BB 101: Physical Biology

TUTORIAL 1: Solutions

1. Let expansion of both dashpot and spring be X_d and extension of spring be X_s

The equation of motion for $t \leq T$ is given by

$kX_s = \gamma V = F$

Thus $X_s = \frac{F}{k}$ and $\frac{dX_d}{dt} = \frac{F}{\gamma}$ with the condition that

At t=0, $X_d = 0$

$$\int_0^{X_d} dX_d = \frac{F}{\gamma}\int_0^t dt$$

$X_d = \frac{F}{\gamma}t$

$X(t) = X_s + X_d = \frac{F}{k} + \frac{F}{\gamma}t$

$x(t) = \frac{F}{k} + \frac{F}{\gamma}t$ for $t \leq T$

At t=T, $X(T) = \frac{F}{k} + \frac{F}{\gamma}T$ τ

For t>T, the equation of motion becomes

$kX_s = \gamma V = 0$

Thus $X_s = 0$ and $\frac{dX_d}{dt} = 0$

With $X_d = \frac{F}{\gamma}$ for t=T

Since $\frac{dX_d}{dt} = 0$, X_d will not change with time for $t \geq T$

$X_d = \frac{F}{\gamma}T$

X(t)=$X_s+X_d=0+\frac{F}{\gamma}T$

or, X(t)=$\frac{F}{\gamma}T$ for t>T

2. Velocity of sedimentation is given by

$$\gamma v = F_{net} = F_g - F_b$$

$$= mg - \rho V g$$

$$= mg - \rho.\frac{4}{3}\pi r^3 g$$

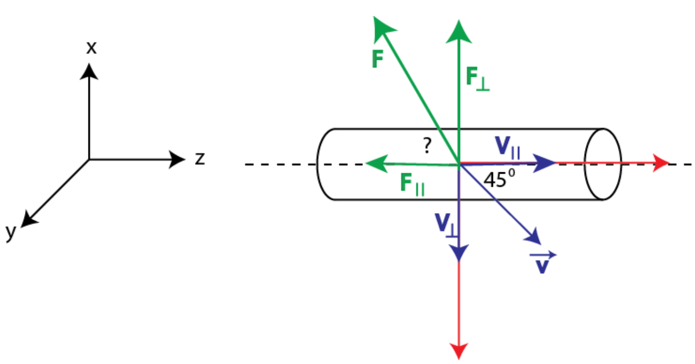
$$=\frac{4}{3}\pi r^3.10\rho.g - \rho.\frac{4}{3}\pi r^3 g$$

$$=3\times 4\pi r^3 \rho g$$

$v=\frac{12\pi\rho g r^3}{6\pi\eta r} = \frac{2\times10^{-18}\times10\times10^{3}}{10^{-3}}=2\times10^{-11}$m/s

Therefore, t=x/v = 31.71 Years

3.



$v_\perp = v_\parallel$ (since angle is 45^0)

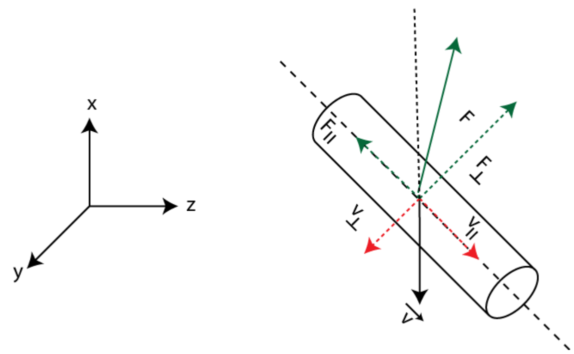
$F^2 = F_\parallel^2 + F_\perp^2 = \gamma_\parallel^2 v_\parallel^2 + \gamma_\perp^2 v_\perp^2 = \gamma_\parallel^2 v_\parallel^2 + \gamma_\parallel^2 v_\perp^2 = 4\gamma_\parallel^2 v_\parallel^2 = 4F_\parallel^2$ since ($\gamma_\perp = \sqrt{3}\gamma_\parallel$)

Therefore, $F = 2F_{\|}$

$$\cos\theta = \frac{F_{\|}}{F}$$

Therefore, $\theta = \cos^{-1}\left(\frac{F_{\|}}{F}\right) = \cos^{-1}\left(\frac{F_{\|}}{2F_{\|}}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^0$

4.



As shown in above figure, if the cylinder is moving with velocity v downward due to cranking of the helix then net drag force F on the cylinder will not be in a direction exactly opposite to v. The direction of the net drag force F will be tilted in the forward direction. This happens since drag force in perpendicular direction is higher drag force in parallel direction when cylinder moves.

One can think of a helix to be consisting of many such cylinders. Components of the drag force of these cylinders in x-y plane will be cancelled for the helix and only z-component survives. Thus, if a bacterial is placed to the right of the helix then it will be propelled due to these non-vanishing z-components of F.