

## On a Heuristic Point of View Concerning the Production and Transformation of Light

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A PROFOUND formal difference exists between the theoretical concepts that physicists have formed about gases and other ponderable bodies, and Maxwell's theory of electromagnetic processes in so-called empty space. While we consider the state of a body to be completely determined by the positions and velocities of an indeed very large yet finite number of atoms and electrons, we make use of continuous spatial functions to determine the electromagnetic state of a volume of space, so that a finite number of quantities cannot be considered as sufficient for the complete determination of the electromagnetic state of space. According to Maxwell's theory, energy is considered to be a continuous spatial function for all purely electromagnetic phenomena, hence also for light, whereas according to the present view of physicists, the energy of a ponderable body should be represented as a sum over the atoms and electrons. The energy of a ponderable body cannot be broken

up into arbitrarily many, arbitrarily small parts, but according to Maxwell's theory (or, more generally, according to any wave theory) the energy of a light ray emitted from a point source continuously spreads out over an ever-increasing volume.

The wave theory of light, which operates with continuous spatial functions, has proved itself superbly in describing purely optical phenomena and will probably never be replaced by another theory. One should keep in mind, however, that optical observations refer to time averages rather than instantaneous values; and it is quite conceivable, despite the complete confirmation of the theory of diffraction, reflection, refraction, dispersion, etc., by experiment, that the theory of light, operating with continuous spatial functions, leads to contradictions when applied to the phenomena of emission and transformation of light.

Indeed, it seems to me that the observations of "black-body radiation," photoluminescence, production of cathode rays by ultraviolet light, and other related phenomena associated with the emission or transformation of light appear more readily understood if one assumes that the energy of light is discontinuously distributed in space. According to the assumption considered here, in the propagation of a light ray emitted from a point source, the energy is not distributed continuously over ever-increasing volumes of space, but consists of a finite number of energy quanta localized at points of space that move without dividing, and can be absorbed or generated only as complete units.

In this paper I wish to present the train of thought and cite the facts that led me onto this path, in the hope that the approach to be presented will prove of use to some researchers in their investigations.

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### 1. ON A DIFFICULTY CONCERNING THE THEORY OF "BLACK-BODY RADIATION"

We shall begin by considering the following case from the perspective of Maxwell's theory and electron theory. Let a space enclosed by completely reflecting walls contain a number of freely moving gas molecules and electrons that exert conservative forces on each other when they come very close, i.e., can collide with each other like molecules according to the kinetic theory of gases.<sup>1</sup> Suppose, further, that a number of electrons are bound to widely separated points in space by forces directed toward these points and proportional to their distances from them. The bound electrons also enter into conservative interactions with the free molecules and electrons when the latter come very close. We call the bound electrons "resonators"; they emit and absorb electromagnetic waves of definite periods.

According to the present view concerning the origin of light, the radiation in the volume we are considering, as is found for the case of dynamic equilibrium based on Maxwell's theory, must be identical with "black-body radiation"—at least if one assumes that resonators of all relevant frequencies are present.

For the time being, we will disregard the radiation emitted and absorbed by the resonators and investigate the condition of dynamic equilibrium corresponding to the interac-

<sup>1</sup> This assumption is equivalent to the supposition that the mean kinetic energies of gas molecules and electrons are equal to each other at thermal equilibrium. As is well known, Mr. Drude used the latter assumption to derive a theoretical expression for the ratio of thermal and electrical conductivities of metals.

tion (collisions between) molecules and electrons. For such an equilibrium, the kinetic theory of gases asserts that the mean kinetic energy of a resonator electron must be equal to the mean translational kinetic energy of a gas molecule. If we decompose the motion of a resonator electron into three mutually perpendicular oscillatory motions, we find for the mean value  $\bar{E}$  of the energy of one such linear oscillatory motion

$$\bar{E} = \frac{R}{N} T,$$

where  $R$  denotes the universal gas constant,  $N$  the number of "real molecules"<sup>[1]</sup> in one gram-equivalent, and  $T$  the absolute temperature. Because of the equality of the time averages of the resonator's kinetic and potential energies, the energy  $\bar{E}$  is two-thirds the kinetic energy of a free monatomic gas molecule. If due to some cause—in our case, radiation processes—the energy of a resonator were to have a time average greater or less than  $\bar{E}$ , then collisions with the free electrons and molecules would lead to an energy loss to or a gain from the gas, different on the average from zero. Thus, in the case we are considering, dynamic equilibrium is possible only if the average energy of each resonator is  $\bar{E}$ .

