UNIT 8 *Arithmetic: Division of Decimals* Activities

Activities

ACTIVITY 8.1 *Magic Sums*

Try out this calculation:

- choose any three different digits from 1 to 9
- write down all possible 2-digit numbers that can be formed from the 3 digits, without repeating any digits in each 2-digit number (you should have 6 of these)
- add all the two 2-digit numbers together; call this T
- add the original three digits together; call this S
- divide T by S: what number do you get?

1. Repeat the procedure with a different set of 3 digits. What can you 'conjecture' about the number obtained at the final stage?

3. Repeat the procedure again with another set of 3 digits.

Extension

Writing the three digits as $a, b, c,$ what are the six possible 2-digit numbers that you can form as actual numbers (*)? Write down expressions for T and S in terms of a, b and c, and simplify them where possible. Divide T by S. What can you now prove?

(* Note that ab is, in fact, the number $10a + b$, etc.)

ACTIVITY 8.2 *Divisibility Rules*

It is easy to see if a number is divisible by 2 or 5 or 10, but what about 3, 4, 7 or 9?

If a number is *even* it is divisible by 2.

If it ends in 0 it is divisible by 10.

1. What is the rule for *divisibility by 5*?

Divisibility by 3 is also easy to test.

For example, to test the number 2772, we add the digits $2+7+7+2=18$ and we know that 2772 is divisible by 3 since 18 is divisible by 3.

Why does this method work?

We can see the reason by considering the number with, for example, three digits 'abc', which can be written as

$$\begin{aligned} a\times 100 + b\times 10 + c\times 1 &= a(99+1) + b(9+1) + c \\ &= 99a + 9b + (a+b+c) \end{aligned}$$

Since $99a + 9b$ is divisible by 3, the number 'abc' is divisible by 3 if $(a+b+c)$ is also divisible by 3.

2. Design a rule for *divisibility by 9*.

Divisibility by 4 is also quite straightforward.

For example, the number 2772 can be written as $2772 = 27\times 100 + 72$.

Since 100 is divisible by 4, and 72 is also divisible by 4, then we see that 2772 is divisible by 4. So the test for divisibility by 4 is to test just the last two digits.

Divisibility by 7 is not so straightforward!

To test the number 2772, we take the first three digits minus twice the last one:

$$2772 \rightarrow 277 - 2\times 2 = 273$$

Now take the first two digits of the answer, minus twice the last digit:

$$273 \rightarrow 27 - 2\times 3 = 21$$

Since 21 is divisible by 7, so is 2772.

3. Test all the following for divisibility by 2, 3, 4, 5, 7 or 9, without using a calculator:

(a) 31 759 (b) 14 376 (c) 553 311 (d) 9130

Extension

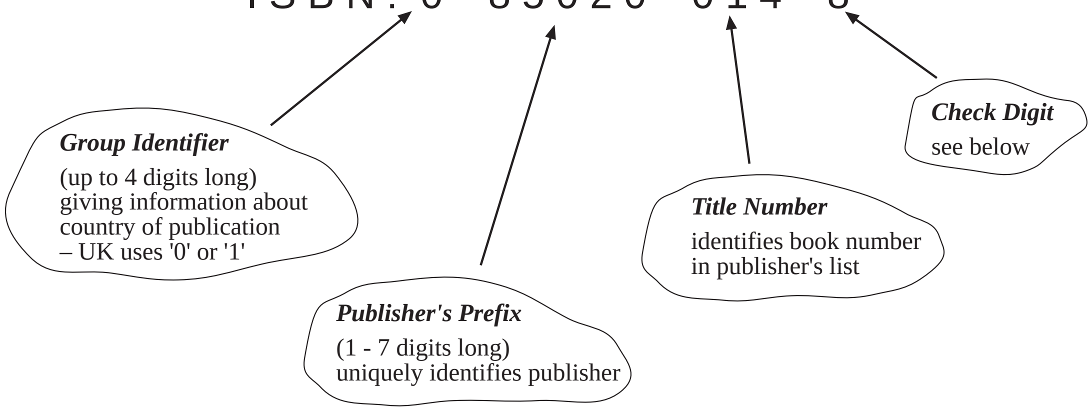
1. Design a test for *divisibility by 11*.
2. Show why the rule for divisibility by 7 works by considering the 4-digit number '$abcd$'.

ACTIVITY 8.3 *ISBN Numbers*

Published books each have an **ISBN** (International Standard Book Numbering) number, usually on their inside front cover. The ISBN system is now used in most countries throughout the world in order to uniquely identify any book.

