

Height of atmosphere

Given that probability of finding oxygen molecules at highest point of atmosphere (p_h) is $\frac{1}{e}$ times of probability of finding oxygen molecules at surface of earth (p_0)

Apply Boltzmann's law

$$\therefore p \propto e^{-\frac{mgh}{k_B T}}$$

(where m is mass of oxygen molecule)

$$\text{and } p_h \propto e^{-\frac{mgh}{k_B T}}$$

$$\Rightarrow \frac{p_h}{p_0} = \frac{e^{-\frac{mgh}{k_B T}}}{e^0} = e^{-\frac{mgh}{k_B T}}$$

$$\therefore \frac{p_h}{p_0} = \frac{1}{e}$$

$$\Rightarrow \frac{mgh}{k_B T} = 1$$

$$\begin{aligned} \text{or } h &= \frac{k_B T}{mg} = \frac{4.14 \times 10^{-21} \text{ J}}{32 \times 10} \\ &= \frac{4.14 \times 6.022 \times 10^{23} \times 10^{-22}}{32} \\ &= \frac{249310.8}{32} \\ &\approx 7791 \text{ m} \\ &\approx 7.8 \text{ km} \end{aligned}$$

