Integrating Advanced Risk-Return Metrics with Efficient Frontier Portfolio Rebalancing

Artem Yalovenko, DePaul University, School of Computing, ayaloven@depaul.edu
Alex Jenner, DePaul University, School of Computing, ajenner1@depaul.edu
Mark Haskins, DePaul University, School of Computing, mhaskin1@depaul.edu
Nidhi Kiran Joshi, DePaul University, School of Computing, njoshi9@depaul.edu

ABSTRACT

In this paper, we explore the ability of various established risk metrics to enhance portfolio rebalancing strategies by leveraging Monte Carlo Simulation and expanding on Efficient Frontier analysis. The paper summarizes risk-return assessment of the best of thousands of portfolio combinations across various sectors through analysis of advanced risk metrics and backtesting on historical data. We seek to develop a portfolio optimization tool that allows for comparison of Mean-Variance optimized portfolio and an equal weight portfolio against a portfolio allocation computed using a composite of risk-return metrics. The results across the tested baskets of stocks, which were picked from various industry sectors, yielded variable returns; some extensively outperforming the S&P 500 benchmark and some not.

KEYWORDS: Portfolio optimization, Monte Carlo simulation, Efficient frontier, Stock performance, Risk-return tradeoff

INTRODUCTION

Asset allocation is a fundamental component of investment strategy which has become some of the most quantitatively researched topics since the introduction of computers. Companies and finance experts rely on established theories and research to provide risk-tailored client investment recommendations. In this paper, we attempt to assess an established portfolio rebalancing approach as well as attempt to enable it to be even more risk aware. This paper will expand on the potential to use Monte Carlo simulations and Efficient Frontier analysis to enhance portfolio rebalancing strategies. We aim to achieve the integration of multiple financial portfolio risk metrics when deriving portfolios that perform way better than traditional approaches, like equal-weight (1/n) allocation or those based on only the Modern Portfolio Theory (MPT) Efficient Frontier. Complete backtesting and simulations on these thousands of different combinations of risk metrics will investigate how they can influence the portfolio outcome, giving a nuanced understanding of the dynamics of risk and return in various market conditions.

The research builds up the work of Markowitz's Modern Portfolio Theory, specifically the Mean-Variance portfolio optimization technique, through the application of Monte Carlo simulations, allowing the generation of 100000 unique portfolio scenarios. By combining various financial risk metrics, we aim to develop a more robust framework for portfolio construction that surpasses the traditional MPT Efficient Frontier and 1/n strategies, facilitating more strategic investment decisions for long-term growth and effective risk management.

LITERATURE REVIEW

The use of Monte Carlo simulation combined with Modern Portfolio Theory has been the basis of research surrounding portfolio optimization for decades. See Table 1 for a summary of the literature on this topic.

Table 1: Summary of Literature Review

YEAR	REFERENCES	JOURNAL
2023	Al Janabi	Studies in Economics and Finance
2017	Al Janabi et al.	European Journal of Operational Research
2018	Ban et al.	Management Science
2014	Bao et al.	Proceedings of the World Congress on Engineering
2023	Cagliero et al.	Expert Systems with Applications
2021	Castro et al.	Journal of Risk and Financial Management
2003	Cesari & Cremonini	Journal of Economic Dynamics and Control
2003	Cvitanić et al.	Journal of Economic Dynamics and Control
2012	Emmanuel	International Journal of Advanced Studies in Computers, Science and Engineering
2024	Gao	Advances in Economics, Management and Political Sciences
2014	Ghodrati & Zahiri	Management Science Letters
2022	Kobets & Savchenko	Information Technology and Implementation
2023a	Li	Highlights in Business, Economics and Management
2023b	Li	Advances in Economics, Management and Political Sciences
2014	Pedersen	S&P Global Market Intelligence
2020	Shadabfar & Cheng	Alexandria Engineering Journal
2019	Siswanah	IOP Conf. Series: Materials Science and Engineering
2017	Song et al.	Neurocomputing
2021	Xiang et al.	International Conference on Developments in eSystems Engineering
2018	Zhang et al.	Quantitative Finance

Application of Monte Carlo Simulation in Portfolio Optimization

Cvitanić et al. used a simple Monte Carlo model and determined that it can be successfully used whether the investor is looking for intertemporal consumption or terminal wealth. They concluded the degree of risk aversion was a key factor of the portfolio's value (2003). Bao et al. used the Least Squares Monte Carlo Method and incorporated transaction costs and investor risk preferences. They found that the approach beat the benchmark portfolio that was passively managed and was successful in optimizing dynamic portfolios (2014). Similarly, Ghodrati & Zahiri combined the Markowitz and Winker methods with Monte Carlo simulation to predict the losses of 26 chemical companies, and they found that they could attain maximum return and least variance using their model (2014).

To maximize returns while controlling risk, ratios like the Sortino and Sharpe ratios have been widely utilized. Cesari & Cremonini compared nine different portfolio optimization strategies employing return at risk, Sortino ratio, and Sharpe ratio using Monte Carlo simulation. They found that different strategies perform better depending on the market. Constant proportion strategies worked better in bear markets and constant mix strategies were better in bull markets (2003). Li computed volatility and returns and created portfolios using Monte Carlo simulation. The portfolio with the lowest return was the one with the minimum variance, while the portfolio with the largest Sharpe ratio produced the best return (2023a). In the same year, Monte Carlo simulation was used to optimize portfolios based on maximum Sharpe ratio and minimum variance. The portfolio constructed from maximum Sharpe ratio performed better than the S&P 500 and the minimum variance portfolio underperformed (Li, 2023b). Very recently, Gao used Monte Carlo simulation to construct the efficient frontier and created three types of portfolios, including equal-weighted, maximum Sharpe ratio, and global minimum variance. Only the maximum Sharpe portfolio and equal-weighted portfolio performed better than the S&P 500 (2024).

Other measures beyond Sharpe Ratio and Sortino Ratio have also been tested. Castro et al. focused on portfolio optimization using the Omega performance measure rather than the Sharpe ratio. Compared to conventional mean-variance models, this produced greater performance (2021). Shadabfar & Cheng used a probability-based method to portfolio optimization, determining the mean and standard deviation of stock returns to feed into a Monte Carlo algorithm (2020).

Expanding on Modern Portfolio Theory and Advanced Techniques

Since the introduction of Modern Portfolio Theory, numerous scholars have challenged the theory and suggested extensions and substitutes, contending that the Markowitz approach to portfolio management has too many restrictions when it comes to actual data (Emmanuel, 2012). In his comparison of the Markowitz and Kelly methods for portfolio optimization, Pedersen (2014) discovered that the geometric mean-based Kelly method outperformed the mean-variance method. Other alternatives to the Mean-Variance Efficient Frontier, such as the Resampled Efficient Frontier method, have also been proposed. This method averages many efficient frontiers created using Monte Carlo simulations. In one instance, portfolios created via Resample Efficient Frontier had higher Sharpe Ratios but suffered greater losses during downward trends in stock prices (Siswanah, 2019).

The most often mentioned flaw in contemporary portfolio theory is that it ignores expenses like transaction fees that come with dealing in illiquid markets. Zhang et al. created a portfolio optimization algorithm using Monte Carlo simulation with the addition of backward recursive programming using least-squares regression. In unstable markets, they discovered that their algorithm could safeguard a portfolio (2018).Al Janabi et al. presented an algorithm for portfolio optimization that accounts for liquidity-adjusted Value-at-Risk constraints. They found that it better captured certain constraints than regular value at risk measures and produced a better efficient frontier than the traditional Markowitz method (2017). In order to further address the shortcomings of conventional value-at-risk approaches, Al Janabi subsequently improved upon this algorithm (2023). Xiang et al. took into account the stochastic character of stock prices when developing a Geometric Brownian Motion model and Monte Carlo simulation to forecast changes in stock prices. They found that it was more accurate in predicting increases than decreases (2021).

Machine Learning in Portfolio Optimization

Various machine learning techniques have also been incorporated into the study of portfolio optimization. Ban et al. Incorporated regularization and cross-validation into mean-variance and mean conditional value at risk portfolio optimization methods with promising results (2018). Kobets & Savchenko used a linear regression model and LSTM neural network to predict closing prices. The model outperformed tests that used historical data on its own (2022). Cagliero et al. proposed combining machine-learning stock strategies with candlestick pattern recognition and found that high-risk strategies are more effective in smaller markets, while more conservative approaches are recommended in large markets (2023). Text sentiment analysis has even been introduced into portfolio optimization strategy. Song et al. used a Naïve Bayes algorithm and combined sentiment analysis of the news and historical price data to predict stock behavior. Their model achieved high accuracy in predicting stock price behavior (2017).

DATA

This paper evaluates and compares portfolio rebalancing strategies under different asset classes using historical price data from various sectors based on the yfinance library. The dataset comprises daily adjusted close prices for chosen lists of publicly traded companies for the following sectors: technology, communication, finance, healthcare, consumer discretionary, value, and growth. The last year of our dataset (year 2023) is our test set, and the data spans from January 1, 2013 to December 31, 2023.

The get_historical_data function makes a call to get the data from yahoo finance python library, yfinance, by taking ticker symbols and returning a DataFrame of adjusted closing prices for the specified time for all tickers.

We then split the data into test and training sets. The training data, stored in variable d, is historical data used to run the simulation. The variable dT stores the test set and contains the adjusted closing prices for the year 2023. It was derived by slicing the dataframe by 252 days, which is the number of trading days in a single calendar year.

We calculate both arithmetic and geometric returns of the stocks over 9 years present in the data. Even though geometric returns are computed, they are only used for results comparison since to calculate volatility, which is required for most of the risk metrics we leverage, we need the returns to be additive. Geometric Average Returns are useful for results comparison because they consider the compounding impact. It is a more precise measure of the performance of investments over an extended period.

We calculate the arithmetic daily and annual returns which are then used to build a daily and annualized covariance matrix. The annualized covariance matrix is used in calculating volatility calculation, an industry standard practice.

Additionally, every basket of stock contains the 10-year treasury rate (^TNX) which we designate as the risk-free rate for certain risk measures such as Sharpe ratio. The ^TNX variable is converted to daily percentage and annualized to get average daily rate. The column is then removed from the data frame since we do not need it for simulation.

Our final data manipulation is calculation of downside-deviation (DD) required for Sortino Ratio calculation. Downside deviation is computed using only negative returns that fall below a target return, which is break-even 0 in our project, but could also be set as risk free rate. We calculate the DD by filtering the daily returns to below the target return and then derive their variance. Lastly, we get the deviation by taking the square root of the variance, annualizing it by multiplying by 252 (the number of trading days in a year), and finally taking the mean across the assets in the sector basket.

