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Part (a): Poisson's ratio

$$\sigma_{ij} = \lambda\,\delta_{ij}\,\varepsilon_{kk} + 2\mu\,\varepsilon_{ij},$$

where λ and μ are the Lamé parameters, and $\varepsilon_{kk} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$.
Consider a uniaxial stress state,

$$\sigma_{xx} = \sigma, \qquad \sigma_{yy} = \sigma_{zz} = 0.$$

Due to symmetry of loading,

$$\varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_t.$$

Apply Hooke's law to the xx-component:

$$\sigma = \lambda(\varepsilon_{xx} + 2\varepsilon_t) + 2\mu\,\varepsilon_{xx}.$$

Apply Hooke's law to the yy-component where stress is zero:

$$0 = \lambda(\varepsilon_{xx} + 2\varepsilon_t) + 2\mu\,\varepsilon_t.$$

Solve the second equation for $\varepsilon_{zz} = \varepsilon_{yy}$:

$$\lambda\varepsilon_{xx} + 2(\lambda + \mu)\varepsilon_{yy} = 0,$$

or

$$\lambda\varepsilon_{xx} + 2(\lambda + \mu)\varepsilon_{zz} = 0,$$
$$\varepsilon_{zz} = \varepsilon_{yy} = -\frac{\lambda}{2(\lambda + \mu)}\,\varepsilon_{xx}.$$

Poisson's ratio (ν) is defined as the negative ratio of lateral strain to longitudinal strain:

$$\nu = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = -\frac{\varepsilon_{zz}}{\varepsilon_{xx}}.$$

Therefore,

$$\boxed{\nu = \frac{\lambda}{2(\lambda + \mu)}}.$$

Part (b): Young's modulus

$$0 = \lambda(\varepsilon_{xx} + 2\varepsilon_t) + 2\mu\,\varepsilon_t \quad \Rightarrow \quad \lambda\varepsilon_{xx} + (2\lambda + 2\mu)\varepsilon_t = 0,$$

so

$$\varepsilon_t = -\frac{\lambda}{2(\lambda + \mu)}\,\varepsilon_{xx}.$$

The xx component gives

$$\sigma = \lambda(\varepsilon_{xx} + 2\varepsilon_t) + 2\mu\varepsilon_{xx}.$$

Substitute ε_t:

$$\sigma = \lambda\varepsilon_{xx} + 2\lambda\varepsilon_t + 2\mu\varepsilon_{xx} = (\lambda + 2\mu)\varepsilon_{xx} + 2\lambda\left(-\frac{\lambda}{2(\lambda + \mu)}\varepsilon_{xx}\right).$$

Simplify:

$$\sigma = \left[(\lambda + 2\mu) - \frac{\lambda^2}{\lambda + \mu}\right]\varepsilon_{xx} = \frac{(\lambda + 2\mu)(\lambda + \mu) - \lambda^2}{\lambda + \mu}\,\varepsilon_{xx}.$$

$$(\lambda + 2\mu)(\lambda + \mu) - \lambda^2 = \lambda^2 + 3\lambda\mu + 2\mu^2 - \lambda^2 = \mu(3\lambda + 2\mu).$$

Therefore

$$\sigma = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}\,\varepsilon_{xx}.$$

By definition, Young's modulus is $E = \dfrac{\sigma}{\varepsilon_{xx}}$ under uniaxial stress, so

$$\boxed{E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}}.$$