18 Speed, Distance and Time

18.1 Speed

In this section we introduce the idea of speed, considering both *instantaneous speed* and *average speed.*

$$\text{Instantaneous speed} = \text{speed at any instant in time}$$

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

If a car travels 100 miles in 2 hours,

$$\text{average speed} = \frac{100}{2}$$

$$= 50 \text{ mph}$$

The car does not travel at a constant speed of 50 mph; its speed varies during the journey between 0 mph and, perhaps, 70 mph. The speed at any time is called the *instantaneous speed.*

The following table lists units in common use for speed and their abbreviations:

Distance	*Time*	*Speed*	*Abbreviation*
mile	hours	miles per hour	mph
kilometres	hours	kilometres per hour	km/h
metres	hours	metres per hour	m/h
metres	seconds	metres per second	m/s
feet	seconds	feet per second	f.p.s. *or* ft. per sec.
centimetres	seconds	centimetres per second	cm/sec *or* cm/s

Example 1

Judith drives from Plymouth to Southampton, a distance of 160 miles, in 4 hours.

She then drives from Southampton to London, a distance of 90 miles, in 1 hour and 30 minutes.

Determine her average speed for each journey.

Solution

Plymouth to Southampton Average speed $= \frac{160}{4}$

$= 40$ mph

Southampton to London Time taken $=$ 1 hour and 30 minutes

$= 1\frac{1}{2}$ hours *or* $\frac{3}{2}$ hours

Average speed $= 90 \div \frac{3}{2}$

$= 90 \times \frac{2}{3}$

$= 60$ mph

Example 2

John can type 960 words in 20 minutes.

Calculate his typing speed in:

(a) words per minute,

(b) words per hour.

Solution

(a) Typing speed $= \frac{960}{20}$

$=$ 48 words per minute

(b) Typing speed $= 48 \times 60$

$=$ 2880 words per hour

Exercises

1. Peter drives 320 miles in 8 hours. Calculate his average speed.

2. Daisy drives from Sheffield to London, a distance of 168 miles, in 4 hours. Calculate her average speed.

3. A snail moves 8 m in 2 hours. Calculate the average speed of the snail in metres per hour.

4. A lorry driver keeps a record of each journey he makes. Calculate the average speed for each journey, using the table below:

Start	*Finish*	*Start Time*	*Finish Time*	*Distance*
Brighton	Norwich	0800	1200	172 miles
Norwich	Carlisle	1400	1900	280 miles
Carlisle	Cardiff	1000	1800	300 miles
Cardiff	Exeter	0700	0930	120 miles
Exeter	Brighton	1030	1530	175 miles

5. Javinda takes $1\frac{1}{2}$ hours to drive 30 km in the rush hour. Calculate his average speed in km/h.

6. Rebecca cycles 20 miles on her bike in 2 hours and 30 minutes. Calculate her average speed in mph.

7. Julie can type 50 words in 2 minutes.
   Debbie can type 300 words in 15 minutes.
   Calculate the typing speed of each of the girls in:
   (a) words per minute,
   (b) words per hour.

8. Fatima, Emma and Andy each drive from London to Brighton, a distance of 60 miles. Fatima takes 1 hour, Emma takes 2 hours and Andy takes $1\frac{1}{2}$ hours. Calculate the average speed for each of the drivers.

9. Eva drives from Edinburgh to Dover in 3 stages:

	Start Time	*Finish Time*	*Distance*
Edinburgh to Leeds	0620	0920	210 miles
Leeds to London	1035	1305	200 miles
London to Dover	1503	1703	78 miles

Calculate her average speed for each stage of her journey.

10. Delia drives 220 km in $3\frac{1}{2}$ hours. Calculate her average speed correct to the nearest km/h.

18.2 Calculating Speed, Distance and Time

In this section we extend the ideas of speed to calculating *distances* and *times*, using the following formulae:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Example 1

Jane drives at an average speed of 45 mph on a journey of 135 miles. How long does the journey take?

