M08/5/MATME/SP1/ENG/TZ1/XX+

22087303

International Baccalaureate®
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Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Wednesday 7 May 2008 (afternoon)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
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SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$.

Find

(a) $\boldsymbol{A} + \boldsymbol{B}$; *[2 marks]*

(b) $-3\boldsymbol{A}$; *[2 marks]*

(c) $\boldsymbol{AB}$. *[3 marks]*

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2. *[Maximum mark: 6]*

Let $p = \sin 40^\circ$ and $q = \cos 110^\circ$. Give your answers to the following in terms of p and/or q.

(a) Write down an expression for

(i) $\sin 140^\circ$;

(ii) $\cos 70^\circ$. *[2 marks]*

(b) Find an expression for $\cos 140^\circ$. *[3 marks]*

(c) Find an expression for $\tan 140^\circ$. *[1 mark]*

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3. *[Maximum mark: 6]*

Consider the arithmetic sequence $2, 5, 8, 11, \ldots$.

(a) Find u_{101}. *[3 marks]*

(b) Find the value of n so that $u_n = 152$. *[3 marks]*

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4. *[Maximum mark: 6]*

Consider $g(x) = 3\sin 2x$.

(a) Write down the period of g. *[1 mark]*

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(b) On the diagram below, sketch the curve of g, for $0 \le x \le 2\pi$. *[3 marks]*

(c) Write down the number of solutions to the equation $g(x) = 2$, for $0 \le x \le 2\pi$. *[2 marks]*

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5. *[Maximum mark: 6]*

(a) Find $\int \frac{1}{2x+3}\,\mathrm{d}x$. *[2 marks]*

(b) Given that $\int_0^3 \frac{1}{2x+3}\,\mathrm{d}x = \ln\sqrt{P}$, find the value of P. *[4 marks]*

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6. *[Maximum mark: 7]*

A particle moves along a straight line so that its velocity, v m s^{-1} at time t seconds is given by $v = 6e^{3t} + 4$. When $t = 0$, the displacement, s, of the particle is 7 metres. Find an expression for s in terms of t.

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7. *[Maximum mark: 7]*

Let $f(x) = \ln(x+5) + \ln 2$, for $x > -5$.

(a) Find $f^{-1}(x)$. *[4 marks]*

Let $g(x) = \mathrm{e}^x$.

(b) Find $(g \circ f)(x)$, giving your answer in the form $ax + b$, where $a, b \in \mathbb{Z}$. *[3 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Consider $f(x) = \frac{1}{3}x^3 + 2x^2 - 5x$. Part of the graph of f is shown below. There is a maximum point at M, and a point of inflexion at N.

(a) Find $f'(x)$. *[3 marks]*

(b) Find the x-coordinate of M. *[4 marks]*

(c) Find the x-coordinate of N. *[3 marks]*

(d) The line L is the tangent to the curve of f at (3 , 12). Find the equation of L in the form $y = ax + b$. *[4 marks]*

9. *[Maximum mark: 15]*

Let $f(x) = 3(x+1)^2 - 12$.

(a) Show that $f(x) = 3x^2 + 6x - 9$. *[2 marks]*

(b) For the graph of f

(i) write down the coordinates of the vertex;

(ii) write down the **equation** of the axis of symmetry;

(iii) write down the y-intercept;

(iv) find both x-intercepts. *[8 marks]*

(c) **Hence** sketch the graph of f. *[2 marks]*

(d) Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the two transformations:

a stretch of scale factor t in the y-direction

followed by

a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t. *[3 marks]*

10. *[Maximum mark: 16]*

A **four-sided** die has three blue faces and one red face. The die is rolled.

Let B be the event a blue face lands down, and R be the event a red face lands down.

(a) Write down

(i) $\mathrm{P}(B)$;

(ii) $\mathrm{P}(R)$. *[2 marks]*

(b) If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the following tree diagram, where p, s, t are probabilities.

Find the value of p, of s and of t. *[2 marks]*

Guiseppi plays a game where he rolls the die. If a blue face lands down, he scores 2 and is finished. If the red face lands down, he scores 1 and rolls one more time. Let X be the total score obtained.

(c) (i) Show that $\mathrm{P}(X=3)=\frac{3}{16}$.

(ii) Find $\mathrm{P}(X=2)$. *[3 marks]*

(d) (i) Construct a probability distribution table for X.

(ii) Calculate the expected value of X. *[5 marks]*

(e) If the total score is 3, Guiseppi wins \$ 10. If the total score is 2, Guiseppi gets nothing.

Guiseppi plays the game twice. Find the probability that he wins exactly \$ 10. *[4 marks]*

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SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

A box contains 100 cards. Each card has a number between one and six written on it. The following table shows the frequencies for each number.

Number	1	2	3	4	5	6
Frequency	26	10	20	k	29	11

(a) Calculate the value of k. *[2 marks]*

(b) Find

(i) the median;

(ii) the interquartile range. *[5 marks]*

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2. *[Maximum mark: 6]*

The following diagram shows part of the graph of f, where $f(x) = x^2 - x - 2$.

(a) Find both x-intercepts. *[4 marks]*

(b) Find the x-coordinate of the vertex. *[2 marks]*

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3. *[Maximum mark: 6]*

Let $\boldsymbol{M} = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}$.

(a) Write down the determinant of $\boldsymbol{M}$. *[1 mark]*

(b) Write down $\boldsymbol{M}^{-1}$. *[2 marks]*

(c) **Hence** solve $\boldsymbol{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$. *[3 marks]*

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4. *[Maximum mark: 6]*

(a) Given that $\cos A = \frac{1}{3}$ and $0 \le A \le \frac{\pi}{2}$, find $\cos 2A$. *[3 marks]*

(b) Given that $\sin B = \frac{2}{3}$ and $\frac{\pi}{2} \le B \le \pi$, find $\cos B$. *[3 marks]*

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5. *[Maximum mark: 6]*

Part of the graph of a function f is shown in the diagram below.

(a) On the same diagram sketch the graph of $y = -f(x)$. *[2 marks]*

(b) Let $g(x) = f(x+3)$.

(i) Find $g(-3)$.

(ii) Describe **fully** the transformation that maps the graph of f to the graph of g. *[4 marks]*

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6. *[Maximum mark: 7]*

There are 20 students in a classroom. Each student plays only one sport. The table below gives their sport and gender.

	Football	**Tennis**	**Hockey**
Female	5	3	3
Male	4	2	3

(a) One student is selected at random.

(i) Calculate the probability that the student is a male or is a tennis player.

(ii) Given that the student selected is female, calculate the probability that the student does not play football. *[4 marks]*

(b) Two students are selected at random. Calculate the probability that neither student plays football. *[3 marks]*

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7. *[Maximum mark: 7]*

Let $\int_1^5 3f(x)\,\mathrm{d}x = 12$.

(a) Show that $\int_5^1 f(x)\,\mathrm{d}x = -4$. *[2 marks]*

(b) Find the value of $\int_1^2 \left(x + f(x)\right)\mathrm{d}x + \int_2^5 \left(x + f(x)\right)\mathrm{d}x$. *[5 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

Consider the points $A(1, 5, 4)$, $B(3, 1, 2)$ and $D(3, k, 2)$, with (AD) perpendicular to (AB).

(a) Find

(i) $\overrightarrow{AB}$;

(ii) $\overrightarrow{AD}$, giving your answer in terms of k. *[3 marks]*

(b) Show that $k = 7$. *[3 marks]*

The point C is such that $\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD}$.

(c) Find the position vector of C. *[4 marks]*

(d) Find $\cos A\hat{B}C$. *[3 marks]*

9. *[Maximum mark: 14]*

Let $f : x \mapsto \sin^3 x$.

(a) (i) Write down the range of the function f.

(ii) Consider $f(x) = 1$, $0 \le x \le 2\pi$. Write down the number of solutions to this equation. Justify your answer. *[5 marks]*

(b) Find $f'(x)$, giving your answer in the form $a \sin^p x \cos^q x$ where $a, p, q \in \mathbb{Z}$. *[2 marks]*

(c) Let $g(x) = \sqrt{3} \sin x (\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of g is revolved through 2π about the x-axis. *[7 marks]*

10. *[Maximum mark: 18]*

The following diagram shows a semicircle centre O, diameter [AB], with radius 2. Let P be a point on the circumference, with $\mathrm{P\hat{O}B} = \theta$ radians.

(a) Find the area of the triangle OPB, in terms of θ. *[2 marks]*

(b) Explain why the area of triangle OPA is the same as the area triangle OPB. *[3 marks]*

Let S be the total area of the two segments shaded in the diagram below.

(c) Show that $S = 2(\pi - 2\sin\theta)$. *[3 marks]*

(d) Find the value of θ when S is a local minimum, justifying that it is a minimum. *[8 marks]*

(e) Find a value of θ for which S has its greatest value. *[2 marks]*

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MATHEMATICS
STANDARD LEVEL
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SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

Consider the infinite geometric sequence $3, 3(0.9), 3(0.9)^2, 3(0.9)^3, \ldots$.

(a) Write down the 10th term of the sequence. Do not simplify your answer. *[1 mark]*

(b) Find the sum of the infinite sequence. *[4 marks]*

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2. *[Maximum mark: 6]*

A particle is moving with a constant velocity along line L. Its initial position is $\mathrm{A}(6, -2, 10)$. After one second the particle has moved to $\mathrm{B}(9, -6, 15)$.

(a) (i) Find the velocity vector, $\overrightarrow{\mathrm{AB}}$.

(ii) Find the speed of the particle. *[4 marks]*

(b) Write down an equation of the line L. *[2 marks]*

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3. *[Maximum mark: 7]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ 3 & p \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -2 & 1 \\ q & \frac{1}{2} \end{pmatrix}$.

(a) Find $\boldsymbol{AB}$ in terms of p and q. *[2 marks]*

(b) Matrix $\boldsymbol{B}$ is the inverse of matrix $\boldsymbol{A}$. Find the value of p and of q. *[5 marks]*

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4. *[Maximum mark: 7]*

Let f be the function given by $f(x) = \mathrm{e}^{0.5x}$, $0 \le x \le 3.5$. The diagram shows the graph of f.

