Section 3.5

Derivatives of Trigonometric Functions

The derivative of the sine is the cosine.

$$\frac{d}{dx}\sin x = \cos x$$

The derivative of the cosine is the negative of the sine.

$$\frac{d}{dx}\cos x = -\sin x$$

Find the derivative of $\frac{\sin x}{(\cos x - 2)}$.

$$\frac{dy}{dx} = \frac{(\cos x - 2)\frac{d}{dx}\sin x - \sin x \frac{d}{dx}(\cos x - 2)}{(\cos x - 2)^2}$$

quotient rule

$$= \frac{(\cos x - 2)(\cos x) - \sin x(-\sin x)}{(\cos x - 2)^2}$$

$$= \frac{\cos^2 x - 2\cos x + \sin^2 x}{(\cos x - 2)^2}$$

$$= \frac{(\sin^2 x + \cos^2 x) - 2\cos x}{(\cos x - 2)^2}$$

$\sin^2 x + \cos^2 x = 1$

$$= \frac{1 - 2\cos x}{(\cos x - 2)^2}$$

The motion of a weight bobbing up and down on the end of a string is an example of *simple harmonic motion.*

A weight hanging from a spring bobs up and down with position function $s = 3\sin t$ (s in meters, t in seconds). What are its velocity and acceleration at time t?

velocity is change in position over change in time

$$s = 3\sin t$$

$$v = \frac{ds}{dt} = 3\cos t$$

$$\frac{\Delta s}{\Delta t} = s'(t)$$

$$a = \frac{dv}{dt} = -3\sin t$$

$$v'(t) = s''(t)$$

Jerk is the derivative of acceleration. If a body's position at time t is

$$j(t) = \frac{da}{dt} = \frac{d^3 s}{dt^3}.$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

Find the equation of a line tangent to $y = \overset{u}{x}\overset{v}{\cos x}$ at $x = 1$.

$$\frac{dy}{dx} = \overset{u'}{(1)}\overset{v}{\cos x} + \overset{u}{x}\overset{v'}{(-\sin x)}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \cos(1) - \sin(1) = m$$

$$\left. y \right|_{x=1} = 1\cos(1) \qquad (1, \cos 1)$$

$$y - \cos(1) = (\cos(1) - \sin(1))(x - 1)$$

$$y - \cancel{\cos(1)} = x\cos(1) - x\sin(1) - \cancel{\cos(1)} + \sin(1)$$

$$y = x(\cos(1) - \sin(1)) + \sin(1)$$

Find the equation of a line tangent to $y=x\cos x$ at $x=1$.

$$y=x\cos x$$

$$m=\frac{d}{dx}(x\cos x)=x(-\sin x)+\cos x(1)$$

Evaluate m when $x=1$

$$m=1(-.8414709848)+(.5403023059)=-.3011686789$$

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When $x=1,\ y=1(\cos 1)=.5403023059$

