Playing with Signals:
Sampling and Interpolation

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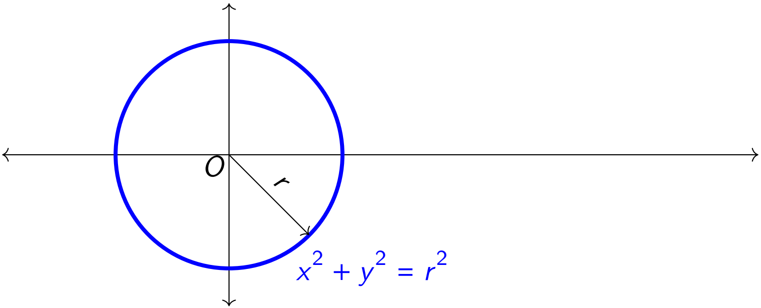
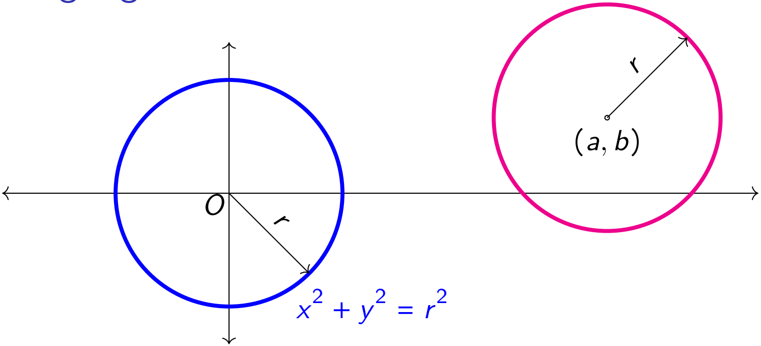
Subject:EE113-RollNo

January 4, 2022

Drawing Signals



Drawing Signals



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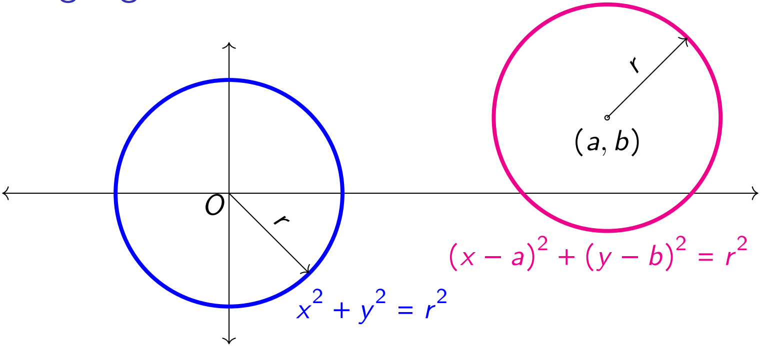

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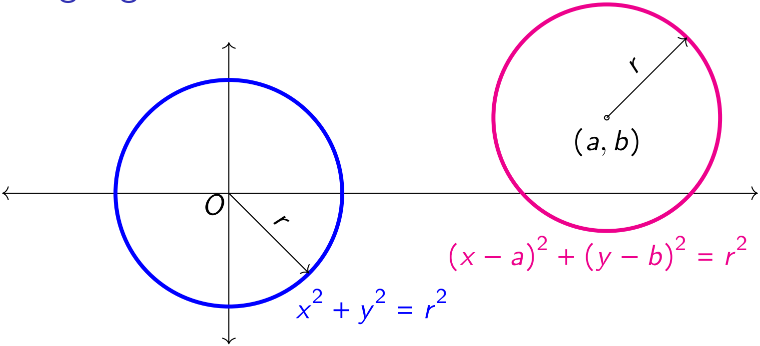