(a) On the same diagram, sketch the graph of f^{-1}. *[3 marks]*

(b) Write down the range of f^{-1}. *[1 mark]*

(c) Find $f^{-1}(x)$. *[3 marks]*

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5. *[Maximum mark: 6]*

Let A and B be independent events, where $\mathrm{P}(A) = 0.6$ and $\mathrm{P}(B) = x$.

(a) Write down an expression for $\mathrm{P}(A \cap B)$. *[1 mark]*

(b) Given that $\mathrm{P}(A \cup B) = 0.8$,

(i) find x;

(ii) find $\mathrm{P}(A \cap B)$. *[4 marks]*

(c) **Hence**, explain why A and B are **not** mutually exclusive. *[1 mark]*

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6. *[Maximum mark: 7]*

The diagram shows part of the graph of $y = f'(x)$. The x-intercepts are at points A and C. There is a minimum at B, and a maximum at D.

(a) (i) Write down the value of $f'(x)$ at C.

(ii) **Hence**, show that C corresponds to a minimum on the graph of f, *i.e.* it has the same x-coordinate. *[3 marks]*

(b) Which of the points A, B, D corresponds to a maximum on the graph of f? *[1 mark]*

(c) Show that B corresponds to a point of inflexion on the graph of f. *[3 marks]*

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7. *[Maximum mark: 7]*

Let $f(x) = \sin^3 x + \cos^3 x \tan x$, $\frac{\pi}{2} < x < \pi$.

(a) Show that $f(x) = \sin x$. *[2 marks]*

(b) Let $\sin x = \frac{2}{3}$. Show that $f(2x) = -\frac{4\sqrt{5}}{9}$. *[5 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.

Let X be the sum of the scores on the two dice.

(a) Find

(i) $\mathrm{P}(X = 6)$;

(ii) $\mathrm{P}(X > 6)$;

(iii) $\mathrm{P}(X = 7 \mid X > 5)$. *[6 marks]*

(b) Elena plays a game where she tosses two dice.

If the sum is 6, she wins 3 points.
If the sum is greater than 6, she wins 1 point.
If the sum is less than 6, she **loses** k points.

Find the value of k for which Elena's expected number of points is zero. *[7 marks]*

9. *[Maximum mark: 16]*

The acceleration, a $\mathrm{m\,s^{-2}}$, of a particle at time t seconds is given by $a = 2t + \cos t$.

(a) Find the acceleration of the particle at $t = 0$. *[2 marks]*

(b) Find the velocity, v, at time t, given that the initial velocity of the particle is $2\,\mathrm{m\,s^{-1}}$. *[5 marks]*

(c) Find $\int_0^3 v\,\mathrm{d}t$, giving your answer in the form $p - q\cos 3$. *[7 marks]*

(d) What information does the answer to part (c) give about the motion of the particle? *[2 marks]*

10. *[Maximum mark: 16]*

Let $f(t) = a\cos b(t-c)+d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.

When $t = 3$, there is a maximum value of 29, at M.
When $t = 9$, there is a minimum value of 15.

(a) (i) Find the value of *a*.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of *d*.

(iv) Write down a value for *c*. *[7 marks]*

The transformation *P* is given by a horizontal stretch of a scale factor of $\frac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$.

(b) Let M' be the image of M under *P*. Find the coordinates of M'. *[2 marks]*

The graph of *g* is the image of the graph of *f* under *P*.

(c) Find $g(t)$ in the form $g(t) = 7\cos B(t-C)+D$. *[4 marks]*

(d) Give a full geometric description of the transformation that maps the graph of *g* to the graph of *f*. *[3 marks]*

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SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

Let $\boldsymbol{A} = \begin{pmatrix} 5 & 1 \\ 6 & 2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & -1 \\ -6 & 5 \end{pmatrix}$.

(a) (i) Find $\boldsymbol{AB}$.

(ii) Write down the inverse of $\boldsymbol{A}$. *[3 marks]*

Let $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$.

(b) Solve the matrix equation $\boldsymbol{AX} = \boldsymbol{C}$. *[4 marks]*

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2. *[Maximum mark: 6]*

The letters of the word PROBABILITY are written on 11 cards as shown below.

P	R	O	B	A	B	I	L	I	T	Y

Two cards are drawn at random without replacement.
Let A be the event the first card drawn is the letter A.
Let B be the event the second card drawn is the letter B.

(a) Find $\mathrm{P}(A)$. *[1 mark]*

(b) Find $\mathrm{P}(B \mid A)$. *[2 marks]*

(c) Find $\mathrm{P}(A \cap B)$. *[3 marks]*

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3. *[Maximum mark: 6]*

Let $f(x) = e^x \cos x$. Find the gradient of the normal to the curve of f at $x = \pi$.

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4. *[Maximum mark: 6]*

The following diagram shows the graphs of the **displacement**, **velocity** and **acceleration** of a moving object as functions of time, t.

(a) Complete the following table by noting which graph A, B or C corresponds to each function. *[4 marks]*

Function	**Graph**
displacement	
acceleration	

(b) Write down the value of t when the velocity is greatest. *[2 marks]*

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5. *[Maximum mark: 6]*

Let $f(x) = x^2$ and $g(x) = 2(x-1)^2$.

(a) The graph of g can be obtained from the graph of f using two transformations. Give a full geometric description of each of the two transformations. *[2 marks]*

(b) The graph of g is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give the graph of h.

The point $(-1, 1)$ on the graph of f is translated to the point P on the graph of h. Find the coordinates of P. *[4 marks]*

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6. *[Maximum mark: 7]*

Let $f(x) = e^{x+3}$.

(a) (i) Show that $f^{-1}(x) = \ln x - 3$.

(ii) Write down the domain of f^{-1}. *[3 marks]*

(b) Solve the equation $f^{-1}(x) = \ln\left(\frac{1}{x}\right)$. *[4 marks]*

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7. *[Maximum mark: 7]*

The graph of $y = \sqrt{x}$ between $x = 0$ and $x = a$ is rotated 360° about the x-axis. The volume of the solid formed is 32π. Find the value of a.

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

A rectangle is inscribed in a circle of radius 3 cm and centre O, as shown below.

The point $\mathrm{P}(x,\ y)$ is a vertex of the rectangle and also lies on the circle. The angle between (OP) and the x-axis is θ radians, where $0 \le \theta \le \frac{\pi}{2}$.

(a) Write down an expression in terms of θ for

(i) x;

(ii) y. *[2 marks]*

Let the area of the rectangle be A.

(b) Show that $A = 18\sin 2\theta$. *[3 marks]*

(c) (i) Find $\frac{\mathrm{d}A}{\mathrm{d}\theta}$.

(ii) Hence, find the exact value of θ which maximizes the area of the rectangle.

(iii) Use the second derivative to justify that this value of θ does give a maximum. *[8 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 16]*

The vertices of the triangle PQR are defined by the position vectors

$$\overrightarrow{OP} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}, \ \overrightarrow{OQ} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ and } \overrightarrow{OR} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}.$$

(a) Find

(i) $\overrightarrow{PQ}$;

(ii) $\overrightarrow{PR}$. *[3 marks]*

(b) Show that $\cos R\hat{P}Q = \frac{1}{2}$. *[7 marks]*

(c) (i) Find $\sin R\hat{P}Q$.

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$. *[6 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 16]*

Let $f(x) = \dfrac{ax}{x^2+1}$, $-8 \le x \le 8$, $a \in \mathbb{R}$. The graph of f is shown below.

The region between $x = 3$ and $x = 7$ is shaded.

(a) Show that $f(-x) = -f(x)$. *[2 marks]*

(b) Given that $f''(x) = \dfrac{2ax(x^2-3)}{(x^2+1)^3}$, find the coordinates of all points of inflexion. *[7 marks]*

(c) It is given that $\int f(x)\,\mathrm{d}x = \dfrac{a}{2}\ln(x^2+1) + C$.

(i) Find the area of the shaded region, giving your answer in the form $p \ln q$.

(ii) Find the value of $\int_4^8 2f(x-1)\,\mathrm{d}x$. *[7 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Thursday 5 November 2009 (afternoon)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 8]*

Let $f(x) = 2x^3 + 3$ and $g(x) = e^{3x} - 2$.

(a) (i) Find $g(0)$.

(ii) Find $(f \circ g)(0)$. *[5 marks]*

(b) Find $f^{-1}(x)$. *[3 marks]*

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2. *[Maximum mark: 6]*

(a) Let $\boldsymbol{u} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix}$. Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{w}$, find the value of p. *[3 marks]*

(b) Let $\boldsymbol{v} = \begin{pmatrix} 1 \\ q \\ 5 \end{pmatrix}$. Given that $|\boldsymbol{v}| = \sqrt{42}$, find the possible values of q. *[3 marks]*

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3. *[Maximum mark: 6]*

Let X be normally distributed with mean 100 cm and standard deviation 5 cm.

(a) On the diagram below, shade the region representing $\mathrm{P}(X > 105)$. *[2 marks]*

(b) Given that $\mathrm{P}(X < d) = \mathrm{P}(X > 105)$, find the value of d. *[2 marks]*

(c) Given that $\mathrm{P}(X > 105) = 0.16$ (correct to two significant figures), find $\mathrm{P}(d < X < 105)$. *[2 marks]*

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4. *[Maximum mark: 5]*

The diagram below shows the graph of a function $f(x)$, for $-2 \le x \le 4$.

(This question continues on the following page)

(Question 4 continued)

(a) Let $h(x) = f(-x)$. Sketch the graph of h on the grid below. *[2 marks]*

(b) Let $g(x) = \frac{1}{2} f(x-1)$. The point $\mathrm{A}(3, 2)$ on the graph of f is transformed to the point P on the graph of g. Find the coordinates of P. *[3 marks]*

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5. *[Maximum mark: 6]*

Consider $f(x) = x^2 + \frac{p}{x}$, $x \neq 0$, where p is a constant.

(a) Find $f'(x)$. *[2 marks]*

(b) There is a minimum value of $f(x)$ when $x = -2$. Find the value of p. *[4 marks]*

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6. *[Maximum mark: 7]*

Solve $\cos 2x - 3\cos x - 3 - \cos^2 x = \sin^2 x$, for $0 \le x \le 2\pi$.

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7. *[Maximum mark: 7]*

Let $f(x) = k \log_2 x$.

(a) Given that $f^{-1}(1) = 8$, find the value of k. *[3 marks]*

(b) Find $f^{-1}\left(\frac{2}{3}\right)$. *[4 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 12]*

In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby.

