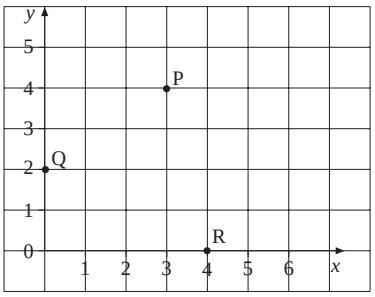
Answers

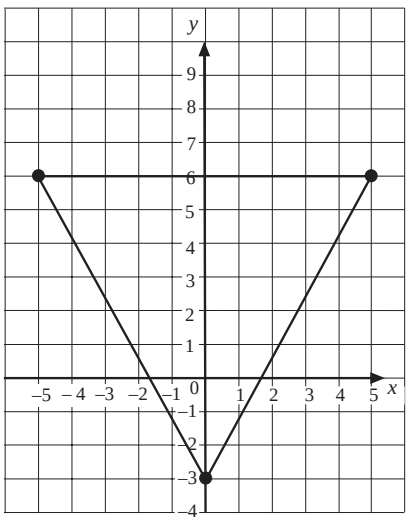
13 Graphs

13.1 Positive Coordinates

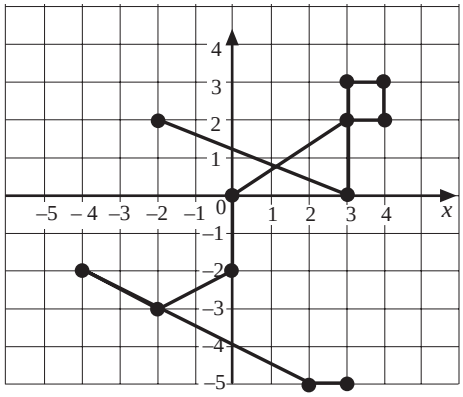
1. A (0, 4), B (1, 3), C (2, 1), D (3, 0), E (4, 3), F (5, 2)
2. Rocky Point (2, 8) , Landing Stage (2, 2) , Old Ben's Cottage (3, 5)
   Old Tower (4, 3) , Café (7, 6) , Sandy Beach (9, 3) , Camp Site (10, 6)
3. (a) rectangle (b) triangle (c) rhombus (d) pentagon (e) hexagon
4. (a) J : (1, 2), (1, 1), (2, 1), (2, 5), (1, 5) and (3, 5)
   S : (4, 1), (6, 1), (6, 3), (4, 3), (4, 5) and (6, 5)
5. (a) (2, 3), (3 2), (4, 3), (5, 2), (6, 3)
6. (a) 

(b) (i) A (2, 1) (ii) (B) (1, 5)

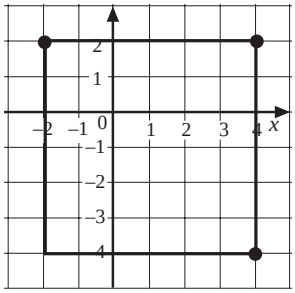
13.2 Coordinates

1. A (2, 5), B (4, 3), C (2, 1), D (2, –2), E (5, –3), F (3, – 4), G (–5, 4),
   H (–3, 3), I (–5, 2), J (– 4, –2), K (–2, –3), L (–6, –5)
2. (a) (5, – 4), (6, –3) and (–6, –5)
   (b) Albany to Alice Springs
   (c) Broome to Perth
3. (a) (0, 4), (12.5, –5.5), (–12, 1), (–6.5, –3.5), (–1, –7) (b) (–1, 1)
   (c) (10, 0) to (–7.5, –10.5) (d) (–17, –2) (e) (10, 7)
4. (a) triangle (b) 

6. (b)

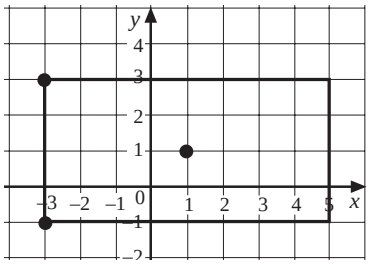


7. (a)



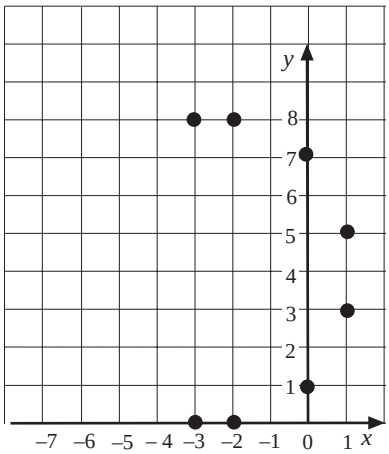
(b) (1, –1)

8. (a)



(b) (5, –1), (5, 3)

9. (b)



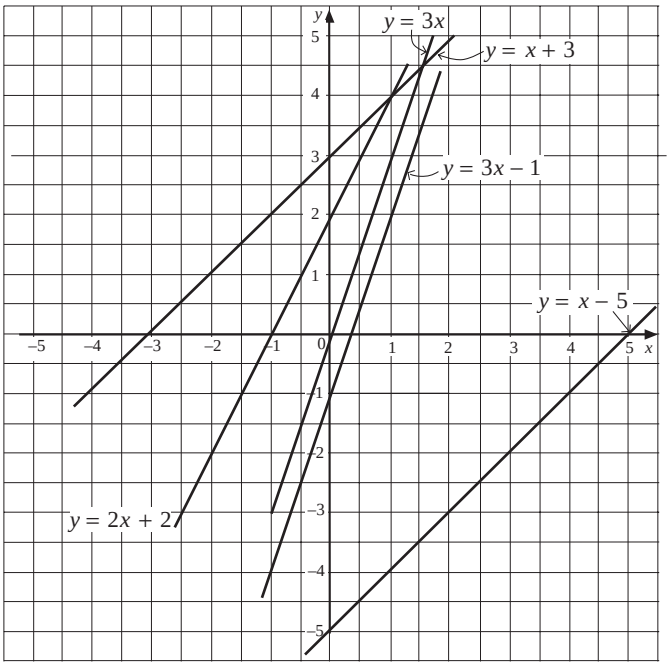
(c) (– 4, 0), (–6, 1), (–7, 3), (–7, 5), (–6 , 7), (– 4, 8)

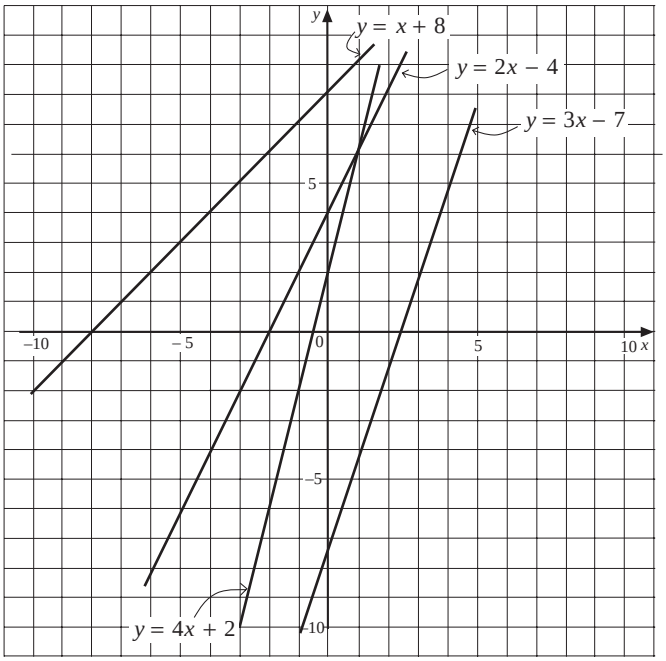
10. (a) 2 units (b) 4 units (c) $\sqrt{32} \approx 5.66$

13.3 Plotting Straight Lines

1. (a) (4, 6), (1, 3), (–3, 1) (e) (0, 2)
2. (a) (3, 7), (1, 3), (–2, –3) (d) Yes

3. (a) (3, 7), (0, –2), (–2, –8) (d) No

4. 

5. 

6. (a)

t	0	2	4
d	0	8	16

(b) (0, 0), (2, 8), (4, 16) (d) x-axis (horizontal)
(e) y-axis (vertical) (f) 3 hours (g) 14 km

7. (a)

m	0	10	25
p	0	40	100

(b) (0, 0), (10, 40), (25, 100)

(d)

	Mark	*Percentage*
John	15	60
Stuart	21	84
Jenny	18	72
Karen	20	80
Mike	15	60

8. (a)

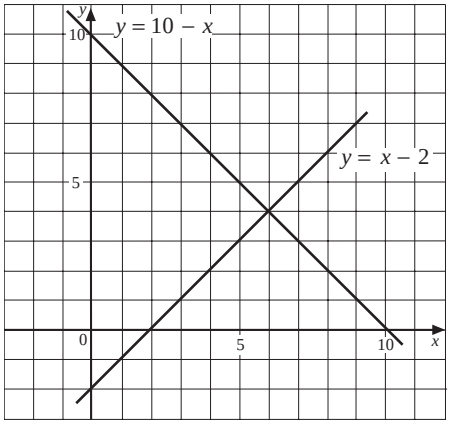
°C	0	20	100
°F	32	68	212

(c) about 27 °C (d) 86 °F

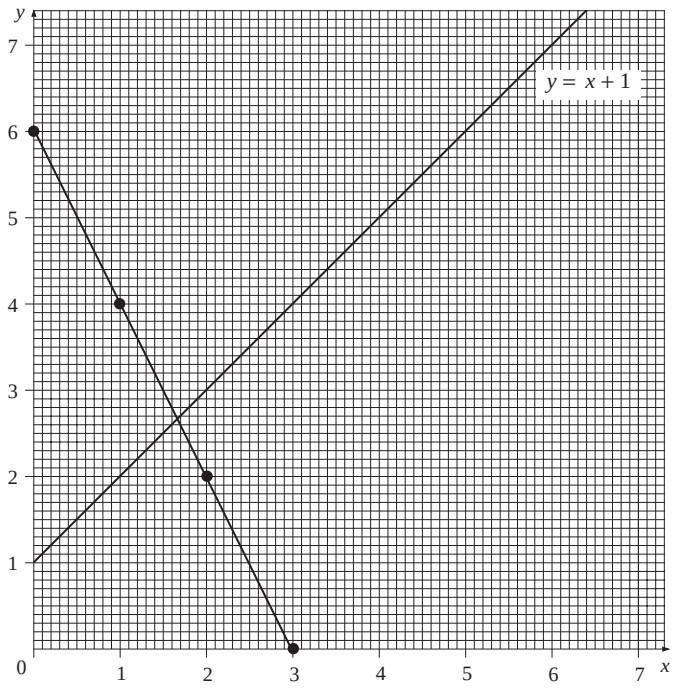
9. (a) (0, 4), (2, 2), (4, 0) (c) (0, 4) and (4, 0)

10. (c) 18 sq. units

11. (a) see graph
(b) (6, 4)
(c) 16 sq. units
(d) (2, 6) , 18 sq. units
(e) (4, 4) , 4 sq. units



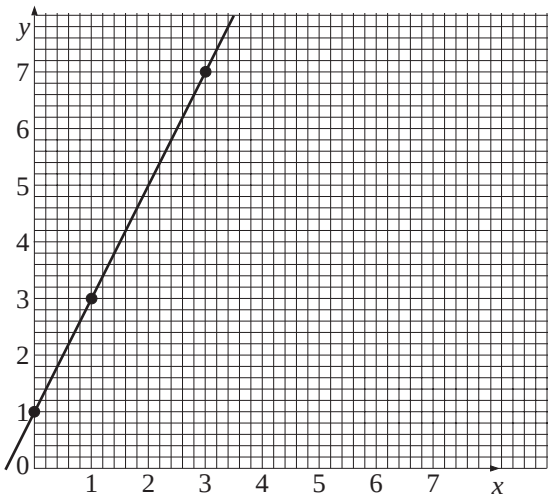
12.



13. (a)

x	y
0	1
1	3
3	7

(b)



(c) $y = 2x + 1$ (d) 4

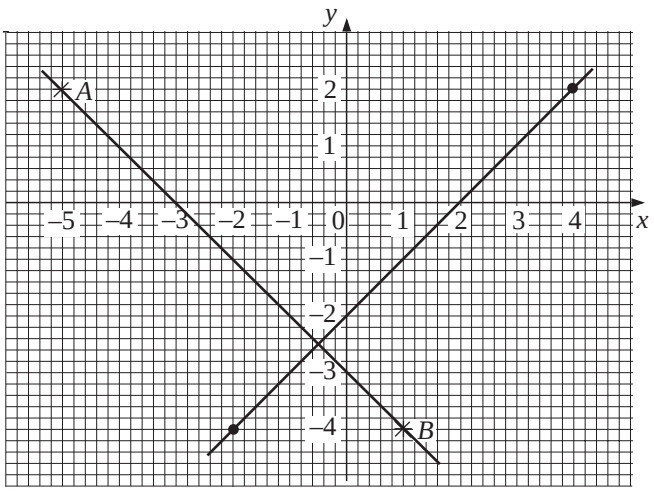
14. (a) $A\ (-5, 2)$, $B\ (1, -4)$

(b)

x	–2	0	2	3	4
y	– 4	–2	0	1	2

(c) see graph

(d) $\left(-\frac{1}{2}, -\frac{5}{2}\right)$



15. (a)

IN	1	2	3	6
OUT	1	7	13	31

(b) (iii) (4, 19)

16. (a)

IN	OUT
3	2
5	8
6	11
8	17

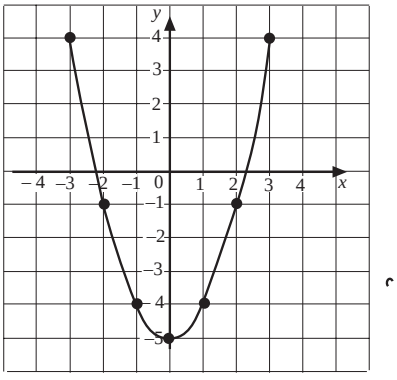
(c) They lie on a straight line

13.4 Plotting Curves

1. (a)

x	–3	–2	–1	0	1	2	3
y	4	–1	– 4	–5	– 4	–1	4

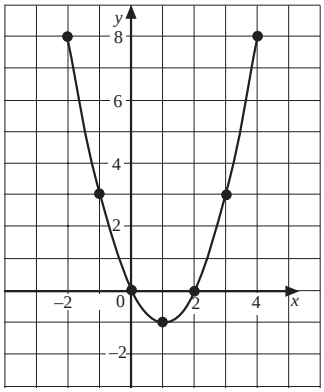
(b) (–3, 4), (–2, –1), (–1, – 4), (0, –5), (1, – 4), (2, –1), (3, 4)



2. (a)

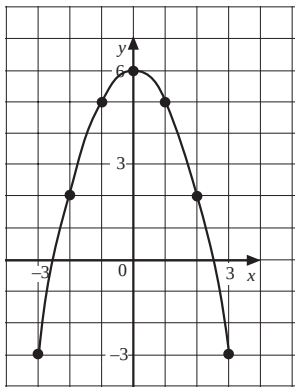
x	–2	–1	0	1	2	3	4
y	8	3	0	–1	0	3	8

(b)



3.

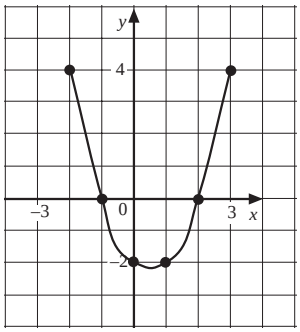
x	–3	–2	–1	0	1	2	3
y	–3	2	5	6	5	2	–3



4. (a)

x	–2	–1	0	1	2	3
y	4	0	–2	–2	0	4

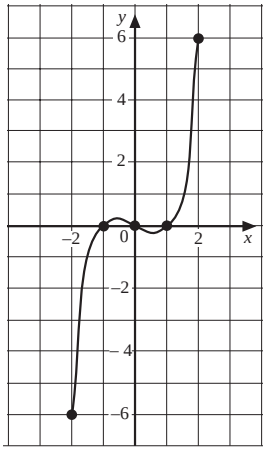
(b) 0.5 (c) –2.25



5. (a)

x	–2	–1	0	1	2
y	–6	0	0	0	6

(b)



(c) (–0.5, 0.375), (0.5, –0.375)

6.

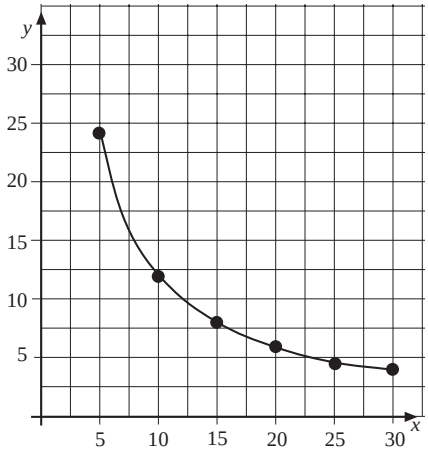
x	1	2	3	4	5	6
y	15	7.5	5	3.75	3	2.5

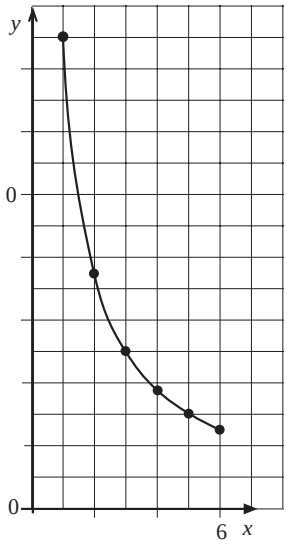
7. (a) $x\,y$ = area = 120 cm^2

(b)

x	5	10	15	20	25	30
y	24	12	8	6	4.8	4

(c)



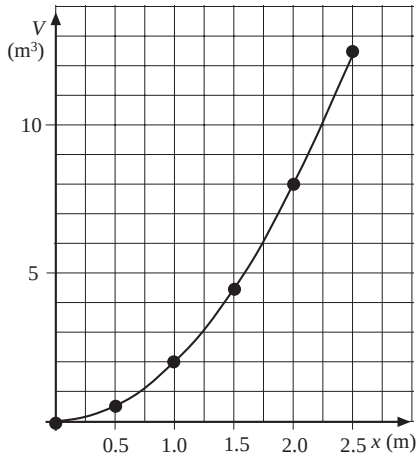


(d) width about 17 cm

(e) about 8.6 cm

8. (a) Volume $= 2 \times x \times x = 2x^2$ (b)

x	0	0.5	1.0	1.5	2.0	2.5
V	0	0.5	2.0	4.5	8.0	12.5

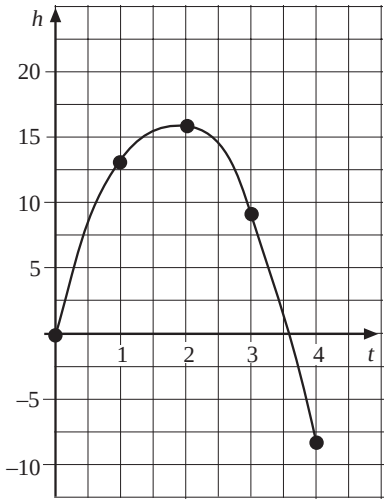


(c) about 9.7 m^2

(d) (i) 2.2 m (ii) 1.6 m

9. (a)

t	0	1	2	3	4
h	0	13	16	9	–8



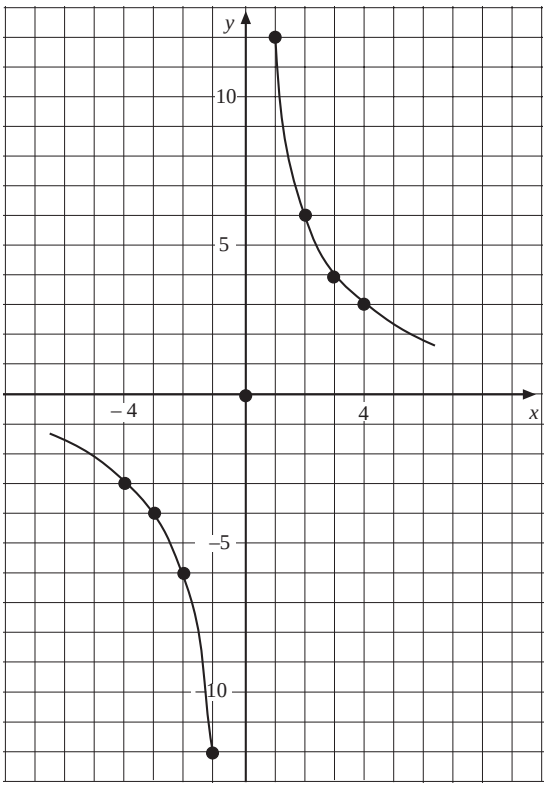
(b) about 3.6 seconds

(c) just over 16 metres

10. (a)

x	– 4	–3	–2	–1	0	1	2	3	4
y	–3	– 4	–6	–12	0	12	6	4	3

(b)



