4.1 Homework questions???

In Exercises 41–48, find the equation of the line tangent to the curve at the point defined by the given value of t.

48. $x = \cos t, \quad y = 1 + \sin t, \quad t = \pi/2$

point (x, y)

$(\cos \pi/2, 1+\sin \pi/2)$

$(0, 2)$

$$m = dy/dx = \frac{dy/dt}{dx/dt} = \frac{0}{-1} = 0$$

$$dy/dt = 0 + \cos t$$

$$\left.\frac{dy}{dt}\right|_{t=\pi/2} = 0$$

$$\frac{dx}{dt} = -\sin t = -1$$

$$\left.\frac{dx}{dt}\right|_{t=\pi/2}$$

Tangent line

$$y = 2$$

In Exercises 29–32, find y''.

32. $y = 9\tan(x/3)$

$$y' = 9\sec^2\left(\frac{x}{3}\right)\left(\frac{1}{3}\right) = 3\sec^2\left(\frac{x}{3}\right)$$

$$y'' = \cancel{3}\left[2\sec\left(\frac{x}{3}\right)\sec\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)\left(\frac{1}{\cancel{3}}\right)\right]$$

$$y'' = 2\sec^2\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)$$

70. True or False $\frac{d}{dx}(\sin x) = \cos x$, if x is measured in degrees or radians. Justify your answer.

$$\frac{d}{dx}(\sin x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} -\sin x\left(\frac{1-\cos h}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)$$

$$= 0 + \cos x\,(1)$$

$$= \cos x$$

True for both because $0_{RAD} = 0^{\circ}$

20. $y = \dfrac{x}{\sqrt{1+x^2}}$ $\Rightarrow$ $y = x(1+x^2)^{-1/2}$

$$y' = \frac{(1)\sqrt{1+x^2} - x\left(\frac{1}{2}\right)(1+x^2)^{-1/2}(2x)}{1+x^2}$$

$$= \frac{\sqrt{1+x^2} - \dfrac{x^2}{\sqrt{1+x^2}}}{1+x^2}\left(\frac{\sqrt{1+x^2}}{\sqrt{1+x^2}}\right)$$

$$= \frac{1+x^2-x^2}{(1+x^2)^{3/2}} = \frac{1}{(1+x^2)^{3/2}}$$

$$y' = (1)(1+x^2)^{-1/2} + x\left(-\frac{1}{2}\right)(1+x^2)^{-3/2}(2x)$$

$$= \frac{1}{(1+x^2)^{1/2}} - \frac{x^2}{(1+x^2)^{3/2}}$$

$$= \frac{1(1+x^2) - x^2}{(1+x^2)^{3/2}}$$

$$= \frac{1}{(1+x^2)^{3/2}}$$

In Exercises 13–24, find dy/dx. If you are unsure of your answer, use NDER to support your computation.

28. $r = \underset{u}{2\theta}\underset{v}{\sqrt{\sec\theta}}$

$r' = u'v + uv'$

$$\frac{dr}{d\theta} = 2\sqrt{\sec\theta} + 2\theta\underbrace{\left(\frac{1}{2}\right)(\sec\theta)^{-1/2}(\sec\theta\tan\theta)}_{\text{Chain Rule}}$$

$$\frac{dr}{d\theta} = 2\sqrt{\sec\theta} + \theta\sqrt{\sec\theta}\tan\theta$$

$\frac{d}{dx}(\sqrt{u}) = \frac{1}{2}u^{\frac{1}{2}-1} \cdot u'(x)$

$\sqrt{u} \downarrow u^{1/2}$

In Exercises 25–28 find $dr/d\theta$.

44. $x = \sec t, \quad y = \tan t, \quad t = \pi/6$

In Exercises 41–48, find the equation of the line tangent to the curve at the point defined by the given value of t.

point

$$\left(\sec \tfrac{\pi}{6}, \tan \tfrac{\pi}{6}\right) \Rightarrow \left(\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

slope

$$m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sec^2 t}{\sec t \tan t} = \frac{\sec t}{\tan t} = \frac{1/\cos t}{\sin t/\cos t} = \frac{1}{\sin t} = \csc t$$

$$m\Big|_{t=\pi/6} = 2$$

$$y - \frac{1}{\sqrt{3}} = 2\left(x - \frac{2}{\sqrt{3}}\right)$$

In Exercises 33–38, find the value of $(f \circ g)'$ at the given value of x.

38. $f(u) = \left(\frac{u-1}{u+1}\right)^2, \quad u = g(x) = \frac{1}{x^2} - 1, \quad x = -1$

$$u\Big|_{x=-1} = 0$$

$$f'(u) = (f \circ g)'(x) = 2\left(\frac{u-1}{u+1}\right)'\left(\frac{u'(u+1)-(u-1)(u')}{(u+1)^2}\right)$$

$$= 2\left(\frac{0-1}{0+1}\right)\left(\frac{u'(1)-(-1)u'}{(1)^2}\right) = 2(-1)(4) = -8$$

$$u' = -2x^{-3} = \frac{-2}{x^3}$$

$$u'\Big|_{x=-1} = 2$$

38. $f(u) = \left(\frac{u-1}{u+1}\right)^2, \quad u = g(x) = \frac{1}{x^2} - 1, \quad x = -1$

$$f(g(x)) = \left(\frac{\frac{1}{x^2} - 2}{\frac{1}{x^2}}\right)^2 = \left(\left(\frac{x^2}{x^2}\right)\left(\frac{\frac{1}{x^2} - 2}{1/x^2}\right)\right)^2 = \left(\frac{1-2x^2}{1}\right)^2$$

$$f(g(x)) = (1-2x^2)^2$$

$$(f \circ g)'(x) = 2(1-2x^2)(-4x)$$

$$(f \circ g)'(-1) = 2(-1)(4) = -8$$

36. $f(u) = u + \frac{1}{\cos^2 u}, \quad u = g(x) = \pi x, \quad x = \frac{1}{4}$

$$f(u) = (f \circ g)(x) = \pi x + \frac{1}{\cos^2 \pi x} = \pi x + (\cos \pi x)^{-2}$$

$$(f \circ g)'(x) = \pi - 2(\cos \pi x)^{-3}(-\sin \pi x) \cdot \pi$$

$$(f \circ g)'\left(\tfrac{1}{4}\right) = \pi + \frac{2\pi \sin \pi/4}{(\cos \pi/4)^3} = \pi + \frac{2\pi}{\left(\frac{1}{\sqrt{2}}\right)^2} = \pi + 4\pi$$

$$= 5\pi$$