

Answers

1. (a) $\psi(\theta) = \frac{1}{\sqrt{2\pi}} e^{\pm i n \theta}$, where $n^2 = \frac{2I}{\hbar^2} E_n$.

(b) $E_n = \frac{\hbar^2 n^2}{2I}$

(c) $n = 0, \pm 1, \pm 2, \dots$
(n is an integer)

2. $A = \sqrt{\frac{3}{4\pi}}$

3. Y_1^0 is an eigen function of operator \hat{L}^2 ;
Corresponding eigen value = $2\hbar^2$

4. (a) For hydrogen atom problem, Hamiltonian can be expressed as a sum of Hamiltonian for centre of mass motion and relative motion between electron and neutron.

$$\hat{H}_{\text{total}} = \hat{H}_{\text{CM}} + \hat{H}_{\text{rel}}$$

$$\hat{H}_{\text{rel}} = \underbrace{-\frac{\hbar^2}{2\mu} \nabla_r^2}_{\text{K.E.}} - \underbrace{\frac{QZe^2}{r}}_{\text{P.E.}}$$

In potential energy operator there is a factor (r) in the denominator.
 $r = \sqrt{x^2 + y^2 + z^2}$

It is not possible to solve the Schrödinger equation by 'separation of variable' method in x, y, z co-ordinate system, which lead to invoke spherical polar co-ordinates (r, θ, ϕ) ;

4.(b) For, rigid rotor, internuclear distance is constant, However in case of Hydrogen atom problem radius is not constant. In both cases there is contribution of angular part (θ, ϕ) ;

$$5. \quad \hat{\nabla}^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) - \frac{\hat{L}^2}{\hbar^2 r^2}$$

$\hat{L}^2 \rightarrow$ Total angular momentum operator.