Solution

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{135}{45}$$

$$= 3 \text{ hours}$$

Example 2

Chris cycles at an average speed of 8 mph. If he cycles for $6\frac{1}{2}$ hours, how far does he travel?

Solution

Distance = speed $\times$ time

$$= 8 \times 6\frac{1}{2}$$

$$= 52 \text{ miles}$$

Example 3

Nikki has to travel a total of 351 miles. She travels the first 216 miles in 4 hours.

(a) Calculate her average speed for the first part of the journey.

(b) If her average speed remains the same, calculate the total time for the complete journey.

Solution

(a) Average speed $= \dfrac{\text{distance}}{\text{time}}$

$$= \frac{216}{4}$$

$$= 54 \text{ mph}$$

(b) Time $= \dfrac{\text{distance}}{\text{speed}}$

$$= \frac{351}{54}$$

$$= 6.5 \text{ hours}$$

Exercises

1. Calculate the distance that you would travel if you drove for:

   (a) 3 hours at 20 mph
   (b) 8 hours at 60 mph
   (c) $\frac{1}{2}$ hour at 76 mph
   (d) $1\frac{1}{2}$ hours at 42 mph
   (e) $6\frac{1}{4}$ hours at 40 mph
   (f) 30 minutes at 33 mph
   (g) 45 minutes at 60 mph
   (h) 90 minutes at 45 mph

2. How long does it take to travel:

   (a) 120 miles at 40 mph
   (b) 300 miles at 50 mph
   (c) 240 miles at 60 mph
   (d) 385 miles at 70 mph
   (e) 60 miles at 40 mph
   (f) 360 miles at 30 mph
   (g) 390 miles at 60 mph
   (h) 253 miles at 46 mph

3. A car travels 300 miles in 5 hours. Calculate the average speed of the car in:

   (a) mph,
   (b) miles per minute.

   How long does it take for the car to travel 82 miles?

4. Janet and Bill leave their home at the same time. Janet has 60 miles to travel and drives at 40 mph. Bill has 80 miles to travel and also drives at 40 mph.

   (a) How long does Janet's journey take?
   (b) How much longer does Bill spend driving than Janet?

5. An athlete can run long distances at 4 metres per second. How far can she run in:

   (a) 50 seconds,
   (b) 3 minutes,
   (c) 1 hour,
   (d) $2\frac{1}{2}$ hours ?

6. Andrew rows at an average speed of 2 metres per second.

   (a) How long does it take him to row:

   (i) 70 m, (ii) 800 m, (iii) $1\frac{1}{2}$ km ?

(b) How far can Andrew row in:

(i) 12 seconds, (ii) $3\frac{1}{2}$ minutes, (iii) 4 hours ?

7. A snail moves 5 m in 2 hours, If the snail moves at the same speed, calculate:

(a) the time it takes to move 20 m,

(b) the distance it would move in $3\frac{1}{2}$ hours,

(c) the time it takes to moves 1 m,

(d) the distance that it moves in 15 minutes.

8. Laura drives for 3 hours at 44 mph.
Clare drives 144 miles in 4 hours.

(a) Who travels the greater distance?

(b) Whose speed is the slower?

(c) How far would Laura travel if she drove for 3 hours at the same speed as Clare?

9. A lorry travels for 3 hours at 48 mph and then for 2 hours at 53 mph.

(a) What is the total distance travelled by the lorry?

(b) What is the average speed for the whole journey?

10. Sally drives for $2\frac{1}{2}$ hours at 50 mph, then drives 80 miles at 40 mph, and finally drives for 30 minutes at 60 mph.

(a) Calculate the total distance that Sally drives.

(b) Calculate the time that Sally takes for the journey.

(c) Calculate her average speed for the whole journey.

18.3 Problems with Mixed Units

In this section we consider working with mixed units, and with changing units used for speeds.

Example 1

(a) Convert 1 hour 24 minutes to hours (decimal).

(b) Write 2.32 hours in hours and minutes.

Solution

(a) $\frac{24}{60} = 0.4$

Therefore,

1 hr 24 mins = 1.4 hours

(b) $0.32 \times 60 = 19.2$

Therefore,

2.32 hours = 2 hrs 19.2 mins

Example 2

A car travels 200 miles in 3 hours and 20 minutes. Calculate the average speed of the car in mph.

