

Tut problems May 4, 2022

(1) A matrix A is said to be nilpotent if $A^k = 0$ for some $k \in \mathbb{N}$

(a) Show that if A is nilpotent,

$I - A$ is invertible

(b) What can you say about eigenvalues of A ?

What is the Char. Eqn of a nilpotent matrix?

(c) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

are all nil ptt. Find geom. Mult of its

eigen value in each case

(d) product of ~~nil ptt~~ commuting nil ptt matrices is nilpotent. Show that the result fails if the matrices do not commute.

(2) A matrix P is said to be an idempotent

or a projection if $P^2 = P$.

a) Show that P has this property so does $I - P$

b) What can you say about eigen values of P ?

c) If P is invertible then $P = I$

d) Suppose P is not invertible and

v_1, \dots, v_k is a basis for $\text{Null sp. } P$.

Complete it to a basis $\{v_1, \dots, v_k, v_{k+1}, \dots, v_n\}$

Prove or disprove Pv_{k+1}, \dots, Pv_n Lin Indep.

Can you deduce from this that P is diagonalizable?

(3) Consider the matrix of Reflection about a plane encountered in Tut Sheet 1

$$H = I - 2nn^T ; (\|n\| = 1)$$

and

$$H_0 = I - nn^T \quad (\text{both } H, H_0 \text{ were discussed})$$

Find Eigen values of H and H_0 .

Are they diagonalizable? Give Reasons.

Is H_0 an idempotent? Try this

Can you ~~have~~ ^{work} out in two ways
Using geometric reasoning and verifying algebraically

(4) Are the matrices $\begin{bmatrix} 2 & 10^5 & 10^9 \\ 0 & 1 & \pi \\ 0 & 0 & 3 \end{bmatrix}$ and

$\begin{bmatrix} 3 & 0 & 0 \\ e & 2 & 0 \\ 10^8 & 10^{10} & 1 \end{bmatrix}$ similar? Why?

(5) What can you say about the eigen values of a Skew Symmetric matrix?

Is a Skew Symm matrix diagonalizable.

(6) Let $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a function s.t.
 $f(0) = 0$ and $\|f(x)\| = \|x\|$

Is it true that $f(x) = Ax$ for some
3x3 matrix A ? What kind of matrix is A ?