

1.

[Maximum mark: 7]

Let $f(x) = \ln(x+5) + \ln 2$, for $x > -5$.

(a) Find $f^{-1}(x)$. [4 marks]

Let $g(x) = e^x$.

(b) Find $(g \circ f)(x)$, giving your answer in the form $ax + b$, where $a, b \in \mathbb{Z}$. [3 marks]

2.

Let $f(x) = 3(x+1)^2 - 12$.

(a) Show that $f(x) = 3x^2 + 6x - 9$. [2 marks]

(d) Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the two transformations:

a stretch of scale factor t in the y -direction

followed by

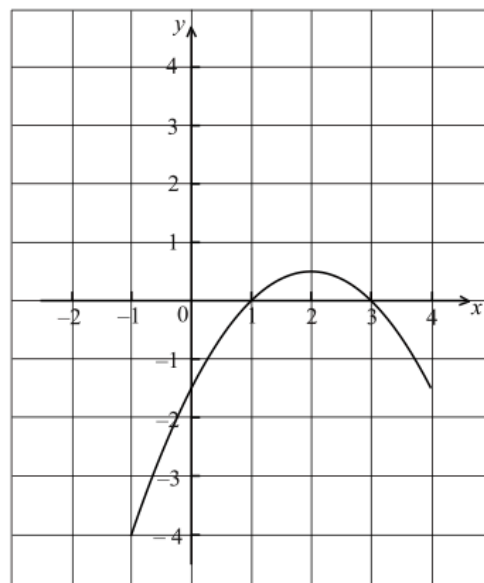
a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t . [3 marks]

3.

[Maximum mark: 6]

Part of the graph of a function f is shown in the diagram below.



(a) On the same diagram sketch the graph of $y = -f(x)$. [2 marks]

(b) Let $g(x) = f(x+3)$.

(i) Find $g(-3)$.

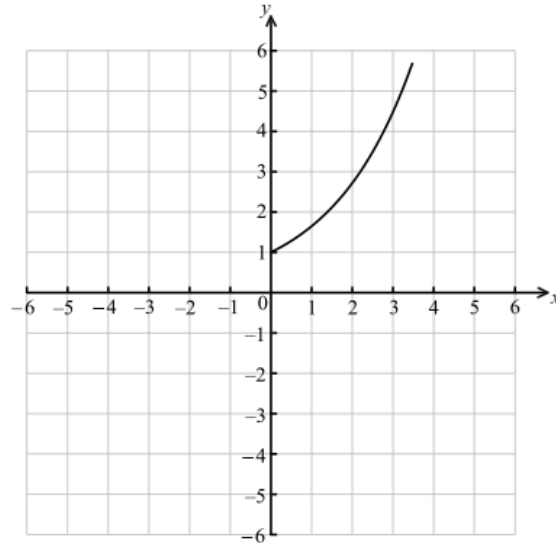
(ii) Describe **fully** the transformation that maps the graph of f to the graph of g .

[4 marks]

4.

[Maximum mark: 7]

Let f be the function given by $f(x) = e^{0.5x}$, $0 \leq x \leq 3.5$. The diagram shows the graph of f .



- (a) On the same diagram, sketch the graph of f^{-1} . [3 marks]
- (b) Write down the range of f^{-1} . [1 mark]
- (c) Find $f^{-1}(x)$. [3 marks]

5.

[Maximum mark: 6]

Let $f(x) = x^2$ and $g(x) = 2(x-1)^2$.

- (a) The graph of g can be obtained from the graph of f using two transformations. Give a full geometric description of each of the two transformations. [2 marks]
- (b) The graph of g is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give the graph of h .
The point $(-1, 1)$ on the graph of f is translated to the point P on the graph of h . Find the coordinates of P. [4 marks]

6.

[Maximum mark: 7]

Let $f(x) = e^{x+3}$.

(a) (i) Show that $f^{-1}(x) = \ln x - 3$.

(ii) Write down the domain of f^{-1} .

[3 marks]

(b) Solve the equation $f^{-1}(x) = \ln\left(\frac{1}{x}\right)$.

[4 marks]

7.

[Maximum mark: 8]

Let $f(x) = 2x^3 + 3$ and $g(x) = e^{3x} - 2$.

(a) (i) Find $g(0)$.

(ii) Find $(f \circ g)(0)$.

[5 marks]

(b) Find $f^{-1}(x)$.

[3 marks]

8.