The equation of the tangent line is

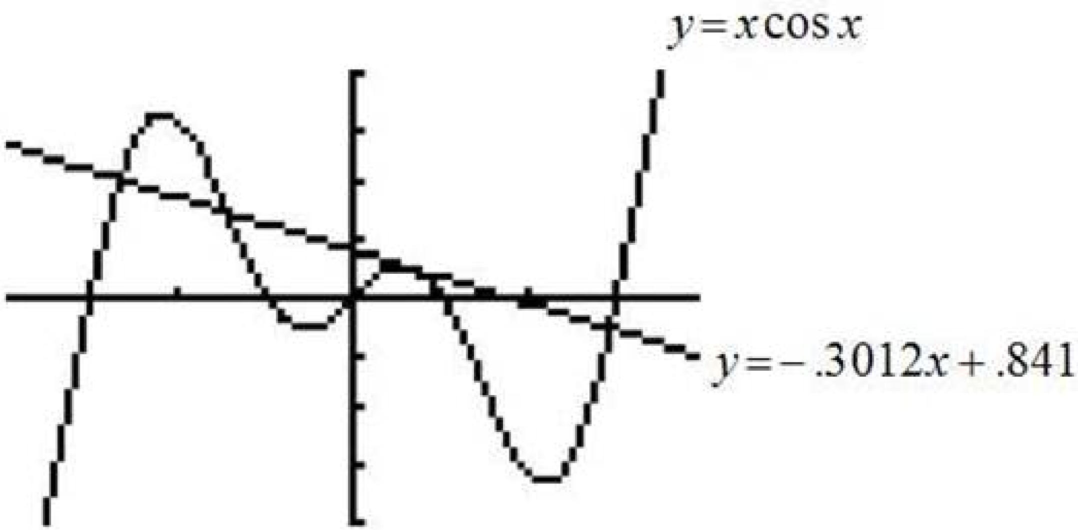
$$y-.5403023059=-.3011686789(x-1)$$

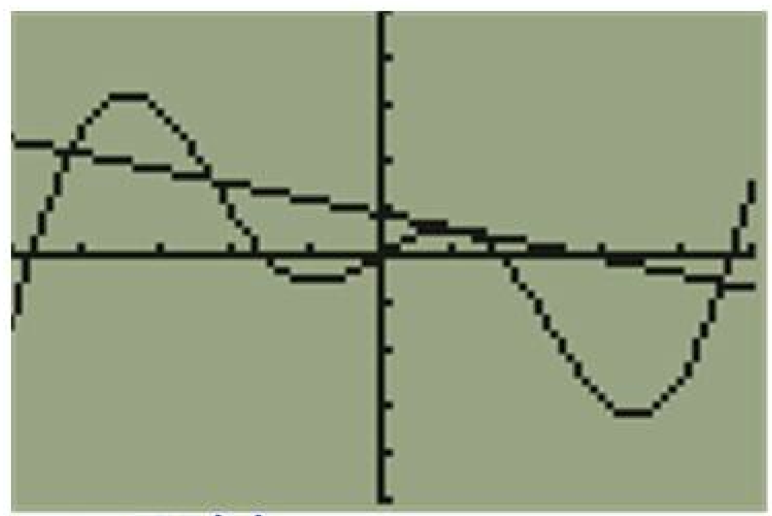
$$y=-.3011686789x+.3011686789+.5403023059$$

$$y=-.3011686789x+.8414709848$$

After rounding the equation is

$$\mathbf{y=-.3012x+.841}$$





$$Y_2 = \underbrace{(\cos(1) - \sin(1))}_{m}X + \underbrace{\sin(1)}_{b}$$

In Exercises 1–10, find dy/dx.

5. $y = 4 - x^2 \sin x$

7. $y = \dfrac{4}{\cos x}$

9. $y = \dfrac{\cot x}{1 + \cot x}$

5) $\dfrac{dy}{dx} = 0 - \left[2x \sin x + x^2 \cos x\right]$

$\dfrac{dy}{dx} = -2x \sin x - x^2 \cos x$

7) $y = 4 \sec x$; $\dfrac{dy}{dx} = 4 \sec x \tan x$

9) $\dfrac{dy}{dx} = \dfrac{-\csc^2 x(1+\cot x) - (-\csc^2 x)\cot x}{(1+\cot x)^2}$

$= \dfrac{-\csc^2 x - \csc^2 x \cancel{\cot x} + \csc^2 x \cancel{\cot x}}{(1+\cot x)^2} = \dfrac{-\csc^2 x}{(1+\cot x)^2}$

In Exercises 13–16, a body is moving in simple harmonic motion with position function $s = f(t)$ (s in meters, t in seconds).

(a) Find the body's velocity, speed, and acceleration at time t.

(b) Find the body's velocity, speed, and acceleration at time $t = \pi/4$.

(c) Describe the motion of the body.

13. $s = 2 + 3\sin t$

(a) $v(t) = 3\cos t$

speed $= |3\cos t|$

$a(t) = 3(-\sin t) = -3\sin t$

(b) $v\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2} =$ speed

$a\left(\frac{\pi}{4}\right) = -\frac{3\sqrt{2}}{2}$

(c)

Starts at position 2 moving toward the positive side slowing down. At t=pi/2 stops at position 5 and starts to move backwards until t= 3pi/2 were it reaches position -1 and starts to move again toward the positive side. Basically, the object is ocsillating between -1 and 5.

