1. The discrete random variable x has probability distribution given by

x	-1	0	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{5}$	a	$\frac{1}{10}$	a	$\frac{1}{5}$

where a is a constant.

(a) Find the value of a.
(2)

(b) Write down $\mathrm{E}(X)$.
(1)

(c) Find $\mathrm{Var}(X)$.
(3)

The random variable $Y = 6 - 2X$

(d) Find $\mathrm{Var}(Y)$.
(2)

(e) Calculate $\mathrm{P}(X \geq Y)$.
(3)
(Total 11 marks)

2. The probability function of a discrete random variable x is given by

$$\mathrm{p}(x) = kx^2 \qquad x = 1, 2, 3$$

where k is a positive constant.

(a) Show that $k = \frac{1}{14}$
(2)

Find

(b) $P(X \geq 2)$

(2)

(c) $E(X)$

(2)

(d) $\text{Var}(1 - X)$

(4)
(Total 10 marks)

3. The discrete random variable X has probability function

$$P(X = x) = \begin{cases} a(3-x) & x = 0,1,2 \\ b & x = 3 \end{cases}$$

(a) Find $P(X = 2)$ and complete the table below.

x	0	1	2	3
$P(X = x)$	$3a$	$2a$		b

(1)

Given that $E(X) = 1.6$

(b) Find the value of a and the value of b.

(5)

Find

(c) $P(0.5 < X < 3)$,

(2)

(d) $E(3X - 2)$.

(2)

(e) Show that the Var(X) = 1.64

(3)

(f) Calculate Var($3X - 2$).

(2)
(Total 15 marks)

4. When Rohit plays a game, the number of points he receives is given by the discrete random variable X with the following probability distribution.

x	0	1	2	3
P($X = x$)	0.4	0.3	0.2	0.1

(a) Find E(X).

(2)

(b) Find F(1.5).

(2)

(c) Show that Var(X) = 1

(4)

(d) Find Var($5 - 3X$).

(2)

Rohit can win a prize if the total number of points he has scored after 5 games is at least 10.
After 3 games he has a total of 6 points.
You may assume that games are independent.

(e) Find the probability that Rohit wins the prize.

(6)
(Total 16 marks)

5. The random variable X has probability distribution given in the table below.

X	-1	0	1	2	3
$P(X = x)$	p	q	0.2	0.15	0.15

Given that $E(X) = 0.55$, find

(a) the value of p and the value of q,
(5)

(b) $\text{Var}(X)$,
(4)

(c) $E(2X - 4)$.
(2)
(Total 11 marks)

6. The discrete random variable X can take only the values 2, 3 or 4. For these values the cumulative distribution function is defined by

$$F(x) = \frac{(x+k)^2}{25} \text{ for } x = 2,3,4$$

where k is a positive integer.

(a) Find k.
(2)

(b) Find the probability distribution of X.
(3)
(Total 5 marks)

7. The random variable X has probability distribution

x	1	3	5	7	9
$P(X = x)$	0.2	p	0.2	q	0.15

(a) Given that $E(X) = 4.5$, write down two equations involving p and q.
(3)

Find

(b) the value of p and the value of q,

(3)

(c) $P(4 < X \leq 7)$.

(2)

Given that $E(X^2) = 27.4$, find

(d) $Var(X)$,

(2)

(e) $E(19 - 4X)$,

(1)

(f) $Var(19 - 4X)$.

(2)
(Total 13 marks)

8. The random variable X has the discrete uniform distribution

$$P(X = x) = \frac{1}{5}, \qquad x = 1, 2, 3, 4, 5.$$

(a) Write down the value of $E(X)$ and show that $Var(X) = 2$.

(3)

Find

(b) $E(3X - 2)$,

(2)

(c) $Var(4 - 3X)$.

(2)
(Total 7 marks)

9. The random variable X has probability distribution

x	1	2	3	4	5
$\mathrm{P}(X = x)$	0.10	p	0.20	q	0.30

(a) Given that $\mathrm{E}(X) = 3.5$, write down two equations involving p and q.
(3)

Find

(b) the value of p and the value of q,
(3)

(c) $\mathrm{Var}(X)$,
(4)

(d) $\mathrm{Var}(3 - 2X)$.
(2)
(Total 12 marks)

10. The random variable X has probability function

$$\mathrm{P}(X = x) = \begin{cases} kx, & x = 1, 2, 3, \\ k(x+1), & x = 4, 5, \end{cases}$$

where k is a constant.

