

Counting Problems

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1) word: unusal

of letters = 7

3 occurrences of u
1 occurrence of n
1 occurrence of s
1 occurrence of a
1 occurrence of l

$${}^4C_4 \quad 1u \rightarrow 1$$

$${}^4C_3 \quad 2u \rightarrow 4$$

$${}^4C_2 \quad 3u \rightarrow 6$$

unique subsets of 5 letters

1 u
u n s a l

$4C_4$

1 way

2 u

u u s a l
u u s n l
u u s n a
u u l n a

$4C_3$

4 ways

3 u

u u u s n
u u u s a
u u u s l
u u u n a
u u u n l
u u u a l

$4C_2$

6 ways



of different strings (unique)
5 letters

set

u u
u
n s
a l

120

+

240

+

for strings, order matters

120

$$1 \cdot \frac{5!}{1!}$$

$$+ 4 \cdot \frac{5!}{2!}$$

$$+ 6 \cdot \frac{5!}{3!} = 480$$

1 u
u n s a l

2 u
u u s a l
u u s n l
u u a l
u u s n a

3 u
u u u s n
u u u s a
u u u s l
u u u n a
u u u n l
u u u a l

of strings =

2) 52 card deck
4 suites
13 cards per suite

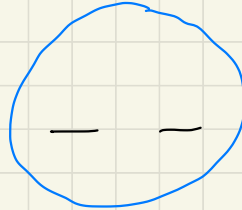
pair = cards have same rank

how many ways to form 5 card hand with 2 pairs

visual



pair



pair



other value

for pairs what matters is value

A 1 2 3 4 5 6 7 8 9 10 11 12

from this we draw a card for each pair

↳ $13 C_2$ ways to do this = 78

?
— — —

3 spots to fill

for the suits we need to do $4 C_2 \cdot 4 C_2 = 36$

and finally for the last card we have to choose from remaining values which are $13-2=11$

$11 C_1 = 11$ · suit $\rightarrow 4 C_1 = 4$

$$78 \cdot 36 \cdot 44 = 123552$$

3) Violinist 16 songs in an hour
7 couples

1 couple having right
will only listen to 1 song MAX

6 couples are okay

how many ways can songs be distributed amongst couples?

songs!

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

Stars & bars combinations

— | xxx xxx xxx xxx xxx x

7 couples of which

16 stars

1st couple can have at MOST 1

Case 1: 1st couple no songs

$$6 - 1 + 16 = 21$$

16

$$\binom{21}{16}$$

Case 2: 1st couple 1 song so 15 songs spread across the 6 couples

$$5 - 1 + 16 = 20$$

$$\binom{20}{15}$$

$$\text{add } C1 + C2 = \binom{21}{16} + \binom{20}{15} = 35853$$

4) Binary Search Trees

Distinct Values 1 - 12

1 2 3 4 5 6 7 8 9 10 11 12

how many ways to build tree of 2 nodes?

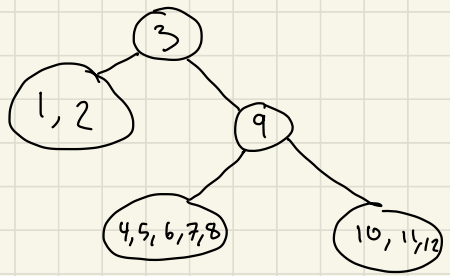
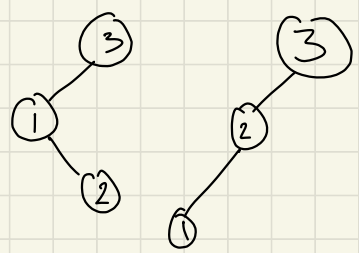


