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State of the art estimates of the Swiss value of travel time savings

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Abstract

This paper presents the findings of a study looking into the valuation of travel time savings (VTTS) in Switzerland, across modes as well as across purpose groups. The study makes several departures from the usual practice in VTTS studies, with the main one being a direct representation of the income and distance elasticity of the VTTS measures. Here, important gains in model performance and significantly different results are obtained through this approach. Additionally, the analysis shows that the estimation of robust coefficients for congested car travel time is hampered by the low share of congested time in the overall travel time, and the use of an additional rate-of-congestion coefficient, in addition to a generic car travel time coefficient, is preferable. Finally, the analysis demonstrates that the population mean of the indicators calculated is quite different from the sample means and presents methods to calculate those, along with the associated variances. These variances are of great interest as they allow the generation of confidence intervals, which can be extremely useful in cost-benefit analyses.

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1 Introduction

The justification of road projects and the ranking of the respective project variants is being reconsidered in Switzerland. The Swiss Department of Transport has defined a system of sustainability indicators and has chosen a multicriteria approach to operationalise them (ASTRA, 2003). Part of this overall system is a cost-benefit analysis which will be identical to that of the new Swiss cost-benefit norms that are being developed under the auspices of the Swiss norming institute for transport (VSS). These guidelines (VSS, 2006) set out the overall framework, but delegate the detailed parameters, such as discount rate, values of travel time savings, value of reliability to a set of subsidiary norms. The clear need for current values had led to a series of studies providing state-of-the-art and current estimates of the relevant values.

This paper describes the survey methods and results of this recent study to estimate the value of travel time savings (VTTS) for passenger transport¹. It implements the recommendation of the scoping study on Swiss VTTS (Abay and Axhausen, 2001). Previous Swiss practice had drawn on older local revealed preference studies, values transferred from abroad or more recent stated preference estimates, which were derived from studies not focusing on the values of travel time savings (see Vrtic *et al.* 2003, Koenig *et al.* 2004 or Axhausen *et al.* 2004).

This study pursued a number of new departures with respect to the choice contexts, the estimation of the VTTS, in particular through the inclusion of directly estimated income and trip distance elasticities and the estimation of VTTS variances. These new departures are of general interest, as they address implicitly the problem of brief time savings on short trips and the question of the appropriate ranges of the VTTS in sensitivity analyses. We are not aware of a similar comprehensive set of variance estimates of VTTS measures.

The structure of the paper is as follows. Section 2 presents a description of the data, while Section 3 discusses model specification. The results of the main modelling analysis are summarised in Section 4, and trade-offs calculated from these results are presented in Section 5. Finally, Section 6 extrapolates the results to the population level, and Section 7 presents the conclusions of the analysis.

¹The study was conducted by the Institute of Transport Planning and Systems (IVT), ETH Zürich and Rapp Trans AG, Zürich in collaboration with J.J. Bates and M. Bierlaire on behalf of the Swiss Association of Transport Engineers (SVI).

2 Data

2.1 Survey design

In line with current practice (see Louviere *et al.* 2000 or recent European studies such as HCG 1990, 1999, Algiers *et al.* 1995, Kurri and Pursula 1995, Ramjerdi *et al.* 1997, Jovicic and Hansen 2003 and Mackie *et al.* 2003), the SP survey was based on information from observed trips. This information was readily available because the basis of recruitment was the ongoing and continuous survey (KEP²) of the Swiss Federal Railways (SBB). The socio-demographic characteristics of the respondents and the information about their trips were made available for all respondents of the KEP.

The questionnaire consisted of three parts: two SP experiments with six or nine choice situations each plus a third part covering various socio-demographic and trip-related questions, which had not been raised during the KEP-interview. Out of the two SP surveys,

SP 1 is a mode choice experiment (car and bus or rail) presented only to respondents who have a car available.

SP 2 is a route choice experiment, where a choice was offered between two routes on the current mode, where, in addition, some car users were presented with a choice between two public transport routes.

An example choice situation for the two surveys is shown in Figure 1.

Prior to the design of the final survey, two pretests were conducted, and the estimation results from these tests led to various modifications in the survey design, the wording of the questions and the variable characteristics. As such, the final specification of the car route choice survey uses three explanatory variables; travel cost, uncongested (free flow) travel time, and congested travel time. This final specification was decided upon after initial attempts showing only the combined travel time in conjunction with an indication of the share of congestion (first version) or the total congested time only (second version). Either of these two approaches led to an overestimation of the ratio between congested and uncongested VTTS, an issue that does not arise when presenting respondents with actual values for both congested and uncongested time.

²KEP stands for “*Kontinuierliche Erhebung zum Personenverkehr*”. The KEP is collected on behalf of the SBB (Swiss Federal Railways) and covers the travel behaviour of adults in Switzerland. The survey has been conducted since the early 1980s, with around 17,000 respondents interviewed each year. The survey collects information about the personal situation of the travellers and about their trips with a distance of more than three kilometres during the week preceding the interview.

Mode choice car – rail (main study version)

Car		Rail	
Travel costs:	18 Fr.	Travel costs:	23 Fr.
Total travel time:	40 minutes	Travel time:	30 minutes
... congested:	10 minutes	Headway:	30 minutes
... uncongested:	30 minutes	No. of changes:	0 times

☐ ← Your choice → ☐

Route choice rail (main study version)

Rail		Rail	
Travel costs:	20 Fr.	Travel costs:	23 Fr.
Travel time:	40 minutes	Travel time:	30 minutes
Headway:	15 minutes	Headway:	30 minutes
No. of changes:	1 times	No. of changes:	0 times

☐ ← Your choice → ☐

Figure 1: Types of SP experiments

The pretest had included a destination choice experiment offering a trade-off between travel time, travel costs and the costs of a basket of goods at two shopping locations. The basket was described as equivalent at the two locations. The VTTS obtained were unusually high and due to the time pressure of the project, it was not possible to reconcile these results with the other experiments. A later experiment with shoppers in the Basel region using an expanded experiments offering basically the same trade off confirmed these high valuations (Erath, 2006) raising interesting issues for future research.

2.2 Field work experiences

Six different combinations of the SPs were dispatched depending on the personal car availability and the mode chosen for the reference trip. As mentioned above, one subgroup of car drivers received rail route choice experiments to balance possible biases arising from only considering experiments based on chosen modes. The respondents received between 9 and 15 choice situations. The overall response rate was 53%, which is considered satisfactory, particularly given that the time between the recruiting interview and the dispatch of the SP surveys varied from 7 to 25 weeks. While there is about a 10% reduction in the response rates

between the fastest and slowest dispatch, the pattern is stronger for the response speed (the mean time to return the surveys after dispatch), where the response speed is nearly twice as fast for those facing a long wait between recruitment and SP - survey. A reasonable interpretation of these two trends is that one obtains the answers of the committed respondents even when the wait is extended, but less committed respondents display a declining tendency to respond with time.

The socio-demographic structure of the final sample is considerably different from the Swiss mean represented by the recent national travel survey ([Bundesamt für Raumentwicklung und Bundesamt für Statistik, 2001](#)), or *Mikrozensus* of the year 2000 (MZ'00). Table 1 shows this comparison between the KEP, the national travel survey, those willing to participate, those receiving a survey and those responding. The gap between the recruited sample and the sample receiving a survey arises from quota considerations or the lack of relevant trips. The quota was imposed to concentrate the survey resources on rarer, i.e. longer trips and business trips.

