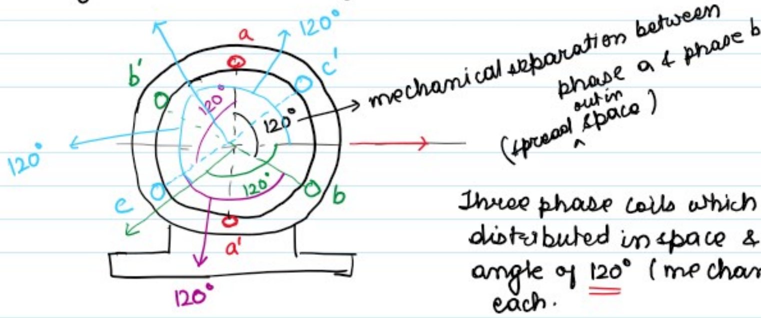
Lecture 32

Friday, July 16, 2021 11:54 AM

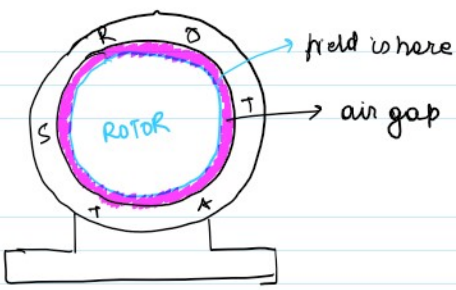
Rotating magnetic field is generated / created

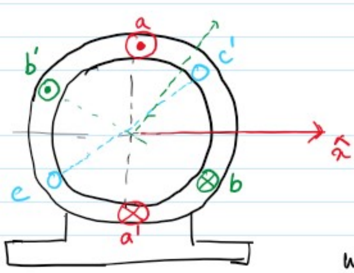


Three phase coils which are distributed in space & make an angle of 120° (mechanical) wrt to each.

These are three phase coils → they are connected to three phase balanced power supply
&
electrically phased shifted by 120° wrt to each other

Three phase windings which are mechanically separated by 120° & electrical excited by supply which has 120° phase shift between phases.





- mechanically displaced by 120°
- magnetic field is along the axis of the coil

$B_a = \qquad \hat{x}$

we are going to inject a current given by

→ $i_a(t) = I_m \cos(\omega t)$ in to coil aa'

$mmf = Ni \qquad H = \frac{Ni}{l}$

$B = \mu H \qquad B = \frac{\mu N i}{l}$

$B_a(t) = k \cdot I_m \cos(\omega t)\,\hat{x}$

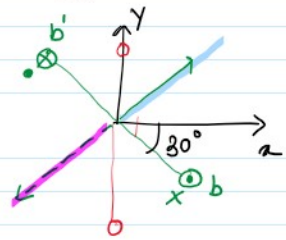
$$\boxed{B_a(t) = B_m \cos(\omega t)\,\hat{x}}$$

current injected in coil bb' $\quad i_b(t) = I_m \cos(\omega t - 120°)$

$\boxed{B_a(t) = B_m \cos(\omega t)\ \hat{x}}$

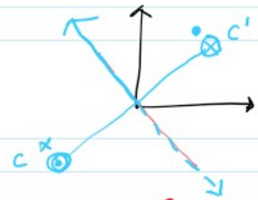
current injected in coil bb' $i_b(t) = I_m \cos(\omega t - 120°)$

