COMPUTATIONAL PHYSICS

HOMEWORK #4

Numerical Methods for Physics , chapter 7.

Reminder of hydrodynamics:
The time evolution of the gas density $\rho(x,t)$ and velocity $v(x,t)$,

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho v)}{\partial x}$$

$$\frac{\partial v}{\partial t} = -v\frac{\partial v}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{f}{\rho} \quad ,$$

where $p(x,t)$ is the gas pressure, and f an external force.

1. Write down the discretised version of the above equations using the FTCS method, the Lax method, and the leap-frog method. Bonus: try and derive a second order version using the Taylor expansion to second order (the equivalent of the Lax-Wendroff method).

2. Find the solution $\rho(x,t)$, and $v(x,t)$ in the range $0 \le x \le 1$, for the following initial conditions: $\rho = 1$ at $0.4 \le x \le 0.6$, $\rho = 0$ at $x \le 0.4$ and $x \ge 0.6$, and $v = 0$ for all x. Assume $f = 0$ and $p = k\rho$, where $k = 1$. This corresponds to expansion of gas, initially confined to a small region in a container. What is the initial sound speed c_s in the gas? What is the system sound speed crossing time t_s? The boundary conditions are $v = 0$ at $x = 0$ and 1 (hard walls which confine the gas). Solve numerically for $\rho(x,t)$ and $v(x,t)$ until $t = 3t_s$. Use the numerical solution method of your choice, and the appropriate resolution in space and time. Explain physically the time evolution you get. Bonus: compare the numerical results of the three different methods.

3. Now start with a uniform density static gas in the same container, i.e. $\rho = 1$ and $v = 0$ for $0 \le x \le 1$, at $t = 0$. Now the container is vertical and subject to gravity, where $f = -10\rho$. Derive analytically the solution for the static case ($v = 0$), i.e. the hydrostatic solution. Solve numerically the time evolution of the system, with the appropriate resolution in space and time, and explain what you get.

Please include a listing of your code.
Note that similar HW sets will not be graded.
Please submit in writing by December 31st, 2017.
Behatzlacha!