Vector r $\vec{r}$

reference coordinate system $\mathcal{A}$ (italicized A)

$(e_x^{\mathcal{A}}, e_y^{\mathcal{A}}, e_z^{\mathcal{A}}) :=$ orthonormal basis of $\mathbb{R}^3$

Vector from point B to point P

$${}_{\mathcal{A}}r_{BP}$$

read as vector r from point B to point P in reference frame $\mathcal{A}$

$${}_{\mathcal{A}}r_{AP} = {}_{\mathcal{A}}r_{AB} + {}_{\mathcal{A}}r_{BP}$$

vector addition is only possible in same reference frame

X indicates coordinates in a system

Cartesian coordinates

$$X_{P_c} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

read as cartesian coordinates of point P (c indicates Cartesian coordinates)

cylindrical coordinates

$$X_{P_z} = \begin{bmatrix} \rho \\ \theta \\ z \end{bmatrix}$$

read as cylindrical coordinates of point P (z stands for cylindrical)

Spherical coordinates

$$X_{P_s} = \begin{bmatrix} r\cos\theta \sin\phi \\ r\sin\theta \sin\phi \\ r\cos\phi \end{bmatrix}$$

$$r_{AP} = r_{AB} + r_{BP}$$

is only true in cartesian coordinates

in spherical coord. and cylindrical coord. we need special algebra

$$\underbrace{{}_{A}(r_{BP})'}_{\mathrm{I}} \neq \underbrace{({}_{A}r_{BP})'}_{\mathrm{II}}$$

because,

in I A is fixed and P is only moving

in II A itself may be moving hence absolute velocity of P is velocity of coord. frame A + velocity of point P

What is the relation between $\dot{r}$ and $\dot{X}$?

(What is the relation between metric space & $r = r(X)$ some nonlinear vector function

$$\Rightarrow \dot{r} = \frac{\partial r}{\partial X}\dot{X}$$

$$\dot{r} = E_p(X)\,\dot{X}$$

Example:

Cartesian coordinates

$$r = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\dot{r} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} \qquad \dot{x} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}$$

$$\Rightarrow \dot{r} = \mathbb{I}\, \dot{x}$$

Cylindrical coordinates

$$r = \begin{pmatrix} \rho\cos\theta \\ \rho\sin\theta \\ z \end{pmatrix} \qquad x = \begin{pmatrix} \rho \\ \theta \\ z \end{pmatrix}$$

$$\dot{r} = \begin{pmatrix} \dot{\rho}\cos\theta - \rho\sin\theta\,\dot{\theta} \\ \dot{\rho}\sin\theta + \rho\cos\theta\,\dot{\theta} \\ \dot{z} \end{pmatrix} \qquad \dot{x} = \begin{pmatrix} \dot{\rho} \\ \dot{\theta} \\ \dot{z} \end{pmatrix}$$

$$E_{\rho z}(x_{\rho z}) = \frac{\partial r(x_{\rho z})}{\partial x_{\rho z}} = \begin{bmatrix} \cos\theta & -\rho\sin\theta & 0 \\ \sin\theta & \rho\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$_{\mathcal{A}} r_{AP}$ position of P wrt A in $\mathcal{A}$

$_{\mathcal{B}} r_{AP}$ position of P wrt A in $\mathcal{B}$

unit vectors of $\mathcal{B}$ expressed in $\mathcal{A}$ frame:

$$\left[{}_{\mathcal{A}} e_x^{\mathcal{B}} \quad {}_{\mathcal{A}} e_y^{\mathcal{B}} \quad {}_{\mathcal{A}} e_z^{\mathcal{B}} \right]$$

$$\Rightarrow {}_{\mathcal{A}} r_{AP} = {}_{\mathcal{A}} e_x^{\mathcal{B}} \cdot {}_{\mathcal{B}} r_{AP_x} + {}_{\mathcal{A}} e_y^{\mathcal{B}} \, {}_{\mathcal{B}} r_{AP_y} + {}_{\mathcal{A}} e_y^{\mathcal{B}} \, {}_{\mathcal{B}} r_{AP_z}$$

