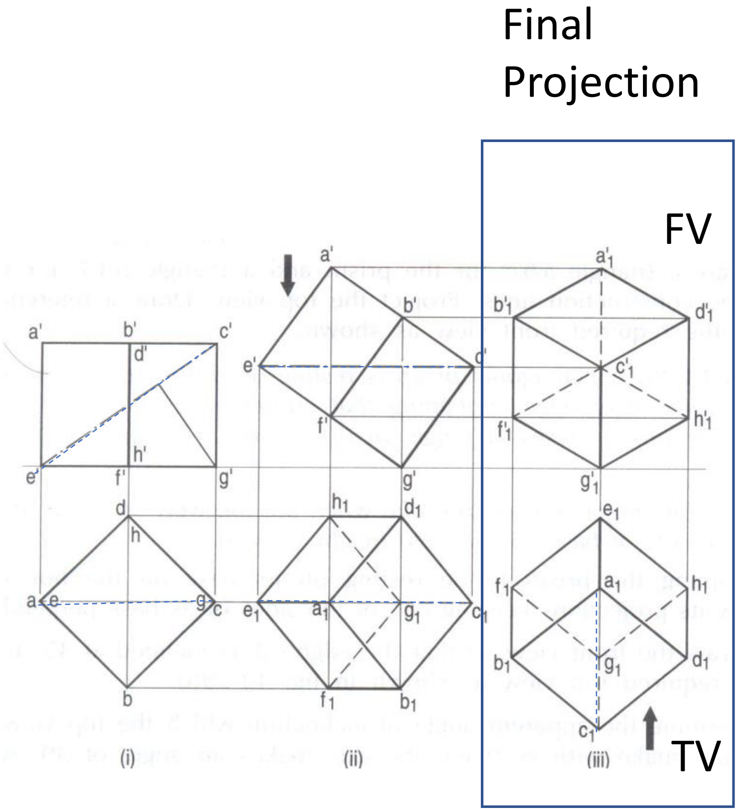
Projection of Point and Line

Sheet 2 ME119-P1-P3-P5

Projection of Points and Lines

- Motivation: Helps with drawing solid objects at arbitrary orientation
  - E.g. cube with solid diagonal (blue dashed line) perpendicular to VP
- We need to know how to:
  - Construct projection of line at arbitrary orientation
  - Find true length and orientation of line given its projection

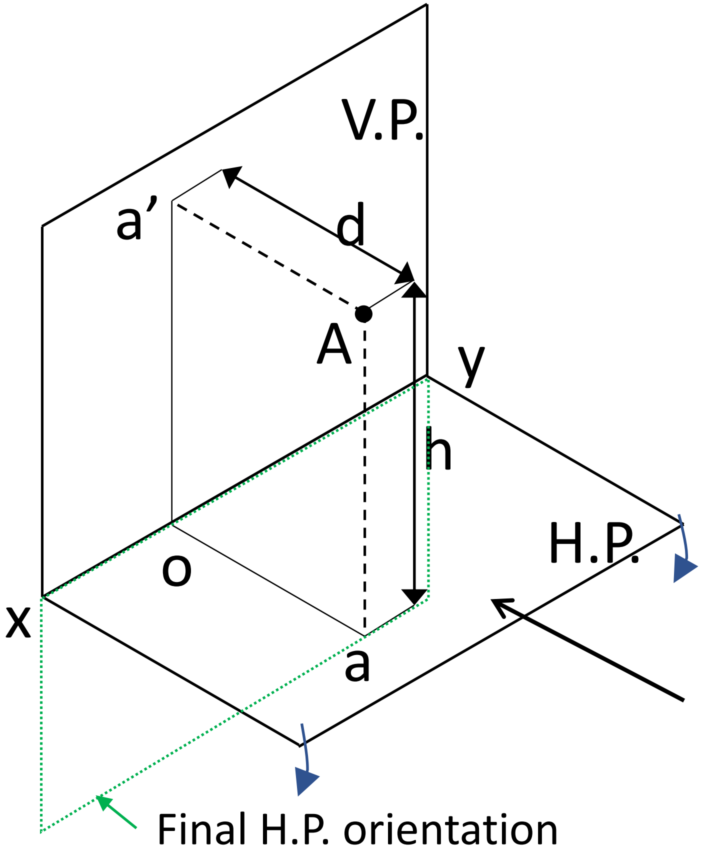


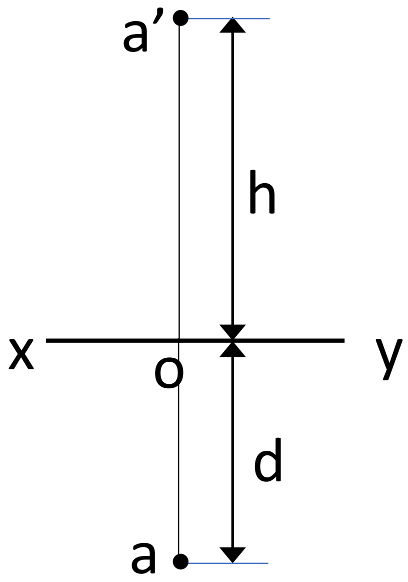
Stage-wise construction of projection of inclined cube

Projection of Points

- A point may be located in any of the four quadrants formed by the principal planes OR it may lie on the planes
- To the draw the projections, use the projectors to get a, a' and rotate one of the planes so as to open the first and the third quadrants

1st Angle Projection of point situated in the 1st quadrant



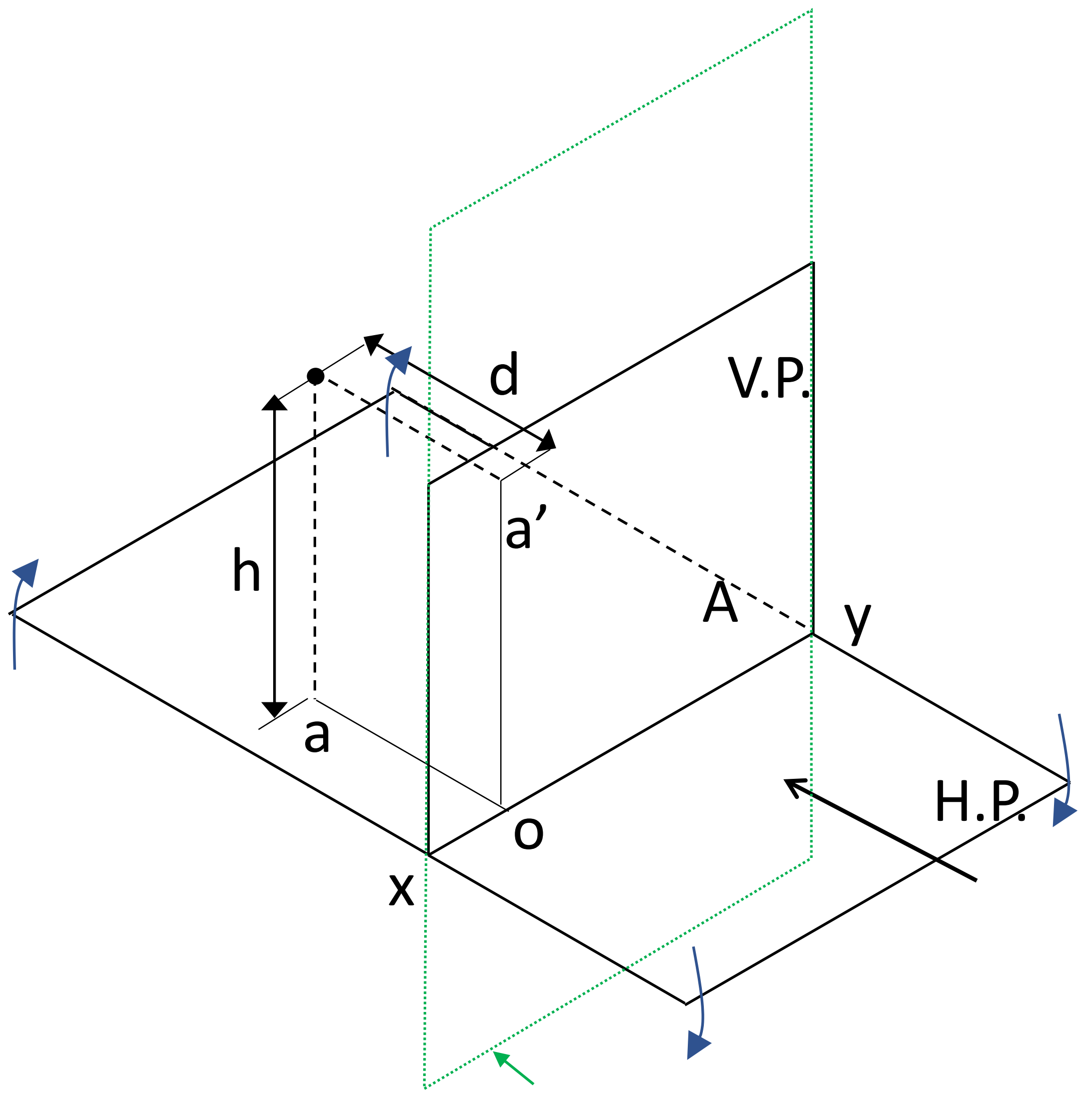


2D projection: obtained after rotating H.P. downwards, around xy

Front view above the xy line
Top view below the xy line

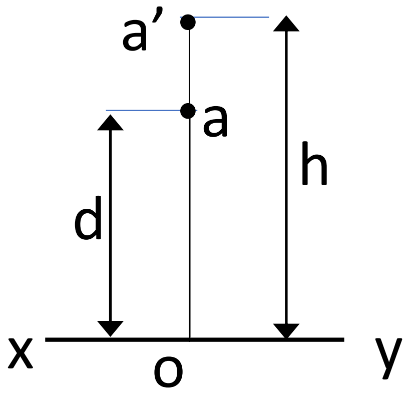
Projection of Points

1st Angle Projection of point situated in the 2^{nd} quadrant



Final H.P. orientation

2D projection: obtained after rotating H.P. around xy

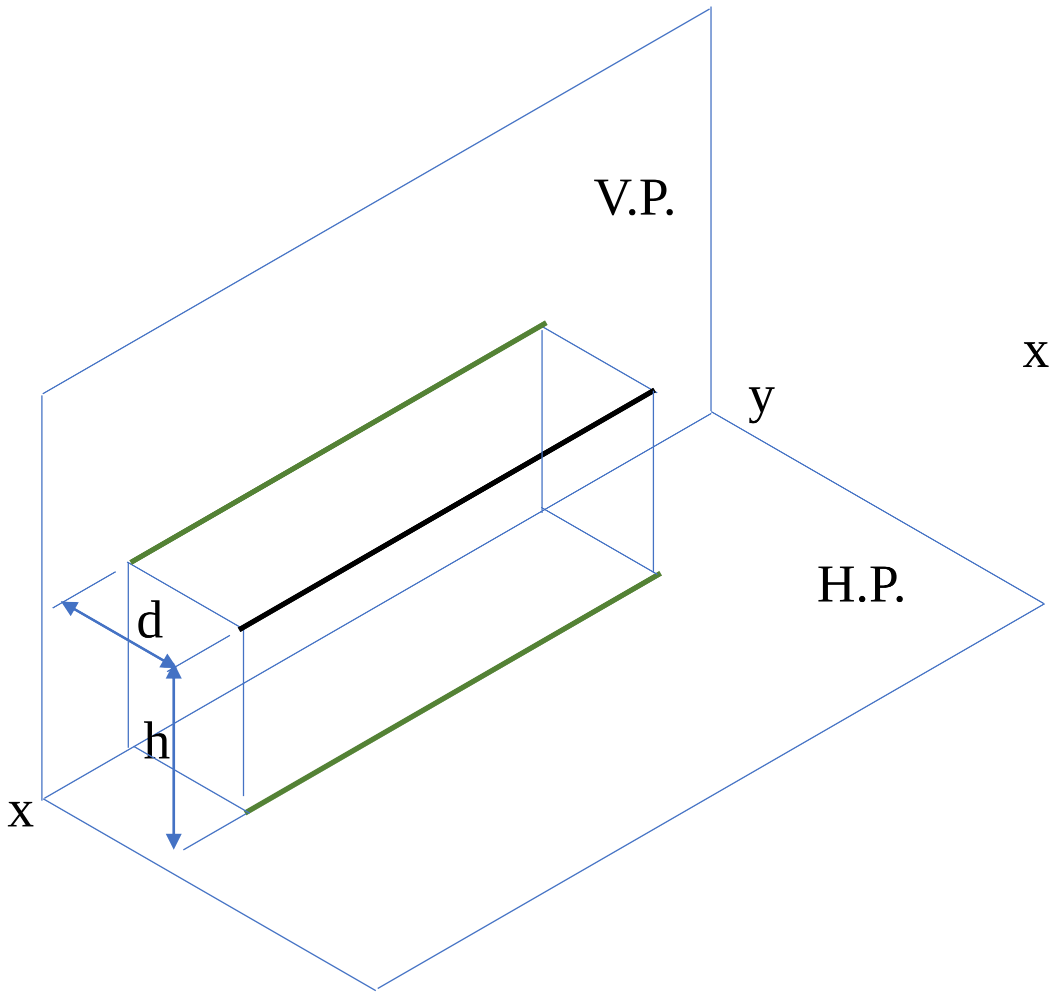


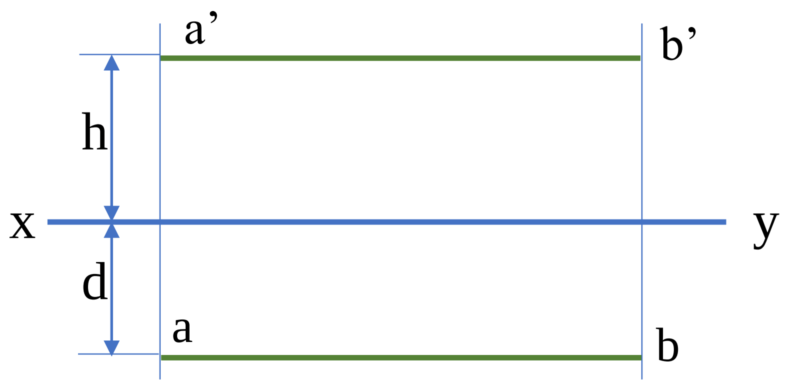
Front view above the xy line
Top view above the xy line

General Comments about Projection of Points

- The front view and the top view of a point are always on the same vertical line
- The distance of the front view of a point from the xy line is equal to the distance of the point from the H.P.
- The distance of the top view of a point from the xy line is equal to the distance of the point from the V.P.
- If a given point is above the H.P., its front view is above the xy line
- If a given point is below the H.P., its front view is below the xy line
- If a given point is in front of the V.P., its top view is below the xy line
- If a given point is behind the V.P., its top view is above the xy line

Projections of a line parallel to both the V.P. and the H.P.



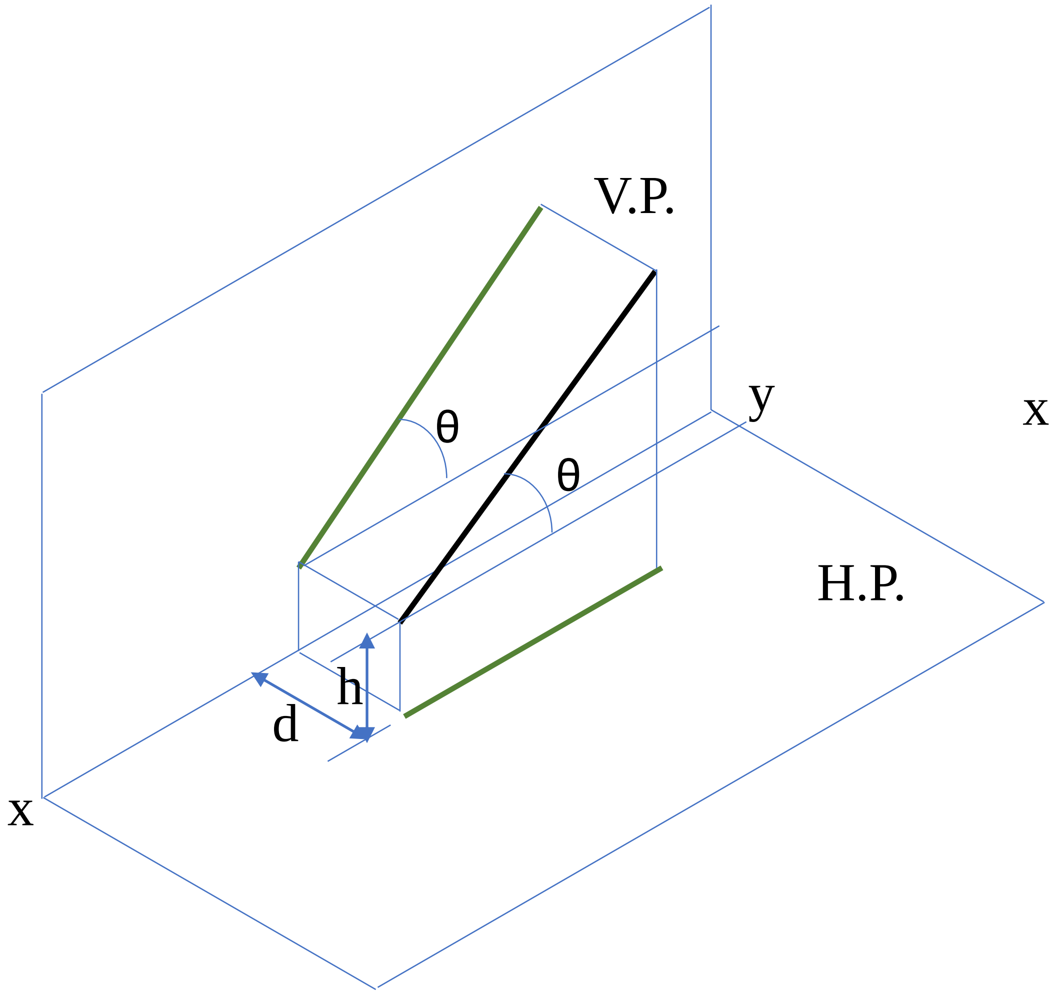


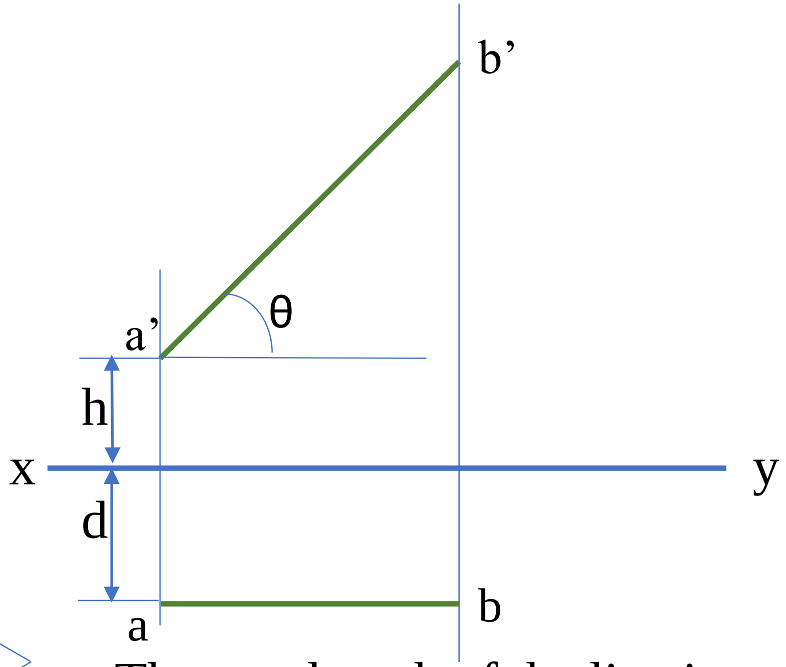
- The true length of the line is seen in both the front view and the top view
- The front view is parallel to the xy line
- The top view is parallel to the xy line

