Deriving formula to recover costs

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The formula we used to recover costs comes from the assumption that the price of each car model is set to maximize profits from that model. Consider the profit maximization problem of a firm facing a log-log demand function:

$$\pi = \max_P \exp(\beta_0 + \beta_1 \log(P) + X + \text{Fixed effects})(P - C),$$

where π stands for profit and C stands for unit cost (hence $P - C$ is unit margin). Note the "exp": because we measure demand in $\log(Q)$ sense, we need to make it back to Q by exponentiate it. Maximizing profit is equivalent to taking the first-order-condition as follows.

$$\exp(\beta_0 + \beta_1 \log(P) + X + \text{Fixed effects}) + \exp(\beta_0 + \beta_1 \log(P) + X + \text{Fixed effects})\frac{\beta_1(P - C)}{P} = 0,$$
$$\Rightarrow 1 + \frac{\beta_1(P - C)}{P} = 0.$$

Note that the exp term cancels out in the first-order condition. Rearranging, we have

$$C = P\left(\frac{1 + \beta_1}{\beta_1}\right),$$

and we have the formula we used.