$$= \left[{}_{\mathcal{A}} e_x^{\mathcal{B}} \quad {}_{\mathcal{A}} e_y^{\mathcal{B}} \quad {}_{\mathcal{A}} e_z^{\mathcal{B}} \right] {}_{\mathcal{B}} r_{AP}$$

$${}_{\mathcal{A}} r_{AP} = C_{\mathcal{A}\mathcal{B}} \cdot {}_{\mathcal{B}} r_{AP}$$

where C is the coordinate transformation of $\mathcal{B}$ frame with respect to $\mathcal{A}$ frame

$${}_{A}u = C_{AB}\, {}_{B}u$$

where, $$C_{AB} = \left[{}_{A}e_x^{B} \;\; {}_{A}e_y^{B} \;\; {}_{A}e_z^{B} \right]$$

$$C_{BA} = C_{AB}^{-1} = C_{AB}^{T}$$

$C \in SO(3)$ and not $\mathbb{R}^3$

$$C_{AC} = C_{AB} \cdot C_{BC}$$

passive rotation : (coordinate frame of reference is different)

$${}_{A}u = C_{AB} \cdot {}_{B}u$$

Represent same vector in different coordinate frame

active rotation : Transformative (coordinate frame of reference is same)

$${}_{A}v = R\, {}_{A}u$$

Transform the vector in the same coordinate frame to another vector

in most robotic applications, in kinematics & dynamics we are only dealing with passive rotations

Generalized coordinates

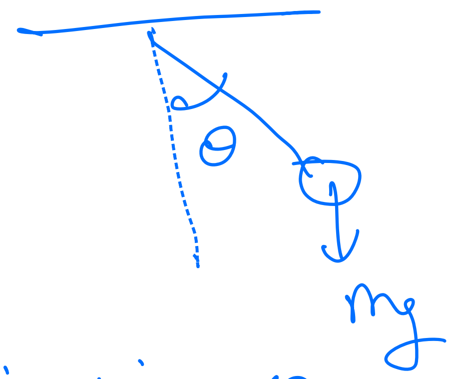
① States of the system in

configuration space of the robot

What is a configuration space?

What is Task space?

What is work space?



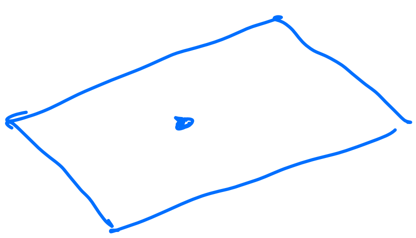
$$ml\ddot{\theta} + b\dot{\theta} + k\theta = 0$$

S^1

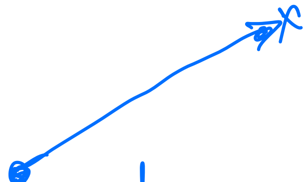
configuration space = Spec of each joint of the robot

= topological

(x, y)



E^2



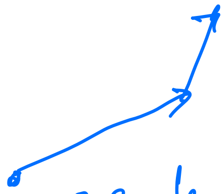
universal joint

S^2

long lat

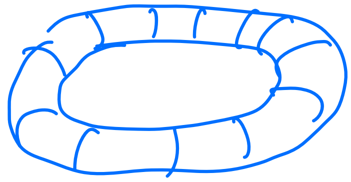
$[-\pi, \pi] \times [\pi/2, \pi/2]$

1 link - 2R manipulator



2R planar/spatial

$S^1 \times S^1$

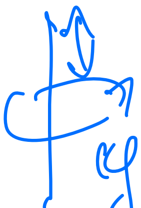


$[0, 2\pi) \times [0, 2\pi)$

prismatic link

E^1

$\mathbb{R}^1$



cylindrical sliding

$E^1 \times S^1$

constraint

$\mathbb{R}^1 \times [0, 2\pi)$

Task space : space in which robot task can be described fully $\in \underbrace{\mathbb{R}^n \times S^m}_{\text{typically}}$

Workspace : Spec of configuration that end effector can reach. independent of task space

for example annulus for 1R manipulator

Workspace $\subset$ Task space

Floating base systems

$$q = \begin{pmatrix} q_b \\ q_j \end{pmatrix} \quad \text{where, } q_b = \mathbb{R}^3 \times SO^3$$

$$\dim(q_{pos}[\text{freejoint}]) = \dim(q_b)$$ [orientation expressed in quaternion]

