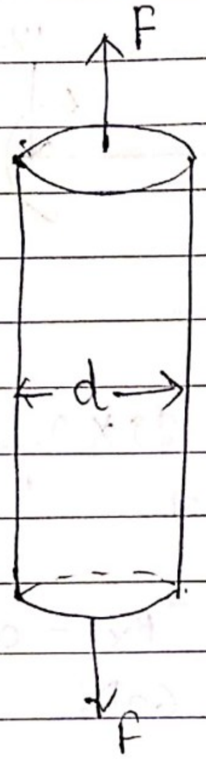
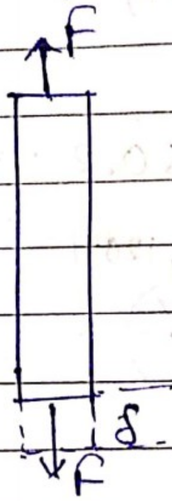
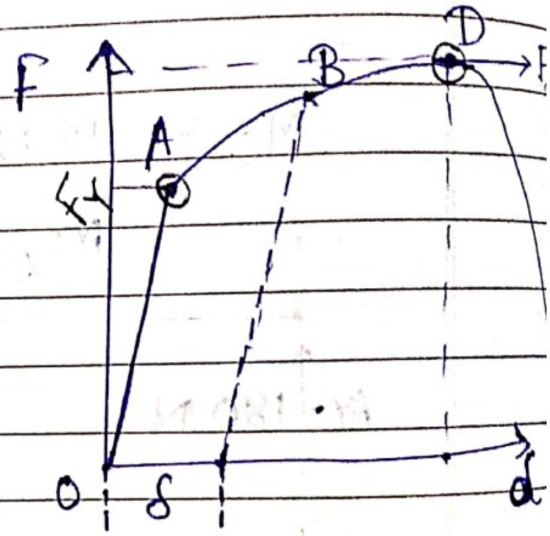
★ Mechanics for Deformable Bodies.

Stress $(\sigma) = \frac{F}{A}$; $A = \frac{\pi d^2}{4}$



A ⇒ perpendicular to F.



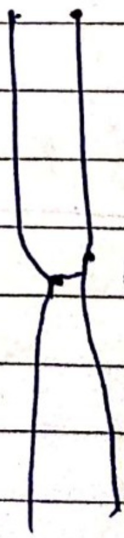


OA → elastic deformation

AB → plastic deformation.

δ → permanent set.

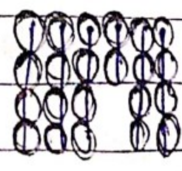
{ force proportional to deformation.
if force is removed, body regains
its size and shape. }



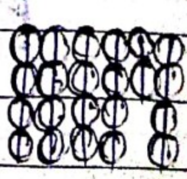
Necking.

★ { Dislocation (Atomic defect) causes plastic deformation.

⇓



On Application of force ⇒

 }

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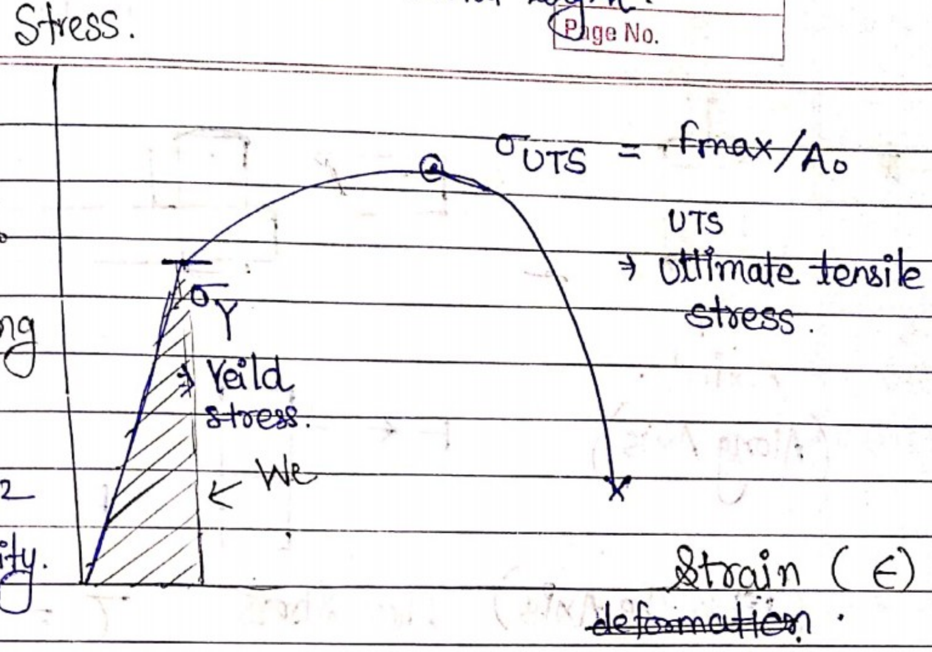
$A_0 \Rightarrow$ Initial Area

$L_0 \Rightarrow$ Initial Length.



$\sigma = \frac{F}{A_0}$

Engineering Stress.

unit → N/m^2

vector quantity.

$\therefore \sigma_{all} = \dfrac{\sigma_{UTS}}{\text{Factor of Safety}} \implies$ eg. for aeroplane, F.O.S $\cong$ 1.2.

$\sigma_{all} \Rightarrow$ allowable stress.

▨ $\Rightarrow$ Reversible Work done (W_e)

(Resilliance of material) $W_e = \frac{1}{2}\sigma_Y \epsilon_Y$. $\qquad \epsilon = \frac{\delta}{L_0}$.

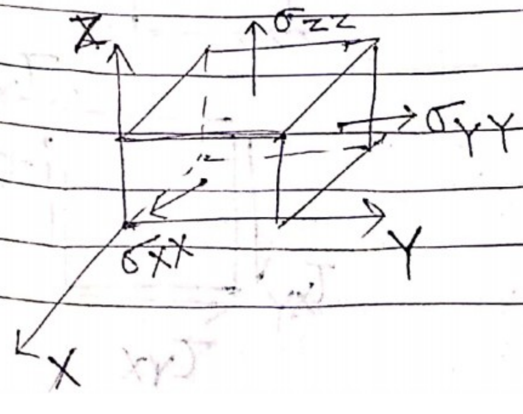
$= \frac{1}{2}\sigma_Y \frac{\sigma_Y}{E}$

$\sigma \propto \epsilon$.

$\sigma = E\epsilon$.

$\epsilon = \frac{\sigma}{E}$

$$\boxed{W_e = \frac{1}{2}\frac{\sigma_Y^2}{E}}$$



AXIAL STRESS.

σ_{XX} σ_{YY} σ_{ZZ}.

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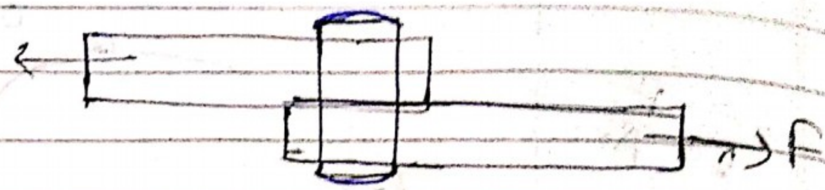
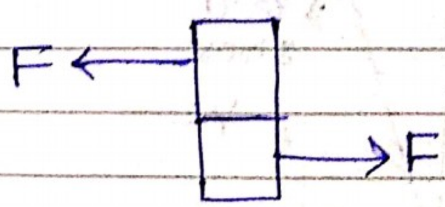
★

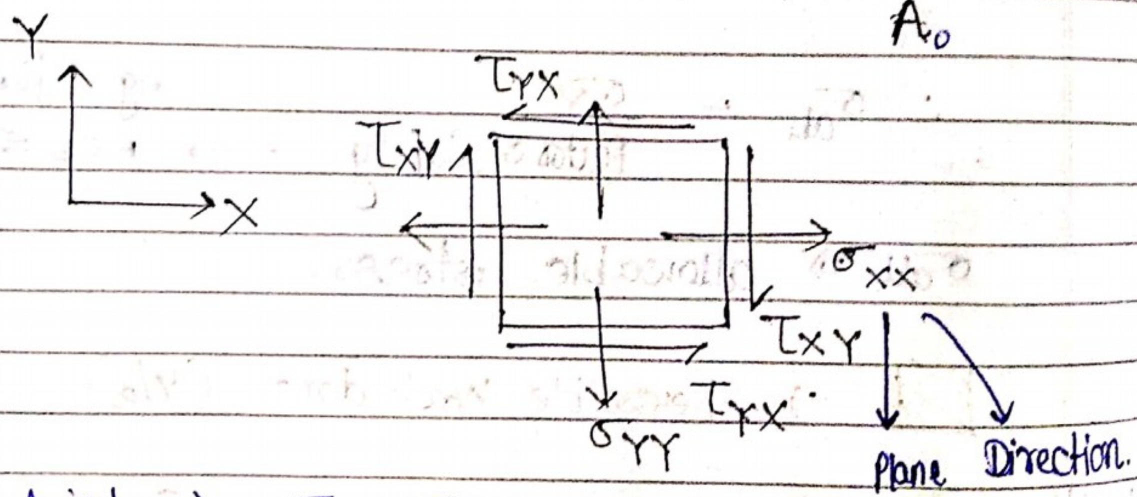


Axial.
(Along Axis)



($\perp^{er}$ to the Axis) . Shear Stress $\tau = \frac{F}{A_0}$

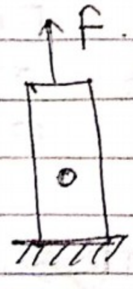
★

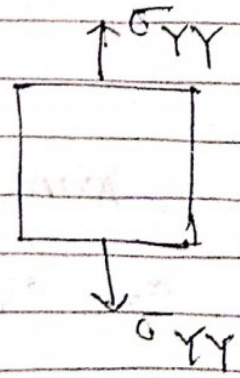


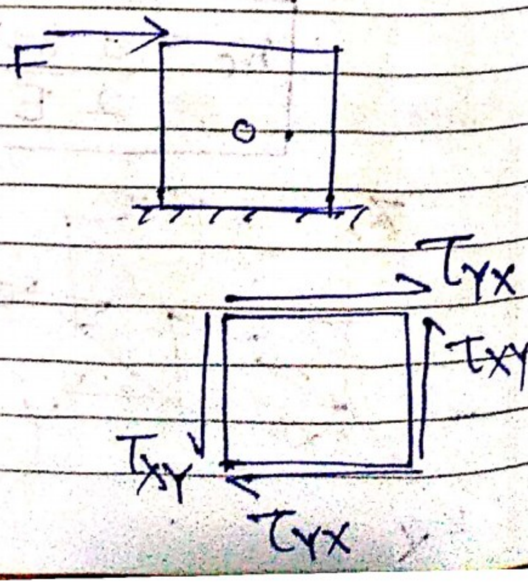
Axial → σ_{xx}, σ_{YY}
Shear → $\tau_{xY} = \tau_{Yx}$.

★ Shear always exist in form of couple.

★







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$\mu \Rightarrow$ Poisson Ratio.

$\sigma \Rightarrow$ Stress

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Date: $\epsilon \Rightarrow$ strain.
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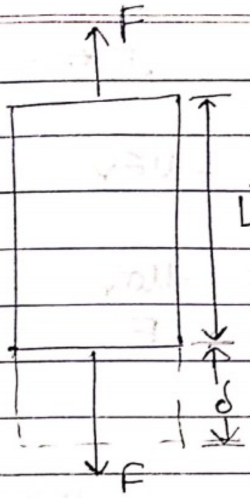
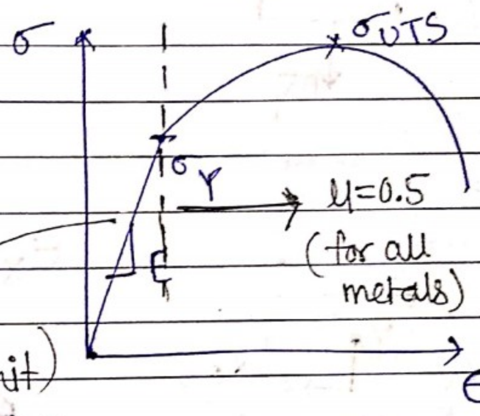
☆ AXIAL $\sigma = \dfrac{F}{A_0}$ MPa.



$\epsilon = \dfrac{\delta}{L}$

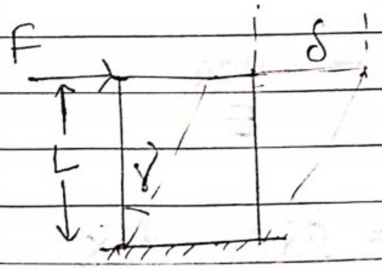
$E \Rightarrow$ Young's Modulus of Elasticity

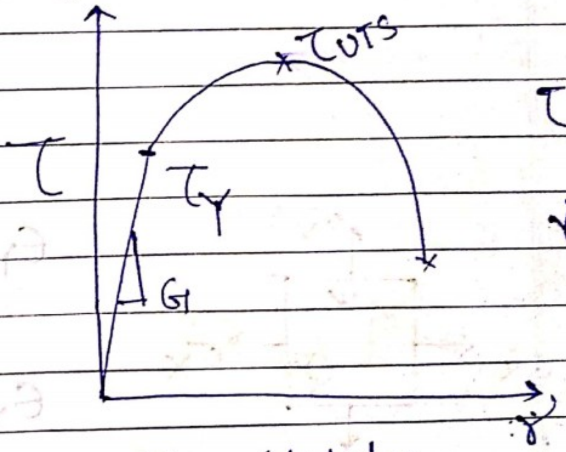


$\mu = 0.3$
(for steel upto elastic limit)

SHEAR

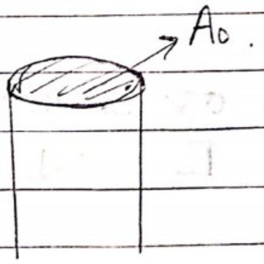
☆





$\tau = \dfrac{F}{A_0}$

$\gamma = \dfrac{\delta}{L}$

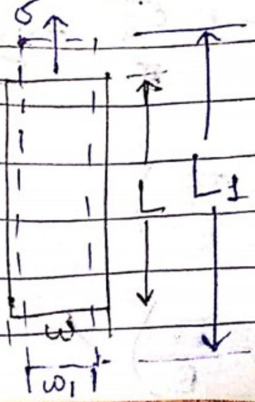


$G \Rightarrow$ Shear Modulus of Rigidity.

$\sigma_Y = 2\tau_Y$; $\sigma_{UTS} = 2\tau_{UTS}$.

$E = 2G(1+\mu)$. $\mu \Rightarrow$ poisson ratio.

Poisson Ratio. ☆



$\Delta L = L_1 - L \oplus$ $\epsilon_L = \dfrac{\Delta L}{L}$

$\Delta w = w_1 - w \ominus$

$\epsilon_w = \dfrac{\Delta w}{w}$

$\therefore \mu = -\dfrac{\epsilon_w}{\epsilon_L} = \dfrac{\text{lateral strain}}{\text{longitudinal strain}}$.

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All formulae derived will be applicabled upto Yeild point (Elastic limit).

