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MODEL UPDATING IN STRUCTURAL DYNAMICS: A SURVEY

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It is well known that finite element predictions are often called into question when they are in conflict with test results. The area known as *model updating* is concerned with the correction of finite element models by processing records of dynamic response from test structures. Model updating is a rapidly developing technology, and it is intended that this paper will provide an accurate review of the state of the art at the time of going to press. It is the authors' hope that this work will prove to be of value, especially to those who are getting acquainted with the research base and aim to participate in the application of model updating in industry, where a pressing need exists.

1. INTRODUCTION

In the modern analysis of systems in engineering and science, a great deal of effort has been invested in the development of sophisticated computer models. The main purpose of such models is to predict both the response of the system to disturbances and the design advantage which might be obtained from modifications in the configuration of the system. Digital computers have been applied particularly in the generation of numerical predictions from discrete models. However, when predictions are compared with experimental results it is often discovered that the degree of correlation is not good enough to allow the application of the model with confidence.

To explain the lack of correlation between predictions and observations it is necessary to consider the likely causes of inaccuracy in numerical models. It should be mentioned that experimental measurements are not taken without error, but we will return to that issue later. Here we consider three commonly encountered forms of model error which may give rise to inaccuracy in the model predictions: (i) *model structure errors*, which are liable to occur when there is uncertainty concerning the governing physical equations—such errors might occur typically in the modelling of neurophysiological processes and strongly non-linear behaviour in certain engineering systems; (ii) *model parameter errors*, which would typically include the application of inappropriate boundary conditions and inaccurate assumptions used in order to simplify the model; (iii) *model order errors*, which arise in the discretization of complex systems and can result in a model of insufficient order—the model order may be considered to be a part of the model structure.

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When the problems of inaccuracy in analytical models are considered, it is perhaps not surprising that researchers have turned their attention to the development of modelling methods based on experimental observation. This area, known as *system identification*, has been particularly vibrant in the control engineering community over the past 30 years. The model to be identified may be a parametric or a non-parametric model and, in addition, it may be non-linear. The celebrated paper by Astrom and Eykhoff [1] is essential reading for beginners. The more accomplished reader who wishes to develop his background knowledge of system identification in the control/systems area should refer to the papers of Strejc [2], Rake [3], and Wellstead [4]. Schetzen [5] has provided the details of the Volterra and Wiener series which have been widely applied in the non-parametric identification of non-linear systems.

The problem of identifying a parametric model involves first determining the most appropriate specific mathematical structure. When the model structure and model order have been decided upon, the problem of system identification reduces to one of *parameter estimation*. The most universal class of estimation methods is that which has its basis in the least squares approach developed by Karl Friedrich Gauss at the turn of the nineteenth century. Since that time the original least squares formulation has been developed to a high degree of sophistication, so as to account for non-linearities (Gauss–Newton), varying uncertainty, bias (instrumental variable) and cross-correlation between responses (minimum variance). More recently, the filtering estimators of Wiener, Kolmogorov and Kalman have been developed. References [1, 6–9] may be consulted for further details on model parameterization by using least squares estimators. The maximum likelihood method [10] is based upon the maximization of the probability density function of the observations conditioned upon the unknown parameters. This latter approach has been widely applied in the estimation of parameters in stochastic process models of the ARMA (Auto Regressive Moving Average) type [11]. In such models the output is regressed on its own past values and mixed with a moving average of the uncorrelated, zero-mean, additive noise. The order of ARMA type models may be determined by using the Akaike information criterion [12], the residual test [13] and/or the pole-zero cancellation test [14]. The excellent modern texts by Soderstrom and Stoica [15] and Ljung [16] provide details of system identification methods in control engineering.

In control engineering, the purpose of system identification (and parameter estimation) is usually the "on line" construction of models which may be applied recursively in model-reference control schemes. In structural dynamics, experimental modal analysis may be considered as a special area of system identification for the determination of modal data (natural frequencies, mode shapes, generalized masses and loss factors) from vibration tests. The underlying principles of experimental modal analysis were established in the seminal paper by Kennedy and Pancu [17], which seems to have been the first paper on experimental modal analysis (phase resonance) in English. At a natural frequency the phase resonance criterion is fulfilled when the real part of the response vector vanishes such that the imaginary part can be regarded as the eigenvector. In a physical test this generally requires a complicated arrangement of exciters at many degrees of freedom. Stahle and Forlifer [18] and Stahle [19] removed the complexity from the experimental arrangement by introducing the phase separation approach, whereby the normal modes were separated from the measured quadrature responses having contributions from the many modes stimulated simultaneously by the excitation. It was demonstrated that the measured quadrature responses were related to the normal mode responses through a matrix containing only the modal damping coefficients and ratios of the system natural frequencies.

With the rapid development of digital computers since the 1960s, experimental modal analysis became (and remains) an extremely active research topic, the details of which have been reported by Ewins [20]. In modern structural design, the economic value of model-based techniques is well proven, and finite elements have become established as the universally accepted analysis method. The validation of finite element models is usually performed by comparing numerical eigendata with natural frequencies and mode shapes acquired from modal tests. It has been mentioned previously that significant discrepancies are often found to exist when numerical predictions are compared with experimental results. The finite element model structure is a discrete arrangement of mass, stiffness and damping terms, and the order of the model is generally well defined and somewhat greater than the number of modes in the frequency range of interest. Thus, in *model updating* we seek to correct the inaccurate parameters in the model so that the agreement between predictions and test results is improved.

In contrast with system identification in control engineering, model updating in structural dynamics is usually performed "off line" by using batch processing techniques. The aim is to generate improved numerical models which may be applied in order to obtain predictions for alternative loading arrangements and modified structural configurations. This aim places a demand upon model updating techniques which does not occur in control system identification. The demand is that the mass, stiffness and damping parameters in the updated model should be physically meaningful.

Before considering in detail the various methods available for model updating, it is worthwhile to turn our attention briefly to the subject of *direct* system identification. In doing so, and in the remainder of this paper, we restrict our attention to the identification of linear models in structural elastodynamics. It appears that the term "direct" was first used by Natke [21] to describe the identification of a system without the updating of a reference model. Natke referred to model updating as "indirect" system identification. Gersch and his colleagues [22, 23] were the first to consider the construction of ARMA time series models by using vibration measurements. Gawronski and Natke [24, 25] considered the classification of ARMA models for vibrating systems, and estimated the order of AR (Auto Regressive) and ARMA models by using system balancing theory. The ARMAX model (Auto Regressive Moving Average with eXogenous variable) includes the input, which is known in the econometric literature as the exogenous variable. Fassois and his co-workers [26–28] have used a sub-optimal maximum likelihood approach for the estimation of a multi-variable ARMAX model in modal identification. Zamirowski [29] has defined the relationships between the physical mass, stiffness and damping parameters and the coefficients in an ARMAX model.

Spatial parameters may be estimated directly from measured modal data when a square, non-singular modal matrix is available. However, it usually happens that the number of measurement stations is greater than the number of modes in the frequency range of interest, and then the modal matrix will be rectangular. Thoren [30] limited the number of degrees of freedom of the model to be equal to the number of modes determined in the test, and obtained useful results with a seven-degree-of-freedom model of the Saturn VS-II LOX bulkhead engine support system. Ross [31] added extra arbitrary, linearly independent vectors to the modal matrix in order to make it invertible. Luk [32] applied pseudo-inversion to the rectangular modal matrix.

A number of techniques exist for the direct identification of spatial parameters from frequency response function (FRF) data. Shye and Richardson [33] extended a curve fitting method and demonstrated that a diagonal mass matrix and a symmetric stiffness matrix can be recovered when the total mass is known. A more common approach is that which results in an over-determined least squares problem. Link and Vollan [34] have proposed a pseudo-inverse method by which the stiffness and damping arrays may be estimated when the mass matrix is known. Leuridan *et al.* [35] made use of a Householder transformation, which has the effect of retaining the conditioning of the original over-determined system of equations. It should be explained here that the usual least squares process requires the inversion of the positive definite Hessian matrix, and by that approach the condition number is given by the square of the condition number of the original equations. Minas and Inman [36] developed a pseudo-inverse method for the identification of a non-proportional damping matrix from incomplete modal data and known mass and stiffness matrices.

