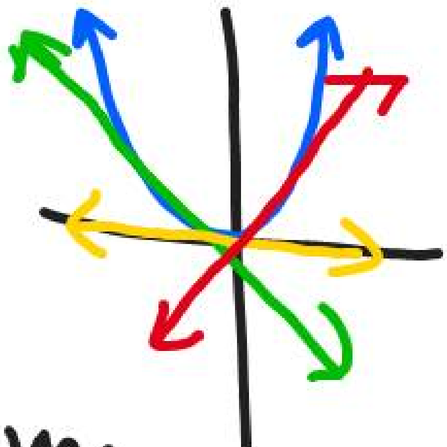
Section 3.1

Derivative of a Function

What is the slope of the line tangent to $f(x)=x^2$ at any point?



many possible slopes ???

The solution is a function called $f'(x)$ or the derivative.

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \text{at } x = a$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example: $f(x) = x^2$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} 2x + h = \boxed{2x = f'(x)}$$

The derivative gives the value of the slope of the tangent line to a curve at a point.

The **derivative** of the function f with respect to the variable x is the function $f'(x)$ whose value at x is

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

provided the limit exists.

The domain of f', the set of points in the domain of f for which the limit exists, may be smaller than the domain of f. If $f'(x)$ exists, we say that f **has a derivative (is differentiable)** at x. A function that is differentiable at every point in its domain is a **differentiable function**.

The derivative of the function f at the point where $x=a$ is the limit

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

provided the limit exists.

There are many ways to denote the derivative of a function $y = f(x)$.

Besides $f'(x)$, the most common notations are:

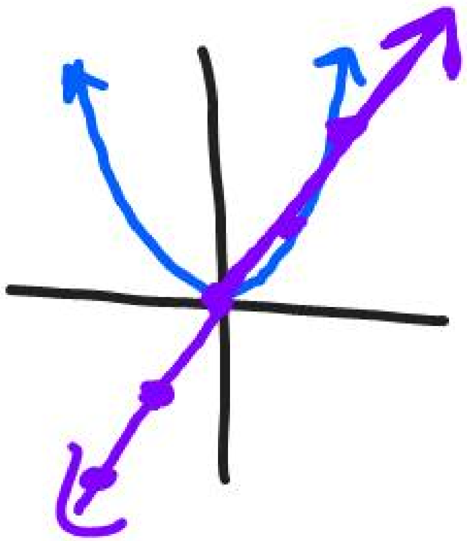
y'	"y prime"	Nice and brief, but does not name the independent variable.
$\dfrac{dy}{dx}$	"dy dx" or "the derivative of y with respect to x"	Names both variables and uses d for derivative.
$\dfrac{df}{dx}$	"df dx" or "the derivative of f with respect to x"	Emphasizes the function's name.
$\dfrac{d}{dx}\boxed{f(x)}$	"d dx of f at x" or "the derivative of f at x"	Emphasis that differentiation is an operation performed on f.

Given:

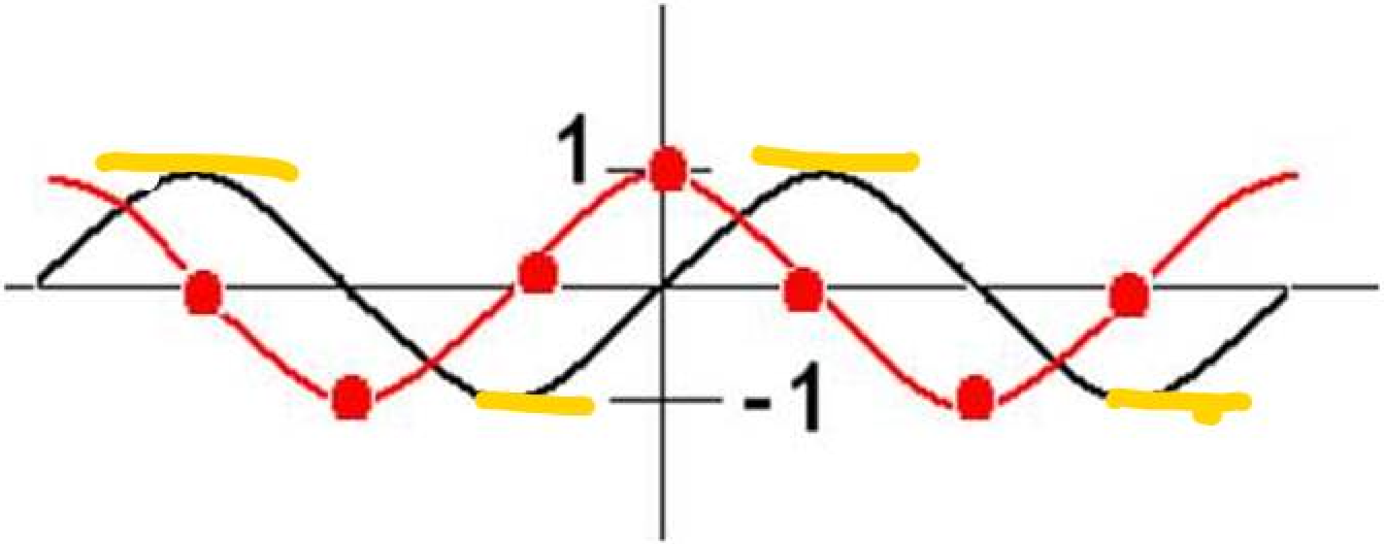
$y = f(x)$

Sketch

$y = f'(x)$

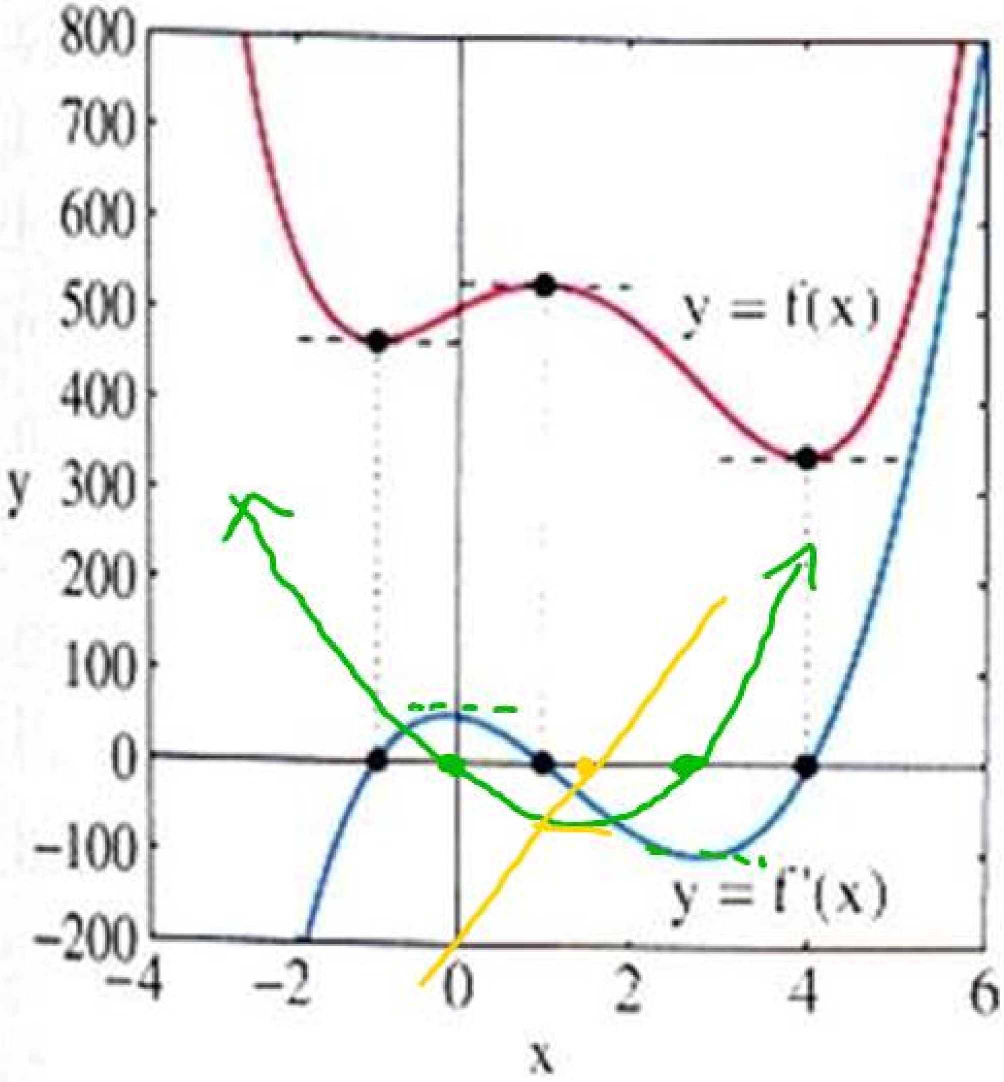


Sketching $f'(x)$ given the graph of $f(x)$.



Estimate the slope at some points.
Connect with a smooth curve.

$$\frac{d}{dx}[\sin x] = \cos x$$



$f(x)$

$f'(x)$

$f''(x)$

$f'''(x)$

FIGURE 3.1.21 Correspondence between the function graph $y = f(x)$ and the derivative graph $y = f'(x)$.