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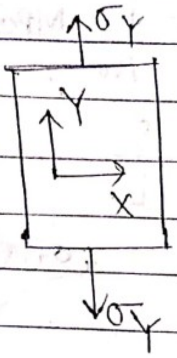
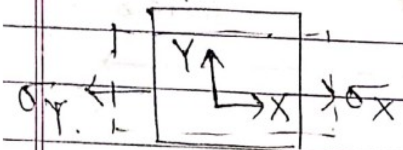


$$\epsilon_Y = \frac{\sigma_Y}{E} \quad ; \quad \mu = -\frac{\epsilon_X}{\epsilon_Y}$$

$$\epsilon_X = -\mu\epsilon_Y$$

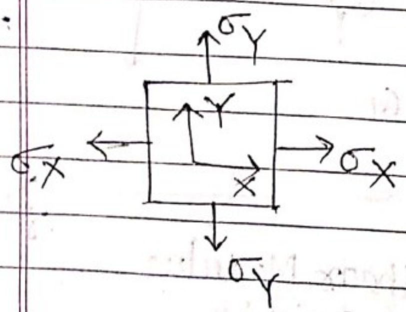
$$\epsilon_X = \frac{-\mu\sigma_Y}{E}$$



$$\epsilon_X = \frac{\sigma_X}{E} \quad ; \quad \mu = \frac{-\epsilon_Y}{\epsilon_X}$$

$$\epsilon_Y = -\mu\epsilon_X$$

$$= \frac{-\mu\sigma_X}{E}$$



$$\epsilon_Y = \frac{\sigma_Y}{E} - \frac{\mu\sigma_X}{E}$$

$$\epsilon_X = \frac{\sigma_X}{E} - \frac{\mu\sigma_Y}{E}$$

3D: $\sigma_x, \sigma_Y, \sigma_Z$

$$\epsilon_Y = \frac{\sigma_Y}{E} - \mu\frac{\sigma_X}{E} - \mu\frac{\sigma_Z}{E} = \frac{\sigma_Y}{E} - \mu\left(\frac{\sigma_X}{E} + \frac{\sigma_Z}{E}\right)$$

$$\epsilon_X = \frac{\sigma_X}{E} - \mu\left(\frac{\sigma_Y}{E} + \frac{\sigma_Z}{E}\right)$$

$$\epsilon_Z = \frac{\sigma_Z}{E} - \mu\left(\frac{\sigma_X}{E} + \frac{\sigma_Y}{E}\right)$$

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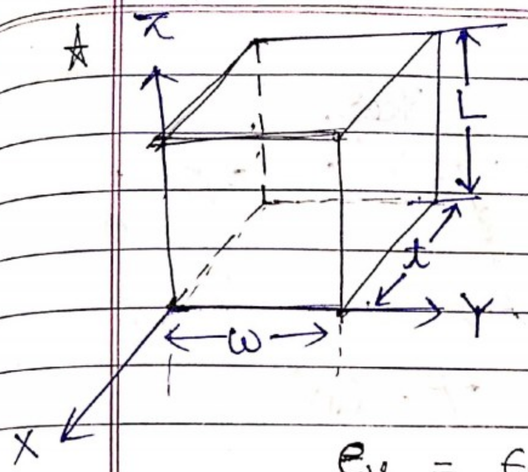
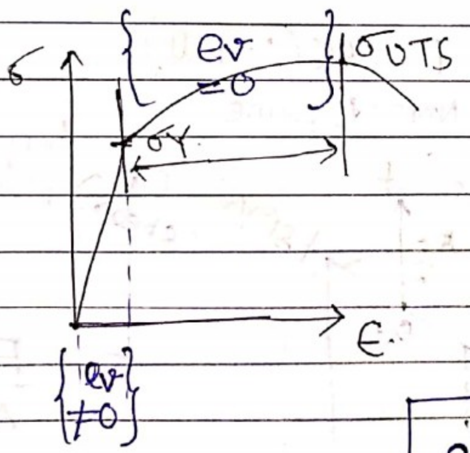


$$V = Lwt$$

$$dV = Lw\,dt + Lt\,dw + tw\,dL$$

$$\frac{dV}{V} = \frac{dt}{t} + \frac{dw}{w} + \frac{dL}{L}$$

$$e_v = \epsilon_x + \epsilon_y + \epsilon_z$$



$$e_v = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E} - 2\mu\left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)$$

$$\boxed{e_v = (\sigma_x + \sigma_y + \sigma_z)\frac{(1-2\mu)}{E}}$$

$e_v = 0$; for plastic part
$e_v \neq 0$; for elastic part.

$$\sigma_x + \sigma_y + \sigma_z = -3P \Rightarrow P = -\frac{(\sigma_x + \sigma_y + \sigma_z)}{3}$$

$$\boxed{e_v = \left(\frac{1-2\mu}{E}\right)(-3P)}$$

Axial stress ⇒ Can change volume & also plastic deformation.

$$E = \frac{\sigma}{\epsilon} \; ; \quad K = \frac{-P}{e_v}$$

K ⇒ Bulk modulus ⇒ $\boxed{E = 3K(1-2\mu)}$

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$E = 2G(1+\mu)$

$E = 3K(1-2\mu)$

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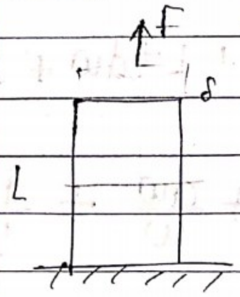
☆ <u>Recall</u>

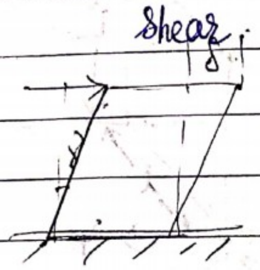
Axial



$\sigma = \frac{F}{A}$

$\epsilon = \frac{\delta}{L}$

Shear



$\tau = \frac{F}{A}$

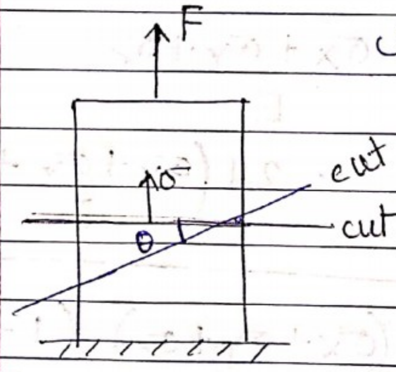
$\gamma = \frac{\delta}{L}$.

$\tau = G\gamma$

$\sigma = E\epsilon$.

$E = 2G(1+\mu)$; $E = 3K(1-2\mu)$.

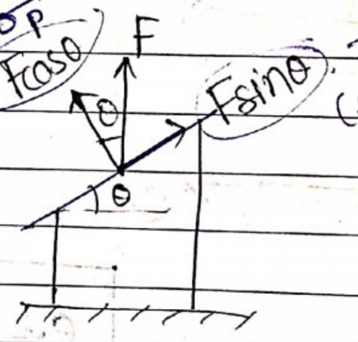
*



(Axial) σ_P Normal to plane.

$F\cos\theta$

$F\sin\theta$ τ_P (shear) Along the plane.



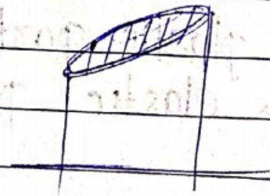
$\sigma_P = \frac{F\cos\theta}{A_\theta}$

$\sigma = \frac{F}{A}$

A_θ.

$\boxed{\sigma_P = \frac{F}{A}\cos^2\theta}$

$A = \frac{\pi d^2}{4}$



$\tau_P = \frac{F\sin\theta}{A_\theta}$

$A_\theta \cos\theta = A$

$A_\theta = \frac{A}{\cos\theta}$.

$\boxed{\tau_P = \frac{F}{A}\sin\theta\cos\theta}$

$\boxed{\sigma_P = \sigma\cos^2\theta}$; $\boxed{\tau_P = \sigma\sin\theta\cos\theta}$

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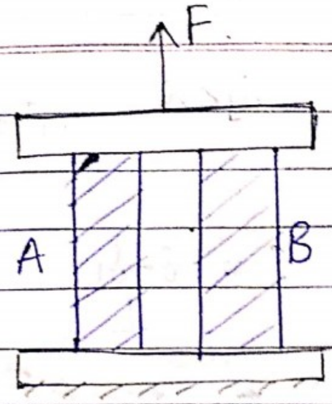
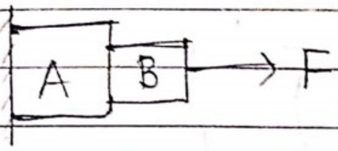


$\sigma, \epsilon \,/\, F, \delta.$

$\delta_A = \delta_B.$

$F = F_A + F_B.$

§.

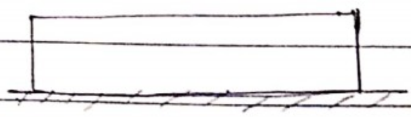


$F_A = F_B \;;\; \delta = \delta_A + \delta_B.$

☆ Initial temp. ⇒ T_1, α ← Coeffi. of thermal expansion

Final temp ⇒ T_2.

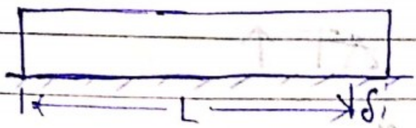
$\Delta T = T_2 - T_1$.



$\epsilon = \alpha \Delta t.$ (Thermal strain).

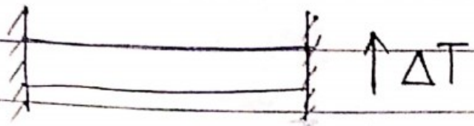
$\delta = \epsilon L$

$\boxed{\delta = L \alpha \Delta T}$

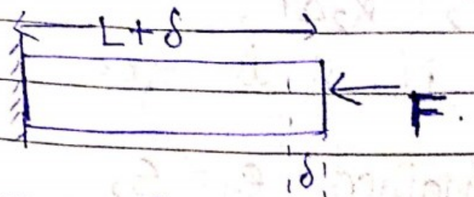


$\boxed{\sigma = ? = 0}$ As $F_{ext} = 0$.

☆.



$\boxed{\epsilon = 0}$



$\boxed{\sigma = E \alpha \Delta T}$

$\boxed{F = \sigma A}$

Here, to calculate the reaction we assume there is no fixed at one end and compare the change in length relating it to find reaction force.

$\epsilon = \frac{\delta}{L}$

$= \frac{\alpha \Delta T L}{L}$

$\epsilon = \alpha \Delta T.$

$\sigma = E\epsilon = E \alpha \Delta T$

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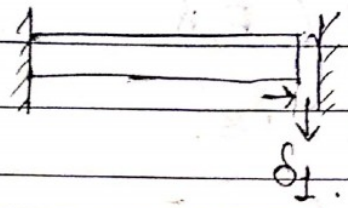
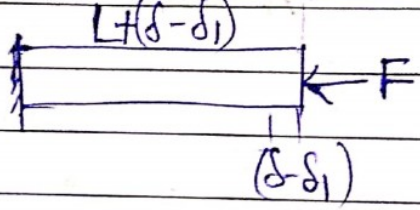


$\uparrow \Delta T$ $\delta_1 < \delta$ $\boxed{\delta = L\alpha\Delta T}$

$$\epsilon = \frac{\delta - \delta_1}{L}$$



* $\sigma = \epsilon E$

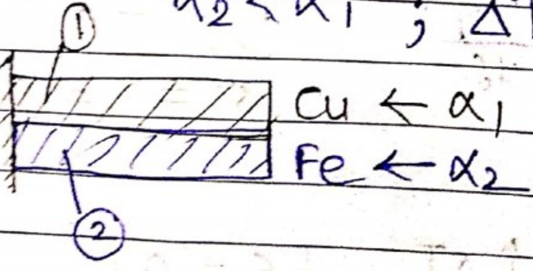
$\rightarrow \frac{F}{A} = \epsilon E$

$$\Rightarrow \boxed{F = EA\left(\frac{L\alpha\Delta T - \delta_1}{L}\right)}$$

If $\delta_1 = 0$; previous case $F = EA\alpha\Delta T$.

* Bimetallic Strip :-

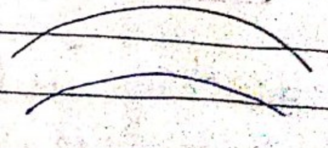
$\alpha_2 < \alpha_1$; $\Delta T \uparrow$.



$\epsilon_1 = \alpha_1 \Delta T$ $\epsilon_2 = \alpha_2 \Delta T$.

($\epsilon_1 > \epsilon_2$) if it is free.

But, we have maintained $\epsilon_1 = \epsilon_2$



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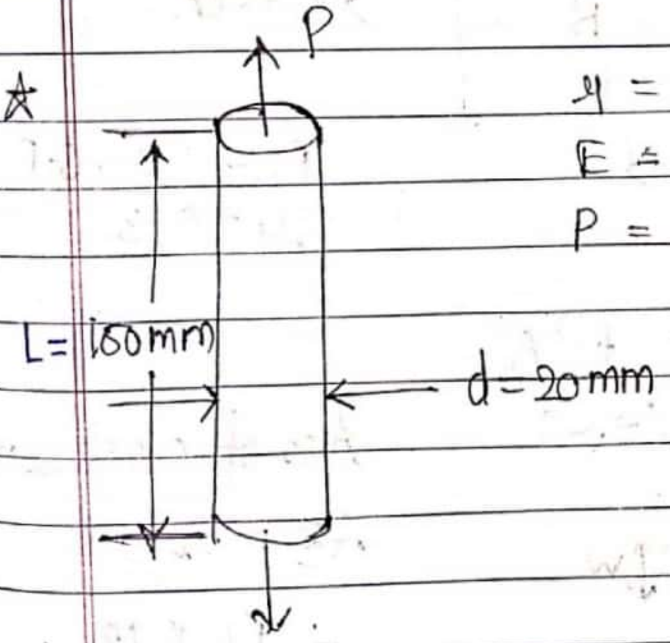


$\mu = 0.35$ $\qquad \delta L = ?$

$E = 70\,GPa$ $\qquad \delta d = ?$

$P = 30\,kN$.

$\sigma = E\epsilon$.

$\epsilon = \frac{\delta L}{L}$ $\qquad \sigma = \frac{P}{A}$

$\delta L = \epsilon L$ $\qquad \epsilon = \left(\frac{P}{A}\right)\left(\frac{1}{E}\right)$

$$\delta L = \left(\frac{P}{A}\right)\left(\frac{1}{E}\right) L$$

$$\Rightarrow \delta L = \left(\frac{30 \times 10^3}{\frac{\pi}{4} \times 400 \times 10^{-6}}\right)\left(\frac{1}{70 \times 10^9}\right)(150)\ mm$$

$$= \frac{120 \times 150}{28 \times 10^3 \times \pi}\ mm$$

$$= \frac{15 \times 12}{280\pi}\ mm = \frac{180}{280\pi}\ mm$$

$$\delta L = \frac{18 \times 7}{28 \times 22} = 0.2045\ mm$$

$$\boxed{\delta L = 0.2045\ mm}$$

$$\Rightarrow \mu = -\frac{\delta d/d}{\delta L/L}$$

$$\Rightarrow \delta d = -\frac{\mu\, \delta L (d)}{L}$$

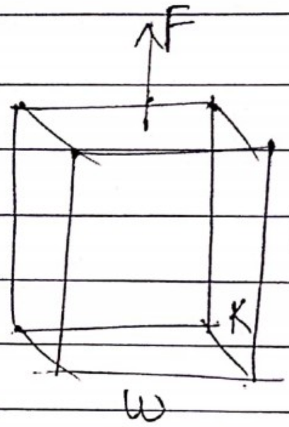
$$= -(0.35)(0.2045)\left(\frac{d}{L}\right)$$

$$\boxed{\delta d = -0.0715\left(\frac{2}{15}\right) mm}$$

contraction.

★ For isotropic material, μ is same always

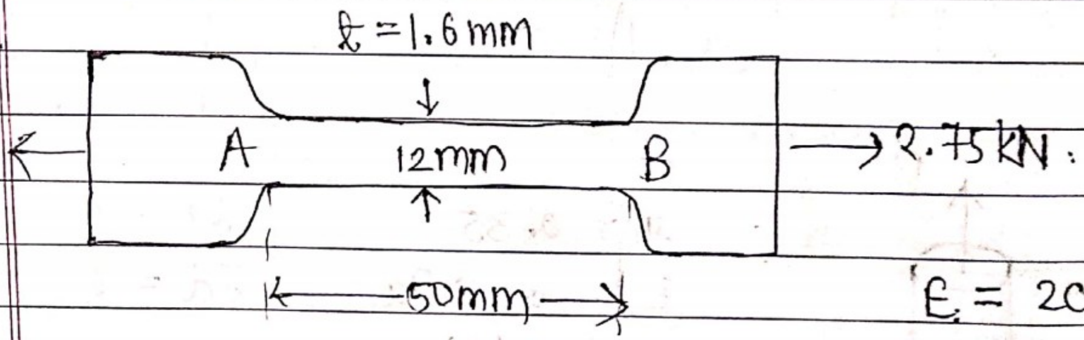
★



$$\mu_K = -\frac{\epsilon_K}{\epsilon_L}$$

$$\mu_W = \frac{-\epsilon_W}{\epsilon_L}.$$

Q.



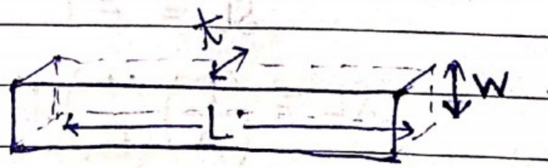
$E = 200\,GPa$

$\mu = 0.3$

Determine..

① Change in L, W, t, A

② Dilatation ($\Delta V / e_v$)



Area of cross section

$A_0 = tw$

$= 1.6 \times 12.$

$$\sigma_x = \frac{F}{A_0} = \frac{2.75 \times 10^3}{1.6 \times 12} = 143.22 \frac{N}{mm^2}$$

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{\Delta L}{L}$$

$$\Delta L = \frac{\sigma_x}{E} L = \frac{143.22 \times 50 \times 10^{-3}}{200 \times 10^9 \times 10^{-6}}$$

$$\Delta L = 35.80 \times 10^{-6}\ m$$

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$$\epsilon_y = \epsilon_z = \frac{-\mu\sigma_x}{E} = \frac{\Delta t}{t} = \frac{\Delta w}{w}.$$

$$\Delta t = \frac{-\mu\sigma_x}{E} t \quad , \quad \Delta w = \frac{-\mu\sigma_x}{E} w.$$

$$\Delta t = \frac{-(0.3)(143.22 \times 10^{6})}{200 \times 10^{9}\ 10^{3}}(1.6 \times 10^{-3})$$

$$= \frac{-(0.3)(143.22)}{200} \times 10^{-6}$$

$$\Delta t = -0.21468 \times 10^{-6}\ m \times 1.6 = -0.3437 \times 10^{-6}\ m$$

$$\Delta w = \frac{-(0.3)(143.22 \times 10^{6})}{200 \times 10^{9}}(12 \times 10^{-3})$$

$$\Delta w = -\frac{(0.3)(143.22)(12)}{200} \times 10^{-6} = -2.57 \times 10^{-6}\ m$$

$$A_0 = t\ w \qquad V = twL$$

$$A + \Delta A = (t + \Delta t)(w + \Delta w). \qquad (V + \Delta V)$$

$$\boxed{\Delta A = ?} \qquad = (t + \Delta t)(w + \Delta w)(L + \Delta L).$$

$$\Delta A = -0.00825\ mm^2 \qquad \boxed{\Delta V = 0.275\ mm^3}$$

(or) $$e_v = \epsilon_x + \epsilon_y + \epsilon_z = \frac{\sigma_x}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_x}{E}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\sigma_x}{E}(1 - 2\mu).$$

$$\Rightarrow \Delta V = \frac{\sigma_x}{E}(1 - 2\mu)V.$$

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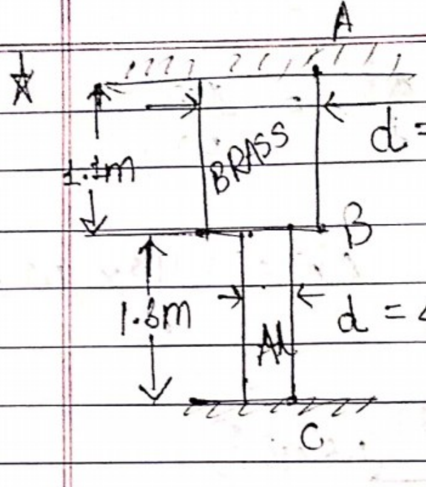
★



$d = 60mm$

$d = 40mm$

$E_b = 105 \, GPa$ — b = brass

$\alpha_b = 20.9 \times 10^{-6} /°C$ — a = aluminium

$E_a = 72 \, GPa$

$\alpha_a = 23.9 \times 10^{-6} /°C$.