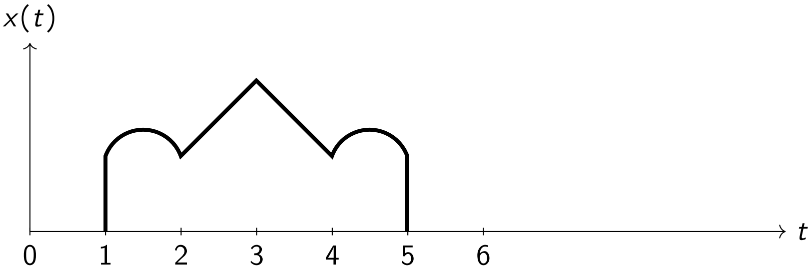

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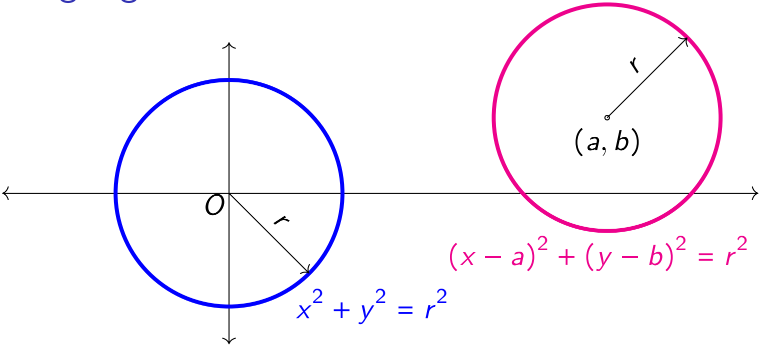

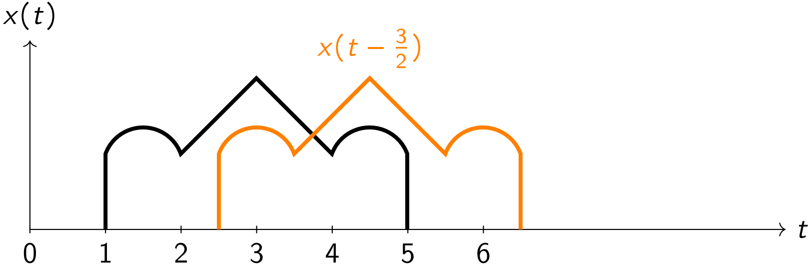

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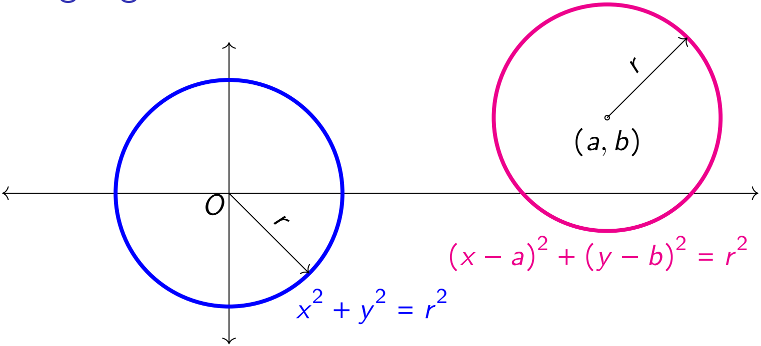