There is a clear shift to male, well educated and employed public transport users, a group of people that is particularly motivated to contribute to the improvement of their daily transport system. This point is further underlined by the fact that nearly all participants answered all questions, including those concerning the household income. Given these differences between the population and sample quotas, the question of sample reweighting needs to be addressed. For descriptive and linear analysis, data sets have to be weighted to emphasise less represented person groups. [Ben-Akiva and Lerman \(1985\)](#) drawing on McFadden have shown that for the estimation of logit choice models, no reweighting is required provided that constants representing the variables relevant for the selectivity are included in the model.

2.3 Estimation data

Respondents were presented with between 9 and 15 choice situations, where the average across the six datasets was 12.95. Table 2 gives a brief overview of the data, in terms of the division into the four separate purpose groups, and the six separate SP surveys. The table also gives the average income and trip distance in each of the purpose groups, for use in the elasticity formulations described in Section 3.2.

3 Model specification

Four main sets of models were estimated during the analysis, with the various models differing along two dimensions, namely the presence or absence of a seg-

	KEP	Recruited sample	Dispatched to	Main study	MZ'00
PT-discount:					
Halbtax ¹ -ownership	36	43.1	52.3	47.4	34.8
GA ² -ownership	6.9	13.9	10.7	11.8	6
Car availability:					
always	61	59.2	73.1	66.7	77.3
sometimes	15	23.8	13.9	18.4	13.9
never	24	17	14	14.9	7.1
Education:					
Primary + lower secondary	21	11	9.9	10.4	34
Vocational training	52	48.3	46.2	50.6	40.7
A-Level, tertiary	26	40.7	43.9	39	25.3
Working Status:					
None	41	30.7	28.3	31.8	47.4
Part-time	15	18.6	15.7	16.3	13.8
Fulltime	37	42.7	49.2	45.3	33
Self-employed	6.7	9	6.8	6.6	5.8
Household income [CHF/Year]:					
less than 20 000				5.8	3.1
20 000 40 000				8.3	14.8
40 000 60 000				12.9	22.5
60 000 80 000				16.3	16.2
80 000 100 000				16.7	9.7
100 000 125 000				10.8	5.2
125 000 150 000				5.3	2.6
more than 150 000				7	4
no response				16.9	21.9

¹Discount ticket giving a 50% reduction in fare.

²*Generalabonnement*, season ticket allowing for free travel on the entire transport network.

Table 1: Socio-demographic characteristics of the different samples

mentation by trip purpose, and the use of a generic car travel time coefficient, or separate coefficients for congested and uncongested car travel time. In this paper, we concentrate on the model used for the VTTS measure included in the relevant subsidiary norm SN 631 822 (VSS, 2006), which uses a generic car travel time coefficient in conjunction with segmentation by trip purpose. Detailed results for the other three models are given by Hess (2006).

The main methodological interest in the present work lies in the use of continuous interactions between tastes and socio-demographic attributes, namely trip

	Business		Commuters		Leisure		Shopping		Total	
	Resp.	Obs.	Resp.	Obs.	Resp.	Obs.	Resp.	Obs.	Resp.	Obs.
Mode choice: car vs bus	1	15	27	405	31	465	21	315	80	1,200
Mode choice: car vs rail	71	987	286	3,867	423	5,757	184	2,430	964	13,041
Route choice: bus for bus users	0	0	18	162	19	171	17	153	54	486
Route choice: car for car users	0	0	0	0	0	0	0	0	0	0
Route choice: rail for car users	0	0	0	0	0	0	0	0	0	0
Route choice: rail for rail users	5	45	33	297	75	675	14	126	127	1,143
Total	77	1,047	364	4,731	548	7,068	236	3,024	1,225	15,870
Average income (CHF)	98,224		84,656		75,182		76,704		79,816	
Average trip distance (km)	97.64		23.22		60.27		14.18		42.91	

Table 2: Description of data

distance and income (see also [Mackie *et al.* 2003](#)). This approach was used as an alternative to simple (and arbitrary) segmentations into different income and distance classes with separate coefficients in different classes. In the present work, no additional efforts were made to allow for random taste heterogeneity, given the already high cost of estimating the models when using the continuous interaction formulation. However, it must also be said that, while continuous mixture models such as Mixed Multinomial Logit (MMNL) do offer great gains in flexibility, it is clearly preferable to, whenever possible, explain taste heterogeneity in a deterministic fashion. The increasing popularity of the MMNL model has however led to a reduction in attempts to do this.

In the following two subsections, we look at the general specification of the utility function in the final model, and give some more details on the continuous interaction functions.

3.1 Utility specification

The majority of estimated parameters are purpose-specific, and while all attributes enter the utility function linearly, they potentially interact non-linearly with a number of socio-demographic attributes. The detailed exploration of non-linear transforms of actual attributes, such as with the help of Box-Cox transforms, is the topic of ongoing work. In the model reported here, a common coefficient is used for congested and uncongested car travel time, but an additional coefficient is associated with the degree of congestion, DC^3 . No interactions with socio-demographic attributes were observed for this coefficient. Finally, it is worth noting that attempts to include additional alternative specific constants in

³ DC was calculated as $\frac{TT_c}{TT_c + TT_u}$, with TT_c and TT_u giving the congested and uncongested components of travel time respectively.

the route choice subsets⁴ did not lead to satisfactory results. A common utility function was used across the six surveys. Given the use of six separate subsets of the data in estimation, it is important to account for potential differences in scale. To this extent, different scale parameters were associated with the different subsets, where the scale for the final route choice experiment ($\mu_{RC,rail}$), was normalised to 1. The general utility function is given by:

$$\begin{aligned}
U = & \beta_{car\ inertia} \delta_{car\ inertia} + \beta_{car\ available} \delta_{car\ available} + \beta_{car\ male} \delta_{car\ male} \\
& + \beta_{bus\ discount} \delta_{bus\ discount} + \beta_{bus\ GA} \delta_{bus\ GA} + \beta_{rail\ discount} \delta_{rail\ discount} \\
& + \beta_{rail\ GA} \delta_{rail\ GA} + \sum_{p=1}^4 \beta_{bus,p} \delta_{bus} \delta_p + \sum_{p=1}^4 \beta_{rail,p} \delta_{rail} \delta_p + \sum_{p=1}^4 \beta_{DC,p} DC \\
& + \sum_{p=1}^4 \beta_{TT_{PT},p} TT_{PT} \delta_p f(inc, TT_{PT}, p) f(dist, TT_{PT}, p) \\
& + \sum_{p=1}^4 \beta_{TT_{car},p} TT_{car} \delta_p f(inc, TT_{car}, p) f(dist, TT_{car}, p) \\
& + \sum_{p=1}^4 \beta_{HW,p} HW \delta_p f(inc, HW, p) f(dist, HW, p) \\
& + \sum_{p=1}^4 \beta_{TC,p} TC \delta_p f(inc, TC, p) f(dist, TC, p) \\
& + \sum_{p=1}^4 \beta_{IC,p} IC \delta_p f(inc, IC, p) f(dist, IC, p)
\end{aligned}$$

where

- $\delta_{car\ inertia}$ is set to 1 if the respondent was observed to choose car in the revealed preference survey. This term is included only for the car alternative in the two mode choice experiments.
- $\delta_{car\ available}$ is set to 1 if a car is generally available to the respondent. This term is included only for the car alternative in the two mode choice experiments.
- $\delta_{car\ male}$ is set to 1 if the respondent is male. This term is included only for the car alternative in the two mode choice experiments.
- $\delta_{bus\ discount}$ is set to 1 if the respondent has a discount ticket. This term is included only for the bus alternative in the first mode choice experiment.

