

3 Graphs

This is a key building block in in mathematics that is used both to illustrate data and algebraic formulae.

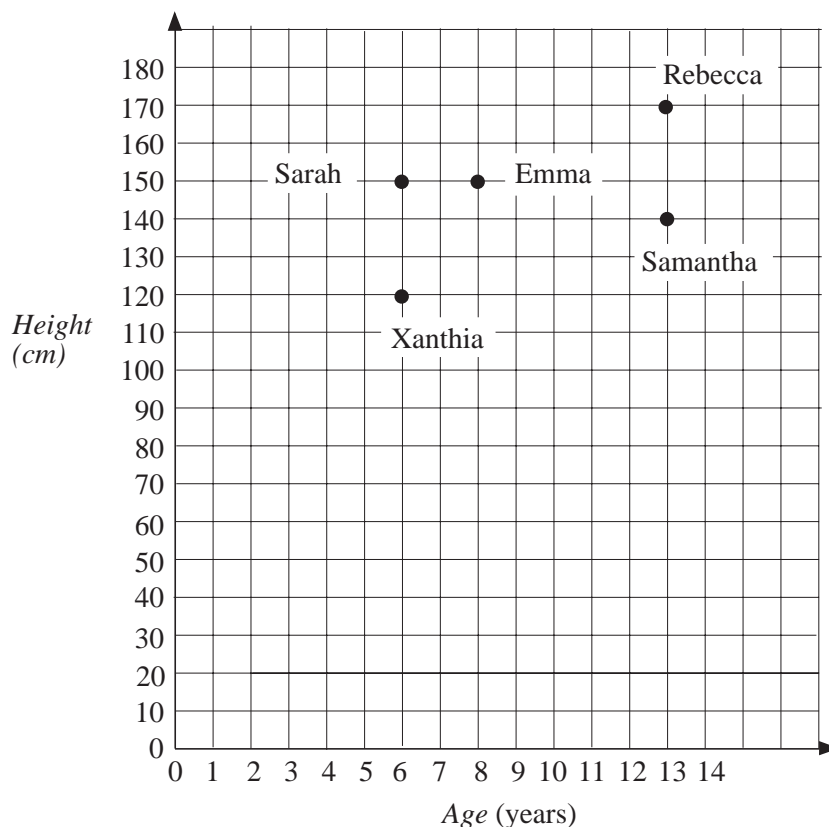
3.1 Scatter graphs

This is a particularly useful way of illustrating "paired" data - that is, information which has *two* related values e.g. height and weight; price and sales; latitude and longitude; rain and sunshine.



Example

The scatter graph below shows height and age. Information for five girls has been plotted on the graph.



- (a) Who is the tallest and how tall is she?
- (b) Who is the youngest and how old is she?
- (c) How much taller is Rebecca than Emma?
- (d) How much younger is Sarah than Samantha?
- (e) Is it true that older people are taller?



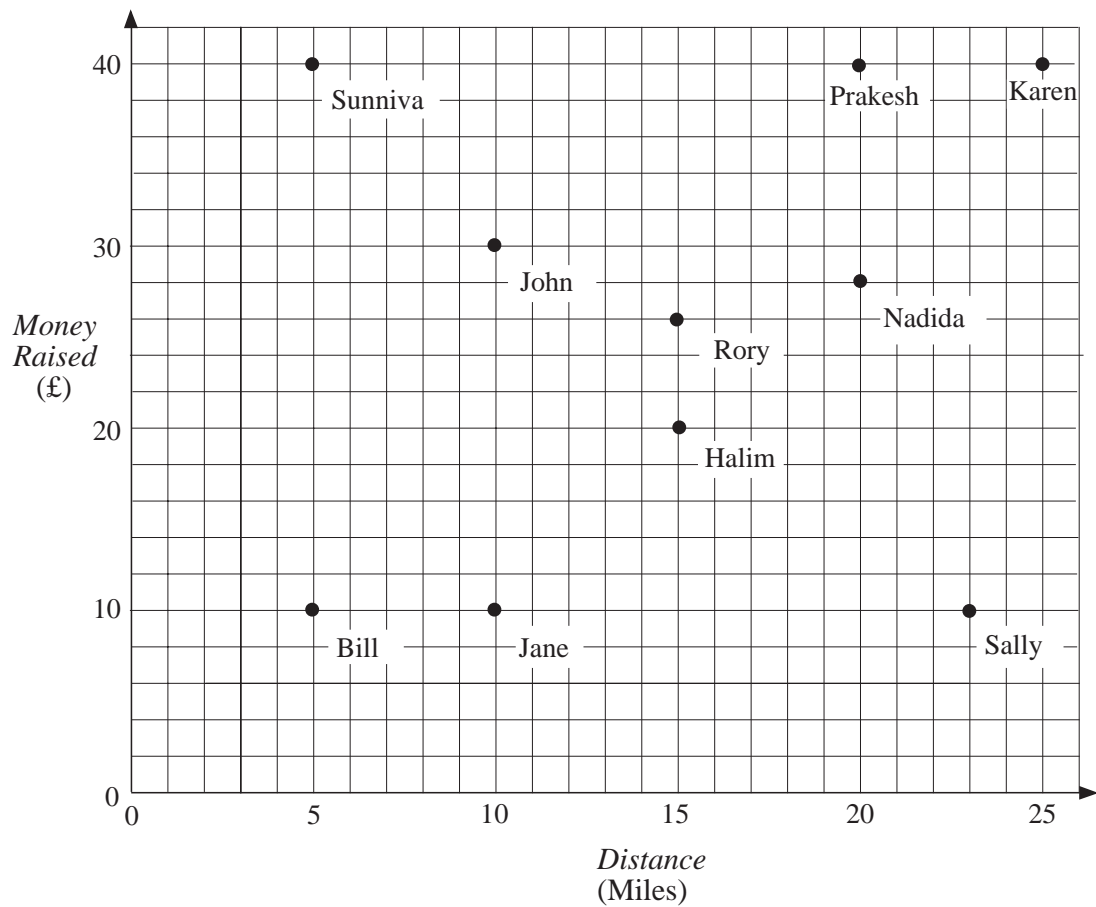
Solution

- (a) Rebecca is the tallest, and her height is 170 cm.
- (b) Sarah and Xanthia are the youngest - they are both 6 years old.
- (c) Emma is 150 cm in height, so Rebecca is
$$170 - 150 = 20 \text{ cm}$$
taller than Emma.
- (d) Samantha is 13 years old, so Sarah is
$$13 - 6 = 7 \text{ years}$$
younger than Samantha.
- (e) There is some evidence from the graph to deduce that older people are usually taller, but it is not true in general for girls less than 14 years old.

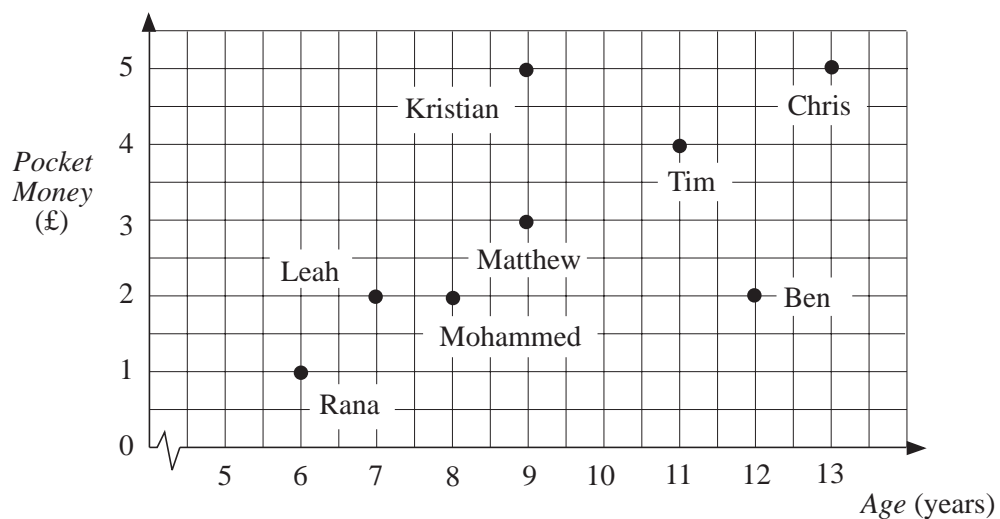


Exercises

1. Ten children took part in a sponsored walk. The scatter plot below shows how much money they raised and how far they walked.
 - (a) (i) How far did Karen walk?
 - (ii) How much money did she raise?
 - (b) Two children walked 15 miles. What are their names?
 - (c) Who walked 20 miles and raised £40?
 - (d) Explain how to work out how much Bill was sponsored for each mile.
 - (e) How much was Sunniva sponsored for each mile?
 - (f) How much money did Rory raise?
 - (g) How far did Sally walk?
 - (h) Generally, was more money raised by walking further?



