Gyroscope Optimization & Analysis

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ABSTRACT

Historically, mariners used gyroscopes as highly accurate navigational tools. Since gyroscopes did not rely on the Earth's magnetic field to determine direction, "gyrocompasses" were the preferable option. The advent of MEMS devices has allowed key inventions, namely the gyroscope, to be integrated into more compact devices, such as smartphones, for various navigational and directional uses [1]. This project focuses on optimizing a MEMS single-axis gyroscope for maximum sensitivity while adhering to stringent design constraints.

I. DESIGN PARAMETERS

Initially, the provided gyroscope design was larger than the maximum design area, so I calculated the expected values based on a design with smaller dimensions.

The dimensions I chose for the proof mass were: 1.6 mm by 1.6 mm. These values are small enough to be within the design area requirements while also providing extra room for the comb-drive and folded spring structures on the edges of the proof mass. Additionally, increasing the proof mass size as much as possible within the design constraints will increase the gyroscope's maximum sensitivity.

Other optimizations include reducing the gap between each finger and increasing the number, N, of fingers. This increases the drive force which also increases the sense direction's velocity. As a result, the current i_{ac} will be larger.

This current is used to calculate the final output voltage, V, for each rotation through Ohm's Law. The maximum transimpedance gain is 10 kΩ which means that the current must be optimized to be as large as possible.

<u>Drive & Sense Spring Constants</u>

$$k_d = \frac{Ew^3h}{l^3} = \frac{(170\ GPa)(100\ \mu m)(20\ \mu m)^3}{(800\ \mu m)^3}$$
$$= 265.625\ N/m$$

$$k_s = \frac{Ehw^3}{l^3} = \frac{(170\ GPa)(30\ \mu m)(100\ \mu m)^3}{(70\ \mu m)^3}$$
$$= 459000\ N/m$$

<u>Drive & Sense Resonant Frequencies</u>

$$f_d = \frac{1}{2\pi}\sqrt{\frac{k_d}{m}} = 3358\ Hz$$

$$f_s = \frac{1}{2\pi}\sqrt{\frac{k_s}{m}} = 139576\ Hz$$

<u>Drive Force</u>

$$F_{drive} = \frac{2\varepsilon_0 N V_{DD} v_{ac}}{g} = \frac{(2)(8.85e-12)(6)(10\ V)(1.2\ V)}{(2.5\ \mu m)}$$

$$F_{drive} = 5.0976 \times 10^{-4}\ N$$

$$b = \frac{A\mu}{h} = \frac{(1600\ \mu m)^2(1.73\times10^{-5})}{(50\ \mu m)}\ [1]$$
$$= 8.8576 \times 10^{-7}$$

<u>Drive Motion Response v. Frequency</u>

$$x_{max} = \frac{F_d}{k_d}Q_d = \frac{(5.0976\times10^{-4})(14214)}{(265.625\ N/m)} = 0.0273\ m$$

