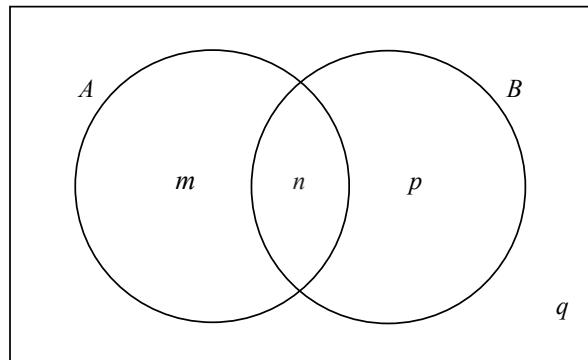


Stat & Prob Pl

2. [Maximum mark: 6]

The Venn diagram below shows events A and B where $P(A)=0.3$, $P(A \cup B)=0.6$ and $P(A \cap B)=0.1$. The values m , n , p and q are probabilities.



- (a) (i) Write down the value of n .
(ii) Find the value of m , of p , and of q . [4 marks]

(b) Find $P(B')$. [2 marks]



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9. [Maximum mark: 14]

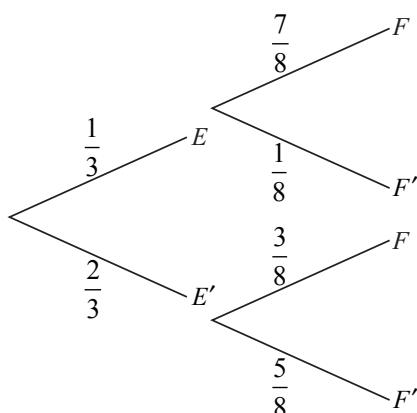
José travels to school on a bus. On any day, the probability that José will miss the bus is $\frac{1}{3}$.

If he misses his bus, the probability that he will be late for school is $\frac{7}{8}$.

If he does not miss his bus, the probability that he will be late is $\frac{3}{8}$.

Let E be the event “he misses his bus” and F the event “he is late for school”.

The information above is shown on the following tree diagram.



(a) Find

(i) $P(E \cap F)$;

(ii) $P(F)$.

[4 marks]

(b) Find the probability that

(i) José misses his bus and is not late for school;

(ii) José missed his bus, given that he is late for school.

[5 marks]

The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday.

(c) Copy and complete the probability distribution table.

[3 marks]

X (cost in euros)	0	3	6
$P(X)$	$\frac{1}{9}$		

(d) Find the expected cost for José for both days.

[2 marks]



4. [Maximum mark: 7]

The probability distribution of a discrete random variable X is given by

$$P(X=x) = \frac{x^2}{14}, \quad x \in \{1, 2, k\}, \text{ where } k > 0.$$

- (a) Write down $P(X = 2)$. [1 mark]

(b) Show that $k = 3$. [4 marks]

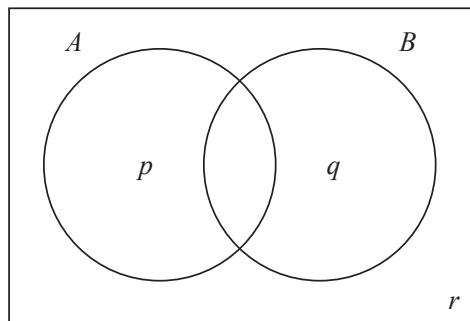
(c) Find $E(X)$. [2 marks]



5. [Maximum mark: 6]

Consider the events A and B , where $P(A) = 0.5$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$.

The Venn diagram below shows the events A and B , and the probabilities p , q and r .



- (a) Write down the value of

 - (i) p ;
 - (ii) q ;
 - (iii) r . [3 marks]

(b) Find the value of $P(A|B')$. [2 marks]

(c) Hence, or otherwise, show that the events A and B are **not** independent. [1 mark]

2210-7303



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

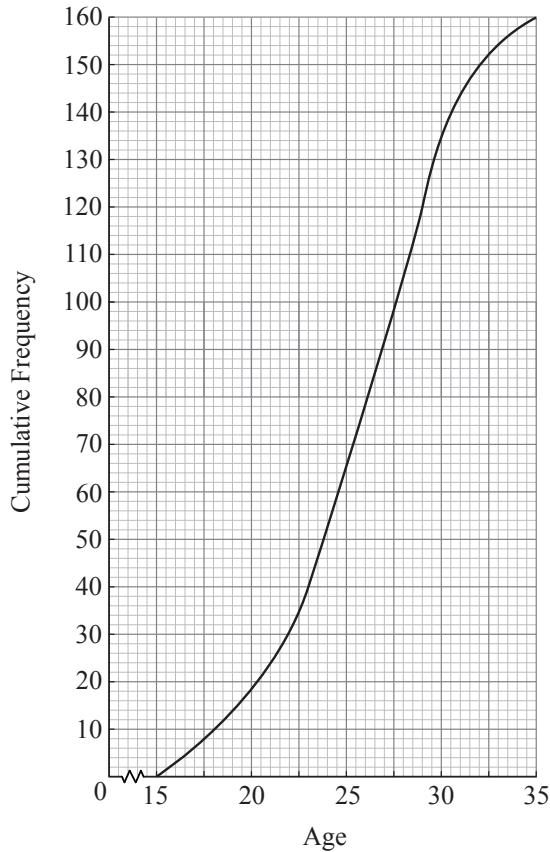
1. [Maximum mark: 7]

The ages of people attending a music concert are given in the table below.

Age	$15 \leq x < 19$	$19 \leq x < 23$	$23 \leq x < 27$	$27 \leq x < 31$	$31 \leq x < 35$
Frequency	14	26	52	52	16
Cumulative Frequency	14	40	92	p	160

- (a) Find p . [2 marks]

The cumulative frequency diagram is given below.



(This question continues on the following page)



(Question 1 continued)

- (b) Use the diagram to estimate

- (i) the 80th percentile;
 - (ii) the interquartile range.

[5 marks]

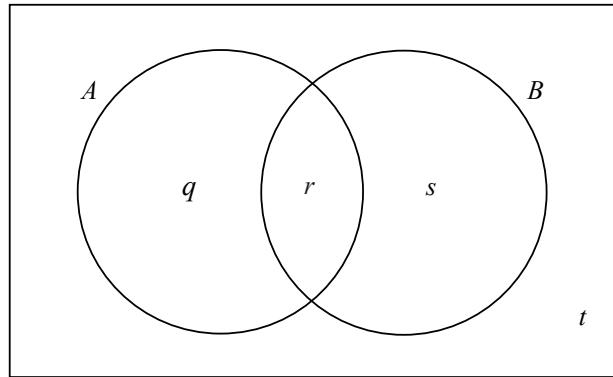


0312

Turn over

4. [Maximum mark: 7]

Events A and B are such that $P(A) = 0.3$, $P(B) = 0.6$ and $P(A \cup B) = 0.7$.



The values q , r , s and t represent probabilities.

- (a) Write down the value of t . [1 mark]

(b) (i) Show that $r = 0.2$.

(ii) Write down the value of q and of s . [3 marks]

(c) (i) Write down $P(B')$.

(ii) Find $P(A|B')$. [3 marks]



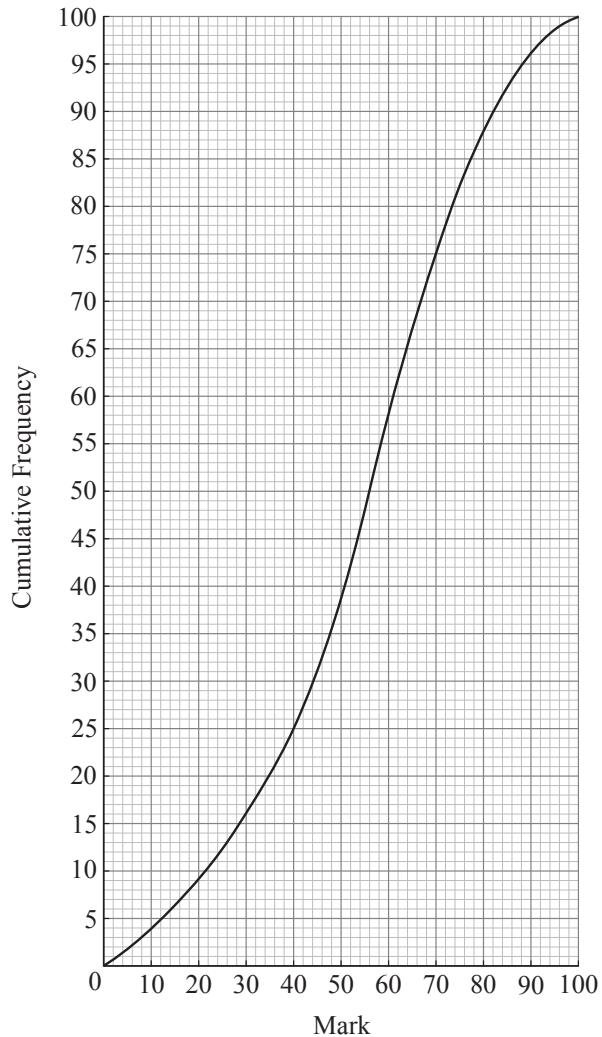
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

The cumulative frequency curve below represents the marks obtained by 100 students.