⁴The inclusion of constants for unlabelled alternatives would allow us to capture effects such as respondents reading the questionnaire from left to right.

- $\delta_{bus\ GA}$ is set to 1 if the respondent has a national season ticket. This term is included only for the bus alternative in the first mode choice experiment.
- $\delta_{rail\ discount}$ is set to 1 if the respondent has a discount ticket. This term is included only for the rail alternative in the first mode choice experiment.
- $\delta_{rail\ GA}$ is set to 1 if the respondent has a season ticket. This term is included only for the rail alternative in the first mode choice experiment.
- δ_p is set to 1 if the respondent falls into purpose group p , where p is an index defining the 4 different purpose segments.
- δ_{bus} is set to 1 for the bus alternative in the first mode choice experiment.
- δ_{rail} is set to 1 for the rail alternative in the second mode choice experiment.
- TT_{PT} is the travel time attribute used for public transport alternatives (bus and rail).
- TT_{car} is the travel time attribute used for the car journeys (car alternatives only).
- DC is the degree of congestion for car journeys (car alternatives only).
- HW is the headway attribute used for public transport alternatives.
- TC is the cost attribute (all alternatives).
- IC is the interchanges attribute used for public transport attributes.
- $f(dist, x, p)$ is the distance elasticity formulation associated with attribute x in purpose segment p (see Section 3.2).
- $f(inc, x, p)$ is the income elasticity formulation associated with attribute x in purpose segment p (see Section 3.2).

3.2 Continuous interactions

While the majority of modelling analyses allow for some interactions between estimated parameters and socio-demographic attributes, these generally come in the form of a segmentation using separate models, or the use of separate coefficients in the same model. The treatment of such interactions in a continuous fashion is relatively rare, with the same applying for interactions between multiple explanatory variables. However, it is clear that such continuous treatments of interactions

have advantages in terms of flexibility when compared to the more assumption-bound segmentation approaches. On the other hand, they pose greater demands in terms of the quality of auxiliary data as well as computational cost.

In this work, continuous interactions of the type shown in equation (1) were used.

$$f(y, x) = \beta_x \left(\frac{y}{\hat{y}} \right)^{\lambda_{y,x}} x, \quad (1)$$

where y is the observed value for a given socio-demographic variable, such as income or trip distance, and \hat{y} is a reference value for this attribute, such as the mean value across a sample population. In this example, the sensitivity to an alternative's attribute x varies with y . The choice of the reference value \hat{y} is arbitrary, and has no effect on model fit, or the estimate for $\lambda_{y,x}$. However, the use of the mean value, \bar{y} , guarantees that the estimate β_x gives the sensitivity to x at the average value of y in the sample population⁵, and helps to stabilise the estimation. The estimate of $\lambda_{y,x}$ gives the elasticity of the sensitivity to x with respect to changes in y ; with negative values for $\lambda_{y,x}$, the (absolute) sensitivity decreases with increases in y , with the opposite applying in the case of positive values for $\lambda_{y,x}$. Finally, the rate of the interaction is determined by the absolute value of $\lambda_{y,x}$, where a value of 0 indicates a lack of interaction. This approach was suggested by Mackie *et al.* (2003) in the context of the reanalysis of the UK value of time study.

At this point, it should be said that a problem with this approach in the present context is caused by the fact that income information is presented in the form of a set of separate income-classes, as opposed to absolute income information, leading to a requirement for using class-midpoints, with the obvious averaging error this involves. Here, it should however be noted that similar averaging error occurs in the case where different income classes are grouped together in a segmentation approach, while the use of separate coefficients in each group risks leading to problems with parameter significance.

4 Estimation results

The estimation results for the final model are summarised in Table 3. All estimated parameters are of the expected sign, and aside from a few constants (and $\lambda_{dist.,TT_{car,business}}$), attain high levels of statistical significance⁶. No bus constant

⁵With $y = \hat{y}$, the term $\left(\frac{y}{\hat{y}} \right)^{\lambda_{y,x}}$ disappears from equation 1.

⁶For the scale parameters, the asymptotic t-ratio is calculated with respect to 1, rather than 0.

Observations	15,870
Null log-likelihood	-11,000.2
Final log-likelihood	-7,242.48
Adjusted ρ^2	0.3370

	estimate	asy. t-rat.		estimate	asy. t-rat.
$\beta_{TT_{PT},business}$	-0.1086	-10.72	$\beta_{car\ inertia}$	1.9076	8.18
$\beta_{TT_{PT},commuters}$	-0.1353	-12.89	$\beta_{car\ available}$	0.5880	3.19
$\beta_{TT_{PT},leisure}$	-0.0571	-14.34	$\beta_{car\ male}$	-0.3295	-2.05
$\beta_{TT_{PT},shopping}$	-0.1066	-7.90	$\beta_{bus\ discount}$	1.1958	1.37
$\beta_{TT_{car},business}$	-0.1100	-6.29	$\beta_{rail\ discount}$	1.6995	7.54
$\beta_{TT_{car},commuters}$	-0.1491	-9.80	$\beta_{bus\ GA}$	3.6511	1.66
$\beta_{TT_{car},leisure}$	-0.0764	-7.96	$\beta_{rail\ GA}$	1.7218	5.46
$\beta_{TT_{car},shopping}$	-0.1462	-6.24	$\lambda_{dist.,TT_{PT},business}$	-0.3135	-3.89
β_{DC}	-0.0572	-6.98	$\lambda_{dist.,TT_{PT},commuters}$	-0.2368	-5.93
$\beta_{TC,business}$	-0.1314	-6.78	$\lambda_{dist.,TT_{PT},leisure}$	-0.2837	-8.70
$\beta_{TC,commuters}$	-0.2920	-12.54	$\lambda_{dist.,TT_{PT},shopping}$	-0.2009	-3.56
$\beta_{TC,leisure}$	-0.1570	-12.63	$\lambda_{dist.,TT_{car},business}$	-0.3573	-1.90
$\beta_{TC,shopping}$	-0.3607	-10.63	$\lambda_{dist.,TT_{car},commuters}$	-0.1321	-2.48
$\beta_{IC,business}$	-1.0318	-8.69	$\lambda_{dist.,TT_{car},leisure}$	-0.3744	-5.48
$\beta_{IC,commuters}$	-1.4274	-17.44	$\lambda_{dist.,TT_{car},shopping}$	-0.1905	-2.52
$\beta_{IC,leisure}$	-1.1492	-21.24	$\lambda_{dist.,TC}$	-0.5949	-26.42
$\beta_{IC,shopping}$	-1.2692	-13.02	$\lambda_{inc.,TC,business}$	-0.8922	-5.76
$\beta_{HW,business}$	-0.0326	-6.25	$\lambda_{inc.,TC,commuters}$	-0.1697	-4.12
$\beta_{HW,commuters}$	-0.0544	-15.07	$\mu_{MC,car,bus}$	0.4082	-2.97
$\beta_{HW,leisure}$	-0.0350	-16.02	$\mu_{MC,car,rail}$	0.5051	-14.57
$\beta_{HW,shopping}$	-0.0510	-11.43	$\mu_{RC,bus}$	0.9878	-0.16
$\beta_{bus,commuters}$	2.9428	4.16	$\mu_{RC,car}$	1.2254	1.57
$\beta_{bus,leisure}$	-1.0980	-0.67	$\mu_{RC,rail\ by\ car}$	0.7688	-4.62
$\beta_{bus,shopping}$	2.4257	2.64	$\mu_{RC,rail}$	1.0000	-
$\beta_{rail,business}$	1.4948	1.23			
$\beta_{rail,commuters}$	1.3332	3.27			
$\beta_{rail,leisure}$	0.2886	0.57			
$\beta_{rail,shopping}$	1.2117	2.80			

