Probability Problems

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1) 15 students, randomly select students to answer question

8 questions during lecture

$\frac{15}{15}$ $\frac{14}{15}$ $\frac{13}{15}$ $\frac{12}{15}$ $\frac{11}{15}$ $\frac{10}{15}$ $\frac{9}{15}$ $\frac{8}{15}$

8 questions

probability that 1st person not asked a question

probability that 8th person not asked a question

same as $\frac{{}_{15}P_8}{15^8} = .101$

2) range of integers 00000 - 99999
so 100 000 integers

only want even integers, must have 2 odd digits
all digits must be unique

generate 8 of these numbers in succession, what is probability that exactly 5 meet criteria

$$\frac{5}{\text{odds}} \times \frac{4}{\text{odd}} \times \frac{7}{-1 \text{ be unique}} \times \frac{6}{} \times \frac{5}{\text{even}}$$

odds: 1, 3, 5, 7, 9

odd: can't be same

even: 2, 4, 6, 8, 0

all digits unique so 10 - 3 = 7 digits

probability integer meets criteria

$$= \frac{5 \times 4 \times 7 \times 6 \times 5}{100000} = \frac{4200}{100,000}$$

for exactly 5 integers $= \left(\frac{4.2}{100}\right)^5 \left(1 - \frac{4.2}{100}\right)^3 \cdot {}_8C_5$

3) 3 fair dice

6 sides

4 or above → 4, 5, 6

3/6

Event A → 2 dice show 4 or above

$P(A) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\cdot\frac{1}{2}\cdot {}_3C_2 + \left(\frac{1}{2}\right)^3 = .5$

2 case → 2 dice sho 4,5,6 or → dice sho 4,5,6

Event B → all three dice show same value

roll 1 → $\frac{6}{6}$ ← can be anything

roll 2 → $\frac{1}{6}$

roll 3 → $\frac{1}{6}$

$P(B) = \frac{6}{6}\times\frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$

$P(A)\cdot P(B) = \frac{1}{2}\cdot\frac{1}{36} = \frac{1}{72}$

$P(A \cap B)$

probability that all 3 dice are greater than 4 and have same value

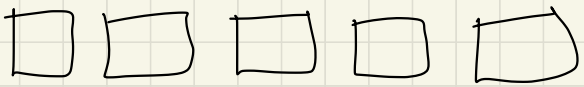
4 4 4 5 5 5 6 6 6

$\frac{3}{216} = \frac{1}{72}$

Since $P(A)\cdot P(B) = P(A \cap B)$

These events are independent

4) Probability of getting a flush



out of 4 suits, choose 1, then choose 5 values

$$\frac{{}_4C_1 \cdot {}_{13}C_5}{{}_{52}C_5} = 0.00198$$

how many hands to get one?

$$E(x) = \frac{1}{0.00198} = 505.05 \text{ hands}$$

505 hands

5)

Superstar plays is .7 win

Superstar doesn't play .5 win

$P(W \mid S) = .7$

$P(W \mid NS) = .5$

$P(S1) = .75$ plays next 5 games

$P(S2) = .25$

team won $\frac{4}{5}$ games

$$P(S \mid w4) = \frac{P(w4 \mid S)\, P(S)}{P(w4)}$$

↓ what we want to know

S1 = superstar plays

S2 = superstar doesn't play

Baye's Thm

$$P(S \mid w4) = \frac{P(w4 \mid S)\, p(S)}{p(w4 \mid S1) \cdot p(S1) + p(w4 \mid S2) \cdot p(S2)}$$

binomial theorem

$$= \frac{((0.7)^4 (0.3) \cdot {}_5C_4) \cdot 0.75}{((0.7)^4 (0.3) \cdot {}_5C_4) \cdot 0.75 + ((0.7)^4 (0.3) \cdot {}_5C_4) \cdot .25}$$

$$= .874$$