# CSE 4705 - Artificial Intelligence

## Multiple Regression

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# Table of Contents

# Table of Contents

# Table of Contents

# Training Set and Notation

Our dataset for the univariate linear regression case:

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Our dataset for the univariate linear regression case:

| Size in feet$^2$ ($x$) | Price ($) in 1000's ($y$) |
|---|---|
| 2104 | 400 |
| 1416 | 232 |
| 1534 | 315 |
| 852 | 178 |
| ... | ... |

$$f_{w,b}(x) = wx + b$$

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## Multiple features (variables)

| Size in feet$^2$ | Number of bedrooms | Number of floors | Age of home in years | Price ($) in $1000's |
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| 2104 | 5 | 1 | 45 | 460 |
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- $\vec{x}^{(i)}$ = vector of features for the $i^{th}$ training example
- $x_j^{(i)}$ = value of feature $j$ for the $i^{th}$ training example

# Table of Contents

# Table of Contents

# Multiple Regression Model

Model Notation:

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  - $b$
- Model (Vector Notation): $f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$

# Table of Contents

# Univariate Regression Example: House Prices Model

Recall gradient descent for the univariate case, $f_{w,b}(x)$:

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Recall gradient descent for the univariate case, $f_{w,b}(x)$:

1. Initialize $w, b$ to random values.
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$$w := w - \alpha \frac{\partial J(w,b)}{\partial w}$$
$$b := b - \alpha \frac{\partial J(w,b)}{\partial b}$$

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$$w := w - \alpha \underbrace{\frac{1}{m} \sum_{i=1}^{m} (f_{w,b}(x^{(i)}) - y^{(i)}) \cdot x^{(i)}}_{\frac{\partial J(w,b)}{\partial w}}$$

$$b := b - \alpha \underbrace{\frac{1}{m} \sum_{i=1}^{m} (f_{w,b}(x^{(i)}) - y^{(i)})}_{\frac{\partial J(w,b)}{\partial b}}$$

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Now, for the multiple regression case, $f_{\vec{w},b}(\vec{x})$, we have:

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2. Repeat until convergence (simultaneous update):

$$w_1 := w_1 - \alpha \frac{\partial J(\vec{w}, b)}{\partial w_1}$$
$$w_2 := w_2 - \alpha \frac{\partial J(\vec{w}, b)}{\partial w_2}$$
$$\vdots$$
$$w_n := w_n - \alpha \frac{\partial J(\vec{w}, b)}{\partial w_n}$$
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$$\vdots$$

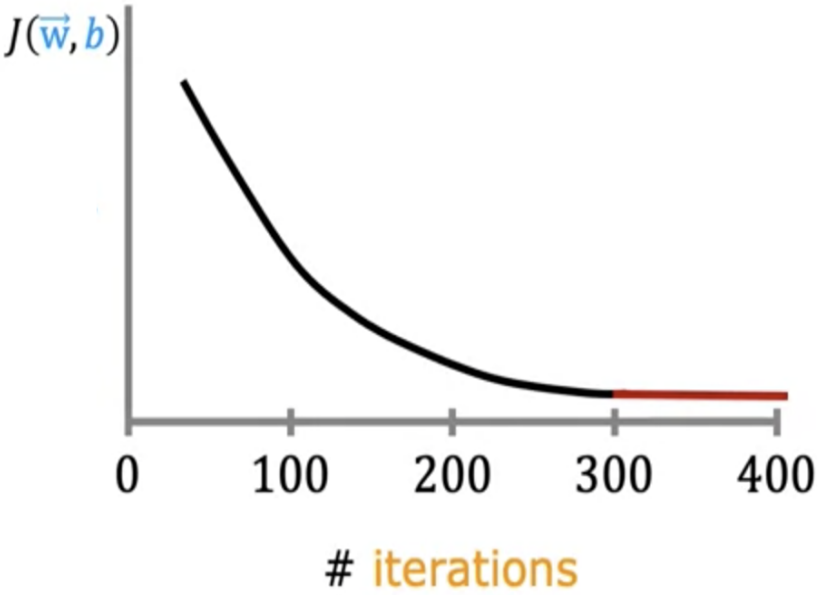
$$w_n := w_n - \alpha \underbrace{\frac{1}{m}\sum_{i=1}^{m}(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}) \cdot x_n^{(i)}}_{\frac{\partial J(\vec{w},b)}{\partial w_n}}$$

$$b := b - \alpha \underbrace{\frac{1}{m}\sum_{i=1}^{m}(f_{\vec{w},b}(x^{(i)}) - y^{(i)})}_{\frac{\partial J(\vec{w},b)}{\partial b}}$$

