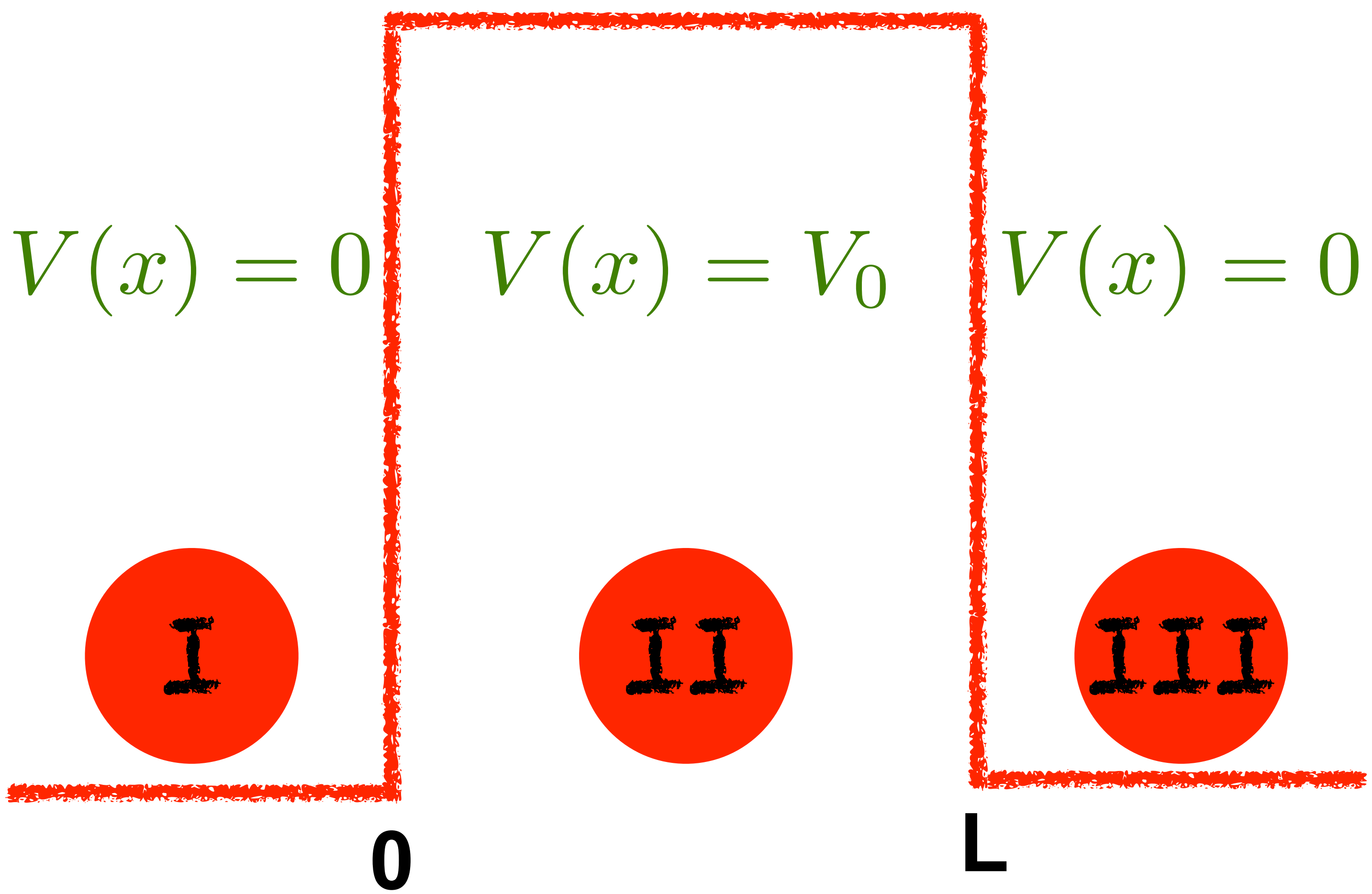
PH-107

Step Potential with finite width

Step Potential with Finite Width



Finite Step Potential

$$V(x) = 0 \;\forall\; x \leq 0$$
$$= V_0 \;\forall\; 0 < x < L$$
$$= 0 \;\forall\; x \geq L$$

Step Potential with Finite Width

$E < V_0$ **and** $E > V_0$

Finite Step Potential

$$V(x) = 0 \;\forall\; x \leq 0$$
$$= V_0 \;\forall\; 0 < x < L$$
$$= 0 \;\forall\; x \geq L$$

$$\phi_I(x) = Ae^{ikx} + Be^{-ikx};\; k^2 = \frac{2mE}{\hbar^2}$$

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$$\phi_{II}(x) = Ce^{-\alpha x} + De^{\alpha x};\; \alpha^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

$$\phi_{II}(x) = Ce^{-ik'x} + De^{ik'x};\; (k')^2 = \frac{2m(E - V_0)}{\hbar^2}$$

$$\phi_{III}(x) = Fe^{ikx} + Ge^{-ikx};\; k^2 = \frac{2mE}{\hbar^2}$$

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Step Potential with Finite Width

