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Research Article

A multicountry comparison of cryptocurrency *vs* gold: Portfolio optimization through generalized simulated annealing

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ABSTRACT

The last few years have seen a paradigm shift in the financial sector with the development of cryptocurrencies as an alternative mode of payment as well as an investment scheme. The aim of this study is two-fold. The first is to quantify the volatility of cryptocurrencies in terms of the dynamics of tail-end behavior using different approaches and choose the one with the lowest value-at-risk. The second is to investigate the effect of its inclusion in a portfolio with and without gold, to see if Bitcoin is indeed the “digital gold”. This paper uses the generalized simulated annealing optimization technique to compare portfolios for ten countries across the world. The data provide convincing evidence in favor of the inclusion of Bitcoin in the optimized portfolios. Rolling-window analyses (three-year and five-year) confirm the same. However, for some countries, the empirical pattern suggests that instead of replacing gold from the portfolio, both should be comprised. Our results are robust in terms of the inclusion of non-linear constraints.

1. Introduction

As a direct consequence of unparalleled technological evolution going hand-in-hand with internationalization, the modern world witnesses a degree of financial penetration like never before. During the global financial crisis and the European debt crisis, investors found gold to be a safe haven for diversifying their portfolios. However, the growth of the internet, which rapidly interconnected smart devices across the world, led to the development of various virtual currencies as an alternative route for both transactions and speculative purposes. As of April 2021, cryptocurrencies claimed a market capitalization worth more than USD 2.3 trillion[1]. To put this into perspective, the big four, namely Apple, Amazon, Facebook, and Microsoft are worth USD 2.2 trillion, USD 1.7 trillion, USD 0.9 trillion, and USD 1.9 trillion, respectively. Several studies have directly compared gold and bitcoin, either based on volatility using GARCH models [1,2], or by contrasting their safe-haven properties [3]. The idea of bitcoin as a possible replacement for gold continues to be a source of much debate. The aim of this paper is to extend this field of research by performing portfolio analysis with equities, gold, and cryptocurrencies across the largest economies in the world. Given the capricious nature of virtual currencies, in the present study, we seek to determine the cryptocurrency with the lowest volatility in terms of tail-end attributes and employ a non-linear optimization methodology to find the risk-adjusted returns of the constructed portfolios. Note that it is not within the scope of this paper to take into consideration the behavioral aspects of investors.

In 1999, Nobel laureate Milton Friedman foresaw that the internet would facilitate digital fund transfers without revealing the participants of the transaction, reducing the role played by the government which would eventually revolutionize the financial systems of the world. This idea was brought to life by Nakamoto[2] in 2008, who introduced the world to the first cryptocurrency Bitcoin (BTC) in the article “Bitcoin: Peer to Peer Electronic Cash System” [5]. Since then, the popularity of cryptocurrencies has risen rapidly with newer cryptocurrencies coming into the foray every year[3]. Currently, there exist more than 2000 different actively traded cryptocurrencies in circulation with Bitcoin (59.42%),

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[1] Source: https://coinmarketcap.com/
[2] Satoshi Nakamoto, is a “nom de plume”, where “Sa” stands for Samsung, “Toshi” means Toshiba, “Naka” refers to Nakamushi and “Moto” is from the Motorola [4].
[3] In the first ever bitcoin trade, Laszlo Hanyecz purchased two pizzas worth USD 25 with 10,000 BTC. It is perhaps the most expensive transaction in the world with 1 BTC worth approximately USD 60,000 till now.

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Ethereum (11.09%), Binance Coin (2.36%), Tether (2.30%), Litecoin (0.81%), and Dogecoin (1.76%) being the relatively popular ones.

While most countries show a negative attitude towards cryptocurrencies, they are legal in countries like France, Russia, Canada, Argentina, Brazil, Republic of Korea, Thailand, and China. Japan was one of the first countries to legalize bitcoins, with the adoption of the Payment Services Act in 2010. Bitcoin was accepted as a decentralized currency in the USA in 2013, with the Commodity Futures Trading Commission (CFTC) categorizing it as a commodity in 2015. Germany actively participates in the development of blockchain solutions, while the Netherland has a special region called “Bitcoin City”. In recent times, cryptocurrencies have been hailed as the New Gold [6] due to their appeal, not only for computer scientists and venture capitalists but also for investors, with the world's largest firms like Tesla and Google deciding to invest in them. Their attractiveness lies in the fact that they are a decentralized mode of exchange, maintaining the pseudonymity of transactors along with a high security architecture and uncomplicated functionality. At present, retail investors all over the world can trade cryptocurrencies via several global exchange platforms. This increased accessibility and limelight are the motivation behind the research objective. However, owing to its lack of supervision and limited supply fixed at 21 million, bitcoin shows highly volatile bubble-like price movements and, as such, is considered a speculative asset at present, with prices being propelled by the greater fool's theory.

Value-at-Risk, widely known as VaR, is a measurement of risk exposure used extensively in financial services. VaR quantifies the probable depletion in the valuation of a risky asset or portfolio, given a confidence interval for a predefined time period. It focuses on tail-end events of any distribution, underlining the downside risk and potential losses. Although risk measures like VaR have been in use since 1922, they were given a mathematical abutment by Harry Markowitz and other economists in the 1950s in the context of portfolio optimization. However, it was not until the 1990s that VaR started being adopted to measure market-risks by financial institutions. Fundamentally, there are three procedures to determine VaR, with variations within each approach. It can be found from the historical data of the financial instrument. The second measure is to fit a parametric distribution to the empirical data and estimate VaR from the moments of the distribution. The third method relies on the Monte-Carlo simulation.

The first objective of the paper is to choose the cryptocurrency with the lowest VaR by employing the aforementioned VaR approaches. The second objective of the paper is to determine whether gold or cryptocurrency has better potential for investors. Some research suggests that though the volatility fluctuations of bitcoin are similar to that of gold and silver, they cannot be included in a portfolio as a diversifier [2]. Others claim that bitcoin performs better than gold when included in a minimum variance portfolio, weights being determined through multivariate time-series analysis [1]. This paper moves away from the simplistic mean-variance analysis propounded by Markowitz [7] and employs the generalized simulated annealing (GenSA) technique [8] for a multi-constraint portfolio optimization model using tail-end risk measurements to choose the efficient frontier. The incitement to use a non-linear optimization methodology like GenSA is to reconcile with the quadratic programming problem that this paper deals with, given the constraints of liquidity and leverage. A 3-year and 5-year rolling window approach is considered for robust results. Additionally, an analysis for 2020 is performed to account for possible variation due to the COVID-19 pandemic.

The quantitative analysis in this paper reveals Bitcoin to have the lowest VaR compared to other cryptocurrencies, which is used in the portfolios for all ten countries. For all countries, the performance of the portfolio including only Bitcoin and equities, in terms of Sharpe, Sortino, and Information ratios, is unparalleled from both 3-year and 5-year dynamic analysis. During 2020, however, a portfolio including both bitcoin and gold alongside equities showed better performance in five out of ten countries. The general picture emerging from the analysis is that despite its high volatility, Bitcoin included in the portfolio, even in small proportions, enhances the returns significantly and counteracts the associated risks. This study also validates the importance of gold, given that for countries where portfolio 3 outweighs the others, gold constitutes a high percentage of the optimized portfolios given its negative or negligible correlation with both equities and cryptocurrencies.

The paper has the following structure: We initially present a brief literature review in Section 2. In Section 3, we discuss the data and describe the methodology in Section 4. We discuss the results in Section 5 and conclude in Section 6.

2. Literature review

Conventionally, gold has always been used to counter inflation and preserve capital due to its little or negative correlation with orthodox paper assets like bonds and stocks [9,10]. However, whether or not gold is indeed a safe haven or hedge for investors is debatable. Before moving forward, it is important to distinguish the terms. A hedge is an instrument that is uncorrelated or negatively correlated with other assets or portfolios on average, while a safe haven is the same but during extreme market conditions [11]. Further, a diversifier is an asset that is positively (but not perfectly) correlated with other instruments in the portfolio. Although there is no theoretical premise behind gold being a safe haven, anecdotal commentaries and financial media seem to suggest so, maybe because gold is one of the first modes of currency to exist. This is well witnessed in traditional investors' mentality across the world. A rigorous econometric analysis by Baur and Lucey [11] studying gold returns in the USA, UK, and Germany found that gold is neither a hedge nor a safe-haven for bonds, but acts as both against equities. Note that its safe-haven properties are considerably short-lived, about fifteen days only. This is justified since gold-returns are linked to several fluctuating factors such as oil prices, currency movements, and monetary policy as a whole, and price expectations are not aligned with income flows can alter rapidly [12]. In the end, gold is a speculative investment as it does not earn any income on its own and only provides a return to investors through increasing prices.

Bitcoin was also identified as a speculative asset rather than an alternative mode of currency, as was intended when it was first introduced [13]. Both Bitcoin and gold are limited in supply and derive their valuation from the interaction of demand-supply forces. As the oldest and most popular cryptocurrency in the market, Bitcoin has also become a safe-haven, especially for millennial investors who put their trust in technology. Another aspect in common with gold is that it shares little or no correlation with traditional investment classes and, as such, can be used as a diversifier during both normal and extreme times [14]. Hong [15] built a time-series model in Bitcoin returns to empirically justify its use as an alternative investment vehicle. His work concluded that trading strategies based on the momentum of Bitcoin returns increase yields and decrease volatility, resulting in consistent profitability of the portfolios comprising equities along with Bitcoin.