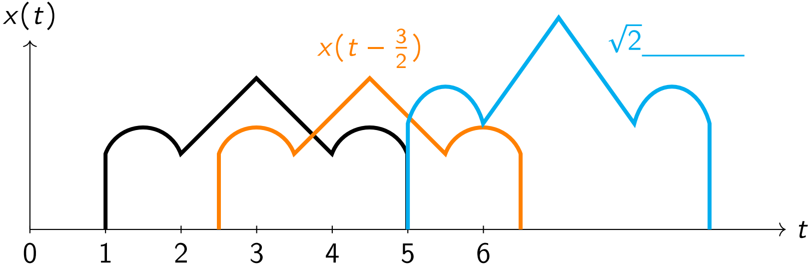

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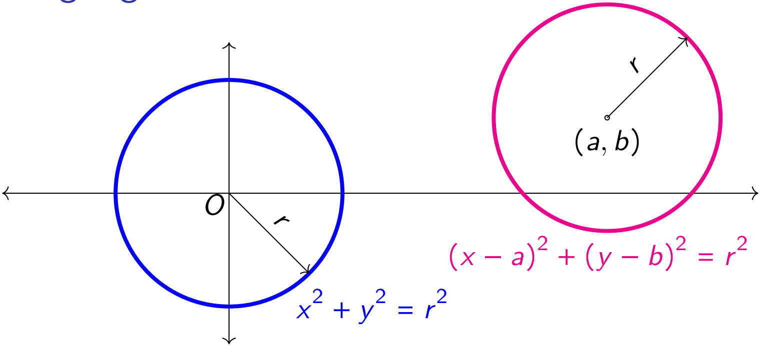
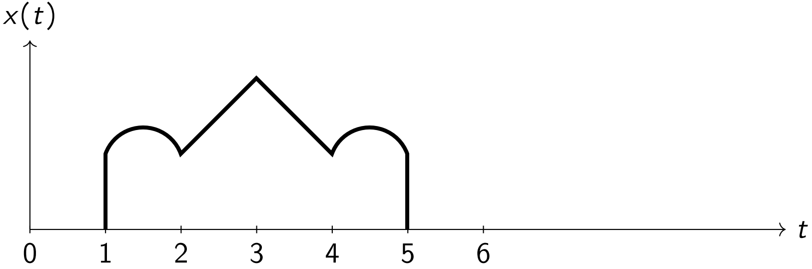
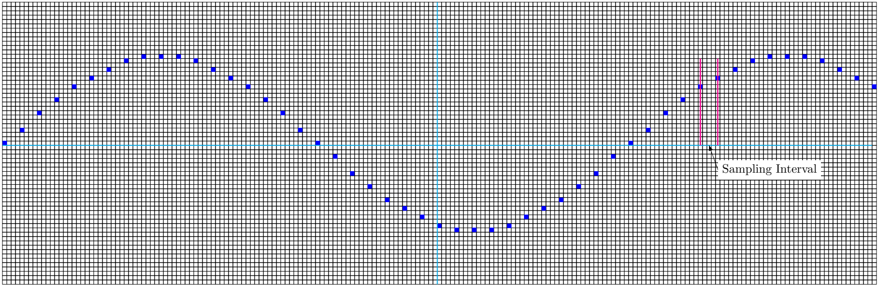




Digital Oscilloscope

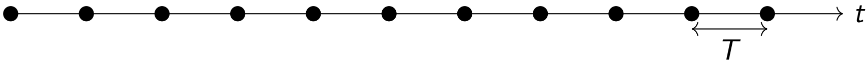


Digital-to-Analog :- “Sufficiently many discrete dots interpolated”

Analog-to-Digital :- “Sample enough to preserve its identity”

GNURADIO: Square wave from sines (Homework in Python).

Sampling Idea: All Zero Signal



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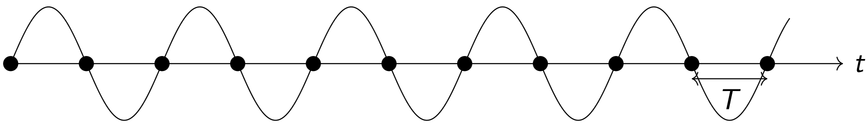

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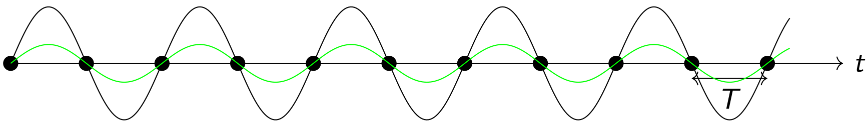

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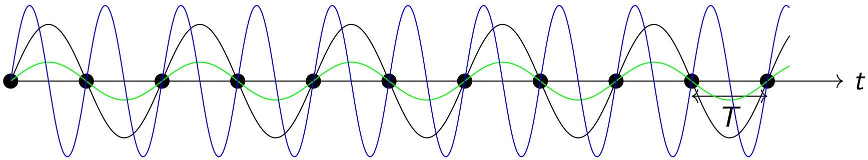