METHODOLOGY

Theoretical Development

Our research attempts to extend Markowitz Portfolio Theory (MPT), which derived the concept of 'efficient frontier' of optimal portfolios. The efficient frontier theory allows for simple identification of optimal investment allocation while maximizing expected return for a given risk level. The theory rates portfolios on scale of return vs risk, where risk is often considered stock volatility *(Mangram 2013).* The efficient frontier is a relatively simple calculation and can be expressed by the below equation:

$$\min_{w}(w^T \Sigma w) \text{ subject to: } w^{T\mu} = R, \quad w^{T1} = 1$$

where:

- w is the vector of portfolio weights
- Σ is the covariance matrix of the portfolio returns
- μ is the vector of expected returns
- *R* is the desired return
- 1 is a vector of ones

Since efficient frontier is based on risk and return profile of a portfolio, the naturally complementing calculation is a Sharpe Ratio. Sharpe ratio is a common portfolio performance metric that tells how much extra return can be expected per unit of volatility *(Sep, 2022) (Benhamou,Guez,Paris,2019).* This simple metric becomes especially useful when holding riskier assets (assets with higher volatility) such as health care stocks. A higher Sharpe value is preferred as it indicates better risk adjusted performance which is generally considered "good" Sharpe ratio being 1 or higher.

Sharpe Ratio can be Defined with this formula:

Sharpe Ratio=$\frac{R_p - R_f}{\sigma_p}$

- where *Rp is* the asset return,
- *Rf is* the risk-free rate,
- and σp the standard deviation of the portfolio's **excess** return.

Our extension of the MPT's Efficient Frontier lies with the addition of multiple advanced risk metrics alongside Sharpe ration. While not directly recreating the similar looking efficient frontier visualization which would account for the additional risk metrics, we are comparing the results of Maximum and Minimum risk adjusted portfolios to portfolios optimized for the additional risk calculations. Our hypothesis suggests that by using Monte Carlo simulation derived portfolio weights we can locate the best investment portfolio by optimizing on these advanced risk metrics and outperform the Efficient Frontier-derived portfolio as well as an equal weight (1/n) portfolio. The performance will be judged based on how well they balance risk and return and how they perform in different sector-market conditions.

Advanced Risk and Performance Metrics

To expand on the MPT optimization method, our optimization used several additional metrics to evaluate each portfolio's risk-return profile. Some of them were strategically picked to ensure investors have multidimensional risk protection. Sortino ratio is one of the most known, such risk-return evaluation metrics, often used to convey the riskiness of hedge funds to potential investors (technically all of our metrics are commonly used to evaluate hedge funds). Sortino Ratio focuses on downside risk and is more relevant for investors concerned with negative returns. The ratio formula is the same as Sharpe with exception of the denominator

Sortino Ratio=$\frac{R_p - R_f}{\sigma_d}$

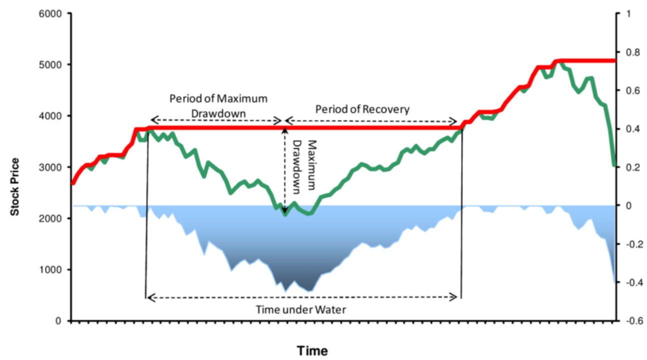
- where *Rp is* the asset return,
- *Rf is* the risk-free rate,
- σd is the standard deviation of **negative** asset returns.

Unlike the Sharpe Ratio, the Sortino Ratio ignores the overall ups and downs and only looks at how much the investment can lose. *(Sep, 2022)* While being a great risk indicator since it allows for the identification of investments during market downturns, a high Sortino Ratio may not be good for very aggressive investors. In general, a Sortino ratio of 2 or greater is considered very good. The ratio does have limitations like not being very appropriate for portfolios where investments have symmetric returns because of the sole downside risk focus. It may also be inefficient when comparing investments with very different return profiles (e.g. aggressive growth tech stock vs low P/E and high dividend value stock).

Another metric we used for portfolio evaluation and optimization was Calmar Ratio. It is also a risk adjusted return measurement of an investment but focuses specifically on maximum drawdown.

Maximum drawdown is one of the most important metrics in algorithmic and quantitative trading to track. The concept behind max drawdown is simple. Maximum Drawdown measures the largest drop from peak to trough before a new peak in the stock price *(Fischer & Lind-Braucher, 2009)*. It is critical because it indicates the potential for recoverable losses and worst-case scenarios probability. Figure 1 provides a graphical representation of the Max Drawn metric.

Figure 1: Max Drawdown



(Fischer & Lind-Braucher, 2009)

We wanted to use Calmar Ratio because of its relation to maximum drawdown calculation, but we also used the raw max drawdown value as one of variables within the composite score optimization. Calmar Ratio formula is simple:

Calmar Ratio= $\frac{\text{Annual Return}}{\text{Maximum Drawdown}}$

Therefore, a high Calmar Ratio indicates that the investment has generated substantial returns in relation to worst risks within a given market timeline, which in turn would tell us the long-term investment stability and growth of the portfolio. (Sep,2022)

We used more comprehensive portfolio profile measures such as Omega Ratio and various derivatives of Value at Risk calculations.

Omega Ratio does not assume a normal distribution of returns unlike MPT's efficient frontier and instead places increased emphasis on the tail end of returns distribution. The ratio captures the probability weighted ratio of possible gains vs losses and since many of assets' distribution of returns aren't symmetrical, and tail risks can be significant, Omega Ratio is one most robust metrics to optimize their asset allocation *(Benhamou,Guez,Paris,2019).* Omega can, however, be sensitive to outliers and thus sensitive to extreme market events and like the other metrics is sensitive to threshold return level. In general, Omega Ratio of 1 is expected to be satisfactory and below 1 suggests a portfolio is more likely to underperform.

Omega Ratio= $\frac{\int_{\text{threshold}}^{\infty}(1-F(x))\,dx}{\int_{-\infty}^{\text{threshold}}F(x)\,dx}$

where

- X is a threshold return which is typically risk-free rate or 0.
- *F*(*x*) is the cumulative distribution function of returns x.

Another non-return-to-volatility to risk measure is utilized to assess portfolios, and one that doesn't assume normal distribution is Value at Risk or VaR. We specifically calculate VaR using the historical method; this was we do not rely on any return's distribution assumption, but instead use actual return data of our assets to estimate potential losses. The obvious advantage of this method is that is non-parametric (no distribution assumption) and easy to implement. However, VaR on its own is sensitive to extreme events (*Sarykalin, Serraino, Uryasev 2014)* which is why we leverage two derivatives of VaR: Conditional Value at Risk (CVaR), and Worst Conditional Value at Risk (WCVaR). CVaR, which is also known as Expected Tail Loss considers the tail end of the loss distribution beyond the VaR threshold thus giving us an ability to evaluate the average of potential losses exceeding VaR *(Rockafellar, Uryasev 2014)*. WCVaR provides the worst-case scenario beyond VaR. We wanted to include this measure because it represents the most severe potential loss in the tail *(Zhu, Fukushima, 2009)*.

An example of these metrics' interpretation:

VaR: At 95% confidence interval and VaR is -3%, means that there is a 5% chance that the portfolio will lose more than 3% of its value at any day.

CVaR: Assume that average of all returns that are worse than -3% (tail aka 5% percentile) is -4%. Thus, CVaR = -4%. So if you consider worst 5% of days, you can expect and average loss of 4%.

WCVaR: Assume that the worst return within the worst 5% of returns is –6%. WCVaR is –6%.

The formulas for VaR and VaR derivatives are:

VaR*α* = $- \inf\{x \mid P(X \leq x) \geq 1 - \alpha\}$

where

- α is the confidence level (e.g., 95% or 99%),

CVaR provides the expected loss exceeding VaR:

CVaR*α* = $\mathrm{CVaR}_\alpha = \mathbb{E}\left[R \mid R \leq \mathrm{VaR}_\alpha\right]$

Where:

- R represents the returns
- VaRα is the Value at Risk at the α confidence level.

WCVaRa = $\mathrm{WCVaR}_\alpha = \min(R \mid R \leq \mathrm{VaR}_\alpha)$

MODEL

After identifying 15 tickers within 7 different sector categories (technology, communication, finance, consumer discretionary, health, value, growth), we pulled the data using yfinance library and made multiple manual calculations to assist in the results evaluation and further computations as described in the data section of the paper.

Our portfolio optimization leverages Monte Carlo simulation to generate 100,000 random unique portfolio weight combination allowing us to calculate the above-described risk metrics within the MC simulation itself issuing a single table containing risk metrics alongside returns, volatility and the individual stock weight allocations. We ensure the uniqueness of each portfolio weighting by used a unique_weights set and ensure that each portfolio weights sum up to 1 by normalizing the weights.

Within the MC simulation loop we calculate multiple critical metrics. First, the expected annual return of the portfolio with a dot product of assigned weights and individual asset annual returns minus the risk-free rate derived from the 10-year treasury bond rate. For each simulated weight, we calculate volatility as a square root of the variance derived from covariance matrix of returns described in the data section. Sharpe ratio is calculated by dividing the portfolio returns calculation (which was already adjusted for risk free rate) by the volatility. Sortino ratio is calculated by diving the portfolio returns calculation by downslide deviation variable calculated before the MC loop started, described in the data section of the paper.
Maximum Drawdown is derived through multiple steps. First, we calculate the portfolio with new weights' daily returns by multiplying the matrix of daily returns for each of the tickers by the weights giving us a vector of weighted sums of returns for each day in the training set. We calculate cumulative returns next by converting the previously derived portfolio daily returns to growth factors using a simple 1+daily return operation. Then we used the .cumprod() function to get the cumulative product of these growth factors over time. Now we can get the drawdown for a given day by using .cummax() which is then used then subtracted from cumulative_returns and used as a denominator. Finally, the max drawdown is located by finding the minimum drawdown value within an absolute function. Max drawdown by itself is a valuable measure, but also use it to calculate Calmar Ratio which simply divides the newly weighted portfolio returns by the max drawdown value.

