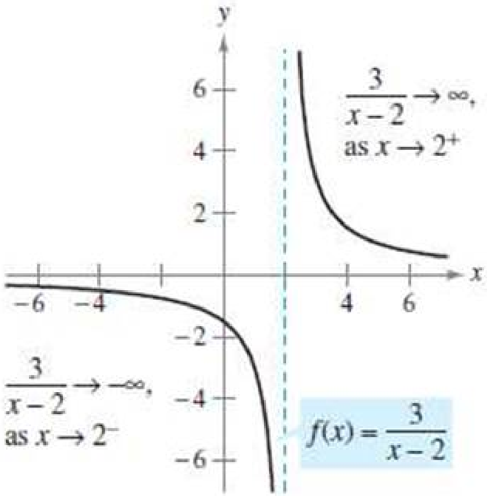
2.2 Limits involving infinity



$f(x)$ increases and decreases without bound as x approaches 2.

Figure 1.39

Let f be the function given by

$$f(x) = \frac{3}{x-2}.$$

From Figure 1.39 and the table, you can see that $f(x)$ *decreases without bound* as x approaches 2 from the left, and $f(x)$ *increases without bound* as x approaches 2 from the right. This behavior is denoted as

$$\lim_{x \to 2^-} \frac{3}{x-2} = -\infty$$

$f(x)$ decreases without bound as x approaches 2 from the left.

and

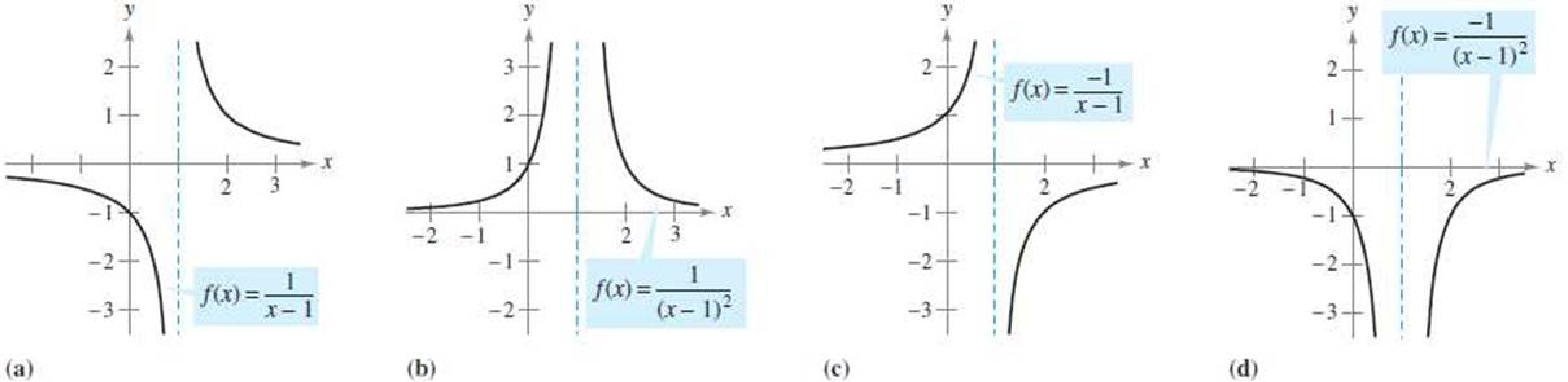
$$\lim_{x \to 2^+} \frac{3}{x-2} = \infty$$

$f(x)$ increases without bound as x approaches 2 from the right.

x approaches 2 from the left. $\rightarrow$ $\leftarrow$ x approaches 2 from the right.

x	1.5	1.9	1.99	1.999	2	2.001	2.01	2.1	2.5
$f(x)$	-6	-30	-300	-3000	?	3000	300	30	6

$f(x)$ decreases without bound. $\rightarrow$ $\leftarrow$ $f(x)$ increases without bound.



a. $\lim_{x \to 1^-} \frac{1}{x-1} = -\infty$ and $\lim_{x \to 1^+} \frac{1}{x-1} = \infty$

b. $\lim_{x \to 1} \frac{1}{(x-1)^2} = \infty$ Limit from each side is ∞.

c. $\lim_{x \to 1^-} \frac{-1}{x-1} = \infty$ and $\lim_{x \to 1^+} \frac{-1}{x-1} = -\infty$

d. $\lim_{x \to 1} \frac{-1}{(x-1)^2} = -\infty$ Limit from each side is $-\infty$.