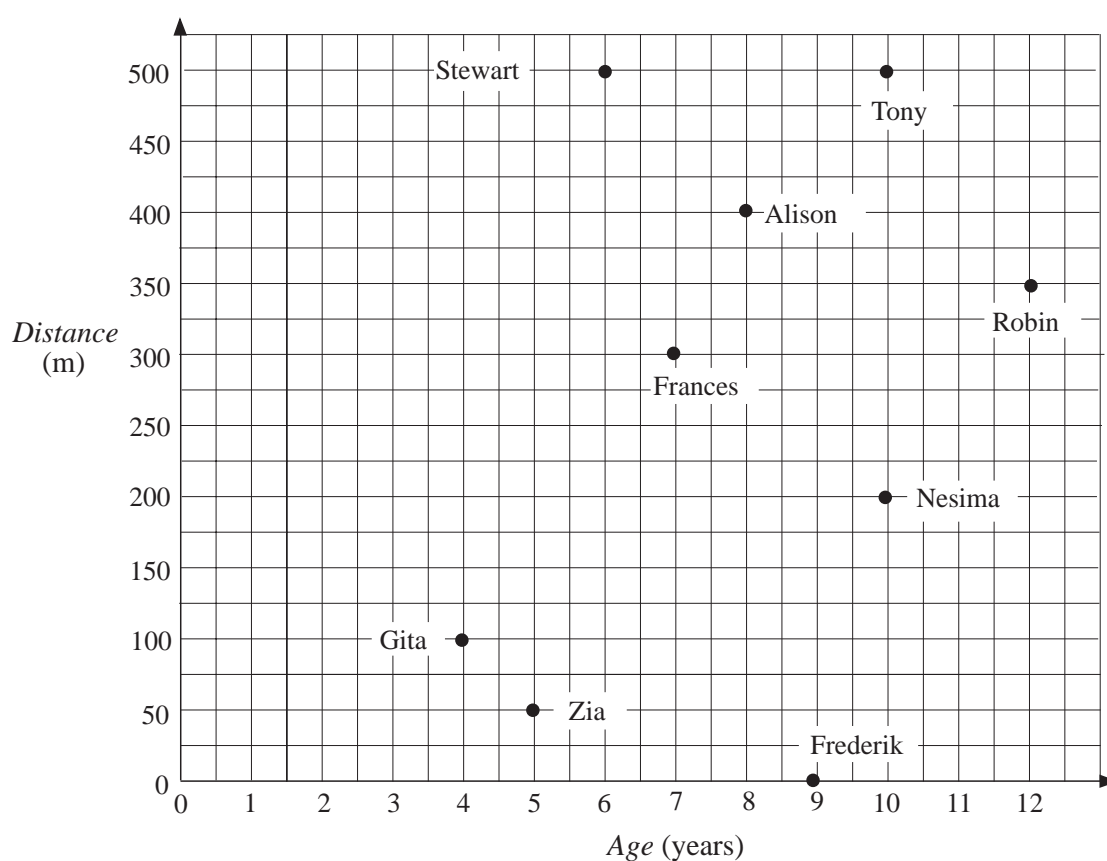
2. The scatter plot below shows the ages of the children who live in one street and how much pocket money they get each week.



- (a)
 - (i) How old is Tim?
 - (ii) How much pocket money does he get?
- (b) Which children are older than Tim?
- (c) Who gets more pocket money than Tim?

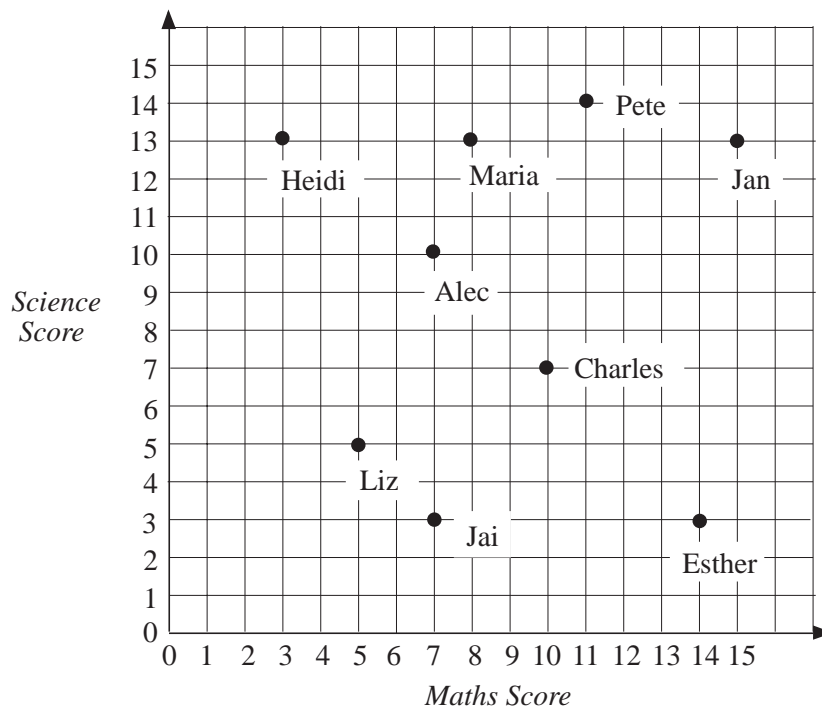
- (d) Who is 9 and gets £ 3 pocket money?
- (e) Who gets the same amount of pocket money as Kristian?
- (f) Who is the same age as Kristian?
- (g) Who gets twice as much pocket money as Ben?
- (h) Who gets half the pocket money Ben gets?
- (i) Ben is trying to persuade his parents to give him more pocket money. How would Ben use this graph to support his claim?
What would be a reasonable amount for Ben to have?

3. The scatter plot shows the ages of some children and the greatest distance they can swim.



- (a) Who is the youngest person that can swim 500 m?
- (b) Who cannot swim at all?
- (c) Who is 8 and can swim 400 m?
- (d) Who is 12 and can swim 350 m?
- (e) Who can swim the furthest?
- (f) Who is the youngest child?
- (g) Who can swim further than Robin?

4. The maths and science teachers at a school gave the Year 7 pupils two tests. The results for 9 pupils are shown on the scatter plot.



- Who had the highest score in science?
- Who had the highest score in maths?
- Who is good at science and poor at maths?
- Who is good at maths and poor at science?

The two test scores are added together.

- Who has the highest total?
- Who has the lowest total?

The results for six other pupils are given in the table below.

Copy the set of axes used above. Draw a scatter plot for these pupils.

<i>Name</i>	<i>Maths Score</i>	<i>Science Score</i>
Rola Reesh	10	8
Karen Eccles	5	14
Jenny Sharp	15	13
Zia Uddin	7	4
Adrian Smith	8	6
Wendy Maull	5	9

5. In a gymnastics competition the performance of each competitor is given a mark out of 10 by two different judges. The results for a competition are given below.

<i>Competitor</i>	<i>Judge 1</i>	<i>Judge 2</i>
Jane	4	5
Nishi	6	3
Julie	8	9
Andrea	2	1
Veronica	5	4
Chiori	3	2

Plot the scores in a scatter plot.

The two scores are added together to give a total.

Use your scatter plot to find:

- Who has the highest total?
- Who is second in the competition?

6. In a sponsored swim, the children raised the money listed in the table below.

<i>Name</i>	<i>Distance (m)</i>	<i>Money Raised</i>
Mark	300	£12
Kingsley	500	£20
Caroline	200	£4
Kevin	400	£10
Pushpa	300	£8
Alex	250	£15
Jai	300	£20
Zahra	50	£10
Lynda	450	£18

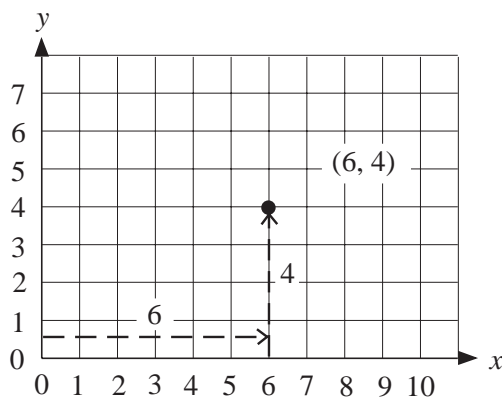
- (a) Draw a scatter plot for the distance and the money raised.
 (b) Copy and complete this sentence using one of these words

"more" "less" "no"

The children who swam the furthest raised money.

3.2 Plotting Points

We will now see how to plot points on a graph.



The x number comes first then the y number:

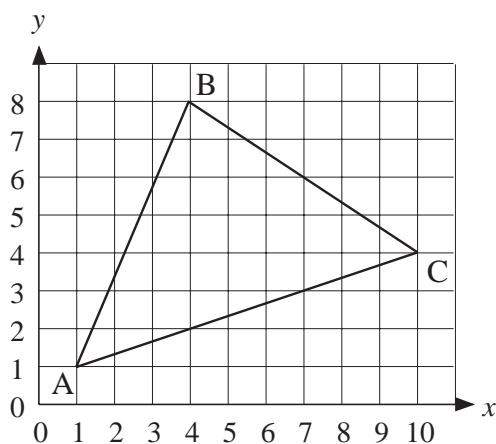
(x, y)

These number are called coordinates.

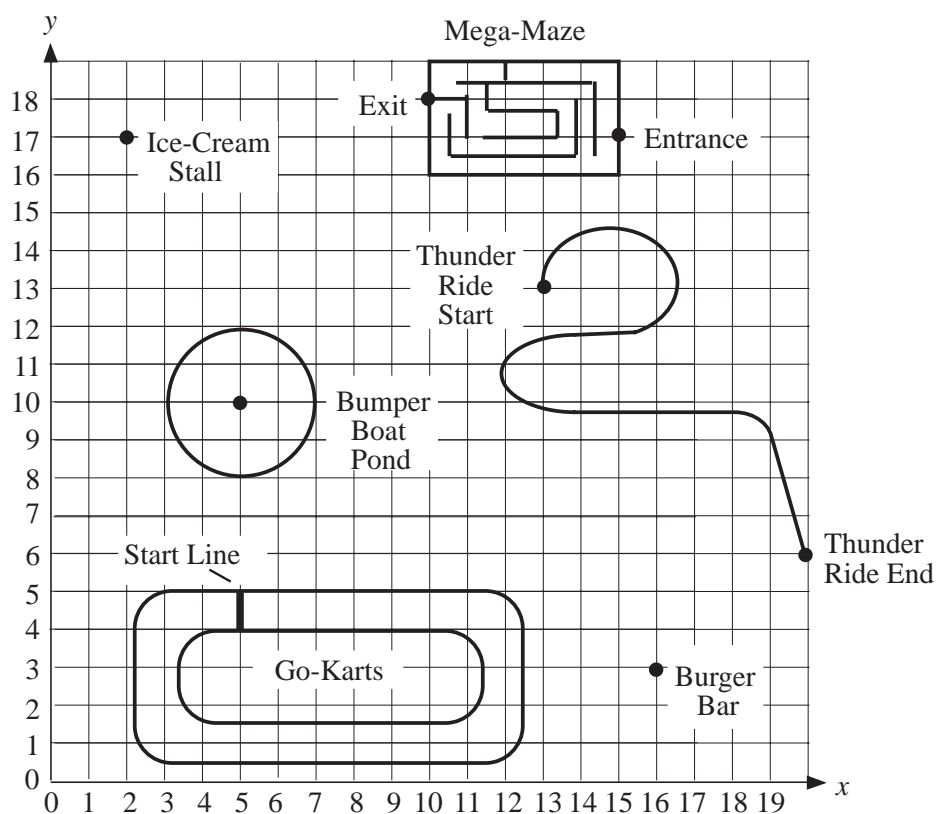


Exercises

1. Write down the coordinates of the three corners of this triangle.



2. The diagram shows a map of a theme park drawn on a set of axes.



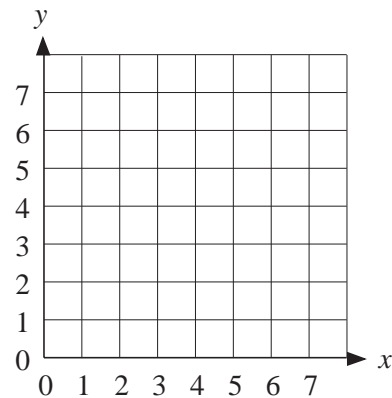
Write down the coordinates of:

- the Burger Bar
- the start of the Thunder Ride
- the end of the Thunder Ride
- the centre of the Bumper Boat Pond

- (e) the Ice Cream Stall
- (f) the Mega-Maze entrance
- (g) the Mega-Maze exit
- (h) both ends of the Go-Karts start line.