1.1. PREVIOUS REVIEW ARTICLES ON SYSTEM IDENTIFICATION AND UPDATING

Survey papers on system identification and model updating, which were specifically intended for a readership of structural dynamicists, began to appear in the early 1970s. Pilkey and Cohen [37] edited a collection of papers presented at the 1972 ASME Winter Annual Meeting. Early survey papers include those of Ibanez [38, 39], Gersch [40] and Hart and Yao [41]. The treatment given in the early papers remains surprisingly up to date; for instance, Ibanez [38] gave a detailed account of pseudo-inverse techniques, including singular value decomposition (SVD), and discussed the modification of *a priori* models. Gersch [40] catalogued identification methods including equation error, output error, maximum likelihood, Bayesian estimation, etc. Hart and Yao [41] presented technology trees for time domain and frequency domain system parameter estimation.

More recently, Natke and his colleagues have presented a series of review papers [42–48] dealing with various specialized aspects of identification and updating. The use of an *a priori* model within an extended, weighted least squares (EWLS) approach was considered in reference [44]. Kozin and Natke [42] and Natke [43] provided wide-ranging surveys of papers in the areas of time series modelling, modal identification and Bayesian estimation of structural parameters. A more recent survey covering similar topics has been given in reference [47]. Natke and Yao [45] and Natke [46] considered the identification of faults and damage detection. Natke [48] reviewed the error localization and regularization techniques for model updating.

Other recent review papers include those of Ibrahim [49], Imregun and Visser [50], and Inman and Minas [51]. Ibrahim [49] discussed model reduction and eigenvector expansion in the context of modal and FRF sensitivity techniques and Lagrange multiplier approaches. Imregun and Visser [50] discussed error matrix methods and the Lin and Ewins FRF method which is based on a matrix identity [52]. Inman and Minas [51] considered updating techniques based upon eigenstructure assignment.

Several papers deal with the experience of applying updating techniques, and valuable comparisons are made. Link [53] and Caesar [54] compared sensitivity, Lagrange multiplier and matrix mixing techniques. Natke [55] considered specific applications in aerospace and structural engineering. Brughmans and his co-workers [56] considered the application of sensitivity techniques to improve the dynamic model of a Boeing DeHavilland DASH 8-300A aircraft by using ground vibration test data. Link and Zhang [57] compared methods with measured eigendata and FRF data from simulated and physical experiments. Allen and Martinez [58] discussed the implementation and application of sensitivity approaches. A thorough study of model updating procedures has recently been completed at the universities of Besançon and Kassel, and at Dornier GmbH [59].

2. THE PHILOSOPHY OF MODEL UPDATING

In the construction of finite element models it is usual to make simplifying assumptions. Often there are detailed features which would require an unduly fine mesh for an accurate geometric representation of the structure. Boundary conditions and connections between components (such as bolted joints, welds, press fits, etc.) are seldom understood with certainty. In such cases the analyst may test the sensitivity of the numerical results to changes in the mesh configuration or boundary constraints, but ultimately he settles for a model which, according to his engineering judgement, will be likely to provide acceptable results. Ewins and Imregun [60] have presented an interesting comparison of finite element results produced by 12 analysts working independently with six commercial finite element packages. The results from the independent analysts are found to differ significantly from each other and from the experimental modal data. The purpose of model updating is to modify the mass, stiffness and damping parameters of the numerical model in order to obtain better agreement between numerical results and test data. If the updated model is to be used predictively, for untested loading conditions or modified structural configurations, then it is important that the improved agreement in results is achieved by correcting the inaccurate modelling assumptions and not by making other (physically meaningless) alterations to the model. This is a tall order.

The reader should perhaps be reminded that errors are associated not only with numerical models but also with experimental testing. It is well known that the dynamics of structures may be affected by the masses and stiffnesses of equipment used to excite or measure the vibrations. Errors in natural frequencies due to the mass of a roving accelerometer are especially common in modal analysis. Piezoelectric transducers tend to lack linearity at low frequencies and may be sensitive to temperature, and to magnetic and acoustic fields. Electronic systems generally introduce low levels of instrument noise, and problems are often encountered at mains frequency where small currents flow between earth lines. Transverse motion and base bending of accelerometers, and accelerometer cable noise, are common sources of error in test data. In the processing of vibration measurements, especially with multi-frequency excitation and FFT techniques, errors can arise as a result of aliasing, spectral leakage and linearization of non-linear effects. The practical difficulties of experimental testing generally result in imprecise measurement data. Further processing, such as modal analysis, introduces additional errors, especially with popular curve fitting methods which require manual intervention by the user.

There can be no doubt that in structural system identification the direction of research was strongly influenced towards model updating by the early contributions of Berman. Berman [61–63] pointed to problems associated with imprecision and incompleteness in the measured data and inaccuracy in the numerical model, and in particular to the following.

(i) Stiffness matrices are dominated by terms associated with the higher modes of the numerical matrix system. Since the matrix of influence coefficients (which may be determined experimentally) is dominated by the lower modes, its inverse is unlikely to provide a physically realistic stiffness model.

(ii) The parameters of the numerical model are directly related to the geometry and material properties of the structure and are therefore physically meaningful. However, the effect of discretization is to cause the measured eigenfrequencies to be over-estimated in the numerical model. From considerations of "exact", dynamic reduction it can be demonstrated that any discrete model may be thought of as a linearization (i.e., a Guyan, statically reduced model) with a limited frequency range. An infinite number of discrete models is capable of accurately predicting the test results in a limited range of frequencies.

Berman concluded that is is impossible to identify a physically meaningful model by using a direct approach. He advocated the use of model updating.

It is well known that finite elements are based upon the material properties (Young's modulus, Poisson ratio, mass density, etc.) and the physical dimensions of the system under test. The shape function of the chosen elements determines the distribution of the mass and stiffness properties, so that the terms (in the mass and stiffness matrices) can be understood physically. However, alternative elements are available with different shape functions and, for that reason, it must be concluded the finite element models are meaningful but non-unique.

Mottershead and his colleagues [64, 65] considered the problem of shape function discretization. When models are produced with increasing mesh refinement the eigenvalues tend to converge (from above with displacement-type elements). The lower eigenvalues converge most readily and as the number of elements is increased new eigenvalues appear which migrate towards a limit with further mesh refinement. For model updating it is important that the eigenvalues are fully converged in the frequency range of interest. Otherwise, it can be expected that errors will be induced in the updated terms as a result of shape function discretization. It is a commonly applied rule of thumb that the first third of the eigenvalues of a finite element model should be accurate enough for design purposes. However, it is generally the case that the number of fully converged eigenvalues is considerably fewer than the first third. Consequently, a larger model (i.e., containing more elements and degrees of freedom) is required for updating than for design. Mottershead *et al.* [65] applied an iterative approach whereby a matrix of sensitivity ratios is used to adjust the measured data. The adjusted measurements are subsequently processed to provide an updated design model which is unaffected by the finite element discretization.

Berman's result concerning the difficulty of inverting a flexibility matrix to obtain stiffness terms is effectively illustrated in two recent contributions on the identification of mechanical joint parameters. Wang and Liou [66] and Hong and Lee [67] expressed the joint stiffness and damping parameters by the difference between two dynamic stiffness matrices. The expression thus defined is premultiplied and postmultiplied by measured FRF matrices, so that the dynamic stiffness matrices disappear. The resulting equation can be rearranged to obtain a least squares estimate of the joint parameters, and the need to invert the measured FRF data is avoided. Small amounts of measurement noise on FRF measurements can lead to substantial errors in a dynamic stiffness matrix obtained by inversion. The problem of inverting a matrix with terms containing measurement errors leads us to the general area of conditioning and uniqueness in parameter estimation and updating.