Solution

3 hours 20 minutes $= 3\frac{20}{60}$

$= 3\frac{1}{3}$ hours

Speed = distance ÷ time

$= 200 \div 3\frac{1}{3}$

$= 200 \div \frac{10}{3}$

$= 200 \times \frac{3}{10}$

= 60 mph

Example 3

An athlete runs 1500 m in 3 minutes and 12 seconds. Calculate the average speed of the athlete in m/s.

Solution

3 minutes 12 seconds $= 3 \times 60 + 12$

= 192 seconds

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{1500}{192}$$

$$= 7.8 \text{ m/s to 1 decimal place}$$

Example 4

A bus travels at a speed of 40 km/h. Calculate the speed of the bus in:

(a) m/s

(b) mph.

Solution

(a) $1 \text{ km} = 1000 \text{ m}$

$40 \text{ km/h} = 1000 \times 40 \text{ m/hr}$

$1 \text{ hour} = 60 \times 60$

$= 3600 \text{ seconds}$

$$40 \text{ km/h} = \frac{1000 \times 40}{3600}$$

$= 11.1 \text{ m/s to 1 decimal place}$

(b) $1 \text{ km} = \frac{5}{8} \text{ mile}$

$$\text{So } 40 \text{ km/h} = \frac{5}{8} \times 40$$

$= 25 \text{ mph}$

Example 5

Convert a speed of 8 m/s to mph.

Solution

$8 \text{ m/s} = 8 \times 3600 \text{ m/h}$

$= 28\,800 \text{ m/h}$

$= 28.8 \text{ km/h}$

$$28.8 \times \frac{5}{8} = 18 \text{ mph}$$

Exercises

1. Convert the following times from hours and minutes to hours, giving your answers as mixed numbers and decimals, correct to 2 decimal places.

   (a) 1 hour 40 minutes (b) 3 hours 10 minutes

   (c) 1 hour 6 minutes (d) 2 hours 18 minutes

   (e) 3 hours 5 minutes (f) 6 hours 2 minutes

   (g) 1 hour 7 minutes (h) 2 hours 23 minutes

2. Change the following times to hours and minutes:

   (a) $1\frac{1}{4}$ hours (b) 1.2 hours

   (c) 3.7 hours (d) 4.4 hours

   (e) 1.45 hours (f) 3.65 hours

3. A car travels 60 miles in 50 minutes. Calculate the average speed of the car in mph.

4. Jane drives 80 miles in 1 hour and 40 minutes. Calculate her average speed.

5. Convert the following speeds to km/h:

   (a) 60 mph (b) 43 m/s

   (c) 66 m/s (d) 84 mph

6. Convert the following speeds to mph:

   (a) 16 m/s (b) 82 km/h

   (c) 48 km/h (d) 7 m/s

7. Alec drives 162 km in 2 hours and 12 minutes. Calculate his average speed in:

   (a) km/h (b) m/s (c) mph

   Give your answers to 2 decimal places.

8. Jai drives 297 miles in 5 hours and 24 minutes.

   (a) Calculate his average speed in mph.

   (b) He then drives for a further 1 hour and 28 minutes at the same average speed. How far has he travelled altogether?

   Give your answers to 2 decimal places.

9. A train travels at 40 m/s. Calculate the time it takes to travel:

   (a) 30 000 m,

   (b) 50 km,

   (c) 200 miles.

