1 Lecture-2

Some notes for the second lecture. Slide-1 only had the titular details.

1.1 Slide-2

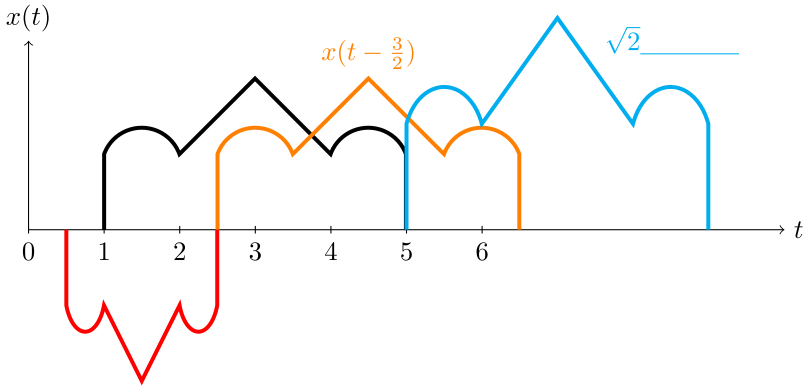

Figure 1: Playing with Signals

The red waveform above is $-x(2t)$. The minus sign was used to obtain a non-overlapping plot.

Exercise 1 *Draw the waveform of $x(\frac{t}{2})$ for the above $x(t)$. Will the signal be compressed in duration or stretched?*

Exercise 2 *Draw the waveform $-x(-2t)$ for the $x(t)$ given in figure.*

1.2 Slide-3

In this slide we showed a sampled sine wave form and talked about GNURADIO. In the sampled waveform, notice the parameter called sampling interval, which we denote as T. For uniform samples (which we will take throughout) the value $\frac{1}{T}$ denotes the number of samples per second, known as the **sample-rate** in GNURADIO and elsewhere.
GNURADIO Installation: In ubuntu the following command will do everything.

```
sudo apt install gnuradio
```

In windows, please use the installers provided at www.gnuradio.org. We use the 3.8 version of gnuradio, but any version will do. Windows people having trouble should go through the trouble shooting guidelines. In the past, creating the correct dll directory and copying the files have helped most people who had initial trouble with installation.

After installing you can search for gnuradio-companion in Windows/Linux, or type gnuradio-companion in a linux terminal. Double clicking will open in the former case, and a window like the one below will appear. Ignore the Windows warnings (first time it complains about xterm etc, do not worry and press OK),

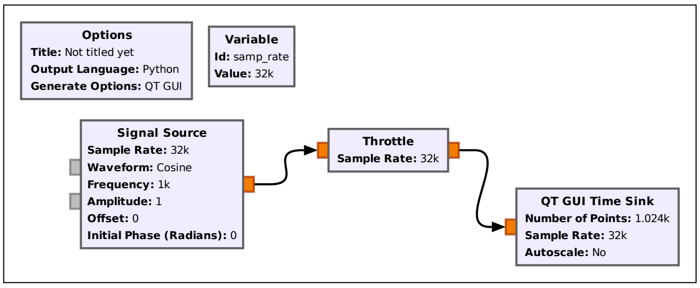

Figure 2: First GNURADIO flowgraph

1. Once inside the window (which we call flowgraph from now), please type Ctrl+F. This will place the cursor in the top right of the flowgraph, inside a search window. Type 'Source' and several options will then be displayed. Choose *Signal Source* by double-clicking or dragging it into the flowgraph window. This is nothing but an equivalent function generator program, please explore the various options by checking the properties of this block. Remember to set the output type as 'float'. The sample-rate $\frac{1}{T}$ is 32000, implying 32000 samples a second. If you choose 1000Hz as the frequency, then one cycle takes 1 millisecond. This will lead to 32 samples in one cycle, and 1000 cycles a second, thus matching the sampling rate. You can change the sampling rate to any number, but as we showed in class, it is best chosen to be a number higher than twice the highest frequency we are dealing with.

2. Now search for a 'Sink', and choose QT GUI Time Sink. Again set the input type to 'float'. The orange color at the terminals signify floats, whereas blue color is for 'complex' data types. Find out the colors for other data types, like integer, character etc.

3. Connect the two blocks. This can be done by clicking the input and output terminals of the two blocks, one after the other (in any order). Remember, the colors of the terminal should be matched for them to be connected correctly, otherwise the wire color will turn red.

4. In general, it is best to add a 'Throttle' block in a flowgraph which does not contain external hardware. If you have not inserted the throttle as in the above picture, then first delete the source-sink direct connection, and rewire through a throttle as shown in the figure, always ensure matching of the terminal colors when they are directly connected.