We will now apply a similar argument to the interaction between the resonators and the radiation present in space. Mr. Planck has derived the condition for dynamic equilibrium in this case<sup>2</sup> on the assumption that the radiation can be treated as the most completely conceivable disordered

<sup>2</sup>M. Planck, *Ann. d. Phys.* 1 (1900): 99.

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process.<sup>3</sup> He found:

$$\bar{E}_\nu = \frac{L^3}{8\pi\nu^2} \rho_\nu.$$

$\bar{E}_\nu$  is here the mean energy of a resonator with eigenfrequency  $\nu$  (per unit frequency interval),  $L$  is the velocity of light,  $\nu$  the frequency, and  $\rho_\nu d\nu$  the energy per unit volume of that part of the radiation whose frequency lies between  $\nu$  and  $\nu + d\nu$ .

<sup>3</sup>This assumption can be formulated as follows. We expand the Z-component of the electrical force ( $Z$ ) at an arbitrary point of the space being considered during the time interval between  $t = 0$  and  $t = T$  (where  $T$  denotes a time period very large relative to all relevant oscillation periods) in a Fourier series,

$$Z = \sum_{\nu=1}^{\nu=\infty} A_\nu \sin\left(2\pi\nu\frac{t}{T} + \alpha_\nu\right),$$

where  $A_\nu \geq 0$  and  $0 \leq \alpha_\nu \leq 2\pi$ . If one imagines that at the same point of space such an expansion is made arbitrarily often at randomly chosen starting times, then one will obtain different sets of values for the quantities  $A_\nu$  and  $\alpha_\nu$ . For the frequency of occurrence of the various combinations of values of the quantities  $A_\nu$  and  $\alpha_\nu$ , there will then exist (statistical) probabilities  $dW$  of the form

$$dW = f(A_1 A_2 \dots \alpha_1 \alpha_2 \dots) dA_1 dA_2 \dots d\alpha_1 d\alpha_2 \dots$$

The radiation is then most conceivably disordered if

$$f(A_1, A_2 \dots \alpha_1, \alpha_2 \dots) = F_1(A_1) F_2(A_2) \dots f_1(\alpha_1) \cdot f_2(\alpha_2) \dots,$$

i.e., when the probability of a specific value of one of the quantities  $A$  or  $\alpha$ <sup>[2]</sup> is independent of the values taken by the other quantities  $A$  and  $\alpha$  respectively. Hence, the more closely fulfilled the condition is that individual pairs of quantities  $A_\nu$  and  $\alpha_\nu$  depend on the emission and absorption processes of particular groups of resonators, the more closely can radiation in our case be considered to be "most conceivably disordered."

If, on the whole, the radiation energy of frequency  $\nu$  does not continually decrease or increase, the following relations must hold:

$$\frac{R}{N} T = \bar{E} = \bar{E}_\nu = \frac{L^3}{8\pi\nu^2} \rho_\nu,$$

$$\rho_\nu = \frac{R}{N} \frac{8\pi\nu^2}{L^3} T. [3]$$

These relations, found as the conditions of dynamic equilibrium, not only fail to agree with experiment but also imply that in our model a definite distribution of energy between ether and matter is out of the question. Indeed, the wider the range of the resonator frequencies chosen, the greater the total radiation energy of the volume, and in the limit we obtain

$$\int_0^\infty \rho_\nu d\nu = \frac{R}{N} \frac{8\pi}{L^3} T \int_0^\infty \nu^2 d\nu = \infty.$$

## 2. ON PLANCK'S DETERMINATION OF THE ELEMENTARY QUANTA<sup>[4]</sup>

We now wish to show that Mr. Planck's determination of the elementary quanta is to a certain extent independent of his theory of "black-body radiation."

Planck's formula<sup>4</sup> for  $\rho_\nu$ , which is satisfied by all experiments up to now, reads

$$\rho_\nu = \frac{\alpha \nu^3}{e^{\frac{\beta \nu}{T}} - 1},$$

<sup>4</sup>M. Planck, *Ann. d. Phys.* 4 (1901): 561.