23. Find equations for the lines that are tangent and normal to the graph of $y = x^2 \sin x$ at $x = 3$.

$y' = 2x \sin x + x^2 \cos x$

$y'(3) = 6 \sin 3 + 9 \cos 3$

$y(3) = 9 \sin 3$

tangent

$y - 9\sin 3 = (6\sin 3 + 9\cos 3)(x-3)$

Normal

$$y - 9\sin 3 = \frac{-1}{6\sin 3 + 9\cos 3}(x-3)$$

25. Assuming that $(d/dx)(\sin x) = \cos x$ and $(d/dx)(\cos x) = -\sin x$, prove each of the following.

(a) $\dfrac{d}{dx}\tan x = \sec^2 x$

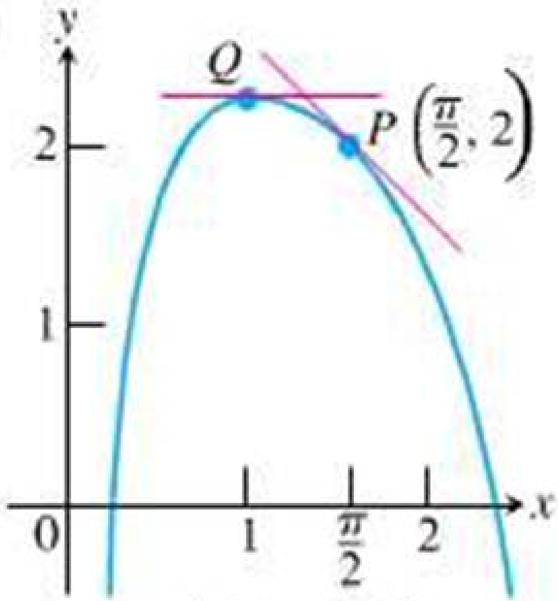
$$\frac{d}{dx}\left[\frac{\overset{u}{\sin x}}{\underset{v}{\cos x}}\right] = \frac{\overset{u'v}{\cos^2 x} \overset{-}{-} \overset{uv'}{\sin x(-\sin x)}}{\underset{v^2}{\cos^2 x}}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

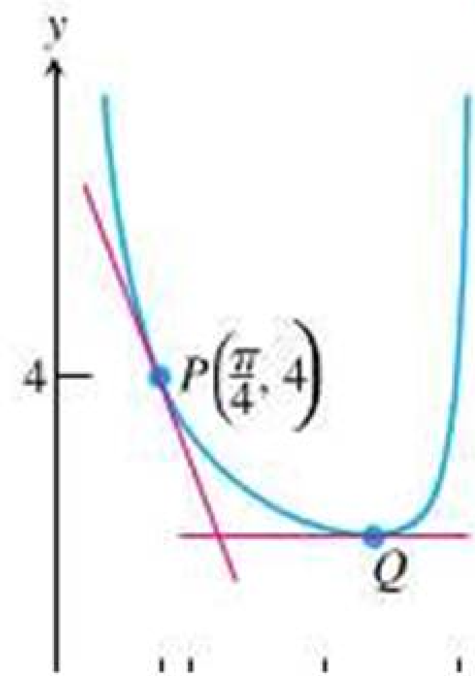
In Exercises 31 and 32, find an equation for **(a)** the tangent to the curve at P and **(b)** the horizontal tangent to the curve at Q.

31.



$y = 4 + \cot x - 2\csc x$

32.



$$Y\left(\tfrac{\pi}{3}\right) = 4 + \frac{1}{\sqrt{3}} - 2\left(\frac{2}{\sqrt{3}}\right)$$

$$Y = \frac{4\sqrt{3} - 3}{\sqrt{3}}$$

$$y' = -\csc^2 x - 2(-\csc x \cot x)$$

$$y'\left(\tfrac{\pi}{2}\right) = -1 - 2(-1)(0) = -1$$

$$y - 2 = -1\left(x - \tfrac{\pi}{2}\right)$$

$$y' = -\csc^2 x + 2\csc x \cot x = 0$$

$$-\cancel{\csc x}\,(\csc x - 2\cot x) = 0$$

$$\csc x = 2\cot x$$

$$\frac{1}{\sin x} = \frac{2\cos x}{\sin x}$$

$$1 = 2\cos x$$

$$\cos x = \tfrac{1}{2}$$

$$x = \tfrac{\pi}{3}$$

39. Find $\dfrac{d^{725}}{dx^{725}}(\sin x)$. $= \cos x$

y

$y = \sin x$

$y' = \cos x$

$y'' = -\sin x$

$y''' = -\cos x$

$y^{(4)} = \sin x$

repeat

$$
\begin{array}{r}
181 \\
4\overline{)725} \\
-4 \\
\hline
32 \\
-32 \\
\hline
05 \\
-4 \\
\hline
1
\end{array}
$$

35. Find y'' if $y = \csc x$.

$$y' = -[\underbrace{\csc x}_{u} \underbrace{\cot x}_{v}]$$

$$y'' = -[-\csc x \cot^2 x + \csc x(-\csc^2 x]$$

$$y'' = \csc x \cot^2 x + \csc^3 x$$

Homework 147
4-48 (4x)