(d) y gets larger and larger

(e) y gets negatively larger and larger

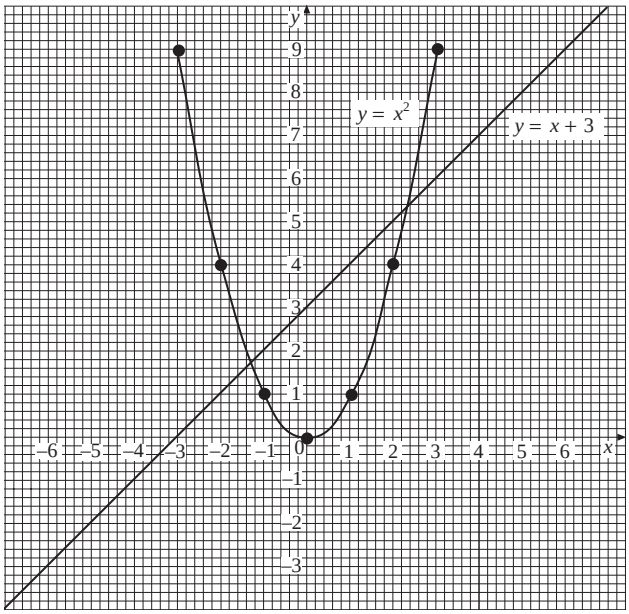
11. (a)

x	–3	–2	–1	0	1	2	3
y	0	1	2	3	4	5	6

(b)

x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9

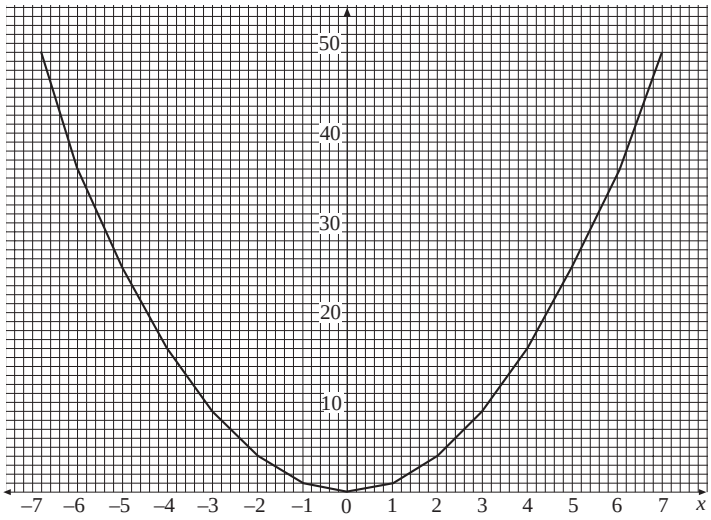
(c)



12. (a)

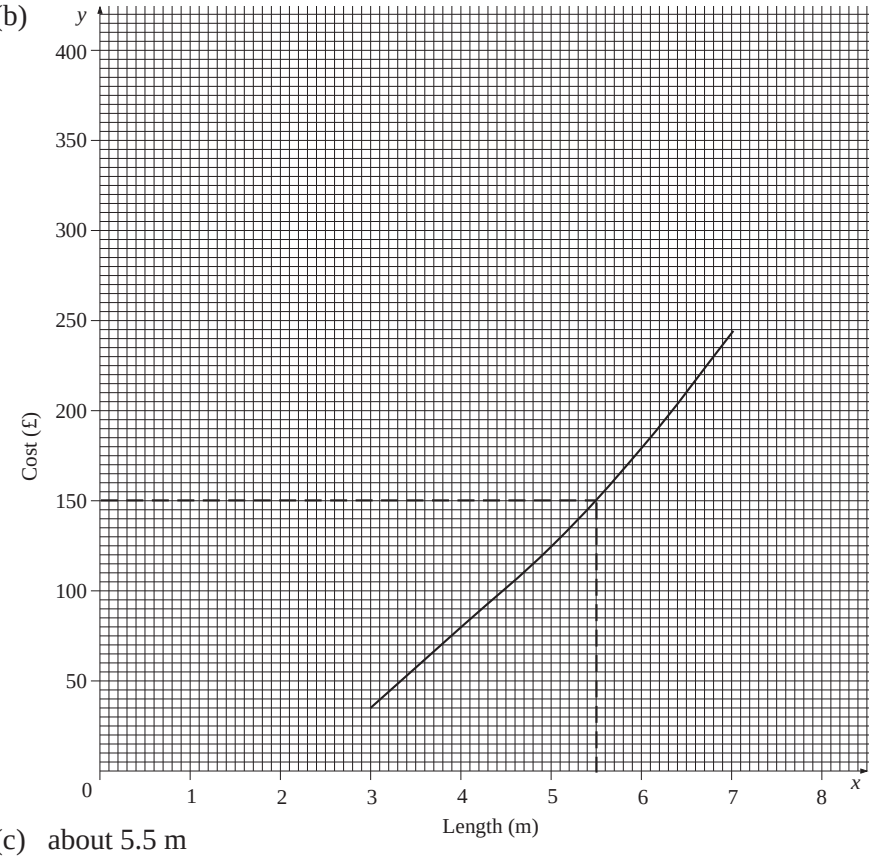
x	–5	5	6
y	25	25	36

(b)



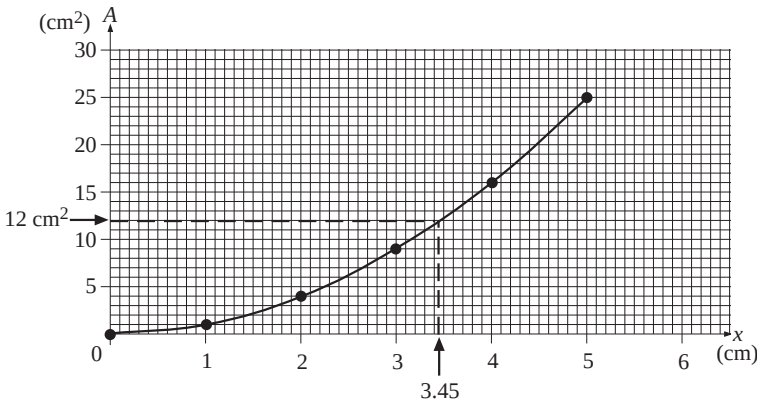
13. (a) £320

(b)



(c) about 5.5 m

14. (a)



(b) $A = x^2$ (c) about 3.45 cm

13.5 Gradient

1. 3
2. AB : 1, CD : 2, EF : 4
3. (a) CD, AB, KL, GH (b) EF, I J
4. $\frac{2}{10} = \frac{1}{5}$ (= 0.2)
5. 2, 1, $\frac{1}{5}$

6. (a) $\frac{1}{2}$ (b) 5 (c) 4 (d) $\frac{4}{5}$ (e) 1 (f) 7

7. (a) $-\frac{1}{2}$ (b) $-\frac{5}{4}$ (c) –1 (d) $-\frac{5}{4}$ (e) 1 (f) $\frac{1}{6}$

8. AB: $-\frac{1}{3}$, BC: 0, CD: $-\frac{6}{5}$, DA: 5

9. (a) (2, 5), (0, 1), (–2, –3) (c) 2 (e) same gradient

10. (c) 3 (d) gradient of $y = 4x - 1$ is 4; gradient of line $y = 5x + 1$ is 5
    (e) coefficient of x is the gradient of a straight line (f) 7

11. (a) 1 (b) –1 (c) 4 (d) –2

13.6 Applications of Graphs

1. (a) 2.7 kg (b) 3.6 kg (c) 11 lbs (d) 6.6 lbs

2. (a) 22.5 litres (b) 6.7 gallons

3. (a) 3 (b) AB: 20 m/s ; CD: 5 m/s ; EF: 10 m/s ; GH:10 m/s (c) AB

4. (a) 150 s (b) AB and CD; 5 m/s (c) EF; $\frac{10}{3}$ m/s

5. 16 + 88 + 4 = 108 metres

6. (a) 20 m (b) 80 m (c) 120 m

7. AB: 0.5 m/hour ; BC: not moving ; CD: 2 m/hour

8. 14 m

9. (a) 7 m/s ; 2 m/s (b) 4.78 m/s

10. Jodie ran faster for the first 10 s but then slowed down until Wendy caught up at the end of the school field. While Wendy rested, Jodie returned at a constant speed until reaching the starting point, whilst Wendy (after her rest) ran faster, reaching the starting point at the same time.

11. (a) 12.5 miles (b) 56.25 miles (c) 6600 mm (d) 35 m

12. (a) 75 km/hour; 20.83 m/s (b) 0.4375 mm/s; 0.04375 m/s
    (c) 60 m/hour ; $\frac{1}{60}$ m/s (d) 0.5 m/min ; $\frac{1}{120}$ m/s

13. (a) 09.36 (b) 7 km (c) 90 mins (d) 4 km/hour

14. (a) 82 seconds from the start (b) 8.5 m (c) Robert – steeper slope

15. (a) about 4300 (b) metres

13.7 Scatter Plots and Lines of Best Fit

1. (c) maths and science (d) not for maths and French

2. (c) 70 miles (d) 4.3 hours

3. (c) £300 (d) 6.5 hours

4. (c) 120 cm (d) 57 kg
5. (b) 17 s (c) 7 s
6. (c) men: about 210 s; women about 220 s
   (d) it looks as if the women will catch up the men , but this is probably not realistic
8. (b) positive correlation
9. (b) the value decreases as they get older
10. (b) 68
11. (b) (i) 68.5 cm (ii) the data points do not fit exactly on a straight line
12. (a) there is positive correlation between the marks (b) 6 or 7
13. (a) negative correlation (b) about 115 mm

13.8 The Equation of a Straight Line

1. (a) $y = 2x + 4$ (b) $y = 3x - 5$ (c) $y = \frac{1}{2}x + 2$
   (d) $y = -2x + 1$ (e) $y = \frac{3}{4}x - 3$
2. (a) gradient = 2, y-intercept = 3 (b) gradient = 4, y-intercept = –2
   (c) gradient = $\frac{1}{2}$, y-intercept = 1 (d) gradient = $\frac{2}{3}$, y-intercept = – 4
   (e) gradient = 4, y-intercept = 8 (f) gradient = 3, y-intercept = –21
   (g) gradient = $\frac{1}{2}$, y-intercept = $\frac{5}{2}$ (h) gradient = $\frac{1}{4}$, y-intercept = $-\frac{5}{2}$
3. (a) 1 (b) –1 (c) $y = x - 1$
4. A : $y = x + 7$ B : $y = x + 6$ C : $y = 2x + 2$
   D : $y = \frac{8}{5}x + 2$ E : $y = \frac{3}{10}x + 1$ F : $y = \frac{1}{4}x + 1$
5. (a) 2, –8 (b) –3, 2 (c) 4, –3 (d) $\frac{1}{2}$, 2
   (e) –2, 8 (f) –3, 4 (g) –1, 8 (h) –3, 15
6. (a) $y = 2 + 2x$ (b) $y = \frac{1}{2}x$ (c) $y = 6 - \frac{1}{3}x$ (d) $y = 6 - 2x$
7. A : $y = 5 - \frac{1}{2}x$ B : $y = 4 + \frac{4}{3}x$ C : $y = -3 + 2x$
   D : $y = -2 - \frac{2}{5}x$ E : $y = -7 + x$ F : $y = -5 + \frac{2}{5}x$
8. (a) $y = 3 + \frac{1}{50}x$ (b) $y = 10 + \frac{1}{25}x$
9. (b) $y = 19 + \frac{3}{4}x$ (y = temperature, x = height)

10. (a) $y = 4.5x$ (c) $y = \frac{11}{20}x$ (y = litres, x = pints)

11. (b) $y = 20 - 10x$ (y = velocity, t = time) (c) 20 ms^{-1}

12. $c = 3$

13. $c = -12$

14. $m = 5$

15. (a) £32.50 (b) (i) $\frac{1}{2}$ (ii) increase in charge for unit increase in time

(c) $c = 10 + \frac{1}{2}t$ (d) 148 minutes

16. (b) $y = 5 + w$

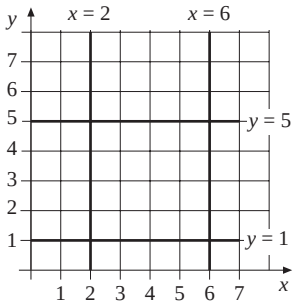
17. (b) (i) 32.5 °C (ii) 68 grams (c) (i) $a = 0.4$, $b = 50$ (ii) 88 grams

18. (a) $T = 20 + 30P$ (b) (i) (4, 140) (ii) about 2.7 lbs

13.9 Horizontal and Vertical Lines

1. A : $x = -7$; B : $x = -4$; C : $x = 3$; D : $x = 8$; E : $y = 3$; F : $y = -5$

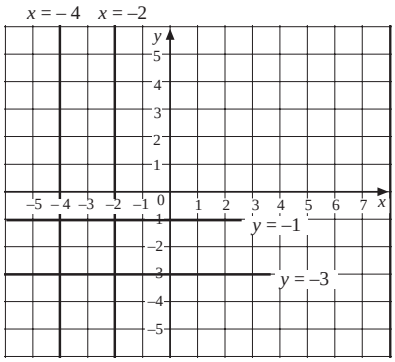
2. (a)



(b) (2, 1) , (2, 5) , (6, 1) , (6, 5)

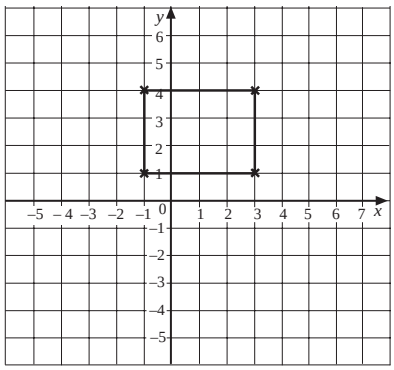
(c) 16 sq. units

3. (a)



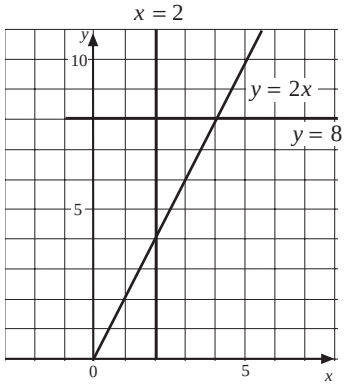
(b) $(-3, -2)$

4. (a)



(b) $x = -1$, $x = 3$, $y = 1$, $y = 4$

5. (a)



(b) 4 sq. units

13.10 Solution of Simultaneous Equations by Graphs

1. (a) (3, 5) (b) (7, 1) (c) (0, –1) (d) (6, –2)
   (e) (–3, –7) (f) (–6, 0)
2. (c) (2, 1)
3. (a) (0, 2) , (2, 4) , (6, 8) (c) (1, –1) , (2, 2) , (4, 8)
   (e) (3, 5)
4. (d) (i) (3, 2) (ii) (–1, –2) (iii) (1, 4)
5. (a) (4, 8) (b) (2, 2) (c) (4, –2)
6. (a) $y = 1 + 3x$, $y = 6 - 2x$ (c) (1, 4)
7. (a) (5, 2) (b) (2, 1) (c) $\left(-\frac{3}{2}, \frac{11}{2}\right)$
8. (a) $x + y = 20$, $y - x = 14$ (b) $y = 20 - x$, $y = 14 + x$
   (c) (3, 17)
9. (a) $2x + 4y = 40$ (b) $y = 10 - \frac{1}{2}x$ (c) $3x + 2y = 36$
   (d) $y = 18 - \frac{3}{2}x$ (f) £8 (g) £6
10. (a) $x + y = 28$, $2x + 5y = 80$
    (b) $y = 28 - x$, $y = 16 - \frac{2}{5}x$ (c) 8
11. $x = 2,\ y = 5$
12. (b) $y = x - 3$ (c)

x	3	7	10
y	0	4	7

(d) $x = \frac{17}{2}$, $y = \frac{11}{2}$

13. (a) (i)

x	100	200	300
y	15	18	21

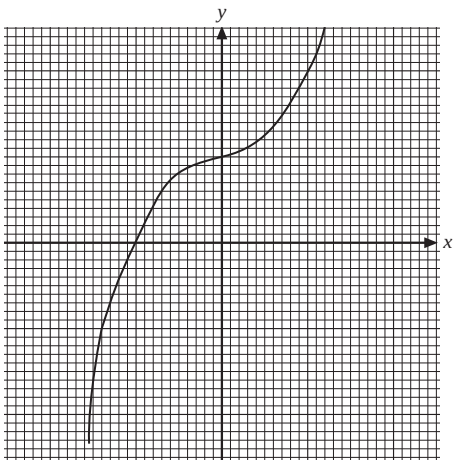
(b) (i) 400 (ii) £24 (c) B, 400

14. (a) (ii) $x = 1.5,\ y = 3.1$ (b) $x = \frac{20}{13},\ y = \frac{40}{13}$
15. $x \approx 1.9$, $y \approx 2.8$

13.11 Graphs of Common Functions

1. (a) reciprocal (b) quadratic (c) linear (d) cubic
   (e) quadratic (f) reciprocal
2. (a) cubic (b) reciprocal (c) linear (d) cubic
   (e) quadratic (f) reciprocal
3. B
4. B and C
5. (a) C (b) D (c) A (d) B
6. (a) D (b) A (c) B (d) C
7. (a) (i) $y = 1 - x^2$ (ii) $2y = 2 + x$ (iii) $y = x^2$ (iv) $xy = 1$

   (b)



13.12 Graphical Solutions of Equations

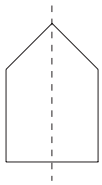
1. 1.26 and 1.46
2. $x = -2$ or 1
3. $x = 3$ or -2 ; $x^2 - x - 6 = 0$
4. (a) $x = 0, -2, 1$ (b) no solutions (c) $x \approx \pm 1.5$
   (d) $x \approx 2.1$ and -0.9
6. (a) $x = -3, 1$ (b) $x \approx 0.73, -2.73$ (c) $-0.6, -3.4$; $-1, a < -1$
7. about 1.3
8. (a) $x = -2, -1, 1$ (b) about $x = 1.5$ (c) $x = -2.25, -0.5, 0.8$

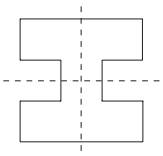
14 Loci and Transformations

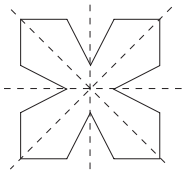
14.1 Drawing and Symmetry

2. (a) 5 cm (b) 7.8 cm (c) 3.2 cm (d) 7.8 cm

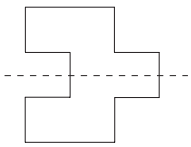
3. (a) 2 cm (b) 2.5 cm (c) 3 cm (d) 5 cm

