TECHNION
Israel Institute
of Technology

Undergraduate Project Report

Cheng-ching Lin

Advisor:
Enrique Kajomovitz

August 16, 2018

1 Abstract

In colliding experiment, we study the interaction between particles occurring during the collision. However, the detector can only detect the particles which are already stable, which means there are some unstable process or particles cannot be detected due to short life-time. Therefore, we want to simulate parton shower, one of undetectable process. Moreover, we want to know the jets of this process.

In simple toy parton shower simulation, we based on *Monte Carlo and Jet Tutorial* written by *Samuel Meehan*[3], and try to simulate the most simple parton shower. In second simulation, We are based on *PYTHIA* manual wriiten by *Torbjörn Sjöstrand*[5], one of parton shower simulation using in *CERN* now, to simulate and try to figure out what happened in parton shower and study the relation between jety mass and parton shower.

2 Introduction

2.1 Parton Shower

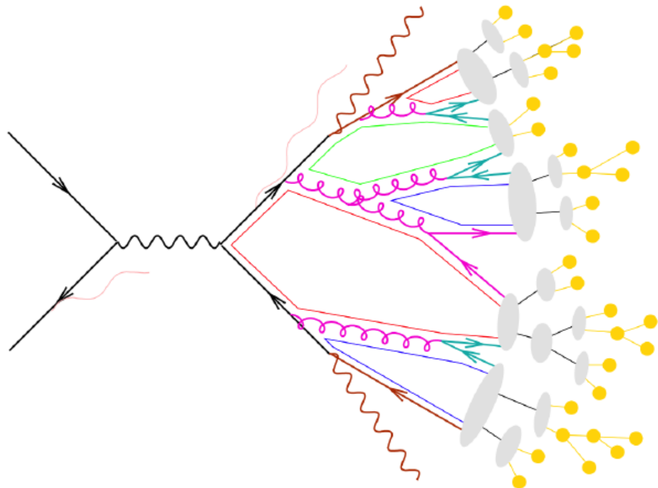


Figure 1: Illustration of the dynamics of parton shower(red and purple lines) and hadronization(gray clusters and yellow circles).[3]

Highly energtic partons(gluons and quarks) can be produced during a collision. These partons undergo one-to-two scattering process, in which partons are produced. These secondary partons undergo the same one-to-two scattering process and produced more partons. This process is called *parton shower*.

The process continues until the certain energy cut-off. When QCD is non-perturbative, partons under go the fragmentation, and after that is hadronizaion. Figure 1 illustrate the process of parton shower and hardonization.

The dynamics of those process, quarks and gluons are described in QCD.

2.2 The energy fraction

A shower can be seen as a sequence of one-to-two branchings $a \to b + c$. Here we call a mother and b, c are the daughters of a. In parton shower, there is an important variable called z, which is related to share the energy of a with daughters, b with fraction z and c with fraction $1 - z$.

Therefore, with energy fraction z and conservation of energy and momentum, we can find all particles energy and momentum in one scattering event.

3 Simple Toy Parton Shower Simulation

This simulation is based on *Monte Carlo and Jet Tutorial* written by *Samuel Meehan*[3].

3.1 The choice of variables

In this simulation, we only have two variables, z and θ. θ is the angle between two particles after scattering, we call scattering angle.

Because of possibility distribution function(PDF) of z, we cannot choose the range $[0, 1]$ on z. We need to avoid the pole in PDF. Therefore, we can only choose $[\epsilon, 1]$, where $\epsilon \ll 1$. For θ, it is the same as z, but the maximum of angle is $\frac{\pi}{2}$, that is, the range of θ is $[\epsilon, \frac{\pi}{2}]$.

3.2 Evolution function

There are two properties of scattering in this simulation. One is called soft, which means energy won't be sperated a lot in every event. It also means that energy fraction z will follow this possibility distribution function(PDF): $\frac{1}{z}$

The other property is colliner, and it means that scattering angle θ won't be very big, just like linear dynamics. θ is followed this PDF: $\frac{1}{\theta}$

3.3 Process of simulation

First, I use Monte Carlo to create a pair of variable (z, θ). With these variables, I also get the energy of two scattering particles.

