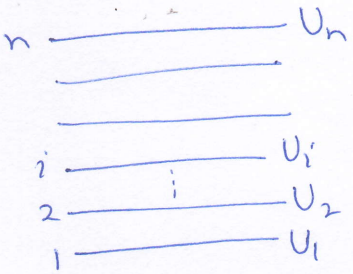
Expression for partition function

Suppose a biomolecule can be found in n possible states where energies of each states are given below



Let's apply Boltzmann's law to each of these states

$$p_1 \propto e^{-U_1/k_BT} \Rightarrow p_1 = A e^{-U_1/k_BT}$$
$$p_2 \propto e^{-U_2/k_BT} \Rightarrow p_2 = A e^{-U_2/k_BT}$$
$$\vdots$$
$$p_i \propto e^{-U_i/k_BT} \Rightarrow p_i = A e^{-U_i/k_BT} \quad \text{—(1)}$$
$$\vdots$$
$$p_n \propto e^{-U_n/k_BT} \quad p_n = A e^{-U_n/k_BT}$$

Take sum of $p_1+p_2+p_i+p_n$

$$p_1+p_2+\cdots p_i+\cdots p_n = A\left(e^{-U_1/k_BT}+e^{-U_2/k_BT}+\cdots e^{-U_i/k_BT}+\cdots e^{-U_n/k_BT}\right)$$

$$1 = A\sum_{i=1}^{n} e^{-U_i/k_BT}$$

$$\Rightarrow A = \frac{1}{\sum e^{-U_i/k_BT}}$$

put value of A in equation (1)

$$\Rightarrow p_i = \frac{e^{-U_i/k_BT}}{\sum_{i=1}^{n} e^{-U_i/k_BT}} = \frac{e^{-U_i/k_BT}}{Z}$$

where $\boxed{Z = \sum_{i=1}^{n} e^{-U_i/k_BT}}$