deBroglie Hypothesis

In Newtonian Mechanics, we usually tend to think of objects as **mass points**, that is *point particles with mass.* A macroscopic object is viewed as a collection of such mass points. Then we try to answer questions about how such objects move when they are subjected to forces and torques etc.

Radiation was, of course, considered different. Initially, there was a debate about the nature of radiation, with Newton arguing that it is made of "corpuscles" and Huygens arguing that it is made of "waves". Young's double slit interference experiment "settled" the question in favour of waves. In particular, it was shown that **diffraction and interference** can be explained only if the radiation is assumed to be a wave. A number of other observations, made over the 19th century, gave additional support to this hypothesis. Maxwell's theory of "electromagnetism" interpreted radiation as **electromagnetic waves**.

But, as we saw in the last two classes, photo-electric effect and Compton effect, re-opened the debate. The various features seen in these two effects **could not be explained** by the wave theory of radiation but **could be explained** if one assumes that electromagnetic radiation consists of **bundles of energy and momenum**. This leads to the question: **Does radiation have both wave-like and particle-like properties?**

Meanwhile, the Newtonian idealization was shown to be true. Existence of atoms was experimentally demonstrated and it was shown that matter indeed consists of Avogadro number of point-like massive particles.

As the wave vs particle question was being debated in the case of radiation, de-Broglie (1924) jumped into the debate and **proposed that particles should have wave properties**. We saw in Compton effect, that a photon with energy $E = h\nu$ has momentum $p = E/c = h\nu/c = h/\lambda$. de-Broglie inverted this relation and declared that a massive particle with momentum $|\vec{p}| = p$, has a **de-Broglie wavelength**

$$\lambda_{\text{dB}} = \frac{h}{p}.$$

If that is so, how to demonstrate it? In particular, can we observed diffraction and interference with massive particles? At the time of the de-Broglie's proposal, electrons were the particles with the smallest mass. So, they are likely to have the largest λ_{dB}. Davisson and Germer (1927) tried to observe electron diffraction.

Key points of Davisson-Germer experiment:

1. Electrons, essentially at rest were produced by heating a filament.

2. These electrons were accelerated through a potential difference V because of which they acquire a kinetic energy eV, where e is the magnitude of the charge of the electron.

3. The kinetic energy is non-relativistic. Hence we calculate the momentum of the electron to be $p = \sqrt{2m_e eV}$.

4. By de-Broglie hypothesis, they should have $\lambda_{\mathrm{dB}} = h/p$. For $V = 54$ volts, $\lambda_{\mathrm{dB}} = 1.67$ Angstroms.

5. Diffraction of X-rays by the planes of a crystalline solid was proposed in 1912. This diffraction leads to **Bragg Peaks** which occur when the wavelength λ of the X-rays and the distance d between the atoms satisfy the relation
$$n\lambda = 2d\sin(90^\circ - \theta),$$
where n is a positive integer.

6. X-ray diffraction experiment, with $\lambda = 1.65$ Angstrom, is done on a nickel crystal. Bragg peak was observed at $\theta = 50^\circ$. Assuming $n = 1$, we calculate $d = 0.91$ Angstrom.

7. If the electrons, accelerated through 54 volts, have the same wavelength as the X-rays in the previous experiment, then they must show the same Bragg peak when such an electron beam is directed at a nickel plate.

8. Davisson and Germer found such a Bragg peak in their experiment. The Bragg formula implies that the electrons in their experiment had a wavelength of 1.65 Angstrom, confirming de-Broglie hypothesis.

Note: The derivation of diffraction peak formula for light is beautifully explained in the YouTube video "Diffraction grating — Light waves — Physics — Khan Academy". The diffraction formula given above contains an extra factor of 2. This was explained by Prof. Gopal in the lecture.