[Maximum mark: 6]

Let $f(x) = 2x - 1$ and $g(x) = 3x^2 + 2$.

(a) Find $f^{-1}(x)$.

[3 marks]

(b) Find $(f \circ g)(1)$.

[3 marks]

9.

[Maximum mark: 7]

Find the value of

(a) $\log_2 40 - \log_2 5$;

[3 marks]

(b) $8^{\log_2 5}$.

[4 marks]

10.

[Maximum mark: 7]

Let $\log_3 p = 6$ and $\log_3 q = 7$.

(a) Find $\log_3 p^2$.

[2 marks]

(b) Find $\log_3 \left(\frac{p}{q} \right)$.

[2 marks]

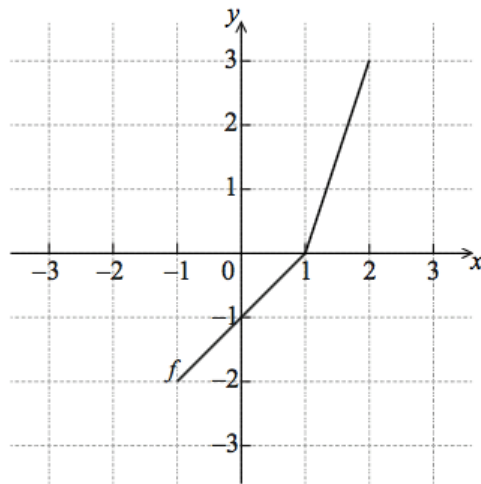
(c) Find $\log_3 (9p)$.

[3 marks]

11.

[Maximum mark: 6]

The diagram below shows the graph of a function f , for $-1 \leq x \leq 2$.



(a) Write down the value of

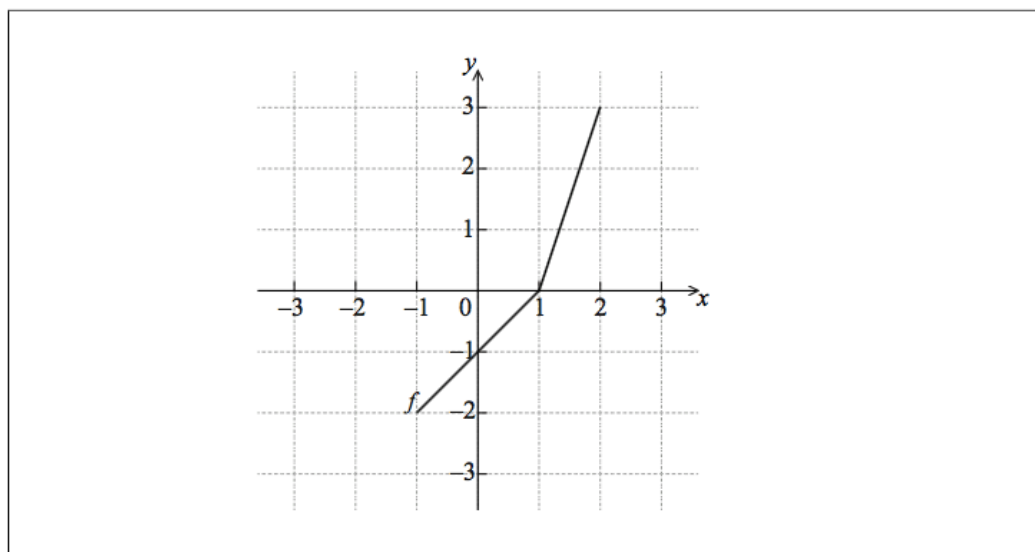
(i) $f(2)$;

(ii) $f^{-1}(-1)$.

[3 marks]

(b) Sketch the graph of f^{-1} on the grid below.

[3 marks]



12.

[Maximum mark: 14]

Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

(a) Find $f^{-1}(x)$.

[2]

(b) Show that $(g \circ f^{-1})(x) = \frac{5}{x+2}$.

[2]

Let $h(x) = \frac{5}{x+2}$, for $x \geq 0$. The graph of h has a horizontal asymptote at $y = 0$.

(c) (i) Find the y -intercept of the graph of h .

(ii) Hence, sketch the graph of h .

[5]

(d) For the graph of h^{-1} ,

(i) write down the x -intercept;

(ii) write down the equation of the vertical asymptote.

[2]

(e) Given that $h^{-1}(a) = 3$, find the value of a .

[3]