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Finite Step Potential

$$V(x) = 0 \;\forall\; x \leq 0$$
$$= V_0 \;\forall\; 0 < x < L$$
$$= 0 \;\forall\; x \geq L$$

Boundary Conditions:

1.

$$\phi_{III}(x) = Fe^{ikx} + Ge^{-ikx};\; G = 0$$

$$\phi_{III}(x) = Fe^{ikx} + Ge^{-ikx};\; G = 0$$

2.

$$\phi_I(0) = \phi_{II}(0) \implies A + B = C + D$$ **and** $$A + B = C + D$$

3.

$$\phi'_I(0) = \phi'_{II}(0) \implies ik(A - B) = \alpha(D - C)$$

and $$k(A - B) = k'(D - C)$$

Step Potential with Finite Width

$E < V_0$ **and** $E > V_0$

Finite Step Potential

$$V(x) = 0 \ \forall \ x \leq 0$$
$$= V_0 \ \forall \ 0 < x < L$$
$$= 0 \ \forall \ x \geq L$$

Boundary Conditions:

4.

$$\phi_{II}(L) = \phi_{III}(L) \Longrightarrow$$

$$Ce^{-\alpha L} + De^{\alpha L} = Fe^{ikL}$$ **and** $$Ce^{-ik'L} + De^{ik'L} = Fe^{ikL}$$

5.

$$\phi'_{II}(L) = \phi'_{III}(L) \Longrightarrow \alpha(De^{\alpha L} - Ce^{-\alpha L}) = ikFe^{ikL}$$

and $$k'(De^{ik'L} - Ce^{-ik'L}) = kFe^{ikL}$$

We need to solve four equations, **in either case**, to get the ratios

B/A, C/A, D/A, and F/A.

Step Potential with Finite Width

The Reflection and the Transmission coefficients are given as

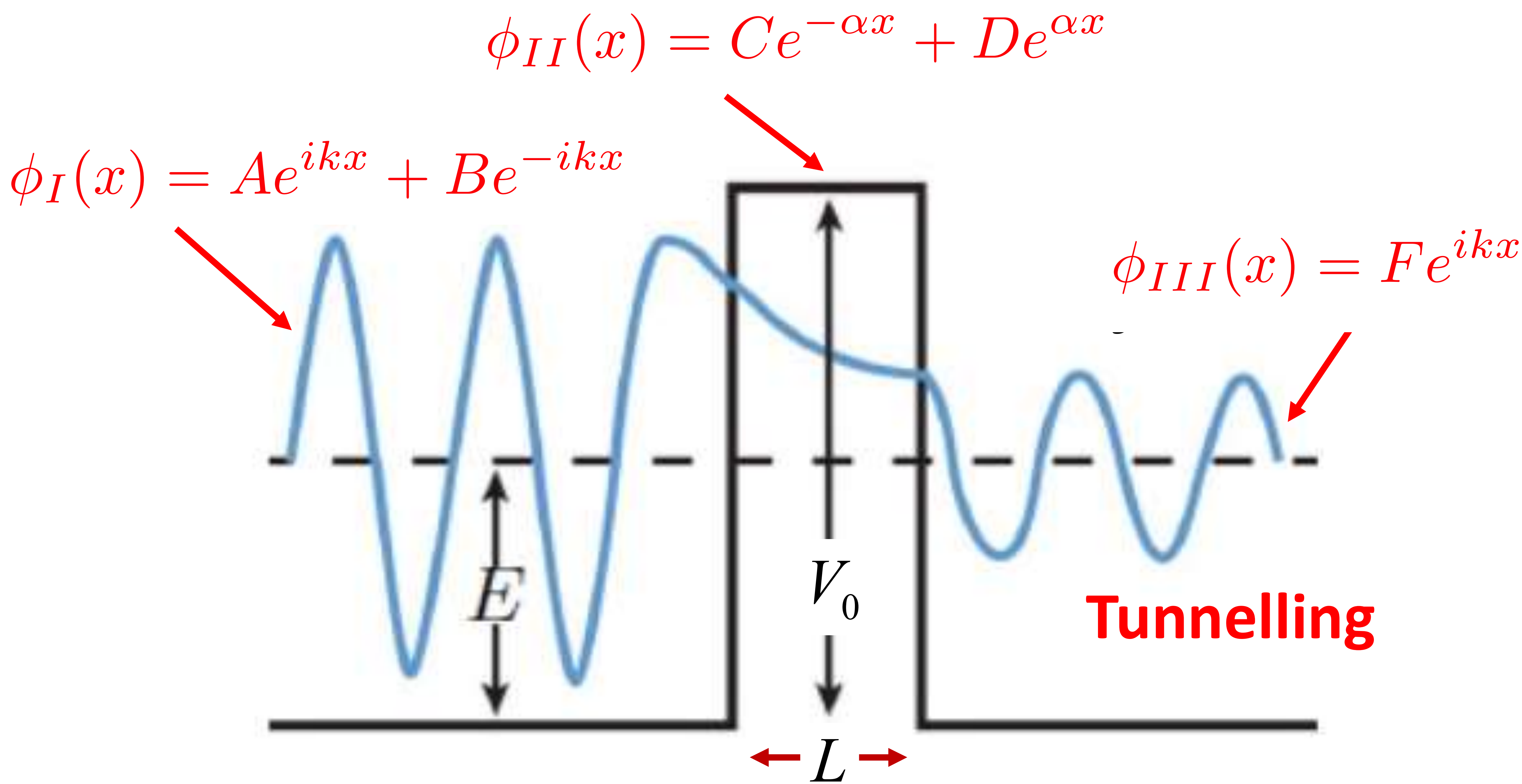
$$R = \left|\frac{B}{A}\right|^2 \text{ and } T = \left|\frac{F}{A}\right|^2$$

