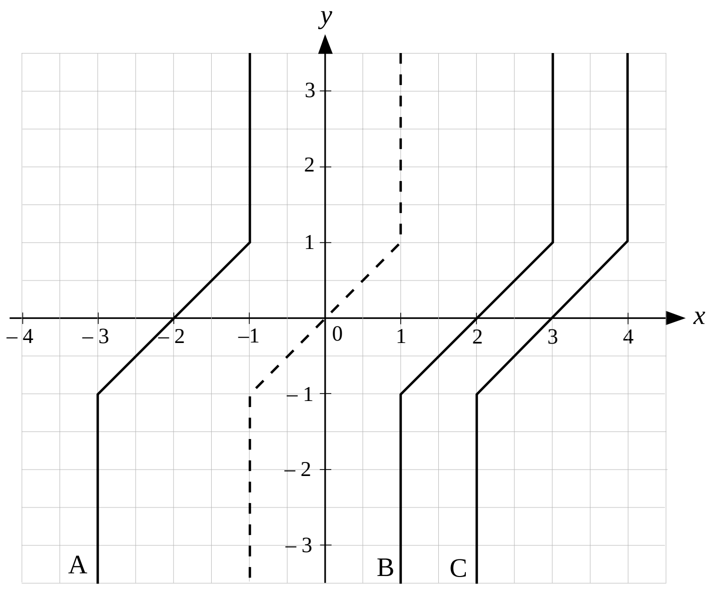
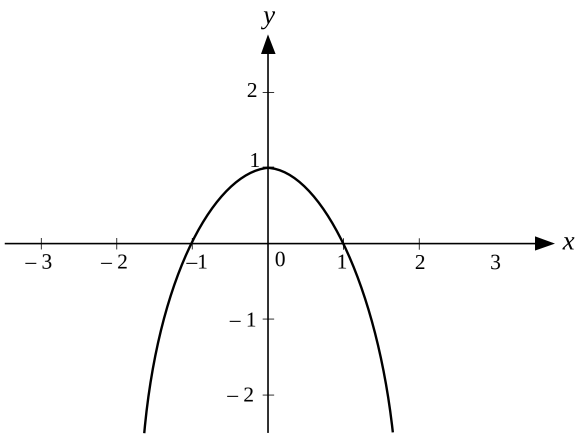
17 Using Graphs

17.1 Transformations of Graphs

1. The graph below illustrates the function $y = f(x)$ by a dashed line. Write down the equation of each of the other functions.



2. The graph below shows $y = g(x)$.



On separate diagrams show:

(a) $y = g(x)$, $y = g(x) + 1$ and $y = g(x) - 1$

(b) $y = g(x)$ and $y = 2g(x)$

(c) $y = g(x)$ and $y = g(2x)$

(d) $y = g(x)$ and $y = g(x - 1)$

(e) $y = g(x)$ and $y = g(x + 1) + 1$

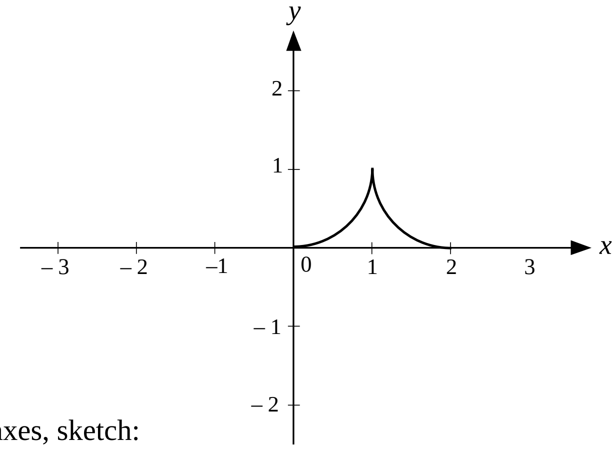
3. On the same set of axes, sketch

$$y = x, \quad y = x + 1, \quad y = x - 1 \quad \text{and} \quad y = x - 2.$$

4. Draw the graphs of:

$$y = x^3, \quad y = (x + 1)^3 \quad \text{and} \quad y = (x - 2)^3.$$

5. The graph below shows $y = f(x)$.



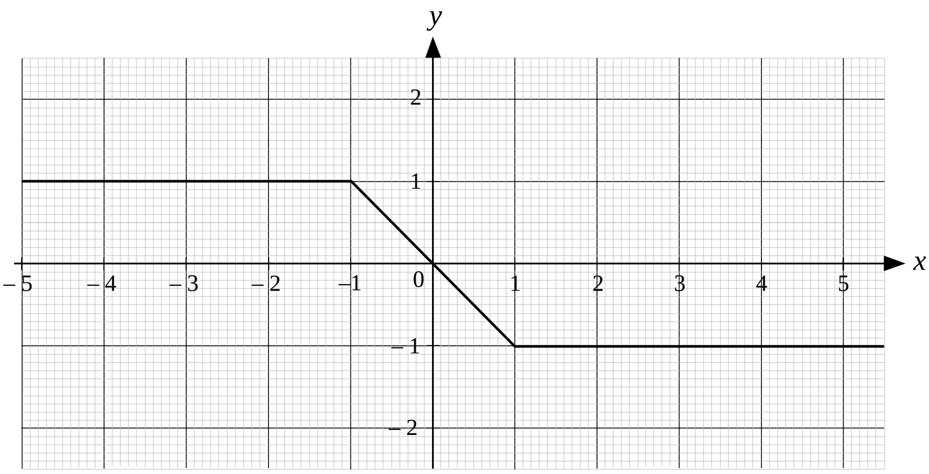
On separate axes, sketch:

(a) $y = f(x)$ and $y = -f(x)$

(b) $y = f(x)$ and $y = f(2x)$

(c) $y = f(x)$ and $y = 2f\left(\frac{1}{2}x\right)$

6. The function $y = f(x)$ is sketched below.



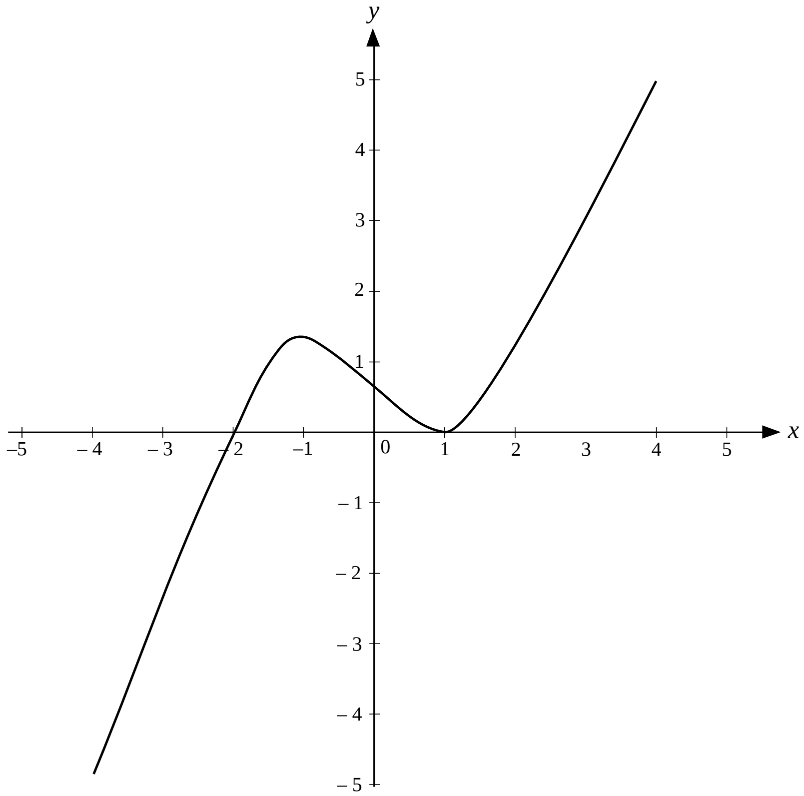
On similar diagrams, sketch

(a) $y = f(x - 1)$

(b) $y = 2f\left(\frac{x}{2}\right)$

(SEG)

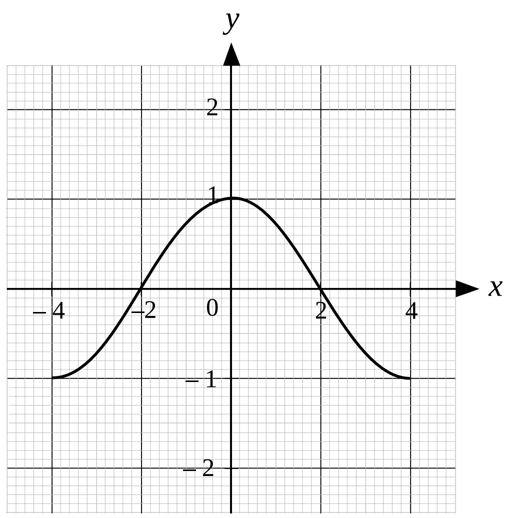
7. The graph of $y = f(x)$ has been drawn on the grid.