<u>Sense Motion Response v. Rotation Rate</u>

$$v_y = 2mQ_sQ_d\omega_s\omega_d\frac{F_d}{k_sk_d}\Omega$$
$$= 0.98\ m/s$$

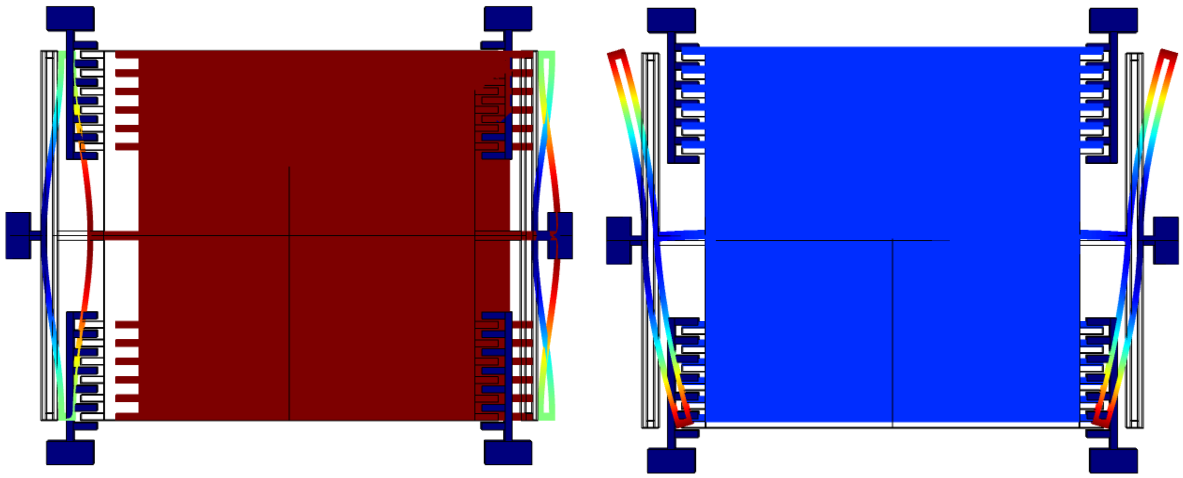


Figure 1: Eigenfrequency in drive (left) and sense (right) mode.

II. CALCULATIONS

Eigenfrequencies

$f_{drive} = 4952\ Hz$

$f_{sense} = 24676\ Hz$

Drive Force

$\alpha = \frac{b_{drive}}{m} = \frac{8.8576\times10^{-7}}{(2330)(1600\ \mu m)^{2}(100\ \mu m)} = 1.485$

This is applied on the gyroscope going in the positive x-direction.

Coriolis Force

$v_x = x_{max} \bullet f_{drive}$

$= (140.37 \times 10^{-6})(4952)$

$= 0.695\ m/s$

$F_{Coriolis} = 2m\Omega v_x = 8.291 \times 10^{-7}\ N$

Sense Mode Displacement w.r.t. Rotation Rate (Ω)

$y_{max} = 0.0792 \times 10^{-6}\ m;\ \Omega = 1\ rad/sec$

Output Voltage Sensitivity w.r.t. Rotation Rate (Ω)

$V = iR;\ R_{max} = 10\ k\Omega,\ i = \frac{V_{DD}\varepsilon_0 A}{g^2} v_y$

$v_y = y_{max} \bullet f_{sense} = 1.954 \times 10^{-3} m/s$

$i = 8.3 \times 10^{-11}\ A$

$V = 830\ nV$

The sensitivity achieved is **830** µV/Ω.