(a) (i) Find the number of boys who play both sports.

(ii) Write down the number of boys who play only rugby. *[3 marks]*

(b) One boy is selected at random.

(i) Find the probability that he plays only one sport.

(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby. *[4 marks]*

Let A be the event that a boy plays football and B be the event that a boy plays rugby.

(c) Explain why A and B are **not** mutually exclusive. *[2 marks]*

(d) Show that A and B are **not** independent. *[3 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 16]*

Let $f(x) = 3 + \dfrac{20}{x^2 - 4}$, for $x \neq \pm 2$. The graph of f is given below.

The y-intercept is at the point A.

(a) (i) Find the coordinates of A.

(ii) Show that $f'(x) = 0$ at A. *[7 marks]*

(b) The second derivative $f''(x) = \dfrac{40(3x^2 + 4)}{(x^2 - 4)^3}$. Use this to

(i) justify that the graph of f has a local maximum at A;

(ii) explain why the graph of f does **not** have a point of inflexion. *[6 marks]*

(c) Describe the behaviour of the graph of f for large $|x|$. *[1 mark]*

(d) Write down the range of f. *[2 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 17]*

Let $f(x) = \sqrt{x}$. Line L is the normal to the graph of f at the point $(4, 2)$.

(a) Show that the equation of L is $y = -4x + 18$. *[4 marks]*

(b) Point A is the x-intercept of L. Find the x-coordinate of A. *[2 marks]*

In the diagram below, the shaded region R is bounded by the x-axis, the graph of f and the line L.

(c) Find an expression for the area of R. *[3 marks]*

(d) The region R is rotated 360° about the x-axis. Find the volume of the solid formed, giving your answer in terms of π. *[8 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Thursday 7 May 2009 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

Let $f(x) = x^2$ and $g(x) = 2x - 3$.

(a) Find $g^{-1}(x)$. *[2 marks]*

(b) Find $(f \circ g)(4)$. *[3 marks]*

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2. *[Maximum mark: 6]*

Find the cosine of the angle between the two vectors $3\boldsymbol{i}+4\boldsymbol{j}+5\boldsymbol{k}$ and $4\boldsymbol{i}-5\boldsymbol{j}-3\boldsymbol{k}$.

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3. *[Maximum mark: 6]*

The fifth term in the expansion of the binomial $(a+b)^n$ is given by $\binom{10}{4} p^6 (2q)^4$.

(a) Write down the value of n. *[1 mark]*

(b) Write down a and b, in terms of p and/or q. *[2 marks]*

(c) Write down an expression for the sixth term in the expansion. *[3 marks]*

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4. *[Maximum mark: 5]*

(a) Find $\log_2 32$. *[1 mark]*

(b) Given that $\log_2\left(\frac{32^x}{8^y}\right)$ can be written as $px + qy$, find the value of p and of q. *[4 marks]*

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5. *[Maximum mark: 6]*

A matrix $\boldsymbol{M}$ has **inverse** $\boldsymbol{M}^{-1} = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$.

(a) Find $\boldsymbol{M}$. *[3 marks]*

(b) Solve the matrix equation $\boldsymbol{MX} = \boldsymbol{B}$, where $\boldsymbol{B} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ and $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$. *[3 marks]*

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6. *[Maximum mark: 5]*

A function f has its first derivative given by $f'(x) = (x-3)^3$.

(a) Find the second derivative. *[2 marks]*

(b) Find $f'(3)$ and $f''(3)$. *[1 mark]*

(c) The point P on the graph of f has x-coordinate 3. Explain why P is not a point of inflexion. *[2 marks]*

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7. *[Maximum mark: 6]*

Let $f(x) = \sqrt{3}e^{2x}\sin x + e^{2x}\cos x$, for $0 \le x \le \pi$. Given that $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, solve the equation $f(x) = 0$.

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8. *[Maximum mark: 6]*

Let $f(x) = \mathrm{e}^{-3x}$ and $g(x) = \sin\left(x - \frac{\pi}{3}\right)$.

(a) Write down

(i) $f'(x)$;

(ii) $g'(x)$. *[2 marks]*

(b) Let $h(x) = \mathrm{e}^{-3x}\sin\left(x - \frac{\pi}{3}\right)$. Find the exact value of $h'\left(\frac{\pi}{3}\right)$. *[4 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

9. *[Maximum mark: 12]*

Two boxes contain numbered cards as shown below.

Two cards are drawn at random, one from each box.

(a) Copy and complete the table below to show all nine equally likely outcomes. *[2 marks]*

3 , 9		
3 , 10		
3 , 10		

Let S be the sum of the numbers on the two cards.

(b) Write down all the possible values of S. *[2 marks]*

(c) Find the probability of each value of S. *[2 marks]*

(d) Find the expected value of S. *[3 marks]*

(e) Anna plays a game where she wins \$50 if S is even and loses \$30 if S is odd. Anna plays the game 36 times. Find the amount she expects to have at the end of the 36 games. *[3 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 16]*

The line L_1 is parallel to the z-axis. The point P has position vector $\begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix}$ and lies on L_1.

(a) Write down the equation of L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[2 marks]*

The line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$. The point A has position vector $\begin{pmatrix} 6 \\ 2 \\ 9 \end{pmatrix}$.

(b) Show that A lies on L_2. *[4 marks]*

Let B be the point of intersection of lines L_1 and L_2.

(c) (i) Show that $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 8 \\ 1 \\ 14 \end{pmatrix}$.

(ii) Find $\overrightarrow{\mathrm{AB}}$. *[7 marks]*

(d) The point C is at $(2, 1, -4)$. Let D be the point such that ABCD is a parallelogram. Find $\overrightarrow{\mathrm{OD}}$. *[3 marks]*

*Do **NOT** write on this page.*

11. *[Maximum mark: 17]*

In this question s represents displacement in metres and t represents time in seconds.

The velocity v m s^{-1} of a moving body is given by $v = 40 - at$ where a is a non-zero constant.

(a) (i) If $s = 100$ when $t = 0$, find an expression for s in terms of a and t.

(ii) If $s = 0$ when $t = 0$, write down an expression for s in terms of a and t. *[6 marks]*

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by $v = 40 - at$, where $t = 0$ at P. The station is 500 m from P.

(b) A train M slows down so that it comes to a stop at the station.

(i) Find the time it takes train M to come to a stop, giving your answer in terms of a.

(ii) Hence show that $a = \frac{8}{5}$. *[6 marks]*

(c) For a different train N, the value of a is 4.
Show that this train will stop **before** it reaches the station. *[5 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Wednesday 5 May 2010 (afternoon)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

Let $f(x) = 8x - 2x^2$. Part of the graph of f is shown below.

(a) Find the x-intercepts of the graph. *[4 marks]*

(b) (i) Write down the equation of the axis of symmetry.

(ii) Find the y-coordinate of the vertex. *[3 marks]*

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2. *[Maximum mark: 6]*

Let $\boldsymbol{W} = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ and $\boldsymbol{P} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$.

(a) Find $\boldsymbol{WP}$. *[3 marks]*

(b) Given that $2\boldsymbol{WP} + \boldsymbol{S} = \begin{pmatrix} 26 \\ 12 \\ 10 \end{pmatrix}$, find $\boldsymbol{S}$. *[3 marks]*

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3. *[Maximum mark: 6]*

(a) Expand $(2+x)^4$ and simplify your result. *[3 marks]*

(b) Hence, find the term in x^2 in $(2+x)^4\left(1+\frac{1}{x^2}\right)$. *[3 marks]*

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4. *[Maximum mark: 7]*

The straight line with equation $y = \frac{3}{4}x$ makes an acute angle θ with the x-axis.

(a) Write down the value of $\tan\theta$. *[1 mark]*

(b) Find the value of

(i) $\sin 2\theta$;

(ii) $\cos 2\theta$. *[6 marks]*

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5. *[Maximum mark: 6]*

Consider the events A and B, where $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.7$ and $\mathrm{P}(A \cap B) = 0.3$.

The Venn diagram below shows the events A and B, and the probabilities p, q and r.

(a) Write down the value of

(i) p;

(ii) q;

(iii) r. *[3 marks]*

(b) Find the value of $\mathrm{P}(A \mid B')$. *[2 marks]*

(c) Hence, or otherwise, show that the events A and B are **not** independent. *[1 mark]*

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6. *[Maximum mark: 6]*

The graph of $f(x) = \sqrt{16 - 4x^2}$, for $-2 \le x \le 2$, is shown below.

The region enclosed by the curve of f and the x-axis is rotated 360° about the x-axis. Find the volume of the solid formed.

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7. *[Maximum mark: 7]*

Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.

(a) Show that $f^{-1}(x) = 3^{2x}$. *[2 marks]*

(b) Write down the range of f^{-1}. *[1 mark]*

Let $g(x) = \log_3 x$, for $x > 0$.

(c) Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer. *[4 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Let $f(x) = \frac{1}{3}x^3 - x^2 - 3x$. Part of the graph of f is shown below.

There is a maximum point at A and a minimum point at $\mathrm{B}(3, -9)$.

(a) Find the coordinates of A. *[8 marks]*

(b) Write down the coordinates of

(i) the image of B after reflection in the y-axis;

(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$;

(iii) the image of B after reflection in the x-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$. *[6 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 13]*

Let $f(x) = \dfrac{\cos x}{\sin x}$, for $\sin x \neq 0$.

(a) Use the quotient rule to show that $f'(x) = \dfrac{-1}{\sin^2 x}$. *[5 marks]*

(b) Find $f''(x)$. *[3 marks]*

In the following table, $f'\left(\dfrac{\pi}{2}\right) = p$ and $f''\left(\dfrac{\pi}{2}\right) = q$. The table also gives approximate values of $f'(x)$ and $f''(x)$ near $x = \dfrac{\pi}{2}$.

x	$\dfrac{\pi}{2} - 0.1$	$\dfrac{\pi}{2}$	$\dfrac{\pi}{2} + 0.1$
$f'(x)$	–1.01	p	–1.01
$f''(x)$	0.203	q	–0.203

(c) Find the value of p and of q. *[3 marks]*

(d) Use information from the table to explain why there is a point of inflexion on the graph of f where $x = \dfrac{\pi}{2}$. *[2 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 18]*

The line L_1 is represented by the vector equation $\boldsymbol{r} = \begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$.

