3.3 Rules for Differentiation

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(mx) = \mathrm{m}$$

Proofs on book

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}\big(f(x) \pm g(x)\big) = f'(x) \pm g'(x)$$

$$\frac{d}{dx}(cf(x)) = cf'(x)$$

That's the end of the easy short cuts!

"The next part of this recipe will involve some calculus."

In Exercises 1–6, find dy/dx.

5. $y = \dfrac{x^3}{3} + \dfrac{x^2}{2} + x$

$dy/dx = \dfrac{3x^2}{3} + \dfrac{2x}{2} + 1 = x^2 + x + 1$

In Exercises 7–12, find the horizontal tangents of the curve.

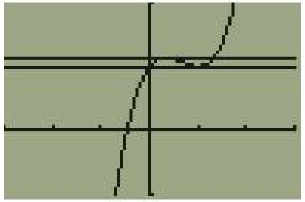
7. $y = x^3 - 2x^2 + x + 1$

$y' = 3x^2 - 4x + 1 = 0$

$(3x-1)(x-1) = 0$

$x = 1/3 \qquad x = 1$

$y = 31/27 \qquad y = 1$



$y\big|_{x=1/3} = \frac{1}{27} - \frac{2}{9} + \frac{1}{3} + 1$

$y = \dfrac{1-6+9+27}{27} = \dfrac{31}{27}$

$y\big|_{x=1} = 1-2+1+1 = 1$

11. $y = 5x^3 - 3x^5$

$\frac{dy}{dx} = 15x^2 - 15x^4 = 0$

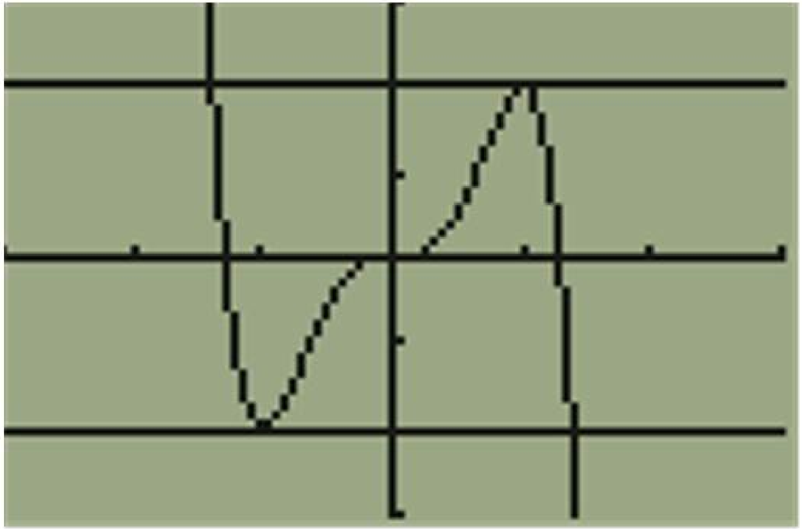
$15x^2(1 - x^2) = 0$

$x = 0$; $x = \pm 1$

$y = 0$; $y = 2$; $y = -2$

$5(1)^3 - 3(1)^5$

$5(-1)^3 - 3(-1)^5$



$$Example:$$

$$f(x) = 5x^5 - x^3 + 8x^2 - x^{-2} + 6x + 45$$

$$f'(x) = 25x^4 - 3x^2 + 16x + 2x^{-3} + 6$$

$$f(x) = \sqrt{x} = x^{1/2}$$

$$f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$\frac{d}{dx}(f(x)g(x))$ = ?

Example: $h(x) = \underline{x^3} \cdot \underline{x^6} = x^9$ so $h'(x) = 9x^8$

If we try to do them separately we get
$3x^2 \cdot 6x^5 = 18x^7$ which is what we call **WRONG!**

Instead we need the PRODUCT RULE

$$\boxed{\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + g(x)f'(x)}$$

See poof in book

$x^3 \cdot 6x^5 + x^6 \cdot 3x^2$

$6x^8 + 3x^8 = 9x^8$

13. Let $y = (\overset{u}{x+1})(\overset{v}{x^2+1})$. Find dy/dx **(a)** by applying the Product Rule, and **(b)** by multiplying the factors first and then differentiating.

a) $\frac{dy}{dx} = u'v + v'u$

$= (1)(x^2+1) + 2x(x+1)$

$= x^2+1+2x^2+2x$

$= 3x^2+2x+1$

b) $y = x^3+x+x^2+1$

$\frac{dy}{dx} = 3x^2+1+2x$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = ?$$

Example: $h(x) = \frac{x^{16}}{x^5} = x^{11},\ so\ h'(x) = 11x^{10}$

Separately we get: $\cancel{\frac{16x^{15}}{5x^4} = 3.2x^{11}}$

Instead we need the QUOTIENT RULE

$$\boxed{\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}}$$

See proof in book

In Exercises 15–22, find dy/dx. (You can support your answer graphically.)

not differentiable at x=0

19. $y = \dfrac{(x-1)(x^2+x+1)}{x^3}$

rewrite

$$= \frac{x^3 + \cancel{x^2} + \cancel{x} - \cancel{x^2} - \cancel{x} - 1}{x^3} = \frac{x^3 - 1}{x^3}$$

u

v

Quotient rule

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{3x^2(x^3) - (x^3-1)3x^2}{(x^3)^2} = \frac{\cancel{3x^5} - \cancel{3x^5} + 3x^2}{x^6}$$

$$= \frac{3x^2}{x^6} = \frac{3}{x^4}$$

$$y = 1 - x^{-3}$$

$$\frac{dy}{dx} = 3x^{-4}$$

23. Suppose u and v are functions of x that are differentiable at $x = 0$, and that $u(0) = 5$, $u'(0) = -3$, $v(0) = -1$, $v'(0) = 2$. Find the values of the following derivatives at $x = 0$.

