# ACTIVITIES 16.1 – 16.2 *Notes and Solutions*

*Notes and solutions given only where appropriate.*

**16.1** 1. (a) 2 (b) 4

2. $2 < \pi < 4$

3. (a) $36°$ (b) $\cos 36° \approx 0.8090$, $2\sin 36 \approx 1.1756°$ (c) 2.3777

(d) 3.6327 (e) $2.3777 < \pi < 3.6327$

*Extension* $\alpha = \dfrac{360}{n}$, $n\sin\left(\dfrac{180}{n}\right)\cos\left(\dfrac{180}{n}\right)$, $n\tan\left(\dfrac{180}{n}\right)$,

$$n\sin\left(\frac{180}{n}\right)\cos\left(\frac{180}{n}\right) < \pi < n\tan\left(\frac{180}{n}\right)$$

(a) $3.139350203 < \pi < 3.1427146$

(b) $3.141571983 < \pi < 3.141602989$

(c) $3.141592654 < \pi < 3.141592654$ (correct to at least 9 decimal places)

**16.2** 1. (a) $\dfrac{\pi}{4} \approx 0.785$ (b) $\dfrac{2\pi}{9} \approx 0.698$ (c) $\dfrac{5\pi}{38} \approx 0.413$ (d) $\dfrac{\pi}{3\sqrt{3}} \approx 0.605$

(e) $\dfrac{\pi}{6} \approx 0.524$ (f) $\dfrac{2\pi^2}{(\pi+2)^2} \approx 0.747$ (g) $\dfrac{2\pi}{15} \approx 0.419$

(h) $\dfrac{\pi \tan 54°}{5} \approx 0.865$

2. One possible conjecture is that $\text{IQ} \to 1$ as the shape becomes closer to a circle.

*Extension* For an $n$-sided polygon, $\text{IQ} = \dfrac{\pi}{n\tan\left(\dfrac{\pi}{n}\right)}$.

(For large $n$, $\dfrac{\pi}{n}$ is small and $\tan\left(\dfrac{\pi}{n}\right)$ can be approximated by $\dfrac{\pi}{n}$,
giving the result that $\text{IQ} \to 1$ as $n \to \infty$.)

# ACTIVITY 16.3 *Notes and Solutions*

**16.3** 1. (a) $A = 3, \quad G \approx 2.60517, \quad H = 2.18978$ (b) $A = 3, \quad G = 3, \quad H = 3$

3. $(a-b)^2 \geq 0 \quad \Rightarrow \quad a^2 - 2ab + b \geq 0^2$

$$\Rightarrow \quad a^2 + b^2 \geq 2ab$$
$$\Rightarrow \quad a^2 + 2ab + b^2 \geq 4ab$$
$$\Rightarrow \quad (a+b)^2 \geq 4ab$$
$$\Rightarrow \quad a + b \geq 2\sqrt{ab}$$
$$\Rightarrow \quad \frac{a+b}{2} \geq \sqrt{ab}$$
$$\Rightarrow \quad A \geq G$$

*Extension* Note that $H = \dfrac{2}{\dfrac{1}{a} + \dfrac{1}{b}} = \dfrac{2ab}{a+b}$, so to show that $G \geq H$,

we need to show that $\sqrt{ab} \geq \dfrac{2ab}{a+b}$

or $a + b \geq 2\sqrt{ab}$ (mutiplying both sides by $\dfrac{a+b}{\sqrt{ab}}$ )

or $\dfrac{a+b}{2} \geq \sqrt{ab}$,

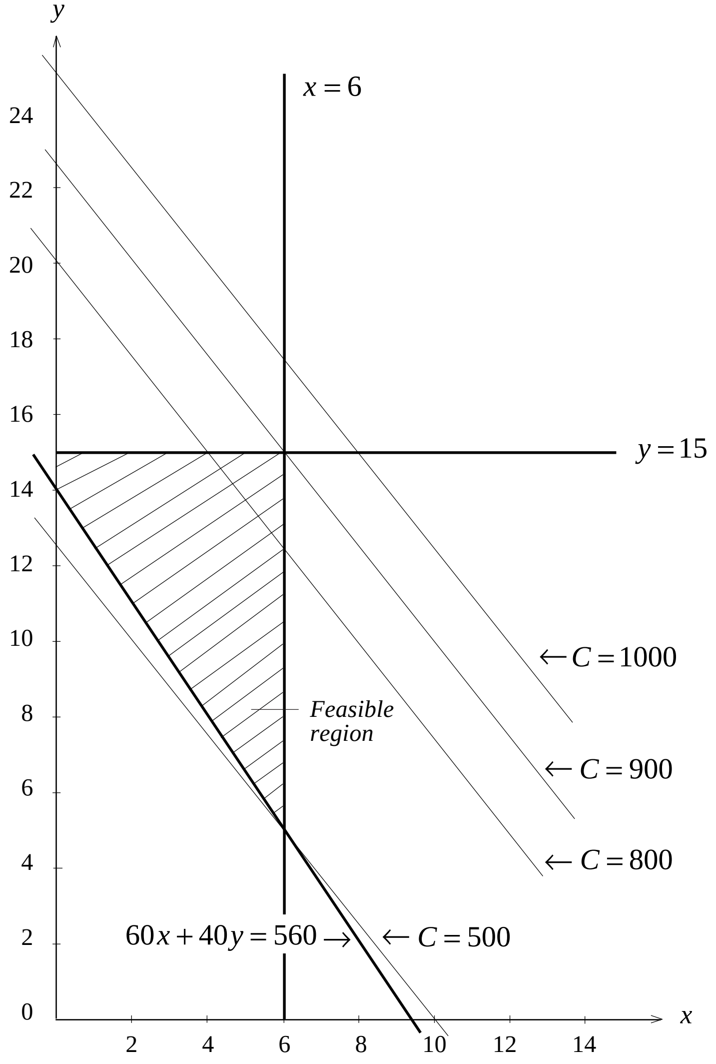
which we know is true.

# ACTIVITY 16.4 *Notes and Solutions*

**16.4** 1. $0 \le y \le 15$

2. (a) $60x + 40y$ (b) $60x + 40y \ge 560$

3.




4. The lines $C =$ constant will attain minimum value at the point $x = 6$, $y = 5$.

This gives $C = 500$ and tells us that we should have

6 double deckers,
5 single deckers.