On a copy of the diagram above, sketch the graph of $y = f(x - 2)$

(LON)

8. On the grid below, the graph of $y = f(x)$ for $-4 \le x \le 4$ is drawn.

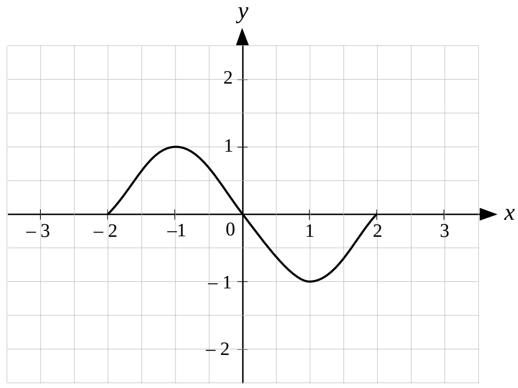


(a) On a copy of the grid, sketch the graph of $y = 2f(x)$.

(b) On another copy of the grid, sketch the graph of $y = f(x - 1)$.

(MEG)

9. The function $y = f(x)$ is illustrated below.

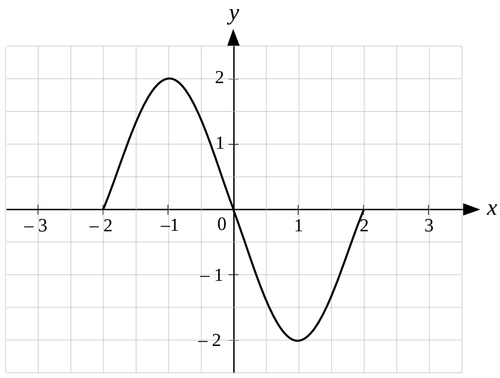


(a) On a copy of the axes, sketch:

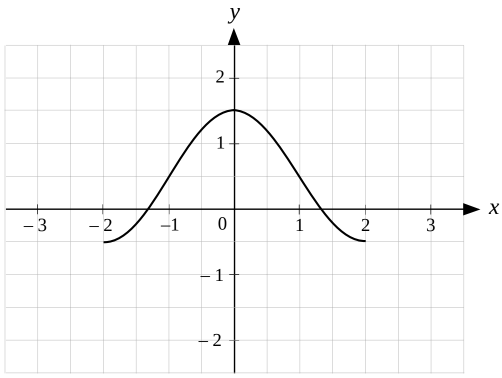
(i) $y = 2f(x)$ (ii) $y = f(x-1)$

(b) Which one of these sketches is of the form $y = f(x) + a$, where a is a constant? What is the value of a?

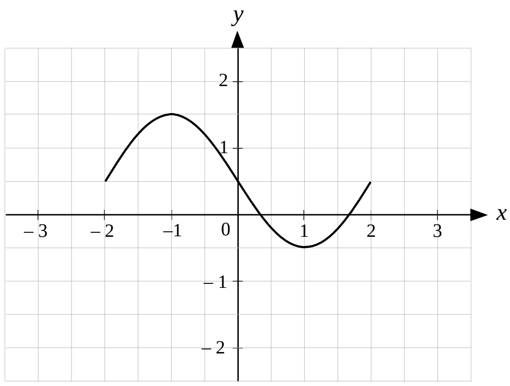
A



B



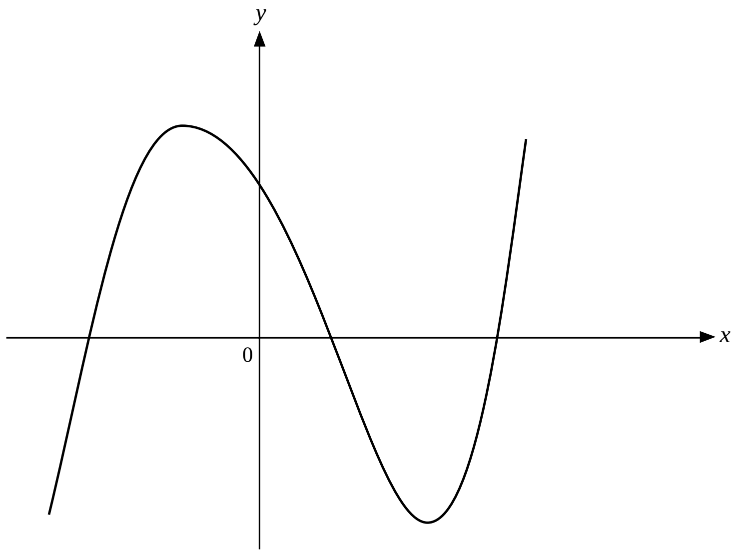
C



(SEG)

10. This is a sketch of the graph of $y = f(x)$, where

$$f(x) = (x+3)(x-2)(x-4).$$



(a) Calculate the value of $f(0)$.