The key issue regarding Bitcoin as an individual investment instrument is its volatile price movements. Baek and Elbeck [16] found that Bitcoin is 26 times more volatile than the S&P 500 index, but this was bound to decline as the market matured. Osterrieder and Lorenz [17] found that Bitcoin is 6–7 times as volatile as G-10 currencies. Bitcoin's first price jump occurred in 2010 when it moved from USD 0.0008 to USD 0.08 within a short span. It went through several price bubbles in the following years until 2017, when it started at USD 1000 in January and climbed to USD 20000 in December. In 2018, Bitcoin traded at an all-time high of USD 20000 per coin, falling to USD 4000 in 2019 and reaching its present peak of USD 60000 in 2021. This return behavior is believed to have been driven fundamentally by the volume of trade and sentiment expressed in the media [18]. But this is not much different from gold. In the 1970s, when the gold standard was abolished by US president Nixon, gold price (per ounce) rose from USD 35 in 1971 to USD 180 in 1974 to USD 850 in 1980, and since then, it has had a volatile journey. Despite that, people continued using it in their portfolios due to its compensating returns and, at present, it has the status of a safe-haven. People are of the general opinion that, with considerable time, Bitcoin will tend to a similar status.

There is an extensive literature on the volatility modeling of cryptocurrencies. While some use variations of GARCH models [3,19,20], our paper intends to approximate the entire distribution of different cryptocurrencies using popular theoretical distributions. This is in line with Osterrieder and Lorenz's [17] work who characterized bitcoin return distribution as non-normal and heavy-tailed. The study of the distribution and extreme-tail behavior of cryptocurrencies is of utmost importance due to its implications in the fields of financial engineering, portfolio-management, and risk-assessment. Operating on parametric distributions, this paper further tries to estimate the VaR and compare several cryptocurrencies on that basis. Furthermore, VaR is used as the risk objective in the portfolio optimization to compare gold and Bitcoin portfolios.

Since it gained prominence, Bitcoin has been termed "digital-gold" and several works have drawn its comparison with gold itself. Dyhrberg [21] investigated the hedging capabilities of Bitcoin in comparison to gold using asymmetric GARCH methodology. He concluded that Bitcoin could be used alongside gold as a hedge against the FTSE index and the US Dollar. Henriques and Sadorsky [1] again investigated the claim of whether or not to include Bitcoin in place of gold in one's portfolio with the use of different GARCH models (dynamic conditional correlation (DCC), asymmetric DCC (ADCC), generalized orthogonal GARCH (GO-GARCH)) to minimize the portfolio variance. Their robust results on trading costs conclude that risk-averse investors would be willing to pay a premium to switch from a portfolio including gold to that including Bitcoin. Other works claim that Bitcoin can never replace gold in a portfolio due to its bubble-like nature [2,22]. One major constraint regarding the usage of Bitcoin is due to its liquidity. It demonstrates less liquidity compared to most investment classes [23]. However, a recent paper by Trimborn et al. [24] shows that despite low liquidity, optimizing a portfolio with Bitcoin using the Liquidity Bounded Risk-Return Technique to control for risk and liquidity issues improves performance significantly. Ghabri et al. [25] showed that adding bitcoin to a portfolio can mitigate liquidity risk under the Mean-Variance-Liquidity framework using various multivariate-GARCH models. They also found that the gold market demonstrates the least liquidity.

The basic Markowitz mean-variance optimization model of portfolio choice considers risk-averse individuals looking to maximize their risk-adjusted returns. Despite its simplicity and acclaimed status in financial economics, it fails to include real-world constraints such as dimensions of portfolio, trading restrictions, etc. Crama and Schyns [27] tried to solve this non-linear mixed integer programming problem through a metaheuristic simulated annealing approach. In our effort towards the choice of portfolio selection between gold and Bitcoin, we employ the aforementioned generalized simulated annealing technique for portfolio optimization, using VaR as a constraint, besides adding other constraints such as box, leverage, transaction cost, etc.

3. Data sources

For our first objective, historical prices for the four most widely traded cryptocurrencies (as of February 2021), namely Bitcoin, Ethereum, Litecoin, and Dogecoin, were collected from Yahoo finance. All four cryptocurrencies were issued in different years, however, this paper includes data from September 17, 2014 to December 31, 2020, as they are available on the public portal. The rationale behind the selection of these four cryptocurrencies was to represent the asset-class as a whole, given the market shares of Bitcoin, Ethereum, and Litecoin. Additionally, Dogecoin was selected due to its recent recognition in the financial media. Further, XRP and Binance coins were considered but avoided due to erratic data. Tether was also analyzed but doesn't align with the objective of minimizing volatility by being a stable coin. We evaluate VaR for each, using daily returns. The adjusted close prices for each of the four cryptocurrencies are plotted below in Fig. 1.

In order to construct portfolios, we collect price data of the four largest companies (market capitalization) as of February 2021 for each of the ten countries, along with historical spot price for gold (GC=F COMEX Delayed Price) as a benchmark representation for the equity market. Specifically, we calibrate different countries from different continents to incorporate investors' behavior from across the world. The choice of the countries is based on the distribution of bitcoin mining hash rate among G20 nations. Daily adjusted-close-price data were collected from open-source Yahoo finance for the period September 17, 2014 to December 31, 2020. The risk-free rate used for portfolio optimization is the ten-year sovereign yield rate for each country. The respective index prices are also considered the benchmark. Since Bitcoin trades seven days a week but others do not, the missing values are omitted for the analysis. To get a sense of volatility, Fig. 2 plots the returns of Bitcoin and Gold.

4. Methodology

To choose the cryptocurrency with the lowest VaR, we employ three broad measures, namely historical simulation, Monte Carlo simulation, and variance-covariance method, which are discussed in Section 4.1. In Section 4.2, we try to determine whether to include gold or cryptocurrency in one's portfolio using the generalized simulated annealing model.

4.1. VaR estimation

VaR is a tail-end risk-adjusted measure of the potential loss in the value of a risky asset or portfolio for a pre-defined confidence interval over a period of time. Thus, VaR for a financial instrument i at the quantile level $\tau \in (0,1)$ is defined as:

$$P(X_{i,t} \leq VaR_{i,t,\tau}) \stackrel{\text{def}}{=} \tau \tag{1}$$

where $X_{i,t}$ represents the log-returns of the asset or portfolio at time t.

This measure is extensively used by commercial and investment banks to quantify downside risk, leading to liquidity crisis from adverse market movements in prices or rates. Although it has been in use since the mid-20th century, it gained impetus in 1995 when J.P. Morgan allowed public access to the data on various securities and asset classes, along with the development of a service called "RiskMetrics™". Since then, several developments in the measurement of VaR have evolved and are liberally used by regulators and institutions in accord[4].

There exist three fundamental approaches to estimating VaR [28], with several modifications within each procedure. VaR can be calculated using values of past returns in the historical method (Section 4.1.1) by running hypothetical portfolios. In Monte Carlo simulation (Section 4.1.2), one randomly generates multiple trials to compute a formula without any closed (analytical) form. Finally, VaR can also be measured by approximating the empirical distribution with a parametric distribution (Section 4.1.3) to find the scale and location parameters from the data. We explicitly explore this particular approach in our study as we expect cryptocurrencies to be a major area of focus for financial literature in the near future, and hence knowing the underlying distribution would be of vital importance.

[4] Expected shortfall (ES) or Conditional Value-at-Risk (CVaR) is another risk-metric based on extreme-tail behavior. The expected shortfall at q% level of significance gives the expected return on the asset or portfolio in the worst q% scenarios. The ES at time t for financial institution j, given X_i at quantile level $\tau \epsilon (0,1)$ is given by: $P(X_{j,t} \leq ES_{j|i,t,\tau} | R_{i,t}) \stackrel{\text{def}}{=} \tau$, where $R_{i,t}$ denotes the information set which includes the event of $X_{i,t} = VaR_{i,t,\tau}$.

