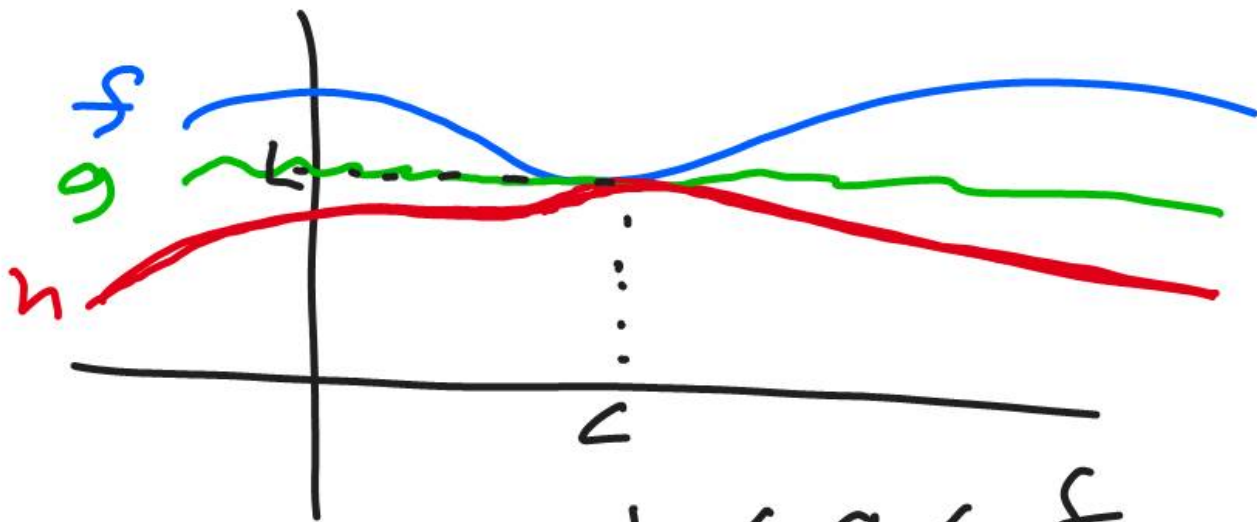


Squeeze Theorem



$$h(x) \leq g(x) \leq f(x)$$

$$\lim_{x \rightarrow c} h(x) = \lim_{x \rightarrow c} f(x) = L \therefore \lim_{x \rightarrow c} g(x) = L$$

V EXAMPLE 11 Show that $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0$.

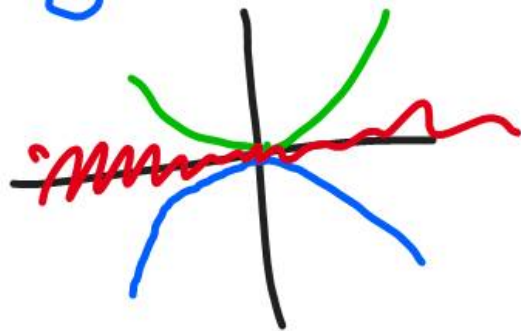
$$-1 \leq \sin \frac{1}{x} \leq 1 \rightarrow -x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$$

$$\lim_{x \rightarrow 0} -x^2 = 0$$

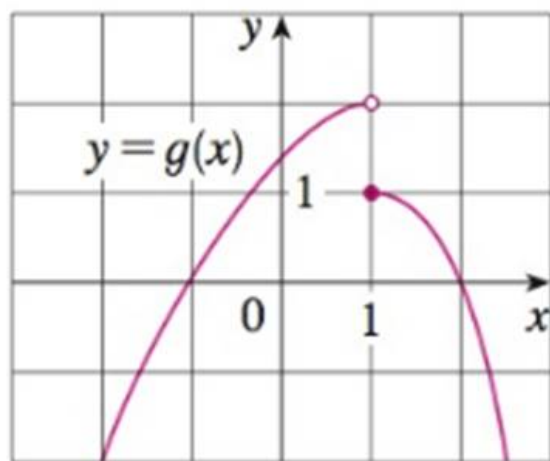
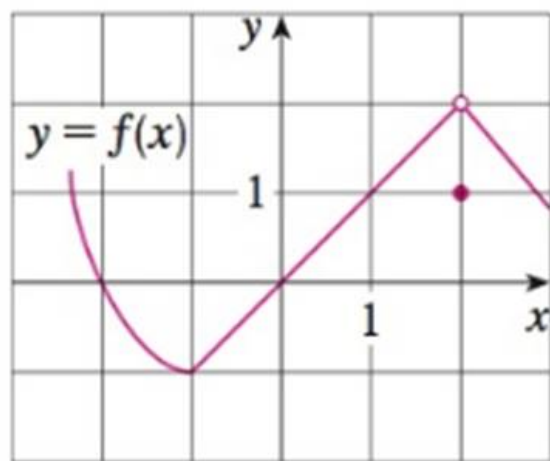
$$\lim_{x \rightarrow 0} x^2 = 0$$