$\left\{ \begin{array}{l} \sigma_{AB} = ? \\ \sigma_{BC} = ? \end{array} \right.$ $\quad \Delta T = 42°C \uparrow$

DEFLECTION OF POINT 'B' = ?

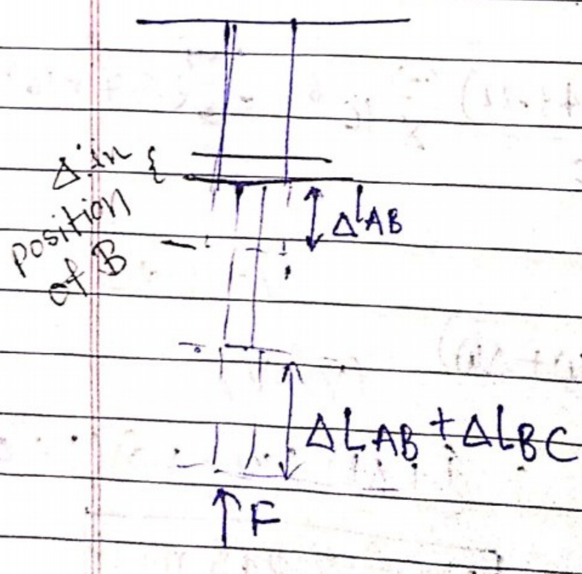
Ans ⇒

$$\Delta L_{AB} = \alpha_b \Delta T L_b$$

$$\Delta L_{BC} = \alpha_a \Delta T L_a$$

$$\delta_T = \Delta T (\alpha_b L_b + \alpha_a L_a)$$

DUE TO 'F'



$$\Delta L_{AB}\Big|_F = \frac{F \cdot L_{AB}}{A_{AB} E_{AB}}$$

$$= \frac{F \times (1.1)}{\frac{\pi}{4} \times (0.06)^2 \times E_b}.$$

$$\Delta L_{BC}\Big|_F = \frac{F \times L_{BC}}{A_{BC} \cdot E_{BC}}$$

$$= \frac{F \times (1.3)}{\pi/4 (0.04)^2 \times E_a}$$

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Page No.

$$\Delta L_{AB}|_T + \Delta L_{BC}|_T = \Delta L_{AB}|_F + \Delta L_{BC}|_F$$

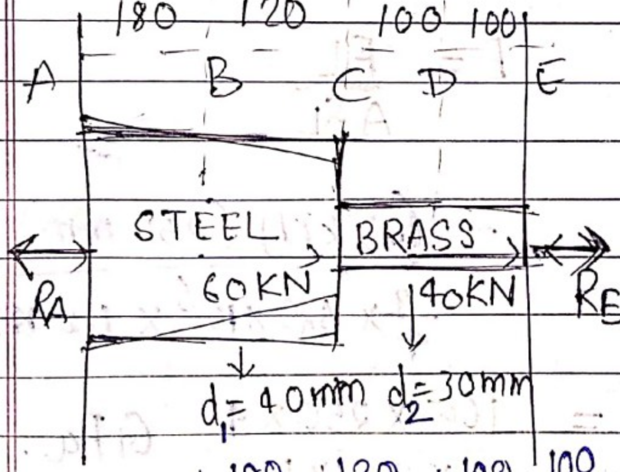
$$\Delta T(\alpha_b L_b + \alpha_a L_a) = F\left(\frac{1.1}{\frac{\pi}{4}\times(0.06)^2\times E_b} + \frac{1.3}{\frac{\pi}{4}(0.04)^2 E_a}\right)$$

$$\Rightarrow \boxed{F = 125.62 \times 10^3\ N.}$$

$\therefore$ Deflection of $B = \Delta L_{AB}|_T + \Delta L_{AB}|_F$

$= 0.5\ mm \downarrow$

★

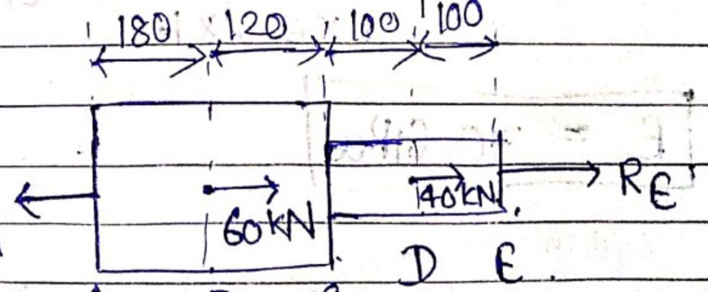


$E_S = 200\ GPa$
$E_B = 105\ GPa$

Deflection of C = ?

Reaction at A, E = ?

$\Rightarrow$

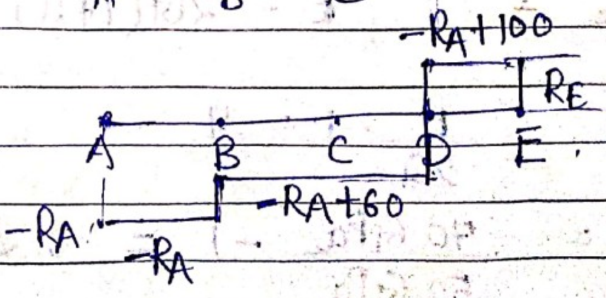


$R_A, R_E = ?$

$\delta_C = ?$

$A_S = \frac{\pi}{4}d_1^2$

$A_B = \frac{\pi}{4}d_2^2$



$\delta_{Total} = 0$

$\delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 0.$

$$\Rightarrow \frac{-R_A L_{AB}}{A_S E_S} + \frac{(-R_A+60)L_{BC}}{A_S E_S} + \frac{(-R_A+60)L_{CD}}{A_B E_B} + (-R_A+100)L_{DE}/A_B E_B = 0. \Rightarrow R_A = ?$$

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Page No.

$R_A =$

$R_E = +R_A - 100.$

$\delta_C = \delta_{AB} + \delta_{BC}.$

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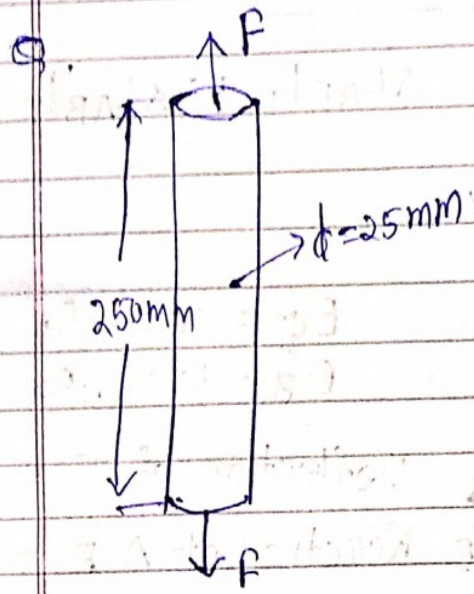

$F = 165\,kN;\ \delta L = 1.2\,mm;$

$E = ?\ ;\ \delta d = ?\ ;\ G = 26\,GPa.$

$\sigma_Y = 440\,MPa$

$$E = \frac{FL}{A\,\delta L}$$

$$= \frac{(165\,kN)(250\,\cancel{mm})}{\frac{\pi}{4} \times 625 \times 10^{-6} \times 1.2\,\cancel{mm}}$$

$$= \frac{165 \times 250 \times 4}{\pi \times 625 \times 1.2}\ GPa.$$

$$\boxed{E \simeq 70\ GPa}$$

$\mu = -\frac{\epsilon_x}{\epsilon_Y}$

$E = 2G(1+\mu)$

$\Rightarrow \frac{E}{2G} - 1 = \mu$

$\Rightarrow \mu = \frac{70\,GPa}{52\,GPa} - 1 = 0.3461$

$\mu = -\frac{\delta d}{\delta L} \rightarrow \delta d = -\mu\,\delta L$

$= -(0.341)(1.2) = -0.41\ mm.$

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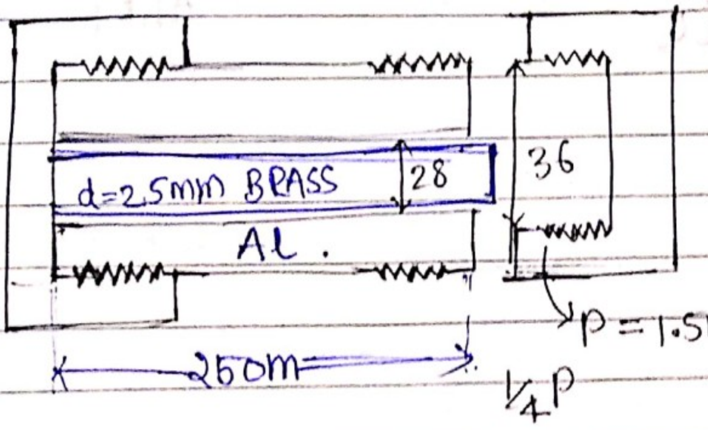
$\left(\sigma = \frac{F}{A} < \sigma_Y \right.$. then only above formulae can be applied $\left. \right)$

★



$\Delta T \uparrow = 40°C$.

$\sigma = ?$

$\sigma_a = ?$

$\sigma_b = ?$

BRASS

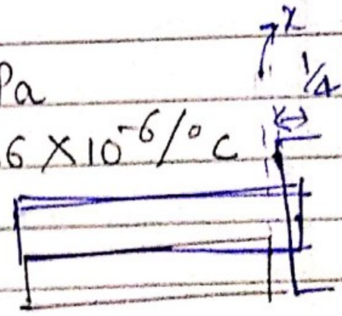
$E_b = 105\ GPa$

$\alpha_b = 20.9 \times 10^{-6}/°C$.

Al

$E_a = 70\ GPa$

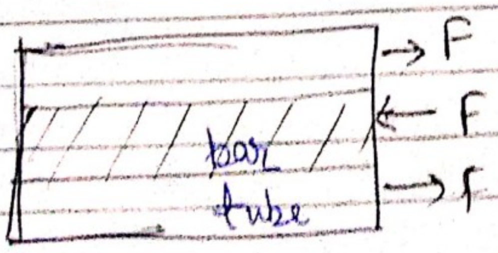
$\alpha_a = 23.6 \times 10^{-6}/°C$



Ans:-

$$\delta^* = \frac{1}{4}P = \frac{1}{4} \times 1.5mm$$

Tube (Al) $\Rightarrow$ $\delta_{tube} = \delta_{BAR} + \delta^*$



$$\delta_{tube} = + \frac{FL}{AE}$$

$$= \frac{F \times 250}{A_t \times 70 \times 10^9}$$

$$\delta_{bar} = -\frac{FL}{A_b E}$$

$$\delta^* = \frac{1.5}{4} mm.$$

$\delta_{tube} = \delta_{bar} + \delta^+$

find F, the only unknown.

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Date:
Page No.

When $\Delta T = 40°C$ is considered ;

$$\delta_a = \frac{F l_a}{A_a E_a} + \alpha_a \Delta T l_a \qquad \delta_b = \frac{F l_b}{A_b E_b} + \alpha_b \Delta T l_b$$

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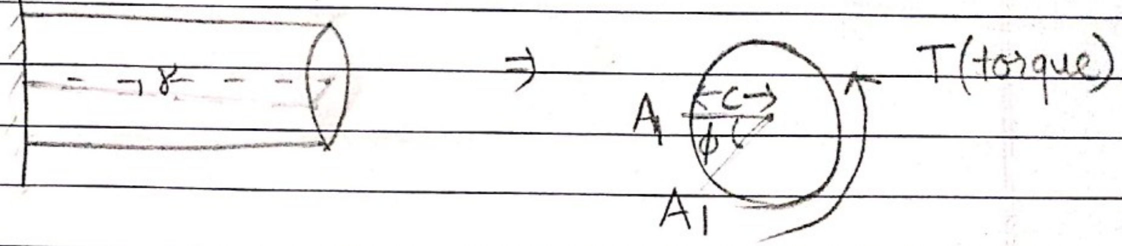
☆ Torsional strain (γ)

side view   front view

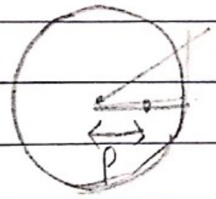


$$\gamma = \frac{c\phi}{L}$$

$AA' = c\phi$

Torsion $\tau = G\gamma$. ⇒ G ⇒ Rigidity Modulus.

Note: $E = 2G(1+\mu)$; $E = 3K(1-2\mu)$



$dF = \tau dA$.

$dT = dA\tau\rho$.

$$\Rightarrow T = \int_0^{2\pi} dT = \int \frac{\tau_{max}}{c} \cdot \rho \cdot \rho dA.$$

$$= \frac{\tau_{max}}{c} \left(\int \rho^2 dA \right) \rightarrow J$$

Area moment of inertia.

$$\Rightarrow T = \frac{\tau_{max}}{c} J$$

$P = T\omega$

Torque, Torsion

$$\boxed{\frac{T}{J} = \frac{\tau}{\rho} = \frac{G\phi}{L}} \rightarrow \text{Twist}$$

length

area moment of inertia   distance from centre.

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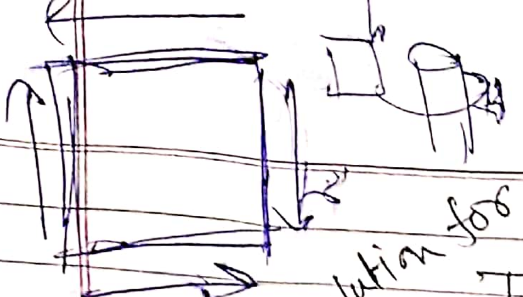

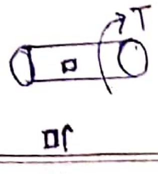

Date:
Page No.

$J = \frac{\pi}{2} c^4$

Modified solution for

$$\frac{T}{J} = \frac{\tau_{max}}{c} \qquad \frac{T}{J} = \frac{G\phi}{L}$$

$$\Rightarrow c = \left(\frac{2T}{\pi \tau_{max}}\right)^{1/3} \qquad \Rightarrow c = \left(\frac{2TL}{\pi G \phi_{max}}\right)^{1/4}$$

For a steel shaft. max c will be the answer.

★ $P = 150$ KW ; 360 rpm ; $G = 77.2$ GPa ;
$\tau_{max} = 50$ MPa ; $L = 2.5$ m ; $\phi_{max} = 3°$
$d = ?$

$$\frac{T}{J} = \frac{\tau}{\rho} = \frac{\tau_{max}}{c} = \frac{G\phi}{L}$$

$P = T\omega$.

$$c = \frac{\tau_{max} L}{G\phi} \qquad \frac{60 \times 150 \times 10^3}{360 \times 2\pi} = T$$

$$= \frac{50 \times 10^6 \times 2.5}{77.2 \times 10^9 \times 3 \times \frac{\pi}{\cancel{180}\,60}} \qquad T = 3979.6 \text{ N}$$

$$= \frac{50 \times 10^6 \times 2.5 \times 60}{77.2 \times 10^9 \times \pi} \qquad \frac{T}{\frac{\pi}{2}c^4} = \frac{G\phi_{max}}{L}$$

$$= \frac{3 \times 2.5}{\pi \times 77.2} \qquad \Rightarrow c^4 = \frac{2TL}{\pi G \phi_{max}}$$

$$\boxed{d = 6.18 \text{ mm}}$$

$$\frac{T}{\frac{\pi}{2}c^3} = \frac{\tau_{max}}{\rho}$$

$$\Rightarrow \left(\frac{2T}{\pi \tau_{max}}\right)^{1/3} = c.$$

$$\Rightarrow c = \left(\frac{2 \times 3979.6}{\pi \times 50 \times 10^6}\right)^{1/3}$$

$$\Rightarrow c = 37 \text{ mm}$$

$$\Rightarrow c = \left(\frac{2 \times 3979.6 \times 2.5}{\pi \times 50 \times 10^6 \times 3 \times \frac{\pi}{180}}\right)^{1/4}$$

$$\Rightarrow \boxed{c = 221 \text{ mm}}$$

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* $L = 2.5m$, $d = 30\ mm$, $f = 30Hz$, $P_{max} = ?$;
$G = 77.2\ GPa$, $\tau_{max} = 50MPa$, $\phi_{max} = 7.5°$.
Find the maximum (allowable) Torque ?

$$\Rightarrow \underset{(i)}{\frac{T}{J}} = \underset{(ii)}{\frac{\tau}{\rho}} = \underset{(iii)}{\frac{G\phi}{L}}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(15\times10^{-3})^4$$

(i, ii)

$$T = \frac{J}{\rho}\tau = \frac{\frac{\pi}{2}(15\times10^{-3})^{\cancel{4}\,3} \times 50\times10^6}{\cancel{(15\times10^{-3})}}$$

$$T = \frac{\pi}{2} \times 225\times15\times10^{-3}\times50$$

$$T = 265.07\ N\cdot m$$

(i, iii) $$T = \frac{JG\phi}{L} = \frac{\frac{\pi}{2}(15\times10^{-3})^4 \times 77.2\times10^9 \times \frac{7.5\pi}{180}}{2.5}$$

$$T = \frac{803.60}{2}\ Nm$$

$\therefore$ T_{max} (allowable) $= 265.07\ Nm$

$$P_{max} = T_{max} \times 2\pi f$$
$$= (265.07)\times 2\pi \times 30$$
$$=$$

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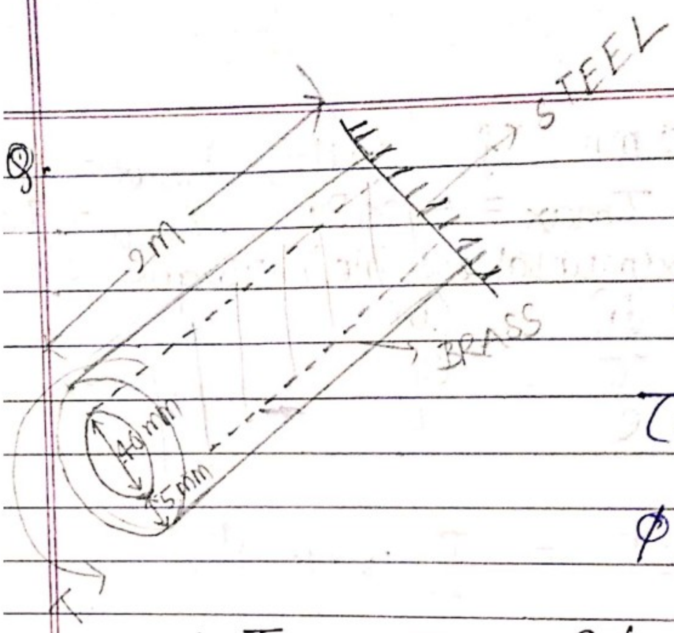
Date:

Page No.