Labeling convention: H.P. projections a,b,c,..
V.P. projections a',b',c',d'

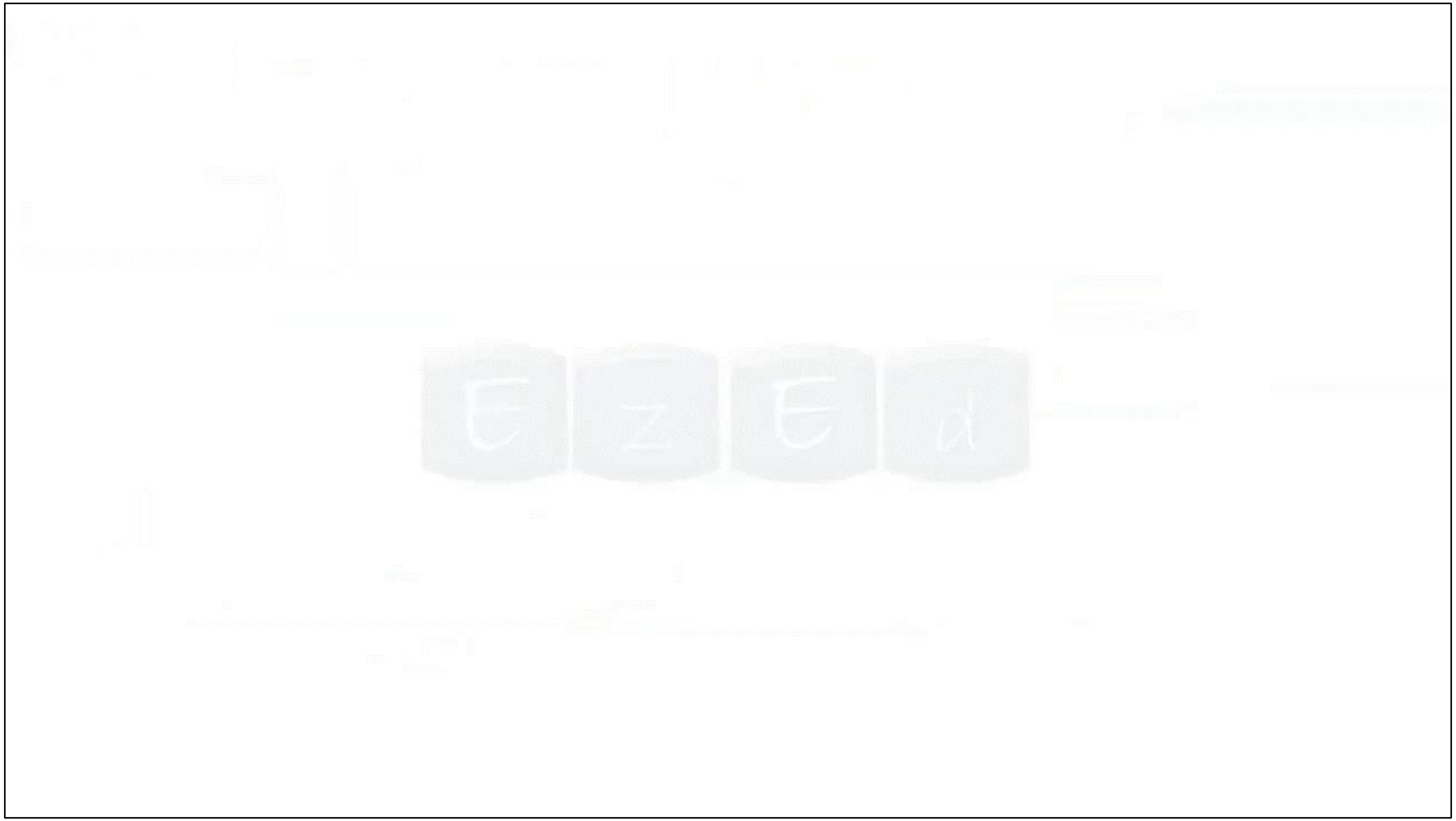
Projections of a line parallel to the V.P. and inclined to H.P.

The inclination of a line to a plane is the angle that the line makes with its projection on that plane



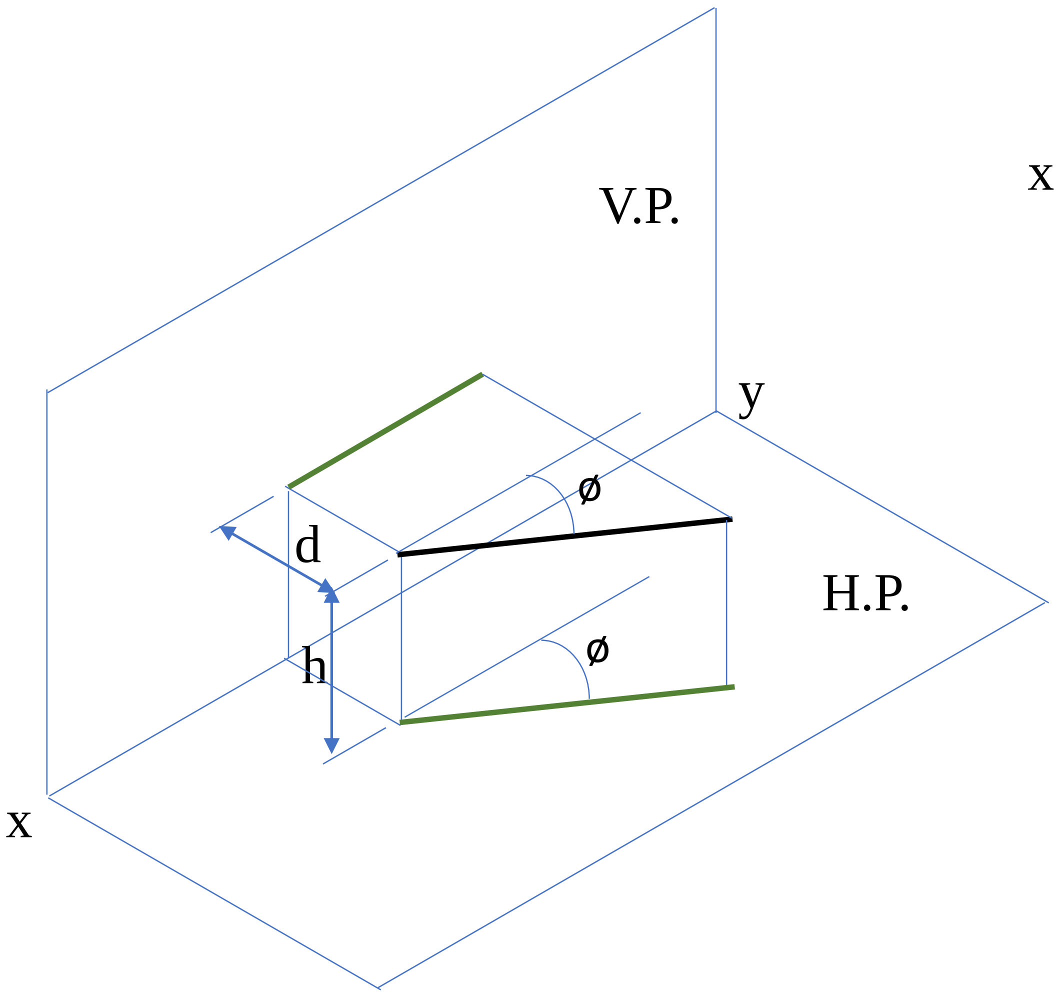


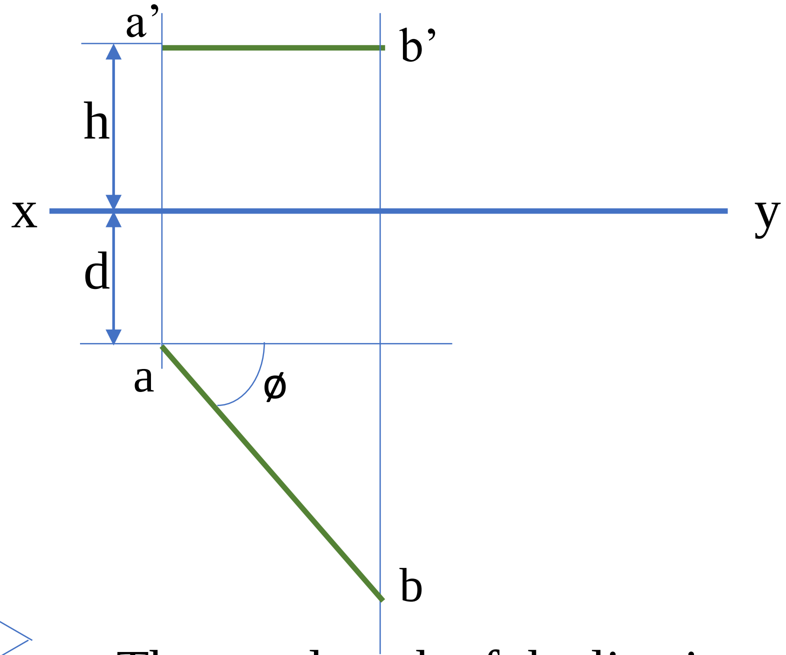
- The true length of the line is seen in the front view
- The angle which the line makes with the H.P. is seen in the front view
- The top view is parallel to the xy line



https://www.youtube.com/watch?v=mStD1NN42tE

Projections of a line parallel to the H.P. and inclined to the V.P.





- The true length of the line is seen in the top view
- The angle which the line makes with the V.P. is seen in the top view
- The front view is parallel to the xy line

General Comments about Projection of Lines

- When a line is parallel to a plane, its projection on that plane is equal its true length, while its projection on the other plane is parallel to the xy line

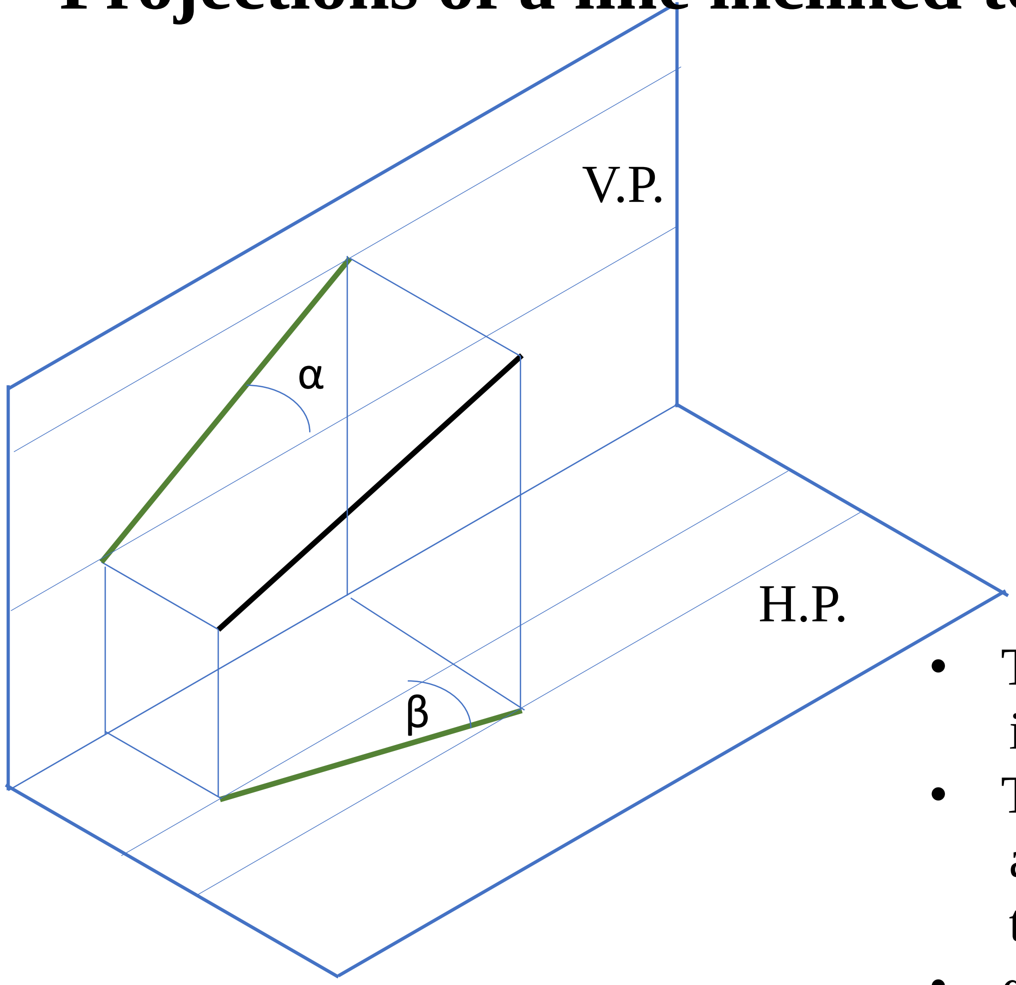
- When a line lies in a plane, its projection on that plane is equal its true length, while its projection on the other plane is in the xy line

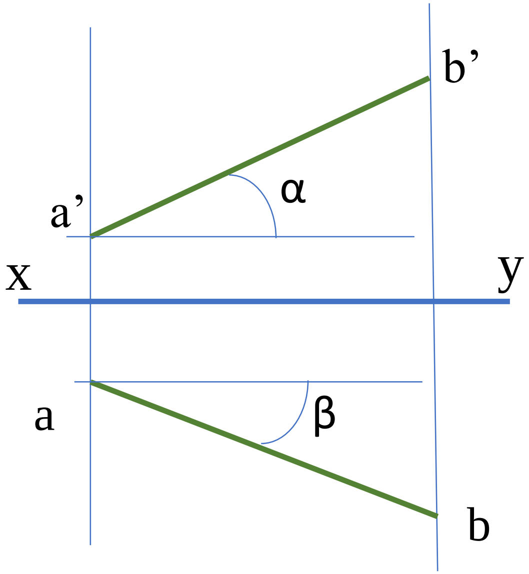
- When a line is perpendicular to a plane, its projection on that plane is a point, while its projection on the other plane is perpendicular to the xy line and equal to its true length

- Inclination with the H.P. can be seen in the **front view** while the inclination with the V.P. can be seen in the **top view**

  i.e. if line is parallel to either V.P. or H.P.

Projections of a line inclined to both H.P. and the V.P.





- The top view and the front view are inclined to the xy line
- The length of the line in the top view and the front view is less than the true length
- α and β are the ***apparent* angle of inclination** with the H.P. and the V.P. and are greater than the true angles of inclination

Projections of a line inclined to both H.P. and the V.P.