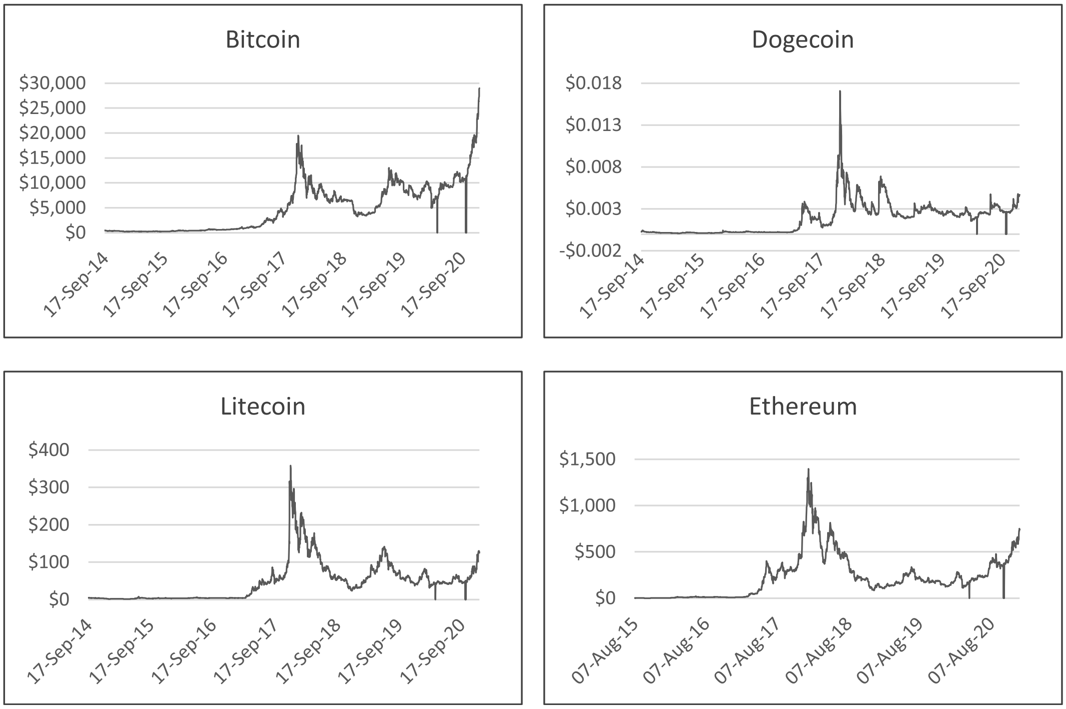

Fig. 1. Bitcoin, Dogecoin, Litecoin (from Sep 14, 2015 to Dec 31, 2020) and Ethereum prices (from Aug 7, 2015 to Dec 31, 2020).
All major cryptocurrencies had a bull run during 2017 where capital flowed from fiat into digital currency. However, in 2018, the mirage dissipated into what came to be known as the Bitcoin crash. Since then, cryptos have been developed with streamlined objectives and have seen a rally since 2020 to reach the highest market capitalization in 2021.

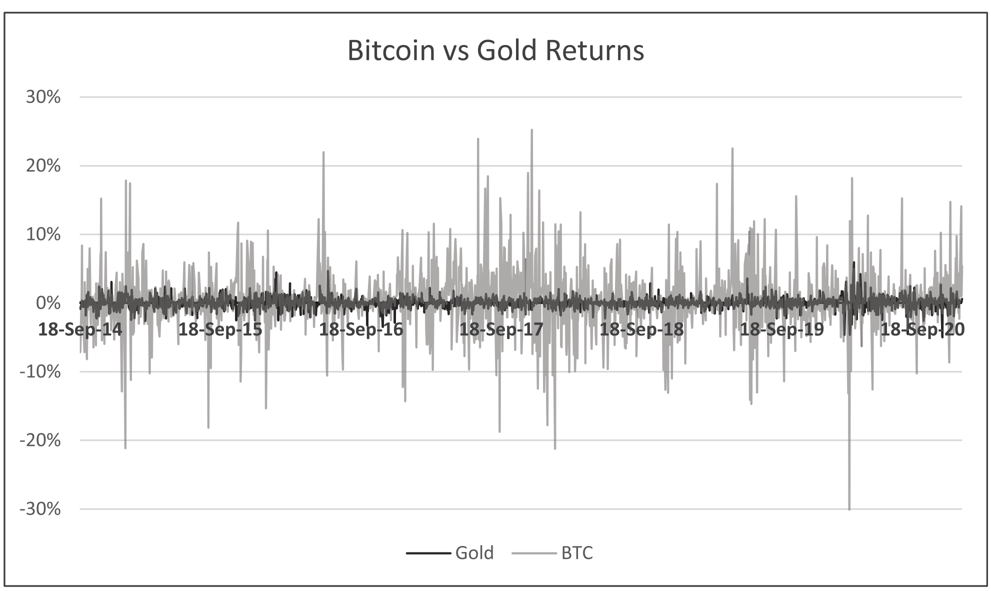

Fig. 2. Daily returns of Bitcoin and Gold.
Both Bitcoin and Gold prices are stationary at first difference according to ADF tests. Noteworthy is the comparison of price volatility between the two, with Bitcoin showing noticeably more eccentric returns.

4.1.1. *Historical simulation*

In this approach, VaR for an asset is calculated from actual returns by organizing them in ascending order (worst return to best return) using Eq. (1). For example, if we want to use 100 days of data to find the 1 day-99% VaR for a financial instrument, it corresponds to the worst day of return, i.e., the 99th percentile of the worst 1% returns. The percentage return is given as

$$X_{i,t} = \ln \frac{V_{i,t}}{V_{i,t-1}} \tag{2}$$

where $V_{i,t}$ denotes the prevalent market value of the i-th instrument at time t.

In this method, there are no underlying distributional assumptions and VaR is measured through actual price movements. However, it assumes that history will repeat itself in terms of volatility, and past performance is an appropriate estimate of future performance. Further, this method gives equal weightage to all returns irrespective of whether there is a trend in the variability, and hence is slow to react to changing market environments.

Hence, we use a hybrid method for historical VaR [29], which estimates VaR by applying exponentially diminishing weights to past returns followed by finding the appropriate percentile for the weighted empirical distribution. We define the weighing factor for the *i*-th return, arranged in ascending order as:

$$W_i = \frac{(1-\lambda)}{(1-\lambda^K)}\lambda^i, i = (0,1,2,...K-1) \tag{3}$$

where K is the most recent return and $\lambda \in [0,1]$ corresponds to the exponential moving average denoting the decay of relevance as we move from the most recent to past observations. Mostly, λ values lie between 0.97 and 0.995. Note that $\sum_{i=0}^{K-1} W_i \approx 1$.

4.1.2. Monte Carlo simulation

The idea of VaR is to ascertain that an asset will not lose more than a given percentage in the following N days with a given probability. Under Monte Carlo simulation, the objective is to generate random hypothetical trials continually. The algorithm engenders random numbers from which we obtain a formula without any analytical form. It is different from historical simulation in that the former uses original historical values.

We try to model the daily log returns (X_1, X_2, ….) of the financial instrument on a continuously compounding basis.

$$X_{t+1} = \mu_t + \sigma_t \varepsilon_{t+1} \tag{4}$$

where μ_t is the expected daily return conditional on the information set $R_{i,t}$; σ_t is the standard deviation (SD) conditional on $R_{i,t}$; ε_{t+1} is the white noise process, denoting shocks which are independent and identically distributed (i.i.d) with mean zero and SD of one.

Duffie and Pan [30] introduced this model to estimate VaR, assuming price movements follow a normal distribution. They introduced a jump-diffusion model (where kurtosis is a declining function of time horizon) and a stochastic-volatility process (where it is an increasing function of time). However, as we know, returns are not necessarily normally distributed and hence Hull and White [31] relaxed the normality assumption and found an efficient way to approximate the risk on an asset or portfolio.

Let us denote the vector of risk factors as $S_i(t) = \begin{bmatrix} S_1(t) \\ \vdots \\ S_n(t) \end{bmatrix}$.

We assume that $S_i(t)$, i=1,2,3, …, n obey a couples stochastic differential equation where dZ_i denotes Brownian motions correlated with each other unlike Eq. (4), whereby a drift and volatility parameter could approximate the needful.

$$d \ln S_i(t) = \mu_i dt + dZ_i \tag{5}$$

Note: $dZ_i \sim N(0, d\Sigma)$, where $d\Sigma_{ij} = \sigma_i \rho_{ij} \sigma_j dt$ denotes the variance-covariance matrix for dZ_i which is assumed to be multivariate-normally distributed.

The objective of this method is to expand the value of the portfolio V, as a function of its risk factors S_i, in terms of Taylor series up to its second order:

$$\begin{aligned} \delta V(S(t)) &= V(S(t)+\delta S(t)) - V(S(t)) \\ &\approx \sum_i^n \frac{\partial V}{\partial S_i}\delta S_i(t) + \frac{1}{2}\sum_{i,j}^n \delta S_i(t)\frac{\partial^2 V}{\partial S_i \partial S_j}\delta S_j(t) \\ &= \sum_i^n \Delta_i \delta S_i(t) + \frac{1}{2}\sum_{i,j}^n \delta S_i(t)\Gamma_{ij}\delta S_j(t) \\ &\approx \sum_i^n \tilde{\Delta}_i[\mu_i \delta t + \delta Z_i] + \frac{1}{2}\sum_{i,j}^n [\mu_i \delta t + \delta Z_i]\tilde{\Gamma_{ij}}[\mu_j \delta t + \delta Z_j] \\ &\approx \sum_i^n \tilde{\Delta}_i \delta Z_i + \frac{1}{2}\sum_{i,j}^n \delta Z_i \tilde{\Gamma_{ij}} \delta Z_j \end{aligned} \tag{6}$$

After using Taylor's expansion, we input the linear approximation of risk factors, and ignore the average returns. We finally obtain Eq. (6), which is the delta-gamma approximation. Note that Δ_i and Γ_{ij} denote the sensitivity of market value V, with respect to risk factors. Γ_{ij} is also known as the Hessian matrix.

$$\Delta_i = \frac{\partial V}{\partial S_i}, \ \Gamma_{ij} = \frac{\partial^2 V}{\partial S_i \partial S_j}, \ i,j = 1,2,\dots,n \tag{7}$$

In this paper, we intend to estimate VaR using Monte-Carlo simulation after computationally determining delta and gamma.