Definition of Vertical Asymptote

If $f(x)$ approaches infinity (or negative infinity) as x approaches c from the right or the left, then the line $x = c$ is a **vertical asymptote** of the graph of f.

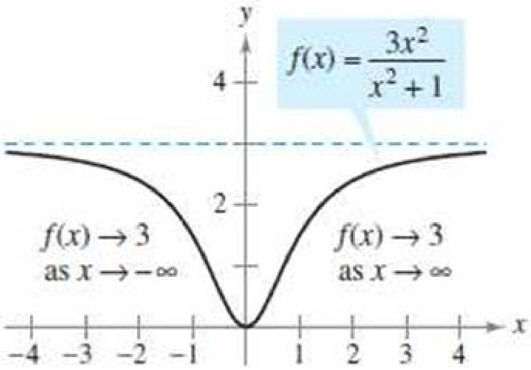
THEOREM 1.14 Vertical Asymptotes

Let f and g be continuous on an open interval containing c. If $f(c) \neq 0$, $g(c) = 0$, and there exists an open interval containing c such that $g(x) \neq 0$ for all $x \neq c$ in the interval, then the graph of the function given by

$$h(x) = \frac{f(x)}{g(x)}$$

has a vertical asymptote at $x = c$.

Limits at Infinity



The limit of $f(x)$ as x approaches $-\infty$ or ∞ is 3.

Figure 3.33

This section discusses the "end behavior" of a function on an *infinite* interval. Consider the graph of

$$f(x) = \frac{3x^2}{x^2 + 1}$$

as shown in Figure 3.33. Graphically, you can see that the values of $f(x)$ appear to approach 3 as x increases without bound or decreases without bound. You can come to the same conclusions numerically, as shown in the table.

x decreases without bound. | x increases without bound.

x	$-\infty \leftarrow$	-100	-10	-1	0	1	10	100	$\rightarrow \infty$
$f(x)$	$3 \leftarrow$	2.9997	2.97	1.5	0	1.5	2.97	2.9997	$\rightarrow 3$

$f(x)$ approaches 3. | $f(x)$ approaches 3.

Definition of a Horizontal Asymptote

The line $y = L$ is a **horizontal asymptote** of the graph of f if

$$\lim_{x \to -\infty} f(x) = L \quad \text{or} \quad \lim_{x \to \infty} f(x) = L.$$

THEOREM 3.10 Limits at Infinity

If r is a positive rational number and c is any real number, then

$$\lim_{x \to \infty} \frac{c}{x^r} = 0.$$

Furthermore, if x^r is defined when $x < 0$, then

$$\lim_{x \to -\infty} \frac{c}{x^r} = 0.$$

Find each limit.

a. $\lim_{x\to\infty} \frac{2x+5}{3x^2+1}$ **b.** $\lim_{x\to\infty} \frac{2x^2+5}{3x^2+1}$ **c.** $\lim_{x\to\infty} \frac{2x^3+5}{3x^2+1}$

Solution In each case, attempting to evaluate the limit produces the indeterminate form ∞/∞.

a. Divide both the numerator and the denominator by x^2.

$$\lim_{x\to\infty} \frac{2x+5}{3x^2+1} = \lim_{x\to\infty} \frac{(2/x)+(5/x^2)}{3+(1/x^2)} = \frac{0+0}{3+0} = \frac{0}{3} = 0$$

b. Divide both the numerator and the denominator by x^2.

$$\lim_{x\to\infty} \frac{2x^2+5}{3x^2+1} = \lim_{x\to\infty} \frac{2+(5/x^2)}{3+(1/x^2)} = \frac{2+0}{3+0} = \frac{2}{3}$$

c. Divide both the numerator and the denominator by x^2.

