In Exercises 9–12, find the limit and confirm your answer using the Sandwich Theorem.

2.2

10. $\lim_{x\to -\infty} \frac{1-\cos x}{x^2} = 0$

$0 \le 1-\cos x \le 2$

$\frac{0}{x^2} \le \frac{1-\cos x}{x^2} \le 2/x^2$

$\lim_{x\to -\infty} 0 = 0$

$\lim_{x\to -\infty} 2/x^2 = 0$

$\therefore \lim_{x\to -\infty} \frac{1-\cos x}{x^2} = 0$

In Exercises 25–34, determine the limit graphically. Confirm algebraically.

2.1

27. $\lim_{x\to 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2}$ $\begin{matrix} \div x^2 \\ \div x^2 \end{matrix} \Rightarrow \lim_{x\to 0} \frac{5x+8}{3x^2-16} = \frac{8}{-16} = -1/2$

In Exercises 19–22, **(a)** find the slope of the curve at $x = a$.
(b) Writing to Learn Describe what happens to the tangent at $x = a$ as a changes.

2.4

b) The slope is always negative.
At a=1, there is no tangent line.
As |a| increases, the slope decreases.

21. $y = \dfrac{1}{x-1}$

$$\lim_{h\to 0} \frac{\frac{1}{a+h-1} - \frac{1}{a-1}}{h} \Rightarrow \lim_{h\to 0} \frac{\frac{a-1-(a+h-1)}{(a+h-1)(a-1)}}{h}$$

$$\lim_{h\to 0} \frac{-h}{(a+h-1)(a-1)} \cdot \frac{1}{h} \Rightarrow \lim_{h\to 0} \frac{-1}{(a+h-1)(a-1)} = \frac{-1}{(a-1)^2}$$

$$m\Big|_{x=a} = \frac{-1}{(a-1)^2}$$

pt $\left(a, \frac{1}{a-1}\right)$

tangent Line $\quad y - \frac{1}{a-1} = \frac{-1}{(a-1)^2}(x-a)$

50. *Continuous Function* Find a value for a so that the function

$$f(x) = \begin{cases} x^2 + x + a, & x < 1 \\ x^3, & x \geq 1 \end{cases}$$

is continuous.

$$\lim_{x \to 1^-} f(x) = 1^2 + 1 + a = 2 + a$$

$$\lim_{x \to 1^+} f(x) = 1$$

$$2 + a = 1$$

$$a = -1$$

In Exercises 15–18, determine whether the curve has a tangent at the indicated point. If it does, give its slope. If not, explain why not.

17. $f(x) = \begin{cases} 1/x, & x \le 2 \\ \dfrac{4-x}{4}, & x > 2 \end{cases}$ at $x = 2$

$\lim_{x \to 2^-} f(x) = \frac{1}{2}$

$\lim_{x \to 2^+} f(x) = \frac{2}{4} = \frac{1}{2}$

$\therefore f(x)$ is cont.

$m_{x \to 2^-}: \lim_{h \to 0} \dfrac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \dfrac{x - x - h}{(x+h)x} \cdot \dfrac{1}{h}$

$= \lim_{h \to 0} \dfrac{-1}{(x+h)(x)} = -1/2x = \dfrac{-1}{4}$

$m_{x \to 2^+}$

$f(x) = 1 - \frac{x}{4}$

$b = 1$

$m = -1/4$

f(x) has a tangent line at $x = 2$

33. ***Horizontal Tangent*** At what point is the tangent to $f(x) = x^2 + 4x - 1$ horizontal?

$$m = \lim_{h \to 0} \frac{(x+h)^2 + 4(x+h) - 1 - (x^2 + 4x - 1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{4x} + 4h \cancel{- 1} \cancel{- x^2} \cancel{- 4x} \cancel{+ 1}}{h}$$

$$= \lim_{h \to 0} 2x + h + 4 = 2x + 4$$

When is $2x + 4 = 0$?

$x = -2$

$(-2, f(-2)) = (-2, -5)$

vertex at $x = -b/2a = \frac{-4}{2} = -2$