Space Parametrization:

Explicit parametrization: minimal rep of space

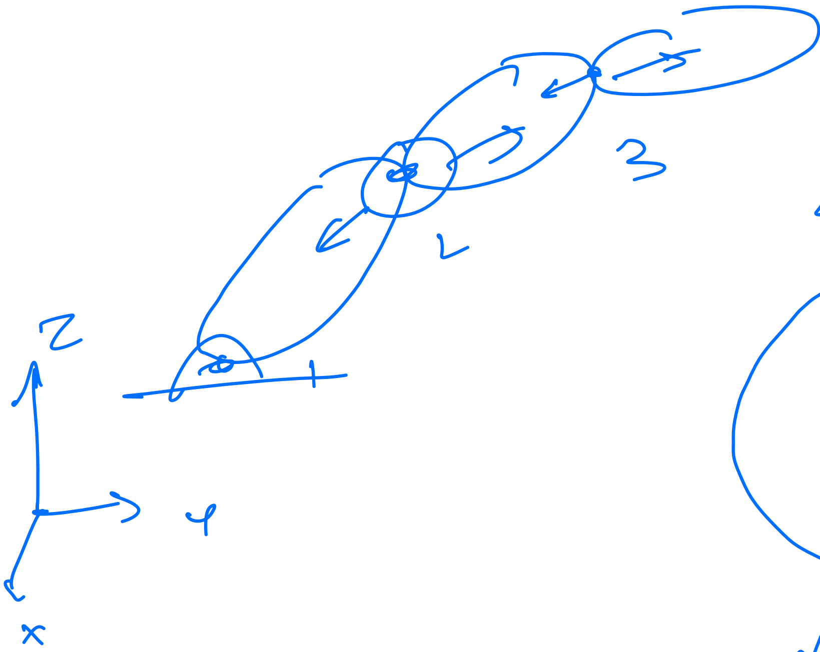
Implicit parametrization: maximal rep of space st constr

n dim space as embedded in high dimension

Euclidean space

DoF: (sum of freedoms - independent constraints)

m dim $\quad m \geq n \quad m-n$ ind constr



Assumption

1 Construction of linear angular velocities for each link n-dof

2 link 1 link 2 link 3

+

constraint for each link

$\boxed{\text{Equation of Motion}}$

Newton-Euler

Recursive - newton - euler

1 hard to do on paper for large systems

Contact selection

2 combinatorics problem exists

Controls   Planning   Model of the system

Euler-Lagrange dynamics

* Energy based

1. Evaluation of system energy expressed in generalized coordinates

KE   PE

V U

$$\boxed{\mathcal{L} = V - U}$$

scalar

2. Construct a PDE around the Lagrangian

3. Get system dynamics

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) + F(\dot{q}) = \tau + J^T f_{\text{tip/intera}}$$

$$+ A$$

Spada sir's papers

Vectorizing dynamics

$M(q)$ = Mass Matrix   Symmetric Positive Definite

$C(q,\dot{q})$ = Coriolis Centripetal vector

(q)

$G(q)$ = (G)PE vector

$F(q)$ = friction/damping

τ = joint torque vector

$J^T f$ = interaction joint torques

guaranteeing stability

Single Stance Phase

$\boxed{\text{LQR}}$

$$\boxed{\dot{M} - 2C = 0}$$

① BIBO Stability

② Lyapunov Stability (SL)