$$\therefore \lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0$$

by the
sandwich
theorem



2. The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.



$$\begin{aligned} f(2) + g(2) &= 1 + 0 = 1 \\ 2^3(f(2)) &= 8 \end{aligned}$$

(a) $\lim_{x \rightarrow 2} [f(x) + g(x)] = 2 + 0 = 2$

(b) $\lim_{x \rightarrow 1} [f(x) + g(x)] = 1 + \cancel{\text{undefined}} \therefore \cancel{\text{undefined}}$

(c) $\lim_{x \rightarrow 0} [f(x)g(x)] = (0)(1.2) = 0$

(d) $\lim_{x \rightarrow -1} \frac{f(x)}{g(x)} = \frac{-1}{0} = \cancel{\text{undefined}}$

(e) $\lim_{x \rightarrow 2} [x^3 f(x)] = 8(2) = 16$ (f) $\lim_{x \rightarrow 1} \sqrt{3 + f(x)} = \sqrt{3 + 1} = 2$

34. (a) Use a graph of

$$f(x) = \frac{\sqrt{3+x} - \sqrt{3}}{x}$$

Plot1 Plot2 Plot3
Y1=(sqrt(3+X)-sqrt(3))/X
Y2=
Y3=
Y4=
Y5=
Y6=

Y1=(f(3+X)-f(3))/X
X=-.0212766 Y=.28818879

Y1=(f(3+X)-f(3))/X
X=.0212766 Y=.28816511

to estimate the value of $\lim_{x \rightarrow 0} f(x)$ to two decimal places.

.29

(b) Use a table of values of $f(x)$ to estimate the limit to four decimal places.

.2887

X	Y1
-1E-4	.28868
1E-5	.28867
0	ERROR
1E-5	.28867
.001	.28865
4.1	.22745
5.01	.21919
X=.001	

(c) Use the Limit Laws to find the exact value of the limit.

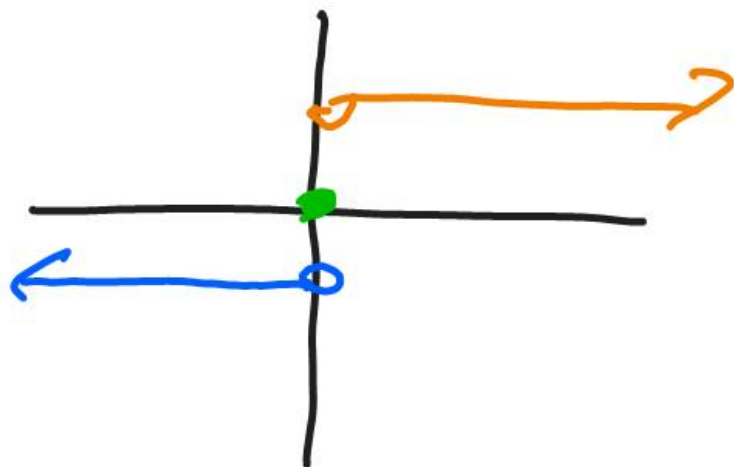
$$\lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3}}{x} \left(\frac{\sqrt{3+x} + \sqrt{3}}{\sqrt{3+x} + \sqrt{3}} \right) = \lim_{x \rightarrow 0} \frac{\cancel{3+x} - \cancel{3} (1)}{x(\sqrt{3+x} + \sqrt{3})}$$

$$\lim_{x \rightarrow 0} \frac{1}{\sqrt{3+x} + \sqrt{3}} = \frac{1}{2\sqrt{3}}$$

1/(2*sqrt(3))
.2886751346

47. The *signum* (or sign) function, denoted by sgn , is defined by

$$\text{sgn } x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$



(a) Sketch the graph of this function.

(b) Find each of the following limits or explain why it does not exist.

(i) $\lim_{x \rightarrow 0^+} \text{sgn } x = /$ (ii) $\lim_{x \rightarrow 0^-} \text{sgn } x = -1$

(iii) $\lim_{x \rightarrow 0} \text{sgn } x$ (iv) $\lim_{x \rightarrow 0} |\text{sgn } x| = 1$

$= \nexists$