5. You have to save the flowgraph now. It is best to have a filename which starts with 3-4 customized letters which are not confused with common python commands or other utilities. For example myfirstprogram.grc is okay, but audio.grc or numpy.grc or 123.grc should not be used, and avoid using audioprogram.grc or audiofile.grc.

6. After saving, execute the flowgraph. Either click on the run icon, or press the second to last play button shown in the figure below.

Sometimes, problems do appear after execution. Alt+Tab can show you if your window is hiding behind. Otherwise, check the name you have given, and also verify the top left block inside the flowgraph is set to QT GUI, and no block is showing any red colored highlights. If the flowgraph has incorrect connection or wrong parameters, the execute button may not show up. In that case, click on the red icon two places to the left of the the play button and then correct the shown errors. If you have trouble, please set up a meeting with me, by sending an email to srbpteach@gmail.com

If things are all fine, you will observe a window like the one shown below, please resize to your convenience.

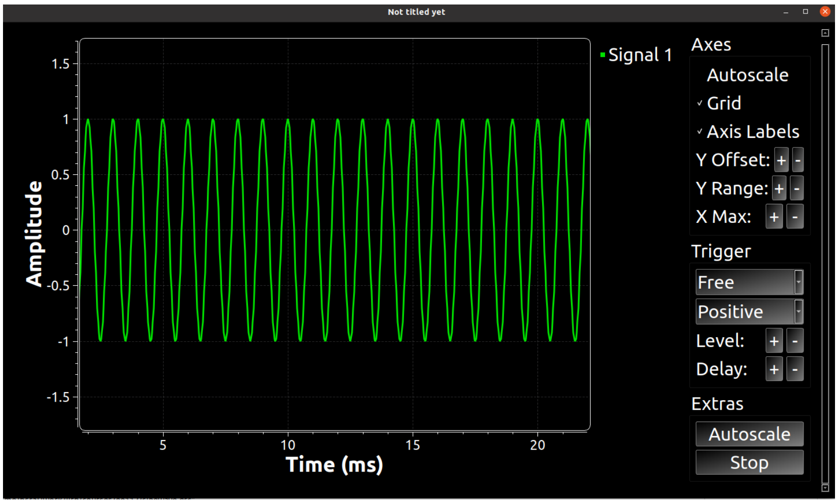

Figure 3: GRC: output terminal

Click the middle mouse button on the image to see various options, or go back to flowgraph, click on the Properties of QT GUI Time Sink and set the option ‘control-panel’. Scrolling on the display will vary the size of the signal, and choosing a rectangle with the mouse will zoom that portion. Observe that what you see is a continuous/analog waveform. In reality, a discrete set of time samples are interpolated to make it look continuous. Gnuradio employs simple line drawings between successive points to complete a plot. This is also called **linear interpolation**. **Caution: You have to close a plot before running the same flowgraph again. Check for hidden running windows, if execute button fails to show up.**

You can set the ‘stem-plot’ option to see the actual samples of the wave. This can be done by setting the flowgraph Sink properties, or middle mouse click options. The display will look like below, once you zoom to a cycle.

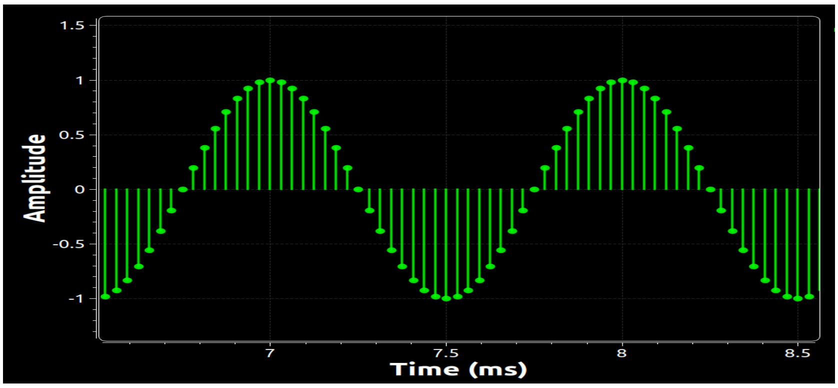

Figure 4: GRC: output with stem plot

Exercise 3 *Count the number of samples in a cycle and look at the time duration of a cycle. Suppose we now change the sampling rate to* 3200000*. How many samples do you expect to see in a cycle of the sine-wave. Run the program with this change, and figure out how to observe one cycle of the wave.*

Question 1 *“Can we define the source and sink by ourself or it is predefined?”*

Answer: The grc search window has a large number of blocks, in fact most blocks available from costly packages like matlab/simulink, labview etc are all available. This is more than enough for most purposes at this level. Nevertheless, with a little bit of python/C++ knowhow, we can write our own blocks, say functions sources, signals, sinks, etc. To illustrate, you can suggest a signal you wish to generate, and I will try to generate that.

