6 200 контролируемых людей

1 раз: 181 чел

2 раза: 9 чел $\quad P(A)=p$

$1-P(A)=q$

H_0: $\xi \sim Bi(2, p)$ $\quad$ A — заболел

H_1: $\overline{H_0}$ $\quad$ A_i — i раз болел

$\sum A_i = \Omega$ — полная группа событий

$$1 = C_2^0 q^2 + C_2^1 pq + C_2^2 p^2$$

$P(A_0) = q^2$ $\quad$ $P(A_1) = 2pq$ $\quad$ $P(A_2) = p^2$

$\nu_0 = \frac{10}{200}$ $\quad$ $\nu_1 = \frac{181}{200}$ $\quad$ $\nu_2 = \frac{9}{200}$

$$\Delta = n \sum_{i=1}^{2} \frac{(P(A_i) - \nu_i)^2}{P(A_i)}$$

$$= n\left(\frac{(q^2 - 10/200)^2}{q^2} + \frac{(2pq - 181/200)^2}{2pq} + \frac{(p^2 - 9/200)^2}{p^2} \right)$$

$$L = \prod p_i = (q^2)^{10} (2pq)^{181} (p^2)^9 = 2^{181} p^{199} q^{201} =$$

$$= 2^{181} p^{199} (1-p)^{201}$$

$$\ln L = 199 \ln p + 201 \ln(1-p) + 181 \ln 2$$

$$\frac{\partial \ln L}{\partial p} = \frac{199}{p} - \frac{201}{1-p} = 0 \qquad 199 - 199p = 201p$$

$$\tilde{p} = \frac{199}{400}$$

$$\frac{\partial^2 \ln L}{\partial p^2} = -\frac{199}{p^2} - \frac{201}{(1-p)^2} < 0 \quad \to \quad \max$$

$$\Delta(\tilde{p}) = 131{,}23466 \quad \sim \quad \chi^2(3-1-1) = \chi^2(1)$$

$$f_{\chi^2(1)}(x) = \frac{(1/2)^{1/2}}{\Gamma(1/2)} x^{(1/2-1)} e^{-1/2 x} = \frac{e^{-1/2 x}}{\sqrt{2\pi x}}$$

$$\Gamma(1/2) = \int_0^\infty \frac{e^{-x}}{\sqrt{x}} dx = 2\int_0^\infty e^{-t} dt = \frac{\sqrt{\pi}}{2} \cdot 2 = \sqrt{\pi}$$

$$\text{p-value} = \int_{131{,}2}^\infty f_{\chi^2(1)}(x) dx = \int_{131{,}2}^\infty \frac{e^{-1/2 x}}{\sqrt{2\pi x}} dx =$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{131{,}2}}^\infty \frac{e^{-t^2/2}}{t} 2t dt = \sqrt{\frac{2}{\pi}} \int_{\sqrt{131{,}2}}^\infty e^{-t^2/2} dt =$$

$$= 2{,}2 \cdot 10^{-30}$$

$$\text{p-value} = 2{,}2 \cdot 10^{-30} \ll a = 0{,}05 \Rightarrow$$

очень уверенно отвергаем H_0

7 2 партии по 100 деталей

1 партия: 25 — ниже
50 — точно
25 — выше

2 партия: 52 — ниже
41 — точно
7 — выше

H_0: независимость номера партии детали и размера деталей

H_1: $\overline{H_0}$

ξ — размер детали

η — номер партии

ξ \ η	1	2	
ниже	25	52	77/200
точно	50	41	91/200
выше	25	7	32/200
	1/2	1/2	

$$\Delta = \sum \left(\frac{m_{ij} - n p_i q_j}{n p_i q_j} \right)^2 \sim \chi^2((3-1)(2-1)) = \chi^2(2)$$

