UNIT 1 Indices

NC: Number 2a

		St	Ac	Ex	Sp
TOPICS (Text and Practice Books)					
1.1	*Multiplication and Division*	✓	-	-	-
1.2	*Square, Cube, Square Root and Cube Root*	✓	✓	-	-
1.3	*Index Notation*	✓	✓	✓	✓
1.4	*Factors*	✓	✓	-	-
1.5	*Prime Factors*	✓	✓	✓	✓
1.6	*Further Index Notation*	✗	✗	✓	✓
1.7	*Standard Form*	✗	✓	✓	✓
1.8	*Calculations with Standard Form*	✗	✓	✓	✓
Activities (* particularly suitable for coursework tasks)					
1.1	*Multiplication Table*	✓	-	-	-
1.2	*Secret Letter*	✓	-	-	-
1.3*	*Last Digit*	✓	✓	-	-
1.4*	*Diagonals*	✓	✓	✓	-
1.5*	*Stepping Stones*	✓	✓	✓	-
1.6	*Factors*	✓	✓	-	-
1.7	*Sieve of Eratosthenes*	✓	✓	✓	✓
1.8*	*Chain Letters*	✗	✓	✓	✓
1.9	*Define*	✓	✓	✓	✓
1.10	*Chess Towers*	✓	✓	✓	✓
1.11	*Standard Index Form*	✗	✗	✓	✓
1.12	*Division Networks*	✓	✓	-	-
OH Slides					
1.1	*Square, Cube, Square Root, Cube Root*	✓	✓	-	-
1.2	*Time Scale*	✗	✓	✓	✓
1.3	*Space Scale*	✗	✗	✓	✓
1.4	*Prefixes and Abbreviations*	✗	✗	✓	✓
Mental Tests					
1.1		✓	✓	-	-
1.2		✓	✓	-	-
1.3		✓	✓	✓	✓
1.4		✓	✓	✓	✓
1.5		✗	✗	✓	✓
1.6		✗	✗	✓	✓
1.7		✗	✗	✓	✓
1.8		✗	✗	✓	✓
Revision Tests					
1.1		✓	-	-	-
1.2		✗	✓	-	-
1.3		✗	✗	✓	✓

UNIT 1 *Indices* Teaching Notes

Background and Preparatory Work

General use of decimal notation for whole numbers and decimal fractions dates from 1585 when **Simon Stevin** (1548–1620) published his book, *Die Thiende*. Stevin used powers of 10 to introduce place value and showed how the algebra of powers (the *index laws*) led to relatively simple ways of doing arithmetic. We write a number such as

three hundred and sixteen and a quarter

in terms of powers of 10 as

$$3 \times 10^2 + 1 \times 10^1 + 6 \times 10^0 + 2 \times 10^{-1} + 5 \times 10^{-2}$$

and shorten this to 316.25.

Here $10^n = \underbrace{10 \times 10 \times \ldots \times 10}_{(n \text{ times})}$, when $n > 0$,

$$10^0 = 1, \text{ and } 10^{-n} = \frac{1}{10^n}.$$

When we multiply 316.25 by 10 we use the index law,

$$10^n \times 10 = 10^{n+1}$$

(and the distributive law) to obtain the quick answer 3162.5.

The two basic *index laws*,

$$10^a \times 10^b = 10^{a+b} \quad \text{and} \quad \left(10^a\right)^b = 10^{ab},$$

can be easily checked from the definitions when a and b are positive integers. A little more thought is needed when a and/or b are negative integers (or fractions!) The definitions of $10^0 \,(=1)$ and $10^{-n}\left(=\dfrac{1}{10^n}\right)$, and later of $10^{\frac{1}{2}}\left(=\sqrt{10}\right)$, are chosen to ensure that the basic index laws

$$x^a \times x^b = x^{a+b}, \quad \left(x^a\right)^b = x^{ab} \quad \text{and} \quad x^a . y^a = (xy)^a$$

remain true.

[Note: some care is needed when $x \le 0$, 0^a is not defined when $a \le 0$, and x^a may have no meaning when $x < 0$ and a is fractional.]

The index laws allow us to write with very large numbers in a compact and manageable form. For example, the number of atoms in the universe is frighteningly large but elementary arguments show that this number is approximately 10^{50}. Scientific notation provides an agreed way of giving in standard form the approximate value of very large numbers which occur in science, e.g.

$$2^{10} = 1024 = 1.024 \times 10^3 \approx 1 \times 10^3$$

$$2^{20} = 1\,048\,576 \approx 1.05 \times 10^6.$$

Writing numbers in this form makes it easy to do rough calculations.

F. Land
The Language of Mathematics
ISBN 0719530725
(Murray)

E. T. Bell
Men of Mathematics
ISBN 0671628186
(Schuster)

G. Stephenson
Inequalities and Optimum Problems in Mathematics and the Sciences
ISBN 0 582 4423 3
(Longman)

A. Gardiner
Infinite Processes
(Springer) 1982

For example,

$$2^{40} = \left(2^{20}\right)^2 \approx \left(1.05 \times 10^6\right)^2$$

$$\left(1.05 \times 10^6\right)^2 = (1.05)^2 \times \left(10^6\right)^2 = 1.1025 \times 10^{12}$$

$$2^{-20} = \frac{1}{2^{20}} \approx \frac{1}{1.05 \times 10^6} \approx 0.95 \times 10^{-6}$$

$$0.95 \times 10^{-6} = 9.5 \times 10^{-7}$$

Teaching Points

Introduction

As this is the first unit in Y10, it is important to strike the right balance from the beginning. It is in fact an ideal topic to engage in whole class activities, using

- revision mental tests
- interactive activities.