(a) $\dfrac{d}{dx}(uv)$ (b) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)$

(c) $\cancel{\dfrac{d}{dx}\left(\dfrac{u}{v}\right)}$ (d) $\dfrac{d}{dx}(7v - 2u)$

a) $\dfrac{d}{dx}(uv) = uv' + u'v = (5)(2) + (-3)(-1) = 10 + 3 = 13$

b) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{u'v - uv'}{v^2} = \dfrac{(-3)(-1) - (5)(2)}{(-1)^2} = 3 - 10 = -7$

d) $\dfrac{d}{dx}(7v - 2u) = 7v' - 2u' = 7(2) - 2(-3) = 14 + 6 = 20$

Example: $f(x) = \frac{8}{x^2+1} = \frac{u}{v}$

Quotient Rule

$$f'(x) = \frac{0(x^2+1) - 8(2x)}{(x^2+1)^2} = \frac{-16x}{(x^2+1)^2}$$

RECIPROCAL RULE

$$\frac{d}{dx}\left(\frac{c}{g(x)}\right) = \frac{-cg'(x)}{(g(x))^2}$$

$$f'(x) = \frac{-8(2x)}{(x^2+1)^2}$$

Problems

37. Find an equation of the line perpendicular to the tangent to the curve $y = x^3 - 3x + 1$ at the point $(2, 3)$

$$\frac{dy}{dx} = 3x^2 - 3 \Big|_{x=2} = 3(2^2) - 3 = 9 = m \text{ for tangent}$$

$$\perp m = -1/9$$

$$y - 3 = -1/9(x - 2)$$

39. Find the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ where the tangent is parallel to the x-axis.

$$\frac{dy}{dx} = 6x^2 - 6x - 12 = 0 = m$$

horizontal Lines

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \quad ; \quad x = -1$$

$$y = 2(2)^3 - 3(2)^2 - 12(2) + 20$$

$$16 - 12 - 24 + 20$$

$$y = 0$$

$$(2, 0)$$

$$y = 2(-1) - 3(1) + 12 + 20$$

$$= 27$$

$$(-1, 27)$$

Higher Order Derivatives

We can compute the derivative of a derivative.

It turns out that such **higher order** derivatives have important applications.

We can compute the derivative of f', called the **second derivative** of f and written f''. We can then compute the derivative of f'', called the **third derivative** of f, written f'''.

For example:

$$f(x) = position$$

$$f'(x) = rate\ of\ change\ in\ position = velocity$$

$$f''(x) = rate\ of\ change\ in\ velocity = acceleration$$

$$f'''(x) or\ f^{(3)}(x) = rate\ of\ change\ in\ acceleration = jerk$$

$$f^{(4)}(x) = rate\ of\ change\ in\ jerk\ (no\ special\ name)$$

$$f^{(5)}(x) = rate\ of\ change\ in\ f^{(4)}(x)\ (no\ special\ name)$$

and so on forever.....


You're a third derivative
I don't even know what that means yet...

Order	*Prime Notation*	*Leibniz Notation*
1	$y' = f'(x)$	$\dfrac{df}{dx}$
2	$y'' = f''(x)$	$\dfrac{d^2 f}{dx^2}$
3	$y''' = f'''(x)$	$\dfrac{d^3 f}{dx^3}$
4	$y^{(4)} = f^{(4)}(x)$	$\dfrac{d^4 f}{dx^4}$
5	$y^{(5)} = f^{(5)}(x)$	$\dfrac{d^5 f}{dx^5}$

In Exercises 33–36, find the first four derivatives of the function.

33. $y = x^4 + x^3 - 2x^2 + x - 5$

$y' = 4x^3 + 3x^2 - 4x + 1$

$y'' = 12x^2 + 6x - 4$

$y''' = 24x + 6$

$y^{(4)} = 24$

51. ***Orchard Farming*** An apple farmer currently has 156 trees yielding an average of 12 bushels of apples per tree. He is expanding his farm at a rate of 13 trees per year, while improved husbandry is boosting his average annual yield by 1.5 bushels per tree. What is the current (instantaneous) rate of increase of his total annual production of apples? Answer in appropriate units of measure.

$$P = T \cdot A$$

$$P' = T' \cdot A + T \cdot A'$$

$$= \underset{\frac{T}{Y}}{(13)}\underset{\frac{BA}{T}}{(12)} + \underset{\frac{T}{Y}}{(156)}\underset{\frac{BA}{T}}{(1.5)} = 390 \ \frac{\text{bushels of apples}}{\text{year}}$$

HW (3.3)
pg 124
4-20 (4x)
24-26 all
28-40 even
53-58 all
Quiz AP 1-4