Table 3: Estimation results for purpose-specific model with generic car travel time coefficient

could be estimated for business travellers ($\beta_{bus,business}$). Generic (cross-purpose) interaction parameters were used for $\lambda_{dist.,TC}$, while an effect of income on cost-sensitivity ($\lambda_{inc.,TC}$) was only observed for business travellers and commuters. The actual estimates for the interaction parameters show a decreasing sensitivity to travel time and travel cost as a function of trip distance, and a decreasing sensitivity to travel cost as a function of income.

It should be noted that this model obtains a very similar model fit to the one using a segmentation of car travel time into the congested and uncongested part

(cf. [Hess, 2006](#)). As such, there is little gain in using separate coefficients for congested and uncongested travel time with the present data. This can partly be explained by the low share for the uncongested part ($< 10\%$), such that the additional congestion coefficient β_{DC} captures most of the penalty.

To give an indication of the effect of using the elasticity formulation, a separate model was estimated in which all interaction parameters were fixed to a value of 0, corresponding to an absence of an income or distance effect. This led to a drop in log-likelihood by 316.84 units, offering significant evidence of the advantages of the elasticity formulation⁷.

5 Trade-offs

This section describes the calculation of the various willingness to pay (WTP) indicators such as the VTTS. Given the non-linear nature of the utility function, and the impact this has on the calculation of trade-offs, a rather detailed presentation of the calculation is given in each case, placing particular emphasis on the effect of changes in income and/or trip distance on the value of a given trade-off. This is then in some cases followed by a graphical representation of the trade-off as a function of income and trip distance. Finally, a brief comparison is given between the mean indicators in the models incorporating income and distance elasticity, and the fixed indicators from a base model estimated without the elasticity formulation.

5.1 Indicators as a function of income and trip distance

On the basis of the parameter estimates from [Table 3](#), it is possible to obtain values for the various trade-offs at the mean income and trip distance across the sample of respondents. Appropriate values for these indicators with a specific income and trip distance can then be obtained with the help of a set of multipliers that take into account the continuous interactions with income and trip distance. This approach is summarised in [Table 4](#), which shows the sample mean for the indicators, along with the appropriate multipliers. This takes into account the fact that not all interactions were significant in all purpose segments (i.e., no income effect on cost sensitivity for leisure and shopping), while some interaction parameters were generic rather than purpose specific⁸.

⁷Detailed results available on request.

⁸Here, the differences in the mean trip distance across purpose segments still leads to differences in the multipliers.

WTP at sample mean	Trip purpose			
	Business	Commuting	Leisure	Shopping
PT travel time (CHF/hour)	49.57	27.81	21.84	17.73
Car travel time (CHF/hour)	50.23	30.64	29.2	24.32
Headway red.(CHF/hour)	14.88	11.18	13.38	8.48
Interchange red. (CHF/change)	7.85	4.89	7.32	3.52
Multipliers				
Income on cost (all WTP)	$\left(\frac{inc}{98223.5}\right)^{0.8922}$	$\left(\frac{inc}{84656}\right)^{0.1697}$	-	-
Dist. on cost (all WTP)	$\left(\frac{dist}{97.64}\right)^{0.5949}$	$\left(\frac{dist}{23.22}\right)^{0.5949}$	$\left(\frac{dist}{60.27}\right)^{0.5949}$	$\left(\frac{dist}{14.18}\right)^{0.5949}$
Dist. on travel time (PT VTTS)	$\left(\frac{dist}{97.64}\right)^{-0.3135}$	$\left(\frac{dist}{23.22}\right)^{-0.2368}$	$\left(\frac{dist}{60.27}\right)^{-0.2837}$	$\left(\frac{dist}{14.18}\right)^{-0.2009}$
Dist. on travel time (car VTTS)	$\left(\frac{dist}{97.64}\right)^{-0.3573}$	$\left(\frac{dist}{23.22}\right)^{-0.1321}$	$\left(\frac{dist}{60.27}\right)^{-0.3744}$	$\left(\frac{dist}{14.18}\right)^{-0.1905}$

Table 4: Calculation of WTP indicators

5.2 Plots

While the estimates for the interaction parameters shown in Table 3 give an indication of the link between socio-demographic indicators and the travel time and travel cost sensitivities, it is of more interest to look at the effect of these indicators on the various trade-offs. Although some idea of these effects can be obtained from the tables earlier on in this section, the easiest way to gain insights into these relationships is through a graphical representation. With this in mind, we now several contour plots, showing the impact of income and trip distance on the various trade-offs presented earlier on⁹.

Figure 2 shows the effect of income and trip distance on the VTTS for public transport. A significant interaction between income and the sensitivity to travel cost could only be identified for business travellers and commuters, such that the plots show a *flat* surface along the income dimension for leisure travel and shopping trips. It can also be seen that the income effect is much less significant for commuters than for business travellers, as could have been inferred from the estimates in Table 3. The estimates also indicated decreasing sensitivities to travel cost and travel time on longer trips. Here, the size of the effect on the cost sensitivity outweighs that on the travel time sensitivity, meaning that the VTTS actually increases on longer trips (as the decreasing effects are more marked in the denominator of the trade-off).

Figure 3 shows the effect of income and trip distance on the VTTS for car travel. Again, given the lack of interaction between income and cost sensitivity for leisure and shopping travel, the plots show a *flat* surface along the income dimension in these two population segments. There is again a decreasing sensitivity to travel time on longer trips, but this is again offset by the more significant in-

⁹Corresponding 3-D surface plots are shown in Hess (2006).

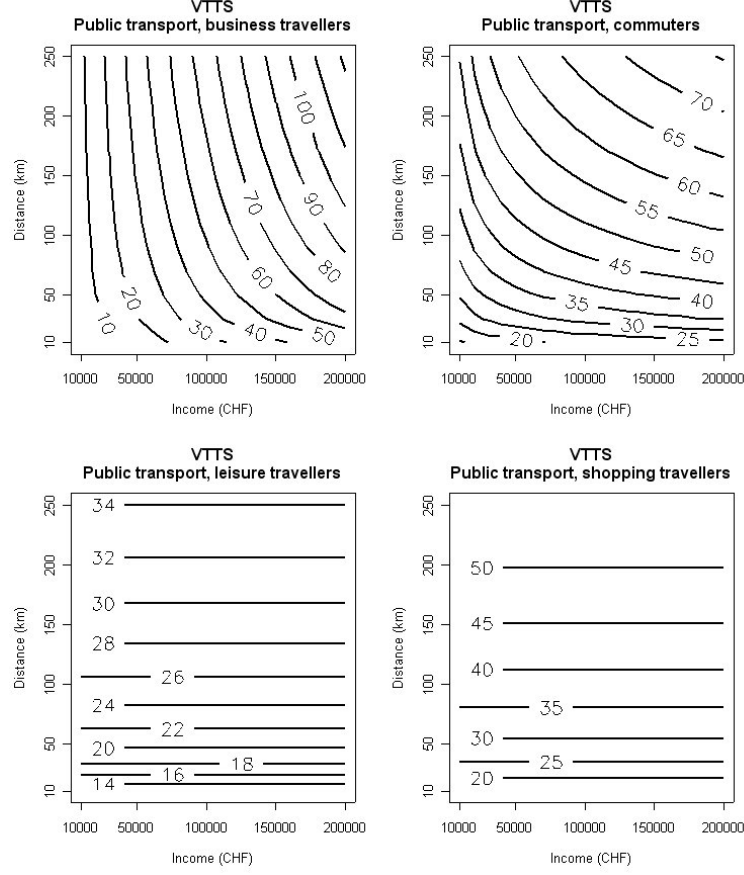


Figure 2: VTTs for PT travel

interaction of trip distance with the travel cost sensitivity, leading to higher VTTs measures on longer trips, especially for commuters and shopping travellers.