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where

$$\alpha = 6.10 \times 10^{-56}$$

$$\beta = 4.866 \times 10^{-11}.$$

For large values of  $T/\nu$ , i.e., for high wavelengths and radiation densities, this equation takes the following limiting form:

$$\rho_\nu = \frac{\alpha}{\beta} \nu^2 T.$$

One can see that this formula agrees with that derived in section 1 from Maxwell's theory and electron theory. By equating the coefficients of the two formulas, we obtain

$$\frac{R}{N} \frac{8\pi}{L^3} = \frac{\alpha}{\beta}$$

or

$$N = \frac{\beta}{\alpha} \frac{8\pi R}{L^3} = 6.17 \times 10^{23},$$

i.e., one atom of hydrogen weighs  $1/N$  gram  $= 1.62 \times 10^{-24}$  g. This is exactly the value found by Mr. Planck, which in turn is in satisfactory agreement with the values found by other methods.

We therefore arrive at the following conclusion: the higher the energy density and wavelength of radiation, the more reasonable the theoretical foundations we have been using prove to be; however, they fail completely in the case of low wavelengths and low radiation densities.

In the following, we shall consider "black-body radiation" together with the experimental facts, without establishing any model for the emission and propagation of radiation.

### 3. ON THE ENTROPY OF RADIATION

The following treatment is contained in a well-known study by Mr. Wien and is presented here only for the sake of completeness.

Consider radiation occupying a volume  $v$ . We assume that the observable properties of this radiation are completely determined when the radiation density  $\rho(\nu)$  is given for all frequencies.<sup>5</sup> Since radiations of different frequencies can be regarded as separable from each other without performing any work or transferring any heat, the entropy of the radiation can be represented by

$$S = v \int_0^{\infty} \varphi(\rho, \nu) d\nu,$$

where  $\varphi$  is a function of the variables  $\rho$  and  $\nu$ . One can reduce  $\varphi$  to a function of a single variable by asserting that adiabatic compression of radiation between reflecting walls does not change its entropy. However, we shall not pursue this, but will immediately investigate how the function  $\varphi$  can be obtained from the black-body radiation law.

In the case of "black-body radiation,"  $\rho$  is such a function of  $\nu$  that the entropy is a maximum for a given energy, i.e.,

$$\delta \int_0^{\infty} \varphi(\rho, \nu) d\nu = 0$$

if

$$\delta \int_0^{\infty} \rho d\nu = 0.$$

<sup>5</sup>This assumption is arbitrary. We shall naturally keep this simplest assumption as long as experiment does not force us to abandon it.



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From this it follows that for every choice of  $\delta\rho$  as a function of  $\nu$ ,

$$\int_0^\infty \left( \frac{\partial\varphi}{\partial\rho} - \lambda \right) \delta\rho \, d\nu = 0,$$

where  $\lambda$  is independent of  $\nu$ . Thus, for black-body radiation,  $\partial\varphi/\partial\rho$  is independent of  $\nu$ .

The following equation applies when the temperature of black-body radiation of volume  $v = 1$  increases by  $dT$ :

$$dS = \int_{\nu=0}^{\nu=\infty} \frac{\partial\varphi}{\partial\rho} \, d\rho \, d\nu,$$

or, since  $\partial\varphi/\partial\rho$  is independent of  $\nu$ ,

$$dS = \frac{\partial\varphi}{\partial\rho} \, dE.$$

Since  $dE$  is equal to the heat added and the process is reversible, we also have

$$dS = \frac{1}{T} \, dE.$$

Comparison yields

$$\frac{\partial\varphi}{\partial\rho} = \frac{1}{T}.$$

This is the law of black-body radiation. Thus, one can derive the law of black-body radiation from the function  $\varphi$ , and, inversely, the function  $\varphi$  can be determined by integration, remembering that  $\varphi$  vanishes for  $\rho = 0$ .

#### 4. LIMITING LAW FOR THE ENTROPY OF MONOCHROMATIC RADIATION AT LOW RADIATION DENSITY

The existing observations of "black-body radiation" show that the law

$$\rho = \alpha \nu^3 e^{-\beta \frac{\nu}{T}}$$

originally postulated by Mr. W. Wien for "black-body radiation" is not strictly valid. However, it has been fully confirmed by experiment for large values of  $\nu/T$ . We shall base our calculations on this formula, bearing in mind, however, that our results are valid only within certain limits.

