Comparison and characterization of Bragg Curve equation utilizing Monte-Carlo simulation (Geant4) and numerical approximation (Python Script) vs Beer and Lambert equation.

Abstract

This effort is intended to understand the Bragg Curve as an avenue to implement Proton Beam Therapy (PBT) to deliver a high dose of radiation to a much-localized cancer tissue after penetrating through the normal tissues in a human body. An alternative approach has been to use photon (X-ray) radiation therapy. Firstly, a Monte Carlo experiment is constructed using Geant4 simulating a Proton gun traveling through a water tank. The energy deposit of the Proton particle with kinetic energy level from 25 MeV to 75 MeV is evaluated and they are plotted versus its penetration distance in figure (3). Secondly, the Bragg Curve [Ref 1] is implemented using Python script and the collision stopping power is calculated and is plotted versus the penetrating distance in figure (2). Thirdly, the Beer-Lambert equation [Ref 2] is plotted using the MATLAB for the same penetration range. This study demonstrates that the Proton Beam Therapy, detailed in the Bragg Curve equation, is in indeed an effective method to treat cancer that has not spread (not metastatic) and the tumors are in or near critical areas such as the brain, heart, and lungs. In conclusion protons, unlike X-rays, decelerate faster than photons and deposit more energy right before they slowdown in figure (3). These phenomena, characterized in the interaction heavy charged particles with matter, makes PBT a great candidate for radiation cancer treatment.

Introduction

A high dose of photon's radiation energy is deposited to the healthy normal tissues when treating tumors utilizing X-Ray physics. This is true since some portion of the radiation energy is absorbed by the nearby organs as the radiation energy is continuously absorbed while the photons travel through the normal tissue to reach the cancer tissue.

The cancer treatment based on X-rays is fundamentally implement by shooting radiation from multiple direction to reduce the impact on the surrounding normal tissues while focusing the deposited energy to the tumors only. The undesired impact is caused by a low amount of radiation; however, it is widely distributed throughout the surrounding normal tissues.

The side effects are significantly reduced in proton therapy. This is true since the radiation instantly disappears after the "Bragg Peak". Consequently, the normal tissues surrounding the targeted cancer tissues are not expose to radiation.

Bragg Curve Governing Equations

The brag Curve characterizes the interaction (energy loss) of heavy charged particles (ionizing radiation) during its travel through matter. The peak occurs immediately before the particles come to rest for the protons, alpha-rays, and other ion rays. When heavy charged particles travel through matter with high kinetic energy, they ionize atoms of the target material and deposited a dose along the path. The cross section increases as the charged particle kinetic energy decreases. The stopping energy is inversely proportional to the square of their velocity. The formula for the mass collision stopping power for heavy charged particles is equation (1).

$$(dT / \rho\, dx) = (0.3071)\, Zz^2 / (A\, \beta^2))\, [13.8373 + \ln (\beta^2 / (1 - \beta^2)) - \beta^2 - \mathrm{Ln}\, (I)] \qquad (1)$$

Equation (1) represents several important features. The target medium has an impact on the collision stopping power. For example, the collision stopping power decrease as the Z is increased. The term Ln (I) in the bracket decreases the stopping power as Z is increased. The collision stopping power has a strong dependency on the particle kinetic energy. The strong dependence comes from the inverse of β^2 term. The collision stopping power is dependent on the particle charge. The factor Z^2 the double charged particle of any given kinetic energy has four times the collision stopping power as a single charged particle. The collision stopping power has no dependence on the incoming particle mass [Ref 1]. Equation (1) is implemented in Python script and plotted in figure (1).