10. A long distance runner runs at an average speed of 7 mph. How long will it take the runner to run:

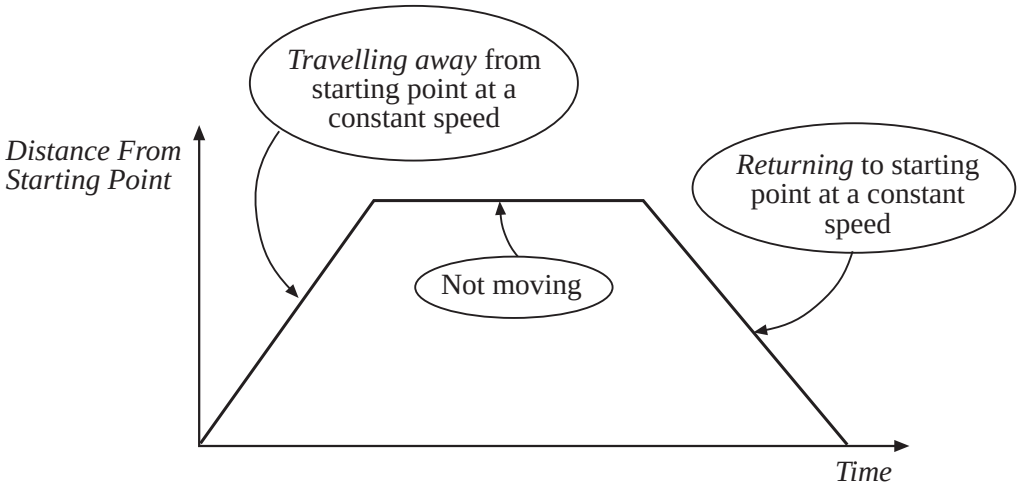
   (a) 20 miles,

   (b) 15 km,

   (c) 10 000 m ?

18.4 Distance-Time Graphs

Graphs that show distance against time can be used to describe journeys. The vertical scale shows the distance from the starting point or reference point.

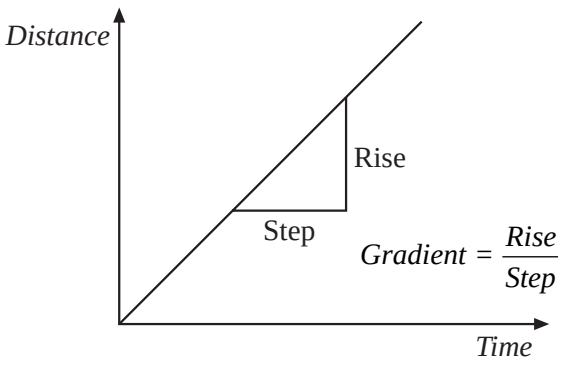


The graph above illustrates 3 parts of a journey.

The *gradient of a straight line* gives the *speed* of the moving object.

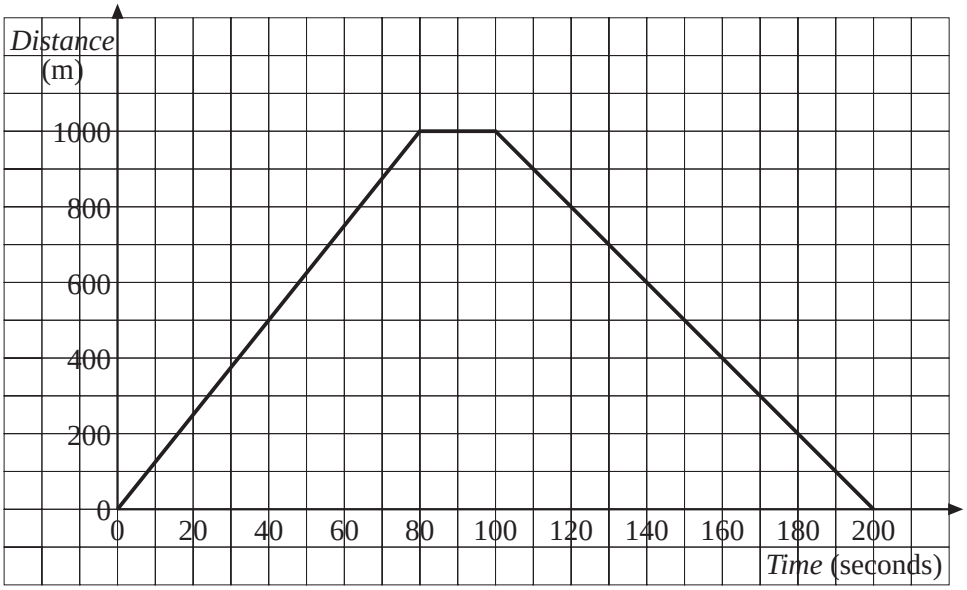
Gradient is a measure of the speed.

