Questions? 4.2

In Exercises 27–30, use implicit differentiation to find dy/dx and then d^2y/dx^2.

28. $x^{2/3} + y^{2/3} = 1$

$$\frac{d}{dx}\left[x^{2/3} + y^{2/3} = 1\right]$$

$$\frac{\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\left(\frac{dy}{dx}\right) = 0}{2/3}$$

$$x^{-1/3} + y^{-1/3}\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = \frac{-x^{-1/3}}{y^{-1/3}} = \frac{-y^{1/3}}{x^{1/3}}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{1}{3}y^{-2/3}\left(\frac{dy}{dx}\right)x^{1/3} - \left(-y^{1/3}\right)\frac{1}{3}x^{-2/3}}{x^{2/3}}$$

$$= \left[\frac{-\frac{1}{3}y^{-2/3}\left(\frac{-y^{1/3}}{\cancel{x^{1/3}}}\right)\cancel{x^{1/3}} + \frac{y^{1/3}x^{-2/3}}{3}}{x^{2/3}}\right]\frac{x^{2/3}(3)}{x^{2/3}(3)}$$

$$= \frac{\frac{x^{2/3}}{y^{1/3}} + y^{1/3}}{3x^{4/3}}\left(\frac{y^{1/3}}{y^{1/3}}\right)$$

$$= \frac{x^{2/3} + y^{2/3}}{3x^{4/3}y^{1/3}} = \frac{1}{3x^{4/3}y^{1/3}}$$

In Exercises 17–26, find the lines that are **(a)** tangent and **(b)** normal to the curve at the given point.

24. $x \sin 2y = y \cos 2x, \quad (\pi/4, \pi/2)$

$$\frac{d}{dx}\left[x \sin 2y = y \cos 2x\right]$$

$$(1)\,\underset{\pi/2}{\cancel{\sin 2y}}^{\,0} + \underset{\pi/4}{x}\,\cos \underset{\pi/2}{2y}\,(2y') = y'\,\cancel{\cos \underset{\pi/4}{2x}}^{\,0} + \underset{\pi/2}{y}\,(-\sin 2\underset{\pi/4}{x})(2)$$

$$\frac{\pi}{4}(-1)(2y') = \frac{\pi}{2}(-1)(2)$$

$$+y'\left(\cancel{\frac{\pi}{2}}\right) = \cancel{\frac{\pi}{2}}\,\cancel{(-1)}\,(2)$$

$$y' = 2$$

TAN: $y - \frac{\pi}{2} = 2\left(x - \frac{\pi}{4}\right)$

NOR: $y - \frac{\pi}{2} = -\frac{1}{2}\left(x - \frac{\pi}{4}\right)$

4. Free Response A curve in the xy-plane is defined by

$\frac{d}{dx}[xy^2 - x^3y = 6.]$

(a) Find $\frac{dy}{dx}$.

(b) Find an equation for the tangent line at each point on the curve with x-coordinate 1.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

a) $(1)y^2 + x2y\left(\frac{dy}{dx}\right) - \left[3x^2y + x^3\left(\frac{dy}{dx}\right)\right] = 0$

$\frac{dy}{dx}\left[2xy - x^3\right] = 3x^2y - y^2$

$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

b) $x = 1 \rightarrow y^2 - y = 6$

$y^2 - y - 6 = 0 \rightarrow (y-3)(y+2) = 0$

$(1,3)$ $(1,-2)$ $y = 3, y = -2$

$m = \frac{3(1)(3) - 9}{2(1)(3) - 1} = \frac{0}{5} = 0$

$y = 3$

$m = \frac{3(1)(-2) - 4}{2(1)(-2) - 1} = \frac{-10}{-5} = 2$

$y + 2 = 2(x - 1)$

c) vertical line

$dy/dx = $ und.

$2xy - x^3 = 0$

$x(2y - x^2) = 0$

$x = 0$ $2y = x^2$

$y = x^2/2$

original $x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$

$\frac{x^5}{4} - \frac{x^5}{2} = 6$; $\frac{-x^5}{4} = 6$; $x^5 = -24$; $x = \sqrt[5]{-24}$

In Exercises 9–12, find dy/dx and find the slope of the curve at the indicated point.

12. $(x + 2)^2 + (y + 3)^2 = 25, \ (1, -7)$

$$2(x+2)(1) + 2(y+3)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2(x+2)}{2(y+3)} = \frac{-(x+2)}{y+3}$$

At $(1,-7)$

$$m = \frac{-(1+2)}{-7+3} = \frac{-3}{-4} = 3/4$$

In Exercises 13–16, find where the slope of the curve is defined.

16. $x^2 + 4xy + 4y^2 - 3x = 6$

$$\frac{d}{dx}\left[x^2 + 4xy + 4y^2 - 3x = 6\right]$$

$$2x + 4\left((1)y + x\frac{dy}{dx}\right) + 8y\frac{dy}{dx} - 3 = 0$$

$$4x\frac{dy}{dx} + 8y\frac{dy}{dx} = 3 - 2x - 4y$$

$$\frac{dy}{dx}(4x + 8y) = 3 - 2x - 4y$$

$$\frac{dy}{dx} = \frac{3 - 2x - 4y}{4x + 8y}$$

$$4x + 8y \neq 0$$

$$8y \neq -4x$$

$$y \neq -x/2$$

Slope is defined $\forall (x, y)$ such that $y \neq -x/2$