Q.



$G_b = 39 GPa$

$G_s = 77.2 GPa$

$T = 600 N.m.$

$\tau_b = ?$ $\tau_s = ?$

$\phi = ?$

$$\frac{T}{J} = \frac{\tau}{\rho} = \frac{G\phi}{L}$$

$$\frac{T}{J} = \frac{G\phi}{L}$$

$T = T_s + T_b$; $\phi_s = \phi_b$

$$\frac{T_s}{J_s} = \frac{G_s \phi}{L}$$

$$T = G_s J_s \frac{\phi}{L} + G_b J_b \cdot \frac{\phi_b}{L}$$

$$= \frac{\phi}{L}\left(G_s J_s + G_b J_b\right)$$

$$\frac{\phi}{L} = \frac{T}{G_s J_s + G_b J_b}$$

$$\Rightarrow \phi = \frac{TL}{G_s J_s + G_b J_b}$$

$$= \frac{(600 Nm)(2m)}{10^9\left[77.2 \times \frac{\pi}{2}(20 \times 10^{-3})^4 + 39 \times \frac{\pi}{2}\left[(25\times 10^{-3})^4 - (20 \times 10^{-3})^4\right]\right]}$$

$\Rightarrow \phi =$

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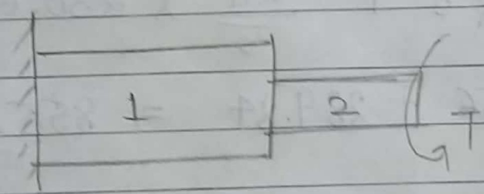
$$\tau_s = G_s \rho_s \left(\frac{\phi}{L}\right) \qquad \tau_b = G_b \rho_b \left(\frac{\phi}{L}\right)$$

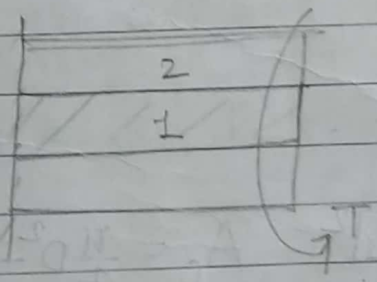
20 mm (20+5) m

27.6 MPa 17.45 MPa

Important considerations :-



$T = T_1 = T_2$; $\phi = \phi_1 + \phi_2$



$T = T_1 + T_2$; $\phi_1 = \phi_2$

* In the same question,

$G_b = 39$ GPa ; $G_s = 77.2$ GPa ;

$\tau_b = 20$ MPa ; $\tau_s = 45$ MPa .

$T_{max} = ?$; $\phi = ?$

688 N m 2.36

$$\phi_b = \frac{\tau_b L}{G_b \rho_b} = \frac{(20 \times 10^6) \times 2}{39 \times 10^9 \times 25 \times 10^{-3}} = 2.35^\circ$$

$$\phi_s = \frac{\tau_s L}{G_s \rho_s} = \frac{45 \times 10^6 \times 2}{77.2 \times 10^9 \times 20 \times 10^{-3}} = 3.339^\circ$$

$\phi = 2.35^\circ$ (As min. of two will be correct answer).

Date:

Page No.

$\frac{T}{J} = \frac{\tau}{e} = \frac{G\phi}{L}$

$\phi_s = \phi_b$ $\quad T = T_s + T_b$

$T = T_s + T_b$

$= \frac{\tau_s J_s}{e_s} + \frac{\tau_b J_b}{e_b}$

$= \frac{G_s J_s \phi}{L} + \frac{G_b J_b \phi}{L}$

$= \frac{45 \times 10^6 \times \frac{\pi}{2}(20\times10^{-3})^{\cancel{4}\,3}}{\cancel{20 \times 10^{-3}}} + \frac{20\times10^6 \times \frac{\pi}{2}((25)^4 - (20\times10^{-3})^4)}{25\times10^{-3}}$

$= \frac{\phi}{L}(G_s J_s + G_b J_b)$

$= \frac{45\pi}{2} \times (20)^3 \times 10^{-3} + \frac{20\pi}{50} \times 10^9 \times 10^{-12}[(25)^4 - (20)^4]$

$= \frac{2.35 \times \frac{\pi}{180}}{2}\left[\left(77.2\times10^9\left(\frac{\pi}{2}\right)(20\times10^{-3})^4\right) + \left(39\times10^9\left(\frac{\pi}{2}\right)\left((25\times10^{-3})^4 - (20\times10^{-3})^4\right)\right)\right]$

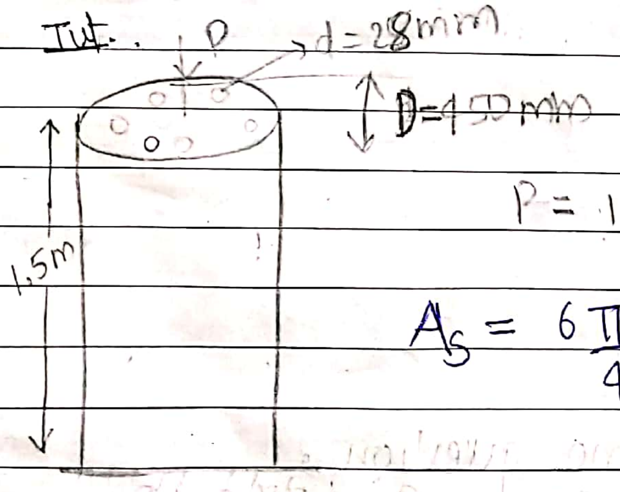
$= \frac{45\pi}{2} \times 10^{-3} \times 8000 + \frac{20\pi}{50} \times 10^{-3} \times 230{,}625$

$= \frac{45\pi}{2} \times 8 + \frac{2\pi}{5} \times 230.625$

$= 688$ Nm

$T = 565.56 + 289.84 = 855.40$ Nm ✗

Q. Tut.



$E_s = 200$ GPa

$E_c = 25$ GPa.

$P = 1550$ kN $\quad \sigma_c = ? \quad \sigma_s = ?$

$A_s = \frac{6\pi d^2}{4} \quad A_c = \frac{\pi}{4}D^2 - A_s$

Ans: $200\times10^9 = E_s = \frac{F_c \times 1.5}{\pi(14\times10^{-3})^2 \times \delta}$ — (1)

$25\times10^9 = E_c = \frac{(1550\text{ kN} - 6F_c)\times 1.5}{\pi[(225\times10^{-3})^2 - 6(14\times10^{-3})^2] \times \delta}$ — (2)

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eq(1) $\div$ eq(2)

$$8 = \frac{F_C}{(1550 \times 10^3 - 6F_C)} \times \frac{0.155}{6.1575 \times 10^{-4}}$$

$(20 \times 10^{-3})^4)$

$$\Rightarrow 0.0318 = \frac{F_C}{1550 \times 10^3 - 6F_C}$$

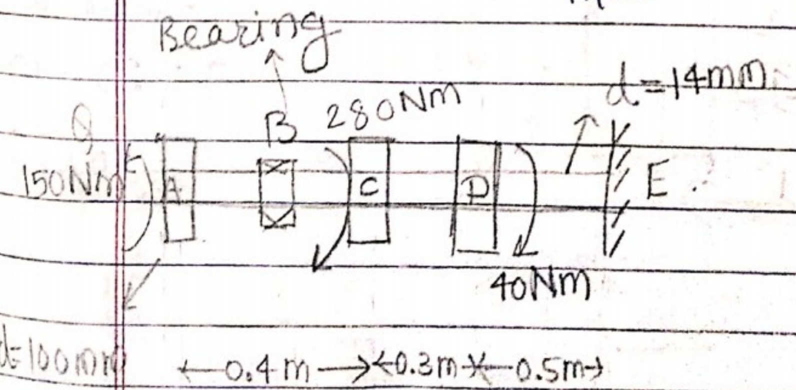
$-(20)^4]$

$$\Rightarrow F_C(1.191) = 49260$$

$$\Rightarrow \boxed{F_C = 41.36\,KN}$$

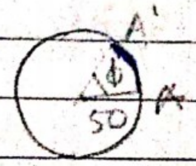
$$\sigma_C = \frac{(1550 - 6(41.36)) \times 10^3}{0.155} \quad \sigma_S = \frac{41.36 \times 10^3}{6.1575 \times 10^{-4}}$$

$$\sigma_C = 8.39\ \frac{MN}{m^2} \ ; \ \sigma_S = 67.2\ MN/m^2$$

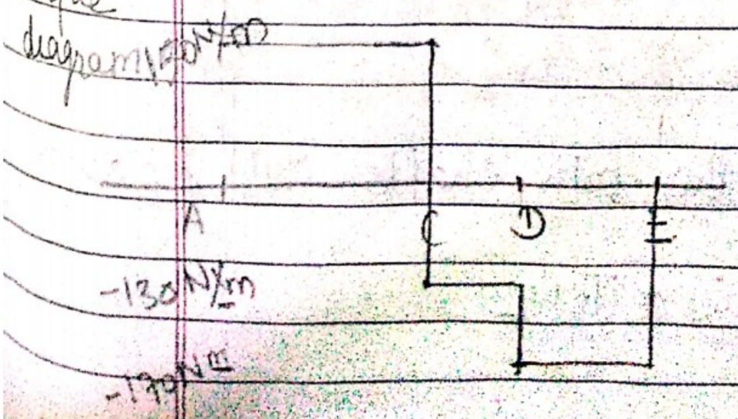


$G_S = 80MPa$

Displacement of A Gear = ?



Torque diagram



$\frac{150}{50} = \frac{x}{1}$

$$\phi = \phi_1 + \phi_2 + \phi_3$$

$T_1 = 150$, $T_2 = -130$, $T_3 = -170$.

$$\frac{T}{J} = \frac{G\phi}{L} \Rightarrow \phi = \frac{TL}{GJ}$$

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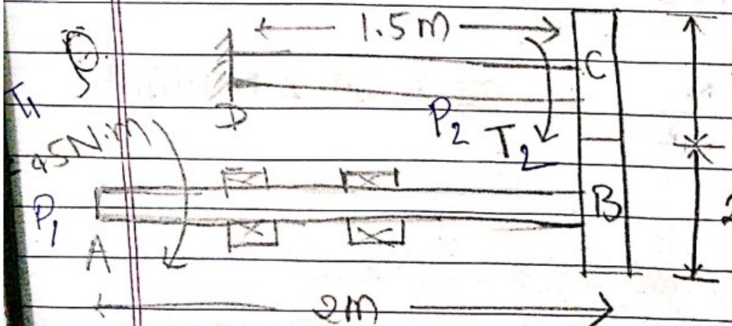
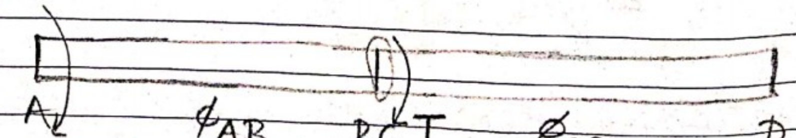
$$\phi = \frac{T_1 L_1}{G_s J} + \frac{T_2 L_2}{G_s J} + \frac{T_3 L_3}{G_s J}$$

$$\phi = \left(\frac{1}{J}\right)\left[\frac{(150)(\ )}{80M} + \frac{(-130)(\ )}{80M} + \frac{(-170)(\ )}{80M}\right]$$



$G = 80 GPa$
$d = 20 mm$
$= 0.02 m$

$\phi_A = ?$



(factor of multiplication)

$\phi = \phi_A = \phi_{AB} + X \phi_{CD}$

Power/Energy in the both shafts will be same

$P_1 = P_2 = T_1 \omega_1 = T_2 \omega_2$

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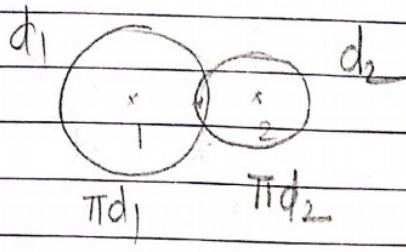
Date:
Page No.



$$\frac{T_1}{T_2} = \frac{\omega_2}{\omega_1} = \frac{d_1}{d_2}$$

(OR) $F.(0.15) = 45\ N\cdot m$

$\Rightarrow T_2 = F.(0.075)$

$\Rightarrow = \frac{45}{0.15}(0.075)$

$$T_2 = \frac{45}{2}\ N\cdot m.$$

$$\pi d_1 \omega_1 = \pi d_2 \omega_2$$

$$\frac{\omega_1}{\omega_2} = \frac{d_2}{d_1}$$

$$\frac{T}{J} = \frac{\tau}{e} = \frac{G\phi}{L}.$$

$$\phi = \frac{TL}{GJ} \qquad \phi_{AB} = 0.0716\ Rad.$$

$$\phi_{AB} = \frac{(45)(2)}{80\times10^9 \times \frac{\pi}{2}(10^{-2})^4}$$

$$= \frac{90 \times 2}{\cancel{80\times10^9}\times\pi\times\cancel{10^{-8}}}$$

$$= \frac{18}{80\pi}$$

$$= 0.716\ Rad$$

$$\phi_{CD} = \frac{\left(\frac{45}{2}\right)\left(\frac{3}{2}\right)}{80\times10^9 \times \frac{\pi}{\cancel{2}} \times 10^{-8}}$$

$$= \frac{135 \times 7}{160 \times 10 \times 22}$$

$$\phi_{CD} = 0.0268\ Rad$$

$$x\phi_{CD} = \phi_{CD} \cdot \frac{T_2}{T_1}$$

$$= \frac{1}{2}\phi_{CD} \quad \boxed{x = 1/2}$$

$$\Rightarrow \phi_A = (0.716) + \frac{1}{2}(0.0268) = \underline{0.7294\ Rad}$$

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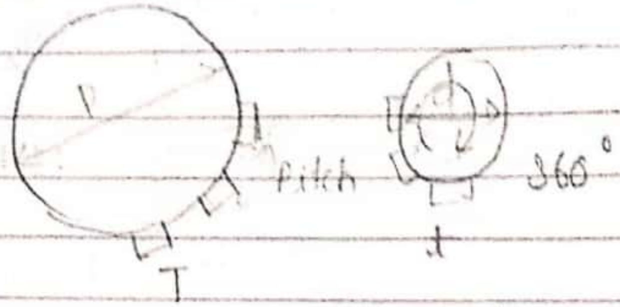
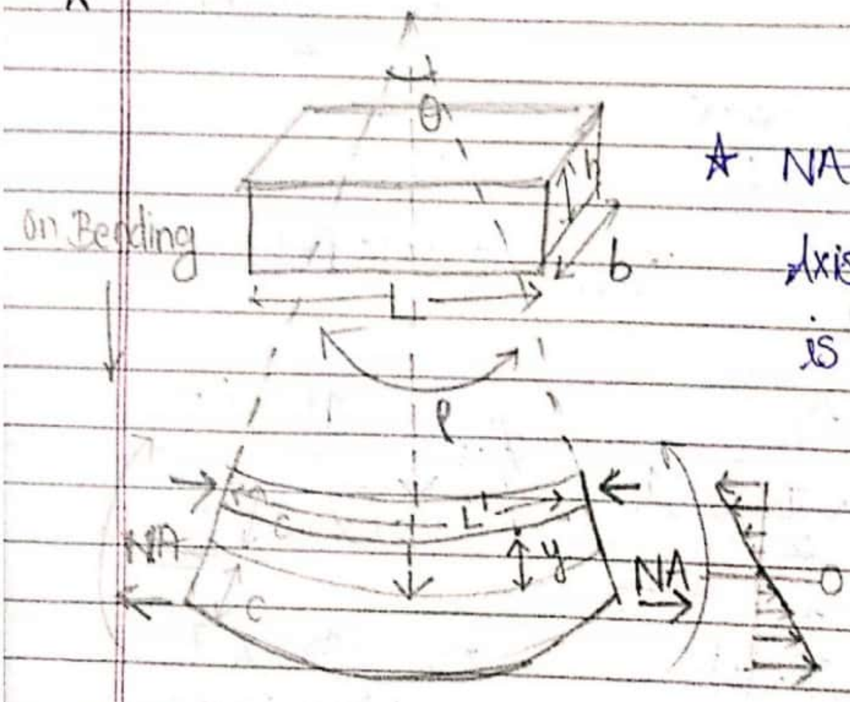


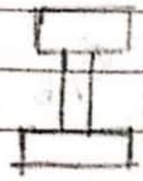
$\frac{T}{t} = \frac{D}{d}$

$T = \frac{\pi D}{P}$ $t = \frac{\pi d}{P}$

☆



☆ NA (Neutral Axis).
Axis along which strain is zero. (centroid).



