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JOURNAL OF Econometrics
Journal of Econometrics 135 (2006) 349–376
www.elsevier.com/locate/jeconom

# A regime switching long memory model for electricity prices

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Available online 22 August 2005

**Abstract**

In this paper we develop a regime switching model which can generate long memory (fractional integration) in each of the regime states. This property is relevant, e.g., for the deregulated market for electricity power in the Nordic countries, which is characterized by electricity spot prices with a high degree of long memory. It occurs that in some time periods bilateral prices are identical whereas in other periods the prices differ. If the price series are fractionally integrated, then in the former regimes, an extreme form of fractional cointegration amongst prices will exist. The latter regimes occur when a capacity congestion exists across regions and multiple price areas will result. We define a Markov switching fractional integration model from which the fractional orders of integration in separate states can be estimated using maximum likelihood techniques. The model is adapted to data for the Nordic electricity spot market, and we find that regime switching and long memory are empirically relevant to co-exist. In particular, we find that the price behaviour for single markets can be very different depending upon the presence or absence of bottlenecks in electricity transmission. Using Monte Carlo forecasting we find that the regime switching model appears to be especially attractive in forecasting relative prices.


*JEL classification:* C2; C22; C32

*Keywords:* Cointegration; Electricity prices; Forecasting; Fractional integration and cointegration; Long memory; Markov switching

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0304-4076/$ - see front matter 
doi:10.1016/j.jeconom.2005.07.021

## 1. Introduction

It has been argued in some studies, for instance Granger and Ding (1996), Diebold and Inoue (2001), and Granger and Hyung (2004), that under certain conditions time series variables can spuriously have long memory when measured in terms of their fractional order of integration (see Granger and Joyeux, 1980; Beran, 1994; Baillie, 1996), when in fact the series exhibit non-linear features such as regime switching. In the present paper we define a model, which allows *both* regime switching *and* long memory in the separate regime states.

The model is motivated by some interesting features characterizing electricity prices when physical interconnections in the exchange of electricity exist bilaterally across regions. For instance, the Nordic power exchange, Nord Pool, is organized such that when no bottlenecks or congestions exist bilaterally at exchange points, the prices across regions will be identical, whereas the market mechanism makes prices depart in situations with capacity constraints. It is thus natural to consider price processes which accommodate regime switching subject to the presence or absence of congestion.

The model we consider is of the Markov switching type originally defined by Hamilton (1989). However, because the defining property of a non-congestion state is that prices are identical, the state variable is observable as opposed to being a latent variable. An important feature of the model is that the price processes in the different regimes can have different degrees of long memory. This gives rise to a number of interesting possibilities. For instance, consider the state with non-congestion and assume that the associated bivariate prices are fractionally integrated of a given order. It follows that prices are fractionally cointegrated in this case, i.e. extending the notion of Granger (1981) and Engle and Granger (1987) in the sense that individual prices are fractionally integrated but price differences are identically zero. Thus, an extreme form of cointegration occurs in this situation because the prices are identical and hence are governed by exactly the same price shocks. The price behavior in the congestion state can (and typically will) be very different. That is, the bivariate prices can be fractionally cointegrated in a more conventional way or the prices can appear not to cointegrate. Hence, our model can potentially exhibit state-dependent fractional cointegration. In the literature, Markov switching (integer-valued) cointegration models have been suggested by a number of authors, see *inter alia* Krolzig (1997), Krolzig et al. (2002), and Hansen and Seo (2002). It is our conjecture that by not conditioning on the congestion state, i.e. when having a model with no regime switching, fractional cointegration can or cannot be found in the full sample. What we do observe in the full sample with no separation into regimes is likely to be a convex combination of the behavior in the single states and hence misleading inference is likely to result.

The appropriate modelling of electricity price processes is of interest for several reasons, see e.g. Engle et al. (1989) and Ramanathan et al. (1997). First of all, the forecasting of such prices is of interest by itself for the management and trading in electricity markets. Because the operation of electricity markets is similar to the operation of financial markets with electricity power derivatives being priced and traded in highly competitive markets, dynamic modelling of means and variances is essential. In the present paper, we focus only on the first moment behavior of electricity prices, but the modeling and forecasting of second moments is clearly of separate interest for price hedging and risk management in such markets.

Furthermore, the price dynamics is of interest with respect to competition analysis of electricity markets. Market delineation is a central issue in competition analysis, see e.g. Motta (2004). Even though the Nordic power markets, for instance, are highly liberalized, there is still a scope for regulating authorities to closely follow the market behavior, see also Fabra and Toro (2002). When there is no congestion, there is obviously a single price existing in the market and the relevant geographical market consists of the regions with identical prices. However, when there is congestion, it is of interest to follow the price dynamics closely because suppliers can have a dominating position and the geographical market delineation becomes less straightforward. The fact that the definition of the relevant geographical market can have a temporal aspect is a particular feature of the electricity market.

The plan of the paper is as follows. In the subsequent section the functioning of the Nordic power market is described. Its organization is explained and the price setting behavior, which is necessary to understand the regime switching property of the market, is presented. Furthermore, some stylized facts about the electricity prices in the Nordic region are discussed, and, in particular, it is shown how seasonality, long memory, and regime switching are important features to consider when building models of the price dynamics. In Section 3, the regime switching multiplicative seasonal ARFIMA model is presented, and in Section 4 it is estimated for prices and relative prices of neighboring regions within the Nordic area. Generally, the price behavior in the different states appears to be rather different. The analysis also demonstrates the importance of allowing for regime switching since non-switching models can generate very misleading inference with respect to the fractional integration and cointegration properties of the data. In particular, two misclassifications of the model dynamics are likely to occur. First, a non-switching model may indicate that the price series are fractionally cointegrated even when the phenomenon only applies to one of the regime states. Secondly, the opposite can happen in which case it is concluded from a non-switching model that the data are not integrated of the same fractional order and/or are not cointegrated, although the series are in fact cointegrated in one of the states. In Section 5 the out-of-sample performance of the switching model is evaluated and compared with the non-switching model. For both the regime switching model and the non-switching model a Monte Carlo forecasting methodology is used. We find that for relative prices the switching model is superior to a non-switching model and the advantages improve the more persistent the regime states appear to be. Furthermore, with respect to one-step ahead forecasting of both the individual price series and the relative price series, the regime switching model is superior in the sense that a larger concentration of density around the actual outcome can be found. Section 6 concludes.

## 2. The Nordic power market

### 2.1. The Nordic power area

The motivation behind the present paper concerns the functioning of competitive power markets which are physically connected for exchange of electricity. Typically, such markets have capacity barriers which tend to affect the relevant market delineation, depending upon the existence or absence of bottlenecks across neighboring regions. The Nordic power market has undergone a remarkable development towards liberalization in recent years. Norway, Sweden, Finland, and Denmark have cooperated for several years to provide their 24 million population with an efficient and reliable power supply. Since 1991 market reforms and deregulation in all the countries have increased competition, and today all Nordic power markets have adapted to the new competitive environment and serves as a model for the restructuring of other power markets.[1]

The supply of electricity power in Norway is almost 100% hydropower whereas Sweden and Finland use nuclear plants, fossil-fuel powered plants, and hydropower. Approximately 90% of the Danish electricity is produced from conventional thermal plants and combined heating and power facilities; a minor proportion (10–12%) of Danish supply is from wind power turbines.[2] The hydropower production is mainly found in the northern parts of the Nordic power web whereas thermal power plants are located in the south. In general, the relatively cheap hydropower generation is transmitted to the heavily populated southern regions, which of course requires a well established power grid transmission capacity to facilitate the flow. When the reservoir levels are adequate, the less costly hydropower production causes the market to prefer this energy source and thus causes low spot prices. In these cases, national and cross-border transmission systems will be used to their capacity in order to level out price discrepancies across regions. On the other hand, when reservoir levels are low there will be a net flow from south to north, and the market will see relatively high prices for thermally generated electricity.

The physical connections of different areas within the Nordic countries are displayed in Table 1. When capacity constraints exist such that demand and supply do not clear the markets across neighboring regions, then congestion occurs. The operation of the power spot market is designed to deal with this problem.

### 2.2. The functioning of the power spot market

In the establishment of a joint Nordic power market, an important ingredient has been the construction of the Nordic power exchange which in fact is the world's

[1] See Nord Pool (2003a) which provides a detailed description of the Nordic power market.

[2] The total power supply for the Nordic area is 55% hydro, 24% nuclear, 20% thermal and combined heating, and 1% renewable.

Table 1
Gridpoints of Nordic power market with physical exchange of electricity

| | NNO | MNO | SNO | FIN | SWE | EDK | WDK |
|---|---|---|---|---|---|---|---|
| North Norway (NNO) | | | | | | | |
| Mid Norway (MNO) | √ | | | | | | |
| South Norway (SNO) | — | √ | | | | | |
| Finland (FIN) | — | — | | | | | |
| Sweden (SWE) | √ | √ | √ | √ | | | |
| East Denmark (EDK) | — | — | — | — | √ | | |
| West Denmark (WDK) | — | — | √ | — | √ | — | |

Source: Nord Pool (2003a).

first multinational power exchange. The spot market[3]—operated by Nord Pool Spot A/S—is an exchange where market participants trade power contracts for physical delivery the next day and is thus referred to as a day-ahead market. The spot market is based on an auction with bids for purchase and sale of power contracts of 1-h duration covering the 24 h of the following day. At the deadline for the collection of all buy and sell orders the information is gathered into aggregate supply and demand curves for each power-delivery hour. From these supply and demand curves the equilibrium spot prices—referred to as the system prices—are calculated.

