16 Inequalities

16.1 Inequalities on a Number Line

1. Represent each of the inequalities below on a number line.
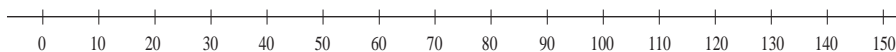
   (a) $x > 2$ (b) $x < 5$ (c) $x > -2$

   (d) $x < -1$ (e) $x \geq 3$ (f) $x \geq -1$

   (g) $x \leq 6$ (h) $x \geq -2$ (i) $1 \leq x \leq 3$

   (j) $-2 \leq x < 1$ (k) $-1 < x < 2$ (l) $-5 \leq x \leq -2$

2. Write down the inequality which describes the region shown in each diagram.

   (a)

   (b)

   (c)

   (d)

   (e)

   (f)

   (g)

   (h)

   (i)
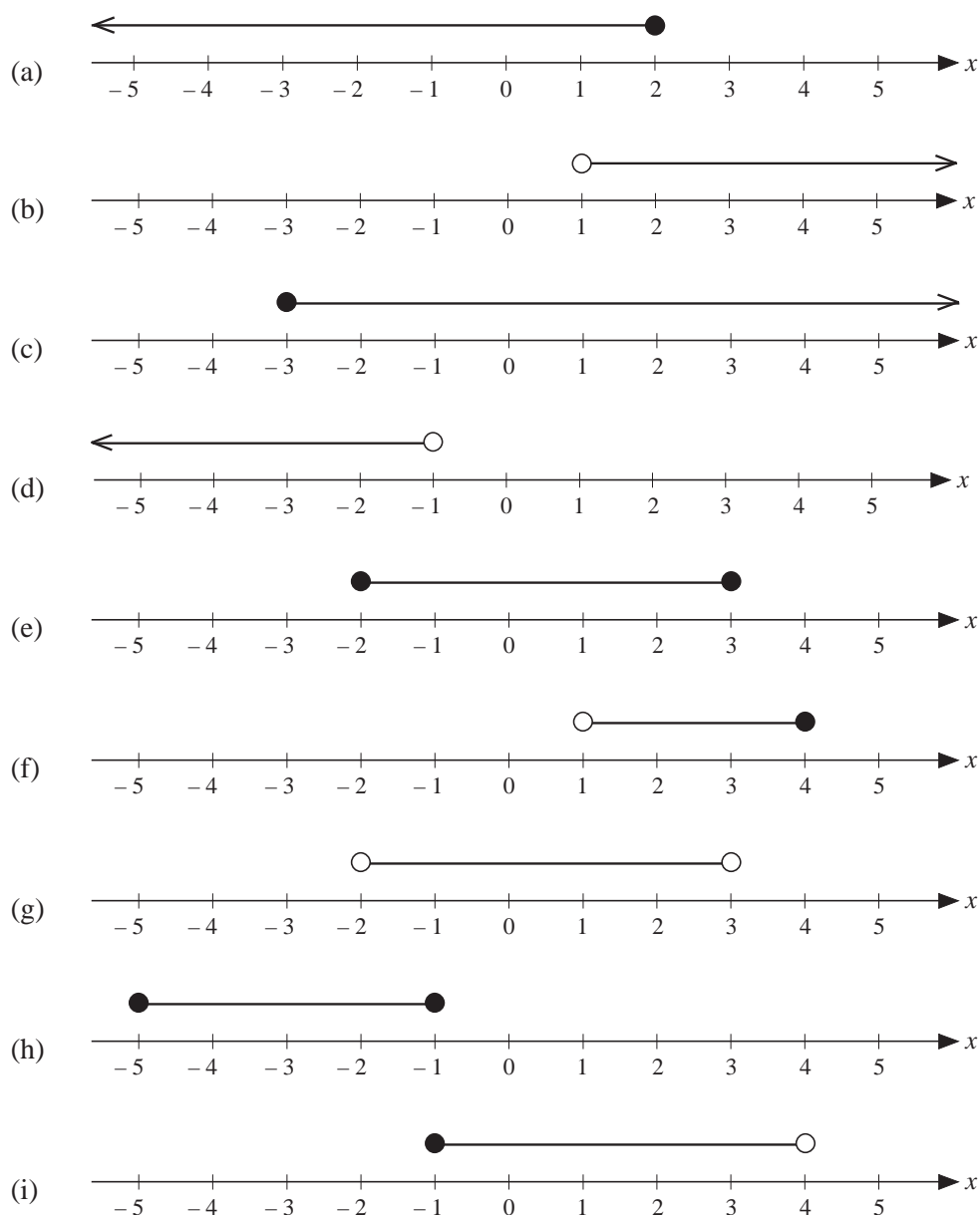
3. In Hungary, on some motorways there is a minimum speed of 40 km per hour and a maximum speed of 120 km per hour.