(b) On a copy of the axes above, sketch the graph of $y = f(-x)$.

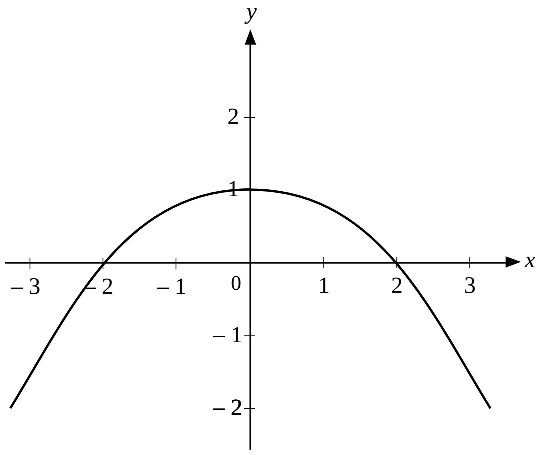
(c) Describe fully the single geometric transformation which maps the graph of $y = f(x)$ onto the graph of $y = f(-x)$.

The equation $f(x) = f(-x)$ has a solution $x = 0$. It also has a positive solution, x, such that $n < x < n+1$, where n is a positive integer.

(d) Write down the value of n.

(LON)

11. The function $y = f(x)$ is illustrated.



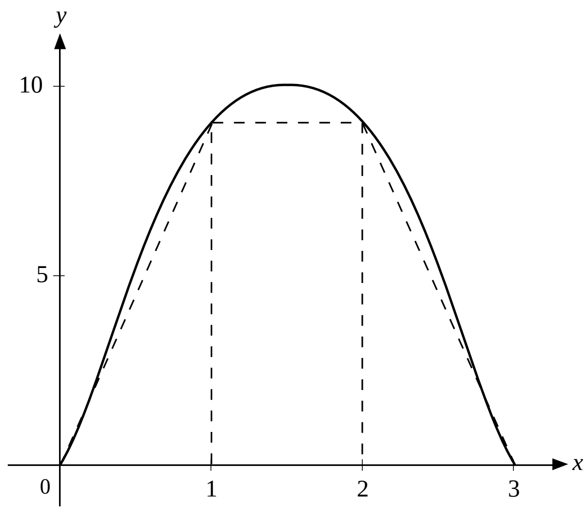
Sketch the graphs of the functions below. Label each graph clearly.

(a) $y = f\left(\frac{x}{2}\right)$ (b) $y = f(x-1)$

(SEG)

17.2 Area under Graphs

1. The graph of $y = 4x(3 - x)$ between $0 \le x \le 3$ is shown below.



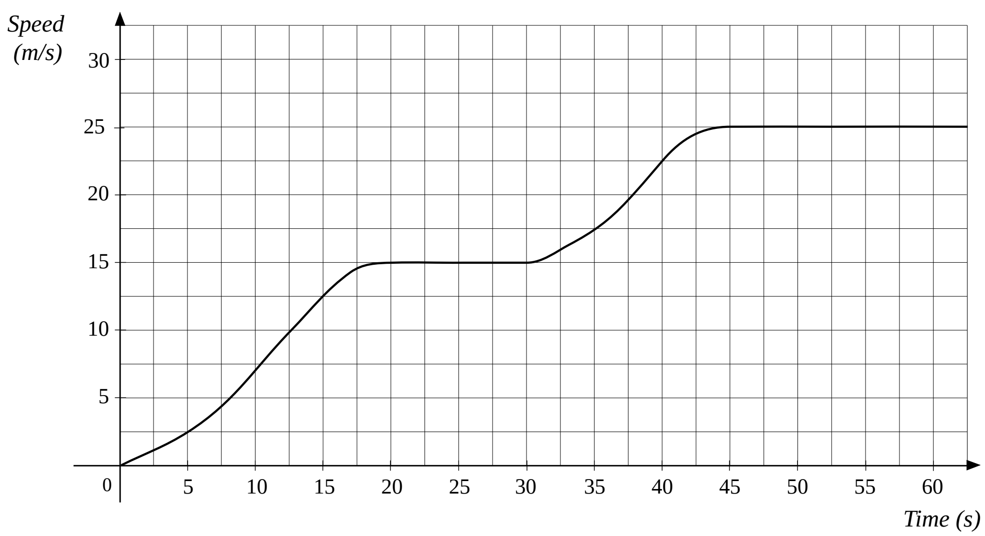
Use the two triangles and rectangle shown to estimate the area between the curve and the *x*-axis. Is your answer an overestimate or underestimate of the actual area?

