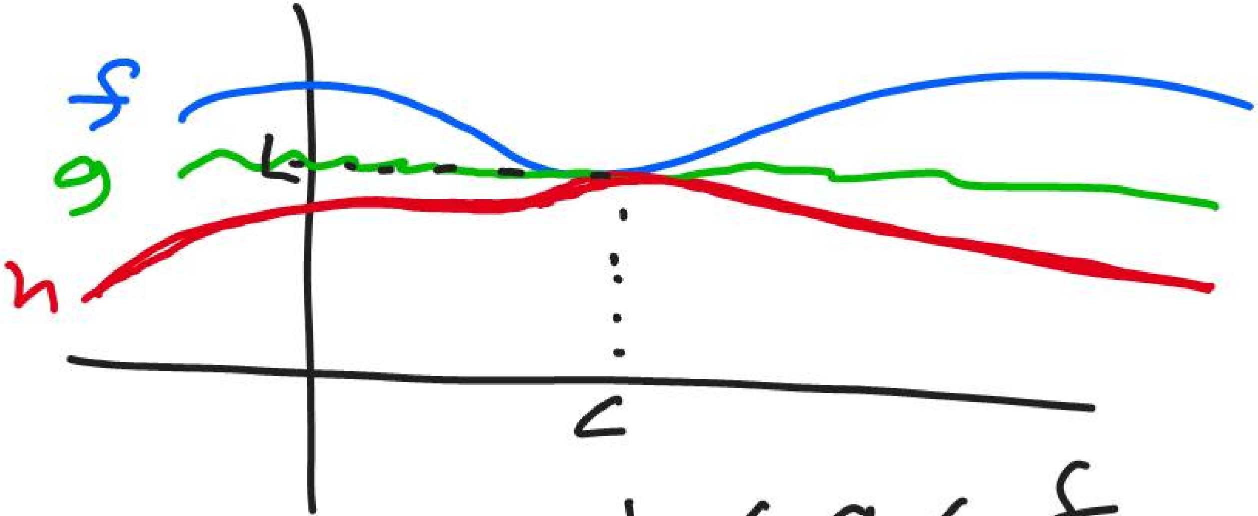
Squeeze Theorem



$$h(x) \leq g(x) \leq f(x)$$

$$\lim_{x \to c} h(x) = \lim_{x \to c} f(x) = L \therefore \lim_{x \to c} g(x) = L$$

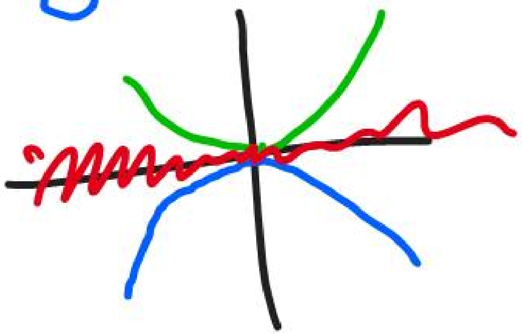
V **EXAMPLE 11** Show that $\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$.

$-1 \leq \sin \frac{1}{x} \leq 1 \Rightarrow -x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$

- $\lim_{x \to 0} -x^2 = 0$
- $\lim_{x \to 0} x^2 = 0$

$\therefore \lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$

by the sandwich theorem

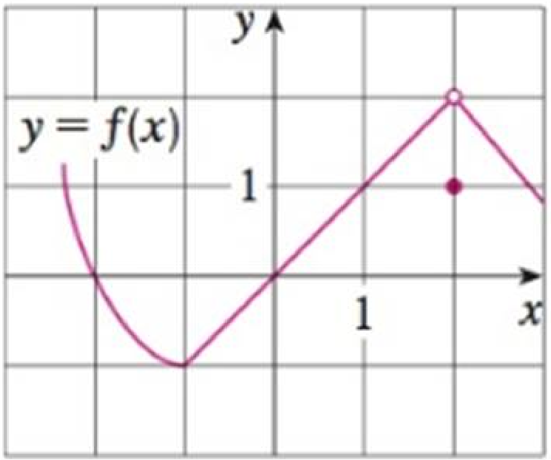


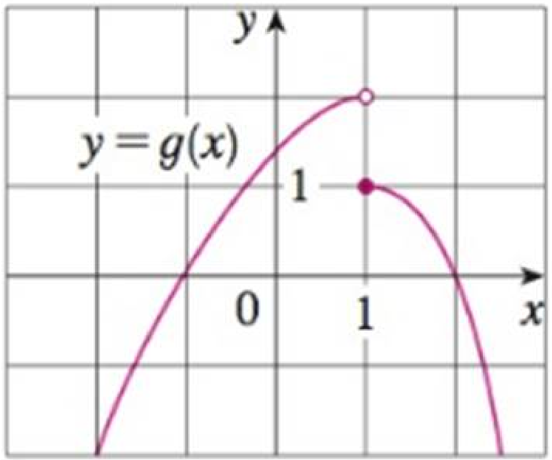
2. The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.

