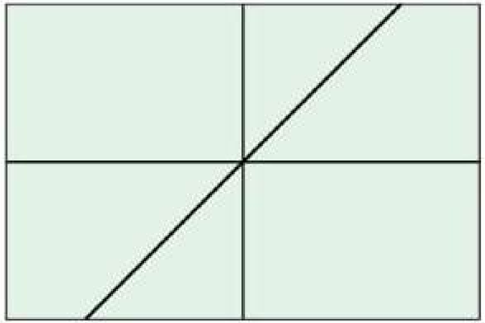
40. *Local Linearity* This is the graph of the function $y = \sin x$ close to the origin. Since $\sin x$ is differentiable, this graph resembles a line. Find an equation for this line.



$y = \sin x$

pt. $(0, 0)$

$m = \frac{dy}{dx} = \cos x$

$\left.\frac{dy}{dx}\right|_{x=0} = \cos 0 = 1$

$y - y_0 = m(x - x_0)$

$y - 0 = 1(x - 0)$

$y = x$

24. Use the definition of the derivative to prove that $(d/dx)(\cos x) = -\sin x$. (You will need the limits found at the beginning of this section.)

$f(x) = \cos x$

$$f'(x) = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cosh - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} -\cos x\left(\frac{1-\cos h}{h}\right) - \sin x\left(\frac{\sin h}{h}\right)$$

0 $\qquad$ 1

$$= 0 - \sin x = -\sin x$$

$$\lim_{h \to 0} \frac{1-\cos h}{h} \left(\frac{1+\cos h}{1+\cos h} \right)$$

$$\lim_{h \to 0} \frac{1-\cos^2 h}{h(1+\cos h)} = \lim_{h \to 0} \frac{\sin^2 h}{h(1+\cos h)}$$

$$\lim_{h \to 0} \left(\frac{\sin h}{h} \right) \left(\frac{\sin h}{1+\cos h} \right)$$

$$(1)\left(\frac{0}{1+1} \right) = 0$$

$P\left(\frac{\pi}{4}, 4\right)$

$m = \frac{dy}{dx} = \sqrt{2}(-\csc x \cot x) + (-\csc^2 x)$

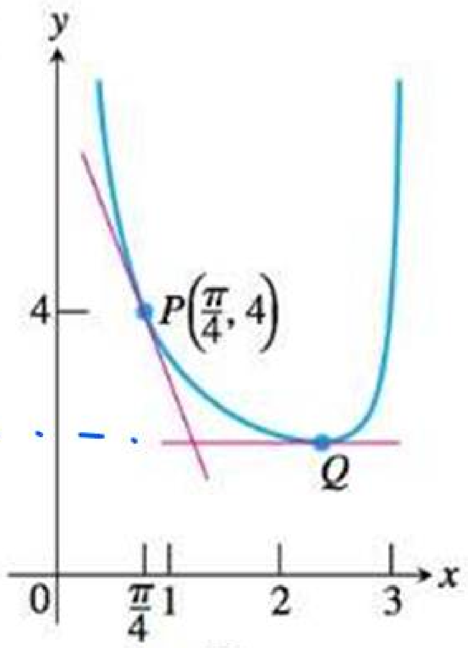
$m = -\sqrt{2}\left(\csc\frac{\pi}{4}\cot\frac{\pi}{4}\right) - \left(\csc\frac{\pi}{4}\right)^2$

$= -\sqrt{2}\sqrt{2}(1) - (\sqrt{2})^2$

$= -2 - 2 = -4$

$y - 4 = -4\left(x - \frac{\pi}{4}\right)$

32.



$y = 1 + \sqrt{2}\csc x + \cot x$

$y = c$

$$\frac{\sqrt{2}(-\csc x \cot x) - \csc^2 x}{-\csc x} = \frac{0}{-\csc x}$$

$\sqrt{2}\cot x + \csc x = 0$

$\sqrt{2}\frac{\cos x}{\sin x} + \frac{1}{\sin x} = 0$

$\frac{1}{\sin x}(\sqrt{2}\cos x + 1) = 0$

$\frac{1}{\sin x} \neq 0$

$\sqrt{2}\cos x + 1 = 0$

$\cos x = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$x = \frac{3\pi}{4}$

$y = 1 + \sqrt{2}\csc\frac{3\pi}{4} + \cot\frac{3\pi}{4}$

$y = 1 + (\sqrt{2})^2 + (-1)$

$y = 2$

36. Find y'' if $y = \theta \tan \theta$.

$$y' = \frac{dy}{d\theta} = (1)\tan\theta + \theta\sec^2\theta$$

$$y'' = \sec^2\theta + (1)\sec^2\theta + \theta(2)\sec\theta[\sec\theta\tan\theta]$$

$$y'' = 2\sec^2\theta + 2\theta\sec^2\theta\tan\theta$$

$$y'' = 2\sec^2\theta(1 + \theta\tan\theta)$$