$$\Delta=\frac{\left(25-200\frac{1}{2}\frac{77}{200}\right)^2}{200\frac{1}{2}\frac{77}{200}}+\frac{\left(50-200\frac{1}{2}\frac{91}{200}\right)^2}{200\frac{1}{2}\frac{91}{200}}+$$

$$+\frac{\left(25-200\frac{1}{2}\frac{32}{200}\right)^2}{200\frac{1}{2}\frac{32}{200}}+\frac{\left(52-200\frac{1}{2}\frac{77}{200}\right)^2}{200\frac{1}{2}\frac{77}{200}}+$$

$$+\frac{\left(41-200\frac{1}{2}\frac{91}{200}\right)^2}{200\frac{1}{2}\frac{91}{200}}+\frac{\left(7-200\frac{1}{2}\frac{32}{200}\right)^2}{200\frac{1}{2}\frac{32}{200}}\approx$$

$$\approx 20{,}486$$

$$p_{\chi^2(2)}(x)=\frac{(1/2)^{2/2}}{\Gamma(2/2)}x^{\left(\frac{2}{2}-1\right)}e^{-\frac{1}{2}x}=\frac{1}{2}e^{-\frac{1}{2}x}$$

$$p\text{-value}=\int_{20{,}486}^{\infty}p_{\chi^2(2)}(x)\,dx=\frac{1}{2}\int_{20{,}5}^{\infty}e^{-x/2}\,dx=$$

$$=0{,}0000356059 \ll a=0{,}05$$

$\Rightarrow$ очень уверенно отвергаем H_0

8 2 потока по 300 человек $\Sigma = 600$

	2	3	4	5
1 поток:	33	43	80	144
2 поток:	39	35	72	154
	$\frac{72}{600}$	$\frac{78}{600}$	$\frac{152}{600}$	$\frac{298}{600}$

H_0: потоки однородные

H_1: $\overline{H_0}$

$$\Delta_1 = \sum \frac{(n_{ij} - n_i \hat{P}(A_j))^2}{n_i \hat{P}(A_j)} \sim \chi^2((4-1)(2-1)) = \chi^2(3)$$

$$\Delta_1 = \frac{\left(33 - 300\frac{72}{600}\right)^2}{300\frac{72}{600}} + \frac{\left(43 - 300\frac{78}{600}\right)^2}{300\frac{78}{600}} + \frac{\left(80 - 300\frac{152}{600}\right)^2}{300\frac{152}{600}} + \frac{\left(144 - 300\frac{298}{600}\right)^2}{300\frac{298}{600}} \approx 1{,}038$$

$$\Delta_1 = \frac{\left(39 - 300\frac{72}{600}\right)^2}{300\frac{72}{600}} + \frac{\left(35 - 300\frac{78}{600}\right)^2}{300\frac{78}{600}} +$$

$$+ \frac{\left(72 - 300\frac{152}{600}\right)^2}{300\frac{152}{600}} + \frac{\left(154 - 300\frac{298}{600}\right)^2}{300\frac{298}{600}} \approx 1{,}038$$

$$\Delta = \Delta_1 + \Delta_2 \approx 2{,}076$$

$$\rho_{\chi^2(3)}(x) = \frac{(1/2)^{3/2}}{\Gamma(3/2)} x^{\left(\frac{3}{2}-1\right)} e^{-\frac{1}{2}x} = \frac{2\sqrt{x}}{2\sqrt{2\pi}} e^{-\frac{1}{2}x} =$$

$$= \frac{\sqrt{x}}{\sqrt{2\pi}} e^{-1/3x}$$

$$\text{p-value} = \int_{2{,}076}^{\infty} \rho_{\chi^2(3)}(x)\,dx =$$

$$= \int_{2{,}078}^{\infty} \frac{\sqrt{x}}{\sqrt{2\pi}} e^{-\frac{1}{2}x} dx = 0{,}556376 > \alpha = 0{,}05$$

$\Rightarrow$ нет оснований отвергать H_0