# Checking for Convergence

## Make sure gradient descent is working correctly

objective: $\min_{\vec{w},b} J(\vec{w},b)$

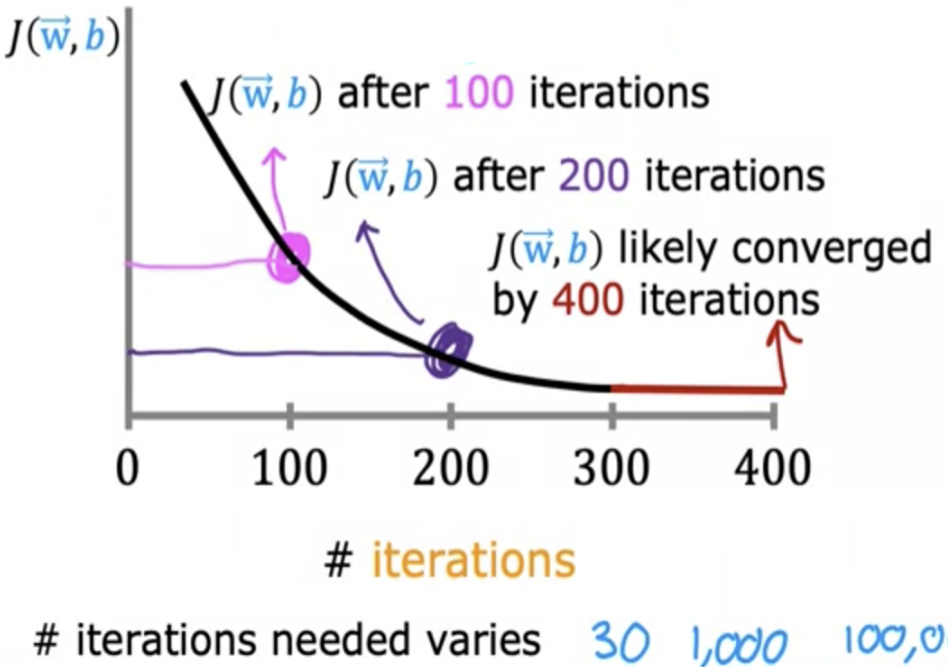


# Checking for Convergence

## Make sure gradient descent is working correctly

objective: $\min_{\vec{w},b} J(\vec{w},b)$

$J(\vec{w},b)$ should decrease after every iteration




# iterations needed varies 30 1,000 100,000

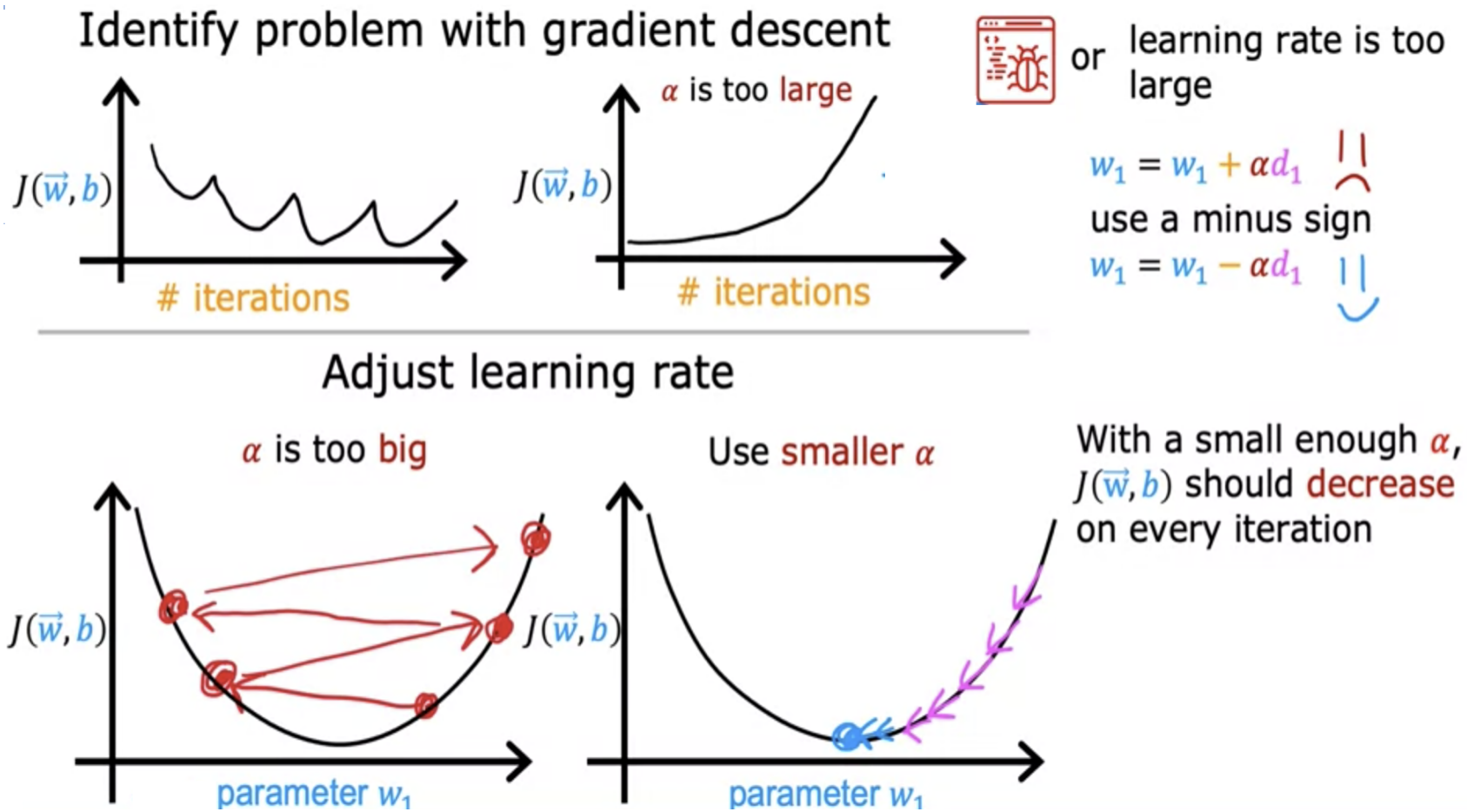
Automatic convergence test

Let $\varepsilon$ "epsilon" be $10^{-3}$.

If $J(\vec{w},b)$ decreases by $\leq \varepsilon$ in one iteration, declare convergence.

(found parameters $\vec{w},b$ to get close to global minimum)

# Checking Learning Rate

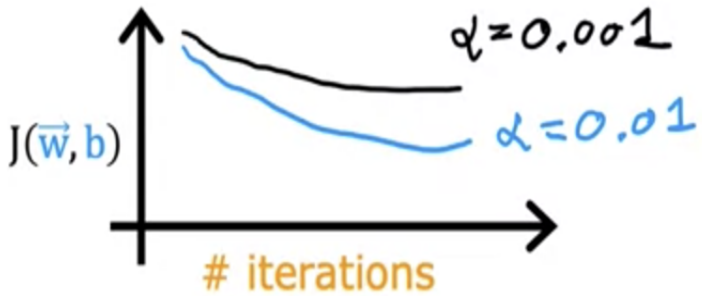


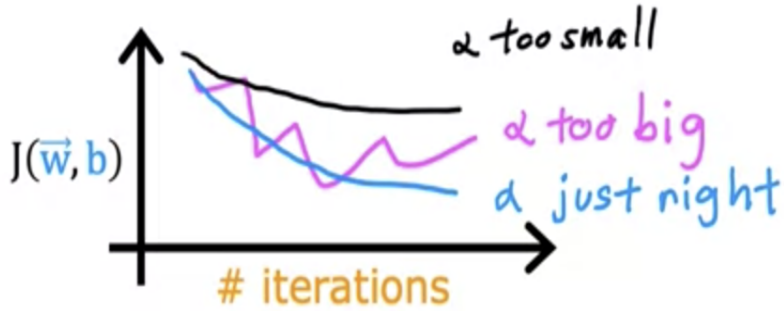
# Learning Rate Optimization

Values of $\alpha$ to try:

... 0.001 0.003 0.01 0.03 0.1 0.3 1 ...

3X ≈3X 3X ≈3X 3X ≈3X



# Table of Contents

# Table of Contents

# Feature Scaling

## Feature and parameter values

$\hat{price} = w_1 x_1 + w_2 x_2 + b$

size #bedrooms

$x_1$: size (feet$^2$) range: 300 – 2,000 large

$x_2$: # bedrooms range: 0 – 5 small

House: $x_1 = 2000$, $x_2 = 5$, $price = \$500k$ one training example

size of the parameters $w_1, w_2$?

$w_1 = 50$, $w_2 = 0.1$, $b = 50$

$\hat{price} = \underbrace{50 * 2000}_{100{,}000\,k} + \underbrace{0.1 * 5}_{0.5\,k} + \underbrace{50}_{50k}$

$\hat{price} = \$100{,}050.5k = \$100{,}050{,}500$

$w_1 = 0.1$, $w_2 = 50$, $b = 50$

$w_1$ small, $w_2$ large

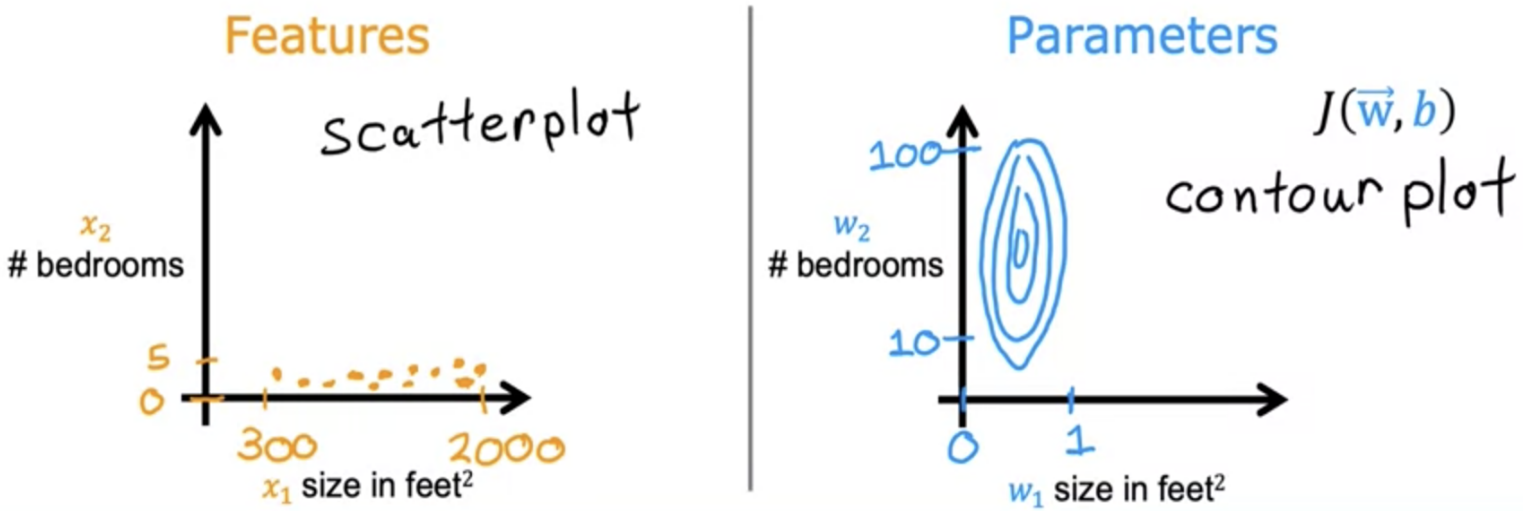
$\hat{price} = \underbrace{0.1 * 2000k}_{200k} + \underbrace{50 * 5}_{250k} + \underbrace{50}_{50k}$

$\hat{price} = \$500k$ more reasonable

# Feature Scaling

## Feature size and parameter size

| | size of feature $x_j$ | size of parameter $w_j$ |
|---|---|---|
| size in feet$^2$ | ⟷ | ⟷ |
| #bedrooms | ⟷ | ⟷ |