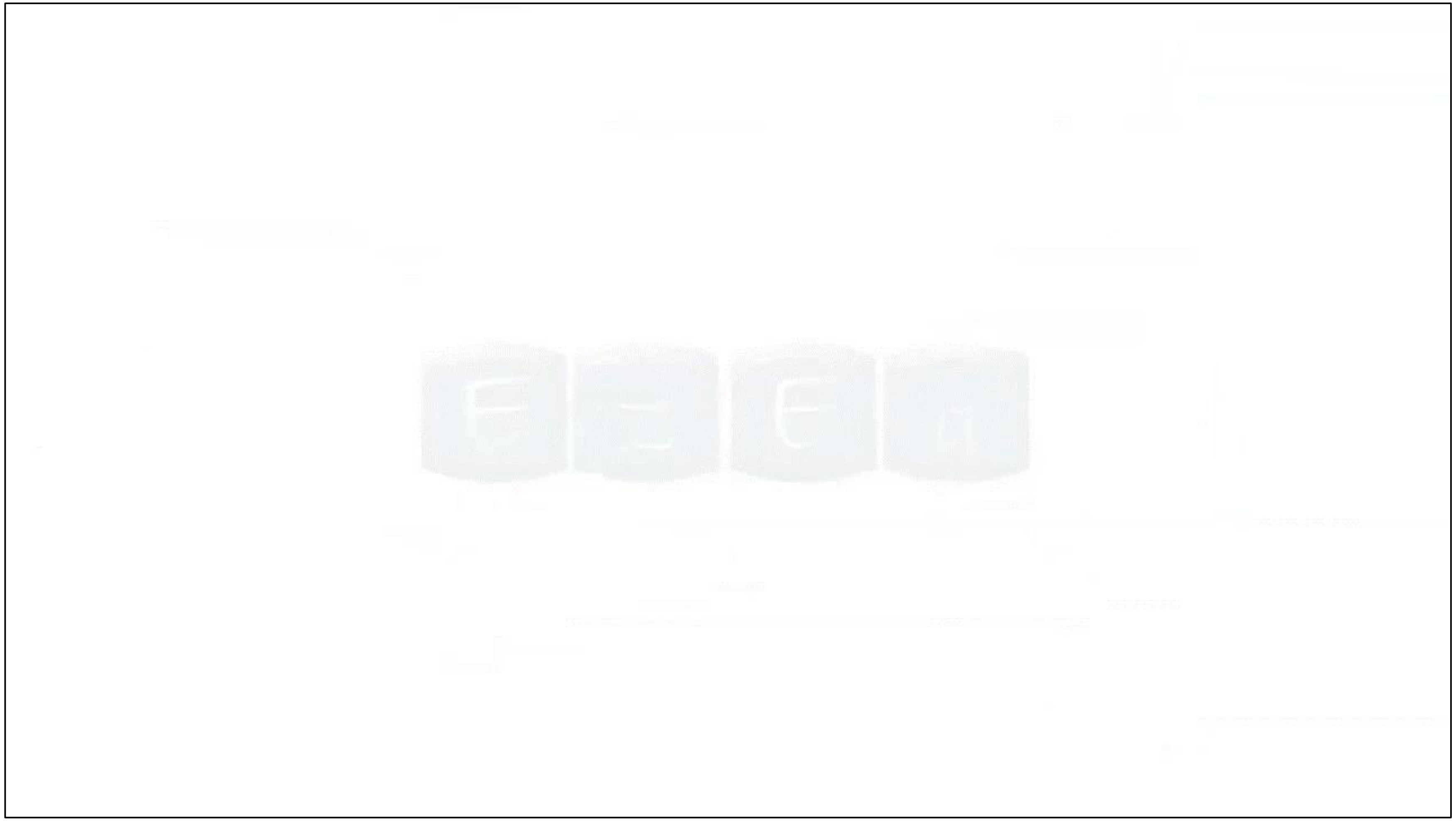
Typical problems include finding the true length of the line and the true angle of inclination with the principal planes once the projections are given

Important Observation: When a line is parallel to a plane, its projection on that plane will show its true length and its true inclination with the other plane

The true length of a line can be obtained by:

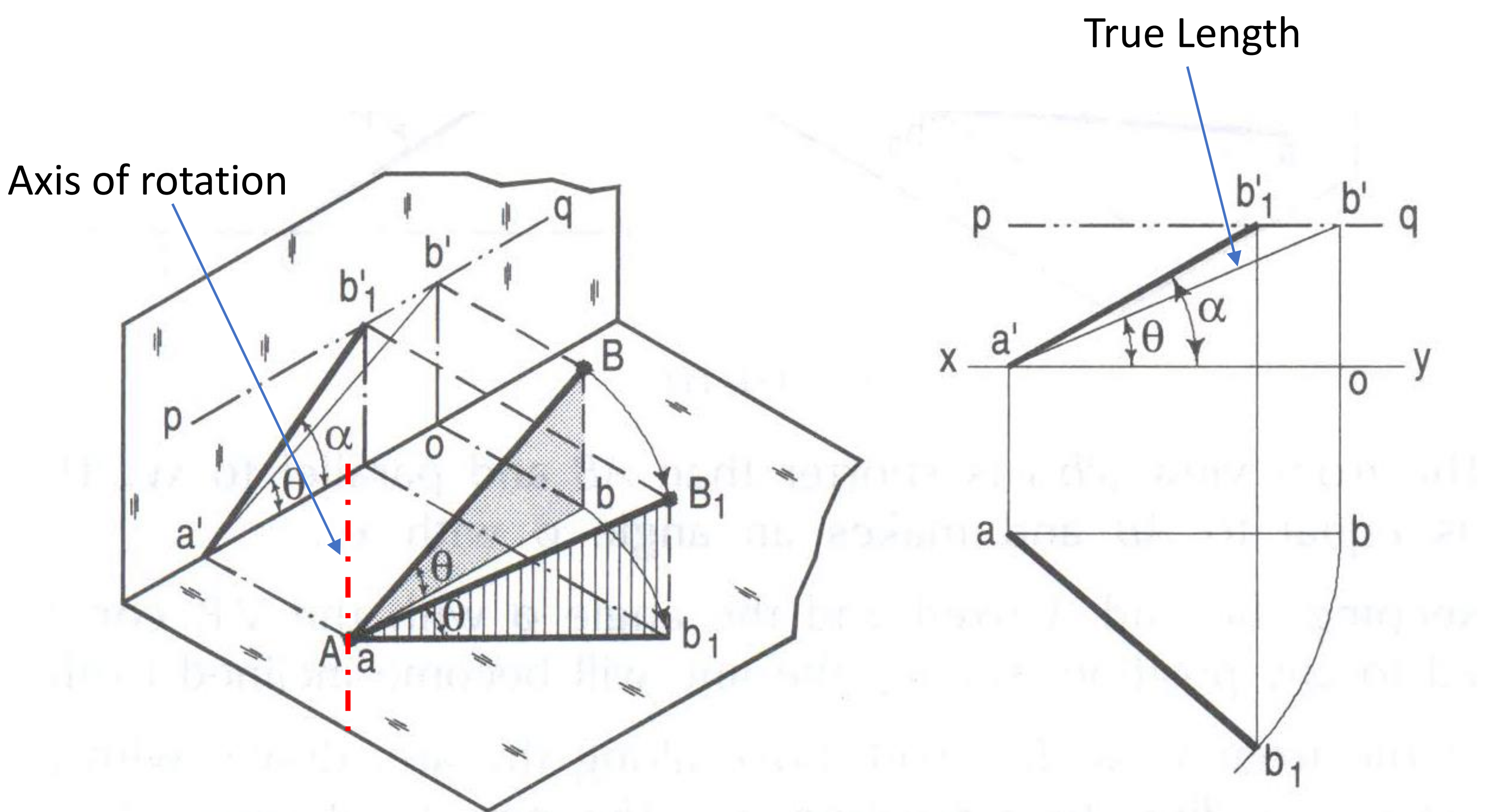
- Rotating the line so as to make it parallel to one of the principal planes (covered in this lecture)
- Projecting the views on auxiliary planes parallel to each view (covered in next lecture)

Line inclined to both the planes



https://www.youtube.com/watch?v=mStD1NN42tE

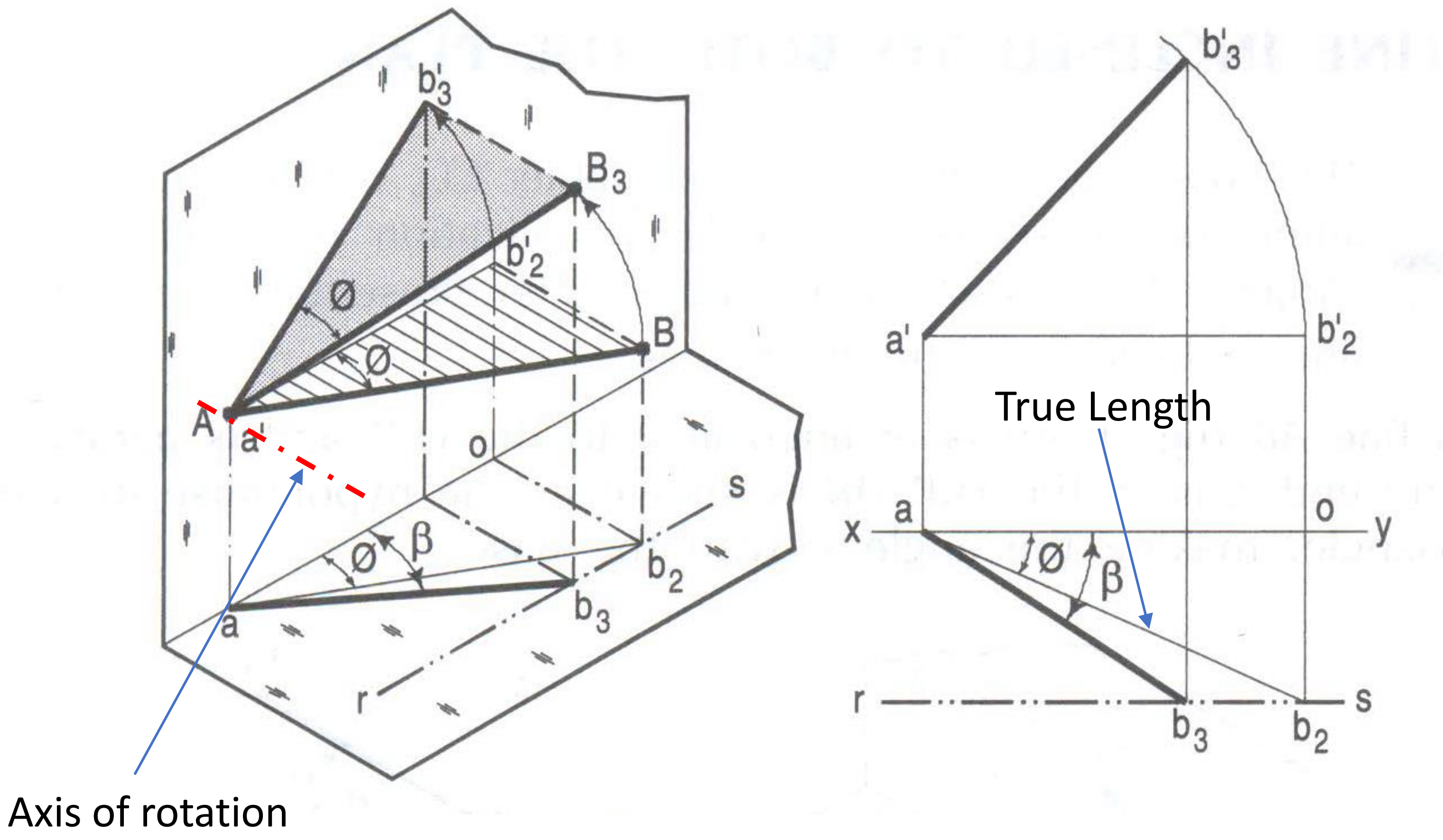
Line Inclined to Both the Planes



Given: Projections of line AB. **Find:** True length of AB and inclination w.r.t. H.P.

- Rotate AB_1 around A (axis of rotation given by vertical red line)
- Height B_1b_1 same as Bb
- After rotation, projection on H.P. (ab) is parallel to xy
- So projection on V.P. (a'b') must be the true length and θ must be true angle w.r.t. H.P.

Procedure to find the True Length and True Angle of Inclination with the V.P. (ø)



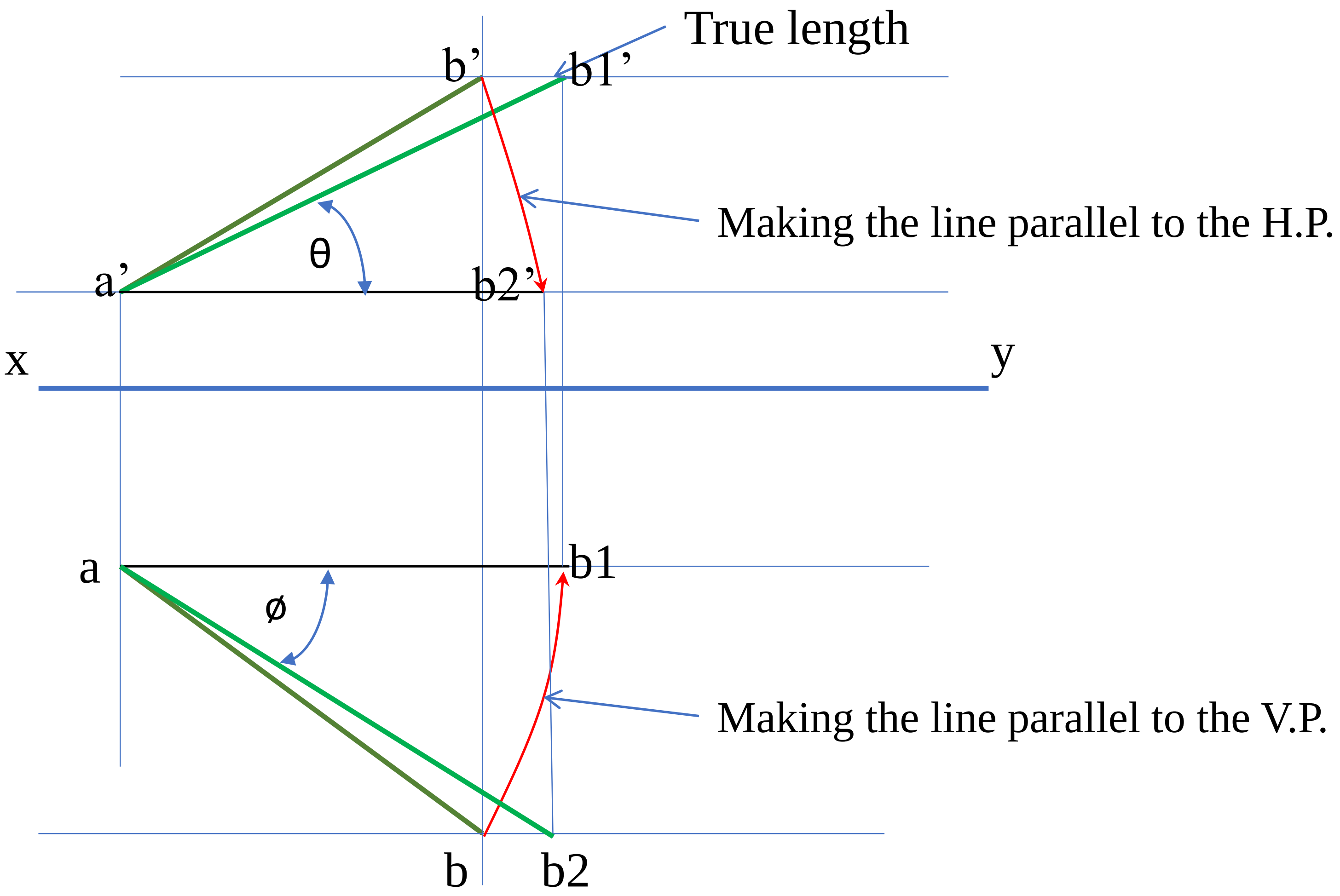
Typical Problem

Given the two projections of a line on the principal planes, find the angle of inclination of the line with the planes and the true length of the line

The two projections can be specified in multiple ways, e.g.

- The location of two endpoints of the line with respect to the principal planes and the distance between the vertical projections
- The location of one end point of the line with respect to the principal planes, the distance between the end projectors and the angle the projectors make with the xy line

Solution



True length
b'
b1'
Making the line parallel to the H.P.
θ
a'
b2'
x
y
a
b1
ø
Making the line parallel to the V.P.
b
b2

Points to Remember

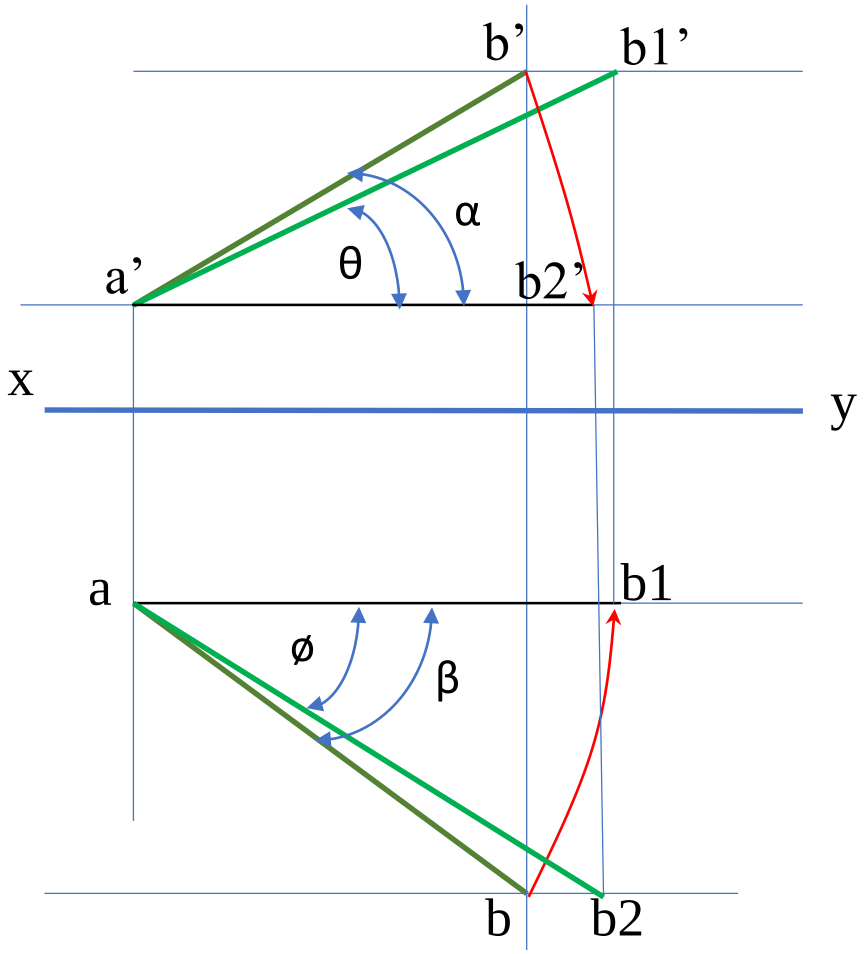
- The true length of a line can only be obtained by projecting it on a plane which is parallel to the line

- The angle of inclination with one of the principal planes can be obtained by making the line parallel to the other principal plane and then measuring the angle between the projection showing its true length and a line parallel to the xy line

- When a line is rotated keeping its angle of inclination with the H.P. constant, the length of the top view does not change. The distance of the end points of the line from the H.P. also remain constant

- When a line is rotated keeping its angle of inclination with the V.P. constant, the length of the front view does not change. The distance of the end points of the line in from the V.P. also remain constant

Steps to Solve a Typical Problem



- Make a rough sketch as shown on the left
- Identify all the parameters that are known
- Find out which of the lines in the sketch can be drawn given the problem parameters
- Find out which lines are required to be drawn to find out the required answers
- Based on the different relations between the true angles, true length, projections, etc. locate the end points of the lines which are required
- After locating the endpoints, construct the lines and measure the required quantities

Example: Finding Projections

The top view of a 75 mm long line AB measures 65 mm, while the length of its front view is 50 mm. Its one end A is in the H.P. and 12 mm in front of V.P. Draw the projections of AB and determine its inclinations with HP and VP.

