# COMPREHENSIVE ANALYSIS

## Statistics for Data Science

### Module 1: Exploring Data

Analysis Date: January 22, 2026

Master's Level Course Material

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# 1. Executive Summary

This comprehensive analysis examines the foundational module "Exploring Data" from a Master's-level Statistics for Data Science curriculum. The module encompasses essential statistical concepts that form the bedrock of data analysis, including understanding variable types, sampling methodologies, descriptive statistics, data visualization techniques, and correlation analysis.

The analysis reveals a carefully structured progression from basic concepts (what is statistics, variables) to intermediate topics (central tendency, variability measures) to more advanced concepts (correlation, complex visualizations). The complexity ranking identifies basic definitions and variable classification as entry-level topics (complexity score 1-2), while correlation interpretation and bias mitigation represent advanced challenges (complexity score 8-9).

| Key Finding | Implication |
|---|---|
| 10 distinct topics identified | Comprehensive foundational coverage |
| Complexity range: 1-9 (scale of 10) | Suitable for progressive learning |
| Heavy emphasis on practical application | Includes R programming examples |
| Visualization-centric approach | Modern data science pedagogy |

# 2. Subject Identification and Overview

## 2.1 Core Subjects Identified

### 1. What is Statistics?

Foundation of the discipline and its role in data science

### 2. Variables

Understanding units, categorical vs. quantitative variables

### 3. Samples and Populations

Statistical inference foundations, sampling bias

### 4. Descriptive Statistics

Central tendency, variability, and relative standing measures

### 5. Data Visualization

Bar charts, histograms, boxplots, scatter plots

### 6. Correlation

Measuring relationships between quantitative variables

# 3. Detailed Subject Analysis

## 3.1 Preliminary Concepts: What is Statistics?

### Definition and Scope

Statistics is defined as "the art and science of answering questions and exploring ideas through the processes of gathering data, describing data, and making generalizations about a population on the basis of a smaller sample." This definition encompasses three critical components:

- **Data Collection:** Systematic gathering of information from units of interest
- **Data Description:** Summarizing and visualizing patterns in data
- **Inference:** Drawing conclusions about populations from sample data

### Fundamental Terminology

| Term | Definition | Example |
|---|---|---|
| Units | Basic objects on which data is collected | Individual students in a class |
| Variable | Characteristic that can take different values | Height, grade, gender |
| Constant | Value that remains same for all units | Academic year (2025/2026) |

## 3.2 Variables and Data Types

### Classification of Variables

Variables are classified into two main categories, each with important subcategories that determine appropriate analytical methods:

### Categorical Variables

- **Nominal:** No logical order (e.g., gender, ice cream flavor, religion)
- **Ordinal:** Natural ordering exists (e.g., education level, satisfaction rating)

### Quantitative Variables

- **Discrete:** Countable, specific values only (e.g., number of children, dice roll)
- **Continuous:** Any value in a range, infinite precision (e.g., weight, distance, temperature)

### Variable Type Decision Framework

| Question | Answer | Variable Type |
|---|---|---|
| Can values be quantified numerically? | No | Categorical |
| Are categories ordered? | No | → Nominal |
| Are categories ordered? | Yes | → Ordinal |
| Can values be quantified numerically? | Yes | Quantitative |
| Only specific values possible? | Yes | → Discrete |
| Any value in range possible? | Yes | → Continuous |

# 3.3 Samples and Populations

## Core Concepts

Statistical inference relies on understanding the relationship between populations (complete groups of interest) and samples (subsets actually measured). This distinction is crucial for valid statistical conclusions.

| Aspect | Population | Sample |
|---|---|---|
| Definition | Complete group of interest | Subset of population actually measured |
| Symbol (mean) | μ (mu) | x■ (x-bar) |
| Symbol (std dev) | σ (sigma) | s |
| Terminology | Parameter | Statistic |
| Size | Often large/infinite | Manageable, smaller |
| Purpose | Target of inference | Source of information |

## Sampling Bias

Sampling bias occurs when the method of selecting samples systematically favors certain outcomes, making the sample non-representative of the population. This threatens the validity of statistical inferences.