A second line L_2 is parallel to L_1 and passes through the point $\mathrm{B}(-8, -5, 25)$.

(a) Write down a vector equation for L_2 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[2 marks]*

A third line L_3 is perpendicular to L_1 and is represented by $\boldsymbol{r} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q\begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$.

(b) Show that $k = -2$. *[5 marks]*

The lines L_1 and L_3 intersect at the point A.

(c) Find the coordinates of A. *[6 marks]*

The lines L_2 and L_3 intersect at point C where $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix}$.

(d) (i) Find $\overrightarrow{\mathrm{AB}}$.

(ii) Hence, find $|\overrightarrow{\mathrm{AC}}|$. *[5 marks]*

M10/5/MATME/SP1/ENG/TZ2/XX+

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Wednesday 5 May 2010 (afternoon)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 6]*

Let $f(x) = p(x-q)(x-r)$. Part of the graph of f is shown below.

The graph passes through the points $(-2, 0)$, $(0, -4)$ and $(4, 0)$.

(a) Write down the value of q and of r. *[2 marks]*

(b) Write down the **equation** of the axis of symmetry. *[1 mark]*

(c) Find the value of p. *[3 marks]*

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2. *[Maximum mark: 8]*

Let $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

(a) Find $\overrightarrow{BC}$. *[2 marks]*

(b) Find a unit vector in the direction of $\overrightarrow{AB}$. *[3 marks]*

(c) Show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AC}$. *[3 marks]*

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3. *[Maximum mark: 5]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$.

(a) Find $\boldsymbol{AB}$. *[3 marks]*

(b) Solve $\boldsymbol{A}^{-1}\boldsymbol{X} = \boldsymbol{B}$. *[2 marks]*

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4. *[Maximum mark: 7]*

Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

(a) Find $f\left(\frac{\pi}{2}\right)$. *[2 marks]*

(b) Find $(g \circ f)\left(\frac{\pi}{2}\right)$. *[2 marks]*

(c) Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of k, $k \in \mathbb{Z}$. *[3 marks]*

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5. *[Maximum mark: 6]*

Let $f(x) = kx^4$. The point $\mathrm{P}(1, k)$ lies on the curve of f. At P, the normal to the curve is parallel to $y = -\frac{1}{8}x$. Find the value of k.

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6. *[Maximum mark: 7]*

Solve $\log_2 x + \log_2 (x-2) = 3$, for $x > 2$.

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7. *[Maximum mark: 6]*

A function f is defined for $-4 \le x \le 3$. The graph of f is given below.

The graph has a local maximum when $x = 0$, and local minima when $x = -3,\ x = 2$.

(a) Write down the x-intercepts of the graph of the **derivative** function, f'. *[2 marks]*

(b) Write down all values of x for which $f'(x)$ is positive. *[2 marks]*

(c) At point D on the graph of f, the x-coordinate is -0.5. Explain why $f''(x) < 0$ at D. *[2 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Consider the function f with second derivative $f''(x) = 3x - 1$. The graph of f has a minimum point at $\mathrm{A}(2, 4)$ and a maximum point at $\mathrm{B}\left(-\frac{4}{3}, \frac{358}{27}\right)$.

(a) Use the second derivative to justify that B is a maximum. *[3 marks]*

(b) Given that $f'(x) = \frac{3}{2}x^2 - x + p$, show that $p = -4$. *[4 marks]*

(c) Find $f(x)$. *[7 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 14]*

José travels to school on a bus. On any day, the probability that José will miss the bus is $\frac{1}{3}$.
If he misses his bus, the probability that he will be late for school is $\frac{7}{8}$.
If he does not miss his bus, the probability that he will be late is $\frac{3}{8}$.
Let E be the event "he misses his bus" and F the event "he is late for school".
The information above is shown on the following tree diagram.

(a) Find

(i) $\mathrm{P}(E \cap F)$;

(ii) $\mathrm{P}(F)$. *[4 marks]*

(b) Find the probability that

(i) José misses his bus and is not late for school;

(ii) José missed his bus, given that he is late for school. *[5 marks]*

The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday.

(c) **Copy** and complete the probability distribution table. *[3 marks]*

***X* (cost in euros)**	0	3	6
P(*X*)	$\frac{1}{9}$		

(d) Find the expected cost for José for both days. *[2 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 17]*

Let $f(x) = 6 + 6\sin x$. Part of the graph of f is shown below.

The shaded region is enclosed by the curve of f, the x-axis, and the y-axis.

(a) Solve for $0 \le x < 2\pi$

(i) $6 + 6\sin x = 6$;

(ii) $6 + 6\sin x = 0$. *[5 marks]*

(b) Write down the exact value of the x-intercept of f, for $0 \le x < 2\pi$. *[1 mark]*

(c) The area of the shaded region is k. Find the value of k, giving your answer in terms of π. *[6 marks]*

Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g.

(d) Give a full geometric description of this transformation. *[2 marks]*

(e) Given that $\int_p^{p+\frac{3\pi}{2}} g(x)\,\mathrm{d}x = k$ and $0 \le p < 2\pi$, write down the two values of p. *[3 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Thursday 4 November 2010 (afternoon)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

The first three terms of an infinite geometric sequence are 32, 16 and 8.

(a) Write down the value of r. *[1 mark]*

(b) Find u_6. *[2 marks]*

(c) Find the sum to infinity of this sequence. *[2 marks]*

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2. *[Maximum mark: 7]*

Let $g(x) = 2x\sin x$.

(a) Find $g'(x)$. *[4 marks]*

(b) Find the gradient of the graph of g at $x = \pi$. *[3 marks]*

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3. *[Maximum mark: 6]*

The diagram shows two concentric circles with centre O.

diagram not to scale

The radius of the smaller circle is 8 cm and the radius of the larger circle is 10 cm. Points A, B and C are on the circumference of the larger circle such that $A\hat{O}B$ is $\frac{\pi}{3}$ radians.

(a) Find the length of the arc ACB. *[2 marks]*

(b) Find the area of the shaded region. *[4 marks]*

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4. *[Maximum mark: 7]*

The diagram below shows the probabilities for events A and B, with $\mathrm{P}(A') = p$.

(a) Write down the value of p. *[1 mark]*

(b) Find $\mathrm{P}(B)$. *[3 marks]*

(c) Find $\mathrm{P}(A' | B)$. *[3 marks]*

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5. *[Maximum mark: 7]*

(a) Show that $4-\cos 2\theta+5\sin\theta=2\sin^2\theta+5\sin\theta+3$. *[2 marks]*

(b) **Hence**, solve the equation $4-\cos 2\theta+5\sin\theta=0$ for $0\le\theta\le 2\pi$. *[5 marks]*

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6. *[Maximum mark: 6]*

The graph of the function $y = f(x)$ passes through the point $\left(\frac{3}{2}, 4\right)$. The gradient function of f is given as $f'(x) = \sin(2x - 3)$. Find $f(x)$.

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7. *[Maximum mark: 7]*

Let $\boldsymbol{A} = \begin{pmatrix} 9e^{x} & e^{x} \\ e^{x} & e^{3x} \end{pmatrix}$.

(a) Find an expression for $\det \boldsymbol{A}$. *[2 marks]*

(b) Find the value of x for which $\boldsymbol{A}$ has no inverse. Express your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$. *[5 marks]*

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*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 17]*

The diagram shows quadrilateral ABCD with vertices $A(1, 0)$, $B(1, 5)$, $C(5, 2)$ and $D(4, -1)$.

diagram not to scale

(a) (i) Show that $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.

(ii) Find $\overrightarrow{BD}$.

(iii) Show that $\overrightarrow{AC}$ is perpendicular to $\overrightarrow{BD}$. *[5 marks]*

The line (AC) has equation $\boldsymbol{r} = \boldsymbol{u} + s\boldsymbol{v}$.

(b) (i) Write down vector $\boldsymbol{u}$ and vector $\boldsymbol{v}$.

(ii) Find a vector equation for the line (BD). *[4 marks]*

The lines (AC) and (BD) intersect at the point $P(3, k)$.

(c) Show that $k = 1$. *[3 marks]*

(d) **Hence** find the area of triangle ACD. *[5 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9. *[Maximum mark: 12]*

Let $f(x) = x^2 + 4$ and $g(x) = x - 1$.

(a) Find $(f \circ g)(x)$. *[2 marks]*

The vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ translates the graph of $(f \circ g)$ to the graph of h.

(b) Find the coordinates of the vertex of the graph of h. *[3 marks]*

(c) Show that $h(x) = x^2 - 8x + 19$. *[2 marks]*

(d) The line $y = 2x - 6$ is a tangent to the graph of h at the point P. Find the x-coordinate of P. *[5 marks]*

10. *[Maximum mark: 16]*

Let $f(x) = x^3$. The following diagram shows part of the graph of f.

diagram not to scale

The point $\mathrm{P}(a, f(a))$, where $a > 0$, lies on the graph of f. The tangent at P crosses the x-axis at the point $\mathrm{Q}\left(\frac{2}{3}, 0\right)$. This tangent intersects the graph of f at the point $\mathrm{R}(-2, -8)$.

(This question continues on the following page)

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

(Question 10 continued)

(a) (i) Show that the gradient of [PQ] is $\dfrac{a^3}{a-\frac{2}{3}}$.

(ii) Find $f'(a)$.

(iii) Hence show that $a=1$. *[7 marks]*

The equation of the tangent at P is $y=3x-2$. Let T be the region enclosed by the graph of f, the tangent [PR] and the line $x=k$, between $x=-2$ and $x=k$ where $-2<k<1$. This is shown in the diagram below.

diagram not to scale

(b) Given that the area of T is $2k+4$, show that k satisfies the equation $k^4-6k^2+8=0$. *[9 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Wednesday 4 May 2011 (afternoon)

Examination code

2	2	1	1	–	7	3	0	3

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
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- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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11 pages

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SECTION A

*Answer **all** questions in the boxes provided.*

1. *[Maximum mark: 5]*

Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

(a) Find $(g \circ f)(x)$. *[2 marks]*

(b) Write down $g^{-1}(x)$. *[1 mark]*

(c) Find $(f \circ g^{-1})(5)$. *[2 marks]*

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2. *[Maximum mark: 6]*

A line L passes through $\mathrm{A}(1, -1, 2)$ and is parallel to the line $\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$.

(a) Write down a vector equation for L in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[2 marks]*

The line L passes through point P when $t = 2$.