In a situation where no grid congestion (or grid bottlenecks) exist across neighboring interconnectors, there will be a single identical price across the areas with no congestions. However, when there is insufficient transmission capacity in a sector of the grid, a grid congestion will arise and the market system will establish different price areas. This is because the Nordic market is partitioned into separate bidding areas which become separate price areas when the contractual flow between bidding areas exceeds the capacity allocated by the transmission system operators for spot contracts. On the other hand, when no such capacity constraints exist in a given hour, the spot system price is also the spot price for the entire Nordic power exchange area, i.e. the system price. The situation where different price areas arise due to bilateral congestions is relevant within the Norwegian power system and the border interconnectors between the Nordic countries.[4]

The fact that separate prices may coexist depending upon regional supply and demand causes the relevant market definition to vary with time. For instance, for the data set we are going to analyze there are six different geographical areas (North Norway is excluded for the reasons given below) for the 33,308 hourly observations. However, 34.24% of the time all the prices for the entire Nordic region were in fact

[3]Since only the spot market will be relevant for the present study, only this market will be described here, see also Nord Pool (2003b). In Nord Pool (2003c) a description is given of the futures and forward markets of the Nordic power exchange which are used for price hedging and risk management.

[4]Within Sweden, Finland, and Denmark grid congestion is managed by counter trade in case of excess supply (demand). In this case the transmission system operators ask generators to reduce (increase) production or large buyers to increase (decrease) demand until excess supply or demand is eliminated.

identical and thus a single price existed for more than one-third of the time. Two price areas existed in 34.55% of the time and three price areas existed in 20.86% of the time. In only 11 h there was complete congestion and six different price areas existed, i.e. there was a different price for each geographical market. Obviously many price area combinations occur with a very small probability.

### 2.3. On the modelling of electricity prices

Analyzing electricity data is interesting for several purposes. The functioning of the electricity market makes it of relevance to build statistical models useful for, e.g. competition analysis and forecasting of electricity prices. Moreover, adequate models for price behavior is also of interest due to the nature of deregulated electricity markets which have similarities with the operation of financial markets. Options, futures, and forward markets exist and act as the financial markets for price hedging and risk management. In the case of the Nordic power exchange—Nord Pool—exchange members can hedge purchases and sales of power with a time horizon of up to four years using continuously traded power derivatives. The development of models for pricing of power derivatives is therefore of importance. Even though the electricity power market is similar to financial markets, and electricity prices have properties similar to financial data, there are also features of electricity price data that are somewhat different, in particular the long memory and strong seasonal variation existing in the data as will be shown.

In microeconomics, the development of peak-load pricing models has received considerable interest, and frequently these models are studied in the context of optimal governmental regulation of public utilities such as electricity companies, see e.g. Steiner (1957), Sherman (1989), and the review by Crew et al. (1995). The peak-load pricing problem is typically characterized by the supply of a public utility good being non-storable and which exhibits periodic and stochastic demand fluctuations. The literature explains how prices will always exceed marginal costs in these cases whereby a scope for public regulation exists. The basic text-book models originating from Boiteux (1949) and Steiner (1957) have been made increasingly advanced to account for features such as multiple technologies and time periods (e.g. Crew and Kleindorfer, 1976), and the introduction of supply-side uncertainty (e.g. Kleindorfer and Fernando, 1993). All these features characterize the electricity markets in particular. Taking these models to the data requires, in addition to price data, detailed information on marginal costs of the various power plants, cost shares, etc. Similar data requirements exist when examining the potential collusion of market participants, see e.g. Fabra and Toro (2002).

In the present case, we only have hourly price data available and hence structural analyses as such cannot be made. However, the price data can be analyzed to examine for instance whether the price behavior is significantly different in congestion and non-congestion periods as described above. Different price behavior in the separate states can be a first indication of abuse of dominant position or collusion in the electricity market. Seeing how the relevant market for electricity in competition analysis changes depending upon the presence or absence of bottlenecks further points to the interest of this issue. The fact that the (geographical) market definition may have a temporal aspect seems to be a particular feature of the electricity market.

### 2.4. Some stylized facts about Nord Pool electricity prices

The data used for analysis in the present paper are hourly spot electricity prices for the Nord Pool area, Mid Norway (MNO), South Norway (SNO), West Denmark (WDK), East Denmark (EDK), Sweden (SWE), and Finland (FIN), for the period 3 January 2000–25 October 2003. This yields a total of 33,308 observations. For East Denmark the sample period starts 29 September 2000 and thus covers 26,828 sample points. Data for North Norway is not included because most of the year this market is merged with Mid Norway.

In Fig. 1, the electricity log price series are displayed. As seen, most price series are characterized by huge fluctuations and outliers, however, the general level of these series tend to be highly persistent possibly with mean reversion. The dominant features of electricity price series have also been discussed by, among others, Escribano et al. (2002) and Carnero et al. (2003). In the latter paper, analyzing a number of European electricity markets including parts of Nord Pool, it is argued that volatility clustering is likely to be a periodic phenomenon and hence pointing to an important difference in electricity price behavior compared to financial assets. Also the fact that huge jumps and outliers in the series seem to exist has caused authors to suggest modelling derivative prices by use of jump diffusions, see for instance Knittel and Roberts (2005) and Atkins and Chen (2002). These effects are likely to occur because there is no easy smoothing of supply and demand shocks since storage of electricity is difficult and expensive. Also, the fact that the individual market prices are subject to periods with switches between congestion and non-congestion is likely to produce jumps in prices.

Because weather is a dominant factor influencing equilibrium prices through changes in demand (and to some degree also supply), it seems reasonable that prices will exhibit mean reversion, see e.g. Knittel and Roberts (2005) and Lucia and Schwartz (2001). Also, the year-to-year variation in water reservoirs is rather significant and the fact that more than 50% of total electricity supply is from hydropower plants explains an important part of the within year seasonal variation.

In Table 2, we report Phillips–Perron tests for a unit root, see Phillips (1987) and Phillips and Perron (1988), using the Andrews (1991) automated data-dependent bandwidth choice and the Bartlett kernel. Also, Table 2 reports the KPSS test of Kwiatkowski et al. (1992), which tests the null of a stationary I(0) process, using the same bandwidth and kernel. The tests were applied to deseasonalized (using hourly, daily, and monthly seasonal dummies) and detrended series. The 5% and 1% critical values are $-3.41$ and $-3.96$ for the Phillips–Perron test and 0.146 and 0.216 for the KPSS test, see Fuller (1976), Kwiatkowski et al. (1992), and Phillips and Jin (2002). The last row of the table displays the bandwidth choice for each series using the Andrews (1991) data-dependent bandwidth. Interestingly, a unit root can be very strongly rejected for all series while the KPSS test of I(0)-ness also strongly rejects.

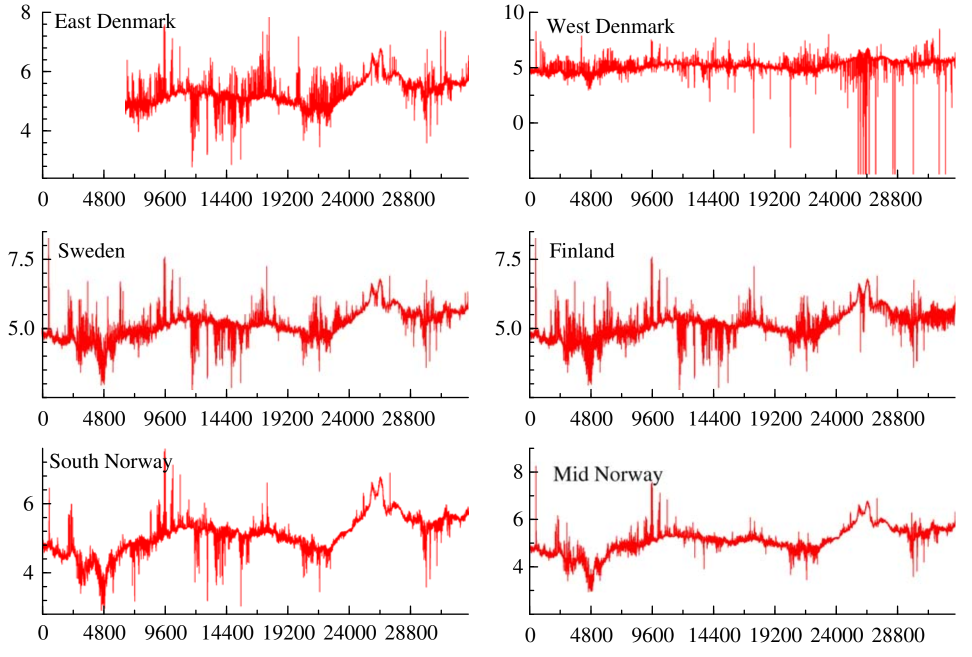


Fig. 1. Hourly log spot electricity prices for the Nord Pool area. *Note*: Observations are from 3 January 2000 to 25 October 2003.

Table 2
Unit root and stationarity tests in hourly electricity log prices

| | EDK | WDK | SWE | FIN | SNO | MNO |
|---|---|---|---|---|---|---|
| Phillips–Perron test | $-58.39^{**}$ | $-64.31^{**}$ | $-51.02^{**}$ | $-63.91^{**}$ | $-35.90^{**}$ | $-36.19^{**}$ |
| KPSS test | $0.8414^{**}$ | $0.7039^{**}$ | $0.4552^{**}$ | $0.5752^{**}$ | $0.3581^{**}$ | $0.3401^{**}$ |
| Bandwidth | 214 | 129 | 310 | 238 | 495 | 493 |

*Note*: The tests were applied to deseasonalized (using hourly, daily, and monthly seasonal dummies) and detrended series and used the Bartlett kernel. The bandwidth is the Andrews (1991) data-dependent bandwidth. One and two asterisks denote significance at the 5% and 1% level.