Example: Finding Projections

The top view of a 75 mm long line AB measures 65 mm, while the length of its front view is 50 mm. Its one end A is in the H.P. and 12 mm in front of V.P. Draw the projections of AB and determine its inclinations with HP and VP.

Given:

- Positions of a and a'
- ab=65
- $a'b'_1=50$
- $a'b'=ab_1=75$

To solve, note that b' and b'_2 must lie on cd.
Similarly, b_1 and b_2 must lie on ef

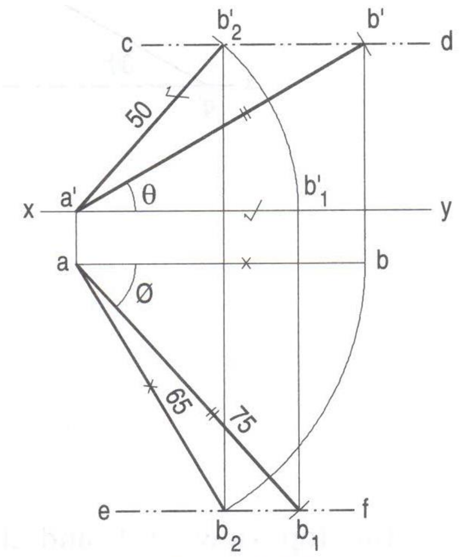
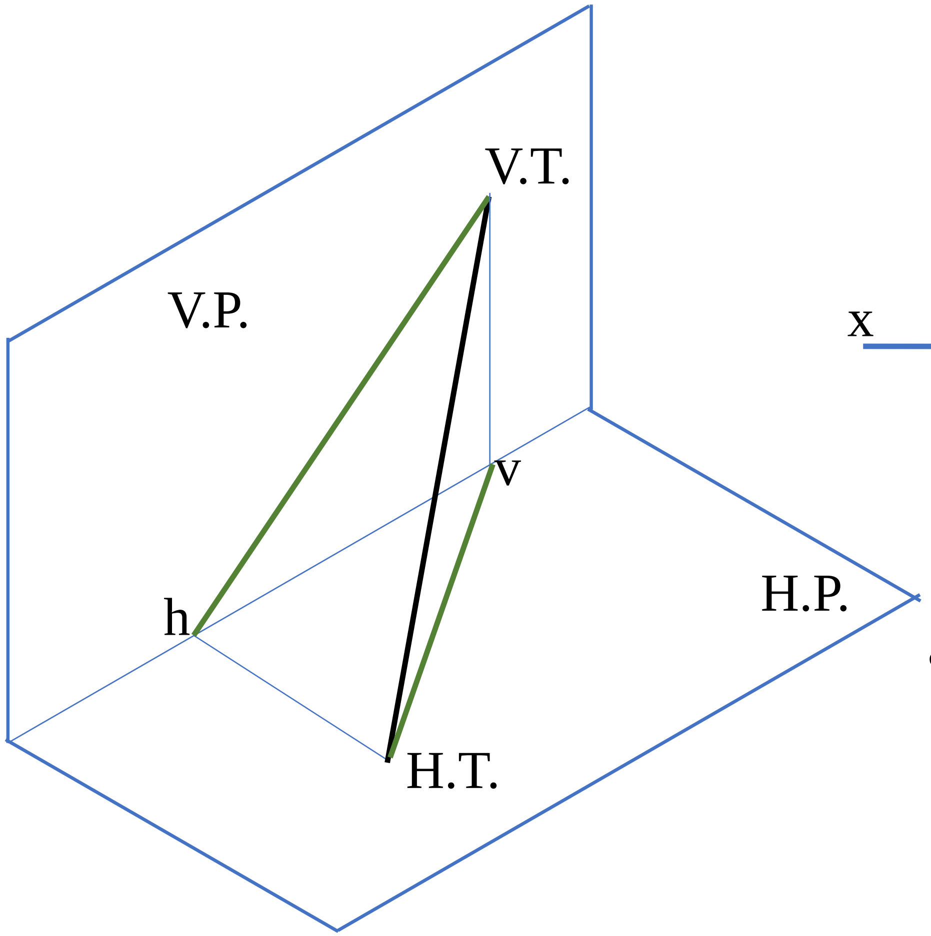
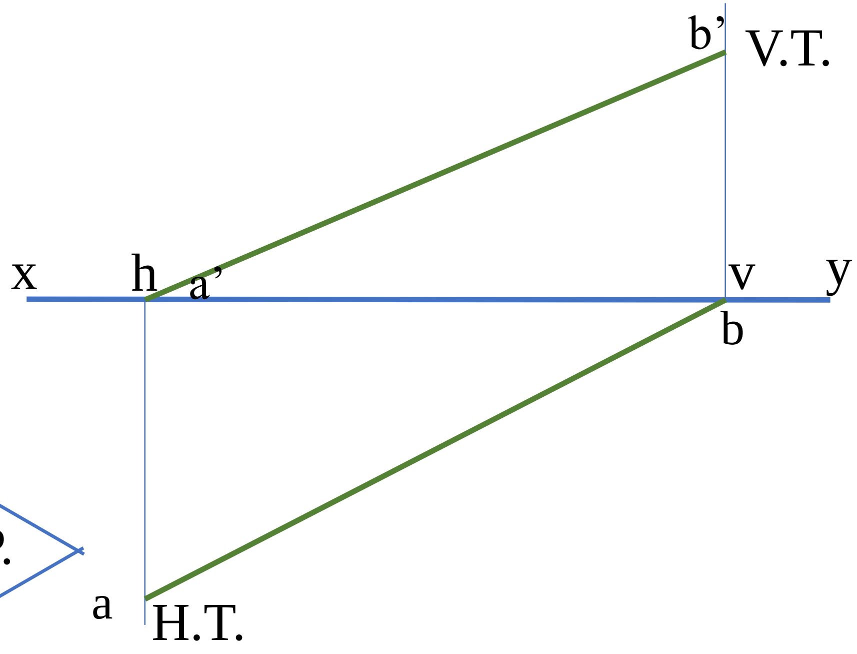


FIG. 10-35

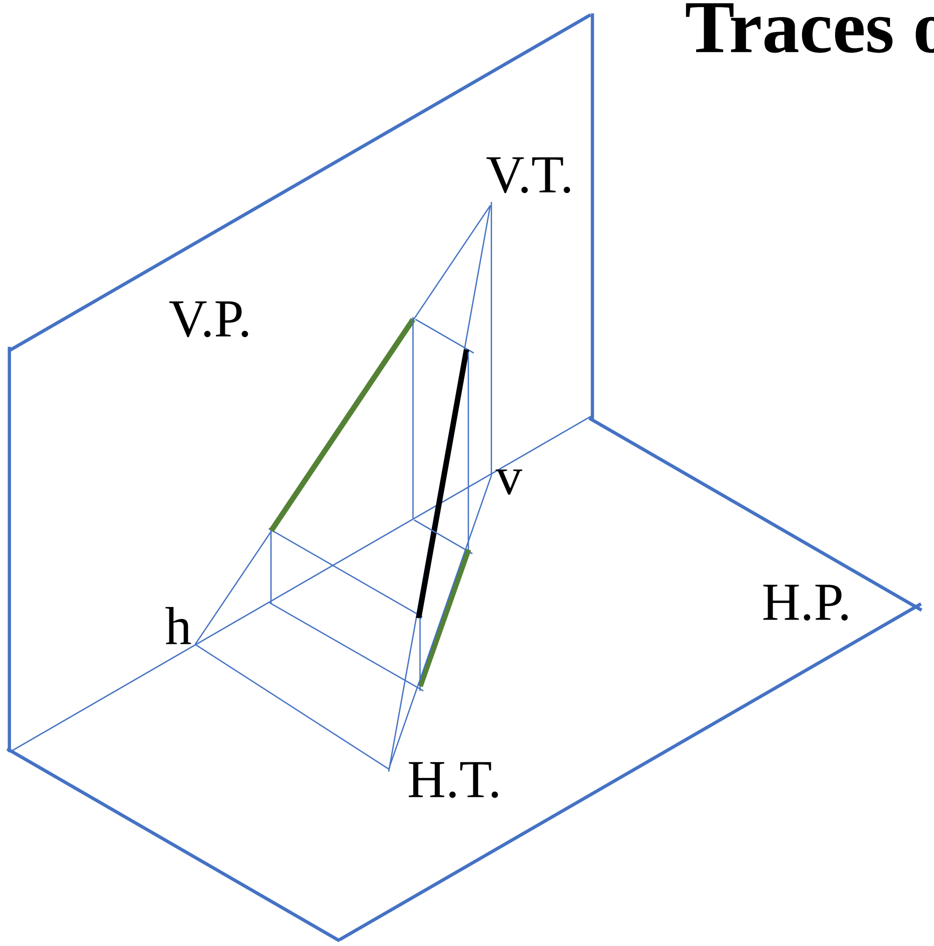
Traces of a Line

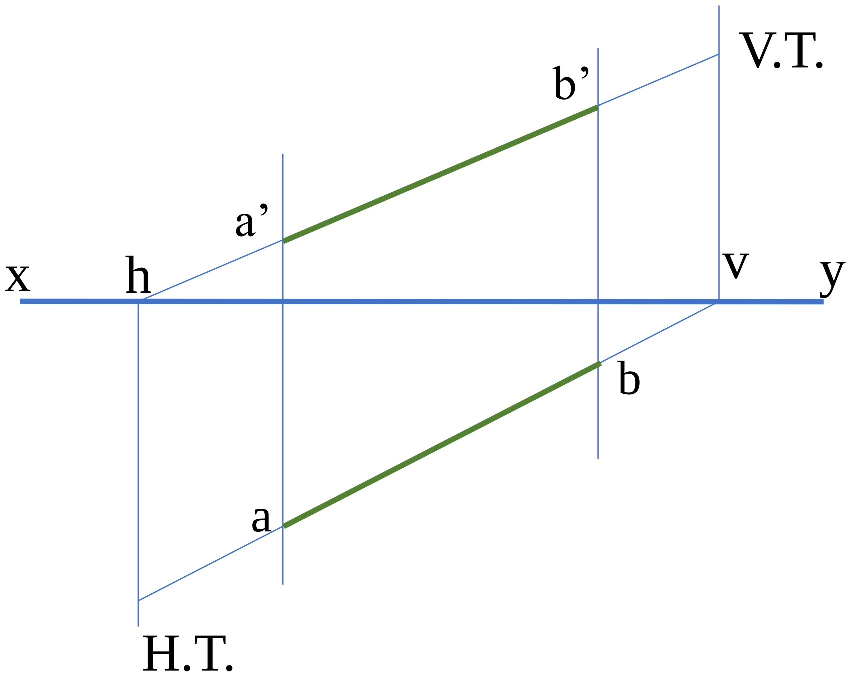
- The point at which a line or a line-produced meets the plane is called the trace
- The point of intersection of the line with the H.P. is called the horizontal trace (H.T.)
- The point of intersection of the line with the V.P. is called the vertical trace (V.T.)





Traces of a Line

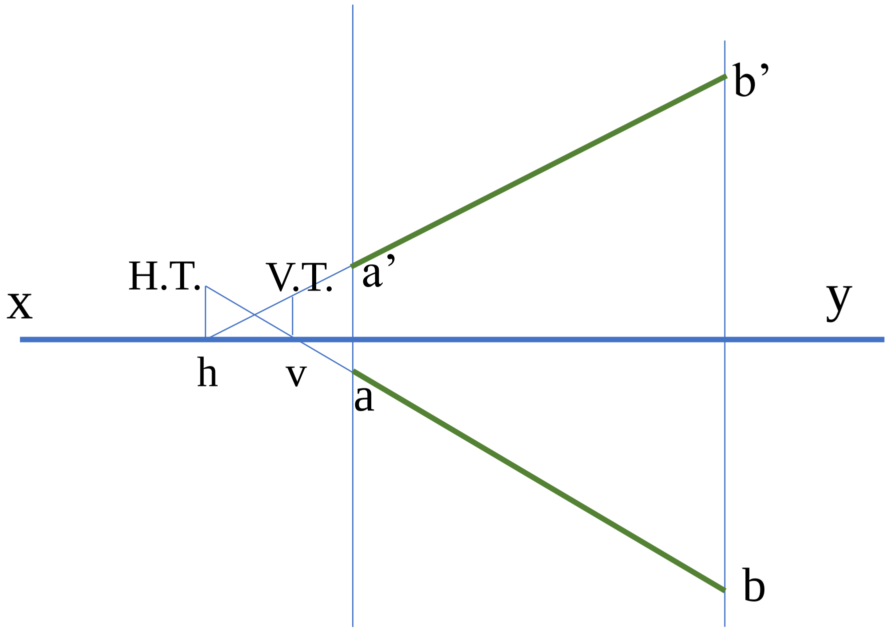


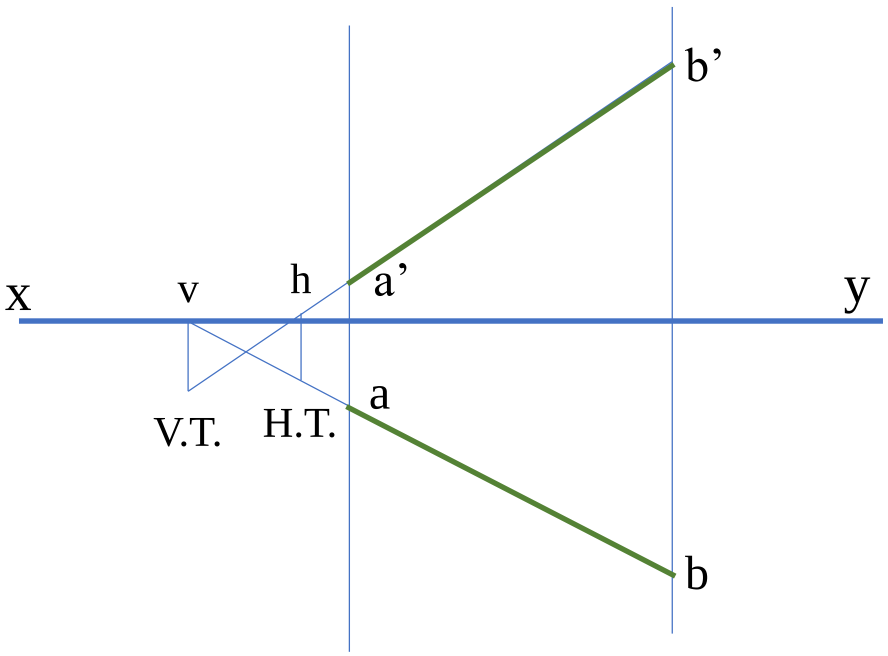


- H.T. is front of the V.P.
- V.T. is above the H.P.

- H.T. lies at the intersection of the top view extended and the projector through point h – the point of intersection of the front view extended and the xy line
- V.T. lies at the intersection of the front view extended and the projector through point v – the point of intersection of the top view extended and the xy line

Representative Positions of the Traces

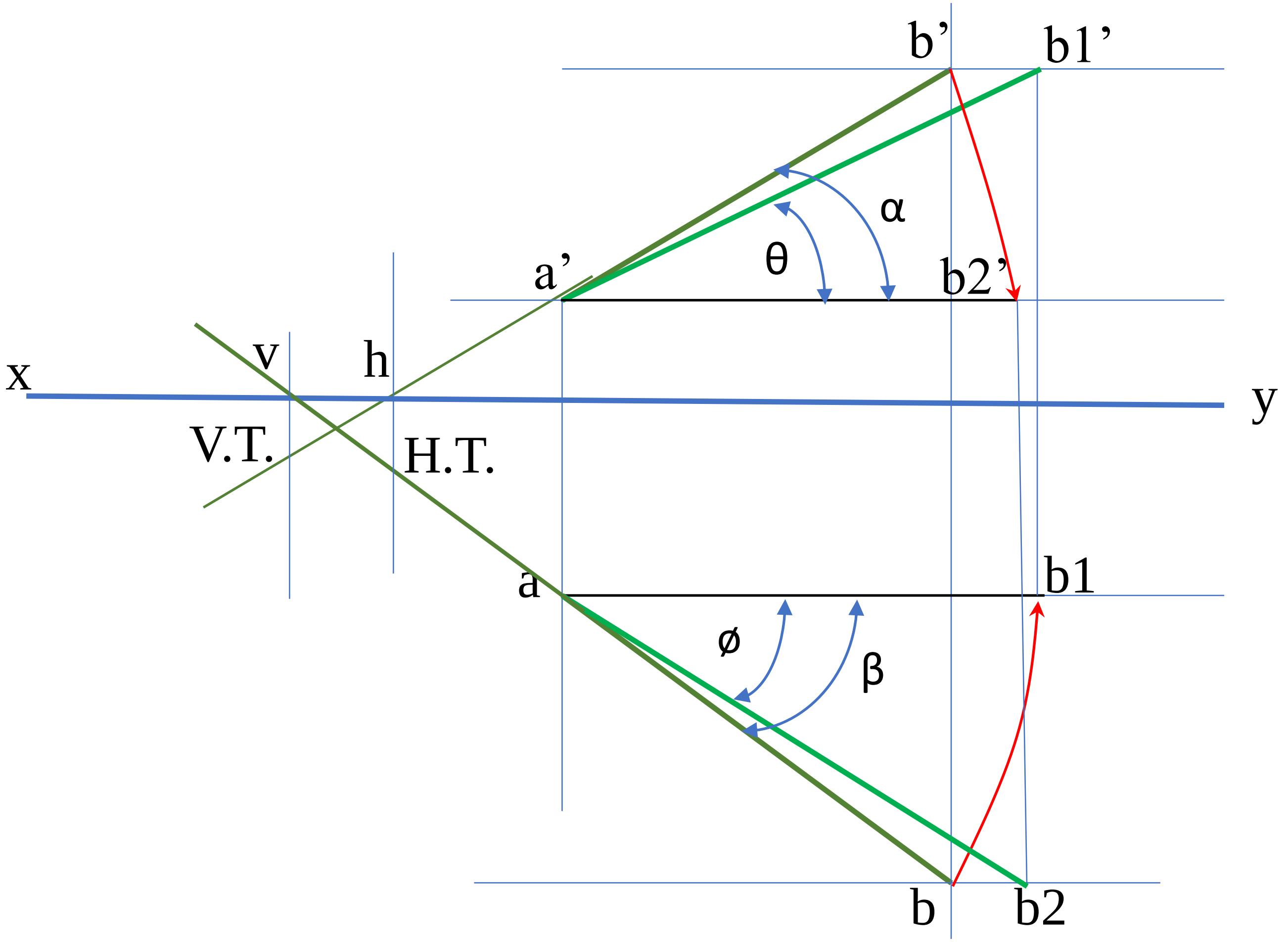




- H.T. is behind the V.P.
- V.T. is above the H.P.