Reason for using I section in rail tracks

→ $L = \rho\theta$; $L' = (\rho - y)\theta$

$\delta = L' - L = (\rho - y)\theta - \rho\theta = -y\theta$.

$\Rightarrow \epsilon = \frac{\delta}{L} = \frac{-y\theta}{\rho\theta} = \frac{-y}{\rho}$

$\Rightarrow \epsilon_{max} = \frac{c}{\rho}$ $\rightarrow \sigma_{max} = E\epsilon_{max} = Ec/\rho$

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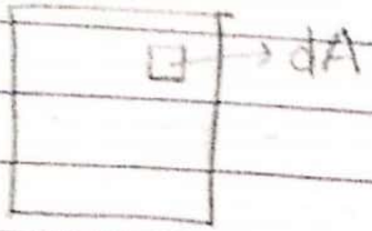


$$dM = dF\, y$$
$$= \sigma\, dA\, y$$
$$= \frac{y}{c}\sigma_{max}\, y . dA$$
$$= \frac{\sigma_{max}}{c} . y^2 dA$$

$$\sigma = \frac{Ey}{\rho}$$
$$= \frac{Ec}{\rho} . \frac{y}{c}$$

$$M = \frac{\sigma_{max}}{c}\int y^2 dA$$ second moment of area

$$\sigma = \frac{y}{c} . \sigma_{max}$$

$$= \frac{\sigma_{max}}{c} . I$$

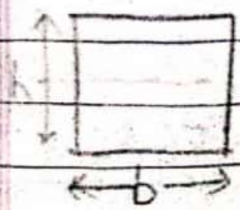
$$\Rightarrow \boxed{\frac{M}{I} = \frac{\sigma_{max}}{c} = \frac{\sigma}{y}} \quad \text{—— (I)}$$

$$\sigma_{max} = E\, \epsilon_{max} .$$

$$\frac{Mc}{I} = \frac{Ec}{\rho} \Rightarrow \frac{M}{I} = \frac{E}{\rho} \quad \text{—— (2)}.$$

$$\Rightarrow \boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{\rho}}$$

Flexural equation for homogenous body.



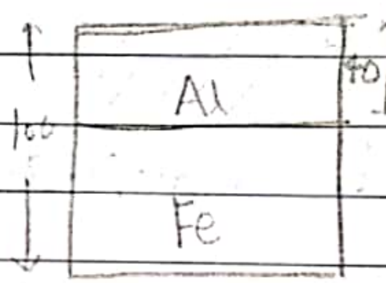
NA

$$\frac{M}{I} = \frac{\sigma_{max}}{c\left(=\frac{h}{2}\right)} \Rightarrow \sigma_{max} = \frac{M}{(I/c)}$$

$$\sigma_{max} = \frac{M}{Z}$$ $Z \Rightarrow$ SECTION MODULUS

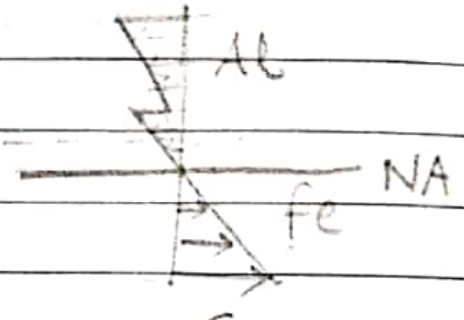
$$Z = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^2 .$$

★ SECTION MODULUS $Z = \frac{I}{c}$

★ 

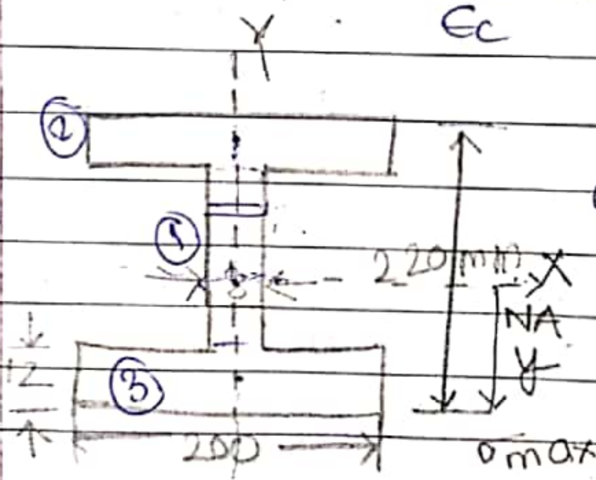
$E_{Al} = 70\,GPa$

$E_{Fe} = 200\,GPa$



$\epsilon_t = \frac{40}{\rho}$; $\epsilon_c = \frac{60}{\rho}$

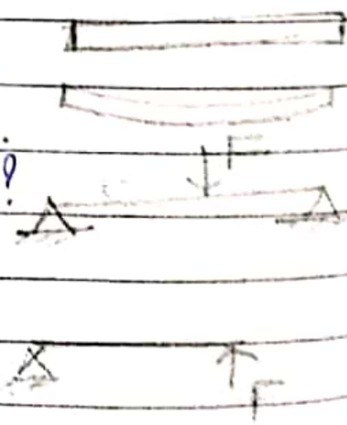
$\Rightarrow \frac{\epsilon_t}{\epsilon_c} = \frac{2}{3}$

★ 

$\sigma_{all} = 155\,MPa$

$M_x = ?$; $M_y = ?$

σ_{max} . $I_{NA} = ?$



$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{\rho}$. $\frac{\sigma_{max}}{y} = \frac{M_x}{(I_{NA})_x}$

$\Rightarrow$ $I_2 = I_3$

$I_{NA} = I_1 + I_2 + I_3$

$= \frac{1}{12}(8)(220-24)^3 + 2\left[\frac{1}{12}(200)(12)^3 + (12\times 200)(104)\right]$

$\boxed{I_{NA} = 56.94\times 10^{-6}\,m^4}$

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Date:
Page No.

$$\Rightarrow \quad M_x = \frac{\sigma_{max}}{y} \times I_{NA}$$

$$= \frac{(155 \times 10^6 \; Pa)}{110 \; \cancel{m} \times 10^{-3}} \times 56.94 \times 10^{-6} \; m^4$$

$$M_x = 80.2 \, kN.m$$

$$\frac{\sigma_{max}}{x} = \frac{M_Y}{(I_{NA})_Y} \qquad \cancel{(M_Y = 248 \, kN.m)}$$

$$M_Y = \frac{\sigma_{max}}{x} \times (I_{NA})_Y$$

$$= \frac{155 \, MPa}{100} \times (I_{NA})_Y$$

$$(I_{NA})_Y = \frac{1}{12}(190)(8)^3 + \left[\frac{1}{12}(200)^3(12)\right] 2$$

$$M_Y =$$

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Date:
Page No.

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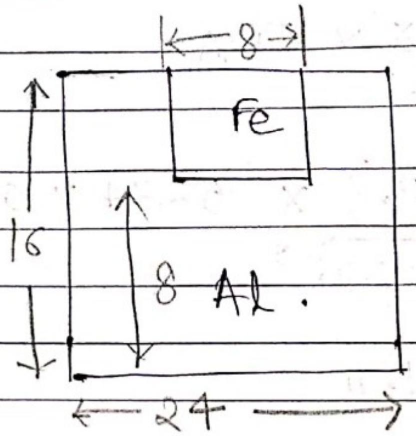
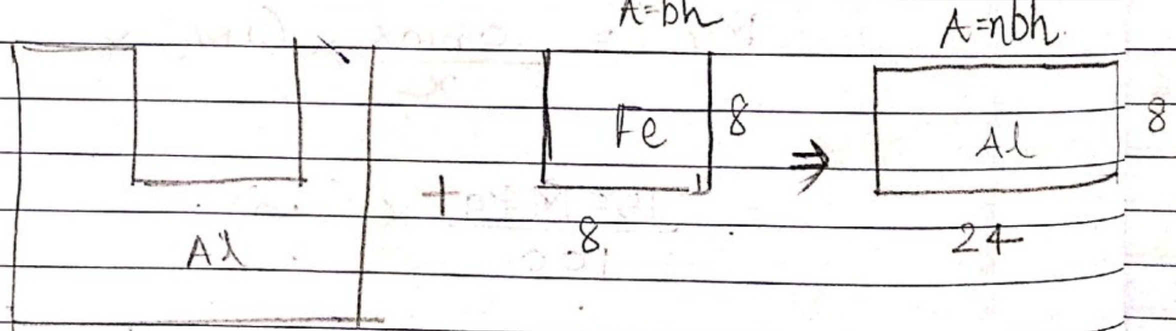


$E_{Fe} = 210\ GPa$;
$E_{Al} = 70\ GPa$;
$M_x = 60\ N \cdot m$;
$\sigma_{Al} = ?$; $\sigma_{Fe} = ?$

Fe = STEEL

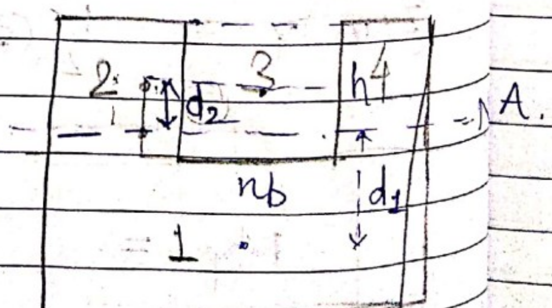
Convert the above to homogenous material.

$$n = \frac{210}{70} = 3.$$



$\sigma_a = E_a \epsilon$
$F = \sigma_a A_a = \sigma_s A_s$
$= E_a \epsilon A_a = E_s \epsilon A_s$.

$$\frac{F_s}{F_a} = \frac{A_s}{A_a}$$



$$y = \frac{\Sigma yA}{A} = \frac{y_1A_1 + y_2A_2 + \overset{ny_3A_3}{y_3A_3} + y_4A_4}{A_1 + A_2 + \underset{nA_3}{A_3} + A_4}$$

$y_{cm} = 9\ mm.$

$\sigma = E \frac{y}{\rho}$

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$J_{NA} = 10.4106 \times 10^{-9}\ m^4$.

$= I_{1NA} + (I_2)_{NA} \times 2 + (I_3)_{NA}$.

$= I_{1C} + A_1 d_1^2 + (I_{2C} + nA_2 d_2^2) + I_{3C} + nA_3 d_2^2$.

$\frac{M}{I} = \frac{\sigma}{y}$. $\rightarrow \sigma = \frac{My}{I}$.

$y_a = 9mm$

STEEL $\sigma_s = \frac{nMy_s}{I_{NA}}$.

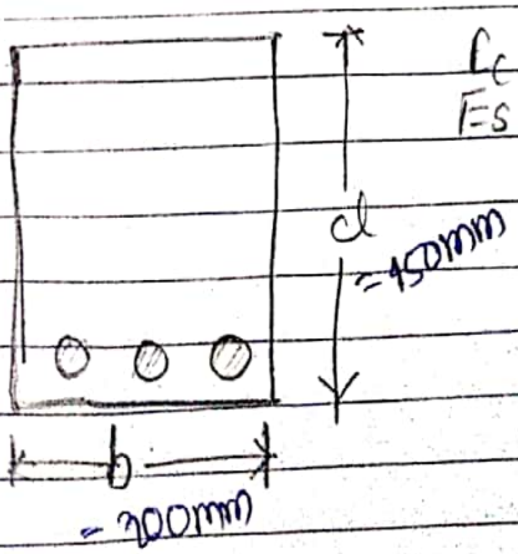
Al. $\sigma_a = \frac{My_a}{I_{NA}}$.

$y_s = 7mm$.

$\sigma_a = 51.9 MPa$.

STEPS
① $A_s \rightarrow nA_s$
② $NA \rightarrow$
③ $I_{NA} \rightarrow nb \dashrightarrow I\ h$.
④ $\sigma_s \rightarrow nMy/I$

☆



$E_c = 25 GPa$
$E_s = 200 GPa$

$\sigma_{c\,all} = 12.5 MPa$.
$\sigma_{s\,all} = 140 MPa$

$A_s = ?$; $M = ?$

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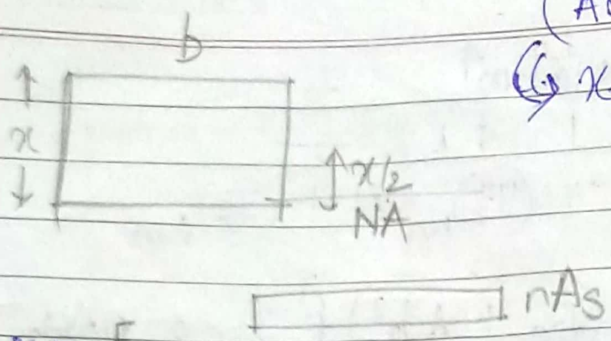
Date:
Page No.

(Area balance against NA)



$$x \cdot b \cdot \frac{x}{2} = nA_s(d-x).$$

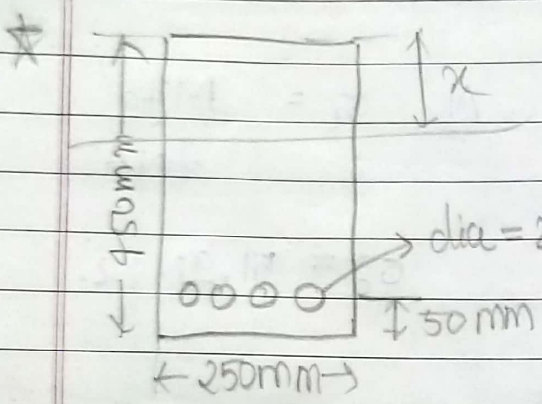
$x = ?$

$$\eta = \frac{E_s}{E_c}$$

$$I = I_c + Ad^2 = nA_s(d-x)^2$$

negligible

$$A_s = \frac{3\pi}{4} D^2$$



$E_c = 25 GPa$; $E_s = 200 GPa$.

$M = 170 kN \cdot m$.

$\sigma_s = ?$, $\sigma_c = ?$

① Locate the NA by using Area balance.

$$(x \times 250) \cdot \frac{x}{2} = nA_s(450-x)$$

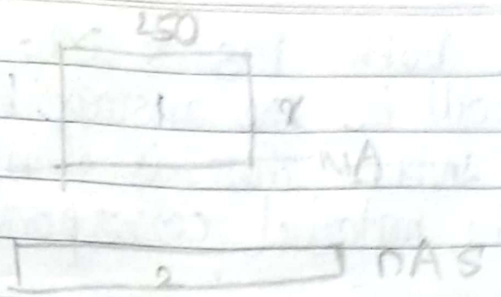
$$n = \frac{E_s}{E_c} = \frac{200}{25} = 8$$

$$A_s = 4 \frac{\pi}{4} (22)^2$$

$$x^2 \cdot \frac{250}{2} = 8\pi(22)^2(450-x)$$

$x = 154.55$ mm

$I = ?$



$$I_{NA} = I_1 + I_2$$

$$= \frac{1}{12} x^3 (250) + (250x)\left(\frac{x}{2}\right)^2 + nA_s(450-x)$$

$$= 1.0404 \times 10^{-3}\ m^4$$

$$\sigma_c = \frac{M\, y_c}{I_{NA}} \qquad \sigma_s = \frac{n M y_s}{I_{NA}}$$

$$y_c = 154.55 \qquad y_s = 450 - 154.55$$

$$\sigma_c = -20\,MPa. \qquad \sigma_s = 330\,MPa.$$

compression

Q.4. $n = \frac{E_s}{E_c} = 8$ $\qquad nA_s(450-x) = 200x \cdot \frac{x}{2}$ —(1).

$$\sigma_s = \frac{nM}{I}(450-x).$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(450-x)}{x}$$

$$\sigma_c = \frac{Mx}{I}$$

$$\frac{140}{12.5} = \frac{8(450-x)}{x}$$

$$\Rightarrow A_s = \frac{200x^2}{16.(450-x)}$$

$$x = \frac{450}{2.4}$$

$$A_s = 1674.105\ mm^2 \qquad x = 187.5\ mm.$$

$$M_c = \frac{\sigma_c I_{NA}}{x} \qquad M_s = \frac{\sigma_s I_{NA}}{n(450-x)}$$

