Tut 1: Q 11

Tut 2: Q 2, 3, 8, 9

Handwritten Notes Question

T.1 Q 11

→ 0 pivots → 0 matrix

→ 1 pivot there are 4 possibilities

$$\begin{bmatrix} 1 & * & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

↓ 3 degrees of freedom, ↓ 2 D.O.F, ↓ 1 DOF, ↓ 0 DOF

→ 2 pivots → 6 possibilities.

$$\begin{bmatrix} 1 & 0 & \times & \times \\ 0 & 1 & \times & \times \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & * & 0 & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & * & * & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

↓ 4, ↓ 3, ↓ 2, ↓ 2

$$\begin{bmatrix} 0 & 1 & * & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

↓ 1 D.OF, ↓ 0 DOF

For 3 pivots, 4 possibilities

$$\begin{bmatrix} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & * & 0 \\ 0 & 1 & * & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & * & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\downarrow$ 3, $\quad\downarrow$ 2 $\quad\downarrow$ 1 $\quad\downarrow$ 0

For 4 pivots → 1 possibility

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Tut 2 - Q2

i) $[1,1,-1] + [-1,1,1] = [0,2,0] = 2[0,1,0]$

ii) $a[1,9,9,8] + b[2,0,0,3] + c[2,0,0,8] = 0$

$9a = 0 \Rightarrow a = 0$

$2b + 2c = 0$
$3b + 8c = 0$ $\Rightarrow 5b = 0 \Rightarrow b = 0$

$\Downarrow$

$c = 0$

Tut 2 - Q3

(1) $\begin{bmatrix} 8 & -4 \\ -2 & 1 \\ 6 & -3 \end{bmatrix} \xrightarrow[R_{1/3}]{R_{1/4}} \begin{bmatrix} 2 & -1 \\ -2 & 1 \\ 2 & -1 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_2]{} \begin{bmatrix} 2 & -1 \\ -2 & 1 \\ 0 & 0 \end{bmatrix}$

$\downarrow R_2 \to R_2 + R_1$

$\begin{bmatrix} 2 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

1 Pivot, Rank = 1

~~Q9)~~

Q2)

i) ~~$[T, D, -U] \& [\ldots]$~~

2) $\begin{bmatrix} m & n \\ n & m \\ p & p \end{bmatrix} \xrightarrow{R_1 \to R_1 + R_2} \begin{bmatrix} m+n & m+n \\ n & m \\ p & p \end{bmatrix}$

$(m^2 + n^2)$

$\xrightarrow{R_3 \to R_3 - \frac{p}{m+n} R_1} \begin{bmatrix} m+n & m+n \\ n & m \\ 0 & 0 \end{bmatrix} \xrightarrow{\frac{1}{m+n} R_1} \begin{bmatrix} 1 & 1 \\ n & m \\ 0 & 0 \end{bmatrix}$

$\longrightarrow \begin{bmatrix} 1 & 1 \\ 0 & m-n \\ 0 & 0 \end{bmatrix}$

$\therefore$ Rank $= 2$

3) $\begin{bmatrix} 0 & 8 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 4 & 5 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 8 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 4 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 4 & 0 \end{bmatrix}$

$\therefore$ Rank $= 3$

Q8)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & 2 \\ a^2 & b^2 & 4 \end{bmatrix} \qquad C = \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$$

solving $AX = C$

$\det(A) = (b-a)(2-a)(2-b)$

For infinitely many solutions ~~we need~~ $\det(A) = 0$ is necessary

$\therefore a = 2$ or $b = 2$

as $b > a$

Case 1 $a = 2$

$$[A : C] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & b & 2 & 3 \\ 4 & b^2 & 4 & 9 \end{array}\right]$$

as $\text{rank}(A) = 2$ if rank of $[A : C] = 3$ then 0 solutions (as that would say C is not in the column space of A) $\therefore \det \begin{bmatrix} 1 & 1 & 1 \\ b & 2 & 3 \\ b^2 & 4 & 9 \end{bmatrix} = 0 \Rightarrow b = 3$ (as $b > a = 2$)

