

1.3 Exponential Functions

DEFINITION Exponential Function

Let a be a positive real number other than 1. The function

$$f(x) = a^x$$

is the **exponential function with base a .**

2. For what values of x is it true that $2^x < 3^x < 5^x$?

$$x > 0$$

3. For what values of x is it true that $2^x > 3^x > 5^x$?

$$x < 0$$

4. For what values of x is it true that $2^x = 3^x = 5^x$?

$$x = 0$$

Rules for Exponents

If $a > 0$ and $b > 0$, the following hold for all real numbers x and y .

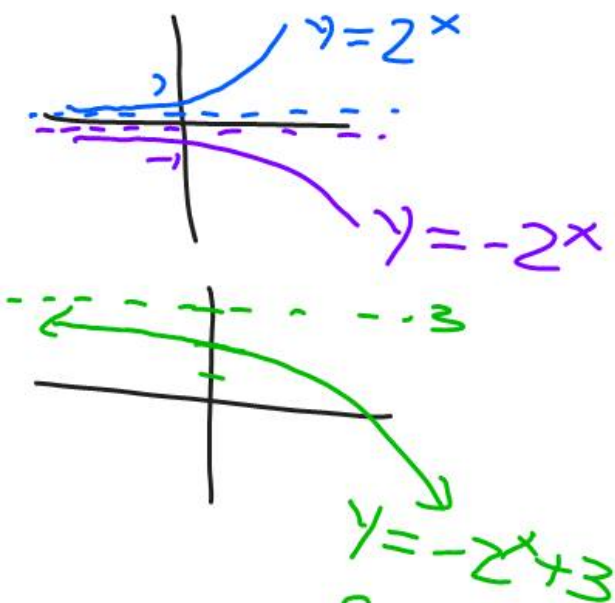
1. $a^x \cdot a^y = a^{x+y}$
2. $\frac{a^x}{a^y} = a^{x-y}$
3. $(a^x)^y = (a^y)^x = a^{xy}$
4. $a^x \cdot b^x = (ab)^x$
5. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

DEFINITIONS Exponential Growth, Exponential Decay

The function $y = k \cdot a^x$, $k > 0$ is a model for **exponential growth** if $a > 1$, and a model for **exponential decay** if $0 < a < 1$.

In Exercises 1–4, graph the function. State its domain and range.

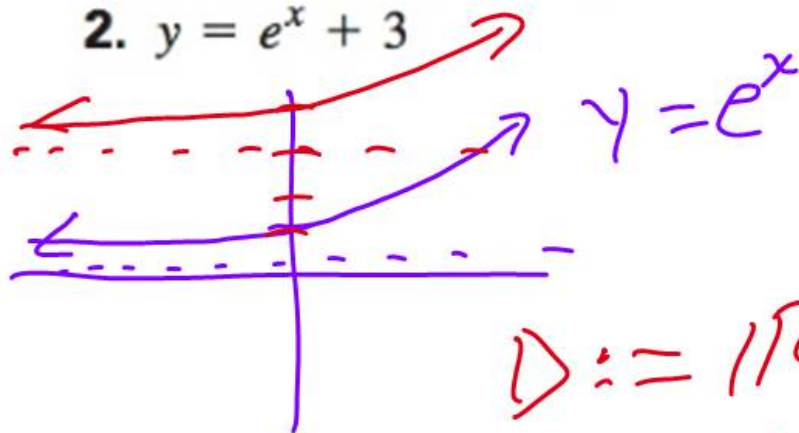
1. $y = -2^x + 3$



$$D := \mathbb{R}$$

$$\text{Range} := (-\infty, 3)$$
$$y < 3$$

2. $y = e^x + 3$



$$D := \mathbb{R}$$

$$\text{Range} := (3, \infty)$$
$$y > 3$$

In Exercises 5–8, rewrite the exponential expression to have the indicated base.

5. 9^{2x} , base 3

$$(3^2)^{2x}$$

$$3^{4x}$$

6. 16^{3x} , base 2

$$(2^4)^{3x}$$

$$2^{12x}$$

In Exercises 13–18, match the function with its graph. Try to do it without using your grapher.

13. $y = 2^x$ (a)

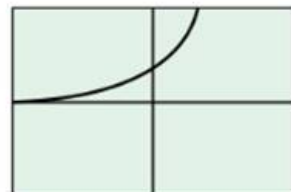
14. $y = 3^{-x}$ (d)

15. $y = -3^{-x}$ (e)

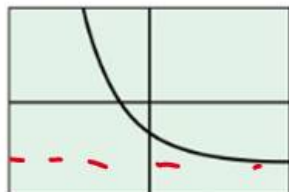
16. $y = -0.5^{-x}$ (c)

17. $y = 2^{-x} - 2$ (b)

18. $y = 1.5^x - 2$ (f)



(a)



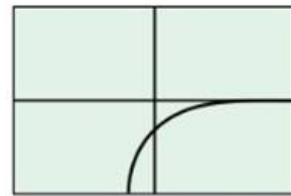
(b)



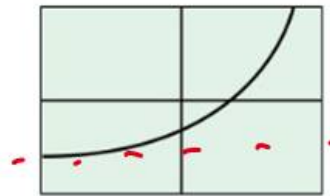
(c)



(d)



(e)



(f)

(16) $y = -\left(\frac{1}{2}\right)^{-x}$
 $= -2^x$

25. Doubling Your Money Determine how much time is required for an investment to double in value if interest is earned at the rate of 6.25% compounded annually.

$$A = P(1 + r)^t$$

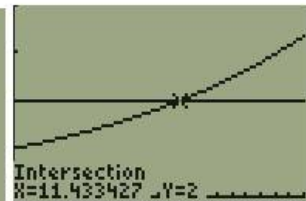
$$2 = (1)(1.0625)^t$$

$$2 = 1.0625^t \rightarrow t = \log_{1.0625} 2$$

$$= \frac{\log 2}{\log 1.0625} =$$

```
Plot1 Plot2 Plot3
Y1=2
Y2=1.0625^X
Y3=
Y4=
Y5=
Y6=
Y7=
```

```
WINDOW
Xmin=0
Xmax=20
Xscl=1
Ymin=0
Ymax=4
Yscl=1
Xres=1
```



Solve by Graph

Solve using Log

```
log(2)/log(1.0625)
11.43342688
ln(2)/ln(1.0625)
11.43342688
```


31. Cholera Bacteria Suppose that a colony of bacteria starts with 1 bacterium and doubles in number every half hour. How many bacteria will the colony contain at the end of 24 h?

$$A = P(1 + r)^t$$

$$A = 1(1 + 1)^t$$

$$A = 2^{48} \text{ bacteria}$$

$$A = 4^{24} \text{ bacteria}$$

Homework

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41-47 all

Quick quiz AP Practice

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