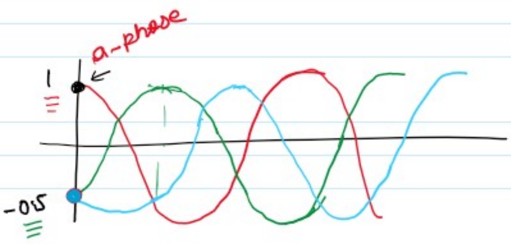
$B_b(t) = B_m \cos(\omega t - 120°)$

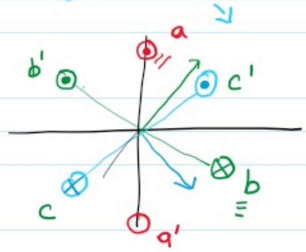


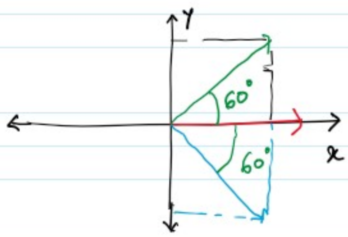
current injected in coil cc' to $i_c(t) = I_m \cos(\omega t + 120°)$

$B_c(t) = B_m \cos(\omega t + 120°)$









$B_{Resultant} = B_a(t) + B_b(t) + B_c(t)$

$\bar{B}_{Res}(t) = B_m \cos \omega t\ \hat{x} + B_m \cos(\omega t - 120°)\left[\cos 60°\ \hat{x} + \sin 60\ \hat{y}\right]$

$+ B_m \cos(\omega t + 120°)\left[\cos(-60°)\hat{x} + \sin(-60°)\hat{y}\right]$

$= B_m \cos \omega t\ \hat{x} + B_m \cos(\omega t - 120°)(0.5\hat{x} + 0.8660°\hat{y})$

$+ B_m \cos(\omega t + 120°)\left[0.5\hat{x} - 0.8660\hat{y}\right]$

$= B_m \cos \omega t\ \hat{x} + B_m \cos(\omega t - 120°)(0.5)\hat{x} + B_m \cos(\omega t + 120°)(0.5)\hat{x}$

$+\left[0.8660\, B_m \cos(\omega t - 120°) - 0.8660\, B_m \cos(\omega t + 120°)\right]\hat{y}$

$= B_m\left[\cos \omega t + 0.5(\cos \omega t \cos 120 + \sin \omega t \sin 120°) + 0.5(\cos \omega t \cos 120° - \sin \omega t \sin 120°)\right]\hat{x}$

$+ B_m\left[0.8660(\cos \omega t \cos 120 + \sin \omega t \sin 120°) - 0.8660(\cos \omega t \cos 120 - \sin \omega t \sin 120°)\right]\hat{y}$

$= B_m\left[\cos \omega t + 0.25 \cos \omega t + \cancel{\frac{0.8660}{2}} \sin \omega t + 0.25 \cos \omega t - \cancel{\frac{0.8660}{2}} \sin \omega t\right]\hat{x}$

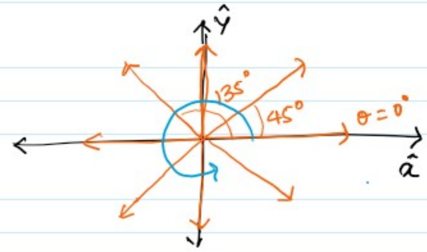
$+ B_m\left[-\cancel{\frac{0.8660}{2}} \cos \omega t + \frac{3}{2} \sin \omega t + \cancel{\frac{0.8660}{2}} \cos \omega t + \frac{3}{2} \sin \omega t\right]\hat{y}$

$= B_m\left[1.5 \cos \omega t\ \hat{x} + 1.5 \sin \omega t\ \hat{y}\right]$

$$\boxed{\bar{B}_{Res}(t) = \underbrace{\tfrac{3}{2}}\, B_m \left[\cos\omega t\, \hat{x} + \sin \omega t\, \hat{y} \right]} = \bar{B_a}(t) + \bar{B_b}(t) + \bar{B_c}(t)$$

Resultant magnetic field changes its direction as time changes. we can say field is rotating

$\theta = \omega t$, $\theta = 0$ to $360°$ $\rightarrow$ t from 0 to T (50Hz 20ms)



$$\bar{B}_{Res}(t) = \tfrac{3}{2} B_m \left[\cos\omega t\, \hat{x} + \sin\omega t\, \hat{y}\right]$$
$$= \tfrac{3}{2} B_m \left[\cos\theta\, \hat{x} + \sin\theta\, \hat{y}\right]$$

