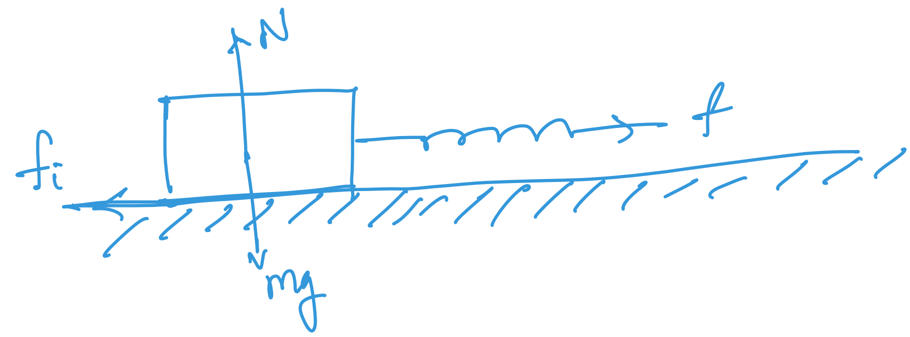
Coulomb Friction Model



$f_i \le \mu N$ $\quad$ μ coefficient of friction

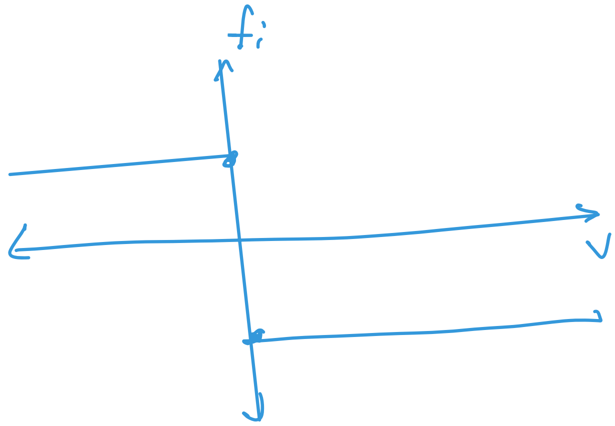
if $v = 0$ $\quad |f_i| < \mu N$

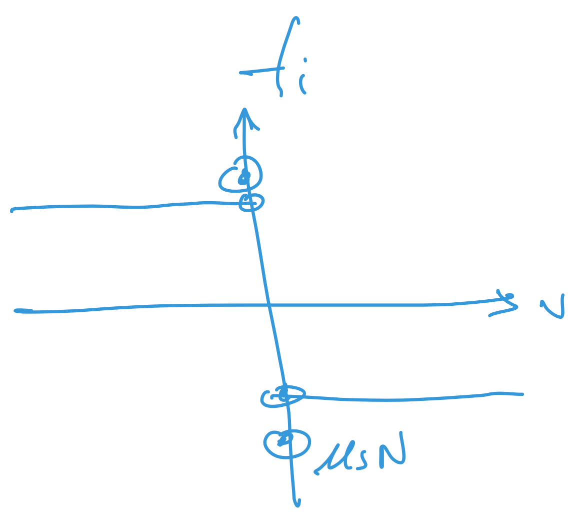
$v > 0$ $\quad |f_i| = \mu N$

model (2) static + dynamic friction

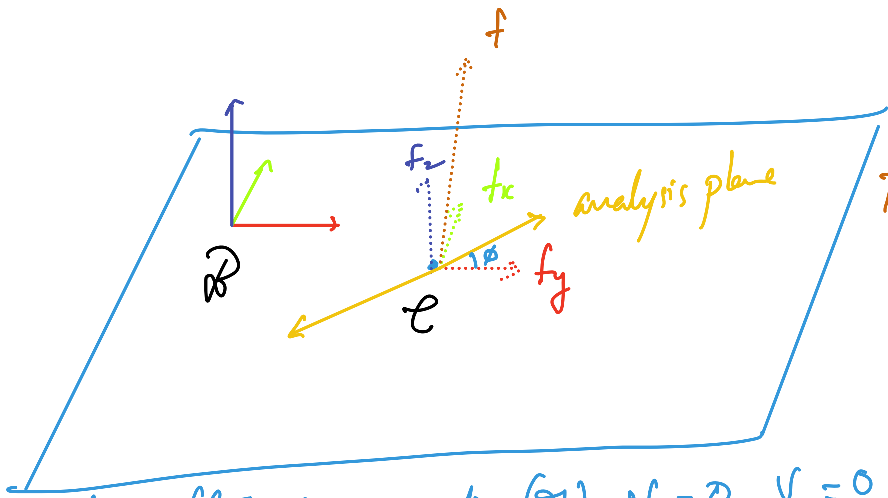
$v = 0$ $\quad f_i = \mu_s N$

$v > 0$ $\quad f_i = \mu_d N$

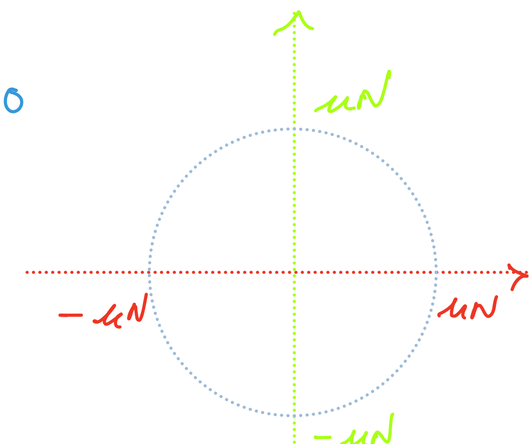




$\mathcal{B}$ = body frame of reference
$\mathcal{C}$ = contact frame of reference



f = force applied on the surface at the point of contact



when there is no slip (or) $v_x = 0$, $v_y = 0$

$$f_x \leq \mu N = \mu f_z$$

$$f_y \leq \mu N = \mu f_z$$

$$f_z > 0$$

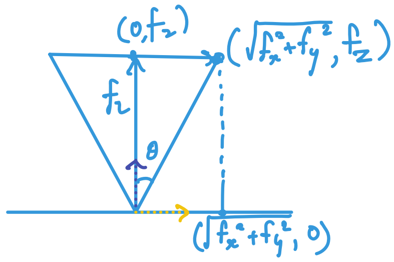
Friction Cone

$$K_c = \left\{ f \;\middle|\; f_z \geq 0,\ \sqrt{f_x^2 + f_y^2} \leq \mu f_z \right\}$$

a cone defining the set of forces that do not allow relative motion at the point of contact.

Let us project the project the spatial friction cone onto a surface to get a planar friction cone, for the sake of analysis