Using the portfolio daily returns values (explanation of calculation of which is in the above paragraph), we calculate the VaR, CVaR, WCVaR inside the MC simulation loop. The portfolio's daily returns are sorted and VaR is located by selecting the returns at the position in the sorted array corresponding to the value at 5% tail of the returns. Then, the CVaR is calculated by locating the returns below that value and computing their average. Finally, WCVaR is calculated just like the CVaR, but instead of average we find the worst value within the 5% tail of the returns.

At last Omega Ratio value is calculated by using our threshold return value of 0. Here we didn't use the risk-free rate and instead opted for a different threshold. We compute omega ratio by finding all the returns in the newly weighted portfolio that are above the threshold and returns above the threshold. Then we get the omega ratio value by summing the excess returns (above threshold) and divide it by the sum of the short falling returns (below threshold).

After all the calculations are completed, each value for each portfolio's weights is appended their preset lists as specified in the data section of the paper. These lists are later used to create a pandas data frame from dictionary that was set to use the lists.

After running the Monte Carlo simulation to produce 100,000 different weighted portfolios we created a single composite score for each according to the 7 metrics listed in the methodology section. We accomplished this by creating a dictionary of weights for the Sharpe, Sortino, and Calmar Ratios and assigning a value of 1 as we wanted to maximize these metrics as they are indicative of better risk-adjusted returns. We then assigned weights of –1 to the Max Draw Down, VaR, CVaR, and WCVaR metrics as we want to minimize the metrics as they measure potential losses or risk. We then standardized these metrics to normalize them for comparison. Next, we created a composite score by multiplying each of the weights by the new standardized metrics and adding them together to generate a single score that looks at both return and risk characteristics of each portfolio. Lastly, we select the portfolio with the highest composite score as our optimal portfolio. Table 2 lists the optimal portfolio selected for the Technology Sector and its corresponding metrics.

Table 2: Tech Sector Optimal Portfolio Metrics	
Returns	0.343321
Volatility	0.30353
Sharpe Ratio	1.131095
Sortino Ratio	1.416526
Calmar Ratio	0.692353
Max Drawdown	0.43762
VaR 95%	-0.030167
CVaR 95%	-0.044328
WCVaR 95%	-0.154923
Omega Ratio	1.237269
AAPL Weight	0.026201
CAN Weight	0.079022
ADBE Weight	0.050061
AMAT Weight	0.015793
AMD Weight	0.173112
AVGO Weight	0.214529
CRM Weight	0.022573
CSCO Weight	0.066693
INTU Weight	0.021019
MSFT Weight	0.017415
NVDA Weight	0.273872
ORCL Weight	0.030044
QCOM Weight	0.00121
TXN Weight	0.008457

Standardized Max Drawdown	0
Standardized VaR 95%	-0.4566
Standardized CVaR 95%	-0.369714
Standardized WCVaR 95%	-0.307601
Composite Score	4.373889

To create the equal weight portfolio, we took the returns of the individual stocks for the historical training data set and assigned each of them an equal weight. For example, if we had 10 stocks in the portfolio, each would be assigned a weight of 10%. We then calculated the cumulative returns of the portfolio by multiplying the weights by the returns to produce the total return over the 9-year period. We then calculated the annualized returns by scaling the cumulative returns to a single year based on the 252 trading days in the dataset. We then calculated the annualized volatility by taking the standard deviation of the daily weighed returns and multiplying them by the square root of the of the 252 trading days to scale to a year. We then calculated the Sharpe, Sortino, and Calmar ratios, along with the VaR, CVaR, and WCVaR metrics for each 1/n portfolio within each sector basket of stocks. Table 3 shows the Equal Weight Portfolio along with the corresponding metrics.

Table 3: Equal Weight Portfolio Metrics	
Returns	0.255318
Volatility	0.239467
Sharpe Ratio	1.024434
Sortino Ratio	0.967031
Calmar Ratio	0.752005
Max Drawdown	0.339517
VaR 95%	-0.023256
CVaR 95%	-0.035454
WCVaR 95%	-0.148786
AAPL Weight	0.066667
ACN Weight	0.066667
ADBE Weight	0.066667
AMAT Weight	0.066667
AMD Weight	0.066667
AVGO Weight	0.066667
CRM Weight	0.066667
CSCO Weight	0.066667
INTU Weight	0.066667
MSFT Weight	0.066667
NVDA Weight	0.066667
ORCL Weight	0.066667
QCOM Weight	0.066667

TXN Weight	0.066667

To create the Minimum Risk and Maximum Risk Portfolios, we used Efficient Frontier analysis which selected the Max Sharpe Ratio and the minimum Volatility portfolio. Figure 2 shows the output of the analysis and the portfolios selected.

Figure 2: Tech Sector Efficient Frontier Portfolios

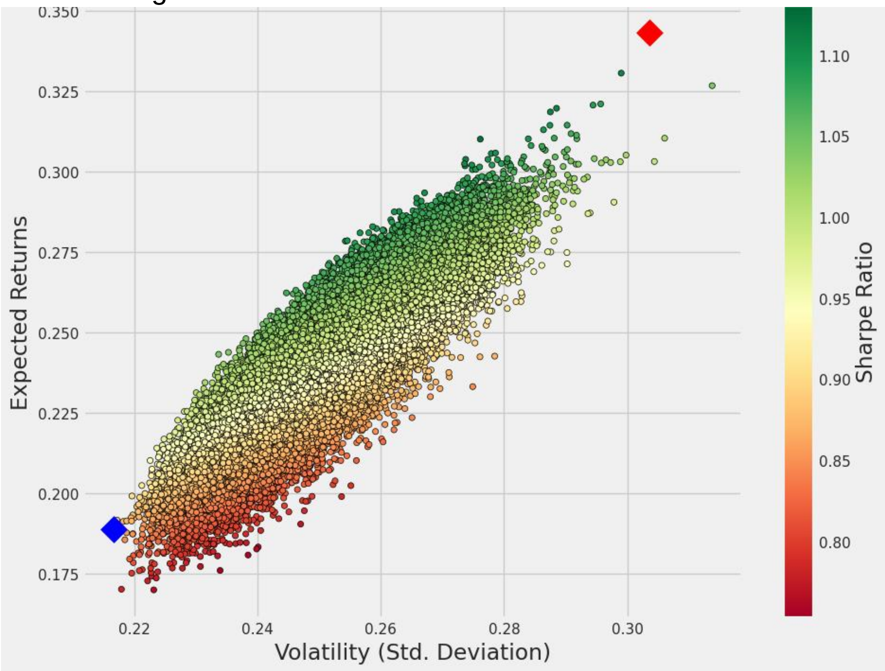


Table 4 below shows the metrics and weight allocations for Minimum Risk and Maximum Risk portfolios selected.

Table 4: Tech Sector Min and Max Risk Portfolios		
Returns	0.18907	0.343321
Volatility	0.21657	0.30353
Sharpe Ratio	0.87302	1.131095
Sortino Ration	0.78008	1.416526
Calmar Ratio	0.55516	0.692353
Max Drawdown	0.43762	0.43762
VaR 95%	-0.0208	-0.030167
CVaR 95%	-0.0324	-0.044328
WCVaR 95%	-0.1255	-0.154923
Omega Ratio	1.19882	1.237269
AAPL Weight	0.10859	0.026201
ACN Weight	0.13359	0.079022

ADBE Weight	0.08394	0.050061
AMAT Weight	0.0131	0.015793
AMD Weight	0.00147	0.173112
AVGO Weight	0.0226	0.214529
CRM Weight	0.03277	0.022573
CSCO Weight	0.10273	0.066693
INTU Weight	0.05728	0.021019
MSFT Weight	0.13613	0.017415
NVDA Weight	0.004	0.273872
ORCL Weight	0.146	0.030044
QCOM Weight	0.03323	0.00121
TXN Weight	0.12458	0.008457

Separate data tables were made to collect and understand the results and tendencies of the portfolio optimizations between the various equal weight, efficient frontier's max, efficient frontier's minimum risk, and above-mentioned composite score portfolio.

Each of these portfolios were back tested using the test set to mimic its performance in actual market conditions. This was done by allocating a hypothetical $1000 for each of the optimized portfolios to trade, calculating the percentage change of each ticker within the test data set, the two are multiplied by each other and by the portfolio weights and finally aggregated together. This allowed us to evaluate the best performing portfolio in the year 2023.

RESULTS

Table 5 shows a table of the results showing actual gain/loss of each portfolio type: equal weight, minimum risk, maximum, risk, and Monte Carlo Composite Score optimized by each sector and sorted by Actual Gain/Loss.

Table 5: All Portfolio Results				
Portfolio Type	Sector	Exp % Gain/Loss	Act % Gain/Loss	|% Difference|
MC Combined Metrics	Tech	34.33%	125.04%	90.71%
Maximum Risk	Tech	34.33%	125.00%	90.67%
MC Combined Metrics	Growth	35.63%	115.77%	80.14%
Maximum Risk	Growth	35.51%	108.23%	72.71%
Equal Weight	Growth	77.42%	80.96%	3.54%
MC Combined Metrics	Comms	22.98%	68.32%	45.34%
Equal Weight	Tech	62.94%	66.62%	3.68%
Minimum Risk	Growth	23.88%	62.38%	38.50%
Minimum Risk	Tech	18.91%	42.70%	23.80%

Maximum Risk	Financial	19.29%	36.42%	17.13%
Equal Weight	Comms	33.27%	34.56%	1.29%
Maximum Risk	Comms	22.01%	31.74%	9.72%
Equal Weight	Consumer	29.28%	29.41%	0.13%
Maximum Risk	Consumer	13.33%	24.10%	10.78%
MC Combined Metrics	Value	14.06%	23.10%	9.04%
MC Combined Metrics	Consumer	10.72%	22.72%	12.00%
Minimum Risk	Consumer	11.05%	22.35%	11.30%
MC Combined Metrics	Financial	19.01%	21.75%	2.73%
Minimum Risk	Comms	11.27%	20.76%	9.49%
Minimum Risk	Financial	17.00%	19.48%	2.48%
Equal Weight	Financial	19.78%	18.91%	0.87%
Equal Weight	Value	13.24%	13.81%	0.57%
Maximum Risk	Health	20.28%	7.80%	12.47%
Equal Weight	Health	4.83%	4.84%	0.01%
MC Combined Metrics	Health	20.63%	4.81%	15.82%
Maximum Risk	Value	13.26%	3.75%	9.51%
Minimum Risk	Value	9.45%	0.07%	9.39%
Minimum Risk	Health	15.41%	-4.34%	19.75%

Our best performing portfolio was the Monte Carlo Composite Score Optimized Technology Portfolio which gained 125.04%. We can also see that the Monte Carlo Composite Score Optimized Technology Portfolio had the biggest delta between expected and actual returns of 90.71%. Another interesting result was that our Monte Carlo Composite Score Optimized Growth Portfolio also returned 115%. Both portfolios contained the same 8 out of the 15 stocks.