(a) Find the value of k.
(2)

(b) Find the exact value of $\mathrm{E}(X)$.
(2)

(c) Show that, to 3 significant figures, $\mathrm{Var}(X) = 1.47$.
(4)

(d) Find, to 1 decimal place, Var(4 – 3X).

(2)
(Total 10 marks)

11. The random variable X has probability function

$$P(X = x) = kx, \qquad x = 1, 2, ..., 5.$$

(a) Show that $k = \frac{1}{15}$.

(2)

Find

(b) $P(X < 4)$,

(2)

(c) E(X),

(2)

(d) E(3X – 4).

(2)
(Total 8 marks)

12. A discrete random variable X has a probability function as shown in the table below, where a and b are constants.

x	0	1	2	3
$P(X = x)$	0.2	0.3	b	a

Given that E(X) = 1.7,

(a) find the value of a and the value of b.

(5)

Find

(b) $P(0 < X < 1.5)$,

(1)

(c) $E(2X - 3)$.

(2)

(d) Show that $Var(X) = 1.41$.

(3)

(e) Evaluate $Var(2X - 3)$.

(2)
(Total 13 marks)

13. A discrete random variable X has the probability function shown in the table below.

x	0	1	2	3
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{12}$

Find

(a) $P(1 < X \leq 3)$,

(2)

(b) $F(2.6)$,

(1)

(c) $E(X)$,

(2)

(d) $E(2X - 3)$,

(2)

(e) Var(X)

(3)
(Total 10 marks)

14. A fairground game involves trying to hit a moving target with a gunshot. A round consists of up to 3 shots. Ten points are scored if a player hits the target, but the round is over if the player misses. Linda has a constant probability of 0.6 of hitting the target and shots are independent of one another.

(a) Find the probability that Linda scores 30 points in a round.

(2)

The random variable X is the number of points Linda scores in a round.

(b) Find the probability distribution of X.

(5)

(c) Find the mean and the standard deviation of X.

(5)

A game consists of 2 rounds.

(d) Find the probability that Linda scores more points in round 2 than in round 1.

(6)
(Total 18 marks)

15. The random variable X has the discrete uniform distribution

$$P(X = x) = \frac{1}{n}, \qquad x = 1, 2, ..., n.$$

Given that E(X) = 5,

(a) show that $n = 9$.

(3)

Find

(b) $P(X < 7)$,

(2)

(c) Var (X).

(4)
(Total 9 marks)

16. The discrete random variable X has probability function

$$P(X = x) = \begin{cases} k(x^2 - 9), & x = 4, 5, 6 \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

(a) Show that $k = \dfrac{1}{50}$.

(3)

(b) Find E(X) and Var(X).

(6)

(c) Find Var($2X - 3$).

(2)
(Total 11 marks)

17. The random variable X represents the number on the uppermost face when a fair die is thrown.

(a) Write down the name of the probability distribution of X.

(1)

(b) Calculate the mean and the variance of X.

(3)

Three fair dice are thrown and the numbers on the uppermost faces are recorded.

(c) Find the probability that all three numbers are 6.

(2)

(d) Write down all the different ways of scoring a total of 16 when the three numbers are added together.

(4)

(e) Find the probability of scoring a total of 16.

(2)
(Total 12 marks)

18. The discrete random variable X has probability function

$$P(X = x) = \begin{cases} k(2-x), & x = 0, 1, 2, \\ k(x-2), & x = 3, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

(a) Show that $k = 0.25$.

(2)

(b) Find E(X) and show that E(X^2) = 2.5.

(4)

(c) Find Var($3X - 2$).

(3)

Two independent observations X_1 and X_2 are made of X.

(d) Show that $P(X_1 + X_2 = 5) = 0$.

(1)

(e) Find the complete probability function for $X_1 + X_2$.

(3)

(f) Find $P(1.3 \le X_1 + X_2 \le 3.2)$.

(3)
(Total 16 marks)

19. A customer wishes to withdraw money from a cash machine. To do this it is necessary to type a PIN number into the machine. The customer is unsure of this number. If the wrong number is typed in, the customer can try again up to a maximum of four attempts in total. Attempts to type in the correct number are independent and the probability of success at each attempt is 0.6.

(a) Show that the probability that the customer types in the correct number at the third attempt is 0.096.

(2)

The random variable A represents the number of attempts made to type in the correct PIN number, regardless of whether or not the attempt is successful.

(b) Find the probability distribution of A.

(2)

(c) Calculate the probability that the customer types in the correct number in four or fewer attempts.

(2)

(d) Calculate $E(A)$ and $\text{Var}(A)$.

(6)

(e) Find $F(1 + E(A))$.

(2)

(Total 14 marks)