Figure 4 shows the effect of income and trip distance on the willingness to pay for headway reductions. As can be seen from Table 4, only two interaction parameters play a role, namely those linking income and trip distance to travel cost sensitivity. The negative effect of these interactions on cost sensitivity translates into increasing VTTs measures with higher income (only for business travellers and commuters) and on longer trips (all purpose segments).

The conclusions for the willingness to accept increases in travel cost in return for reductions in the number of interchanges are exactly the same as for the willingness to pay for headway reductions, as the same interactions are used (cf.

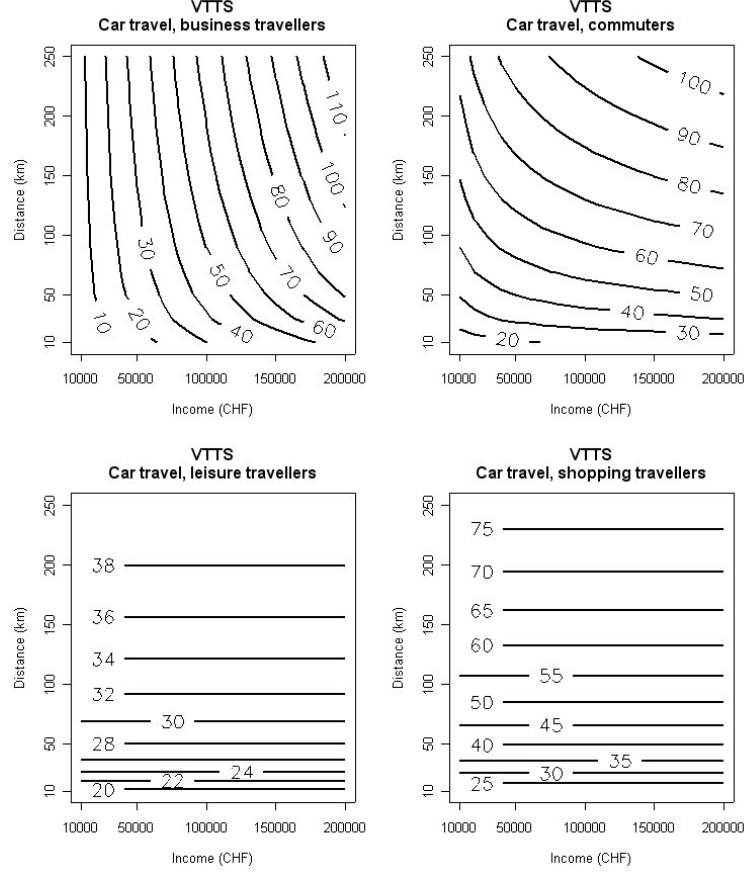


Figure 3: VTTs for car travel

Table 4). As such, the only difference between them is one of scale. Consequently, we only show the figure for the willingness to pay for headway reductions.

5.3 Comparison between elasticity model and base model

To give an indication of the effects of using the elasticity formulation on actual model results (other than model fit), Table 5 shows the indicators from the general model at the average income and distance, along with the fixed indicators from the base model. While there are significant differences between the two models for some of the indicators, the overall differences are smaller than might perhaps have been expected. However, aside from giving insights into the changes in trade-offs depending on socio-demographic indicators, the elasticity formulation

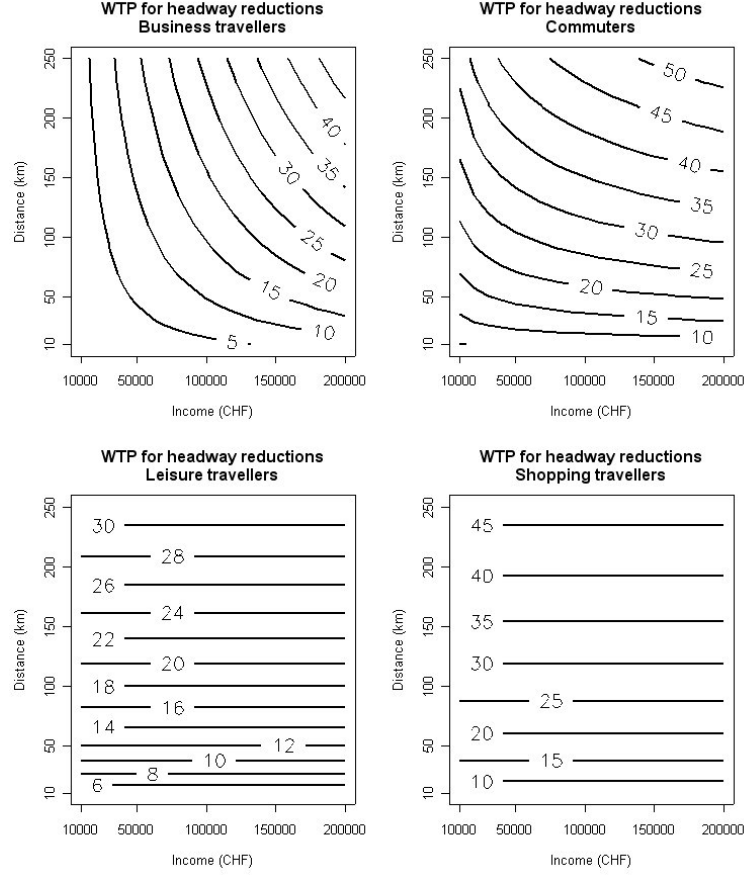


Figure 4: WTP for headway reductions

has another key advantage. Indeed, unlike the base formulation, it allows for the calculation of weighted population level indicator values, as described in Section 6.

6 Calculation of population level values

This section describes the calculation of population level values for the different trade-offs discussed in Section 5. We first look at the reweighting required to obtain values representative of the population level in Section 6.1, before describing the calculation of appropriate measures of spread in Section 6.2. The results of the calculations are presented in Section 6.3.