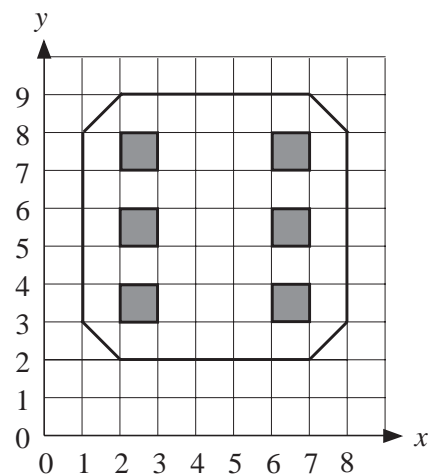
3. Draw a grid like this one.

- (a) Join the points with coordinates $(0, 3)$, $(5, 6)$ and $(5, 0)$ to draw a triangle.
- (b) On the same diagram join the points with coordinates $(2, 0)$, $(2, 6)$ and $(7, 3)$ to draw a second triangle.
- (c) Describe the shape you have now drawn.

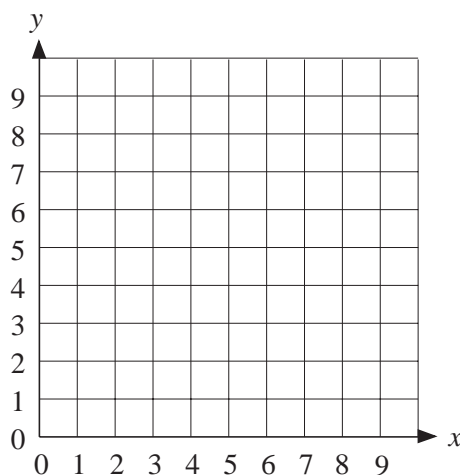


4. The diagram shows the face of a dice showing a 6.

Write a set of instructions that would give the face of the dice that shows a 1.



5. Draw a grid like this.

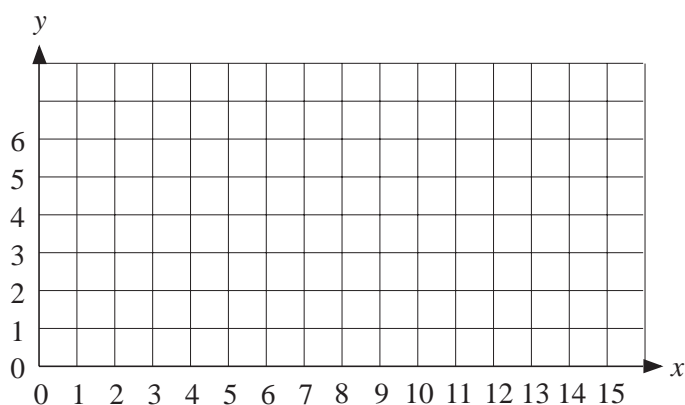


Join these points in order.

Use the same grid for all four parts.

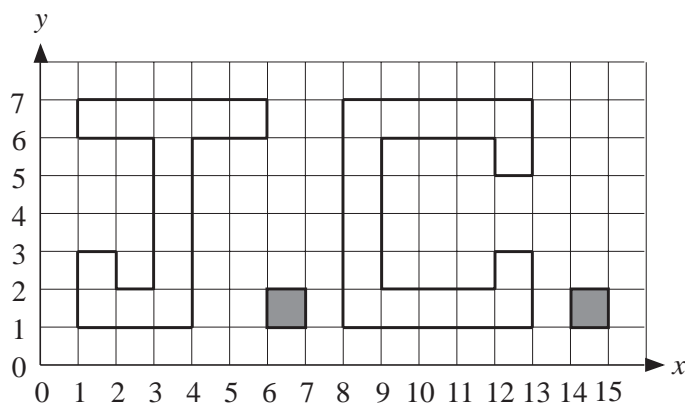
- (a) $(4, 6)$, $(5, 7)$, $(6, 6)$, $(4, 6)$.
- (b) $(5, 8)$, $(4, 8)$, $(4, 7)$, $(5, 8)$, $(6, 8)$, $(6, 7)$, $(5, 8)$.
- (c) $(4, 5)$, $(5, 4)$, $(6, 5)$, $(5, 3)$, $(4, 5)$.
- (d) $(5, 2)$, $(3, 4)$, $(3, 5)$, $(2, 5)$, $(2, 8)$, $(3, 8)$, $(3, 9)$, $(7, 9)$, $(7, 8)$, $(8, 8)$, $(8, 5)$, $(7, 5)$, $(7, 4)$, $(5, 2)$.

6. For this question you will need a grid like the one below.



Join each set of points in order to discover a message.

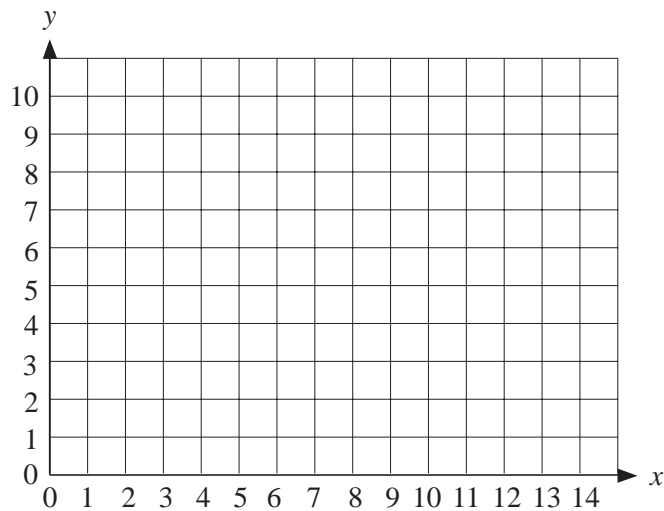
- (a) $(14, 4), (14, 5), (15, 5), (15, 4), (14, 4)$
 - (b) $(5, 1), (5, 6), (8, 6), (8, 5), (6, 5), (6, 4), (8, 4), (8, 3), (6, 3), (6, 2), (8, 2), (8, 1), (5, 1)$
 - (c) $(9, 6), (10, 6), (10, 2), (12, 2), (12, 1), (9, 1), (9, 6)$
 - (d) $(14, 1), (14, 3), (16, 3), (16, 6), (13, 6), (13, 1), (14, 1)$
 - (e) $(4, 6), (3, 6), (3, 4), (2, 4), (2, 6), (1, 6), (1, 1), (2, 1), (2, 3), (3, 3), (3, 1), (4, 1), (4, 6)$
7. The picture shows a set of initials.



- (a) Write out a set of instructions to draw these initials.
- (b) Draw your initials on a grid in a similar way.

Write out a set of instructions. Give the instructions to a friend and see if they can use them to draw your initials.

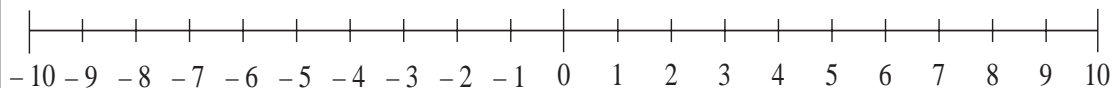
8. Answer this question on a copy of this grid.



- Shade in the square with corners at the points with coordinates (5, 3), (5, 4), (6, 4), (6, 3).
 - Join these points in order: (2, 4), (4, 8), (6, 9).
 - Join these points in order: (6, 4), (7, 5), (8, 5), (9, 4), (9, 3), (8, 2), (7, 2) and (6, 3).
 - Complete the picture to show a pair of glasses and write down the coordinates of the extra points that you use.
9. Draw a picture on a grid. Write a set of instructions using the coordinates so that a friend can draw the picture.

3.3 Negative Numbers

We extend our number system now to include negative numbers. It is useful to use a number line to illustrate this concept.



You can see for example

$$-2 < 4$$

$$-6 < -3$$

$$5 > -4$$

$$-1 > -7$$

You can check all these by looking at their positions on the number line.



Example

Make each statement below true by using the symbols $<$ or $>$.

(a) -5 4

(b) 3 7

(c) -6 -9

(d) 2 -2



Solution

(a) $-5 < 4$ (b) $3 < 7$ (c) $-6 > -9$ (d) $2 > -2$



Exercises

1. What temperature is:

- (a) 3°C warmer than -1°C
- (b) 6°C colder than -3°C
- (c) 5°C warmer than -5°C
- (d) 8°C warmer than -7°C
- (e) 5°C colder than -2°C
- (f) 3°C colder than 1°C
- (g) 6°C colder than 2°C
- (h) 8°C warmer than -12°C
- (i) 10°C colder than -2°C
- (j) 20°C warmer than -12°C ?

2. What number is;

- (a) 3 more than -2
- (b) 6 less than 1
- (c) 5 more than -7
- (d) 6 more than -10
- (e) 5 less than -4
- (f) 16 less than 3
- (g) 5 more than -20
- (h) 6 more than 5
- (i) 12 less than 10
- (j) 20 more than -8 ?

