The modeling of the Robust Model

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Introduction

In this exercise we are going to model the MDF supply chain. We assume that there are 3 already established factories and there exists 10 potential places to establish distribution center. We also assume that all 30 province are the customer of us and there are 20 probable demand scenario which may happen in real world. In Section1 we are going to describe the notation we use next in section 2 we model the problem with Mulvey and leung method and in the final section we compare the answer of the 2 stage stochastic programming with the Robust solutions we get here.

1 Notation Clearance

In this part all the notations that are going to be used are described. Note that subscripts are the sets that the variables and parameters are described in.

1.1 Sets

i : factory index
j : Distribution center index
k : customer zone index
s : scenario index
s' : also scenario index

1.2 Parameters

d_{ks} : demand of customer k under scenario s
c_{ij} : transportation cost of unit product from factory i to DC j
a_{jk} : transportation cost of unit product from DC j to customer zone k
b_k : shortage costs
λ : how important it is for us to decrease the expected objective
γ : how important it is for us to decrease the variance of objective values
ω : how bad it is not to satisfy constraints
g_j : fixed cost of opening cost
cw_i : capacity of factory i
cy_j : capacity of DC j
π_s : probability of occurence of scenario s
b_k : shortage costs

1.3 Variables

x_{ijs} : amount of products shipped from factory i to DC j under scenario s
u_{jks} : amount of products shipped from DC j to customer zone k under scenario s
ξ_s : excess of a constraint
$\mathcal{O}_s$: objective function for each senario
h_s : A variable to monitor the variance ish...
y_j : binary variable representing activation of DC
Z : Objective

2 Modeling

In this part we use both mulvey and leung models to give our model some robustness in facing uncertainty.

2.1 Mulvey

$$\min Z := \lambda \mathbb{E}(\mathcal{O}_s) + \gamma Var(\mathcal{O}_s) + \omega \sum_{s \in S} \pi_s \xi_s \tag{1}$$

Subject to: (2)

$$\sum_j u_{jks} + \xi_s \geq d_{ks}; \qquad \forall k, s \tag{3}$$

$$\sum_i x_{ijs} = \sum_k u_{jks} \qquad \forall j, s \tag{4}$$

$$\sum_j x_{ijs} \leq cw_i \qquad \forall i, s \tag{5}$$

$$\sum_i x_{ijs} \leq cy_j * y_j \qquad \forall j, s \tag{6}$$

$$\mathcal{O}_s = \sum_j g_j y_j + \sum_{i,j} c_{ij} x_{ijs} + \sum_{j,k} a_{jk} u_{jks} \qquad \forall s \tag{7}$$

2.2 Leung

$$\min Z := \lambda \mathbb{E}(\mathcal{O}_s) + \gamma \mathcal{O}_s - \sum_{s'} \pi_{s'} \mathcal{O}_{s'} + 2h_s + \omega \sum_{s \in S} \pi_s \xi_s \tag{8}$$

Subject to: (9)

$$\sum_j u_{jks} + \xi_s \geq d_{ks}; \qquad \forall k, s \tag{10}$$

$$\sum_i x_{ijs} = \sum_k u_{jks} \qquad \forall j, s \tag{11}$$

$$\sum_j x_{ijs} \leq cw_i \qquad \forall i, s \tag{12}$$

$$\sum_i x_{ijs} \leq cy_j * y_j \qquad \forall j, s \tag{13}$$

$$\mathcal{O}_s = \sum_j g_j y_j + \sum_{i,j} c_{ij} x_{ijs} + \sum_{j,k} a_{jk} u_{jks} \qquad \forall s \tag{14}$$

$$0 \leq \mathcal{O}_s - \sum_{s'} \pi_{s'} \mathcal{O}_{s'} + h_s \qquad \forall s \tag{15}$$

3 Solutions and its analysis

After solving both models in extreme conditions($\lambda = 1, \gamma = 0, \omega = 1000000$(almost infinite) we reach the same answer as we had from two stage stochastic modeling which validates our modeling. By decreasing ω it can be seen that the objective value will decrease since model will withdraw some demands in exchange of lowering the objective value which is one of the characteristics of realistic robust modeling.

4 Numerical Results

```
----     177 VARIABLE z.L                    =  3817641.689  objective function va
                                                             lue

----     177 VARIABLE y.L  binary variable representing activation of DC

4  1.000,    6  1.000,    7  1.000,    10 1.000


----     177 VARIABLE x.L  amount of products shipped from factory i to DC j unde
                           r scenario s

              1           2           3           4           5           6

1.4    4678.000    4678.000    4678.000    4678.000    4678.000    4678.000
1.6    3889.000    3889.000    3889.000    3853.987    3600.988    3888.106
1.7    3966.000    3966.000    3966.000    3966.000    3966.000    3966.000
1.10   3560.984                3687.984                            4030.878
3.10               3907.984                3603.996    3839.996

  +           7           8           9          10          11          12

1.4    4678.000    4678.000    4678.000    4678.000    4678.000    4678.000
1.6    3390.984    3636.984    3568.996    3214.988    3889.000    3610.987
1.7    3966.000    3966.000    3966.000    3966.000    3966.000    3966.000
1.10               4031.000                3849.996
3.10   4031.000                3455.987                3401.984    3371.996

  +          13          14          15          16          17          18
```