M1.1 to 1.8

A1.1, 1.6, 1.7, 1.9, 1.10, 1.11

Topics and concepts should be reinforced at all times; it is important to spend more time now ensuring that pupils understand the principles (such as the *laws of indices*) and are proficient in their use, than moving on too fast and allowing pupils to remain confused throughout Y10 and Y11 (and, for many, throughout A- Level!) When you go over the mental tests, make sure that mistakes (and misconceptions) are discussed with the whole class.

- For *Standard* route pupils, the use of multilink cubes can help in clarifying what is meant by square, cube, square root and cube root, etc.

OS1.1

- For all other routes, it is important to stress the need for both index notation and standard form. It is essentially a very efficient way of representing and calculating very large and very small numbers.

OS1.2 and 1.3

- For the *Express* and *Special* routes the use of standard form in all the sciences should be stressed.

Language / Notation

- It is important that students are clear what the mathematical terms used really mean, e.g. square, cube, power, root, index, factor, multiple, common factor, common multiple, prime number, reciprocal, standard form.
- Note that the plural of index is indices.
- Keep referring to these terms, checking with the whole class their understanding of the meaning.

Key Points

- 'Standard form', 'scientific notation', 'standard index notation' all mean the same thing.
- A number written in standard form must consist of **three** parts:
  (i) a decimal number in which there is one non-zero number before the decimal point,
  (ii) a multiplication sign,
  (iii) 10 to the power of an integer (including 0, ± 1, etc.)
- Some calculator displays for standard form are of the form
  3.25 E – 0.6 or 3.25^{-06}, which actually mean 3.25×10^{-6}.

Misconceptions

- a^n does **not** mean $a \times n$
  $\sqrt{a}$ does **not** mean $\frac{1}{2} \times a$
  $\sqrt[3]{a}$ does **not** mean $\frac{1}{3} \times a$
- Students also often have difficulty understanding that a^1 is actually a, and the definition that $a^0 = 1$ might give rise to problems.
- They may well have problems with $a^{\frac{1}{2}} = \sqrt{a}$ and might confuse $\left(a^n\right)^m$ with $a^n \times a^m$.
  For these later manipulations, always convince your students by writing out examples fully, e.g.

$$\begin{aligned}\left(a^4\right)^3 &= a^4 \times a^4 \times a^4 \\ &= (a \times a \times a \times a) \times (a \times a \times a \times a) \times (a \times a \times a \times a) \\ &= a^{12}.\end{aligned}$$

Key Concepts

Laws of Indices	**St**	**A**	**E**	**Sp**
1. $a^n = a \times a \times a \times ... \times a$ (n times)	✓	✓	✓	✓
2. $a^1 = a$	✓	✓	✓	✓
3. $a^0 = 1$	✓	✓	✓	✓
4. $a^m \times a^n = a^{m+n}$	✗	✓	✓	✓
5. $a^m \div a^n = a^{m-n}$	✗	✓	✓	✓
6. $\left(a^n\right)^m = a^{nm}$	✗	✓	✓	✓
7. $a^{-n} = \dfrac{1}{a^n}$	✗	✗	✓	✓
8. $a^{\frac{1}{n}} = \sqrt[n]{a}$	✗	✗	✓	✓
9. $a^{\frac{n}{m}} = \sqrt[m]{a^n}$	✗	✗	✓	✓

Activities

1.1 Multiplication Table

This is particularly suitable for an introduction or as revision of earlier work for the *Standard* route. It could be used either as a whole class activity or individually.

1.2 Secret Letters

Good revision/homework activity for the *Standard* and *Academic* routes.

1.3, 1.4, 1.5

Suitable starters for coursework (see below).

1.6 Factors

Suitable for revision of earlier work and can be used for whole class activity or homework – again particularly suitable for *Standard* but also *Academic*.

1.7 Sieve of Eratosthenes

Excellent activity for discovering primes – suitable for *Standard* and *Academic*, and *Express* could use it for finding primes between 100 and 200, etc.

1.8 Chain Letters

Suitable tasks for coursework (see below).

1.9 Define

Good whole class interactive activity which could be used with all pupils.

1.10 Chess Towers

Shows the advantage of index notation – again, it could be used either for whole class discussion or individually.

1.11 Standard Index Form

Particularly suitable for whole class discussion for *Express* and *Special* routes.

1.12 Division Networks

This is a straightforward activity to check on factors.

Applications

The use of index notation, particularly standard form, is crucial in nearly all areas of science, including engineering, astronomy, biology and medicine. For many applications, prefixes are commonly used as shown below. 0S 1.4

Prefix	Abbrev.	Multiples
mega	M	10^6
kilo	k	10^3
. . . .	. . . .	. . . .
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}

In mathematics, we tend to restrict this type of use to, for example, kg (kilogram), km (kilometre), cm (centimetre), mm (millimetre).

Coursework

This particular topic, on its own, is not really suitable for coursework except for

C1/1 *Public Key Cryptographs*

in which index notation is used with prime numbers. However, this is essential for the *Express* and *Special* routes.