Question 2 *How are 'sufficient' and 'enough' contradicting?*

Answer: To clarify, the better word is 'just enough'. We wish to keep the minimum number of samples so as to correctly identify a signal, in other words 'just enough'. However, having more samples is better when you want to show the signal in an efficient manner, say by simply connecting consecutive points using lines (this is what GNURADIO does). Having 'sufficiently many' makes the display/playout task easier.

Question 3 *How about a straight line joining the points?*

Answer: This is called piece-wise polynomial interpolation of the first order, or linear interpolation. This technique is used by GNURADIO displays. To understand this, try reducing the sampling rate in the first flowgraph to 8000 then 4000 and 2000 to see how the sinewave looks under linear interpolation.

1.3 Slide-4

This is the most important slide for today's lectures, but I might have run a bit fast through this. Feel free to come back and discuss, if the following explanations do not satisfy you.

Our main concern is how to connect a set of samples with sampling interval T seconds. We start with all zero samples. This is like knitting through the holes using a needle and spool. If the string is very flexible, we can knit through the holes which are close, however a hard string may not wiggle enough to sequentially go through all the holes. Frequency can be intuitively related to the ability to *wiggle*.

Using a sine waveform $\alpha \sin(2\pi \frac{1}{2T} t)$ we can interpolate the all zero waveform. The waveform $\alpha \sin(2\pi \frac{k}{2T} t)$ will also do the job for any $k \geq 1$. But, the story changes dramatically if we only allow frequencies which are *strictly* below $\frac{1}{2T}$. In fact, one can show then that the trivial signal $x(t) = 0, \forall t$ is the only continuous signal which can pass through all the holes. We take this as a fact, a more rigorous proof will be covered in future courses.

We next covered the so called Shannon interpolator. To get the idea through, let us assume that only frequencies less than or equal to $\frac{1}{2T}$ is allowed, and you have to interpolate the 'simplest' non-trivial set of values, given by a single sample of unit height at time 0, and all other samples are zero. The cardinal sine wave (i.e. $\text{sinc}(\frac{t}{T})$) will do the job, where

$$\text{sinc}(t) := \frac{\sin(\pi t)}{\pi t}. \tag{1}$$

Exercise 4 *Show that* $sinc(\frac{t}{T})$ *has the value* 1 *at* $t = 0$*, and it is zero for all* $t = nT, n \in \mathbb{Z}, n \neq 0$*. Here* $\mathbb{Z}$ *represents the set of integers.*

Now, how do we know that the Shannon interpolator does not contain frequencies greater than $\frac{1}{2T}$. To understand this, we came up with an idea of frequency content of a signal $x(t)$. This was termed as Fourier Transform, which has the definition

$$X(f) := \langle x(t), \exp(j2\pi f t) \rangle = \int_{\mathbb{R}} x(t) \exp(-j2\pi f t) dt \tag{2}$$

1.4 Slide-5

Question 4 *Why you took conjugate of* $p(t)$ *alone in the above definition?*

Answer: Notice that for an $n-$dimensional real valued vector $\bar{x}$, the dot product (or inner product) $\langle \bar{x}, \bar{x} \rangle$ is nothing but the squared magnitude of the vector. But in signal processing, we usually deal with complex valued vectors. In this case, the definition

$$\langle \bar{x}, \bar{y} \rangle = \sum_{i=1}^{n} x_i y_i^*$$

is more appropriate for dot product. Firstly, it will get us the original definition for real vectors. Secondly $\langle \bar{x}, \bar{x} \rangle$ correctly represents the magnitude now. Without the conjugate, you may even end up having a negative value for the squared magnitude, and we may not get an unambiguous distance metric. Please read about Hilbert Spaces (Wikipedia) for more details.

Question 5 *What was special with the waveform Shannon used - was it that it contains NO frequencies above 1/2T?*

Answer: The Shannon waveform was independently derived by the physics parties and other places as well, see for example the Whitaker formula. The beauty is that once you interpolate that simple one valued signal (we will define this in future as an **impulse** measure), we can interpolate any set of samples by repeating the idea and superposition. This is pretty much like your superposition theorem in circuit analysis. You are guaranteed that no frequencies above $\frac{1}{2T}$ is present.

Question 6 *How do we get the integral in the dot product equation?*

Answer: For two vectors it is the summation of the element-wise products. Now, intuitively think of the signal as a collection of ordered pairs $(t, x(t))$, indexed by values of $t \in \mathbb{R}$ (i.e. real axis). For computing the dot product $\langle x(t), y(t) \rangle$, we are effectively taking the point-wise products and adding up, leading to the integral formula. Please see above for the need of taking conjugate of one of the signals.

