Year 8 - Probability

Exercise 1

The set of all possible outcomes is known as the _____.

$$P(event) = \frac{outcomes\ matching\ event}{total\ outcomes}$$

List out all the possible outcomes given each description, underline or circle the outcomes that match, and hence work out the probability.

	Event	Sample Space	Probability
1	Getting one heads and one tails on the throw of two coins.	HH, **HT**, **TH**, TT	$P(H\ and\ T) = \frac{1}{2}$
2	Getting two tails after two throws.		$P(TT) =$
3	Getting at least 2 heads after 3 throws.		$P(\geq 2\ H) =$
4	Getting exactly 2 heads after 3 throws.		
5	Rolling a prime number and throwing a head.		
6	In three throws of a coin, a heads never follows a tails.		
7	For a randomly chosen meal with possible starters **A**vacado, **B**eans and **C**auliflower, and possible main courses **D**og, **E**scalopes or **F**ish, ending up with neither Avacado nor Dog.		

Exercise 2

Again, work out the probabilities of the following, but you now no longer need to list the outcomes, merely *count* them.

	Event	Num matching outcomes	Num total outcomes	Probability
1	Drawing a Jack from a pack of cards.	4	52	$P(J) = \frac{4}{52} = \frac{1}{13}$
2	Drawing a club from a pack of cards.			
3	Drawing a card which is either a club or is an even number.			
4	Throwing two sixes on a die in a row.			
5	Throwing an even number on a die followed by an odd number.			
6	Throwing three square numbers on a die in a row.			
7	Seeing exactly two heads in four throws of a coin.			
8	Seeing the word 'BOB' when arranging two plastic Bs and an O on a sign.			
☠	Seeing the word SHELL when arranging a letter S, H, E and two letter Ls on a sign.			
☠☠	After shuffling a pack of cards, the cards in each suit are all together.			

Exercise 3

Imagine you have four cards numbered 1 to 4, and by considering (a) all possible outcomes and (b) outcomes matching the event described, work out the probability of the following, ensuring you use appropriate "P(..) =" notation.

Event	Matching outcomes	Total Outcomes	Probability
One number randomly picked being even.	2	4	$P(Even) = \frac{2}{4}$
The four numbers, when randomly placed in a line, reads 1-2-3-4			
Two numbers, when placed in a line, contain a two and a three.			
Three numbers, when placed in a line, form a descending sequence.			
Two numbers, when placed in a line, give a sum of 5.			
When you pick a number out a bag, look at the value then put it back, then pick a number again, both numbers are 1.			
☠ When you pick a number from a bag, put the number back, and do this 4 times in total, the values of your numbers form a 'run' of 1 to 4 in any order.			

2D sample spaces

Example: I throw two dice and add up the scores. By filling in the sample space table, determine the probability that:

a) My total is 10? $P(total = 10) =$ _____
b) My total is at least 10? $P(total > 10) =$ _____
c) My total is at most 9? $P(total \leq 9) =$ _____

First Die \ Second Die	+ 1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Exercise 4

For the following, form an appropriate sample space table, and use the table to answer the questions.

1) After throwing two fair coins...

   a. The probability of throwing two heads. _____

   b. The probability of throwing a heads and a tails.

      _____

2) After throwing two fair die and adding the two outcomes...

   a. The total is prime. _____

   b. The total is less than 4. _____

   c. The total is odd. _____

3) After throwing two dice and **multiplying** the outcomes...

   a. The product is 6. _____

   b. The product is at most 6. _____

   c. The product at least 7. _____

   d. The product is odd. _____

4) After spinning two spinners, one 3-sided labelled A, B and C, and one 4-sided labelled A, B, C and D...

   a. The letters are both vowels. _____

   b. At least one letter is a vowel. _____

   c. We see the letters B and C. _____

Events and Laws of Probability

An event is _____
(More formally, it is a subset of the sample space)

If two events are **mutually exclusive** then

_____

and P(A or B) = _____

A' means that _____

and $P(A') =$ _____

Examples:

- A and B are mutually exclusive events and P(A) = 0.3, P(B) = 0.2

  P(A or B) = _____
  P(A') = _____
  P(B') = _____

- C and D are mutually exclusive events and P(C') = 0.6, P(D) = 0.1

  P(C or D) = _____

- E, F and G are mutually exclusive events and P(E or F) = 0.6
  P(F or G)=0.7 (and P(E or F or G) = 1), then

