Derivation Integrating factors

21 February 2021 23:22

$$L\frac{di}{dt} + Ri = V$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

$$e^{R/Lt}\frac{di}{dt} + \frac{R}{L}e^{R/Lt}i = \frac{V}{L}e^{R/Lt}$$

$$\frac{d}{dt}\left[e^{R/Lt}i\right] = \frac{V}{L}e^{R/Lt}$$

$$e^{R/Lt}i(t)\Big|_0^t = \frac{V}{L}\quad\frac{L}{R}e^{R/Lt}\Big|_0^t$$

$$e^{R/Lt}i(t) - i(0) = \frac{V}{L}\cdot\frac{L}{R}\left[e^{R/Lt} - 1\right]$$

$$e^{R/Lt}i(t) - i(0) = \frac{V}{R}\left[e^{R/Lt} - 1\right]$$

$$i(t) - i(0)e^{-R/Lt} = \frac{V}{R}\left[1 - e^{-R/Lt}\right]$$

$$\overline{e^{\alpha t}}\; i(t) - i(0) = \frac{V}{L}\int_0^t e^{\alpha t}\cos\omega t\,dt \leftarrow$$

$u = \cos(\omega t + \phi)$ $\frac{du}{dt} = -\omega\sin(\omega t + \phi)$

$u = \cos\omega t$ $\quad \frac{du}{dt} = -\omega\sin\omega t$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$dv = \underline{e^{\alpha t}\,dt}$ $\quad v = \int e^{\alpha t}\,dt = \frac{1}{\alpha}e^{\alpha t}$

$$= \frac{V}{L}\left[\frac{1}{\alpha}e^{\alpha t}\cos\omega t - \int\frac{1}{\alpha}e^{\alpha t}\,-\omega\sin\omega t\,dt + \text{const}\right]$$

$$i(t)e^{\alpha t} - i(0) = \frac{V}{L}\left[\frac{1}{\alpha}e^{\alpha t}\cos\omega t + \frac{\omega}{\alpha}\underbrace{\int e^{\alpha t}\sin\omega t\,dt}\ + \text{const}\right]$$

$$\rightarrow \int e^{\alpha t}\sin\omega t\,dt = \frac{1}{\alpha}e^{\alpha t}\sin\omega t - \frac{\omega}{\alpha}\int e^{\alpha t}\cos\omega t\,dt + \text{const}$$

$$\int e^{\alpha t}\cos\omega t\,dt = \left[\frac{1}{\alpha}e^{\alpha t}\cos\omega t + \frac{\omega}{\alpha}\left\{\frac{1}{\alpha}e^{\alpha t}\sin\omega t - \frac{\omega}{\alpha}\int e^{\alpha t}\cos\omega t\right\} + \text{const}\right]$$

$$\int e^{\alpha t}\cos\omega t\,dt = \frac{1}{\alpha}e^{\alpha t}\cos\omega t + \frac{\omega}{\alpha^2}e^{\alpha t}\sin\omega t - \frac{\omega^2}{\alpha^2}\underbrace{\int e^{\alpha t}\cos\omega t\,dt}\ + \text{const}$$

$$\left(1+\frac{\omega^2}{\alpha^2}\right)\int e^{\alpha t}\cos\omega t\,dt = \frac{1}{\alpha}e^{\alpha t}\cos\omega t + \frac{\omega}{\alpha^2}e^{\alpha t}\sin\omega t + \text{const}$$

$$\int e^{\alpha t}\cos\omega t\,dt = \frac{\alpha^2}{\alpha^2+\omega^2}\cdot\frac{1}{\alpha}e^{\alpha t}\cos\omega t + \frac{\alpha^2}{\alpha^2+\omega^2}\cdot\frac{\omega}{\alpha^2}e^{\alpha t}\sin\omega t + \text{const}$$

$$\rightarrow \int e^{\alpha t}\cos\omega t\,dt = \boxed{\frac{\alpha}{\alpha^2+\omega^2}}e^{\alpha t}\cos\omega t + \boxed{\frac{\omega}{\alpha^2+\omega^2}}e^{\alpha t}\sin\omega t + \text{const}$$

$$i(t)e^{\alpha t} - i(0) = \frac{V_m}{L}e^{\alpha t}\left[\frac{RL}{R^2+\omega^2L^2}\cos\omega t + \frac{\omega L^2}{R^2+\omega^2L^2}\sin\omega t\right] + \text{const}$$

$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\left[\frac{R}{\sqrt{R^2+\omega^2L^2}}\cos\omega t + \frac{\omega L}{\sqrt{R^2+\omega^2L^2}}\sin\omega t\right] + \text{const}\,e^{-\alpha t} + i(0)e^{-\alpha t}$$

$$i(t) = \frac{V_m}{Z}\left[\cos\omega t\cos\theta + \sin\omega t\sin\theta\right] + \text{const}\,e^{-\alpha t} + i(0)e^{-\alpha t}$$

$$\boxed{i(t) = \frac{V_m}{Z}\cos(\omega t+\theta) + \text{const}\,e^{-\alpha t} + i(0)e^{-\alpha t}}$$