UNIT 16 *Algebra: Linear Equations* Teaching Notes

Historical Background and Introduction

In one sense, people use 'formulae' all the time, but often without realising it: a glazier may use tables to work out the cost of panes of glass, and a decorator will have his own 'rule of thumb' for estimating how much paint is needed for the outside of a house. The 'routines' calculate a *required number* (the price of glass, or the number of litres of paint needed), but they are often difficult to understand because they are *not* expressed mathematically.

A mathematical formula should take the form of an *equation* which expresses some required quantity in terms of other, easily measured quantities. For example,

$$\text{area of rectangle} = \text{length} \times \text{breadth},$$

and although this is a good beginning, it is *only* a beginning. The ancient Babylonians and the Egyptians (c. 2000 BC) used many approximate calculational 'routines' and 'formulae' of this kind, but mathematics cannot be done efficiently with words.

To go further we have to replace friendly words with abstract symbols and extend the familiar arithmetic numbers with an 'arithmetic of letters': that is, we need to develop *algebra.*

The word 'algebra' comes from the title of a book

Al - jabr w'al muqabala

written in 830 AD by the Arabic astronomer Al - Khurarizuri. The exact translation of the title is disputed: 'al - jabr' means something like "restoring and balancing" (which refers to the idea of shifting things from one side of an equation to the other), and 'w'al muqabala' means something like "cancelling and simplifying". These two ideas reflect the central art of algebra – namely that of '*rearranging and simplifying expressions*'.

Full-blooded elementary algebra in which, for example, the general quadratic equation can be written as

$$ax^2 + bx + c = 0$$

and its solution given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

was only developed after 1600 AD: the effect was dramatic. After waiting 3500 years for an effective language, the next century saw an explosive growth – with the rise of coordinate geometry (Descartes 1637) and calculus (Newton, Leibniz 1660 - 1684).

Around 1600, Galileo observed that "The Book of Nature is written in the language of mathematics": it soon became clear that the language of *all* mathematics is algebra!

Routes	**Standard**	**Academic**	**Express**
16.1 Fundamental Algebraic Skills	✓	✓	✓
16.2 Function Machines	✓	✓	✓
16.3 Linear Equations	(✓)	✓	✓
Language			
Formulae	✓	✓	✓
Codes	✓	✓	✓
Function Machines	✓	✓	✓
Input	✓	✓	✓
Output	✓	✓	✓
Operation and Inverse Operation	✓	✓	✓
Equation	✓	✓	✓

(✓) *denotes extension work for these pupils*

Misconceptions

Misconceptions abound in algebra as pupils do not stick to the rules; for example, when solving equations, common misconceptions are:

- $\frac{x}{2} = 4 \Rightarrow x = 2$
- $x + 3 = 5 \Rightarrow x = 5 + 3 = 8$
- $x - (-4) = 0 \Rightarrow x = 4$