Compton Effect

Consider a free moving particle with **rest mass** m_0, moving with momentum $\vec{p}$. According to special relativity, its energy is given by

$$E = \sqrt{|\vec{p}|^2 c^2 + m_0^2 c^4},$$

where c is the speed of light. For non-relativistic motion, the speed of the particle is much less than the speed of light. One can show that this is equivalent to $|\vec{p}| \ll m_0 c$. Then the expression for the energy of the particle becomes

$$\begin{aligned} E &= m_0 c^2 \sqrt{1 + \frac{|\vec{p}|^2}{m_0^2 c^2}} \\ &\approx m_0 c^2 + \frac{|\vec{p}|^2}{2m_0}, \end{aligned}$$

through the use of binomial theorem.

From the relativistic point of view, the energy of a free moving non-relativistic particle is $m_0 c^2 + |\vec{p}|^2/(2m_0)$. Since $m_0 c^2$ is a constant which is always there, we ignore it and denote the energy of such a particle to be $|\vec{p}|^2/(2m_0)$.

The relation between E and $\vec{p}$ holds for massless particles also. For photons, we have $E = |\vec{p}|c$. A photon necessarily has non-zero energy and non-zero momentum (which are proportional to each other).

In both photoelectric effect and Compton effect, we have photons interacting with charged particles. They are light photons in the case of photoelectric effect and X-ray photons in the case of Compton effect. Both light and X-rays are **electromagnetic radiation**. That is, they consist of electric and magnetic fields. From your study of electricity and magnetism, you know that electric field originates due to electric charge (Coulomb's law) and magnetic field originates due to electric current (Ampere's law). Therefore, one needs an electric charge or at least a charge distribution for the interaction of an electromagnetic radiation.

A full-fledged understanding of both photoelectric effect and Compton effect requires **Quantum Electrodynamics** which describes all aspects of photon-electron interactions. In the discussions of this course, we make some simplifying assumptions to understand these two effects. We are specially interested in understanding how they establish that electromagnetic radiation consists of **Photons**.

The energies of light photons are of the order of a few eV. So their momenta are of the order of eV/c. The electron rest mass energy is 0.511 MeV/c^2. So, when a light photon hits an electron, its momentum changes very little but its energy changes enough for it to escape the metal. So we do not bother about the momentum of the electron or the photon in photoelectric effect and worry only about the energy.

The X-ray photons have energies of the order of keV, which means they have momenta of the order of keV/c. When an X-ray photon hits an electron in a metal, we can't ignore the momentum of the photon. If we assume that the electron absorbs the X-ray photon (as it we do in photoelectric effect) then such an electron will be "unstable" and it regains "stability" by emitting a photon of slightly lower energy. This last statement is somewhat vague. But, making it quantitative requires a lot of details of photon-electron interactions.

Key points in the derivation of Compton wavelength shift $\Delta\lambda$ expression are

- Intial photon energy and momentum are known.
- Intial electron is assumed to be at rest (we will come back to this assumption later).
- Final state photon energy and momentum **are measured** in the experiment.
- Final state electron is **not detected** in the experiment. So its energy and momentum are **not known** apriori.
- Find final state electron energy by applying energy conservation
- Find final state electron momentum by applying momentum conservation **in both x and y directions**
- Finally, demand that final state electron energy and momentum should satisfy $E^2 - |\vec{p}|^2 c^2 = m_0^2 c^4$.

The algebra, which was carried out by Prof. Gopal in video lecture and also in the Modern Physics textbook, gives you the expression for $(\lambda' - \lambda)$.

There are some features in the figure which are not explained by the above derivation.

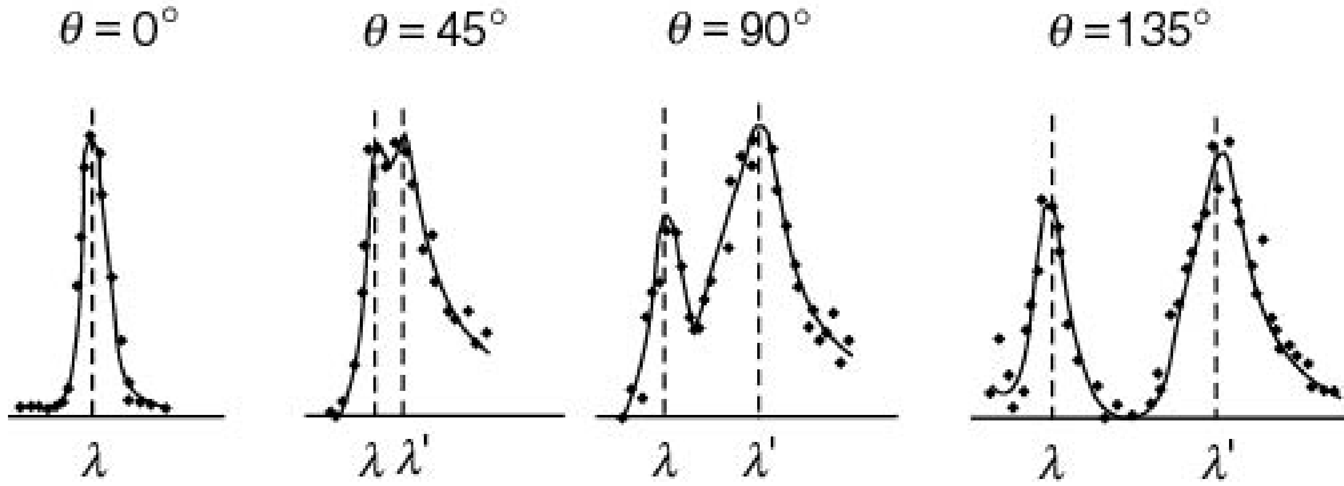

Figure 1: Compton Effect Figures

1. Why is there a peak at λ even when $\theta \neq 0$? We discussed in the class and realized that this occurs when the X-ray photon scatters off the Carbon ion. The Compton wavelength of electron is of the order of 10^{-12} meters and the Compton wavelength of Carbon ion will be about 10^{-16} meters. So, wavelength shift off X-rays, due to scattering of Carbon ions is negligibly small.

2. Why do we get a continuum distribution? There are three possible reasons:

   - The original source emits X-rays, not of a fixed energy (or wavelength) but in a small range of energies.
   - The detector has a non-zero resolution so the measured value is not exactly the value of the scattered X-ray but is spread around it a bit.
   - The assumption that the initial electron is at rest is not fully correct. The electrons in a conductor move about and they have non-zero momentum. To a large extent, this momentum is negligible. However, a spread in the values of initial electron momentum will lead to a spread in the values of the final X-ray photon energy.

3. Why do we have data points with $\lambda' < \lambda$? If the scattered photon has

smaller wavelength, it has more energy. Where does this extra energy come from? If the initial electron has moderate kinetic energy, it is possible that the scattered X-ray photon absorbs this energy and will emerge with larger energy. This is called **Inverse Compton Effect**.

Fundamental Constants

Relativity assumes that the speed of light c is a fundamental constant. Quantum Theory introduced another fundamental constant $\bar{h}$. Because of these two fundamental constants, we have the following **important** consequence: **Every mass defines an energy, a length and a time scale**.