As *A*, *B* and *F* are functions of momentum (energy) *k* (*E*) of the particle, so *R* and *T* are also functions of *k* (*E*).

Note that no additional factor such as $\dfrac{k_2}{k_1}$ need to be multiplied to $\left|\dfrac{F}{A}\right|^2$

It can be shown that $R(E) + T(E) = 1$

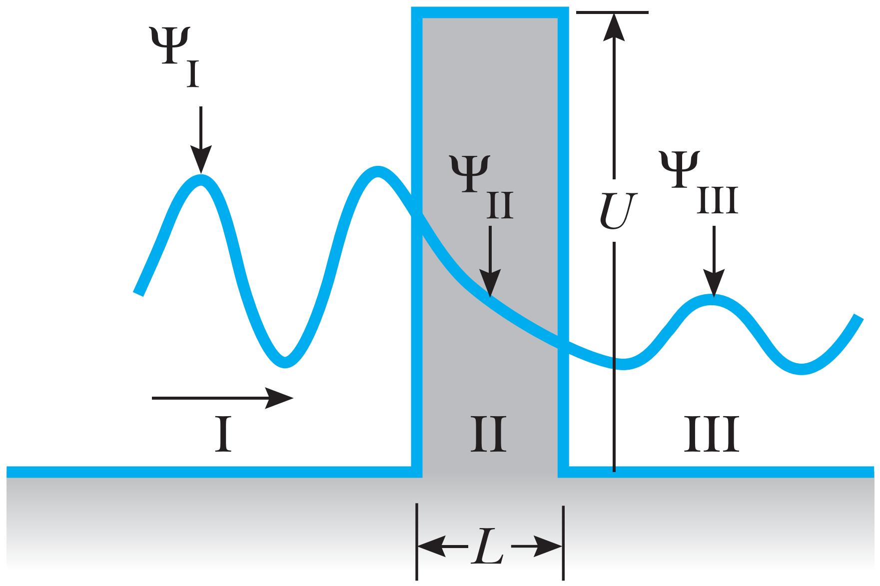
Step Potential with Finite Width

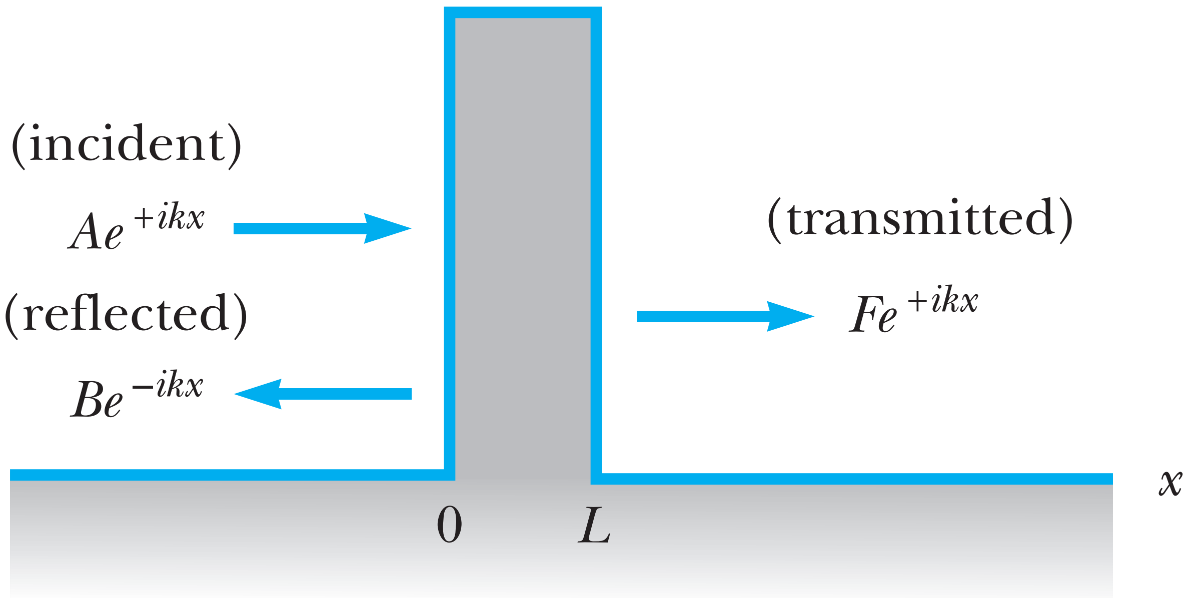