2. Estimate the area under the curve $y = 4 - x^2$ between $x = -2$ and $x = 2$ using:

   (a) 2 trepezia (b) 4 trapezia (c) 8 trapezia.

3. Draw the graphs of $y = 2x$ and $y = x^2 - 5x + 6$. Find the area between the curve and the line by using 5 trapezia.

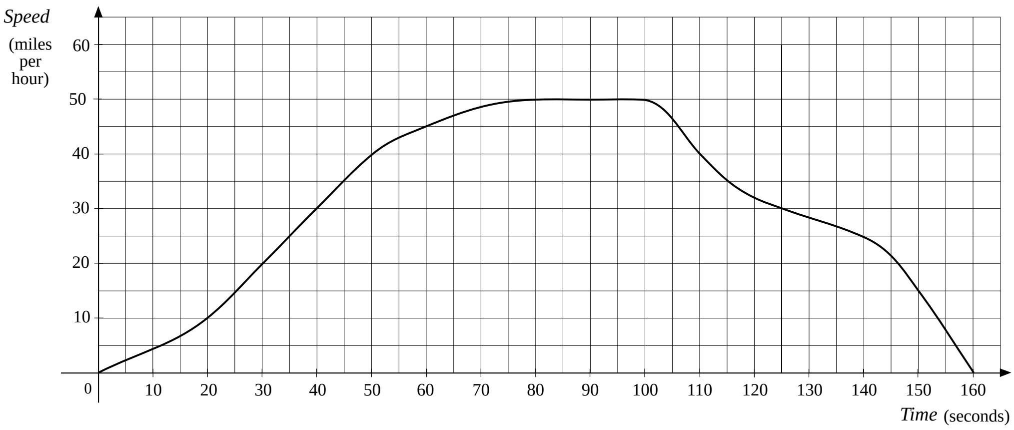
4. The graph below shows how the speed of a car varies as it sets off from rest.



   (a) Estimate the distance travelled in the first (i) 30 seconds
   (ii) 60 seconds?

   (b) What is the average speed for the first (i) 30 seconds
   (ii) 60 seconds?

5. The graph below illustrates a speed-time graph for a local train journey from one station to the next.



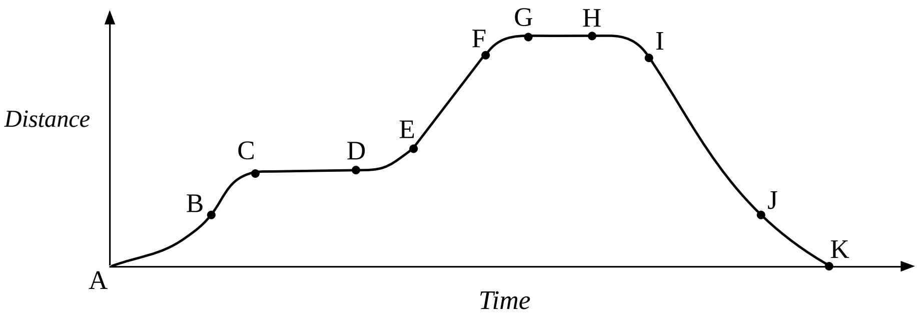
(a) What is the maximum speed achieved?

(b) Estimate the total distance between the two stations.

(c) Explain in words what happens to the speed during the journey.

17.3 Tangents to Curves

1. The distance-time graph of a journey made on a bicycle is shown.



(a) Between which two points is the bicycle slowing down?

(b) Between which two points is the bicycle moving at a constant speed and travelling away from the starting point?

(c) Between which two points is the bicycle accelerating and travelling back to the starting point?

(SEG)

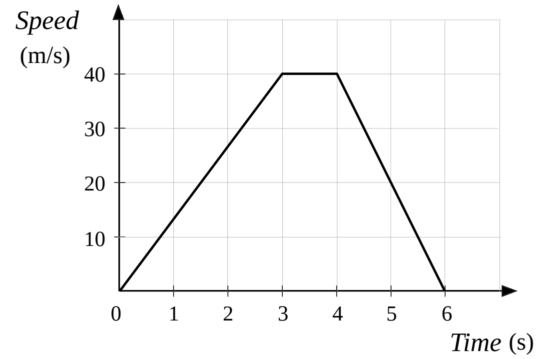
2. (a) Draw the graph of $y = \frac{x^2}{2}$ for $0 \le x \le 4$.