$$f \triangleq \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$



$$\boxed{\tan\theta = \frac{\sqrt{f_x^2+f_y^2}}{f_z}}$$

$\theta \leq \alpha$ but if $\theta = \alpha$ $\quad \tan\alpha = \frac{\mu \cancel{f_z}}{\cancel{f_z}}$

** $\boxed{\alpha = \tan^{-1}\mu}$

and at α $f_{1p} = f_1$ & $f_{2p} = f_2$

from the above analysis we can come up with

Alternative definition of Friction Cone $= pos(\{{}_\phi f_1, {}_\phi f_2\})$

doesn't apply to spatial friction cones because there exist new $\{{}_\phi f_1, {}_\phi f_2\}$ for each angle $\phi \in [0, \pi] \subseteq \mathbb{R}$

and $K = \{ \oplus \{{}_\phi f_1, {}_\phi f_2\} \; \forall \phi \in [0, \pi] \}$

You can see how the set has infinite elements

for the sake of computation we define Pyramidal K_p which is an underestimator of K_e and call it $\underline{K_p}$

$$K_p = pos(\{f_1, f_2, f_3, f_4\}) = \{ f \in \mathbb{R}^{(2n-1)} \mid f \geq 0 \}$$

where, $f_1 = (\mu, 0, 1)$

$f_2 = (0, \mu, 1)$

$f_3 = (-\mu, 0, 1)$

$f_4 = (0, -\mu, 1)$

In mujoco the contact frame orientation is slightly different X - contact normal YZ - contact tangential plane