(This question continues on the following page)



(Question 1 continued)

- (a) Find the median mark. [2 marks]

(b) Find the interquartile range. [3 marks]



4. [Maximum mark: 8]

The random variable X has the following probability distribution, with $P(X > 1) = 0.5$.

x	0	1	2	3
$P(X = x)$	p	q	r	0.2

(a) Find the value of r . [2 marks]

(b) Given that $E(X) = 1.4$, find the value of p and of q . [6 marks]



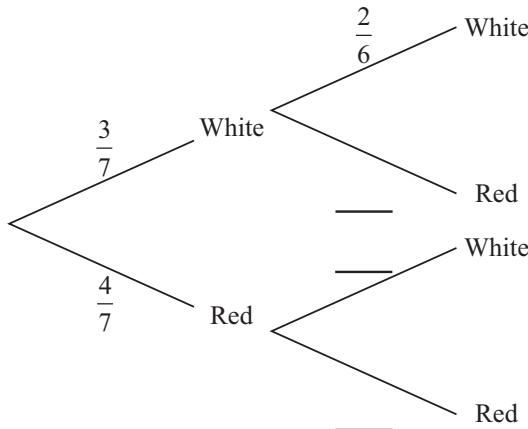
0616

Do **NOT** write solutions on this page.

9. [Maximum mark: 14]

Bag A contains three white balls and four red balls. Two balls are chosen at random without replacement.

- (a) (i) Copy and complete the following tree diagram. (Do **not** write on this page.)



- (ii) Find the probability that two white balls are chosen.

[5 marks]

Bag B contains four white balls and three red balls. When two balls are chosen at random without replacement from bag B, the probability that they are both white is $\frac{2}{7}$.

A standard die is rolled. If 1 or 2 is obtained, two balls are chosen without replacement from bag A, otherwise they are chosen from bag B.

- (b) Find the probability that the two balls are white.

[5 marks]

- (c) Given that both balls are white, find the probability that they were chosen from bag A.

[4 marks]



Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

- 1.** [Maximum mark: 6]

A discrete random variable X has the following probability distribution.

x	0	1	2	3
$P(X=x)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	p

- (a) Find p . [3]

(b) Find $E(X)$. [3]



12EP02

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

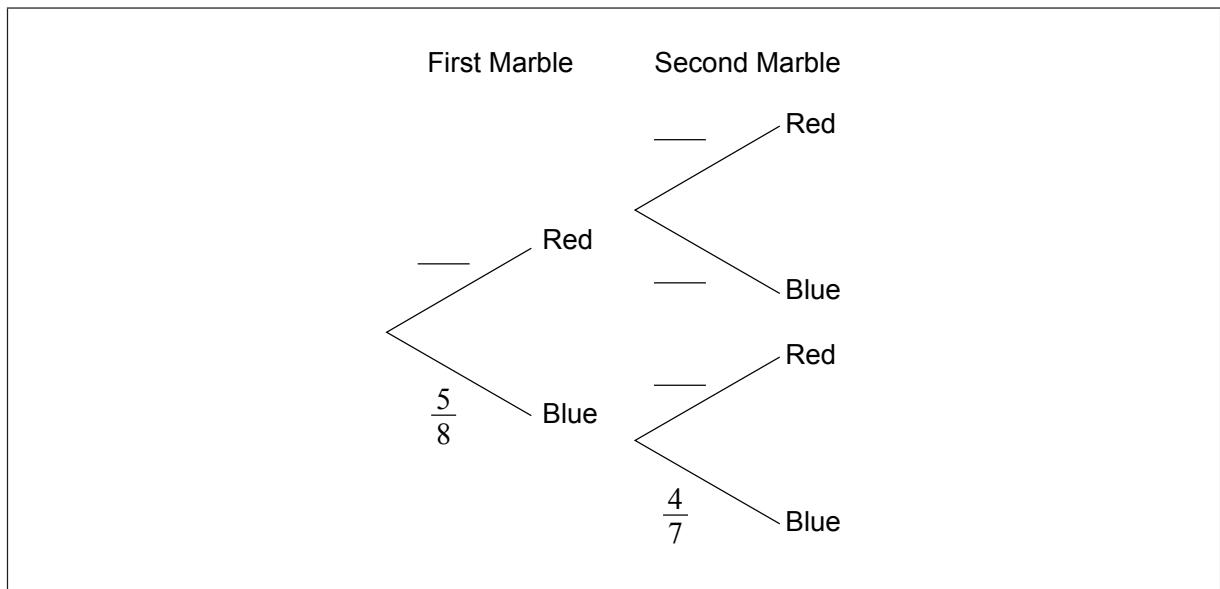
1. [Maximum mark: 6]

A bag contains eight marbles. Three marbles are red and five are blue. Two marbles are drawn from the bag without replacement.

- (a) Write down the probability that the first marble drawn is red. [1]

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- (b) Complete the following tree diagram. [3]



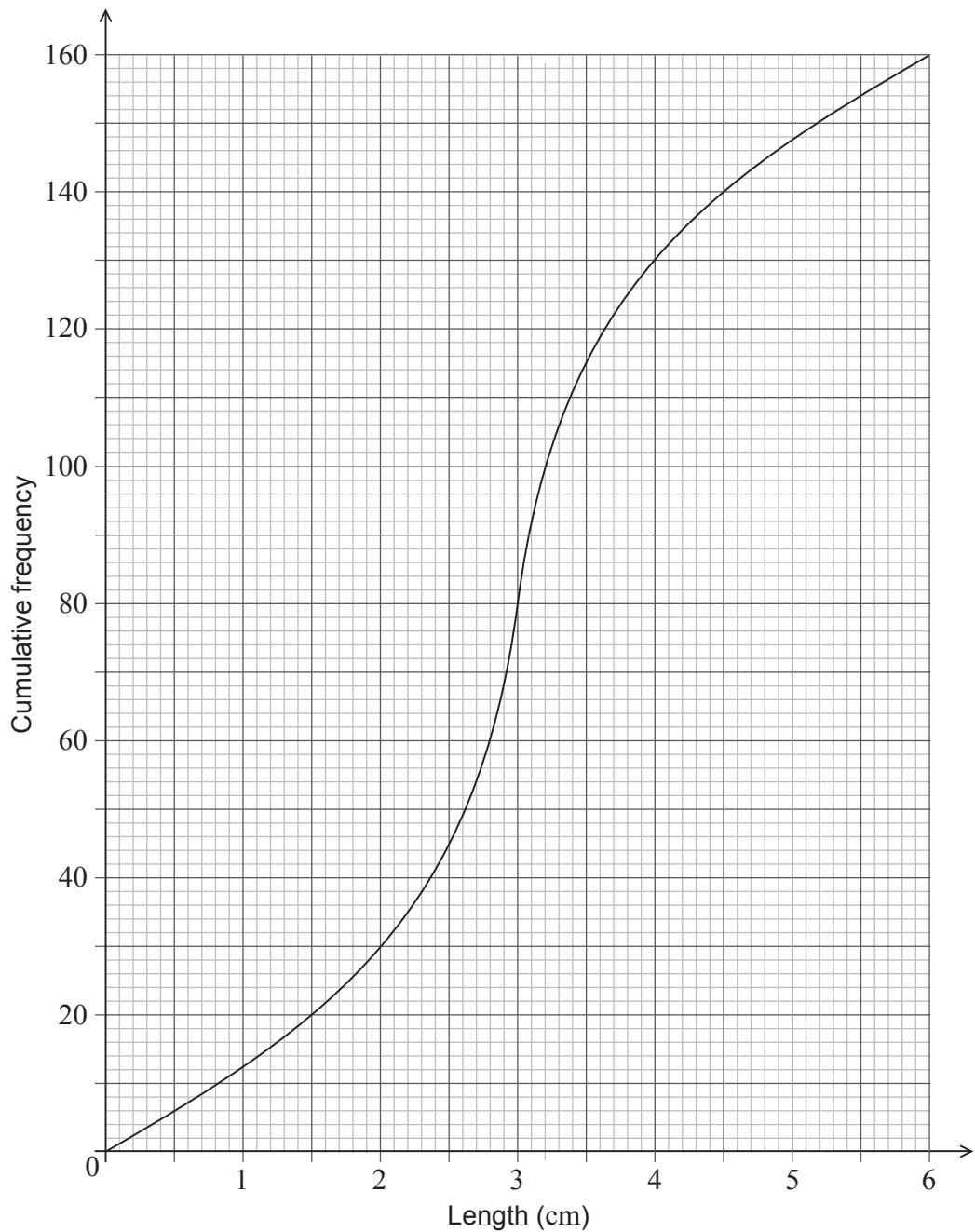
- (c) Find the probability that both marbles are blue. [2]

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3. [Maximum mark: 6]

The following cumulative frequency diagram shows the lengths of 160 fish, in cm.