Note that, in this simulation, we need to make sure that[4]

$$|\theta E_{scattering}| \geq E_{threshold} \tag{1}$$

Where $E_{threshold}$ is the least energy that particles can be scattered. This function is not only to prevent the energy will be less than threshold energy, but also to prevent the scattering angle cannot be collinear at high energy.

After getting energy of scattering particles, we can use the conservation of momentum to produce other side of particle. When energy of all particles decrease to critical energy, then the simulation is finished.

3.4 Result

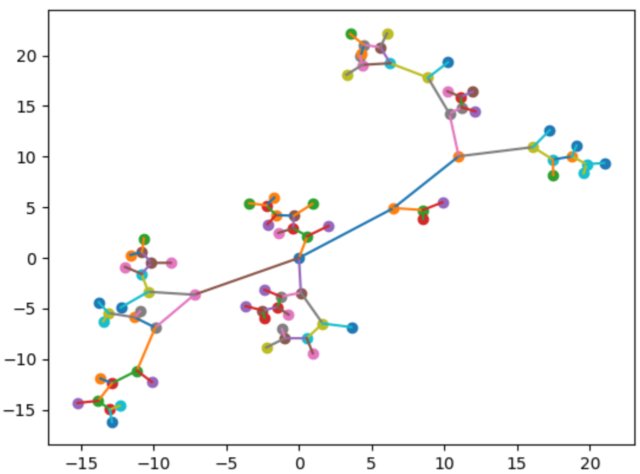


Figure 2: This is the result of first simulation. Axises don't mean anything. Every dots means one event, and the length of lines presented energy of every events. The hard scattering is on (0,0)

Figure 2 is the result of first simulation. As we said, since the constrain formula 1, we can see when the energy of events is big, or the position of events is closer to hard scattering $(0, 0)$, then the scattering angle is small. On the other hand, if the energy of events is small, or the position of events is far away from hard scattering $(0, 0)$, then the scattering angle is big.

4 Depth in strong coupling

In the previous section, we only consider about how the highly energy unstable particles scattering, and ignore all the strong force. In this section, we will discuss more about strong coupling α_s and its evoluation.

4.1 QCD and coupling constant

Here, we introduce another important variable Q^2 in the parton shower, which decide the kinematics of each branching. Q^2 has dimensions of squared mass and is related to the mass and transverse scale of branching.

We define α_s is coupling variable, which means coupling strength in the strong interaction. In QCD, α_s is not a constant. It is the function of energy, is showed belowed.

$$\alpha_s(Q^2) = \frac{\alpha_s(\Lambda^2)}{1 + \ln\left(\frac{Q^2}{\Lambda^2}\right) b_0 \alpha_s(\Lambda^2)}$$

Where Λ^2 is the energy threshold between perturbation and non-perturbation regime. b_0 is a constant which depends on the number of fermionic and bosonic loop. For N_c quark colors and N_f flavors,

$$b_0 = \frac{11N_c - 2N_f}{12\pi}$$

For $N_c = 3$ colors and $N_f \leq 6$ quarks, $b_0 > 0$ and hence α_s decreases with increasing Q^2. From Figure 3, we know