We can define solvers which will determine what the contact dimensions mean

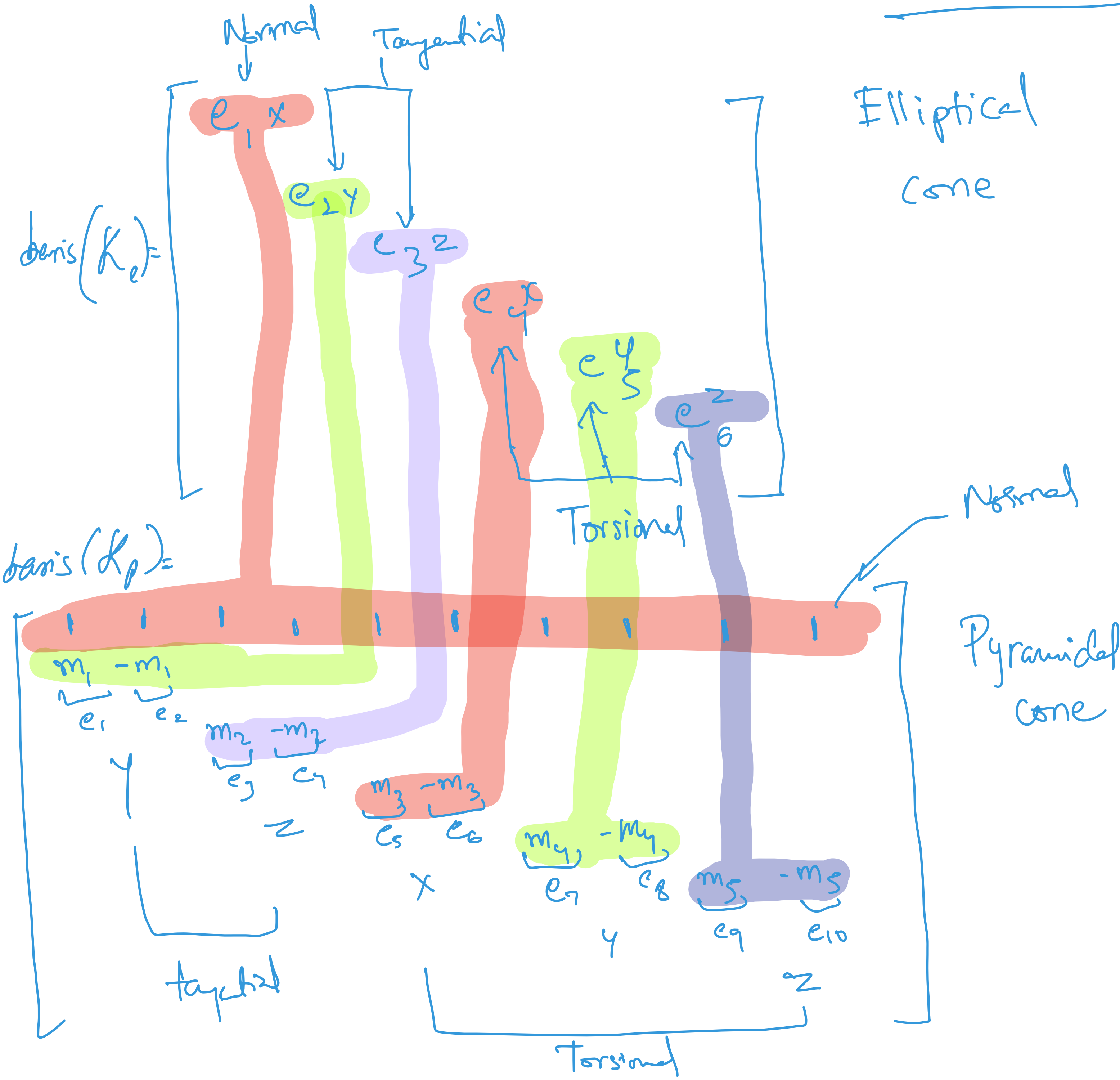
condim	Elliptic solvers	Pyramidal Solvers
1	1 frictionless contact	1 frictionless contact
3	3 Normal + Tangential	4 Normal + tangential
4	4 Normal + Tangential + Torsional	6 Normal + Tangential + Torsional
6	6 oppose motion in all relative dof	10 oppose motion in all relative dof

$$K_e = \{ f \in \mathbb{R}^n : f_1 \geq 0, f_1^2 \geq \sum f_i^2/\mu_{i-1}^2 \}$$

$$K_p = \{ f \in \mathbb{R}^{2(n-1)} : f \geq 0 \}$$

Basis for Friction Cones

note that for condim = 1 we are talking only about e_3^z of row(1)



note that m is scaled basis

x axis is the contact normal direction
YZ are the contact tangential directions