$$\lim_{x\to\infty} \frac{2x^3+5}{3x^2+1} = \lim_{x\to\infty} \frac{2x+(5/x^2)}{3+(1/x^2)} = \frac{\infty}{3}$$

You can conclude that the limit *does not exist* because the numerator increases without bound while the denominator approaches 3.

x	x^2	x^3
1	1	1
2	4	8
3	9	27
4	16	64
10	100	1000

Determine the following limits, if they exist.

$$\lim_{x\to\infty}\left(5-\frac{2}{x^2}\right) = 5$$

$$\lim_{x\to\infty}\frac{2x-1}{x+1} = 2$$

$$\lim_{x\to\infty}\frac{\sin x}{x} = 0$$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

Find each limit.

a. $\lim\limits_{x\to\infty} \dfrac{3x-2}{\sqrt{2x^2+1}}$ **b.** $\lim\limits_{x\to-\infty} \dfrac{3x-2}{\sqrt{2x^2+1}}$

$$\lim_{x\to\infty} \frac{\dfrac{3x-2}{\sqrt{x^2}=x}}{\sqrt{\dfrac{2x^2+1}{x^2}}} =$$

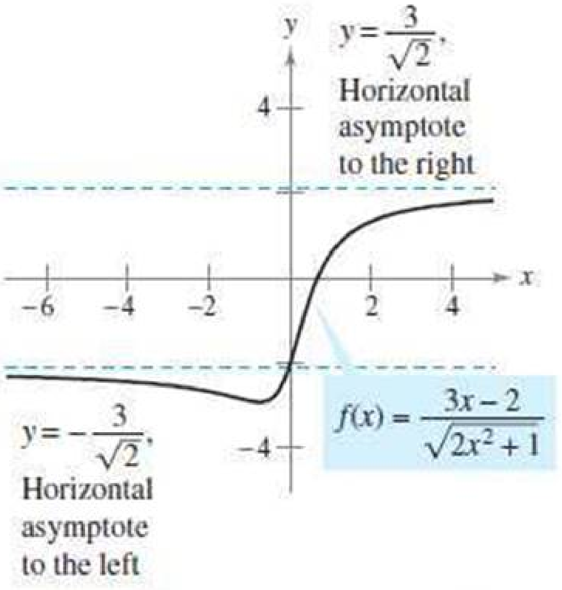
$$\lim_{x\to\infty} \frac{3-2/x}{\sqrt{2+1/x^2}} = \frac{3}{\sqrt{2}}$$

$$\lim_{x\to-\infty} \frac{\dfrac{3x-2}{\sqrt{x^2}=-x}}{\sqrt{\dfrac{2x^2+1}{x^2}}}$$

$$\lim_{x\to-\infty} \frac{-3+2/x}{\sqrt{2+1/x^2}}$$

$$= -3/\sqrt{2}$$

$$\sqrt{x^2} = |x| = \begin{cases} x \;; & x \geq 0 \\ -x \;; & x < 0 \end{cases}$$



Functions that are not rational may have different right and left horizontal asymptotes.

$$\sqrt{(2)^2} = 2$$

$$\sqrt{x^2} = x$$

True only for $x \geq 0$

$$\sqrt{(-2)^2} = 2$$

$-(-2)$

$$\sqrt{x^2} = |x|$$

*** End Behavior can be found studying other simpler functions***

Find an end behavior model. Identify any horizontal asymptote.

$$f(x) = 3x^4 - 2x + 5$$

w

$$\lim_{x \to \pm\infty} f(x) = \infty$$

$$f(x) = \frac{x-3}{2x+1}$$

$$\lim_{x \to \pm\infty} f(x) = 1/2$$

Find models for right and left end behavior

$$f(x) = e^x - 2x$$

As x goes to infinity, e^x dominates.
But as x goes to negative infinity e^x goes to zero and -2x dominates.

HW
4-48 (4x)
53,54, 61-64
Quiz AP 1-4