Algebra: Simplifying Algebraic Expressions, Expanding Brackets, Solving Linear Equations, Applications. KS3, KS4. Non-Calculator.

A. Simplify the following expressions:

1. $5a + 3a$
2. $6a - 4a$
3. $4a + a$
4. $x + x + x + x$
5. $a - a$
6. $3a + 2a - 5a$
7. $3a + 5c - a + 2c$
8. $3x + 2x + 3y - y$
9. $3x - x + 3 - 2$
10. $3x + y - x + 4y$
11. $3x + 2y - 3x + 4y$
12. $2x + 5y - 3y + x$
13. $p + q - p - q$
14. $p + q + p + q + p$
15. $4p - 5p$
16. $5c + 2d - 3c - 4d$
17. $5x - 3y + 2x - 4y$
18. $5p - 3q + 2 - 4p + 5 + 4q$
19. $2ac + 3ac - 4ac$
20. $xy + yx$
21. $2xy - 4ac + 5yx + 4ac$
22. $3xy + 4xy - xy$
23. $3cd - 4cd + cd$
24. $xy + yx - 2xy + 1$
25. $2ab + 3cd - 4ab - 3cd$
26. $4y^2 - 3y^2$
27. $4\,x^3 - x^3$
28. $3x^2 + y^3 - x^2 - y^3$
29. $4y^2 + 5y - 3y^2 - 4y$
30. $2x^2 + 3x - 5x^2 - x + 8$
31. $x^2 + x^2 + x^2$
32. $x^2 + x^2 + 3x^2$
33. $x^2 + x^2 - x^2$
34. $x^2y + xy + x^2y$
35. $x^2y + xy^2 - x^2y + 2y\,^2x$

B. Expand the brackets and simplify where possible.

1. $4(x-3)$
2. $4(2x-3)$
3. $2(3-4y)$
4. $x(x+1)$
5. $x(x-2)$
6. $x(x^2+4x-3)$
7. $y(x-y^2)$
8. $4(p+2)+3(2p-3)$
9. $2(3p+2)+3(2p-3)$
10. $3(2p-5)+2(3p-3)$
11. $2p(p+2)+3p(2p-3)$
12. $3p(p-2)+2p(3p-2)$
13. $2p(p-3)+3p(3p-2)$
14. $x(x^2-2y)+3x^2(x+2y)$
15. $-(x-3)$
16. $-4(2x-3)$
17. $-2(3-4y)$
18. $-x(x+1)$
19. $-x(x-2)$
20. $-x(x^2+4x-3)$
21. $-y(x-y^2)$
22. $7(p+2)-3(2p-3)$
23. $2(3p+2)-3(2p-3)$
24. $3(2p-5)-2(3p-3)$
25. $2p(p+2)-3p(2p-3)$
26. $3p(p-2)-2p(3p-2)$
27. $2p(p-3)-3p(3p-2)$
28. $3(x-2y)-2(x-3y)$
29. $2(3x+1)-5(2x-3)$
30. $x(x^2-2y)-3x^2(x+2y)$
31. $2(3x+1)-(2x-3)$
32. $2(p-4)+3(2p-1)$
33. $a(a+2b-3c)+3c(a-2b+3c)-2b(a-b-3c)$
34. $a(b-c+d)-a(b-c+d)$
35. $3a(2b-3c+4d)-2a(3b-c+6d)$
36. $5-2(x-3)$
37. $6+4(3-x)$
38. $6+(2x+6)$
39. $6-(2x+6)$
40. $2x^2(4xy-5)-8yx^3+9x^2$

C. Solve the following equations:

1. $x + 3 = 9$
2. $2x = 6$
3. $4 - x = 5$
4. $2x + 3 = 13$
5. $2x = 1$
6. $3x = 2$
7. $4x = 20$
8. $4x - 1 = 19$
9. $4x = -20$
10. $2x = -6$
11. $4x = -8$
12. $4x = -1$
13. $2x + 3 = -5$
14. $2x - 3 = 5$
15. $2x - 3 = x + 2$
16. $7x - 3 = 2x + 12$
17. $7y - 8 = 5y + 2$
18. $4x + 5 = 2x - 11$
19. $5x - 6 = 2x - 15$
20. $x + 2x = -15$
21. $3x - 5 = 4x - 7$
22. $2x + 7 = 5x - 3$
23. $2x + 7 = 12 - 3x$
24. $6y - 2 = 8y - 5$
25. $8 - 4x = 10 - 2x$
26. $12 = 3x - 6$
27. $3(x - 5) = 12$
28. $5(2x - 3) = 15$
29. $5(3 - 2x) = 30$
30. $3(2x - 4) = 8$
31. $7x + 2 = 5(x - 2)$
32. $22 - 3x = 2(x + 6)$
33. $13 - 3x = 4(x - 2)$
34. $x - 18 = 2(2x - 3)$
35. $4(2x - 3) = 3x - 27$
36. $3(2x - 5) = 6 + 2(x - 3)$
37. $4 - (3x - 5) = 6 - (2x + 7)$
38. $x(x + 5) = x^2 - 15$
39. $3x(2 + x) = x(3x - 2) - 24$
40. $3(x - 4) - 2(x - 5) = 6x - 2(x - 5)$