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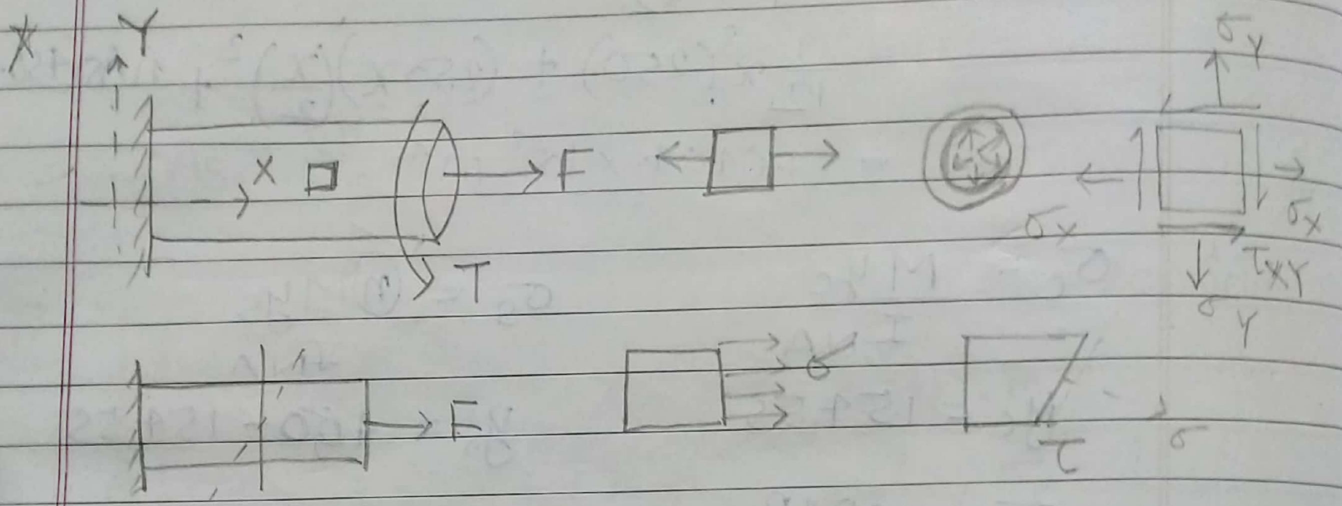
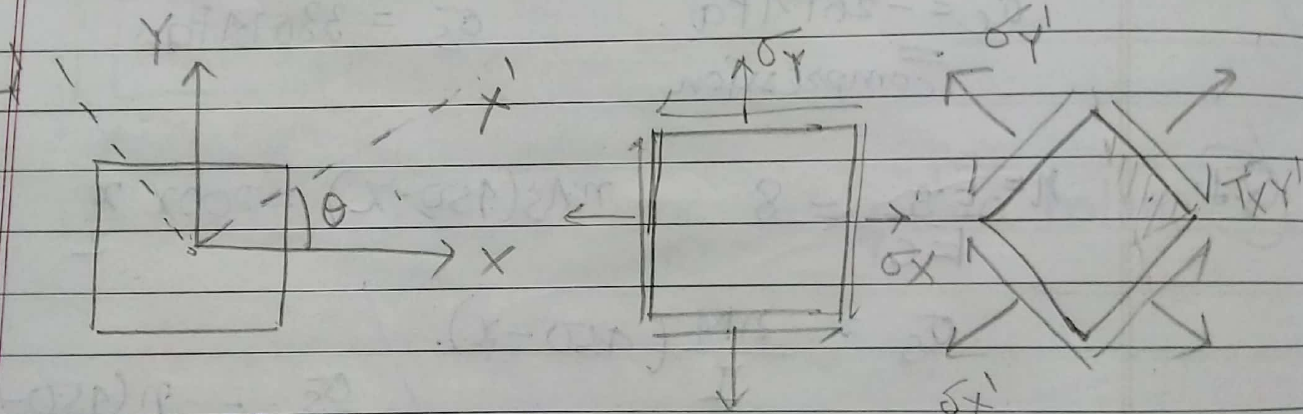
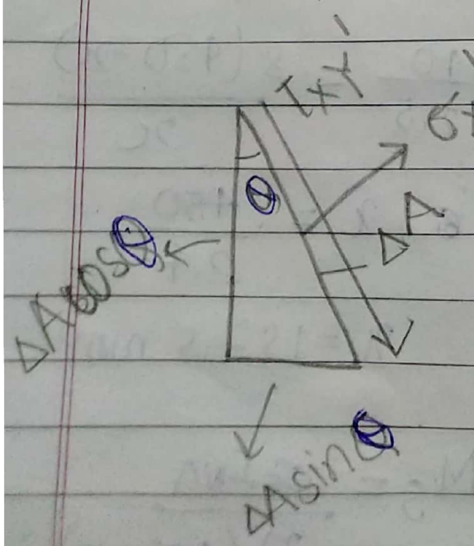
Calculate both Ms & Mc when lower will be the answer because higher is consider ~~when~~ there is chance that it will break the material corresponding to lower one.

Here, $M_c = M_s = 90.8$ kN.m.





$\sigma_x' = f(\sigma_x, \sigma_y, \tau_{xy})$; $\sigma_y' = -//-$; $\tau_{xy}' = -//-$;



Date:
Page No.

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\sin 2\theta\right) + \tau_{xy}\cos 2\theta$$

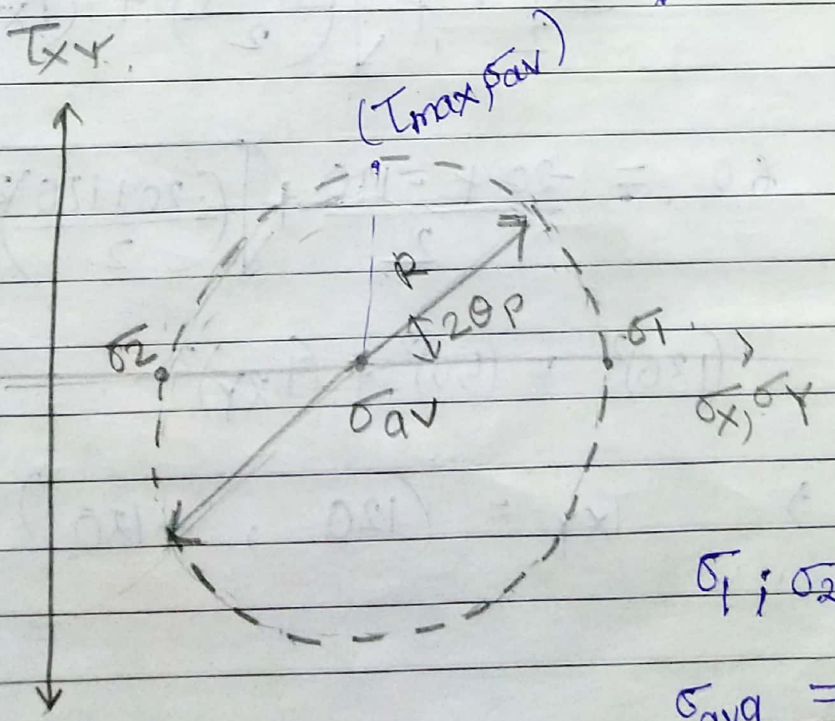
* Principle stress direction $\tau_{x'y'} = 0$.

$$0 = -\left(\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right)$$

$$\Rightarrow \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

$$\sigma_1 = \cancel{\sigma_x'} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \cancel{\sigma_y'} = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



$\tau_{max} = R$

~~$\sigma_1 = \sigma_2 = \sigma_{av}$~~

$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 ; \sigma_2 = \sigma_{avg} \pm R.$$

$$\sigma_{avg} = \frac{\sigma_x+\sigma_y}{2}.$$

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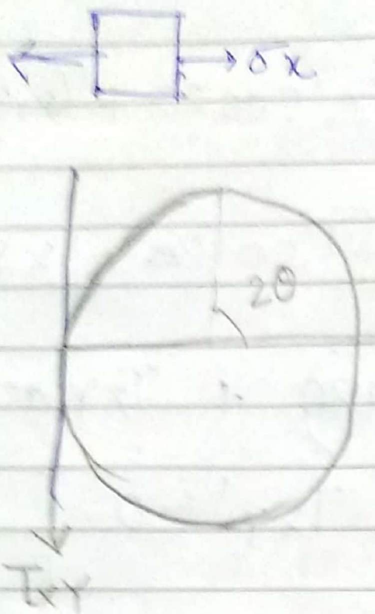


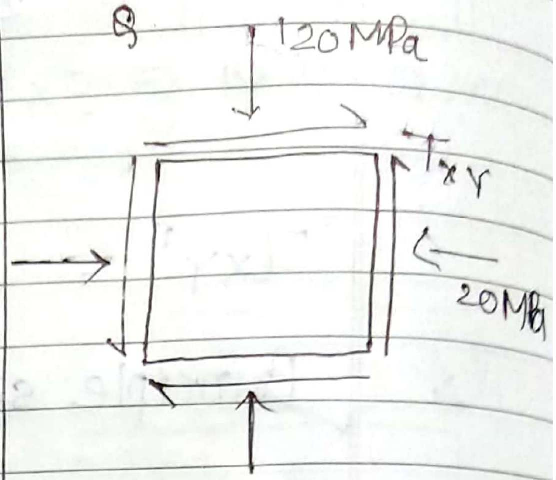
$\sigma_1 = \sigma_x$

$\sigma_2 = 0$

$\tau_{max} = R = \frac{\sigma_x}{2}$

$\theta = 45^\circ$

Q



$\sigma_{max} \leq 60 MPa$

$\tau_{xy} = ?$ (RANGE)

Tensile +ve

Compressive −ve

Anticlockwise +ve

Clockwise −ve

$\sigma_x = -20 MPa$

$\sigma_y = -120 MPa$

$$\sigma_1 = \sigma_{avg} + R$$

$$= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\Rightarrow \quad 60 = \frac{-20 + -120}{2} + \sqrt{\left(\frac{-20+120}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \quad (130)^2 = (50)^2 + (\tau_{xy})^2$$

$$\Rightarrow \quad \tau_{xy} = (120 \; , \; -120) \; MPa.$$

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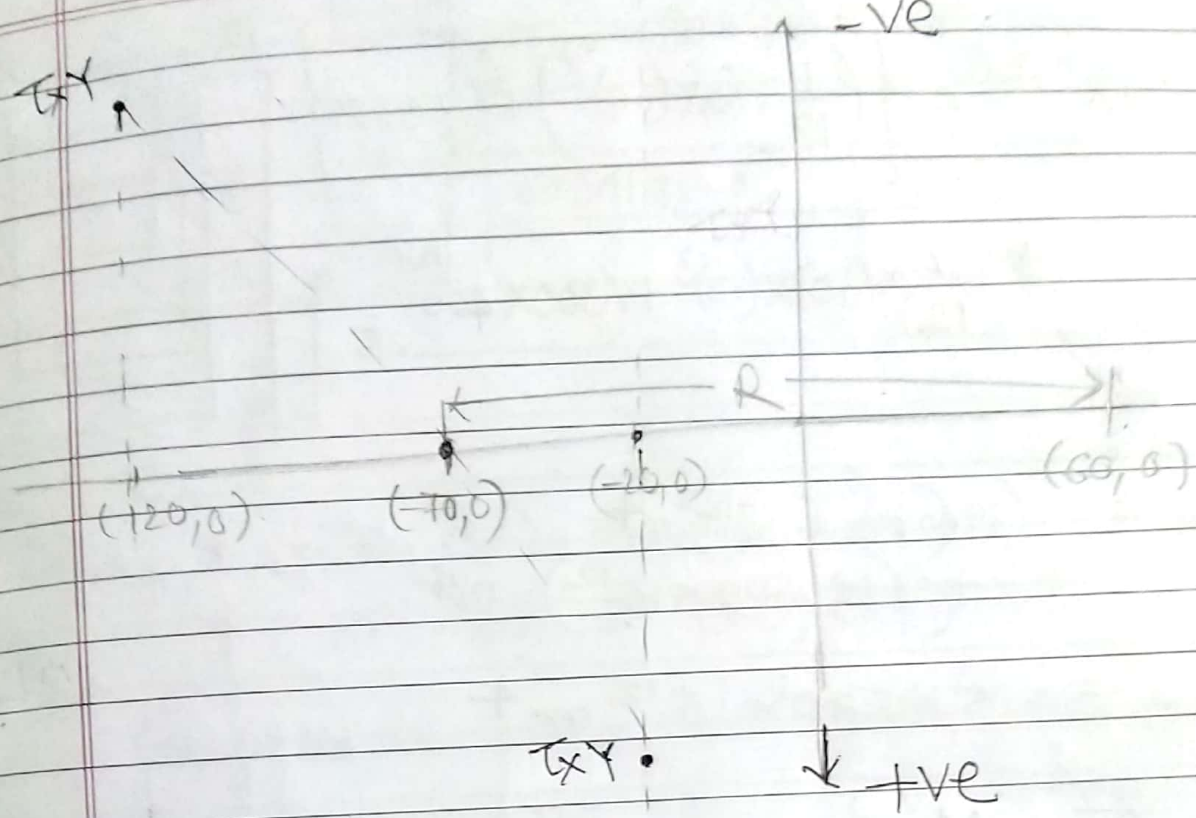
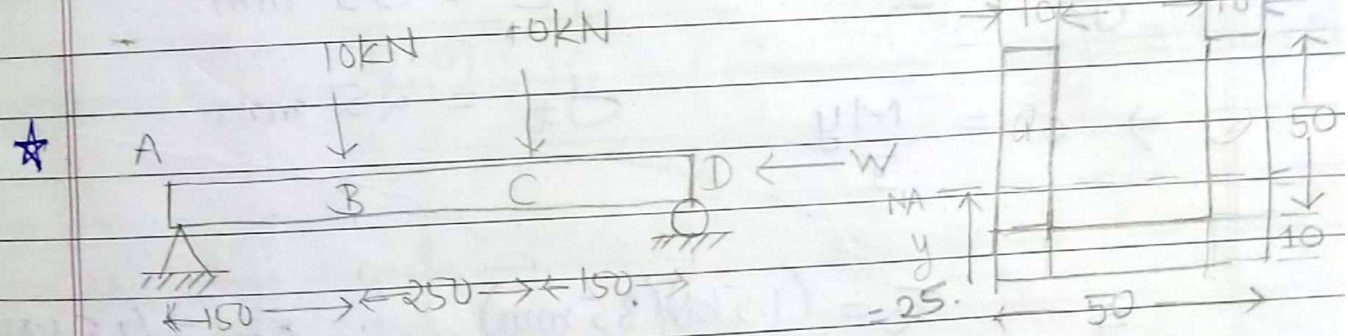
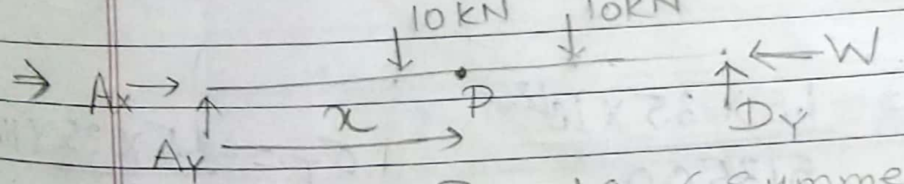




Find all the stress in BC (σ_c, σ_t)



BM at P

$A_Y = D_Y = 10$ (symmetric)

$= +A_y x - 10(x - 0.15)$

$= 10 \times 0.15 \times 10^3$

$= 1500$ Nm

$= 1.5$ kNm

1.5

BM

$$y = \frac{(30)(600) + (30)(600) + (5)(300)}{1500}$$

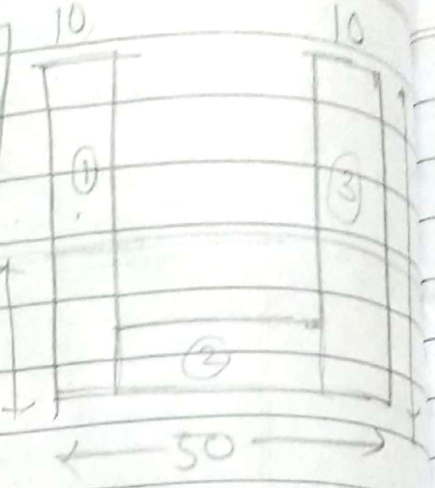
$$y = \frac{375}{15} = 25 = 25$$

Date:
Page No.

$M = 1.5 kN \cdot m \;;\; y = 25 mm$

$$I_{NA} = 2 \times \left[\left(\frac{1}{12} \times 10 \times (60)^3 \right) + (600 \times 25) \right]$$

$$+ \frac{1}{12} \times (30) \times (10)^3 + (300 \times 400)$$



$$I_{NA} = (390000 \times 10^{-12})\, m^4 + (122500 \times 10^{-12})\, m^4$$

$$I_{NA} = 512500 \times 10^{-12}\, m^4$$

$$\frac{\sigma}{y} = \frac{M}{I} \qquad y_c = 35\, mm$$

$$y_t = 25\, mm$$

$$\Rightarrow \sigma = \frac{My}{I}$$

$$\Rightarrow \sigma_c = \frac{(1.5 kNm)(35\, mm)}{512500 \times 10^{-12}\, m^4} \qquad \sigma_T = \frac{(1.5\, kNm)(25\, mm)}{512500 \times 10^{-12}\, m^4}$$

$$\Rightarrow \sigma_c = \frac{1.5 \times 35 \times 10^{12}}{512500} \qquad \sigma_T = \frac{1.5 \times 25 \times 10^{12}}{512500}$$

$$\Rightarrow \sigma_c = 102.439\ MPa. \qquad \sigma_T = 73.17\ MPa.$$

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Date:
Page No.