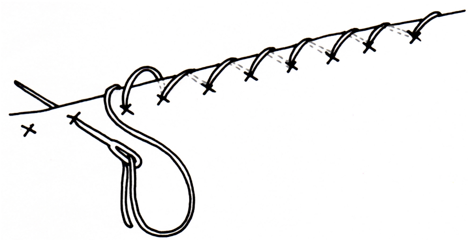

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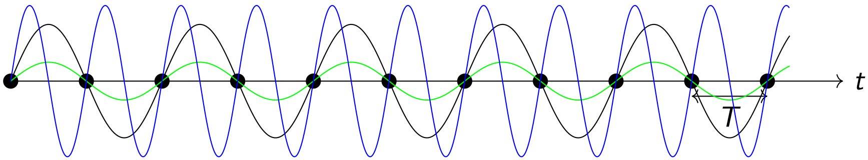
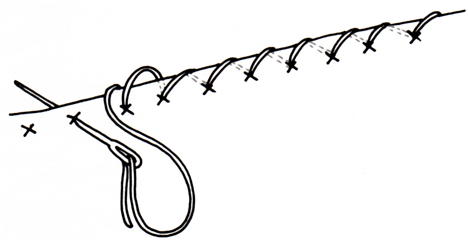


No non-zero continuous interpolator having only frequencies below $\frac{1}{2T}$.



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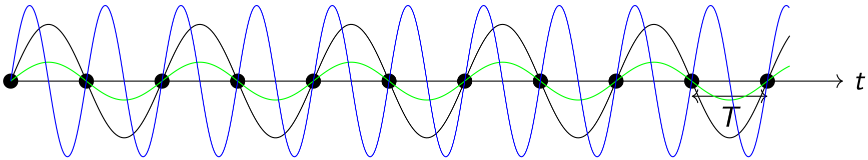

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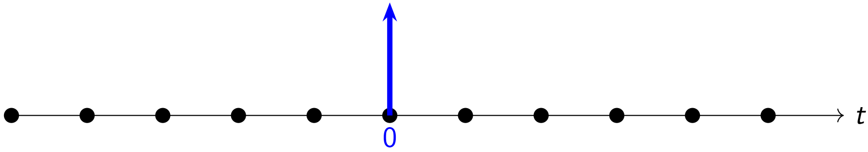
Shannon Interpolator: $\mathrm{sinc}\left(\frac{t}{T}\right)$



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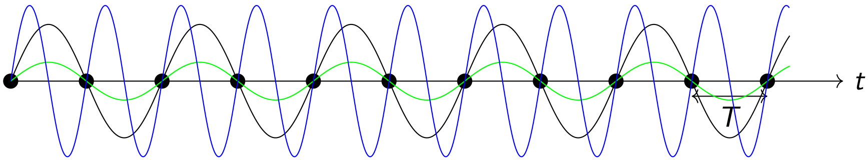
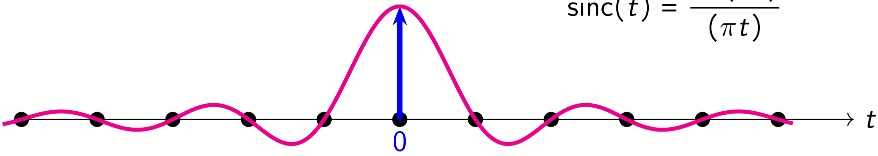


No non-zero continuous interpolator having only frequencies below $\frac{1}{2T}$.

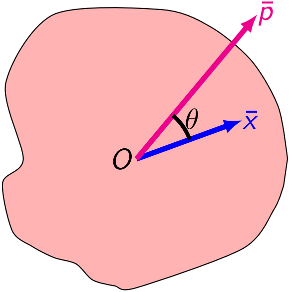
Shannon Interpolator: $\text{sinc}\left(\frac{t}{T}\right)$

$$\text{sinc}(t) = \frac{\sin(\pi t)}{(\pi t)}$$



Frequency, Linearity etc

The dot product $\langle \bar{x}, \bar{p} \rangle$ of two vectors $\bar{x}, \bar{p}$ measures their overlap.



