Problems for April 13

1) Let A be a $2\times n$ matrix of real numbers

Is there a relation between

$\det(AA^T)$ (which is a Grammian)

and the sum of the (2×2) principal minors

of A^TA

<u>Note:</u> Given a square Matrix $[p_{ij}]$

the sum of the 2×2 principal minors are

$$\begin{vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{vmatrix} + \begin{vmatrix} p_{11} & p_{13} \\ p_{31} & p_{33} \end{vmatrix} + \dots + \begin{vmatrix} p_{n-1,n-1} & p_{n-1,n} \\ p_{n,n-1} & p_{nn} \end{vmatrix}$$

Is there a similar result when A is $3\times n$ matrix?

Any Guess?

2) Show that

$$\begin{vmatrix} \cos\alpha & 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 2\cos\alpha & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 2\cos\alpha & 1 & 0 & \cdots & 0 \\ \cdots & & \cdots & & \cdots & \cdots & 1 \\ & & & & & 1 & 2\cos\alpha \\ 0 & 0 & \cdots & & & & \end{vmatrix} = \cos n\alpha.$$

3) Suppose $\langle\,,\rangle$ is a hermitian product

$$4\langle x,y\rangle = \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2.$$

4) Do Q8 in Sheet 4

Q4

5) Show that if A is a $(n\times n)$ complex matrix whose rows are orthonormal then so are its cols.

6) $V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ i \\ -1 \end{bmatrix}$; $W = \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ -2i \\ -1 \end{bmatrix}$. Find $u\in\mathbb{C}^3$ s.t $\{v, w, u\}$ orthonormal

Any way other than Gram-Schmidt?