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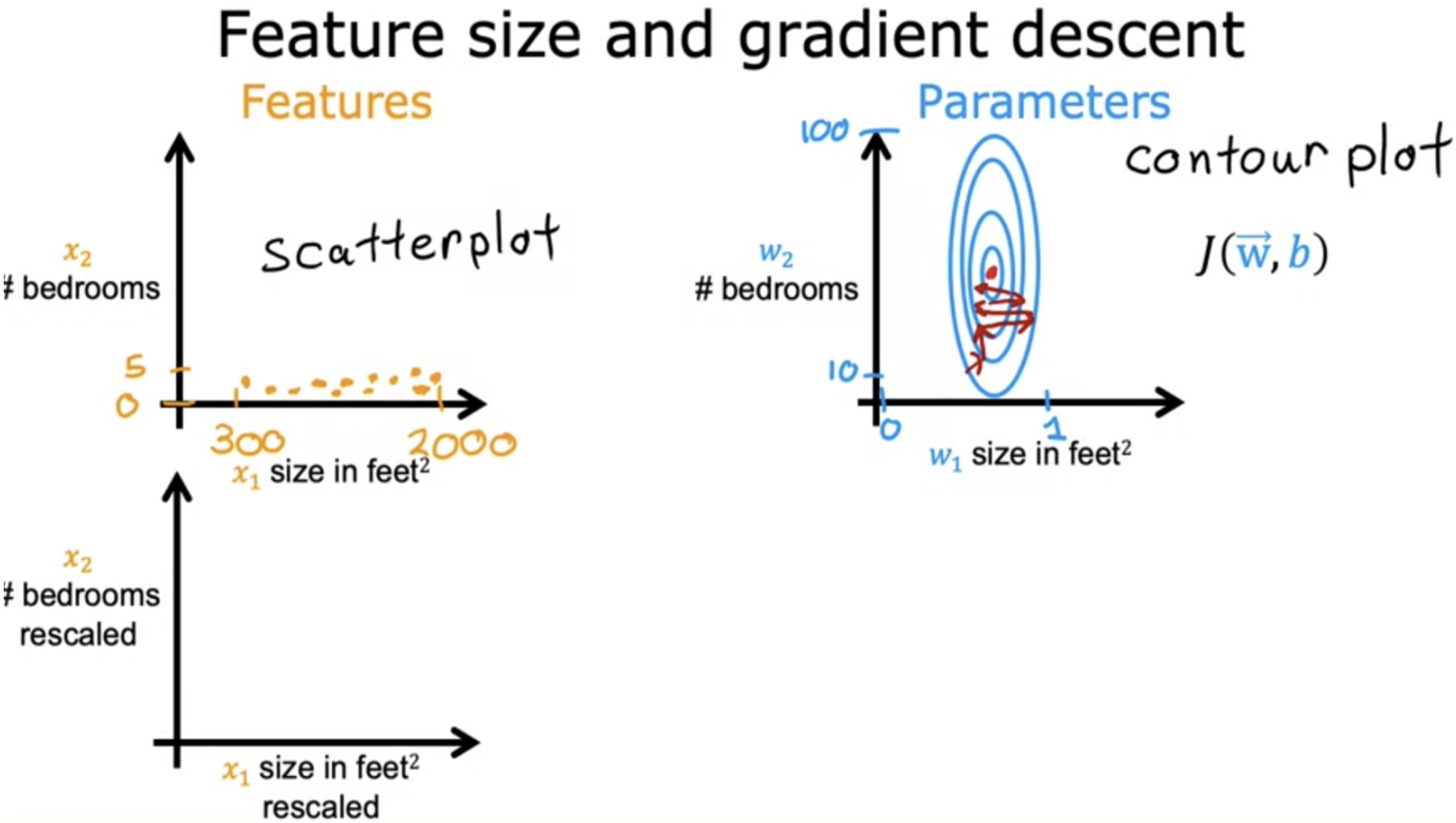

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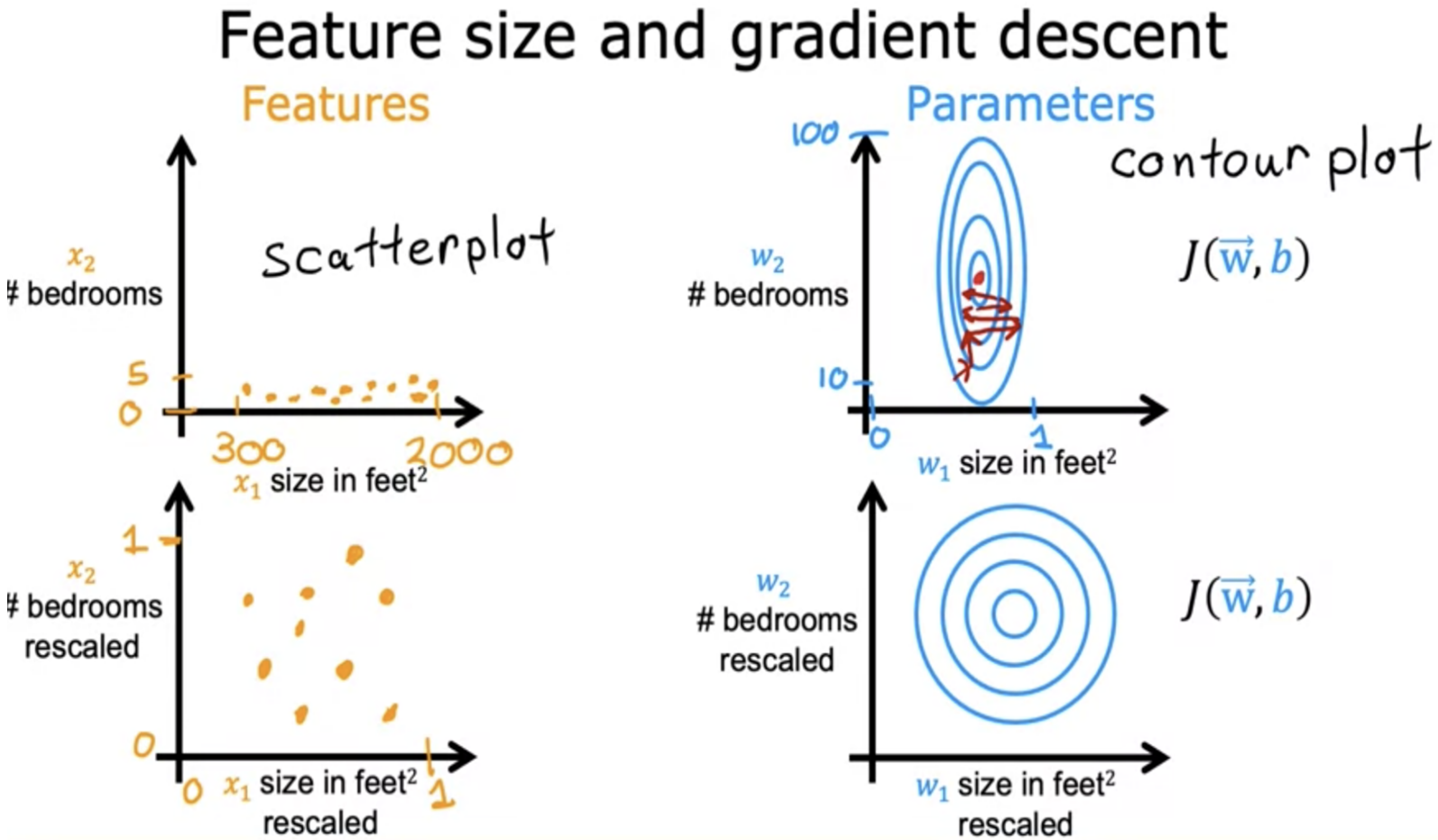
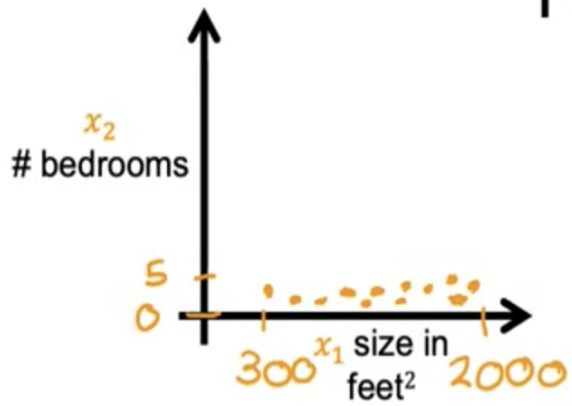


