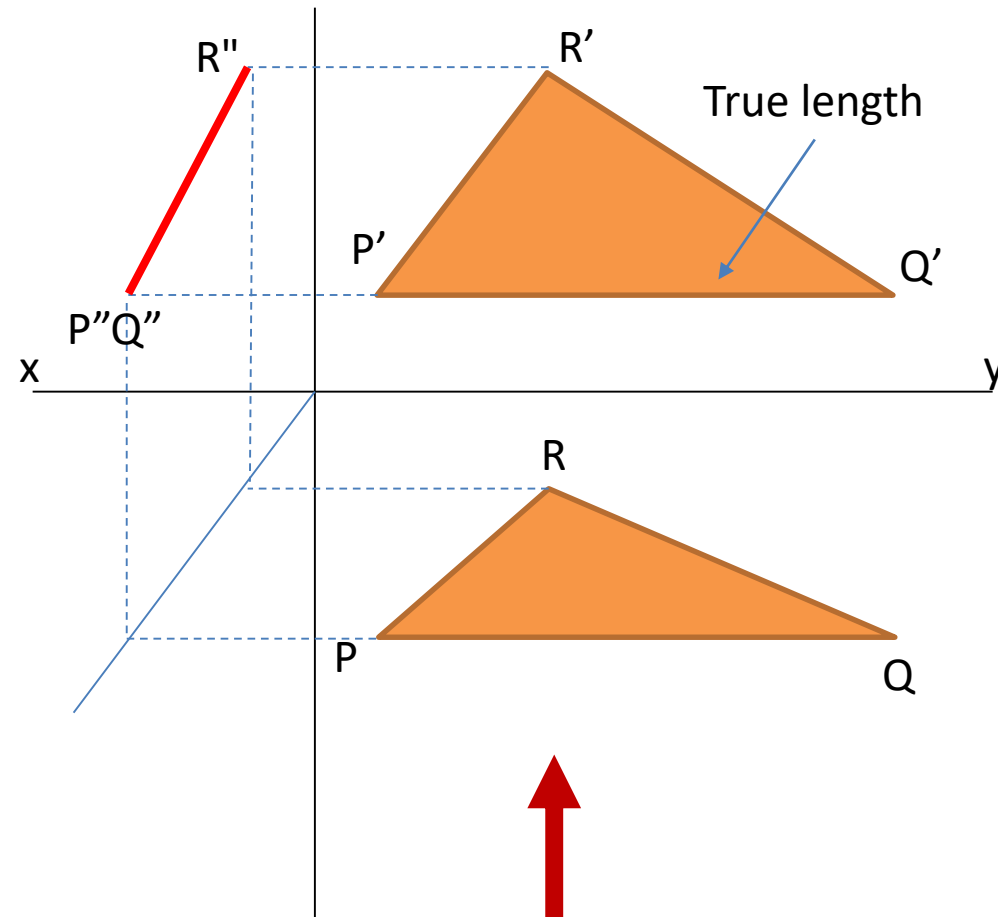
Given: True shape of the plane, the angle of inclination with one of the reference planes, the angle an edge makes with the reference plane

1. Assume the plane parallel to that reference plane to which it is to be made inclined. Draw one edge perpendicular to the xy axis. The true shape will be seen in one of the views while the other view will appear as a line.
2. Tilt the plane about the perpendicular edge to the required inclination
3. Rotate the appropriate view so that the edge makes the required angle with the reference axis. Make sure that the angle of rotation is drawn taking into account the true length of the edge. The size and shape of this view does not change during the rotation. Project the other view so that the distances of the corners from the xy axis are preserved

To Find an Edge View of an Oblique Plane

Given: The front view and top view of plane PQR

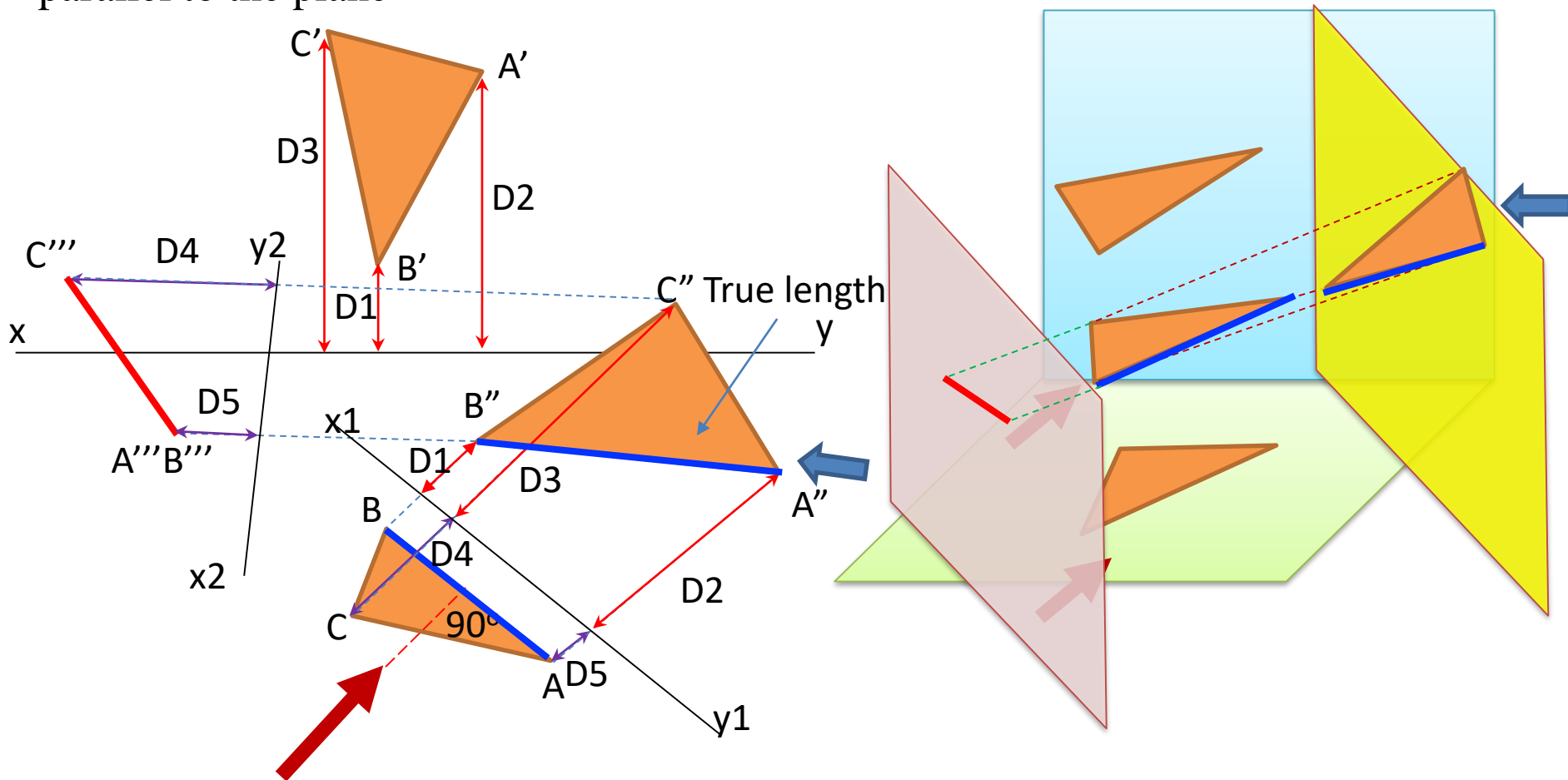
Edge view of a plane is obtained by looking at the plane with the direction of viewing parallel to the plane