This supports our visual impression from Fig. 1 which suggests high persistence but mean reversion.

The results of Table 2 suggest that neither an I(1) nor an I(0) description of the price series is appropriate. An alternative way of measuring long memory and mean reversion is by estimation of fractionally integrated processes for the price series. Lee and Shie (2004) show that the Phillips–Perron test is consistent against fractional alternatives if the fractional order is less than unity. Also, Lee and Schmidt (1996) show that the KPSS test is consistent against fractional long memory alternatives, such as I($d$) processes for $d \neq 0$. Given the evidence of the Phillips–Perron tests and the KPSS tests, the I($d$) specification seems attractive.

The estimated fractional $d$ parameters are reported in Table 3 using a parametric ARFIMA model. The specification is the multiplicative seasonal ARFIMA (SARFIMA) model

$$A(L)(1-aL^{24})(1-L)^d(y_t-\mu)=\varepsilon_t, \quad \varepsilon_t \sim nid(0,\sigma_\varepsilon^2), \tag{1}$$

where $A(L)$ is a lag polynomial of order 8 capturing the within-the-day effects, the polynomial $(1-aL^{24})$ corresponds to a daily quasi-difference filter, and $\mu$ is a mean. All the series have been corrected for deterministic seasonality by regression on seasonal dummy variables (hour-of-day, day-of-week, and month-of-year) prior to the estimation of (1). Several other specifications were experimented with, e.g. longer $A(L)$ polynomial and weekly stochastic seasonality instead of daily, but (1) was found to be the superior model in terms of in-sample fit and whiteness of the residuals.

Model (1) was estimated by conditional maximum likelihood, i.e. by minimizing the residual sum-of-squares. Note that the estimation method and asymptotic normality of the estimates do not require Gaussianity of the errors, but only that they are i.i.d. or martingale differences, and furthermore the data can be both stationary or non-stationary, for details see Tanaka (1999) and Nielsen (2004).

As seen in Table 3, there is a clear indication of both long memory, border-line non-stationarity, and mean reversion because the fractional $d$ is estimated to be in the interval $0.41<d<0.52$. West Denmark is an exception, however. Hence, compared with the Phillips–Perron and KPSS tests, there seems to be a strong support for long memory and fractional integration to appropriately describe the price dynamics. It is interesting to note that Carnero et al. (2003) also find long memory in Norwegian electricity data but less so in electricity markets of The Netherlands, Germany, and France. One possible explanation of this is the fact that a significant amount of electricity supply in Nord Pool is from hydropower plants and it is a classical empirical finding that, e.g., river flows and water reservoir levels exhibit long memory, see Hurst (1951, 1956).

Price persistence within the year can be partially explained by seasonal variation due to hydropower reservoir levels. General economic and business activities may be other sources of this property. However, seasonality at the high frequencies, that is hour-of-day, and day-of-week effects, appear to be rather important as can be seen

Table 3
Univariate estimates of fractional $d$ for hourly log prices

| | EDK | WDK | SWE | FIN | SNO | MNO |
|---|---|---|---|---|---|---|
| SARFIMA | 0.41 (0.0122) | 0.31 (0.0152) | 0.44 (0.0113) | 0.41 (0.0113) | 0.52 (0.0101) | 0.52 (0.0112) |

*Note*: Standard errors are in parentheses. The SARFIMA is the parametric model specification (1) estimated by conditional maximum likelihood. The series have been corrected for deterministic seasonality prior to estimation of $d$.

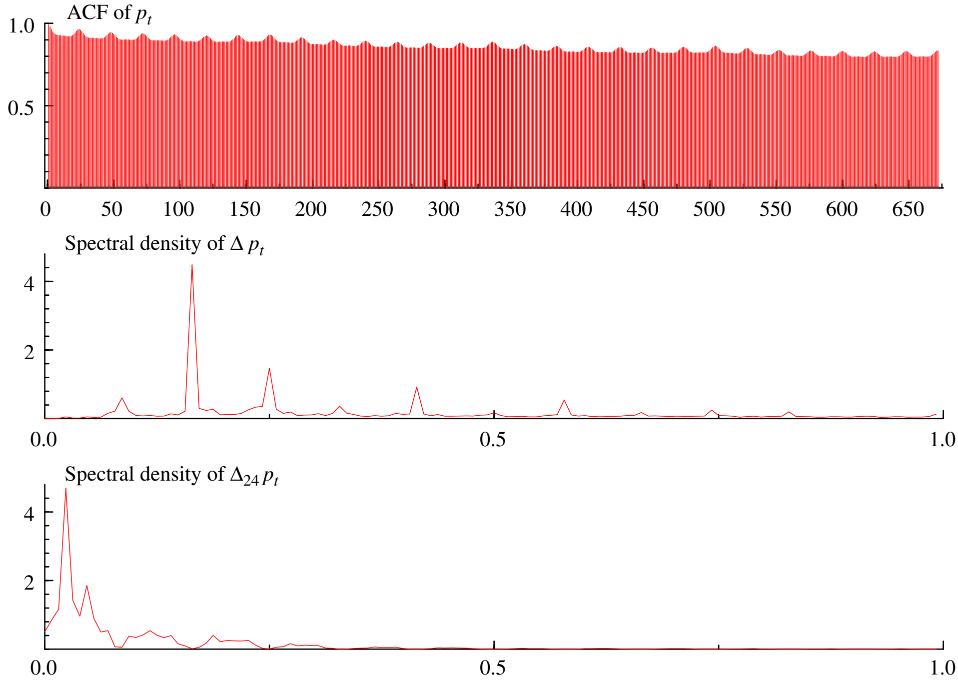


Fig. 2. Autocorrelation function and spectral densities. *Note*: The first panel is the autocorrelation function, ACF, of $p_t$, the second is the spectral density of hourly differences, $\Delta p_t$, and the third panel is the spectral density of daily differences, $\Delta_{24}p_t$, for hourly log prices of South Norway. 672 lags were included corresponding to 4 weeks of observations.

from Fig. 2 which displays the autocorrelation function[5] of a representative series (the log of the South Norwegian price series) together with the spectral density of the hourly and daily differences respectively. The long memory of the series is also apparent given the slow decay of the autocorrelation function. Clearly, the seasonal variation in the data needs particular focus when analyzing these data. The strong seasonal variation is a stylized fact of most electricity price series. For data measured at a daily frequency Carnero et al. (2003) favor periodic models, whereas Lucia and Schwartz (2001) and Escribano et al. (2002) prefer deterministic seasonal models. For the analysis undertaken in the present paper, periodic models are simply infeasible given that an hourly sampling frequency is relevant. Instead, we prefer a model specification with a mixture of both deterministic and stochastic seasonality.

The descriptive measures presented so far do not discriminate between the regime switching features of the data, i.e. the fact that in certain hours capacity constraints prevent electricity from flowing freely across grid points. When there is congestion, the market prices across neighboring regions with a physical cable connection will

[5]The autocorrelation function includes 672 lags of hourly observations which corresponds to 4 weeks of sample points.

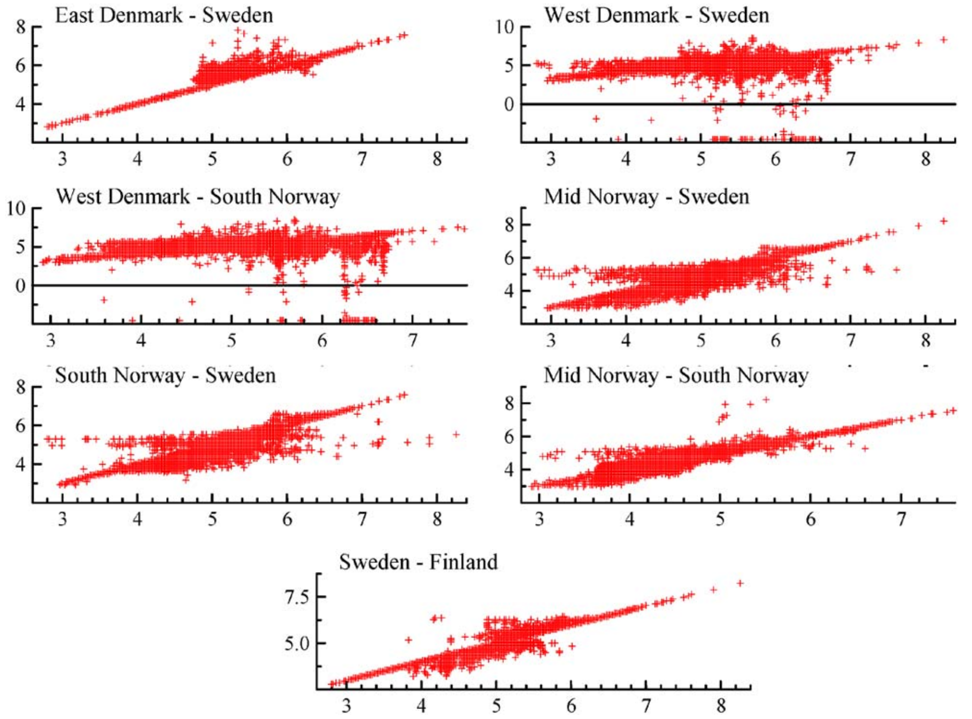


Fig. 3. Scatter plots of hourly log prices across Nord Pool regions.

differ. When no congestion exists, the prices will be identical which in fact is the defining property of non-congestion. Fig. 3 displays scatter plots for each of the seven grid points within the Nord Pool area. The clear tendency for a significant number of observations to lie on a 45 degree line is rather obvious from these plots.