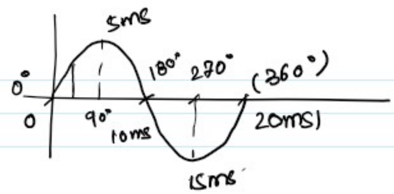
$t = 0$ $\theta = 0$ $\bar{B}_{Res}(0°) = \tfrac{3}{2} B_m\, \hat{x}$

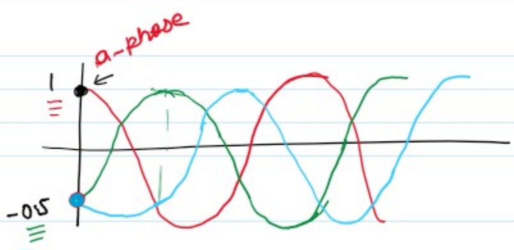
$t = 2.5ms$ $\theta = 45°$ $\bar{B}_{Res}(45°) = \frac{3}{2} B_m \left[\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}\right]$

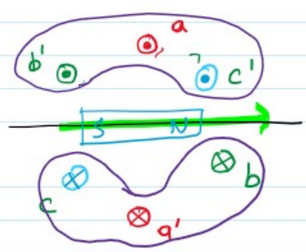
$t = 5ms$ $\theta = 90°$ $\bar{B}_{Res}(90°) = \frac{3}{2} B_m\, \hat{y}$

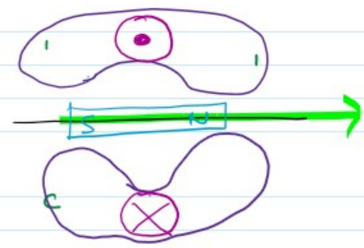
$t = 7.5ms$ $\theta = 135°$ $\bar{B}_{Res}(135°) = \frac{3}{2} B_m \left[-\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}\right]$

$t = 10ms$ $\theta = 180°$ $\bar{B}_{Res}(180°) = \frac{3}{2} B_m\, (-\hat{x})$





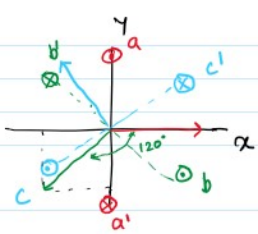




speed of rotation of equivalent bar magnet is equal to the electrical frequency (two pole)

$$\underline{\underline{f_e = f_m}} \quad \text{(two poles)}$$

Two poles / one pole pair
↓ magnetic poles



$\bar{B_a}(t) = B_m \cos(\omega t)\, \hat{x}$
$\bar{B_b}(t) = B_m \cos(\omega t - 120°)\left[-0.5\hat{x} - 0.8660\hat{y}\right]$
$\bar{B_c}(t) = B_m \cos(\omega t + 120°)\left[-0.5\hat{x} + 0.8660\hat{y}\right]$

Unit vector along axis of coil B is
$= \cos(-120°)\, \hat{x} + \sin(-120°)\, \hat{y}$
$= -0.5\hat{x} - 0.8660\hat{y}$

unit vector along axis of coil c is
$= \cos(120°)\, \hat{x} + \sin(120°)\, \hat{y}$
$= -0.5\hat{x} + 0.8660\hat{y}$