Table 6 shows that 50% or 11 out of our 22 portfolios beat the non-optimized equal weight portfolios. These mostly included the Maximum Risk and Monte Carlo Composite Score Optimized Portfolios.

Table 6: Comparison to Equal Weight Portfolios				
Type	Sector	EWQ % Gain	Act % Gain	% Diff EQW
MC Combined Metrics	Tech	66.62%	125.04%	58.42%
Maximum Risk	Tech	66.62%	125.00%	58.38%
MC Combined Metrics	Growth	80.96%	115.77%	34.81%
MC Combined Metrics	Comms	34.56%	68.32%	33.76%
Maximum Risk	Growth	80.96%	108.23%	27.27%
Maximum Risk	Financial	18.91%	36.42%	17.51%
MC Combined Metrics	Value	13.81%	23.10%	9.29%
Maximum Risk	Health	4.84%	7.80%	2.96%

MC Combined Metrics	Financial	18.91%	21.75%	2.84%
Minimum Risk	Financial	18.91%	19.48%	0.57%
MC Combined Metrics	Health	4.84%	4.81%	-0.03%
Maximum Risk	Comms	34.56%	31.74%	-2.82%
Maximum Risk	Consumer	29.41%	24.10%	-5.31%
MC Combined Metrics	Consumer	29.41%	22.72%	-6.69%
Minimum Risk	Consumer	29.41%	22.35%	-7.06%
Minimum Risk	Health	4.84%	-4.34%	-9.18%
Maximum Risk	Value	13.81%	3.75%	-10.06%
Minimum Risk	Value	13.81%	0.07%	-13.74%
Minimum Risk	Comms	34.56%	20.76%	-13.80%
Minimum Risk	Growth	80.96%	62.38%	-18.58%
Minimum Risk	Tech	66.62%	42.70%	-23.92%

Table 7 shows that Our MC Combined Metrics technology sector, Maximum Risk technology sector, and MC Combined Metrics portfolios all performed the best generating over 100% returns.

Table 7: Best Performing Portfolios				
Portfolio Type	Sector	Ending Bal	Gain/Loss	% Gain/Loss
MC Combined Metrics	Tech	$2,250.43	$1,250.43	125.04%
Maximum Risk	Tech	$2,250.04	$1,250.04	125.00%
MC Combined Metrics	Growth	$2,157.68	$1,157.68	115.77%

We believe these were the best models because the individual stocks in each of these sectors all have high growth potential compared to stocks in the broader market. The stocks in these portfolios mostly had consistent revenue growth quarter over quarter, a high Price to Earnings ratio (P/E), and no or low dividends as the companies re-invested profits back into the business to obtain higher growth. These returns were higher than we expected based upon the 10 year trading data model but were still in line with our expected geometric annual returns for 2023. Below figures show how these expected geometric returns shaped the portfolio allocation. For example, NVDA, AMD, and AVGO made up over 66% of our best performing portfolio. See Figure 3 and Figure 4 below.

Figure 3: Tech Sector Expected Geometric Returns

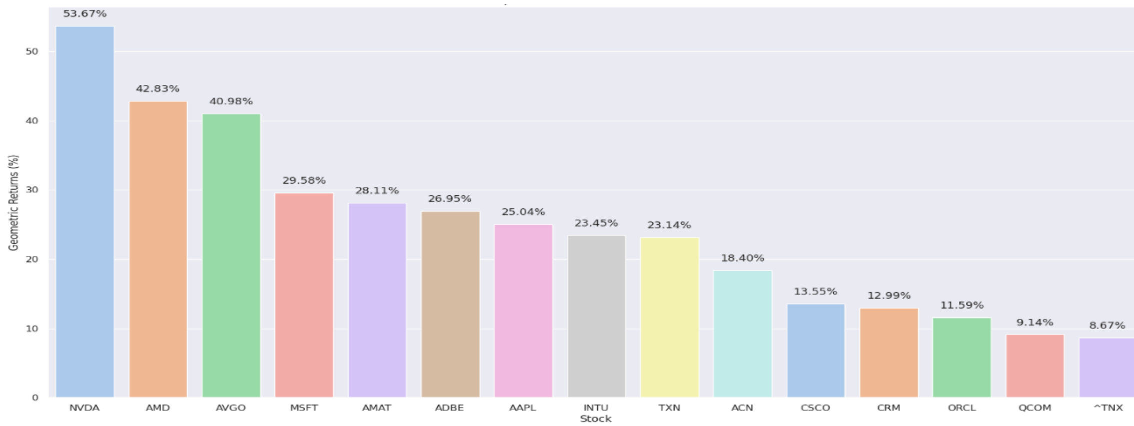


Figure 4: Tech Sector Monte Carlo Optimized Metric Portfolio Weight Allocation

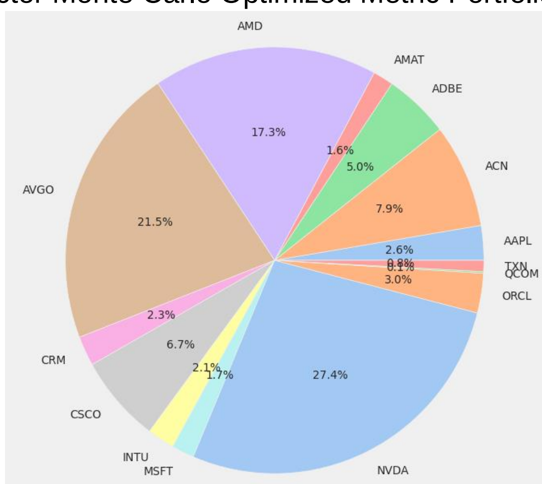


Table 8 shows are worst performing portfolios. They were Minium Risk Health Sector which lost -4.34 % over the same period. Followed by the Minimum Risk Value (.07% gain) and the Maximum Risk Value (3.75% gain)

Table 8: Worst Performing Portfolios

Portfolio Type	Sector	Ending Bal	Gain/Loss	% Gain/Loss
Minimum Risk	Health	$956.57	-$43.43	-4.34%
Minimum Risk	Value	$1,000.66	$0.66	0.07%
Maximum Risk	Value	$1,037.48	$37.48	3.75%

Healthcare underperformed the broader market in 2023.his was due to investors re-allocating portfolios for a higher interest rate environment and the healthcare industry still normalizing after the pandemic. We believe the Value sector also underperformed compared to other portfolios because the stocks in these portfolios tend to have lower returns due to stable and predictable earnings, low P/E ratios, and High Dividend Yields that prioritize paying dividends back to shareholders versus reinvesting the money.

We can see the inverse of our best performing model versus our worst performing model in that the expected geometric returns for stocks like UNH, ELV, LLY generated expected geometric returns of 49.8%, 46.12%, and 45.3% yet only made-up weights of .5%, 5.1%, and .5% in our minimized portfolio as we prioritized limiting risk and volatility versus returns. See Figure 5 and Figure 6 below.

Figure 5: Health Sector Expected Geometric Returns

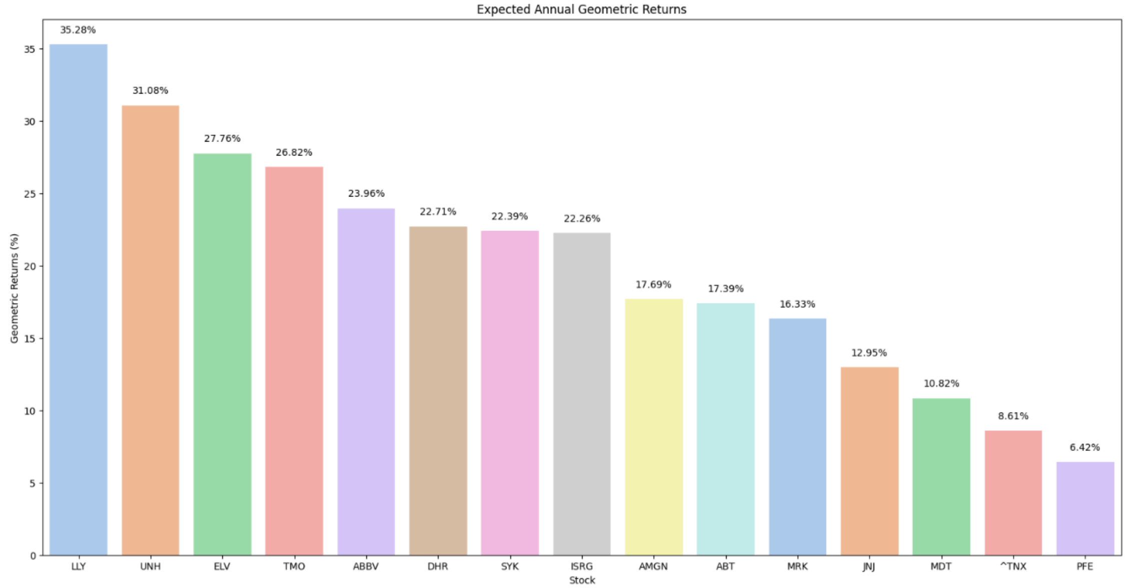


Figure 6: Health Sector Minimum Risk Portfolio Weight Allocation

Another aspect of our models was how it was rebalanced according to metrics. We rebalanced portfolios based upon creating the composite score for selecting a portfolio weighting with the highest Sharpe, Sortino, and Calmar Ratios while minimizing Max Drawdown, VaR, CVaR, and WCVaR scores to account for potential losses. Using a different sector for Financial we can see how the portfolios were adjusted. Figure 6 shows a 10% drop in weight for Blackstone between Max Risk and Optimized.

