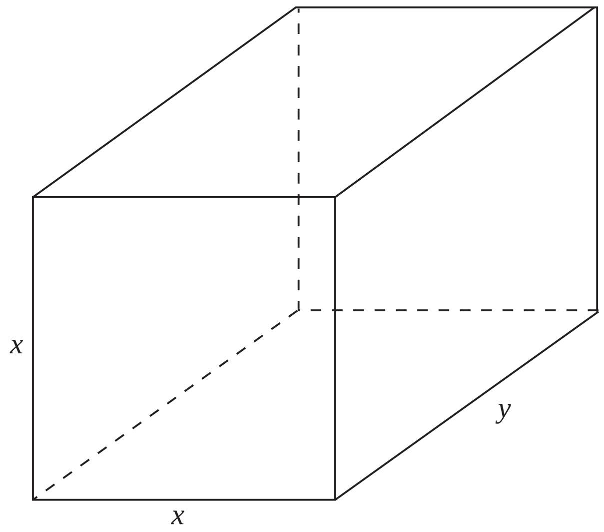
UNIT 13 *Graphs, Equations and Inequalities* Activities

Activities

ACTIVITY 13.1 *Maximum Volume of Cuboid*



This cuboid has a square end and a volume of 100 cm^3.

1. Show that the surface area of the cuboid is given by

$$A = 2x^2 + \frac{400}{x}$$

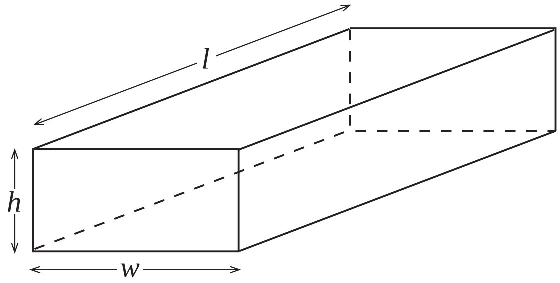
2. Use a graph to determine the values of x and y for which the surface area is a minimum.

3. What shape would this give?

ACTIVITY 13.2 *Posting Parcels*

The *Royal Mail, Data Post International* parcel service accepts parcels up to a maximum size as given in the following rules:

1. *Length + height + width* must not exceed *900 mm.*
2. None of the *length, height, width* must exceed *600 mm.*

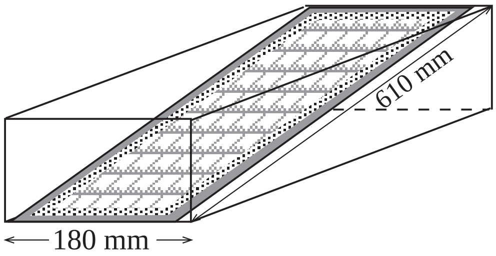


1. Which of the following parcels would be accepted for this service:

   (a) $l = 620$ mm, $h = 120$ mm, $w = 150$ mm

   (b) $l = 500$ mm, $h = 350$ mm, $w = 150$ mm

   (c) $l = 550$ mm, $h = 100$ mm, $w = 150$ mm

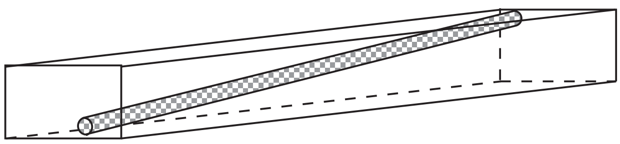
2. 

   A picture with frame, 610 mm by 180 mm, is to be placed diagonally in a rectangular box as shown.

   Find suitable dimensions for the box so that it would be accepted for the Data Post service.

3. A long, thin tube is to be sent by *Data Post.*

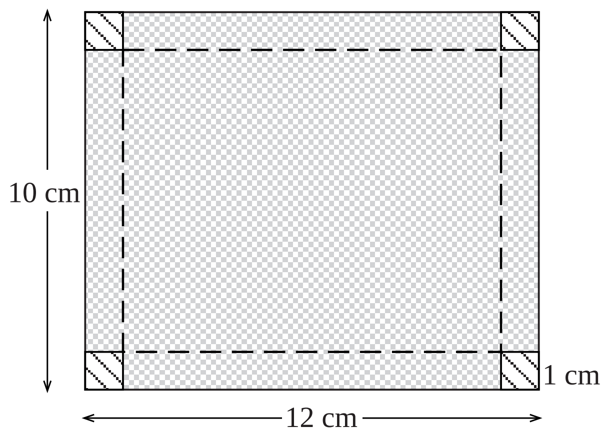
   What is the largest possible length that can be sent?



Extension

What are the dimensions of the rectangular box of *maximum volume* that can be sent through the *Data Post* service?

ACTIVITY 13.3 *Maximum Volume of Open-Topped Box*



We want to make cardboard trays for displaying home-made goods at a local bazaar.

The sheets of card available are of size

$$10 \text{ cm} \times 12 \text{ cm}$$

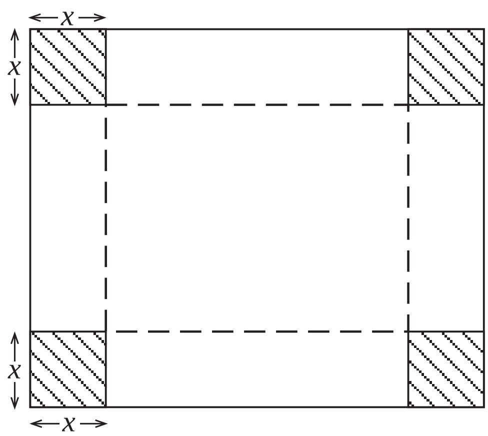
If we cut out a one cm^2 square from each corner and fold the sides up, we have a tray of volume

$$8 \times 10 \times 1 = 80 \text{ cm}^3$$

Can you form a tray with a larger volume?

1. Construct trays by cutting out squares of side 2 cm, 3 cm, etc.
   For each tray, determine the volume. Which tray has *maximum* volume?

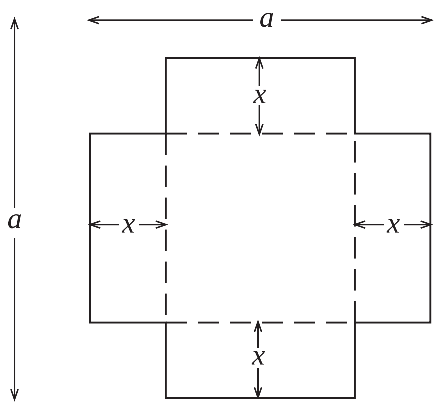
We can find an even larger volume by considering cutting a square of side x cm from the card, as shown opposite.



2. What is the length and breadth of the base of the tray?
   Show that the volume is given by
   $$V = 4x^3 - 44x^2 + 120x$$

3. Draw a graph of V against x, plotting V for
   $$x = 0.\ 0.5,\ 1,\ 1.5,\ \ldots,\ 5$$
   Estimate the value of x which gives the maximum value.

We can generalise these results further by considering the same problem of finding the tray of maximum volume that can be cut from a square of side a cm.



4. Show that the volume of the tray is given by
   $$V = 4x^3 - 4ax^2 + a^2 x$$
   We can non-dimensionalise the problem by taking $a = 1$.

5. With $a = 1$, plot a graph of V against x for
   $$x = 0,\ 0.1,\ 0.2,\ \ldots,\ 0.5$$
   What value of x gives a maximum value for V ?