quiz question-1 soln

The solution of TISE ~~is~~ for a free particle is:-

$$\phi(x) = A e^{ikx} + B e^{-ikx} \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

The time dependent solution is just the above solution multiplied by $e^{-\frac{iEt}{\hbar}}$

$$\phi(x,t) = A e^{i\left(kx - \frac{\hbar k^2}{2m}t\right)} + B e^{-i\left(kx + \frac{\hbar k^2 t}{2m}\right)}$$

we see that it is superposition of waves moving right and left.

The speed of these waves (coefficient of t over coeff. of x)

$$V_{quantum} = \frac{\hbar k}{2m} \Rightarrow \sqrt{\frac{E}{2m}}$$

On the other hand in classical mechanics;

$$E = \frac{1}{2} m V_c^2$$

$$\boxed{V_c = \sqrt{\frac{2E}{m}} = 2V_{quantum}}$$

The speed of the wave is half of the speed obtained from classical mechanics.

But this quantum velocity is not actually the particle velocity but its the velocity of the ripples inside wave packet or which is called the phase velocity which can be more than or less than the group velocity which is the actual speed of the particle.

quiz question - 2 soln

In case of a free electron, the wavefunction $\phi(x)$ is not normalised.

$$\int_{-\infty}^{\infty} \phi^*(x)\, \phi^*(x)\, dx = |A|^2 \int_{-\infty}^{\infty} dx$$

~~they~~ the separable solutions donot represent actual physical states i.e. a free particle cannot exist in a stationary state

The problem occurs because we have a single value of k corresponding to the particle.

~~If we~~ The general solution of TDSE is still a combination of its solutions. So with that, if we superimpose a spread of values of k such that the resulting wavepacket closely resembles the free particle, then it can be normalised.