    (a) Copy the number line below and represent this information on it.

    (b) Write down the inequality to describe this situation.

4. List all the whole numbers which satisfy each of the inequalities below.

    (a) $2 \le x \le 5$ (b) $1 < x \le 8$ (c) $2 \le x < 9$ (d) $4 < x < 9$

5. List all the integers which satisfy each inequality below.

    (a) $-2 \le x \le 3$ (b) $-7 \le x < -1$

    (c) $-3 < x < 2$ (d) $-4 < x \le 2$

6. Write down one fraction which satisfies each inequality below.

    (a) $\dfrac{1}{4} < x < \dfrac{3}{4}$ (b) $-\dfrac{1}{2} < x < 0$

    (c) $1 < x < \dfrac{3}{2}$ (d) $-\dfrac{1}{4} < x < \dfrac{1}{4}$

7. x is a whole number such that $-3 \le x < 5$ and y is a whole number such that $-4 \le y \le 2$. What is the greatest possible value of:

    (a) $x + y$ (b) $x - y$ (c) xy?

8. x is an integer. List all the values of x such that $-1 < 2x \le 8$.

    (AQA)

16.2 Solutions of Linear Inequalities

1. Solve each inequality below and illustrate your solution on a number line.

    (a) $2x + 3 \le 5$ (b) $3x - 4 > 11$ (c) $5x + 3 > 28$

    (d) $5 - 2x \ge 11$ (e) $\dfrac{3x - 5}{2} < 2$ (f) $3(4x + 1) \ge -9$

2. Solve the following inequalities.

    (a) $3x - 4 < 26$ (b) $6 - 4x > 18$ (c) $7x - 2 \le 12$

    (d) $5x + 7 > -13$ (e) $\dfrac{1 + 2x}{5} > 3$ (f) $\dfrac{4 - 5x}{2} \le 7$

3. Solve each of the following inequalities and illustrate each solution on a number line.

    (a) $9 \le 2x - 1 \le 15$ (b) $5 \le 3x + 14 \le 29$

    (c) $13 \le 5 - 4x < 25$ (d) $-2 \le 2x + 1 \le 5$

4.   (a)   Solve the inequality $7x + 3 > 2x - 15$.

      (b)   Solve the inequality $2(3x - 2) < 11$.

5.   Find all integer values of n which satisfy the inequality

$$1 \leq 2n - 5 < 10 .$$

6.   Solve the following inequalities for x.

      (a) $1 + 3x < 7$ (b) $4x - 3 > 3x - 2$

7.   (a)   List all the integer values of n for which $-4 < n + 1 \leq 2$.

      (b)   Solve the inequality
$$3x + 5 < 1 - 2x .$$

8. x is a whole number such that $-5 \leq x < 2$.

      (a)   (i)   Write down all the possible values of x.

            (ii) y is a whole number such that $-3 < y \leq -1$. Write down the
                  greatest possible value of xy.

      (b)   Solve $5n + 6 < 23$.

9.   (a)   A sequence is generated as shown.

Term	1st	2nd	3rd	4th	5th
Sequence	3	5	7	9	11

            What is the nth term in the sequence?

      (b)   Another sequence is generated as shown.

Term	1st	2nd	3rd	4th
Sequence	4	7	12	19

            What is the nth term in the sequence?

      (c)   The nth term of a different sequence is $5n + 7$.
            Solve the inequality $5n + 7 < 82$.

10.   Solve the inequality $7y < 3y + 6$.

11. (a)   Solve the inequality $3x + 7 \geq 13$.

     (b)   A mathematics teacher says

            I am thinking of an integer. I double the integer and add 1.
            *The result is **less than** – 7.*

            What is the **largest** integer the teacher could have thought of?

                                                                                    (AQA)

