Ground rules

https://slideplayer.com/slide/5291168/

Time dependent Schrodinger Equation

$$i\hbar \frac{\partial}{\partial t}\Psi(x,y,z,t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x,y,z)\right]\Psi(x,y,z,t)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Classical wave equation for de Broglie waves

Separation of variables:

$$\Psi_n(x,y,z,t) = \psi_n\,(x,y,z)\phi(t)$$

Separation of variables

$$i\hbar\frac{\partial}{\partial t}\psi_n\,(x,y,z)\phi(t)=\left[-\frac{\hbar^2}{2m}\nabla^2+V(x,y,z)\right]\psi_n\,(x,y,z)\phi(t)$$

$$\psi_n\,(x,y,z).\,i\hbar\frac{\partial\phi(t)}{\partial t}=\phi(t)\left[-\frac{\hbar^2}{2m}\nabla^2\psi_n\,(x,y,z)+V(x,y,z).\,\psi_n\,(x,y,z)\right]$$

$$\frac{i\hbar}{\phi(t)}\frac{\partial\phi(t)}{\partial t}=\left[-\frac{\hbar^2}{2m.\,\psi_n\,(x,y,z)}\nabla^2\psi_n\,(x,y,z)+V(x,y,z).\,\psi_n(x,y,z)\right]\qquad=W$$

Separation constant

$$\Psi_n(x,y,z,t)=\psi_n\,(x,y,z)\phi(t)$$

Separation of variables

$$i\hbar\frac{\partial}{\partial t}\psi_n\,(x,y,z)\phi(t) = \boxed{\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x,y,z)\right]}\psi_n\,(x,y,z)\phi(t)$$

$\hat{H}$,**Hamiltonian operator**

$$\psi_n\,(x,y,z).\,i\hbar\frac{\partial\phi(t)}{\partial t} = \phi(t)\left[-\frac{\hbar^2}{2m}\nabla^2\psi_n\,(x,y,z) + V(x,y,z).\psi_n\,(x,y,z)\right]$$

$$\frac{i\hbar}{\phi(t)}\frac{\partial\phi(t)}{\partial t} = \left[-\frac{\hbar^2}{2m.\psi_n\,(x,y,z)}\nabla^2\psi_n\,(x,y,z) + V(x,y,z).\psi_n(x,y,z)\right] \qquad = W$$

$$\frac{i\hbar}{\boxed{\phi(t)}}\frac{\partial\phi(t)}{\partial t} = W; \quad \left[-\frac{\hbar^2}{2m.\psi_n\,(x,y,z)}\nabla^2\psi_n\,(x,y,z) + V(x,y,z).\psi_n(x,y,z)\right] = W$$

$e^{-iWt/\hbar}$

$$\boxed{\Psi_n(x,y,z,t) = \psi_n\,(x,y,z)e^{-iWt/\hbar}}$$

Stationary states

In classical mechanics $\hat{H}$ represents total energy

We can therefore write

$$\widehat{H}\psi = W\psi \qquad \text{as} \qquad \widehat{H}\psi = E\psi$$

$$\boxed{\Psi_n(x,y,z,t) = \psi_n\,(x,y,z)e^{-iE_n t/\hbar}}$$

Each $\boldsymbol{\psi_n(x,y,z)}$: a particular value of energy $\boldsymbol{E_n}$

☞ a particular **Energy eigenstate (Stationary state)**

Quantization? Not yet!!

Eigenvalues and Eigenfunctions

In classical mechanics $\hat{H}$ represents total energy

We can therefore write

$$\widehat{H}\psi = W\psi \qquad \text{as} \qquad \widehat{H}\psi = E\psi$$

$$\Psi_n(x, y, z, t) = \psi_n\,(x, y, z)e^{-iE_n t/\hbar}$$

Schrodinger equation is an eigenvalue equation

There can be many solutions $\boldsymbol{\psi_n(x)}$ each corresponding to different energy $\boldsymbol{E_n}$

Laws of Quantum Mechanics

Mathematical description of Quantum mechanics: built upon the concept of operators