# Feature Scaling

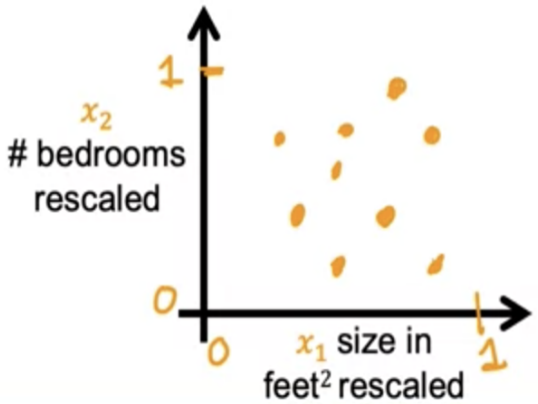
## Feature scaling




$300 \leq x_1 \leq 2000$ $\quad 0 \leq x_2 \leq 5$

$$x_{1,scaled} = \frac{x_1}{2000} \quad x_{2,scaled} = \frac{x_2}{5}$$

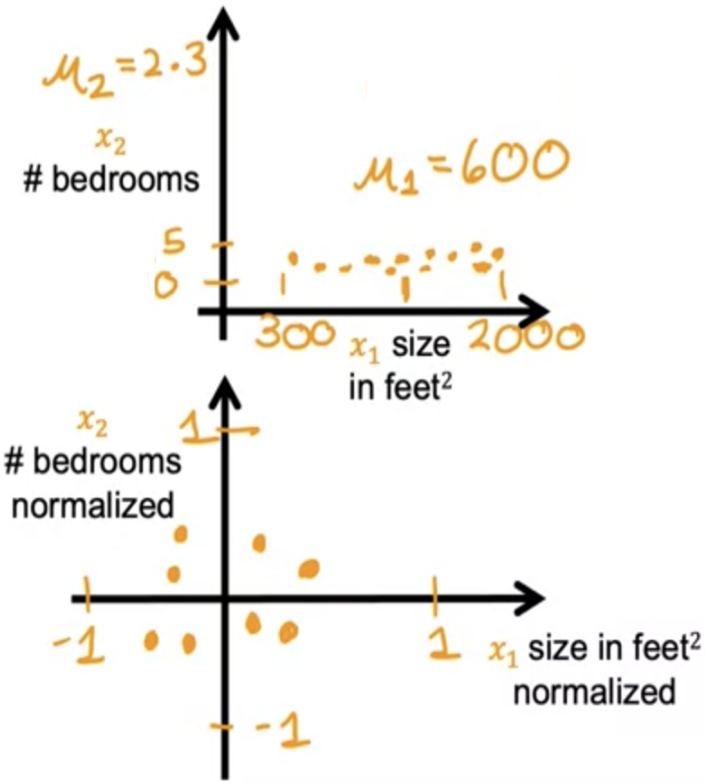
max max




$0.15 \leq x_{1,scaled} \leq 1$ $\quad 0 \leq x_{2,scaled} \leq 1$

# Feature Scaling

## Mean normalization




$$300 \le x_1 \le 2000 \qquad 0 \le x_2 \le 5$$

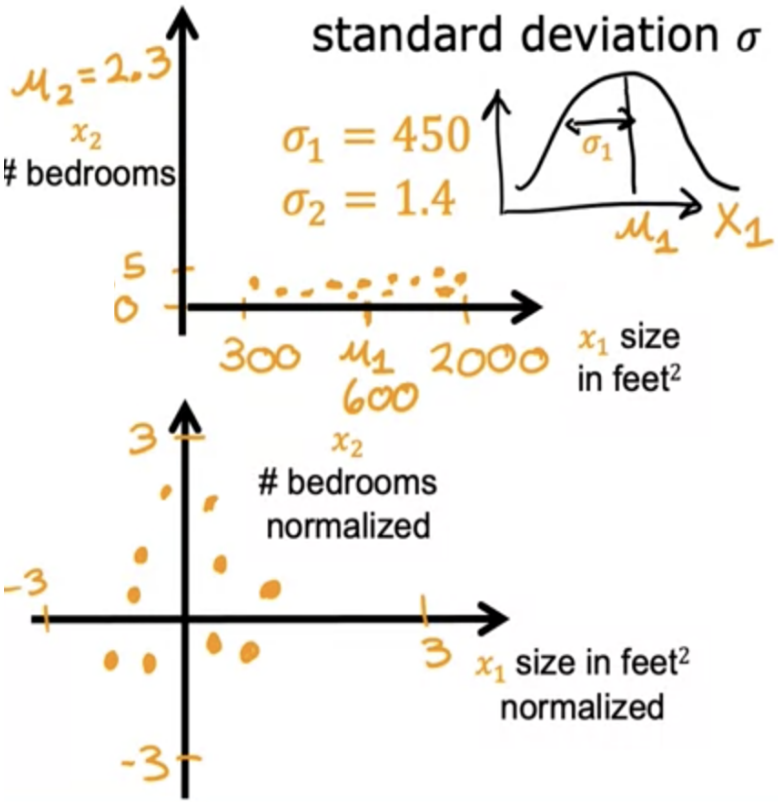
$$x_1 = \frac{x_1 - \mu_1}{2000 - 300} \qquad \text{max-min}$$

$$x_2 = \frac{x_2 - \mu_2}{5 - 0} \qquad \text{max-min}$$

$$-0.18 \le x_1 \le 0.82 \qquad -0.46 \le x_2 \le 0.54$$

# Feature Scaling

## Z-score normalization




$300 \leq x_1 \leq 2000 \qquad 0 \leq x_2 \leq 5$

$$x_1 = \frac{x_1 - \mu_1}{\sigma_1} \qquad x_2 = \frac{x_2 - \mu_2}{\sigma_2}$$

$-0.67 \leq x_1 \leq 3.1 \quad -1.6 \leq x_2 \leq 1.9$

# Feature Scaling

## Feature scaling

aim for about $-1 \leq x_j \leq 1$ for each feature $x_j$

$-3 \leq x_j \leq 3$

$-0.3 \leq x_j \leq 0.3$

} acceptable ranges

$0 \leq x_1 \leq 3$ okay, no rescaling

$-2 \leq x_2 \leq 0.5$ okay, no rescaling

$-100 \leq x_3 \leq 100$ too large → rescale

$-0.001 \leq x_4 \leq 0.001$ too small → rescale

$98.6 \leq x_5 \leq 105$ too large

# Table of Contents

# Feature Engineering - Example

## Feature engineering

$$f_{\vec{w},b}(\vec{x}) = w_1 x_1 + w_2 x_2 + b$$

frontage depth

$$area = frontage \times depth$$

$$x_3 = x_1 x_2$$

new feature

$$f_{\vec{w},b}(\vec{x}) = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$

# Feature Engineering - Definition

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Feature Engineering: is the use of intuition and problem domain knowledge

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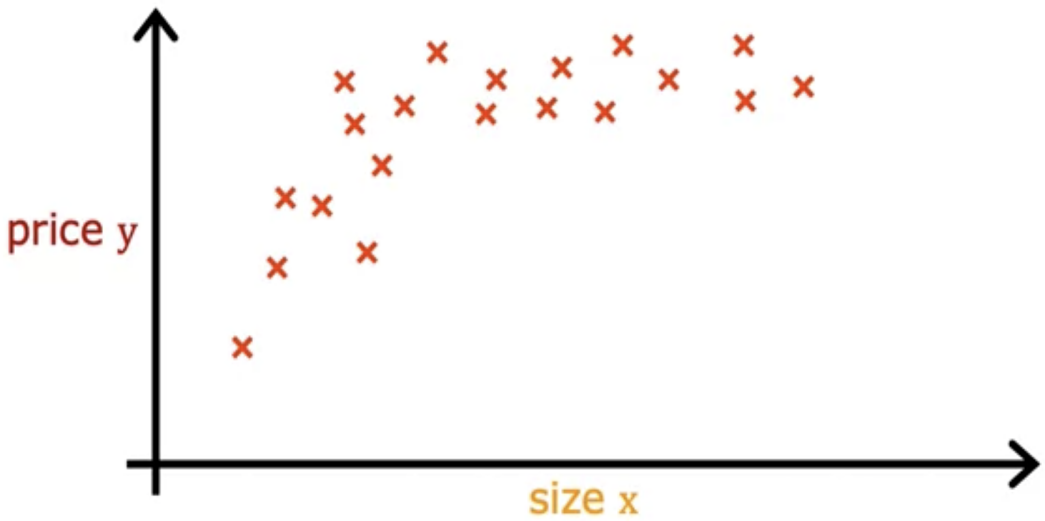
Feature Engineering: is the use of intuition and problem domain knowledge to design new features by transforming or combining original features in order to improve the predictive power of the model

# Table of Contents

# Polynomial Regression



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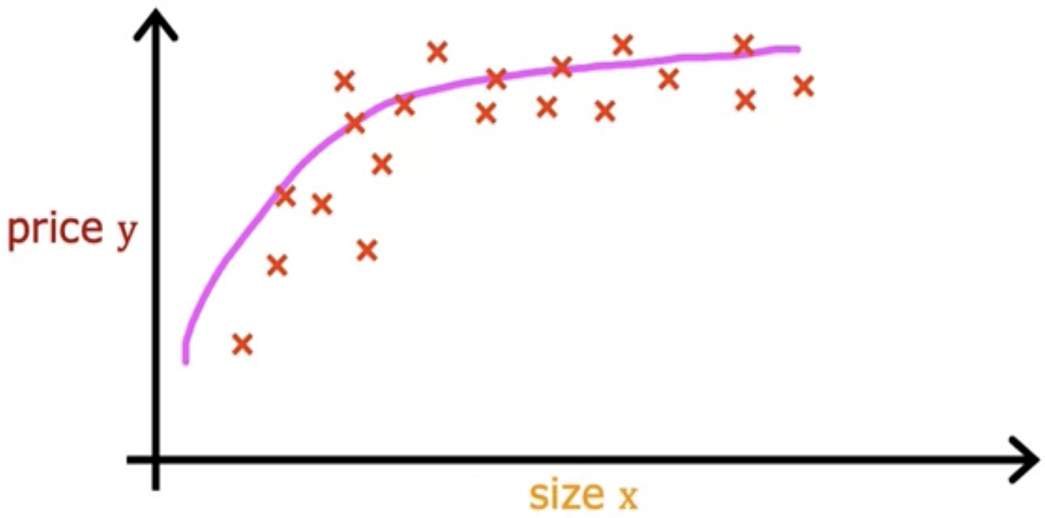