3. Write each set of numbers in order with the smallest first.
- (a) 6, -7, 8, -2, -5, -10, 3
 (b) 3, -2, 8, 0, -1, 1, -3
 (c) 5, -7, -20, 100, -50, -90, 60
4. Put either a $<$ or $>$ sign between each pair of numbers to give a true statement.
- (a) 4 2 (b) -6 -2
 (c) -3 4 (d) 2 -4
 (e) -6 -7 (f) -6 -5
 (g) 0 1 (h) -1 0
5. Is each statement below true or false?
- (a) $6 > 7$ (b) $4 > 3$
 (c) $8 > -1$ (d) $5 > -6$
 (e) $-6 < -7$ (f) $-1 > 0$
 (g) $-3 < 2$ (h) $-7 < 6$
 (i) $-4 > -3$ (j) $-5 < -2$
6. Write down any integer that could go in the boxes below.
- (a) $5 < \square < 7$
 (b) $-5 < \square < -3$
 (c) $-3 > \square > -7$
 (d) $-6 < \square < 0$
 (e) $-1 < \square < 2$

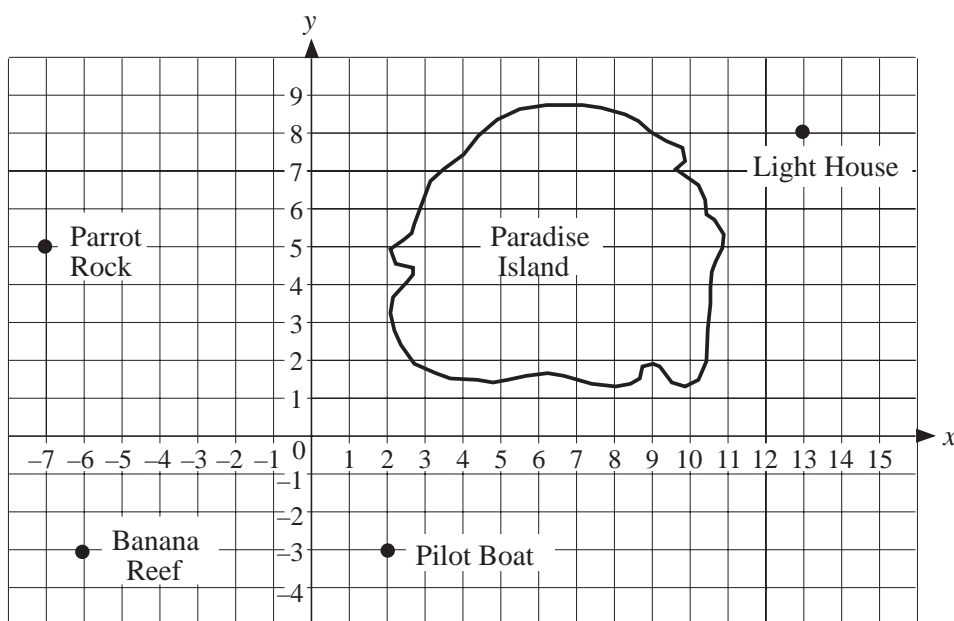
3.4 Coordinates

With the introduction of negative numbers, we can bring in coordinate axes with positive and negative numbers.



Example

A map of Paradise Island is drawn on the grid below.



What are the coordinates of;

- (a) the Lighthouse
- (b) the Pilot Boat
- (c) Parrot Rock
- (d) Banana Reef?



Solution

- (a) $x = 13, y = 8$ which is written as (13, 8)
- (b) (2, -3)
- (c) (-6, 5)
- (d) (-6, -3)



Exercises

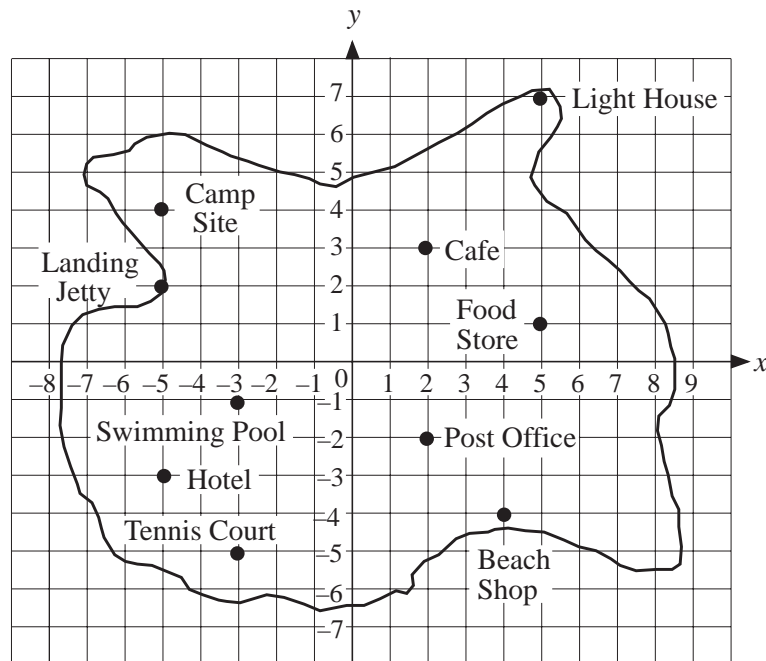
1. The map below shows a small island.
 - (a) What are the coordinates of;
 - (i) the cafe
 - (ii) the beach shop
 - (iii) the hotel
 - (iv) the campsite
 - (v) the swimming pool?

- (b) Nisha moves from the place with coordinates $(-3, -5)$ to the place with coordinates $(-5, 4)$.

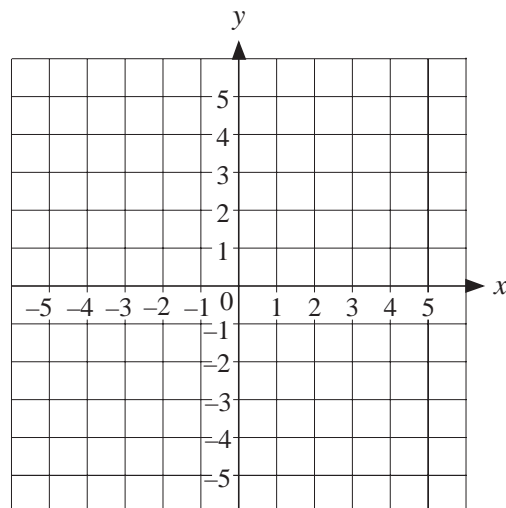
Where did she start?

Where did she finish?

- (c) Explain why Jacob cannot walk in a straight line from the place with coordinates $(5, 7)$ to the place with coordinates $(-5, 4)$.

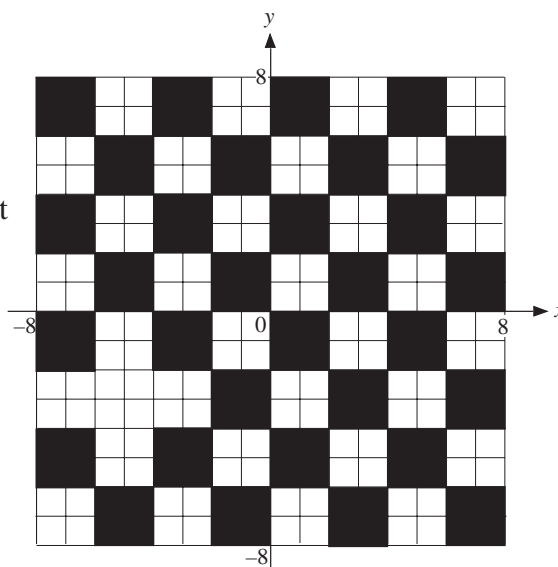


2. (a) Draw a set of axes like those in the diagram.
- (b) Mark the points with coordinates $(4, 0)$, $(-4, 0)$, $(0, 4)$, $(0, -4)$, $(1, 2)$, $(1, -2)$, $(3, 3)$, $(3, -3)$, $(2, 1)$, $(2, -1)$, $(-1, 2)$, $(-1, -2)$, $(-3, 3)$, $(-3, -3)$, $(-2, 1)$, $(-2, -1)$.
- (c) Join the points to form an 8 pointed star.



3. The picture shows a giant chess board that has been drawn on the surface of a playground.

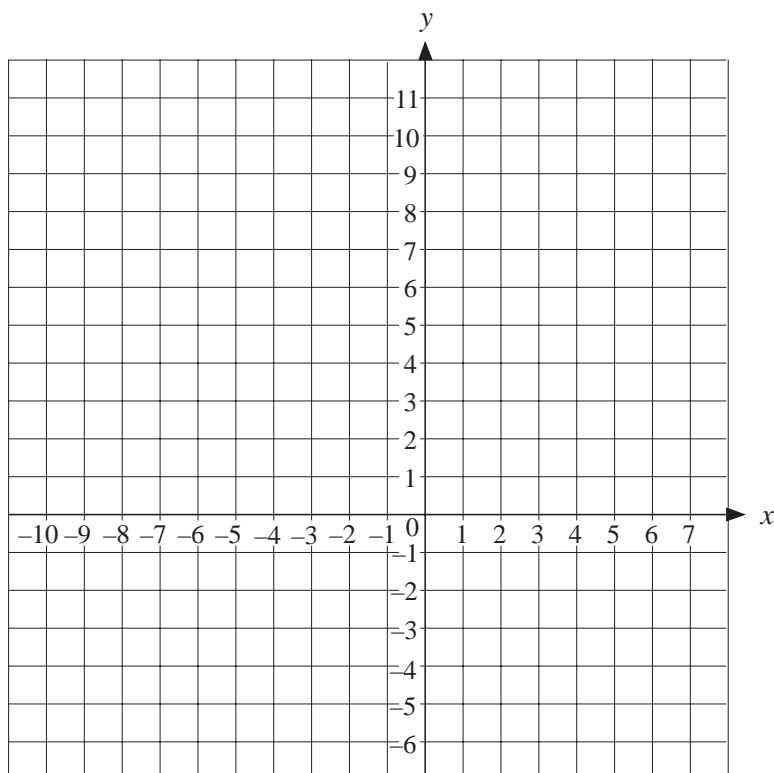
- (a) What are the coordinates of the corners of the square that is the wrong colour?
- (b) John always looks in the direction of the y -axis. When he starts at the point with the coordinates $(0, 7)$, he has one foot on a black square and one foot on a white square.



