Question 8. Marks: 6.5/7.0

Consider an infinite cylinder of radius a with its axis along the z-direction. The space outside the cylinder is filled with an electromagnetic wave, with the electric field given by

$$\vec{E}(\rho, \phi, z, t) = \frac{A}{\rho} \cos(kz - \omega t)\hat{\rho}$$

There are no electric and magnetic fields inside the cylinder ($r < a$).

a) Calculate the magnetic field $\vec{B}$ associated with the EM wave.[2]

b) Calculate the time averaged Poynting vector.[1]

c) Calculate the surface charge density on the cylinder.[2]

d) Calculate the surface current density on the cylinder.[2]

Rubrics: Correct direction of magnetic field (1.00), Correct magnitude of magnetic field (1.00), Correct direction of time averaged

Question 7. Marks: 6.0/6.0

Consider a charge density given by

$$\rho(\vec{r}, t) = \frac{q}{4\pi r^2}\delta(vt - r) - q\delta^3(\vec{r})$$

for all time $t > 0$, where δ denotes the one-dimensional Dirac delta function, and δ^3 denotes the three-dimensional Dirac delta function.

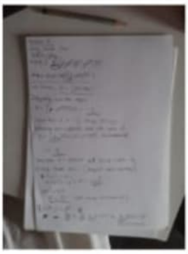
a) Calculate the electric field $\vec{E}(\vec{r}, t)$. [2]

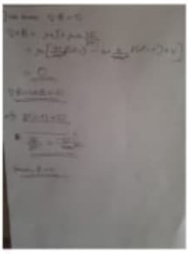
b) Calculate the current density $\vec{J}(\vec{r}, t)$. [1]

c) Calculate the magnetic field $\vec{B}(\vec{r}, t)$. [3]

Rubrics: 6 marks (6.00)

Comments: None





Question 6. Marks: 7.0/7.0

A charge $+Q$ is uniformly distributed over a non-conducting ring of radius R and mass M. The ring lies in the xy-plane with the center of the ring at $(0, 0, H)$ at time $t = 0$. The ring is released from rest at time $t = 0$. Assume that the ring remains in the xy-plane throughout its fall. The region in which the ring falls contains a magnetic field $\vec{B}(\vec{r}) = B_0 [(y/H)\hat{y} + (1 - z/H)\hat{z}]$. Calculate the angular velocity of the ring just before it hits the ground. [7]

Rubrics: All correct (7.00)

Comments: None

answer explanation can be mentioned here

Question 5. Marks: 7.0/7.0

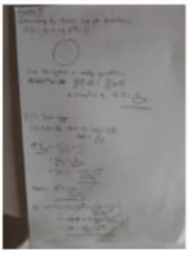
A sphere of radius R, made of a linear dielectric material of dielectric constant ϵ_r, initially has a polarization $\vec{P} = k\vec{r}$, where k is a positive constant. Now, a point charge $+q$ is introduced at the center of this sphere.

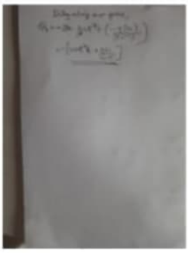
a) Find the electric displacement $\vec{D}$ of the new configuration. [4]

b) Calculate the total volume bound charge.[3]

Rubrics: All correct (7.00)

Comments: None

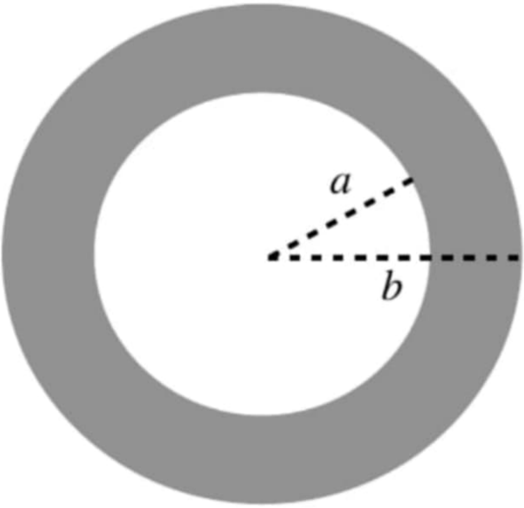




answer explanation can be mentioned here

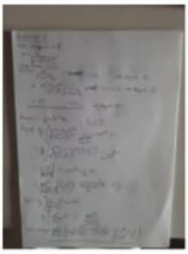
Question 4. Marks: 6.0/6.0

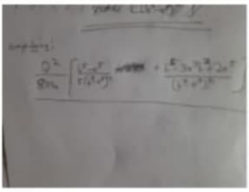
Consider a spherical shell with inner radius a and outer radius b with uniform charge density as shown in the figure. If the total charge is $+Q$, calculate the electrostatic energy U_E of this system. [6]



