2.1 Questions????

In Exercises 65–68, find the limit graphically. Use the Sandwich Theorem to confirm your answer.

68. $\lim_{x \to 0} x^2 \cos \frac{1}{x^2}$

$\lim_{x \to 0} -x^2 = 0$

$\lim_{x \to 0} x^2 = 0$

$\left[-1 \leq \cos\left(\frac{1}{x^2}\right) \leq 1\right] x^2$

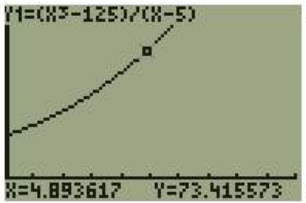
$-x^2 \leq x^2 \cos\left(\frac{1}{x^2}\right) \leq x^2$

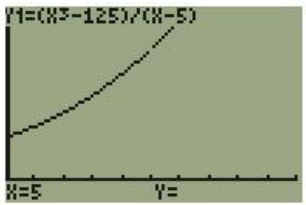
$\therefore$ by the Sandwich Theorem

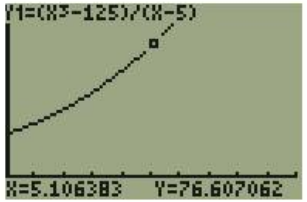
$\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right) = 0$

In Exercises 25–34, determine the limit graphically. Confirm algebraically.

34. $\lim_{x \to 5} \dfrac{x^3 - 125}{x - 5}$ $= 75$







X	Y1
4.99	74.85
4.999	74.985
4.9999	74.999
5	ERROR
5.0001	75.002
5.001	75.015
5.01	75.15

X=5.01

5 | 1 0 0 -125
5 25 125
1 5 25 | 0

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$\lim_{x \to 5} \frac{x^3 - 125}{x - 5} = \lim_{x \to 5} x^2 + 5x + 25$$

$$= 25 + 25 + 25 = 75$$

In Exercises 61–64, complete parts (a)–(d) for the piecewise-defined function.

(a) Draw the graph of f.

(b) At what points c in the domain of f does $\lim_{x \to c} f(x)$ exist?

(c) At what points c does only the left-hand limit exist?

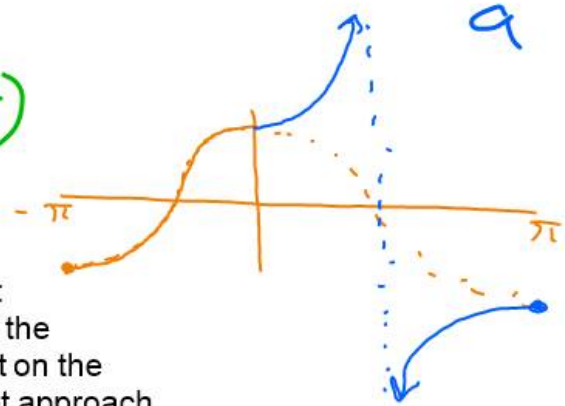
(d) At what points c does only the right-hand limit exist?

62. $f(x) = \begin{cases} \cos x, & -\pi \le x < 0 \\ \sec x, & 0 \le x \le \pi \end{cases}$

b) $\lim_{x \to c} f(x)$ exist $\forall\ c \in (-\pi, \pi)$ except $c = \pi/2$

c) $\lim_{x \to \pi^-} f(x) = -1$

d) $\lim_{x \to -\pi^+} f(x) = -1$



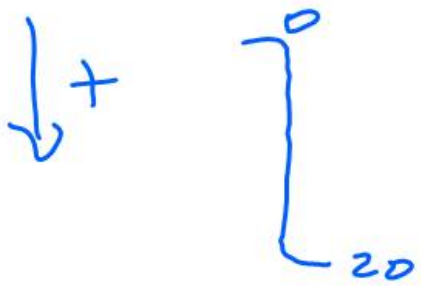
NOTE:
PI and -PI are not included because the limit does not exist on the end points (cannot approach from both sides)

70. *Free Fall on a Small Airless Planet* A rock released from rest to fall on a small airless planet falls $y = gt^2$ m in t sec, g a constant. Suppose that the rock falls to the bottom of a crevasse 20 m below and reaches the bottom in 4 sec.

(a) Find the value of g.

(b) Find the average speed for the fall.

(c) With what speed did the rock hit the bottom?



$y = gt^2$

$20 = g(4)^2$

$\therefore g = 5/4 \text{ m/s}^2$

Average speed: $\bar{v} = \frac{20 - 0}{4 - 0} = 5 \text{ m/s}$

$v(4) = \lim_{\Delta t \to 0} \frac{g(4+\Delta t)^2 - g(4)^2}{4 + \Delta t - 4} = \lim_{\Delta t \to 0} \frac{g(16 + 8\Delta t + (\Delta t)^2) - 16g}{\Delta t}$

$\lim_{\Delta t \to 0} \frac{8g\Delta t + (\Delta t)^2 g}{\Delta t} = \lim_{\Delta t \to 0} 8g + \Delta t g = 8g$

$= 8(5/4) = 10 \text{ m/s}$