(b) Find

(i) $\overrightarrow{\mathrm{OP}}$;

(ii) $|\overrightarrow{\mathrm{OP}}|$. *[4 marks]*

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3. *[Maximum mark: 6]*

Let $\boldsymbol{A} = \begin{pmatrix} 2 & -4 \\ -1 & 3 \end{pmatrix}$.

(a) Find $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Solve the matrix equation $\boldsymbol{AX} = \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}$. *[4 marks]*

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4. *[Maximum mark: 7]*

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X = x) = \frac{x^2}{14},\ x \in \{1,\ 2,\ k\}\text{, where } k > 0.$$

(a) Write down $\mathrm{P}(X = 2)$. *[1 mark]*

(b) Show that $k = 3$. *[4 marks]*

(c) Find $\mathrm{E}(X)$. *[2 marks]*

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5. *[Maximum mark: 7]*

Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

(a) Use the quotient rule to show that $g'(x) = \frac{1 - 2\ln x}{x^3}$. *[4 marks]*

(b) The graph of g has a maximum point at A. Find the x-coordinate of A. *[3 marks]*

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6. *[Maximum mark: 7]*

Solve the equation $2\cos x = \sin 2x$, for $0 \le x \le 3\pi$.

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7. *[Maximum mark: 7]*

Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

(a) Find the value of k. *[5 marks]*

(b) The line $y = p$ intersects the graph of f. Find all possible values of p. *[2 marks]*

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*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

In a group of 16 students, 12 take art and 8 take music. One student takes neither art nor music. The Venn diagram below shows the events art and music. The values p, q, r and s represent numbers of students.

(a) (i) Write down the value of s.

(ii) Find the value of q.

(iii) Write down the value of p and of r. *[5 marks]*

(b) (i) A student is selected at random. Given that the student takes music, write down the probability the student takes art.

(ii) **Hence**, show that taking music and taking art are **not** independent events. *[4 marks]*

(c) Two students are selected at random, one after the other. Find the probability that the first student takes **only** music and the second student takes **only** art. *[4 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9. *[Maximum mark: 16]*

The following diagram shows the obtuse-angled triangle ABC such that $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 0 \\ -4 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ 2 \\ -6 \end{pmatrix}$.

diagram not to scale

(a) (i) Write down $\overrightarrow{BA}$.

(ii) Find $\overrightarrow{BC}$. *[3 marks]*

(b) (i) Find $\cos A\hat{B}C$.

(ii) Hence, find $\sin A\hat{B}C$. *[7 marks]*

The point D is such that $\overrightarrow{CD} = \begin{pmatrix} -4 \\ 5 \\ p \end{pmatrix}$, where $p > 0$.

(c) (i) Given that $|\overrightarrow{CD}| = \sqrt{50}$, show that $p = 3$.

(ii) Hence, show that $\overrightarrow{CD}$ is perpendicular to $\overrightarrow{BC}$. *[6 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10. *[Maximum mark: 16]*

The velocity $v\ \mathrm{m\,s^{-1}}$ of a particle at time t seconds, is given by $v = 2t + \cos 2t$, for $0 \le t \le 2$.

(a) Write down the velocity of the particle when $t = 0$. *[1 mark]*

When $t = k$, the acceleration is zero.

(b) (i) Show that $k = \frac{\pi}{4}$.

(ii) Find the exact velocity when $t = \frac{\pi}{4}$. *[8 marks]*

(c) When $t < \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$ and when $t > \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$.

Sketch a graph of v against t. *[4 marks]*

(d) Let d be the distance travelled by the particle for $0 \le t \le 1$.

(i) Write down an expression for d.

(ii) Represent d on your sketch. *[3 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Wednesday 4 May 2011 (afternoon)

Examination code

2	2	1	1	–	7	3	0	5

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
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- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

0112

11 pages

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SECTION A

*Answer **all** questions in the boxes provided.*

1. *[Maximum mark: 6]*

In an arithmetic sequence, $u_1 = 2$ and $u_3 = 8$.

(a) Find d. *[2 marks]*

(b) Find u_{20}. *[2 marks]*

(c) Find S_{20}. *[2 marks]*

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2. *[Maximum mark: 6]*

The Venn diagram below shows events A and B where $\mathrm{P}(A) = 0.3$, $\mathrm{P}(A \cup B) = 0.6$ and $\mathrm{P}(A \cap B) = 0.1$. The values m, n, p and q are probabilities.

(a) (i) Write down the value of n.

(ii) Find the value of m, of p, and of q. *[4 marks]*

(b) Find $\mathrm{P}(B')$. *[2 marks]*

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3. *[Maximum mark: 7]*

The following diagram shows quadrilateral ABCD, with $\overrightarrow{AD} = \overrightarrow{BC}$, $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

diagram not to scale

(a) Find $\overrightarrow{BC}$. *[2 marks]*

(b) Show that $\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$. *[2 marks]*

(c) Show that vectors $\overrightarrow{BD}$ and $\overrightarrow{AC}$ are perpendicular. *[3 marks]*

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4. *[Maximum mark: 6]*

Let $h(x) = \dfrac{6x}{\cos x}$. Find $h'(0)$.

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5. *[Maximum mark: 7]*

Let $f(x) = 3\ln x$ and $g(x) = \ln 5x^3$.

(a) Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$. *[4 marks]*

(b) The graph of g is a transformation of the graph of f. Give a full geometric description of this transformation. *[3 marks]*

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6. *[Maximum mark: 5]*

A scientist has 100 female fish and 100 male fish. She measures their lengths to the nearest cm. These are shown in the following box and whisker diagrams.

Female fish

Male fish

(a) Find the range of the lengths of **all** 200 fish. *[3 marks]*

(b) Four cumulative frequency graphs are shown below.

Which graph is the best representation of the lengths of the **female** fish? *[2 marks]*

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7. *[Maximum mark: 8]*

Let $\boldsymbol{A} = \begin{pmatrix} 3 & x \\ -2 & -3 \end{pmatrix}$.

(a) Find the value of x for which $\boldsymbol{A}^{-1}$ does not exist. *[3 marks]*

(b) Given that $\boldsymbol{A} = \boldsymbol{A}^{-1}$, find x. *[5 marks]*

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*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 16]*

The following diagram shows part of the graph of the function $f(x) = 2x^2$.

diagram not to scale

The line T is the tangent to the graph of f at $x = 1$.

(a) Show that the equation of T is $y = 4x - 2$. *[5 marks]*

(b) Find the x-intercept of T. *[2 marks]*

(c) The shaded region R is enclosed by the graph of f, the line T, and the x-axis.

(i) Write down an expression for the area of R.

(ii) Find the area of R. *[9 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9. *[Maximum mark: 15]*

The following diagram shows part of the graph of a quadratic function f.

The x-intercepts are at $(-4, 0)$ and $(6, 0)$, and the y-intercept is at $(0, 240)$.

(a) Write down $f(x)$ in the form $f(x) = -10(x-p)(x-q)$. *[2 marks]*

(b) Find another expression for $f(x)$ in the form $f(x) = -10(x-h)^2 + k$. *[4 marks]*

(c) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. *[2 marks]*

A particle moves along a straight line so that its velocity, $v \text{ m s}^{-1}$, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \le t \le 6$.

(d) (i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero. *[7 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10. *[Maximum mark: 14]*

The following diagram represents a large Ferris wheel, with a diameter of 100 metres.

Let P be a point on the wheel. The wheel starts with P at the lowest point, at ground level. The wheel rotates at a constant rate, in an anticlockwise (counterclockwise) direction. One revolution takes 20 minutes.

(a) Write down the height of P above ground level after

(i) 10 minutes;

(ii) 15 minutes. *[2 marks]*

Let $h(t)$ metres be the height of P above ground level after t minutes. Some values of $h(t)$ are given in the table below.

t	$h(t)$
0	0.0
1	2.4
2	9.5
3	20.6
4	34.5
5	50.0

(b) (i) Show that $h(8) = 90.5$.

(ii) Find $h(21)$. *[4 marks]*

(c) **Sketch** the graph of h, for $0 \le t \le 40$. *[3 marks]*

(d) Given that h can be expressed in the form $h(t) = a\cos bt + c$, find a, b and c. *[5 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Wednesday 2 November 2011 (afternoon)

Examination code

8	8	1	1	–	7	3	0	1

1 hour 30 minutes

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0112

11 pages

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Let f be a quadratic function. Part of the graph of f is shown below.

The vertex is at $\mathrm{P}(4, 2)$ and the y-intercept is at $\mathrm{Q}(0, 6)$.

(a) Write down the equation of the axis of symmetry. *[1 mark]*

The function f can be written in the form $f(x) = a(x-h)^2 + k$.

(b) Write down the value of h and of k. *[2 marks]*

(c) Find a. *[3 marks]*

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2. *[Maximum mark: 5]*

Let $\boldsymbol{P} = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ and $\boldsymbol{Q} = \begin{pmatrix} 4 & -2 \\ -10 & 6 \end{pmatrix}$.

(a) Find $\boldsymbol{PQ}$. *[3 marks]*

(b) Hence, or otherwise, write down $\boldsymbol{P}^{-1}$. *[2 marks]*

3. *[Maximum mark: 6]*

A box contains six red marbles and two blue marbles. Anna selects a marble from the box. She replaces the marble and then selects a second marble.

(a) Write down the probability that the first marble Anna selects is red. *[1 mark]*

(b) Find the probability that Anna selects two red marbles. *[2 marks]*

(c) Find the probability that one marble is red and one marble is blue. *[3 marks]*

4. *[Maximum mark: 6]*

Let $f'(x) = 3x^2 + 2$. Given that $f(2) = 5$, find $f(x)$.

5. *[Maximum mark: 6]*

The random variable X has the following probability distribution.

x	1	2	3
$\mathrm{P}(X = x)$	s	0.3	q

Given that $\mathrm{E}(X) = 1.7$, find q.

6. *[Maximum mark: 8]*

Let $\sin\theta = \frac{2}{\sqrt{13}}$, where $\frac{\pi}{2} < \theta < \pi$.

(a) Find $\cos\theta$. *[3 marks]*

(b) Find $\tan 2\theta$. *[5 marks]*

7. *[Maximum mark: 8]*

Let $f(x) = \frac{1}{2}x^2 + kx + 8$, where $k \in \mathbb{Z}$.