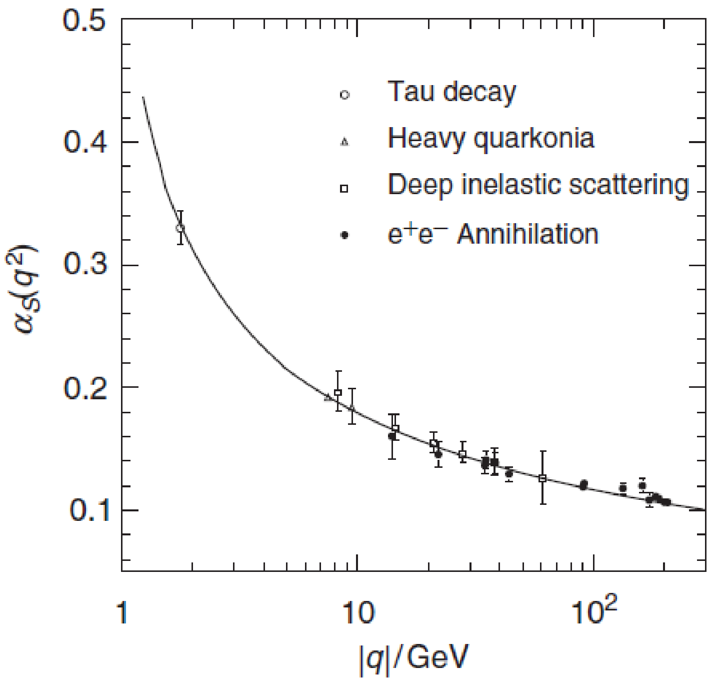


Figure 3: Measurements of α_s at different enrgy scale $|q|$. X-axis is energy scale, and Y-axis is the strong couplings.[1]

that if Q^2 is smaller than Λ^2, then coupling constant α_s is too big to use perturbation theory. In that case, we need to use more complicated QCD to solve it. However, the coupling constant α_s will be small enough that we can use perturbation theory to solve it, if Q^2 is larger than Λ^2.[7]

4.2 The evolution in parton shower

We want to write down the probability of branching, and we write down the differential t first.

$$dt = d\ln(Q^2) = \frac{dQ^2}{Q^2}$$

They probability for a parton to branch is given by evolution equations, which is also called DGFAP. We write down the differential probability dP, for parton a branching,

$$dP_a = \Sigma_{b,c} \left(\frac{\alpha_s}{2\pi}\right) P_{a\to bc} dt dz$$

There are different kinds of branching with different particles. In this report, we only discuss one branching, $q \to qg$. Here is splitting kernel.

$$P_{q\to qg}(z) = C_F \frac{1+z^2}{1-z}$$

For $C_F = \frac{4}{3}$.

The splitting kernel is actually the function of probability of branching in some z. The variable t in the process of evolution is like the time of function. It does not mean that a single parton can run through a range of t. Every parton has fixed t, and t will keep change as evolution go on.

For given t value, we can define the integral of the branching probability all over allowed z values.

$$\mathcal{L}_{a\to bc}(t) = \int_{z_-(t)}^{z_+(t)} dz \left(\frac{\alpha_s}{2\pi}\right) P_{a\to bc}(z) \tag{2}$$

In the parton shower, we always evolve from very high energy to low energy. Therefore, if the evolution of parton a starts at t_{max}, the probability not branching at $t < t_{max}$ is given by the product of no-branching probabilities with small intervals δ_t. We let $\delta_t \to 0$, then the no-branching probability exponentiates:

$$P_{no-branching}(t, t_{max}) = \exp\left\{-\int_t^{t_{max}} dt' \Sigma_{b,c}\mathcal{L}_{a\to bc}(t')\right\} = S_a(t) \tag{3}$$

We also call it Sudakov form factor. Thus the actual probability that a branching of a occurs at t is given by

$$\frac{dP_a}{dt} = -\frac{dP_{no-branching}(t, t_{max})}{dt} = (\Sigma_{b,c}\mathcal{L}_{a\to bc}(t)) \exp\left(-\int_t^{t_{max}} dt' \Sigma_{b,c}\mathcal{L}_{a\to bc}(t')\right)$$

The first term is the naïve branching probability, the second suppression due to the conservation of total probability.[5]

4.3 Veto Algorithm

In the simulation, we need to use a technique called Veto algorithm. Veto algorithm is used to solve decaying problem with Monte Carlo method. The math structure of scattering problem is same as decaying, thus we can use this technique in the simulation. Usually, we use inverse transformation method to solve this kind of problems. However, if the origin function is not a good function, then we need to use Veto algorithm. Instead of origin function, Veto algorithm need to find other better function to solve the problem.