Is his left foot on the black square or the white square?

Describe where his feet are if he starts at;

- (i) $(2, 5)$
 - (ii) $(-5, 3)$
 - (iii) $(6, -3)$
 - (iv) $(-8, -1)$
 - (v) $(-2, 3)$
4. Draw the grid shown in the diagram.



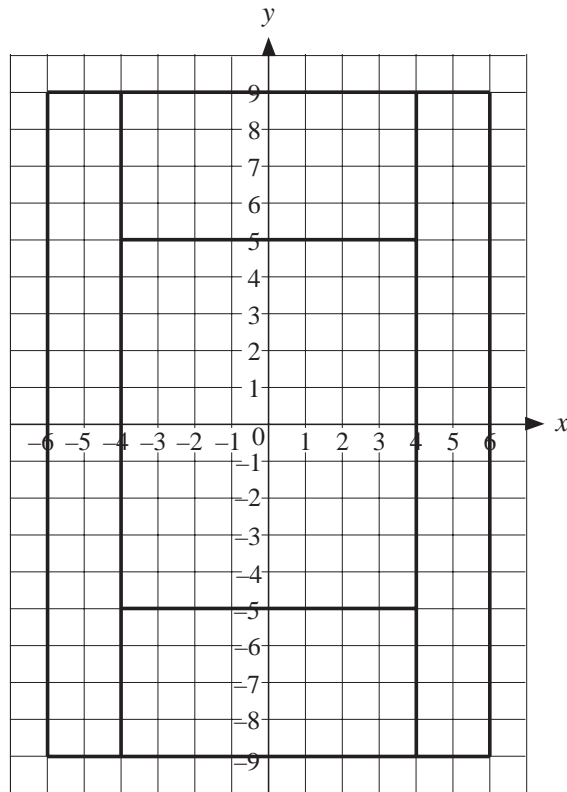
On the grid plot the points below joining each point to the next point.

$(0, 0)$, $(2, -2)$, $(0, -5)$, $(-2, -5)$, $(0, -2)$, $(-2, 0)$, $(-4, -2)$, $(-8, -5)$,
 $(-10, -5)$, $(-6, -2)$, $(-4, 0)$, $(-2, 6)$, $(-5, 5)$, $(-7, 3)$, $(-7, 4)$,
 $(-5, 6)$, $(-2, 7)$, $(-1, 7)$, $(-1, 8)$, $(-2, 9)$, $(-2, 10)$, $(-1, 11)$, $(1, 11)$,
 $(2, 10)$, $(2, 9)$, $(1, 8)$, $(1, 7)$, $(2, 7)$, $(4, 5)$, $(6, 5)$, $(6, 4)$, $(4, 4)$, $(2, 6)$,
 $(0, 0)$.

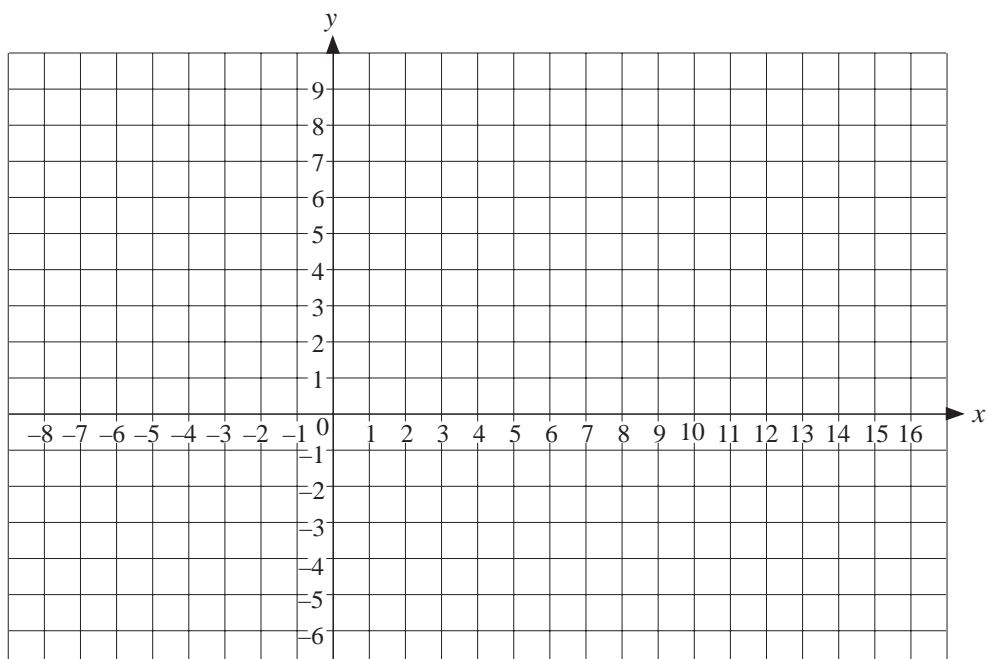
5. The diagram shows a tennis court drawn on a set of axes. The position of the ball is directly above the coordinates given.

The ball is served at $(1, 9)$ and hit at $(-3, -7)$. It travels back over the net and is hit again at $(-3, 8)$. The ball bounces next at $(6, -9)$.

Draw the path of the tennis ball on a copy of this diagram.



6. (a) Draw a copy of this grid.



- (b) On this grid draw the rectangles with corners at the following points with coordinates;

(i) $(-6, 6)$, $(-5, 6)$, $(-5, 4)$, $(-6, 4)$

(ii) $(-2, 1)$, $(-3, 1)$, $(-3, 3)$, $(-2, 3)$

(iii) $(3, 1)$, $(3, 3)$, $(4, 3)$, $(4, 1)$

(iv) $(10, 1)$, $(10, 3)$, $(9, 3)$, $(9, 1)$

(v) $(12, 4)$, $(13, 4)$, $(13, 6)$, $(12, 6)$

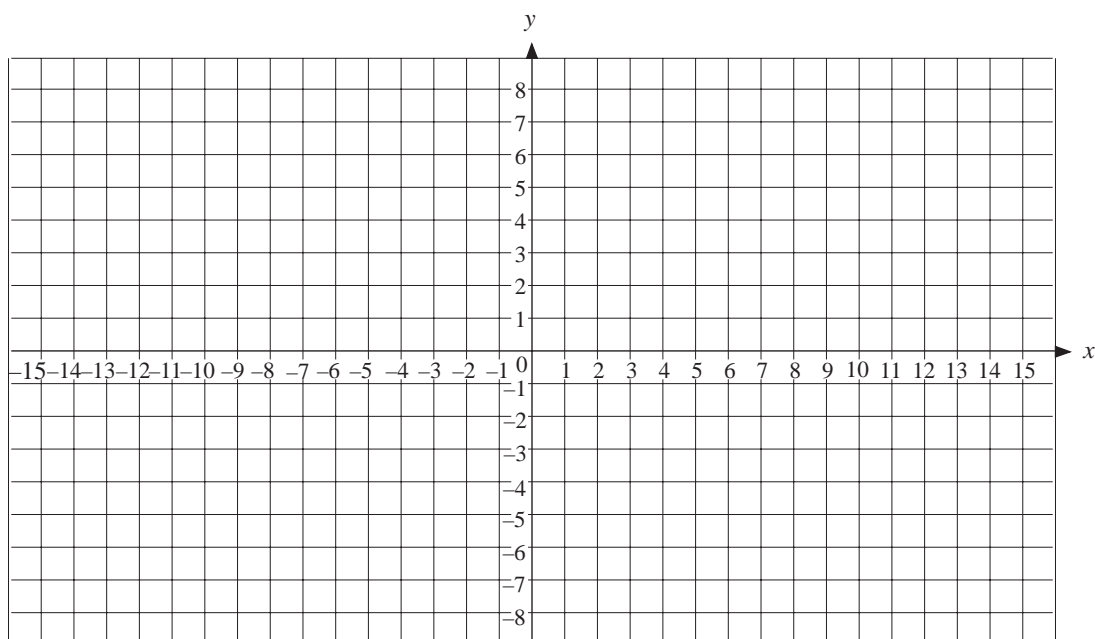
- (c) Join the points with coordinates

$(1, -5)$, $(1, -1)$, $(2, 0)$, $(5, 0)$, $(6, -1)$, $(6, -5)$

- (d) Join the points with coordinates

$(-7, -5)$, $(-7, 5)$, $(-8, 7)$, $(-8, 9)$, $(-7, 9)$, $(-7, 8)$, $(-6, 8)$,
 $(-6, 9)$, $(-5, 9)$, $(-5, 8)$, $(-4, 8)$, $(-4, 9)$, $(-3, 9)$, $(-3, 7)$,
 $(-4, 5)$, $(-3, 5)$, $(-3, 4)$, $(-2, 4)$, $(-2, 5)$, $(-1, 5)$, $(-1, 4)$, $(0, 4)$,
 $(0, 5)$, $(1, 5)$, $(1, 4)$, $(2, 4)$, $(2, 5)$, $(3, 5)$, $(3, 4)$, $(4, 4)$, $(4, 5)$,
 $(5, 5)$, $(5, 4)$, $(6, 4)$, $(6, 5)$, $(7, 5)$, $(7, 4)$, $(8, 4)$, $(8, 5)$, $(9, 5)$,
 $(9, 4)$, $(10, 4)$, $(10, 5)$, $(11, 5)$, $(10, 7)$, $(10, 9)$, $(11, 9)$, $(11, 8)$,
 $(12, 8)$, $(12, 9)$, $(13, 9)$, $(13, 8)$, $(14, 8)$, $(14, 9)$, $(15, 9)$, $(15, 7)$,
 $(14, 5)$, $(14, -5)$, $(-7, -5)$.

7. (a) Draw a picture of your own on a copy of the axes below.



- (b) Write a set of instructions and give them to a friend, so they can draw your picture.

3.5 Plotting Polygons

Here we look at polygons plotted on coordinate axes, but first, we must recap the names of polygons.

