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In Exercises 33–36, use Theorem 7 to show that the given function is continuous.

36. $f(x) = \tan\left(\dfrac{x^2}{x^2+4}\right)$

THEOREM 7 Composite of Continuous Functions

If f is continuous at c and g is continuous at $f(c)$, then the composite $g \circ f$ is continuous at c.

$f(x)$ is discontinuous when $\dfrac{x^2}{x^2+4} = n\pi/2$

$n \in$ odd

Example

$\therefore f(x)$ is continuous

$$\frac{x^2}{x^2+4} = \pi/2$$

$$x^2 = x^2\,\pi/2 + 2\pi$$

$$x^2(1-\pi/2) = 2\pi \rightarrow x^2 = \frac{2\pi}{1-\pi/2}$$

contradiction because $\dfrac{2\pi}{1-\pi/2} < 0$

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In Exercises 33–36, use Theorem 7 to show that the given function is continuous.

36. $f(x) = \tan\left(\dfrac{x^2}{x^2+4}\right)$

THEOREM 7 Composite of Continuous Functions

If f is continuous at c and g is continuous at $f(c)$, then the composite $g \circ f$ is continuous at c.

$f(x) = (g \circ h)(x)$; $g(x) = \tan x$; $h(x) = \dfrac{x^2}{x^2+4}$

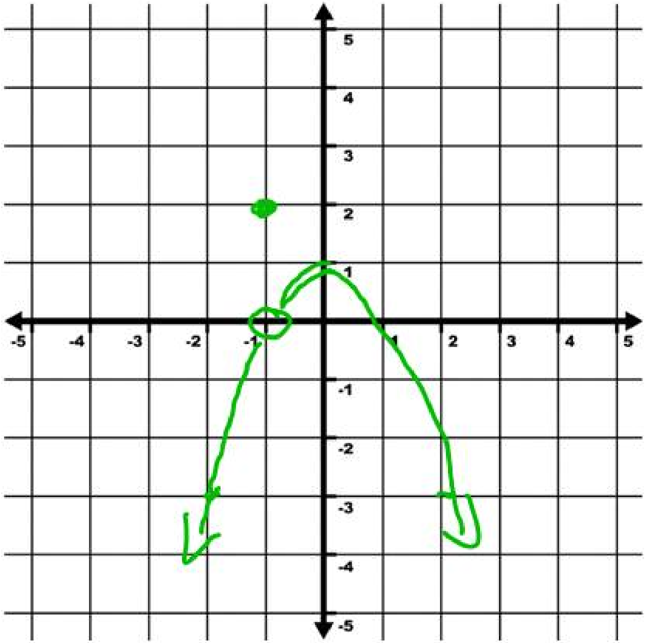
① $h(x)$ is continuos $x^2+4 \neq 0$

② $0 < \dfrac{x^2}{x^2+4} < 1$ and $\tan(c)$, $0 < c < 1$ is continuous

③ $f(x)$ is continuous

In Exercises 19–24, **(a)** find each point of discontinuity. **(b)** Which of the discontinuities are removable? not removable? Give reasons for your answers.

22. $f(x) = \begin{cases} 1 - x^2, & x \neq -1 \\ 2, & x = -1 \end{cases}$



f(x) is discontinuous at $x = -1$

This discontinuity is removable.

redifine f(x)

$$f(x) = \begin{cases} 1 - x^2 & ; \ x \neq -1 \\ 0 & ; \ x = -1 \end{cases}$$

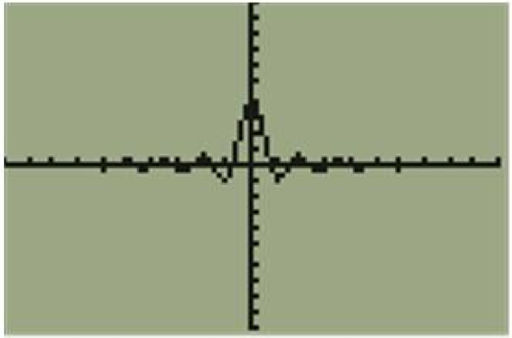
In Exercises 25–30, give a formula for the extended function that is continuous at the indicated point.

28. $f(x) = \dfrac{\sin 4x}{x}, \quad x = 0$

$$\Longrightarrow \quad g(x) = \begin{cases} \dfrac{\sin 4x}{x} ; & x \neq 0 \\ 4 ; & x = 0 \end{cases}$$

$$\lim_{x \to 0} \frac{\sin 4x}{x} = 0/0$$

$$\lim_{x \to 0} 4\left(\frac{\sin 4x}{4x}\right) = 4 \lim_{\substack{x \to 0 \\ 4x \to 0}} \underbrace{\frac{\sin 4x}{4x}}_{1} = 4(1) = 4$$



3. Multiple Choice Which of the following lines is a horizontal asymptote for

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$$f(x) = \frac{3x^3 - x^2 + x - 7}{2x^3 + 4x - 5}?$$

(A) $y = \frac{3}{2}x$ (B) $y = 0$ (C) $y = 2/3$ (D) $y = 7/5$ (E) $y = 3/2$

$$\lim_{x \to \infty} \frac{3x^3 - x^2 + x - 7}{2x^3 + 4x - 5} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{3 - \frac{1}{x} + \frac{1}{x^2} - 7/x^3}{2 + 4/x^2 - 5/x^3} = \frac{3 - 0 + 0 - 0}{2 + 0 - 0}$$

$$= 3/2$$

In Exercises 9–12, find the limit and confirm your answer using the Sandwich Theorem.

12. $\lim_{x \to \infty} \frac{\sin(x^2)}{x}$

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$$-1 \le \sin(x^2) \le 1$$

$$\frac{-1}{x} \le \frac{\sin(x^2)}{x} \le \frac{1}{x} \qquad \text{or} \qquad \frac{-1}{x} \ge \frac{\sin(x^2)}{x} \ge \frac{1}{x}$$

$x > 0$ $\qquad$ $x < 0$

$$\lim_{x \to \infty} \frac{1}{x} = 0 \;;\; \lim_{x \to \infty} \frac{-1}{x} = 0 \;\therefore\; \lim_{x \to \infty} \frac{\sin(x^2)}{x} = 0$$

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55. True or False It is possible to extend the definition of a function f at a jump discontinuity $x = a$ so that f is continuous at $x = a$. Justify your answer.

False, if there is a jump discontinuity at x=a then the limit does not exist at x=a so the function will never be continuous at x=a

In Exercises 33–36, use Theorem 7 to show that the given function is continuous.

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34. $f(x) = \sin(x^2 + 1)$

$f(x) = (h \circ g)(x)$

where $h(x) = \sin x$

and $g(x) = x^2 + 1$

Since $h(x)$ and $g(x)$ are continuous $\forall x \in \mathbb{R}$ then $f(x)$ is continuous.

In Exercises 21–26, find $\lim_{x\to\infty} y$ and $\lim_{x\to-\infty} y$.

22. $y = \left(\frac{2}{x} + 1\right)\left(\frac{5x^2 - 1}{x^2}\right)$ $\div x^2$ / $\div x^2$

$$\lim_{x\to\infty} y = \left(\frac{2}{\infty} + 1\right)\left(\frac{5 - \frac{1}{\infty^2}}{1}\right) = (1)(5) = 5$$

$$\lim_{x\to-\infty} y = 5$$