(This question continues on the following page)



(Question 3 continued)

- (a) Find the median length. [2]

The following frequency table also gives the lengths of the 160 fish.

Length x cm	$0 \leq x \leq 2$	$2 < x \leq 3$	$3 < x \leq 4.5$	$4.5 < x \leq 6$
Frequency	p	50	q	20

- (b) (i) Write down the value of p .
(ii) Find the value of q . [4]



7. [Maximum mark: 7]

A bag contains black and white chips. Rose pays \$10 to play a game where she draws a chip from the bag. The following table gives the probability of choosing each colour chip.

Outcome	black	white
Probability	0.4	0.6

Rose gets no money if she draws a white chip, and gets $\$k$ if she draws a black chip. The game is fair. Find the value of k .



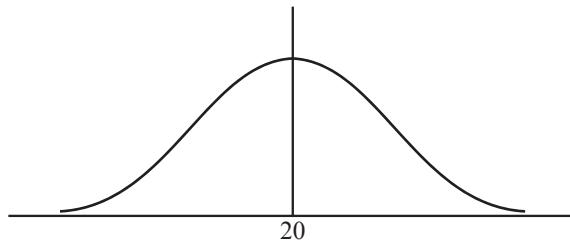
4. [Maximum mark: 8]

A random variable X is distributed normally with a mean of 20 and variance 9.

- (a) Find $P(X \leq 24.5)$. [3 marks]

(b) Let $P(X \leq k) = 0.85$.

(i) Represent this information on the following diagram.



- (ii) Find the value of k . [5 marks]



Turn over

5. [Maximum mark: 7]

A box holds 240 eggs. The probability that an egg is brown is 0.05.

- (a) Find the expected number of brown eggs in the box. [2 marks]

(b) Find the probability that there are 15 brown eggs in the box. [2 marks]

(c) Find the probability that there are at least 10 brown eggs in the box. [3 marks]



Stat & Prob P2

7. [Maximum mark: 7]

A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.
90 % of the boxes made by machine B pass inspection.

A box is selected at random.

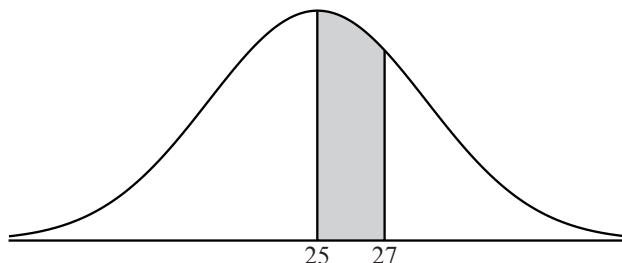
- (a) Find the probability that it passes inspection. [3 marks]

(b) The company would like the probability that a box passes inspection to be 0.87.
Find the percentage of boxes that should be made by machine B to achieve this. [4 marks]



6. [Maximum mark: 7]

Let the random variable X be normally distributed with mean 25, as shown in the following diagram.



The shaded region between 25 and 27 represents 30 % of the distribution.

- (a) Find $P(X > 27)$. [2 marks]

(b) Find the standard deviation of X . [5 marks]



Turn over

7. [Maximum mark: 7]

The probability of obtaining “tails” when a biased coin is tossed is 0.57. The coin is tossed ten times. Find the probability of obtaining

- (a) **at least** four tails; [4 marks]
(b) the fourth tail on the tenth toss. [3 marks]



0916

Turn over

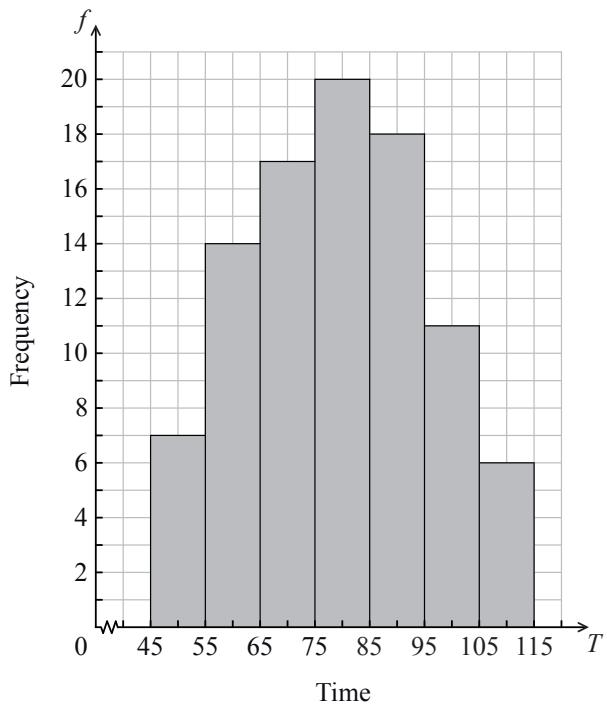
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SECTION B

Answer all questions on the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 13]

The histogram below shows the time T seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for T .

Time	$45 \leq T < 55$	$55 \leq T < 65$	$65 \leq T < 75$	$75 \leq T < 85$	$85 \leq T < 95$	$95 \leq T < 105$	$105 \leq T < 115$
Frequency	7	14	p	20	18	q	6

- (a) (i) Write down the value of p and of q .

- (ii) Write down the median class.

[3 marks]

- (b) A child is selected at random. Find the probability that the child takes less than 95 seconds to solve the puzzle.

[2 marks]

(This question continues on the following page)



Do NOT write solutions on this page.

(Question 8 continued)

Consider the class interval $45 \leq T < 55$.

(c) (i) Write down the interval width.

(ii) Write down the mid-interval value.

[2 marks]

(d) Hence find an estimate for the

(i) mean;

(ii) standard deviation.

[4 marks]

John assumes that T is normally distributed and uses this to estimate the probability that a child takes less than 95 seconds to solve the puzzle.

(e) Find John's estimate.

[2 marks]



Turn over

4. [Maximum mark: 6]

The heights of a group of seven-year-old children are normally distributed with mean 117 cm and standard deviation 5 cm. A child is chosen at random from the group.

- (a) Find the probability that this child is taller than 122.5 cm. [3 marks]

(b) The probability that this child is shorter than k cm is 0.65. Find the value of k . [3 marks]



Turn over

7. [Maximum mark: 8]

A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested.

- (a) Find the probability that there is at least one defective lamp in the sample. [4 marks]
- (b) Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps. [4 marks]

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2. [Maximum mark: 6]

The following table shows the Diploma score x and university entrance mark y for seven IB Diploma students.

Diploma score (x)	28	30	27	31	32	25	27
University entrance mark (y)	73.9	78.1	70.2	82.2	85.5	62.7	69.4

- (a) Find the correlation coefficient. [2]

The relationship can be modelled by the regression line with equation $y = ax + b$.

- (b) Write down the value of a and of b . [2]

Rita scored a total of 26 in her IB Diploma.

- (c) Use your regression line to estimate Rita's university entrance mark. [2]