4. (a)  (i) One line of symmetry (ii) 1

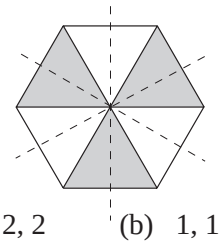
(b)  (i) Two lines of symmetry (ii) 2

(c)  (i) Four lines of symmetry (ii) 4

(d) (i) No lines of symmetry (ii) 2

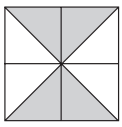
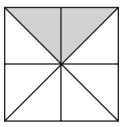
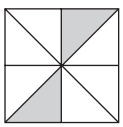
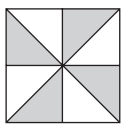
(e)  (i) One line of symmetry (ii) 1

(f) (i) No lines of symmetry (ii) 2

5. (a)  (b) 3

6. (a) 2, 2 (b) 1, 1 (c) 4, 4 (d) 1, 1

7. (a) A (b) D, E (c) C, F (d) B, D, E (e) C, F

8. (a)  (b)  (c)  (d) 

14.2 Scale Drawings

1. (a) 6 m by 5 m (b) 32.5 m^2 (c) 10.5 m

2. (a) 3.6 m, 2.4 m (b) 60 cm by 60 cm (c) 60 cm by 180 cm
   (d) 3.78 m^2

4. (a) 3 m by 3.25 m (b) 1.75 m (c) 0.9375 m^2 (d) 9.75 m^2

5. (b) 4.2 m, 5.8 m

6. (a) 4.8 m (b) 3.6 m (c) 2.5 cm by 1.875 cm (d) 0.75 cm

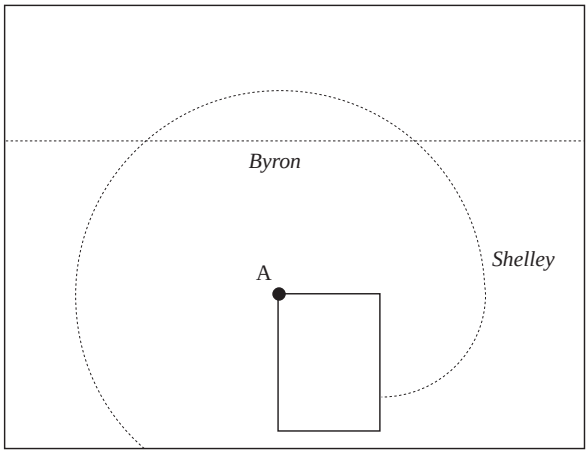
7. (a) 8 cm by 10 cm (b) 16 cm by 20 cm (c) 4 cm by 5 cm

8. (a) 1 : 175 m^2 , 2 : 162.5 m^2 , 3 : 400 m^2 , 4 : 237.5 m^2, 5 : 350 m^2

9. (a) (i) 5 m (ii) 16 cm (b) $8\frac{1}{2}$ feet

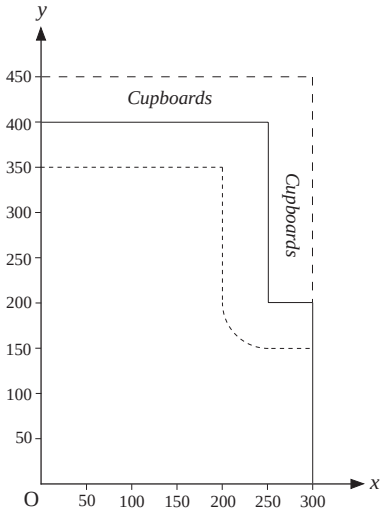
10. (a) 2 cm (b) 13.4 cm

11. (a)



(b) 7000 cm^2

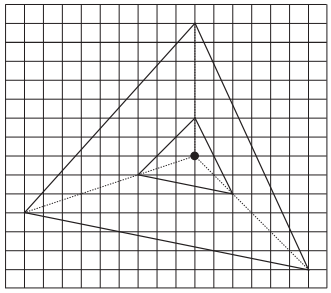
12. (b) (i) (300, 0, 250) (ii) (0, 400, 100) (c)

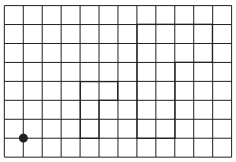


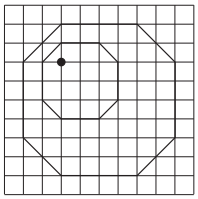
14.3 Constructing Triangles and other Shapes

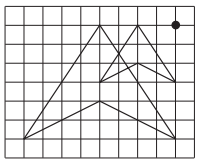
2. (a) 4.4 (b) AC ≈ 3.1 cm, BC ≈ 4.4 cm (c) AB ≈ 10.4 cm
   (d) 46.6° (e) AC ≈ 3.6 cm (f) BC ≈ 11.6 cm
3. 4.7 cm
4. 29.0°, 75.5°, 75.5°
7. (b) 4.6 cm, 7.8 cm
9. 2.6 cm
10. 48°
13. (a) 26 cm

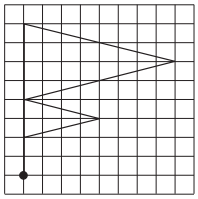
14.4 Enlargements

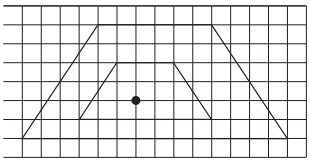
1. 

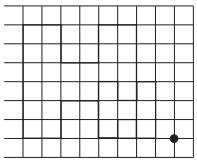
2. (a) 
   Scale factor: 2

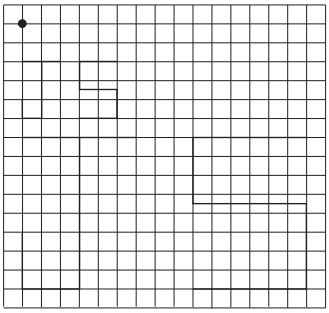
   (b) 
   Scale factor: 2

   (c) 
   Scale factor: 2

   (d) 
   Scale factor: 2

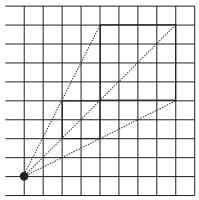
   (e) 
   Scale factor: 3

   (f) 
   Scale factor: 3

3. 

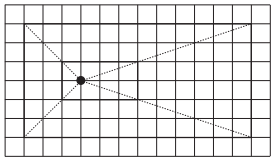
4. (a) 2 (b) 5 (c) 3 (d) 2.5 (e) 4 (f) 1.5

5. (a)



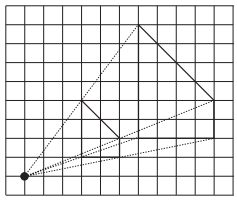
Scale factor: 2

(b)



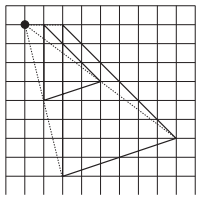
Scale factor: 2

(c)



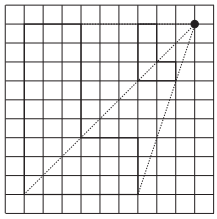
Scale factor 2

(d)



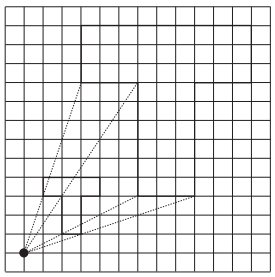
Scale factor: 2

(e)



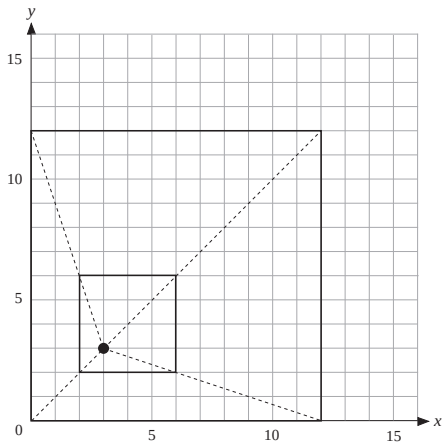
Scale factor: 3

(f)

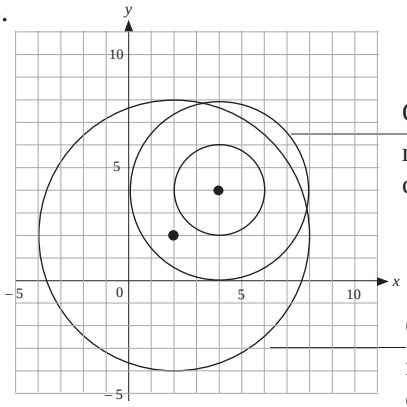


Scale factor: 3

7.



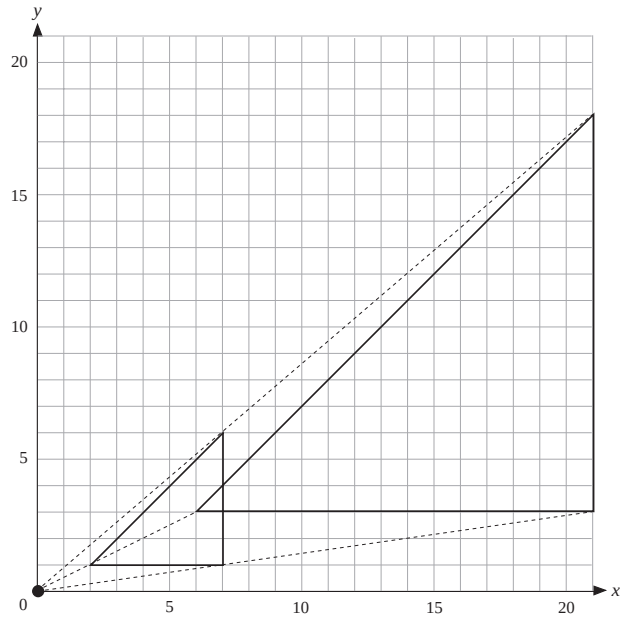
8.



Circle, radius 4, centre (4,4)

Circle, radius 6, centre (2,2)

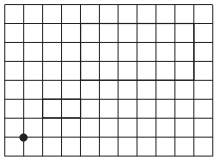
9. (a)



(b) 3

(c) (0, 0)

11.



12. (a)

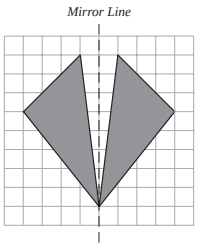
1	2	3	4
4	8	12	16
1	4	9	16

(b)

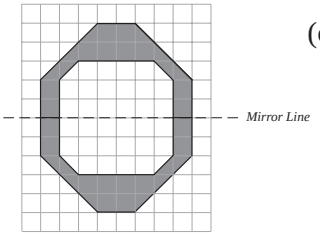
8
32
64

14.5 Reflections

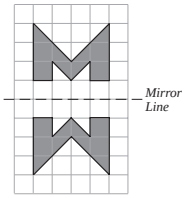
1. (a)



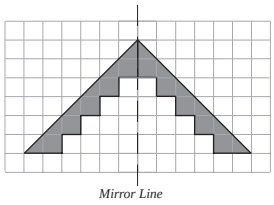
(b)



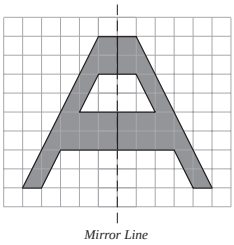
(c)



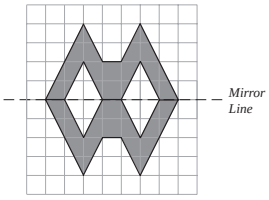
(d)



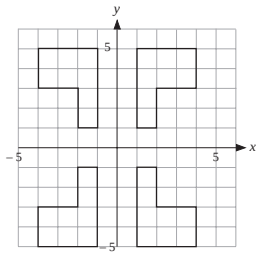
(e)



(f)



3.

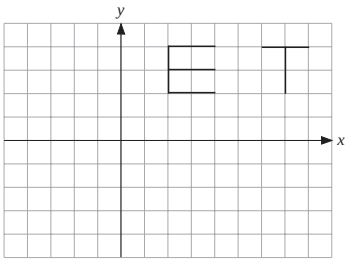


(c) $(-1,-1),\ (-1,5),\ (-4,5),\ (-4,3),\ (-2,3),\ (-2,1)$

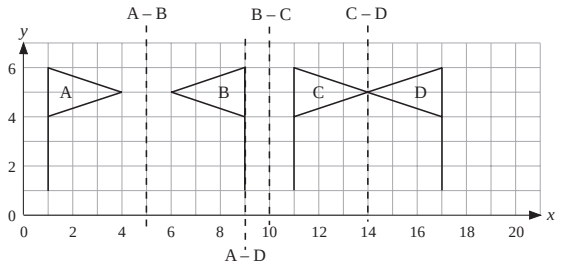
(d) $(-1,-1),\ (-1,-5),\ (-4,-5),\ (-4,-3),\ (-2,-3),\ (-2,-1)$

(e) Reflect original shape in x-axis

4.



5.

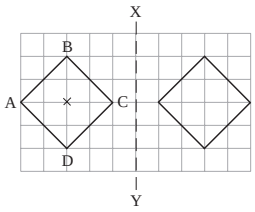


6. (b) (iii) x coordinates become negative, y coordinates stay the same.

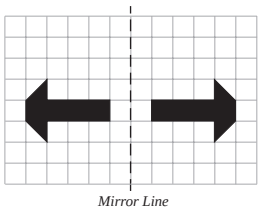
(c) (iii) y coordinates become negative, x coordinates stay the same.

14.5

7.

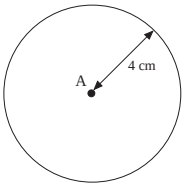


8. (a) 5 square units (b)



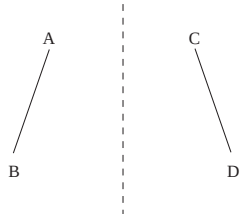
14.6 Construction of Loci

1.

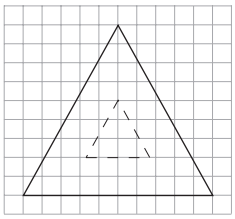


2.

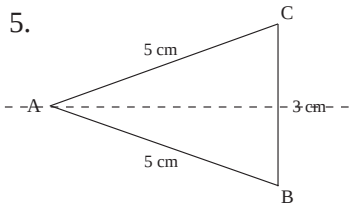
3.



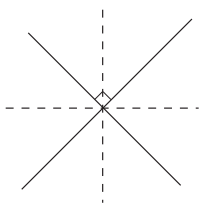
4.



5.

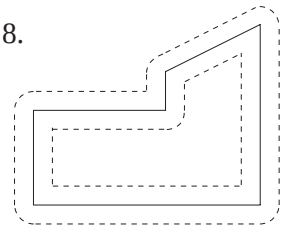


6.

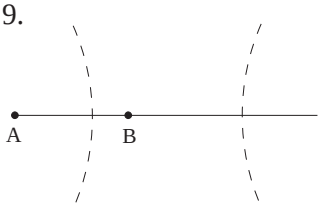


7.

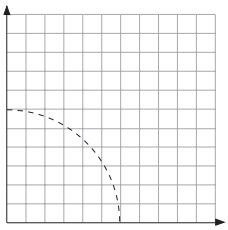
8.



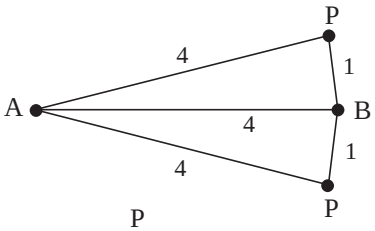
9.



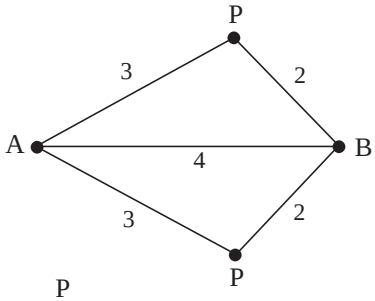
10.



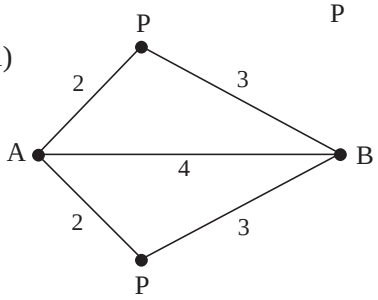
11. (a) (i)



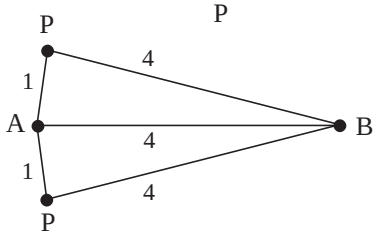
(ii)



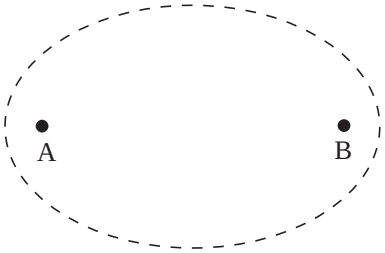
(iii)



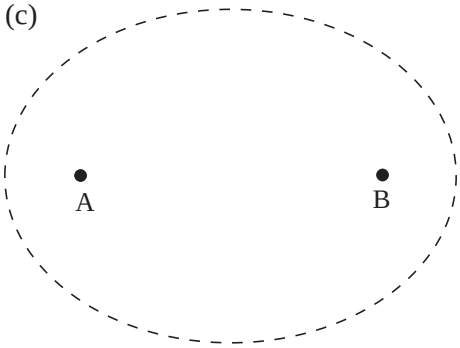
(iv)



(b)



(c)

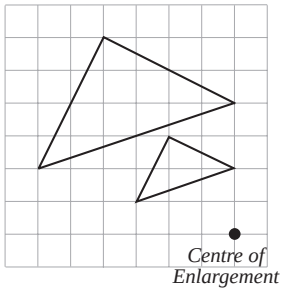


14.7 Enlargements which Reduce

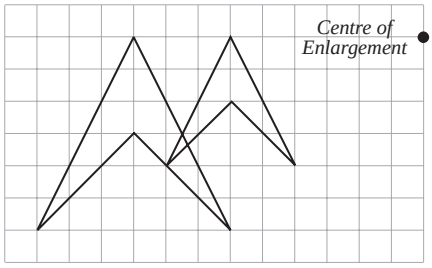
1. (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{3}{4}$ (e) $\frac{2}{3}$ (f) $\frac{1}{2}$

2. (a) $\frac{1}{5}$, (8, 7) (b) $\frac{2}{3}$, (13, 9) (c) $\frac{1}{4}$, (9, 1)

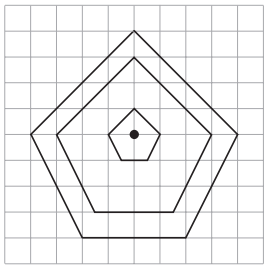
3. (a)



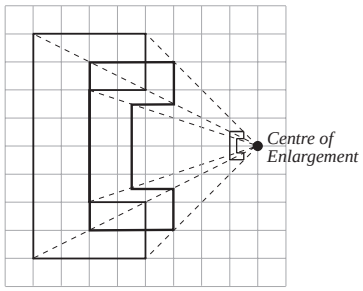
(b)



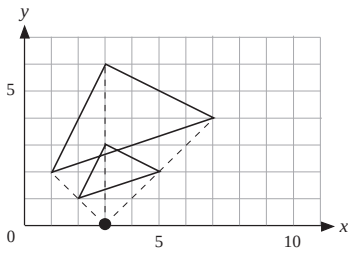
(c)



(d)

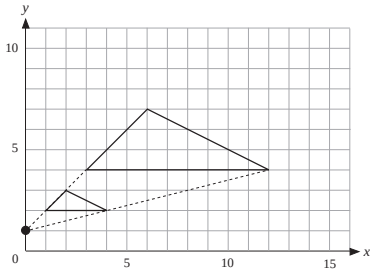


4. (a)



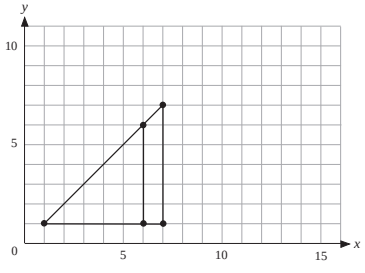
(2, 1), (3, 3), (5, 2)

(b)

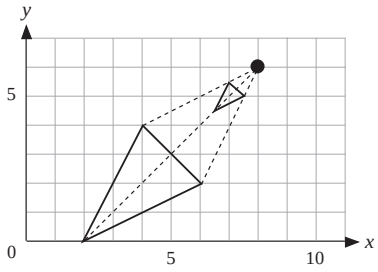


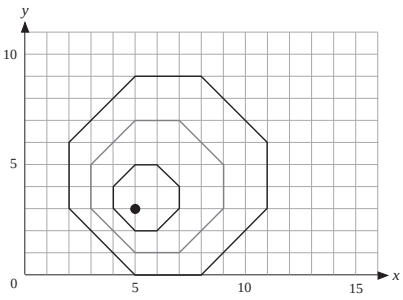
(1, 2), (4, 2), (2, 3)

(c)



(d)



5. 