Barrier penetration depth $\delta = \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m(V_0 - E)}}$

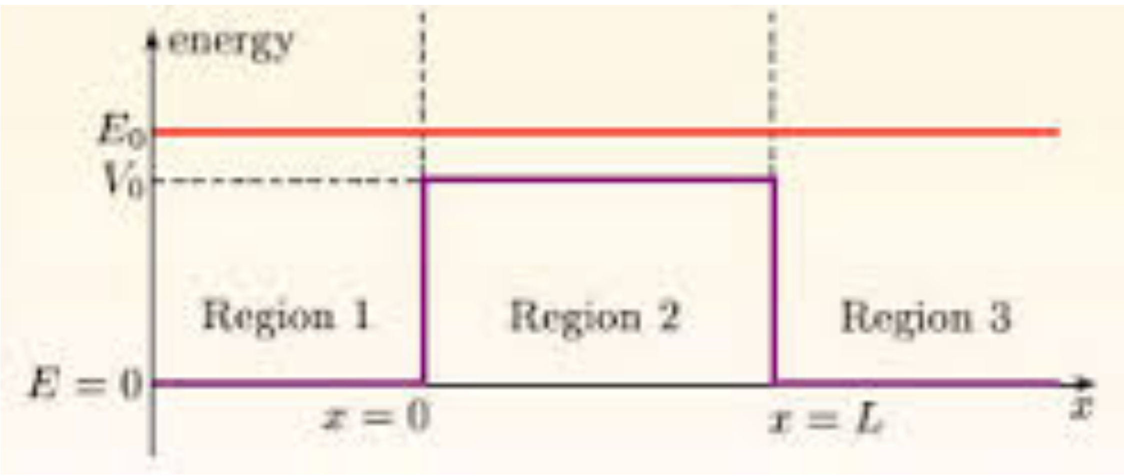
Step Potential with Finite Width

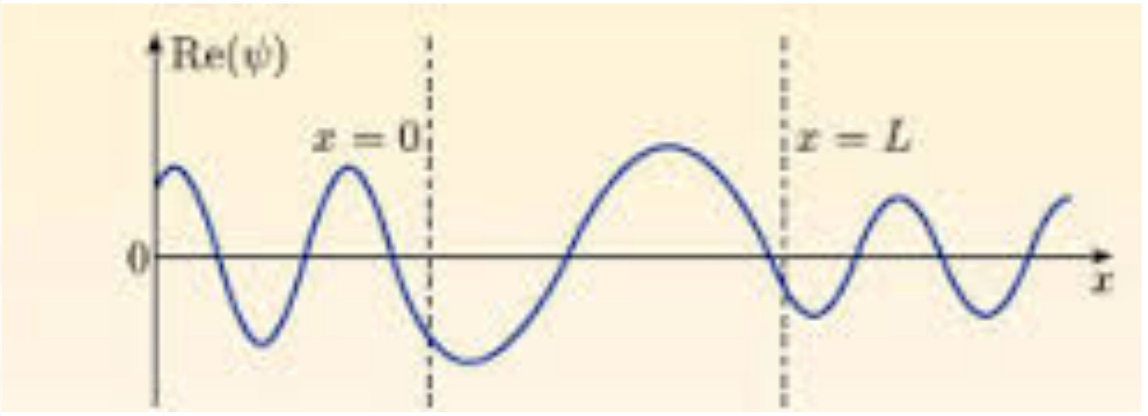
For $E < V_0$





For $E > V_0$





Step Potential with Finite Width

The expression for Transmission coefficients is not straightforward as it was in the previous case and requires some mathematical steps:

$E < V_0$ **and** $E > V_0$

$$T(E) = \left[1 + \frac{1}{4}\left(\frac{V_0^2}{E(V_0 - E)}\right)\sinh^2(\alpha L)\right]^{-1}$$

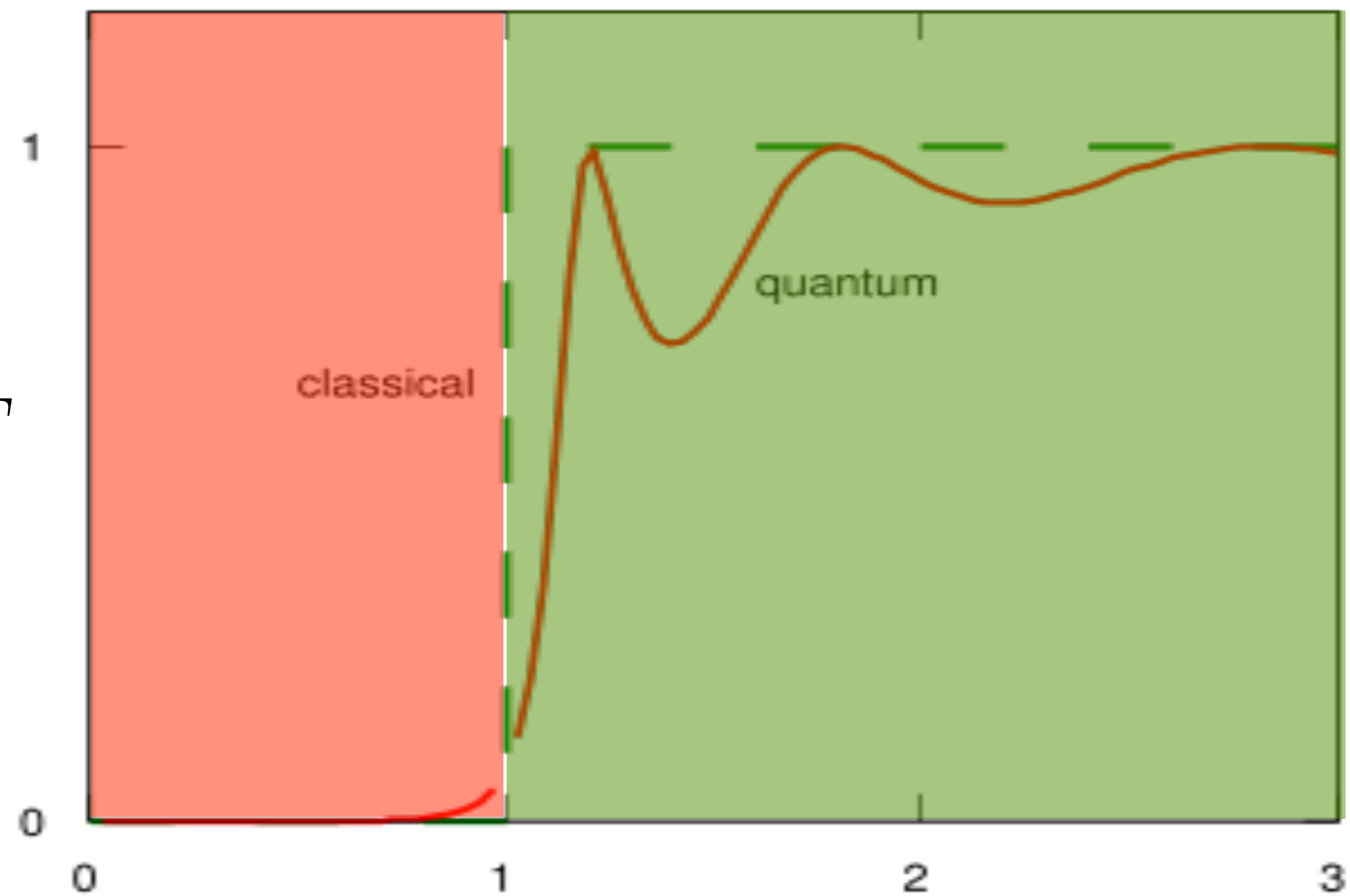
and

$$T(E) = \left[1 + \frac{1}{4}\left(\frac{V_0^2}{E(E - V_0)}\right)\sin^2(k'L)\right]^{-1}$$

Here, $\sinh(\alpha L) = \dfrac{(e^{\alpha L} - e^{-\alpha L})}{2}$

Step Potential with Finite Width

$$T(E) = \left[1 + \frac{1}{4}\left(\frac{V_0^2}{E(V_0 - E)}\right)\sinh^2(\alpha L)\right]^{-1} \qquad T(E) = \left[1 + \frac{1}{4}\left(\frac{V_0^2}{E(E - V_0)}\right)\sin^2(k' L)\right]^{-1}$$