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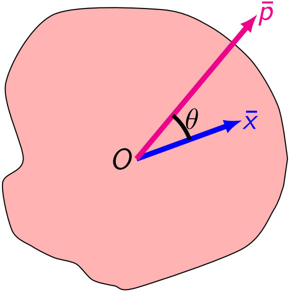

$$\langle x(t), p(t) \rangle = \int_{\mathbb{R}} x(t) p^*(t) dt.$$

$$\langle x(t), p(t) \rangle = 0 \Longrightarrow \text{orthogonal}$$

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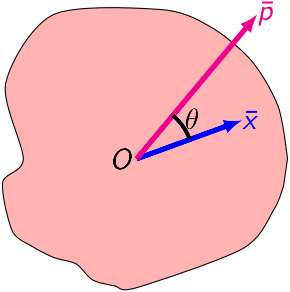

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$$\langle x(t), \cos(2\pi f t) \rangle = \langle x(t), \sin(2\pi f t) \rangle = 0, \forall f \geq f_0 \Longrightarrow x(t) \text{ bandlimited}.$$

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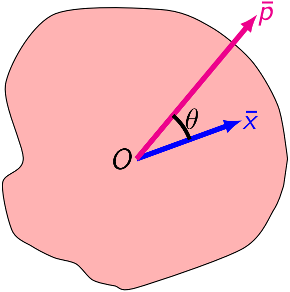

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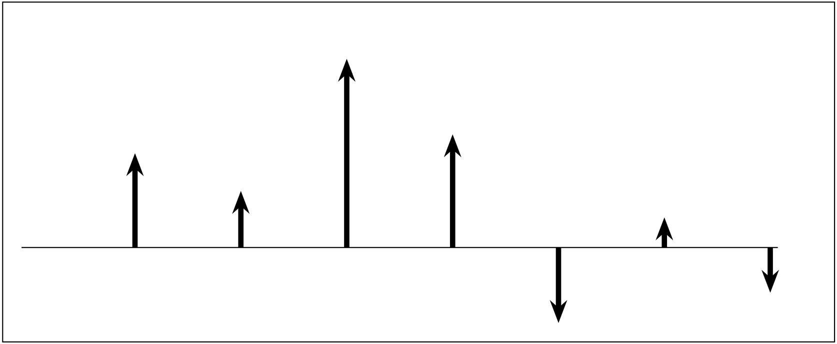
$$\langle x(t), p(t) \rangle = 0 \implies \text{orthogonal}$$

$\langle x(t), \cos(2\pi f t) \rangle = \langle x(t), \sin(2\pi f t) \rangle = 0, \forall f \geq f_0 \implies x(t)$ bandlimited.

The Fourier Transform of the signal $x(t)$ is

$$X(f) := \langle x(t), \exp(j2\pi f t) \rangle = \int_{\mathbb{R}} x(t) \exp(-j2\pi f t) dt$$

Digital-to-Analog



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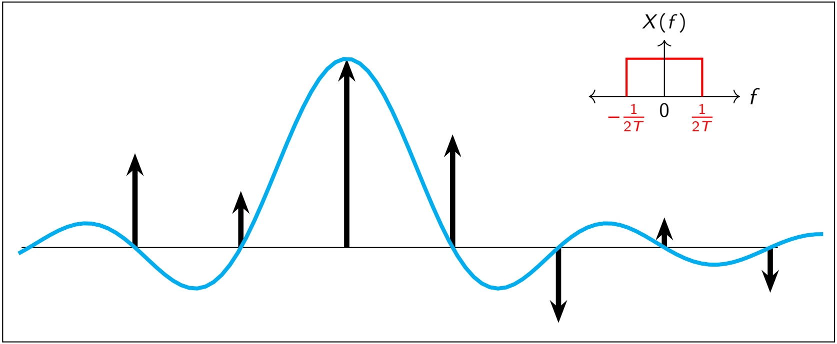