APPLICATIONS:

1. The width of a rectangle is x centimeters and its length is $(x + 5)$ cm.

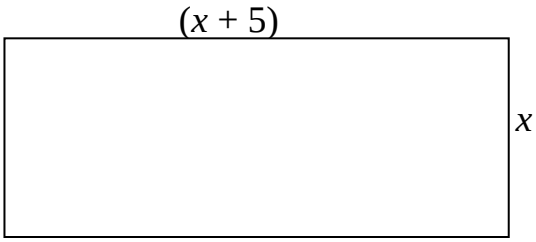


Diagram not drawn to scale

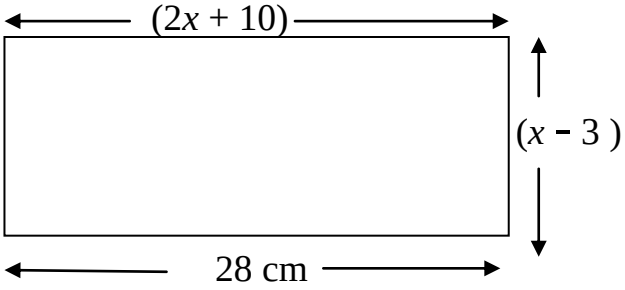
(a) Write down an expression for the perimeter of the rectangle, giving your answer in its simplest form.

The perimeter of the rectangle is 62 cm.

(b) Work out the length of the rectangle.

2. The diagram below is a rectangle. All measurements are in centimeters.

Diagram not drawn to scale



(a) Work out the value of x.

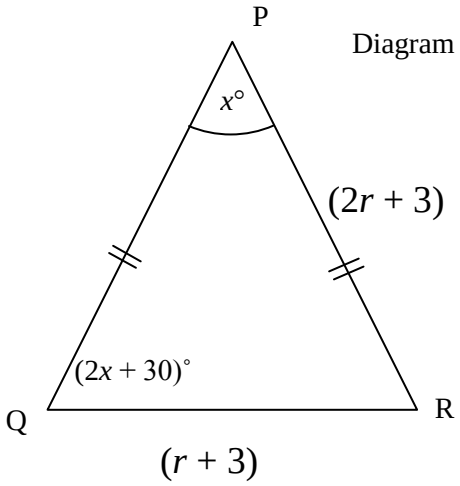
(b) Hence, work out the perimeter and area of the rectangle.

3. PQR is an isosceles triangle with PQ = PR, and angle QPR = x°.

Angle PQR = $(2x + 30)^\circ$

PR = $(2r + 3)$ cm

QR = $(r + 3)$ cm

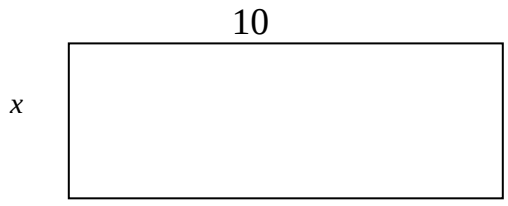


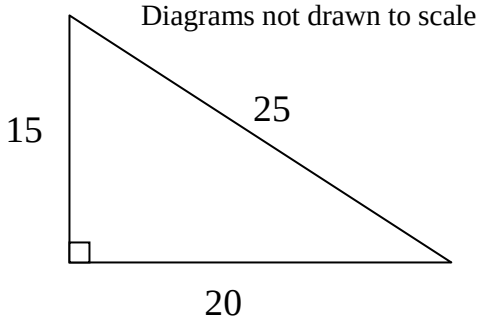
(a) Find an expression for the perimeter of the triangle in terms of r, giving your answer in its simplest form.