We suppose the probability that nothing has happened by time t is expressed by $N(t)$ and the differential probability that something happens at t by $P(t)$, then the basic equation is

$$P(t) = -\frac{dN}{dt} = f(t)N(t)$$

For simplifying, we let $N(0) = 1$. The above equation probability can be solved and thus

$$N(t) = N(0)\exp\left(-\int_0^t f(t')dt'\right) = \exp\left(-\int_0^t f(t')dt'\right)$$

$$P(t) = f(t)\exp\left(-\int_0^t f(t')dt'\right) \tag{4}$$

If we integrate $P(t)$, then

$$\int_0^t P(t')dt' = N(0) - N(t) = 1 - \exp\left(-\int_0^t f(t')dt'\right) = 1 - R$$

which has the solution

$$F(0) - F(t) = \ln(R) \Rightarrow t = F^{-1}(F(0) - \ln(R))$$

For R is random number between 0 and 1, and $F(t)$ is the integration of $f(t)$.

However, if $f(t)$ is not good enough or $F(t)$ does not have inverse function, then we need to find another better function $g(t)$, with $f(t) \leq g(t)$ for all $t > 0$. In this case, we need to use Veto algorithm instead.

1. Start with $i = 0$ and $t_0 = 0$.
2. Add 1 to i and select $t_i = G^{-1}(G(t_{i-1}) - \ln(R))$, with the constraint that $t_i > t_{i-1}$
3. Compare a R'(new random number) with the ratio $f(t_i)/g(t_i)$. If $f(t_i)/g(t_i) \leq R'$, then return to 2. Otherwise t_i is retained as final answer.

Why does this algorithm work? Let's see an example. Suppose the probability that first try works, which means there is no rejected t. Thus,

$$P_0(t) = \exp\Big(-\int_0^t g(t')dt'\Big) g(t)\frac{f(t)}{g(t)} = f(t)\exp\Big(-\int_0^t g(t')dt'\Big)$$

Now we consider the case where t_1 is rejected.

$$P_1(t) = \int_0^t dt_1 \exp\Big(-\int_0^{t_1} g(t')dt'\Big) g(t_1)\left[1-\frac{f(t_1)}{g(t_1)}\right]\exp\Big(-\int_{t_1}^t g(t')dt'\Big) g(t)\frac{f(t)}{g(t)}$$

$$= P_0(t)\int_0^t dt_1\,[g(t_1)-f(t_1)]$$

Considering t_1 and t_2 rejected.

$$P_2(t) = P_0(t)\int_0^t dt_1\,[g(t_1)-f(t_1)]\int_{t_1}^t dt_2\,[g(t_2-f(t_2))]$$

$$= P_0(t)\frac{1}{2}\left(\int_0^t [g(t')-f(t')]dt'\right)^2$$

Therefore, we can calculate the total probability.

$$P(t) = P_0(t)\Sigma_{i=0}^{\infty}\frac{1}{i!}\left(\int_0^t [g(t')-f(t')]dt'\right)^i$$

$$= f(t)\exp\Big(-\int_0^t g(t')dt'\Big)\exp\Big(\int_0^t [g(t')-f(t')]dt'\Big)$$

$$= f(t)\exp\Big(-\int_0^t f(t')dt'\Big)$$

It is just equation (3).[2]

Usually, $f(t)$ is also the function of additional variable x. The method will change like this: we find a suitable function $g(t,x)$ with $f(t,x)\le g(t,x)$, and then

1. We need to find a function $G(t)=\int g(t,x)dx$ that only depend on t.
2. Start with $i=0$ and $t_0=0$.
3. Add 1 to i and select $t_i = G^{-1}(G(t_{i-1})-\ln(R))$, with the constraint that $t_i > t_{i-1}$
4. When t_i is decided, we can decide x_i by $G_x^{-1}(R'(G_x(x_{max})-G_x(x_{min}))+G_x(x_{min})$, for $G_x = G(t_i,x)$.
5. Compare a R'' with the ratio $f(t_i,x_i)/g(t_i,x_i)$. If $f(t_i,x_i)/g(t_i,x_i)\le R''$, then return to 3. Otherwise (t_i,x_i) is retained as final answer. In conclusion, Veto algorithm is just accept-reject algorithm with two different way to evolve. First, we need to find a overestimate function to cover original function. Second, after doing accept-reject process, if we reject the result, it will be put into evoluation again, until we accept the new result.

