

PH108 : Basics of Electricity & Magnetism : Problem Set 1

Only * problems are to be solved in the tut session

- 1.* Calculate the curl and divergence of the following vector functions. If the curl turns out to be zero, construct a scalar function ϕ of which the vector field is the gradient:

(a) $F_x = x + y$; $F_y = -x + y$; $F_z = -2z$

(b) $G_x = 2y$; $G_y = 2x + 3z$; $G_z = 3y$

(c) $H_x = x^2 - z^2$; $H_y = 2$; $H_z = 2xz$

2. Calculate the Laplacian of the following functions:

(a) $T_a = x^2 + 2xy + 3z + 4$

(b) $T_b = \sin(x) \sin(y) \sin(z)$

(c) $T_c = e^{-5x} \sin(4y) \cos(3z)$

(d) $\vec{v} = x^2\hat{i} + 3xz^2\hat{j} - 2xz\hat{k}$

- 3.* Test the Stokes theorem for the vector $\vec{v} = xy\hat{i} + 2yz\hat{j} + 3z\hat{k}$ using a triangular area with vertices at (0,0,0), (0,2,0) and (0,0,2).

4. Compute the unit normal vector \hat{n} to the ellipsoidal surfaces defined by constant values of $\Phi(x, y, z) = V \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \right)$. What is \hat{n} when $a = b = c$?

- 5.* A force defined by $\vec{F} = A(y^2\hat{i} + 2x^2\hat{j})$ is exerted on a particle which is initially at the origin of the co-ordinate system. A is a positive constant. We transport the particle on a triangular path defined by the points (0,0,0), (1,0,0), (1,1,0) in the counterclockwise direction.

- (a) How much work does the force do when the particle travels around the path? Is this a conservative force?

- (b) The particle is placed at rest right at the origin. Is this a stable situation? Give any argument (mathematical, physical, intuitive) to justify the stability (or instability) of this situation.

6. The area bounded by the curve $r = 2R \cos \theta$ has a surface charge density $\sigma(r, \theta) = \sigma_0 \frac{r}{R} \sin^4 \theta$. What is the total amount of charge?

- 7.* Suppose that the height of a certain mountain (in feet) is given by

$$h(x, y) = 10(2xy - 3x^2 - 4y^2 + 14x + 10y + 40),$$

where x is the distance (in km) east, y the distance north of the closest town.

- (a) Where is the top of the mountain located, and how high is it?

- (b) How steep is the slope (in feet per km) at a point 1 km north and 1 km east of the town? In what direction is the slope steepest, at that point?

8. The gradient operator ∇ behaves like a vector in “some sense”. For example, divergence of a curl ($\nabla \cdot (\nabla \times \vec{A}) = 0$) for any \vec{A} , may suggest that it is just like $\vec{A} \cdot (\vec{B} \times \vec{C})$ being zero if any two vectors are equal. Prove that $\nabla \times \nabla \times \vec{F} = \nabla(\nabla \cdot \vec{F}) - \nabla^2 \vec{F}$. To what extent does this look like the well known expansion of $\vec{A} \times \vec{B} \times \vec{C}$?