BB 101: Physical Biology

TUTORIAL 2: Solutions

1. Velocity of the paddle for downward motion of the paddle is given by

$$V_P = (u - v)\hat{\imath}$$

The net force action on the body is zero and hence sum of drag force acting on the paddle and drag force acting on the body should be zero

$$-\gamma_1 u\hat{\imath} - \gamma_2(u - v)\hat{\imath} = 0$$

The, balancing the drag forces for the downward movement of paddle gives

$$-\gamma_1 u = \gamma_2(u - v)$$

$$u = \frac{\gamma_2 v}{\gamma_1 + \gamma_2} \qquad (1)$$

Velocity of the paddle for upward motion of the paddle is given by

$$V_P = (v' - u')\hat{\imath} = -(u' - v')\hat{\imath}$$

The net force action on the body is zero and hence sum of drag force acting on the paddle and drag force acting on the body should be zero

$$\gamma_1 u'\hat{\imath} + \gamma_2(u' - v')\hat{\imath} = 0$$

Thus, balancing drag forces for the upward movement of paddle gives

$$u' = \frac{\gamma_2 v'}{\gamma_1 + \gamma_2}\hat{\imath} \qquad (2)$$

Displacement Δx of the microorganism due to downward movement of paddle

$$\Delta x = tu$$

Since motion of paddle is geometrically reciprocal, therefore distance travelled by paddle during both upward and downward movement must be identical

$$tv = t'v' \qquad (3)$$

Now, displacement $\Delta x'$ of the microorganism due to upward movement of paddle

$$\Delta x' = -t'u'$$

$$= -\left(\frac{tv}{v'}\right)\left(\frac{\gamma_2 v'}{\gamma_1 + \gamma_2}\right) \text{ using (2) and(3)}$$

$$= -\left(\frac{tv}{v'}\right)\left(\frac{\gamma_2 v'}{\gamma_1 + \gamma_2}\right)$$

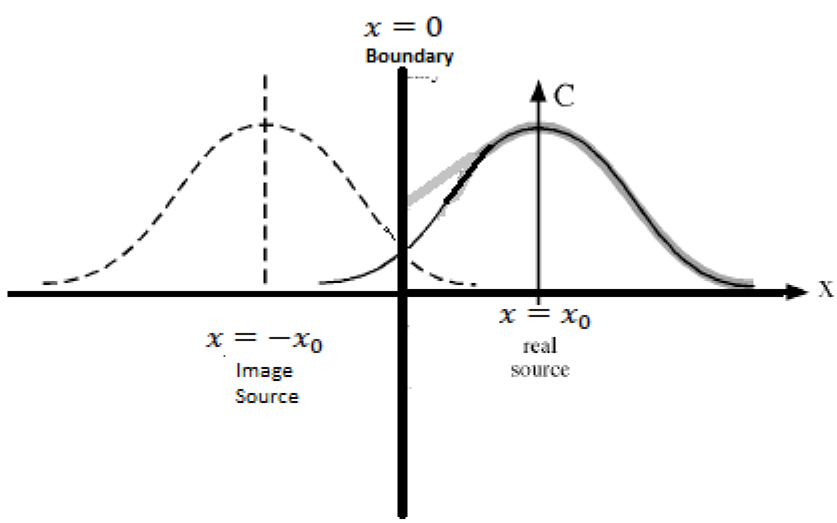
$$= -t(\frac{\gamma_2 v}{\gamma_1 + \gamma_2})$$

$$= -tu = -\Delta x \quad using\ (1)$$

Since $\Delta x' = -\Delta x$, There will not be any net displacement

2. The solution of diffusion equation in presence of a perfectly reflecting boundary located at x=0 can be obtained by adding the concentration due to an imaginary source located at $x=x_0$

$$C(x,t) = \frac{C_0}{\sqrt{4\pi Dt}} e^{\frac{-(x-x_0)^2}{4Dt}} + \frac{C_0}{\sqrt{4\pi Dt}} e^{\frac{-(x+x_0)^2}{4Dt}} \qquad x > 0$$



3. Let the concentration of the drug in the tablet be C_0. In this case there are two processes that are happening, diffusion of the drug and reaction of the drug

Therefore, equation capturing both reaction and diffusion is given by

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} - kC$$

We shall solve this equation under steady state condition i.e. $\frac{\partial C}{\partial t} = 0$

Therefore, equation to solve is

$$0 = D\frac{\partial^2 C}{\partial x^2} - kC$$

Or,

$$\frac{\partial^2 C}{\partial x^2} = \frac{k}{D}C \qquad (1)$$

The general solution of this second order differential equation is given by

$$C(x) = A_1 e^{-B_1 x} + A_2 e^{+B_2 x}$$

Let use the boundary condition $C(x) = 0$ at $x = \infty$ and $C(x) = C_0$ at $x = 0$

Let us the first boundary condition $(x) = 0$ at $x = \infty$

$$\Rightarrow A_2 = 0$$

Therefore,

$$C(x) = A_1 e^{-B_1 x}$$

Now let's use the second boundary condition $C(x) = C_0$ at $x = 0$

$$\Rightarrow A_1 = C_0$$

Therefore,

$$C(x) = C_0 e^{-B_1 x} \qquad (2)$$

To determineB_1, substitute (2) in (1)

$$\Rightarrow B_1^2 = \frac{k}{D}$$

$$\Rightarrow B_1 = \sqrt{\frac{k}{D}}$$

Therefore, $$C(x) = C_0 e^{-\sqrt{\frac{k}{D}}x}$$

To compute the rate at which the drug is being drawn out, we have to calculate flux at $x = 0$

Therefore, $$J(x) = -D\frac{\partial C}{\partial x} = \sqrt{kD}C_0 e^{-\sqrt{\frac{k}{D}}x}$$

$$J(x = 0) = -D\frac{\partial C}{\partial x} = \sqrt{kD}C_0$$

This suggests that flux at $x = 0$ is proportional to both k and D i.e. drug will be drawn out rapidly if either diffusion rate or reaction rate is higher