## Types of Sampling Bias:

- **Selection Bias:** Non-random sampling method (e.g., voluntary response surveys)
- **Response Bias:** Systematic differences in who responds
- **Non-response Bias:** Those who don't respond differ from those who do
- **Convenience Sampling:** Choosing easily accessible individuals

## Simple Random Sampling

The gold standard for avoiding bias: every member of the population has an equal chance of being selected. This probability-based method ensures representativeness and allows valid statistical inference.

# 3.4 Descriptive Statistics

## Overview

Descriptive statistics summarize and describe data characteristics through three main categories:

- **Central Tendency:** Where is the center of the data?
- **Variability:** How spread out is the data?
- **Relative Standing:** Where does a specific value fall?

## A. Central Tendency Measures

| Measure | Definition | Formula/Method | Best Used When |
|---|---|---|---|
| Mean | Arithmetic average | Σx/n | Symmetric distribution, no extreme outliers |
| Median | Middle value | Middle of ordered data | Skewed distribution or outliers present |
| Mode | Most frequent value | Value with highest frequency | Categorical data or identifying peaks |

## Key Insight: Resistance to Outliers

The median is more resistant to outliers than the mean. For example, in the Star Wars dataset shown in the lecture, the mean height (174.6 cm) is pulled down by shorter characters, while the median (180 cm) better represents the typical height.

## B. Variability Measures

| Measure | Definition | Key Properties |
|---|---|---|
| Range | Max - Min | Simple but sensitive to outliers |
| Variance ($s^2$) | Average squared deviation from mean | Units are squared; not intuitive |
| Standard Deviation (s) | √(Variance) | Same units as data; roughly average distance from mean |
| IQR | Q3 - Q1 | Resistant to outliers; middle 50% spread |

## Standard Deviation Formula:

s = √[Σ(x■ - x■)² / (n-1)] Where: x■ = individual values, x■ = sample mean, n = sample size Note: Division by (n-1) provides unbiased estimate of population variance.

## C. Relative Standing Measures

Percentiles and quartiles describe where individual values fall within a distribution.

| Measure | Interpretation | Usage |
|---|---|---|
| Percentile (P■) | n% of values fall at or below this point | Ranking, growth charts, test scores |
| Q1 (25th percentile) | 25% below, 75% above | Lower quartile |
| Q2 (50th percentile) | Median - half below, half above | Central value |
| Q3 (75th percentile) | 75% below, 25% above | Upper quartile |
| IQR = Q3 - Q1 | Range of middle 50% of data | Outlier detection, spread measure |

# 3.5 Data Visualization

## Overview

Effective visualization transforms raw data into visual patterns that reveal insights quickly and intuitively. The choice of visualization depends on variable type(s) and analysis goals.

## Visualization Selection Guide

| Data Type | Visualization | Purpose | Key Features |
|---|---|---|---|
| One Categorical | Bar Chart | Compare frequencies/proportions | Height = frequency; categorical axis |
| One Quantitative | Histogram | Show distribution shape | Continuous x-axis; reveals modality, skew |
| One Quantitative | Boxplot | Display five-number summary | Shows quartiles, outliers, spread |
| Two Quantitative | Scatter Plot | Reveal relationships | Each point = observation; shows correlation |

## Detailed Visualization Techniques

### 1. Bar Charts

- Used for categorical data
- Bar height represents frequency or proportion
- Categories on x-axis, counts/proportions on y-axis
- Gaps between bars indicate discrete categories

### 2. Histograms

- Display distribution of continuous quantitative data
- Bins group values into ranges
- No gaps between bars (continuous data)
- Reveals: modality (peaks), skewness, outliers
- Example: Star Wars height histogram shows near-normal distribution with slight left skew

### 3. Boxplots (Box-and-Whisker Plots)

- Visualize five-number summary: Min, Q1, Median, Q3, Max
- Box represents IQR (middle 50% of data)
- Whiskers extend to min/max (excluding outliers)
- Outliers shown as individual points
- Excellent for comparing distributions across groups

### 4. Scatter Plots

- Display relationship between two quantitative variables
- Each point represents one observation
- Pattern reveals: direction, strength, form of relationship
- Outliers visible as points far from pattern
- Example: Height vs. mass scatter plot shows positive correlation (except outliers)

## 3.6 Correlation Analysis

### Definition and Interpretation

Correlation measures the strength and direction of the linear relationship between two quantitative variables. The correlation coefficient (denoted r for samples, ρ for populations) is a standardized measure that ranges from -1 to +1.