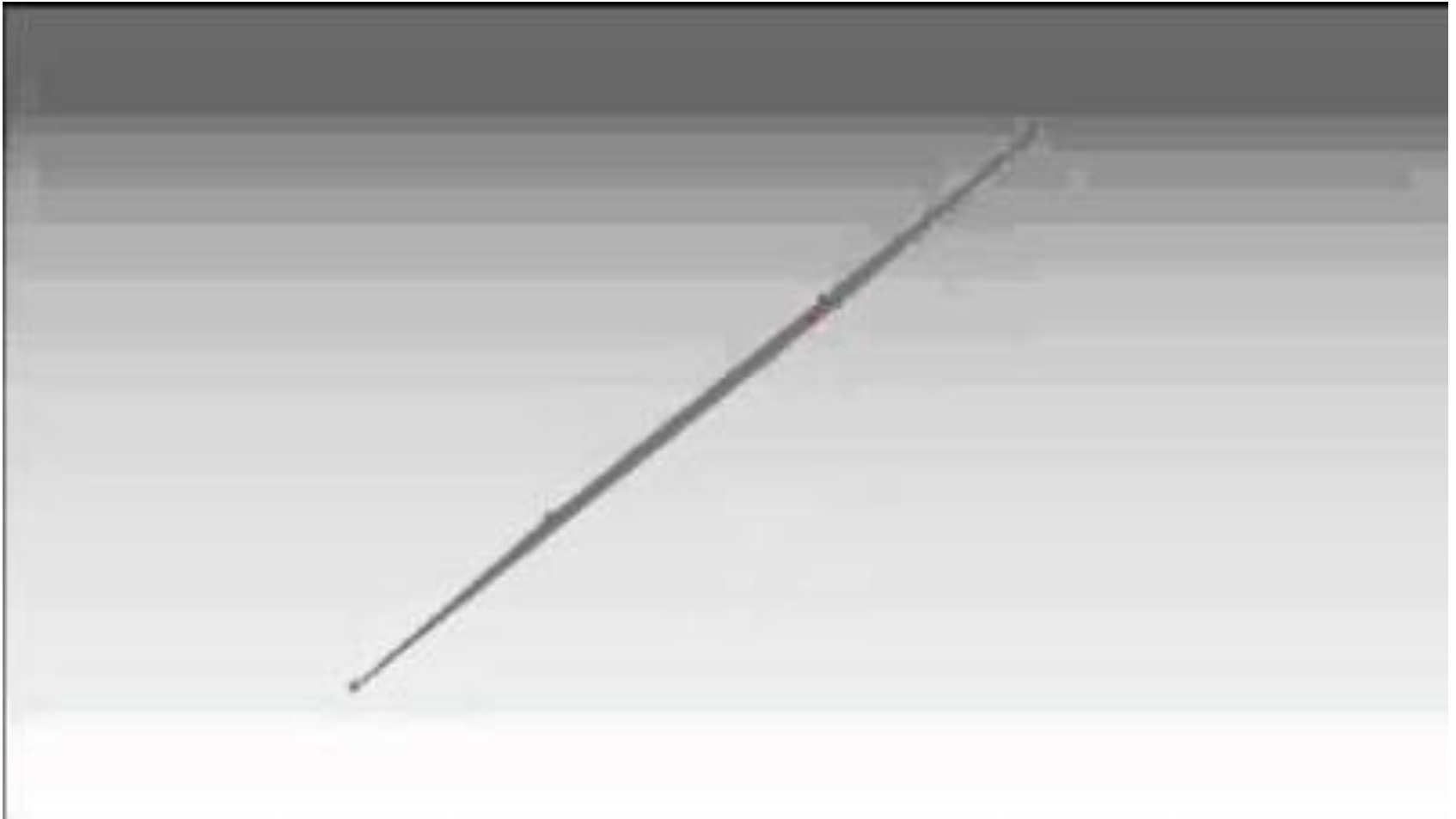
To Find an Edge View of an Oblique Plane

Given: The front view and top view of plane PQR

Edge view of a plane is obtained by looking at the plane with the direction of viewing parallel to the plane