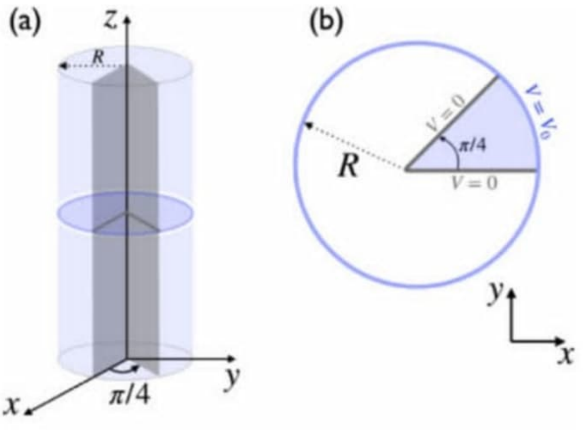
Rubrics: All Correct (6.00)

Comments: None





Consider an infinite cylinder of radius R with its axis along the z-axis. A wedge is formed from this cylinder by two grounded planes at $\phi = 0$ and $\phi = \pi/4$ (see figure (a) below). The face at $\rho = R$ is kept at a constant potential V_0 and is electrically insulated from the $\phi = 0$ and $\phi = \pi/4$ planes. A cross-sectional view of this setup is shown in the figure (b) below.



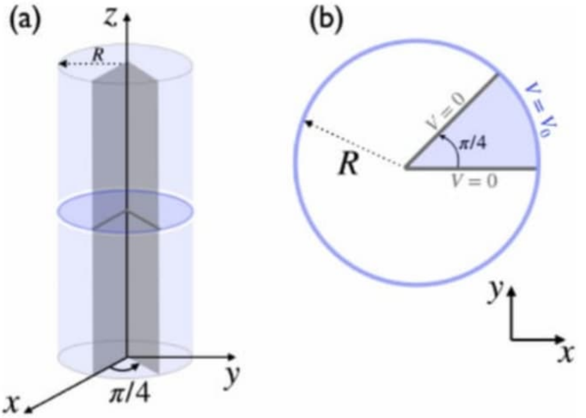
The symmetry of the problem implies that the potential inside the region bounded by these three surfaces is a function of ρ and ϕ only. The general solution to the Laplace's equation in polar coordinates can then be written as

$$V(\rho,\phi) = (a_0 + b_0 \ln\rho)(c_0 + d_0\phi) + \sum_{k\neq 0}(a_k\rho^{-k} + b_k\rho^{k})(c_k\cos k\phi + d_k \sin k\phi),$$

where (a_k, b_k, c_k, d_k) are all constants.

a) Write down all the boundary conditions for $V(\rho,\phi)$.[1]

$\phi = \pi/4$ (see figure (a) below). The face at $\rho = R$ is kept at a constant potential V_0 and is electrically insulated from the $\phi = 0$ and $\phi = \pi/4$ planes. A cross-sectional view of this setup is shown in the figure (b) below.



The symmetry of the problem implies that the potential inside the region bounded by these three surfaces is a function of ρ and ϕ only. The general solution to the Laplace's equation in polar coordinates can then be written as

$$V(\rho, \phi) = (a_0 + b_0 \ln \rho)(c_0 + d_0\phi) + \sum_{k\neq 0} (a_k\rho^{-k} + b_k\rho^k)(c_k \cos k\phi + d_k \sin k\phi),$$

where (a_k, b_k, c_k, d_k) are all constants.

a) Write down all the boundary conditions for $V(\rho, \phi)$.[1]

b) Determine the constants (a_0, b_0, c_0, d_0).[1]

c) Find the potential $V(\rho, \phi)$ in the shaded region in figure (b), subject to the above boundary conditions. [5]