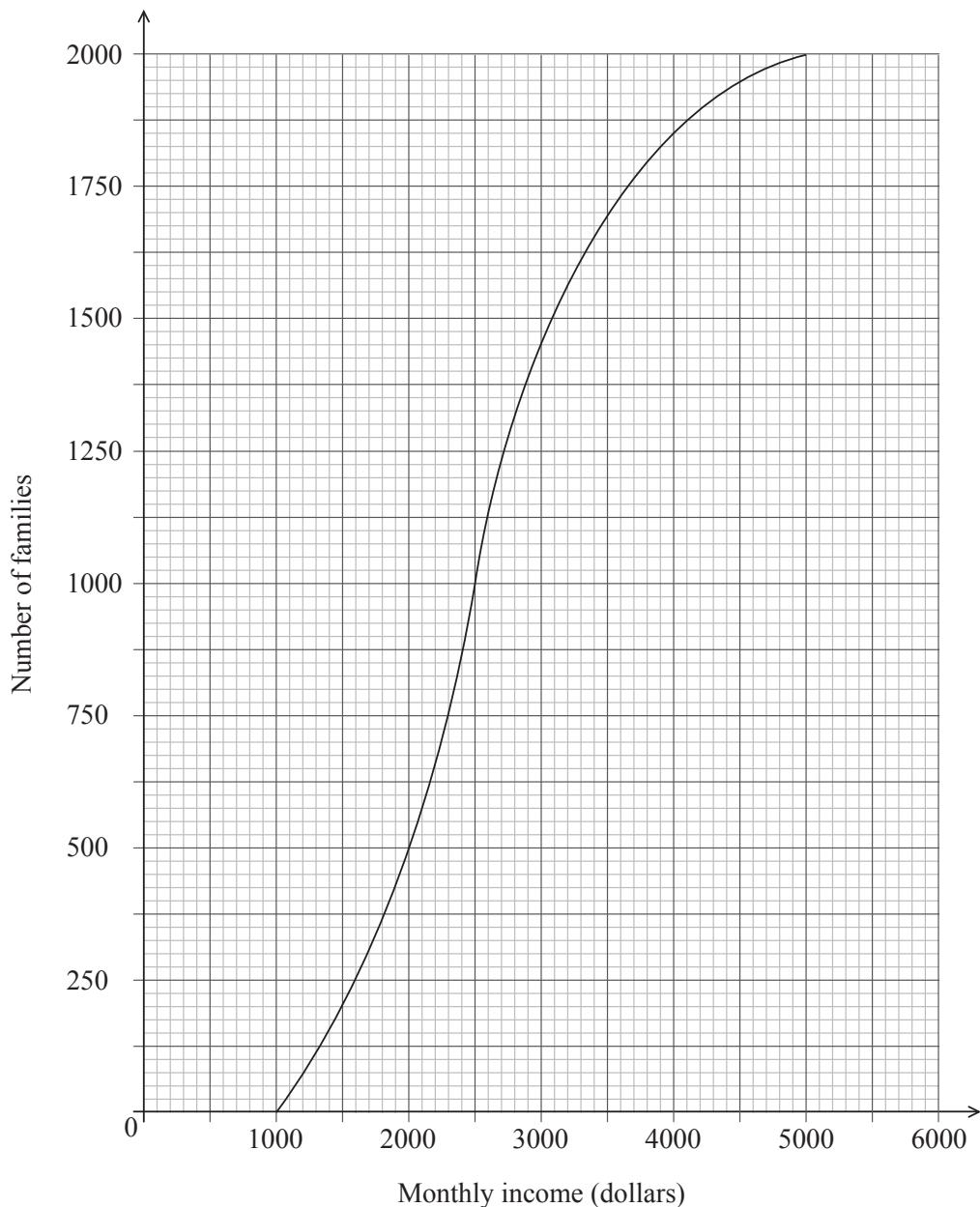
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SECTION B

Answer all questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The following cumulative frequency graph shows the monthly income, I dollars, of 2000 families.



(This question continues on the following page)



Do NOT write solutions on this page.

(Question 8 continued)

- (a) Find the median monthly income. [2]
- (b) (i) Write down the number of families who have a monthly income of 2000 dollars or less.
- (ii) Find the number of families who have a monthly income of more than 4000 dollars. [4]

The 2000 families live in two different types of housing. The following table gives information about the number of families living in each type of housing and their monthly income I .

	$1000 < I \leq 2000$	$2000 < I \leq 4000$	$4000 < I \leq 5000$
Apartment	436	765	28
Villa	64	p	122

- (c) Find the value of p . [2]
- (d) A family is chosen at random.
- (i) Find the probability that this family lives in an apartment.
- (ii) Find the probability that this family lives in an apartment, given that its monthly income is greater than 4000 dollars. [4]
- (e) Estimate the mean monthly income for families living in a villa. [3]



16EP11

Turn over

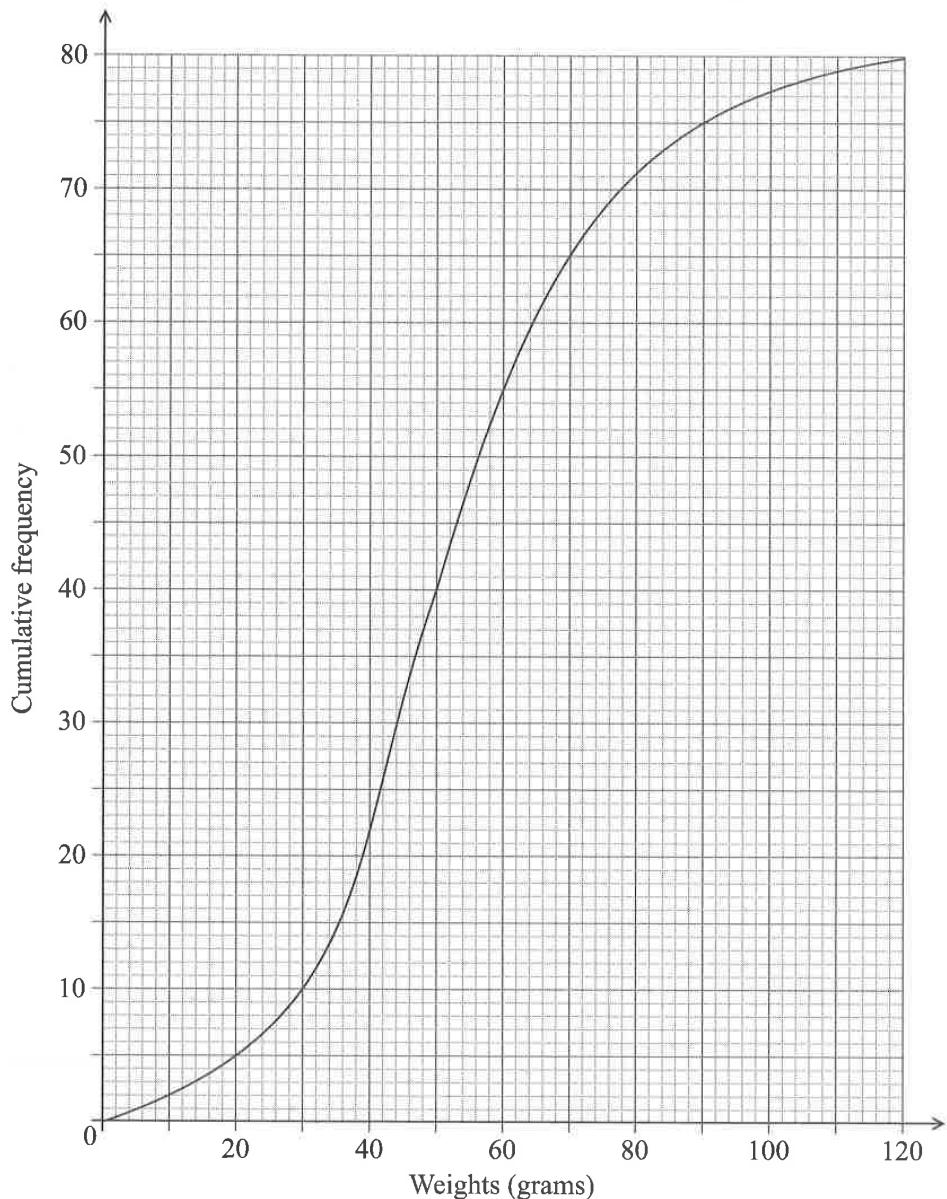
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SECTION B

Answer all questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

The weights in grams of 80 rats are shown in the following cumulative frequency diagram.



(This question continues on the following page)



Do NOT write solutions on this page.

(Question 8 continued)

- (a) (i) Write down the median weight of the rats.
(ii) Find the percentage of rats that weigh 70 grams or less. [4]

The same data is presented in the following table.

Weights w grams	$0 \leq w \leq 30$	$30 < w \leq 60$	$60 < w \leq 90$	$90 < w \leq 120$
Frequency	p	45	q	5

- (b) (i) Write down the value of p .
(ii) Find the value of q . [4]
(c) Use the values from the table to estimate the mean and standard deviation of the weights. [3]

Assume that the weights of these rats are normally distributed with the mean and standard deviation estimated in part (c).

- (d) Find the percentage of rats that weigh 70 grams or less. [2]
(e) A sample of five rats is chosen at random. Find the probability that at most three rats weigh 70 grams or less. [3]



Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

The following table shows the average number of hours per day spent watching television by seven mothers and each mother's youngest child.

Hours per day that a mother watches television (x)	2.5	3.0	3.2	3.3	4.0	4.5	5.8
Hours per day that her child watches television (y)	1.8	2.2	2.6	2.5	3.0	3.2	3.5

The relationship can be modelled by the regression line with equation $y = ax + b$.

- (a) (i) Find the correlation coefficient.

- (ii) Write down the value of a and of b .

[4]

Elizabeth watches television for an average of 3.7 hours per day.

- (b) Use your regression line to predict the average number of hours of television watched per day by Elizabeth's youngest child. Give your answer correct to one decimal place.

[3]



Do **not** write solutions on this page.