Figure 7: Financial Sector Weight Allocation by Portfolio Type

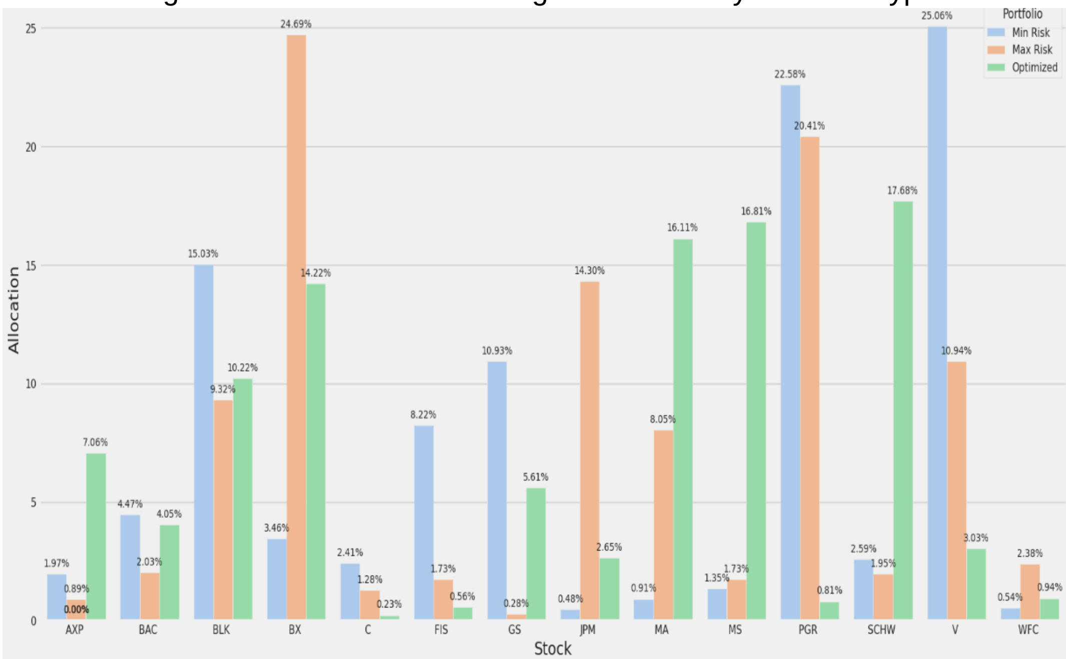
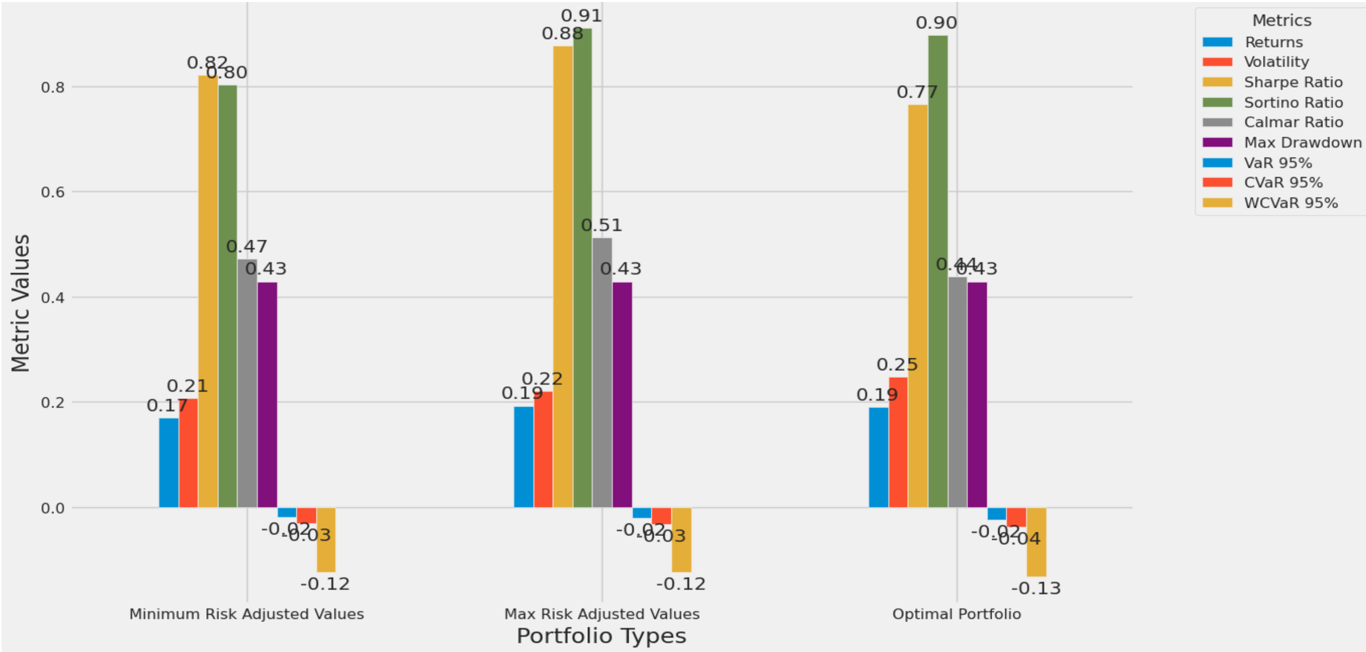


Figure 8 shows a comparison of the portfolio metrics used to evaluate the portfolios. Noticeable differences can be seen in the Sharp and Sortino Ratios for each.

Figure 8: Financial Sector Comparison of Portfolios by Metrics



When investing, a common benchmark to compare returns to is the S&P 500 due to it's broad representation of the US market and it's weight allocation by company market cap. For 2023, our test data time-period, we can see the S&P 500 returned 21.90%.

Table 9 demonstrates that 60% (13 out of 22) of our portfolios beat the S&P 500 for 2023.

Table 9: Portfolios' Performance vs S&P 500 2023			
Type	Sector	Gain vs S&P 500	% Gain vs S&P 500
MC Combined Metrics	Tech	$1,031.43	103.14%
Maximum Risk	Tech	$1,031.04	103.10%
MC Combined Metrics	Growth	$938.68	93.87%
Maximum Risk	Growth	$863.25	86.33%
MC Combined Metrics	Comms	$464.20	46.42%
Minimum Risk	Growth	$404.79	40.48%
Minimum Risk	Tech	$208.04	20.80%
Maximum Risk	Financial	$145.22	14.52%
Maximum Risk	Comms	$98.37	9.84%
Maximum Risk	Consumer	$22.03	2.20%
MC Combined Metrics	Value	$11.98	1.20%
MC Combined Metrics	Consumer	$8.17	0.82%
Minimum Risk	Consumer	$4.48	0.45%
MC Combined Metrics	Financial	-$1.53	-0.15%
Minimum Risk	Comms	-$11.38	-1.14%
Minimum Risk	Financial	-$24.20	-2.42%
Maximum Risk	Health	-$140.98	-14.10%

MC Combined Metrics	Health	-$170.87	-17.09%
Maximum Risk	Value	-$181.52	-18.15%
Minimum Risk	Value	-$218.34	-21.83%
Minimum Risk	Health	-$262.43	-26.24%

CONCLUSION

In conclusion, we were able to answer our original three research questions posed. First, we can see that 50%, or 11 out of our 22 portfolios, beat their equal weight portfolio counterparts. These high performing portfolios were mostly comprised over the higher risk Maximum Risk Adjust portfolios and the Monte Carlo Composite Score Optimized Portfolios. This was in line with our expectations as 2023 was a great year across the board for the US equities market and portfolios that maximized return over risk would generate higher returns. Second, we can see that selected portfolios that using Monte Carlo Simulations combined with additional evaluation metrics were able to outperform models selected based on Efficient Frontier analysis which focused only on returns and volatility (risk) alone. Lastly, we can see that our portfolios all performed relatively well in 2023 as these were favorable market conditions. However, the Value and Health sectors underperformed as these were less favorable sectors for 2023. Investors tend to reposition out of Value stocks during a bull market and the Health Sector was still normalizing post covid 19.

FUTURE WORK

There are multiple areas of research where our proposed rebalancing tool has not been tested or evaluated in. One of the biggest concerns we see with this tool is its relatively poor performance on seemingly diversified portfolios like consumer discretionary sector basket. Our results suggest that even with combined risk metric portfolio optimization, it may struggle to locate the best portfolio weights for stock baskets that have some poor or mediocre historical performance stocks, but it's very efficient at finding the winners within portfolios with high growth or stable growth. This is why we see it imperative to test this tool on entire S&P 500 set of stocks and compare the tool's allocation results to S&P 500 portfolio analysts' (like Vanguard S&P 500 ETF (VOO) or State Streets's SPDR S&P 500 ETF Trust (SPY)).
This also calls for more in-depth examination of our test results' individual stock performances for factors that could be additionally included into the optimization problem. For instance, Pedersen (2014) in his paper (Portfolio Optimization & Monte Carlo Simulation) used accounting metrics to feature engineer advance financial variables (e.g. Present Value of Dividend, Equity Growth Model, and Value Yield) to evaluate underlying company performance rather than just simply relying on stock prices alone. The introduction of such information into the problem could provide a more reliable insight into the company's future. Additionally, the composite score approach requires peer review from subject matter experts to confirm its validity, something we are eager to research further.

AUTHOR CONTRIBUTIONS

Artemiy Yalovenko identified the financial risk metrics, wrote preprocessing code and the MC simulation with risk calculations, created the composite score optimization and wrote code for various visualizations.
Mark Haskins performed the preliminary data analysis, methods, results, sections along with the corresponding visualizations.
Alex Jenner conducted the literature review and researched and selected stocks for analysis.
Nidhi Contributed to the literature review, citation and compilation of the report.
All authors contributed to the writing of the paper.

REFERENCES

Al Janabi, M. A. M. (2023). Constrained optimization algorithms for the computation of investable portfolios analytics: evaluation of economic-capital parameters for performance measurement and improvement. *Studies in Economics and Finance, 40*(1), 112-137. https://doi.org/10.1108/SEF-01-2020-0026

Al Janabi, M. A. M., Hernandez, J. A., Berger, T. & Nguyen, D. K. (2017). Multivariate dependence and portfolio optimization algorithms under illiquid market scenarios. *European Journal of Operational Research, 259*(3), 1121-1131. https://doi.org/10.1016/j.ejor.2016.11.019

Ban, G., El Karoui, N., & Lim, A. E. B. (2018). Machine learning and portfolio optimization. *Management Science, 64*(3), 1136-1154. https://doi.org/10.1287/mnsc.2016.2644

Bao, C., Lee, G., & Zhu, Z. (2014). A Simulation-based portfolio optimization approach with least squares learning. *Proceedings of the World Congress on Engineering, 2*(1), 905-910.

