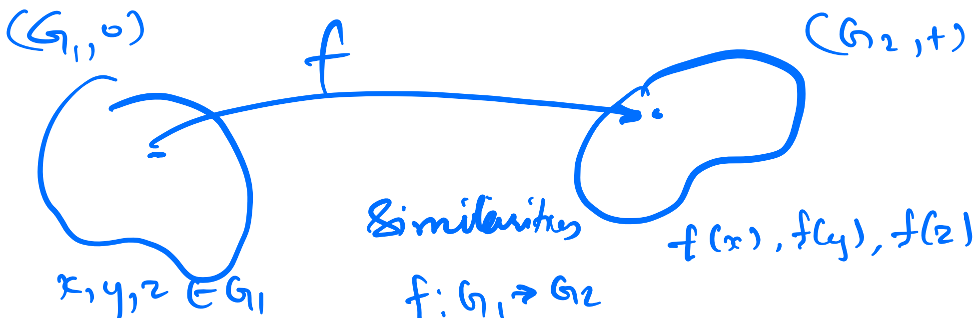
Def Groups - finite / infinite set + binary operator

Prop Groups - Closure Associativity Identity Inverse

Def Homomorphism - f between G_1 & G_2 ident similarities

Prop Homomorphism - Structure preserving Map



Similarities

$f(x), f(y), f(z) \in G_2$

$x \circ y = z$

$f : G_1 \to G_2$

$x \to f(x)$

$y \to f(y)$

$z \to f(z)$

$f(x) + f(y) = f(z)$

$= f(x \circ y)$

Def Injective : 1-1 mapping of dom - codom

Def Surjective : every y has atleast one x

Def Bijective : Injective + Surjective

Def Rigid body : collection of particles + inter Particle distances fixed

Parametrization:

Explicit parametrization: minimal rep of space

Implicit parametrization : maximal rep of space st constr

n dim space as embedded in high dimension

Euclidean space

DoF = (sum of freedoms - Independent constraints)

m dim $m \geq n$ $m-n$ ind constr

Configuration Space:

> the complete space of position of every joint of the robot is called joint space

(or) configuration space - topological

rigid body in plane (x, y, θ)

prone to singularity

$$\left.\begin{array}{l} S^1 \subseteq \mathbb{R}^2 \\ S^2 \subseteq \mathbb{R}^3 \\ S^n \subseteq \mathbb{R}^{n+1} \end{array}\right]$$

circle → surface → hypersurface

point robot on line	$x \in \mathbb{R}^1$
point robot on plane	$x \in \mathbb{R}^2$
point robot on circle	$\theta \in S^1$
robot with rev joint	$\theta \in S^1$
" " 2 rev joints	$\theta \in S^1 \times S^1$
rigid body in plane	$x, \theta \in \mathbb{R}^2 \times S^1$
rigid body in space	$x, \theta \in \mathbb{R}^3 \times S^1 \times S^1 \times S^1$

Joint	dof	Constraints planar	Constraints spatial
Revolute	1	2	5
prismatic	1	2	5
Helical	1	NA	5
Cylindrical	2	NA	4
Universal	2	NA	4
Spherical	3	NA	3

Grubler's Condition

$$DoF = K(N-1-j) + \sum f_i$$

K: parameters of plane or spatial link
N: number of links
j: number of joints
f_i: freedom of jth joint (dof)

(1) Euler Angles
(2) Axis Angle
(3) Rotation Matrices
(4) Quaternions

Abelian Group: $R^T = R^{-1}$

$$R_1 R_2 = R_2 R_1$$

$$ {}^{A}_{B}[R] = R(\hat{Z}_A, \theta) = \begin{array}{c} \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{ccc} \;\;x & \quad\; y & \quad\; z \end{array} \end{array} $$

$$R(\hat{y}_A, \theta) = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$

$$R(\hat{x}_A, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix}$$

$SO(2)$

$SO(3)$

α_{ij} the angle by x_B wrt x_A, y_A, z_A

$$\hat{x}_B = c\alpha_{11}\hat{x}_A + c\alpha_{12}\hat{y}_A + c\alpha_{13}\hat{z}_A$$

$$\hat{y}_B = c\alpha_{21}\hat{x}_A + c\alpha_{22}\hat{y}_A + c\alpha_{23}\hat{z}_A$$

$$\hat{z}_B = c\alpha_{31}\hat{x}_A + c\alpha_{32}\hat{y}_A + c\alpha_{33}\hat{z}_A$$

$$R = \begin{bmatrix} c\alpha_{11} & - & - \\ | & & | \\ | & - & c\alpha_{33} \end{bmatrix} = \begin{bmatrix} | & | & | \\ a & b & c \\ | & | & | \end{bmatrix}$$

$|a| = |b| = |c| = 1$

$r_{11}^2 + r_{12}^2 + r_{13}^2 = 0$

$a^T b = b^T c = c^T a = 0$

$r_{11} r_{12} + r_{21} r_{22} + r_{31} r_{32} = 0$

$R^T R = I$

$\begin{bmatrix} | & | & | \\ a & b & c \\ | & | & | \end{bmatrix} \Rightarrow \det(R) \Rightarrow a^T(b \times c)$

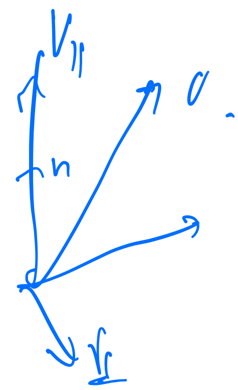
Axis angle theorem:

$$V = V_{\parallel} + V_{\perp}$$

$$V' = V_{\parallel} + V_{\perp}'$$

We have to find $V_{\parallel}$ & $V_{\perp}'$

$$V' = V_{\parallel} + \overbrace{\cos\theta\, V_{\perp} + \sin\theta\,(n \times V_{\perp})}^{V_{\perp}'}$$



$$V' = V_{\parallel} + \cos\theta\,(v - v^{\parallel}) + \sin\theta\,\overset{n \times V_{\perp}}{(n \times v)}$$

$$= v^{\parallel}(1-\cos\theta) + \sin\theta\,(n \times v) + \cos\theta\, v$$

$$= (v \cdot n)\,n\,(1-\cos) + \sin\theta\,(n \times v) + \cos\theta\, v$$

$$v' = \underbrace{\left(I + \sin\theta\, \hat{n} + (1-\cos\theta)\,\hat{n}^2\right)}\, v$$

$R(n, \theta)$

$$\text{trace}(R) = 1 + 2\cos\theta$$

$$\theta = \arccos\left(\frac{\text{tr}(R) - 1}{2}\right)$$

$$\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \frac{1}{2s\theta} \begin{bmatrix} r_{32} & 23 \\ 13 & 31 \\ 21 & 12 \end{bmatrix} = \frac{1}{2s\theta} \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

anticlockwise sequence

Singularity: $R = I \quad \theta = 0 \quad \theta = \pi$

$R \neq I \quad \hat{n}, \theta, -\hat{n}, 2\pi - \theta$

$SE(2) = R_2 \times SO(2)$

$${}^{A}_{B}[T] = \begin{bmatrix} {}^{A}_{B}[R] & {}^{O}A_{B} \\ 0 & 1 \end{bmatrix}$$

$SE(3) = R_3 \times SO(3) = \{(P, R) \mid P \in \mathbb{R}^3, R \in SO(3)\}$

$TT^{-1} = I$

DH

T L O j

x_{i-1} x_{i-1} z_i z_i

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	θ_1
2	$\pi/2$	0	0	θ_2
3	0			