First of all, this formula yields

$$\frac{1}{T} = -\frac{1}{\beta \nu} \ln \frac{\rho}{\alpha \nu^3},$$

and next, using the relation obtained in the preceding section,

$$\varphi(\rho, \nu) = -\frac{\rho}{\beta \nu} \left\{ \ln \frac{\rho}{\alpha \nu^3} - 1 \right\}.$$

Suppose now that we have radiation of energy  $E$ , with frequency between  $\nu$  and  $\nu + d\nu$ , occupying a volume  $v$ . The entropy of this radiation is

$$S = v \varphi(\rho, \nu) d\nu = -\frac{E}{\beta \nu} \left\{ \ln \frac{E}{v \alpha \nu^3 d\nu} - 1 \right\}. [5]$$

If we restrict ourselves to investigating the dependence of the entropy on the volume occupied by the radiation, and denote the entropy of radiation by  $S_0$  at volume  $v_0$ , we obtain

$$S - S_0 = \frac{E}{\beta \nu} \ln \left[ \frac{v}{v_0} \right].$$

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This equation shows that the entropy of monochromatic radiation of sufficiently low density varies with the volume according to the same law as the entropy of an ideal gas or a dilute solution. In the following, we shall interpret this equation on the basis of the principle introduced into physics by Mr. Boltzmann, according to which the entropy of a system is a function of the probability of its state.

### 5. MOLECULAR-THEORETICAL INVESTIGATION OF THE DEPENDENCE ON VOLUME OF THE ENTROPY OF GASES AND DILUTE SOLUTIONS

In calculating entropy by molecular-theoretical methods, the word "probability" is often used in a sense differing from the way the word is defined in probability theory. In particular, "cases of equal probability" are often hypothetically stipulated when the theoretical models employed are definite enough to permit a deduction rather than a stipulation. I will show in a separate paper that, in dealing with thermal processes, it suffices completely to use the so-called statistical probability, and hope thereby to eliminate a logical difficulty that still obstructs the application of Boltzmann's principle. Here, however, I shall give only its general formulation and its application to very special cases.

If it is meaningful to talk about the probability of a state of a system, and if, furthermore, each increase of entropy can be conceived as a transition to a state of higher probability, then the entropy  $S_1$  of a system is a function of the probability  $W_1$  of its instantaneous state. Therefore, if we have two systems  $S_1$  and  $S_2$  that do not interact with each

other, we can set:

$$S_1 = \varphi_1(W_1),$$

$$S_2 = \varphi_2(W_2).$$

If these two systems are viewed as a single system of entropy  $S$  and probability  $W$ , then

$$S = S_1 + S_2 = \varphi(W)$$

and

$$W = W_1 \cdot W_2.$$

The last equation tells us that the states of the two systems are events that are independent of each other.

From these equations it follows that

$$\varphi(W_1 \cdot W_2) = \varphi_1(W_1) + \varphi_2(W_2),$$

and, finally,

$$\varphi_1(W_1) = C \ln(W_1) + \text{const.}$$

$$\varphi_2(W_2) = C \ln(W_2) + \text{const.}$$

$$\varphi(W) = C \ln(W) + \text{const.}$$

The quantity  $C$  is therefore a universal constant; it follows from the kinetic theory of gases that its value is  $R/N$ , where the meaning of the constants  $R$  and  $N$  is the same as above. If  $S_0$  denotes the entropy of a certain initial state and  $W$  is the relative probability of a state of entropy  $S$ , we obtain in general:

$$S - S_0 = \frac{R}{N} \ln W.$$

We now treat the following special case. Let a volume  $v_0$  contain a number ( $n$ ) of moving points (e.g., molecules) to

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which our discussion will apply. The volume may also contain any arbitrary number of other moving points of any kind. No assumption is made about the law governing the motion of the points under discussion in the volume except that, as concerns this motion, no part of the space (and no direction within it) is preferred over the others. Further, let the number of (aforementioned) moving points under discussion be small enough that we can disregard interactions between them.

This system, which, for example, can be an ideal gas or a dilute solution, possesses a certain entropy  $S_0$ . Let us imagine that all  $n$  moving points are assembled in a part of the volume  $v_0$  of size  $v$  without any other changes in the system. It is obvious that this state has a different value of entropy ( $S$ ), and we now wish to determine the difference in entropy with the help of Boltzmann's principle.