9. [Maximum mark: 16]

A company makes containers of yogurt. The volume of yogurt in the containers is normally distributed with a mean of 260 ml and standard deviation of 6 ml.

A container which contains less than 250 ml of yogurt is **underfilled**.

- (a) A container is chosen at random. Find the probability that it is underfilled. [2]

The company decides that the probability of a container being underfilled should be reduced to 0.02. It decreases the standard deviation to σ and leaves the mean unchanged.

- (b) Find σ . [4]

The company changes to the new standard deviation, σ , and leaves the mean unchanged. A container is chosen at random for inspection. It passes inspection if its volume of yogurt is between 250 and 271 ml.

- (c) (i) Find the probability that it passes inspection.

- (ii) Given that the container is **not** underfilled, find the probability that it passes inspection. [6]

- (d) A sample of 50 containers is chosen at random. Find the probability that 48 or more of the containers pass inspection. [4]



Turn over

3. [Maximum mark: 6]

The following table shows the sales, y millions of dollars, of a company, x years after it opened.

Time after opening (x years)	2	4	6	8	10
Sales (y millions of dollars)	12	20	30	36	52

The relationship between the variables is modelled by the regression line with equation $y = ax + b$.

- (a) (i) Find the value of a and of b . [4]
- (ii) Write down the value of r .
- (b) Hence estimate the sales in millions of dollars after seven years. [2]

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12EP05

Turn over

6. [Maximum mark: 7]

Ramiro walks to work each morning. During the first minute he walks 80 metres. In each subsequent minute he walks 90 % of the distance walked during the previous minute. The distance between his house and work is 660 metres. Ramiro leaves his house at 08:00 and has to be at work by 08:15.

Explain why he will not be at work on time.

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12EP08

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9. [Maximum mark: 16]

A machine manufactures a large number of nails. The length, L mm, of a nail is normally distributed, where $L \sim N(50, \sigma^2)$.

- (a) Find $P(50 - \sigma < L < 50 + 2\sigma)$. [3]
- (b) The probability that the length of a nail is less than 53.92 mm is 0.975.
Show that $\sigma = 2.00$ (correct to three significant figures). [2]

All nails with length at least t mm are classified as large nails.

- (c) A nail is chosen at random. The probability that it is a large nail is 0.75.
Find the value of t . [3]
- (d) (i) A nail is chosen at random from the large nails. Find the probability that the length of this nail is less than 50.1 mm.
(ii) Ten nails are chosen at random from the large nails. Find the probability that at least two nails have a length that is less than 50.1 mm. [8]



Turn over

2. [Maximum mark: 6]

Consider the following cumulative frequency table.

x	Frequency	Cumulative frequency
5	2	2
15	10	12
25	14	26
35	p	35
45	6	41

- (a) Find the value of p . [2 marks]
- (b) Find
- (i) the mean;
- (ii) the variance. [4 marks]

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Turn over

7. [Maximum mark: 7]

A random variable X is normally distributed with $\mu = 150$ and $\sigma = 10$.

Find the interquartile range of X .



0812

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

The following table gives the examination grades for 120 students.

Grade	Number of students	Cumulative frequency
1	9	9
2	25	34
3	35	p
4	q	109
5	11	120

- (a) Find the value of
(i) p ;
(ii) q . [4 marks]
- (b) Find the mean grade. [2 marks]
- (c) Write down the standard deviation. [1 mark]

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3. [Maximum mark: 5]

Jan plays a game where she tosses two fair six-sided dice. She wins a prize if the sum of her scores is 5.

- (a) Jan tosses the two dice once. Find the probability that she wins a prize. [3 marks]
- (b) Jan tosses the two dice 8 times. Find the probability that she wins 3 prizes. [2 marks]

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10. [Maximum mark: 15]

The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

- (a) Find the probability that a player weighs more than 82 kg. [2 marks]
- (b) (i) Write down the standardized value, z , for 68 kg.
(ii) Hence, find the standard deviation of weights. [4 marks]

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

- (c) (i) Find the set of all possible weights of players that take part in the tournament.
(ii) A player is selected at random. Find the probability that the player takes part in the tournament. [5 marks]

Of the players in the league, 25 % are women. Of the women, 70 % take part in the tournament.

- (d) Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman. [4 marks]



Quadratic P1

6. [Maximum mark: 6]

Let $f(x) = px^2 + (10 - p)x + \frac{5}{4}p - 5$.

- (a) Show that the discriminant of $f(x)$ is $100 - 4p^2$. [3]
- (b) Find the values of p so that $f(x) = 0$ has two **equal** roots. [3]

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12EP07

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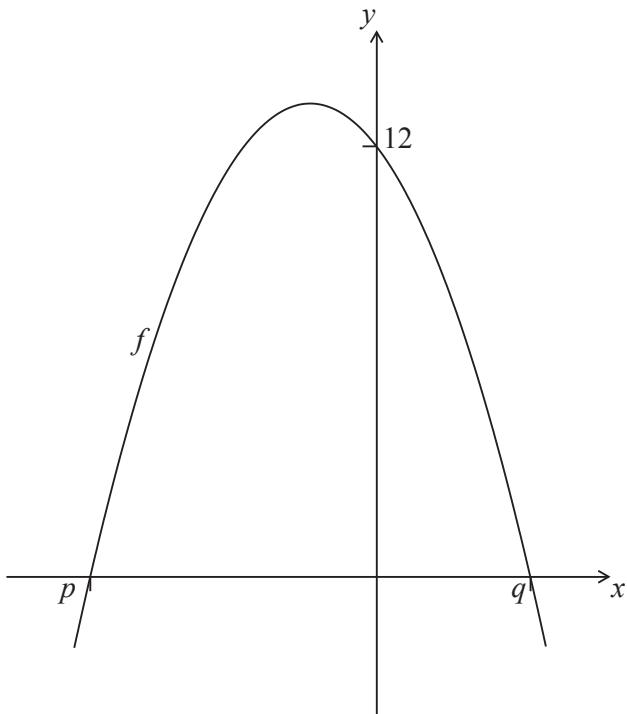
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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Let $f(x) = a(x + 3)(x - 1)$. The following diagram shows part of the graph of f .



The graph has x -intercepts at $(p, 0)$ and $(q, 0)$, and a y -intercept at $(0, 12)$.

- (a) (i) Write down the value of p and of q .

- (ii) Find the value of a .

[6]

- (b) Find the equation of the axis of symmetry of the graph of f .

[3]

- (c) Find the largest value of f .

[3]

The function f can also be written as $f(x) = a(x - h)^2 + k$.

- (d) Find the value of h and of k .

[3]



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

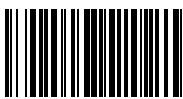
Let $f(x) = x^2 + x - 6$.

(a) Write down the y -intercept of the graph of f . [1]

(b) Solve $f(x) = 0$. [3]

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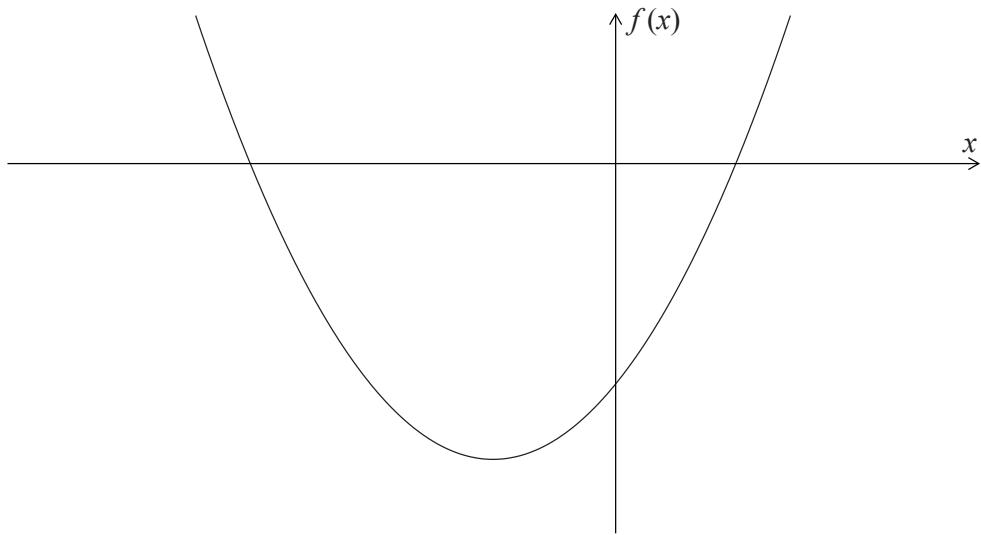
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12EP02

2. [Maximum mark: 6]

The diagram below shows part of the graph of $f(x) = (x-1)(x+3)$.