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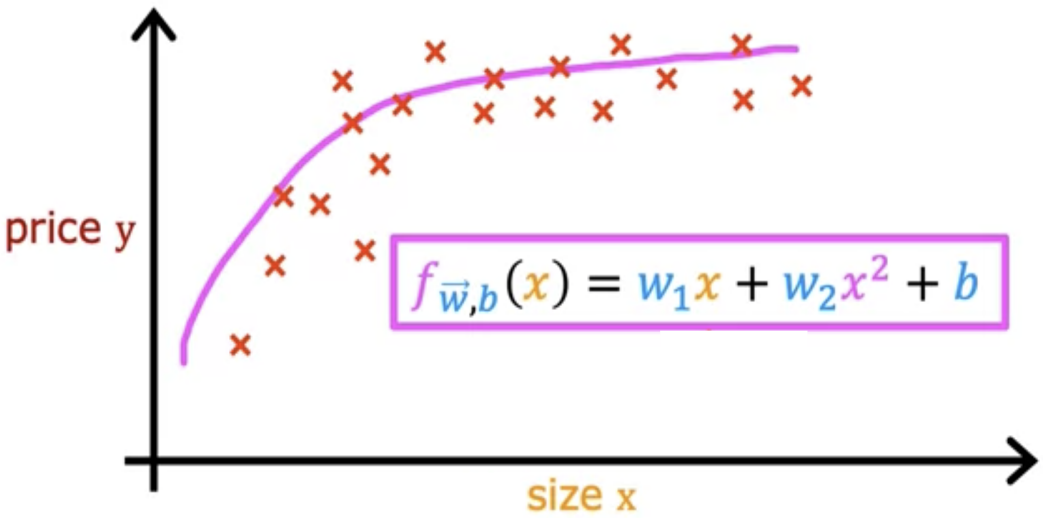

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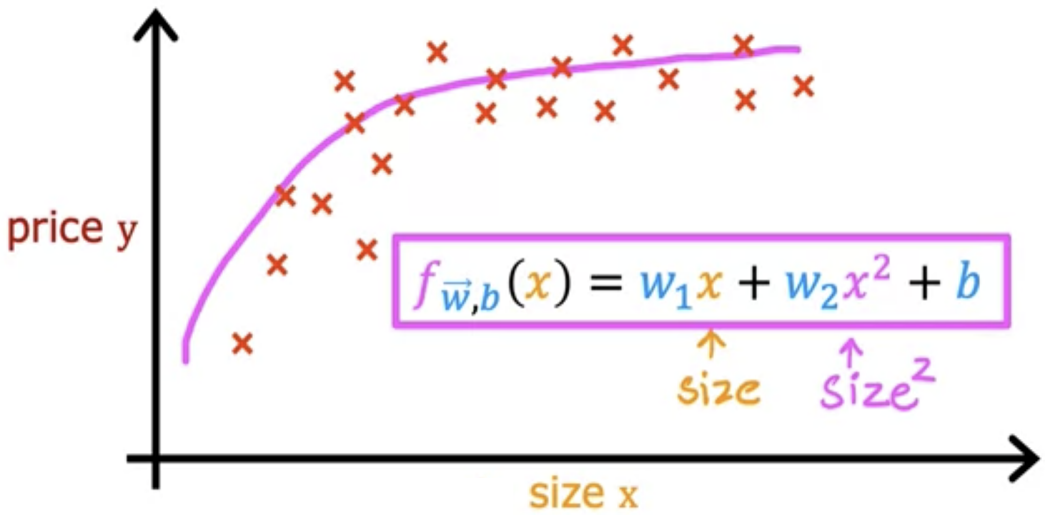

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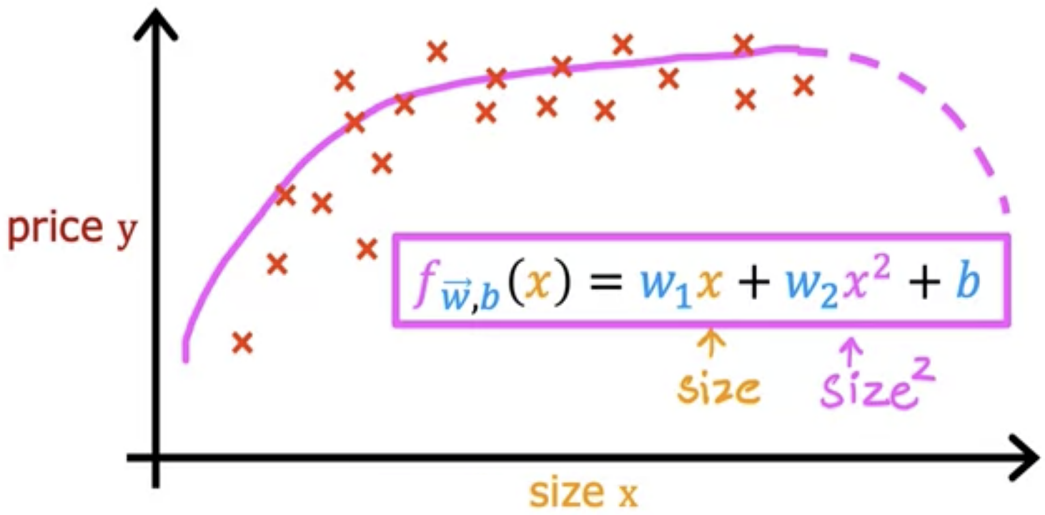
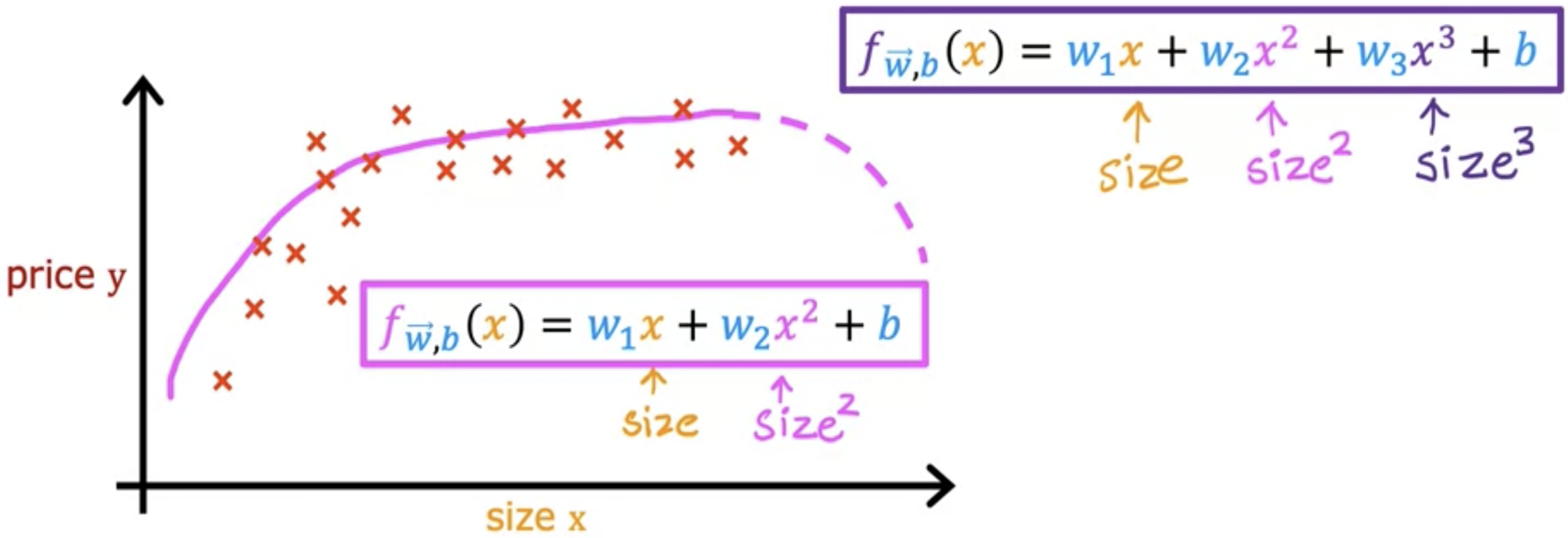