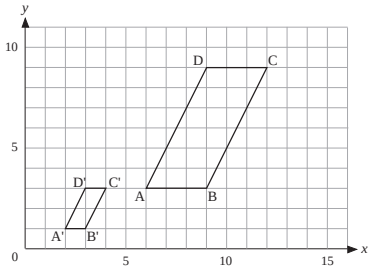
(c) Scale factor: $\frac{1}{3}$

Centre of enlargement: (5, 3)

6. (a) $\frac{1}{5}$ (b) 6 cm, 9 cm

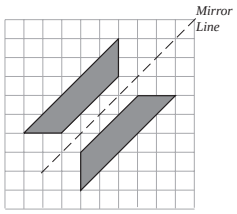
7. (a) $\frac{2}{3}$; $x = 12$ cm, $y = 8$ cm, $z = 6$ cm (b) $\frac{3}{5}$; $x = 24$ cm, $y = 12$ cm

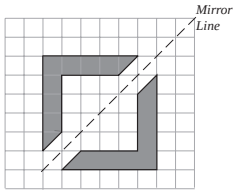
(c) $\frac{1}{9}$; $x = 7$ cm, $y = 6$ cm, $z = 4$ cm (d) $\frac{3}{8}$; $x = 15$ cm, $y = 18$ cm

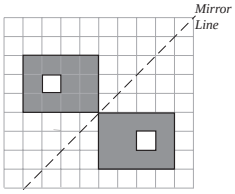
8. (a) (i) 

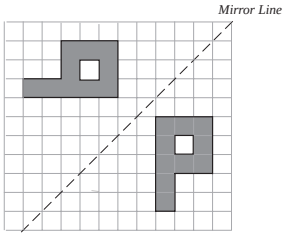
(ii) 2 cm^2 (b) 1.2 cm

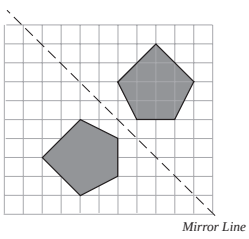
14.8 Further Reflections

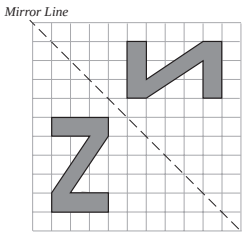
1. (a) 

(b) 

(c) 

(d) 

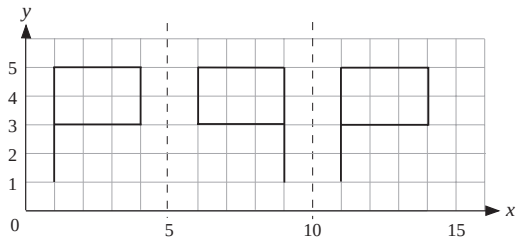
(e) 

(f) 

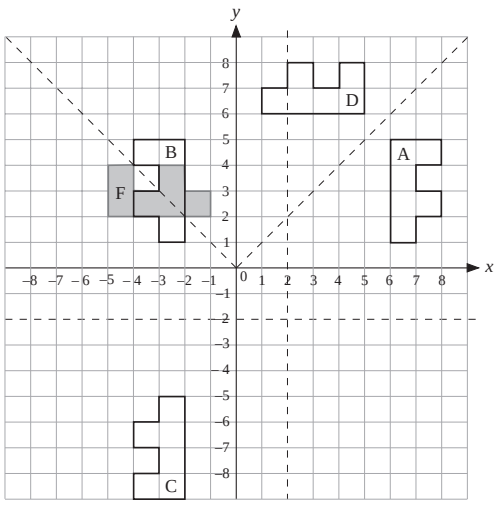
2.

	E	D	C	B
A	$x = 10$	X	$x = 6.5$	X
B	$x = 11.5$	X	$x = 8$	
C	X	$x = 11.5$		
D	$x = 15$			

3.

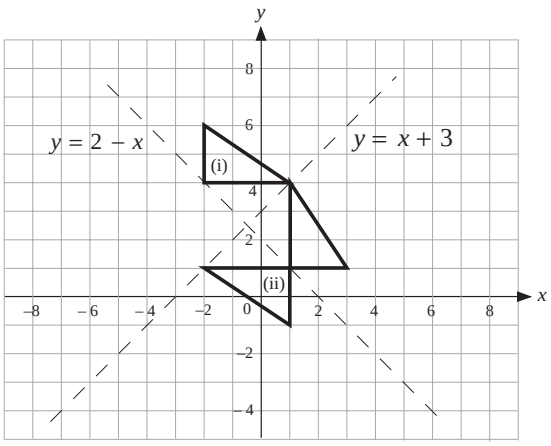


4.

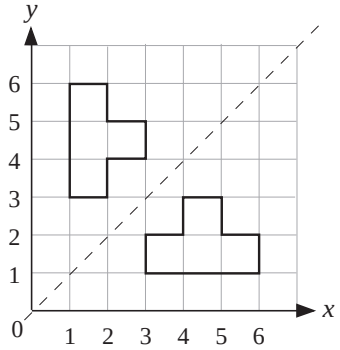


5. (a) $y = x$ (b) $x = 2$ (c) $y = -x$ (d) $y = -x$
(e) $x = -5.5$ (f) $y = x$ (g) $y = 2$

6.



7.



(d)

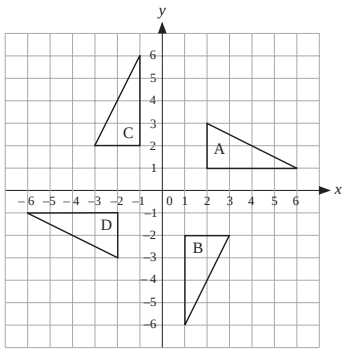
(3, 1)	(1, 3)
(6, 1)	(1, 6)
(6 , 2)	(2, 6)
(5, 2)	(2, 5)
(5, 3)	(3, 5)
(4, 3)	(3, 4)
(4, 2)	(2, 4)
(3, 2)	(2, 3)

(e) Coordinates interchanged

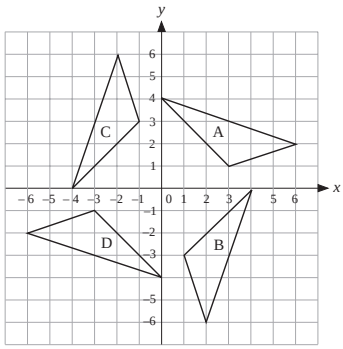
(f) (2, 3), (2, 5), (3, 3) and (3, 5)

14.9 Rotations

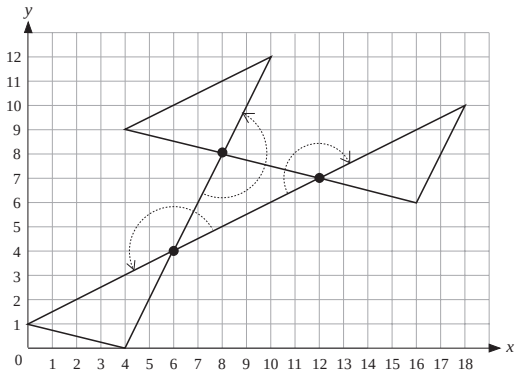
1.



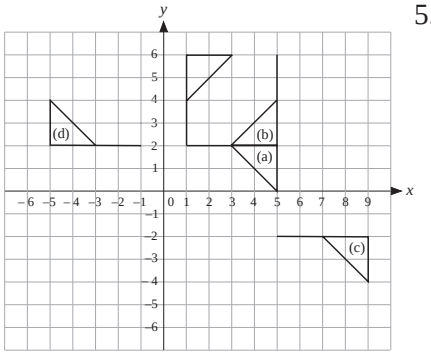
2.



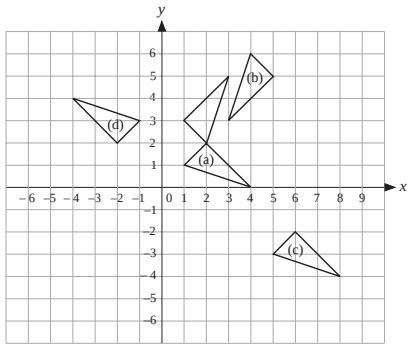
3.



4.



5.



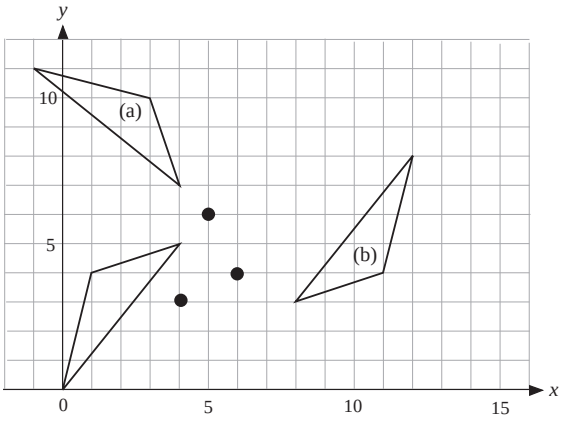
6. (a) A to B: 90° rotation, clockwise, centre (0, 0)
A to C: 180° rotation, centre (0, 0)
A to D: 90° rotation, anticlockwise, centre (0, 0)
A to E: 180° rotation, centre (5, 3)

(b) A, B and D

7. A to B: (9, 9)
A to C: (8, 5)
A to D: (10, 7)
A to E: (6, 9)
A to F: (7, 11)

8. (a) A to B: 180° rotation, centre (5, 7)
A to C: 180° rotation, centre (0, 4.5)
A to D: 90° rotation, clockwise, centre (0.5, 3.5)
A to E: 180° rotation, centre (0, 2)
A to F: 90° rotation, anticlockwise, centre $(-1, 5)$

(b) E and B

9. 

(c) 180° rotation, centre (6, 4)

10. 180° rotation, centre (13, 5), followed by 180° roatation, centre (7, 3)

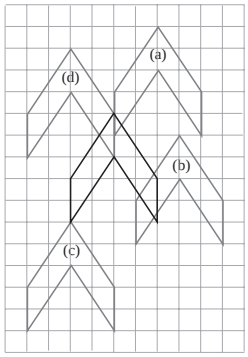
14.10 Translations

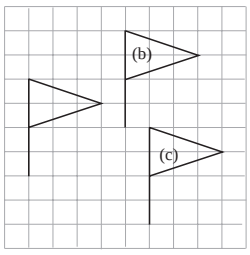
1. A: $\begin{pmatrix}1\\6\end{pmatrix}$ B: $\begin{pmatrix}5\\4\end{pmatrix}$ C: $\begin{pmatrix}5\\-6\end{pmatrix}$ D: $\begin{pmatrix}0\\-6\end{pmatrix}$ E: $\begin{pmatrix}-5\\-5\end{pmatrix}$

   F: $\begin{pmatrix}-4\\2\end{pmatrix}$ G: $\begin{pmatrix}-8\\5\end{pmatrix}$

2. (a) $\begin{pmatrix}9\\3\end{pmatrix}$ (b) $\begin{pmatrix}-5\\-1\end{pmatrix}$ (c) $\begin{pmatrix}-4\\-2\end{pmatrix}$ (d) $\begin{pmatrix}-4\\-7\end{pmatrix}$ (e) $\begin{pmatrix}4\\7\end{pmatrix}$

   (f) $\begin{pmatrix}5\\8\end{pmatrix}$ (g) $\begin{pmatrix}-9\\-8\end{pmatrix}$ (h) $\begin{pmatrix}8\\-3\end{pmatrix}$

3. 

4. 

   (d) $\begin{pmatrix}-5\\2\end{pmatrix}$

5. (a) $\begin{pmatrix}8\\0\end{pmatrix}$

6. 4: $A \to \begin{pmatrix}2\\-2\end{pmatrix}$ and $\begin{pmatrix}3\\-2\end{pmatrix}$ and $\begin{pmatrix}3\\-3\end{pmatrix}$; $B \to \begin{pmatrix}1\\-2\end{pmatrix}$

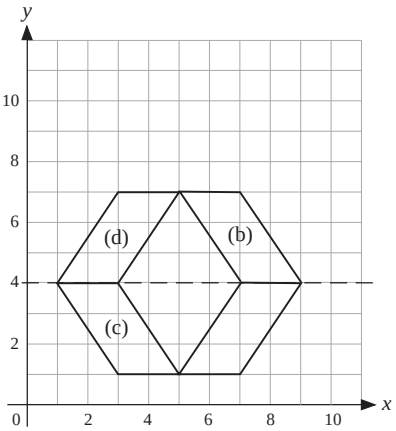
   5: $A \to \begin{pmatrix}4\\-2\end{pmatrix}$ and $\begin{pmatrix}5\\-3\end{pmatrix}$; $B \to \begin{pmatrix}3\\-1\end{pmatrix}$ and $\begin{pmatrix}3\\-2\end{pmatrix}$ and $\begin{pmatrix}3\\-3\end{pmatrix}$

8. (b) $\begin{pmatrix}6\\6\end{pmatrix}$ (c) $\begin{pmatrix}0\\3\end{pmatrix}$ (d) $\begin{pmatrix}-2\\-6\end{pmatrix}$

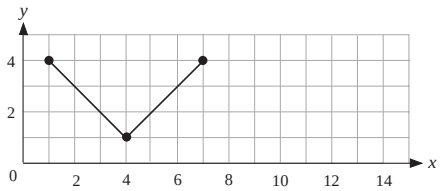
9. (a) $\begin{pmatrix}-2\\-13\end{pmatrix}$ (b) $\begin{pmatrix}3\\13\end{pmatrix}$ (c) $\begin{pmatrix}4\\0\end{pmatrix}$ (d) $\begin{pmatrix}-4\\0\end{pmatrix}$

14.11 Combined Transformations

1.



2. (a)

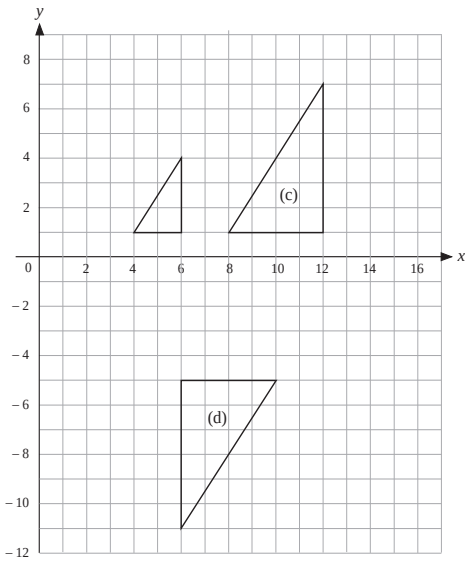


(b) Reflect in line $x = 7$.
Rotate 180° about (7, 4) and then reflect in line $y = 4$.

3. (a) Rotations through 90°, 180° and 270° about centre (2.5, 5.5) and rotation of 180° about centre (1, 4).

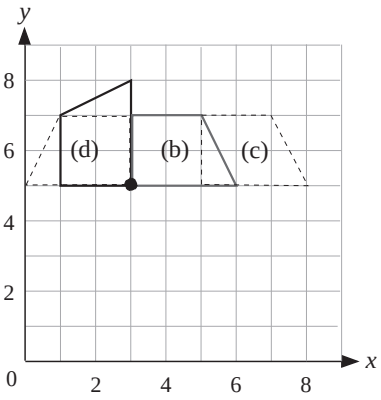
(b) Reflection in $y = 4$, $x = 2.5$, $y = x + 3$, $y = 8 - x$.

4.



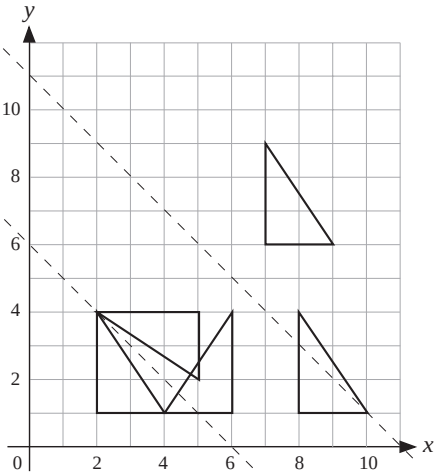
(e) Rotation of 180° about $(7, -2)$ and enlargement of scale factor $\frac{1}{2}$ about (6, 1)

5.



(e) Reflect in line $y = 8 - x$

6.



(c) Translation $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$

(d) Translation $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$

7. (a) A to B: reflection in $y = x$

A to C: rotation of $90°$ clockwise about (0, 0)

A to D: reflection in $y = 0$

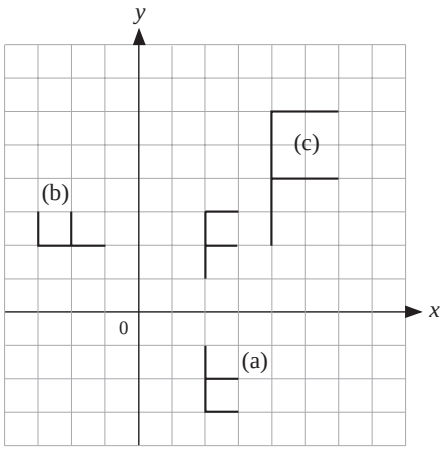
(b) A to E: rotation of $180°$ about (0, 0), followed by translation $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

A to F: reflection in $y = 0$, followed by translation $\begin{pmatrix} -7 \\ 0 \end{pmatrix}$

A to G: rotation of $90°$ anticlockwise about (0, 0), followed by translation $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

A to H: relection in $x = 0$, followed by translation $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

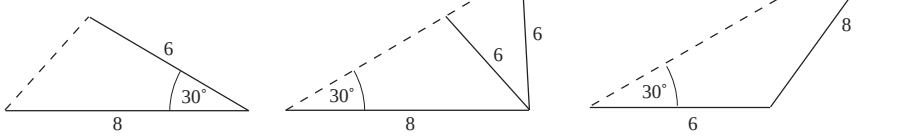
9.