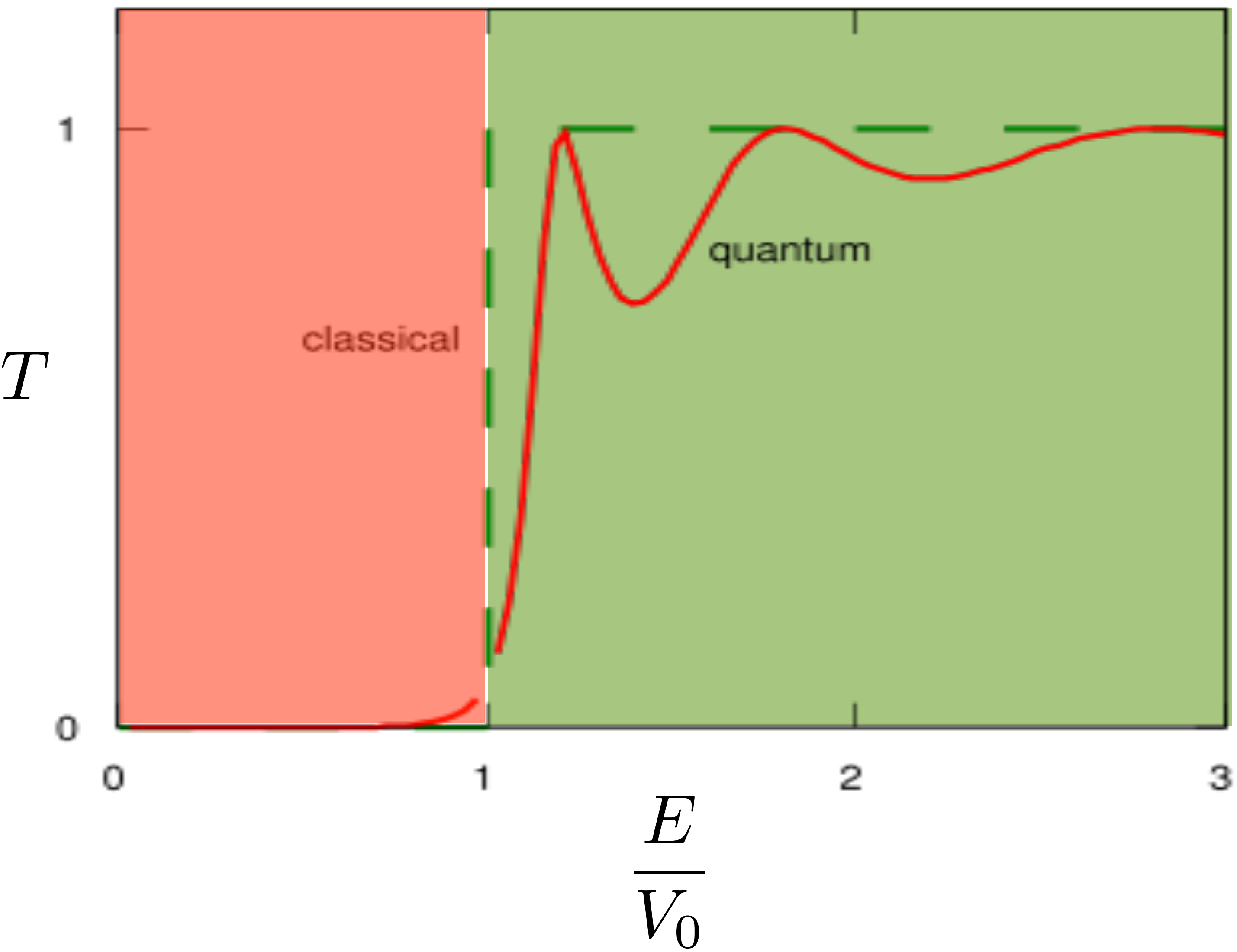


T

$\frac{E}{V_0}$

$E < V_0$ **and** $E > V_0$

Step Potential with Finite Width



$$T(E) = \left[1 + \frac{1}{4}\left(\frac{V_0^2}{E(E-V_0)}\right)\sin^2(k'L)\right]^{-1}$$