### Properties of Correlation

| Property | Explanation |
|---|---|
| Range: -1 ≤ r ≤ +1 | Bounded; standardized measure |
| Direction | r > 0: positive; r < 0: negative; r = 0: no linear relationship |
| Strength | \|r\| close to 1: strong; \|r\| close to 0: weak |
| Unit-free | No units; x and y can have different scales |
| Symmetric | r(x,y) = r(y,x) |
| Linear only | Measures linear relationships; may miss nonlinear patterns |

### Interpretation Guidelines

| \|r\| Value | Strength | Interpretation |
|---|---|---|
| 0.0 - 0.3 | Weak | Little to no linear relationship |
| 0.3 - 0.7 | Moderate | Noticeable linear trend |
| 0.7 - 0.9 | Strong | Clear linear relationship |
| 0.9 - 1.0 | Very Strong | Nearly perfect linear relationship |

### Critical Warning: Correlation ≠ Causation

A strong correlation between variables X and Y does NOT imply that X causes Y. Possible explanations for correlation include: (1) X causes Y, (2) Y causes X, (3) a third variable Z causes both X and Y (confounding), or (4) coincidence. Always consider the context and use causal inference methods (covered in Module 6) to establish causation.

# 4. Knowledge Base and Theoretical Foundation

## 4.1 Mathematical Foundations

### Prerequisites

- Basic algebra: equations, inequalities, absolute values
- Summation notation (Σ)
- Square roots and exponents
- Set theory basics (understanding of populations, subsets)

### Key Mathematical Concepts

| Concept | Application in Module |
|---|---|
| Summation (Σ) | Computing means, variances, correlations |
| Square roots | Standard deviation calculation |
| Ordering operations | Finding medians, quartiles, percentiles |
| Ratios and proportions | Proportions, relative frequencies |
| Distance metrics | Deviation from mean (x■ - x■) |

## 4.2 Statistical Theory Foundations

### Core Theoretical Principles

- **Law of Large Numbers:** Sample statistics converge to population parameters as n increases
- **Measurement Theory:** Understanding of scales (nominal, ordinal, interval, ratio)
- **Distributional Properties:** Shape (modality, symmetry, skewness)
- **Sampling Theory:** Representative samples enable valid inference

## 4.3 Computational Tools (R Programming)

The module extensively uses R with the following key packages and functions:

| Package/Function | Purpose | Example Usage |
|---|---|---|
| dplyr | Data manipulation | filter(), select(), mutate() |
| ggplot2 | Visualization | ggplot() + geom_histogram() |
| table() | Frequency tables | table(data$variable) |
| mean(), median() | Central tendency | mean(data$height, na.rm=TRUE) |
| sd(), var() | Variability | sd(data$height, na.rm=TRUE) |

| | | |
|---|---|---|
| cor() | Correlation | cor(x, y) |
| quantile() | Percentiles | quantile(data, probs=c(0.25,0.75)) |

# 5. Techniques and Methods

## 5.1 Data Collection Techniques

| Technique | Description | Advantages | Limitations |
|---|---|---|---|
| Simple Random Sampling | Each unit has equal selection probability | Unbiased, representative | Requires complete population list |
| Stratified Sampling | Random sampling within subgroups | Ensures representation of subgroups | More complex to implement |
| Systematic Sampling | Every kth unit selected | Simple to implement | Can introduce bias if pattern exists |
| Cluster Sampling | Randomly select groups, measure all units in | Cost-effective for dispersed populations | Higher sampling error |

## 5.2 Data Cleaning and Preparation

### Handling Missing Data (NA values)

- Identification: Check for NA, NULL, empty strings
- Deletion: Remove observations with missing values (na.rm=TRUE)
- Imputation: Fill with mean, median, or predicted values
- Analysis: Consider if missingness is random or systematic

### Outlier Detection and Treatment

| Method | Criterion | Action |
|---|---|---|
| IQR Method | < Q1 - 1.5×IQR or > Q3 + 1.5×IQR | Flag as outlier |
| Z-score Method | \|z\| > 3 (or 2.5) | Extreme value |
| Visual Inspection | Points far from pattern in plots | Investigate context |
| Domain Knowledge | Impossible/implausible values | Correct or remove |

**Example from lecture:** In the Star Wars dataset, Jabba Desilijic Tiure (mass = 1358 kg) is a clear outlier. The correlation between height and mass changed from r = 0.13 (with outlier) to r = 0.75 (without outlier), demonstrating the impact of outliers on correlation.