(b) By drawing tangents, find the gradient of the curve at $x = 0$, 1, 2, 3 and 4.

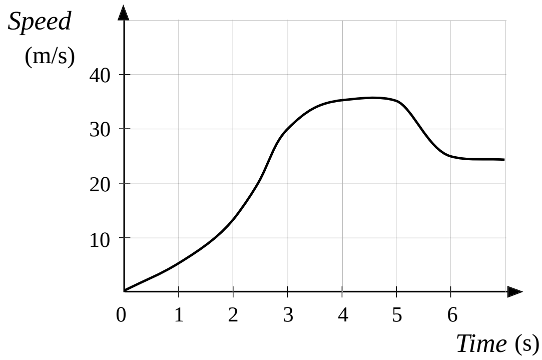
(c) Comment on the pattern of results.

3. For each of the velocity-time graphs below, sketch a corresponding acceleration-time graph.

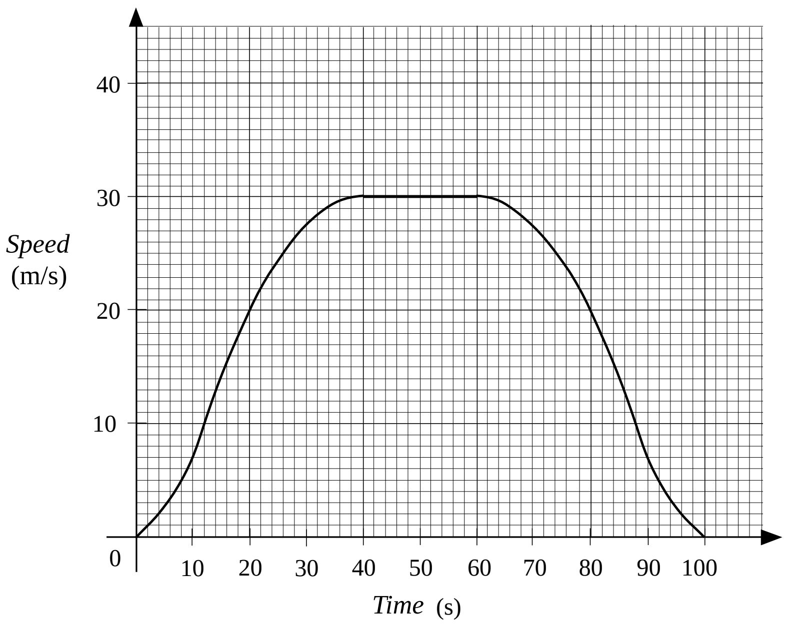
(a)



(b)



4. The graph below illustrates the journey of a London Underground train between two stations. The journey takes 100 seconds.



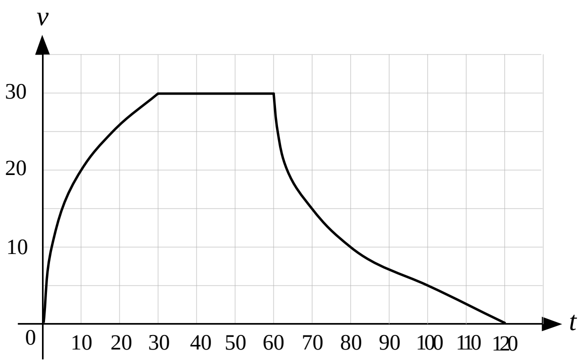
(a) Describe how the speed of the train varies on the journey.

(b) Estimate the maximum acceleration of the train

(c) Estimate the distance, in kilometres, between the stations.