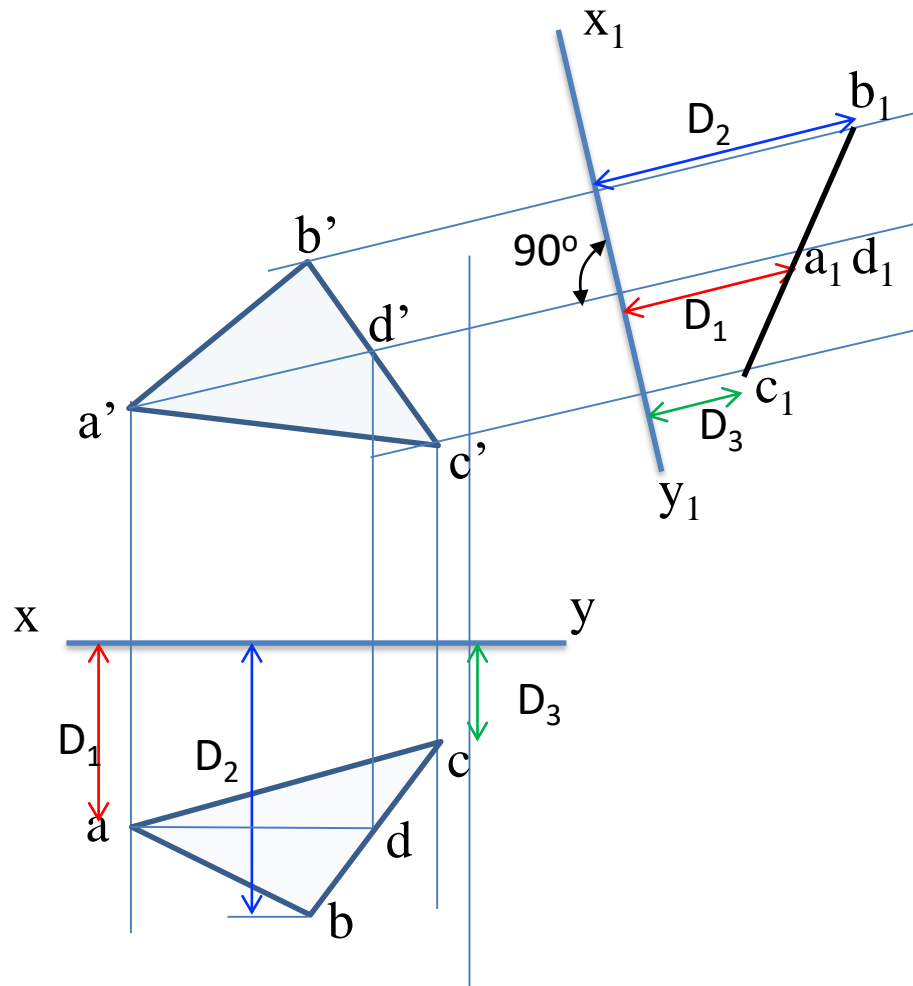
Edge View of a plane



To Find the Edge View of an Oblique Plane

Given: The front view and top view of plane ABC

Edge view of a plane is obtained by looking at the plane with the direction of viewing parallel to the plane



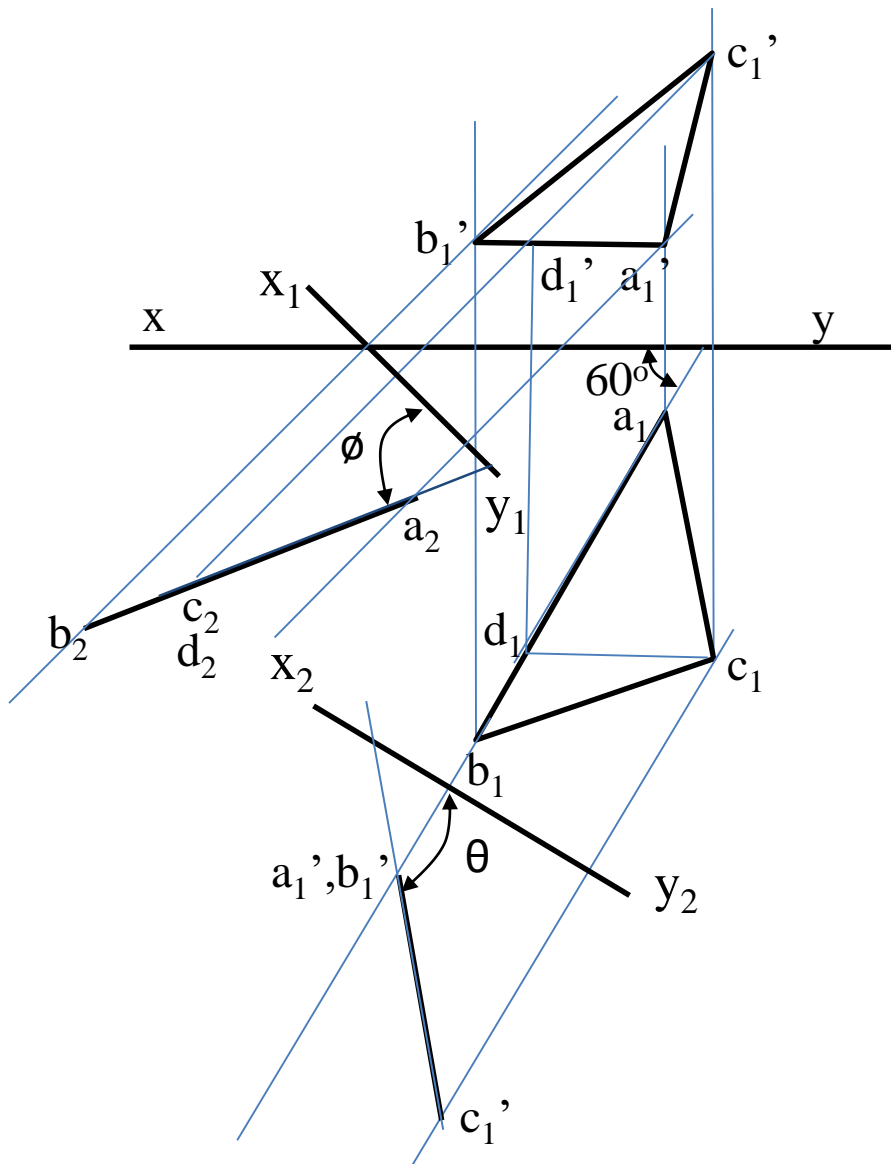
To find the edge view of ABC we need to find a plane perpendicular to it

