COMPUTATIONAL PHYSICS

HOMEWORK #6

Numerical Methods for Physics , chapters 9 & 4.3

1. Use the von Neumann stability analysis and show analytically that the implicit FTSC method for the solution of the Schrodinger equation is stable. Use Taylor expansion to show analytically that the Crank-Nicolson scheme is accurate to second order in space and time.

2. A reminder concerning the calculation of the retarded time $t_{\rm ret}(\vec{r}, \vec{r}_1)$ - the emission time of a signal observed at a position $\vec{r}$ at time t, from a particle moving at a trajectory $\vec{r}_1(t)$. Assume the particle is moving at a constant velocity v_0 along the Y axis, and passes at the origin $x = 0, y = 0$ at $t = 0$. Derive analytically an expression for $t_{\rm ret}$ for an observer at a given position $\vec{r}$ in the $X - Y$ plain, at time t. Solve numerically for $t_{\rm ret}$ using the Newton method. Assume $c = 1$ and $v_0 = 0.5$. Use the numerical solutions to make a plot of $t - t_{\rm ret}$ as a function of t, for three positions: $\vec{r} = (1, -1)$, $\vec{r} = (1, 0)$ and $\vec{r} = (1, 1)$, for t in the range -10 to 10. Add a plot on the same figure of the analytic solutions.

3. A particle with a charge $q = 1$ is moving in a circular orbit at a constant angular velocity ω, such that $\vec{r} = (A\cos(wt), A\sin(wt))$, where $A = 1$. Calculate $t_{\rm ret}(\vec{r}_{i,j})$ at $t = 0$ for the grid points $\vec{r}_{i,j} = (-10 + 0.4 \times (i - 1), -10 + 0.4 \times (j - 1))$, for $i, j = 1 - 51$. Make a plot of the cut $t_{\rm ret}(x, 0)$ and verify the results make sense. Assume $\omega = 0.1$ and $\omega = 0.9$, where $c = 1$.

   Write down the general analytic expression for the retarded potential $\Phi(\vec{r}, t)$ from a charged particle moving at a specified orbit. Calculate numerically and plot $\Phi(\vec{r}_{i,j}, 0)$ for a particle in a the above circular orbit, for $\omega = 0.1$ and $\omega = 0.9$ $(c = 1)$. Either make a 2D plot (using color or grey scale to denote the value of Φ at each point), or plot a 1D cut of $\Phi(x)$ for different values of y.

   Write down the general analytic expression for the retarded electric field $\vec{E}(\vec{r}, t)$ produced by a moving charged particle. Write the expression for the specific orbit in the problem above. Calculate numerically $\vec{E}(\vec{r}_{i,j}, 0)$ and make a 2D vector plot of $\vec{E}$. Explain the difference between the near field and the far field results.

Please include a listing of your code.
Please submit in writing by February 1st, 2018.
Behatzlacha!