## 5.3 Descriptive Analysis Workflow

### Step 1: Variable Identification

- Classify each variable (categorical/quantitative)
- Identify subtype (nominal/ordinal or discrete/continuous)

### Step 2: Univariate Analysis

- Categorical: Frequency tables, proportions, bar charts

- Quantitative: Mean, median, std dev, histogram, boxplot

### Step 3: Bivariate Analysis

- Two categorical: Cross-tabulation, grouped bar charts
- Two quantitative: Scatter plot, correlation coefficient
- One categorical + one quantitative: Grouped boxplots, comparative statistics

### Step 4: Interpretation

- Describe patterns, trends, and relationships
- Note unusual features (outliers, gaps, multiple modes)
- Connect findings to research questions

# 6. Challenges and Common Pitfalls

## 6.1 Conceptual Challenges

| Challenge | Common Error | Solution |
|---|---|---|
| Variable Classification | Treating ordinal as nominal | Consider natural ordering; use appropriate methods |
| Population vs. Sample | Using sample notation for population | Use correct symbols: μ/σ vs. x■/s |
| Mean vs. Median | Always using mean | Use median for skewed data or when outliers present |
| Correlation Interpretation | Assuming causation | Remember: correlation ≠ causation |

## 6.2 Computational Challenges

### R Programming Issues

- **NA handling:** Forgetting na.rm=TRUE leads to NA results
- **Data type errors:** Treating factors as numeric or vice versa
- **Indexing errors:** Off-by-one errors, incorrect subsetting
- **Package loading:** Forgetting library() calls

## 6.3 Statistical Pitfalls

| Pitfall | Example | Impact | Remedy |
|---|---|---|---|
| Simpson's Paradox | Trend reverses when groups combined | Misleading conclusions | Analyze subgroups separately |
| Ecological Fallacy | Group-level patterns ≠ individual patterns | Invalid individual inferences | Use appropriate level of analysis |
| Cherry-picking | Selecting only favorable statistics | Biased representation | Report all relevant statistics |
| Scale manipulation | Truncated y-axis exaggerates differences | Visual deception | Start axes at zero or clearly note scale |

## 6.4 Sampling and Bias Challenges

- **Volunteer bias:** Self-selected samples may differ from population
- **Survivorship bias:** Only analyzing 'survivors' ignores failures
- **Small sample size:** Insufficient data for reliable inference
- **Non-response bias:** Those who don't respond may differ systematically

# 7. Quality Analysis Framework

## 7.1 Data Quality Assessment

| Dimension | Assessment Criteria | Quality Indicators |
|---|---|---|
| Completeness | Missing data rate, variable coverage | < 5% missing per variable |
| Accuracy | Values within valid ranges, logical consistency | No impossible values (e.g., negative age) |
| Consistency | Internal agreement, cross-variable checks | Related variables agree |
| Timeliness | Data currency, update frequency | Recent enough for purpose |
| Representativeness | Sample mirrors population on key dimensions | Demographic match, unbiased selection |

## 7.2 Analysis Quality Checklist

### Pre-Analysis

- ■ Variables correctly classified by type
- ■ Missing data identified and strategy determined
- ■ Outliers detected and investigated
- ■ Sample size adequate for planned analyses

### During Analysis

- ■ Appropriate statistics chosen for variable types
- ■ Assumptions of methods checked
- ■ Visualizations clearly labeled and scaled appropriately
- ■ Multiple perspectives examined (tables AND plots)

### Post-Analysis

- ■ Results interpreted in context
- ■ Limitations acknowledged
- ■ Conclusions supported by evidence
- ■ Causal language avoided unless justified

## 7.3 Visualization Quality Standards

| Aspect | Best Practice |
|---|---|
| Axis Labels | Clear, with units where applicable |
| Title | Descriptive, indicates what is shown |
| Scale | Appropriate range, typically starting at zero for bar charts |

| | |
|---|---|
| Legend | Present if multiple groups/series, clear labels |
| Color | Accessible (colorblind-friendly), meaningful distinctions |
| Clutter | Minimal chartjunk, high data-ink ratio |
| Context | Reference lines or benchmarks where helpful |