4.1.3. Parametric distribution approach

As defined earlier, for a specified time period t and confidence limit p, VaR is the decline in the market value that is exceeded with probability $1-p$. In the variance-covariance method, the volatilities and correlations of risk factors and the sensitivities of asset values with respect to the risk factors are employed to approximate the VaR. It is an analytical way that assumes a bell-shaped distribution of returns.

For a given confidence interval alpha α, VaR can be defined as:

$$VaR_\alpha(X_t) = -\sup\{x \in \mathbb{R} : F_x \le 1-\alpha\} \tag{8}$$

VaR is a closed-form function of the correlation and standard deviation of the returns. Given that returns are multivariate normally distributed (by assumption), the variance-covariance matrix is tallied through “risk mapping”. However, the assumption of the multivariate normal distribution is irrational. Hence, we intend to estimate the parameters of a theoretical distribution that best approximates the empirical data.

We first perform the Tukey-Lambda test to check the conformity of the empirical data to a specific theoretical distribution. Tukey-Lambda is conventionally used to categorize the appurtenant distribution that befits the sample with regard to its quantile function defined in terms of a shape parameter, λ (location parameter, μ and scale parameter, σ might also be used to generalize). The inverse cumulative distribution function, or quantile function $Q(p;\lambda)$ is defined in terms of its cumulative probabilities, p:

$$Q(p;\lambda) = \begin{cases} \ln \frac{p}{1-p}, \text{ if } \lambda = 0 \\ \frac{1}{\lambda}\left(p^\lambda - (1-p)^\lambda\right), \text{ if } \lambda \neq 0 \end{cases} \tag{9}$$

Lower the λ, the tails of the distribution become fatter and heavier. And more likely it is that the distribution is characterized by extreme events, influencing the returns. Hence, we maximize the probability-plot-correlation-coefficient by changing λ. The probability plot correlation coefficient (PPCC) [32] is a diagrammatic approach to identify the shape parameter for a distributional family that approximates the empirical data suitably. The values of lambda and corresponding distributions are provided in Table A.1 in the appendix.

In an effort to classify the distributional family that best describes the returns data for the cryptocurrencies, we model using the above distributions and a few more generalized distributions which have lambda values between Laplace and Cauchy. We explore Johnson's Su distribution, Slash distribution, Generalized Pareto distribution, and Generalized extreme value distribution. The supremum is found using the Quantile functions constructed from the respective cumulative distribution functions. Table A.2 (see appendix) summarizes the distributions and the respective CDFs used in the paper.

Despite CVaR being a more precise risk metric than VaR, we choose VaR as a measure in the optimization objective as it offers more statistical stability than CVaR estimates. VaR disregards the extreme tail-ends (most unstable parts of the distribution), which actually leads to better out-of-sample performance than CVaR, as by definition the CVaR accounts for the most extreme tail-end losses [33]. Estimators based on VaR are also statistically robust and automatically overlook outliers for instruments with mean-reverting properties, and the Augmented-Dicky-Fuller and KPSS tests show that our data are indeed mean-reverting.

4.2. Portfolio optimization

In Markowitz optimization, one has to maximize returns along with minimizing risk [7]. Risk is measured with mean squared deviation from expected returns in the Markowitz model. Despite its theoretical viability, the classical Markowitz model is too unsophisticated to represent real-world scenarios with stricter constraints. Firstly, a better quantile-based risk measure can be VaR. Additionally, liquidity constraints, box constraints, and transaction cost constraints add more depth to the problem. Given the non-linearity of the optimization problem along with the noisy data and additional constraints, this paper intends to extend the model into a multi-constraint portfolio optimization problem with a VaR objective for portfolio construction, with the optimization technique being used as generalized simulated annealing[5]. Simulated annealing [34] is a metaheuristic approach where the objective function can be thought of as the energy function of molten metal and constraints in the form of artificial temperatures are familiarized and steadily cooled, parallel to the annealing technique in metallurgy. The randomness or stochasticity is introduced in the process through the artificial temperature, from which the process tends towards the global optimum towards the end of the annealing process.

Given that cryptocurrency returns are poorly stationarily distributed, with expected returns μ_t and the extent of risk V_t , measured as VaR. The mathematical description of our objective along with the constraints *viz*:

$$\begin{aligned} &\min V_p = VaR_p \text{ objective function} \\ &R_p = w_i \times \mu_i \geq R_{Treshold} \text{ return constraint} \\ &\sum w_i = 1 \text{ budget constraint} \\ &\underline{w_i} \leq w_i \leq \overline{w_i} \text{ box constraint} \\ &d_i = 1 - \sum {w_i}^2 \text{ diversification constraint} \end{aligned} \tag{10}$$

where V_p is the portfolio risk, R_p is the return from the portfolio, $R_{Treshold}$ is the minimum expected return, μ_i is the individual asset returns that constitute the portfolio, w_i weight of the i-th financial asset in the portfolio, $\underline{w_i}$ and $\overline{w_i}$ are the lower and upper bounds of weight, here set as 0.0001 and 0.5, respectively, d_i is the extent of diversification in the portfolio. Given the non-negativity (long-only) constraints, this quadratic programming problem is solved by using the GenSA[6] package in R.

We intend to maximize the Modified Sharpe Ratio in order to find the most efficient frontier solution for our optimized portfolio. One of our objectives was to compare gold and cryptocurrency in a portfolio. In order to explicate that, we consider three portfolios for each country. First, we include the cryptocurrency with the lowest VaR[7] along with the four largest (market capitalization) firms. Second, we consider the returns on gold along with the same four companies. Finally, we consider the portfolio consisting of both cryptocurrency and gold, besides the four stocks. We choose the portfolio that gives the maximum Sharpe, Sortino, and Information ratios[8]. This exercise is performed on a 3-year and 5-year rolling window basis to account for the effects of altering dynamics, parameter variability, and structural reformation. Results are presented as the average of the five 3-year windows and 3–5 years windows. Furthermore, to see the impact of the COVID-19 pandemic, data for 2020 were analyzed separately as well.

Table 1
Tukey Lambda distribution results.

Cryptocurrency	Lambda	PPCC
Bitcoin	−0.24	0.99
Ethereum	−0.29	0.99
Litecoin	−0.34	0.98
Dogecoin	−0.32	0.99

This table provides the empirical findings from fitting the price data of cryptocurrencies into the Tukey-Lambda distribution to find suitable distributions for the parametric estimation of value-at-risk. The probability-plot-correlation-coefficients are significantly high to show that estimated Lambdas are appropriate. PPCC: probability plot correlation coefficient.

5. Results & discussion

To estimate the volatility of cryptocurrencies, we use extreme value theory to statistically model their distributions in order to compute their VaR. This aids our initial objective in including the cryptocurrency with the lowest probability of extreme events in the portfolio. Section 5.1 discusses this analysis in detail. Our further second objective is to compare portfolios including either gold or bitcoin or both, and the results of GenSA optimization are presented in Section 5.2.

5.1. VaR estimation

Given the heavy-tailed nature of cryptocurrencies, intuitively speaking, they do not follow a normal distribution. To further the intuition, normal Q-Q plots for all four cryptocurrencies show substantial deviations from the same. To answer the question of which parametric distribution best fits the empirical data, we start by studying the dynamics of the distribution using the Tukey-Lambda distribution. Table 1 summarizes our findings. Comparing each value with those mentioned in Table A.1 (see Appendix), we can conclude that each of the four distributions falls somewhere between Laplace (λ=−0.12) and Cauchy (λ=−1.00) distributions and can probably be well approximated using more generalized distributions. Also noteworthy are the high values of probability-plot-correlation-coefficients that show the optimal fits into the Tukey-Lambda distribution.

Fig. 3 plots the empirical data for each of the four cryptocurrencies under consideration, along with the cumulative distribution functions of several possible distributions with specifically tuned parameters to fit the data. It is quite visible that, in accordance to our presentiment, Johnson's Su and Generalized extreme value distributions approximate the data well. This result is robust, given that among the supremum calculated for each of the aforementioned distributions, these show the lowest value (Generalized pareto distribution also shows relatively low supremum). In particular, this allows us to reconcile our results with the well-known empirical findings that cryptocurrency distributions are heavy-tailed, and hence all analyses assuming normal distribution should be reconsidered.

Given the fit of the distributions, we then proceed to calculate the value-at-risk for the four cryptocurrencies. Table A.3 (see appendix)

[5] Besides generalized simulated annealing, the data are also analysed using the R Optimization Infrastructure which is an extensive and compendious domain modelling both linear and non-linear problems.