	Trip purpose			
	Business	Commuting	Leisure	Shopping
PT travel time				
Elasticity-based model	49.57	27.81	21.84	17.73
Base model	44.22	24.32	18.39	14.95
Car travel time				
Elasticity-based model	50.23	30.64	29.2	24.32
Base model	41.83	28.87	20.3	21.05
Headway reduction				
Elasticity-based model	14.88	11.18	13.38	8.48
Base model	13.46	10.62	12.85	8.11
Interchange reduction				
Elasticity-based model	7.85	4.89	7.32	3.52
Base model	7.07	4.73	7.33	3.51

Table 5: Comparison between trade-offs from elasticity and base models at sample mean for income and trip distance

6.1 Weighting

The calculation of the population level values is straightforward. Let $Q_{k,i,d}$ give the value of trade-off k in income-class i and distance-class d where income is divided into 8 classes between CHF12,000 and CHF180,000, and where trip distance is divided into 16 classes between 5 and 155 kilometres. Let $w_{i,d}$ give the population-weight for the combination of income-class i and distance-class d , with $\sum_{i=1}^I \sum_{d=1}^D w_{i,d} = 1$, with $I = 8$ and $D = 16$. Then the value of the trade-off reweighted to the population level is given by $\hat{Q} = \sum_{i=1}^I \sum_{d=1}^D w_{i,d} Q_{k,i,d}$.

6.2 Calculation of variances

Individual variances for a trade-off $Q_{k,i,d}$ can be calculated straightforwardly. Using the notation from Section 6.1, the weighted population level variance of the trade-off is given by

$$\text{var} \left(\sum_{i,d} w_{i,d} Q_{k,i,d} \right) = \sum_{i,d} w_{i,d}^2 \text{var} (Q_{k,i,d}). \quad (2)$$

In the present context, two situations arise, one for willingness to pay indicators, and one for the trade-off between interchanges and public transport travel time. The actual calculation of $\text{var} (Q_{k,i,d})$ is made more complicated by the presence of the various interaction terms. The derivation is slightly tedious, and details are given in Appendix A.

6.3 Results

Table 6 presents the weighted values of the various indicators along with their variances (and standard deviations)¹⁰. It is worth stressing that these calculations would not be possible with the base model, where the trade-offs are independent of income and distance. The major differences between the results shown in Table 6 and those in Table 5 are a further indication of the effect of allowing for income and distance effects, hence permitting the calculation of population level values. As an example, there are very significant decreases in the VTTS measures across population groups and modes, which is a direct result of the higher mean income in the estimation sample when compared to the population level. Table 6 also presents the corresponding overall values from a cross-purpose model, showing the loss of information when not accounting for differences across population segments.

7 Conclusions

This paper has presented the findings of a study looking into the valuation of travel time savings in Switzerland, across modes as well as across purpose groups. In summary, the main methodological points are

- the analysis has shown the benefit of a specification allowing for continuous interactions between respondents' tastes and socio-demographic indicators such as trip distance and income.
- the analysis has shown that the estimation of a robust coefficient for congested car travel time is hampered by the low share of congested time in the overall travel time, and the use of an additional rate-of-congestion coefficient, in addition to a generic car travel time coefficient, is preferable.
- the analysis has shown important differences between the four purpose groups in the calculated trade-offs, including but not limited to the VTTS.
- the analysis has demonstrated that the population mean of the indicators calculated is quite different from the sample means and has presented methods to calculate those and the associated variances.

In terms of cost-benefit practice, the results suggest a number of changes. A link-based cost-benefit analysis, as for example suggested in the German EWS

¹⁰In addition to these mean values and their associated variances, marginal values and variances were also calculated for each income class and distance class. These results are presented in detail in [Hess \(2006\)](#).

		Business			Commuters			Leisure			Shopping			Overall		
		mean			18.93			11.90			13.10			14.10		
		var			0.40			0.17			0.52			0.15		
		std.dev.			0.63			0.41			0.72			0.39		
VTTS for PT (CHF/hour)	mean	25.18			18.93			11.90			13.10			14.10		
	var	1.76			0.40			0.17			0.52			0.15		
	std.dev.	1.33			0.63			0.41			0.72			0.39		
VTTS for car travel (CHF/hour)	mean	27.66			19.04			18.83			17.84			20.98		
	var	10.97			0.52			2.09			1.40			0.71		
	std.dev.	3.31			0.72			1.45			1.18			0.84		
WTP for headway reductions (CHF/hour)	mean	4.63			6.29			4.57			5.54			5.37		
	var	0.03			0.02			0.01			0.05			0.01		
	std.dev.	0.17			0.16			0.10			0.23			0.10		
WTP for reductions in interchanges (CHF/change)	mean	2.44			2.75			2.50			2.30			2.61		
	var	0.00			0.00			0.00			0.00			0.00		
	std.dev.	0.07			0.05			0.04			0.06			0.03		
Interchanges versus PT travel time (min/change)	mean	5.01			8.14			11.52			10.12			10.31		
	var	0.09			0.06			0.13			0.27			0.05		
	std.dev.	0.29			0.24			0.36			0.52			0.22		

Table 6: Indicators at the population mean for purpose-specific model with generic car travel time coefficient

(FGSV, 1997) is clearly inappropriate if the VTTS of the link users depend on their respective trip distance. The on-going change to origin-destination specific analyses needs to be accelerated in those countries still employing link-based approaches. While it is inappropriate for a social cost-benefit analysis to distinguish between income groups, the results indicate possibilities for the operators of toll roads and public transport services to differentiate their prices accordingly.

The distance elasticity attenuates the problem of brief savings on short trips, which make up the bulk of all travel. In contrast to proposals to discount brief savings (up to some arbitrary duration) completely, here we obtain much smaller valuations naturally, as is intuitively expected, but maintain that all savings have to be valued.

The full variance estimates will help practical applications to define better informed values for the necessary sensitivity analyses. One can even shift to a proper risk analysis of the results, as one can attach confidence judgements to them. This will help to overcome simple minded best-case and worst-case scenario approaches which suffer from the absence of information about their likelihood of occurrence.

The destination choice experiment mentioned above raises the challenge to current practice in VTTS estimation to move from travel choices to activity choices. The stated-response experiments currently undertaken, say route choice, mode choice and departure time choice, ignore the benefits obtained at the destination (or origin in the case of departure time models). The trip purpose differences are a weak approximation of the true differences. In line with the arguments of activity-based modelling, one should in future move to models of activity scheduling as a fuller and more appropriate base to derive the values of travel time savings.

References

- Abay, G. and K. W. Axhausen (2001) *Zeitkostenansätze im Personenverkehr: Vorstudie*, SVI Forschungsauftrag 42/00, *Schriftenreihe*, **472**, Bundesamt für Strassen, UVEK, Bern.
- Algers, S., J. Lindqvist Dillen and S. Widlert (1995) *The National Swedish Value of Time Study*, paper presented at the PTRC European Transportation Forum, Warwick.
- ASTRA (2003) *NISTRA: Nachhaltigkeitsindikatoren für Strasseninfrastrukturprojekte: Ein Instrument zur Beurteilung von Strasseninfrastrukturprojek-*

ten unter Berücksichtigung der Nachhaltigkeitsziele - Methodenbericht, Astra, Bern.

Axhausen, K. W., A. Koenig, G. Abay, J. J. Bates and M. Bierlaire (2004) *Swiss Value of Travel Time Savings*, paper presented at the European Transport Conference, Strasbourg.

Ben-Akiva, M. and S. R. Lerman (1985) *Discrete Choice Analysis: Theory and Application to Travel Demand*, MIT Press, Cambridge, MA.

Bundesamt für Raumentwicklung und Bundesamt für Statistik (2001) *Mobilität in der Schweiz, Ergebnisse des Mikrozensus 2000 zum Verkehrsverhalten*, Bern and Neuenburg.