5 Simple Pythia Parton Shower Simulation

This simulation is based on *PYTHIA* manual[5] and *Particle Physics Phenomenology Lecture* 3[6] written by *Torbjörn Sjöstrand*. In this simulation, the parton shower take into account evoluation of α_s.

5.1 The choice of variables

In *JETSET*, they choose $Q^2 = m_a^2$, and we also do the same choice. In the choice of splitting variables, there are four different alternative z, 'local' and 'global' z definition combined with 'constrained' or 'unconstrained'. The meaning of 'local' z definition is the energy fractions are defined in the rest frame of grandmother, that is, the mother of particle a; 'global' z definition, however, is defined in hard scattering frame.

In a branching $a\to bc$ the kinematically allowed range of $z=z_a$ values, $z_-\le z\le z_+$, is given by

$$z_\pm = \frac{1}{2}\left[1+\frac{m_b^2-m_c^2}{m_a^2}\pm\frac{|\mathbf{p_a}|}{E_a}\frac{\sqrt{(m_a^2-m_b^2-m_c^2)^2-4m_b^2m_c^2}}{m_a^2}\right]$$

With 'constrained' evolution. For 'unconstained' evolution, the range of z become

$$z_{\pm} = \frac{1}{2}\left[1 \pm \frac{|\mathbf{p_a}|}{E_a}\theta(m_a - m_{min,a})\right] \tag{5}$$

Where $\theta(x)$ is step function. We assume the daughters are massless, and $m_{min,a}$ is the minimum mass of mother to branching. After the calculation, we will get the four-momentum $p_b^{(0)}$, $p_c^{(0)}$ and mass m_b, m_c. In order to fix our assumption, we need to find actual values by

$$p_{b,c} = p_{b,c}^{(0)} \pm \left(r_c p_c^{(0)} - r_b p_b^{(0)}\right) \tag{6}$$

where

$$r_{b,c} = \frac{m_a^2 \pm \left(m_c^2 - m_b^2\right) - \sqrt{\left(m_a^2 - m_b^2 - m_c^2\right)^2 - 4m_b^2 m_c^2}}{2m_a^2} \tag{7}$$

In my simulation, I use 'global' z definition wtih 'unconstrained' evolution.

5.2 Evolution function

In veto algrithm, we need to choose the function $g(t)$ instead of origin one to find t and z. First, we need to change the equation 5.

$$\begin{aligned} z_{\pm} &= \frac{1}{2}\left[1 \pm \frac{|\mathbf{p_a}|}{E_a}\theta(m_a - m_{min,a})\right] \\ &= \frac{1}{2}\left[1 \pm \sqrt{1 - \frac{m_a^2}{E_a^2}}\theta(m_a - m_{min,a})\right] \\ &= \frac{1}{2}\left[1 \pm \sqrt{1 - \frac{\Lambda^2}{E_a^2}e^{t_a}}\theta(m_a - m_{min,a})\right] \end{aligned} \tag{8}$$

Now, we can write down our Sudakov form factor and subtitute equation 2 into 3. Note that, in my case, we only discuss about the process of $q \to qg$.

$$\begin{aligned} P_{no-branching}(t) &= \exp\left\{-\int_t^{t_{max}} dt' \int_{z_-(t)}^{z_+(t)} dz \left(\frac{\alpha_s}{2\pi}\right) P_{q\to qg}(z)\right\} \\ &= \exp\left\{-\int_t^{t_{max}} dt' \int_{z_-(t)}^{z_+(t)} dz \left(\frac{\alpha_s}{2\pi}\right) C_F \frac{1+z^2}{1-z}\right\} \end{aligned} \tag{9}$$

For $C_F = \frac{4}{3}$. In order to find $f(t)$ which will be used in Veto algorithm, we need to do the integration.