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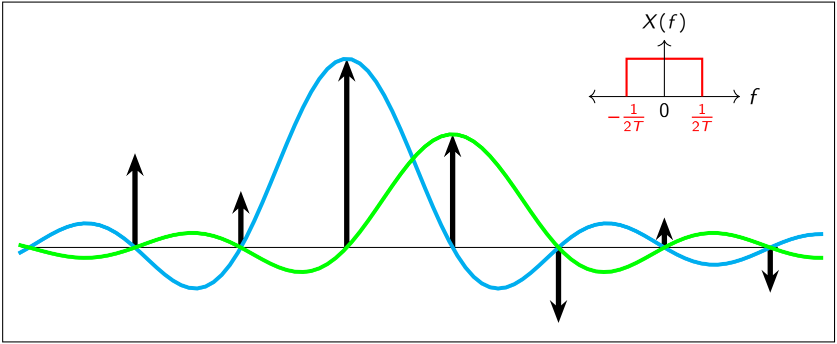

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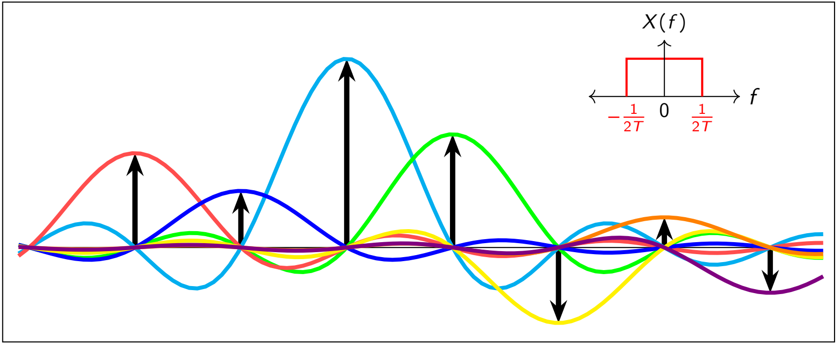

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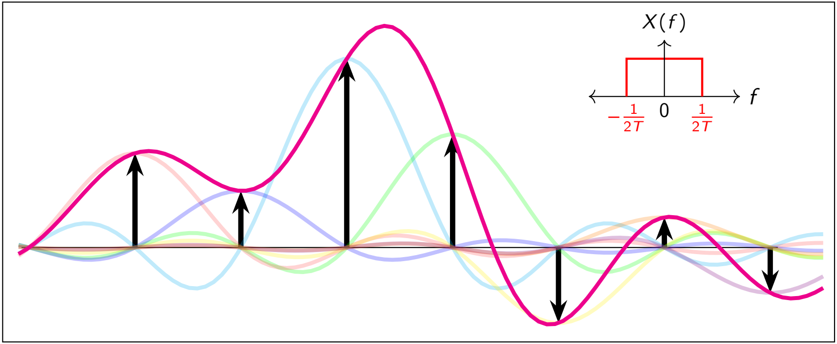

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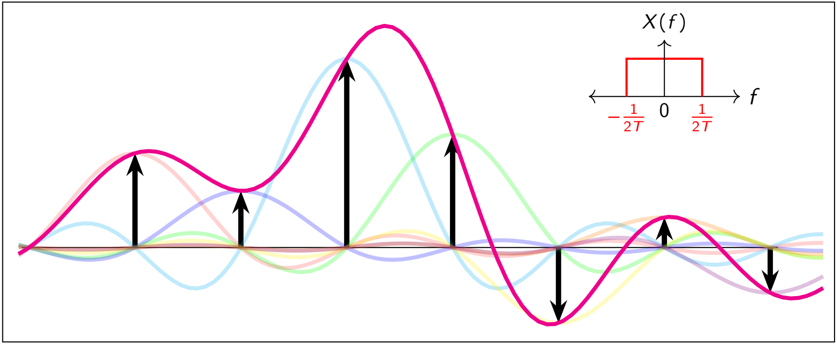

Using superposition: (Shannon Interpolation Formula)

$$x(t) = \sum_{n \in \mathbb{Z}} x[n] \, \mathrm{sinc}\left(\frac{t}{T} - n\right)$$