Test measurements which contain fewer modes than the order of the identified model are said to be incomplete. Incompleteness usually arises as a result of the exciter or transducer location or because of the limited frequency range involved in experimental tests. The problems introduced by incompleteness can become significant in large structures where it is expensive to take measurements at a large number of locations and to process large volumes of data. It is well known that incompleteness can give rise to problems of ill-conditioning and non-uniqueness in system identification. The incompleteness problem can be overcome by taking the missing information from an *a priori* model instead of from the experimental records, and it is this strategy which distinguishes model updating from direct system identification.

A problem unrelated to that described above, but also referred to by some workers as "incompleteness", occurs when there is a need to compare measured and analytical vibration mode shapes. Analytical vectors generally contain more points than are available from measurements. In that case, either the measured eigenvector should be expanded or the analytical model should be reduced. In this paper the authors reserve the word "incomplete" to mean a deficit of modal information. The techniques for eigenvector expansion and model reduction are dealt with in later sections of this paper.

Studies which address the problem of incompleteness include those of Natke [44], who used an EWLS approach, and Mottershead [68], who used SVD. Mottershead and Foster [69] combined the EWLS and SVD techniques in a method which is closely related to the ϵ-decomposition proposed by Ojalvo and his co-workers [70, 71]. Zhang and Ojalvo [72] improved the ϵ-decomposition method by taking a second order approximation to the SVD instead of a first order approximation, as in the original formulation of the ϵ-decomposition method. Rothwell and Drachman [73] provided a unified Tikhonov regularization and SVD approach which has the effect of retaining the condition number of the original equations (i.e. $CN(\mathbf{A})$ rather than $CN(\mathbf{A}^T\mathbf{A}) = CN^2(\mathbf{A})$: $\mathbf{Ax} = \mathbf{b}$). Natke [74, 75] considered the adjacent solutions brought about by truncation in regularization methods.

Regularization has become a central issue in system identification, because the dynamical behaviour is observed in a narrow knowledge space and, consequently, the systems of equations are strongly under-determined. Lallement and Cogan [76] enlarged the knowledge space by using anti-resonance data. Wada, Kuo and Glaser [77] used multiple tests with varying boundary conditions and Nalitolela, Penny and Friswell [78] used added masses. Ben-Haim and Prells [79, 80] used adaptive excitation to enable parameter updating in subsets. Zhang and Natke [81] applied component-mode synthesis to achieve a two-level updating procedure, which has the effect of reducing the number of parameters to be estimated at each level.

The problems of processing noisy, incomplete measurements have stimulated research efforts into locating the most inaccurate model parameters prior to updating. This area of work has become known as *error localization*. The methods applied in error localization are often closely related to those used in updating, and may similarly be affected by incompleteness. The objective of localization is to determine those degrees of freedom in the model which are associated with significant parameterization error. When that has been done, updating can be performed economically. Zhang and Lallement [82] extended the method of Sidhu and Ewins [83], which is based upon a truncated binomial expansion of the flexibility and inverse mass matrices. The unmeasured part of the structural eigenvectors is recovered by using an updated estimate of the homogeneous part of the motion equation. The complete eigenvectors, thus estimated, are used in a spectral decomposition which allows a further updated estimate of the homogeneous equation to be formed. The process continues iteratively until the eigenvector estimates are sufficiently converged. The mass and stiffness terms in the updated homogeneous equation are used only for localization purposes, since they lack the connectivity of the original finite element model. Lallement and Piranda [84] considered three localization methods. In addition to that described immediately above, Lallement and Piranda considered a method based on balancing the structural eigenvalue equation, and a further method based on eigendata sensitivities. In the former it is again necessary to recover the unmeasured part of the structural eigenvectors. The latter requires that the numerical and experimental modes be paired and is said to be the most reliable method. Gysin [85] investigated the application of eigenvector expansion methods in localization. Link and Santiago [86] used a sensitivity based localization approach, which involves the testing of each column of the sensitivity matrix in turn, to locate those parameters which have the greatest influence on the equation error. In the same paper, Link and Santiago introduced localization indicator functions based upon energy concepts. Fissette and Ibrahim [87] used a force balance method for error localization. Shepard and Milani [88] presented a frequency based localization procedure based upon the Rayleigh quotient. To, Lin and Ewins [89] used an eigendynamic constraint method which is based on the structural eigenvalue equation and the mass orthogonality relationship for prescribed modes. The physical connectivity of the analytical model can usually be preserved. Lieven and Ewins [90] created pseudo-flexibility and pseudo-intertance matrices with paired analytical and experimental modes, and used the SVD approach to produce pseudo-inverse matrices of incomplete stiffness and mass. The mass and stiffness errors are localized from the difference between the experimental and analytical pseudo-inverses. Natke and Cempel [91] investigated localization techniques for fault detection. They concluded that short wavelength vibrations are required to detect faults.

3. LAGRANGE MULTIPLIER AND OTHER REPRESENTATION MODEL TECHNIQUES

It was said in the opening paragraph of the previous section that the purpose of model updating is to provide physical meaning to a deficient model from test measurements. An alternative strategy has been to update a numerical model such that it exactly reproduces an incomplete set of measured eigendata. Approaches of this type tend to transform finite element "knowledge" models into so-called "representation" models [82]. In this section, four representation model approaches are considered: (i) reference basis methods, (ii) matrix mixing, (iii) eigenstructure assignment and (iv) inverse eigenvalue methods.

The reference basis methods were introduced by Baruch [92–95] and Berman [96–98] in the late 1970s and early 1980s. According to these methods, the reference basis, which must be one parameter set taken from either the masses, stiffnesses or measured modes, is considered to be inviolate. The two remaining parameter sets are then updated separately by minimizing an objective function, with constraints imposed through Lagrange multipliers. It should be recalled that, in constrained minimization, Lagrange multiplier methods result in the strict imposition of the constraints, whereas penalty methods allow flexibility to a degree which is controlled by the penalty number.

Baruch and Bar Itzhack [92] and Baruch [93] obtained updated eigenvector data by minimizing the mass-weighted norm of distance between the analytical and sought vector. In reference [93] an updated stiffness array, $\mathbf{K}$, is determined by minimizing the norm, $\frac{1}{2}\|\mathbf{M}_A^{-1/2}(\mathbf{K}-\mathbf{K}_A)\mathbf{M}_A^{-1/2}\|$, where $\mathbf{M}_A$ and $\mathbf{K}_A$ represent the analytical mass and stiffness arrays. Lagrange multipliers are used to enforce satisfaction of the equations of motion and stiffness symmetry. Berman [96] questioned the assumption of an exact mass matrix, especially with regard to the application of "static" shape functions in finite element analysis. Three papers appeared subsequently (those of Berman [97], Baruch [94] and Berman and Nagy [98]) in which measured modes were used as the reference basis. The mass and stiffness arrays were manipulated in such a way that the updated model was forced to replicate the test results. The resulting model was representational.

The model reference basis approaches have been the focus of considerable attention in structural dynamics research. Berman and Nagy [98] minimized the objective function

$$J_M = \|\mathbf{M}_A^{-1/2}(\mathbf{M}-\mathbf{M}_A)\mathbf{M}_A^{-1/2}\| + \sum_{i=1}^{m}\sum_{j=1}^{m}\lambda_{ij}(\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}-\mathbf{I})_{ij}, \tag{3.1}$$

where λ_{ij} is a Lagrange multiplier used to enforce orthogonality of the vectors with respect to the updated masses. The minimization procedure results in the expression for the updated mass

$$\mathbf{M} = \mathbf{M}_A + \mathbf{M}_A\mathbf{\Phi}\mathbf{m}_A^{-1}(\mathbf{I}-\mathbf{m}_A)\mathbf{m}_A^{-1}\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}_A, \tag{3.2}$$

where

$$\mathbf{m}_A = \mathbf{\Phi}^{\mathrm{T}}\mathbf{M}_A\mathbf{\Phi}, \tag{3.3}$$