$$\begin{aligned} f(t') &= \int_{z_-(t)}^{z_+(t)} dz \left(\frac{\alpha_s}{2\pi}\right) C_F \frac{1+z^2}{1-z} \\ &= C_F \left(\frac{\alpha_s}{2\pi}\right)\left(-z - z^2 - 2\ln(1-z)\right)\Big|_{z_-(t)}^{z_+(t)} \end{aligned}$$

Now we substitude equation 8. Thus,

$$f(t') = C_F \left(\frac{\alpha_s}{2\pi}\right)\left(-\frac{2}{3}\sqrt{1 - \frac{\Lambda^2}{E_a^2}e^{t'_a}} + 2\ln\left(\frac{1 + \sqrt{1 - \frac{\Lambda^2}{E_a^2}e^{t'_a}}}{1 - \sqrt{1 - \frac{\Lambda^2}{E_a^2}e^{t'_a}}}\right)\right) \tag{10}$$

Because we need to choose the a function that $g(t,z) \geq f(t,z)$. Therefore, we choose $g(t', z')$ that

$$g(t', z') = \left(\frac{C_F}{2\pi b_0}\right)\frac{2}{1-z'} \tag{11}$$

The truth is that $g(t', z') = g(z'(t'))$ is just the function of z, and z is the function of t.

Usually, Veto algorithm is used to find t from small t_0. However, in final-state parton shower, the process start from hard-scattering energy $E_{hard-scattering}$ to cut-off energy E_0. Therefore, if we integrate equation (3) from t to t_{max}, then we will get this.

$$\int_t^{t_{max}} P(t')dt' = N(t_{max}) - N(t) = 1 - \exp\left(-\int_t^{t_{max}} f(t')dt'\right) = 1 - R$$

$$F(t) - F(t_{max}) = \ln(R) \Rightarrow t = F^{-1}(F(t_{max}) + \ln(R))$$

Also, the parton shower will end if all particles energy below the cut-off energy $E_{cut-off}$, we just need to stop evaluation t when the energy of parton is lesser than cut-off energy. Those particles will be on-shell when they reach cut-off energy.

5.3 Process

I create a simulation about parton shower on Python. In this section, I will describe the process of simulation. First, I suppose a overestimated term. In this term, the range of z is $z_\pm = z_\pm(Q^2_{cut})$ and coupling term α_0 is[6]

$$\alpha_0 = \frac{\alpha_{m_z}}{1 + 2b_0\alpha_{m_z}\ln\left(\frac{Q_{cut}}{m_z}\right)}$$

where Q_{cut} is the cut-off Q, $m_z = 91.2GeV$ is the mass of z boson, and $\alpha_{m_z} = 0.12$ is the strong coupling of z boson.[4] Next, I evolve two daughter particles seperately, supposing the other particle is massless in the centre of mass frame. Thus, I can calculate four-momentum by simple relations.

Third, I calculate the splitting probability. If a random number $R \in [0, 1]$ is bigger than accept probability $P_{accept} = 1 - P_{no-branching}$, I reject the result, but I seen rejected Q_{reject} as new Q_{mother} and start new evolution, which means

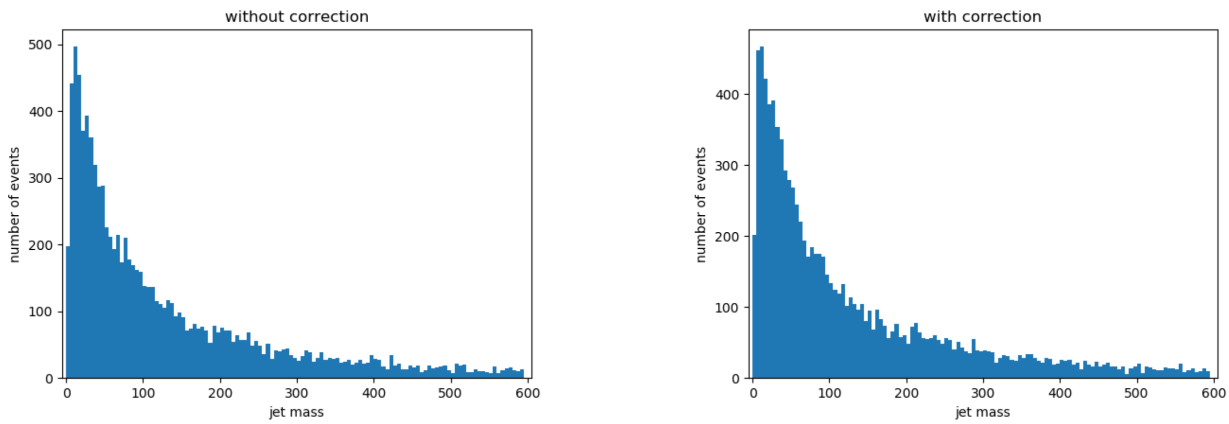
$$\xrightarrow{after \quad evaluation} Q \rightarrow \begin{Bmatrix} \text{accept}, & \text{if } R \leq P_{accept} \\ Q_{mother} = Q, & \text{reject, if } R > P_{accept} \end{Bmatrix}$$