  P(F) = _____
  P(E) = _____
  P(G) = _____

Exercise 5

1. In the following questions, all events are mutually exclusive.
   a. P(A) = 0.6, P(C) = 0.2
      P(A') = _____, P(C') = _____
      P(A or C) = _____

   b. P(A) = 0.1, P(B') = 0.8, P(C') = 0.7
      P(A or B or C) = _____

   c. P(A or B) = 0.3, P(B or C) = 0.9, P(A or B or C) = 1
      P(A) = _____
      P(B) = _____
      P(C) = _____

   d. P(A or B or C or D) = 1. P(A or B or C) = 0.6
      and P(B or C or D) = 0.6 and P(B or D) = 0.45
      P(A) = _____, P(B)= _____
      P(C) = _____, P(D) = _____

2. All Tiffin students are either good at maths, English or music, but not at more than one subject. The probability that a student is good at maths is 1/5. The probability they are are good at English is 1/3. What is the probability that they are good at music?

   _____

3. The probability that Alice passes an exam is 0.3. The probability that Bob passes the same exam s 0.4. The probability that either pass is 0.65. Are the two events mutually exclusive? Give a reason.

   _____

4. The following tables indicate the probabilities for spinning different sides, A, B, C and D, of an unfair spinner. Work out x in each case.

A	B	C	D
0.1	0.3	x	x

$x =$ _____

A	B	C	D
0.5	$2x$	0.2	x

$x =$ _____

A	B	C	D
x	$2x$	$3x$	$4x$

$x =$ _____

A	B
x	$4x + 0.25$

$x =$ _____

5. I am going on holiday to one destination this year, either France, Spain or America. I'm 3 times as likely to go to France as I am to Spain but half as likely to go to America than Spain. What is the probability that I don't go to Spain?

_____

6. ☠☠ P(A or B or C) = 1.
P(A or B) = $4x - 0.1$ and $P(B \text{ or } C) = 4x$. Determine expressions for P(A), P(B) and P(C) and hence determine the range of values for x.
[Hint: think how you did this in Q1c. Now just use the same method, but algebraically!]

_____

Experimental vs Theoretical Probability

Theoretical probability _____

_____

Experimental Probability is also known as _____

and is _____

$$Experimental\ Probability = \text{\_\_\_\_\_}$$

Examples:

A) The table below shows the probabilities for spinning an A, B and C on a spinner. If I spin the spinner 150 times, estimate the number of Cs I will see.

Outcome	A	B	C
Probability	0.12	0.34	

Answer: _____

B) I spin another spinner 120 times and see the following counts:

Outcome	A	B	C
Count	30	45	45

What is the relative frequency of B?

_____

Exercise 6

1. An unfair die is rolled 80 times and the following counts are observed.
   a. Determine the relative frequency of each outcome.

Outcome	1	2	3	4	5	6
Count	20	10	8	4	10	28
R.F.						

   b. Dr Bob claims that the theoretical probability of rolling a 3 is 0.095. Is Dr Bob correct?

_____

_____

2. An unfair coin has a probability of heads 0.68. I throw the coin 75 times. How many **tails** do I expect to see?

_____

3. Dr Laurie throws a fair die 600 times, and sees 90 ones.
   a. Calculate the relative frequency of throwing a 1.

_____

   b. Explain how Laurie can make the relative frequency closer to a $\frac{1}{6}$.

_____

4. The table below shows the probabilities of winning different prizes in the gameshow "I'm a Tiffinian, Get Me Outta Here!". 160 Tiffin students appear on the show. Estimate how many cuddly toys will be won.

Prize	Cockroach Smoothie	Cuddly Toy	Maths Textbook	Skip Next Landmark
Probability	0.37	x	0.18	$2x$

_____

5. A six-sided unfair die is thrown n times, and the relative frequencies of each outcome are 0.12, 0.2, 0.36, 0.08, 0.08 and 0.16 respectively. What is the minimum value of n?

_____

6. A spin a spinner with sectors A, B and C 200 times. I see twice as many Bs as As and 40 more Cs than As. Calculate the relative frequency of spinning a C.

_____

7. ☠ I throw a fair coin some number of times and the relative frequency of Heads is 0.45. I throw the coin a few more times and the relative frequency is now equal to the theoretical probability. What is the minimum number of times the coin was thrown?

_____

8. ☠ I throw an unfair coin n times and the relative frequency of Heads is 0.35. I throw the coin 10 more times, all of which are Heads (just by luck), and the relative frequency rises to 0.48. Determine n.
[Hint: Make the number of heads after the first n throws say k, then form some equations]

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