10 (a) (i) $90°$ (ii) $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ (b) $(2, -2)$

(c) reflection in line $y = -x$

14.12 Congruence

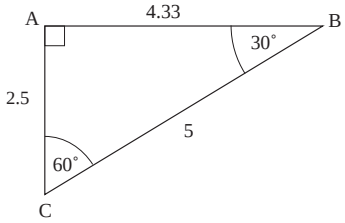
1. ABC and DEF; JKL and GHI; MON and PRQ
2. Could be *SSS*, *SAS*, *ASA* or *RHS*
3. *SAS*
4. *SAS*
5. ABC and AFE; *SAS*
6. *RHS*
7. 

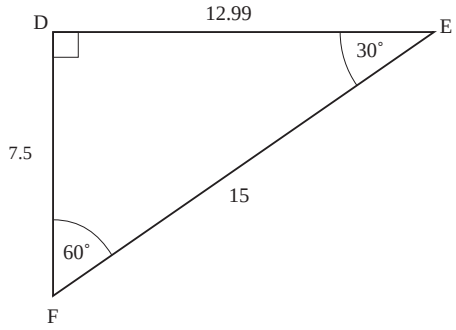
8. AXB and CXD; BXC and DXA
9. (a) *SSS* (b) *SAS*

14.13 Similarity

1. A and E; C and F, B and D

2. (a)





(b) 1 : 9

3. (a) AB = 3 cm, DH = 4 cm (b) AB = 4 cm, EH = 3 cm

4. (a) 1 : 4 (b) 1 : 3

5. (b) (i) 17.5 cm (ii) 10.5 cm (iii) 9 cm (c) (i) 4 : 25 (ii) 4 : 21

6. (b) parallel (c) $x = 10$ cm, $y = 14$ cm (d) 9 : 40

7. (a) 101.25 cm^3 (b) 58.59375 cm^3 (c) 468.75 cm^3

8. (a) 400 cm^3 (b) 4

9. (a) (i) 44% (ii) 72% (b) 14.5%

10. (a) 2 : 3 (b) 4 : 9

11. (a) 12 cm (b) 64 cm^3

12. (a) 63 cm (b) 16.7°

13. (a) 2.5 cm (b) 60° (c) 10.83 cm

14. 280 m

15. (a) 3.44 (b) 40.7° (c) (i) 104.3° (ii) 18 cm

16. (a) (i) APR (ii) $\frac{25}{9}$ m (b) 25.8°

17. (a) 1.63 : 1 (b) 2.08 : 1 (c) Yes – as volume is about half

18. (a) 12 ml (b) 0.9
    (c) *small*: 1.44 ml per penny; *medium*: 1.5 ml per penny; so *medium* is the better offer
    (d) (i) 55 ml (ii) 132.5 ml

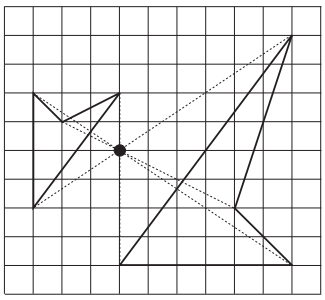
19. (a) 81 cm^3 (b) 4.5 cm

14.14 Enlargements with Negative Scale Factors

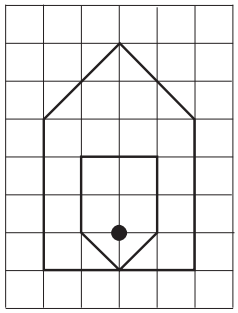
1. A: 2; B: -1; C: -2; D: -3; E: -1.5; F: $\frac{1}{2}$; G: $-\frac{1}{2}$

2. A: 2, (11, 2); B: -1, (9, 7); C: -1, (5.5, 4); D: -2, (10, 5)

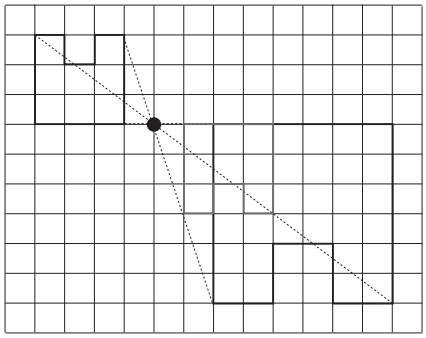
3. (a)



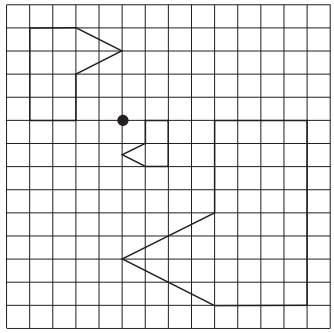
(b)



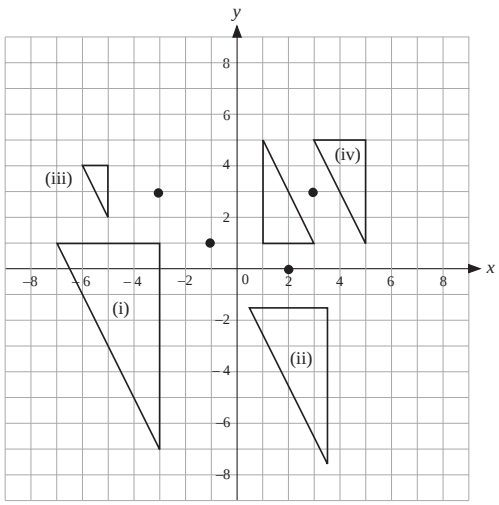
(c)



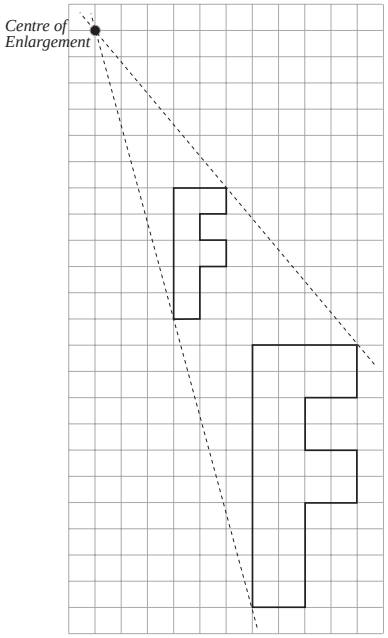
(d)



4.

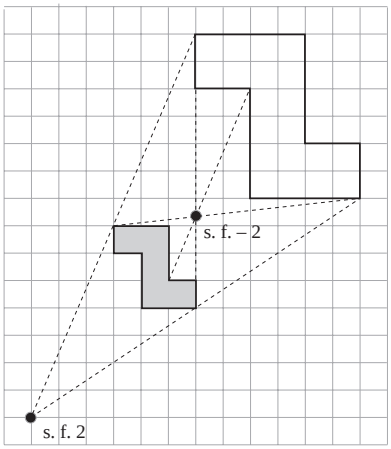


5.

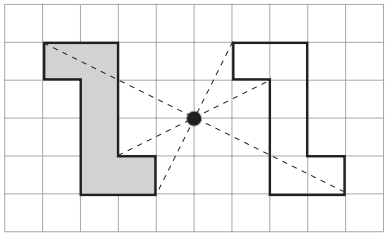


(a) Enlargement, scale factor 2, centre as shown

6. (a)



(b)



Centre of enlargment as shown, scale factor -1

7. (c) Multiplied by 1, 4 and 9 respectively (d) Yes for area

8. 3.2 cm

15 Variation

15.1 Simple Ratios

1. (a) 2 : 1 (b) 4 : 1 (c) 1 : 2 (d) 3 : 4 (e) 1 : 6 (f) 4 : 21
   (g) 1 : 6 (h) 1 : 3 (i) 2 : 25 (j) 1 : 8 (k) 4 : 5 (l) 7 : 5
   (m) 27 : 16 (n) 8 : 25 (o) 50 : 49 (p) 4 : 1 (q) 3 : 1 (r) 2 : 7
2. (a) 15 : 16 (b) 16 : 15
3. (a) 10 : 7 (b) 7 : 10
4. (a) 1 : 3 and 1 : 2 (b) Company A
5. (a) 5 : 9 (b) 11 : 18
6. (a) 3 : 2 (b) 1 : 4
7. 3 : 17
8. (a) 9 : 11 (b) 3 : 7 (c) 10 : 9
9. (a) 800 cm^3 (b) 160 cm^3 (c) 40 cm^3
10. 240
11. 24
12. 80
13. 135

15.2 Proportion and Ratio

1. 6 inches
2. (a) 120 gms (b) 160 gms
3. (a) 161 (b) 48
4. (a) 45p (b) £5.75 (c) £9.90
5. (a) 84p (b) £3.40 (c) 24p
6. (a) £2.85 (b) £6.84 (c) £23.94
7. (a) 500 gms (b) 13
8. (a) £23.06 (b) about 16.95 litres
9. (a) 4, 2, 1 (b) 5, $\frac{5}{2}$ $\frac{5}{4}$ (c) $\frac{3}{2}$, $\frac{3}{4}$, $\frac{3}{8}$ (teaspoonsful)
10. (a) 5 days (b) 5 days (c) 3 men (2.5)

11. (a) 20 teachers (b) 616 teachers
12. (a) $2\frac{1}{2}$ hours (b) 300 boxes
13. (a) 9 hours (b) 20mins (c) 4
14. Employ 7 people and it takes 2 days
15. £1.45
16. 3, 6, 6, $7\frac{1}{2}$
17. (a) 9 litres (b) (i) 16 (c) (ii) 150
18. 4.5 cm
19. (a) 6 kg (b) 64p
20. Small pot - unit cost = 0.1035p (< 0.1071p)

15.3 Map Scales and Ratios

1. (a) 13.5 km (b) 30 km (c) 16.5 km (d) 11.25 km
2. 234 km, 63.2 km, 84 km, 171.2 km
3. 2.4 km
4. 105 km
5. (a) 1.4 cm (b) 4.2 cm (c) 84 cm
6. 18 km; 30 cm
7. (a) 16.5 km (b) 33 cm
8. (a) 1 : 400 000 (b) 1 : 40 000 (c) 1 : 600 000 (d) 1 : 480 000
9. 1 : 350 000 (a) 1 : 700 000 (b) 1 : 525 000 (c) 1 : 393 750
10. 141 m
11. (a) 1 : 540 000 (b) about 19.5 km
12. 0.1125 km^2

15.4 Proportional Division

1. 2 litres of oxygen, 8 litres of nitrogen
2. 18 boys, 45 girls
3. Ben - 80, Emma - 60

4. (a) £7.05, £5.64 (b) £18.65, £14.92 (c) £13.33, £10.67

5. (a) 500 g, 300 g, (b) 125 g, 75 g (c) 187.5 g, 112.5 g

6. £250, £300, £450

7. £8000, £6000, £4000, £2000

8. Ahmed: 90, Afzal: 150

9. apples: 2kg; bananas: 2.4 kg; oranges: 1.6 kg

10. 50

11. 50 g

15.5 Direct Proportion

1. (a) Yes (b) Yes (c) No (d) Yes

2. (a)

x	8	10	15	22
y	24	30	45	66

(b)

p	5	7	9	11
q	30	42	54	66

(c)

r	2	3	5	7
s	12	18	30	42

(d)

w	4	8	9	12
z	28	56	63	84

(e)

x	5	6	7	9
y	17.5	21	24.5	31.5

(f)

t	3	4	6	9
h	29.4	39.2	58.8	88.2

(g)

p	12.6	16.8	24.5	7
q	19.8	26.4	38.5	77

(h)

v	5	12	19	29
p	1.80	4.32	6.84	10.44

3. (a) $C = \frac{500}{3}A$ (b) £4500 (c) 45 m^2

4. (a) $F = \frac{5}{2}x$ (b) 10 cm (c) 32.5 N

5. (a) $d = 60\pi n$ (cm) (b) 150.8 m (c) 1061

6. (a) $d = 70t$ miles (t in hours) (b) (i) 140 miles (ii) $\frac{35}{6}$ miles (iii) 84 miles

   (c) (i) 3 hours (ii) 12 mins (iii) $\frac{12}{7}$ mins

7. (a) $c = 2\pi r$, 2π (b) π

8. $z = \frac{42}{5}x$

15.6 Inverse Proportion

1. (a) False (b) True (c) True (d) False

2. (a)

x	10	20	25	50
y	10	5	4	2

(b)

p	32	8	4	$\frac{1}{2}$
q	2	8	16	128

(c)

r	1	3	5	10
s	45	15	9	4.5

(d)

a	0.9	2.4	4.8	7.2
b	1.6	0.6	0.3	0.2

(e)

g	1	2	9	81
h	16.2	8.1	1.8	0.2

(f)

u	2	4	8	17
v	6.8	3.4	1.7	0.8

3. 16

4. (a) $IR = 500$ (b) $\frac{1}{3}$ (c) current is halved

5. (a) halved (b) decreased by a factor of 4 (c) $PV = 800$ (d) 80 Nm^{-2}

6. (a) 3.5 (b) $yx = 28$ (c) x would increase to 14

7. (a) Yes (b) (i) £2400 (ii) 24 years

8. (a) 425 Hz (b) 0.04 m

15.7 Functional and Graphical Representations

1. (a) $T = 40x$ (b) $P = 40v^2$ (c) $R = \frac{216}{x^3}$

   (d) $Y = \frac{72}{x}$ (e) $V = \frac{2500}{x^2}$

2. (a) y is inversely proportional to the fifth power of x
   (b) y is proportional to the square of x
   (c) y is inversely proportional to the square of x
   (d) y is proportional to x
   (e) y is proportional to the cube of x
   (f) y is inversely proportional to x

3. (a)

x	1	2	5	8
y	4	16	100	256

(b)

x	1	1.1	1.2	1.3
y	2	2.662	3.456	4.394

(c)

x	1	2	5	10
y	5	1.25	0.2	0.05

(d)

x	1	3	9	12
y	27	1	$\frac{1}{27}$	$\frac{1}{64}$

4. (a) 768 (b) 4

5. 1

6. (a) $V = 4.2r^3$ (b) 525 cm^3 (c) 12.6 cm

7. (a) $y = \frac{8}{5}x, \quad y = \frac{x}{2}$ (b) $y = \frac{1}{x}, \quad y = \frac{4}{x}$

   (c) $y = \frac{x^2}{2}, \quad y = \frac{x^2}{8}$ (d) $y = \frac{4}{x^2}, \quad y = \frac{8}{x^2}$

8. (a) $y = \frac{5}{4}x$ (b) $y = \frac{3}{x}$ (c) $y = \frac{x^2}{4}$ (d) $y = \frac{4}{x^2}$

9. z is proportional to $\frac{1}{x^2}$; $k = 144$

10. (a) multiplied by a factor of 4 (b) increased by $\sqrt{2} \approx 1.41$

15.8 Further Functional Representations

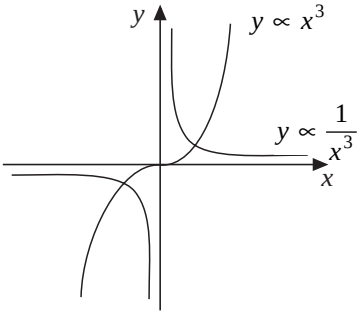
1. (a) $f \propto \frac{1}{\sqrt{v}}$; f is fuel consumption and v is speed

   (b) $r \propto h$; r is rate at which water runs out and h is depth of water

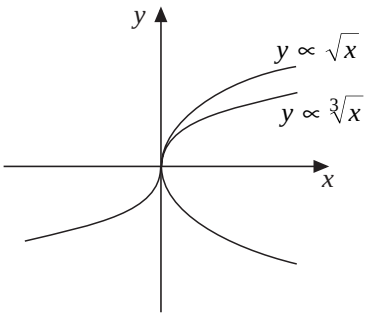
   (c) $r \propto v^{\frac{3}{2}}$; r is air resistance and v is speed

   (d) $P \propto \frac{1}{\sqrt{s}}$; P is period and s is stiffness of spring

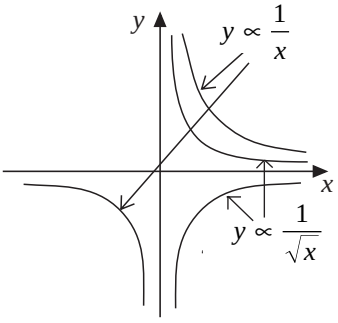
2. (a)



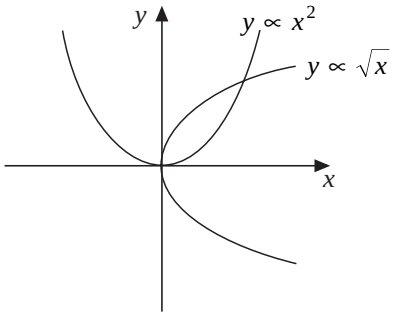
(b)



(c)



(d)



3. (a)

x	4	9	100	144
y	1	1.5	5	6

(b)

x	4	9	25	100
y	1.5	1	0.6	0.3

(c)

x	1	4	9	49
y	5	$\frac{5}{8}$	$\frac{5}{27}$	$\frac{5}{343}$

(d)

x	9	25	36	81
y	54	250	432	1458

4. (a) 1.47 seconds (b) 556 grams (c) multiplied by $\sqrt{2}$ (≈ 1.41)
   (d) multiplied by 4

5. (a) $\frac{1}{3}$ (b) multiplied by 0.794 (c) multiplied by 1.587

6. 1638.4 kg

7. (a) No (b) Yes, $n < 1$ (c) No (d) No (e) Yes, $n < 0$
   (f) Yes, $n > 1$

8. (a) $V = \pi r^2 h$ (b) $V \propto$ height (c) $V \propto (\text{radius})^2$

9. $T \propto \frac{1}{\sqrt{mk}}$

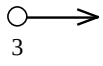
10. (a) 6 (b) -1 (c) $-\frac{3}{2}$

11. $T = 0.2\sqrt{l}$

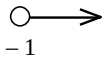
12. (a) graph (iv) (b) graph (ii) (c) graph (v)

16 Inequalities

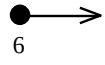
16.1 Inequalties on a Number Line

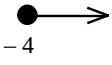
1. (a) 

(b) 

(c) 

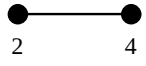
(d) 

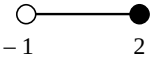
(e) 

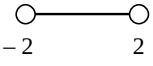
(f) 

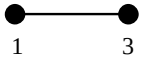
(g) 

(h) 

(i) 

(j) 

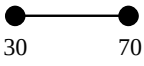
(k) 

(l) 

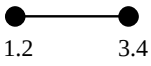
2 (a) $x < 1$ (b) $x \geq -1$ (c) $x \geq -2$ (d) $-2 \leq x \leq 2$

(e) $x > -3$ (f) $-2 < x \leq 2$ (g) $-1 < x < 3$ (h) $-1 < x \leq 3$

(i) $-4 \leq x < 1$ (j) $-5 \leq x < 10$

3. (a) 

(b) $30 \leq v \leq 70$

4. (a) 

(b) $1.2 \leq x \leq 3.4$

5. (a) 1, 2, 3, 4, 5, 6, 7, 8 (b) 4, 5, 6 (c) 2, 3, 4 (d) 4, 5

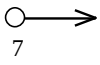
6. (a) $-1, 0, 1$ (b) $-5, -4, -3, -2$ (c) $-1, 0$ (d) $-4, -3$

7. (a) $\frac{3}{4}$, etc (b) $\frac{1}{3}$, etc (c) $\frac{5}{12}$, etc (d) $\frac{1}{4}$, etc