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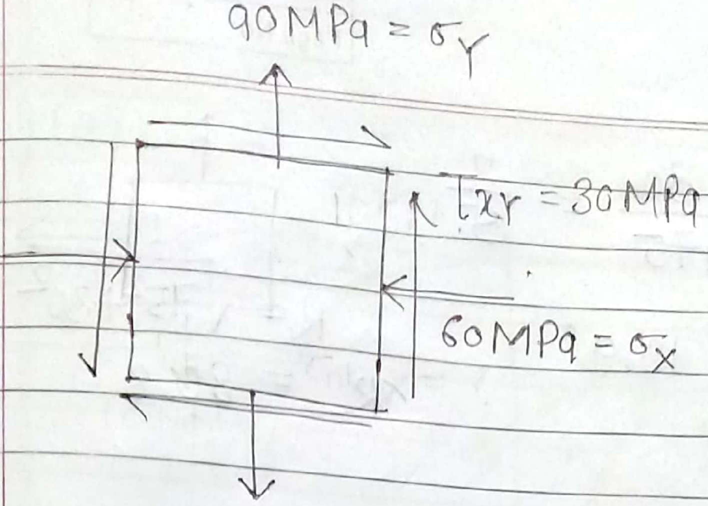


$\theta = 25°$ ↻

$= 10°$ ↻

$\sigma, \tau = ?$

Ans.

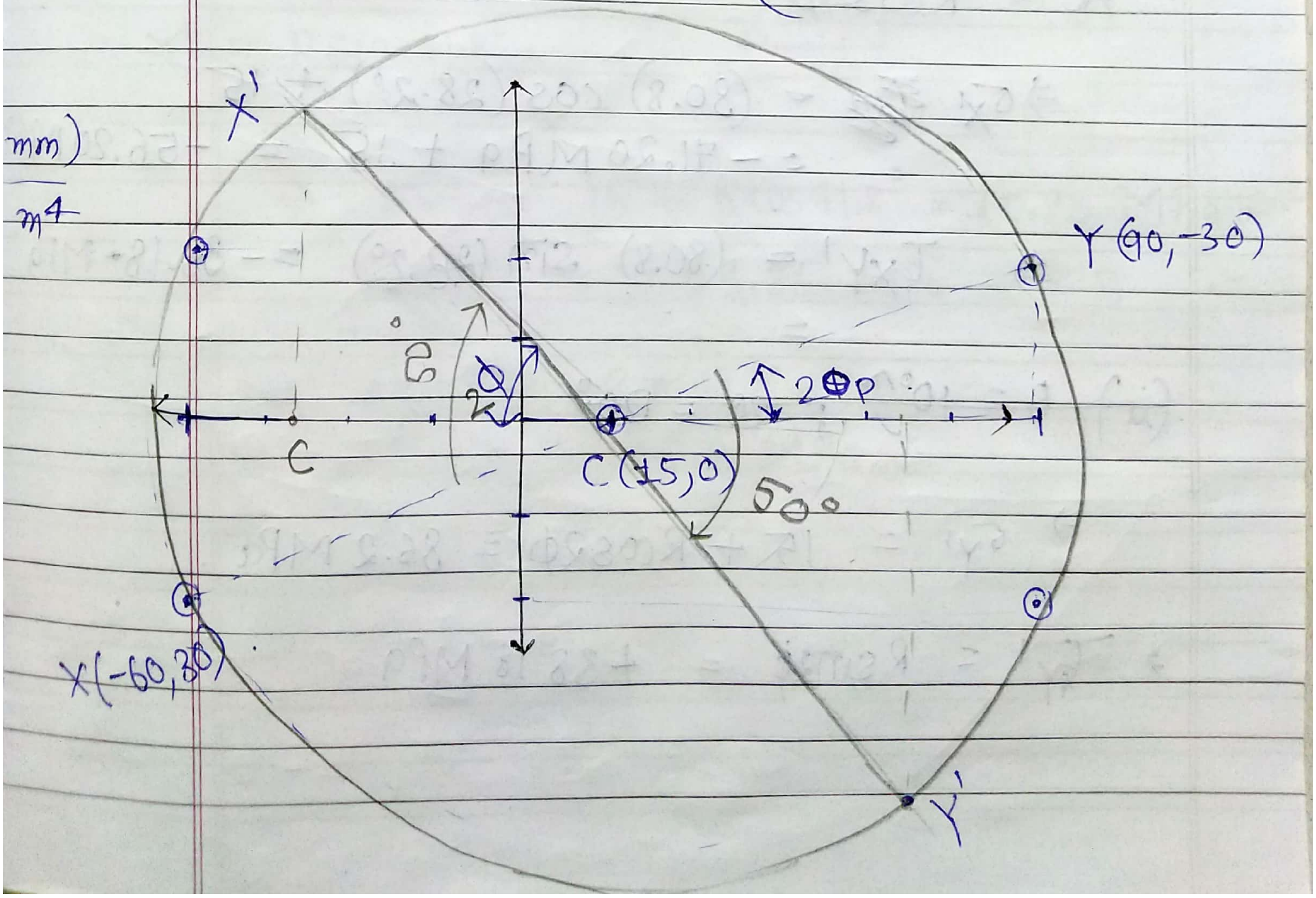
$X(-60, 30) \equiv (\sigma_X, \tau_{XY})$

$Y(90, -30) \equiv (\sigma_Y, \tau_{XY})$

Compressive −ve   CCW +ve

Tensile +ve   CW −ve.

$C(\sigma_{av}, 0) \equiv \left(\frac{-60+90}{2}, 0\right) \equiv (15, 0)$



$\tan 2\theta_p = \frac{30}{75} = \frac{2}{5}$

$\theta_p = 10.9°$ $\quad$ $R = \sqrt{75^2 + 30^2}$

$R = 80.8$

$2\theta_p = 21.8°$

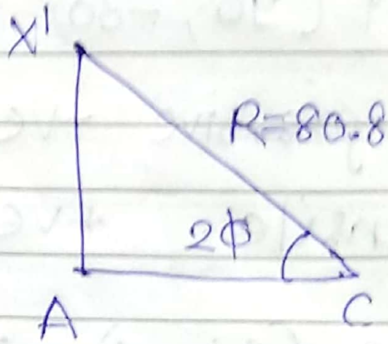
(i) $\theta = 25°$; $2\theta = 50°$

$X'(\sigma_{x'}, \tau_{xy'})$

$2\phi = 2\theta - 2\theta_p$

$= 50 - 21.8$

$2\phi = 28.2°$



$AX' = R\sin 2\phi$

$AC = R\cos 2\phi$

$\Rightarrow \sigma_{x'} = (80.8)\cos(28.2°) + 15$

$= -71.20\,MPa + 15 = -56.20\,MPa$

$\Rightarrow \tau_{xy'} = (80.8)\sin(28.2°) = -38.18\,MPa$

~~(ii) $\theta = 10°$; $2\theta = 120°$.~~

$\Rightarrow \sigma_{y'} = 15 + R\cos 2\phi = 86.2\,MPa$.

$\Rightarrow \tau_{xy'} = R\sin 2\phi = +38.18\,MPa$

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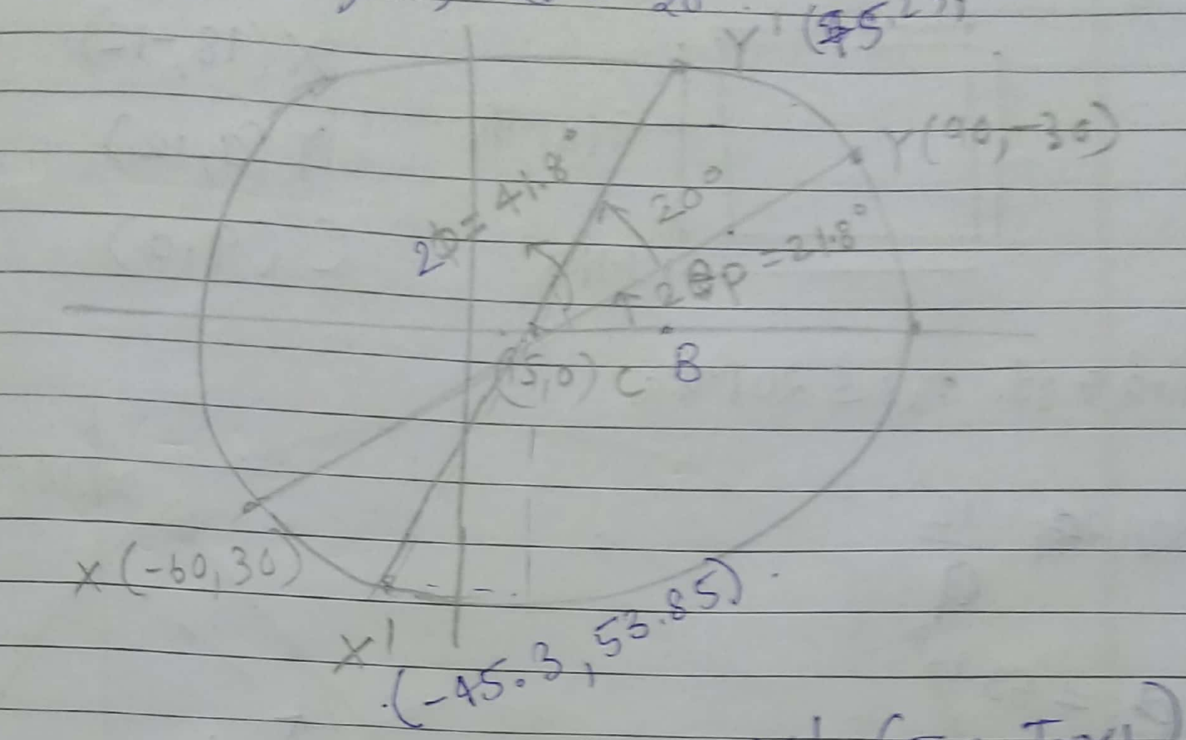
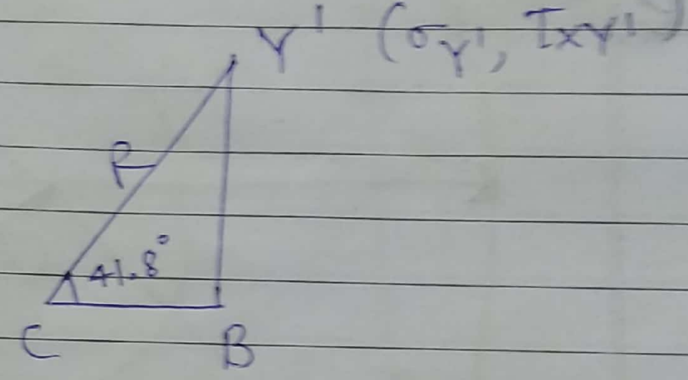
(ii) $\theta = 10^\circ$; $2\theta = 20^\circ$



$\Rightarrow \quad 2\phi = 41.8^\circ$



$CB = R\cos 41.8^\circ$

$Y'B = R\sin 41.8^\circ$.

$\Rightarrow \quad \sigma_{Y'} = 15 + R\cos 41.8^\circ = 75.23\ \text{MPa}$ ✓

$\tau_{xY'} = -53.85\ \text{MPa} = Y'B$ ✓

$\Rightarrow \quad \sigma_{X'} = -R\cos 41.8^\circ + 15$

$= -45.3\ \text{MPa}$ ✓

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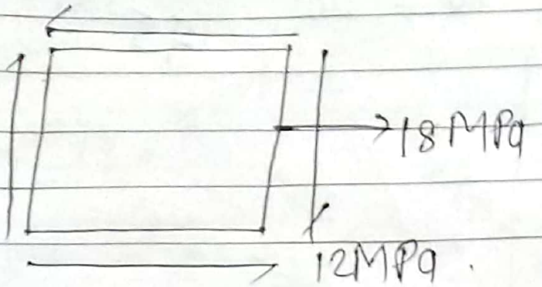
Q.



X (18, −12)

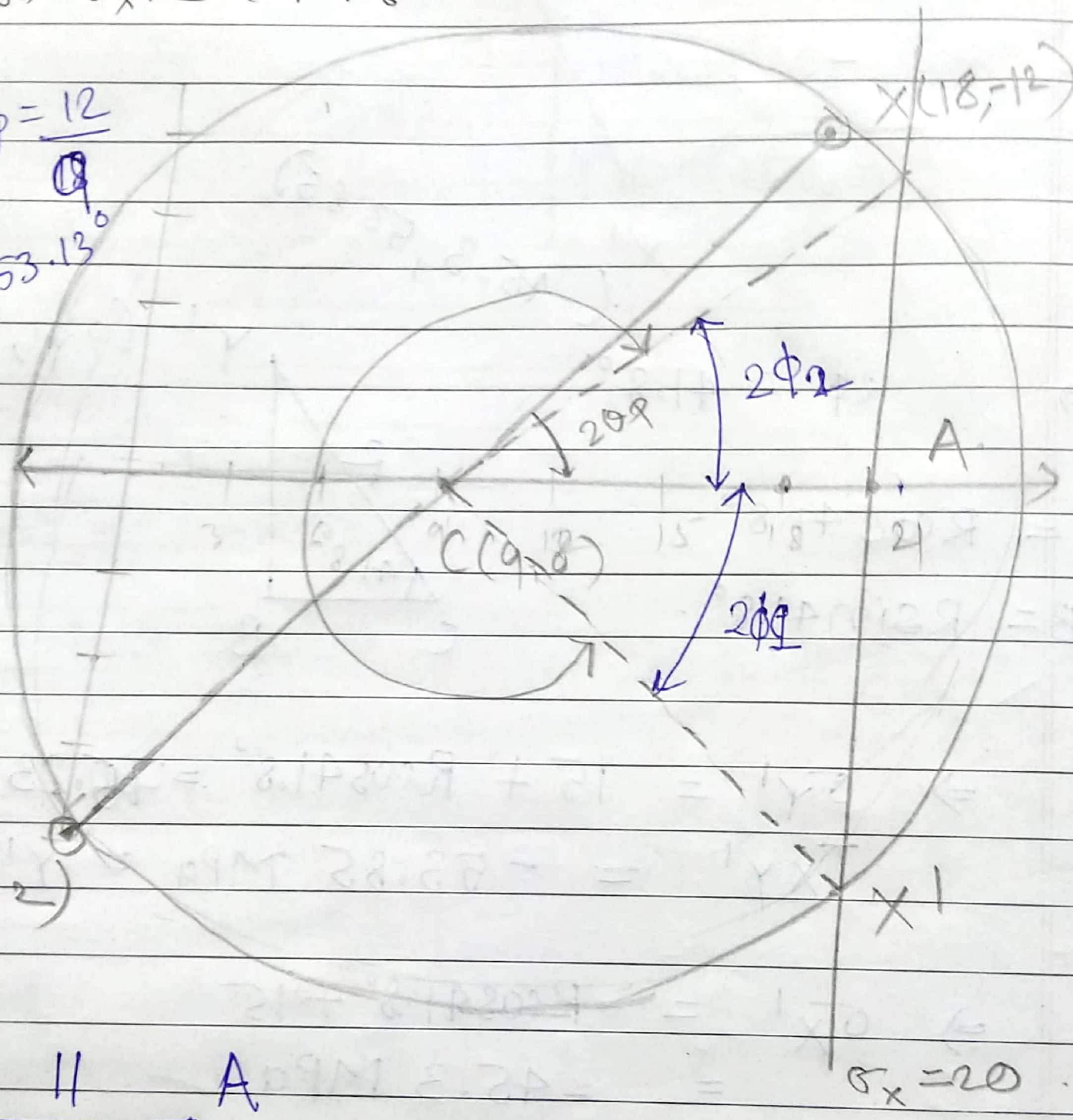
Y (0, 12)

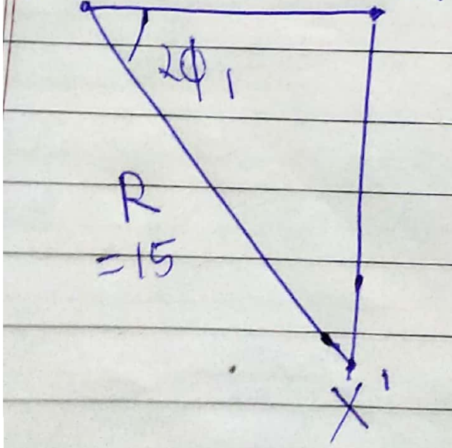
C (9, 0)

find θ s.t. $\sigma_{x'} \leq 20$ MPa

$\tan 2\phi_p = \frac{12}{9}$

$2\phi_p = 53.13^\circ$





$11 = 15 \cos 2\phi_1$

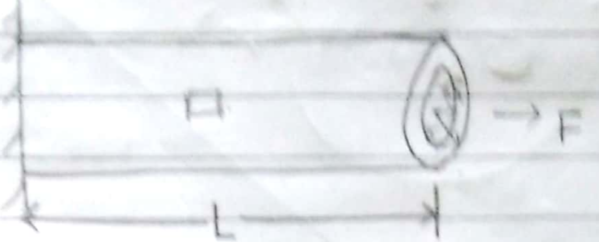
$2\phi_1 = 42.85^\circ$ or -42.85°

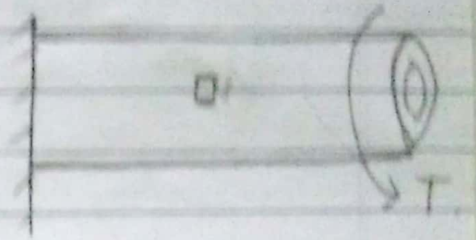
$2\phi_2 = 2\phi_p - 2\phi_1 = -10.28^\circ$

$2\phi_2 = -10.28^\circ$ or 10.28°

$2\theta = -2\phi_2$

$2\pi = 2\phi_p - 2\phi_1$

★ ① 

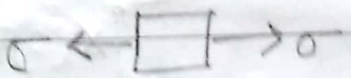
② 

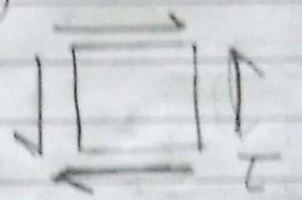
$\Rightarrow \frac{T}{J} = \frac{\tau}{e} = \frac{G\phi}{L}$

$\tau = \frac{Te}{J}$

$\sigma = \frac{F}{A} = \frac{F}{\pi(r_o^2 - r_i^2)}$

If only axial force is on a body, then axial stress throughout the body will be same.

