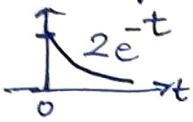
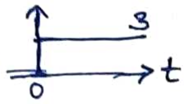
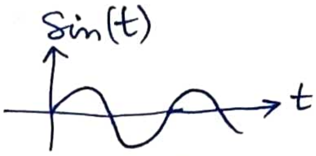
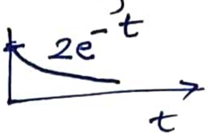
EE113, Tutorial Sheet Control Systems: 17 Feb 2022

Q-1 Convolve the following pairs

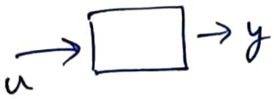
(a)  & 

(b)  & 

(c)  & 

(same functions).

Obtain solution of convolution by first principles (definitions) & match with using Laplace transform.

Q-2: Consider the diff equation 

$$u = \frac{d^3}{dt^3}y + 8\frac{d^2}{dt^2}y + 8\frac{d}{dt}y.$$

Argue that corrective action $u = -ky$, $k > 0$ ensures closed loop has stable poles only for $k \in (0, K_{max})$

find K_{max}.

Q-3: Find poles with u as input & y as output

(a) $\frac{d}{dt}y + 3y = \frac{d}{dt}u - 5u$

(b) $\frac{d^2}{dt^2}y + 3\dot{y} + 2y = \frac{d}{dt}u + 4u.$

Also find more solutions (in addition to)

Check $ce^{3t} = u(t)$, $c \in \mathbb{R}$ gives $y(t) = f(c)e^{3t}$

Find $f(c)$ for (a) & (b)

Also solve for ce^{5t} & ce^{-4t}, $c \in \mathbb{R}$.

Q-4: Plot response (amount in bank) for keeping Rs. 1 in bank on day 0: plot for simple interest & compound interest.

(Think of both continuous time & discrete time)

Q-5

– write difference equation for a summer (input at k^{th} instant $= u(k)$ & $y(0)$ to start.)

for simple interest

for compound interest] money in k^{th} day $= u(k)$

Q-6

Consider the circuit  with capacitance C, Resistance R & $v(0) = v_0 \in \mathbb{R}$ (real numbers).

(a) write differential equation in $v(t)$

(b) Take Laplace transform of both sides (without assuming initial conditions zero) and solve for $V(s)$

(c) Take inverse Laplace tranform of $V(s)$ and solve for $v(t)$

(d) Plot for different values of R, C (but $v_0 = -3$) on the same figure.

Q-7:

Convolve $f_1(t) = e^{-3t}$ for $t \geq 0$ & $f_2(t) = 6e^{5t}$ for $t \geq 0$ } & $= 0$ for $t < 0$.

(a) Convolve using time-domain definition

(b) Use Laplace transforms of f_1 & f_2 & multiply & verify with (a)

Q-8:

Use linearity, express $\sin \omega t$ as linear combination of $e^{j\omega t}$ & $e^{-j\omega t}$ (& complex "signals") and verify that system with transfer function $G(s)$ with input $\sin(\omega t)$ gives output $|G(j\omega)| \sin(\omega t + \angle G(j\omega))$

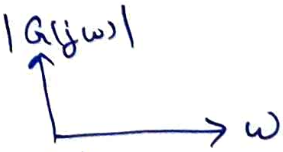
(Note ω is fixed some frequency in rad/s).

Q-9:

Consider following circuit with input as current source $i(t)$ and find transfer fn G from $i(t)$ to $v(t)$.

(a) Find transfer function $G(s)$

(b) Plot $|G(j\omega)|$ versus ω



and conclude this is low pass filter.

(c) obtain impulse response.