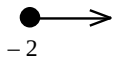
8. $-3, -2, -1, 0, 1$

16.2 Solution of Linear Inequalities

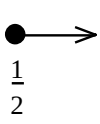
1. (a) $x \leq -3$ 

(b) $x > 7$ 

(c) $x < 5$ 

(d) $x \geq -2$ 

(e) $x < 5$ 

(f) $x \geq \frac{1}{2}$ 

2. (a) $x < 7$ (b) $x \geq -3$ (c) $x \leq 4$ (d) $x \geq \frac{2}{3}$

(e) $x < \frac{4}{5}$ (f) $x > \frac{6}{7}$ (g) $x \leq 6$ (h) $x > 2$

(i) $x \leq 4$ (j) $x \geq 3$ (k) $x < 3\frac{1}{5}$ (l) $x \geq 3\frac{1}{3}$

3. (a) $1 < x \leq 3$ (b) $2 \leq x < 5$ (c) $1 \leq x < 5$
   (d) $2 < x < 6$ (e) $-5 \leq x \leq -1\frac{2}{3}$ (f) $-2 < x < 3$

4. (a) $-1 < x \leq \frac{4}{5}$ (b) $2 \leq x \leq \frac{9}{2}$ (c) $\frac{1}{3} < x < \frac{2}{3}$
   (d) $\frac{1}{2} \leq x \leq \frac{15}{8}$ (e) $-1 \leq x \leq -\frac{3}{4}$ (f) $1 < x < 6$

5. (b) $50 \leq 4x - 20 \leq 120$ (c) $18 \leq x \leq 35$

6. (a) $200 \leq m \leq 320$ (b) $200 \leq \frac{5k}{8} \leq 320$ (c) $320 \leq k \leq 512$

7. (a) $50 \leq C \leq 90$ (b) $C = 0.1n + 10$ (c) $400 \leq n \leq 800$

8. $15\frac{5}{9} \leq C \leq 21\frac{1}{9}$

9. (a) -1, 0, 1, 2 and 3 (b) 17, 18, 19, 20 or 21

10. (a) (i) $-4, -3, -2, -1$, 0 and 1 (ii) 16 (b) $10 \leq x < 20$

16.3 Inequalities Involving Quadratic Terms

1. (a) $-1 \leq x \leq 1$ (b) $x \geq 2$ or $x \leq -2$ (c) $x \geq 5$ or $x \leq -5$
   (d) $-7 < x < 7$ (e) $x > 6$ or $x < -6$ (f) $x > 2$ or $x < -2$
   (g) $x \geq 2.5$ or $x \leq -2.5$ (h) $-0.5 < x < 0.5$ (i) $x \geq 1.5$ or $x \leq -1.5$

2. (a) $x \geq 4$ or $x \leq -4$ (b) $x \geq 2$ or $x \leq -2$ (c) $-5 < x < 5$
   (d) $-\frac{1}{2} < x < \frac{1}{2}$ (e) $x \geq \frac{2}{3}$ or $x \leq -\frac{2}{3}$ (f) $x \geq \frac{2}{5}$ or $x \leq -\frac{2}{5}$
   (g) $-\frac{1}{3} \leq x \leq \frac{1}{3}$ (h) $-3 < x < 3$ (i) $x \geq 10$ or $x \leq -10$
   (j) $-2 < x < 2$ (k) $x \geq 3$ or $x \leq -3$ (l) $x \geq \frac{1}{2}$ or $x \leq -\frac{1}{2}$

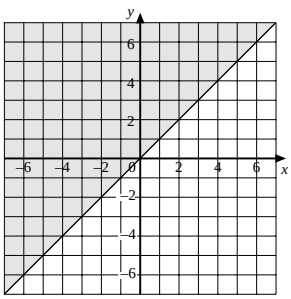
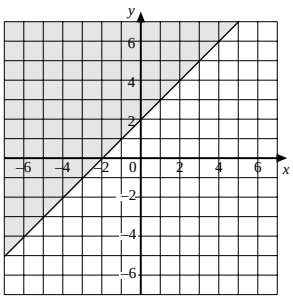
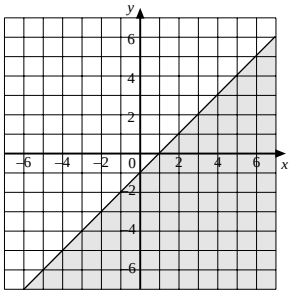
3. (a) $x \geq 2$ or $x \leq -3$ (b) $2 \leq x \leq 5$ (c) $x > 5$ or $x < 0$
   (d) $0 \leq x \leq 6$ (e) $2 < x < 5$ (f) $x > 3$ or $x < -4$
   (g) $x > 1$ or $x < -\frac{1}{2}$ (h) $-2 \leq x \leq \frac{3}{2}$

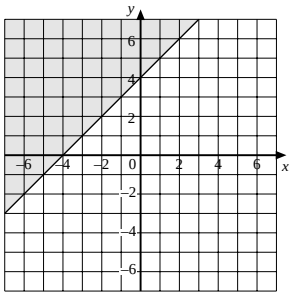
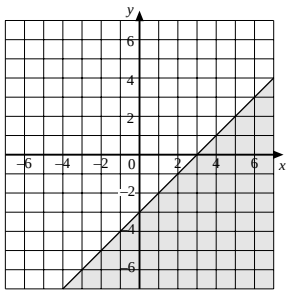
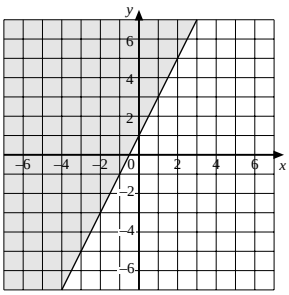
4. (a) $9 \leq x^2 \leq 16$ (b) $3 \leq x \leq 4$

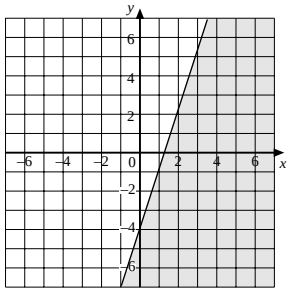
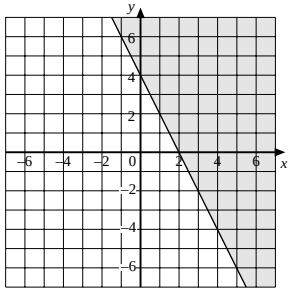
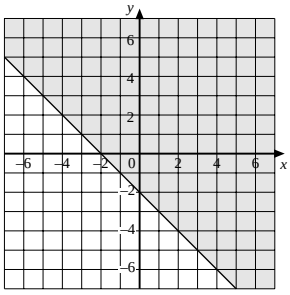
5. (a) $A = 8x^2$ (b) $2 \leq x \leq 5$ (c) 20 (d) 4

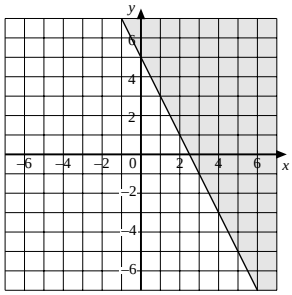
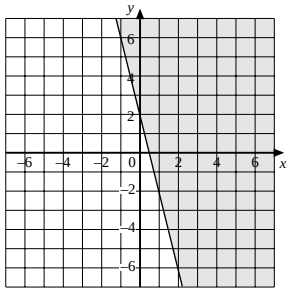
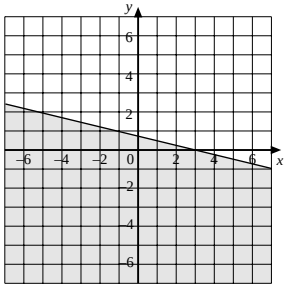
6. (a) $x < 2$ (b) $-1 < x < 1$

16.4 Graphical Approach to Inequalities

1. (a)  (b)  (c) 

(d)  (e)  (f) 

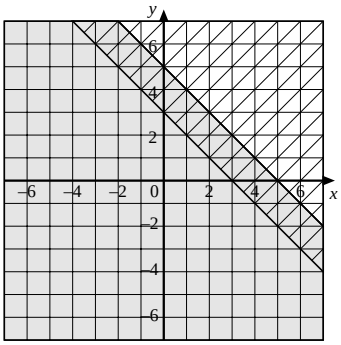
(g)  (h)  (i) 

(j)  (k)  (l) 

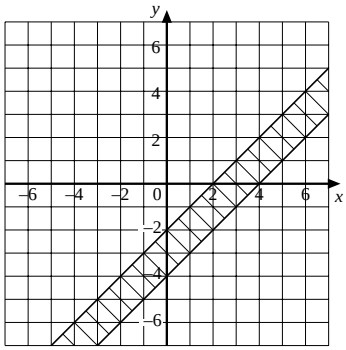
2. (a) (i) $y = x + 1$ (ii) $y \leq x + 1$ (b) (i) $y = 2x$ (ii) $y > 2x$

(c) (i) $x + y = 5$ (ii) $x + y \leq 5$ (d) (i) $x + y = 4$ (ii) $x + y > 4$

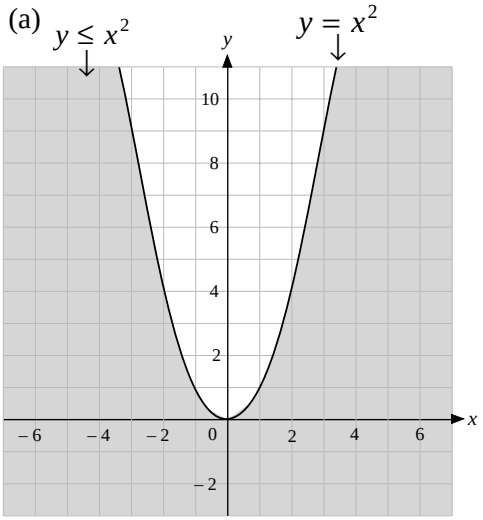
(e) (i) $y = x + 2$ (ii) $y \geq x + 2$ (f) (i) $y = x - 1$ (ii) $y < x - 1$

3. (a) 

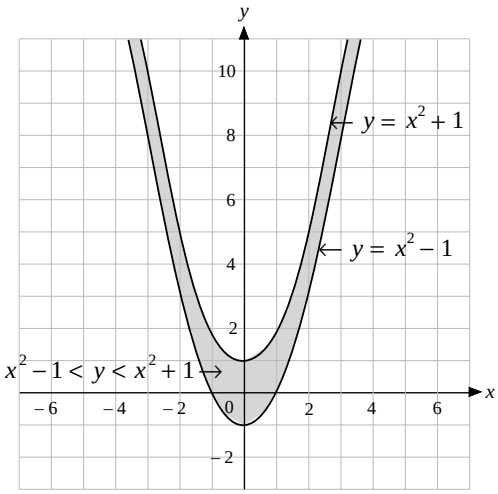
$3 \leq x + y \leq 5$

(b) 

4. (a)

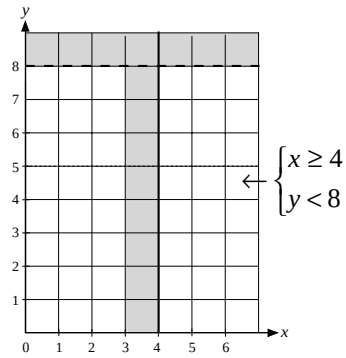


(b)

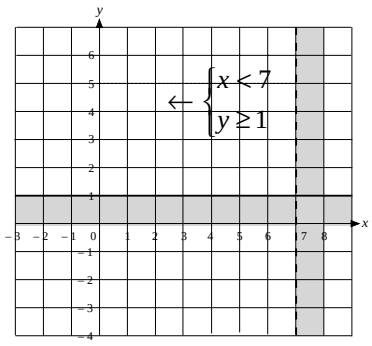


16.5 Dealing With More Than One Inequality

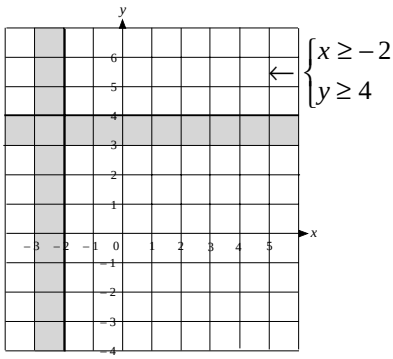
1. (a)



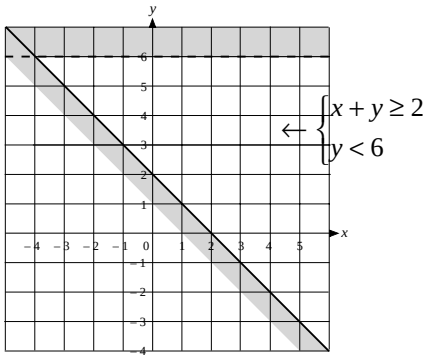
(b)



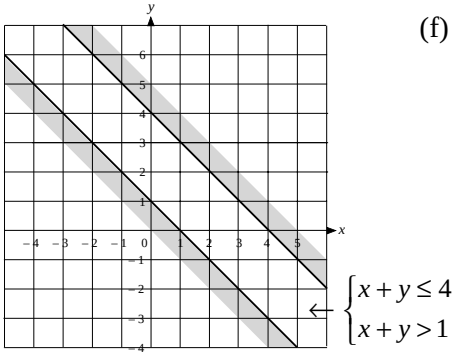
(c)



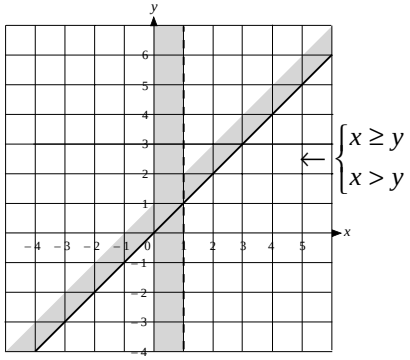
(d)



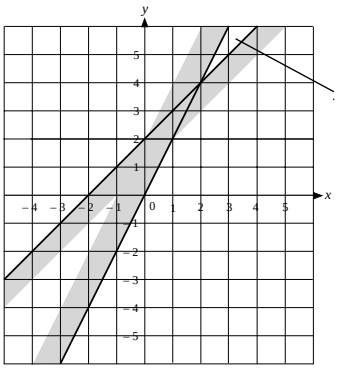
(e)



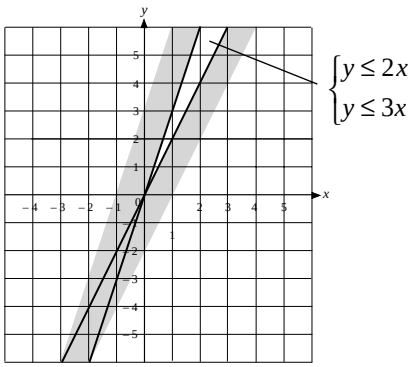
(f)



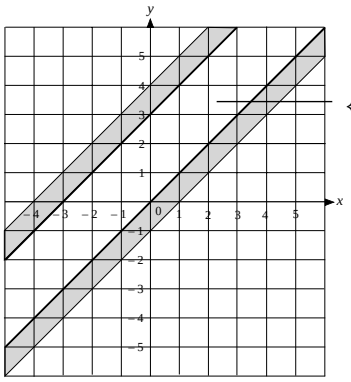
(g)



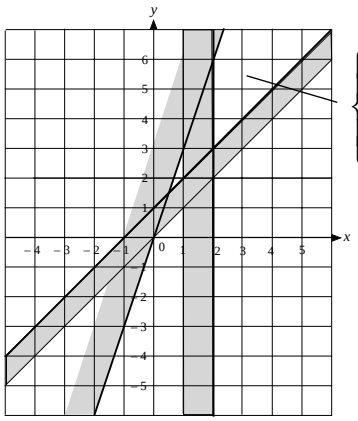
(h)



(i)

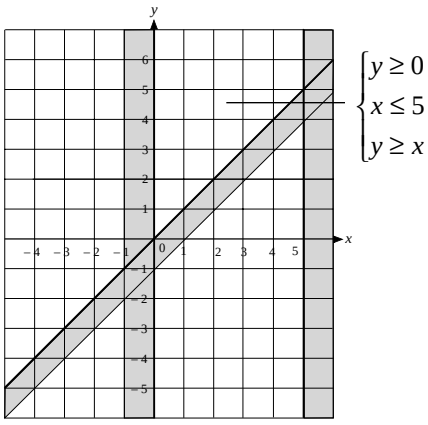


2. (a)



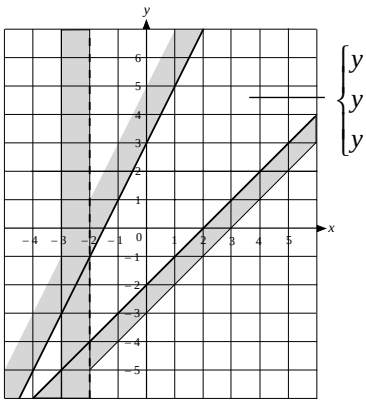
Vertices: (2, 3), (2, 6)

(b)



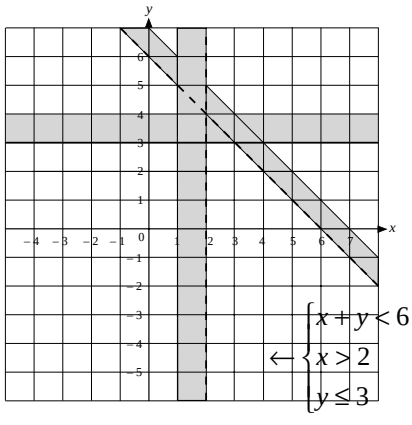
Vertices: (0, 0), (5, 5)

(c)



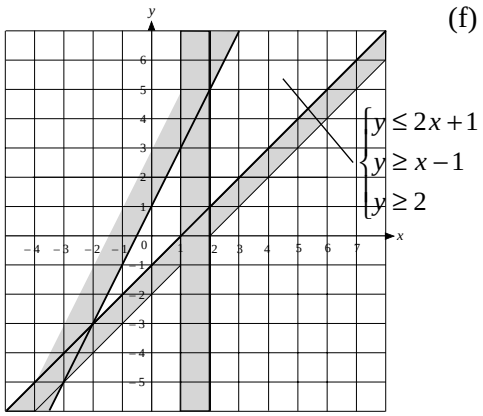
Vertices: $(-2, -4)$, $(-2, -1)$

(d)



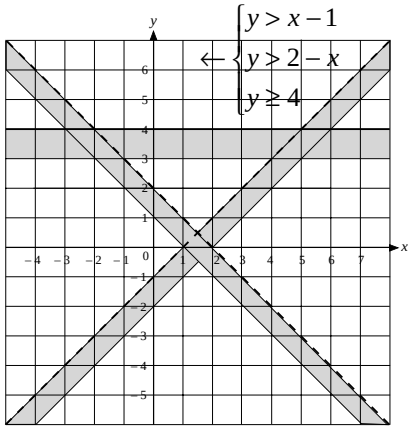
Vertices: (2, 3), (3, 3)

(e)



Vertices: (2, 1), (2, 5)

(f)



Vertices: $(-2, 4)$, (5, 4)

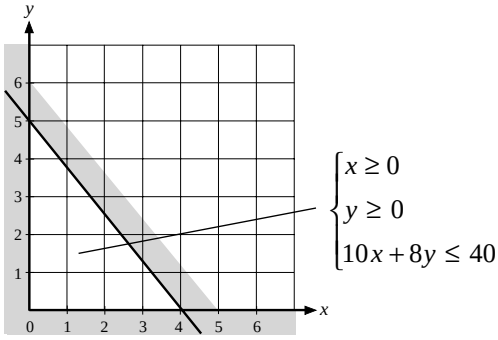
3. (a) $x \ge 1,\ y \ge 2,\ x + y \le 7$ (b) $x \le 5,\ y \ge 2,\ y \le x + 2$