(a) 

(b) 

for any element on outer periphery of rod,

$e = r_o$

$\tau = \frac{T r_o}{J}$

★ Now, Draw Mohr's Circle for case (a) and case (b).

$\Rightarrow$ (a) $\sigma_x = \sigma$, $\tau = 0$, $\sigma_y = 0$

$X(\sigma_x, \tau_{xy}) = (\sigma, 0)$

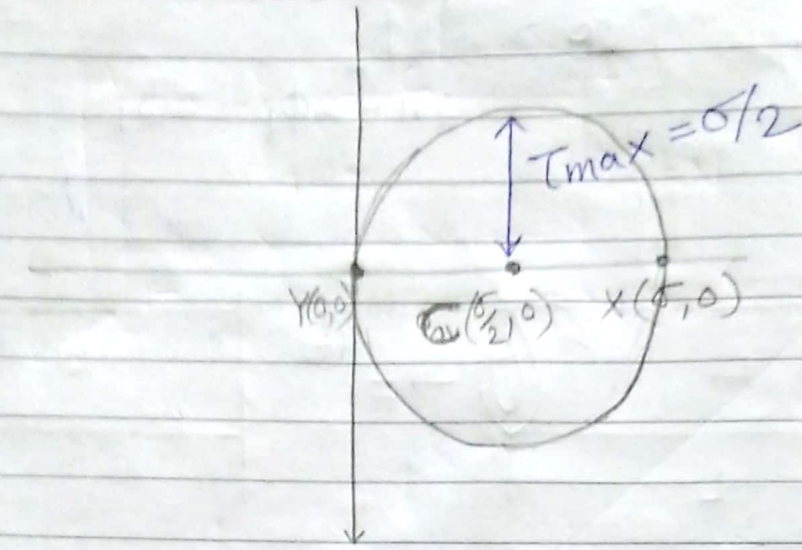
$Y(\sigma_y, \tau_{xy}) = (0, 0)$

$C(\sigma_{av}, 0) = (\sigma/2, 0)$

E & G, μ, K ; $\frac{T_1}{T_2} = e^{\mu\theta}$

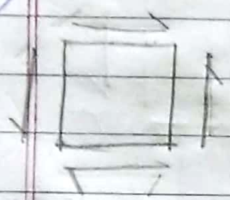
σ_{xx}, σ_{YY}, σ_{zz}, μ, E, ϵ_{xx}, ϵ_{YY}

Date:
Page No. 6

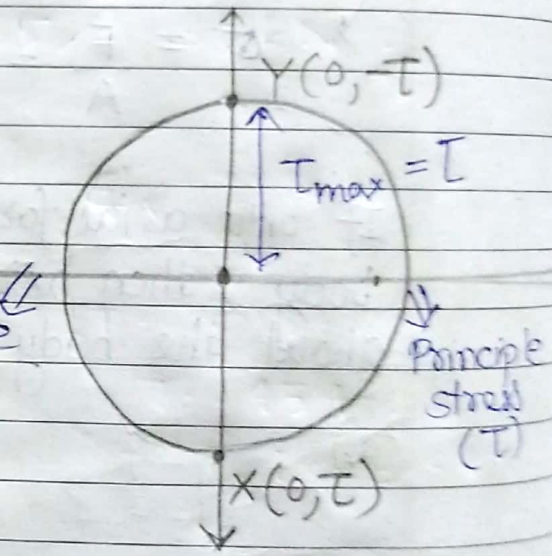


(b) $\sigma_x = 0$; $\sigma_Y = 0$; $\tau_{xY} = \frac{T\rho}{J}$

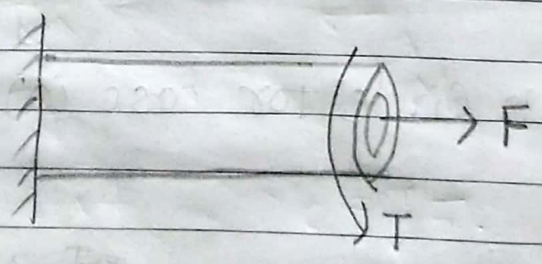
→

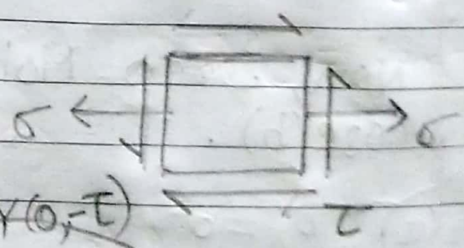


X $(0, T\rho/J)$
Y $(0, -T\rho/J)$
C $(0, 0)$.

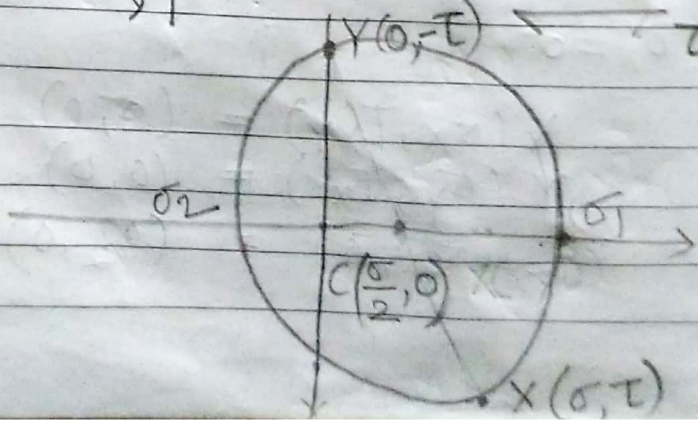


(3)





X(σ, τ)
Y(0, $-\tau$)
C($\frac{\sigma}{2}$, 0)



$\frac{dM}{dx} = V;\ \frac{dV}{dx} = W;\quad \frac{\tau}{\rho} = \frac{T}{J} = \frac{G\phi}{L};\quad \frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R};$

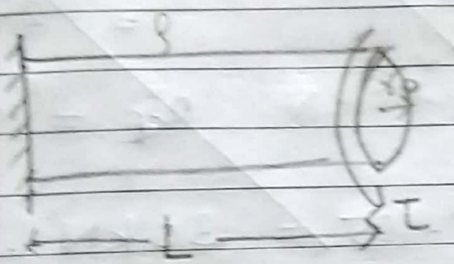
Principle stresses

$$\sigma_1 = \sigma_{avg} + R = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

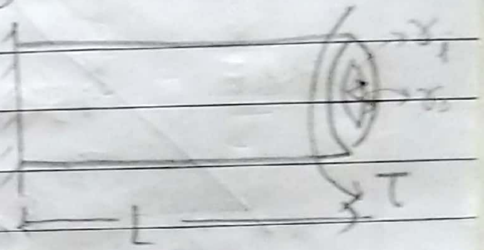
$$= \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

$$\sigma_2 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

(4)



(5)



Ans $\tau = ?\ ;\ \gamma = ?$

$$\frac{\tau}{\rho} = \frac{T}{J}$$

$$\tau_{max} = \frac{T r_0}{J}$$

$$J = \frac{\pi}{2}\left(r_0^4 - r_i^4\right)$$

$$\tau_{4\,max} = \frac{T r_0}{J_4}$$

$$J_4 = \frac{\pi}{2} r_0^4$$

$$\tau_{5\,max} = \frac{T r_0}{J_5}$$

$$J_5 = \frac{\pi}{2}\left(r_0^4 - r_i^4\right)$$

$$\frac{(\tau_4)_{avg}}{mass} = \frac{\left(\frac{T r_0}{\frac{\pi}{2} r_0^4} + 0\right)/2}{d.\pi r_0^2 L}$$

d = density

$$= \frac{2TdL}{r_0(2)}$$

$$\frac{(\tau_4)_{avg}}{mass} = \frac{TdL}{r_0}$$

$$\frac{(\tau_5)_{avg}}{mass} = \frac{\left(\frac{T(r_0 + r_i)}{\frac{\pi}{2}(r_0^4 - r_i^4)}\right)/2}{d.\pi(r_0^2 - r_i^2)L}$$

$$> \quad \frac{(\tau_5)_{avg}}{mass} = \frac{TdL(r_0 + r_i)}{r_0^2 + r_i^2}$$

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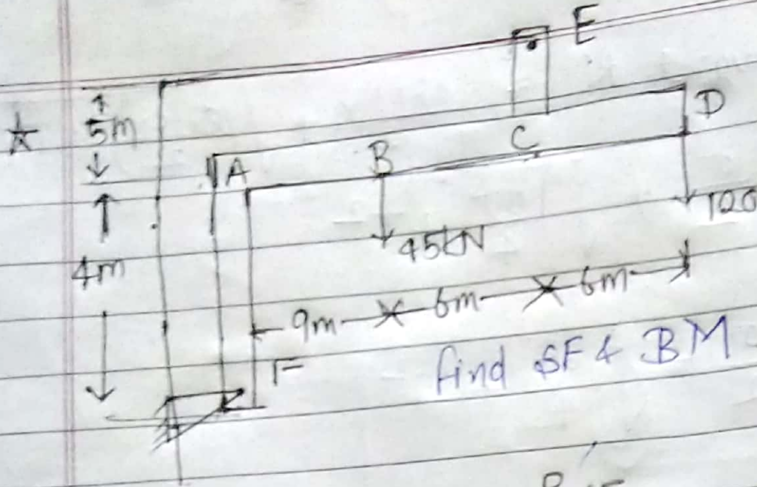
Page No.



find SF & BM.

$R_{CE} = R_{AF} \times 165$

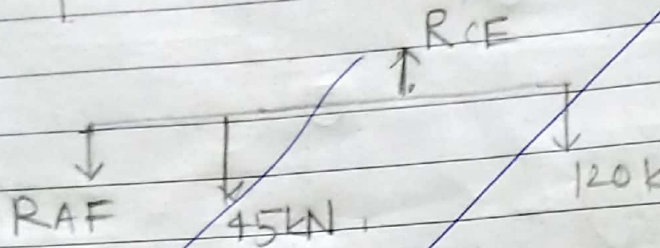
$R_{CE} \times 18$

$= (45 \times 9) + (120)(21)$

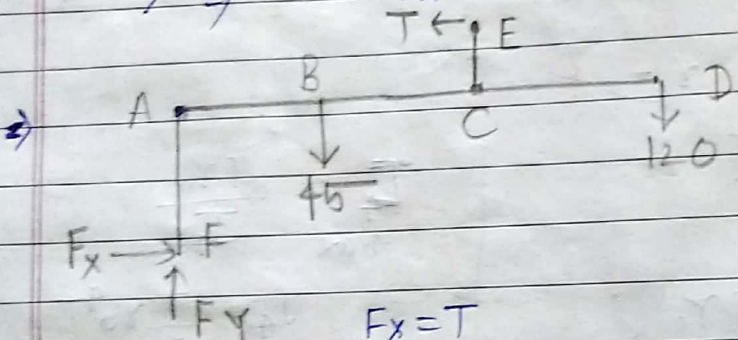
$R_{CE} = \frac{45 \times 9 + 120 \times 21}{18}$

$= 27 + 168$

$R_{CE} = 195$ kN



$R_{AF} = 195 - 165 = 30$ kN.



$\Sigma F_x - (9 \times 45) + 5T - (21 \times 120) = 0$

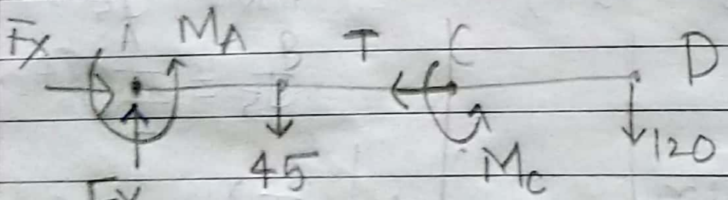
$9T = 45 \times 9 + 21 \times 120$

$F_x = T$

$T = 45 + 280$

$F_Y = 165$ kN.

$T = F_x = 325$ kN

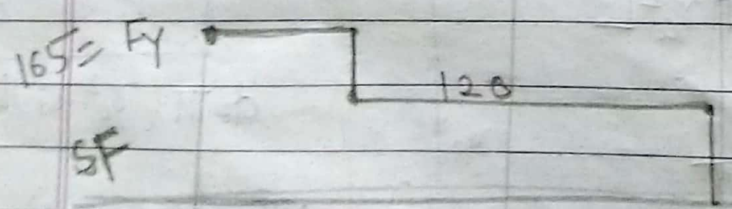


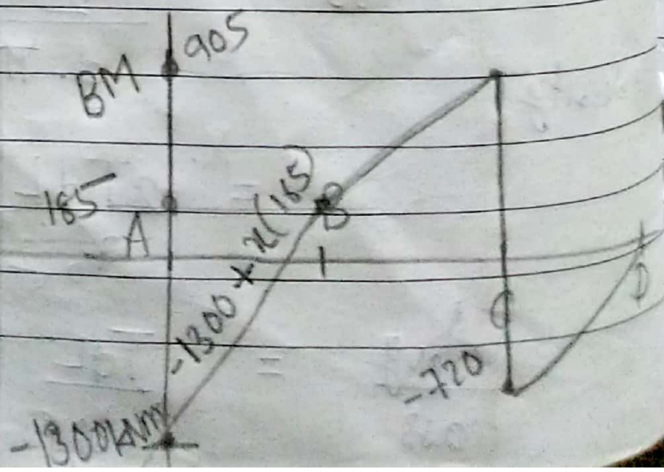
1300

$M_A = \cancel{660}$ kN.m

$M_C = 1625$ kN.m

$M_A = F_x 4$; $M_C = T5$.





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Date:

Page No.

$$BM_{B\to C} = -1300 + x(165) - (x-9)(45)$$

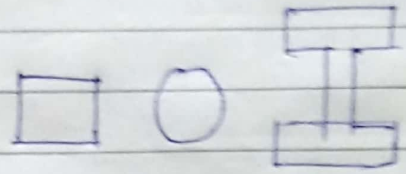
$$BM_C = -1300 + 15(165) - 6(45)$$
$$= 905$$

$$BM_C = 905 - 185 = 720$$
(M_C)

$$BM_{C\to D} = -1300 + x(165) - (x-9)\,45 - 1625$$

$$BM_D = -1300 + 21(165) - (12)45 - 1625$$
$$= 0.$$

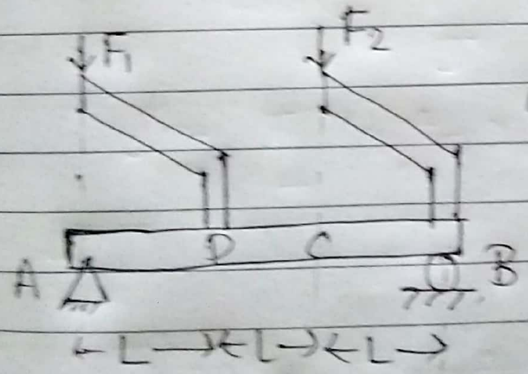
$$\frac{\sigma}{y} = \frac{M}{I}$$

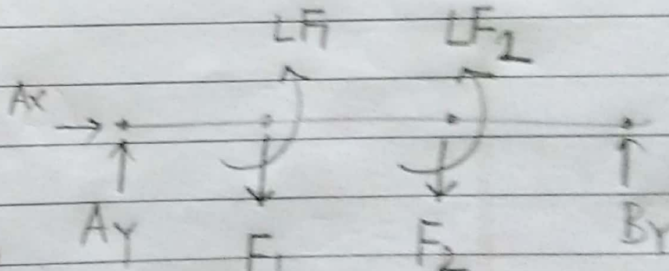


$$\sigma = \frac{My}{I}$$

for σ_{max}; M should be BM max.

*





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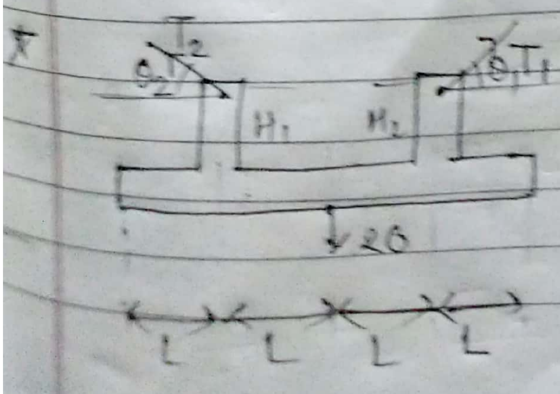

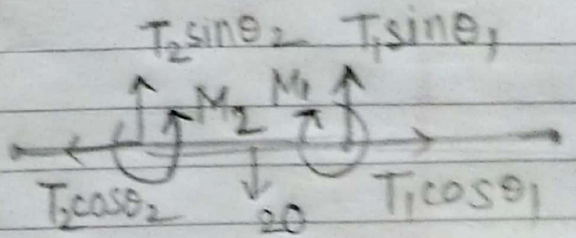

$$M_2 = T_2\cos\theta_2 H_1$$

$$M_1 = T_1\cos\theta_1 H_2$$

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