(b) Work out the value of r, if the perimeter is 49 cm.

(c) Work out the value of x.

4. The area of the right-angled triangle is equal to the area of the rectangle. Work out the value of x. All measurements are in centimeters.





ANSWERS/SOLUTIONS

A. ① $8a$

② $2a$

③ $5a$

④ $4x$

⑤ 0

⑥ 0

⑦ $2a+7c$

⑧ $5x+2y$

⑨ $2x+1$

⑩ $2x+5y$

⑪ $6y$

⑫ $3x+2y$

⑬ 0

⑭ $3p+2q$

⑮ $-p$

⑯ $2c-2d$

⑰ $7x-7y$

⑱ $p+q+7$

⑲ ac

⑳ $2xy$

㉑ $7xy$

㉒ $6xy$

㉓ 0

㉔ 1

㉕ $-2ab$

㉖ y^2

㉗ $3x^3$

㉘ $2x^2$

㉙ y^2+y

㉚ $-3x^2+2x+8$

㉛ $3x^2$

㉜ $5x^2$

㉝ x^2

㉞ $2x^2y+xy$

㉟ $3xy^2$

B.

① $4x-12$ ② $8x-12$ ③ $6-8y$ ④ x^2+x

⑤ x^2-2x ⑥ x^3+4x^2-3x ⑦ $xy-y^3$

⑧ $4p+8+6p-9$
$= 10p-1$

⑨ $6p+4+6p-9$
$= 12p-5$

⑩ $6p-15+6p-6$
$= 12p-21$

⑪ $2p^2+4p+6p^2-9p$
$= 8p^2-5p$

⑫ $3p^2-6p+6p^2-4p$
$= 9p^2-10p$

⑬ $2p^2-6p+9p^2-6p$
$= 11p^2-12p$

⑭ $x^3-2xy+3x^3+6x^2y$
$= 4x^3-2xy+6x^2y$

⑮ $-x+3x$

⑯ $-8x+12$ ⑰ $-6+8y$ ⑱ $-x^2-x$

⑲ $-x^2+2x$ ⑳ $-x^3-4x^2+3x$

㉑ $-xy+y^3$ ㉒ $7p+14-6p+9$
$= p+23$

㉓ $6p+4-6p+9$
$= 13$

㉔ $6p-15-6p+6$
$= -9$

㉕ $2p^2+4p-6p^2+9p$
$= -4p^2+13p$

㉖ $3p^2-6p-6p^2+4p$
$= -3p^2-2p$

㉗ $2p^2-6p-9p^2+6p$
$= -7p^2$

㉘ $3x-6y-2x+6y$
$= 2x$

(29) $6x+2-10x+15$
$= -4x+17$

(30) $x^3-2xy-3x^3-6x^2y$
$= -2x^3-2xy-6x^2y$

(31) $6x+2-2x+3$
$= 4x+5$

(32) $2p-8+6p-3$
$= 8p-11$

(33) $a^2+2ab-3ac+3ca-6cb+9c^2-2ba+2b^2+6bc$
$= a^2+9c^2+2b^2$

(34) $ab-ac+ad-ab+ac-ad$ OR let $b-c+d=x$
$= 0$ $ax-ax=0$

(35) $6ab-9ac+12ad-6ab+2ac-12ad$
$= -7ac$

(36) $5-2x+6$
$= 11-2x$

(37) $6+12-4x$
$= 18-4x$

(38) $6+2x+6$
$= 12+2x$

(39) $6-2x-6$
$= -2x$

(40) $8x^3y-10x^2-8yx^3+9x^2$
$= -x^2$

C.