(c) $x \ge 2,\ y \le 6,\ y \ge x - 1$ (d) $x \le 3,\ y \ge -3,\ y \le x + 1$

(e) $x \ge -2,\ y \ge x - 2,\ y \le 2 - x$

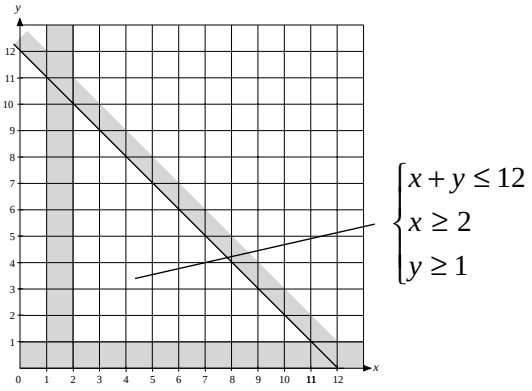
(f) $x + y \ge -3,\ y \ge 2x - 3,\ 2y \le x + 3$

4. (c)



5. (a) (i) $x + y \le 12$ (ii) $x \ge 2$ (iii) $y \ge 1$

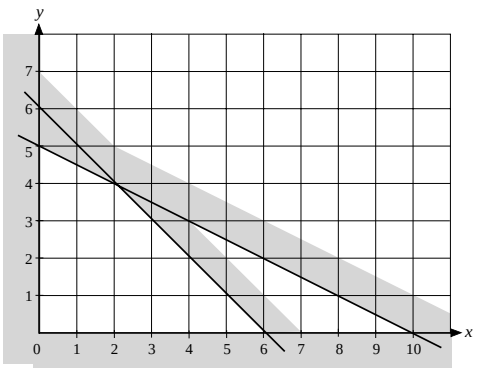
(b)



6. (a) (i) $x + y \le 30$ (ii) $x \le 20$ (iii) $y \le 22$ (b) $x \ge 0,\ y \ge 0$

7. (a) $x + 2y \leq 10$, $x + y \leq 6$, $x \geq 0$, $y \geq 0$

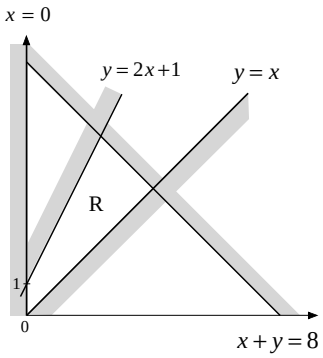
(b)



8. (a) (i) 3 (ii) £8 (iii) £5.50 (iv) 15 (b) 3 or 4

9. (b) inner triangle (c) 67 £2 tickets and 133 £3 tickets; £533

10. (a) 3, 4, 5, 6 and 7 (b)



11. (a) $x < 250$, $x + y \leq 300$, $x \geq 2y$ (c) £1000 (d) 120

17 Using Graphs

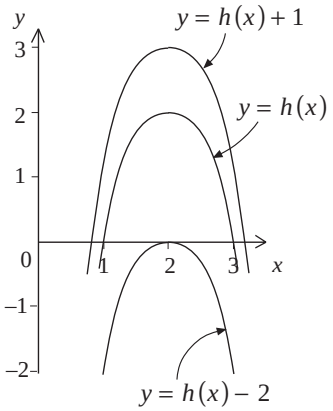
17.1 Transformations of Graphs

1. A : $y = f(x + 2)$

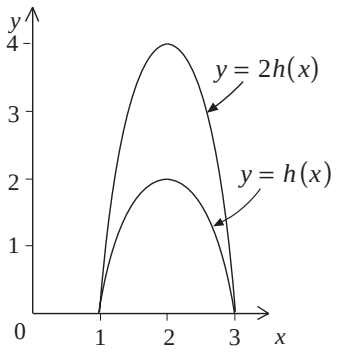
   B : $y = f(x - 3)$

   C : $y = f(x - 5)$

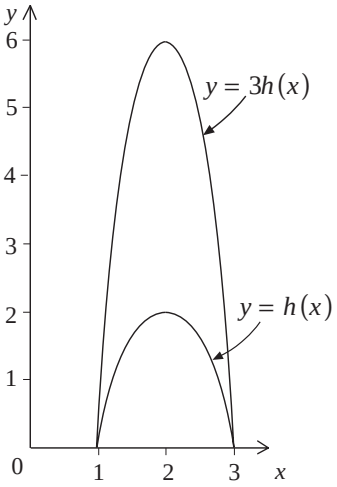
2. (a)



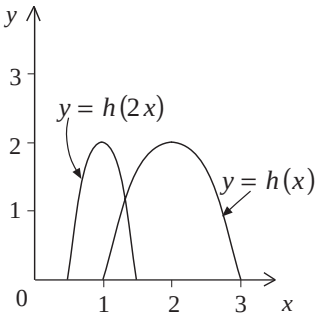
(b)



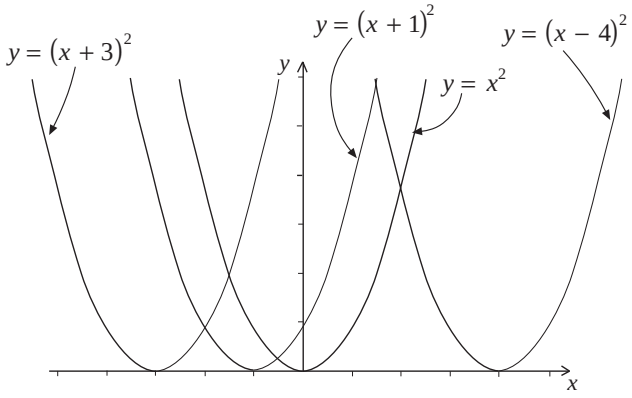
(c)



(d)

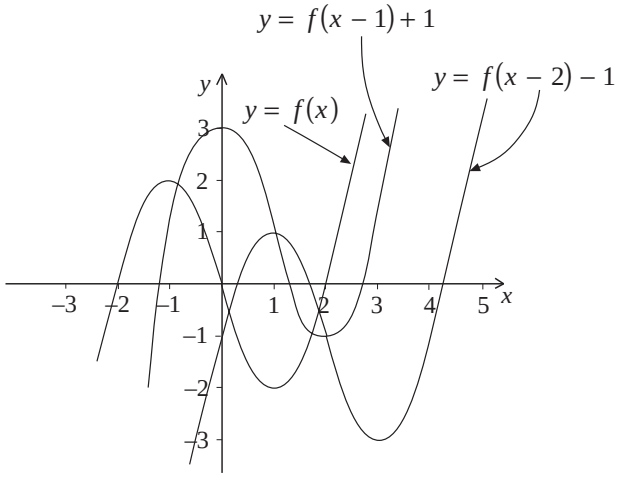


3.

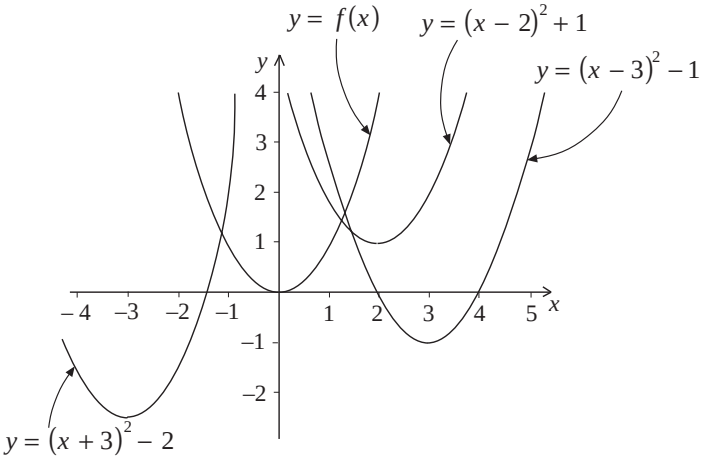


4. (a) Move $x = f(x)$ 2 units along the positive x-axis, and then 2 units up the y-axis.

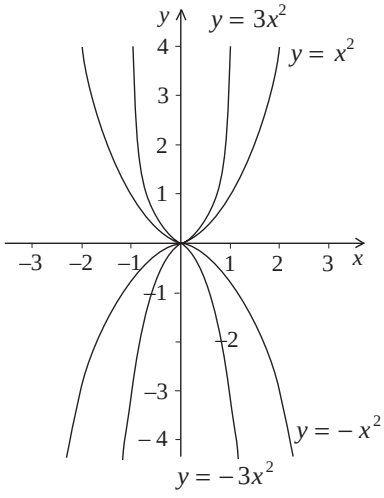
(b)



5.

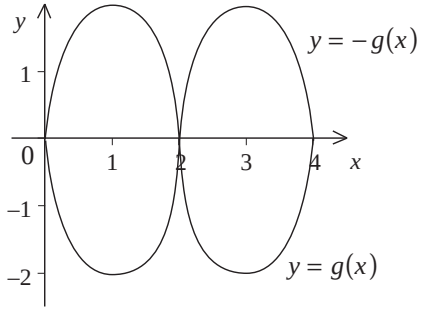


6.

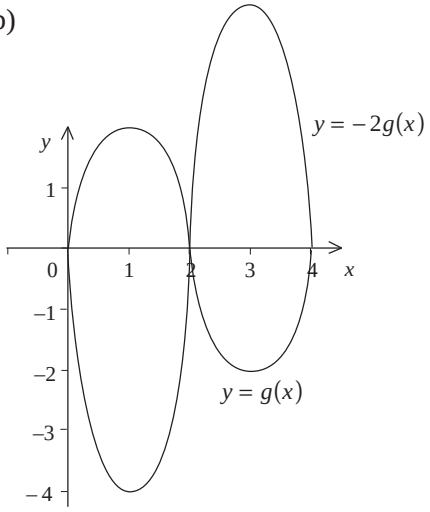


(graphs are reflections of each other)

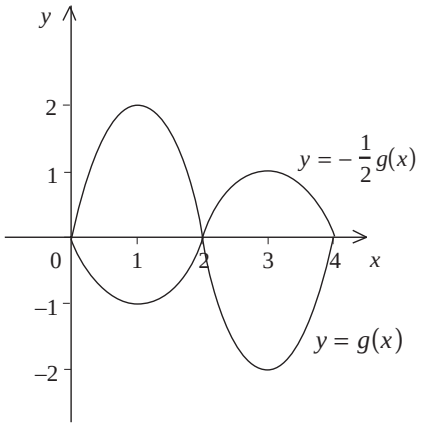
7. (a)



(b)

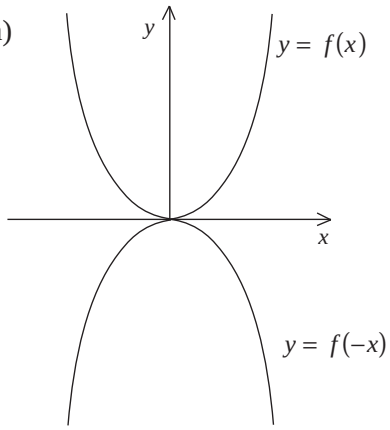


(c)

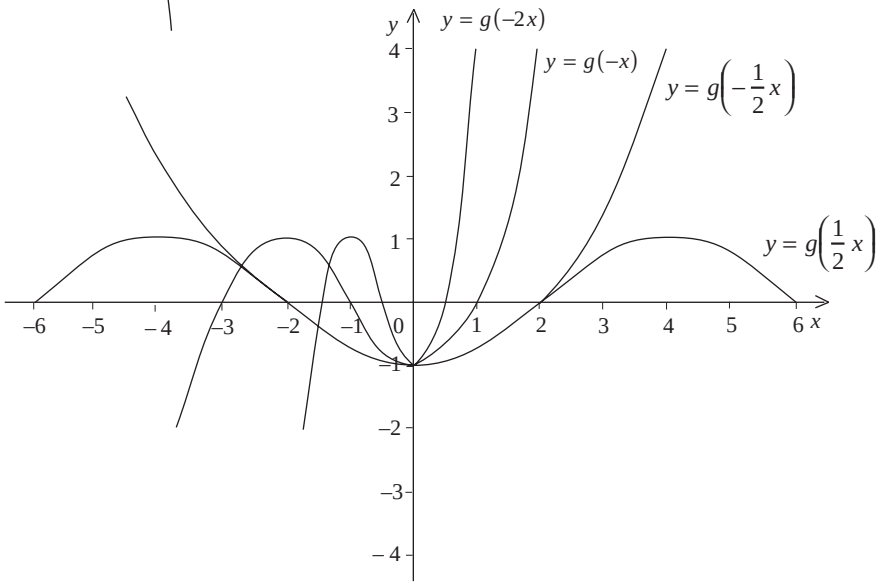


8. (a) $y = f(x)$ and $y = -f(x)$ (b) $y = f(x-1)$ and $y = -f(x-1)+1$

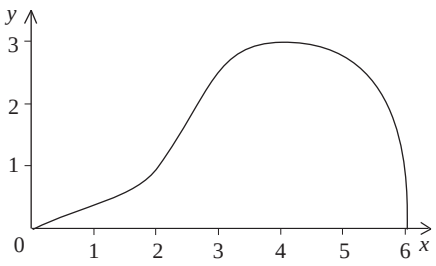
9. (a)



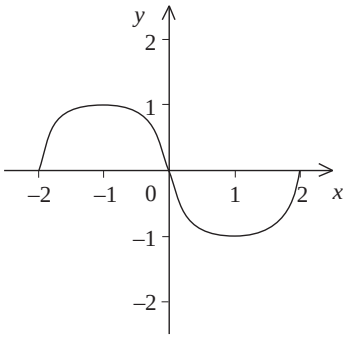
(b)



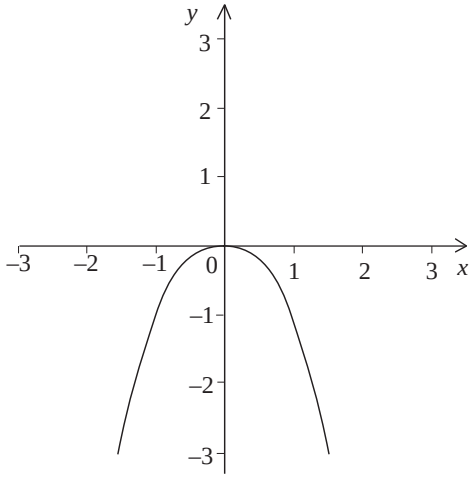
10. (a)



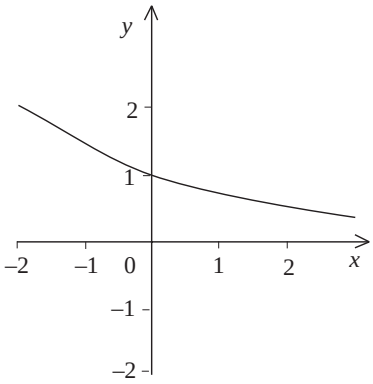
(b)



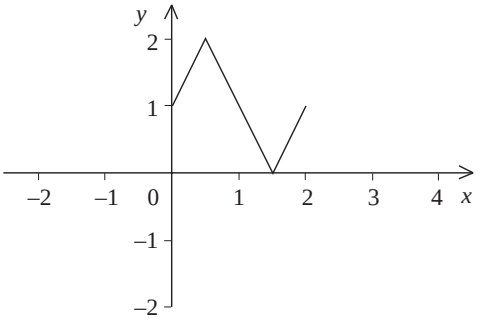
(c)



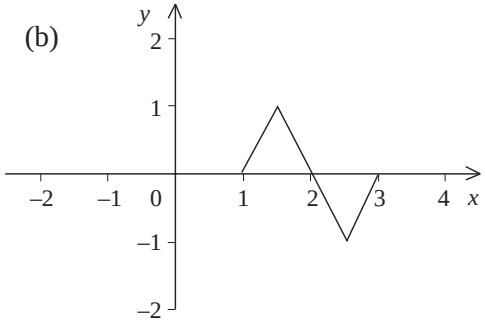
(d)



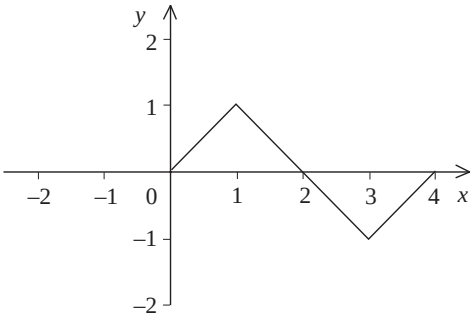
11. (a)



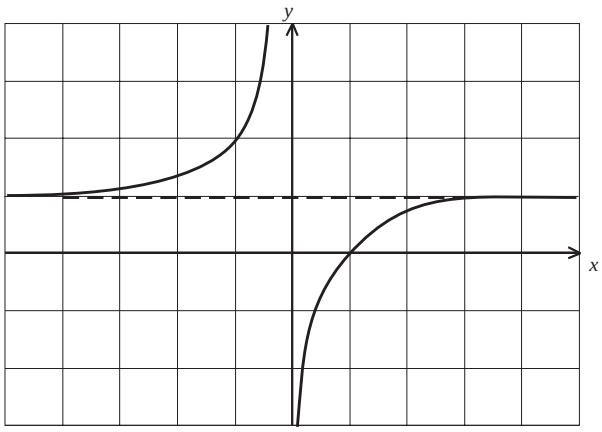
(b)



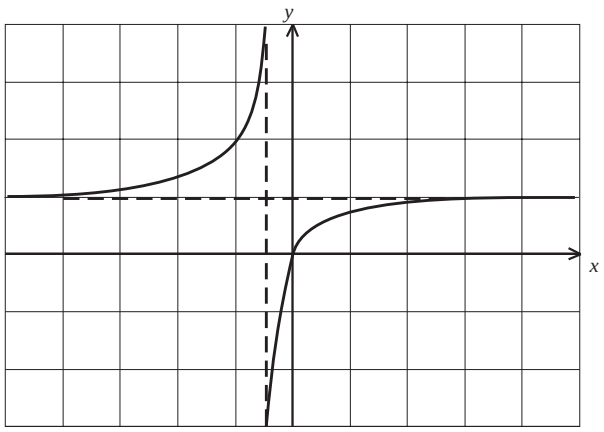
(c)



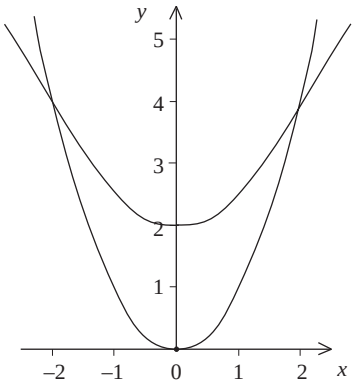
12. (a)



(b)



17.2 Areas Under Graphs

1. 16
2. 10
3. (a) 10 (b) $11\frac{3}{8}$
4. (b) about 42 (c) over-estiamte
5. 9
6. 