A plane which is perpendicular to the true length view of a line is perpendicular to the plane containing the line

Therefore construct a line $a'd'$ which is a true length view. Do this by drawing a line ad parallel to the xy in the other view. Then construct a plane perpendicular to true length view. This is represented by x_1y_1 . Draw projectors from a' , b' and c' perpendicular x_1y_1 . Locate points a_1 , b_1 and c_1 by transferring the depth dimensions. This is the edge view of the plane ABC

Projection of Oblique Planes

Problem: To find the angle of the inclination of the plane with the reference planes



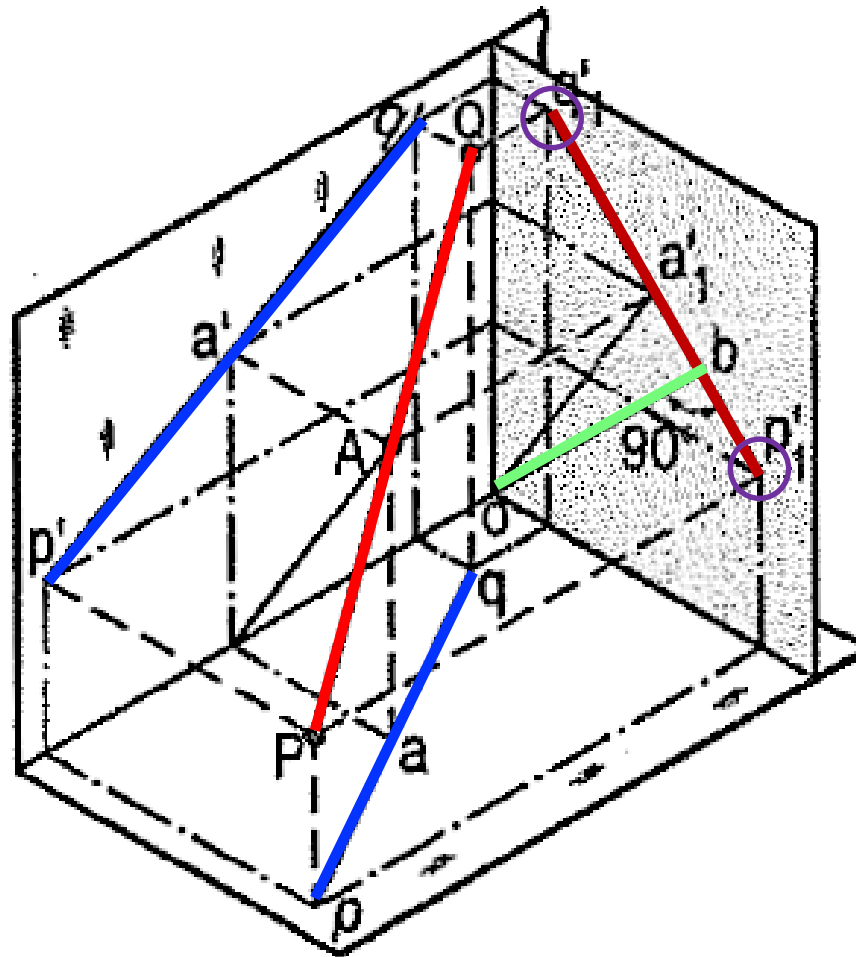
-Draw the front view and the top view of the plane.

-Draw the edge view of the plane using the front view. This gives the angle of inclination with the V.P. (ϕ)

-Draw the edge view of the plane using the top view. This gives the angle of inclination with the H.P. (θ)

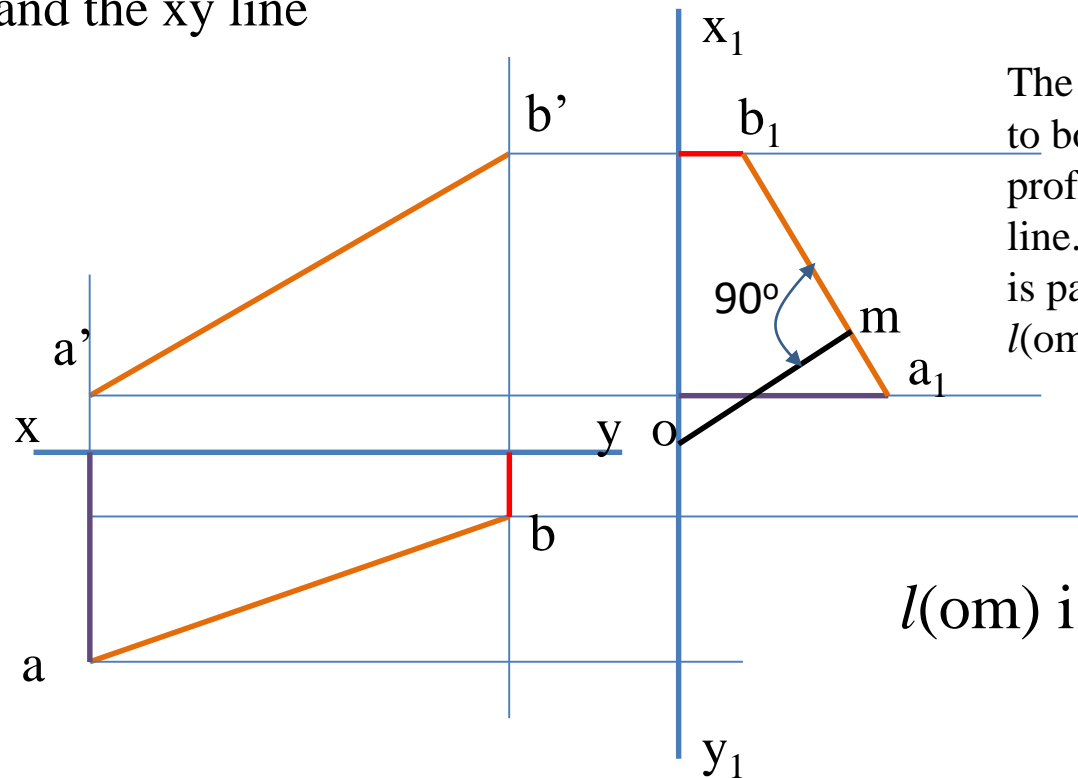
To Determine the Shortest Distance Between a Line and the XY Line: 3D visualization