(a) Find the values of k such that $f(x) = 0$ has two equal roots. *[4 marks]*

(b) Each value of k is equally likely for $-5 \le k \le 5$. Find the probability that $f(x) = 0$ has no roots. *[4 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 18]*

The line L_1 passes through the points $\mathrm{P}(2, 4, 8)$ and $\mathrm{Q}(4, 5, 4)$.

(a) (i) Find $\overrightarrow{\mathrm{PQ}}$.

(ii) Hence write down a vector equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b}$. *[4 marks]*

The line L_2 is perpendicular to L_1, and parallel to $\begin{pmatrix} 3p \\ 2p \\ 4 \end{pmatrix}$, where $p \in \mathbb{Z}$.

(b) (i) Find the value of p.

(ii) Given that L_2 passes through $\mathrm{R}(10, 6, -40)$, write down a vector equation for L_2. *[7 marks]*

(c) The lines L_1 and L_2 intersect at the point A. Find the x-coordinate of A. *[7 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9. *[Maximum mark: 14]*

The following diagram shows the graph of $f(x) = a\sin\left(b(x-c)\right) + d$, for $2 \le x \le 10$.

There is a maximum point at $\mathrm{P}(4, 12)$ and a minimum point at $\mathrm{Q}(8, -4)$.

(a) Use the graph to write down the value of

(i) a;

(ii) c;

(iii) d. *[3 marks]*

(b) Show that $b = \frac{\pi}{4}$. *[2 marks]*

(c) Find $f'(x)$. *[3 marks]*

(d) At a point R, the gradient is -2π. Find the x-coordinate of R. *[6 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10. *[Maximum mark: 13]*

Let $f(x) = \frac{1}{4}x^2 + 2$. The line L is the tangent to the curve of f at $(4, 6)$.

(a) Find the equation of L. *[4 marks]*

Let $g(x) = \frac{90}{3x+4}$, for $2 \le x \le 12$. The following diagram shows the graph of g.

(b) Find the area of the region enclosed by the curve of g, the x-axis, and the lines $x = 2$ and $x = 12$. Give your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$. *[6 marks]*

(c) The graph of g is reflected in the x-axis to give the graph of h. The area of the region enclosed by the lines L, $x = 2$, $x = 12$ and the x-axis is 120 cm^2.

Find the area enclosed by the lines L, $x = 2$, $x = 12$ and the graph of h. *[3 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Thursday 3 May 2012 (afternoon)

Examination code

2	2	1	2	–	7	3	0	3

1 hour 30 minutes

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- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0112

12 pages

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

The ages of people attending a music concert are given in the table below.

Age	$15 \le x < 19$	$19 \le x < 23$	$23 \le x < 27$	$27 \le x < 31$	$31 \le x < 35$
Frequency	14	26	52	52	16
Cumulative Frequency	14	40	92	p	160

(a) Find p. *[2 marks]*

The cumulative frequency diagram is given below.

(This question continues on the following page)

(Question 1 continued)

(b) Use the diagram to estimate

(i) the 80th percentile;

(ii) the interquartile range. *[5 marks]*

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2. *[Maximum mark: 6]*

Let $\boldsymbol{A}$ be a 2×2 matrix and $\boldsymbol{B}$ an $m\times n$ matrix, where $\boldsymbol{A}=\begin{pmatrix}-2 & 0\\ 1 & 3\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}1 & 0 & 3\\ -1 & 3 & 0\end{pmatrix}$.

(a) Write down the value of m and of n. *[2 marks]*

(b) Find $\boldsymbol{AB}$. *[3 marks]*

(c) Let $\boldsymbol{C}$ be a $p\times4$ matrix. Given that the product $\boldsymbol{BC}$ exists, write down the value of p. *[1 mark]*

3. *[Maximum mark: 6]*

Let $f(x) = e^{6x}$.

(a) Write down $f'(x)$. *[1 mark]*

The tangent to the graph of f at the point $P(0, b)$ has gradient m.

(b) (i) Show that $m = 6$.

(ii) Find b. *[4 marks]*

(c) Hence, write down the equation of this tangent. *[1 mark]*

4. *[Maximum mark: 7]*

Events A and B are such that $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \cup B) = 0.7$.

The values q, r, s and t represent probabilities.

(a) Write down the value of t. *[1 mark]*

(b) (i) Show that $r = 0.2$.

(ii) Write down the value of q and of s. *[3 marks]*

(c) (i) Write down $\mathrm{P}(B')$.

(ii) Find $\mathrm{P}(A|B')$. *[3 marks]*

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5. *[Maximum mark: 7]*

The diagram below shows part of the graph of $f(x) = a\cos(b(x-c)) - 1$, where $a > 0$.

The point $P\left(\frac{\pi}{4}, 2\right)$ is a maximum point and the point $Q\left(\frac{3\pi}{4}, -4\right)$ is a minimum point.

(a) Find the value of a. *[2 marks]*

(b) (i) Show that the period of f is π.

(ii) Hence, find the value of b. *[4 marks]*

(c) Given that $0 < c < \pi$, write down the value of c. *[1 mark]*

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6. *[Maximum mark: 6]*

Given that $\int_0^5 \frac{2}{2x+5}\,\mathrm{d}x = \ln k$, find the value of k.

7. *[Maximum mark: 6]*

Let $f(x) = (\sin x + \cos x)^2$.

(a) Show that $f(x)$ can be expressed as $1 + \sin 2x$. *[2 marks]*

The graph of f is shown below for $0 \le x \le 2\pi$.

(b) Let $g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of g for $0 \le x \le 2\pi$. *[2 marks]*

The graph of g can be obtained from the graph of f under a horizontal stretch of scale factor p followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$.

(c) Write down the value of p and a possible value of k. *[2 marks]*

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*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 17]*

A line L_1 passes though points $\mathrm{P}(-1, 6, -1)$ and $\mathrm{Q}(0, 4, 1)$.

(a) (i) Show that $\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.

(ii) Hence, write down an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[3 marks]*

A second line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$.

(b) Find the cosine of the angle between $\overrightarrow{\mathrm{PQ}}$ and L_2. *[7 marks]*

(c) The lines L_1 and L_2 intersect at the point R. Find the coordinates of R. *[7 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 13]*

Let $f(x) = \log_p(x+3)$ for $x > -3$. Part of the graph of f is shown below.

The graph passes through $\mathrm{A}(6, 2)$, has an x-intercept at $(-2, 0)$ and has an asymptote at $x = -3$.

(a) Find p. *[4 marks]*

The graph of f is reflected in the line $y = x$ to give the graph of g.

(b) (i) Write down the y-intercept of the graph of g.

(ii) Sketch the graph of g, noting clearly any asymptotes and the image of A. *[5 marks]*

(c) Find $g(x)$. *[4 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

In this question, you are given that $\cos\frac{\pi}{3} = \frac{1}{2}$, *and* $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

The displacement of an object from a fixed point, O is given by $s(t) = t - \sin 2t$ for $0 \le t \le \pi$.

(a) Find $s'(t)$. *[3 marks]*

In this interval, there are only two values of t for which the object is not moving. One value is $t = \frac{\pi}{6}$.

(b) Find the other value. *[4 marks]*

(c) Show that $s'(t) > 0$ between these two values of t. *[3 marks]*

(d) Find the distance travelled between these two values of t. *[5 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Thursday 3 May 2012 (afternoon)

Examination code

2	2	1	2	–	7	3	0	5

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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- The maximum mark for this examination paper is *[90 marks]*.

0116

14 pages

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

The cumulative frequency curve below represents the marks obtained by 100 students.

(This question continues on the following page)

(Question 1 continued)

(a) Find the median mark. *[2 marks]*

(b) Find the interquartile range. *[3 marks]*

2. *[Maximum mark: 6]*

Let $f(x) = 2x - 1$ and $g(x) = 3x^2 + 2$.

(a) Find $f^{-1}(x)$. *[3 marks]*

(b) Find $(f \circ g)(1)$. *[3 marks]*

3. *[Maximum mark: 6]*

The following diagram shows the graph of $f(x) = a\cos(bx)$, for $0 \le x \le 4$.

There is a minimum point at $\mathrm{P}(2, -3)$ and a maximum point at $\mathrm{Q}(4, 3)$.

(a) (i) Write down the value of a.

(ii) Find the value of b. *[3 marks]*

(b) Write down the gradient of the curve at P. *[1 mark]*

(c) Write down the equation of the normal to the curve at P. *[2 marks]*

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4. *[Maximum mark: 8]*

The random variable X has the following probability distribution, with $\mathrm{P}(X > 1) = 0.5$.

x	0	1	2	3
$\mathrm{P}(X = x)$	p	q	r	0.2

(a) Find the value of r. *[2 marks]*

(b) Given that $\mathrm{E}(X) = 1.4$, find the value of p and of q. *[6 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

5. *[Maximum mark: 6]*

The diagram below shows the graph of a function $f(x)$, for $-2 \le x \le 3$.

(a) Sketch the graph of $f(-x)$ on the grid below. *[2 marks]*

(This question continues on the following page)

(Question 5 continued)

(b) The graph of f is transformed to obtain the graph of g. The graph of g is shown below.

The function g can be written in the form $g(x) = af(x+b)$. Write down the value of a and of b. *[4 marks]*

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6. *[Maximum mark: 7]*

Consider the equation $x^2 + (k-1)x + 1 = 0$, where k is a real number.

Find the values of k for which the equation has two **equal** real solutions.

7. *[Maximum mark: 7]*

Given that $\left(1+\frac{2}{3}x\right)^n(3+nx)^2 = 9+84x+\ldots$, find the value of n.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The following diagram shows the graph of a quadratic function f, for $0 \le x \le 4$.

The graph passes through the point $\mathrm{P}(0, 13)$, and its vertex is the point $\mathrm{V}(2, 1)$.

(a) The function can be written in the form $f(x) = a(x-h)^2 + k$.

(i) Write down the value of h and of k.

(ii) Show that $a = 3$. *[4 marks]*

(b) Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$. *[3 marks]*

(c) Calculate the area enclosed by the graph of f, the x-axis, and the lines $x = 2$ and $x = 4$. *[8 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 14]*

Bag A contains three white balls and four red balls. Two balls are chosen at random without replacement.