# Polynomial Regression

## Polynomial regression



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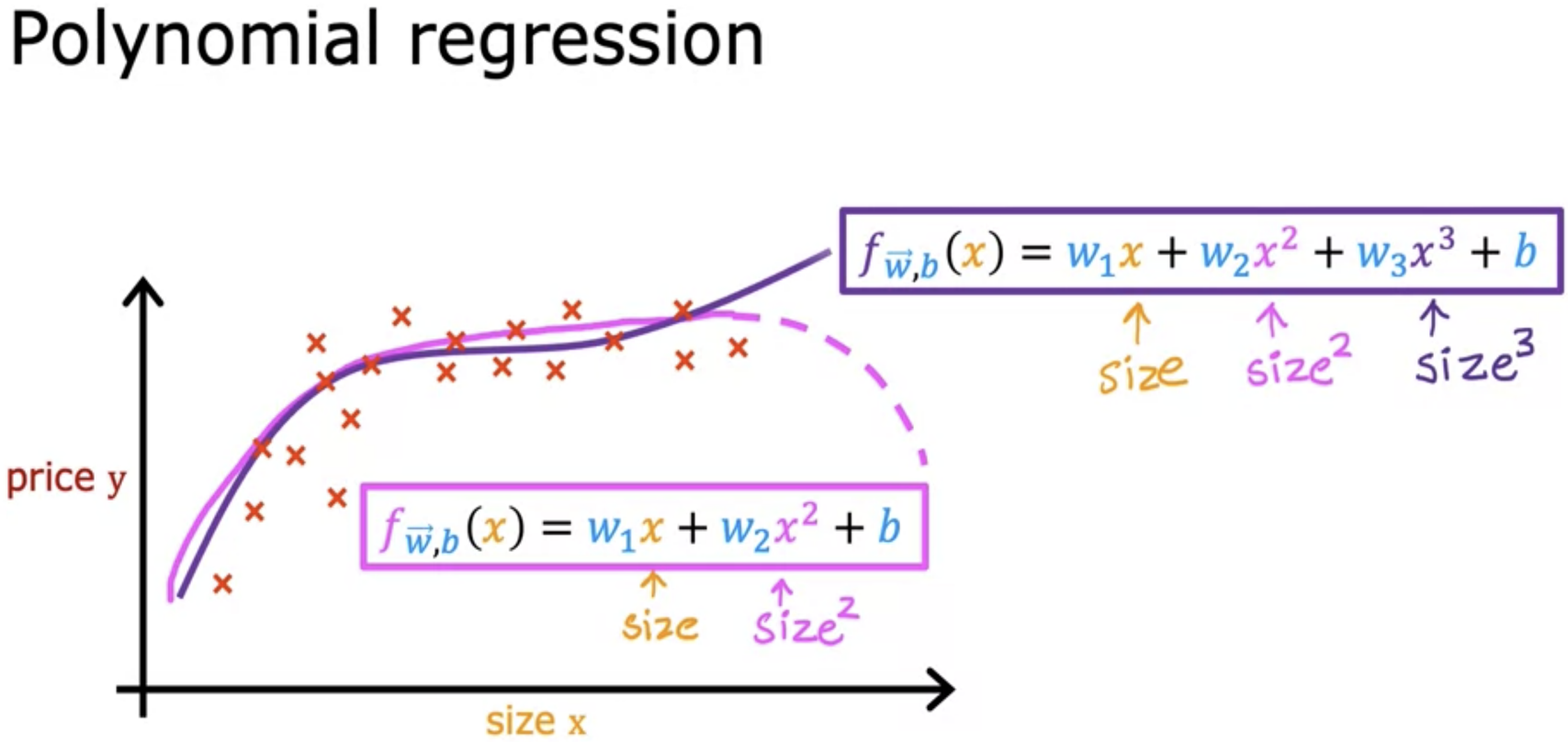

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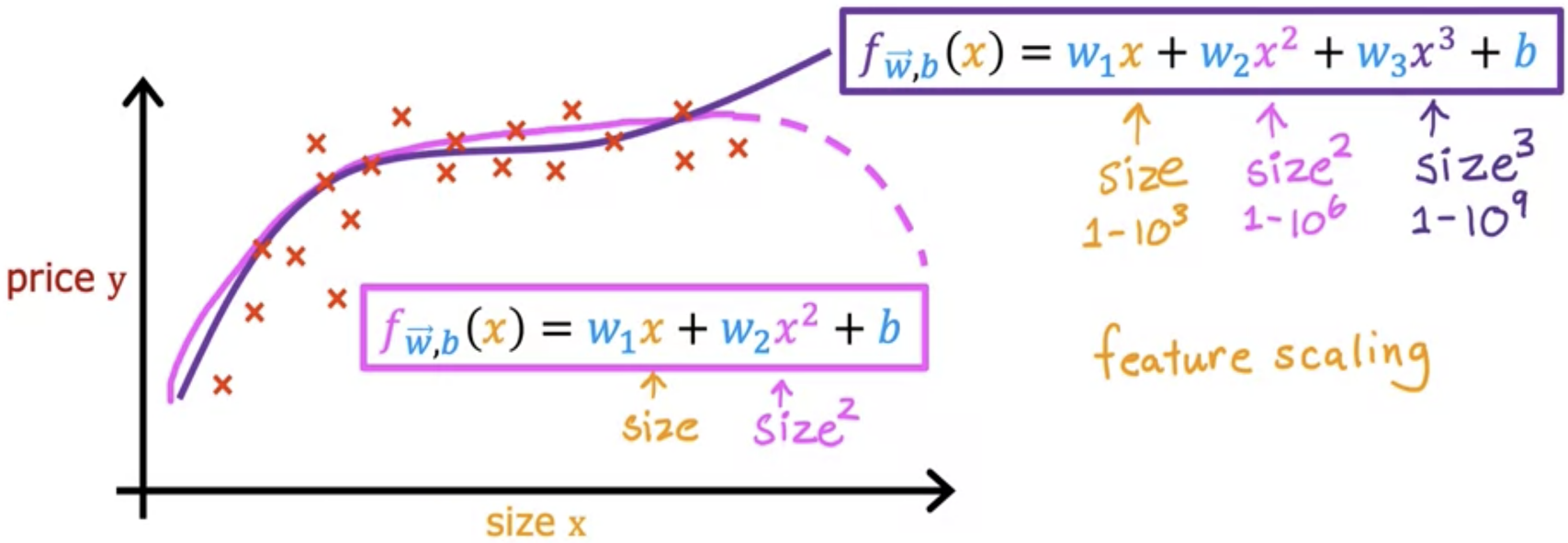

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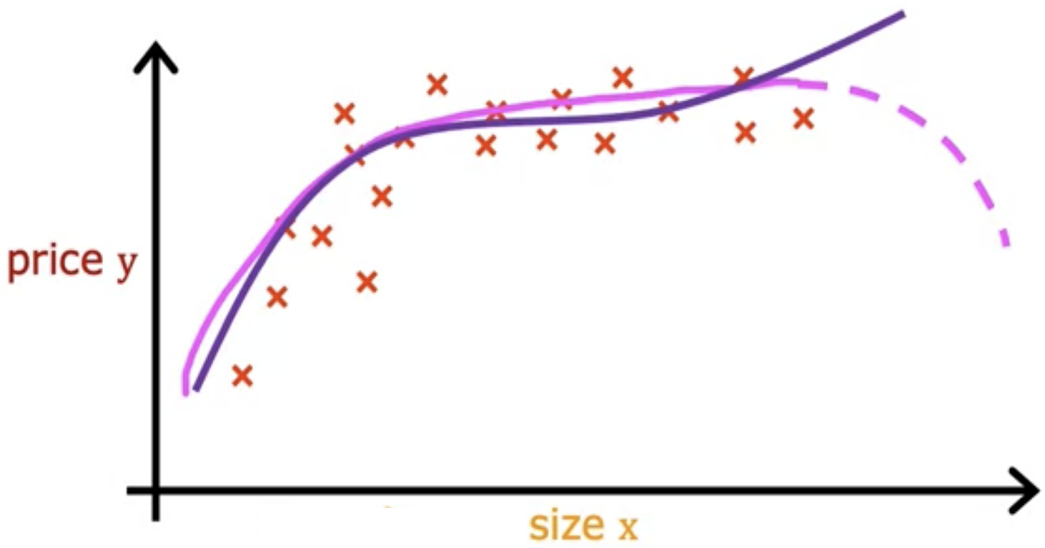

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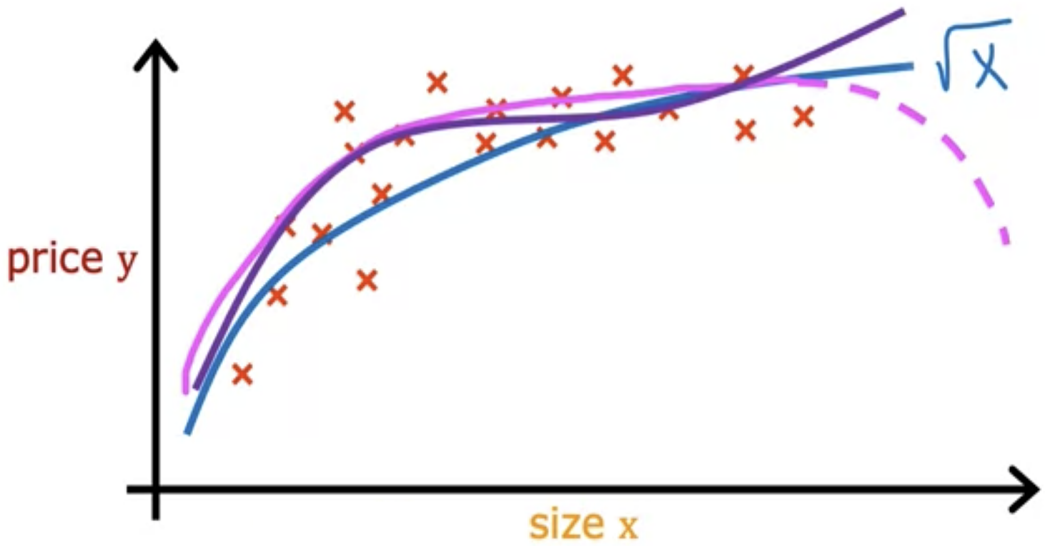