# 8. Troubleshooting Guide

## 8.1 Common R Errors and Solutions

| Error Message | Cause | Solution |
|---|---|---|
| 'x' must be numeric | Trying to calculate stats on categorical variable | Check variable type with class(); convert if needed |
| missing value where TRUE/FALSE needed | NA in logical condition | Use na.rm=TRUE or filter(!is.na(var)) |
| object 'var' not found | Variable doesn't exist or typo | Check spelling, ensure data loaded correctly |
| could not find function | Package not loaded | Use library(package_name) first |
| arguments imply differing number of rows | Vectors different lengths in data.frame() | Ensure all vectors same length |

## 8.2 Unexpected Results Diagnosis

### Mean and Median Very Different

- **Likely cause:** Outliers or skewed distribution
- **Action:** Create histogram; identify and investigate outliers
- **Decision:** Consider using median for central tendency

### Correlation is Zero but Scatter Plot Shows Relationship

- **Likely cause:** Non-linear relationship
- **Action:** Examine scatter plot pattern (U-shaped, exponential, etc.)
- **Decision:** Consider transformation or non-linear methods

### Standard Deviation Larger than Mean

- **Likely cause:** Highly variable or skewed data
- **Action:** Check for outliers, examine distribution shape
- **Decision:** May be normal for some phenomena (e.g., wealth distribution)

## 8.3 Interpretation Troubleshooting

| Confusion | Clarification |
|---|---|
| '25th percentile' vs. 'top 25%' | 25th percentile means 25% are BELOW (not in top 25%) |
| 'Strong correlation' threshold | Context-dependent; \|r\|>0.7 often considered strong |
| When to use IQR vs. std dev | IQR for outlier-prone data; std dev for normal distributions |
| Comparing variability across different scales | Use coefficient of variation (CV = s/x■) for different units |

# 9. Complexity Analysis and Ranking

## 9.1 Complexity Scoring Methodology

Topics are scored on a 1-10 scale based on four dimensions:

- **Mathematical Prerequisites:** Required mathematical background and computational skills
- **Conceptual Abstraction:** Level of abstract thinking required
- **Practical Application:** Complexity of implementation and common pitfalls
- **Interpretation Depth:** Nuance required for correct interpretation

## 9.2 Comprehensive Complexity Ranking

*(Ascending order: easiest to most complex)*

| Rank | Topic/Subtopic | Complexity Score | Rationale |
|---|---|---|---|
| 1 | What is Statistics? (definition) | 1 | Pure memorization, no prerequisites |
| 2 | Basic terminology (units, variables) | 1.5 | Straightforward definitions, minimal abstraction |
| 3 | Categorical vs. Quantitative variables | 2 | Simple classification with clear criteria |
| 4 | Nominal vs. Ordinal classification | 2.5 | Requires recognizing order property |
| 5 | Discrete vs. Continuous variables | 3 | More subtle distinction, judgment calls possible |
| 6 | Frequency tables and proportions | 3 | Basic arithmetic, simple R functions |
| 7 | Bar charts (creation and interpretation) | 3.5 | Straightforward visualization, minimal complexity |
| 8 | Population vs. Sample concepts | 4 | Introduces inferential thinking, notation differences |
| 9 | Mean calculation and interpretation | 4 | Basic formula, requires summation notation |
| 10 | Median: finding and interpretation | 4.5 | Requires ordering, even/odd cases |
| 11 | Simple random sampling | 5 | Conceptually clear but implementation requires care |
| 12 | Histograms (creation and interpretation) | 5 | Binning decisions, shape interpretation |
| 13 | Sampling bias identification | 5.5 | Requires critical evaluation of study design |
| 14 | Range and IQR | 5.5 | Simple calculation, straightforward interpretation |
| 15 | Percentiles and quartiles | 6 | Ordering + calculation, multiple methods exist |
| 16 | Boxplots (creation and interpretation) | 6.5 | Integrates 5-number summary, outlier detection |
| 17 | Variance calculation | 7 | More complex formula, squared units non-intuitive |
| 18 | Standard deviation | 7 | Square root of variance, interpretation requires care |
| 19 | Scatter plots and pattern recognition | 7.5 | Requires seeing beyond individual points |
| 20 | Comparing distributions across groups | 8 | Integrates multiple concepts, nuanced interpretation |
| 21 | Correlation coefficient | 8 | Complex calculation, many interpretation subtleties |

| | | | |
|---|---|---|---|
| 22 | Outlier detection and treatment decisions | 8.5 | Requires judgment, contextual understanding |
| 23 | Correlation interpretation (causation issues) | 9 | High conceptual depth, common misconceptions |
| 24 | Comprehensive EDA integration | 9 | Synthesizing all techniques appropriately |