(1) $x = 9 - 3 = 6$

(2) $x = \frac{6}{2} = 3$

(3) $4 - 5 = x$

$-1 = x$

(4) $2x = 13 - 3$

$2x = 10$

$x = 5$

(5) $x = \frac{1}{2}$

(6) $x = \frac{2}{3}$

(7) $x = 5$

(8) $4x = 20$

$x = 5$

(9) $x = -5$

(10) $x = -3$

(11) $x = -2$

(12) $x = -\frac{1}{4}$

(13) $2x = -8$

$x = -4$

(14) $2x = 8$

$x = 4$

(15) $2x - x = 2 + 3$

$x = 5$

(16) $7x - 2x = 12 + 3$

$5x = 15$

$x = 3$

(17) $7y - 5y = 2 + 8$

$2y = 10$

$y = 5$

(18) $4x - 2x = -11 - 5$

$2x = -16$

$x = -8$

(19) $5x - 2x = -15 + 6$

$3x = -9$

$x = -3$

(20) $3x = -15$

$x = -5$

(21) $-5 + 7 = 4x - 3x$

$2 = x$

(22) $7 + 3 = 5x - 2x$

$10 = 3x$

$3\frac{1}{3} = \frac{10}{3} = x$ both acceptable

(23) $2x+3x = 12-7$

$5x = 5$

$x = 1$

(24) $-2+5 = 8y-6y$

$3 = 2y$

$1.5 = \frac{3}{2} = y$

(25) $8-10 = -2x+4x$

$-2 = 2x$

$-1 = x$

(26) $12+6 = 3x$

$18 = 3x$

$6 = x$

(27) $3x-15 = 12$ OR $(x-5) = \frac{12}{3} = 4$

$3x = 27$ $\quad x = 4+5 = 9$

$x = 9$

(28) $10x-15 = 15$ OR $2x-3 = \frac{15}{5} = 3$

$10x = 30$ $\quad 2x = 6$

$x = 3$ $\quad x = 3$

(29) $15-10x = 30$ OR $3-2x = \frac{30}{5} = 6$

$15-30 = 10x$ $\quad 3-6 = 2x$

$-15 = 10x$ $\quad -3 = 2x$

$-1.5 = -\frac{15}{10} = x$ $\quad -\frac{3}{2} = x$

(30) $6x-12 = 8$

$6x = 20$

$x = \frac{20}{6} = \frac{10}{3} = 3\frac{1}{3}$

(31) $7x+2 = 5x-10$

$7x-5x = -10-2$

$2x = -12$

$x = -6$

(32) $22-3x = 2x+12$

$22-12 = 2x+3x$

$10 = 5x$

$2 = x$

(33) $13-3x = 4x-8$

$13+8 = 4x+3x$

$21 = 7x$

$3 = x$

(34) $x-18 = 2(2x-3)$

$x-18 = 4x-6$

$-18+6 = 4x-x$

$-12 = 3x$

$-4 = x$

(35) $8x-12 = 3x-27$

$8x-3x = -27+12$

$5x = -15$

$x = -3$

(36) $6x - 15 = 6 + 2x - 6$

$6x - 2x = 15$

$4x = 15$

$x = \frac{15}{4}$ OR $3\frac{3}{4}$

(37) $4 - 3x + 5 = 6 - 2x - 7$

$4 + 5 - 6 + 7 = -2x + 3x$

$10 = x$

(38) $x^2 + 5x = x^2 - 15$

$x^2 + 5x - x^2 = -15$

$5x = -15$

$x = -3$

(39) $6x + 3x^2 = 3x^2 - 2x - 24$

$6x + 3x^2 - 3x^2 + 2x = -24$

$8x = -24$

$x = -3$

(40) $3x - 12 - 2x + 10 = 6x - 2x + 10$

$x - 2 = 4x + 10$

$-2 - 10 = 4x - x$

$-12 = 3x$

$-4 = x$

Applications

(1) (a) $x + x + 5 + x + x + 5$ OR $2(x + x + 5)$

$= 4x + 10$ $= 2(2x + 5)$

$= 4x + 10$

(b) $4x + 10 = 62$

$4x = 52$

$x = 13$

OR half the perimeter $= x + x + 5 = 31$

$2x = 26$

$x = 13$

(2) (a) $2x+10 = 28$ opposite sides equal.

$2x = 18$

$x = 9$

(b) Hence the width $= x-3 = 9-3 = 6$

$L = 18,\ W = 6 \quad P = 2(L+W)$

Perimeter $= 2(18+6) = 2(24) = 48\text{ cm}$

Area $= LW = 18 \times 6 = 108\text{ cm}^2$

$18 \times 6 = 108$

(3) (a) Perimeter $= 2r+3+2r+3+r+3$

$= 5r+9$ cm

(b) $5r+9 = 49$

$5r = 40$

$r = 8$

(c) angle $P\hat{R}Q = 2x+30$ isosceles triangle.

Sum of all 3 angles $= 180$

$2x+30+2x+30+x = 180$

$5x+60 = 180$

$5x = 120$

$x = \frac{120}{5} = \frac{240}{10} = 24$ (×2)

(4) Area of triangle $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 20 \times 15 = 150$

area of rectangle $= 10x$

$10x = 150$

$x = \frac{150}{10} = 15$

I hope you find this useful. If you find any errors, please let me know.