## 3. A regime switching model with long memory

Based on the stylized facts presented in the previous section it seems obvious that, in addition to seasonal effects and long memory, an adequate statistical model for electricity prices should include information concerning the co-variation with other markets. In particular, the regime switching feature seems important.

It has been argued in some studies that long memory in the form of fractional integration can easily be interchanged with non-linear models. For instance, Diebold and Inoue (2001) demonstrate that mixture or regime switching models with suitably adapted time varying transition probabilities can generate an autocovariance structure similar to fractionally integrated processes, see also Granger and Ding (1996). In addition, Bos et al. (1999), Haldrup and Nielsen (2003), and Granger and Hyung (2004) argue that level shifts that are not appropriately dealt with can result in spurious indication of long memory and one may conjecture that in fact many types of hidden non-linearity can be expected to generate long memory as a result of model misspecification.

In the present case it is of importance to have a model which accommodates *both* fractional integration *and* regime switching simultaneously in order not to mix up model features. Some interesting scenarios can be considered. Assume that individual electricity prices across two regions are fractionally integrated in the non-congestion state. This means that an extreme form of fractional cointegration will exist in this state because the prices are *identical* across the two areas and thus price differences will be identically zero. On the other hand, the behavior of the two individual price series in the congestion state can be very different. If prices are compared without considering the different regime possibilities it is hard to say what to expect from the data, however, the mixing of the two processes is likely to produce series which have a behavior being a convex combination of the two state processes.

Consider the following model specification, which we denote a regime switching multiplicative SARFIMA or RS-SARFIMA,[6]

$$A_{s_t}(L)(1 - a_{s_t}L^{24})(1 - L)^{d_{s_t}}(y_t - \mu_{s_t}) = \varepsilon_{s_t,t}, \quad \varepsilon_{s_t,t} \sim nid(0, \sigma^2_{s_t}), \tag{2}$$

where $A_{s_t}(L)$ is a eighth order lag polynomial and $s_t = 0, 1$ denotes the regime, determined by a Markov chain with transition probabilities

$$P = \begin{bmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{bmatrix}. \tag{3}$$

Observe that because identical prices means that we are in a non-congestion state, all regimes are observable. Hence, as opposed to a standard Hamilton (1989) regime switching model, the Markov process generating the states is non-latent.

For each pair of prices (i.e. for each physical connection), the (univariate) series $y_t$ may denote one of the two individual log price series or the log relative price series. In each case, $y_t$ has been corrected for deterministic seasonality prior to the estimation of (2), and to reflect the regime switching nature of the model and using the fact that the regimes are observable, the coefficients on the dummy variables are allowed to differ across states. Note that if $y_t$ denotes a log relative price, all parameters are zero when $s_t = 0$ including $\sigma_0^2$, i.e. a deterministic state. Several alternative specifications for the regime switching model (2) were experimented with, e.g., longer $A_{s_t}(L)$ polynomial and weekly instead of daily stochastic seasonality, but (1) was found to be the superior model in terms of in-sample fit.

We now describe the estimation procedure for the model (2). Since the regimes are observable, we may count the number of observations in each regime and the number of transitions between regimes to form an estimate of $P$. Thus, the maximum likelihood estimates (MLEs) of the transition probabilities are simply given as

$$\hat{p}_{00} = \frac{n_{00}}{n_{00} + n_{01}}, \tag{4}$$

[6]Note that model (2) is a regime switching version of the non-switching model (1).

$$\hat{p}_{11} = \frac{n_{11}}{n_{10} + n_{11}}, \tag{5}$$

where $n_{ij}$ is the number of times we observe regime $i$ follow regime $j$, for $i, j = 0, 1$.

Estimation of the remaining parameters is by conditional maximum likelihood following the results of Tanaka (1999) and Nielsen (2004) for the non-switching model (1). By normality of the errors in (2), the likelihood function is

$$L = -\frac{T}{2}\ln\left(\frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_{s_t,t}^2\right) - \frac{T}{2}(1 + \ln(2\pi)), \tag{6}$$

where

$$\hat{\varepsilon}_{s_t,t} = \hat{A}_{s_t}(L)(1 - \hat{a}_{s_t}L^{24})(1 - L)^{\hat{d}_{s_t}}(y_t - \hat{\mu}_{s_t}), \quad s_t = 0, 1,$$

and we use the convention that $\hat{\varepsilon}_{j,t} = 0$ if $s_t \neq j$ for $j = 0, 1$. That is, we take advantage of the fact that the regimes are observable which allows us to extract the residual series to maximize the likelihood function (6).

Equivalently, the estimation procedure can be described as the maximization of

$$L = -\frac{T}{2}\ln\left(\frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t^2\right) - \frac{T}{2}(1 + \ln(2\pi)), \tag{7}$$

where $\hat{\varepsilon}_t = \sum_{j=0}^{1}\hat{\varepsilon}_{j,t}1(s_t = j)$ and $1(\cdot)$ is the indicator function. It is thus seen that, because regimes are observable, this maximization problem can be solved by minimizing the residual sum-of-squares $\sum_{t=1}^{T}\hat{\varepsilon}_t^2$. Finally, the estimate of the variance is

$$\hat{\sigma}_{s_t}^2 = \frac{1}{n_{s_t0} + n_{s_t1}}\sum_{t=1}^{T}\hat{\varepsilon}_{s_t,t}^2, \quad s_t = 0, 1,$$

where $\hat{\varepsilon}_{j,t} = 0$ if $s_t \neq j$ for $j = 0, 1$. As starting values for the numerical maximization of (6) (or equivalently (7)) we choose, for the parameters of both regimes, the estimates from the multiplicative SARFIMA model (1) with no regime switching.[7]

Following the results of Tanaka (1999) and Nielsen (2004) for the non-switching model, the errors do not in fact have to be Gaussian, but may be martingale differences, which is important in our case where the errors are likely to be heavy tailed, c.f. the discussion above. Also following Tanaka (1999), Nielsen (2004), and others, the estimation results do not depend on the stationarity or non-stationarity of the data, and standard asymptotics apply for all values of the fractional parameters.

Notice that, disregarding the matrix $P$ which is not needed for the estimation of the remaining parameters, the RS-SARFIMA model (2) has exactly twice as many parameters as the non-switching SARFIMA model (1). Since the estimation is by conditional maximum likelihood, the significance of the RS-SARFIMA model relative to the simpler and more parsimonious SARFIMA model can be tested by

[7]The Ox programming language, (see Doornik (2001)), was used for estimation.

means of, e.g. a likelihood ratio (LR) test. Such an LR test would thus be asymptotically $\chi^2$ distributed with degrees of freedom equal to the number of parameters in each state. We shall apply such a test in the subsequent empirical analysis to test the significance of the regime switching model for our data.

## 4. Empirical results

In this section, we consider the empirical analysis of the Nordic electricity price data (described above) applying the regime switching long memory model from Section 3. Each data set is a pair of log price series for two physically connected markets and the corresponding log relative price. In particular, we consider the five pairs for Sweden (see Table 1) and the pairs West Denmark—South Norway and Mid Norway—South Norway.

First, the estimated transition probabilities (4) and (5) are displayed in Table 4 for each of our data pairs. The estimates of $\hat{p}_{00}$, the probability of staying in regime 0 (non-congestion), range from 0.874 to 0.987 implying a mean time in regime 0 of 7.93–76.57 h. The smallest estimates of $\hat{p}_{00}$ are obtained for the data sets containing WDK (0.873 resp. 0.877), and are much smaller than the other estimates of $\hat{p}_{00}$ which range from 0.943 to 0.987. The estimates of $\hat{p}_{11}$, the probability of staying in regime 1 (congestion), range from 0.785 to 0.902 implying a mean time in regime 1 of 4.65–10.21 hours. Thus, the mean times in regime 1 are not as different across data sets as the mean times in regime 0. It also appears that both states are rather persistent, and in particular, the non-congestion state generally tends to be more persistent than the congestion state. The persistence of the states will be helpful for forecasting purposes, since more persistent states can be forecasted more accurately, see Section 5 below.

We next turn to the remaining estimation results for the model. The estimation was carried out as described in Section 3 above, but did not use the last 24 h of observations for each data pair. The last 24 h of observations are to be used for comparison with the model out-of-sample forecasts in Section 5. Prior to estimation,

Table 4
Transition probabilities and mean duration of states, $\lambda$ (h)

| Bivariate series | $\hat{p}_{00}$ | $\hat{p}_{11}$ | $\widehat{\lambda}_0$ | $\widehat{\lambda}_1$ |
|---|---|---|---|---|
| EDK-SWE | 0.9869 | 0.7848 | 76.57 | 4.65 |
| WDK-SWE | 0.8739 | 0.8196 | 7.93 | 5.54 |
| WDK-SNO | 0.8773 | 0.8882 | 8.15 | 8.95 |
| MNO-SWE | 0.9549 | 0.8831 | 22.19 | 8.55 |
| SNO-SWE | 0.9528 | 0.8979 | 21.20 | 9.79 |
| MNO-SNO | 0.9428 | 0.9020 | 17.47 | 10.21 |
| SWE-FIN | 0.9795 | 0.8463 | 48.78 | 6.51 |

*Note*: $\widehat{\lambda}_i$ is the estimate of the mean duration of state $i$ in hours.