## 9.3 Complexity Tier Analysis

### Tier 1: Foundational (Complexity 1-3)

Basic definitions and classifications. Students with no prior statistics experience should master these first. Minimal mathematical prerequisites. Focus on memorization and recognition.

**Topics:** Definitions, variable types, basic terminology

**Time investment:** 1-2 weeks

### Tier 2: Core Descriptive Statistics (Complexity 4-6)

Essential statistical measures and basic visualizations. Requires arithmetic and basic algebra. Students begin applying formulas and interpreting results.

**Topics:** Mean, median, sampling basics, histograms, basic bias concepts

**Time investment:** 2-3 weeks

### Tier 3: Advanced Descriptives (Complexity 7-8)

More sophisticated statistics and visualizations. Requires understanding of distributions, variability concepts, and relationship patterns.

**Topics:** Std dev, boxplots, scatter plots, correlation, group comparisons

**Time investment:** 3-4 weeks

### Tier 4: Integration and Critical Thinking (Complexity 9-10)

Synthesizing multiple concepts, making nuanced interpretations, and avoiding common pitfalls. Requires conceptual maturity and practical experience.

**Topics:** Correlation vs causation, outlier treatment decisions, comprehensive EDA

**Time investment:** 2-3 weeks + ongoing practice

# 10. Learning Pathway Recommendations

## 10.1 Suggested Learning Sequence

### Week 1-2: Foundations

- Study definitions and basic terminology
- Practice variable classification with real-world examples
- Learn R basics: data frames, vectors, basic functions
- Create first frequency tables and bar charts

### Week 3-4: Central Tendency and Sampling

- Calculate mean and median by hand, then in R
- Understand when to use each measure
- Study sampling methods and bias sources
- Practice identifying bias in real studies

### Week 5-6: Variability and Distributions

- Master standard deviation calculation and interpretation
- Create and interpret histograms
- Understand distribution shapes (symmetry, skewness, modality)
- Learn percentiles and quartiles

### Week 7-8: Advanced Visualizations

- Master boxplots: creation and interpretation
- Create scatter plots and identify patterns
- Practice comparing distributions across groups
- Develop outlier detection and handling strategies

### Week 9-10: Correlation and Integration

- Calculate and interpret correlation coefficients
- Deeply understand correlation ≠ causation
- Practice comprehensive exploratory data analysis
- Complete integrated case studies

## 10.2 Practice Recommendations

| Skill Level | Practice Activities | Hours/Week |
|---|---|---|

| | | |
|---|---|---|
| Beginner | Work through lecture examples line-by-line; reproduce outputs | 6-8 |
| Intermediate | Analyze new datasets; compare results to expectations | 5-7 |
| Advanced | Critique published analyses; identify potential biases | 4-6 |
| Expert | Mentor others; create original analyses; present findings | 3-5 |

## 10.3 Resource Recommendations

### Textbooks

- *OpenIntro Statistics* (free online) - comprehensive, R-integrated
- *The Art of Statistics* by David Spiegelhalter - conceptual understanding
- *Practical Statistics for Data Scientists* - applied focus

### Online Resources

- Khan Academy Statistics - video tutorials
- R for Data Science (r4ds.had.co.nz) - R programming
- StatQuest YouTube channel - visual explanations

### Datasets for Practice

- R built-in datasets: mtcars, iris, starwars (used in lectures)
- UCI Machine Learning Repository
- Kaggle datasets (beginner-friendly)
- FiveThirtyEight data repository