(SEG)

5. The graph shows the velocity, v metres per second, of a car t seconds after it joins a busy main road.

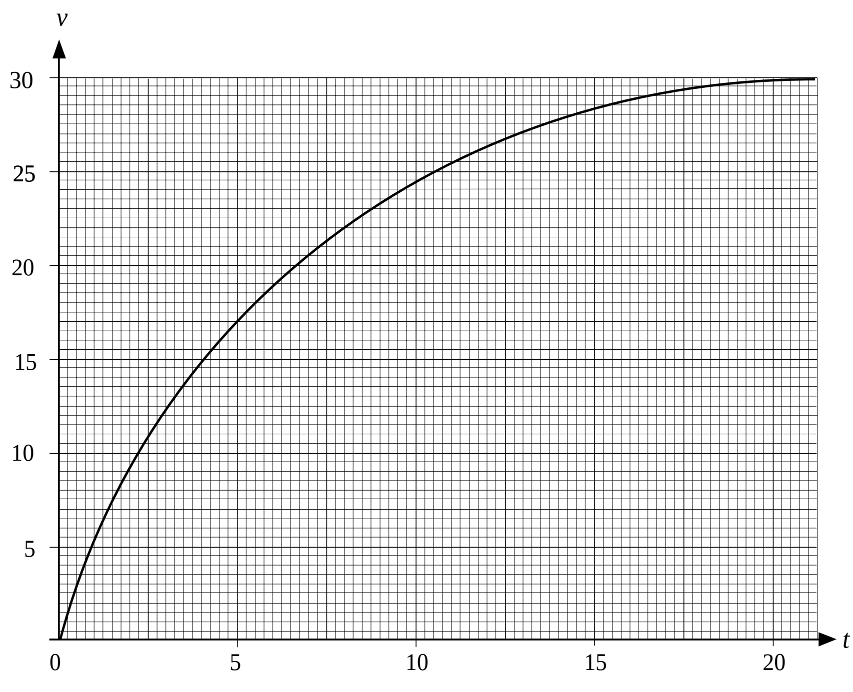


(a) Calculate the acceleration of the car 10 seconds after it joins the busy main road.

(b) Calculate an estimate of the total distance travelled by the car in the two minutes shown in the graph.

(LON)

6. The graph below shows the velocity of a car over the time interval $0 < t < 20$, where t is time in seconds and v is velocity in metres per second.



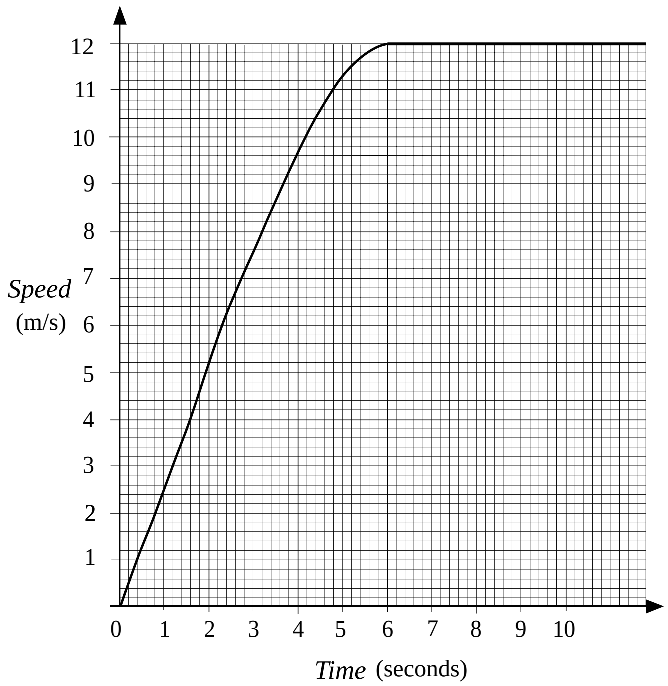
(a) Use the graph to estimate the acceleration of the car at $t = 7.5$.

(b) (i) Estimate the area under the graph for the interval $0 \le t \le 10$.

(ii) What does this area represent?

(SEG)

7. The speed-time graph for an athlete in the first 10 seconds of a race is shown.



(a) Use the graph to estimate the acceleration 4 seconds from the start.

(b) Use the graph to estimate the distance run by the athlete in the first 10 seconds.

(c) Calculate the average speed of the athlete during the first 10 seconds of the race.

(SEG)

8. A firework is launched vertically upwards at time $t = 0$. Its vertical height, h, in metres from its point of launch is given by $h = 20t - 5t^2$, where t is in seconds.

(a) Use a graphical method to estimate:

(i) the first time at which the firework reaches a height of 12 metres

(ii) the maxiumum height reached by the firework.