[6] Simulated annealing is a stochastic process [35] for global optimization problems where constraints are treated as artificial temperature to simulate thermal noise and progressively unheated. Generalized simulated annealing introduces a modified Cauchy-Lorentz visiting function with shape parameter Ω_s to generate a trial jump distance $\Delta x(t)$ under artificial temperature $T_{\Omega_s}(t)$ which is accepted or rejected based on probability of moving downhill or uphill using a generalized Metropolis algorithm. $f_{\Omega_s}(\Delta x(t)) \alpha \frac{[T_{\Omega_s}(t)]^{-\frac{B}{3-\Omega_s}}}{\left[1+(\Omega_s-1)\frac{(\Delta x(t))^2}{[T_{\Omega_s}(t)]^{\frac{2}{3-\Omega_s}}}\right]^{\frac{1}{\Omega_s-1}+\frac{B-1}{2}}}$ The decrement of artificial temperature is specified by $T_{\Omega_s}(t) = T_{\Omega_s}(1)\frac{2^{\Omega_s-1}-1}{(1+t)^{\Omega_s-1}-1}$ For additional information, the reader might refer to Xiang et al. [8].

[7] Expected Shortfall (Conditional Value-at-risk) is alternatively considered as a risk-metric in the optimization exercise to check for robustness of the results.

[8] In the case of different outcomes, the best two out of three are considered.

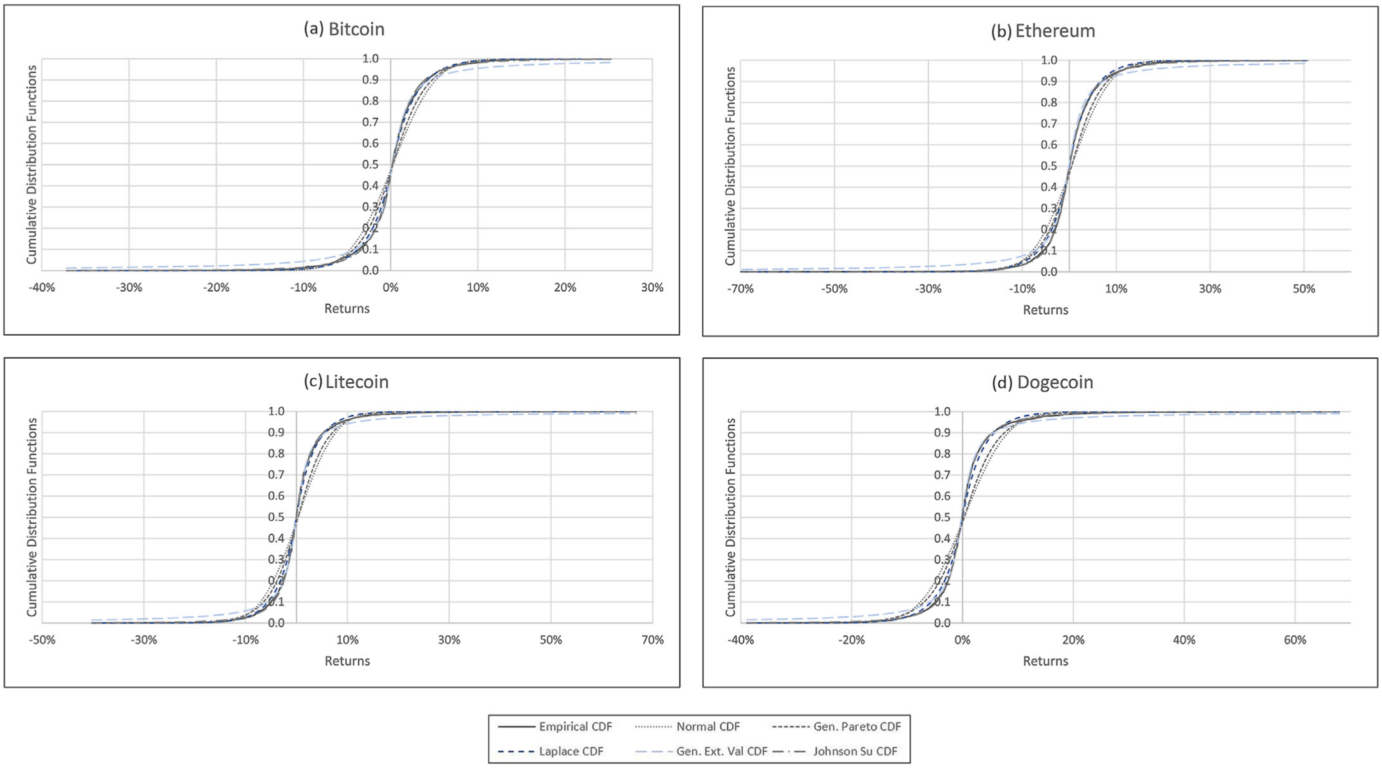

Fig. 3. Cumulative distribution Plots for BTC, ETH, DOGE, LTC.
These graphs depict how empirical data for (a) Bitcoin, (b) Ethereum, (c) Litecoin and (d) Dogecoin fit into the Normal, Generalized pareto (Gen. Pareto), Laplace, Generalized extreme Value (Gen. Ext. Value), and Johnson's Su distributions. Visually, Gen. Ext. Value distribution shows best fit in middle-portion, while Johnson's Su distribution promisingly fits the tails of empirical distribution. CDF: cumulative distribution function.

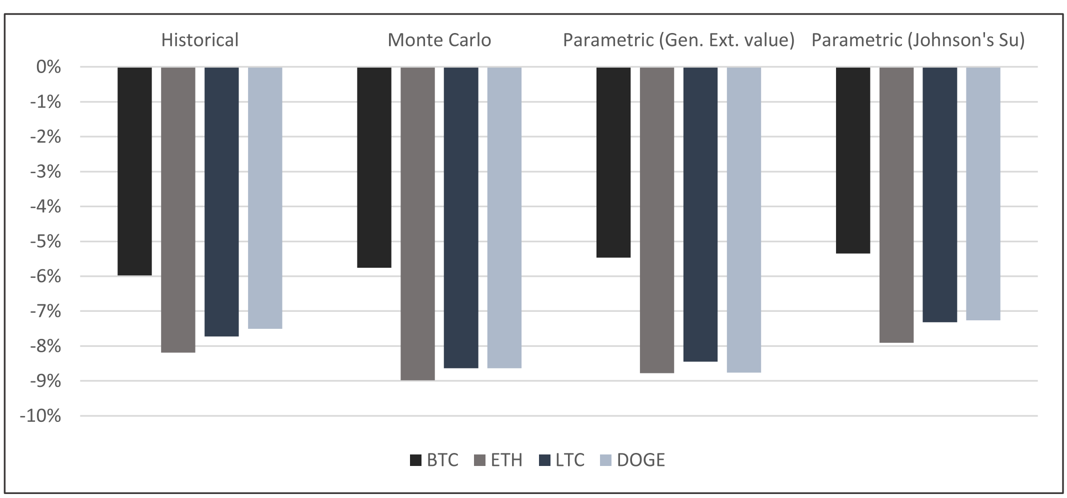

Fig. 4. Cryptocurrency Value-at-risk (5%).
This figure is a graphical summary of Value-at-risk at a 5% level of significance for Bitcoin, Ethereum, Litecoin, and Dogecoin according to (i) Historical simulation; (ii) Monte-Carlo; (iii) Parametric Method-Generalized extreme value distribution; and (iv) Parametric Method-Johnson's Su distribution. Across all methods, BTC shows the least volatility.

summarizes the computation of VaR at 95%, 97.5%, and 99% confidence intervals according to the various methods outlined in Section 4.1 using daily return data. Accordingly, in the worst 5% of cases, an investor will end up losing a maximum of 5.35% (BTC), 7.91% (ETH), 7.32% (DOGE), and 7.26% (LTC) of the total invested amount respectively in the following day, according to Johnson's Su VaR. Similar interpretations can be made for the other procedures. Fig. 4 is a graphical summary of the VaR for Bitcoin, Ethereum, Litecoin, and Dogecoin as measured by the historical simulation, Monte-Carlo simulation, and parametric (Johnson's Su, Generalized extreme value) distribution methods. The data provide clear evidence in favor of our hypothesis that bitcoin, being the oldest and most popular cryptocurrency is more stable than others, despite having poorer fundamentals. Hence, in the following section, we compare portfolios including either gold or bitcoin, or both. However, with the changing market scenario, and more knowledge pertaining to the crypto market, the measurement of volatility warrants further discussion, given that the bitcoin market share has been dwindling since the start of 2021.

Indeed, Bitcoin returns, despite having the lowest VaR among other cryptocurrencies, exhibit considerable volatility in comparison to other financial instruments. For perspective, gold during the same period had a historical VaR of 1.34%, while S&P 500 index had a VaR of 1.48%, at a 5% level of significance. However, it is noteworthy that, with the market maturing, the volatility of bitcoin returns has noticeably decreased. This development incites us to undertake the exercise to compare portfolios including either gold or bitcoin.

