Problems for Wednesday Tutorial

March 23, 2022

Mainly Review.

(1) Let $\hat{a}, \hat{b}$ be unit vectors in $\mathbb{R}^3$. Discuss whether the equation $\hat{a} \times x = \hat{b}$ has solutions in $\mathbb{R}^3$; $\times$ is the cross product.

(2) Let $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $p = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$ $(x_1^2 + x_2^2 = 1)$. What can you say about the set $\{p, Ap, A^2p, \ldots\}$. Is it a finite set or an infinite set?

(3) Consider the equation $x^2 + y^2 - z^2 + 7xy - 3yz + 6xz = 3$. Write it in the form $[x\ y\ z]\, A \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ for some (3×3) symmetric matrix A. Is such a matrix A unique? What if we drop the symmetry requirement??

(4) Recall the notion of an invertible matrix from Class 12. How would you decide whether a 3×3 matrix is invertible or not? Can you decide?

If u is a unit vector in $\mathbb{R}^3$ (column vector), is $I - uu^T$ invertible?

Can you discuss the map $f : \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (I - 2uu^T)x$ geometrically?

(5) Find two mutually $\perp$ unit vectors $\hat{u}, \hat{v}$ s.t. $\hat{u}, \hat{v}$ lie on the plane $x + y + z = 0$. Without a parametrization for circle $x^2 + y^2 + z^2 = 1$, $x + y + z = 0$.