We ask: How great is the probability of the last-mentioned state relative to the original one? Or: How great is the probability that at a randomly chosen instant of time, all  $n$  independently moving points in a given volume  $v_0$  will be found (by chance) in the volume  $v$ ?

Obviously, this probability, which is a "statistical probability," has the value

$$W = \left(\frac{v}{v_0}\right)^n;$$

from this, by applying Boltzmann's principle, one obtains

$$S - S_0 = R \left(\frac{n}{N}\right) \ln \left(\frac{v}{v_0}\right).$$

It is noteworthy that, in the derivation of this equation, from which the Boyle-Gay-Lussac law and the analogous

law of osmotic pressure can easily be derived thermodynamically,<sup>6</sup> no assumptions need be made about the law governing the motion of the molecules.

## 6. INTERPRETATION ACCORDING TO BOLTZMANN'S PRINCIPLE OF THE EXPRESSION FOR THE DEPENDENCE OF THE ENTROPY OF MONOCHROMATIC RADIATION ON VOLUME

In section 4 we found the following expression for the dependence of the entropy of monochromatic radiation on volume:

$$S - S_0 = \frac{E}{\beta\nu} \ln \left( \frac{\nu}{\nu_0} \right).$$

If we write this formula in the form

$$S - S_0 = \frac{R}{N} \ln \left[ \left( \frac{\nu}{\nu_0} \right)^{\frac{N}{R} \frac{E}{\beta\nu}} \right]$$

and compare it with the general formula expressing the Boltzmann principle,

$$S - S_0 = \frac{R}{N} \ln W,$$

we arrive at the following conclusion: If monochromatic radiation of frequency  $\nu$  and energy  $E$  is enclosed (by reflecting

<sup>6</sup>If  $E$  is the energy of the system, we get

$$-d(E - TS) = p d\nu = T dS = R \frac{n}{N} \frac{d\nu}{\nu},^{(6)}$$

thus

$$p\nu = R \frac{n}{N} T.$$

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walls) in the volume  $v_0$ , the probability that at a randomly chosen instant the total radiation energy will be found in the portion  $v$  of the volume  $v_0$  is

$$W = \left( \frac{v}{v_0} \right)^{\frac{N}{R} \frac{E}{\beta \nu}}$$

From this we further conclude that monochromatic radiation of low density (within the range of validity of Wien's radiation formula) behaves thermodynamically as if it consisted of mutually independent energy quanta of magnitude  $R\beta\nu/N$ .<sup>[7]</sup>

We also want to compare the mean value of the energy quanta of black-body radiation with the mean kinetic energy of the center-of-mass motion of a molecule at the same temperature. The latter is  $\frac{3}{2}(R/N)T$ , while the mean value of the energy quantum obtained on the basis of Wien's formula is

$$\frac{\int_0^\infty \alpha \nu^3 e^{-\frac{\beta \nu}{T}} d\nu}{\int_0^\infty \frac{N}{R\beta \nu} \alpha \nu^3 e^{-\frac{\beta \nu}{T}} d\nu} = 3 \frac{R}{N} T.$$

If monochromatic radiation (of sufficiently low density) behaves, as concerns the dependence of its entropy on volume, as though the radiation were a discontinuous medium consisting of energy quanta of magnitude  $R\beta\nu/N$ , then it seems reasonable to investigate whether the laws governing the emission and transformation of light are also constructed as if light consisted of such energy quanta. We will consider this question in the following sections.

## 7. ON STOKES'S RULE

Let monochromatic light be transformed by photoluminescence into light of a different frequency, and, in accordance with the result just obtained, let us assume that both the incident and emitted light consist of energy quanta of magnitude  $(R/N)\beta\nu$ , where  $\nu$  denotes the relevant frequency. The transformation process can then be interpreted as follows. Each incident energy quantum of frequency  $\nu_1$  is absorbed and—at least at a sufficiently low distribution density of the incident energy quanta—generates by itself a light quantum of frequency  $\nu_2$ ; it is possible that the absorption of the incident light quantum may simultaneously give rise to the emission of light quanta of frequencies  $\nu_3, \nu_4$ , etc., as well as to energy of some other kind (e.g., heat). It makes no difference by means of what intermediary processes this end result occurs. If the photoluminescent substance is not regarded as a permanent energy source, then, according to the principle of conservation of energy, the energy of an emitted energy quantum cannot be greater than that of the light quantum that produced it; hence it follows that

$$\frac{R}{N}\beta\nu_2 \leq \frac{R}{N}\beta\nu_1,$$

or

$$\nu_2 \leq \nu_1.$$

This is the well-known Stokes's rule.