Given the front view and the top of a line PQ, determine the shortest distance between the line and the xy line



To Determine the Shortest Distance Between a Line and the XY Line

Given the front view and the top of a line AB, determine the shortest distance between the line and the xy line



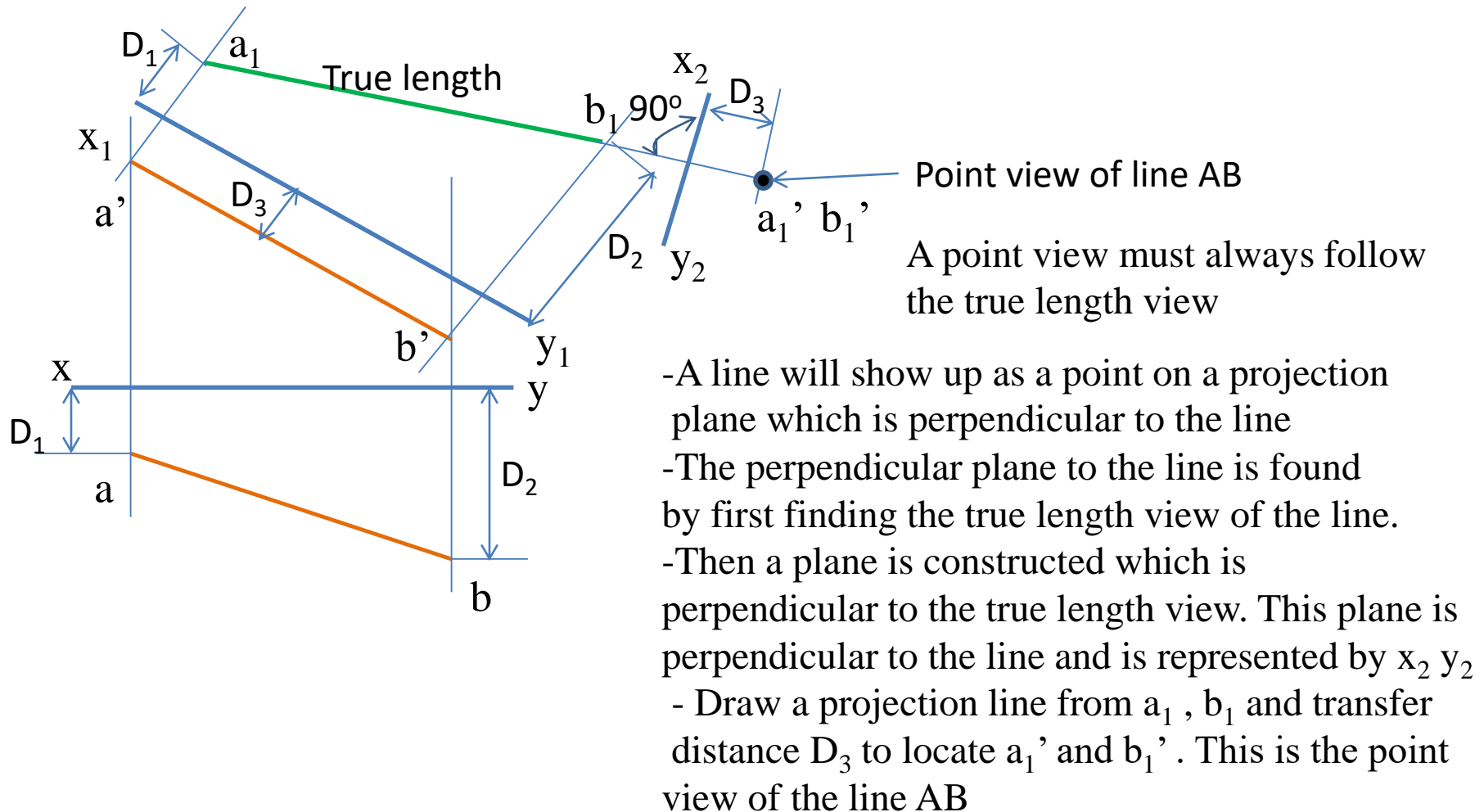
The shortest segment is perpendicular to both the xy line and line AB. The profile plane is perpendicular to the xy line. Hence the shortest segment is parallel to profile plane. Therefore $l(om)$ is the true length

$l(om)$ is the required length

To determine the shortest distance between any two non-intersection lines, it is necessary to find the point view of one of the lines. In the above example 'o' is the point view of the xy line

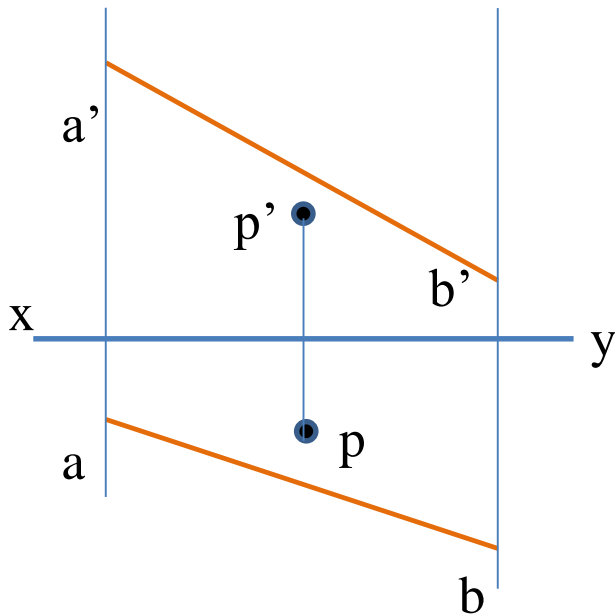
To Find the Point View (Point Projection) of a Line