Resonance occurs when the argument of sin-function satisfies as

$$k'L = n\pi; \quad n = 1, 2, 3...$$

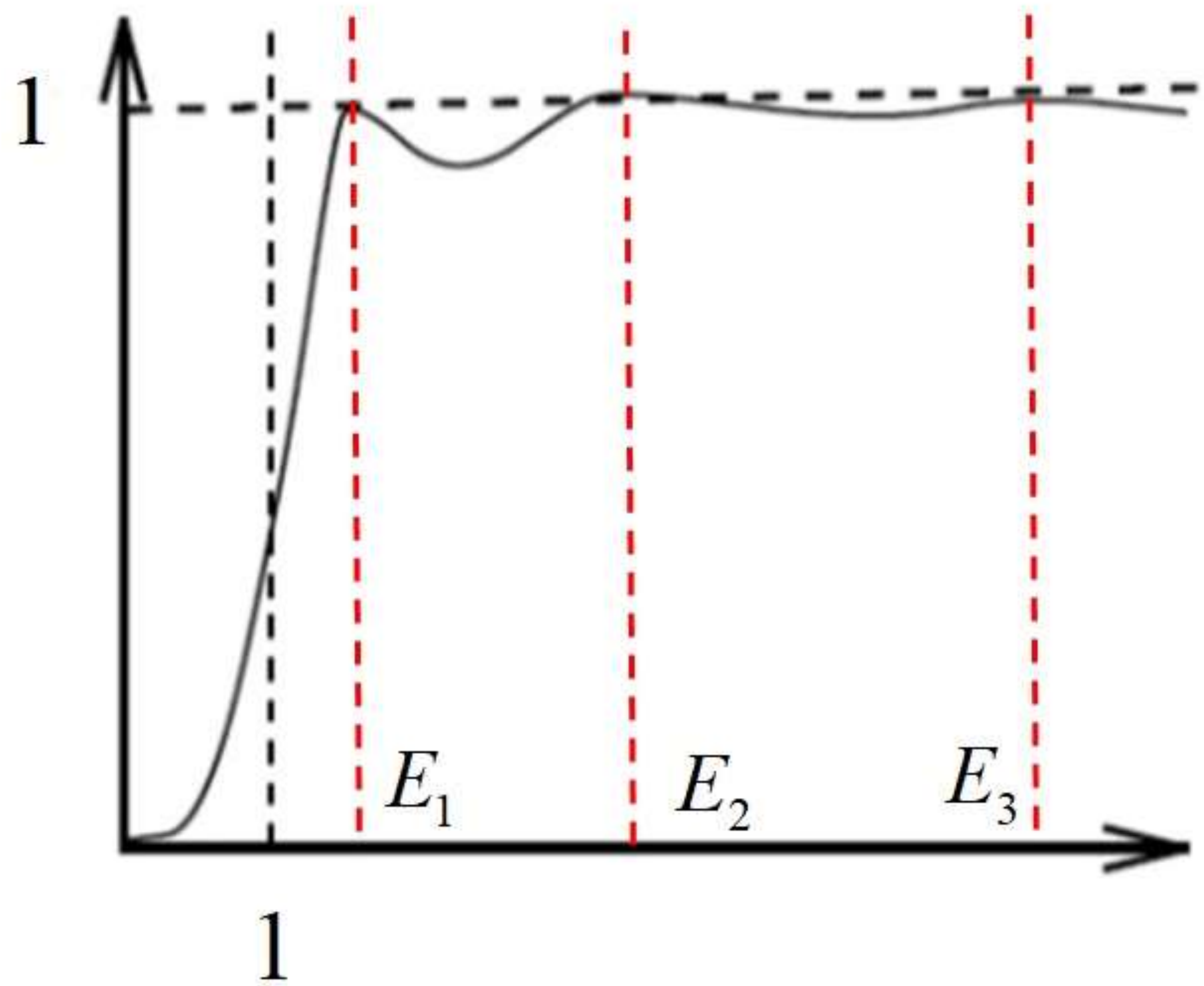
and

$$(k')^2 = \frac{2m(E-V_0)}{\hbar^2}$$

$$\implies E = \frac{n^2\pi^2\hbar^2}{2mL^2} + V_0$$

Step Potential with Finite Width

$$T(E) = \left[1 + \frac{1}{4}\left(\frac{V_0^2}{E(E-V_0)}\right)\sin^2(k'L)\right]^{-1}$$