$\mathbf{\Phi}_{p\times m}$ is an incomplete modal matrix ($m < p$), and p is the order of the analytical and updated models. Following the computation of $\mathbf{M}$ from equation (3.2), an updated stiffness array can be determined by minimizing a further objective function,

$$J_K = \|\mathbf{M}^{-1/2}(\mathbf{K}-\mathbf{K}_A)\mathbf{M}^{-1/2}\| + \sum_{i=1}^{p}\sum_{j=1}^{m}\lambda_{Kij}(\mathbf{K\Phi}-\mathbf{M\Phi\Lambda})_{ij}$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{m}\lambda_{Oij}(\mathbf{\Phi}^{\mathrm{T}}\mathbf{K\Phi}-\mathbf{\Lambda})_{ij}+\sum_{i=1}^{p}\sum_{j=1}^{i}\lambda_{Sij}(\mathbf{K}-\mathbf{K}^{\mathrm{T}})_{ij}, \tag{3.4}$$

where $\mathbf{\Lambda}$ represents the spectral matrix. Here the Lagrange multipliers are used to enforce the equations of motion, orthogonality and stiffness symmetry. The stiffness updating equations can be written as

$$\mathbf{K} = \mathbf{K}_A + (\mathbf{\Delta}+\mathbf{\Delta}^{\mathrm{T}}), \tag{3.5}$$

where

$$\mathbf{\Delta} = \tfrac{1}{2}\mathbf{M\Phi}(\mathbf{\Phi}^{\mathrm{T}}\mathbf{K}_A\mathbf{\Phi}+\mathbf{\Lambda})\mathbf{\Phi}^{\mathrm{T}}\mathbf{M} - \mathbf{K}_A\mathbf{\Phi\Phi}^{\mathrm{T}}\mathbf{M}. \tag{3.6}$$

It should be noted that although $\mathbf{M}^{-1/2}$ appears in the objective functions (3.1) and (3.4), its computation is unnecessary since it is absent from the updating equations (3.2) and (3.5).

Regarding the performance of the algorithm, it has already been stated that the incomplete, measured eigendata are reproduced exactly. However, there is nothing to prevent the updated model from generating spurious modes in the frequency range of interest. In addition, the finite element mass and stiffness matrices may suffer a loss of positive-definiteness in the updating process.

Caesar [99] expanded the modal reference basis technique by introducing additional Lagrange multiplier constraints to account for rigid-body mass information, such as centre of gravity, total mass and moment of inertia. Baruch [95] developed a stiffness reference basis approach. Baruch and his co-workers [100] further investigated the application of reference basis methods in conjunction with mass conservation. Wei [101, 102] developed a modal reference basis method to allow the simultaneous updating of the mass and stiffness arrays. The interaction of mass and stiffness terms were considered by Wei [103]. Fuh and his colleagues [104] developed a reference basis method for representational updating of structural systems with non-proportional damping.

The matrix mixing approach [54, 105] is a development of the methods of Thoren [30] and Ross [31] (discussed in section 1). The problem with assembling mass and stiffness matrices from test data is that the number, m, of measured eigendata is usually significantly smaller than the order, p, of the required model. Structural matrices assembled on the basis of $m < p$ eigenmodes are incomplete. The matrix mixing approach uses finite element modes where test data are unavailable. Thus,

$$\mathbf{M}^{-1} = \sum_{i=1}^{m}\boldsymbol{\phi}_{Ti}\boldsymbol{\phi}_{Ti}^{\mathrm{T}} + \sum_{i=m+1}^{p}\boldsymbol{\phi}_{Ai}\boldsymbol{\phi}_{Ai}^{\mathrm{T}} \quad \text{and} \quad \mathbf{K}^{-1} = \sum_{i=1}^{m}\frac{\boldsymbol{\phi}_{Ti}\boldsymbol{\phi}_{Ti}^{\mathrm{T}}}{\omega_{Ti}^2} + \sum_{i=m+1}^{p}\frac{\boldsymbol{\phi}_{Ai}\boldsymbol{\phi}_{Ai}^{\mathrm{T}}}{\omega_{Ai}^2}, \tag{3.7, 3.8}$$

where the subscripts A and T denote analytical and test data, respectively. The matrix mixing method generally returns fully populated mass and stiffness matrices which bear little relation to physical connectivity. The mode shape vectors from analysis and test, $\boldsymbol{\phi}_{Ai}$ and $\boldsymbol{\phi}_{Ti}$, both must be of dimension p.

To *et al.* [106] and Neidbal *et al.* [107] updated the analytical mass and stiffness matrices by enforcing orthogonality with respect to the measured modal vectors. The method can be extended to include the eigendynamic system equation, together with the orthogonality relations, and this has the advantage of preserving the physical connectivity of the updated model.

The eigenstructure assignment and pole placement techniques for model updating were pioneered by Minas and Inman [108–111] (see also the review article by Inman and Minas [51]). The procedure of eigenstructure assignment was developed by Srinathkumar [112] and adapted to mechanical structures by Andry *et al.* [113]. The pole placement method is well known in modern state space control; see, for instance, the book by Porter and Crossley [114].

In the eigenstructure assignment approach, state feedback is used to describe the right side of the dynamic equation of motion in terms of the displacement and velocity states. The problem then reduces to one of determining the terms in the feedback gain matrix such that the eigenvalues and eigenvectors of the closed loop system are identical to the measured eigendata. The result of this procedure is that modifications are made to the stiffness and damping terms but the analytical mass matrix remains unchanged. The updated stiffness and damping arrays are given by

$$\mathbf{K} = \mathbf{K}_A + \mathbf{B}_0\mathbf{G}\mathbf{C}_0, \qquad \mathbf{C} = \mathbf{C}_A + \mathbf{B}_0\mathbf{G}\mathbf{C}_1, \tag{3.9, 3.10}$$

where $\mathbf{B}_0$ is an input distribution matrix, $\mathbf{C}_0$ and $\mathbf{C}_1$ are the matrices relating the outputs and states, and $\mathbf{G}$ is the feedback gain matrix. The matrices $\mathbf{B}_0$, $\mathbf{C}_0$ and $\mathbf{C}_1$ are derived from the excitation positions and the location and type of measurement. The matrix $\mathbf{B}_0$ may be chosen arbitrarily, and $\mathbf{C}_0$ and $\mathbf{C}_1$ must be chosen such that $\mathbf{C}_1\boldsymbol{\Phi}\boldsymbol{\Lambda} + \mathbf{C}_0\boldsymbol{\Phi}$ is non-singular. The terms in the matrix $\mathbf{G}$ are determined by the eigenstructure assignment method. The matrices $\boldsymbol{\Phi}$ and $\boldsymbol{\Lambda}$ contain the incomplete measured eigenvectors and eigenvalues. In general, the correction matrices $\mathbf{B}_0\mathbf{G}\mathbf{C}_0$ and $\mathbf{B}_0\mathbf{G}\mathbf{C}_1$ will not be symmetric, so that further correction is necessary when no gyroscopic or circulatory forces are present. This may be accomplished by iterating on the choice of $\mathbf{C}_0$ and $\mathbf{C}_1$ until symmetric correction matrices are obtained. Zimmerman and Widengren [115, 116] used eigenstructure assignment combined with a generalized algebraic Riccati equation to calculate symmetric corrections to the stiffness and damping matrices directly. The equivalence relations between various eigenstructure assignment methods were considered by Zimmerman and Widengren [116].

The eigenstructure assignment method requires the measurement of only $m < p$ eigenvector terms. This is a considerable virtue, since rotational and internal degrees of freedom, which are present in finite element models, are largely unmeasurable in experimental modal analysis. The unmeasured mode shape terms are recovered by using the finite element model which, for that purpose, must be a good representation of the structure under test. The issue of eigenvector expansion is central to all of the representation model techniques. Kidder [117] used an inverse Guyan technique. He and Ewins [118] considered this further and extended the method to deal with complex measured modes. O'Callahan and his colleagues [119] assumed that the physical mode shape vectors are given by linear combinations of the finite element eigenvectors. Using a similar assumption, Lipkins and Vandeurzen [120] applied a smoothing approach and Smith and Beattie [121] used an orthogonal procrustes expansion. O'Callahan and Avitabile [122] proposed a system equivalent reduction expansion process (SEREP). Brown [123] and Williams and Green [124] used spline fitting techniques. Lieven and Ewins [125] used analytical modes in conjunction with the MAC (Modal Assurance Criterion) matrix. Gysin [85] offered a performance comparison of several of the above expansion techniques.