Table 5
Switching model estimates of $d$ for log prices and log relative prices

| Bivariate series | SARFIMA (1) | | | RS-SARFIMA (2) | | | | | | $LR_1$ | $LR_2$ | $LR_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{d}_1$ | $\hat{d}_2$ | $\hat{d}_3$ | $\hat{d}_1^0$ | $\hat{d}_1^1$ | $\hat{d}_2^0$ | $\hat{d}_2^1$ | $\hat{d}_3^0$ | $\hat{d}_3^1$ | | | |
| EDK-SWE | 0.43 (0.012) | 0.43 (0.012) | 0.05 (0.018) | 0.46 (0.012) | 0.03 (0.013) | 0.46 (0.011) | 0.03 (0.012) | 0 | −0.26 (0.077) | 1148** | 1000** | 5376** |
| WDK-SWE | 0.31 (0.015) | 0.42 (0.011) | 0.27 (0.017) | 0.38 (0.024) | 0.28 (0.021) | 0.33 (0.013) | 0.46 (0.014) | 0 | 0.37 (0.015) | 144** | 444** | 2982** |
| WDK-SNO | 0.30 (0.015) | 0.44 (0.011) | 0.28 (0.016) | 0.30 (0.026) | 0.31 (0.017) | 0.16 (0.008) | 0.63 (0.017) | 0 | 0.37 (0.015) | 151** | 872** | 2138** |
| MNO-SWE | 0.44 (0.010) | 0.42 (0.012) | 0.31 (0.014) | 0.39 (0.008) | 0.38 (0.018) | 0.43 (0.012) | 0.18 (0.014) | 0 | 0.40 (0.016) | 796** | 498** | 6276** |
| SNO-SWE | 0.45 (0.011) | 0.41 (0.012) | 0.31 (0.016) | 0.38 (0.008) | 0.32 (0.013) | 0.41 (0.012) | 0.21 (0.013) | 0 | 0.39 (0.018) | 1092** | 702** | 5116** |
| MNO-SNO | 0.43 (0.011) | 0.44 (0.011) | 0.29 (0.016) | 0.31 (0.006) | 0.36 (0.009) | 0.32 (0.005) | 0.33 (0.008) | 0 | 0.39 (0.019) | 3250** | 3336** | 3120** |
| SWE-FIN | 0.39 (0.012) | 0.38 (0.012) | 0.24 (0.017) | 0.42 (0.011) | −0.02 (0.012) | 0.43 (0.012) | −0.02 (0.005) | 0 | 0.48 (0.022) | 1070** | 2604** | 6528** |

*Note*: Standard errors are in parentheses. The subscripts denote the price region (3 is the log relative price) and the superscripts denote the state. $LR_i$ is the likelihood ratio test of equal coefficients in states 0 and 1 for price region $i$ ($i = 3$ is the relative price), i.e. a test of the null of no switching. All the LR tests are $\chi^2$ distributed with 12 degrees of freedom (1% critical value is 26.22), and one and two asterisks denote significance at the 5% and 1% level.

each log price series $y_t$ (which for each data pair can be one of the two individual log price series or the log relative price series) had deterministic seasonality removed by regression on dummy variables for hour-of-day, day-of-week, and month-of-year, where the coefficients on the dummy variables may differ across states. If $y_t$ is a log relative price, all dummy variable coefficients in regime 0, the non-congestion state, are estimated to be zero (which is the obvious estimate). Thus, the procedure removes any deterministic seasonality while allowing the seasonal effects to vary across states.

The empirical results from the estimation of the full model in (2) are presented in Table 5. The first three columns of estimates give the estimated $d$ values for the three series (two log price series and the log relative price series) with no regime switching, i.e. constraining the parameters to be equal in both states, and the subscripts denote the series (3 is the log relative price). The next six columns contain the estimates of $d$ for the same series but with regime switching, where superscripts denote regimes.[8] Note that, obviously, the MLEs of all parameters in the non-congestion regime 0 are identically zero when $y_t$ is a log relative price and thus in particular $\hat{d}_3^0 = 0$. For all estimates, standard errors are provided in parentheses. In the final three columns, we present the *likelihood ratio* LR tests of the significance of the regime switching models compared to the models with no switching, i.e. compared to (1). The LR tests

[8] Observe that the estimated price process for a particular region is defined subject to the region with which it is compared, i.e. the regimes vary for different combinations of regions and hence this will affect the estimated price processes.

are asymptotically $\chi^2$ distributed with 12 degrees of freedom (the number of parameters in each regime) and the 1% critical value is 26.22.

First consider the East Denmark–Sweden (EDK–SWE) physical link. There seems to be rather clear evidence from the estimates of the non-switching model that the two series are fractionally cointegrated. Both series are fractionally integrated of the same order (at least to two decimal places both estimates are 0.43), but the relative price is integrated of a much smaller order, estimated at 0.05, which is significantly different from the integration orders of the individual series since the confidence bands based on the reported standard errors are very tight. However, given the special features of the data we may suspect that this finding is in some way spurious in the sense that cointegration only exists for a reduced sample rather than in the full sample. This is indeed confirmed by looking at the estimates for the switching model. Here we see that the model is perfectly cointegrated in the non-congestion regime 0, where the individual prices are integrated of order 0.46 and the relative price is zero by definition. On the other hand, in the congestion regime 1, there is no sign of cointegration, but instead the data appears to be roughly I(0). It thus appears that the results for the non-switching model reflect a kind of convex combination of the results for the two individual regimes and that the finding of fractional cointegration in the non-switching model is indeed spurious in the sense that cointegration really only exists in the non-congestion state.

Secondly, for the WDK-SWE physical link, the estimates from the non-switching model bear no indication of cointegration between the two price series since they appear integrated of rather different orders (0.31 resp. 0.42). However, in the regime switching model the integration orders of the two series are estimated at roughly the same value in the non-congestion regime[9] where the relative prices are identically zero. In the congestion regime there is no indication of cointegration since the two series appear integrated of different orders in that case. This implies that the results for the non-switching model again appear to be a combination of the results for the two regimes, but with a higher weight on the congestion regime making the non-switching estimates appear as though the prices are not cointegrated. Thus, the importance of the regime switching model is very clear in this case, since ignoring the possibility of different regimes actually leads us to falsely conclude that no type of cointegration exists among the two price series.

The MNO-SWE, SNO-SWE, MNO-SNO, and SWE-FIN links are very similar to the first case, i.e. to the EDK-SWE link, in the sense that they appear cointegrated based on the non-switching model whereas the regime switching model reveals that cointegration exists only in the non-congestion regime. Note that the SWE-FIN price behavior is rather extreme under congestion where the individual prices are approximately I(0) whereas the relative price is integrated of order 0.48. Finally, for the WDK-SNO link, there does not appear to be cointegration in the non-switching model, and in the regime switching model the estimates of the fractional integration

[9]Note that even though the prices are identical in the non-congestion state, the estimates of $d$ in the individual price series can be different even in the non-congestion state. This is because the dynamics of the estimated models in the non-congestion state include observations from the congestion state.

orders are quite different across regimes and price series, even though there is cointegration in regime 0 by definition. Since the estimates under regime 0 depend on regime 1 observations through the long lags, this strange finding may be attributed to the rather extreme (the only non-stationary estimate of $d$ in the table) estimate of $d$ under regime 1.

Finally, the LR tests of significance of the regime switching specification take values between 144 and 6528, with the highest values generally being obtained for the log relative price series. Obviously, all these values are highly significant in their asymptotic $\chi^2$ distributions. The tremendous significance of the LR tests thus stress the importance of regime switching for an adequate description of our electricity price data.[10]

Summing up our empirical findings, we have seen that our new regime switching specification, with a potentially deterministic state and observable regimes, is very important in order to reach correct conclusions about the behavior of the electricity prices and relative electricity prices in the bivariate analyses considered here. This is due to the state-dependent cointegration that exists in these price series because of the functioning of the electricity markets, see Section 2. In particular, there are two distinct misclassifications of the behavior of the bivariate price series. First, a non-switching model analysis may indicate that the two price series are (fractionally) cointegrated, as for the EDK-SWE data pair, even though this reflects the fact that the data are cointegrated only in one of the two regimes and not cointegrated in the other regime. Second, the opposite may happen in which case we erroneously conclude from a non-switching model analysis that the data are not cointegrated or are not integrated of the same (fractional) order and hence cannot be cointegrated, as for the WDK-SWE data pair, even though the data are cointegrated in one of the two regimes. This illustrates the importance of allowing regime switching in our data analysis due to the special characteristics of the electricity price data.

## 5. Forecasting

We now consider the forecasting of electricity prices and relative prices for up to 24 h, which is the relevant forecast horizon as discussed in Section 2.3, see also Engle et al. (1989) and Ramanathan et al. (1997). The forecasting of electricity prices is important for a number of reasons as we have previously argued.

Analytical formulae for forecasting regime switching models such as (2) are available, see e.g. Davidson (2004a, b), but are computationally very intensive since

[10]In the estimation of the models we also noticed that there were fewer outliers in the residuals from the regime switching models than in the residuals from the non-switching models, although there were still more outliers than expected if the residuals were normally distributed. However, the heavy tails of the distribution of the residuals (which are present for all our models, both switching and non-switching) and the consequent departure from normality is not critical to the estimation procedure employed since it is robust to non-normal error distributions as discussed above. More importantly, the observation that fewer outliers seem to be present in the residuals from the switching models supports the importance of allowing regime switching to adequately describe our data.

the calculation of analytical forecast error bands for a $k$-step ahead forecast requires $M^{k+1}$ steps, where $M$ is the number of states. The computation time required with $M = 2$ and with over 33,000 observations makes analytical forecasting infeasible even for small to moderate $k$ such as $k = 3$ or 4.