It is to be emphasized that, according to our conception, at low illumination the amount of light emitted must be proportional to the intensity of the incident light, because each incident energy quantum will induce an elementary process of the kind indicated above, independently of other



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incident energy quanta. In particular, there will be no lower limit for the intensity of the incident light, below which the light would be unable to excite the fluorescent effect.

According to the conception of the phenomenon presented here, deviations from Stokes's rule are conceivable in the following cases:

1. If the number of energy quanta per unit volume simultaneously being transformed is so large that an energy quantum of emitted light can obtain its energy from several incident energy quanta.
2. When the incident (or emitted) light does not have the same energy distribution as black-body radiation within the range of validity of Wien's law; if, for example, the incident light is produced by a body of such high temperature that Wien's law is no longer valid for the relevant wavelengths.

The latter possibility deserves special attention. According to the conception developed above, it is indeed not impossible that even at very low densities a "non-Wien radiation" will behave differently as concerns its energy from black-body radiation within the range of validity of Wien's law.

### 8. ON THE GENERATION OF CATHODE RAYS BY ILLUMINATION OF SOLID BODIES

The usual view that the energy of light is continuously distributed over the space through which it travels faces especially great difficulties when one attempts to explain photoelectric phenomena, which are expounded in a pioneering work by Mr. Lenard.<sup>7</sup>

<sup>7</sup>P. Lenard, *Ann. d. Phys.* 8 (1902): 169, 170.

According to the view that the incident light consists of energy quanta of energy  $(R/N)\beta\nu$ , the production of cathode rays by light can be conceived in the following way. The body's surface layer is penetrated by energy quanta whose energy is converted at least partially into kinetic energy of the electrons. The simplest conception is that a light quantum transfers its entire energy to a single electron; we will assume that this can occur. However, we will not exclude the possibility that electrons absorb only a part of the energy of the light quanta.

An electron in the interior of the body having kinetic energy will have lost part of its kinetic energy by the time it reaches the surface. In addition, we will assume that, in leaving the body, each electron must perform some work,  $P$  (characteristic for the body). The electrons leaving the body with the greatest perpendicular velocity will be those located right on the surface and ejected normally to it. The kinetic energy of such electrons is

$$\frac{R}{N}\beta\nu - P.$$

If the body is charged to a positive potential  $\Pi$  and surrounded by conductors at zero potential, and if  $\Pi$  is just sufficient to prevent a loss of electrical charge by the body, it follows that

$$\Pi\epsilon = \frac{R}{N}\beta\nu - P,$$

where  $\epsilon$  denotes the charge of the electron; or

$$\Pi E = R\beta\nu - P',$$

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where  $E$  is the charge of one gram-equivalent of a monovalent ion and  $P'$  is the potential of this quantity of negative charge relative to the body.<sup>8</sup>

If one sets  $E = 9.6 \times 10^3$ , then  $\Pi \cdot 10^{-8}$  is the potential in volts that the body acquires when irradiated in vacuum.

To see whether the derived relation agrees in order of magnitude with experiment, set  $P' = 0$ ,  $\nu = 1.03 \times 10^{15}$  (which corresponds to the ultraviolet limit of the solar spectrum) and  $\beta = 4.866 \times 10^{-11}$ . We obtain  $\Pi \cdot 10^7 = 4.3$  volts, a result that agrees in order of magnitude with the results of Mr. Lenard.<sup>9</sup>

If the formula derived is correct, then  $\Pi$ , when plotted in Cartesian coordinates as a function of the frequency of the incident light, must give a straight line whose slope is independent of the nature of the substance under study.

