Problems for April 27

I) Identify the Quadrics and in atleast one instance find the directions of the principal axes

(i) $2xy + 2yz + 2zx = 1$

(ii) $x^2 - 2y^2 + 4z^2 + 6yz = 1$

(iii) $-x^2 - y^2 + 2z^2 + 8xy - 4xz + 4yz = 1$

II) Compute $\int \exp -(2x^2 + 5y^2 + 2z^2 - 4xy - 2xz + 4yz)\, dxdydz$

III) Show that $ax^2 + by^2 + cz^2 + 2hxy + 2gxz + 2fyz$ factorizes into a product of linear factors (possibly with Complex Coeff) iff
$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0.$$

Hint. First discuss the case when $f = g = h = 0$

(IV) Show that a (3×3) orthogonal matrix has eigen value $+1$ or -1

Further, if det of Matrix is 1 then 1 is necessarily an eigen value.

(V) Let A be a 3×3 Orthogonal real matrix and v be a unit vector with eigen value ± 1 or -1}.

Let $\alpha + i\beta$ be a Complex eigen value and $\rho + i\sigma$ be the Corresp. eigen vector. in $\mathbb{C}^3$

prove that
$$A\rho = \alpha\rho - \beta\sigma$$
$$A\sigma = \beta\rho + \alpha\sigma$$

Is it possible to arrange it so that $\{v, \rho, \sigma\}$ orth. normal?

Call $O = [v\ \rho\ \sigma]$ What is O^TAO ?