Instead, we consider forecasting by Monte Carlo stochastic simulation as advocated and implemented by Davidson (2004a). However, note that our implementation differs in several respects, for instance we have observable states and a deterministic regime (when $y_t$ is a log relative price). Thus, for forecasting, the model is simulated 24 periods ahead assuming Gaussian innovations. Conducting 1,000 Monte Carlo replications, we can extract the median forecast and 95% forecast error bands for each period from the simulated forecasts.

Figs. 4–10 display the forecasting results for the seven data sets also considered above. Each figure contains three panels which display median forecasts and 95% error bands for the two individual log price series (top two panels) as well as for the log relative prices (bottom panel). In each panel, the diamonds are the actually

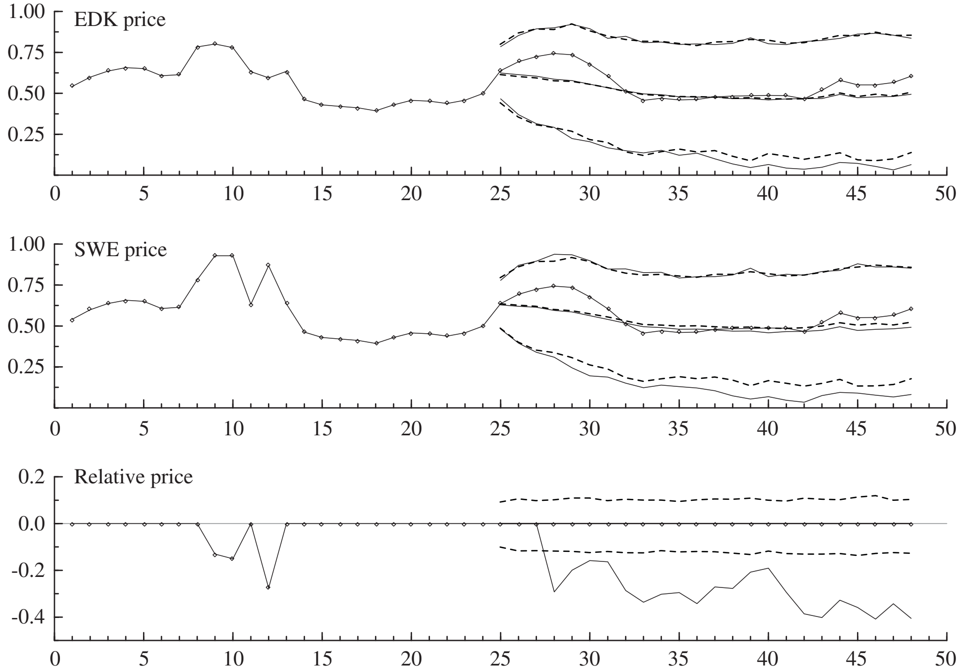


Fig. 4. Forecasts for the EDK-SWE physical link. *Note*: The three panels show forecasts for the EDK log price, the SWE log price, and the log relative price. In each panel, the diamonds are the actually observed values covering the last 24 in-sample observations as well as 24 out-of-sample observations. Each panel also has three solid and three dotted lines. The three solid lines are median forecasts and 95% forecast error bands for the RS-SARFIMA model, and the three dotted lines are the corresponding median forecasts and error bands for the non-switching SARFIMA model. Note that some of the lines may overlap.

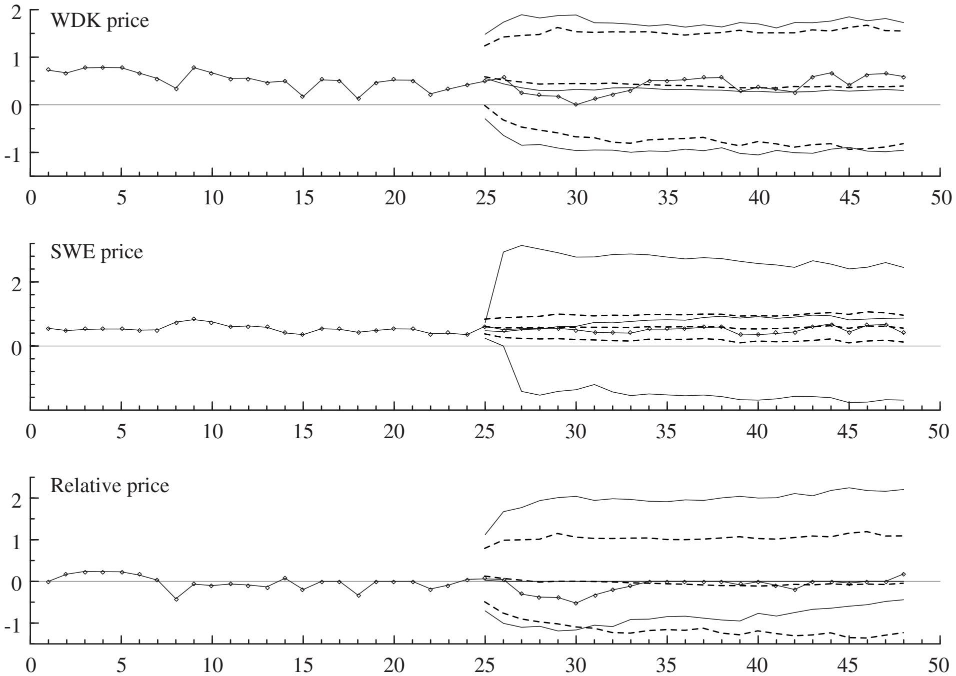


Fig. 5. Forecasts for the WDK-SWE physical link. *Note*: The three panels show forecasts for the WDK log price, the SWE log price, and the log relative price. In each panel, the diamonds are the actually observed values covering the last 24 in-sample observations as well as 24 out-of-sample observations. Each panel also has three solid and three dotted lines. The three solid lines are median forecasts and 95% forecast error bands for the RS-SARFIMA model, and the three dotted lines are the corresponding median forecasts and error bands for the non-switching SARFIMA model. Note that some of the lines may overlap.

observed values covering the last 24 in-sample observations as well as 24 out-of-sample observations. Each panel also has three solid and three dotted lines. The three solid lines are median forecasts and 95% forecast error bands for the RS-SARFIMA model, and the three dotted lines are the corresponding median forecasts and error bands for the non-switching SARFIMA model. Note that some of the lines may overlap. Also note that the forecasts for the relative prices are not defined as the simple difference between the forecasts for the two price series, but rather they are forecasts from the (switching or non-switching) model for the log relative prices which was estimated in the previous section.

In Fig. 4 the forecasts are shown for the EDK-SWE physical link. For all three series, we notice that the median forecasts from both models are very close to the actually observed value. For the two individual log price series in the top two panels of Fig. 4, it appears that the confidence bands for the regime switching model forecasts are initially slightly tighter than the confidence bands for the non-switching model forecasts. After 3–4 h the regime switching model confidence bands become a

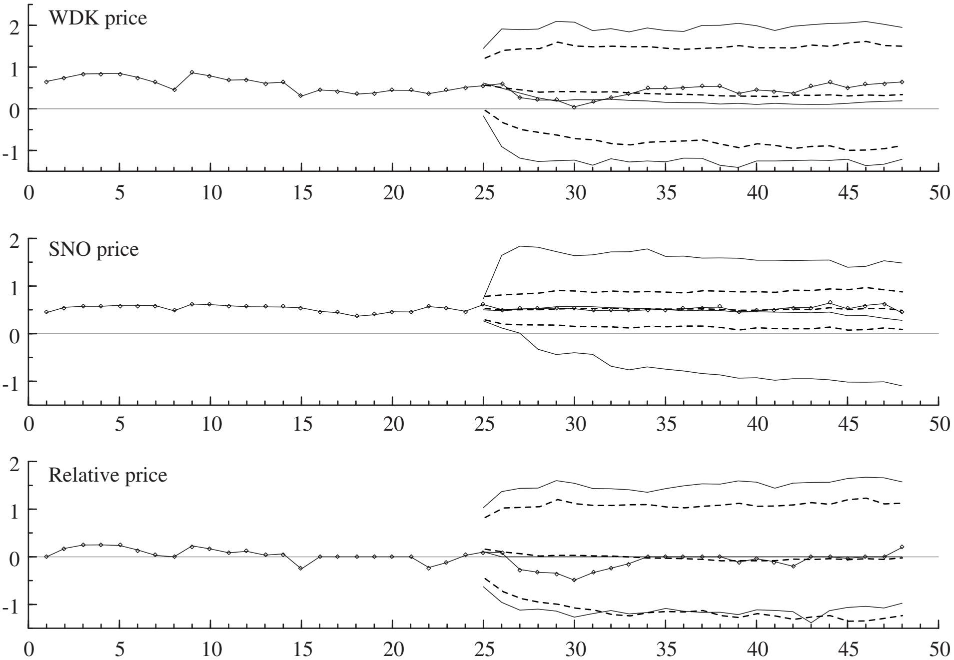


Fig. 6. Forecasts for the WDK-SNO physical link. *Note*: The three panels show forecasts for the WDK log price, the SNO log price, and the log relative price. In each panel, the diamonds are the actually observed values covering the last 24 in-sample observations as well as 24 out-of-sample observations. Each panel also has three solid and three dotted lines. The three solid lines are median forecasts and 95% forecast error bands for the RS-SARFIMA model, and the three dotted lines are the corresponding median forecasts and error bands for the non-switching SARFIMA model. Note that some of the lines may overlap.

little wider, though. This is most likely due to the fact that the uncertainty about the state is increasing as we forecast further into the future. However, the forecast confidence bands for the regime switching model remain similar to those of the non-switching model even for the 24 h ahead forecast.