Benhamou, E., Guez, B., & Paris, N. (2019). Omega and Sharpe Raio. *SSRN Electronic Journal.* DOI:10.2139/ssrn.3469888

Cagliero, L., Foir, J. & Garza, P. (2023). Shortlisting machine learning-based stock trading recommendations using candlestick pattern recognition. *Expert Systems with Applications, 216*(C). https://doi.org/10.1016/j.eswa.2022.119493

Castro, J. G., Tito, E. A., & Brandao, L. E. (2021). Optimization of a portfolio of investment projects: A real options approach using the omega measure. *Journal of Risk and Financial Management, 14*(11). https://doi.org/10.3390/jrfm14110530

Cesari, R., & Cremonini, D. (2003). Benchmarking, portfolio insurance and technical analysis: A Monte Carlo comparison of dynamic strategies of asset allocation. *Journal of Economic Dynamics & Control, 27*(6). 987-1011. https://doi.org/10.1016/S0165-1889(02)00052-0

Cvitanić, J., Goukasian, L., & Zapatero, F. (2003). Monte Carlo computation of optimal portfolios in complete markets. *Journal of Economic Dynamics and Control*, *27*(6), 971-986. https://doi.org/10.1016/S0165-1889(02)00051-9

Emmanuel, R.E. (2012). Portfolio analysis in US stock market using Markowitz model. *International Journal of Advanced Studies in Computers, Science and Engineering, 1*(3).

Fischer, E. O., & Lind-Braucher, S. (2010). Optimal Portfolios with Traditional and Alternative Investments: An Empirical Investigation. *the Journal of Alternative Investments*, *13*(2), 58–77.

Gao, W. (2024). Portfolio optimization based on U.S. stock. *Advances in Economics, Management and Political Sciences, 59*(1), 258-264. DOI:10.54254/2754-1169/59/20231130

Ghodrati, H., & Zahiri, Z. (2014). A Monte Carlo simulation technique to determine the optimal Portfolio. *Management Science Letters, 4*(3), 465-474. DOI:10.5267/j.msl.2014.1.023

Kapsos, M., Christofides, N., & Rustem, B. (2014). Worst-case robust Omega ratio. *European Journal of Operational Research*, *234*(2), 499–507. https://doi.org/10.1016/j.ejor.2013.04.025

Kobets, V., & Savchenko, S. (2022). Building an optimal investment portfolio with Python machine learning tools. *Information Technology and Implementation*, 307-315.

Li, Z. (2023a). Portfolio allocation optimization with US equities. *Highlights in Business, Economics, and Management*, *5*, 185-191. DOI:10.54097/hbem.v5i.5075

Li, A. (2023b). Portfolio optimization by Monte Carlo simulation. *Advances in Economics, Management and Political Sciences, 50*(1), 133-138. DOI: 10.54254/2754-1169/50/20230568

Markowitz, H. (1952). The Utility of Wealth. *Journal of Political Economy*, *60*(2), 151–158. https://doi.org/10.1086/257177

Pedersen, M. (2014). Portfolio optimization and Monte Carlo simulation. *S&P Global Market Intelligence.* https://doi.org/10.2139/ssrn.2438121

Rockafellar, R. T., & Uryasev, S. (2000). Optimization of conditional value-at-risk. *the Journal of Risk*, *2*(3), 21–41. https://doi.org/10.21314/jor.2000.038

Sarykalin, S., Serraino, G., & Uryasev, S. (2014). Value-at-risk vs. conditional value-at-risk in risk management and optimization. *INFORMS TutORials in Operations Research,* 270-294. https://doi.org/10.1287/educ.1080.0052

Sen, J. (2022). A comparative study on the Sharpe ratio, Sortino ratio, and Calmar ratio in portfolio optimization.

Shadabfar, M., & Cheng, L. (2020). Probabilistic approach for optimal portfolio selection using a hybrid Monte Carlo simulation and Markowitz model. *Alexandria Engineering Journal, 59*(5), 3381–3393. https://doi.org/10.1016/j.aej.2020.05.006

Siswanah, E. (2019). Comparative analysis of mean variance efficient frontier and resampled efficient frontier For optimal stock portfolio formation. *IOP Conference Series. Materials Science and Engineering*, *846*(1), 012065. https://doi.org/10.1088/1757-899x/846/1/012065

Song, Q., Liu, A., & Yang, S. Y. (2017). Stock portfolio selection using learning-to-rank algorithms with news sentiment. *Neurocomputing*, *264*, 20–28. https://doi.org/10.1016/j.neucom.2017.02.097

Xiang, J. N. P., Velu, S. R., & Zygiaris, S. (2021). Monte Carlo simulation prediction of stock prices. *International Conference on Developments in eSystems Engineering (DeSE)*. https://doi.org/10.1109/dese54285.2021.9719349

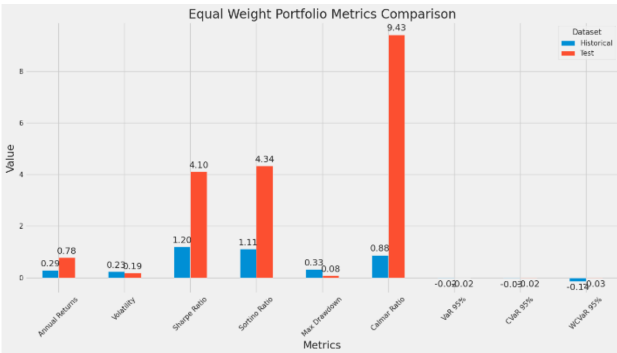
Zhang, R., Langrené, N., Tian, Y., Zhu, Z., Klebaner, F., & Hamza, K. (2018). Dynamic portfolio optimization with liquidity cost and market impact: a simulation-and-regression approach. *Quantitative Finance*, *19*(3), 519–532. https://doi.org/10.1080/14697688.2018.1524155

Zhu, S., & Fukushima, M. (2009). Worst-case conditional value-at-risk with application to robust portfolio management. *Operations Research, 57*(5), 1155-1168. https://doi.org/10.1287/opre.1080.0684

APPENDIX: COMPARISON OF PERFORMANCE

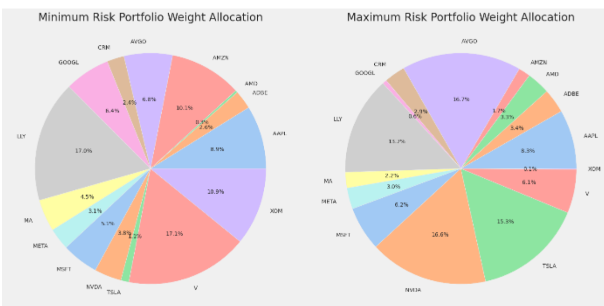
Growth Stock Simulation Results

Equal Weight (1/n) Portfolio Backtest



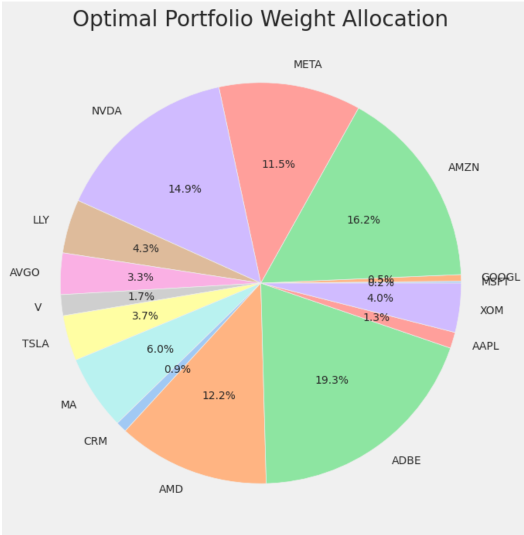

```
----------------------------------------------------------------
                Expected Returns on my Equal Weight Portfolio
----------------------------------------------------------------
Starting balance $ 1000 : Ending with $ 1774.24

----------------------------------------------------------------
Using the Equal Weights Portfolio
    Starting balance $ 1000 : Ending with $ 1809.61
----------------------------------------------------------------
```

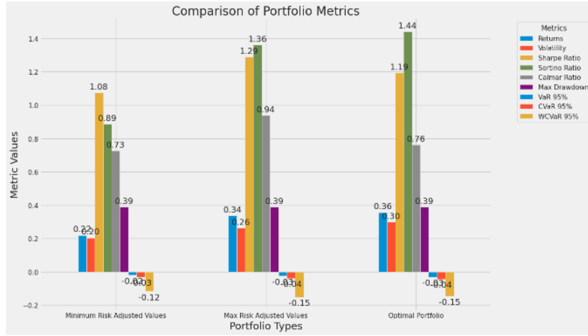
Backtests of Efficient Frontier Optimal Portfolios




```
----------------------------------------------------------------
                Expected Returns on my Portfolio
----------------------------------------------------------------
         Minimum Risk Adjusted Values  Max Risk Adjusted Values
Returns                    1238.79875               1355.126887

----------------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
----------------------------------------------------------------
Using the Portfolio Weights for Minimum Risk Return Portfolio

    Starting balance $ 1000 : Ending with $  1623.79

----------------------------------------------------------------
Using the Portfolio Weights Maximized Risk-Return Portfolio

    Starting balance $ 1000 : Ending with $  2082.25
```

MC-Based Combined Metrics Optimization Results

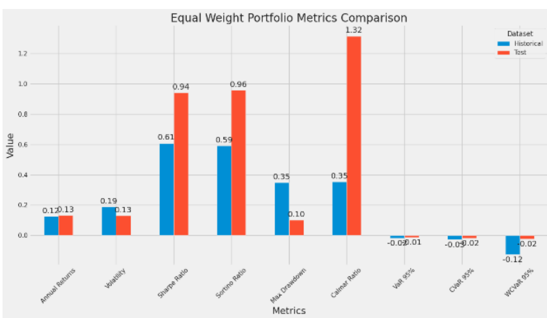





```
------------------------------------------------------------
              Expected Returns on my Portfolio
------------------------------------------------------------
1356.2703969399226

------------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
Using the Portfolio Weights for Optimal Portfolio

    Starting balance $1000: Ending with $ 2157.68
------------------------------------------------------------
```

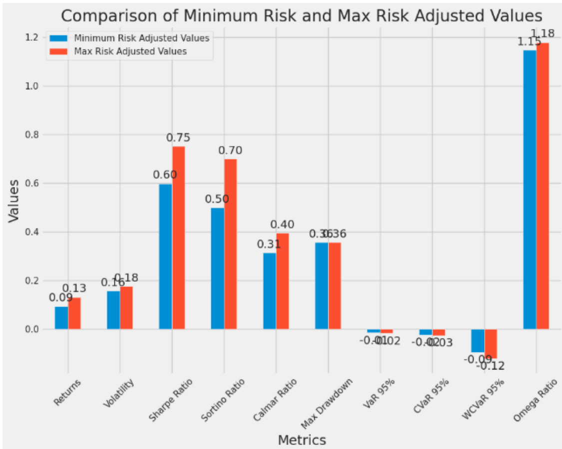
Value Stock Simulation Results:

Equal Weight (1/n) Portfolio Backtest



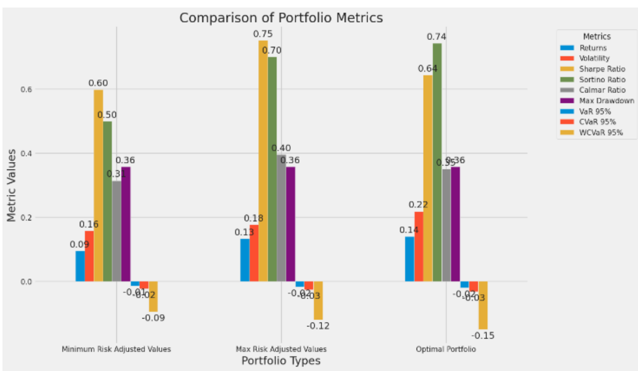

```
------------------------------------------------------------
              Expected Returns on my Equal Weight Portfolio
------------------------------------------------------------
Starting balance $ 1000 : Ending with $ 1132.43

------------------------------------------------------------
Using the Equal Weights Portfolio
    Starting balance $ 1000 : Ending with $ 1138.12
------------------------------------------------------------
```

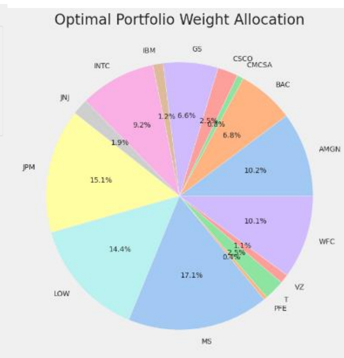
Backtests of Efficient Frontier Optimal portfolios




```
------------------------------------------------------------
              Expected Returns on my Portfolio
------------------------------------------------------------
         Minimum Risk Adjusted Values  Max Risk Adjusted Values
Returns                   1094.532434               1132.604757

------------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
------------------------------------------------------------
Using the Portfolio Weights for Minimum Risk Return Portfolio

    Starting balance $ 1000 : Ending with $  1000.66

------------------------------------------------------------
Using the Portfolio Weights Maximized Risk-Return Portfolio

    Starting balance $ 1000 : Ending with $  1037.48
```

MC-Based Combined Metrics Optimization Results





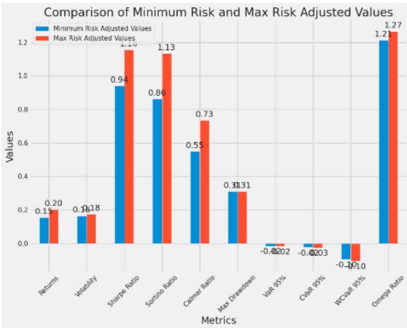

```
-----------------------------------------------------------------
              Expected Returns on my Portfolio
-----------------------------------------------------------------
1140.6069154907457

-----------------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
Using the Portfolio Weights for Optimal Portfolio

    Starting balance $1000: Ending with $ 1230.98
-----------------------------------------------------------------
```

Health Stock Simulation Results

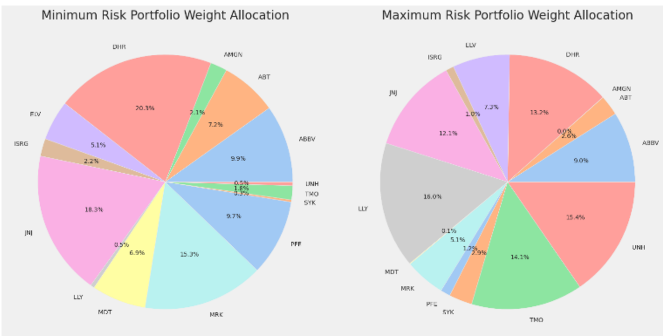
Equal Weight (1/n) Portfolio Backtest


```
-----------------------------------------------------------------
              Expected Returns on my Equal Weight Portfolio
-----------------------------------------------------------------
Starting balance $ 1000 : Ending with $ 1048.29

-----------------------------------------------------------------
Using the Equal Weights Portfolio
    Starting balance $ 1000 : Ending with $ 1048.4
-----------------------------------------------------------------
```

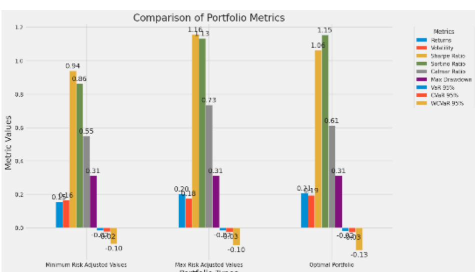
Backtests of Efficient Frontier Optimal Portfolios

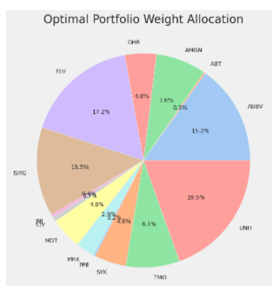



```
-----------------------------------------------------------
              Expected Returns on my Portfolio
-----------------------------------------------------------
        Minimun Risk Adjusted Values  Max Risk Adjusted Values
Returns                  1154.066331               1202.760866

-----------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
-----------------------------------------------------------
Using the Portfolio Weights for Minimum Risk Return Portfolio

    Starting balance $ 1000 : Ending with $  956.57

-----------------------------------------------------------
Using the Portfolio Weights Maximized Risk-Return Portfolio

    Starting balance $ 1000 : Ending with $  1078.02
```



MC-Based Combined Metrics Optimization Results

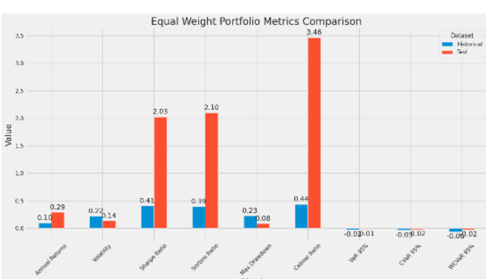





```
-----------------------------------------------------------
              Expected Returns on my Portfolio
-----------------------------------------------------------
1206.2861348743356

-----------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
Using the Portfolio Weights for Optimal Portfolio

    Starting balance $1000: Ending with $ 1048.13
-----------------------------------------------------------
```

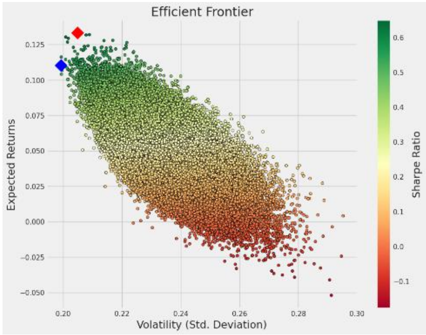
Consumer Discretionary Stock Simulation Results

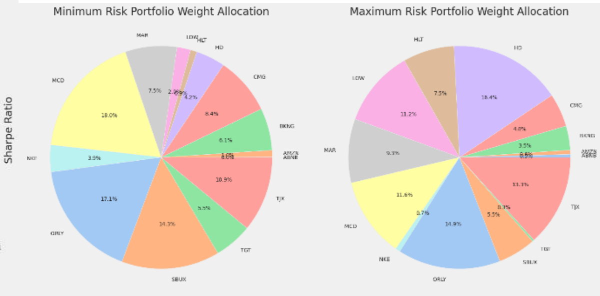
Equal Weight (1/n) Portfolio Backtest

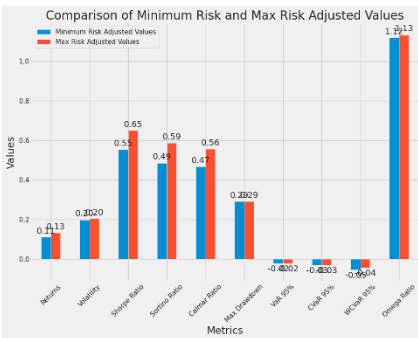



```
-----------------------------------------------------------
              Expected Returns on my Equal Weight Portfolio
-----------------------------------------------------------
Starting balance $ 1000 : Ending with $ 1292.84

-----------------------------------------------------------
Using the Equal Weights Portfolio
    Starting balance $ 1000 : Ending with $ 1294.13
-----------------------------------------------------------
```

Backtests of Efficient Frontier Optimal Portfolios

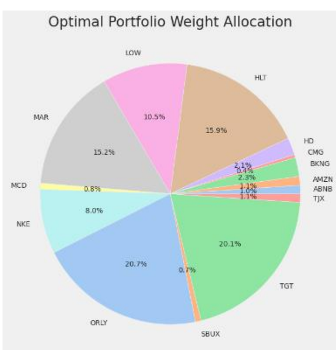







```
------------------------------------------------------------------------
                   Expected Returns on my Portfolio
------------------------------------------------------------------------
          Minimum Risk Adjusted Values  Max Risk Adjusted Values
Returns                    1110.478732               1133.286097

------------------------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
------------------------------------------------------------------------
Using the Portfolio Weights for Minimum Risk Return Portfolio

    Starting balance $ 1000 : Ending with $  1223.48

------------------------------------------------------------------------
Using the Portfolio Weights Maximized Risk-Return Portfolio

    Starting balance $ 1000 : Ending with $  1241.03
```

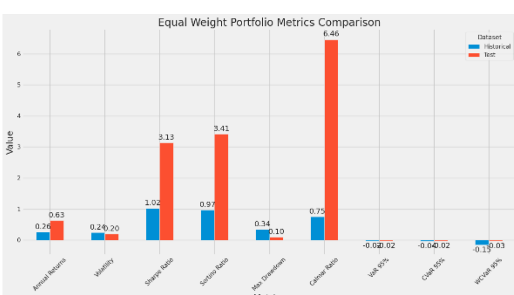
MC-Based Combined Metrics Optimization Results




```
------------------------------------------------------------
              Expected Returns on my Portfolio
------------------------------------------------------------
1107.1984174533313

------------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
Using the Portfolio Weights for Optimal Portfolio

    Starting balance $1000: Ending with $ 1227.17
------------------------------------------------------------
```