5; this should be a reasonable estimate, but slightly an underestimate

7. 950 m
8. (a) about 15 m^3 (b) about 100 cm^3 (c) about 6250 cm^3
9. (a) (ii) about 1.9 m/s^2 (iii) deacceleration (b) about 85 m

17.3 Tangents to Curves

1. (b)

x	0	1	2	3	4
gradient	0	2	4	6	8

(c) gradient $= 2x$

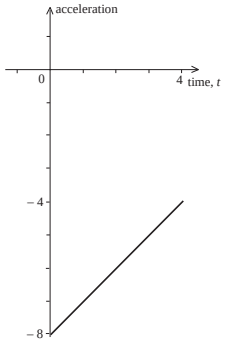
2. (b) 5, –2 (c) 10

3. (a) 1.8 °C/min (b) 0.5 °C /min (c) 0.3 °C/min

4. (b)

t	0	1	2	3	4
gradient	–8	–6	–4	–2	0

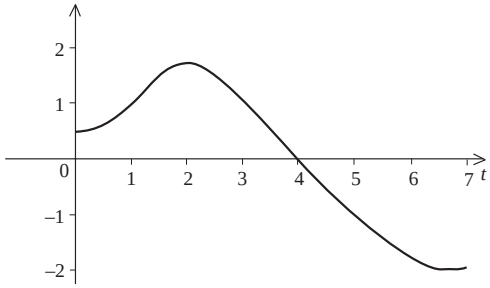
(c)



5. (a)

t	0	1	2	3	4	5	6	7
velocity	0.5	0.8	1.6	0.8	0	–0.9	–2	–2

(b)



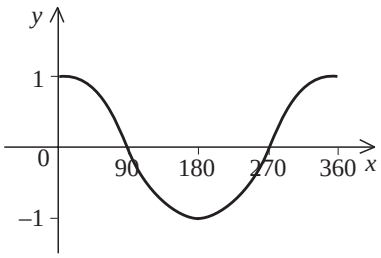
6. (a)

x	–3	–2	–1	0	1	2	3
gradient	27	12	3	0	3	12	27

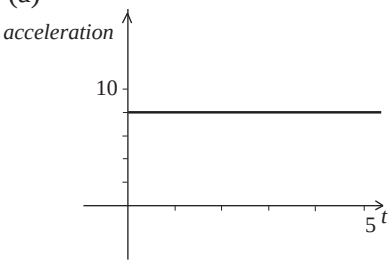
(b) gradient $= 3x^2$

7. (a) 90°, 270° (b) 1 at $x = 0$, 360° : –1 at $x = 180$

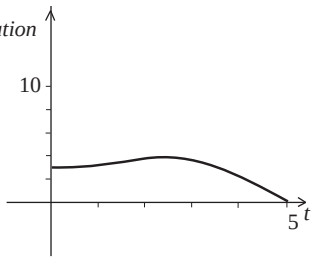
(c)



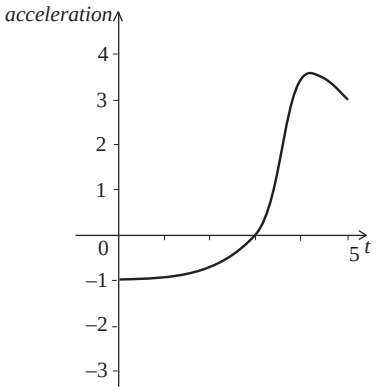
8. (a)



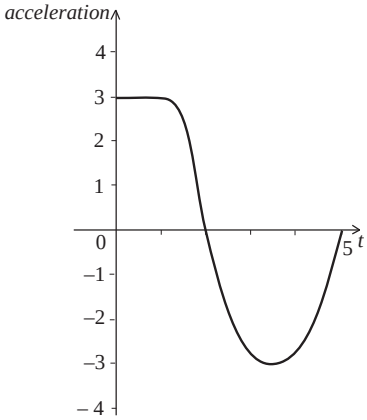
(b)



(c)



(d)



9. 0.6 m/s^2

17.4 Finding Coefficients

1. (a) $a = \frac{1}{2},\ b = \frac{3}{2}$ (b) $a = 3,\ b = -5$ (c) $a = 15,\ b = 0.25$

   (d) $a = 30,\ b = -5$

2. (a) $d = 0.3v$ (b) $d = 0.015v^2$ (c) $d = 0.3v + 0.015v^2$

3. about 0.19

4. about 4.9

5. $R = \frac{10}{I}$

6. Idea probably correct, but data point $(I = 3,\ H = 27.6)$ appears to be incorrect.

7. $A = 4,\ B = 5.1$: 8.96

18 3-D Geometry

18.1 Using Pythagoras' Theorem and Trigonometry in Three Dimensions

1. (a) 6.7 cm (b) 80.6 cm (c) 55.2 cm
   (d) $\sqrt{3}x$ (e) $\sqrt{6}x$ (f) $\sqrt{x^2 + y^2 + z^2}$
2. $\sqrt{56} \approx 7.5$ cm .
3. (a) 52.4° (b) 65.5°
4. $\sqrt{3} \approx 1.73$ cm
5. (a) 4.5 m (b) 2.87 m
6. (a) 2.44 m (b) $d = \sqrt{h^2 + \left(l^2 + \frac{w^2}{4}\right)}$
7. (a) 29.7 m (b) 31.1°
8. 15.6 cm
9. (a) 10.6 cm (b) 28.1°; (a) 11.1 cm (b) 22.6°
10. (a) 4.9 cm (b) 56.3° (c) 64.6°

18.2 Angles and Planes

1. (a) 25.4° (b) 26.6° (c) 18.4°
   (d) 16.6° (e) 16.6° (f) 67.4°
2. (a) 1.66 cm (b) 10.99 cm
3. (a) 23.4° (b) 22.8° (c) 77.1°
4. (a) 36.9° (b) 22.6° (c) 36.9° (d) 71.6°
5. (a) 10.4 cm (b) 74.2° (c) 78.7°
6. 76°, 80.5°
7. (a) 29° (b) 29° (c) 43.3° (d) 61°
8. 77.8°, 76°
9. 64.8°, 71.6°
10. 2 m

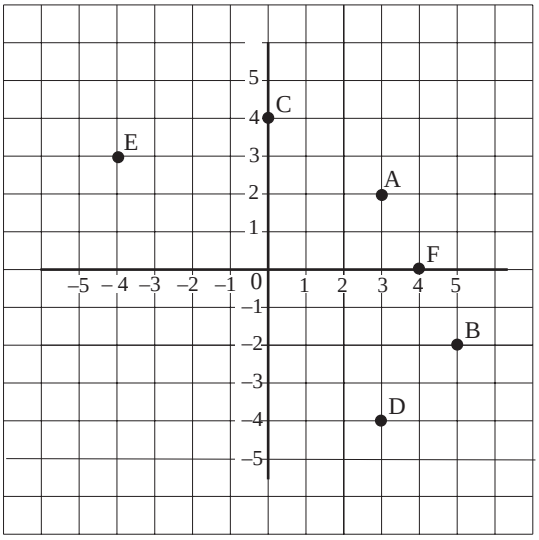
19 Vectors

19.1 Vectors and Scalars

1. Vectors: (b), (d), (e) scalars: (a), (c), (f)

2. (a) $\begin{pmatrix}6\\0\end{pmatrix}$ (b) $\begin{pmatrix}6\\-3\end{pmatrix}$ (c) $\begin{pmatrix}-3\\2\end{pmatrix}$ (d) $\begin{pmatrix}-6\\-3\end{pmatrix}$

   (e) $\begin{pmatrix}6\\3\end{pmatrix}$ (f) $\begin{pmatrix}3\\-5\end{pmatrix}$ (g) $\begin{pmatrix}-3\\-2\end{pmatrix}$ (h) $\begin{pmatrix}3\\2\end{pmatrix}$

3. 

   $\mathrm{EF} = \begin{pmatrix}8\\-3\end{pmatrix}$

4. (a) $\begin{pmatrix}7\\2\end{pmatrix}$ (b) $\begin{pmatrix}3\\-1\end{pmatrix}$ (c) $\begin{pmatrix}4\\11\end{pmatrix}$ (d) $\begin{pmatrix}1\\12\end{pmatrix}$

   (e) $\begin{pmatrix}-1\\-12\end{pmatrix}$ (f) $\begin{pmatrix}-4\\-3\end{pmatrix}$ (g) $\begin{pmatrix}12\\21\end{pmatrix}$ (h) $\begin{pmatrix}-6\\10\end{pmatrix}$

   (i) $\begin{pmatrix}0\\16\end{pmatrix}$ (j) $\begin{pmatrix}17\\-1\end{pmatrix}$ (k) $\begin{pmatrix}-12\\-1\end{pmatrix}$ (l) $\begin{pmatrix}12\\-28\end{pmatrix}$

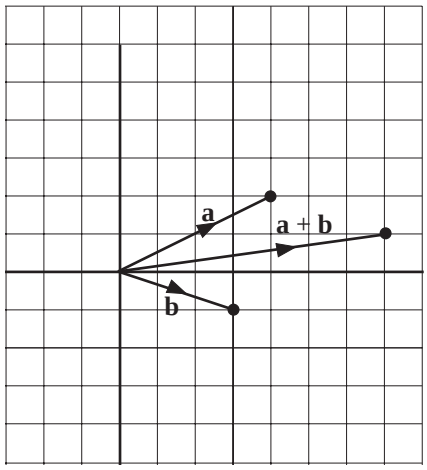
5. (a) $\begin{pmatrix}10\\1\end{pmatrix}$ (b) $\begin{pmatrix}2\\-8\end{pmatrix}$ (c) $\begin{pmatrix}0\\9\end{pmatrix}$ (d) $\begin{pmatrix}-4\\-14\end{pmatrix}$

   (e) $\begin{pmatrix}-10\\-14\end{pmatrix}$ (f) $\begin{pmatrix}24\\21\end{pmatrix}$

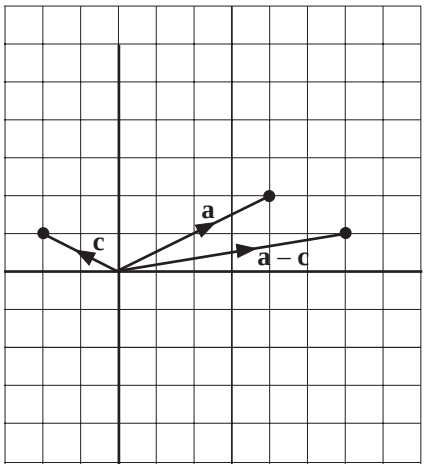
6. (a) $\begin{pmatrix}1\\-3\end{pmatrix}$ (b) $\begin{pmatrix}6\\-2\end{pmatrix}$ (c) $\begin{pmatrix}1\\-1\end{pmatrix}$ (d) $\begin{pmatrix}\frac{1}{2}\\-\frac{3}{2}\end{pmatrix}$ (e) $\begin{pmatrix}-1\\-3\end{pmatrix}$

   (f) $\begin{pmatrix}4\\7\end{pmatrix}$ (g) $\begin{pmatrix}3\\-\frac{11}{2}\end{pmatrix}$ (h) $\begin{pmatrix}-7\\\frac{13}{2}\end{pmatrix}$ (i) $\begin{pmatrix}-5\\3\end{pmatrix}$

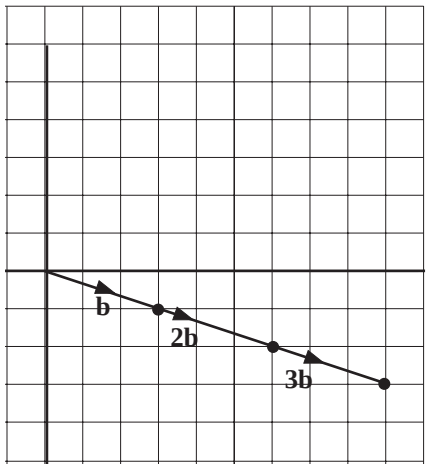
7. (a)



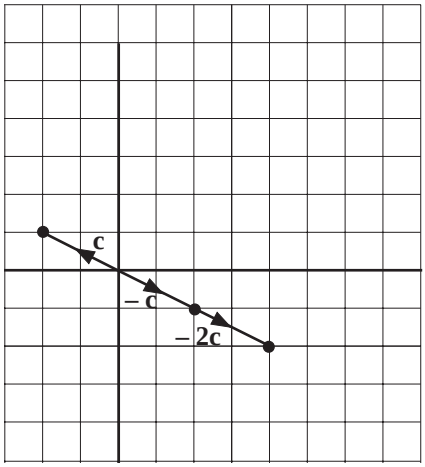
(b)



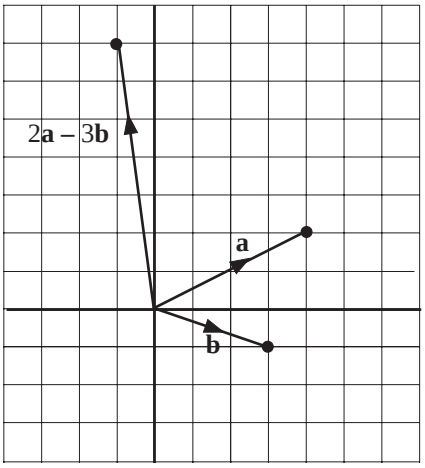
(c)



(d)



(e)



19.2 Applications of Vectors

1. (a) 3.4 m/s, 63° (b) $3\frac{1}{3}$ s (c) 5 m
2. 2.5 m/s, 1.875 m/s
3. (a) 34 m/s (b) 081°, 62.5 m/s
4. 37° to bank; 20 s
5. (a) bearing 343°, speed 209 km/h (b) bearing 017.5°

6. 408 seconds

7. (b) $P = 577$ N, $Q = 289$ N

8. $R = 580$ N, $F = 155$ N

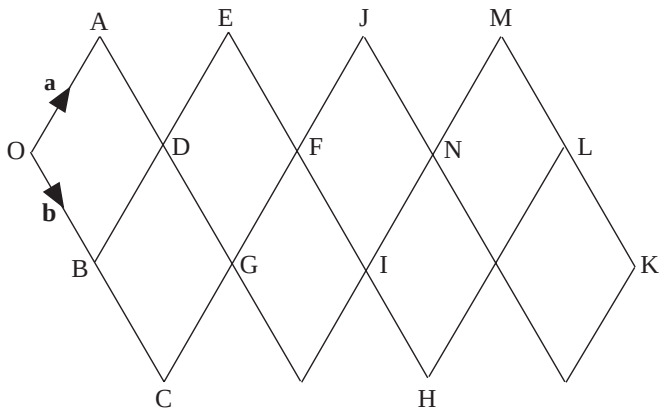
9. 0.64 m/s

10. 93.75 m

11. (a) (i) 0.85 m/s (ii) 28° (b) (i) 8 m (ii) 20 s

19.3 Vectors and Geometry

1. (a) $\mathbf{4a}$ (b) $\mathbf{a}$ (c) $\mathbf{b}$ (d) $-\mathbf{a} + 2\mathbf{b}$
   (e) $\mathbf{b}$ (f) $3\mathbf{a} + 2\mathbf{b}$ (g) $3\mathbf{b}$ (h) $3\mathbf{b}$
   (i) $\mathbf{a} - \mathbf{b}$ (j) $2\mathbf{a} - 2\mathbf{b}$ (k) $-\mathbf{a} - 2\mathbf{b}$ (l) $-3\mathbf{a} - 2\mathbf{b}$
   (m) $-3\mathbf{a} + 3\mathbf{b}$ (n) $-2\mathbf{a}$ (o) $-4\mathbf{a} + \mathbf{b}$ (p) $-3\mathbf{a} - 3\mathbf{b}$

2. 

3. (a) (i) $\mathbf{c}$ (ii) $\mathbf{a}$ (iii) $-\mathbf{a}$ (iv) $-\mathbf{a} + \mathbf{c}$ (v) $\mathbf{a} + \mathbf{c}$ (vi) $\mathbf{a} - \mathbf{c}$
   (b) (i) $\frac{1}{2}\mathbf{c}$ (ii) $\mathbf{a} + \frac{1}{2}\mathbf{c}$ (iii) $\frac{1}{2}\mathbf{a}$ (iv) $\mathbf{c} + \frac{1}{2}\mathbf{a}$ (v) $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$

4. (a) $\frac{1}{2}(\mathbf{q} + \mathbf{p})$ (b) $\frac{1}{2}(\mathbf{p} + \mathbf{q})$ (c) M and N are coincident

5. (a) $\vec{AD} = 6\mathbf{i}$, $\vec{OD} = 6\mathbf{i} + 6\mathbf{j}$
   (b) $\vec{CE} = 4\mathbf{j}$, $\vec{OE} = 8\mathbf{i} + 4\mathbf{j}$
   (c) $\vec{OM} = 7\mathbf{i} + 5\mathbf{j}$

6. (a) $\frac{1}{2}(\mathbf{p} + \mathbf{q})$, $\frac{1}{2}\mathbf{p} + \frac{5}{2}\mathbf{q}$ (b) $2\mathbf{q}$

8. (a) $2\mathbf{a} + \mathbf{b} + \mathbf{c}$ (b) $\mathbf{a} + \frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c}$ (c) $-\mathbf{a}$ (d) $\mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{c}$
   (e) $\frac{1}{2}(\mathbf{c} - \mathbf{b})$

10. (a) $\frac{1}{3}\mathbf{b} - \mathbf{d}$ (b) $\mathbf{b} + \mathbf{d}$ (c) $\alpha = \beta = 3$

11. $\vec{AQ} = \frac{1}{3}(\mathbf{a} + \mathbf{b})$

12. (a) $\vec{AC} = 2\mathbf{p} + 8\mathbf{q}$

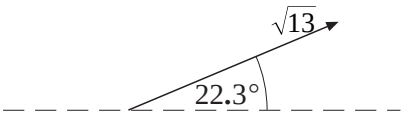
13. (a) (i) $-\mathbf{a} + \frac{1}{2}\mathbf{c}$ (ii) $\mathbf{c} - 2\mathbf{a}$ (b) 1 : 2

19.4 Further Work with Vectors

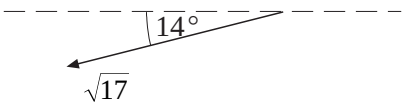
1. (a) $\begin{pmatrix} 40\cos 20° \\ 40\sin 20° \end{pmatrix}$ (b) $\begin{pmatrix} 30\cos 80° \\ 30\sin 80° \end{pmatrix}$ (c) $\begin{pmatrix} 8\cos 30° \\ 8\sin 30° \end{pmatrix}$

(d) $\begin{pmatrix} 7\cos 20° \\ -7\sin 20° \end{pmatrix}$ (e) $\begin{pmatrix} -12\cos 40° \\ 12\sin 40° \end{pmatrix}$ (f) $\begin{pmatrix} -10\cos 38° \\ -10\sin 38° \end{pmatrix}$

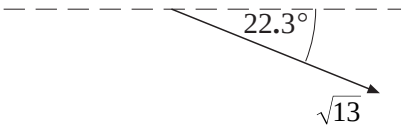
2. (a)



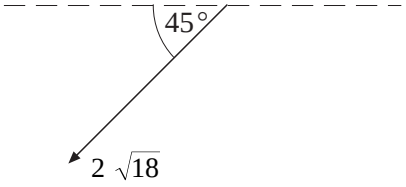
(b)



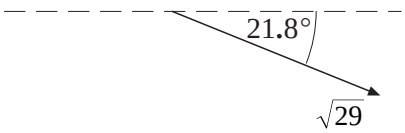
(c)



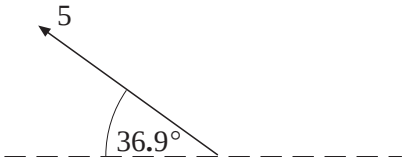
(d)



(e)

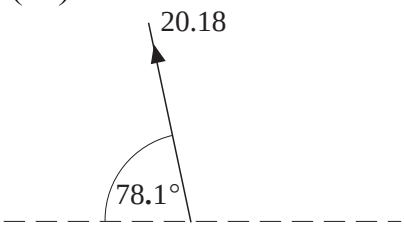


(f)



3. $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$, $\sqrt{34}$

4.



5. $\begin{pmatrix} 3 \\ -1.2 \end{pmatrix}$, 3.23 m/s at 111.8°

6. 242 m/s, $221.4°$

7. 226.4 m/s at $169.2°$

8. $F \approx 7.21$ N, $\theta \approx 43.9°$

9. $\theta \approx 23.6°$, 80.9 N

10. $\theta \approx 59.3°$, 386.5 N

11. (a) (i) $2\cos a$ (ii) $1 - 2\sin\alpha$ (b) $30°$

19.5 Commutative and Associative Properties

1. $\mathbf{a} - \mathbf{b} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$, $\mathbf{b} - \mathbf{a} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$, $\mathbf{a} - \mathbf{b} \neq \mathbf{b} - \mathbf{a}$

5. $(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \begin{pmatrix} 6 \\ 1 \end{pmatrix} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$