39. Let f be the function defined as

$$f(x) = \begin{cases} 3 - x, & x < 1 \\ ax^2 + bx, & x \geq 1 \end{cases}$$

where a and b are constants.

(a) If the function is continuous for all x, what is the relationship between a and b?

(b) Find the unique values for a and b that will make f both continuous and differentiable.

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$3-1 = a(1)^2 + b(1)$$

$$2 = a+b$$

$$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$$

$$-1 = 2ax + b$$

m

1

$$-1 = 2a + b$$

$$2-a = b$$

$$-1 = 2a + 2 - a$$

$$-3 = a$$

$$2 - -3 = b$$

$$5 = b$$

37. Find an equation of the line perpendicular to the tangent to the curve $y = x^3 - 3x + 1$ at the point $(2, 3)$

Tangent

$$\frac{dy}{dx} = 3x^2 - 3 \ ; \ \left.\frac{dy}{dx}\right|_{x=2} = 3(2)^2 - 3 = 9 = m$$

$$y - 3 = 9(x - 2)$$

Perpendicular

$$y - 3 = -\frac{1}{9}(x - 2)$$

38. Find the tangents to the curve $y = x^3 + x$ at the points where the slope is 4. What is the smallest slope of the curve? At what value of x does the curve have this slope?

$\frac{dy}{dx} = 3x^2+1 = 4$

$3x^2 - 3 = 0$

$3(x+1)(x-1) = 0$

$x = -1$	$x = 1$
$y = (-1)^3 + -1$	$y = 2$
$y = -2$	
$(-1,-2)$	$(1,2)$

$(1,2)$ $m = 4$

$y-2 = 4(x-1)$

$(-1,-2)$

$y+2 = 4(x+1)$

tangents

Min $\frac{dy}{dx} = 3x^2+1$

at $x = 0$

$\frac{dy}{dx} = 1$

In Exercises 7–12, find the horizontal tangents of the curve.

12. $y = x^4 - 7x^3 + 2x^2 + 15$

$\frac{dy}{dx} = 4x^3 - 21x^2 + 4x = 0$

$x(4x^2 - 21x + 4) = 0$

$x = 0$ $\quad x = \frac{-(-21) \pm \sqrt{(-21)^2 - 4(4)(4)}}{2(4)}$

$= \frac{21 \pm \sqrt{441 - 64}}{8}$

$x = \frac{21 \pm \sqrt{377}}{8}$

For $x = 0$

① $y = 15$

For $x = \frac{21 - \sqrt{377}}{8}$

$x \approx .198$

② $y = 15.026$

For $x = \frac{21 + \sqrt{377}}{8}$

③ $y = -185.13$

In Exercises 27 and 28, find an equation for the line tangent to the curve at the given point.

28. $y = \dfrac{x^4 + 2}{x^2}, x = -1$

Rewrite

$$y = x^2 + 2x^{-2}$$

$$\frac{dy}{dx} = 2x - 4x^{-3}$$

$$y = \frac{1+2}{1} = 3$$

$$\left.\frac{dy}{dx}\right|_{x=-1} = 2(-1) - 4(-1)^{-3} = -2 - \frac{4}{-1} = -2 + 4 = 2$$

$$y - 3 = 2(x + 1)$$

32. $y = 2\sqrt{x} - \frac{1}{\sqrt{x}} = 2x^{1/2} - x^{-1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(2)x^{1/2-1} - \left(-\frac{1}{2}\right)x^{-1/2-1}$$

$$= x^{-1/2} + \frac{1}{2}x^{-3/2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x}} + \frac{1}{2\sqrt{x^3}}$$