$\bar{B}_{Res}(t) = \bar{B_a}(t) + \bar{B_b}(t) + \bar{B_c}(t)$

$$= -0.5\hat{x} + 0.8660\hat{y}$$

$$\bar{B}_{Res}(t) = \bar{B}_a(t) + \bar{B}_b(t) + \bar{B}_c(t)$$

$$= B_m\cos(\omega t)\,\hat{x} + B_m[\cos\omega t\cos(120^\circ) + \sin\omega t\sin 120^\circ][-0.5\hat{x} - 0.8660\hat{y}]$$
$$+ B_m[\cos\omega t\cos 120^\circ - \sin\omega t\sin 120^\circ][-0.5\hat{x} + 0.8660\hat{y}]$$

$$= B_m\Big[\cos\omega t\,\hat{x} - 0.5\cos\omega t\cos 120^\circ\,\hat{x} - \cancel{0.8660\cos\omega t\cos 120^\circ}\,\hat{y}$$
$$- \cancel{0.5\sin\omega t\sin 120^\circ}\,\hat{x} - 0.8660\sin\omega t\sin 120^\circ\,\hat{y}$$
$$- 0.5\cos\omega t\cos 120^\circ\,\hat{x} + 0.8660\,\cancel{\cos\omega t\cos 120^\circ}\,\hat{y}$$
$$+ \cancel{0.5\sin\omega t\sin 120^\circ}\,\hat{x} - 0.8660\sin\omega t\sin 120^\circ\,\hat{y}\Big]$$

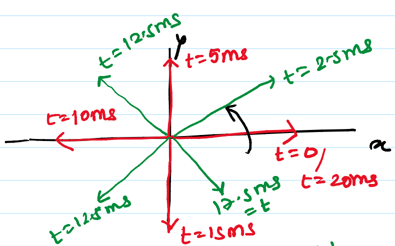
$$= B_m[\cos\omega t\,\hat{x} - 2(0.5\cos\omega t\cos 120^\circ)\hat{x}$$
$$+ 2(0.8660\sin\omega t\sin 120^\circ)\hat{y}]$$

$$= B_m[\cos\omega t\,\hat{x} - 2(0.5)\cos(\omega t)(-0.5)\hat{x}$$
$$+ 2(0.8660)\sin\omega t\,(0.8660)\hat{y}]$$

$$= B_m\left[(\cos\omega t + 0.5\cos\omega t)\hat{x} + 2\cdot\left(\frac{\sqrt{3}}{2}\right)\sin\omega t\cdot\left(\frac{\sqrt{3}}{2}\right)\hat{y}\right]$$

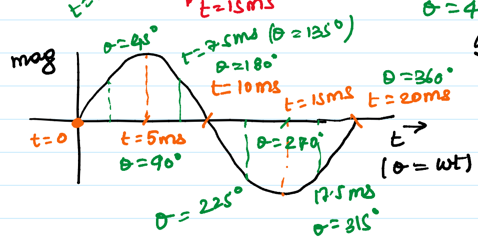
$$= B_m\left[1.5\cos\omega t\,\hat{x} + \frac{3}{2}\sin\omega t\,\hat{y}\right]$$

$$\boxed{\bar{B}_{Res}(t) = 1.5B_m[\cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}]}$$ → Rotating magnetic field



$\theta = \omega t$

	$t = 0$ sec	$\bar{B}_{Res}(0) = 1.5B_m[\hat{x}]$
$(\theta = 90^\circ)$	$t = 5$ms	$\bar{B}_{Res}(5ms) = 1.5B_m[\hat{y}]$
$\theta = 180^\circ$	$t = 10$ms	$\bar{B}_{Res}(10ms) = 1.5B_m[-\hat{x}]$
$\theta = 270^\circ$	$t = 15$ms	$\bar{B}_{Res}(15ms) = 1.5B_m[-\hat{y}]$
$\theta = 360^\circ$	$t = 20$ms	$\bar{B}_{Res}(20ms) = 1.5B_m[\hat{x}]$
$\theta = 45^\circ$	$t = 2.5$ms	$\bar{B}_{Res}(2.5ms) = 1.5B_m\left[\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}\right]$



50Hz → 20ms time period

$$2\pi f t = \frac{\pi}{4}$$

$$f t = \frac{1}{8}$$

$$t = \frac{1}{8\times 50} = \frac{20ms}{8} = 2.5ms$$