The question of structural connectivity, which was mentioned earlier in this section, was discussed in general terms by Berman [126]. Specifically, Kabe [127] proposed the minimization of the objective function

$$\epsilon = \|\hat{\mathbf{I}} - \hat{\mathbf{I}} \odot \mathbf{\Gamma}\| \tag{3.11}$$

where the i, jth term of $\hat{\mathbf{I}}$ is given by

$$\hat{\mathbf{I}}_{ij} = \begin{Bmatrix} 1 & \text{if } \mathbf{K}_{Aij} \neq 0 \\ 0 & \text{if } \mathbf{K}_{Aij} = 0 \end{Bmatrix}. \tag{3.12}$$

The matrix $\mathbf{\Gamma}$ is to be determined such that

$$\mathbf{K} = \mathbf{K}_A \odot \mathbf{\Gamma}, \tag{3.13}$$

where $\odot$ is the element by element (scalar) matrix multiplication operator. The minimization of equation (3.11) is subject to Lagrange multiplier constraints such that $\mathbf{\Gamma}$ is symmetric, and the dynamic equation of motion (with known $\mathbf{M}$ and incomplete measured modes) is satisfied exactly. The problem then reduces to the solution of an equation involving the undetermined Lagrange multipliers. To find the Lagrange multipliers is computationally expensive, involving the calculation of the eigenvalues of a $pm \times pm$ matrix. Kammer [128] used a projector matrix method which was computationally efficient and turns out to be equivalent to Kabe's method in most cases. Smith and Beattie [129] considered quasi-Newton methods for stiffness updating which preserve the structural connectivity. Gordis *et al.* [130] used matrix mapping techniques to investigate the connectivity of substructures in frequency domain structural synthesis.

Inverse eigenvalue techniques have been described in the tutorial text by Gladwell [131]. Lancaster and Maroulas [132] solved the inverse eigenvalue problem for a second order system when the complete spectral data is given. Starek and Inman [133] applied similar techniques to a model correction problem involving rigid body modes, and the same authors [134] developed an inverse approach for generating symmetric coefficient matrices. $\mathbf{M}^{-1}\mathbf{K}$ and $\mathbf{M}^{-1}\mathbf{C}$ are expanded in terms of the Jordan form and the two Jordan chain matrices containing the right and left eigenvectors. The measured and analytical data is mixed to obtain the updated system. Bucher and Braun [135] presented an analysis of the inverse problem whereby mass and stiffness modifications are found by an eigendata assignment technique. The left modal vectors are extracted from test data which takes the form of incomplete (truncated) FRFs. Rather than updating a finite element model, the purpose is to determine structural modifications which can be implemented on a physical system in order to assign particular eigenmodes and natural frequencies.

4. PENALTY AND OTHER METHODS FOR USE ON MODAL DATA

The object of methods based on a penalty function is to maximize the correlation between the measured and analytical modal model. These methods allow one a wide choice of parameters to update, but the requirement to optimize a non-linear penalty function implies an iterative procedure, with the possible attendant convergence problems. Also, an iterative scheme requires the evaluation of the analytical modal model at every iteration. When the change in the parameters between successive iterations is small, a good estimate of the modal model is available and may be used to improve the eigensystem calculation efficiency. The methods generally are based on the use of a truncated Taylor series of the modal data as a function of the unknown parameters. This series is often truncated to produce the linear approximation

$$\delta \mathbf{z} = \mathbf{S} \delta \boldsymbol{\theta} \tag{4.1}$$

where $\delta\boldsymbol{\theta}$ is the perturbation in the parameters, $\delta\mathbf{z}$ is the perturbation in the measured output and $\mathbf{S}$ is the sensitivity matrix.

The sensitivity matrix, S, contains the first derivative of the eigenvalue and mode shapes with respect to the parameters. Calculating these derivatives is computationally intensive and efficient methods for their computation are required. Fox and Kapoor [136] calculated the derivative of the ith eigenvalue, λ_i, with respect to the jth parameter, θ_j, by taking the derivative of the eigenvector equation, to give

$$\left(\frac{\partial \mathbf{K}}{\partial \theta_j} - \lambda_i \frac{\partial \mathbf{M}}{\partial \theta_j}\right)\boldsymbol{\phi}_i - \frac{\partial \lambda_i}{\partial \theta_j}\mathbf{M}\boldsymbol{\phi}_i + (\mathbf{K} - \lambda_i \mathbf{M})\frac{\partial \boldsymbol{\phi}_i}{\partial \theta_j} = 0. \tag{4.2}$$

Premultiplying by the transpose of the eigenvector, $\boldsymbol{\phi}_i$, and using mass orthogonality and the original definition of the eigensystem produces

$$\frac{\partial \lambda_i}{\partial \theta_j} = \boldsymbol{\phi}_i^{\mathrm{T}}\left(\frac{\partial \mathbf{K}}{\partial \theta_j} - \lambda_i \frac{\partial \mathbf{M}}{\partial \theta_j}\right)\boldsymbol{\phi}_i. \tag{4.3}$$

This expression is easy to calculate and requires only the ith eigenvalue and eigenvector. Eigenvector derivatives are more difficult to calculate. Fox and Kapoor [136] have given two methods. The first involves use of equation (4.2) together with an equation based on the derivative of the mass orthogonality equation to generate $n + 1$ equations in the n unknown elements of the eigenvector derivative vector. This set of equations is solved by using a pseudo-inverse technique. Unfortunately, introducing the extra set of equations destroys the banding inherent in the mass and stiffness matrices and hence loses computational efficiency. Nelson [137] suggested a similar algorithm which retains the banded nature of the original problem by fixing one element of the eigenvector derivative at an arbitrary value. The eigenvector derivative, up to a constant, is then calculated by using equation (4.2) and is normalized by using the derivative of the mass orthogonality equation. In the second method given by Fox and Kapoor [136] one expresses the eigenvector derivative as a linear combination of all the eigenvectors. The coefficients in this expression are calculated by using equation (4.2) and the derivative of the mass orthogonality. Lim *et al.* [138] presented an approximation which resulted in only the lower frequency modes being used in this expression for the eigenvector derivative. The reader is referred to the papers by Ojalvo [139], Sutter *et al.* [140], Rudisill and Chu [141], Tan [142] and Tan and Andrew [143] for other, mainly iterative, methods for calculating eigensystem derivatives. Close or repeated eigenvalues can, at best, cause ill-conditioning or slow convergence in these methods, and in these systems the simple algorithms of Fox and Kapoor [136] and Nelson [137] cannot be used satisfactorily. Zhang and Lallement [144] used a structural modification technique to separate close modes to enable the calculation of the sensitivity matrix to be well conditioned.

In penalty function methods the number of parameters to update is very wide and choosing a particular set is difficult. Different sets of parameters may make the identification problem either well-conditioned or ill-conditioned [145]. Methods to overcome ill-conditioned equations are considered later. The unknown parameters may be matrix elements, substructure parameters or physical quantities, such as the Young's modulus. The algorithms considered in this section are not generally used to update matrix elements directly. Updating such parameters reduces the engineer's understanding of the modelling errors and can lead to physically unrealizable models. To update substructure parameters, the mass and stiffness matrices are written in terms of the unknown parameters, as

$$\mathbf{M} = \mathbf{M}_0 + \theta_1 \mathbf{M}_1 + \theta_2 \mathbf{M}_2 + \cdots, \qquad \mathbf{K} = \mathbf{K}_0 + \theta_1 \mathbf{K}_1 + \theta_2 \mathbf{K}_2 + \cdots, \tag{4.4}$$

where the matrix coefficients $\mathbf{M}_i$ and $\mathbf{K}_i$ are specified for a given choice of substructure [44]. By using substructures the unknown parameters, θ_i, are normalized to be unity for the original analytical model. Using physical parameters gives very similar expressions for the mass and stiffness matrices, once the parameters have been normalized for numerical conditioning. The mass and stiffness matrices will be non-linear functions of some physical parameters. Calculating the eigensystem derivatives directly (equation (4.3)) will give the same result as expressing the mass and stiffness matrices in the form of equation (4.4), when using a truncated Taylor series, and then computing the derivatives. Thus the substructure and physical parameter methods are very similar.