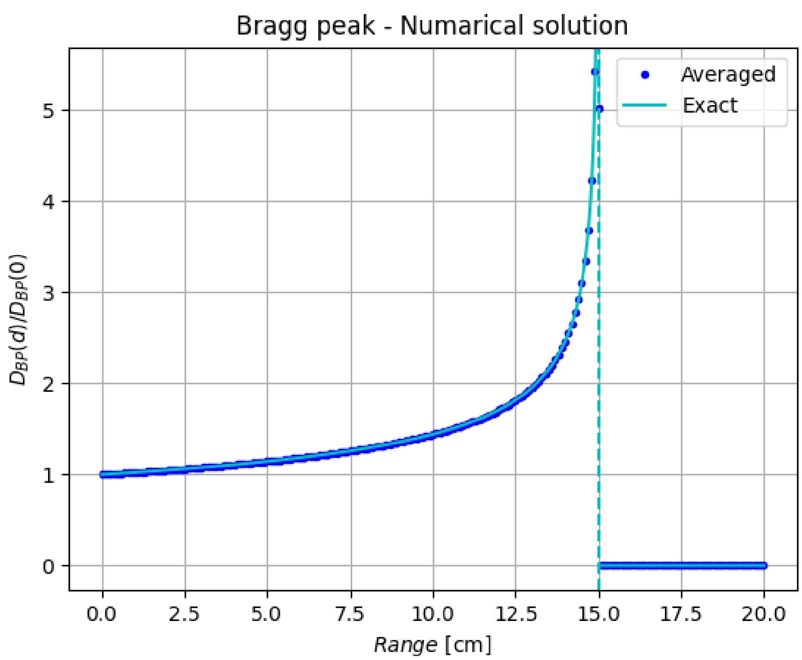

Figure 1: Bragg Curve's numerical solution (Python script)

Beer-Lambert Governing Equation

The X-ray radiation therapy is governed by the Beer and Lambert equation. The amount of the X-ray photons passing through human tissue (water in our case study) drops off exponentially. Photon penetration in matter is stochastic in nature and it is governed by probability per unit distance traveled while photon interacts by one of many physical processes. The dominant physical process are Bremsstrahlung, Photoelectric Effect, Compton Effect and Pair Production. The linear attenuation coefficient is denoted by μ and has the unit of inverse length [1/cm]. Monenergistic photons are attenuated exponentially equation (2).

$$\text{Intensity (out) / Intensity (in)} = \exp(-\mu\, x) \qquad (2)$$

The energy of the X-ray for the beam of photon must be in range of 2 [MeV] to penetrate near five centimeters in water. The linear attenuation and energy-absorption coefficients for this energy range assumes the slab is thin compared with the mean free paths of the incident and secondary photons, so the multiple scattering of photons is negligible and bremsstrahlung photons escape the slab. The energy deposition is caused by the secondary electrons produced by photons [Ref 2].

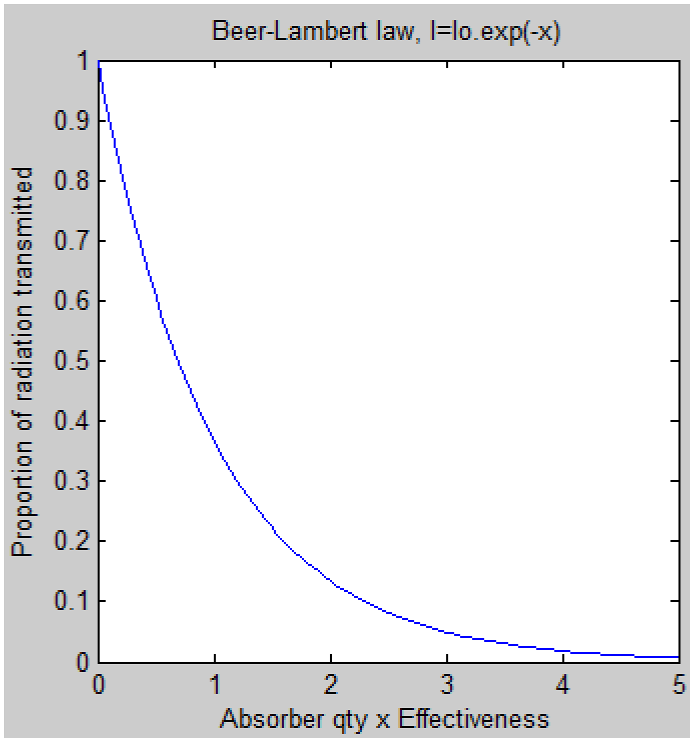

Figure 2: Beer-Lambert numerical solution (MATLAB)

As figure (2) presents the normalized energy deposited in water due to equation (2). As the plot presents the energy despite is much higher near the normal tissue of skin for x in range of 0 to 0.5 [cm]. Furthermore, the energy deposited does not drop off like the Proton therapy and it continues after the cancer tissue.