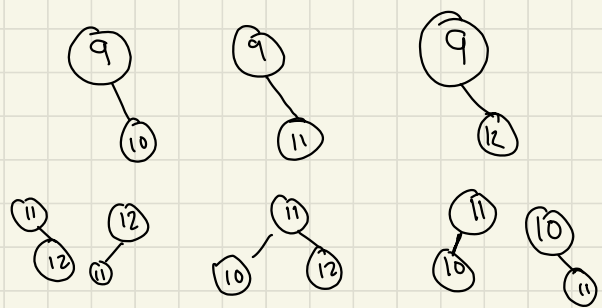
figure out ways to do
 2 children
 5 children
 3 children

for 2 children



2 ways

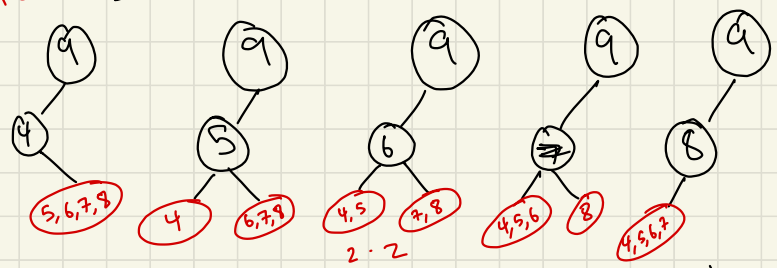
for 3 children



2 + 1 + 2

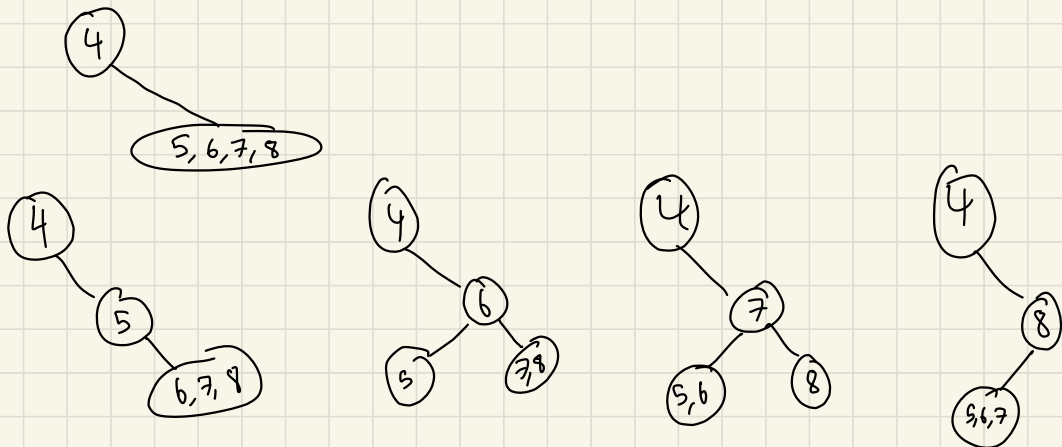
5 ways

for 5 children



$$14 + 5 + 4 + 5 + 14 = 42$$

for 4 children

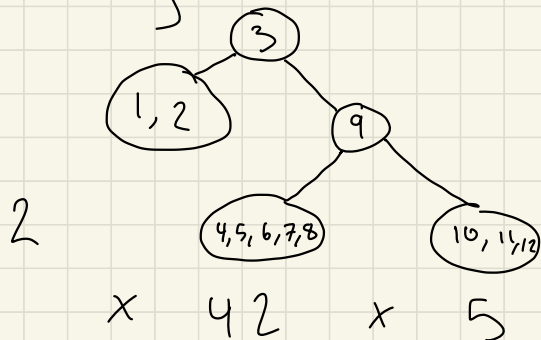


5 ways + 2 ways + 2 ways + 5 ways

14 ways to
organize 4 children

Therefore for 5 children there are
42 ways to organize.

Looking back at original tree:



= 420
ways to
organize

5) COVID vaccine

10 friends same time slot

4 identical nurses, of which 1 may/may not
time of administration varies
guaranteed to serve 1 patient
be scheduled for break

I imagine adding both situations where 4th nurse
break/no break

nurse

nurse

nurse

nurse

[4 Nurses]

[7, 1, 1, 1]

[6, 2, 1, 1]

[5, 3, 1, 1]

[4, 4, 1, 1]

[5, 2, 2, 1]

[4, 2, 2, 2]

[3, 3, 3, 1]

[3, 3, 2, 2]

[3 Nurses]

[8, 1, 1]

[7, 2, 1]

[6, 2, 2]

[6, 3, 1]

[5, 4, 1]

[5, 3, 2]

[4, 4, 2]

[4, 3, 3]

9 + 8

= 17 ways