Both natural frequencies and mode shapes may be used in the updating procedure. Until recently, the mode shape data was not generally used for updating for two reasons; the mode shape often contains significant measurement errors, and it has to be normalized for consistency with the analytical model. Although the general shape of the mode shape vector is likely to be reasonable, elements of the vector may be up to 20% in error [146]. If a method which allows the uncertainty of the measured data to be incorporated is used, then the method will use very little of the mode shape data. In contrast, the natural frequencies can be measured to much better accuracy than 1% and can be used with confidence. The development of scanning laser Doppler measurement systems [147, 148] may improve the quantity and quality of the mode shape data. These systems are able to scan large areas of a structure quickly and give the mode shape at a large number of locations, without introducing errors by modifying the structure. Although the quality of each individual entry in the mode shape vector may not be improved, the increased number of elements with a similar variance gives an overall increase in the quality of the data available. Mode shape data then becomes more significant in the updating process.

When the natural frequencies alone are used to update the analytical model the mode shape data is still required to pair the measured and analytical natural frequencies. The usual method to achieve this match is the Modal Assurance Criterion (MAC) [149]. Distinguishing different modes by using the MAC may be difficult if the measurement locations on a structure are poorly selected. A variety of methods in which a finite element model of the structure tested is used, some involving the optimization of MAC values, are available to help in the choice of measurement locations and to assess the quality of a particular co-ordinate set. Penny *et al.* [150] and Jarvis [151] used Guyan reduction to determine the most important co-ordinates. Kammer [152] chose the sensor locations so as to provide the most independent set of mode shape vectors. Kammer [153, 154] has also considered the effects of noise and model error on the choice of locations. Lallement *et al.* [155, 156] chose the measurement locations so as to optimize the condition of the estimation problem.

In equation (4.1) the number of parameters and measurements are not equal, and therefore the sensitivity matrix $\mathbf{S}$ is not square. Chen and Garba [157] considered the case in which there are more parameters than measurements. The parameter vector closest to the original analytical parameters was sought which reproduced the required measurement change. When the number of measurements exceeds the number of parameters, a least squares or weighted least squares technique may be used. Hart and Yao [41] and Ojalvo *et al.* [158] have given further details. The updated parameters are calculated as

$$\delta\boldsymbol{\theta} = [\mathbf{S}^{\mathrm{T}}\mathbf{S}]^{-1}\mathbf{S}^{\mathrm{T}}\delta\mathbf{z}, \qquad \delta\boldsymbol{\theta} = [\mathbf{S}^{\mathrm{T}}\mathbf{V}_{\varepsilon\varepsilon}^{-1}\mathbf{S}]^{-1}\mathbf{S}^{\mathrm{T}}\mathbf{V}_{\varepsilon\varepsilon}^{-1}\delta\mathbf{z} \tag{4.5}$$

for the least squares and weighted least squares respectively. The matrix $\mathbf{V}_{\varepsilon\varepsilon}$ is a positive definite weighting matrix that is usually related to the estimated variance of the measured data. The parameters and measurements should be scaled to improve the conditioning of the matrix inversion [158]. Piranda *et al.* [156] discussed the practical implementation of these methods, including the choice of measurement locations, the criteria for convergence and automatic mode pairing. Eckert and Caesar [159] used a weighted sum of squares of the eigenvalue differences, the differences in the elements of the eigenvector and the change in the updated parameters as the objective function. The algorithm used to solve the minimization problem allowed the inclusion of linear and non-linear inequality constraints on the parameters. Jung and Ewins [160] discussed the effect of the choice of macro-elements or substructures on the model error localization.

Collins *et al.* [161, 162] obtained a parameter estimate using a statistical technique, sometimes referred to as Bayesian estimation. The relationship between these updating methods and Bayesian estimation is more in spirit than in the detailed calculation. Bayesian estimators are based on Bayes' formula, which gives the probability density function (pdf) of the parameters, including the measured information (the posterior pdf), in terms of the pdf of the parameter before updating (the prior pdf) [163]. The estimator may be generated from the posterior pdf in a variety of ways, such as the mean of the pdf. In the updating literature the pdfs are assumed to be Gaussian and are then given uniquely by their mean and variance. The updated parameters are estimated to be those with the most certainty; that is, the parameters with the least variance. Both the measurements and the current parameter estimates are assumed to have errors given in terms of their estimated variances, $\mathbf{V}_{\varepsilon\varepsilon}$ and $\mathbf{V}_{\theta\theta}$. The updated parameters with the minimum variance are then calculated as

$$\delta\theta = \mathbf{V}_{\theta\theta}\mathbf{S}^{\mathrm{T}}[\mathbf{S}^{\mathrm{T}}\mathbf{V}_{\theta\theta}\mathbf{S} + \mathbf{V}_{\varepsilon\varepsilon}]^{-1}\delta\mathbf{z}, \tag{4.6}$$

and the variance of this updated parameter estimate as

$$\mathbf{V}^{*}_{\theta\theta} = \mathbf{V}_{\theta\theta} - \mathbf{V}_{\theta\theta}\mathbf{S}^{\mathrm{T}}[\mathbf{S}^{\mathrm{T}}\mathbf{V}_{\theta\theta}\mathbf{S} + \mathbf{V}_{\varepsilon\varepsilon}]^{-1}\mathbf{S}\mathbf{V}_{\theta\theta}. \tag{4.7}$$

This updating method has received considerable interest, and is the central part of several commercial updating packages [146, 164, 165]. Many papers give examples of the use of this method, with a wide variety in the choice of unknown parameters. Thomas *et al.* [166] updated elements of the mass and stiffness matrices. The most common parameters to use are physical parameters which allow easier interpretation of the updated model [146, 161, 164, 167, 168]. Dascotte [169] applied the method to update the material constants for a composite structure.

The minimum variance method suggested by Collins *et al.* [161, 162] is based on the assumption that the measured data and the analytical data are statistically independent. In general, this will be true only for the first iteration. After the first iteration the measured data has been used to update the parameters and so the assumption of statistical independence is a gross simplification. The most noticeable effect is that the method reproduces the measured data exactly [146] provided that the parameters chosen have sufficient freedom to do so. Friswell [170] calculated the correlation between the measurements and the updated parameter estimates at every iteration. This correlation matrix was then used to calculate the next parameter vector estimate on the basis of an argument similar to that of Collins *et al.* [161, 162]. The method given by Friswell [170] seems to converge more quickly than the original method of Collins *et al.* [161, 162]. Also, the measured natural frequencies are not reproduced exactly by the updated model unless the variance of the measured natural frequency is zero. Friswell *et al.* [171] demonstrated the method on an experimental example.

During a modal test of a structure, only a limited number of modes within the measured frequency range will be identified. The amount of reliable information available may be insufficient to estimate the parameters satisfactorily. Using a least squares solution requires more measured data than parameters. Wada *et al.* [77, 172] suggested using multiple boundary conditions to increase the available experimental data. Nalitolela *et al.* [78] suggested repeating the tests with different amounts of known mass or stiffness added to the structure. In both cases both the experimental structure and the analytical model are changed to represent the known modifications. Lallement and Cogan [76] considered the use of both parametric and boundary condition modifications to increase the available information. More information from the FRF may be incorporated by imagining that a stiffness is added to the structure [173].

Kuo and Wada [174] suggested using the non-linear or second order sensitivity equations, and produced correction terms to give improved convergence properties compared to that of the linearized algorithm. Brandon [175] discussed the computation of the second order sensitivity terms. Kim *et al.* [176] used a non-linear perturbation approach to reproduce a given modal model in a structural optimization setting rather than model updating. Ojalvo and Pilon [177] presented a method in which the second order sensitivity of the natural frequencies is used.