Given: The front view and top view of the line AB



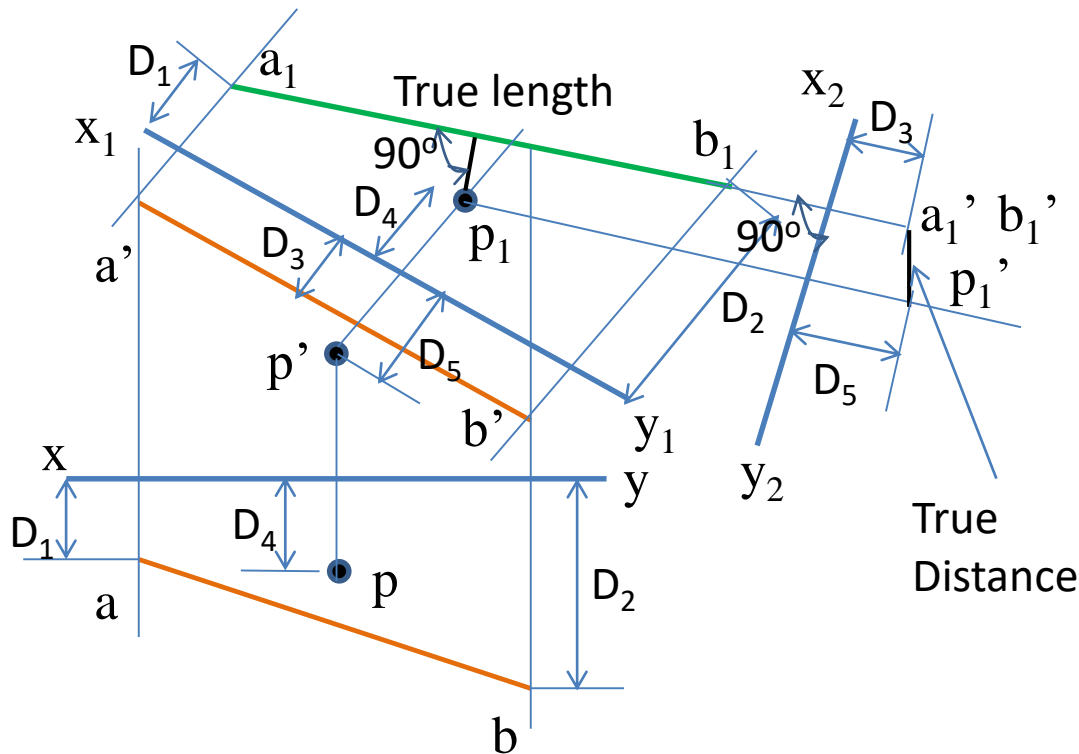
To Find the Shortest Distance Between a Line and a Point

Given: The front view and top view of the line AB and point P



To Find the Shortest Distance Between a Line and a Point

Given: The front view and top view of the line AB and point P



- The shortest distance between a given point and a given line is measured along the perpendicular drawn from the point to the line
- Lines that are perpendicular will have their projections shown perpendicular in a view showing either or both lines in true length

- Draw the true length view of the line AB
- Obtain the projection of the point P in the same view
- The segment which is perpendicular to the line from the point P, will appear perpendicular in this view as the true length of line AB is seen
- Draw a plane perpendicular to the true length view. This is represented by x_2y_2
- Locate the point view of line AB
- Since the segment from the point to the line is parallel to x_2y_2 , its projection in the other view will show its true length. This is the distance of the point P from the line AB