Erath, A. (2006) *Value of travel time savings for shopping trips in Switzerland*, paper presented at the 6th Swiss Transport Research Conference, Monte Verità, Ascona.

FGSV (1997) *Empfehlungen für Wirtschaftlichkeitsuntersuchungen an Strassen*, FGSV, Köln.

HCG (1990) *The Netherlands value of time study*, final report to Dienst Verkeerskunde, Rijkswaterstaat, Hague Consulting Group, Den Haag, Netherlands.

HCG (1999) *The value of travel time*, final report to the Department for Transport, Hague Consulting Group, Den Haag, Netherlands.

Hess, S. (2006) *Estimation of the Swiss Valuation of Travel Time Savings, Arbeitsbericht Verkehrs- und Raumplanung*, IVT, ETH Zürich, Zürich.

Jovicic, G. and C. O. Hansen (2003) A passenger travel demand model for Copenhagen, *Transportation Research Part A: Policy and Practice*, **37** (4) 333–349.

Koenig, A., G. Abay and K. W. Axhausen (2004) *Zeitkostenansätze im Personenverkehr*, *Schriftenreihe*, **1065**, final report for SVI 2001/534, Bundesamt für Strassen, UVEK, Bern.

Kurri, J. and M. Pursula (1995) *Finnish Preliminary Value of Time Studies*, The Swedish Road and Transport Research Institute & The Swedish Transport and Communications Research Board, Linköping, Sweden.

Louviere, J. J., D. A. Hensher and J. Swait (2000) *Stated Choice Models: Analysis and Application*, Cambridge University Press, Cambridge.

- Mackie, P., M. Wardman, A. S. Fowkes, G. Whelan, J. Nellthorp and J. J. Bates (2003) *Values of travel time savings in the UK*, Report to Department of Transport, Institute for Transport studies, University of Leeds and John Bates Services, Leeds and Abingdon.
- Ramjerdi, F., L. Rand, I. Saetermo and K. Saelensminde (1997) *The Norwegian Value of Time Study*, TOI Report 397, Norwegian Centre for Transport Research, Oslo, Norway.
- Vrtic, M., K. W. Axhausen, R. Maggi and F. Rossera (2003) *Verifizierung von Prognosemethoden im Personenverkehr*, on behalf of SBB and Bundesamt für Raumentwicklung (ARE), IVT, ETH Zürich and USI Lugano, Zürich and Lugano.
- VSS (2006) *SN 641 820 Kosten-Nutzen-Analysen (KNA) bei Massnahmen im Strassenverkehr*, VSS, Zürich.

A Formulae for variances of population level trade-offs

In this appendix, we present the derivation of the formulae for the calculation of the variances of population level trade-offs. We first look at the willingness to pay indicators, in Section A.1, before proceeding to the trade-off between the number of interchanges and public transport travel time (Section A.2).

A.1 Willingness to pay indicators

The general formulation for monetary indicators for an individual in income class i and distance class d is given by:

$$Q_{i,d} = K \frac{\beta_x}{\beta_{TC}} \frac{\left(\frac{dist_d}{D}\right)^{\lambda_{x,dist}}}{\left(\frac{dist_d}{D}\right)^{\lambda_{TC,dist}} \left(\frac{inc_i}{I}\right)^{\lambda_{TC,inc}}} \quad (3)$$

where $K = 60$ if β_x is associated with an attribute measured in minutes. The above simplification in the numerator in terms of interaction parameters is made possible by the fact that income interactions are only observed for the cost attribute.

The variance of the trade-off (with income class i and distance class d) is given by:

$$var(Q_{i,d}) = \nabla_{Q_{i,d}}^T \Omega \nabla_{Q_{i,d}}, \quad (4)$$

where

$$\nabla_{Q_{i,d}}^T = \left\langle \frac{\partial Q_{i,d}}{\partial \beta_x}, \frac{\partial Q_{i,d}}{\partial \beta_{TC}}, \frac{\partial Q_{i,d}}{\partial \lambda_{x,dist}}, \frac{\partial Q_{i,d}}{\partial \lambda_{TC,dist}}, \frac{\partial Q_{i,d}}{\partial \lambda_{TC,inc}} \right\rangle, \quad (5)$$

and where the various partial derivatives are given by:

$$\begin{aligned} \frac{\partial Q_{i,d}}{\partial \beta_x} &= K \frac{1}{\beta_{TC}} \frac{\left(\frac{dist_d}{D}\right)^{\lambda_{x,dist}}}{\left(\frac{dist_d}{D}\right)^{\lambda_{TC,dist}} \left(\frac{inc_i}{I}\right)^{\lambda_{TC,inc}}} \\ &= \frac{1}{\beta_x} Q_{i,d} \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{\partial Q_{i,d}}{\partial \beta_{TC}} &= -K \frac{\beta_x}{(\beta_{TC})^2} \frac{\left(\frac{dist_d}{D}\right)^{\lambda_{x,dist}}}{\left(\frac{dist_d}{D}\right)^{\lambda_{TC,dist}} \left(\frac{inc_i}{I}\right)^{\lambda_{TC,inc}}} \\ &= -\frac{1}{\beta_{TC}} Q_{i,d} \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{\partial Q_{i,d}}{\partial \lambda_{x,dist}} &= K \frac{\beta_x}{\beta_{TC}} \frac{\left(\frac{dist_d}{D}\right)^{\lambda_{x,dist}} \ln\left(\frac{dist_d}{D}\right)}{\left(\frac{dist_d}{D}\right)^{\lambda_{TC,dist}} \left(\frac{inc_i}{I}\right)^{\lambda_{TC,inc}}} \\ &= \ln\left(\frac{dist_d}{D}\right) Q_{i,d} \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{\partial Q_{i,d}}{\partial \lambda_{TC,dist}} &= -K \frac{\beta_x}{\beta_{TC}} \frac{\left(\frac{dist_d}{D}\right)^{\lambda_{x,dist}} \left(\frac{dist_d}{D}\right)^{\lambda_{TC,dist}} \ln\left(\frac{dist_d}{D}\right)}{\left(\left(\frac{dist_d}{D}\right)^{\lambda_{TC,dist}}\right)^2 \left(\frac{inc_i}{I}\right)^{\lambda_{TC,inc}}} \\ &= -\ln\left(\frac{dist_d}{D}\right) Q_{i,d} \end{aligned} \quad (9)$$

and

$$\frac{\partial Q_{i,d}}{\partial \lambda_{TC,inc}} = -\ln\left(\frac{inc_i}{I}\right) Q_{i,d}. \quad (10)$$