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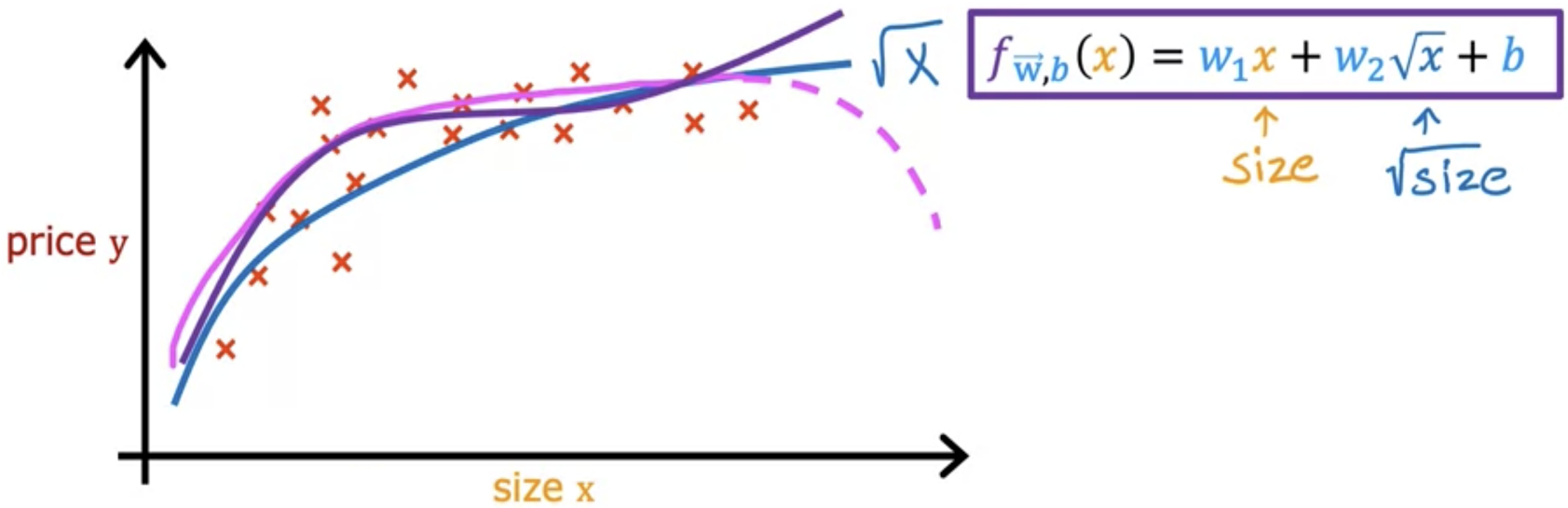

# Table of Contents

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- 0-index in code vs. 1-index in linear algebra

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This is an improvement, but it still computes the terms of $f$ in series (and thus, is relatively slow).

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Each term is computed consecutively, (in series).

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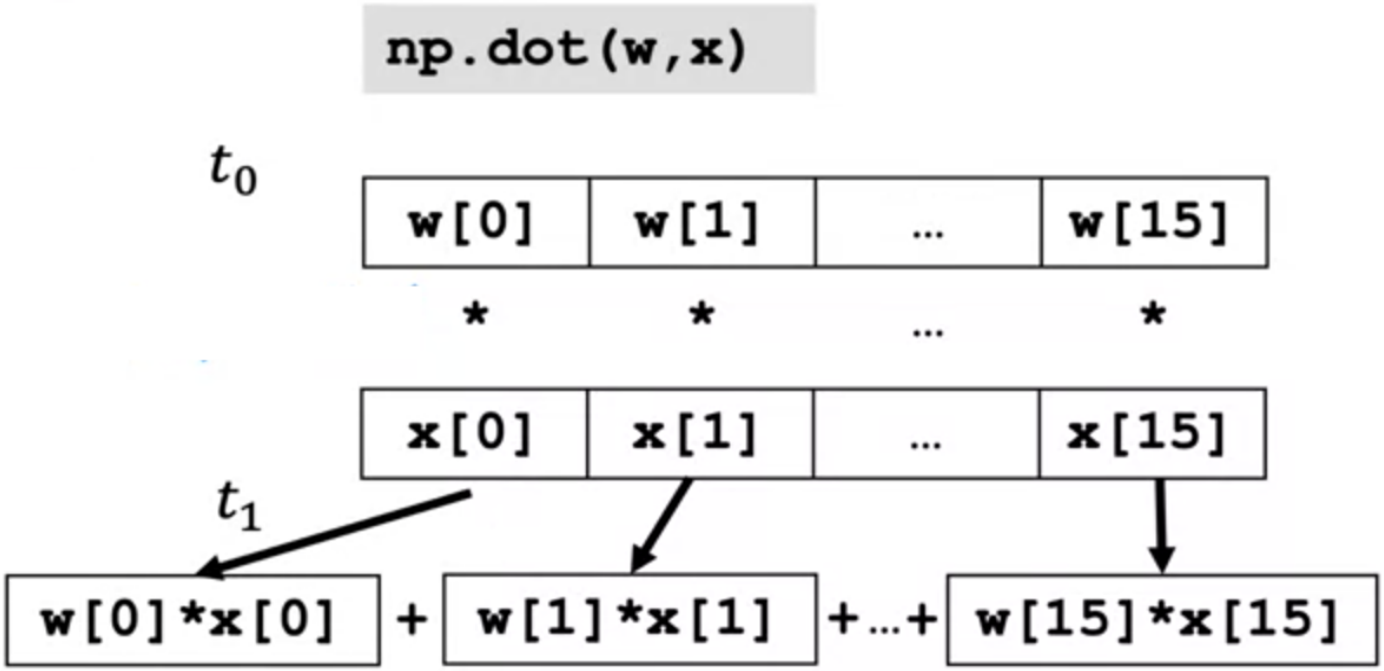

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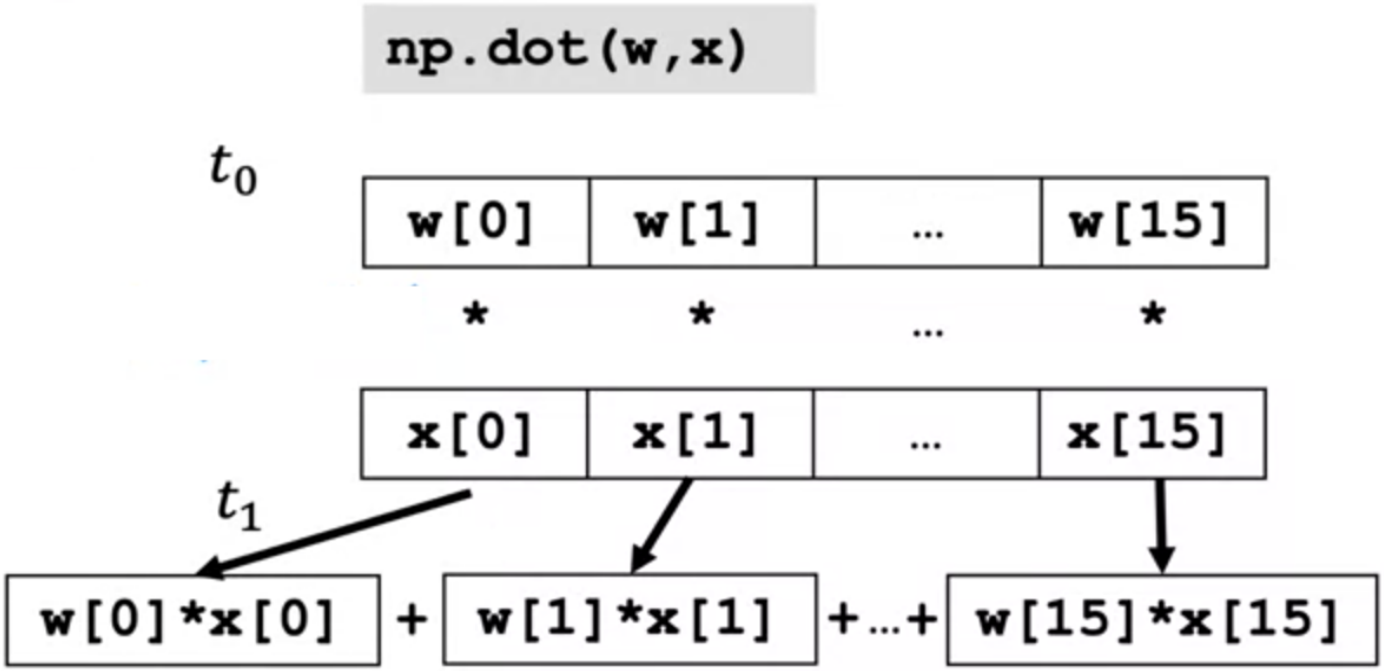