Figure 1: extreme condition on Mulvey

```
----     175 VARIABLE z.L                     =  3817641.950  objective function va
                                                               lue

----     175 VARIABLE y.L  binary variable representing activation of DC

4  1.000,    6  1.000,    7  1.000,    10 1.000


----     175 VARIABLE x.L  amount of products shipped from factory i to DC j unde
                           r scenario s

               1           2           3           4           5           6

1.4     4678.000    4678.000    4678.000    4678.000    4678.000    4678.000
1.6     3889.000    3889.000    3889.000    3889.000    3889.000    3889.000
1.7     3966.000    3966.000    3966.000    3966.000    3966.000    3966.000
1.10                                                                4030.000
3.10    3561.000    3908.000    3688.000    3569.000    3552.000

   +           7           8           9          10          11          12

1.4     4678.000    4678.000    4678.000    4678.000    4678.000    4678.000
1.6     3889.000    3889.000    3889.000    3889.000    3889.000    3889.000
1.7     3966.000    3966.000    3966.000    3966.000    3966.000    3966.000
1.10    3533.000    3779.000                3176.000    3402.000
3.10                            3136.000                            3094.000

   +          13          14          15          16          17          18

1.4     4678.000    4678.000    4678.000    4678.000    4678.000    4678.000
1.6     3889.000    3889.000    3889.000    3889.000    3889.000    3889.000
1.7     3966.000    3966.000    3966.000    3966.000    3966.000    3966.000
1.10    3441.000    3580.000    3574.000
3.10                                        3773.000    3818.000    3555.000
```

Figure 2: extreme condition on Leung

```
----     177 VARIABLE z.L                    =  3749899.863  objective function va
                                                             lue

----     177 VARIABLE y.L  binary variable representing activation of DC

2 1.000,    4 1.000,    7 1.000,    9 1.000


----     177 VARIABLE x.L  amount of products shipped from factory i to DC j unde
                           r scenario s

              1           2           3           4           5           6

1.2                                         3000.113                2995.649
1.4    4624.106    4678.000    4670.445    4678.000    4678.000    4670.881
1.7    3966.000    3966.000    3960.310    3966.000    3966.000    3960.649
1.9    4402.106    4456.000    4448.445    4456.000    4456.000    4448.881
2.2    2983.035    3001.000    2995.310                3000.593
3.2      17.965                   5.690                   0.407       5.351
3.4      50.394                   7.555                               7.119
3.7                               5.690                               5.351
3.9      50.394                   7.555                               7.119

   +          7           8           9          10          11          12

1.2    2985.981                                        2882.173    2558.621
1.4    4678.000    4678.000    4293.952    4678.000    4678.000    4678.000
1.7    3966.000    3966.000    3966.000    3966.000    3966.000    3966.000
1.9    4456.000    4456.000    4455.952    4456.000    4456.000    4456.000
```

Figure 3: Mulvey's solution for $\lambda = 1, \gamma = 1, \omega = 1000$

```
----    175 VARIABLE z.L                    =  3750589.625  objective function va
                                                            lue

----    175 VARIABLE y.L  binary variable representing activation of DC

1 1.000,    2 1.000,    7 1.000,    8 1.000


----    175 VARIABLE x.L  amount of products shipped from factory i to DC j unde
                          r scenario s

                  1           2           3           4           5           6

1.1     4067.000    4067.000    4067.000    4067.000    4067.000    4067.000
1.2     3001.000    3001.000    3001.000
1.7     3966.000    3966.000    3966.000    3966.000    3966.000    3966.000
2.2                                         3001.000    3001.000    3001.000
2.8     4901.000    4901.000    4901.000
3.8                                         4901.000    4901.000    4901.000

   +              7           8           9          10          11          12

1.1     4067.000    4067.000    4067.000    4067.000    4067.000    4067.000
1.2     3001.000    3001.000    3001.000    3001.000
1.7     3966.000    3966.000    3966.000    3966.000    3966.000    3966.000
2.2                                                     3001.000    3001.000
2.8     4901.000    4901.000    4855.262    4893.042
3.8                                                     4901.000    4899.729

   +             13          14          15          16          17          18

1.1     4067.000    4067.000    4067.000    4067.000    4067.000    4067.000
1.2                 3001.000                            3001.000
1.7     3966.000    3966.000    3966.000    3966.000    3966.000    3966.000
2.2     3001.000                3001.000    3001.000                3001.000
2.8                 4901.000                            4901.000
```

Figure 4: Leung's solution for $\lambda = 1, \gamma = 1, \omega = 10000$