Monte-Carlo Simulation

In this case study, the following few items are configured. The world material is a box of air. The human body is assumed to be a tank (box) of water. The volume is constructed using G4VSolids, G4LogicalVolume, and G4PVPlacement volumes. The particle source is proton beam implemented by G4ParticleDefinition. The physics list is consistent of G4EmStandardsPhysics, G4DecayPhysics, G4EmExtraPhysics, G4HadronElasticPhysics, and G4HardonPhysicsFTFP_BERT. The production and cuts are ignored. The user actions and sensitive detectors are G4MultiFunctinalDetector, G4VPrimitiveScorer, and SetSensetiveDector for native scoring [Ref 3]. As figure (3) presents the Bragg Peaks for a proton pencil beam are plotted for different kinetic energy. The cross section of deposited energy is increased right before the all the energy is deposited. The penetration distance in [cm] is a function of kinetic energy.

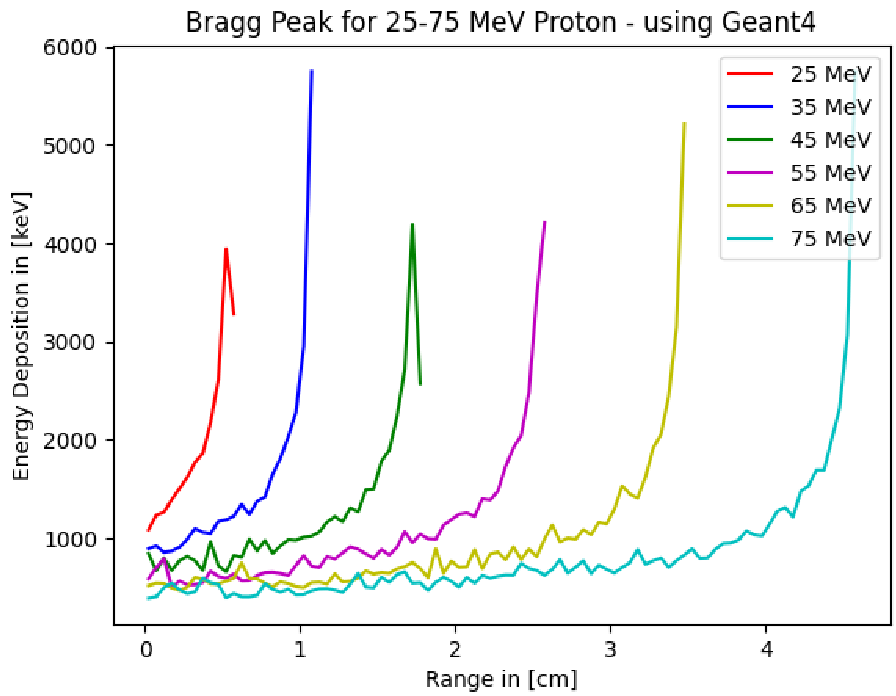

Figure 3: Break Peak (Geant4)

Conclusion

In conclusion, the physics of protons is advantages to X-ray in terms of side effect for treatment of cancer tissue. The Interaction of heavy charged particles with matter, makes PBT a better candidate for radiation cancer treatment. This is true since protons decelerate much faster than photons and deposit more energy right before they slow down. Consequently, the healthy tissue, surrounding the cancer tumor, is exposed to less radiation. Furthermore, the Physics of X-ray follows an exponential model so most of the energy despite occurs for penetration distance of 0 to 0.5 [cm]. X-ray technology is not the best candidate when the cancer tumor is deeper into the human body.

Reference

[Ref 1] Robley D. Evans, Ph. D. (1955). The Atomic Nucleus. McGraw-Hill, Inc (page 665)

[Ref 2] John R. Lamarsh and Antohony J. Baratta. Introduction to Nuclear Engineering. Published: Prentice Hall, hird edition (page 102)

[Ref 3] https://www.geant4.org/docs/