Question 7 *Can u show graph of x that it is orthogonal to both sin and cos*

Answer: Definitely. Notice that the cardinal sine $x(t) = \text{sinc}(t)$ has no frequency higher than $f_{max} = \frac{1}{2}$. Now, the sum $ax(t) + bx(t - \tau)$ also has a similar property. Now mix as many such signals as you want together and make merry. If required we can give a plethora of other examples as well. Notice however, that showing cardinal sine has no higher frequencies than $\frac{1}{2T}$ can be a pain, as Fourier and others encountered at that time. A proper answer was given much later by Cesaro.

1.5 Slide-6

Question 8 *What is the significance of the size of the arrows?*

Answer: The sizes represent the sample values. To illustrate when you speak over the microphone, the mobile device/computer collects voltage values generated as the samples. If you look at a set of consecutive values, they will trace the arrow position shown in the figure.

Question 9 *What is sampling rate in this figure?*

Answer: We assumed successive samples are separated by T units, so sampling rate is $\frac{1}{T}$.

Question 10 *Shouldn't it be sinc$(2\pi t/T)$?*

Answer: Since samples are T time units apart, $\text{sinc}(\frac{t}{T})$ will do the job. Notice that $\text{sinc}(t) = \sin(\pi t)/(\pi t)$, with the π already inside by definition. Furthermore $\text{sinc}(1) = 0$, and $\text{sinc}(\frac{t}{T}) = 0$ at $t = T$.

Question 11 *What is f_{max} and T?*

Answer: We have taken samples which are T seconds apart. Suppose our original signal has only frequencies below f_{max}, i.e. $|f| < f_{max}$ for $x(t)$, and we are given samples of $x(t)$ at $t = nT, n \in \mathbb{Z}$. Two things are implied by this. First, there is no **other** signal $y(t)$ with frequency contents only at $|f| < f_{max}$, having $x(nT) = y(nT), n \in \mathbb{Z}$, i.e. $x(t)$ and $y(t)$ cannot have the sample samples for a sampling interval of T. How do we know this? If $x(t)$ and $y(t)$ have the same set of samples, then $x(t) - y(t)$ have all zero samples at $t = nT, n \in \mathbb{Z}$. By what we learnt above, the only signal with such frequency contents, and passing through all zeros, is the zero signal, whenever $f_{max} \leq \frac{1}{2T}$ or $T \leq \frac{1}{2f_{max}}$. In other words, $x(t) = y(t), \forall t \in R$. So the interpolation leads to a unique signal among the ones which have no frequency at or above f_{max}. Secondly, Shannon's interpolation will create the unique signal for us.

Question 12 *What was $X(f)$ in that graph?*

Answer: In the previous slide, we had defined $X(f)$ as the Fourier Transform of $x(t)$. In particular, $X(f_0)$ will tell us the frequency content at frequency f_0. If $X(f_0) = 0$ then there is no frequency content at $f = f_0$.

1.6 Slide-7

Question 13 ***Answer:*** *What is the meaning of frequency of polynomials?*

Answer: The frequency content of any *reasonable* analog waveform $x(t)$ can be compute by evaluating $X(f)$ (Fourier Transform), which is nothing but the dot product of $x(t)$ and $\exp(j2\pi ft)$. Suppose the signal $x(t) = p_1(t)$ for $t_1 \leq t \leq t_2$ and $x(t) = p_2(t)$ for $t_2 \leq t \leq t_3$. Outside $t \in [t_1, t_3]$, assume any nice waveform for which we can find the above integral for $X(f)$. The frequency content of $x(t)$ can be analyzed by computng $X(f)$, we can break it into multiple regions of time and evaluate. We substitute the correct polynomial when it comes to the intervals of interest. As a thumb rule, unless the signal is very smooth (many times differentiable at each sample point), we cannot assure bandlimited with a maximum frequency. Thus, all those polynomial interpolators, use much higher frequencies (possible unbounded)

Question 14 *So even if we zoom on photos app, are more than one pixel out of 64 filled?*

Answer: Let us do these examples in the lab session on Thursday, and it will be clear then.

Question 15 *What is a 2D line?*

Answer: Linear equations in $2D$ can be thought of as planes. But, for images, it is more like a matrix of values. A simple technique to linearly interpolate is to find a separate line between one sample and each of the neighboring samples in every direction. For an in between point for which you do not know the value, interpolate by picking the value on the closest line. Let us try to do some experiments in the coming lab session.