- H.T. is in front of the V.P.
- V.T. is below the H.P.

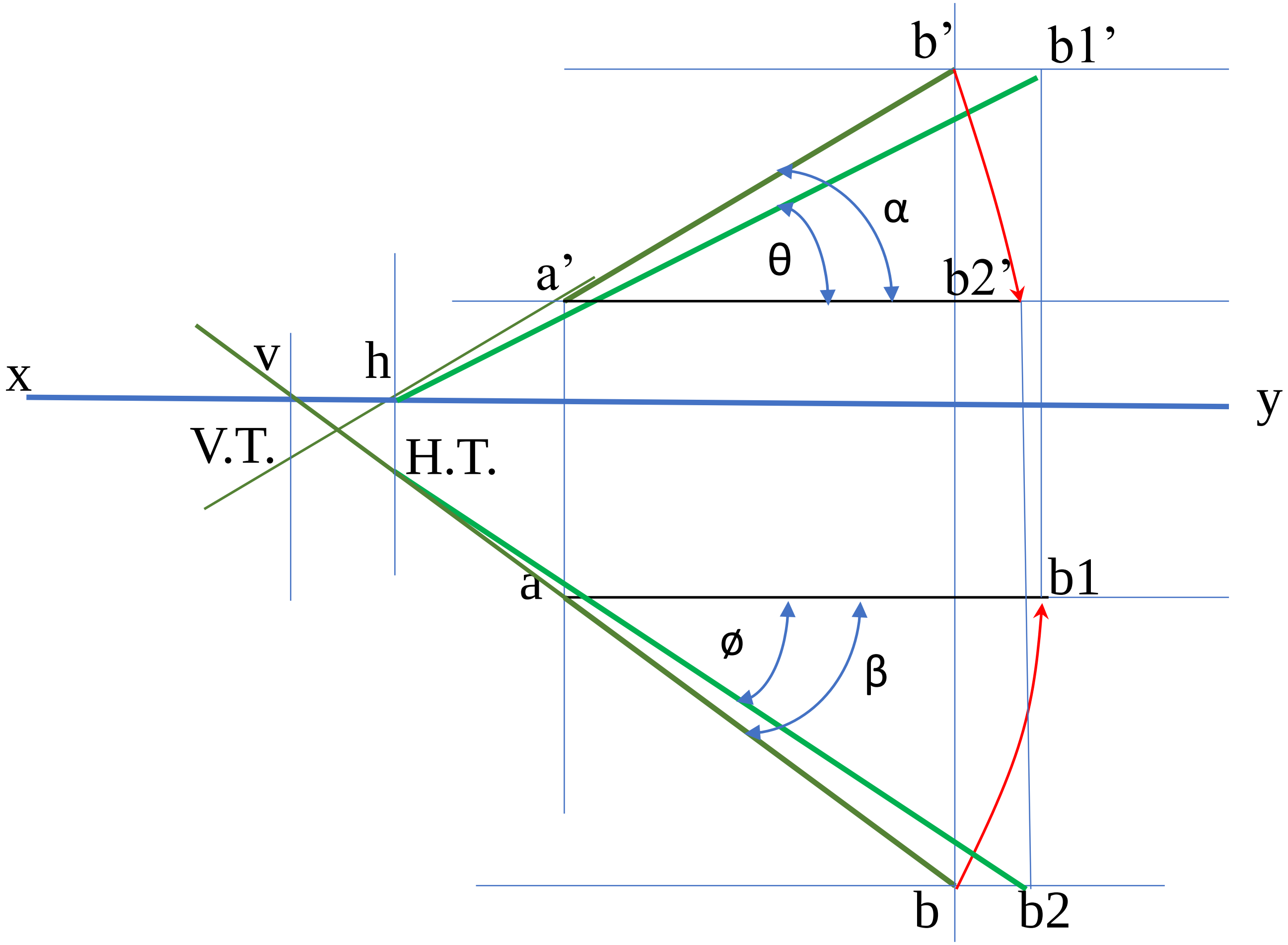
Steps to Solve a Typical Problem



- Here T.L. is still being found by rotating a'b' or ab.
- In some cases, information about VT/HT is given instead
  - We then need to use a different method

- Traces are extra pieces of information in the usual projection of lines problem
- When H.T. or V.T. is given instead of a or a', it helps to work with the projections vb and hb' (instead of ab and a'b')

Steps to Solve a Typical Problem

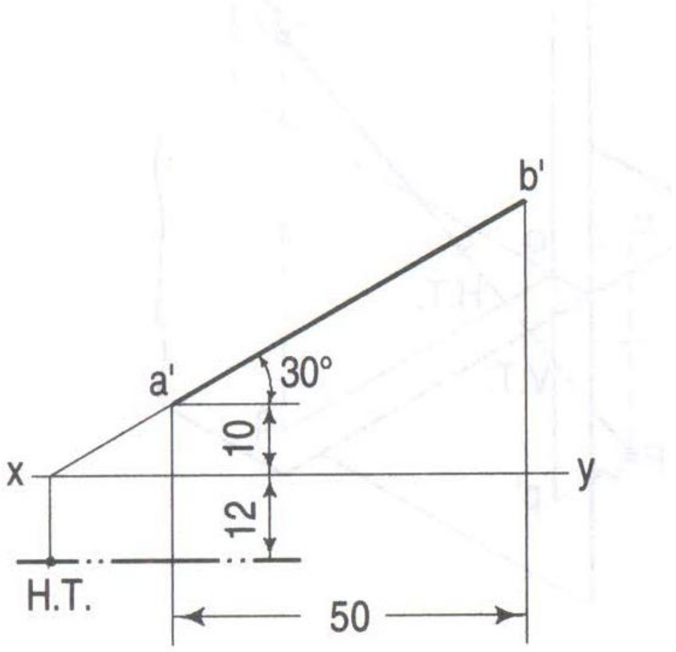


In this case, we use the projections hb' and H.T.-b

- Traces are extra pieces of information in the usual projection of lines problem
- When H.T. or V.T. is given instead of a or a', it helps to work with the projections vb and hb' (instead of ab and a'b')

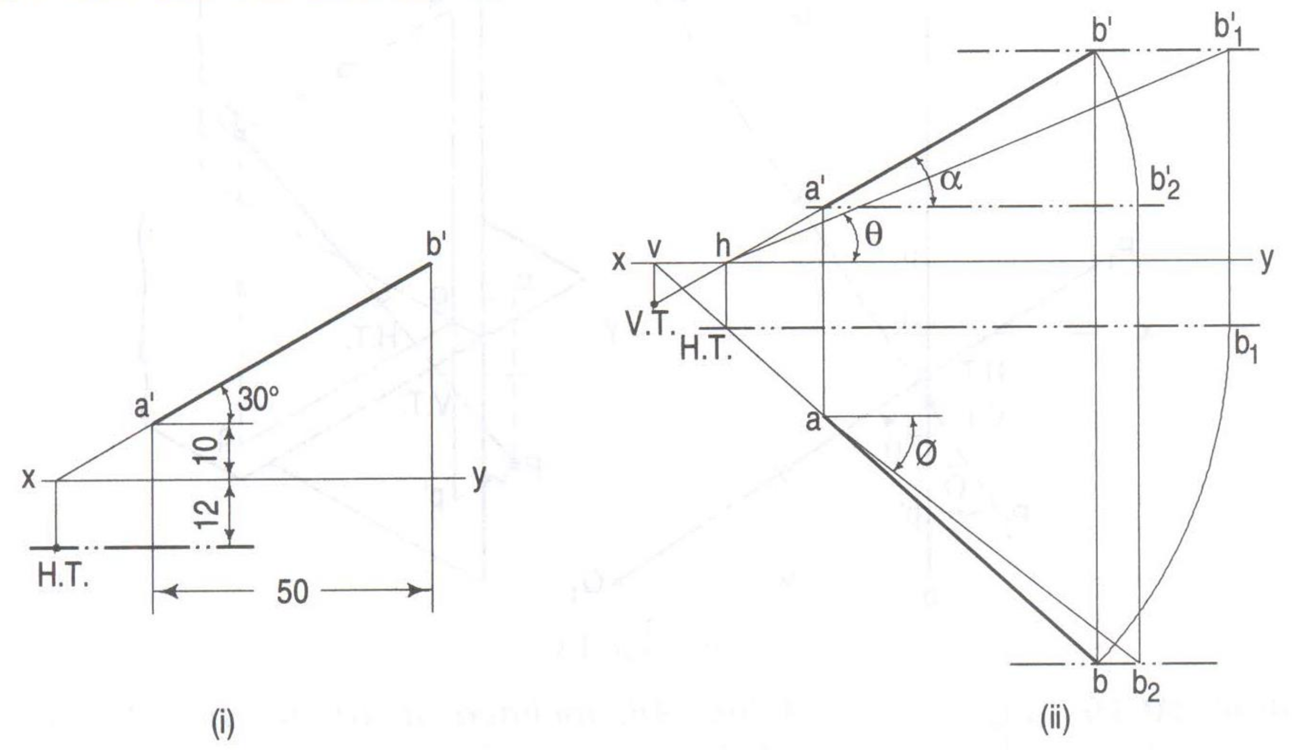
Example: Finding V.T.

The front view a'b' and the H.T. of a line AB, inclined at 23 deg to the H.P. are given. Determine the true length of AB, its inclination with the V.P. and its V.T.



Example: Finding V.T.

The front view a'b' and the H.T. of a line AB, inclined at 23 deg to the H.P. are given. Determine the true length of AB, its inclination with the V.P. and its V.T.



- H.T. is fixed, so B is rotated around H.T. such that $\theta = 23$. Draw hb_1'.
- Draw HTb_1 by drawing normal to xy from b_1'
- Distance between H.T. and b_1 is same as distance between H.T. and b

Auxiliary Planes

- Horizontal Plane (H.P.), Vertical Plane (V.P.) and the Profile Plane (P.P.) are referred to as principal planes

- Projections on the principal planes are called as principal views

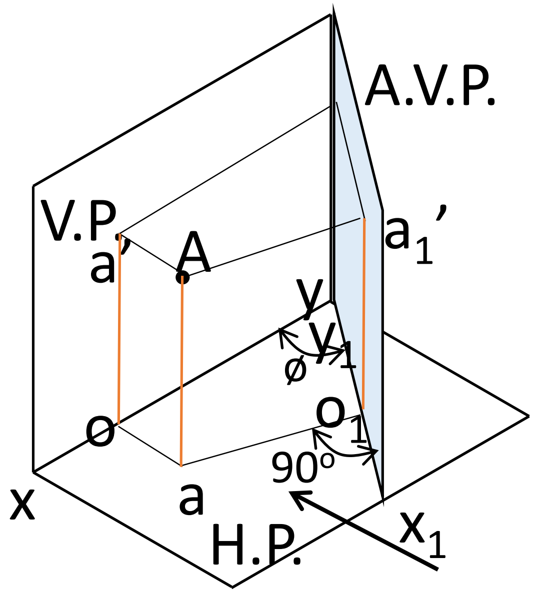
- Additional views called auxiliary views are obtained by projecting on planes called auxiliary planes.

- Auxiliary views help to convey additional information not conveyed by the principal views (e.g. true shape of a surface inclined to two or more principal planes)

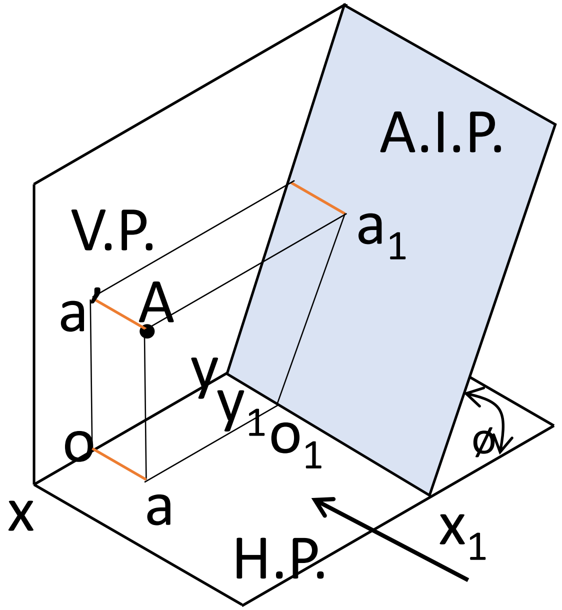
- Sometimes auxiliary planes provide an easier method to solve problems involving projection -
  - Finding the true length of a line
  - Finding the inclination of the line with the H.P and the V.P.
  - Finding the shortest distance between two lines
  - Finding the true shape of a plane object

Types of Auxiliary Planes

- Auxiliary Vertical Plane (A.V.P.)
  It is perpendicular to the H.P. and inclined to the V.P.
  Projection on A.V.P. is called auxiliary front view
  Auxiliary front view and the front view have a common dimension (height)

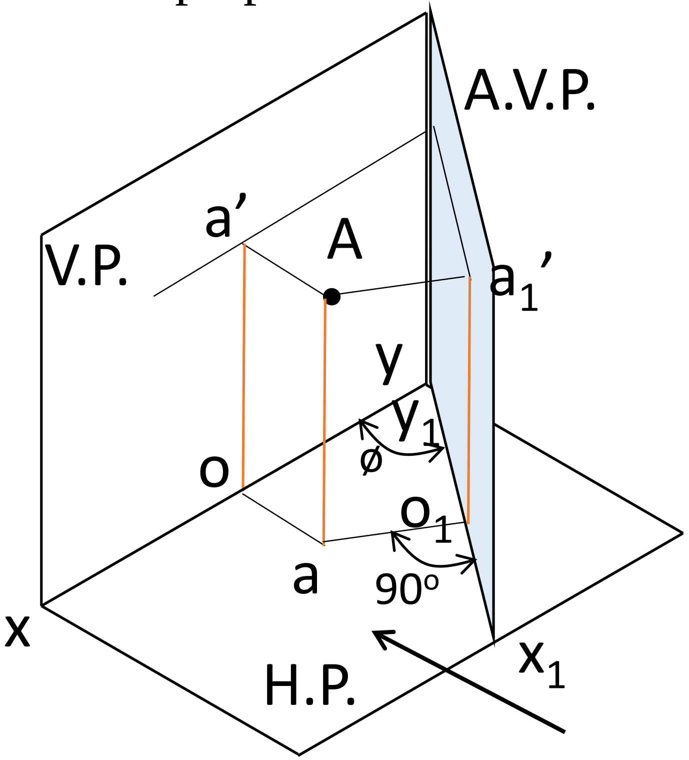


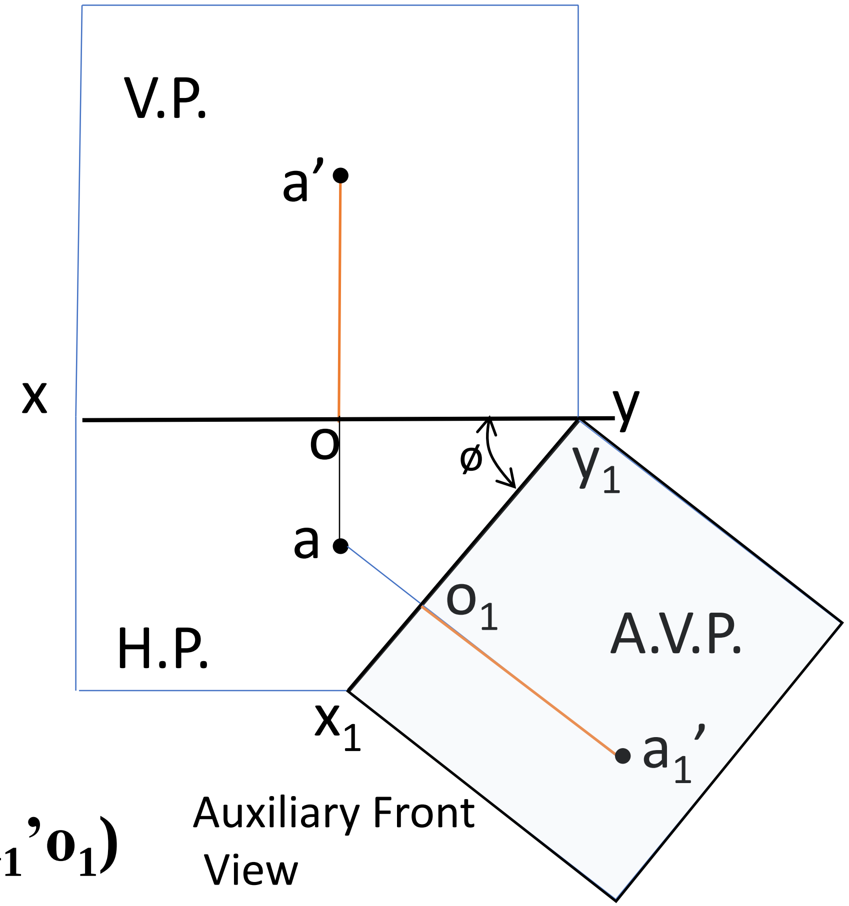
- Auxiliary Inclined Plane (A.I.P.)
  It is perpendicular to the V.P. and inclined to the H.P.
  Projection on A.I.P. is called auxiliary top view
  Auxiliary top view and the top view have a common dimension (depth)



Projection of a Point on the A.V.P.