Classical Variable	QM Operator
Position, x	$\hat{x}$
Momentum, $p_x = mv$	$\hat{p}_x = \frac{\hbar}{i}\frac{d}{dx} = -i\hbar\frac{d}{dx}$
Kinetic Energy, $T_x = \frac{p_x^2}{2m}$	$\hat{T}_x = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2}$
Kinetic Energy, $T = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}$	$\hat{T} = \frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$
Potential Energy, $V(x)$	$\hat{V}(x)$

Laws of Quantum Mechanics

The values which come up as result of an experiment are the eigenvalues of the appropriate operator

In any measurement of observable associated with operator $\hat{A}$, the only values that will be ever observed are the eigenvalues an, which satisfy the eigenvalue equation:

$$\hat{A} \cdot \Psi_n = a_n \cdot \Psi_n$$

$\boldsymbol{\Psi_n}$ are the eigenfunctions of the system and $\boldsymbol{a_n}$ are corresponding eigenvalues

If the system is in state $\boldsymbol{\Psi_k}$, a measurement on the system will yield an eigenvalue $\boldsymbol{a_k}$

Laws of Quantum Mechanics

Only real eigenvalues will be observed, which will specify a number corresponding to the classical variable

If $\quad Y(x) = Sin(cx)$

$$\frac{d}{dx} Y(x) = c \times Cos(cx)$$

$$\frac{d^2}{dx^2} Y(x) = -c^2 \times Sin(cx) = -c^2 \times Y(x)$$

If $\quad Y(x) = e^{ax}$

$$\frac{d}{dx} Y(x) = a \times e^{ax}$$

$$\frac{d^2}{dx^2} Y(x) = a^2 \times e^{ax} = a^2 \times Y(x)$$

There may be, and typically are, many eigenfunctions for the same QM operator!

Laws of Quantum Mechanics

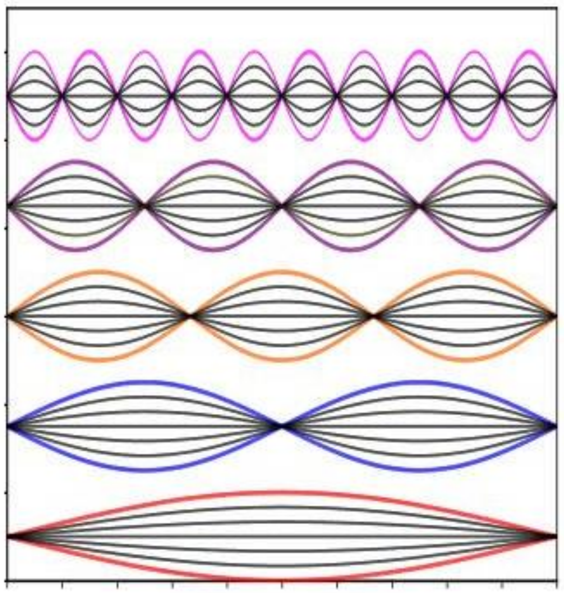
All the eigenfunctions of Quantum Mechanical operators are "Orthogonal"

$$\int_{-\infty}^{+\infty} \psi_m^*(x)\psi_n(x)dx = \langle \psi_m | \psi_n \rangle = 0 \quad \text{for } m \neq n$$

Superposition of states

Schrodinger equation: Classical wave equation for de Broglie waves

General solutions of Classical wave equation: **Standing waves**



Superposition of **Normal modes**
(Length: Half integral multiple of wavelength)

Solutions of Schrodinger equation:
Linear combination of wavefunctions that are **orthogonal** to each other

https://www.slideserve.com/urbana/standing-waves

Before and after measurement

Measurement: Property has a value of P

Before measurement:

Realist: Value = P (Einstein)
⇒ *Quantum theory is incomplete*

Orthodox: Entanglement (Bohr, **Copenhagen interpretation**)
⇒ *Measurement produces the value*

Agnostic: Don’t know, don’t care

Before and after measurement

Measurement: Property has a value of P

Before measurement:

Realist: Value = P (Einstein)
⇒ *Quantum theory is incomplete*

Orthodox: Entanglement (Bohr, **Copenhagen interpretation**)
⇒ *Measurement produces the value*

Agnostic: Don't know, don't care

Immediately after measurement:

Same Value = P

Wavefunction collapse

But what is this wavefunction?

Google Doodle, December 11, 2017