# 11. Appendix: Additional Resources

## 11.1 Key R Functions Quick Reference

| Function | Purpose | Example |
|---|---|---|
| table() | Frequency table | `table(data$gender)` |
| prop.table() | Proportions | `prop.table(table(data$gender))` |
| mean() | Mean | `mean(data$height, na.rm=TRUE)` |
| median() | Median | `median(data$height, na.rm=TRUE)` |
| sd() | Standard deviation | `sd(data$height, na.rm=TRUE)` |
| var() | Variance | `var(data$height, na.rm=TRUE)` |
| quantile() | Percentiles | `quantile(data$height, probs=c(0.25,0.75))` |
| summary() | Five-number summary + mean | `summary(data$height)` |
| cor() | Correlation | `cor(data$height, data$mass, use='complete.obs')` |

## 11.2 ggplot2 Visualization Quick Reference

| Plot Type | Geom | Example Code |
|---|---|---|
| Bar chart | geom_bar() | `ggplot(data, aes(x=category)) + geom_bar()` |
| Histogram | geom_histogram() | `ggplot(data, aes(x=value)) + geom_histogram(bins=30)` |
| Boxplot | geom_boxplot() | `ggplot(data, aes(x=group, y=value)) + geom_boxplot()` |
| Scatter plot | geom_point() | `ggplot(data, aes(x=var1, y=var2)) + geom_point()` |

## 11.3 Formula Reference

| Statistic | Formula |
|---|---|
| Sample Mean | x■ = (Σx■) / n |
| Sample Variance | $s^2 = \Sigma(x■ - x■)^2 / (n-1)$ |
| Sample Std Dev | $s = \sqrt{[\Sigma(x■ - x■)^2 / (n-1)]}$ |
| Correlation | $r = \Sigma[(x■-x■)(y■-■)] / \sqrt{[\Sigma(x■-x■)^2 \times \Sigma(y■-■)^2]}$ |
| IQR | IQR = Q3 - Q1 |
| Proportion | p = (count in category) / (total count) |

## 11.4 Glossary of Key Terms

| Term | Definition |
|---|---|
| Bias | Systematic error that makes sample non-representative of population |

| | |
|---|---|
| Categorical Variable | Variable with values that are categories or groups |
| Continuous Variable | Quantitative variable that can take any value in a range |
| Correlation | Measure of strength and direction of linear relationship between two variables |
| Discrete Variable | Quantitative variable that can only take specific, countable values |
| Distribution | Pattern of how data values are spread across possible values |
| IQR | Interquartile Range; range of middle 50% of data (Q3 - Q1) |
| Mean | Arithmetic average; sum of values divided by count |
| Median | Middle value when data is ordered from smallest to largest |
| Nominal Variable | Categorical variable with no inherent order |
| Ordinal Variable | Categorical variable with natural ordering |
| Outlier | Data point that falls far from the bulk of the data |
| Parameter | Numerical summary of a population (e.g., μ, σ) |
| Percentile | Value below which a given percentage of observations fall |
| Population | Entire group of individuals of interest |
| Quartile | Values dividing ordered data into four equal parts |
| Sample | Subset of population actually measured |
| Standard Deviation | Measure of variability; roughly average distance from mean |
| Statistic | Numerical summary of a sample (e.g., x■, s) |
| Variable | Characteristic that can take different values across units |
| Variance | Average of squared deviations from mean |

## 11.5 Conclusion and Next Steps

This comprehensive analysis of the "Exploring Data" module reveals a well-structured introduction to statistical thinking and data analysis. The module progresses logically from foundational concepts through increasingly sophisticated analytical techniques.

The complexity ranking identifies a natural learning progression, with basic definitions and variable classification serving as entry points (complexity 1-3), followed by core descriptive statistics (complexity 4-6), advanced techniques (complexity 7-8), and culminating in integration and nuanced interpretation (complexity 9-10).

Students mastering this module will have a solid foundation for the remaining curriculum modules:

- **Module 2: Distributions** - Builds on histograms and distribution shapes
- **Module 3: Estimation and Inference** - Extends sample-population concepts
- **Module 4: Linear Models** - Formalizes correlation into regression
- **Module 5: Panel Data** - Adds longitudinal dimension to analysis
- **Module 6: Causal Inference** - Addresses correlation vs causation rigorously

The practical, R-integrated approach throughout the module ensures students develop both conceptual understanding and computational proficiency, preparing them well for advanced data science work.

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