- A.V.P. is perpendicular to the H.P. and inclined to the V.P.





Height is preserved

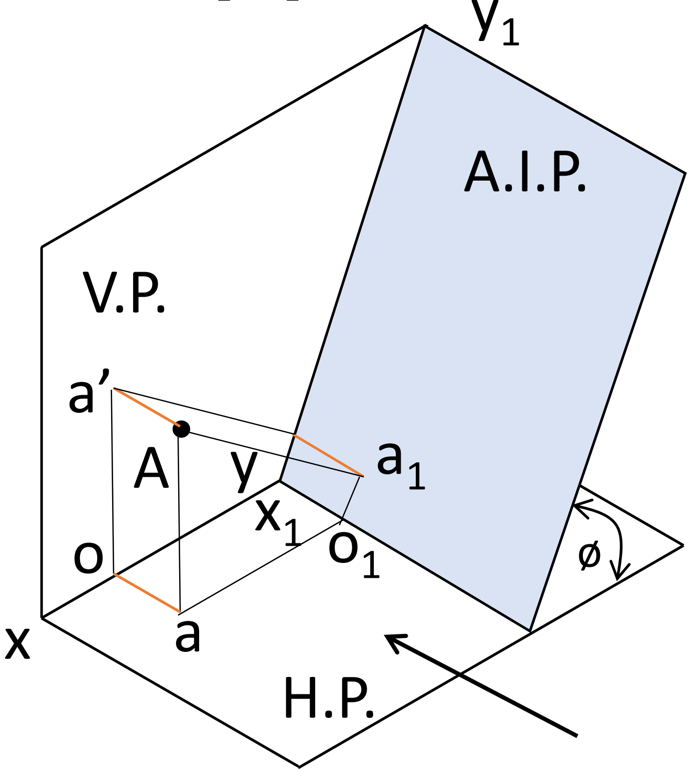
$$l(a'o) = l(a_1'o_1)$$

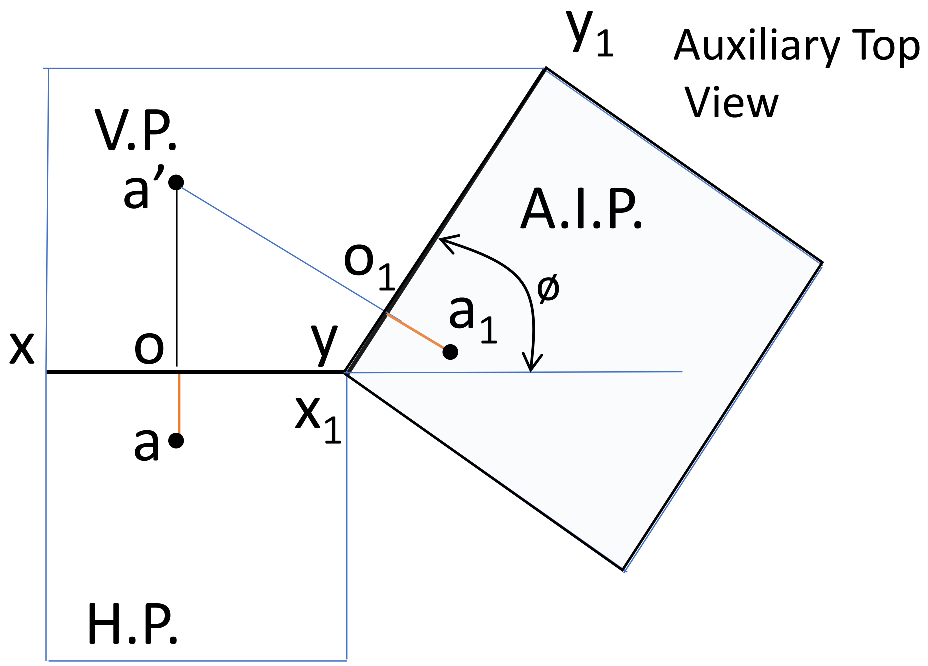
Auxiliary Front View

Auxiliary front view is drawn by rotating the auxiliary vertical plane about the line x_1y_1

Projection of a Point on the A.I.P.

- A.I.P. is perpendicular to the V.P. and inclined to the H.P.



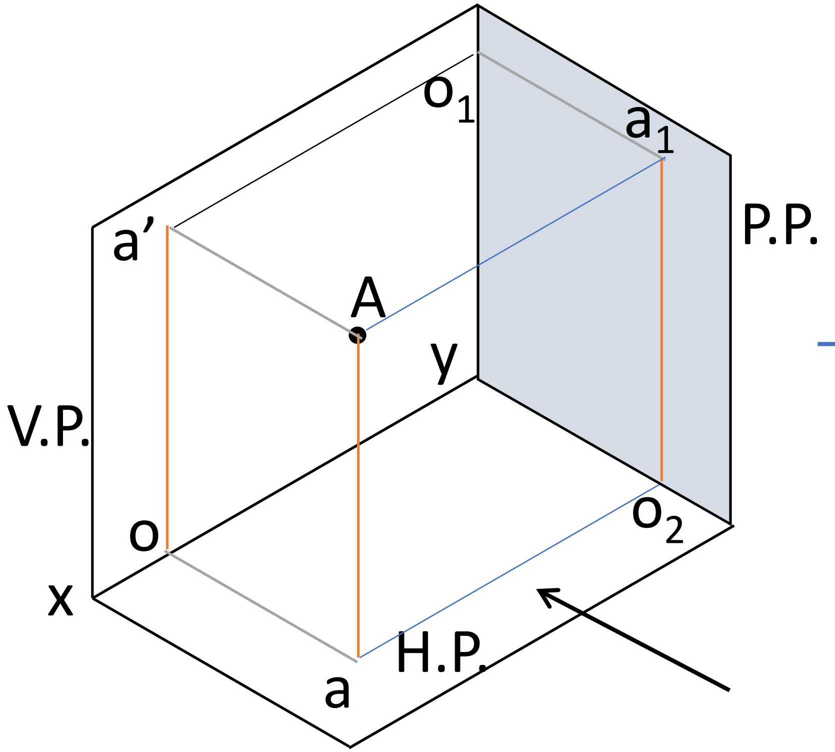


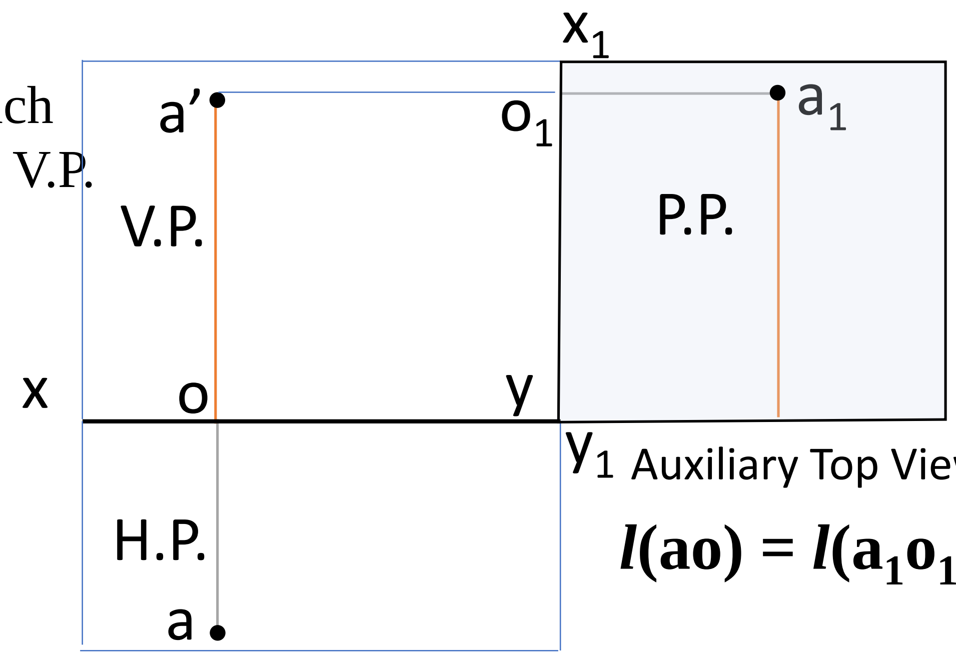
Depth is preserved $\boldsymbol{l(ao) = l(a_1o_1)}$

Auxiliary top view is drawn by rotating the auxiliary inclined plane about the line x_1y_1

Profile Plane

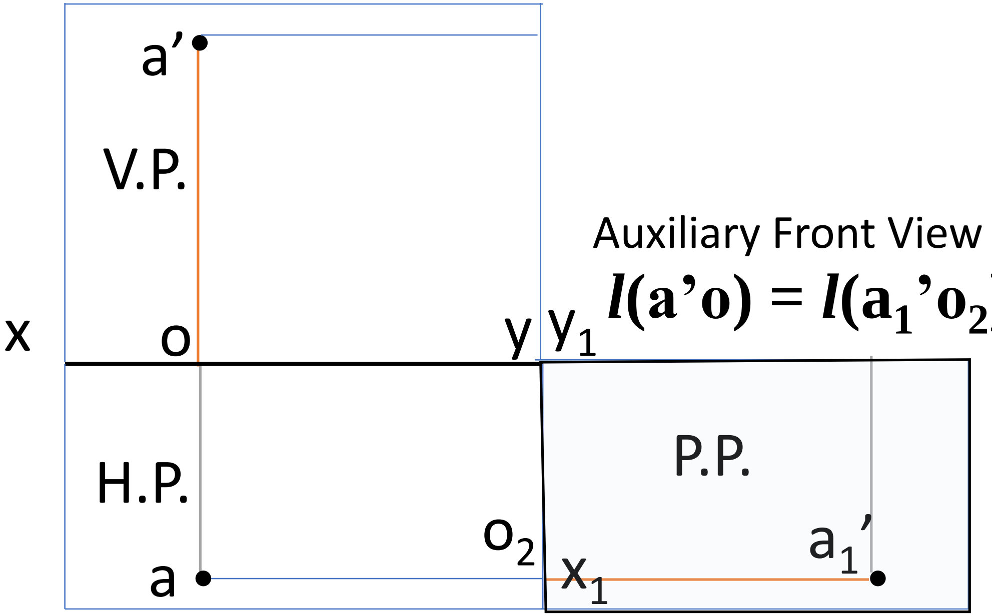
Profile Plane (P.P.): Principal Plane which is perpendicular to both the H.P. and the V.P.





Auxiliary Top View

$$l(ao) = l(a_1o_1)$$



Auxiliary Front View

$$l(a'o) = l(a_1'o_2)$$

Important Points to Remember

- The auxiliary top view of a point lies on a line drawn through the front view, perpendicular to the new reference line x_1y_1 and at a distance from it, equal to the distance of the first top view from own reference line xy

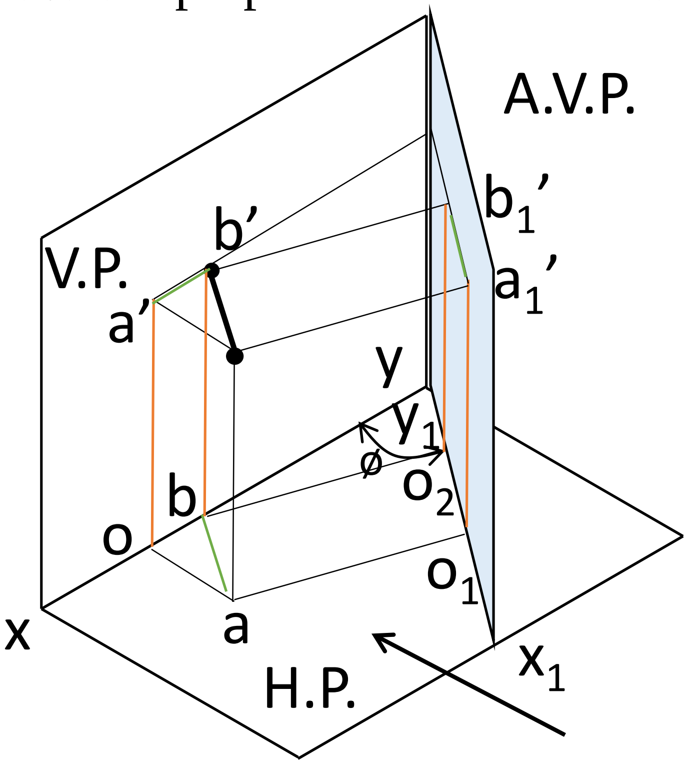
- The auxiliary front view of a point lies on a line drawn through the top view, perpendicular to the new reference line x_1y_1 and at a distance from it, equal to the distance of the first front view from own reference line xy

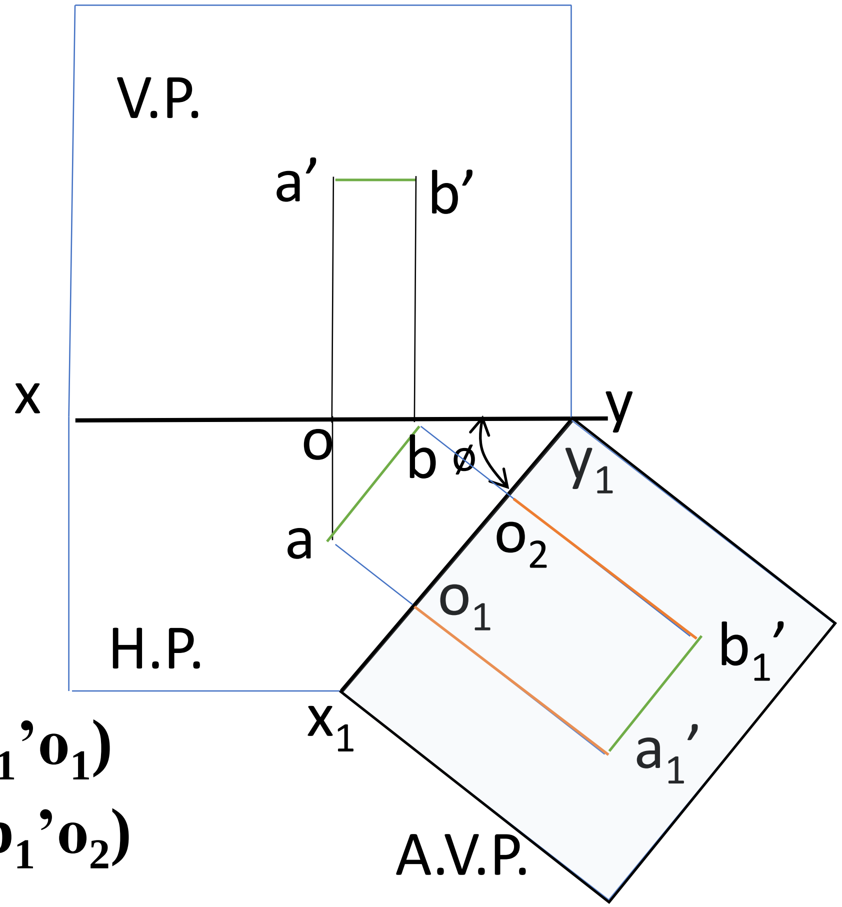
- The distances of all the front views of the same point (projected from the same top view) from their respective reference lines are equal

- The distances of all the top views of the same point (projected from the same front view) from their respective reference lines are equal

Projection of Line Parallel to H.P. on the A.V.P.

- A.V.P. is perpendicular to the H.P. and inclined to the V.P.