5.2. Portfolio optimization

The idea of a comparison between gold and bitcoin arises due to their diversifying characteristics in any portfolio. As visualized from the correlation heatmaps illustrated in Fig. 5, for each of the ten countries under study, the major stocks in the market share a negligible to negative correlation with both gold and bitcoin. Fascinatingly, gold and bitcoin share a low correlation between them, which can hence diversify the portfolio they are both included in. Although both show prominent correlation during times of macroeconomic turmoil, they diverge once markets return to business as usual. The intuitive explanation behind these findings is that gold being positively skewed, acts as an excellent diversifier against the more negatively skewed equity returns. Given a beta value of zero under the capital asset pricing model, gold retains its value irrespective of stock market ups and downs. Alternatively, the whole idea behind Bitcoin was to be an alternative to the traditional instruments in the market, and as such, shares negligible interrelationships with most traditional assets. With this motivation, we proceed with portfolio optimization using a generalized simulated annealing technique with VaR constraints.

Table 2 provides a comparison between the risk-adjusted returns for the three portfolios using a 3-year rolling average. For China, the portfolio consisting of both BTC and GLD alongside equity outperforms the other two in terms of modified Sharpe ratio and Sortino ratio. However, with regard to the Information ratio, which measures returns relative to a benchmark, portfolios including only BTC and equities have a higher value of 1.29, compared to 0.75 (portfolio 2) and 1.11 (portfolio 3). Similar patterns emerge in India, Russia, Germany, and Australia too. In the case of the USA, France, and Brazil, portfolio 1 outweighs portfolios 2 and 3 in terms of all three risk-adjusted return measures. For Japan, we observe portfolio 1 is better in terms of Information ratio and Sortino ratio. Finally, in the case of Canada, portfolio 1 is again the best in terms of Sharpe and Information ratios. The general picture emerging from the analysis is that the inclusion of bitcoin in the portfolios results in higher risk-adjusted returns. The result is robust to relaxing the constraint of long-only trading.

Table A.4 (see appendix) shows the optimal weights of the individual assets for each portfolio as computed using a generalized simulated annealing model. Associating weights calculated from the ROI (R Optimisation Infrastructure Solver) solver validates the robustness of the portfolio results. An interesting side finding is that, for the countries where portfolios including both BTC and GLD, along with equities, outperform the others, the weight associated with gold in the optimal portfolio is significantly high in comparison to others. Also, the weight associated with bitcoin is paltry. It might seem counter intuitive that bitcoin portfolios perform better and yet the weightage allocated to BTC is so low, but indeed the higher than usual returns coupled with excessive volatility are the cause behind it.

Table 3 shows the analysis for the 5-year rolling average. Portfolio 1 betters the other two in terms of all three ratios for the USA, France, China, India, Canada, Germany, and Brazil. In the case of Japan, portfolio 3 outweighs the rest. For Russia, portfolio 1 is again the best in terms of Sharpe and Information ratios. These results are also robust to relaxing the constraint of long-only trading.

To study the effect of the pandemic, the analysis of returns for each of the portfolios during 2020 reflects a similar trend to previous years, as shown in Table 4. For France, Russia, Canada, Germany, and Japan, Portfolio 1 outweighs portfolios 2 and 3 in terms of all risk-adjusted ratios. However, in the USA, China, India, Brazil, and Australia, portfolio 3 seems to be the better performer. This validates the findings that including Bitcoin in one's portfolio is crucial to achieving better risk-adjusted returns, from a pure price perspective.

6. Conclusions

Cryptocurrency is an exciting development in the ever-evolving financial market owing to the global connectivity through technology. As cognizance of its efficacy continues to grow with large institutional investors, including them in their asset classes, it has been termed by the media as “digital gold”. Numerous recent studies have debated Bitcoin's substitutability for gold [3,36]. However, there remains an existential threat to its maturity given that it is in a neonatal stage. The impetus to undertake this study arises from the apprehension among retail investors about the volatile nature of cryptocurrencies and whether to include them in their portfolios.

This paper investigates the dynamics of price movements of the four largest cryptocurrencies (market capitalization), namely Bitcoin, Ethereum, Dogecoin, and Litecoin, in terms of VaR. Three different methods *viz* historical simulation, Monte-Carlo simulation, and parametric approximation, are undertaken to estimate VaR. One key take-away is that price data for cryptocurrencies are well approximated by heavy-tailed distributions such as Johnson's Su and Generalized extreme value distribution. Our results show that Bitcoin has the lowest VaR and is thereby compared to gold as an asset class. This study further undertakes portfolio optimization using the non-linear generalized simulated annealing technique. Different portfolios consisting of permutations and combinations of equities, gold, and Bitcoin are constructed for ten countries across the world. The general conclusion incipient from the analysis is that regardless of its volatility nature, Bitcoin should be contained within the portfolio perhaps in a small percentage, as it augments the returns significantly and counteracs the associated risks. This study does not neglect gold either, given that for most countries, gold constitutes a high proportion of the optimized portfolios owing to its negative correlation with all the other asset classes.

These results suggest the importance of Bitcoin and are in accordance with previous studies by Henriques and Sadorsky [1], who also corroborated the better performance of Bitcoin using various GARCH models. Our findings are aligned with Selmi et al. [36], who show the diversifier properties of Bitcoin alongside gold thereby enhancing the performance of portfolios. In line with Bouoiyour et al. [26], we notice that indeed gold provides the necessary diversification while Bitcoin adds value. This is implied by the high percentage of gold in the portfolios during 2020. However, this study does not disregard the safe-haven characteristics of gold either and suggests the inclusion of both alongside in many cases, as per the analysis using data from 2020. While our findings are indeed encouraging and quite close to real-world functioning by laying importance on both asset classes, there are certain limitations that require further study. Given the limited nature of our data, future research will have to ascertain the precision of measurement of tail-end behavior of cryptocurrencies, perhaps using techniques such as network theory. It also warrants additional discussion in terms of the risk appetite of individual investors, which are not explicitly considered.

The shortage of liquidity in traditional financial assets continues to be a concern for governments, regulators, investors, and speculators. Bitcoin is a pioneering innovation in the financial world on that front. Due to their decentralized nature, cryptocurrencies hold the key to the future of the digitization of payments using blockchain technology.

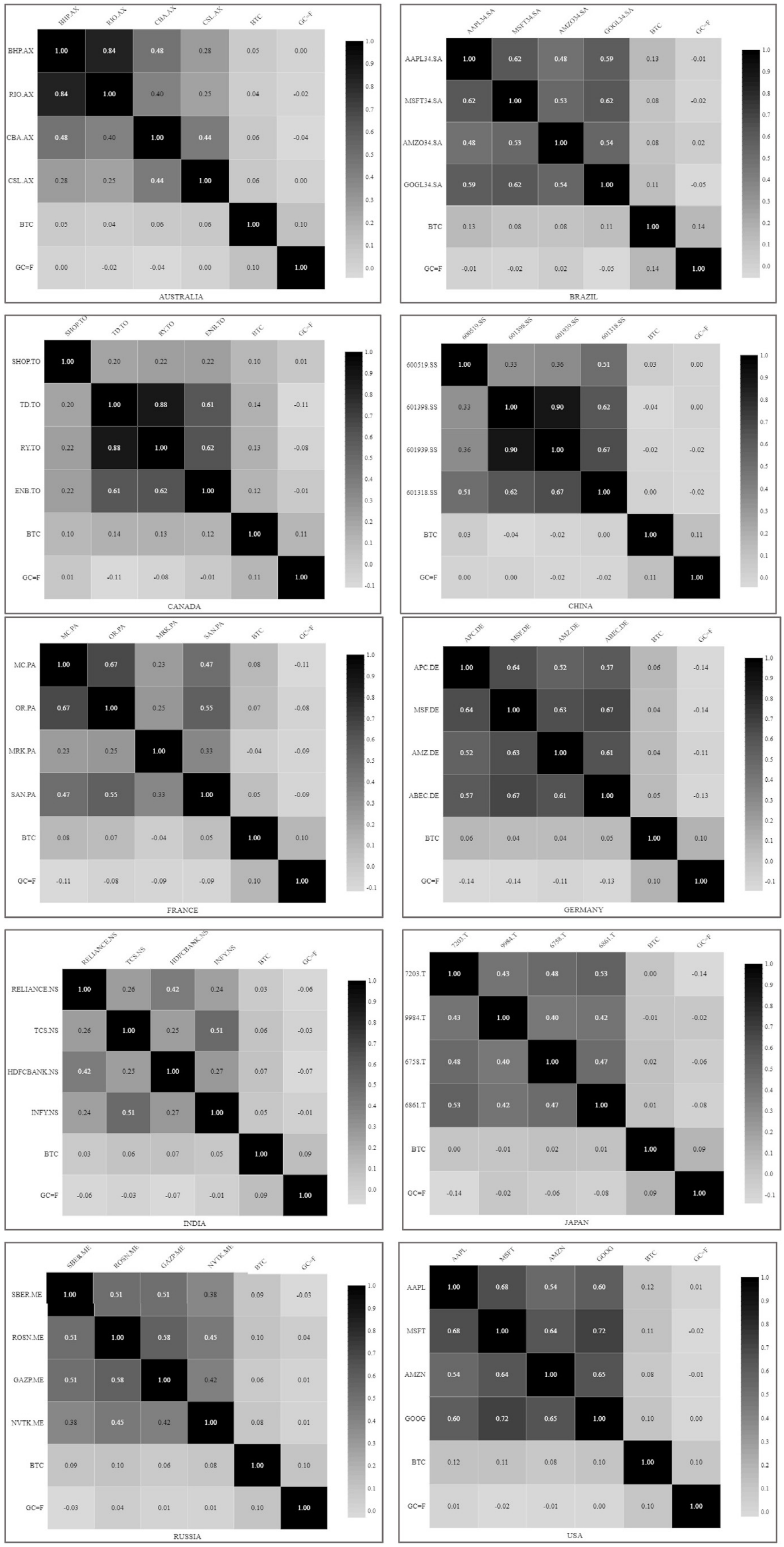

Fig. 5. The correlation heatmaps among the four equities with the largest market capitalization, BTC and gold, respectively, of the ten countries.
For all countries, equities share a negligible, yet positive relationship with BTC, however a small negative correlation with gold. Also, gold can be a diversifier to Bitcoin.