- (a) Write down the x -intercepts of the graph of f . [2 marks]
- (b) Find the coordinates of the vertex of the graph of f . [4 marks]



Turn over

7. [Maximum mark: 6]

The equation $x^2 - 3x + k^2 = 4$ has two distinct real roots. Find the possible values of k .



6. [Maximum mark: 7]

Consider the equation $x^2 + (k - 1)x + 1 = 0$, where k is a real number.

Find the values of k for which the equation has two **equal** real solutions.



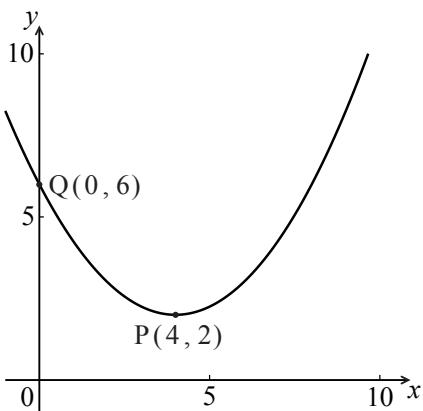
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

Let f be a quadratic function. Part of the graph of f is shown below.



The vertex is at $P(4, 2)$ and the y -intercept is at $Q(0, 6)$.

- (a) Write down the equation of the axis of symmetry.

[1 mark]

The function f can be written in the form $f(x) = a(x - h)^2 + k$.

- (b) Write down the value of h and of k .

[2 marks]

- (c) Find a .

[3 marks]

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7. [Maximum mark: 8]

Let $f(x) = \frac{1}{2}x^2 + kx + 8$, where $k \in \mathbb{Z}$.

- (a) Find the values of k such that $f(x) = 0$ has two equal roots. [4 marks]
- (b) Each value of k is equally likely for $-5 \leq k \leq 5$. Find the probability that $f(x) = 0$ has no roots. [4 marks]



7. [Maximum mark: 7]

Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

- (a) Find the value of k . [5 marks]
- (b) The line $y = p$ intersects the graph of f . Find all possible values of p . [2 marks]

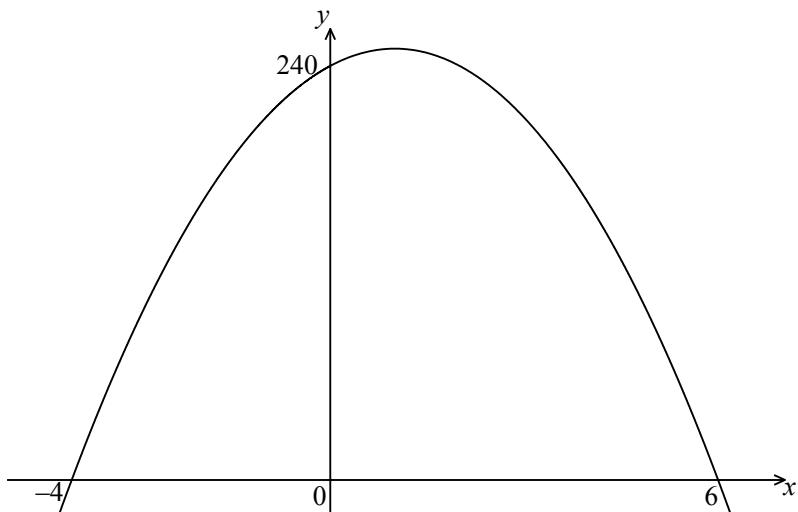
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9. [Maximum mark: 15]

The following diagram shows part of the graph of a quadratic function f .



The x -intercepts are at $(-4, 0)$ and $(6, 0)$, and the y -intercept is at $(0, 240)$.

- (a) Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$. [2 marks]
- (b) Find another expression for $f(x)$ in the form $f(x) = -10(x - h)^2 + k$. [4 marks]
- (c) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. [2 marks]

A particle moves along a straight line so that its velocity, $v \text{ ms}^{-1}$, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \leq t \leq 6$.

- (d) (i) Find the value of t when the speed of the particle is greatest.
(ii) Find the acceleration of the particle when its speed is zero. [7 marks]



Quadratic P1

7. [Maximum mark: 8]

Let $f(x) = kx^2 + kx$ and $g(x) = x - 0.8$. The graphs of f and g intersect at two distinct points. Find the possible values of k .



Do **NOT** write solutions on this page.

9. [Maximum mark: 15]

Consider the function $f(x) = x^2 - 4x + 1$.

- (a) Sketch the graph of f , for $-1 \leq x \leq 5$.

[4 marks]

This function can also be written as $f(x) = (x - p)^2 - 3$.

- (b) Write down the value of p .

[1 mark]

The graph of g is obtained by reflecting the graph of f in the x -axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

- (c) Show that $g(x) = -x^2 + 4x + 5$.

[4 marks]

The graphs of f and g intersect at two points.

- (d) Write down the x -coordinates of these two points.

[3 marks]

Let R be the region enclosed by the graphs of f and g .

- (e) Find the area of R .

[3 marks]



Turn over

2. [Maximum mark: 7]

Let $f(x) = 2x^2 - 8x - 9$.

(a) (i) Write down the coordinates of the vertex.

(ii) Hence or otherwise, express the function in the form $f(x) = 2(x - h)^2 + k$. [4 marks]

(b) Solve the equation $f(x) = 0$.

[3 marks]



Do **NOT** write solutions on this page.

9. [Maximum mark: 15]

Consider the function $f(x) = x^2 - 4x + 1$.

- (a) Sketch the graph of f , for $-1 \leq x \leq 5$.

[4 marks]

This function can also be written as $f(x) = (x - p)^2 - 3$.

- (b) Write down the value of p .

[1 mark]

The graph of g is obtained by reflecting the graph of f in the x -axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

- (c) Show that $g(x) = -x^2 + 4x + 5$.

[4 marks]

The graphs of f and g intersect at two points.

- (d) Write down the x -coordinates of these two points.

[3 marks]

Let R be the region enclosed by the graphs of f and g .

- (e) Find the area of R .

[3 marks]



Turn over

7. [Maximum mark: 7]

Let $f(t) = 2t^2 + 7$, where $t > 0$. The function v is obtained when the graph of f is transformed by

a stretch by a scale factor of $\frac{1}{3}$ parallel to the y -axis,
followed by a translation by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

- (a) Find $v(t)$, giving your answer in the form $a(t-b)^2+c$. [4 marks]
- (b) A particle moves along a straight line so that its velocity in ms^{-1} , at time t seconds, is given by v . Find the distance the particle travels between $t = 5.0$ and $t = 6.8$. [3 marks]



2. [Maximum mark: 6]

Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down. The graph of g is the image of the graph of f after this translation.

- (a) Write down the coordinates of the vertex of the graph of g . [2 marks]

- (b) Express g in the form $g(x) = 3(x - p)^2 + q$. [2 marks]

The graph of h is the reflection of the graph of g in the x -axis.

- (c) Write down the coordinates of the vertex of the graph of h . [2 marks]

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Turn over

Function Transformation Log P1

3. [Maximum mark: 7]

Let $f(x) = 3x - e^{x-2} - 4$, for $-1 \leq x \leq 5$.

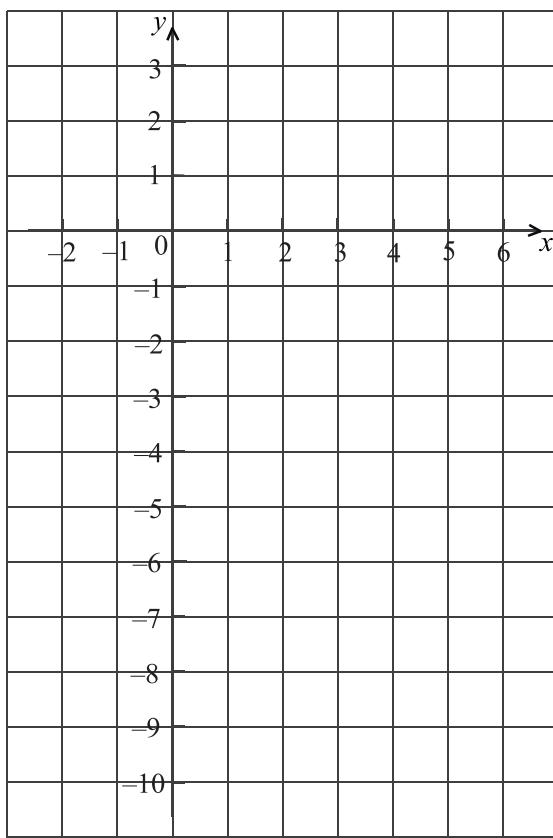
- (a) Find the x -intercepts of the graph of f .

[3 marks]

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- (b) On the grid below, sketch the graph of f .