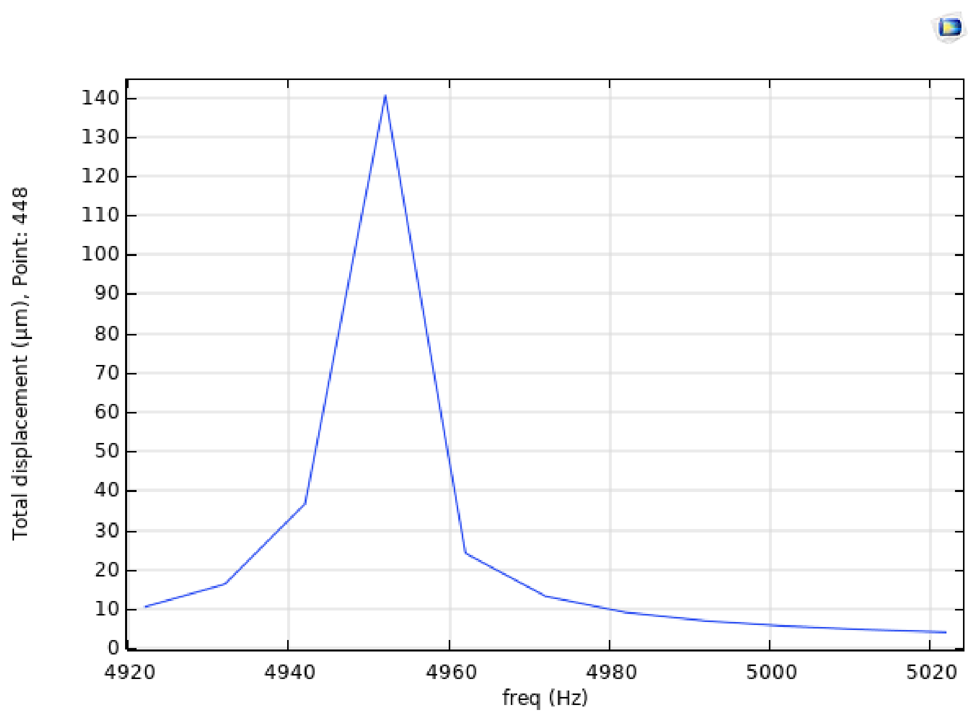
Noise & Resolution of Gyroscope

$F_n = \sqrt{4k_b Tb} = 1.211 \times 10^{-13} \frac{N}{\sqrt{Hz}}$

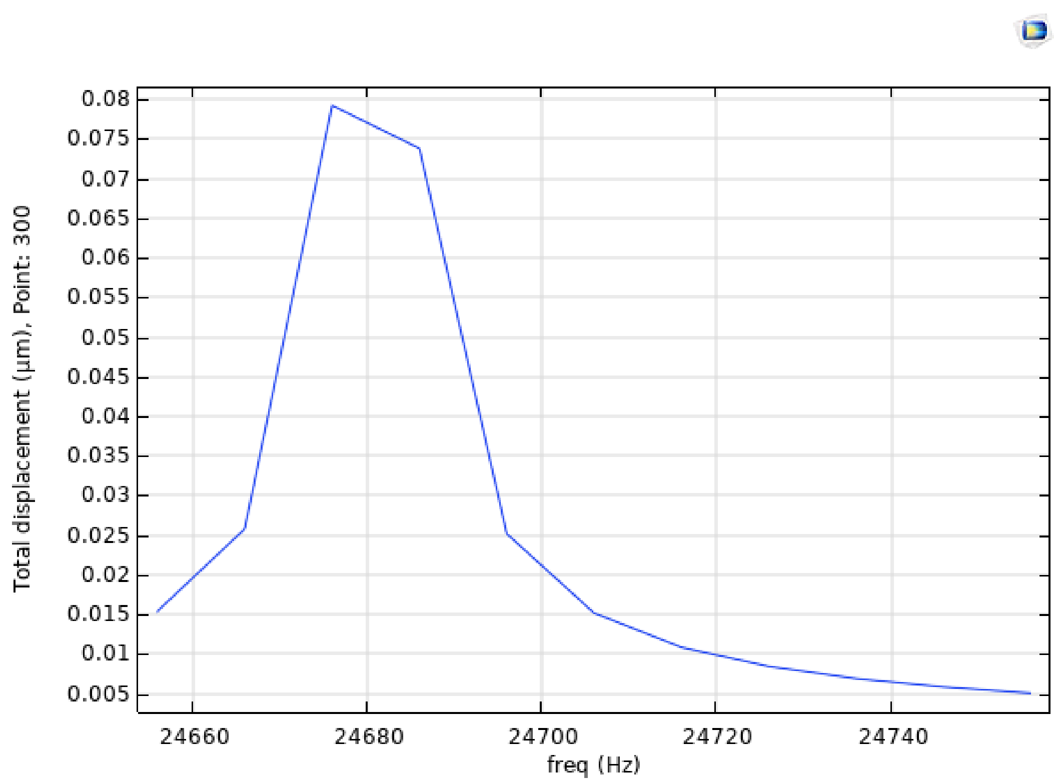
Noise affects the sense mode more significantly than the drive mode which could decrease the resolution of the gyroscope. Mechanical friction and thermal noise are factors within this gyroscope. In fact, the Coriolis force competes specifically with the thermal noise.

However, as calculated above, the force noise density is extremely small, indicating that the gyroscope's sensitivity will not be impacted severely. Thus, the resolution of the signal will be maintained.