Names of Polygons

<i>Number of Sides</i>	<i>Name</i>
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon



Note

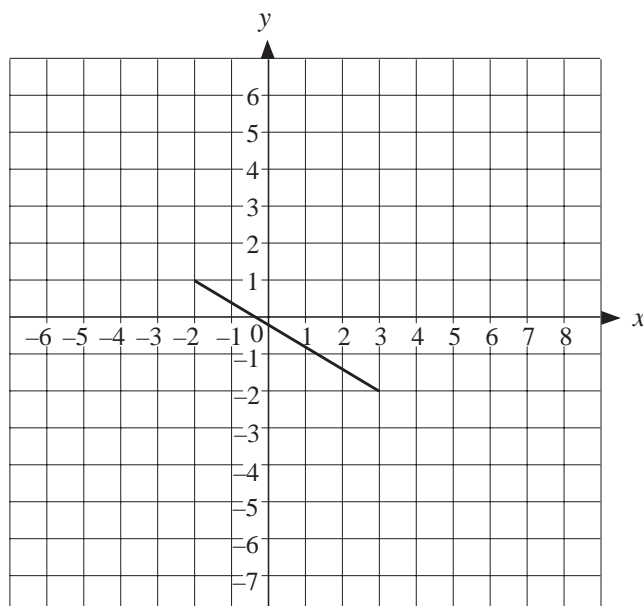
In a *regular* polygon:

- (a) All the sides are the same length.
- (b) All the angles are the same size.



Example

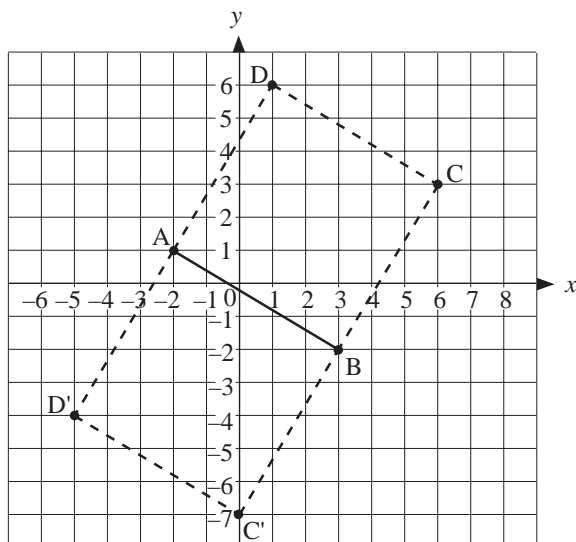
The line is one side of a square. What are the possible coordinates of the corners of the square?





Solution

You can construct a square in two ways using the given lines.



These are shown opposite.

To go from A to B, x increases by 5 units, y by 3 units.

So, to go to C, you increase x by 3 and y by 5 etc.

This gives

$C(6, 3)$, $D(1, 6)$

or, alternatively

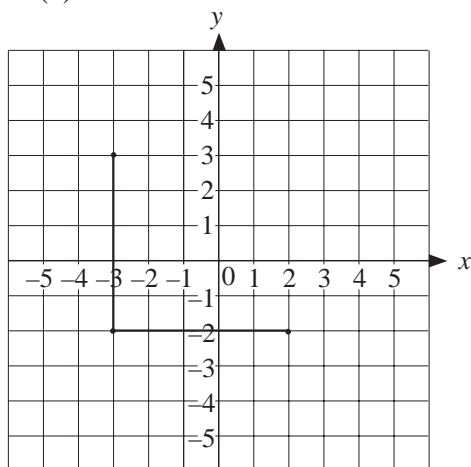
$C'(-3, -10)$, $D'(-2, -7)$



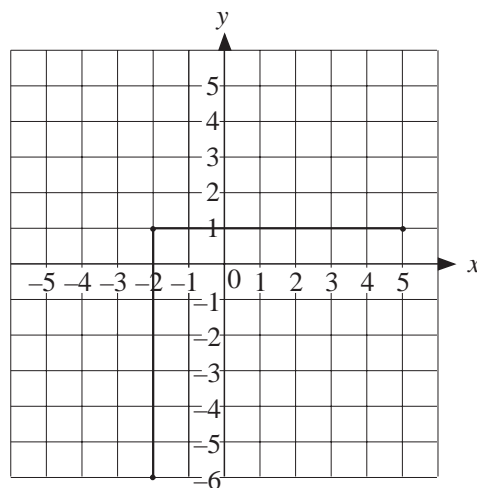
Exercises

- Write down the coordinates of the missing corner of each square.

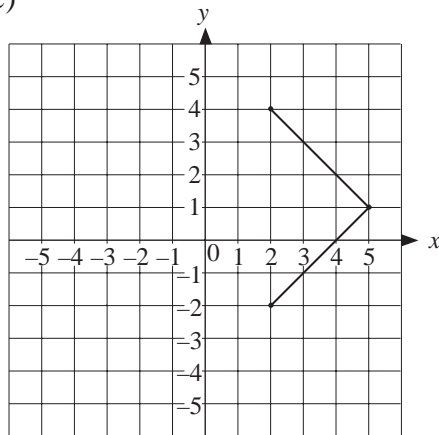
(a)



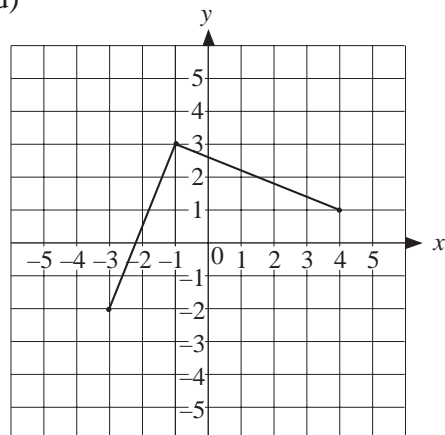
(b)



(c)



(d)

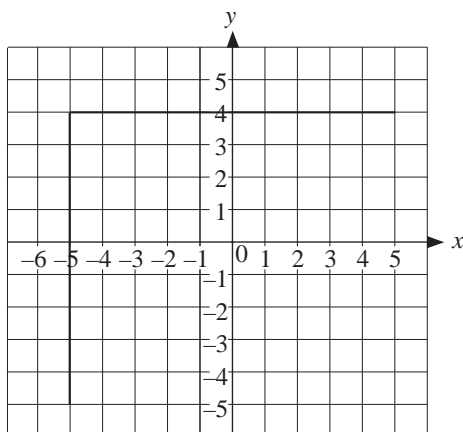


2. In each case the coordinates of 3 corners of a square are given. Find the coordinates of the other corner.

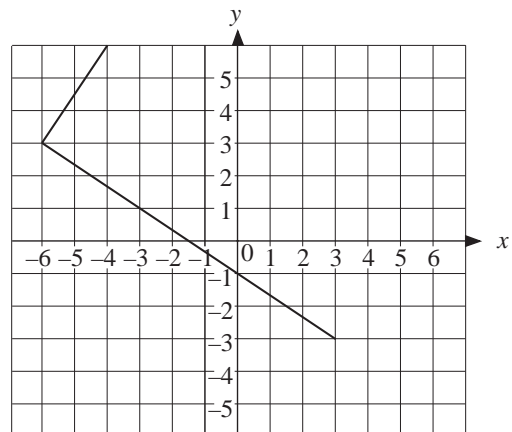
- (a) $(2, -2)$, $(2, 3)$ and $(-3, 3)$
- (b) $(2, 3)$, $(3, 4)$ and $(1, 4)$
- (c) $(2, 2)$, $(4, 4)$ and $(4, 0)$
- (d) $(-6, 2)$, $(-5, -5)$ and $(1, 3)$
- (e) $(-5, -2)$, $(-2, -1)$ and $(-1, -4)$

3. Write down the coordinates of the missing corner of the rectangles.

(a)



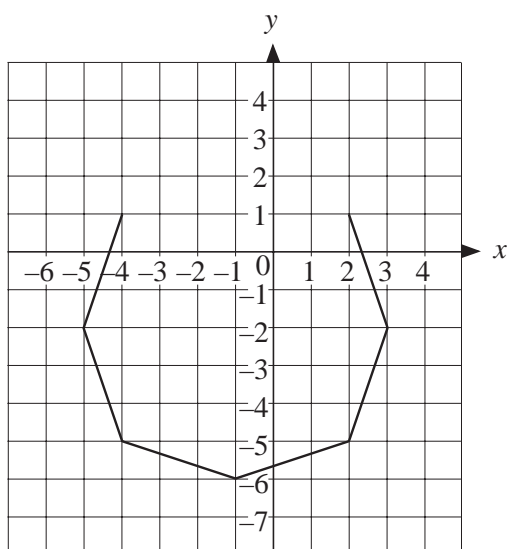
(b)



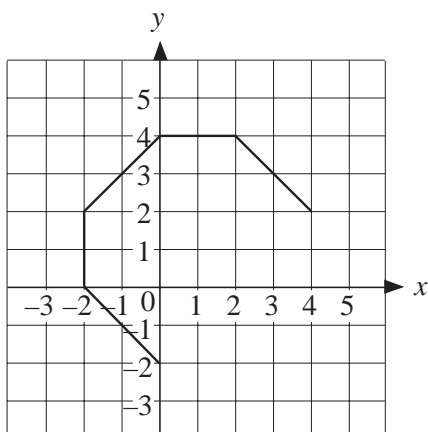
4. The coordinates of 3 corners of a rectangle are given below. Find the coordinates of the other corner of each rectangle.

- (a) $(-4, 2)$, $(-4, 1)$ and $(6, 1)$
- (b) $(0, 2)$, $(-2, 0)$ and $(4, -6)$
- (c) $(-4, 5)$, $(-2, -1)$ and $(1, 0)$
- (d) $(-5, 1)$, $(-2, 5)$ and $(6, -1)$

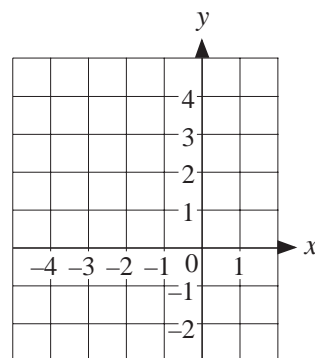
5. (a) The coordinates of 2 corners of a square are $(-4, 4)$ and $(1, -1)$. Explain why it is possible to draw three different squares using these two points.
- (b) Draw the three different squares.
- (c) If the coordinates of the corners had been $(-5, 1)$ and $(1, 3)$ would it still be possible to draw 3 squares? Draw the possible squares.
6. The sides of an octagon are all the same length. The diagram shows part of the octagon. Find the coordinates of the missing corner.