[3 marks]



(This question continues on the following page)



10. [Maximum mark: 17]

A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After n years the number of taxis, T , in the city is given by

$$T = 280 \times 1.12^n.$$

- (a) (i) Find the number of taxis in the city at the end of 2005.
 - (ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000. [6 marks]
 - (b) At the end of 2000 there were 25 600 people in the city who used taxis. After n years the number of people, P , in the city who used taxis is given by
- $$P = \frac{2\ 560\ 000}{10 + 90e^{-0.1n}}.$$
- (i) Find the value of P at the end of 2005, giving your answer to the nearest whole number.
 - (ii) After seven complete years, will the value of P be double its value at the end of 2000? Justify your answer. [6 marks]
 - (c) Let R be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$.
 - (i) Find the value of R at the end of 2000.
 - (ii) After how many complete years will the city first reduce the number of taxis? [5 marks]



7. [Maximum mark: 8]

The number of bacteria, n , in a dish, after t minutes is given by $n = 800e^{0.13t}$.

- (a) Find the value of n when $t = 0$. [2 marks]
- (b) Find the rate at which n is increasing when $t = 15$. [2 marks]
- (c) After k minutes, the rate of increase in n is greater than 10 000 bacteria per minute. Find the least value of k , where $k \in \mathbb{Z}$. [4 marks]
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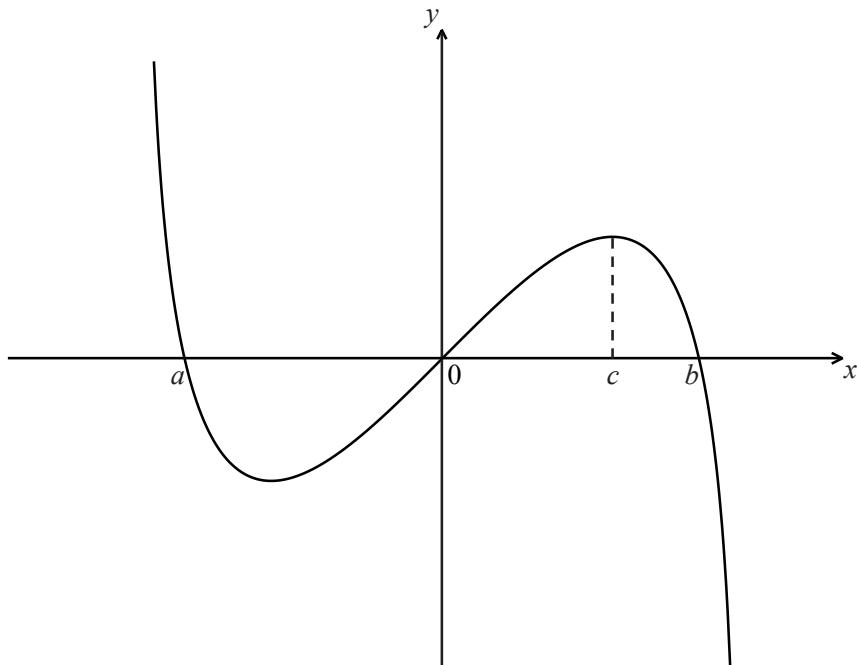
Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

SECTION B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 12]

Let $f(x) = x \ln(4 - x^2)$, for $-2 < x < 2$. The graph of f is shown below.



The graph of f crosses the x -axis at $x = a$, $x = 0$ and $x = b$.

- (a) Find the value of a and of b .

[3 marks]

2. [Maximum mark: 6]

Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down. The graph of g is the image of the graph of f after this translation.

(a) Write down the coordinates of the vertex of the graph of g . [2 marks]

(b) Express g in the form $g(x) = 3(x - p)^2 + q$. [2 marks]

The graph of h is the reflection of the graph of g in the x -axis.

(c) Write down the coordinates of the vertex of the graph of h . [2 marks]



Turn over

Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

10. [Maximum mark: 16]

Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

(a) Show that $f(x) = \log_3 2x$. [2 marks]

(b) Find the value of $f(0.5)$ and of $f(4.5)$. [3 marks]

The function f can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$.

(c) (i) Write down the value of a and of b .

(ii) Hence on graph paper, sketch the graph of f , for $-5 \leq x \leq 5$, $-5 \leq y \leq 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote. [6 marks]

(d) Write down the value of $f^{-1}(0)$. [1 mark]

The point A lies on the graph of f . At A, $x = 4.5$.

(e) On your diagram, sketch the graph of f^{-1} , noting clearly the image of point A. [4 marks]



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** questions in the boxes provided.

1. [Maximum mark: 5]

Let $f(x) = 3x$, $g(x) = 2x - 5$ and $h(x) = (f \circ g)(x)$.

(a) Find $h(x)$. [2 marks]

(b) Find $h^{-1}(x)$. [3 marks]



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

Let $f(x) = 2x + 4$ and $g(x) = 7x^2$.

(a) Find $f^{-1}(x)$. [3 marks]

(b) Find $(f \circ g)(x)$. [2 marks]

(c) Find $(f \circ g)(3.5)$. [2 marks]



6. [Maximum mark: 8]

Jose takes medication. After t minutes, the concentration of medication left in his bloodstream is given by $A(t) = 10(0.5)^{0.014t}$, where A is in milligrams per litre.

- (a) Write down $A(0)$. [1 mark]
- (b) Find the concentration of medication left in his bloodstream after 50 minutes. [2 marks]
- (c) At 13:00, when there is no medication in Jose's bloodstream, he takes his first dose of medication. He can take his medication again when the concentration of medication reaches 0.395 milligrams per litre. What time will Jose be able to take his medication again? [5 marks]

**Turn over**

7. [Maximum mark: 7]

Let $f(t) = 2t^2 + 7$, where $t > 0$. The function v is obtained when the graph of f is transformed by

a stretch by a scale factor of $\frac{1}{3}$ parallel to the y -axis,
followed by a translation by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

- (a) Find $v(t)$, giving your answer in the form $a(t-b)^2+c$. [4 marks]
- (b) A particle moves along a straight line so that its velocity in ms^{-1} , at time t seconds, is given by v . Find the distance the particle travels between $t = 5.0$ and $t = 6.8$. [3 marks]



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Do **NOT** write solutions on this page.

9. [Maximum mark: 15]

Consider the function $f(x) = x^2 - 4x + 1$.

- (a) Sketch the graph of f , for $-1 \leq x \leq 5$.

[4 marks]

This function can also be written as $f(x) = (x - p)^2 - 3$.

- (b) Write down the value of p .

[1 mark]

The graph of g is obtained by reflecting the graph of f in the x -axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

- (c) Show that $g(x) = -x^2 + 4x + 5$.

[4 marks]

The graphs of f and g intersect at two points.

- (d) Write down the x -coordinates of these two points.

[3 marks]

Let R be the region enclosed by the graphs of f and g .

- (e) Find the area of R .

[3 marks]



Turn over

6. [Maximum mark: 6]

Let f and g be functions such that $g(x) = 2f(x+1) + 5$.

- (a) The graph of f is mapped to the graph of g under the following transformations:

vertical stretch by a factor of k , followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

- (i) k ;
- (ii) p ;
- (iii) q .

[3 marks]

- (b) Let $h(x) = -g(3x)$. The point $A(6, 5)$ on the graph of g is mapped to the point A' on the graph of h . Find A' .

[3 marks]



Turn over

10. [Maximum mark: 14]

Let $f(x) = \frac{3x}{x-q}$, where $x \neq q$.

- (a) Write down the equations of the vertical and horizontal asymptotes of the graph of f . [2]

The vertical and horizontal asymptotes to the graph of f intersect at the point $Q(1, 3)$.

- (b) Find the value of q . [2]

- (c) The point $P(x, y)$ lies on the graph of f . Show that $PQ = \sqrt{(x-1)^2 + \left(\frac{3}{x-1}\right)^2}$. [4]

- (d) Hence find the coordinates of the points on the graph of f that are closest to $(1, 3)$. [6]



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9. [Maximum mark: 15]

Let $f(x) = \cos\left(\frac{\pi}{4}x\right) + \sin\left(\frac{\pi}{4}x\right)$, for $-4 \leq x \leq 4$.

(a) Sketch the graph of f . [3]

(b) Find the values of x where the function is decreasing. [5]

(c) The function f can also be written in the form $f(x) = a \sin\left(\frac{\pi}{4}(x+c)\right)$, where $a \in \mathbb{R}$, and $0 \leq c \leq 2$. Find the value of

(i) a ;

(ii) c . [7]

10. [Maximum mark: 14]

Let $f(x) = \frac{3x}{x-q}$, where $x \neq q$.

(a) Write down the equations of the vertical and horizontal asymptotes of the graph of f . [2]

The vertical and horizontal asymptotes to the graph of f intersect at the point $Q(1, 3)$.