Finally, Because I calculate four-momentums of particles when I suppose the other is massless, I need to do the correction by 6 and 7. After these process, I can get the result of first branching in these simulation.

5.4 Results

The purpose of this simulation is to simulate the jet mass we get after colliding. Next, I will discuss results without correction and with correction. Correction means the result is corrected by 6 and 7. Let the energy of hard scattering Q_{hard} is $2000GeV$, and I do 10000 events.

5.4.1 Jet Mass



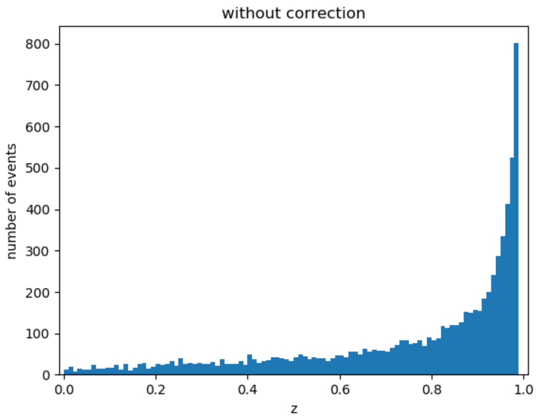
(a) without correction (b) with correction

Figure 4: jet mass(GeV) v.s. number of events

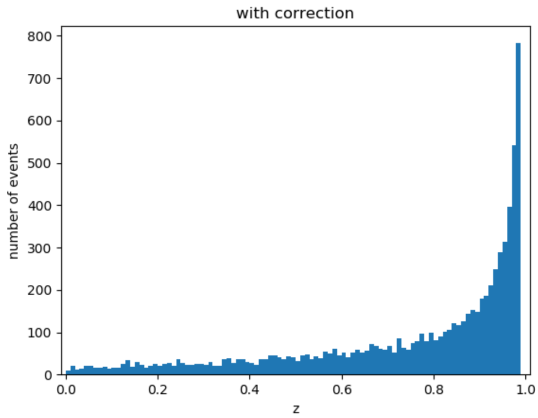
The results of jet mass are in figure 4. The average of jet mass without correction is $153.67GeV$, and the average of jet mass with correction is $150.58GeV$. The shape of curve is almost the same, but if we focus on the peaks, we can see that the number of events on the peak with correction one is smaller than the other one. However, the correction average jet mass is smaller than the one without correction. It means also the number of events on the peak is smaller, or we can say that events are more distributed, the whole events move downward by correction.

5.4.2 z

The results of z are in figure 5. We can see that two figures are almost no different, they show most of z are very close to 1. It means most energy are remain into quarks, only few energy are transformed into gluons. It shows in this simulation, z still allows "soft" property.