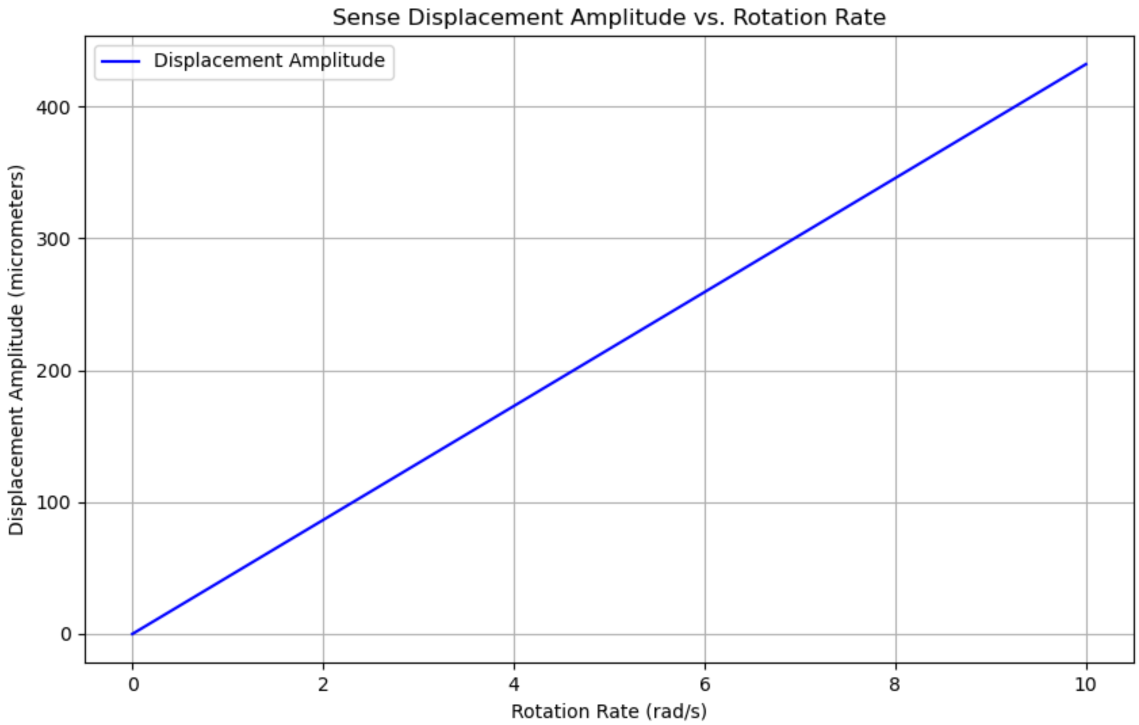
III. PLOTS



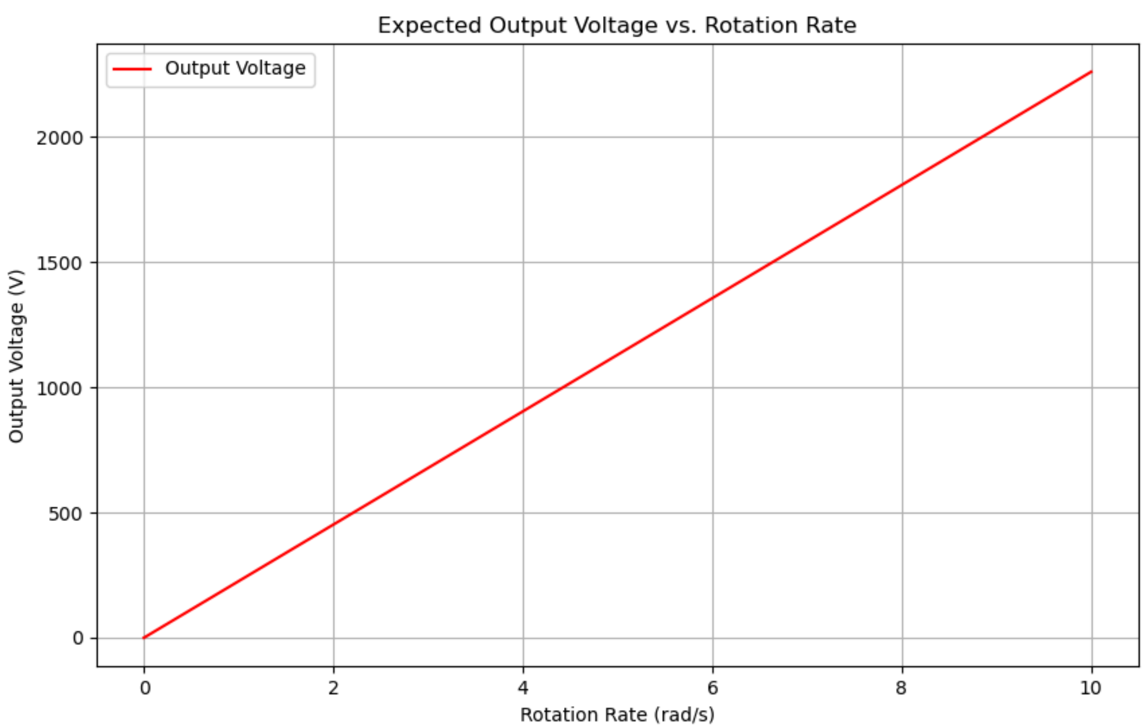
Plot 1: Displacement Amplitude v. Frequency (Drive Mode)