$\boxed{\therefore a = 2, b = 3}$ gives infinitely many solutions

Case 2 $b = 2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ a & 2 & 2 & 3 \\ a^2 & 4 & 4 & 9 \end{array}\right]$$

as rank $\begin{bmatrix} 1 & 1 & 1 \\ a & 2 & 3 \\ a^2 & 4 & 9 \end{bmatrix} = 3$ (as $a < 2$) $\therefore$ 0 solutions

$(a, b) = (2, 3)$ is the only possibility

Q9)

i) The new rows are linear combinations of previous rows and vice versa

ii) $C_1, \dots, C_n$ are columns. With an ERO through E. The new colums are

$E C_1, \dots, E C_n$

If $C_{j_1}, \dots, C_{j_n}$ are Lin Ind then so are $E C_{j_1}, \dots, E C_{j_n}$ and vice versa due to E being invertible

Q10)

Case I)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ a & b & 2 & 3 \\ a^2 & b^2 & 4 & [illegible] \end{array}\right]$$

to have infinitely many solutions

$\det A = 0$

$\Downarrow$

$(b-a)(2-a)(2-b)=0$

$\therefore a=2$ or $b=2$

since $b-a = 0$

$$Ax = b$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 2 & -1 & 1 & -2 & -1 & 1 \\ 2 & 2 & 6 & 0 & 4 & 2 & 4 & 10 \\ 1 & -1 & 1 & -2 & 0 & -5 & -4 & -3 \\ 2 & 2 & 6 & 0 & 4 & 2 & 4 & 10 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 2 & -1 & -1 & -2 & -1 & 1 \\ 0 & 2 & 2 & 2 & 2 & 6 & 6 & 8 \\ 0 & -1 & -1 & -1 & -1 & -3 & -3 & -4 \\ 0 & 2 & 2 & 2 & 2 & 6 & 6 & 8 \end{array}\right]$$

$$\downarrow \quad (R_3 \mapsto R_3 - \tfrac{1}{2} R_2,\ R_4 \mapsto R_4 - R_2)$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 2 & -1 & -1 & -2 & -1 & 1 \\ 0 & 2 & 2 & 2 & 2 & 6 & 6 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

① Rank = 2. ~~As~~ As Rank A + Nullity A = 7

$\therefore$ Nullity A = 5.

② From the row echelon form we see that the consistency criteria is fulfilled and hence it is solvable

③ We choose the columns containing the pivots. ~~in the process of Gaussian Elimination each pivot~~ the row space. Choose rows which generate the row space.

~~Eg~~. In this case $\begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$ works

Sol

4) Null space of A doesn't change with elementary row operations

$$\begin{bmatrix} 1 & 0 & 2 & -1 & +1 & -2 & -1 \\ 0 & 2 & 2 & 2 & 2 & 6 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \\ k_7 \end{bmatrix} = 0$$

then $k_1 = -2k_3 + k_4 - k_5 + 2k_6 + k_7$

$k_2 = -k_3 - k_4 - k_5 - 3k_6 - 3k_7$

General vector of the Null space can be written as

$[-2k_3 + k_4 - k_5 + 2k_6 + k_7, -k_3 - k_4 - k_5 - 3k_6 - 3k_7, k_3, k_4, k_5, k_6, k_7]^t$

Basis for Null space is

$$\begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 6 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

5) Basis for column space (any two Linearly Independent columns of A)

$$\begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \\ 2 \end{bmatrix}$$

(6) Basic solution is

$B = [1, 4, 0, 0, 0, 0, 0]^T$

$\therefore$ General solution is $B + v$

where v is an element of null space of A

$\therefore$ Complete set of solutions

$S = B + \sum_{i=1}^{s} k_i v_i$ where $\{v_1, \dots, v_s\}$ is the basis for null space of A and $k_1, \dots, k_s \in \mathbb{R}$.

(7) Free variables are $\{x_3, x_4, x_5, x_6, x_7\}$