a) continuity

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$3 - 1 = a + b$$

b) differentiability

$$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$$

$$-1 = 2a + b$$

$$-(2 = a + b)$$

$$\boxed{-3 = a}$$

39. Let f be the function defined as

$$f(x) = \begin{cases} 3 - x, & x < 1 \\ ax^2 + bx, & x \geq 1 \end{cases}$$

where a and b are constants.

(a) If the function is continuous for all x, what is the relationship between a and b? $a + b = 2$

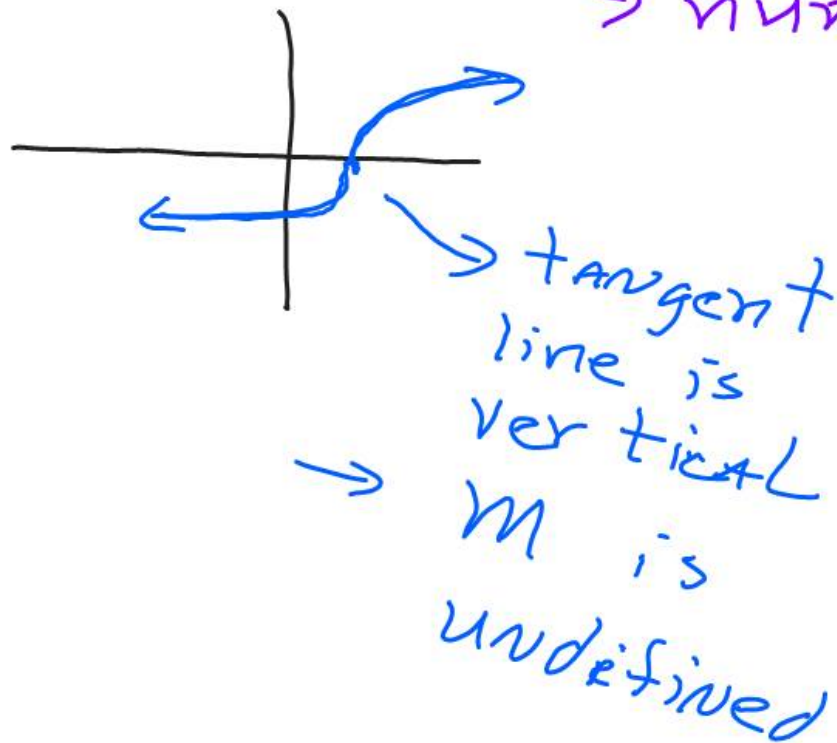**(b)** Find the unique values for a and b that will make f both continuous and differentiable.

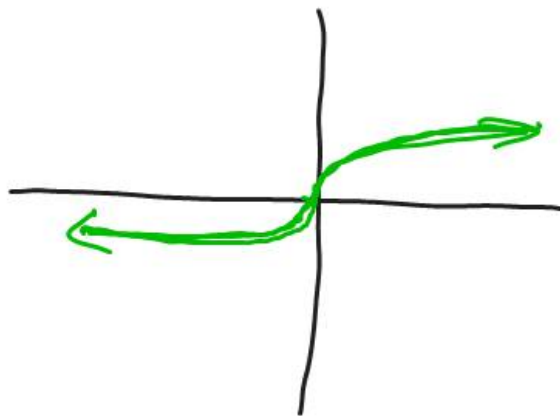$$2 = -3 + b$$

$$\boxed{5 = b}$$

43. Multiple Choice Let $f(x) = \sqrt[3]{x} - 1$. At which of the following points is $f'(a) \neq \text{NDER}\,(f(x), x, a)$?

(A) $a = 1$ (B) $a = -1$ (C) $a = 2$ (D) $a = -2$

(E) $a = 0$

$y = \sqrt[3]{x}$

$\rightarrow$ big number

$\rightarrow$ tangent line is vertical

$\rightarrow$ m is undefined

In Exercises 44 and 45, let

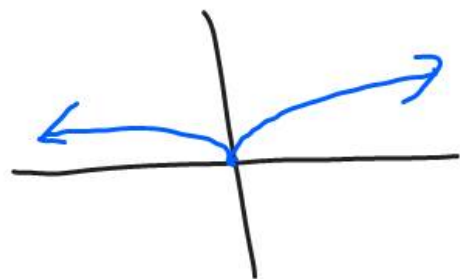$$f(x) = \begin{cases} 2x + 1, & x \leq 0 \\ x^2 + 1, & x > 0. \end{cases}$$

44. Multiple Choice Which of the following is equal to the left-hand derivative of f at $x = 0$?
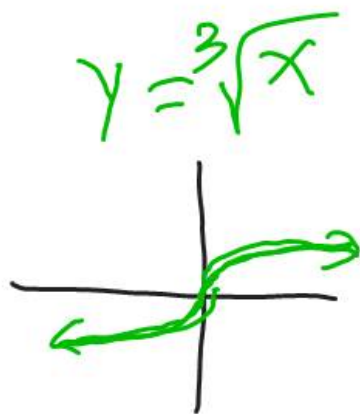
(A) $2x$ (B) 2   (C) 0   (D) $-\infty$ (E) ∞