Plot 2: Displacement Amplitude v. Frequency (Sense Mode)



Plot 3: Sense Displacement Amplitude v. Rotation Rate



Plot 4: Expected Output Voltage v. Rotation Rate

w_spring1	20 [um]	2E-5 m	width of spring 1
w_spring_...	30 [um]	3E-5 m	width of spring connector
w_mass	800 [um]	8E-4 m	width of proof mass
w_finger	100 [um]	1E-4 m	finger width
w_drive_fi...	30[um]	3E-5 m	width of drive finger anchor
w_connec...	50 [um]	5E-5 m	width of anchor-spring connector
w_connec...	200 [um]	2E-4 m	width of mass-spring connector
w_anchor2	100 [um]	1E-4 m	width of anchor2
w_anchor	200 [um]	2E-4 m	width of anchor
thickness	100 [um]	1E-4 m	DO NOT CHANGE - device thickness
offset	50 [um]	5E-5 m	offset on anchor
N	6	6	fingers/comb
m	2330[kg/m^3]*(2*w_mass)*(2*h_mass)*(thickness)	5.9648E-7 kg	
l_ov	70 [um]	7E-5 m	equilibrium finger overlap
h_spring1	800 [um]	8E-4 m	height of spring 1
h_spring_...	30 [um]	3E-5 m	height of spring connector
h_mass	800 [um]	8E-4 m	height of proof mass
h_finger	30 [um]	3E-5 m	finger height
h_drive_fi...	2*N*(h_finger+g)+h_finger+offset	4.7E-4 m	height of drive finger anchor
h_connec...	h_connector	2E-5 m	height of anchor-spring connector
h_connec...	20 [um]	2E-5 m	height of mass-spring connector
h_anchor2	100 [um]	1E-4 m	height of anchor2
h_anchor	100 [um]	1E-4 m	height of anchor
g	2.5 [um]	2.5E-6 m	finger gap
finger_spa...	2*(h_finger+g)	6.5E-5 m	adjacent finger space
b	(2*w_mass)*(2*h_mass)*(1.73E-5)/(offset)	8.8576E-7 m	damping coefficient

Figure 1: List of optimized gyroscope parameters.

```python
import numpy as np
import matplotlib.pyplot as plt

# Constants and parameters (example values from the document)
rotation_rates = np.linspace(0, 10, 100)  # Rotation rates in rad/s
mass = 5.9648e-7  # Proof mass in kg (derived from calculations)
f_drive = 4972  # Drive frequency in Hz
f_sense = 24746  # Sense frequency in Hz
max_displacement = 43.213  # Max displacement in micrometers
sensitivity = 226  # Sensitivity in V/(rad/s)
resistance = 10e3  # Resistance in Ohms

# Plot 3: Sense Displacement Amplitude vs. Rotation Rate
def sense_displacement(rotation_rate):
    return max_displacement * rotation_rate  # Linear relationship implied
by document
```

```
displacements = sense_displacement(rotation_rates)

# Plot 4: Expected Output Voltage vs. Rotation Rate
def output_voltage(rotation_rate):
    return sensitivity * rotation_rate  # Linear relationship

voltages = output_voltage(rotation_rates)
```
Figure 2: Python code generating Plots 3 & 4 based on values obtained from COMSOL file.

REFERENCES

[1] Z. M. Naing, S. Anatolii, H. S. Paing and L. Vinh Thang, "Evaluation of Microelectromechanical System Gyroscope and Accelerometer in the Object Orientation System Using Complementary Filter," 2021 IEEE Conference of Russian Young Researchers in Electrical and Electronic Engineering (ElConRus), St. Petersburg, Moscow, Russia, 2021, pp. 2777-2781, doi: 10.1109/ElConRus51938.2021.9396198.

[2] Hall, N. (Ed.). (2021, May 13). *Air properties definitions*. NASA. https://www.grc.nasa.gov/www/k-12/airplane/airprop.html