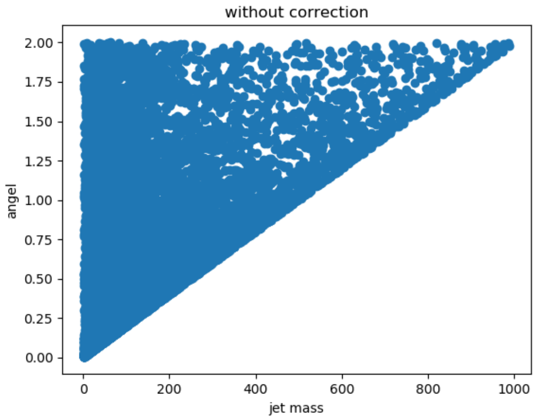
(a) without correction



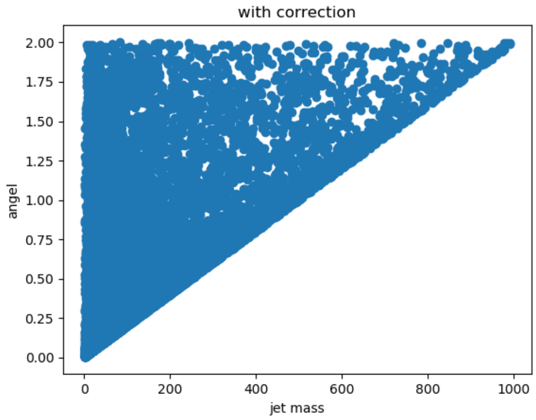
(b) with correction

Figure 5: z v.s. number of events

5.4.3 Scattering Angle



(a) without correction



(b) with correction

Figure 6: jet mass v.s. angle

The results of scattering angles are in figure 6. The formula of scattering angle is

$$\theta \approx \frac{Q}{Q_{hard}} \frac{1}{\sqrt{z(1-z)}}$$

Why the distribution is like triangle, we need to know what happen in boundary. We know that $z_{\pm} = \frac{1}{2}[1 \pm \sqrt{1-(\frac{Q}{Q_{hard}})^2}]$, when $\frac{Q}{Q_{hard}}$ is close to 1, then z is close to $\frac{1}{2}$, and $\theta \to 2$ is the upper bound. When $\frac{Q}{Q_{hard}}$ is far away from 1, then the range of z is more bigger, and it shows in figure like an triangle. If jet mass is smaller, the z range is bigger, and the vertical is wider. This feature is what we said before, "collinear" property in scattering angle.

6 Conculsion

In this project, we want to simulate the jet mass which we get after events. The way we simulate it is to create a simulation about the first branching of parton shower. Therefore, I create two type of parton shower simulation. First, I create Simple Toy Simulation. In this simulation, we can show some simple features of parton shower. Like "soft" z and "collinear" scattering angle θ. From this simulation, we can know the basic mechanism of parton shower. Next, I create more complicated simulation, based on *Pythia*. In that simulation, we consider running strong coupling α_s, and use Veto algorithm to evolve particles. We get the results of jet mass, and compare them between with and without correction. We can also see the properties in simple toy simulation are showed up in this simulation. Therefore, we can easily see the property of jet in simple case in the last simulation. This project can be study further with these simulation, like changing different splitting kernels to show different process and correspounding jets. Moreover, we can also study about the relation between strong coupling and jets. I specially thank Professor Enrique Kajomovitz for helping me a lot in this project.

References

[1] Siegfried Bethke. The 2009 world average of αs. *The European Physical Journal C*, 64(4):689–703, 2009.

[2] Ronald Kleiss and Rob Verheyen. Competing sudakov veto algorithms. *The European Physical Journal C*, 76(7):359, Jun 2016.

[3] Samuel Meehan. Monte carlo and jet tutorial. Technical report, University of Washington, 2017.

[4] Gavin Salam. Ingredients for accurate collider phyiscs. Technical report, CERN, 2016.

[5] Torbjorn Sjostrand. Pythia 5.7 and jetset 7.4 physics and manual. *arXiv preprint hep-ph/9508391*, 1995.

[6] Torbjorn Sjostrand. Particle physics phenomenology lecture 3. Technical report, Lund University, 2018. `http://home.thep.lu.se/~torbjorn/ppp2018/lec3.pdf`.

[7] Mark Thomson. *Modern particle physics*, chapter 10.5 Running of alpha S and asymptotic freedom. Cambridge University Press, 2013.