(b) Find the value of q . [2]

(c) The point $P(x, y)$ lies on the graph of f . Show that $PQ = \sqrt{(x-1)^2 + \left(\frac{3}{x-1}\right)^2}$. [4]

(d) Hence find the coordinates of the points on the graph of f that are closest to $(1, 3)$. [6]



Function Transformation Log P2

1.

[Maximum mark: 7]

Let $f(x) = \ln(x+5) + \ln 2$, for $x > -5$.

(a) Find $f^{-1}(x)$.

[4 marks]

Let $g(x) = e^x$.

(b) Find $(g \circ f)(x)$, giving your answer in the form $ax+b$, where $a, b \in \mathbb{Z}$.

[3 marks]

2.

Let $f(x) = 3(x+1)^2 - 12$.

(a) Show that $f(x) = 3x^2 + 6x - 9$.

[2 marks]

(d) Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the two transformations:

a stretch of scale factor t in the y -direction

followed by

a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

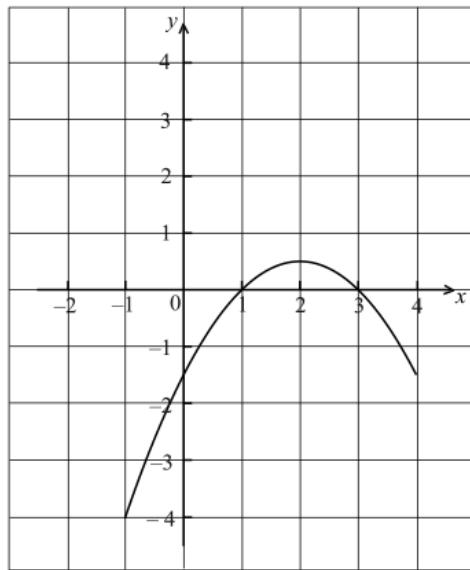
Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t .

[3 marks]

3.

[Maximum mark: 6]

Part of the graph of a function f is shown in the diagram below.

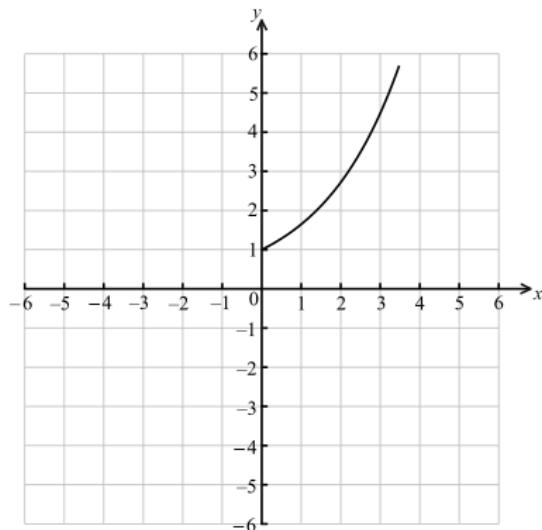


- (a) On the same diagram sketch the graph of $y = -f(x)$. [2 marks]
- (b) Let $g(x) = f(x + 3)$.
- (i) Find $g(-3)$.
- (ii) Describe **fully** the transformation that maps the graph of f to the graph of g . [4 marks]

4.

[Maximum mark: 7]

Let f be the function given by $f(x) = e^{0.5x}$, $0 \leq x \leq 3.5$. The diagram shows the graph of f .



- (a) On the same diagram, sketch the graph of f^{-1} . [3 marks]
- (b) Write down the range of f^{-1} . [1 mark]
- (c) Find $f^{-1}(x)$. [3 marks]

5.

[Maximum mark: 6]

Let $f(x) = x^2$ and $g(x) = 2(x-1)^2$.

- (a) The graph of g can be obtained from the graph of f using two transformations.
Give a full geometric description of each of the two transformations. [2 marks]
- (b) The graph of g is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give the graph of h .
The point $(-1, 1)$ on the graph of f is translated to the point P on the graph of h .
Find the coordinates of P. [4 marks]

6.

[Maximum mark: 7]

Let $f(x) = e^{x+3}$.

(a) (i) Show that $f^{-1}(x) = \ln x - 3$.

(ii) Write down the domain of f^{-1} .

[3 marks]

(b) Solve the equation $f^{-1}(x) = \ln\left(\frac{1}{x}\right)$.

[4 marks]

7.

[Maximum mark: 8]

Let $f(x) = 2x^3 + 3$ and $g(x) = e^{3x} - 2$.

(a) (i) Find $g(0)$.

(ii) Find $(f \circ g)(0)$.

[5 marks]

(b) Find $f^{-1}(x)$.

[3 marks]

8.

[Maximum mark: 6]

Let $f(x) = 2x - 1$ and $g(x) = 3x^2 + 2$.

(a) Find $f^{-1}(x)$.

[3 marks]

(b) Find $(f \circ g)(1)$.

[3 marks]

9.

[Maximum mark: 7]

Find the value of

(a) $\log_2 40 - \log_2 5$;

[3 marks]

(b) $8^{\log_2 5}$.

[4 marks]

10.

[Maximum mark: 7]

Let $\log_3 p = 6$ and $\log_3 q = 7$.

(a) Find $\log_3 p^2$. [2 marks]

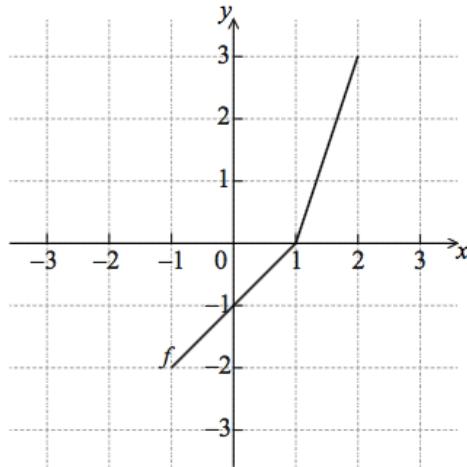
(b) Find $\log_3 \left(\frac{p}{q} \right)$. [2 marks]

(c) Find $\log_3 (9p)$. [3 marks]

11.

[Maximum mark: 6]

The diagram below shows the graph of a function f , for $-1 \leq x \leq 2$.

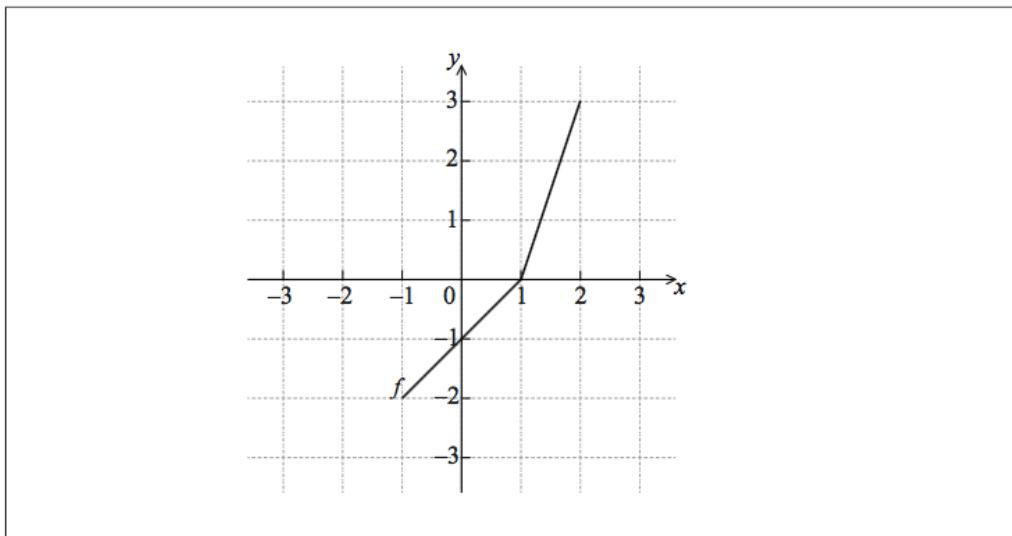


(a) Write down the value of

- (i) $f(2)$;
- (ii) $f^{-1}(-1)$. [3 marks]

- (b) Sketch the graph of f^{-1} on the grid below.

[3 marks]



12.

[Maximum mark: 14]

Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

- (a) Find $f^{-1}(x)$.

[2]

- (b) Show that $(g \circ f^{-1})(x) = \frac{5}{x+2}$.

[2]

Let $h(x) = \frac{5}{x+2}$, for $x \geq 0$. The graph of h has a horizontal asymptote at $y = 0$.

- (c) (i) Find the y -intercept of the graph of h .

- (ii) Hence, sketch the graph of h .

[5]

- (d) For the graph of h^{-1} ,

- (i) write down the x -intercept;

- (ii) write down the equation of the vertical asymptote.

[2]

- (e) Given that $h^{-1}(a) = 3$, find the value of a .

[3]