

UNIT 7 *Ratio and Proportion*

Teaching Notes

Historical Background and Introduction

Ratio and proportion are concepts introduced by the mathematician *Euclid* (about 300 BC) in his Book V. The definitions are reproduced below:

Definitions from Book V

1. A magnitude is a *part* of a magnitude, the less of the greater, when it measures (divides) the greater.
2. The greater is a *multiple* of the less when it is measured by the less.
3. A *ratio* is a sort of the relation in respect to quantity between two magnitudes of the same kind.
4. Magnitudes are said to *have a ratio* to one another which are capable, when multiplied, of exceeding one another.
5. Magnitudes are said to *be in the same ratio*, the first to the second and the third to the fourth, when, if any equal multiples whatever are taken of the first and third, and any equal multiples whatever of the second and fourth, the former multiples alike exceed, are alike to, or alike fall short of, the latter multiples respectively taken in corresponding order.
6. Let magnitudes which have the same ratio be called *proportional*.
7. When, of the equimultiples, the multiple of the first magnitude exceeds the multiple of the second, but the multiple of the third does not exceed the multiple of the fourth, then the first is said to *have a greater ratio* to the second than the third has to the fourth.
9. When three magnitudes are proportional, the first is said to have to the third the *duplicate ratio* of that which it has to the second.
10. When four magnitudes are continuously proportional, the first is said to have to the fourth the *triplicate ratio* of that which it has to the second.

The key part of these definitions is number 5, where the notion of equal ratios is introduced. Today we would think of this as equivalent fractions but the Greeks of Euclid's time did not use fractions in their formal work.

In this unit, we revise the concept of equivalent fractions, and also bring in the concept of map scales in the form $1 : n$. Using real maps and noting their different scales would be a useful introduction. The unit also deals with linear conversion; changes to currency conversion rates could be followed, over a period of time, to illustrate their effects (it should also be noted that rates differ according to whether you are buying or selling, and that most money changers also add commission charges).

Routes

		Standard	Academic	Express
7.1	Equivalent Ratios	✓	✓	✓
7.2	Direct Proportion	✓	✓	✓
7.3	Proportional Division	(✓)	✓	✓
7.4	Linear Conversion	✓	✓	✓
7.5	Inverse Proportion	×	×	✓

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Language

	Standard	Academic	Express
Equivalent ratios	✓	✓	✓
Direct proportion	✓	✓	✓
Proportional division	(✓)	✓	✓
Linear conversion	✓	✓	✓
Inverse proportion	×	×	✓

Misconceptions

- pupils often do not realise that, for example, $2 : 8$ is the same ratio as $1 : 4$.
- it is a misconception that increasing a map scale increases the map distance (in fact, it decreases the map distance).
- mistakes occur when using direct instead of proportional division; for example, if it takes 4 people 2 hours to address 200 envelopes, it takes 2 people 1 hour (incorrect) rather than the correct answer of 4 hours.

Challenging Questions

The following questions are more challenging than others in the same section:

	<i>Section</i>	<i>Question No.</i>	<i>Page</i>
<i>Practice Book Y8A</i>	7.1	8	118
" "	7.3	10	120
" "	7.4	9	123
" "	7.5	7	126