7. The angles between the sides of an octagon are all the same. The sides are not all the same length. Find the coordinates of the missing corners of the octagon.

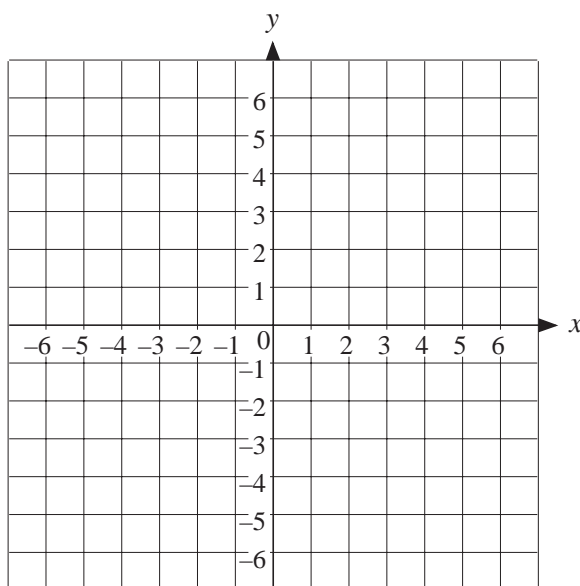


8. (a) Join the points with coordinates $(-2, -1)$, $(-3, -1)$, $(-4, 1)$ and $(-2, 2)$ on this grid.
- (b) This shape is half of a pentagon that has one line of symmetry. Complete the pentagon.
- Write down the coordinates of the extra corners.



9. Half of a heptagon with one line of symmetry can be drawn by joining the points with coordinates: $(2, 4)$, $(-2, 1)$, $(-2, -1)$, $(0, -3)$ and $(2, -3)$.
- Join the coordinates. You have drawn one half of the heptagon. Complete the heptagon. Write down the coordinates.

10. (a) Mark the points with these coordinates on a grid like this one.
- $(0, 5)$, $(4, 3)$, $(-6, 0)$, $(-5, -3)$ and $(2, -5)$.
- (b) Add extra points and draw a nonagon with the y-axis as a line of symmetry.
- (c) Write down the coordinates of the extra points.

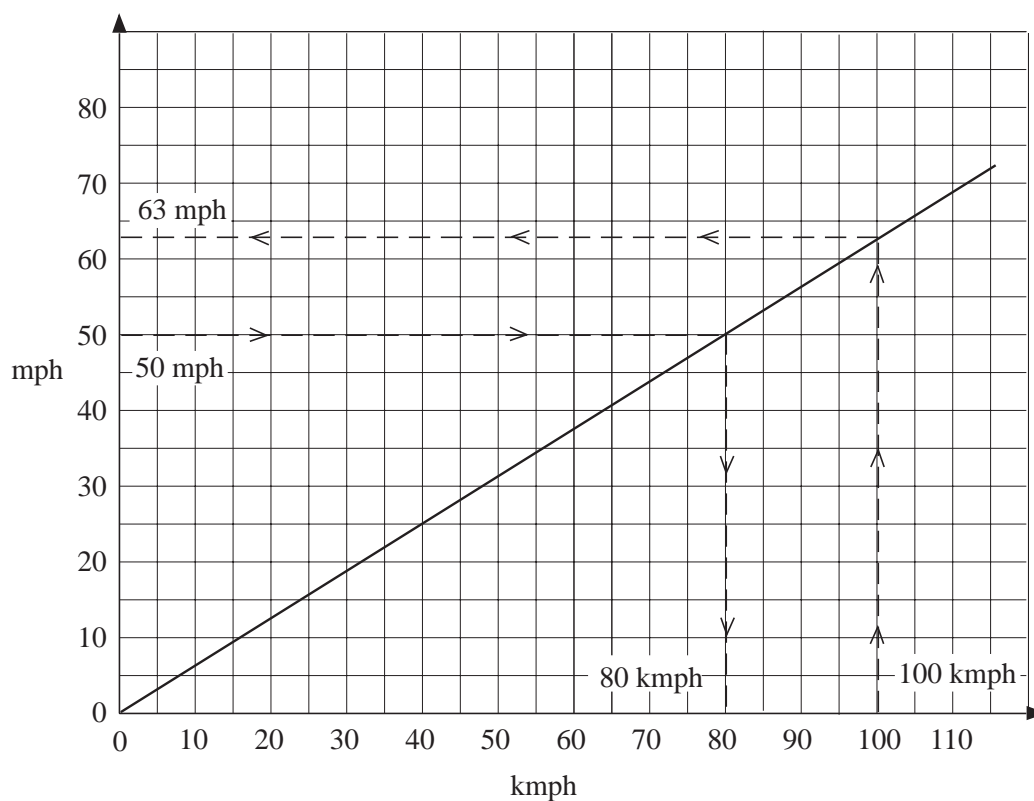


3.6 Conversion Graphs

A conversion graph can be used to change one quantity to another when the units are changed.

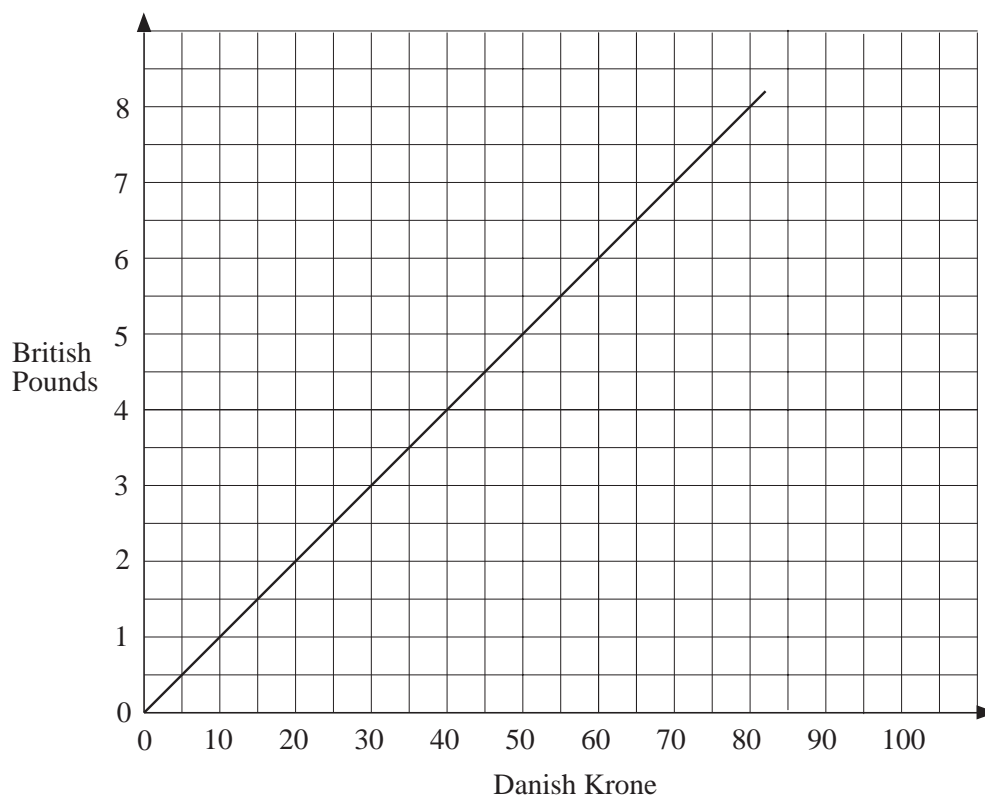
The graph below has been used to

- (a) Convert 50 mph into kmph.
- and
- (b) Convert 100 kmph into mph.



Exercises

- The graph below can be used for converting Danish Krone into British Pounds.



(a) Convert these amounts to Danish Krone.

(i) £ 3.00

(ii) £ 7.00

(iii) £ 4.50

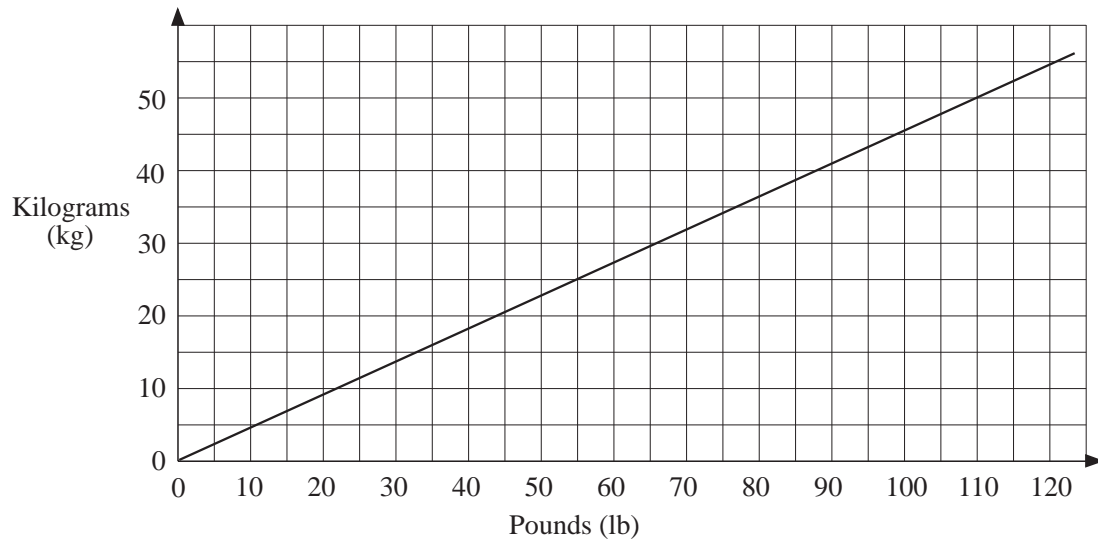
(b) Convert these amounts to British Pounds.