$$\frac{d}{dx}\left[2x+1\right] = 2$$

$$y = \sqrt[3]{x}$$

In Exercises 11–16, the function fails to be differentiable at $x = 0$. Tell whether the problem is a corner, a cusp, a vertical tangent, or a discontinuity.
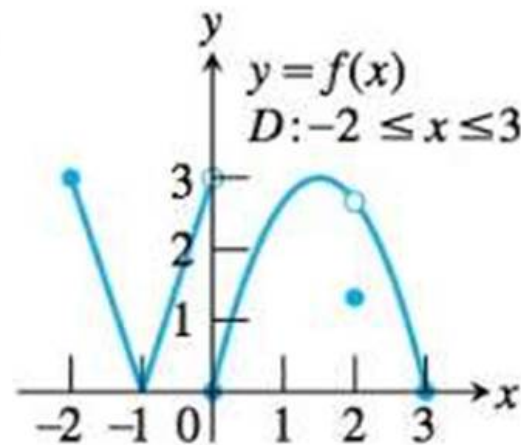
16. $y = \sqrt[3]{|x|}$

cusp at $x = 0$

In Exercises 5–10, the graph of a function over a closed interval D is given. At what domain points does the function appear to be

(a) differentiable?   (b) continuous but not differentiable?

(c) neither continuous nor differentiable?

8.

$y = f(x)$

$D: -2 \leq x \leq 3$

(a) $(-2,-1) \cup (-1,0) \cup (0,2) \cup (2,3)$
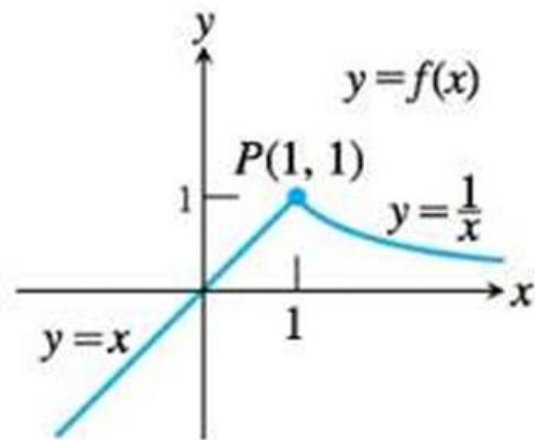
Or $(-2,3)$ except -1, 0, and 2

(b) -1

(c) -2, 0, 2, 3

At end points, only one-sided limits exist.

In Exercises 1–4, compare the right-hand and left-hand derivatives to show that the function is not differentiable at the point P. Find all points where f is not differentiable.

4.

$y = f(x)$, $P(1, 1)$, $y = \frac{1}{x}$, $y = x$

right-hand

$$y = \frac{1}{x} = x^{-1}$$

$$\frac{dy}{dx} = -1x^{-2} = \frac{-1}{x^2}$$

$$\frac{dy}{dx} = \frac{-1}{x^2}$$

$$\frac{dy}{dx}\bigg|_{x=1} = \frac{-1}{1} = -1$$

left-hand

$$\frac{dy}{dx} = 1$$

$1 \neq -1$ ∴ f is not differentiable at $x = 1$
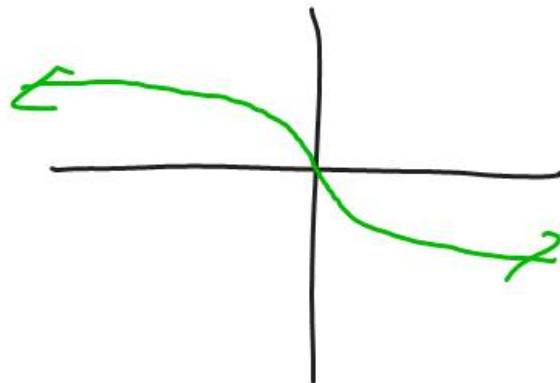
In Exercises 11–16, the function fails to be differentiable at $x = 0$.
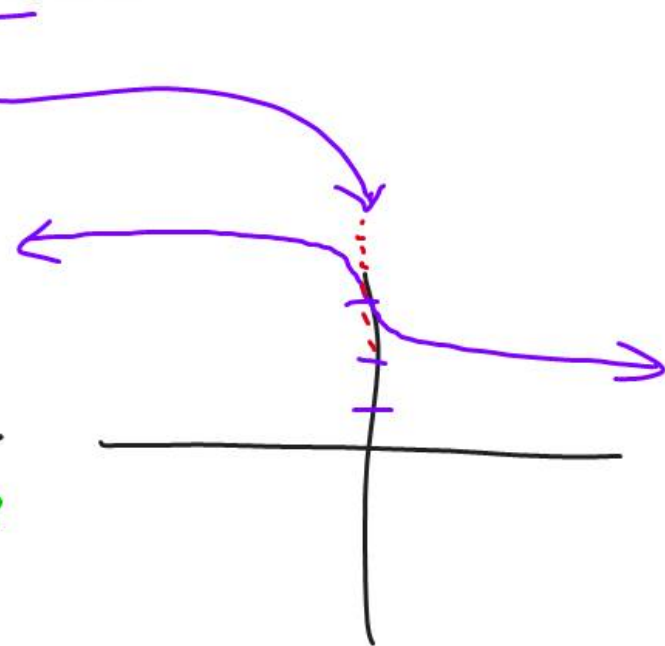Tell whether the problem is a corner, a cusp, a vertical tangent, or a discontinuity.

14. $y = 3 - \sqrt[3]{x}$

$y = \sqrt[3]{x}$

$y = -\sqrt[3]{x}$

$y = 3 - \sqrt[3]{x}$