Differentiation by the Chain Rule - Classwork

Suppose you were asked to take the derivatives of the following. Do so.

a) $f(x)=(x^2+3x)^2$ b) $f(x)=(x^2+3x)^3$ c. $f(x)=(x^2+3x)^6$ d) $f(x)=\sqrt{x^2+3x}$

a) causes no problem. b) is also not a problem but multiplying it out so you can take the derivative is a bit of a pain. You are capable of doing c) but clearly do not wish to. But d) can't be done with the knowledge you have.

We now introduce a method of taking derivatives of more complicated expressions. It is called the **chain rule**. If $y=f(u)$ is a differentiable function of u and $u=g(x)$ is a differentable function of x, then $y=f(g(x))$ is a differentable function of x and $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$ or equivalently, $\frac{d}{dx}\left[f(g(x))\right]=f'(g(x))\cdot g'(x)$

Example 1) If $f(x)=(2x+5)^2$, find $f'(x)$ without and with the chain rule. Show they are equivalent.

a) without chain rule b) with chain rule

Example 2) If $f(x)=(2x+5)^3$, find $f'(x)$ without and with the chain rule. Show they are equivalent.

a) without chain rule b) with chain rule

Example 3) If $f(x)=(2x+5)^{10}$, find $f'(x)$ Example 4) If $f(x)=\sqrt{2x+5}$, find $f'(x)$

Example 5) Find y' if $y=\frac{1}{4x-3}$ Example 6) Find y' if $y=(3x^2-2x+1)^3$

Find the derivatives of the following:

7) $y=(7-4x^2)^{2/3}$ 8) $y=-5\sqrt{x^2-4x+1}$ 9) $y=\frac{-2}{\sqrt[3]{6x-1}}$

More difficult problems: We now have 3 basic rules. Power rule, product rule, and quotient rule. Note that the chain rule is not a basic rule of differentiation. The chain rule is always in effect. Even when you find the derivative of $y=7x$, your answer is 7 times the derivative of x which is 7(1) = 7.

Find the derivatives of the following:

10) $y=x^2(2x-3)^4$ 11) $y=x\sqrt{4-x^2}$ 12) $y=\left(\frac{2x-1}{2x+1}\right)^5$

13) $y=\frac{x}{\sqrt{x^2-1}}$ 14) $y=\sqrt{\frac{x}{4x-1}}$ 15) $y=\sqrt[3]{4x-\sqrt{2x+1}}$

Given that $f(2)=-3, f'(2)=6, g(2)=3, g'(2)=-2, f'(3)=4,$ find the derivatives of the following at $x=2$.

16) $f(x)\cdot g(x)$ 17) $\frac{f(x)}{g(x)}$ 18) $[f(x)]^3$ 19) $f(g(x))$

Differentiation by the Chain Rule - Homework

Find the derivatives of the following:

1. $y=(3x-8)^4$ 2. $y=(3x^2+2)^5$ 3. $y=4(x^2+x-1)^{10}$

4. $y=-5(4-9x)^{3/2}$ 5. $y=\frac{1}{3x-2}$ 6. $y=\frac{-1}{(x^2-5x-6)^2}$

7. $y=\left(\frac{2}{2-x}\right)^2$ 8. $y=\frac{4x}{(x+1)^2}$ 9. $y=\frac{-3}{(x^3-x^2+3)^3}$

10. $y=x^3(5x-1)^4$ 11. $y=\sqrt{1-t}$ 12. $y=\sqrt[3]{3x^3-4x+2}$

13. $y=\frac{2}{\sqrt{2x+3}}$ 14. $y=\frac{-1}{\sqrt{x+1}}$ 15. $y=\sqrt{\frac{3x}{2x-3}}$

16. $y=\sqrt{x}(1-2x)^2$ 17. $y=\sqrt[3]{\frac{2t}{t^2-4}}$ 18. $y=(x^2+2x-6)^2(1-x^3)^2$

For each of the following, find the equation of the tangent line at the indicated point. Verify by calculator.

19. $y=\sqrt{x^2+2x+8}$ at $(2,4)$ 20. $y=\sqrt[5]{3x^3+4x}$ at $(2,2)$ 21. $y=\sqrt{\frac{3x-1}{2x+1}}$ at $(-1,2)$

Given the following information, find the value of the derivative of the functions at $x=3$. Be careful, not all the information is needed to calculate these. Answers are next to the problem.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	1	8	-3	-5
6	3	-2	4	5
8	-1	3	π	4
1	2	-6	5	0

22. $f(x)+g(x)$ (Ans: -8) 23. $f(x)g(x)$ (Ans: -29) 24. $\frac{f(x)}{g(x)}$ (Ans: $\frac{-19}{64}$)

25. $\frac{g(x)}{f(x)}$ (Ans: 19) 26. $(f(x))^2$ (Ans: -6) 27. $\frac{1}{g(x)}$ (Ans: $\frac{5}{64}$)

28. $\sqrt{f(x)}$ (Ans: $\frac{-3}{2}$) 29. $\sqrt{f(x)+g(x)}$ (Ans: $\frac{-4}{3}$) 30 $f^3(x)g(x)$ (Ans: -77)