30. ***A Whitewater River*** Bear Creek, a Georgia river known to kayaking enthusiasts, drops more than 770 feet over one stretch of 3.24 miles. By reading a contour map, one can estimate the elevations (y) at various distances (x) downriver from the start of the kayaking route (Table 3.4).

TABLE 3.4 Elevations Along Bear Creek

Distance Downriver (miles)	River Elevation (feet)
0.00	1577
0.56	1512
0.92	1448
1.19	1384
1.30	1319
1.39	1255
1.57	1191
1.74	1126
1.98	1062
2.18	998
2.41	933
2.64	869
3.24	805

Mp slopes

.28 → −116

.74 → −178

(a) Sketch a graph of elevation (y) as a function of distance downriver (x).

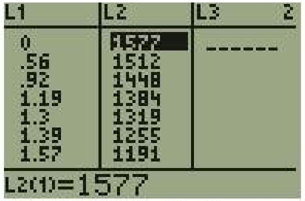
(b) Use the technique of Example 5 to get an approximate graph of the derivative, dy/dx.

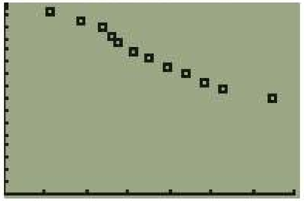
(c) The average change in elevation over a given distance is called a *gradient*. In this problem, what units of measure would be appropriate for a gradient? feet/miles

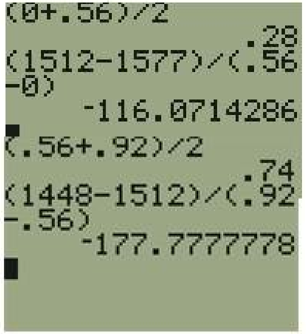
(d) In this problem, what units of measure would be appropriate for the derivative? feet/miles

(e) How would you identify the most dangerous section of the river (ignoring rocks) by analyzing the graph in (a)? Explain.

(f) How would you identify the most dangerous section of the river by analyzing the graph in (b)? Explain.



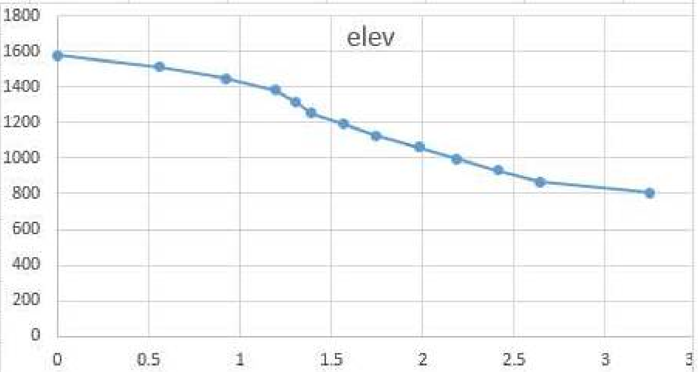




e) Since slopes are negative, the smallest slope will give you the most dangerous section.

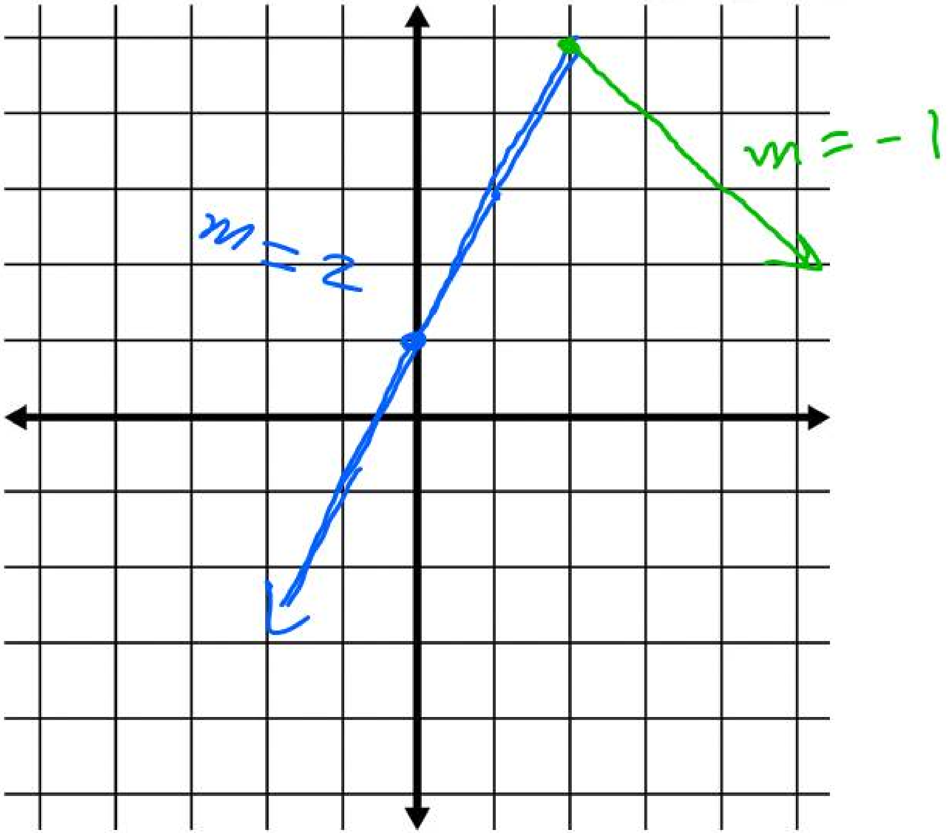
f) It will be the lowest points on the derivative graph.