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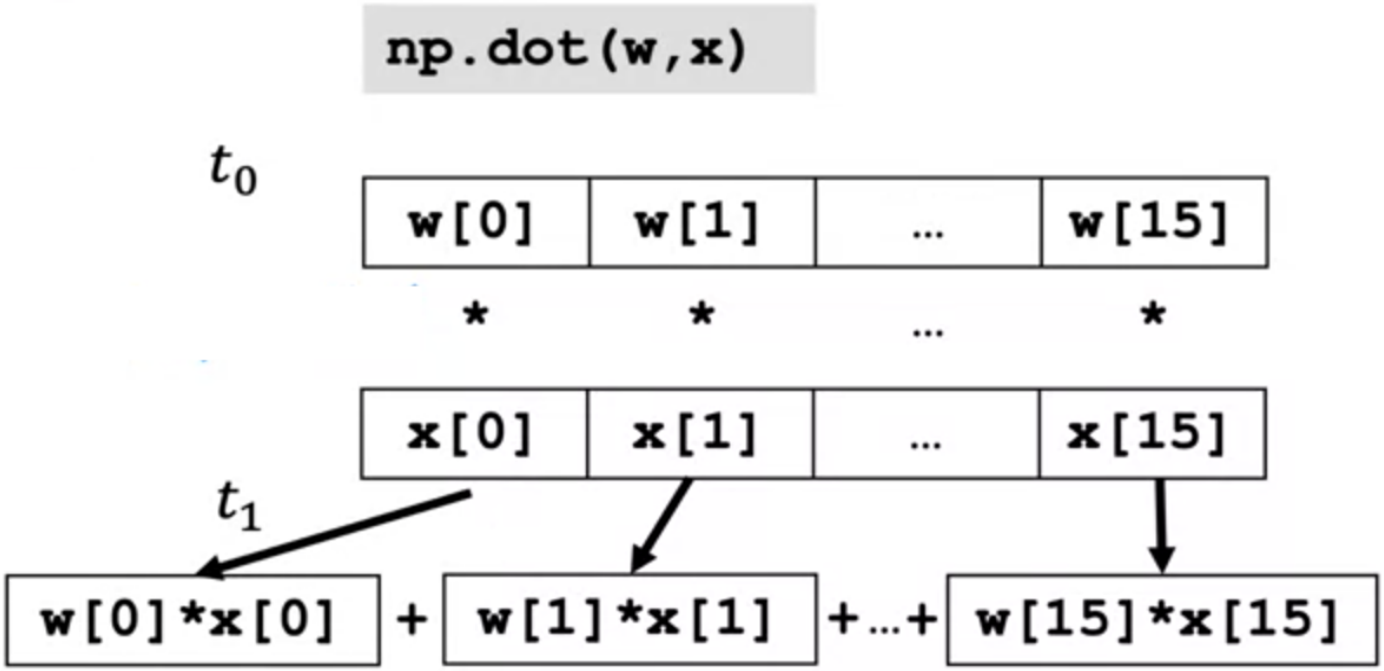


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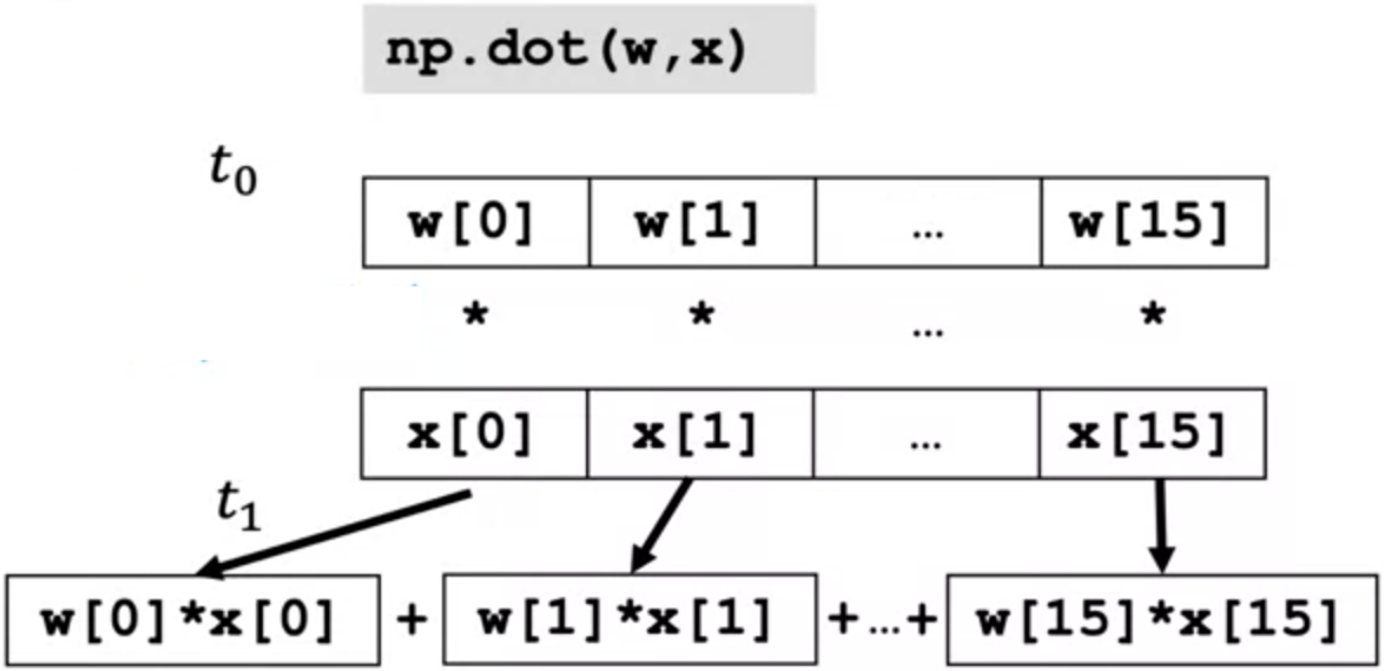


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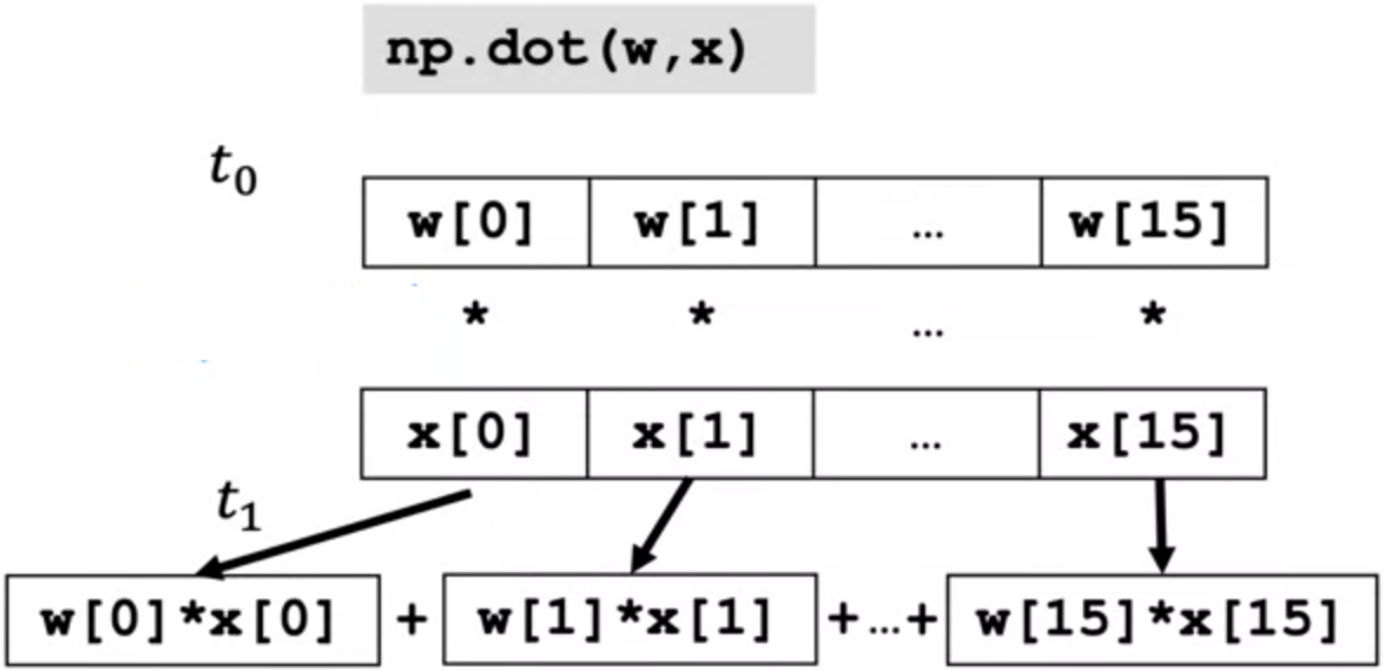


Terms are computed in parallel, using parallel processing features of the underlying computer chip, leading to a dramatic improvement in performance. (Scales well to large datasets.)

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  - $\vec{d} = (d_1, d_2, \cdots d_{16})$ - vector of partial derivatives wrt $w$'s
- We store these vectors in numpy arrays:

```
w = np.array([0.5, 1.3, … 3.4])
d = np.array([0.3, 0.2, … 0.4])
```

# Vectorization

- Non-vectorized approach using a for-loop (poor performance):

$$w_1 = w_1 - 0.1d_1$$
$$w_2 = w_2 - 0.1d_2$$
$$\vdots$$
$$w_{16} = w_{16} - 0.1d_{16}$$

```
for j in range(0,16):
    w[j] = w[j] - 0.1 * d[j]
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  - Less code
  - Flexible - handles vectors of different sizes with not change to code