Table 2
3-year average risk-adjusted return measures for portfolios.

Country	PORTFOLIO 1 (Equity + BTC)			PORTFOLIO 2 (Equity + Gold)			PORTFOLIO 3 (Equity + Gold + BTC)		
	Sharpe	Sortino	IR	Sharpe	Sortino	IR	Sharpe	Sortino	IR
USA	0.0684	0.1358	1.3143	0.0612	0.1298	0.4916	0.0630	0.1300	0.7391
Canada	0.0857	0.1575	2.0890	0.0498	0.1090	0.8336	0.0837	0.1581	1.8354
Germany	0.0797	0.1415	1.4326	0.0556	0.1246	0.8997	0.1011	0.1499	1.2036
France	0.0891	0.1205	1.1712	0.0566	0.1018	0.4902	0.0952	0.1267	0.9928
China	0.0470	0.1142	1.2959	0.0412	0.1120	0.7544	0.0858	0.1349	1.1198
Japan	0.0711	0.1243	1.2785	0.0321	0.0679	0.4134	0.0956	0.1128	0.8111
India	0.0862	0.1405	1.2544	0.0370	0.1284	0.3332	0.0808	0.1428	1.1415
Australia	0.0613	0.1040	1.2116	0.0404	0.0907	0.5763	0.1312	0.1088	0.9369
Brazil	0.0977	0.1990	1.1362	0.0683	0.1540	0.0573	0.0960	0.1949	0.2663
Russia	0.0647	0.1347	1.1938	0.0334	0.1135	0.3636	0.0769	0.1385	0.8878

This table highlights the Sharpe, Sortino, and Information ratios (IR) to compare the three portfolios constructed for each of the ten countries; the first portfolio consists of only equities and Bitcoin, the second contains gold in place of Bitcoin, and the third combines both. Portfolio 1 outperforms portfolios 2 and 3 according to 3-year dynamic averaging.

Table 3
5-year average risk-adjusted return measures for portfolios.

Country	PORTFOLIO 1 (Equity + BTC)			PORTFOLIO 2 (Equity + Gold)			PORTFOLIO 3 (Equity + Gold + BTC)		
	Sharpe	Sortino	IR	Sharpe	Sortino	IR	Sharpe	Sortino	IR
USA	0.0561	0.1145	0.9715	0.0549	0.1256	0.5123	0.0517	0.1107	0.5170
Canada	0.0616	0.1265	1.4149	0.0566	0.1256	1.2295	0.0557	0.1225	1.1476
Germany	0.0587	0.1262	1.1817	0.0526	0.1183	0.7808	0.0548	0.1099	0.9763
France	0.0545	0.0993	0.9248	0.0559	0.0947	0.3873	0.0521	0.0926	0.7857
China	0.0545	0.1257	1.3521	0.0509	0.1331	0.9872	0.0519	0.1168	1.0914
Japan	0.0437	0.0921	0.9790	0.0379	0.0828	0.4731	0.0502	0.1010	1.0207
India	0.0506	0.1129	0.9494	0.0414	0.1163	0.4101	0.0492	0.1077	0.7991
Australia	0.0511	0.1034	1.0861	0.0398	0.0738	0.5895	0.0568	0.1211	0.9542
Brazil	0.0620	0.1436	0.5469	0.0534	0.1361	0.3010	0.0588	0.1322	0.4628
Russia	0.0449	0.1068	0.9086	0.0325	0.1114	0.3853	0.0403	0.1090	0.5829

This table highlights the Sharpe, Sortino, and Information ratios (IR) to compare the three portfolios constructed for each of the ten countries; based on a 5-year rolling average method. Portfolio 1 outperforms portfolios 2 and 3 in most cases.

Table 4
Risk-adjusted return measures for portfolios during 2020.

Country	PORTFOLIO 1 (Equity + BTC)			PORTFOLIO 2 (Equity + Gold)			PORTFOLIO 3 (Equity + Gold + BTC)		
	Sharpe	Sortino	IR	Sharpe	Sortino	IR	Sharpe	Sortino	IR
USA	0.0533	0.1239	0.6431	0.0618	0.149	0.9111	0.0599	0.1454	0.3175
Canada	0.0624	0.1314	2.3906	0.0576	0.1259	0.9647	0.0546	0.1087	0.7496
Germany	0.091	0.2107	2.9141	0.0504	0.1132	0.8038	0.084	0.1988	2.1768
France	0.0394	0.0761	0.9875	0.0365	0.0633	0.6488	0.0277	0.0442	0.6032
China	0.0651	0.156	1.2329	0.0558	0.1397	0.8731	0.1106	0.2596	3.5099
Japan	0.0867	0.2054	2.3514	0.0695	0.1495	0.7222	0.0797	0.1708	0.9783
India	0.0567	0.1465	1.7947	0.0491	0.1296	0.6981	0.0876	0.2181	2.179
Australia	0.0578	0.1331	2.0429	0.0249	0.056	0.4492	0.0823	0.1908	2.3529
Brazil	0.0857	0.2196	3.0001	0.0772	0.2012	2.0101	0.0867	0.2221	2.1011
Russia	0.0368	0.0986	1.0336	0.0166	0.0661	0.3475	0.0174	0.0648	0.3507

This table highlights the risk-adjusted ratios during the COVID-19 pandemic. Returns analyzed for 2020 showcases for France, Russia, Canada, Germany, and Japan; portfolio 1 is most suited. While portfolio 3 outperforms the other two in the USA, China, India, Brazil, and Australia.

However, owing to its volatile nature, it continues to be the epicenter of debate on whether or not to include it in one's portfolio. Our study attempts to provide some insight for investors by showing that indeed Bitcoin should be a part of one's portfolio owing to its high risk-adjusted returns.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability statement

The data that support the findings of this study are openly available at https://in.finance.yahoo.com/. The simulation code and data that support the findings of this study are available from the corresponding author upon reasonable request.

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APPENDIX

Table A.1
Standard values for Tukey-Lambda distribution

Lambda	Approximate Distribution
−1.00	Cauchy
−0.12	Laplace
−0.06	Hyperbolic Secant
0.00	Logistic
0.14	Normal
1.00	Uniform

This table depicts the standard values of Lambda after the empirical data have been fitted into the Tukey distribution. An approximate distribution is chosen according to the closest Lambda value.

Table A.2
Summary of distributions used in estimating value-at-risk (VaR)

Distribution	CDF
Normal	$F_X(x) = P(X \le x) = \Phi(\frac{x-\mu}{\sigma})$
Logistic	$F_X(x) = \frac{1}{1+e^{-\frac{x-\mu}{a}}}$
Hyperbolic Secant	$F_X(x) = \frac{2}{\pi}\arctan\left(e^{\frac{\pi}{2}\left(\frac{x-\mu}{\sigma}\right)}\right)$
Laplace	$F_X(x) = \frac{1}{2}e^{\frac{x-\mu}{b}}, \ x \le \mu$
Cauchy	$F_X(x) = \frac{1}{\pi}\arctan\left(\frac{x-\mu}{\gamma}\right) + \frac{1}{2}$
Log-normal	$F_X(x) = \Phi(\frac{\ln(x)}{\sigma})$
Slash	$F_X(x) = \begin{cases} \frac{1}{2}, \ x = a \\ \Phi\left(\frac{x-a}{b}\right) - b\frac{\varphi(0) - \varphi(\frac{x-a}{b})}{x-a}, \ x \ne a \end{cases}$
Johnson's Su	$F_X(x) = F\left(\gamma + \delta \, asinh(\frac{x-\xi}{\lambda})\right)$
Generalized Normal	$F_X(x) = \frac{1}{2} + sign(x-m)\frac{\gamma\left(\frac{1}{b}, \left(\frac{\lvert x-m \rvert}{a}\right)^b\right)}{2\Gamma(\frac{1}{b})}$
Generalized Pareto	$F_{(\varepsilon,\mu,\delta)}(x) = \begin{cases} 1 - \left[1 + \frac{\varepsilon(x-\mu)}{\delta}\right]^{-\frac{1}{\varepsilon}}, \ \varepsilon \ne 0 \\ 1 - e^{-\frac{x-\mu}{\delta}}, \ \varepsilon = 0 \end{cases}$
Generalized extreme value	$F_{(\mu,\sigma,\varepsilon)}(x) = \begin{cases} e^{-\left[1+\varepsilon(\frac{x-\mu}{\sigma})\right]}, \ \varepsilon \ne 0 \\ e^{-exp(-\frac{x-\mu}{\sigma})}, \ \varepsilon = 0 \end{cases}$

This table provides the cumulative distribution functions (CDFs) used to obtain quantile functions for all the distributions considered in this paper.