(a) (i) **Copy** and complete the following tree diagram. *(Do **not** write on this page.)*

(ii) Find the probability that two white balls are chosen. *[5 marks]*

Bag B contains four white balls and three red balls. When two balls are chosen at random without replacement from bag B, the probability that they are both white is $\frac{2}{7}$.

A standard die is rolled. If 1 or 2 is obtained, two balls are chosen without replacement from bag A, otherwise they are chosen from bag B.

(b) Find the probability that the two balls are white. *[5 marks]*

(c) Given that both balls are white, find the probability that they were chosen from bag A. *[4 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

Let $f(x) = \dfrac{x}{-2x^2 + 5x - 2}$ for $-2 \le x \le 4$, $x \ne \dfrac{1}{2}$, $x \ne 2$. The graph of f is given below.

The graph of f has a local minimum at $\mathrm{A}(1, 1)$ and a local maximum at B.

(a) Use the quotient rule to show that $f'(x) = \dfrac{2x^2 - 2}{(-2x^2 + 5x - 2)^2}$. *[6 marks]*

(b) Hence find the coordinates of B. *[7 marks]*

(c) Given that the line $y = k$ does not meet the graph of f, find the possible values of k. *[3 marks]*

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N12/5/MATME/SP1/ENG/TZ0/XX

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Tuesday 6 November 2012 (afternoon)

Examination code

8	8	1	2	–	7	3	0	1

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0116

13 pages

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Let $\boldsymbol{A} = \begin{pmatrix} 0 & 3 \\ -2 & 4 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -4 & 0 \\ 5 & 1 \end{pmatrix}$.

(a) Find $\boldsymbol{AB}$. *[3 marks]*

(b) Given that $\boldsymbol{X} - 2\boldsymbol{A} = \boldsymbol{B}$, find $\boldsymbol{X}$. *[3 marks]*

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2. *[Maximum mark: 6]*

The following table shows the probability distribution of a discrete random variable X.

x	0	2	5	9
$\mathrm{P}(X = x)$	0.3	k	$2k$	0.1

(a) Find the value of k. *[3 marks]*

(b) Find $\mathrm{E}(X)$. *[3 marks]*

3. *[Maximum mark: 7]*

(a) Find $\int_4^{10} (x-4)\,\mathrm{d}x$. *[4 marks]*

(b) Part of the graph of $f(x)=\sqrt{x-4}$, for $x \geq 4$, is shown below. The shaded region R is enclosed by the graph of f, the line $x=10$, and the x-axis.

The region R is rotated 360° about the x-axis. Find the volume of the solid formed. *[3 marks]*

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4. *[Maximum mark: 6]*

Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.

The point P lies on the graph of f. At P, $x = 1$.

(a) Find $f'(x)$. *[2 marks]*

(b) The graph of f has a gradient of 3 at the point P. Find the value of a. *[4 marks]*

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5. *[Maximum mark: 6]*

Let $\sin 100^\circ = m$. Find expressions for each of the following in terms of m

(a) $\cos 100^\circ$; *[3 marks]*

(b) $\tan 100^\circ$; *[1 mark]*

(c) $\sin 200^\circ$. *[2 marks]*

6. *[Maximum mark: 8]*

The line L passes through the point $(5, -4, 10)$ and is parallel to the vector $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$.

(a) Write down a vector equation for line L. *[2 marks]*

(b) The line L intersects the x-axis at the point P. Find the x-coordinate of P. *[6 marks]*

7. *[Maximum mark: 6]*

The equation $x^2 - 3x + k^2 = 4$ has two distinct real roots. Find the possible values of k.

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Answers written on this page
will not be marked.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The weekly wages (in dollars) of 80 employees are displayed in the cumulative frequency curve below.

(This question continues on the following page)

*Do **NOT** write solutions on this page.*

(Question 8 continued)

(a) (i) Write down the median weekly wage.

(ii) Find the interquartile range of the weekly wages. *[4 marks]*

The box-and-whisker plot below displays the weekly wages of the employees.

(b) Write down the value of

(i) a;

(ii) b;

(iii) c. *[3 marks]*

Employees are paid \$20 per hour.

(c) Find the median number of **hours** worked per week. *[3 marks]*

(d) Find the number of employees who work more than 25 hours per week. *[5 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 14]*

Let A and B be points such that $\overrightarrow{OA} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}$.

(a) Show that $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. *[1 mark]*

Let C and D be points such that ABCD is a **rectangle**.

(b) Given that $\overrightarrow{AD} = \begin{pmatrix} 4 \\ p \\ 1 \end{pmatrix}$, show that $p = 3$. *[4 marks]*

(c) Find the coordinates of point C. *[4 marks]*

(d) Find the area of rectangle ABCD. *[5 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

Let $f(x) = \dfrac{6x}{x+1}$, for $x > 0$.

(a) Find $f'(x)$. *[5 marks]*

Let $g(x) = \ln\left(\dfrac{6x}{x+1}\right)$, for $x > 0$.

(b) Show that $g'(x) = \dfrac{1}{x(x+1)}$. *[4 marks]*

(c) Let $h(x) = \dfrac{1}{x(x+1)}$. The area enclosed by the graph of h, the x-axis and the lines $x = \dfrac{1}{5}$ and $x = k$ is $\ln 4$. Given that $k > \dfrac{1}{5}$, find the value of k. *[7 marks]*

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will not be marked.

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M13/5/MATME/SP1/ENG/TZ1/XX

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Thursday 9 May 2013 (afternoon)

Examination code

2	2	1	3	–	7	3	0	3

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
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- Section A: answer all questions in the boxes provided.
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- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0112

11 pages

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

(a) Find

(i) $2\boldsymbol{a} + \boldsymbol{b}$;

(ii) $|2\boldsymbol{a} + \boldsymbol{b}|$. *[4 marks]*

Let $2\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \boldsymbol{0}$, where $\boldsymbol{0}$ is the zero vector.

(b) Find $\boldsymbol{c}$. *[2 marks]*

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2. *[Maximum mark: 6]*

The diagram below shows part of the graph of $f(x) = (x-1)(x+3)$.

(a) Write down the x-intercepts of the graph of f. *[2 marks]*

(b) Find the coordinates of the vertex of the graph of f. *[4 marks]*

3. *[Maximum mark: 7]*

Consider $f(x) = x^2 \sin x$.

(a) Find $f'(x)$. *[4 marks]*

(b) Find the gradient of the curve of f at $x = \frac{\pi}{2}$. *[3 marks]*

4. *[Maximum mark: 7]*

Let $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{X}$ be square matrices, such that $\boldsymbol{XA}+\boldsymbol{B}=\boldsymbol{C}$.

(a) Find an expression for $\boldsymbol{X}$ in terms of $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$. *[2 marks]*

(b) Given that $\boldsymbol{A}=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$, $\boldsymbol{B}=\begin{pmatrix}0 & 1\\ 1 & 2\end{pmatrix}$ and $\boldsymbol{C}=\begin{pmatrix}1 & 3\\ -3 & 4\end{pmatrix}$, find $\boldsymbol{X}$. *[5 marks]*

5. *[Maximum mark: 6]*

Let $f(x) = \sqrt{x-5}$, for $x \geq 5$.

(a) Find $f^{-1}(2)$. *[3 marks]*

(b) Let g be a function such that g^{-1} exists for all real numbers. Given that $g(30) = 3$, find $(f \circ g^{-1})(3)$. *[3 marks]*

6. *[Maximum mark: 6]*

Let $f(x) = \int \frac{12}{2x-5}\,\mathrm{d}x$, for $x > \frac{5}{2}$. The graph of f passes through $(4, 0)$.

Find $f(x)$.

7. *[Maximum mark: 7]*

Find the value of

(a) $\log_2 40 - \log_2 5$; *[3 marks]*

(b) $8^{\log_2 5}$. *[4 marks]*

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Consider points $A(1, -2, -1)$, $B(7, -4, 3)$ and $C(1, -2, 3)$. The line L_1 passes through C and is parallel to $\overrightarrow{AB}$.

(a) (i) Find $\overrightarrow{AB}$.

(ii) Hence, write down a vector equation for L_1. *[4 marks]*

A second line, L_2, is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s\begin{pmatrix} 3 \\ -3 \\ p \end{pmatrix}$.

(b) Given that L_1 is perpendicular to L_2, show that $p = -6$. *[3 marks]*

(c) The line L_1 intersects the line L_2 at point Q. Find the x-coordinate of Q. *[7 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 16]*

Jar A contains three red marbles and five green marbles. Two marbles are drawn from the jar, one after the other, without replacement.

(a) Find the probability that

(i) none of the marbles are green;

(ii) exactly one marble is green. *[5 marks]*

(b) Find the expected number of green marbles drawn from the jar. *[3 marks]*

Jar B contains six red marbles and two green marbles. A fair six-sided die is tossed. If the score is 1 or 2, a marble is drawn from jar A. Otherwise, a marble is drawn from jar B.

(c) (i) Write down the probability that the marble is drawn from jar B.

(ii) Given that the marble was drawn from jar B, write down the probability that it is red. *[2 marks]*

(d) Given that the marble is red, find the probability that it was drawn from jar A. *[6 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

Consider $f(x) = \ln(x^4 + 1)$.

(a) Find the value of $f(0)$. *[2 marks]*

(b) Find the set of values of x for which f is increasing. *[5 marks]*

The second derivative is given by $f''(x) = \dfrac{4x^2(3 - x^4)}{(x^4 + 1)^2}$.

The equation $f''(x) = 0$ has only three solutions, when $x = 0$, $\pm\sqrt[4]{3}$ $(\pm 1.316\ldots)$.

(c) (i) Find $f''(1)$.

(ii) **Hence**, show that there is no point of inflexion on the graph of f at $x = 0$. *[5 marks]*

(d) There is a point of inflexion on the graph of f at $x = \sqrt[4]{3}$ $(x = 1.316\ldots)$. Sketch the graph of f, for $x \geq 0$. *[3 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Thursday 9 May 2013 (afternoon)

Examination code

2	2	1	3	–	7	3	0	5

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
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- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0116

15 pages

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Let $f(x) = 4x - 2$ and $g(x) = -2x^2 + 8$.

(a) Find $f^{-1}(x)$. *[3 marks]*

(b) Find $(f \circ g)(1)$. *[3 marks]*

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2. *[Maximum mark: 6]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 2 & 1 \\ 1 & q \end{pmatrix}$, $\boldsymbol{C} = \begin{pmatrix} p & -1 \\ 6 & 3 \end{pmatrix}$, so that $\boldsymbol{AB} = \boldsymbol{C}$.