dist	elev		midpoints	slopes
0	1577		0.28	-116.071
0.56	1512		0.74	-177.778
0.92	1448		1.055	-237.037
1.19	1384		1.245	-590.909
1.3	1319		1.345	-711.111
1.39	1255		1.48	-355.556
1.57	1191		1.655	-382.353
1.74	1126		1.86	-266.667
1.98	1062		2.08	-320
2.18	998		2.295	-282.609
2.41	933		2.525	-278.261
2.64	869		2.94	-106.667
3.24	805			



28. *Graphing f from f′* Sketch the graph of a continuous function f with $f(0) = 1$ and

$$f'(x) = \begin{cases} 2, & x < 2 \\ -1, & x > 2. \end{cases}$$



20. Find the lines that are (a) tangent and **(b)** normal to the curve $y = \sqrt{x}$ at $x = 4$.

point $(4,2)$

$\sqrt{x} \Rightarrow x^{1/2}$

$\Rightarrow \frac{1}{2} x^{1/2 - 1}$

$\Rightarrow \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$; $\left.\frac{dy}{dx}\right|_{x=4} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$m = \frac{1}{4}$ $pt = (4, 2)$

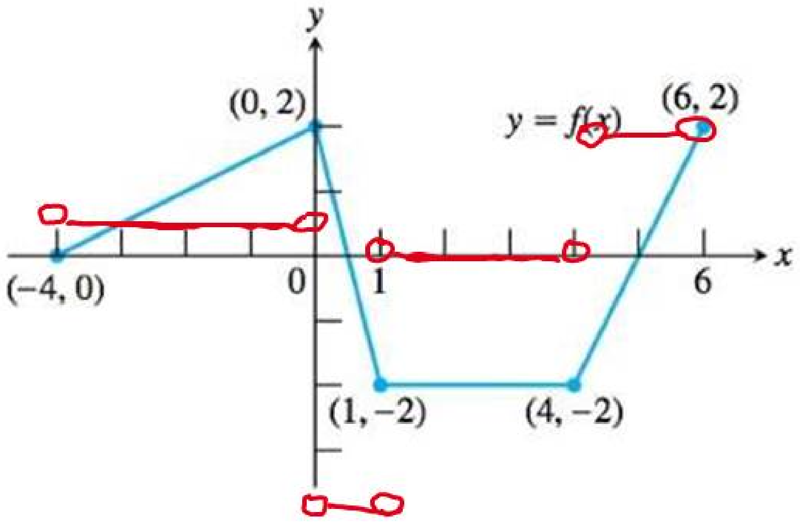
tangent line

$y - 2 = \frac{1}{4}(x-4)$

normal line

$y - 2 = -4(x-4)$

26. The graph of the function $y = f(x)$ shown here is made of line segments joined end to end.



(a) Graph the function's derivative.

(b) At what values of x between $x = -4$ and $x = 6$ is the function not differentiable?

$-4, 0, 1, 4, 6$

12. Find $\frac{d}{dx} f(x)$ if $f(x) = 3x^2$.

$\frac{d}{dx} f(x) = 6x$

Extending the Ideas

44. Find the unique value of k that makes the function

$$f(x) = \begin{cases} x^3, & x \le 1 \\ 3x + k, & x > 1 \end{cases}$$

differentiable at $x = 1$.

continuity

$$\lim_{x \to 1^-} f(x) = 1^3 = 1$$

$$\lim_{x \to 1^+} f(x) = 3 + K$$

$$3 + K = 1$$

$$K = -2$$

diff

$$\lim_{x \to 1^-} f'(x) = 3(1)^2 = 3$$

$$\lim_{x \to 1^+} f'(x) = 3$$