16.3   Inequalities Involving Quadratic Terms

1.   Illustrate the solutions to the following inequalities on a number line.

     (a) $x^2 \leq 4$ (b) $x^2 \geq 1$ (c) $x^2 \geq 9$

     (d) $x^2 < 36$ (e) $x^2 \leq 2.25$ (f) $x^2 > 0.25$

2.   Find the solutions of the following inequalities.

     (a) $x^2 + 5 \leq 6$ (b) $2x^2 - 5 \geq 27$ (c) $5x^2 - 4 \leq 16$

     (d) $9x^2 \leq 1$ (e) $4x^2 \geq 25$ (f) $16x^2 - 12 \geq 13$

     (g) $2(x^2 - 4) < 10$ (h) $\dfrac{x^2 - 3}{2} \geq 23$ (i) $20 - 2x^2 \leq 2$

3.   Find the solutions of the following inequalities.

     (a) $(x - 1)(x - 2) \geq 0$ (b) $(x + 2)(x - 3) \leq 0$

     (c) $(x - 1)(x - 2) < 0$ (d) $(x + 5)(x - 4) > 0$

     (e) $x(x + 5) \geq 0$ (f) $(x - 1)x < 0$

4.   By factorising, solve each of the following inequalities.

     (a) $x^2 + x - 2 \geq 0$ (b) $x^2 - 5x + 6 \leq 0$

     (c) $x^2 - 4x < 0$ (d) $2x^2 + 3x - 2 > 0$

     (e) $x^2 + 6x + 8 \leq 0$ (f) $5x^2 - 15x \geq 0$

     (g) $6x - 2x^2 > 0$ (h) $1 - 5x - 6x^2 \leq 0$

5.   The area, A, in cm^2, of a square satisfies the inequality $9 \leq A \leq 36$. What is the:

     (a)   maximum   (b)   minimum

     possible length of its sides?

6.   (a)   Factorise completely $14n - 4n^2$.

     (b)   Find the integer values of n for which $14n - 4n^2 > 0$.

                                                                                    (MEG)

7.   Solve the inequality $x^2 < 25$.   *(OCR)*
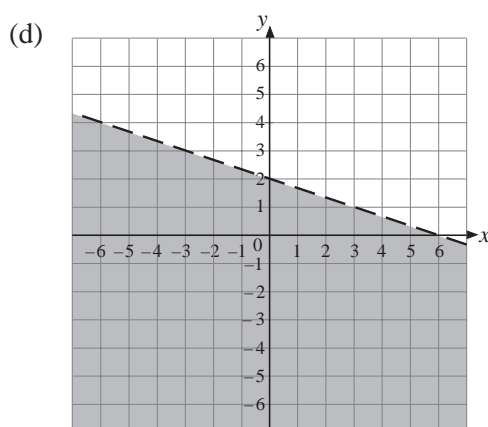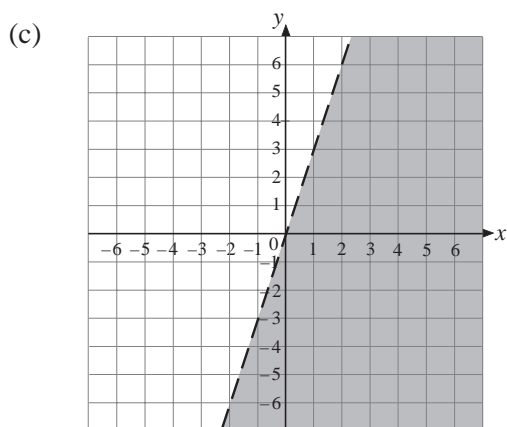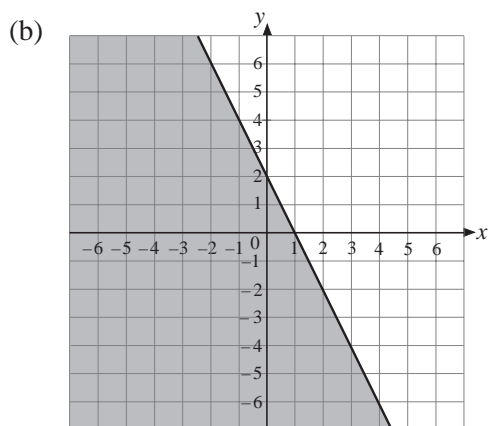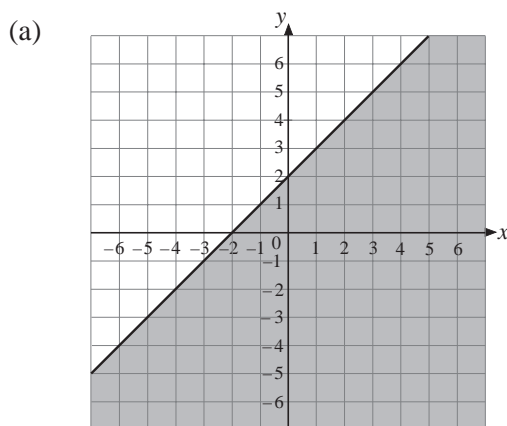
16.4 Graphical Approach to Inequalities

1. Illustrate on a set of coordinate axes each of the following inequalities.

   (a) $y \le x$ (b) $y > x + 1$ (c) $y < x - 2$

   (d) $y \le x + 4$ (e) $y > 3 - 2x$ (f) $y \le 3x - 3$

   (g) $2x + y \ge 4$ (h) $x - y \ge 2$ (i) $x + 2y < 3$

2. For each region below, find:

   (i)   the equation of the line which forms the boundary

   (ii)   the inequality represented by the shaded region.

   (a)

   (b)

   (c)

   (d)

3. On the same set of axes, shade the regions $x + y \ge 1$ and $x - y \le 2$.
   Indicate the region satisfied by both inequalities.

4. Shade the region which satisfies $2 \le x + y \le 4$.

5. Shade the region which satisfies $-1 \le 2x + y < 2$.