③ UUB

④ Nagumo principle

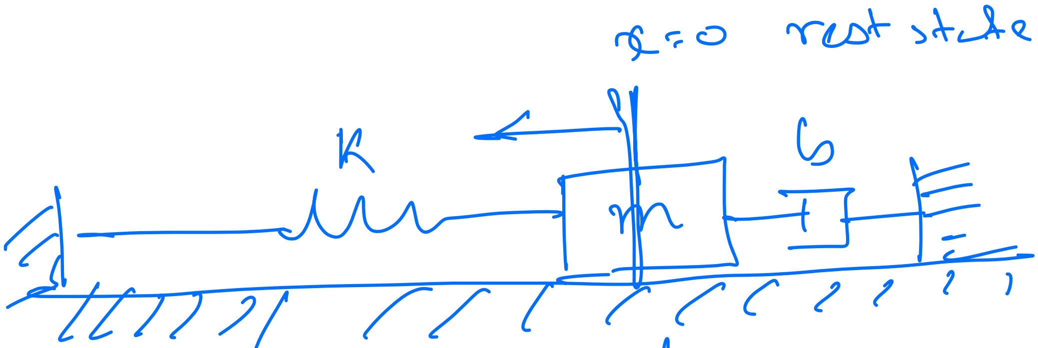
⑤ Barrier function

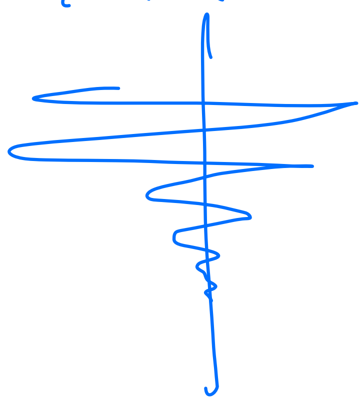
(opt → stability)

Aaron Ames

Jessy Grizzle

$$\boxed{m\ddot{x} + b\dot{x} + Kx = 0}$$





rest

Dynamic Stability = Set point tracking

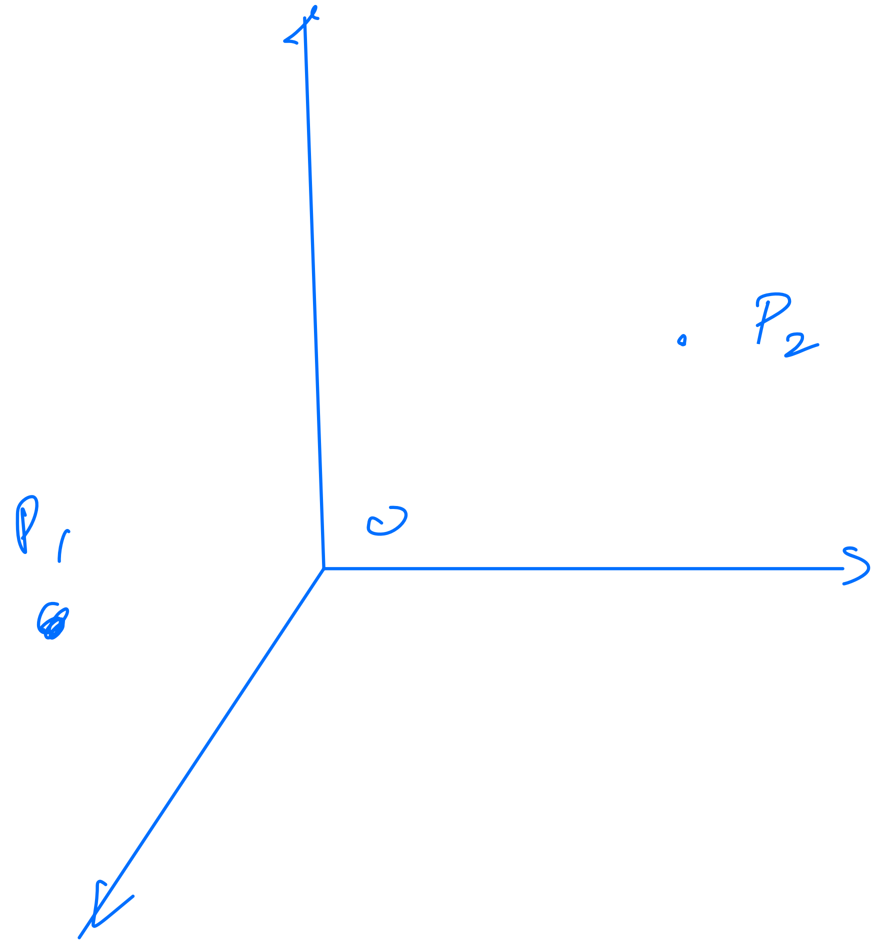
Static Stability

Regulation

State space of the System

$$P = \begin{bmatrix} x \\ y \end{bmatrix}$$

Pose



$$P_1 \rightarrow P_2 = \text{Task}$$

<u>Task space</u>