For the log relative prices in the bottom panel of Fig. 4, the switching model produces vastly superior forecasts compared to the non-switching model. Whereas the forecast confidence bands for the non-switching model remain of the same order of magnitude as for the individual log price series, the switching model is capable of exploiting the structure of the data to produce much tighter confidence bands, at least for the first 3 h. For the entire 24 h forecast horizon, the median forecast for the switching model is exactly equal to the observed value (which is zero throughout the forecast period). Indeed, for the first three forecasts, the forecast confidence bands for the switching model are just points, i.e. all three solid lines are equal to zero, meaning that at least 95% of the probability mass for those forecasts are located at that point, see also Fig. 11. For the remaining forecasts up to 24 h ahead, the

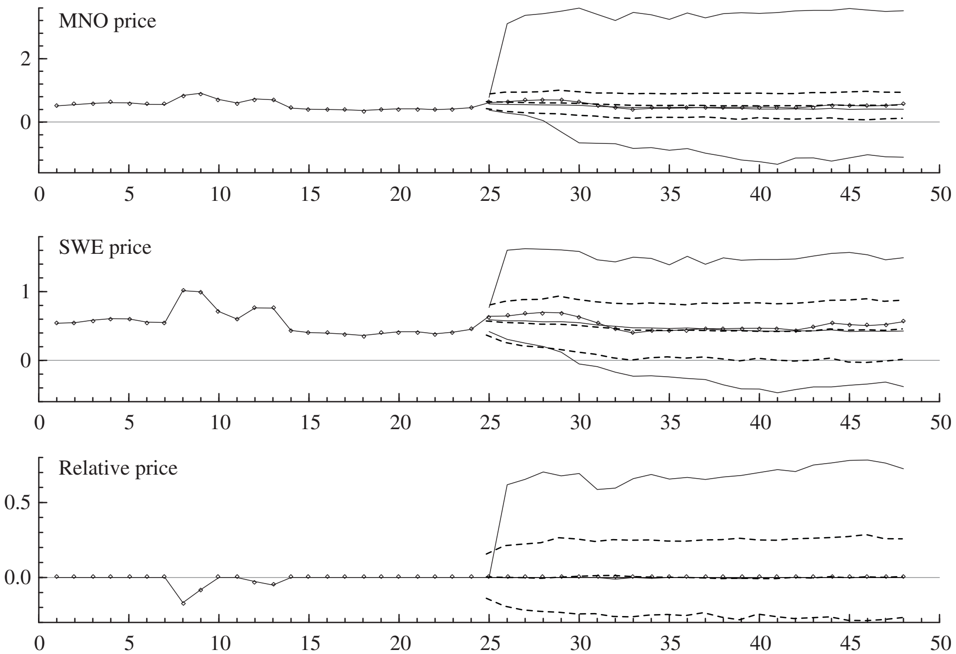


Fig. 7. Forecasts for the MNO-SWE physical link. *Note*: The three panels show forecasts for the MNO log price, the SWE log price, and the log relative price. In each panel, the diamonds are the actually observed values covering the last 24 in-sample observations as well as 24 out-of-sample observations. Each panel also has three solid and three dotted lines. The three solid lines are median forecasts and 95% forecast error bands for the RS-SARFIMA model, and the three dotted lines are the corresponding median forecasts and error bands for the non-switching SARFIMA model. Note that some of the lines may overlap.

switching model upper confidence band is still zero meaning that with 95% confidence we can predict that the East Denmark prices will be no higher than the Sweden prices.

Figs. 5 and 6 display the forecasts for the WDK-SWE and WDK-SNO physical links, respectively. In both cases, the forecasting performance for WDK is similar to that for EDK in Fig. 4, i.e. the regime switching model seems to give very similar but slightly wider confidence bands compared to the non-switching model. For the SWE prices in both Figs. 5 and 6, the forecast error bands for the regime switching model are somewhat wider than for the non-switching model. However, for all four individual price series in Figs. 5 and 6 the median forecasts are very close to the actually observed value. For the relative prices, both pairs end the estimation period in the congestion regime, and we thus do not expect the forecast performance of the regime switching model to be as great as in Fig. 4 where the estimation period ended in the deterministic state. Indeed, this is confirmed in the bottom graphs of Figs. 5 and 6 where the forecasting performances of the non-switching and regime switching models are very similar.

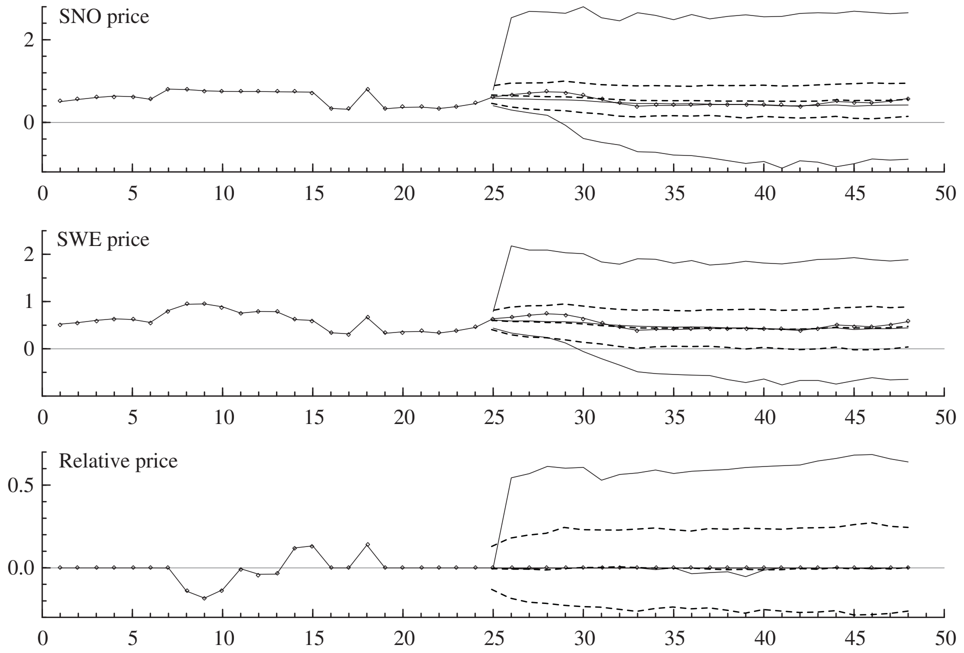


Fig. 8. Forecasts for the SNO-SWE physical link. *Note*: The three panels show forecasts for the SNO log price, the SWE log price, and the log relative price. In each panel, the diamonds are the actually observed values covering the last 24 in-sample observations as well as 24 out-of-sample observations. Each panel also has three solid and three dotted lines. The three solid lines are median forecasts and 95% forecast error bands for the RS-SARFIMA model, and the three dotted lines are the corresponding median forecasts and error bands for the non-switching SARFIMA model. Note that some of the lines may overlap.

In Figs. 7–9 the forecasts for the MNO-SWE, SNO-SWE, and MNO-SNO physical links are displayed. The forecasts for each of the six individual prices in these three figures are very similar to the SWE and SNO forecasts in Figs. 5 and 6 where the switching model provides somewhat wider confidence bands than the non-switching model. In all six cases, the median forecasts are very close to the actually observed values though. All three log relative price series in Figs. 7–9 end in the non-congestion regime, and thus the regime switching model seems to outperform the non-switching model in the prediction of the log relative prices.

Finally, Fig. 10 displays the forecasts for the SWE-FIN link. For this data pair, the regime-switching model seems to perform at least as well as the non-switching model in terms of out-of-sample forecasting, except maybe the lower confidence band for the log relative prices which is a little wide. However, the upper confidence band is very tight even though the data pair ends in the congestion regime.

The relatively poor forecast performance of regime switching models, and non-linear models in general, is well known in the literature, see e.g. Clements and Hendry (1999) and Dacco and Satchell (1999). On that background it is somewhat

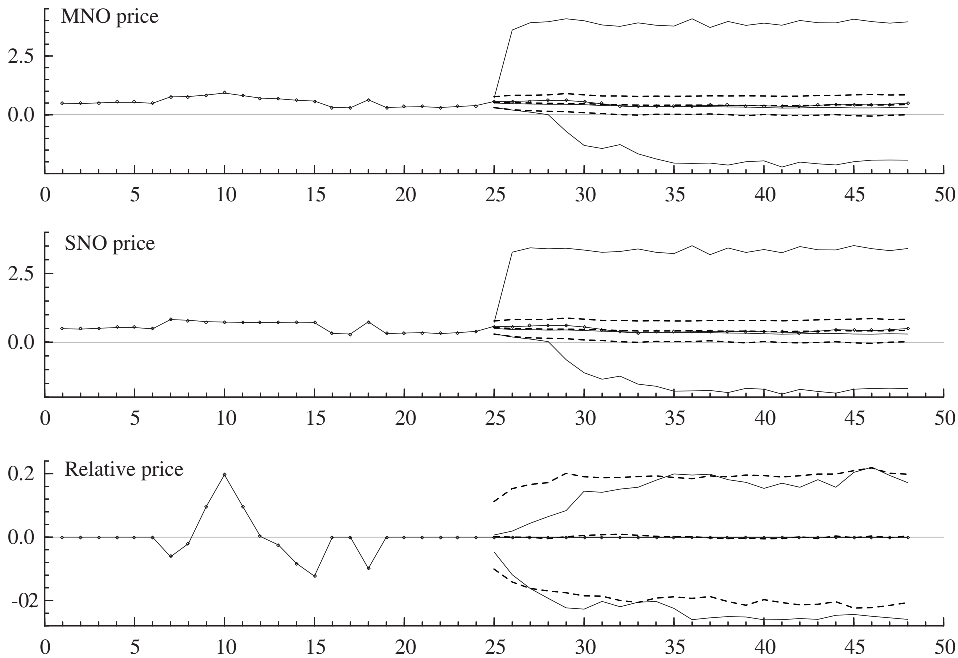


Fig. 9. Forecasts for the MNO-SNO physical link. *Note*: The three panels show forecasts for the MNO log price, the SNO log price, and the log relative price. In each panel, the diamonds are the actually observed values covering the last 24 in-sample observations as well as 24 out-of-sample observations. Each panel also has three solid and three dotted lines. The three solid lines are median forecasts and 95% forecast error bands for the RS-SARFIMA model, and the three dotted lines are the corresponding median forecasts and error bands for the non-switching SARFIMA model. Note that some of the lines may overlap.

surprising that our regime switching model seems to perform similarly to the linear SARFIMA model in terms of out-of-sample forecasting for the individual series, and even outperforms the linear model in the forecasting of relative prices. Note also that the forecasting performance of the switching model could be significantly improved upon if the forecasts were conditional on the regime. Thus, if the regime was perfectly predictable the switching model would clearly outperform the non-switching model in terms of out-of-sample forecasting as well as in-sample fit.