Important Points to Remember

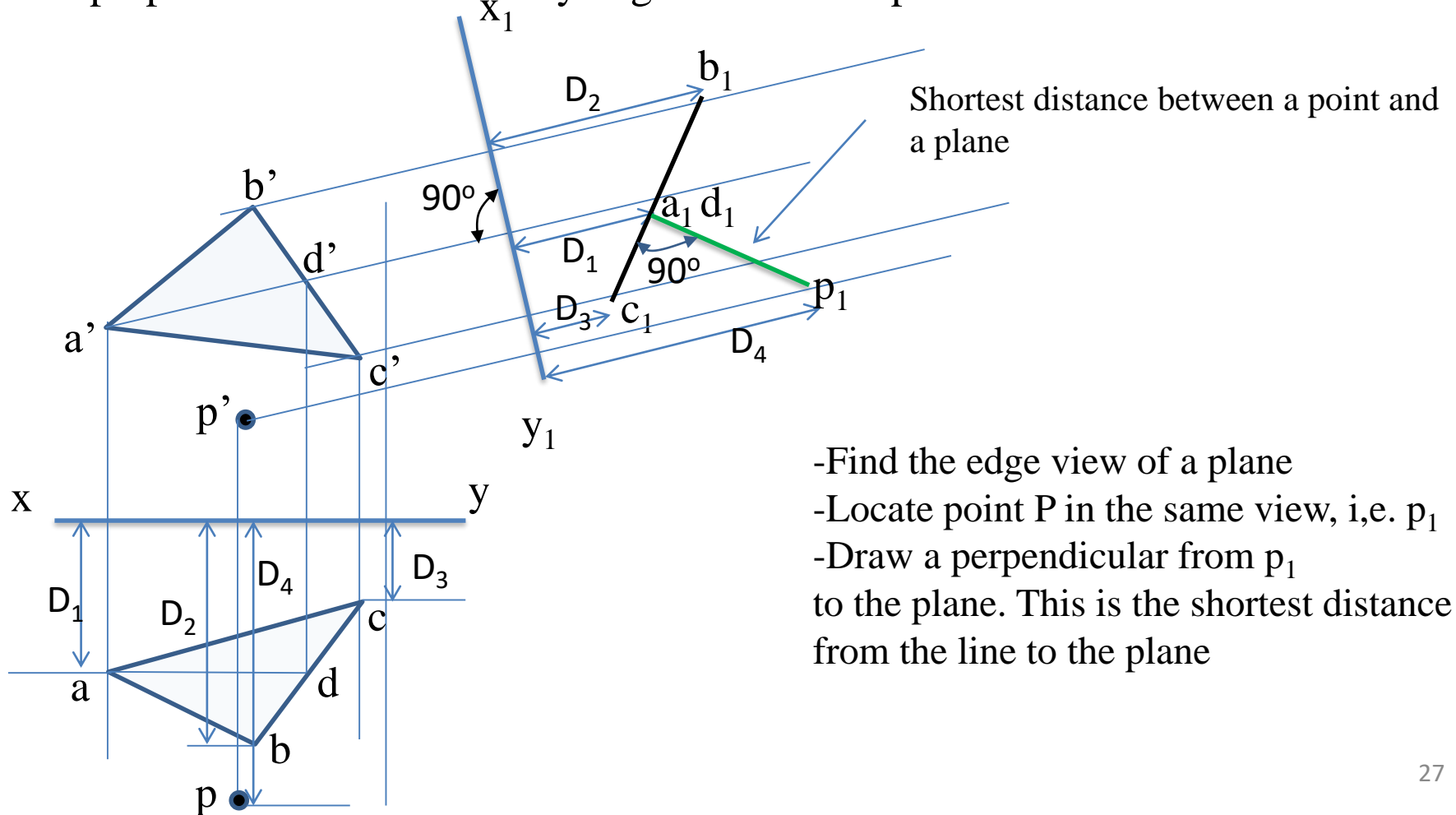
- When one view of a line is parallel to a reference line, the other view represents the true length
- A line will appear as a point when it projected on a plane perpendicular to it
- The shortest distance between a given point and a given line is measured along the perpendicular drawn from the point to the line
- Lines that are perpendicular in space will have their projections perpendicular in any view which shows either or both of the lines in true lengths

To Find the Shortest Distance from a Point to an Oblique Plane

Given: The front view and top view of plane ABC and a point P

The shortest distance is measured along the perpendicular from the point to the plane

The perpendicular is seen in any edge view of the plane

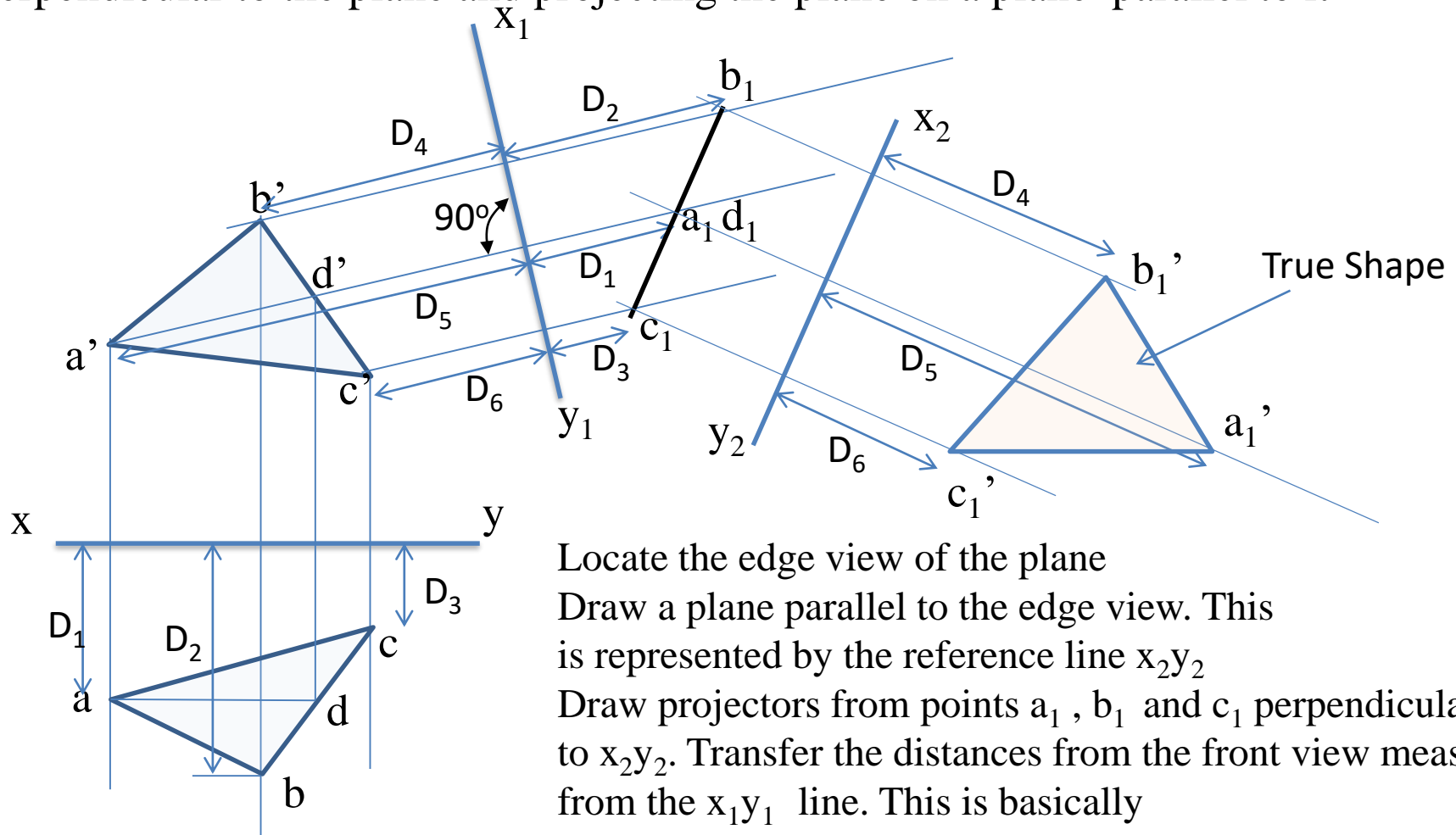


- Find the edge view of a plane
- Locate point P in the same view, i.e. p_1
- Draw a perpendicular from p_1 to the plane. This is the shortest distance from the line to the plane