Table A.3
Value-at-risk at 1%, 2.5%, and 5% levels of significance

1.0%				
	Bitcoin	Ethereum	Dogecoin	Litecoin
Historical	10.58%	15.64%	15.44%	13.44%
Monte-Carlo	9.85%	16.59%	14.08%	15.23%
BRW	8.52%	9.12%	16.85%	13.54%
Normal	8.69%	14.57%	14.04%	13.14%
Hypersecant	9.91%	16.63%	16.00%	14.97%
Laplace	9.47%	16.00%	13.83%	13.49%
Cauchy (MLE)	44.36%	75.95%	59.99%	59.03%
Johnson's Su	12.27%	16.51%	17.00%	16.89%
Slash	9.86%	14.57%	14.21%	12.34%
Generalized Normal	8.98%	15.21%	14.74%	13.69%
Generalized Pareto	9.75%	14.23%	14.98%	13.14%
Generalized Extreme value	12.85%	15.14%	17.54%	17.21%

2.5%				
	Bitcoin	Ethereum	Dogecoin	Litecoin
Historical	7.89%	11.64%	10.86%	10.08%
Monte-Carlo	8.75%	13.69%	12.44%	12.05%
BRW	4.73%	8.10%	11.43%	10.26%
Normal	7.28%	12.20%	11.78%	11.02%
Hypersecant	7.66%	12.85%	12.40%	11.60%
Laplace	7.20%	12.25%	10.59%	10.34%
Cauchy (MLE)	17.59%	30.37%	24.06%	23.57%
Johnson's Su	7.90%	11.14%	10.81%	10.74
Slash	7.16%	11.67%	11.31%	10.64%
Generalized Normal	5.88%	13.81%	11.44%	10.19%
Generalized Pareto	5.55%	11.13%	13.08%	9.94%
Generalized Extreme value	10.55%	13.44%	15.84%	11.31%
5.0%				
	Bitcoin	Ethereum	Dogecoin	Litecoin
Historical	5.98%	8.19%	7.51%	7.73%
Monte Carlo	5.76%	8.98%	8.74%	8.64%
BRW	4.21%	6.56%	5.83%	8.22%
Normal	6.07%	10.16%	9.84%	9.20%
Hypersecant	5.96%	9.98%	9.67%	9.05%
Laplace	5.49%	9.41%	8.14%	7.96%
Cauchy (MLE)	8.64%	15.13%	12.04%	11.71%
Johnson's Su	5.35%	7.91%	7.32%	7.26%
Slash	4.16%	8.67%	8.31%	7.64%
Generalized Normal	2.88%	10.81%	8.44%	7.19%
Generalized Pareto	4.55%	8.13%	10.08%	6.94%
Generalized Extreme value	5.47%	8.78%	8.76%	8.45%

This table show the results of quantifying value-at-risk at 99%, 97.5%, and 95% confidence intervals, respectively, for Bitcoin, Ethereum, Dogecoin, and Litecoin according to the three methodologies described in Section 4.1. Of the four, Bitcoin has the lowest value-at-risk.

Table A.4
Optimized weights of asset classes.

Country	Index	Rf	Portfolio 1		Portfolio 2		Portfolio 3	
			Ticker	Weight	Ticker	Weight	Ticker	Weight
USA	NASDAQ Composite (^IXIC)	1.62%	AAPL	0.3016	AAPL	0.2348	AAPL	0.0794
			MSFT	0.2977	MSFT	0.2139	MSFT	0.2476
			AMZN	0.0773	AMZN	0.2639	AMZN	0.1219
			GOOG	0.3153	GOOG	0.2108	GOOG	0.0502
			BTC	0.0080	GC = F	0.0766	BTC	0.2593
							GC = F	0.2416
Canada	S&P/TSX (^GSPTSE)	0.52%	SHOP.TO	0.0646	SHOP.TO	0.0242	SHOP.TO	0.1860
			TD.TO	0.3438	TD.TO	0.2557	TD.TO	0.2248
			RY.TO	0.1554	RY.TO	0.2637	RY.TO	0.1101
			ENB.TO	0.3985	ENB.TO	0.0235	ENB.TO	0.0219
			BTC	0.0377	GC = F	0.4329	BTC	0.2796
							GC = F	0.1777
Germany	DAX Performance Index (^GDAXI)	0.80%	APC.DE	0.0390	APC.DE	0.0102	APC.DE	0.2597
			MSF.DE	0.3509	MSF.DE	0.1213	MSF.DE	0.1499
			AMZ.DE	0.3707	AMZ.DE	0.0156	AMZ.DE	0.2396
			ABEC.DE	0.1832	ABEC.DE	0.4097	ABEC.DE	0.1484
			BTC	0.0562	GC = F	0.4431	BTC	0.0275
							GC = F	0.1747
France	CAC 40 (^FCHI)	−0.45%	MC.PA	0.0430	MC.PA	0.0715	MC.PA	0.3651
			OR.PA	0.3055	OR.PA	0.2890	OR.PA	0.1544
			MRK.PA	0.3557	MRK.PA	0.2090	MRK.PA	0.2078
			SAN.PA	0.1957	SAN.PA	0.0713	SAN.PA	0.1726
			BTC	0.1000	GC = F	0.3591	BTC	0.0885
							GC = F	0.0116
China	SSE Composite Index (000001.SS)	3.24%	600519.SS	0.3726	600519.SS	0.1435	600519.SS	0.1424
			601398.SS	0.3898	601398.SS	0.2167	601398.SS	0.1554
			601939.SS	0.1241	601939.SS	0.0288	601939.SS	0.0077
			601318.SS	0.0994	601318.SS	0.3686	601318.SS	0.1415
			BTC	0.0141	GC = F	0.2425	BTC	0.0645
							GC = F	0.4885
Japan	NIKKEI 225 (^N225)	0.02%	7203.T	0.4955	7203.T	0.1418	7203.T	0.1246
			9984.T	0.0211	9984.T	0.1013	9984.T	0.1114
			6758.T	0.1653	6758.T	0.2111	6758.T	0.2321
			6861.T	0.2461	6861.T	0.2205	6861.T	0.0093
			BTC	0.0719	GC = F	0.3253	BTC	0.1922
							GC = F	0.3305
India	NIFTY 50 (^NSEI)	5.90%	RELIANCE.NS	0.0126	RELIANCE.NS	0.3782	RELIANCE.NS	0.2253
			TCS.NS	0.2241	TCS.NS	0.0403	TCS.NS	0.2552

(*continued on next column*)

(*continued*)

Country	Index	Rf	Portfolio 1		Portfolio 2		Portfolio 3	
			Ticker	Weight	Ticker	Weight	Ticker	Weight
			HDFCBANK.NS	0.1772	HDFCBANK.NS	0.0849	HDFCBANK.NS	0.1033
			INFY.NS	0.2170	INFY.NS	0.1405	INFY.NS	0.1333
			BTC	0.3691	GC = F	0.3562	BTC	0.0180
							GC = F	0.2648
Australia	S&P/ASX 200 (ˆAXJO)	0.98%	BHP.AX	0.1708	BHP.AX	0.2020	BHP.AX	0.1292
			RIO.AX	0.0452	RIO.AX	0.0848	RIO.AX	0.1885
			CBA.AX	0.3730	CBA.AX	0.1940	CBA.AX	0.2153
			CSL.AX	0.4060	CSL.AX	0.3784	CSL.AX	0.1630
			BTC	0.0045	GC = F	0.1408	BTC	0.0815
							GC = F	0.2226
Brazil	IBOVESPA (ˆBVSP)	6.92%	AAPL34.SA	0.2208	AAPL34.SA	0.1930	AAPL34.SA	0.1924
			MSFT34.SA	0.3279	MSFT34.SA	0.3465	MSFT34.SA	0.1622
			AMZO34.SA	0.1984	AMZO34.SA	0.1921	AMZO34.SA	0.1003
			GOGL34.SA	0.2296	GOGL34.SA	0.0724	GOGL34.SA	0.2171
			BTC	0.0234	GC = F	0.1960	BTC	0.0760
							GC = F	0.2520
Russia	MOEX Russia Index (IMOEX.ME)	7.14%	SBER.ME	0.1885	SBER.ME	0.0016	SBER.ME	0.1860
			ROSN.ME	0.1507	ROSN.ME	0.1103	ROSN.ME	0.1540
			GAZP.ME	0.2030	GAZP.ME	0.3878	GAZP.ME	0.2309
			NVTK.ME	0.2947	NVTK.ME	0.2954	NVTK.ME	0.0799
			BTC	0.1661	GC = F	0.2050	BTC	0.1070
							GC = F	0.2420

The proportions in the table represent the weightage to be allocated to each asset class according to GenSA portfolio optimization. Note that for portfolio 3, in most cases, gold has the highest percentage in the portfolio.

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