The switching model does seem to provide superior forecasts for the relative prices, and it appears to be particularly successful when the post-sample observations belong to the relatively more persistent regime 0 (non-congestion). Another way to see this is to consider the forecast densities. As an example, we provide in Fig. 11 the one-step ahead (smoothed) forecast densities for the EDK-SWE physical link also considered in Fig. 4. The three panels in Fig. 11 display the forecast densities for both the non-switching and the regime switching model for each of the three time series (two individual prices and the relative price). The vertical lines in the graphs are the actually observed values.

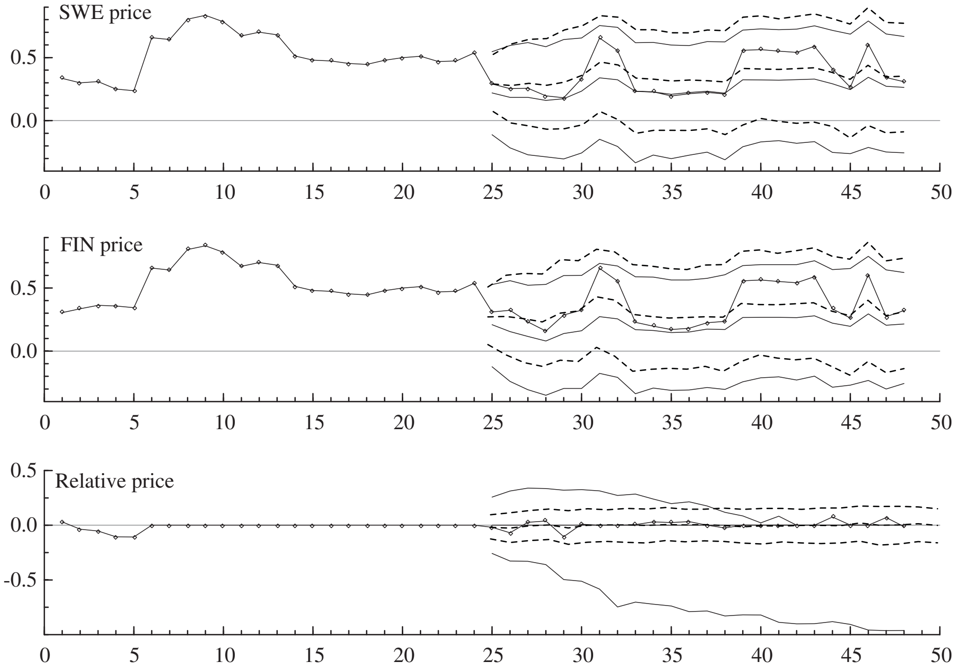


Fig. 10. Forecasts for the SWE-FIN physical link. *Note*: The three panels show forecasts for the SWE log price, FIN log price, and log relative price. In each panel, the diamonds are the actually observed values covering the last 24 in-sample observations as well as 24 out-of-sample observations. Each panel also has three solid and three dotted lines. The three solid lines are median forecasts and 95% forecast error bands for the RS-SARFIMA model, and the three dotted lines are the corresponding median forecasts and error bands for the non-switching SARFIMA model. Note that some of the lines may overlap.

It appears that, for the two individual log price series, the forecasts from the regime switching model has a higher concentration of density around the actually observed value. Furthermore, when we look at the bottom panel which displays the one-step ahead forecast densities for the log relative prices, the regime switching model is, not surprisingly, greatly superior to the non-switching model in terms of out-of-sample forecasting. The (smoothed) density of the simulated forecasts from the regime switching model is highly concentrated around the observed value of zero (indeed, 98.6% of the density is concentrated on this point), whereas the density of the non-switching model appears in the figure as an almost flat line close to the horizontal axis (due to the scaling of the vertical axis and the concentration of the regime switching forecasts at zero).

## 6. Conclusion

We have suggested a Markov regime switching model with long memory in the separate states, which appears to well describe the dynamics of electricity prices

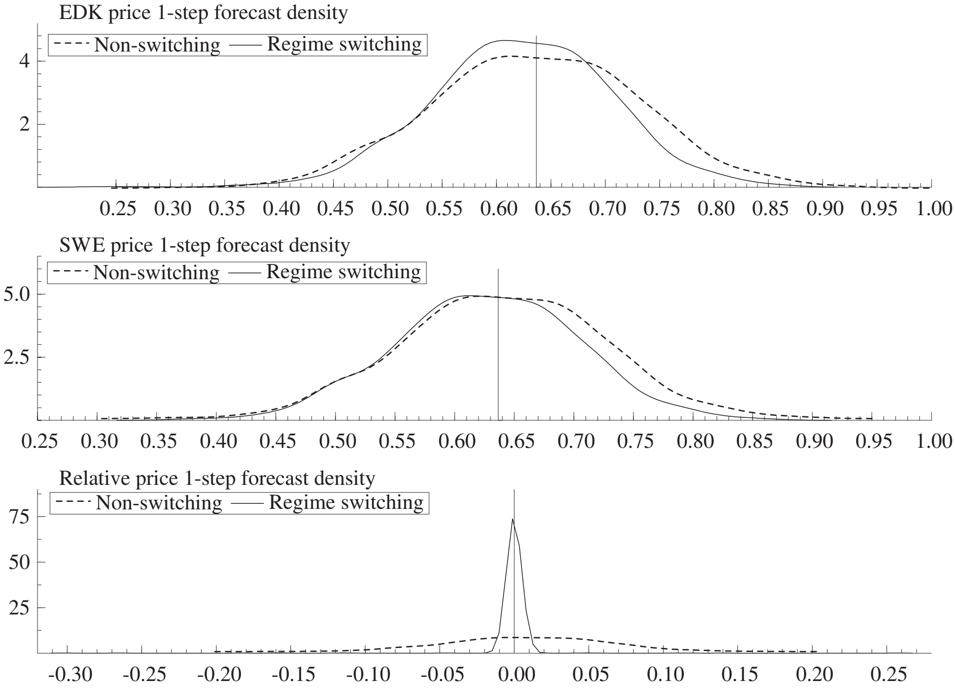


Fig. 11. One-step ahead forecast densities for the EDK-SWE physical link. *Note*: The vertical lines are the actually observed values.

within the Nord Pool area. The model is motivated by the functioning of the Nordic electricity power market where natural switches between different regimes exist reflecting the possible presence or absence of bottlenecks in the transmission of electricity across exchange grid points.

In our empirical analysis, we have seen that our new regime switching specification, with a potentially deterministic state and observable regimes, is very important in order to reach correct conclusions about the behavior of the electricity prices and relative electricity prices. Furthermore, the switching model seems to provide better forecasts for the relative prices, and it appears to be particularly successful when regime persistence is high and the post-sample observations belong to the non-congestion regime.

A number of generalizations are obvious for future research. The analysis undertaken in the present study considers bivariate comparisons of the price series. However, because all the separate regions in the Nord Pool regions are interconnected directly or indirectly one can in fact define multiple regime states where more than two price area combinations are considered. For instance, East Denmark and West Denmark have no direct transmission line for electricity exchange, but a connection exists via Sweden and possibly Sweden plus South Norway. So in some periods prices in East Denmark and West Denmark are identical due to their linkages via Sweden and Norway. An extension of our model set up to include multiple regions and price areas would be interesting to pursue in the future. Another interesting extension concerns the inclusion of other relevant variables in the model like weather data, the flow quantity of electricity, and other variables. In particular, the influence of such factors on the transition probabilities seems interesting. Also, the direction of a possible congestion is potentially of relevance, i.e. the question of whether the bottleneck is from region 1 to region 2 or opposite.

In this paper, we have focused on models for the level of electricity prices. With respect to risk management and the pricing of power derivatives, models for the volatility are essential and a similar framework with regime switching and long memory seems natural for this case as well. The development and analysis of such models remain for the future.

## Acknowledgements

Previous versions of this paper were presented at the "Predictive Methodology and Application in Economics and Finance" conference in honor of Clive W.J. Granger in San Diego, January 2004, at the "Aarhus Econometrics" conference in Svinkløv, Denmark, May 2004, the EC$^2$-meeting in Marseilles, December 2004, and in seminars at the Copenhagen Business School, Cornell University, and Université de Montréal. We are grateful to the participants as well as two anonymous referees for their very helpful comments. M.Ø. Nielsen is grateful for financial support from the Danish Social Science Research Council (Grant No. 24-02-0181). We also thank Dennis Andersen for research assistance.

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