To Determine the True Shape and Size of an Oblique Plane

Given: The front view and top view of plane ABC

The true shape and size of a plane can be seen by viewing in the direction perpendicular to the plane and projecting the plane on a plane parallel to it



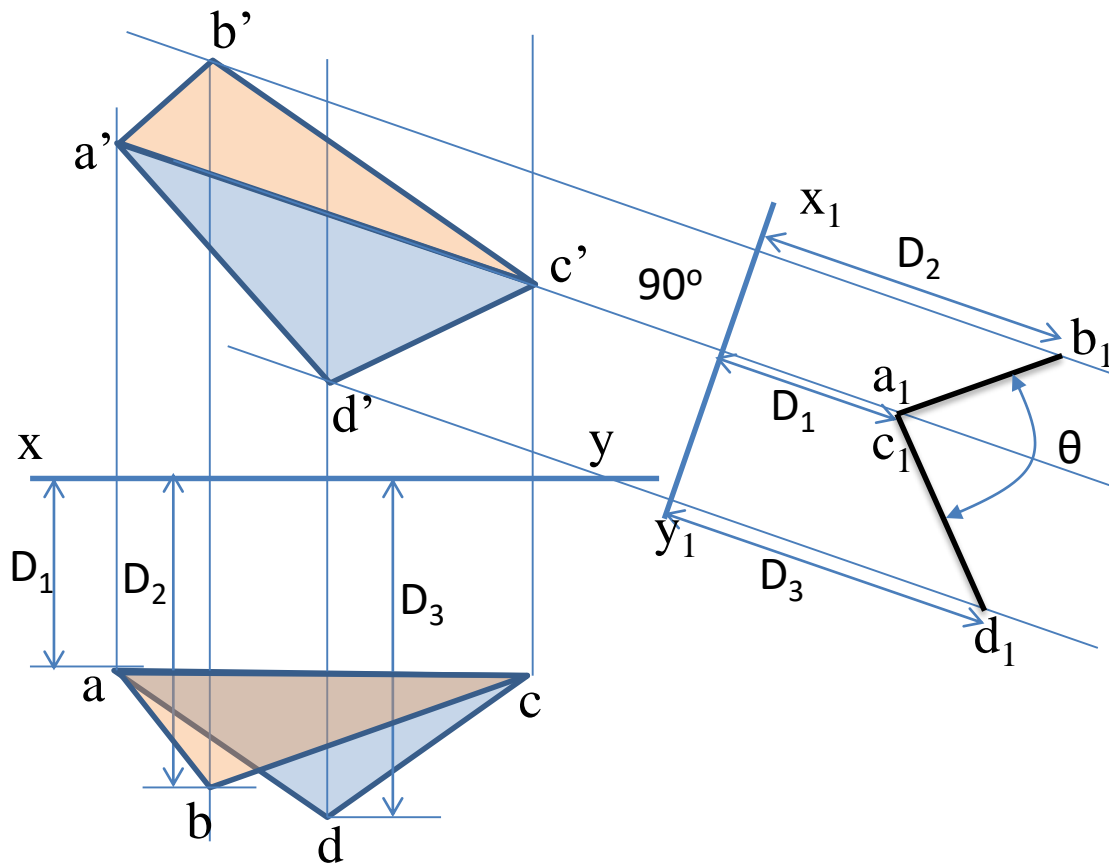
Locate the edge view of the plane

Draw a plane parallel to the edge view. This is represented by the reference line x_2y_2

Draw projectors from points a_1 , b_1 and c_1 perpendicular to x_2y_2 . Transfer the distances from the front view measured from the x_1y_1 line. This is basically equivalent to projecting ABC on a plane parallel to it. This represents the true shape of the plane

To Find the Angle Between Two Intersecting Planes

Given: The front view and top view of two intersecting planes, ABC and ABD



- The angle of intersection can be seen on a plane perpendicular to both the planes
- The two planes will appear as two intersecting lines with the angle between them equal to the angle between the planes

- To find a plane perpendicular to both the planes, find a plane perpendicular to the intersection line

- This is done by finding the true length view of the intersection line.

- Then a plane is constructed perpendicular to the true length view.

This is represented by $x_1 y_1$

- Draw projectors from $a', b' c'$ and d' perpendicular x_1y_1 . Locate points $a_1, b_1 c_1$ and d_1 by transferring the depth dimensions.

This gives the edge views of the planes ABC and ADC. The angle can now be measured

In this particular figure as ac , the line of intersection, is parallel to the xy line, $a'c'$ is the true length view of the line

Points to Remember

1. If the true shape and size of a plane is seen in its front (top) view, then it is parallel to the V.P. (H.P.).
2. If the front (top) view is not the true shape, then the plane is inclined to the V.P. (H.P.)
3. If a line is in the V.P. (H.P.), its true length is seen in the front (top) view
4. If a plane or a line does not change its relation with the reference plane (e.g. a plane or a line which is perpendicular to a plane remains perpendicular), the projection on that reference plane does not change in size and shape.
4. The angle of inclinations of the plane can be obtained by drawing edge view using the front view and the top view. The edge views are the auxiliary front and auxiliary top views
5. To obtain the true shape of a plane given its projections, one needs to proceed by first drawing the edge view of the plane. Then the true shape is obtained by drawing an auxiliary view using a reference line drawn parallel to the edge view

END