7. Suppose that we would like to get an idea of how much coffee is consumed by the entire University of Rochester each day. We take a sample of 100 days and find that the average amount of coffee consumed by the University of Rochester per day is 580 gallons.

   a. Assume that coffee consumption comes from a normal distribution with $\sigma = 90$. Find a two-sided 95% confidence interval for the average amount of coffee consumed by the University of Rochester each day.
   b. Assuming the same information as part a, suppose that we now only want a upper-bound confidence interval. Calculate a one-sided 95% upper-bound confidence interval for the average amount of coffee consumed by the University of Rochester each day.
   c. Now, suppose that we do not know the variance of the true distribution of coffee consumption. However, in our sample, we see that $s = 80$. Find a two-sided 95% confidence interval for the average amount of coffee consumed by the University of Rochester each day.
   d. Assuming the same information as part c, suppose that we now only want a upper-bound confidence interval. Calculate a one-sided 95% upper-bound confidence interval for the average amount of coffee consumed by the University of Rochester each day.
   e. Assuming the same information as part a (i.e., known population variance), calculate the number of samples needed in order to get a two-sided 95% confidence interval for the average amount of coffee consumed by the University of Rochester each day of length 16.

A7). a) $\left(\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} , \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right)$

$\sigma = 90$

$n = 100.$

$z_{\alpha/2} = 1.96$

$\bar{x} = 580$

a). $\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 580 - 1.96 * \frac{90}{\sqrt{100}}$

$= 580 - 1.96 * 9$

$= 562.36.$

$\bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 580 + 1.96 * \frac{90}{\sqrt{100}}$

$= 580 + 1.96 * 9$

$= 597.64$

Conf interval = $(562.36, 597.64)$

b) $\left(-\alpha, \bar{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}}\right)$

$z_{\alpha} = 1.645$

$\bar{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}} = 580 + 1.645 \times \frac{90}{10}$

$= 594.805$

$(-\infty, 594.805)$

c) $s = 80$

$\left(\bar{x} - z_{\alpha/2} \frac{s}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \frac{s}{\sqrt{n}}\right)$

$\bar{x} - z_{\alpha/2} \frac{s}{\sqrt{n}} = 580 - 1.96 \times \frac{80}{\sqrt{100}}$

$= 580 - 1.96 \times 8$

$= 564.32.$

$\bar{x} + z_{\alpha/2} \frac{s}{\sqrt{n}} = 580 + 1.96 \times \frac{80}{\sqrt{100}}$

$= 595.68$

$(564.32, 595.68).$

d) $\left(-\infty\ ,\ \bar{x} + z_{\alpha} \frac{s}{\sqrt{n}}\right)$

$$\bar{x} + z_{\alpha} \frac{s}{\sqrt{n}} = 580 + 1.645 \times \frac{80}{\sqrt{100}}$$

$$= 593.16$$

$$(-\infty\ ,\ 593.16).$$

e) $m = \frac{l}{2} = \frac{16}{2} = 8$

$$n = \left[\frac{z^2_{\alpha/2} \cdot \sigma^2}{m^2} \right]$$

$$= \left[\frac{1.96^2 \times 90^2}{8^2} \right] = 486.2025$$

After rounding $\underline{486}$