The number is always 10 digits in length and is divided into four parts as shown in the example below.

ISBN: 0 85020 014 8



The first three parts of the number must use up the first 9 digits, and the tenth digit is the *check* digit. This is designed so that any error in the previous nine digits is spotted. The check digit is calculated in the following way:

> *Multiply the first nine digits by 10, 9, 8, . . ., 2 respectively.*
> *The check number is the smallest number that needs to be added to this total so that it is exactly divisible by 11.*

For the example above, we have

$$0 \times 10 + \left(8 \times 9 + 5 \times 8 + 0 \times 7 + 2 \times 6 + 0 \times 5\right) + \left(0 \times 4 + 1 \times 3 + 4 \times 2\right) = 135$$

so the check digit must be 8, since 143, $(135 + 8)$, is divisible by 11.

(note that if the number 10 is needed for the check digit, the symbol X is used)

1. Find the check digit for the first 6 books of the publisher who uses the prefix 869931.

(a) 1 869931 00	(b) 1 869931 01	(c) 1 869931 02
(d) 1 869931 03	(e) 1 869931 04	(f) 1 869931 05

2. The ISBN 0 7135 2272 5 has an error in it.

   What is the most likely true number?

Extension

A publisher uses the prefix 1 869931. Develop an algorithm (or a computer program) to find the check digit for the first 100 possible books with title numbers from 00 to 99.

ACTIVITIES 8.1 - 8.2 *Notes and Solutions*

Notes and solutions are given only where appropriate.

8.1 1.
2. } The number 22 should have been obtained each time.
3.

Extension

$$T = (10a + b) + (10b + a) + (10b + c) + (10c + b) + (10c + a) + (10a + c)$$

$$= 22c + 22b + 22c$$

$$= 22\,(a + b + c)$$

$$S = (a + b + c)$$

so $T \div S = 22$ (and this has to be true whatever digits are chosen at the first stage)

8.2 This activity is designed to help pupils with mental arithmetic and to gain confidence in dealing with numbers.

1. The number must end in 0 or 5.
2. A number is divisible by 9 if the sum of its digits is divisible by 9.
3.

Divisible by	2	3	4	5	7	9
31 759	×	×	×	×	✓	×
14 376	✓	✓	✓	×	×	×
553 311	×	✓	×	×	×	✓
9130	✓	×	×	✓	×	×

Extension

1. The number $'abcd'$ is divisible by 11 if $(a - b + c - d)$ is either 0 or 11.
   You can see this by looking at the number 2673, for example.
   We can write

$$2673 = 2 \times 1000 + 6 \times 100 + 7 \times 10 + 3 \times 1$$

$$= 2 \times 1001 + 6 \times 99 + 7 \times 11 + 3 \times 1 - 2 \times 1 + 6 \times 1 - 7 \times 1$$

$$= 2 \times 1001 + 6 \times 99 + 7 \times 11 - \underbrace{(2 - 6 + 7 - 3)}_{\text{equal to } 0} \times 1$$

divisible by 11

   This can easily be extended to numbers with more digits.

2. You can write the number $'abcd'$ as

$$1000a + 100b + 10c + d$$

$$= 10(100a + 10b + c) + d$$

$$= 10(100a + 10b + c - 2d) + 21d$$

   Since $21d$ is divisible by 7, the number $'abcd'$ is divisible by 7, provided $'abc' - 2d$ is divisible by 7.

ACTIVITY 8.3 *Notes and Solutions*

8.3 More information and details of the ISBN system can be obtained from:

Standard Book Numbering Agency Ltd
12 Dyatt Street
London
WC1A 1DF

1. (a) 9 (b) 7 (c) 5

   (d) 3 (e) 1 (f) X

2. The total number comes to 170.

   Possible numbers that have just one digit different from the incorrect number are:

   0 7135 1272 5 and 0 7136 2272 5

   but there are others!

Extension

m \ n	0	1	2	3	4	5	6	7	8	9
0	9	7	5	3	1	X	8	6	4	2
1	6	4	2	0	9	7	5	3	1	X
2	3	1	X	8	6	4	2	0	9	7
3	0	9	7	5	3	1	X	8	6	4
4	8	6	4	2	0	9	7	5	3	1
5	5	3	1	X	8	6	4	2	0	9
6	2	0	9	7	5	3	1	X	8	6
7	X	8	6	4	2	0	9	7	5	3
8	7	5	3	1	X	8	6	4	2	0
9	4	2	0	9	7	5	3	1	X	8