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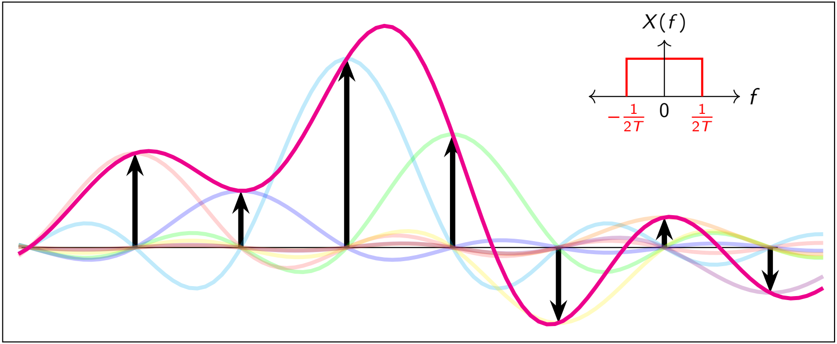
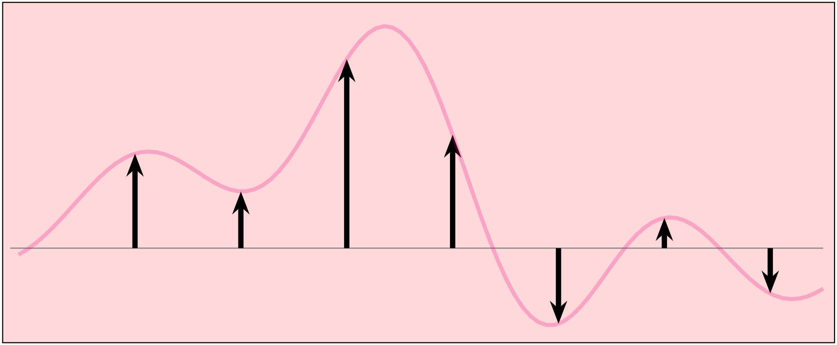


Using superposition: (Shannon Interpolation Formula)

$$x(t) = \sum_{n \in \mathbb{Z}} x[n] \operatorname{sinc}\left(\frac{t}{T} - n\right) \quad \text{“Convolution”}$$

Piece-wise Polynomials



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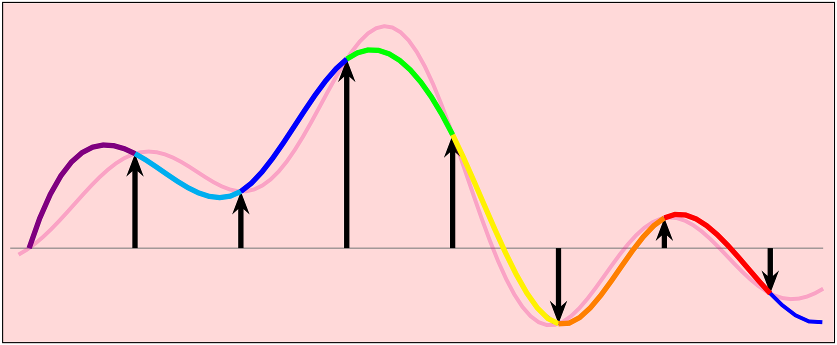

2D Demo: Interpolating Images; 1D : Audio (MP3 or WAV)

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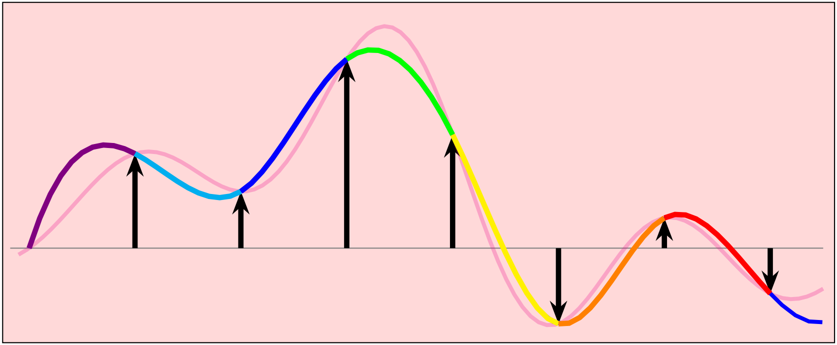

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How to deal with Colors (multi-dimensional)