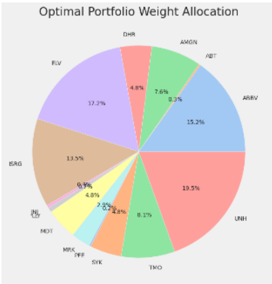
Technology Stock Simulation Results

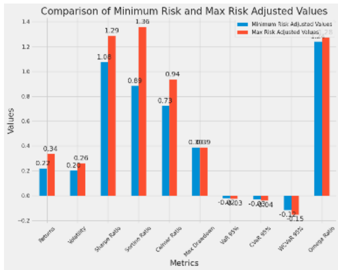
Equal Weight (1/n) Portfolio Backtest

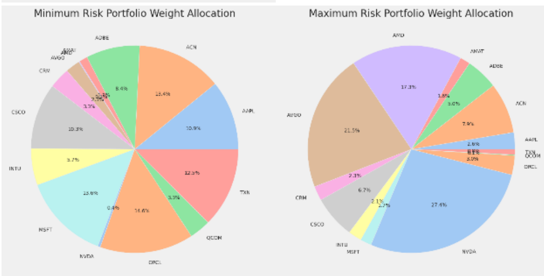



```
------------------------------------------------------------
           Expected Returns on my Equal Weight Portfolio
------------------------------------------------------------
Starting balance $ 1000 : Ending with $ 1629.4

------------------------------------------------------------
Using the Equal Weights Portfolio
    Starting balance $ 1000 : Ending with $ 1666.23
------------------------------------------------------------
```

Backtests of Efficient Frontier Optimal Portfolios

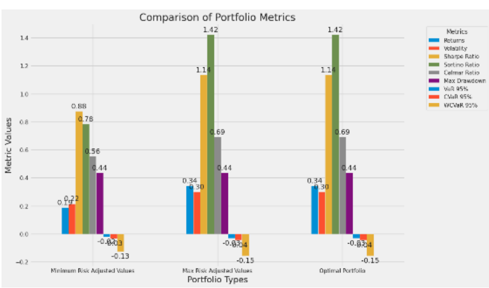


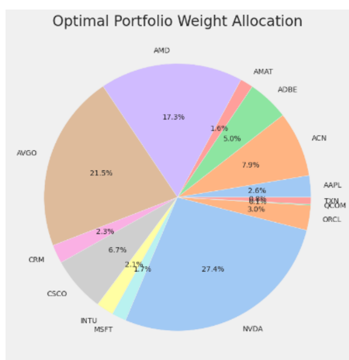





```
--------------------------------------------------------------
                 Expected Returns on my Portfolio
--------------------------------------------------------------
        Minimum Risk Adjusted Values  Max Risk Adjusted Values
Returns                  1189.067473               1343.321206

--------------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
--------------------------------------------------------------
Using the Portfolio Weights for Minimum Risk Return Portfolio

    Starting balance $ 1000 : Ending with $  1427.04

--------------------------------------------------------------
Using the Portfolio Weights Maximized Risk-Return Portfolio

    Starting balance $ 1000 : Ending with $  2250.43
```

<u>MC-Based Combined Metrics Optimization Results</u>

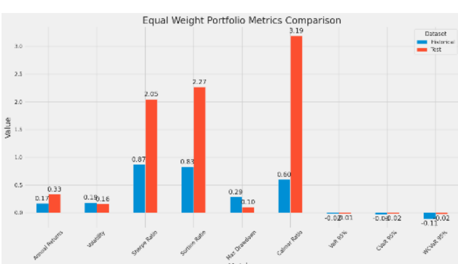





```
--------------------------------------------------------------
                 Expected Returns on my Portfolio
--------------------------------------------------------------
1343.3212060748635

--------------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
Using the Portfolio Weights for Optimal Portfolio

    Starting balance $1000: Ending with $ 2250.43
--------------------------------------------------------------
```

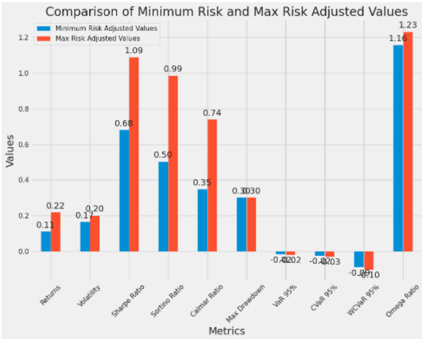
Communications Stock Simulation Results

<u>Equal Weight (1/n) Portfolio Backtest</u>

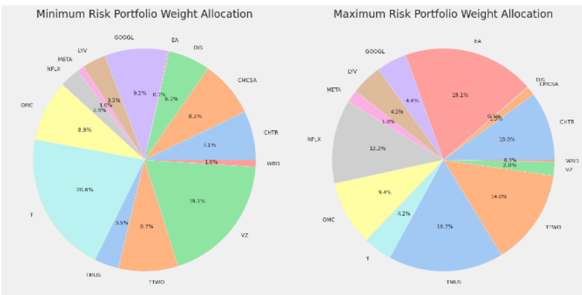



```
--------------------------------------------------------------
            Expected Returns on my Equal Weight Portfolio
--------------------------------------------------------------
Starting balance $ 1000 : Ending with $ 1332.71

--------------------------------------------------------------
Using the Equal Weights Portfolio
    Starting balance $ 1000 : Ending with $ 1345.64
--------------------------------------------------------------
```

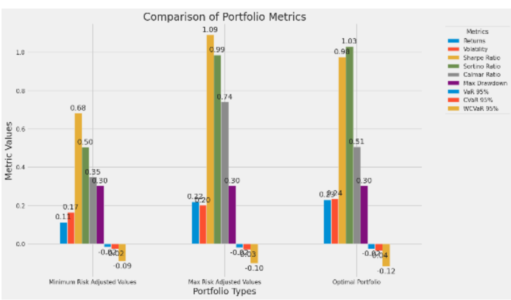
<u>Backtests of Efficient Frontier Optimal Portfolios</u>

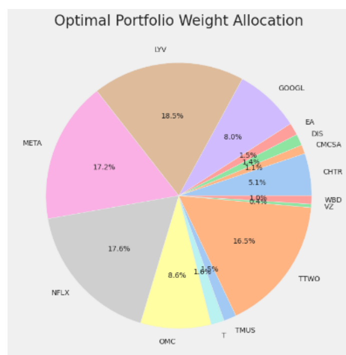



```
-------------------------------------------------------------------
                    Expected Returns on my Portfolio
-------------------------------------------------------------------
         Minimum Risk Adjusted Values  Max Risk Adjusted Values
Returns                   1112.690372               1220.134519

-------------------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
-------------------------------------------------------------------
Using the Portfolio Weights for Minimum Risk Return Portfolio

    Starting balance $ 1000 : Ending with $  1207.62

-------------------------------------------------------------------
Using the Portfolio Weights Maximized Risk-Return Portfolio

    Starting balance $ 1000 : Ending with $  1317.37
```



MC-Based Combined Metrics Optimization Results

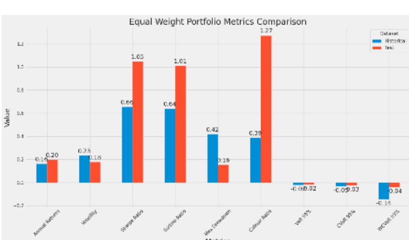





```
-------------------------------------------------------------------
                 Expected Returns on my Portfolio
-------------------------------------------------------------------
1229.8379108657869

-------------------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
Using the Portfolio Weights for Optimal Portfolio

    Starting balance $1000: Ending with $ 1683.2
-------------------------------------------------------------------
```

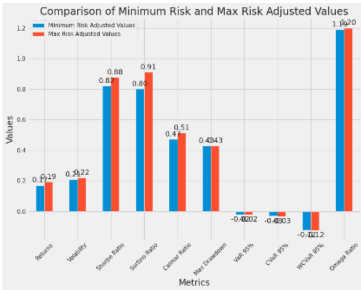
Financial Stock Simulation Results

Equal Weight (1/n) Portfolio Backtest

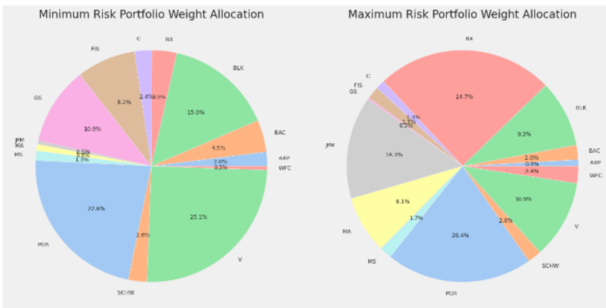



```
-------------------------------------------------------------------
              Expected Returns on my Equal Weight Portfolio
-------------------------------------------------------------------
Starting balance $ 1000 : Ending with $ 1197.83

-------------------------------------------------------------------
Using the Equal Weights Portfolio
    Starting balance $ 1000 : Ending with $ 1189.13
-------------------------------------------------------------------
```

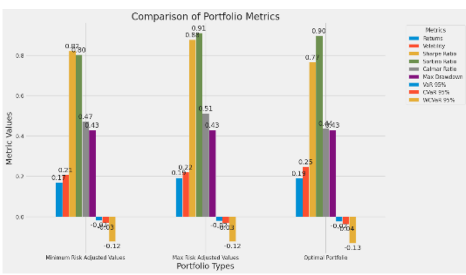
Backtests of Efficient Frontier Optimal Portfolios

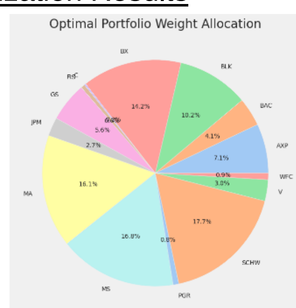



```
-------------------------------------------------------------
                  Expected Returns on my Portfolio
-------------------------------------------------------------
        Minimum Risk Adjusted Values  Max Risk Adjusted Values
Returns                 1169.960174               1192.87785

-------------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
-------------------------------------------------------------
Using the Portfolio Weights for Minimum Risk Return Portfolio

    Starting balance $ 1000 : Ending with $  1194.8

-------------------------------------------------------------
Using the Portfolio Weights Maximized Risk-Return Portfolio

    Starting balance $ 1000 : Ending with $  1364.22
```



MC-Based Combined Metrics Optimization Results






```
-------------------------------------------------------------
              Expected Returns on my Portfolio
-------------------------------------------------------------
1190.1310430489866

-------------------------------------------------------------
If I invested 1000 USD on | 2022-12-29 00:00:00 | I would have...
Using the Portfolio Weights for Optimal Portfolio

    Starting balance $1000: Ending with $ 1217.47
-------------------------------------------------------------
```