(a) Find the value of p. *[3 marks]*

(b) Find the value of q. *[3 marks]*

3. *[Maximum mark: 7]*

Let $\log_3 p = 6$ and $\log_3 q = 7$.

(a) Find $\log_3 p^2$. *[2 marks]*

(b) Find $\log_3\left(\frac{p}{q}\right)$. *[2 marks]*

(c) Find $\log_3(9p)$. *[3 marks]*

4. *[Maximum mark: 6]*

The diagram below shows the graph of a function f, for $-1 \le x \le 2$.

(a) Write down the value of

(i) $f(2)$;

(ii) $f^{-1}(-1)$. *[3 marks]*

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(Question 4 continued)

(b) Sketch the graph of f^{-1} on the grid below. *[3 marks]*

5. *[Maximum mark: 6]*

The diagram below shows part of the graph of a function f.

The graph has a maximum at $\mathrm{A}(1, 5)$ and a minimum at $\mathrm{B}(3, -1)$.

The function f can be written in the form $f(x) = p\sin(qx) + r$. Find the value of

(a) p; *[2 marks]*

(b) q; *[2 marks]*

(c) r. *[2 marks]*

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6. *[Maximum mark: 7]*

A rocket moving in a straight line has velocity v km s^{-1} and displacement s km at time t seconds. The velocity v is given by $v(t) = 6e^{2t} + t$. When $t = 0$, $s = 10$. Find an expression for the displacement of the rocket in terms of t.

7. *[Maximum mark: 7]*

The following is the graph of a function f, for $0 \le x \le 6$.

The first part of the graph is a quarter circle of radius 2 with centre at the origin.

(a) Find $\int_0^2 f(x)\,\mathrm{d}x$. *[4 marks]*

(b) The shaded region is enclosed by the graph of f, the x-axis, the y-axis and the line $x = 6$. The area of this region is 3π.

Find $\int_2^6 f(x)\,\mathrm{d}x$. *[3 marks]*

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Answers written on this page
will not be marked.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

A running club organizes a race to select girls to represent the club in a competition. The times taken by the group of girls to complete the race are shown in the table below.

Time t minutes	$10 \le t < 12$	$12 \le t < 14$	$14 \le t < 20$	$20 \le t < 26$	$26 \le t < 28$	$28 \le t < 30$
Frequency	50	20	p	40	20	20
Cumulative Frequency	50	70	120	q	180	200

(a) Find the value of p and of q. *[4 marks]*

(b) A girl is chosen at random.

(i) Find the probability that the time she takes is less than 14 minutes.

(ii) Find the probability that the time she takes is at least 26 minutes. *[3 marks]*

A girl is selected for the competition if she takes less than x minutes to complete the race.

(c) Given that 40 % of the girls are not selected,

(i) find the number of girls who are not selected;

(ii) find x. *[4 marks]*

(This question continues on the following page)

*Do **NOT** write solutions on this page.*

(Question 8 continued)

Girls who are not selected, but took less than 25 minutes to complete the race, are allowed another chance to be selected. The new times taken by these girls are shown in the cumulative frequency diagram below.

(d) (i) Write down the number of girls who were allowed another chance.

(ii) Find the percentage of the **whole** group who were selected. *[4 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 16]*

Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \le x \le \pi$.

(a) Find $f'(x)$. *[3 marks]*

Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g.

(b) Find $g(4)$. *[3 marks]*

The function g can be expressed in the form $g(x) = a(x-h)^2 + 3$.

(c) (i) Write down the value of h.

(ii) Find the value of a. *[4 marks]*

(d) Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g. *[6 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 14]*

Consider the functions $f(x)$, $g(x)$ and $h(x)$. The following table gives some values associated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	–14	–18
$f'(x)$	1	1
$g'(x)$	–5	–3
$h''(x)$	–6	0

(a) Write down the value of $g(3)$, of $f'(3)$, and of $h''(2)$. *[3 marks]*

The following diagram shows parts of the graphs of h and h''.

There is a point of inflexion on the graph of h at P, when $x = 3$.

(b) Explain why P is a point of inflexion. *[2 marks]*

Given that $h(x) = f(x) \times g(x)$,

(c) find the y-coordinate of P; *[2 marks]*

(d) find the equation of the normal to the graph of h at P. *[7 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

N13/5/MATME/SP1/ENG/TZ0/XX

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

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Monday 11 November 2013 (afternoon)

Examination code

8	8	1	3	–	7	3	0	1

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

12EP01

12 pages

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

In the following diagram, $\overrightarrow{OP} = \boldsymbol{p}$, $\overrightarrow{OQ} = \boldsymbol{q}$ and $\overrightarrow{PT} = \frac{1}{2}\overrightarrow{PQ}$.

diagram not to scale

Express each of the following vectors in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$,

(a) $\overrightarrow{QP}$; *[2]*

(b) $\overrightarrow{OT}$. *[3]*

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2. *[Maximum mark: 6]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 2 & -4 \\ 0 & 1 \end{pmatrix}$.

(a) Find $\boldsymbol{AB}$. *[3]*

(b) Find $\det(\boldsymbol{AB} + \boldsymbol{C})$. *[3]*

3. *[Maximum mark: 7]*

The following is a cumulative frequency diagram for the time t, in minutes, taken by 80 students to complete a task.

(This question continues on the following page)

(Question 3 continued)

(a) Find the number of students who completed the task in less than 45 minutes. *[2]*

(b) Find the number of students who took between 35 and 45 minutes to complete the task. *[3]*

(c) Given that 50 students take less than k minutes to complete the task, find the value of k. *[2]*

4. *[Maximum mark: 6]*

Consider a function $f(x)$ such that $\int_1^6 f(x)\mathrm{d}x = 8$.

(a) Find $\int_1^6 2f(x)\mathrm{d}x$. *[2]*

(b) Find $\int_1^6 \left(f(x)+2\right)\mathrm{d}x$. *[4]*

5. *[Maximum mark: 7]*

Let $f(x) = \sin\left(x + \frac{\pi}{4}\right) + k$. The graph of f passes through the point $\left(\frac{\pi}{4}, 6\right)$.

(a) Find the value of k. *[3]*

(b) Find the minimum value of $f(x)$. *[2]*

Let $g(x) = \sin x$. The graph of g is translated to the graph of f by the vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

(c) Write down the value of p and of q. *[2]*

6. *[Maximum mark: 6]*

Let $f(x) = e^{2x}$. The line L is the tangent to the curve of f at $(1, e^2)$.

Find the equation of L in the form $y = ax + b$.

7. *[Maximum mark: 8]*

The equation $x^2 + (k+2)x + 2k = 0$ has two distinct real roots.

Find the possible values of k.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

(a) Find $f^{-1}(x)$. *[2]*

(b) Show that $\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$. *[2]*

Let $h(x) = \frac{5}{x+2}$, for $x \geq 0$. The graph of h has a horizontal asymptote at $y = 0$.

(c) (i) Find the y-intercept of the graph of h.

(ii) Hence, sketch the graph of h. *[5]*

(d) For the graph of h^{-1},

(i) write down the x-intercept;

(ii) write down the equation of the vertical asymptote. *[2]*

(e) Given that $h^{-1}(a) = 3$, find the value of a. *[3]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 16]*

The first three terms of a infinite geometric sequence are $m-1$, 6, $m+4$, where $m \in \mathbb{Z}$.

(a) (i) Write down an expression for the common ratio, r.

(ii) Hence, show that m satisfies the equation $m^2 + 3m - 40 = 0$. *[4]*

(b) (i) Find the two possible values of m.

(ii) Find the possible values of r. *[6]*

(c) The sequence has a finite sum.

(i) State which value of r leads to this sum **and** justify your answer.

(ii) Calculate the sum of the sequence. *[6]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.

(a) Show that $f'(x) = \frac{\ln x}{x}$. *[2]*

(b) There is a minimum on the graph of f. Find the x-coordinate of this minimum. *[3]*

Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g.

The graph of f' has an x-intercept at $x = p$.

(c) Write down the value of p. *[2]*

The graph of g intersects the graph of f' when $x = q$.

(d) Find the value of q. *[3]*

(e) Let R be the region enclosed by the graph of f', the graph of g and the line $x = p$. Show that the area of R is $\frac{1}{2}$. *[5]*

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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

Tuesday 13 May 2014 (afternoon)

1 hour 30 minutes

Examination code

2	2	1	4	–	7	3	0	3

INSTRUCTIONS TO CANDIDATES

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11 pages

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2. *[Maximum mark: 7]*

In an arithmetic sequence, the third term is 10 and the fifth term is 16.

(a) Find the common difference. *[2]*

(b) Find the first term. *[2]*

(c) Find the sum of the first 20 terms of the sequence. *[3]*

6. *[Maximum mark: 7]*

Let $\int_{\pi}^{a} \cos 2x \, \mathrm{d}x = \frac{1}{2}$, where $\pi < a < 2\pi$. Find the value of a.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 17]*

The line L_1 passes through the points $\mathrm{A}(2, 1, 4)$ and $\mathrm{B}(1, 1, 5)$.

(a) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$. *[1]*

(b) Hence, write down

(i) a direction vector for L_1;

(ii) a vector equation for L_1. *[3]*

Another line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$. The lines L_1 and L_2 intersect at the point P.

(c) Find the coordinates of P. *[6]*

(d) (i) Write down a direction vector for L_2.

(ii) Hence, find the angle between L_1 and L_2. *[7]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

The sides of a square are 16 cm in length. The midpoints of the sides of this square are joined to form a new square and four triangles (diagram 1). The process is repeated twice, as shown in diagrams 2 and 3.

diagram 1 diagram 2 diagram 3

Let x_n denote the length of one of the equal sides of each new triangle.
Let A_n denote the area of each new triangle.

(a) The following table gives the values of x_n and A_n, for $1 \le n \le 3$. **Copy** and complete the table. *(Do **not** write on this page.)* *[4]*

n	1	2	3
x_n	8		4
A_n	32	16	

(b) The process described above is repeated. Find A_6. *[4]*

(c) Consider an initial square of side length k cm. The process described above is repeated indefinitely. The total area of the shaded regions is $k\,\text{cm}^2$. Find the value of k. *[7]*