Other penalty functions have been tried. For example, Grossman [178] maximized a correlation coefficient based on a weighted average of the ratio of measured to analytical natural frequencies and the modal assurance criterion values between the measured and analytical mode shapes. The frequency ratio is arranged to be less than one. Nash [179] minimized a weighted sum of eigenvalues and eigenvectors. Upper and lower bounds were applied to the parameter values and confidence factors were calculated for the updated parameters. Heylen [180, 181] introduced a combined method. The sensitivity equations (4.1) are combined with the mass and stiffness orthogonality equations to form a set of simultaneous equations for the parameter updating increments. This set of equations was solved by a pseudo-inverse technique. Nobari *et al.* [182] transformed the equations of motion to modal co-ordinates using the low frequency modes and then optimized the equation error based on the measured modal data. The method does not require that the analytical and experimental modes are paired, but iteration is required because of incompleteness in the measured data. Berger *et al.* [183] minimized the residual forces obtained when the measured natural frequencies and mode shapes are introduced into the equations of motion. Roy *et al.* [184] minimized a weighted sum of the difference in the kinetic and strain energies between the analytical model and measured data. The method reduces the mass and stiffness matrices of the analytical model, and the parameters are updated by using quadratic programming techniques, which allows inequality constraints on the parameters.

More advanced optimization schemes using non-linear programming may be used to incorporate inequality constraints. The method suggested by Janter *et al.* [185–187] tries to reproduce the measured frequencies, reduce the off-diagonal terms in the orthogonality matrices and maintain the parameters in predetermined ranges. See also the internal reports for Fokker Space and Systems by Rekers [188, 189].

Cawley and Adams [190] used the ratio of natural frequencies to identify a damage location in a structure. To first order, this frequency ratio is dependent on the damage location but not the level of damage. The theoretical changes in frequency ratio for all the potential damage sites are estimated and the statistically most likely location identified.

One major practical difficulty with all schemes involving the optimization of a penalty function is the assignment of the weighting matrices. Methods using the minimum variance estimation scheme require the variance of both the measured data and the initial analytical parameters to be specified. The statistical origins of these methods should not be given too much significance, because only one set of measurements and initial parameters is available in general. The assumption is that these measurements and parameters are one sample from the statistical distribution of possible measurements and parameters. The uncertainty in the measurements and parameters can be estimated and converted to a variance. Although this is more art than science, especially for the finite element parameter uncertainties, the alternative is to assume an arbitrary accuracy for the data: for example, the measurements may be assumed to be exact. Blakely and Walton [191] and Friswell [170] have given some guidelines on the choice of measurement and parameter uncertainties.

The penalty function methods usually require solving a set of linear simultaneous equations. In the equations to update the parameter estimate and variance in the statistical method this is hidden by including a matrix inverse. Increasing the weight given to the initial analytically estimated parameters or choosing a different set of unknown parameters [145] will improve the conditioning of this set of equations, although there may be occasions on which this is undesirable. In any case, the parameters should be scaled correctly to improve the problem conditioning. The best methods for solving these sets of equations use the Singular Value Decomposition (SVD) [192]. Rothwell and Drachman [73] suggested an improved method which uses the SVD but gives superior solutions.

Error matrix methods are a group of techniques that estimate the error in the mass and stiffness matrices, on the assumption that this error is small. Although the updated model usually has an eigensystem that is close to the measured system, reproduction of the measured eigensystem is not a constraint in the method. Sidhu and Ewins [83] obtained an expression for the error in the stiffness matrix by expanding the unknown, updated flexibility matrix in terms of the error matrix. Assuming that second order terms in the error matrix are negligible and rearranging produces an estimate of the error matrix, $\Delta \mathbf{K}$, given by,

$$\Delta \mathbf{K} \simeq \mathbf{K}_A[\mathbf{K}_A^{-1} - \mathbf{K}_T^{-1}]\mathbf{K}_A, \tag{4.8}$$

where the subscripts A and T represent the analytical and measured matrices. In practice, the measurements contain an incomplete set of modes. Furthermore, the modes are measured at fewer points than are available on the analytical model. The latter problem is solved by reducing the analytical model degrees of freedom or by expanding the mode shape data (see section 3). The incompleteness in the measured modes is solved implicitly by the method. The stiffness matrix elements are strongly influenced by the high frequency—usually unmeasured—modes, whereas the flexibility matrix depends predominantly on the low frequency modes. An accurate measured flexibility matrix is easily computed from the measured modal model, but this flexibility matrix cannot be inverted to give a stiffness matrix because it is rank deficient due to the lack of high frequency modes. In equation (4.8) the analytical model is used, in effect, to fill in for the unmeasured data. A similar approach may be applied to the mass matrix. Gysin [193], Lawrence [194] and Park *et al.* [195] have given further details and examples of the use of the error matrix method. Zhang and Lallement [82] also considered using some of the second order terms in the expansion of the flexibility matrix. Lieven and Ewins [196] discussed the effect of incompleteness and noise on the quality of the results obtained from the error matrix method. Lieven and Ewins [90] calculated the error matrix by directly inverting the rank deficient flexibility matrix using a singular value decomposition.

5. PENALTY FUNCTION METHODS FOR USE ON MEASURED FRF DATA

The methods for using measured FRF data optimize a penalty function involving the FRF data directly. Extracting natural frequency and mode shapes for structures with close modes or a high modal density can be difficult, and avoiding the requirement for a modal model sidesteps these problems [197]. Friswell and Penny [198] discussed the use of typical algorithms for structures with close modes. There are two different types of error function definition in the penalty functions, both based on the equations of motion in the frequency domain given, for viscous damping, by

$$[-\omega^2\mathbf{M} + \mathrm{j}\omega\mathbf{C} + \mathbf{K}]\mathbf{x}(\omega) = \mathbf{f}(\omega), \tag{5.1}$$

or, alternatively, in terms of the dynamic stiffness matrix $\mathbf{B}(\omega)$,

$$\mathbf{B}(\omega)\mathbf{x}(\omega) = \mathbf{f}(\omega), \tag{5.2}$$

where $\mathbf{B}(\omega) = -\omega^2\mathbf{M} + \mathrm{j}\omega\mathbf{C} + \mathbf{K}$. The equation error approach minimizes the error in the equations of motion given in equation (5.2):

$$\boldsymbol{\epsilon}_{EE} = \mathbf{f}(\omega) - \mathbf{B}(\omega)\mathbf{x}(\omega). \tag{5.3}$$

One slight difficulty is that FRFs, receptances for example, are usually measured rather than the displacement and force individually. In this case the excitation force is assumed to be white and hence the vector $\mathbf{f}$ has unity force magnitude at all frequencies, and the displacement is replaced by the receptance. The alternative approach is to minimize the output error, defined as the difference between the measured and estimated response given by

$$\boldsymbol{\epsilon}_{OE} = \mathbf{B}(\omega)^{-1}\mathbf{f}(\omega) - \mathbf{x}(\omega). \tag{5.4}$$

Once again, measured FRFs are usually used as for the equation error. Fritzen [199], Natke [44], Fritzen and Zhu [200] and Cottin *et al.* [201] have discussed the equation and output error quantities in more detail. The output error may also be minimized by using a logarithmic scale, and the errors in the modal data may be included in any of the penalty functions [200].

The major advantage of the equation error approach is that the error is a linear function of the parameters, if one assumes that typical physical parameters such as flexural rigidity are used. Its major disadvantages are the need to measure all the co-ordinates and the fact that the parameter estimates are biased. Care must also be taken that the FRF data contains sufficient information to obtain physically sensible parameters. Although the FRFs have far more data points than the modal model, it should not be thought that they contain a proportionally increased amount of information. Because the FRFs may be reproduced very closely by the complete modal model in the frequency range of interest, the FRFs contain only a little information concerning the out-of-band modes, which is easily masked by measurement noise.

To overcome the requirement to measure all degrees of freedom, the number of degrees of freedom in the analytical model may be reduced. Finite element models of realistic structures are generally high order and produce a correspondingly high number of natural frequencies, damping coefficients and mode shapes. The natural frequency of most of these modes will be outside of the frequency range of interest in practical applications. For example, when measurements of the structure are taken by using a computerized data acquisition system, the resulting FRFs have an upper limit on the usable frequency range determined by the sampling rate through the Nyquist frequency. Thus it should be possible to reduce the number of degrees of freedom in the theoretical model for little loss of accuracy over the measured frequency range. This is on the assumption that sufficient degrees of freedom are included to provide at least the same number of modes, within the frequency range, in the reduced model as were in the original model. The accuracy of the FRF of the reduced order model within the frequency range of interest will be improved by including a reasonable number of modes outside the measured frequency range.