$f(2) + g(2)$

$= 1 + 0 = 1$

$2^3(f(2)) = 8$





(a) $\lim\limits_{x\to 2}\,[f(x) + g(x)]$

$= 2 + 0 = 2$

(b) $\lim\limits_{x\to 1}\,[f(x) + g(x)]$ $= 1 + \nexists \quad \therefore \quad \nexists$

(c) $\lim\limits_{x\to 0}\,[f(x)g(x)]$

$= (0)(1.8) = 0$

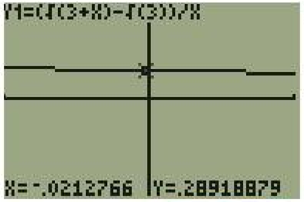
(d) $\lim\limits_{x\to -1}\dfrac{f(x)}{g(x)}$ $= \dfrac{-1}{0} = \nexists$

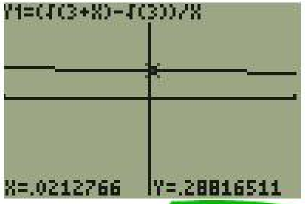
(e) $\lim\limits_{x\to 2}\,[x^3 f(x)]$ $= 8(2) = 16$

(f) $\lim\limits_{x\to 1}\sqrt{3 + f(x)}$ $= \sqrt{3+1} = 2$

34. (a) Use a graph of

$$f(x) = \frac{\sqrt{3+x} - \sqrt{3}}{x}$$





to estimate the value of $\lim_{x\to 0} f(x)$ to two decimal places.

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(b) Use a table of values of $f(x)$ to estimate the limit to four decimal places.

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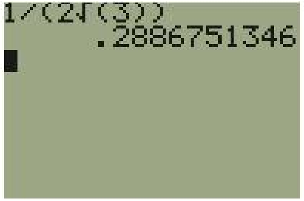
(c) Use the Limit Laws to find the exact value of the limit.

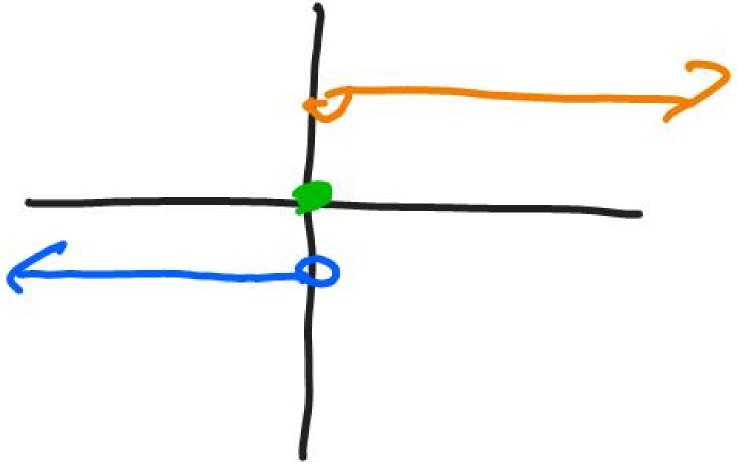
$$\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x}\left(\frac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}}\right) = \lim_{x\to 0} \frac{\cancel{3}+\cancel{x}-\cancel{3}\ (1)}{\cancel{x}\left(\sqrt{3+x}+\sqrt{3}\right)}$$

$$\lim_{x\to 0} \frac{1}{\sqrt{3+x}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$$



47. The *signum* (or sign) *function*, denoted by sgn, is defined by

$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$



(a) Sketch the graph of this function.

(b) Find each of the following limits or explain why it does not exist.

(i) $\lim_{x \to 0^+} \operatorname{sgn} x = 1$ (ii) $\lim_{x \to 0^-} \operatorname{sgn} x = -1$

(iii) $\lim_{x \to 0} \operatorname{sgn} x$ (iv) $\lim_{x \to 0} |\operatorname{sgn} x| = 1$

$= \nexists$

$|\operatorname{sgn} x|$