Height is preserved

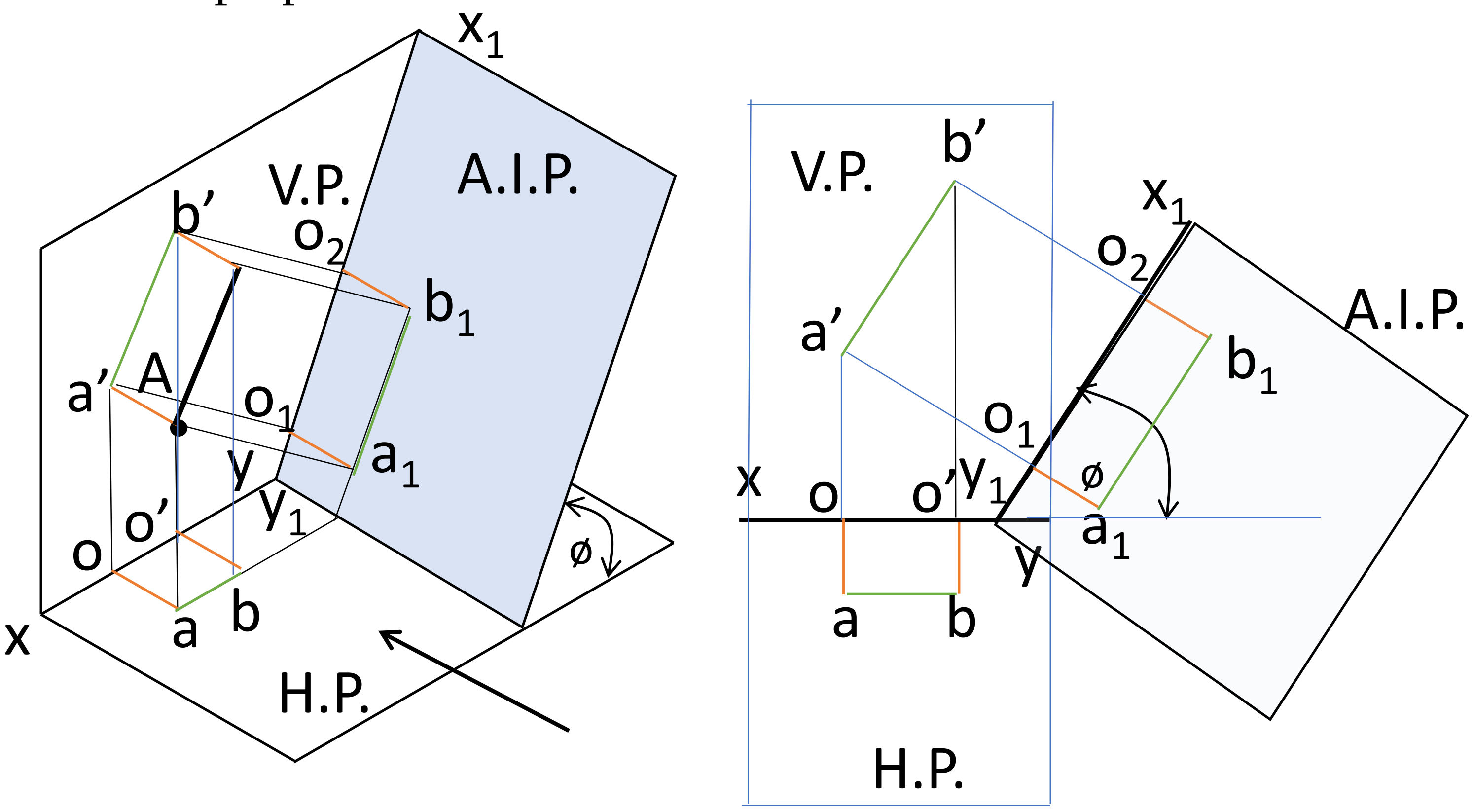
$$\boldsymbol{l(a'o) = l(a_1'o_1)}$$

$$\boldsymbol{l(b'b) = l(b_1'o_2)}$$

Auxiliary front view is drawn by rotating the auxiliary vertical plane about the line x_1y_1 .The true length of the line is seen in the A.V.P.

Projection of Line Parallel to V.P. on the A.I.P.

- A.I.P. is perpendicular to the V.P. and inclined to the H.P.

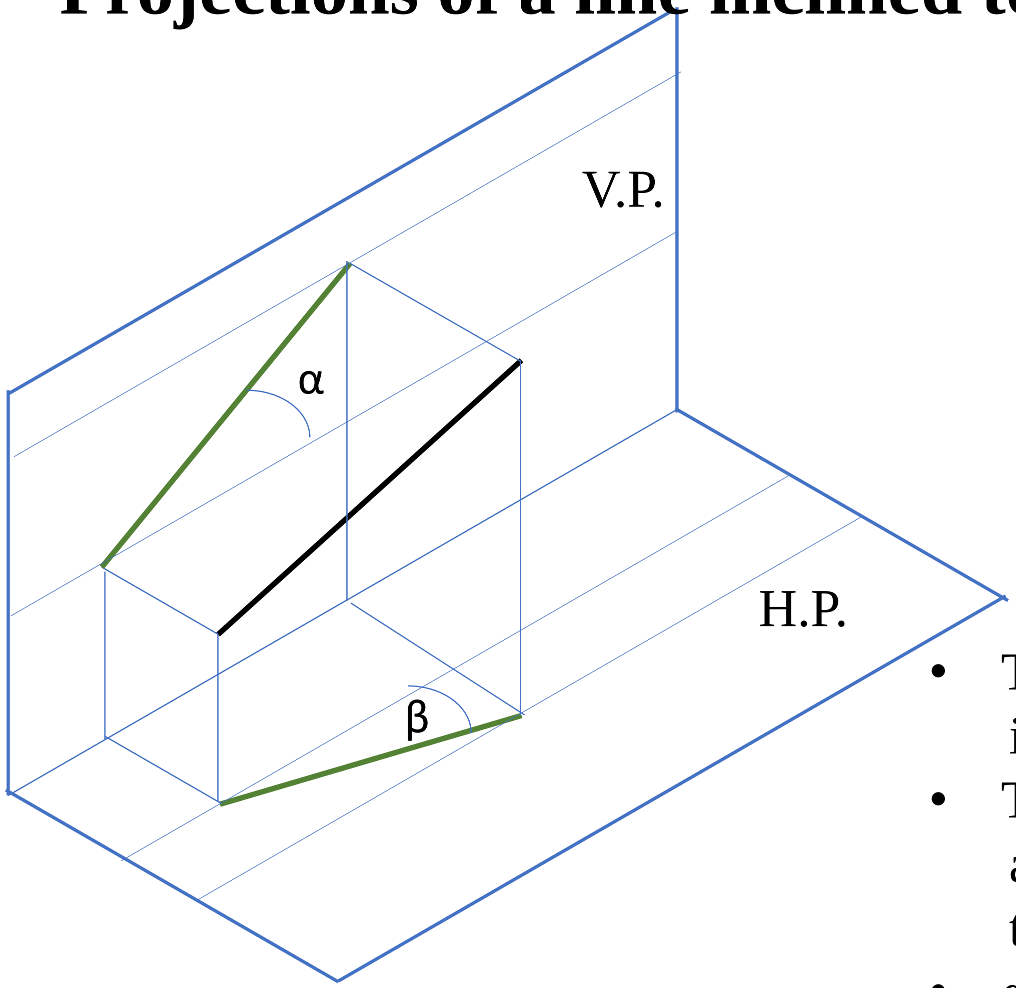


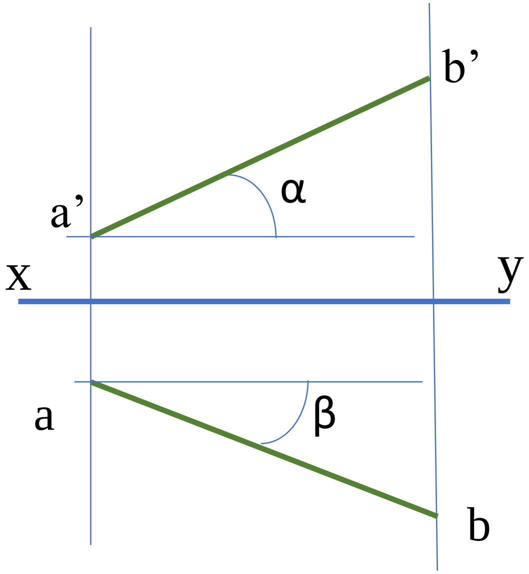
Depth is preserved

$$l(ao) = l(a_1o_1)$$
$$l(bo') = l(b_1o_2)$$

Auxiliary top view is drawn by rotating the auxiliary inclined plane about the line x_1y_1 . The true length of the line is seen in the A.I.P

Projections of a line inclined to both H.P. and the V.P.



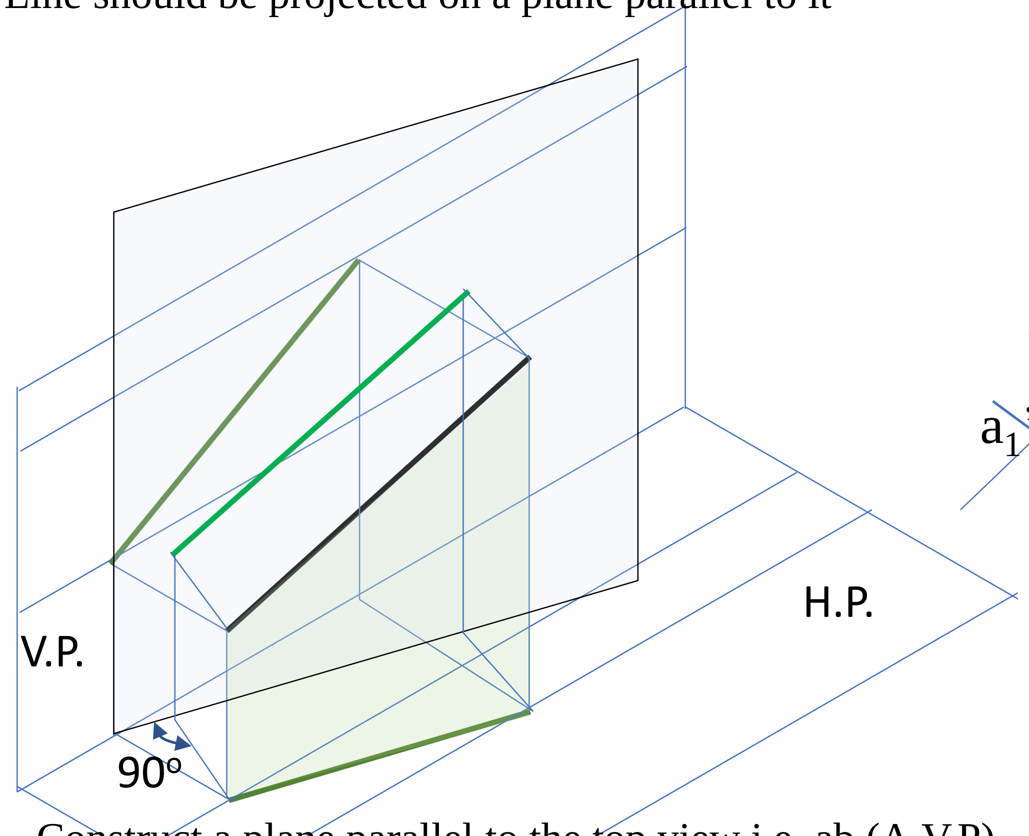


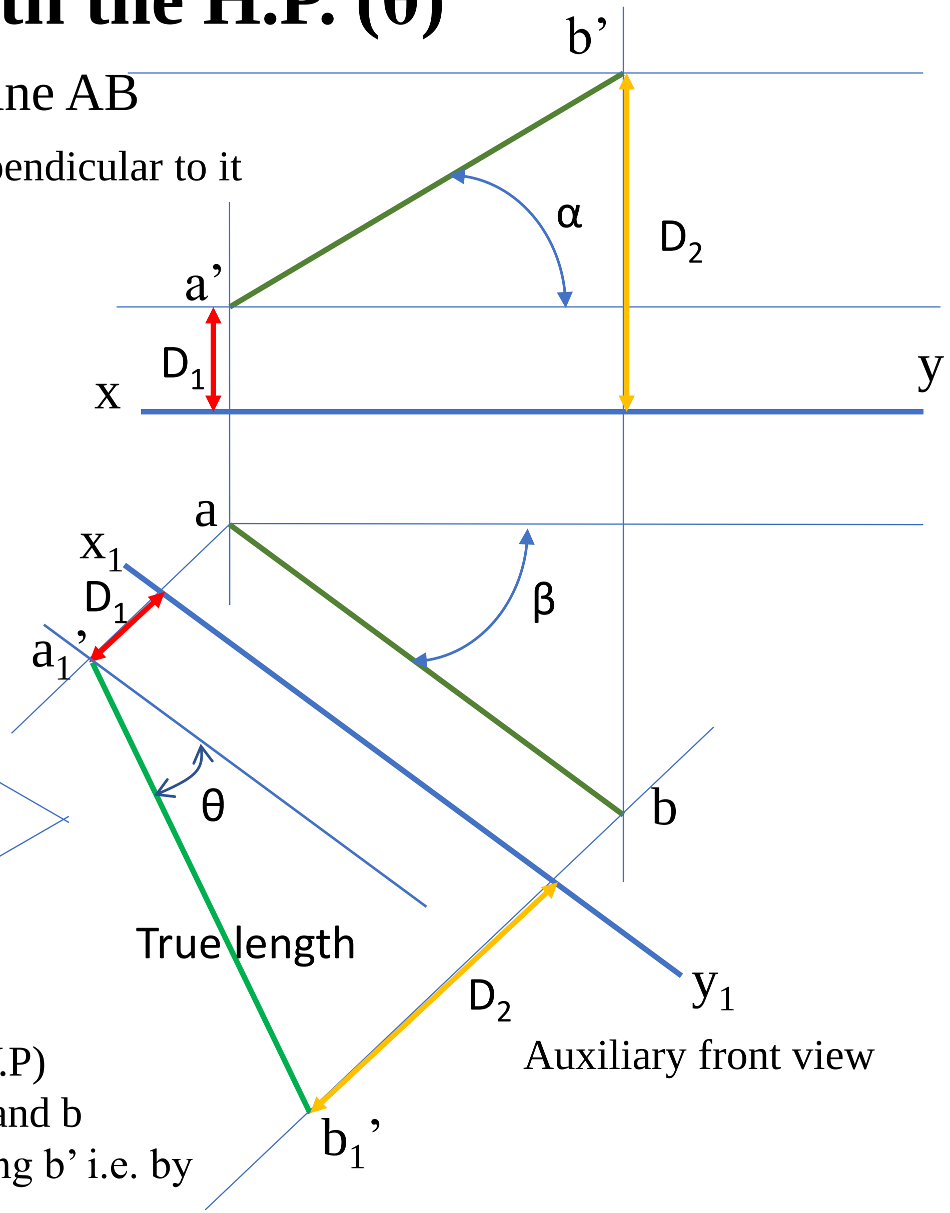
- The top view and the front view are inclined to the xy line
- The length of the line in the top view and the front view is less than the true length
- α and β are the apparent angle of inclination with the H.P. and the V.P. and are greater than the true angles of inclination

To find the True Length and True Angle of Inclination with the H.P. (θ)

Given: The front view and top view of the line AB

The true length of line can be observed by looking perpendicular to it
Line should be projected on a plane parallel to it





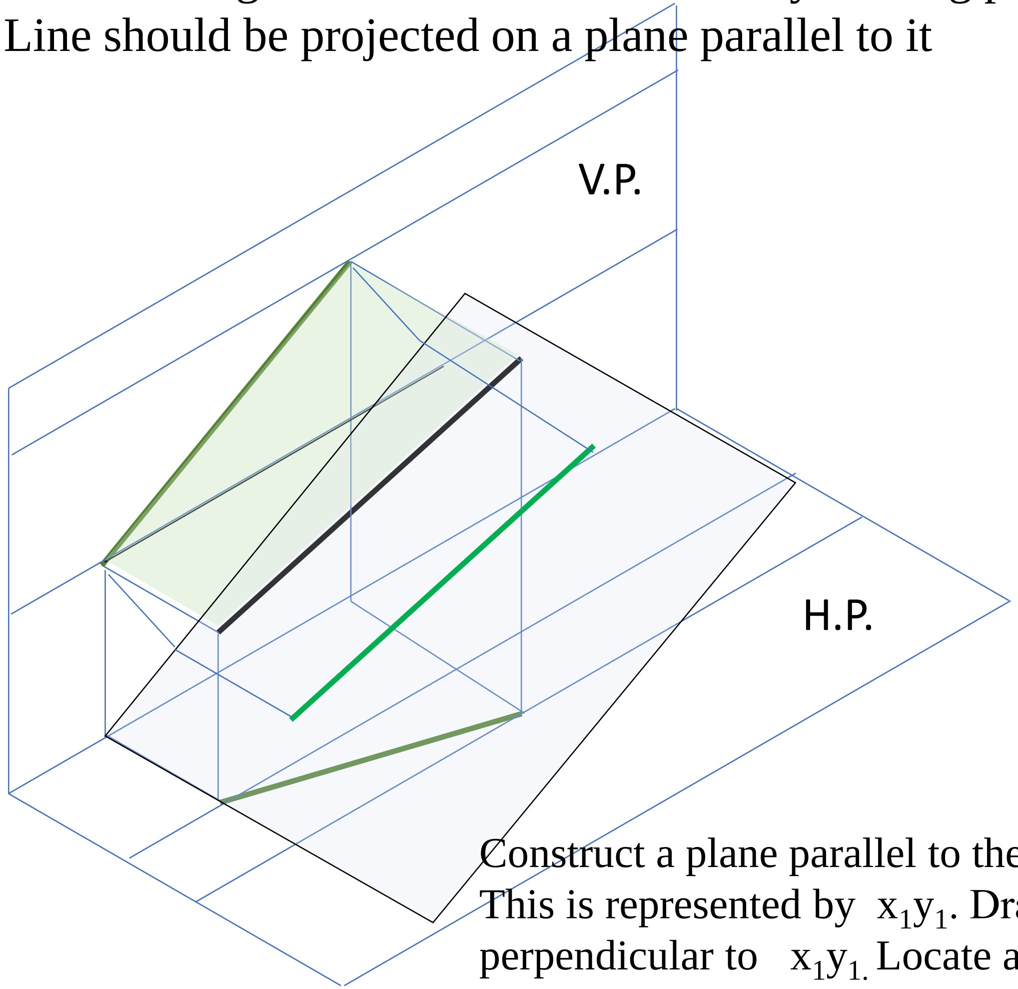
Construct a plane parallel to the top view i.e. ab (A.V.P)
This is represented by x_1y_1. Draw projectors from a and b perpendicular to x_1y_1. Locate a_1' using a' and b_1' using b' i.e. by transferring the height dimension