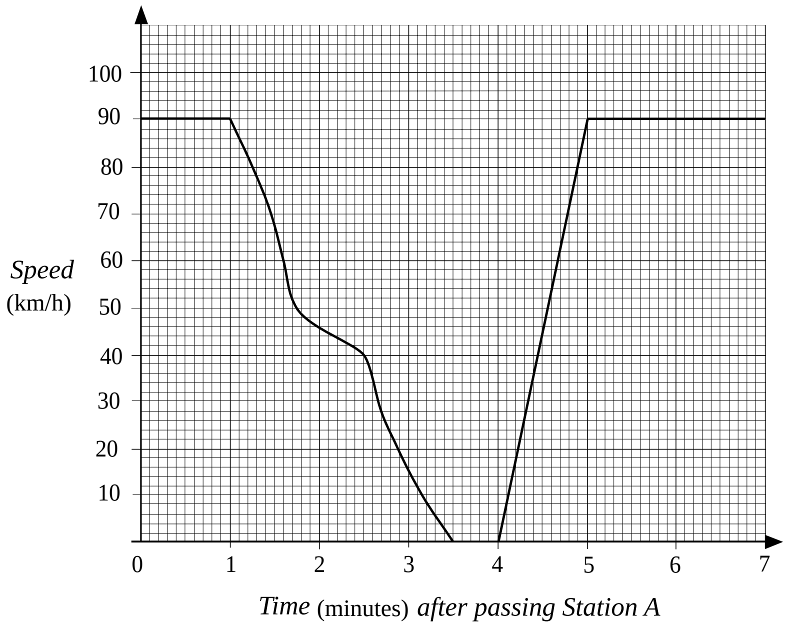
(b) Use your graph to estimate the speed of the firework when it first reaches a height of 12 metres.

(c) Solve an appropriate quadratic equation to calculate the second time that the firework reaches a height of 12 metres. *Give your answer correct to two decimal places.*

(SEG)

9. A train normally travels between two stations, A and B, at a steady speed of 90 kilometres/hour.

On a particular day, the driver had to stop at a red signal between the stations. The speed/time graph is shown below.



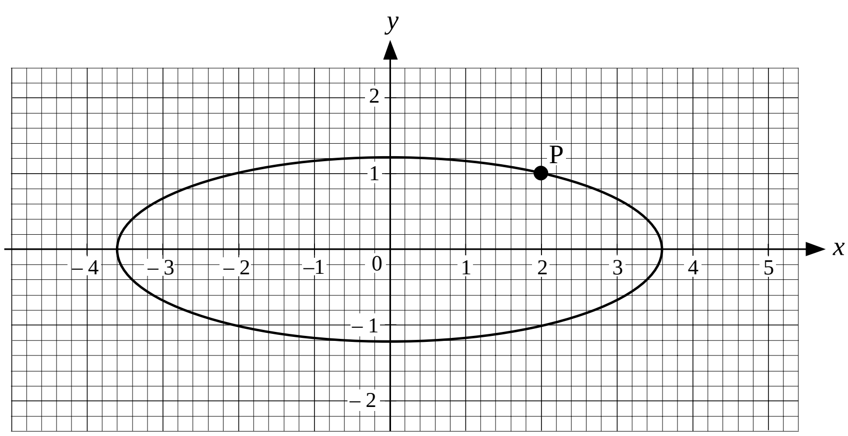
(a) (i) From the graph, estimate the deceleration of the train, in kilometres/hour/minute, two minutes after passing station A.

(ii) Express this deceleration in metres/second^2.

(b) Estimate the distance of the signal from station A.

(NEAB)

10.



(a) Draw the tangent at P on a copy of the curve.

(b) Calculate the gradient of the tangent you have drawn. *Give your answer correct to 1 decimal place.*

(LON)

17.4 Finding Coefficients

1. Each table below gives a set of data and the form of the relationship between the variables. In each case, find the value of the constants a and b.

(a)

x	1	2	4	10
y	1.5	1	0	-3

$y = ax + b$

(b)

x	2	4	6	8
y	5	29	69	125

$y = ax^2 + b$

(c)

x	1	2	3	4
y	22	10	6	4

$y = \dfrac{a}{x} + b$

(d)

x	1	4	9	16
y	8	6	4	2

$y = a\sqrt{x} + b$

(e)

x	0	1	2	3
y	20	19	12	-7

$y = ax^3 + b$

2. For a simple pendulum, experimental data for the time period and the length of the pendulum are given below.

Length (cm)	16	25	49	64	100
Time (s)	0.8	1.0	1.4	1.6	2.0

It is expected that the time is proportional to the square root of the length.
Is this confirmed by this data? If so, find the constant of proportionality.

3. The height of the tide, h metres, measured at a particular time, t hours, is given by the data below.

Time (hours)	0	3	6	9	12
Height (m)	10	4	-2	4	10

Assuming that the data follow a relationship of the form

$$h = a\cos(30t) + b,$$

estimate the value of the constants a and b.