As far as I can tell, this conception of the photoelectric effect does contradict its properties as observed by Mr. Lenard. If each energy quantum of the incident light transmits its energy to electrons, independently of all others, then the velocity distribution of the electrons, i.e., the nature of cathode rays produced, will be independent of the intensity of the incident light; on the other hand, under otherwise identical circumstances, the number of electrons leaving the body will be proportional to the intensity of the incident light.<sup>10</sup>

<sup>8</sup>If one assumes that the individual electron can only be detached by light from a neutral molecule by the expenditure of some work, one does not have to change anything in the relation just derived; one need only consider  $P'$  as the sum of two terms.

<sup>9</sup>P. Lenard, *Ann. d. Phys.* 8 (1902): 165, 184, table I, fig. 2.

<sup>10</sup>P. Lenard, *loc. cit.*, pp. 150, 166-168.

Remarks similar to those regarding possible deviations from Stokes's rule apply to the possible limits of validity of the laws set forth above.

In the foregoing it has been assumed that at least some of the quanta of the incident light transmit their energies completely to one electron each. If this likely assumption is not made, one obtains the following equation in place of the last one:

$$\Pi E + P' \leq R\beta\nu.$$

For cathode luminescence, which is the inverse of the process discussed above, one obtains by analogous considerations:

$$\Pi E + P' \geq R\beta\nu.$$

For the substances investigated by Mr. Lenard,  $PE^{(8)}$  is always considerably greater than  $R\beta\nu$  because the potential difference, which the cathode rays must traverse in order to produce visible light, amounts in some cases to hundreds, in others to thousands of volts.<sup>11</sup> We must therefore assume that the kinetic energy of one electron goes into the production of many energy quanta of light.

## 9. ON THE IONIZATION OF GASES BY ULTRAVIOLET LIGHT

We have to assume that, in ionization of a gas by ultraviolet light, one energy quantum of light serves to ionize one gas molecule. From this it follows that the ionization energy (i.e., the work theoretically required for its ionization) of a

<sup>11</sup> P. Lenard, *Ann. d. Phys.* 12 (1903): 469

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molecule cannot be greater than the energy of an absorbed light quantum capable of producing this effect. If  $J$  denotes the (theoretical) ionization energy per gram-equivalent, it follows that

$$R\beta\nu \geq J.$$

According to Lenard's measurements, the largest effective wavelength for air is about  $1.9 \times 10^{-5}$  cm, hence

$$R\beta\nu = 6.4 \times 10^{12} \text{ erg} \geq J.$$

An upper limit for the ionization energy can also be obtained from the ionization potentials of rarefied gases. According to J. Stark<sup>12</sup> the smallest measured ionization potential (for platinum anodes) for air is about 10 volts.<sup>13</sup> Thus, one obtains  $9.6 \times 10^{12}$  as the upper limit for  $J$ , which is almost equal to the value found above. There is still another consequence, verification of which by experiment seems to me of great importance. If each absorbed energy quantum of light ionizes one molecule, then the following relation must hold between the quantity of light absorbed  $L$  and the number  $j$  of gram-molecules of ionized gas:

$$j = \frac{L}{R\beta\nu}.$$

If our conception is correct, this relation must be valid for all gases that (at the relevant frequency) show no appreciable absorption unaccompanied by ionization.

(*Annalen der Physik* 17 [1905]: 132-148)

<sup>12</sup>J. Stark, *Die Elektrizität in Gasen*, p. 57 (Leipzig, 1902).

<sup>13</sup>In the interior of the gases, however, the ionization potential of negative ions is five times greater.

## EDITORIAL NOTES

<sup>[1]</sup>"Real molecules" are presumably those that are not dissociated.

<sup>[2]</sup> $x$  should be  $\alpha$ .

<sup>[3]</sup>Expressions equivalent to Einstein's equation were obtained in 1905 by Rayleigh and Jeans without the use of material resonators.

<sup>[4]</sup>Here "elementary quanta" refers to fundamental atomic constants. In 1901, Planck determined the mass of the hydrogen atom, Loschmidt's number ( $N$ ), Boltzmann's constant, and the elementary electric charge.

<sup>[5]</sup> $S$  refers to radiation with frequencies between  $\nu$  and  $\nu + d\nu$ , and  $E = p\nu d\nu$ .

<sup>[6]</sup>The last term should be multiplied by  $T$ .

<sup>[7]</sup> $R\beta/N$  corresponds to Planck's " $h$ ."

<sup>[8]</sup> $PE$  should be  $\Pi E$ .