Note that the term $Q_{i,d}$ is common to all these partial derivatives. If we now write $\nabla_{Q_{i,d}} = Q_{i,d} \mathbf{X}^{11}$, then $var(Q_{i,d}) = Q_{i,d}^2 [\mathbf{X}^T \Omega \mathbf{X}]$, where Ω is defined as in equation (11). The quadratic form of $[\mathbf{X}^T \Omega \mathbf{X}]$ can be evaluated as:

$$\begin{aligned} [\mathbf{X}^T \Omega \mathbf{X}] &= \left(\frac{\sigma(\beta_x)}{\beta_x}\right)^2 + \left(\frac{\sigma(\beta_{TC})}{\beta_{TC}}\right)^2 + \left(\sigma(\lambda_{x,dist}) \ln\left(\frac{dist_d}{D}\right)\right)^2 \\ &+ \left(\sigma(\lambda_{TC,dist}) \ln\left(\frac{dist_d}{D}\right)\right)^2 + \left(\sigma(\lambda_{TC,inc}) \ln\left(\frac{inc_i}{I}\right)\right)^2 \\ &- 2 \frac{cov(\beta_{TC}, \beta_x)}{\beta_x \beta_{TC}} + 2 \frac{cov(\lambda_{x,dist}, \beta_x) \ln\left(\frac{dist_d}{D}\right)}{\beta_x} \\ &- 2 \frac{cov(\lambda_{TC,dist}, \beta_x) \ln\left(\frac{dist_d}{D}\right)}{\beta_x} - 2 \frac{cov(\lambda_{TC,inc}, \beta_x) \ln\left(\frac{inc_i}{I}\right)}{\beta_x} \\ &- 2 \frac{cov(\lambda_{x,dist}, \beta_{TC}) \ln\left(\frac{dist_d}{D}\right)}{\beta_{TC}} + 2 \frac{cov(\lambda_{TC,dist}, \beta_{TC}) \ln\left(\frac{dist_d}{D}\right)}{\beta_{TC}} \\ &+ 2 \frac{cov(\lambda_{TC,inc}, \beta_{TC}) \ln\left(\frac{inc_i}{I}\right)}{\beta_{TC}} - 2 cov(\lambda_{TC,dist}, \lambda_{x,dist}) \ln\left(\frac{dist_d}{D}\right)^2 \\ &- 2 cov(\lambda_{TC,inc}, \lambda_{x,dist}) \ln\left(\frac{dist_d}{D}\right) \ln\left(\frac{inc_i}{I}\right) \\ &+ 2 cov(\lambda_{TC,inc}, \lambda_{TC,dist}) \ln\left(\frac{dist_d}{D}\right) \ln\left(\frac{inc_i}{I}\right) \end{aligned} \quad (12)$$

A.2 Trade-off between interchanges and travel time

The base formulation for an individual in distance class d and income class i (not used) is now given by:

$$Q_{i,d} = \frac{\beta_{changes}}{\beta_{TT,PT}} \frac{1}{\left(\frac{dist_d}{D}\right)^{\lambda_{TT,dist}}}, \quad (13)$$

where the simplification in terms of interaction parameters is made possible by the fact that no interactions with income or trip distance are observed for the interchanges attribute.

Again, the variance is given by:

$$var(Q_{i,d}) = \nabla_{Q_{i,d}}^T \Omega \nabla_{Q_{i,d}} \quad (14)$$

¹¹ \mathbf{X} is a vector containing the terms that $Q_{i,d}$ multiplies in the various partial derivatives.

$$\Omega = \begin{bmatrix} \sigma(\beta_x)^2 & cov(\beta_{TC}, \beta_x) & cov(\beta_{TC}, \beta_x) & cov(\beta_{TC}, \beta_x) & cov(\lambda_{TC, inc}, \beta_x) \\ cov(\beta_{TC}, \beta_x) & cov(\lambda_{x, dist}, \beta_x) & \sigma(\beta_{TC})^2 & cov(\lambda_{TC, dist}, \beta_{TC}) & cov(\lambda_{TC, inc}, \beta_{TC}) \\ cov(\lambda_{x, dist}, \beta_x) & cov(\lambda_{x, dist}, \beta_x) & \sigma(\lambda_{x, dist})^2 & cov(\lambda_{TC, dist}, \lambda_{x, dist}) & cov(\lambda_{TC, inc}, \lambda_{x, dist}) \\ cov(\lambda_{TC, dist}, \beta_x) & cov(\lambda_{TC, dist}, \beta_{TC}) & cov(\lambda_{TC, dist}, \lambda_{x, dist}) & \sigma(\lambda_{TC, dist})^2 & \sigma(\lambda_{TC, inc})^2 \\ cov(\lambda_{TC, inc}, \beta_x) & cov(\lambda_{TC, inc}, \beta_{TC}) & cov(\lambda_{TC, inc}, \lambda_{x, dist}) & cov(\lambda_{TC, inc}, \lambda_{TC, dist}) & \sigma(\lambda_{TC, inc})^2 \end{bmatrix} \quad (11)$$

where

$$\nabla_{Q_{i,d}}^T = \left\langle \frac{\partial Q_{i,d}}{\partial \beta_{changes}}, \frac{\partial Q_{i,d}}{\partial \beta_{TT,PT}}, \frac{\partial Q_{i,d}}{\partial \lambda_{TT,dist}} \right\rangle \quad (15)$$

with partial derivatives given by:

$$\frac{\partial Q_{i,d}}{\partial \beta_{changes}} = \frac{1}{\beta_{changes}} Q_{i,d} \quad (16)$$

$$\frac{\partial Q_{i,d}}{\partial \beta_{TT,PT}} = -\frac{1}{\beta_{TT,PT}} Q_{i,d} \quad (17)$$

$$\begin{aligned} \frac{\partial Q_{i,d}}{\partial \lambda_{TT,dist}} &= -\frac{\beta_{changes}}{\beta_{TT,PT}} \frac{\left(\frac{dist_d}{D}\right)^{\lambda_{TT,dist}} \ln\left(\frac{dist_d}{D}\right)}{\left(\left(\frac{dist_d}{D}\right)^{\lambda_{TT,dist}}\right)^2} \\ &= -\ln\left(\frac{dist_d}{D}\right) Q_{i,d} \end{aligned} \quad (18)$$

Again, the term $Q_{i,d}$ is common to all these partial derivatives. By again writing $\nabla_{Q_{i,d}} = Q_{i,d} \mathbf{X}$, we have $var(Q_{i,d}) = Q_{i,d}^2 [\mathbf{X}^T \Omega \mathbf{X}]$, where Ω is defined as:

$$\Omega = \left[\begin{pmatrix} \sigma(\beta_{changes})^2 & cov(\beta_{TT,PT}, \beta_{changes}) & cov(\lambda_{TT,dist}, \beta_{changes}) \\ cov(\beta_{TT,PT}, \beta_{changes}) & \sigma(\beta_{TT,PT})^2 & cov(\lambda_{TT,dist}, \beta_{TT,PT}) \\ cov(\lambda_{TT,dist}, \beta_{changes}) & cov(\lambda_{TT,dist}, \beta_{TT,PT}) & \sigma(\lambda_{TT,dist})^2 \end{pmatrix} \right] \quad (19)$$

The quadratic form of $[\mathbf{X}^T \Omega \mathbf{X}]$ can then be evaluated as:

$$\begin{aligned} [\mathbf{X}^T \Omega \mathbf{X}] &= \left(\frac{\sigma(\beta_{changes})}{\beta_{changes}}\right)^2 + \left(\frac{\sigma(\beta_{TT,PT})}{\beta_{TT,PT}}\right)^2 + \left(\sigma(\lambda_{TT,dist}) \ln\left(\frac{dist_d}{D}\right)\right)^2 \\ &\quad - 2 \frac{cov(\beta_{TT,PT}, \beta_{changes})}{\beta_{changes} \beta_{TT,PT}} \\ &\quad - 2 \frac{cov(\lambda_{TT,dist}, \beta_{changes}) \ln\left(\frac{dist_d}{D}\right)}{\beta_{changes}} \\ &\quad + 2 \frac{cov(\lambda_{TT,dist}, \beta_{TT,PT}) \ln\left(\frac{dist_d}{D}\right)}{\beta_{TT,PT}} \end{aligned} \quad (20)$$