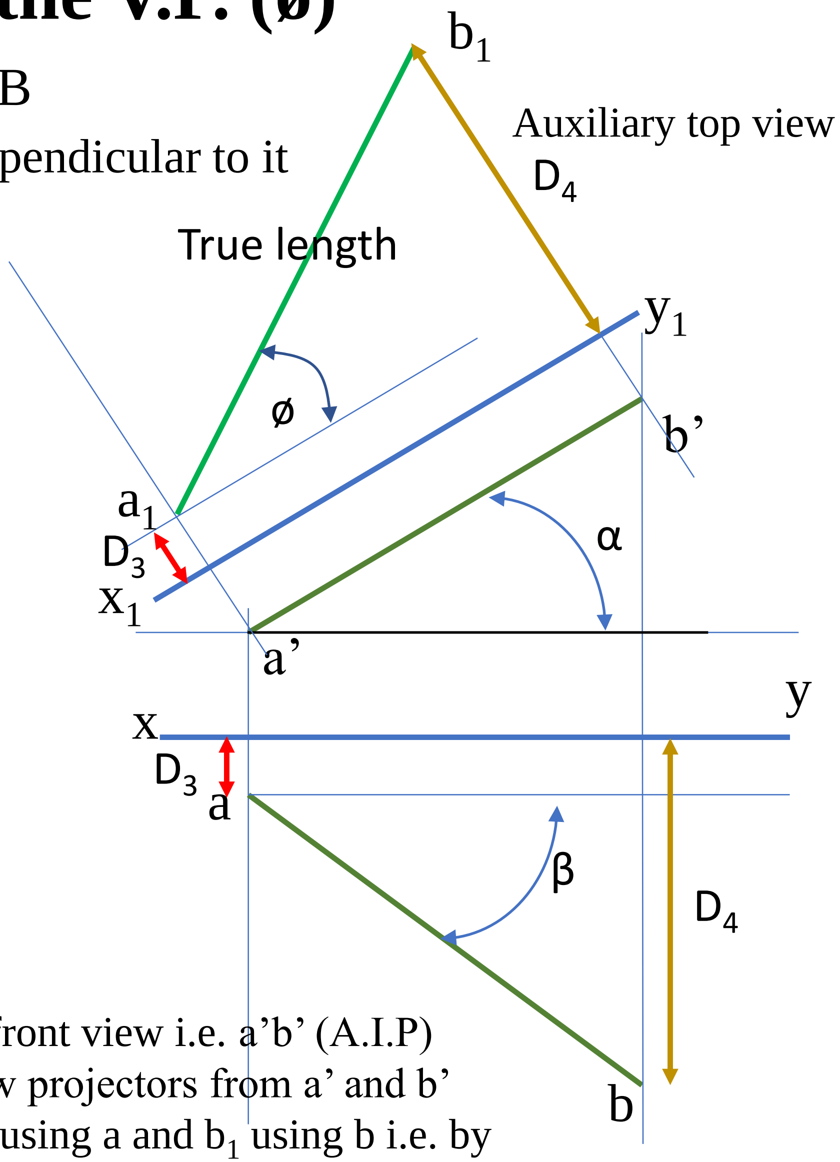
To find the True Length and True Angle of Inclination with the V.P. (ø)

Given: The front view and top view of the line AB

The true length of line can be observed by looking perpendicular to it

Line should be projected on a plane parallel to it



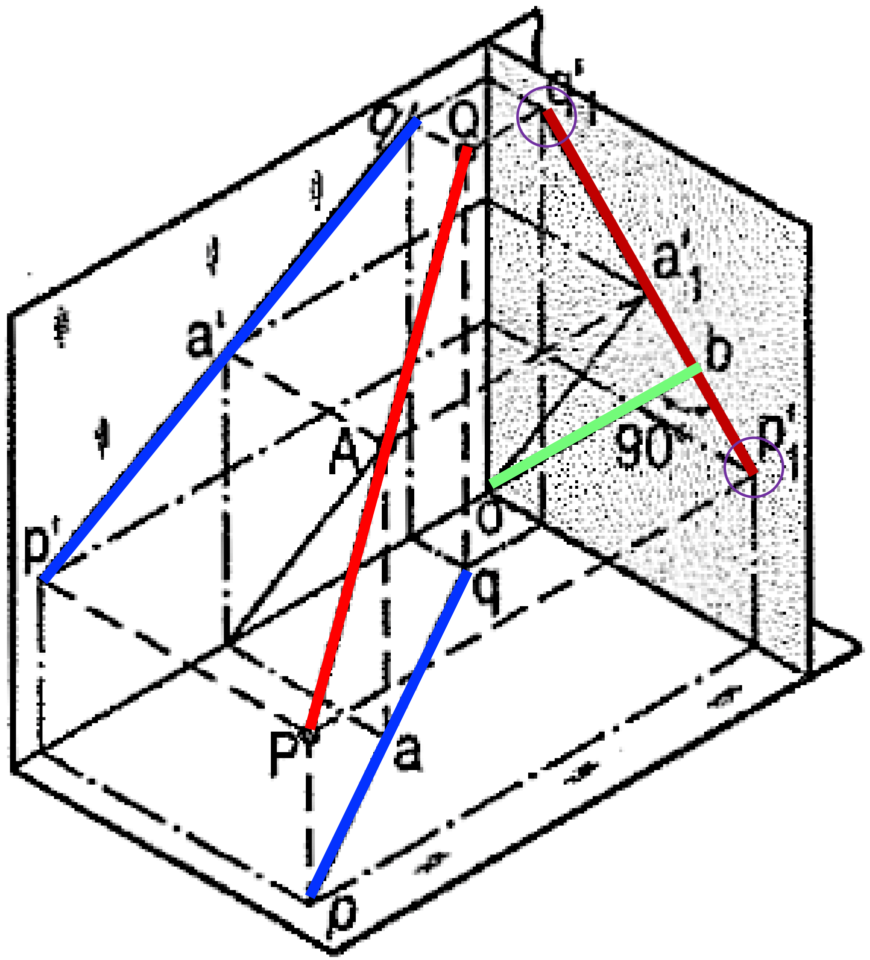


Construct a plane parallel to the front view i.e. a'b' (A.I.P) This is represented by x_1y_1. Draw projectors from a' and b' perpendicular to x_1y_1. Locate a_1 using a and b_1 using b i.e. by transferring the depth dimension

END

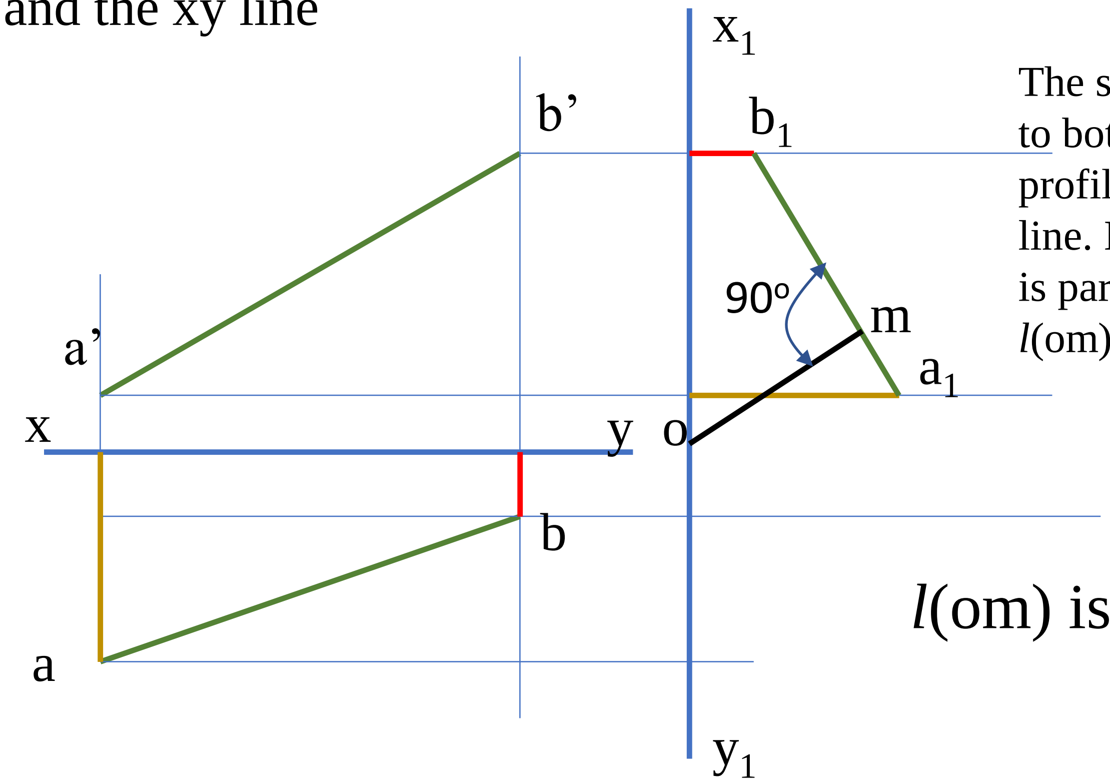
To Determine the Shortest Distance Between a Line and the XY Line: 3D visualization

Given the front view and the top of a line PQ, determine the shortest distance between the line and the xy line



To Determine the Shortest Distance Between a Line and the XY Line

Given the front view and the top of a line AB, determine the shortest distance between the line and the xy line



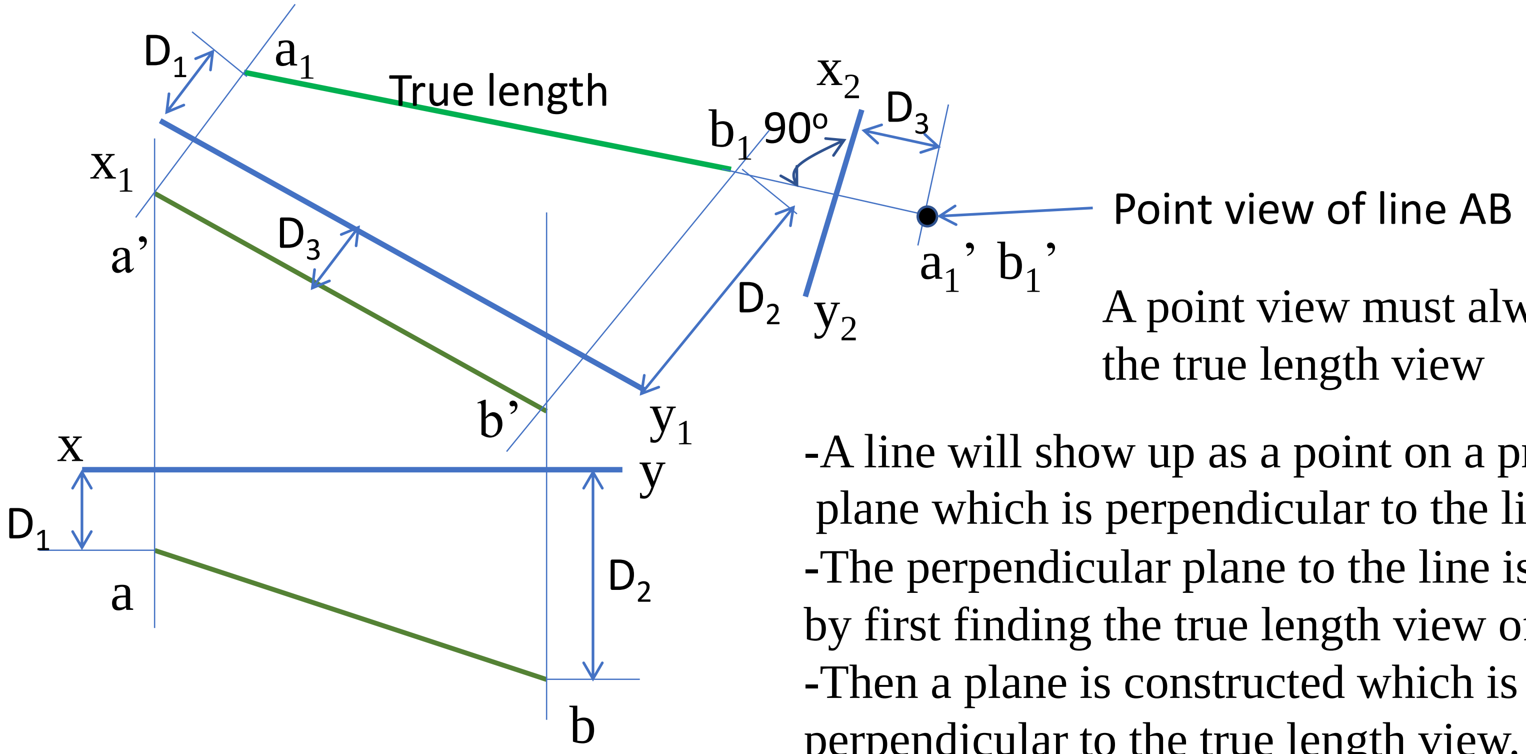
The shortest segment is perpendicular to both the xy line and line AB. The profile plane is perpendicular to the xy line. Hence the shortest segment is parallel to profile plane. Therefore *l*(om) is the true length

l(om) is the required length

To determine the shortest distance between any two non-intersection lines, it is necessary to find the point view of one of the lines. In the above example 'o' is the point view of the xy line

To Find the Point View (Point Projection) of a Line

Given: The front view and top view of the line AB

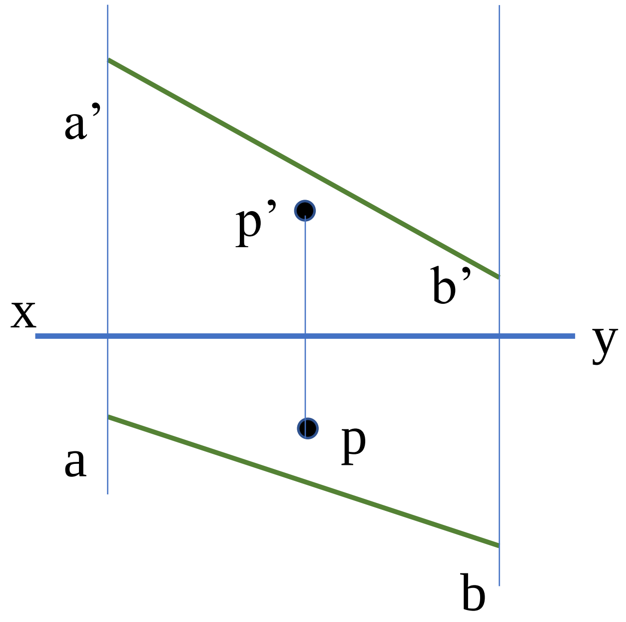


A point view must always follow the true length view

-A line will show up as a point on a projection plane which is perpendicular to the line
-The perpendicular plane to the line is found by first finding the true length view of the line.
-Then a plane is constructed which is perpendicular to the true length view. This plane is perpendicular to the line and is represented by x_2 y_2
- Draw a projection line from a_1 , b_1 and transfer distance D_3 to locate a_1' and b_1' . This is the point view of the line AB

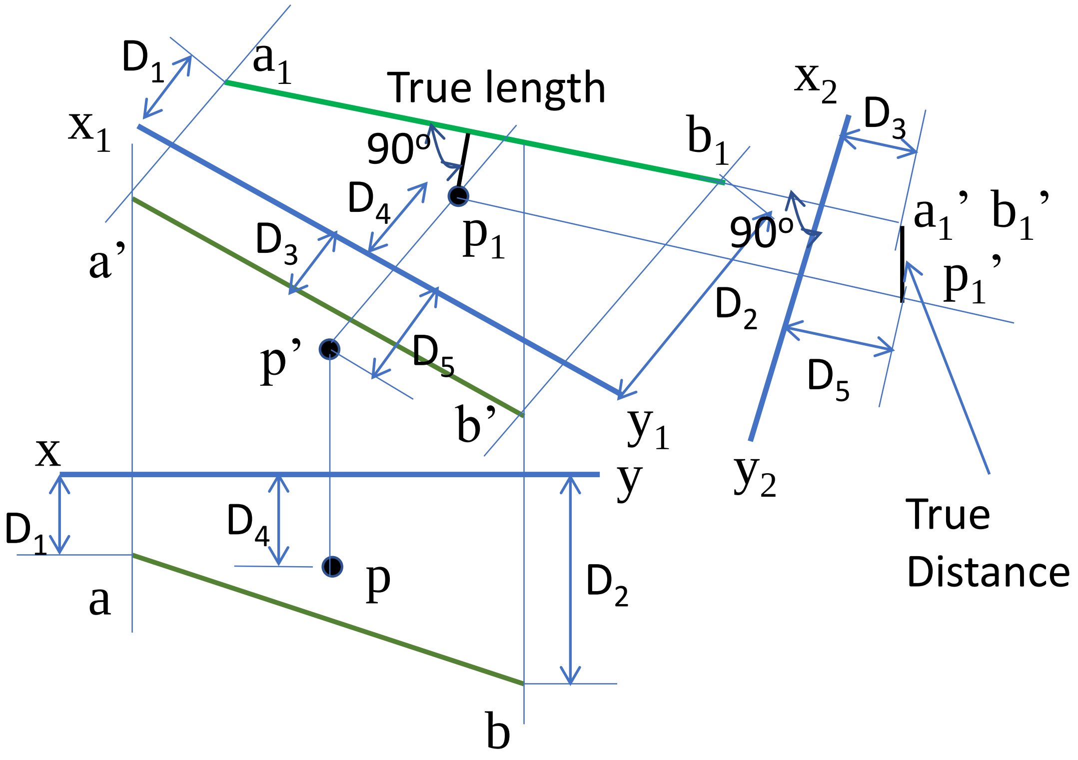
To Find the Shortest Distance Between a Line and a Point

Given: The front view and top view of the line AB and point P



To Find the Shortest Distance Between a Line and a Point

Given: The front view and top view of the line AB and point P



- The shortest distance between a given point and a given line is measured along the perpendicular drawn from the point to the line
- Lines that are perpendicular will have their projections shown perpendicular in a view showing either or both lines in true length

-Draw the true length view of the line AB
-Obtain the projection of the point P in the same view
-The segment which is perpendicular to the line from the point P, will appear perpendicular in this view as the true length of line AB is seen
-Draw a plane perpendicular to the true length view. This is represented by x_2y_2
-Locate the point view of line AB
-Since the segment from the point to the line is parallel to x_2y_2 , its projection in the other view will show its true length. This is the distance of the point P from the line AB

Important Points to Remember

- When one view of a line is parallel to a reference line, the other view represents the true length
- A line will appear as a point when it projected on a plane perpendicular to it
- The shortest distance between a given point and a given line is measured along the perpendicular drawn from the point to the line
- Lines that are perpendicular in space will have their projections perpendicular in any view which shows either or both of the lines in true lengths