(i) 80 DK

(ii) 60 DK

(iii) 25 DK

2. The graph below can be used for converting weight from kilograms to pounds.



(a) Convert these weights from kilograms to pounds.

(i) 30 kg

(ii) 10 kg

(iii) 45 kg

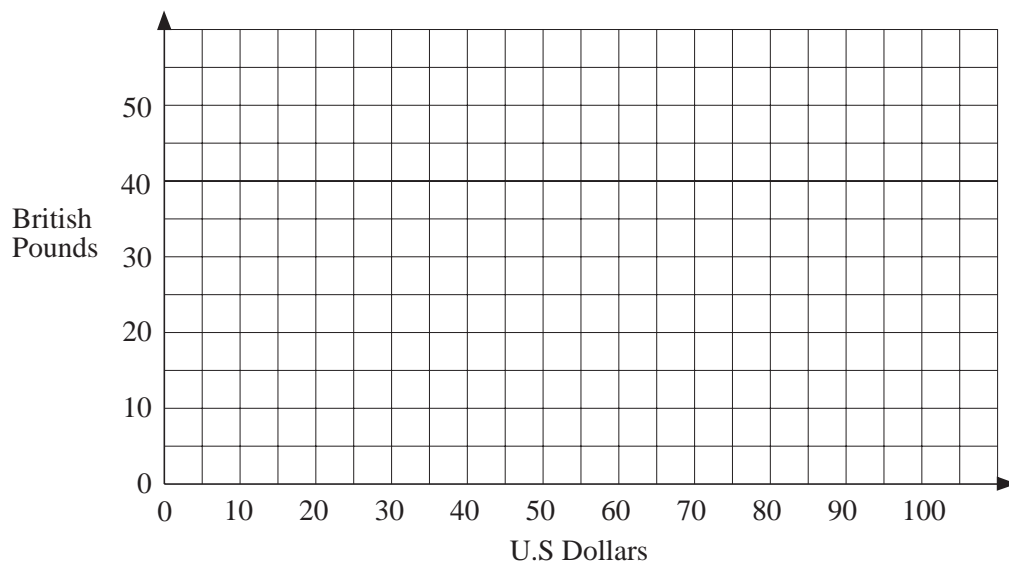
(b) Convert these weights from pounds to kilograms.

(i) 110 lb

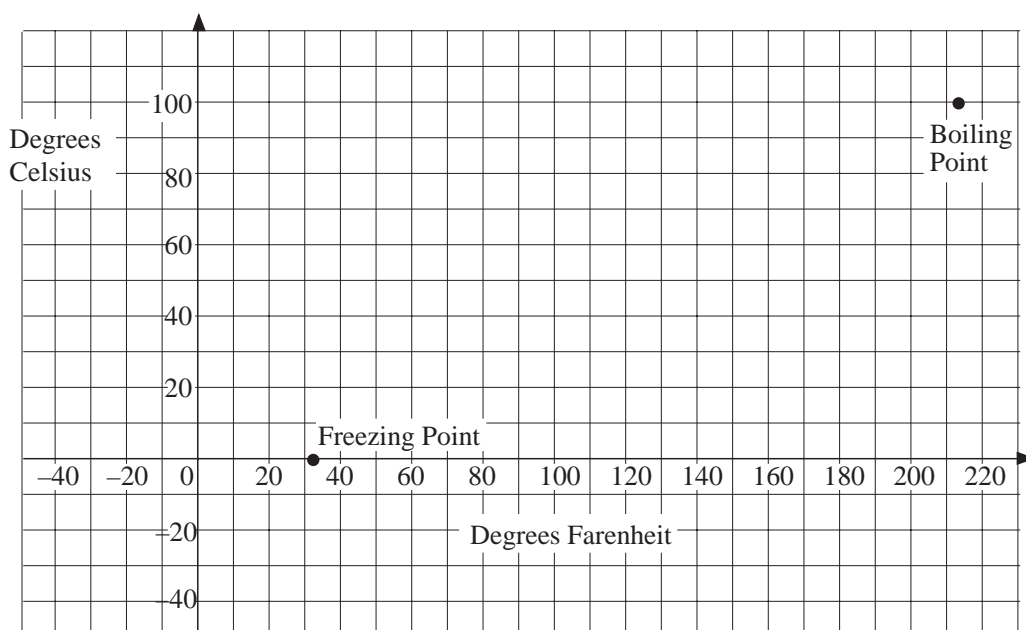
(ii) 20 lb

(iii) 85 lb

3. (a) Copy the set of axes shown below, ready to draw a graph for converting British Pounds to US Dollars.



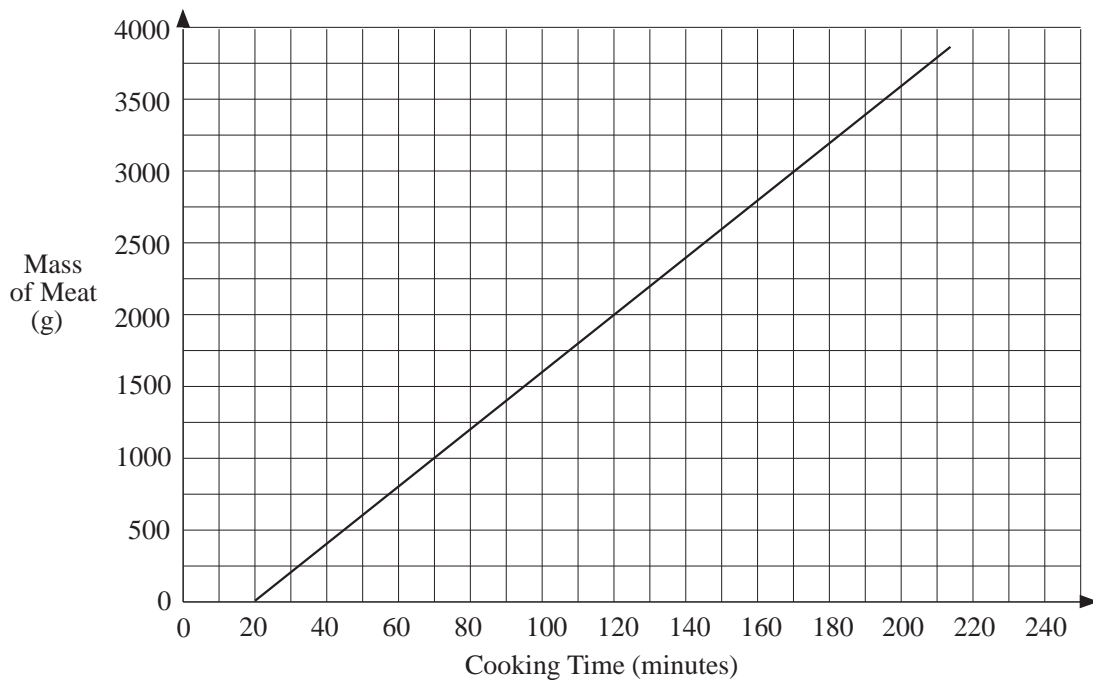
- (b) If £ 50 is equivalent to \$ 80, plot a point on the graph and draw a straight line to use for conversions.
- (c) Convert these amounts to US Dollars.
- (i) £ 20 (ii) £ 35 (iii) £ 15
- (d) Convert these amounts to British Pounds.
- (i) \$ 50 (ii) \$ 45 (iii) \$ 25
4. (a) If £ 100 is equivalent to 250 Australian Dollars, draw a conversion graph.
- (b) Use your graph to convert these amounts to British Pounds.
- (i) 200 Australian Dollars (ii) 50 Australian Dollars
- (c) Use your graph to convert these amounts to Australian Dollars.
- (i) £ 75 (ii) £ 80 (iii) £ 45
5. The axes below are to be used for a graph for converting temperatures between degrees Fahrenheit and Celsius. The two points shown on the graph represent the freezing and boiling points of water.



- (a) Copy the graph and draw a line through the two points given.
- (b) Use your graph to convert these temperatures to Celsius.
- (i) 160°F (ii) 60°F (iii) 70°F (iv) 95°F
- (c) Use your graph to convert these temperatures to Fahrenheit.
- (i) 60°C (ii) 20°C (iii) 30°C (iv) 45°C

- (d) Explain how to use your graph to convert $0^{\circ}\text{Fahrenheit}$ to Celsius.
- (e) What temperature is the same in both Fahrenheit and Celsius?

6. The graph below can be used to find the time needed to cook a piece of meat.



- (a) How long is needed to cook 2000 grams of meat?
 - (b) How much longer does it take to cook 2500 grams than 2000 grams?
 - (c) What mass of meat could be cooked in $1\frac{1}{2}$ hours?
 - (d) The mass of a piece of meat is 1500 grams. It has been cooking for 30 minutes. How long is it until the meat will be cooked?
- 7.
- (a) If 22 gallons is equivalent to 100 litres, draw a conversion graph.
 - (b) Use your graph to convert 20 gallons to litres.
 - (c) Use your graph to convert 40 litres to gallons.
 - (d) John has 12 gallons of petrol in his car. He uses 4 gallons on a journey. How many litres of petrol does he have left?
 - (e) Rachel uses 8 litres of petrol per week going to work. How many gallons of petrol would she use in 5 weeks?
 - (f) Ted's car uses 30 gallons of petrol on an 800 mile journey. How many litres would be used in a 400 mile journey?
 - (g) A can contains $\frac{1}{4}$ of a litre of a drink. How many cans would be needed to hold 2 gallons of the drink?

- (h) A modern toilet uses 8 litres of water per flush. An old toilet uses 2 gallons of water per flush.

Which toilet uses the most water?

8. If 10 metres is the same as 33 feet, draw a graph to convert between metres and feet.

Use your graph to answer the following questions.

- (a) In a long jump competition Mohammed jumps 4 m and James jumps 12 feet. Who wins?
- (b) Which is longer, 20 feet or 6.5 m?
- (c) Philip says that 8 metres is less than 28 feet. Is he right?
- (d) A rope is 9 metres long. What is the distance to the nearest foot?
- (e) A new flagpole arrives at a school. It is 1 metre taller than the old one. The old flagpole was 18 feet. How tall is the new flagpole?