

$$\psi(x) = C_0 e^{-\frac{m\omega x^2}{2\hbar}}$$

$$\int_{-\infty}^{\infty} \psi^*(x) \psi(x) dx = 1$$

$$\Rightarrow C_0^2 \int_{-\infty}^{\infty} e^{-\frac{2m\omega}{2\hbar} x^2} dx = 1$$

gaussian integral.

$$\Rightarrow C_0^2 \cdot \sqrt{\frac{\pi \hbar}{2m\omega}} = 1$$

$$C_0 = \left( \frac{m\omega}{\pi \hbar} \right)^{1/4}$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$\omega^2 m = k$$

$$\psi(x) = \left( \frac{2\alpha}{\pi} \right)^{1/4} e^{-\alpha x^2}$$

$$\Delta \cdot (\psi^* | x^2 | \psi)$$

$$= \left( \frac{2\alpha}{\pi} \right)^{1/2} \cdot \int_{-\infty}^{\infty} x^2 e^{-2\alpha x^2} dx$$

$$= \frac{2}{\pi} \cdot \frac{\sqrt{\pi}}{2\alpha} = \frac{1}{\alpha}$$

→ using gamma function,

$$\Delta x = \sqrt{\frac{\hbar}{2m\omega} - 0^2}$$

$$= \sqrt{\frac{\hbar}{2m\omega}}$$

$$\Delta p = \sqrt{\frac{m\hbar\omega}{2}}$$

$$\therefore \Delta x \cdot \Delta p = \frac{1}{2}$$