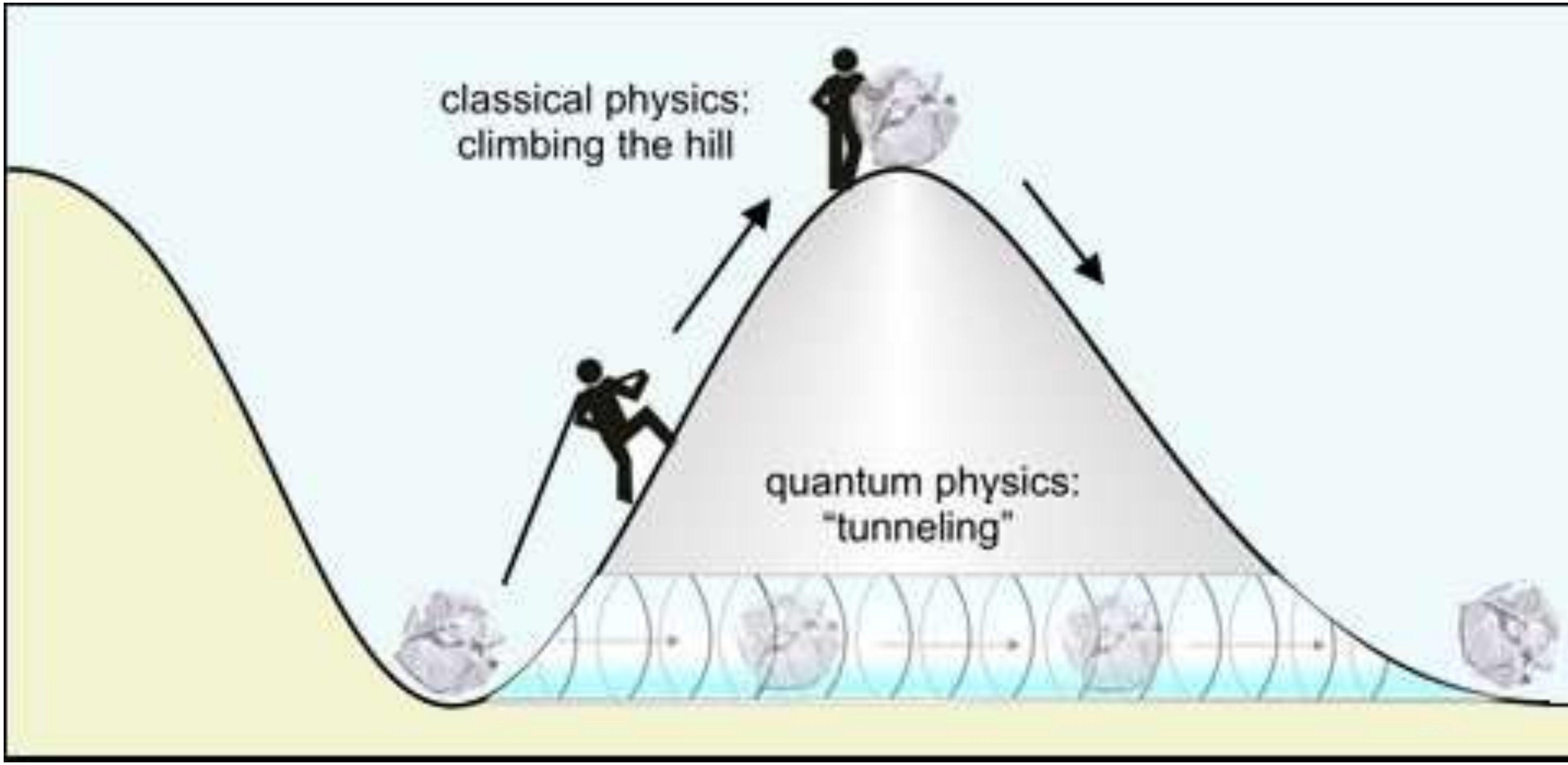


(1) Ramsaur effect in atomic physics: Noble gases become nearly transparent to electrons of specific energies.

(2) Size resonance: MeV energy neutrons pass transparently through nuclei at resonant energies.

Transmission resonances at E_1, E_2 and E_3.

Step Potential with Finite Width



Tunnelling: Non-intuitive, intellectually fascinating and technologically important process.

- Natural phenomena such as radioactive alpha-decay
- Scanning tunnelling microscope (STM)

Quantum Tunneling

Quantum Mechanical Tunneling is very essential concept and has widespread applications.

Tunneling is exploited in several practical applications: Scanning Tunneling Microscope (STM), Tunnel Diodes, Field emission electron sources.

Tunneling is also known for light waves

Before, I discuss the applications of quantum tunneling, let us look some puzzling observations related to tunnelling:

No power transmission or energy transfer across the potential step. Yet the probability of finding the particle is non-zero in region II

1. There's no energy transfer, but we can find the particle. How do we understand this ?

2. If we try to calculate the transmission coefficient, will it be zero?

3. Some ideas on trying to measure the energy of the particle in region II.

Let us try to understand !!

Problem 1

An electron with total energy $E = 6$ eV approaches a potential barrier with height $V_0 = 12$ eV. If the width of the barrier is $L = 0.18$ nm, what is the probability that the electron will tunnel through the barrier?

Show that:

1. For $\alpha L \gg 1$, $\quad T \simeq \dfrac{16E(V_0 - E)}{V_0^2} e^{-2\alpha L}$

2. For $\alpha L \ll 1$, $\quad T = \left[1 + \dfrac{m^2 V_0^2 L^2}{\hbar^4 k^2}\right]^{-1}$