Where an algorithm updates physical parameters the approach is slightly different from the usual reduction process, as the reduced model must retain the dependence on the unknown parameters. Methods of model order reduction have been used extensively in control and filter applications to reduce the cost of designing or implementing a high order controller or filter. The oldest and least computationally demanding algorithms are based on Padé approximations or continued fractions [202, 203]. These methods are not suitable for reducing the order of structural models because it is difficult to incorporate unknown physical parameters, and they substantially alter the eigenvalues of the system, which can usually be measured quite accurately. When the full model is predicting the system natural frequencies adequately, the reduced order model should also predict the lower natural frequencies adequately.

Static condensation, for example Guyan [204] and Irons [205], has been used to reduce the order of structural problems. Equations that do not include an external force term are used to eliminate spatial variables. Generally, these methods must be handled with extreme care, as important natural frequencies may be changed considerably, or omited altogether [206]. Paz [207] suggested a method of dynamic condensation that is really limited to solving the theoretical eigenproblem. The condensation techniques have been generalized and also used to expand measured mode shapes by using a variety of techniques such as the Improved Reduced System (IRS) [208], the System Equivalent Reduction Expansion Process (SEREP) [122] and a hybrid method [209]. The methods are generalized by defining a transformation matrix between the analytical and measured degrees of freedom.

Reducing the order by retaining only the modes with the lowest natural frequencies is slightly more complex and computationally more demanding. It has the advantage that the lower natural frequencies remain unchanged and, provided that enough modes are included, the reduced model can approximate the full model sufficiently accurately. Natke and Zhang [210], Friswell [211, 212] and Hoff and Natke [213] have used this method formulated as an incomplete modal transformation to reduce the model order. Link [86, 214], Larsson and Sas [215] have used a similar method based on exact reduction.

There has been considerable interest in control engineering on methods based on balanced realizations and the Hankel singular values of a system. Moore [216] proposed the balanced realization approach based on the transformation given by Laub [217]. Glover [218] developed optimal Hankel-norm approximations for multi-variable systems. Gawronski and Natke [219, 220] also considered the balancing approach, and Yae and Inman [221] extended the method to retain physical states in the reduced system. These methods of reduction are inappropriate for the identification of structural parameters for three reasons. First, the large dimension of a finite element model makes the computation times involved prohibitive. Second, the methods do not allow for unknown parameters. Finally, the lower eigenvalues of the system are changed although the methods do incorporate error bounds. In practical cases in structural dynamics, the end result is little different from that of the incomplete modal transformation.

The bias problem has been discussed by Frizten [199], who suggested an instrumental variable approach to eliminate the bias. A similar approach was adopted by Wang [222].

Most parameter estimation algorithms generate an over-determined set of linear equations in the unknown parameters, usually written in matrix form, which are usually solved by a pseudo-inverse technique. Even though the measurement noise may have zero mean, the parameter estimates may be biased because the coefficients of the parameters in the equations may be corrupted by noise. Unbiased estimates may be obtained by using the instrumental variable approach, in which the equation is multiplied by the transpose of a matrix of the same size as the coefficient matrix which is uncorrelated with the measurement noise. Eykhoff [223] and Ljung [16] have given more details of the method and have discussed the choice of matrix, or instruments. Cottin *et al.* [201] showed that with significant measurement noise the results from an equation error formulation are more biased than results from an output error formulation. An alternative method used to reduce the bias in the equation error formulation is to weight the error by using the current analytical model. Goyder [224] presented this method for modal extraction based on a single-degree-of-freedom model. Friswell and Penny [225] presented the extension of the method to parameter updating in multi-degree-of-freedom systems. The weights are chosen so that on convergence the penalty function is similar to the output error. Link [86, 214] and Yang and Park [226] have used similar methods.

The output error approach has the advantage of minimizing the error between the measured data and the analytical prediction of that data. If the measured data are contaminated with noise having zero mean, then the resulting parameter estimates are unbiased. Also, no reduction in the model is required. The disadvantage of output error algorithms is that they require the minimization of a non-linear penalty function, with the associated problems of convergence and computational time. The penalty function may be minimized directly as a non-linear function of the parameters [197]. Mottershead and Shao [227] used least squares to calculate the parameter changes to minimize a penalty based on a linearized output error. Arruda [228] optimized the output error of a rotor model using a line search. The sensitivity of the FRFs to the parameters may be calculated directly from the dynamic matrix [227] or by using the eigenvalue and eigenvector sensitivity, as done, for example, by Sharp and Brooks [229] and Nalecz and Wicher [230].

An alternative to direct minimization is to introduce a frequency domain filter. Simonian [231, 232] developed a filter based on measured power spectral densities for the estimation of wind forces. Hart and Martinez [233] used an iterated extended Kalman filter to extract unknown structural parameters. Mottershead and Stanway [234] modified the algorithm of Detchmendy and Sridhar [235] to update the structural parameters by minimizing the output error. A similar approach has been developed for the equation error by Mottershead *et al.* [68, 236, 237], and also by using the instrumental variables method by Mottershead [237] and Mottershead and Foster [238]. Mottershead *et al.* [239] included the added constraint of a positive definite mass matrix. In the practical implementation of these methods, the singular value decomposition is used to solve the equation, and the solution is sought that is closest to that of the original analytical model [68, 69, 240].

Santos and Arruda [241] and Zhang and Natke [81] considered the updating of a complex structure by minimizing the output error using component mode synthesis. Zhang and Natke [81] reduced the model size using an incomplete modal transformation. Lin and Ewins [242] presented a method in which the difference between the measured and analytical receptances is written as a linear function of the parameters, by using a matrix identity. These equations are then solved for the unknown parameters. The method could be considered to be a weighted equation error algorithm. Visser and Imregun [243] considered the problems of incompleteness of the experimental data.

6. CLOSURE

In this paper, a thorough description of the state of the art in finite element model updating has been provided. It is the authors' opinion that the most promising techniques now need to be tested in an industrial environment. In the aerospace industry the need for test-verified finite element models has long been recognized. However, the approach used in practice is still based on fitting model parameters to test data by trial and error methods. In the motor industry the speed with which new models can be brought to the market has proved to be increasingly critical. A particular design obstacle is the interaction of structural vibration modes with acoustic modes in the passenger compartment. The application of model updating methods can lead to the rapid development of improved vibration and acoustic models.

While successful results have been reported for many of the approaches described previously in this paper, it is vital that such approaches are capable of handling problems of industrial dimensions. The reported results are often obtained by using data from laboratory test structures to update models with no more than a few hundred degrees of freedom. Finite element models with tens, or hundreds, of thousands of degrees of freedom are commonplace in industry, and the updating of such large models now poses a major intellectual challenge to the researchers.

The authors hesitate to recommend "preferred" updating approaches. However, certain directions appear to be sensible in view of the strongly under-determined nature of the updating task. The preliminary elimination, through localization, of certain members of the set of candidate updating parameters is clearly helpful. So too is the use of test data from multiple test configurations (static as well as dynamic), and regularization techniques can be used to improve the conditioning of the problem, and may result in a solution closely neighbouring a solution with physical meaning. The research conducted in these areas has, in the main, been closely coupled to the use of sensitivity based approaches. Those methods in which eigendata sensitivities are used (rather than FRF sensitivities) are particularly robust, and appear to be well suited to industrial model updating problems.

It is considered that a need exists for the use of confidence levels which might be assigned to quantified mesh, or test data, uncertainties. Such confidence levels should be used together with the sensitivities at both the localization and the updating stages. At a more basic level, guidance needs to be provided in the preparation of finite element meshes for updating. In present updating procedures one assumes that the initial model is fixed with respect to the mesh configuration, boundary conditions and type of element. Methods need to be put in place which will allow the selection of a "most suitable" initial model from a set of candidates. Further work needs to be conducted on the elimination of systematic errors from both the measurements and the finite element model.

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