Note that a *negative gradient* indicates that the object is moving *towards the starting point* rather than away from it.



Example 1

The graph shows how far a child is from home.



(a) Describe how the child moves.

(b) Calculate the speed of the child on each part of the journey.

Solution

(a) The first part of the graph shows the child moving away from home at a constant speed.

The second (horizontal) part of the graph shows that the child remains in the same position.

The third part of the graph shows the child returning to the starting point at a steady speed.

(b) During the first stage the child travels 1000 m in 80 seconds.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{1000}{80}$$

$$= 12.5 \text{ m/s}$$

During the second stage the speed of the child is zero.

During the third stage as the child returns, he travels 1000 m in 100 seconds.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{1000}{100}$$

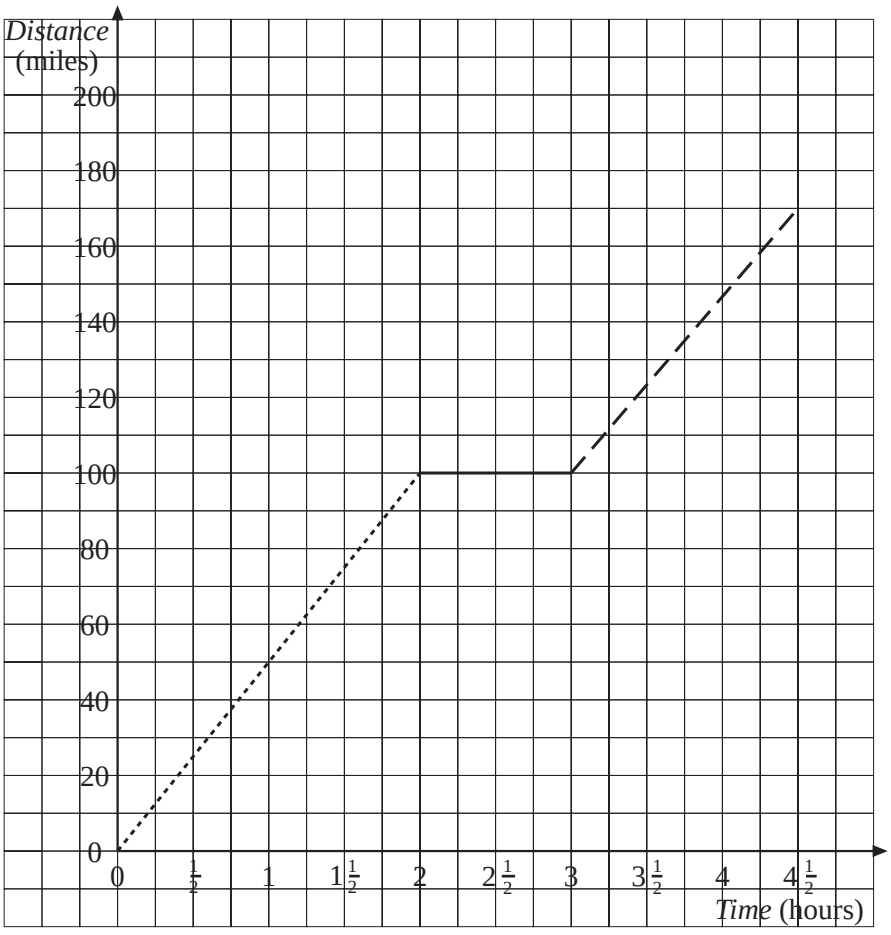
$$= 10 \text{ m/s}$$

Example 2

On a journey, Rebecca drives at 50 mph for 2 hours, rests for 1 hour and then drives another 70 miles in $1\frac{1}{2}$ hours.

Draw a distance-time graph to illustrate this journey.

Solution



First stage ·····

Travels 100 miles in 2 hours.

Second stage ——

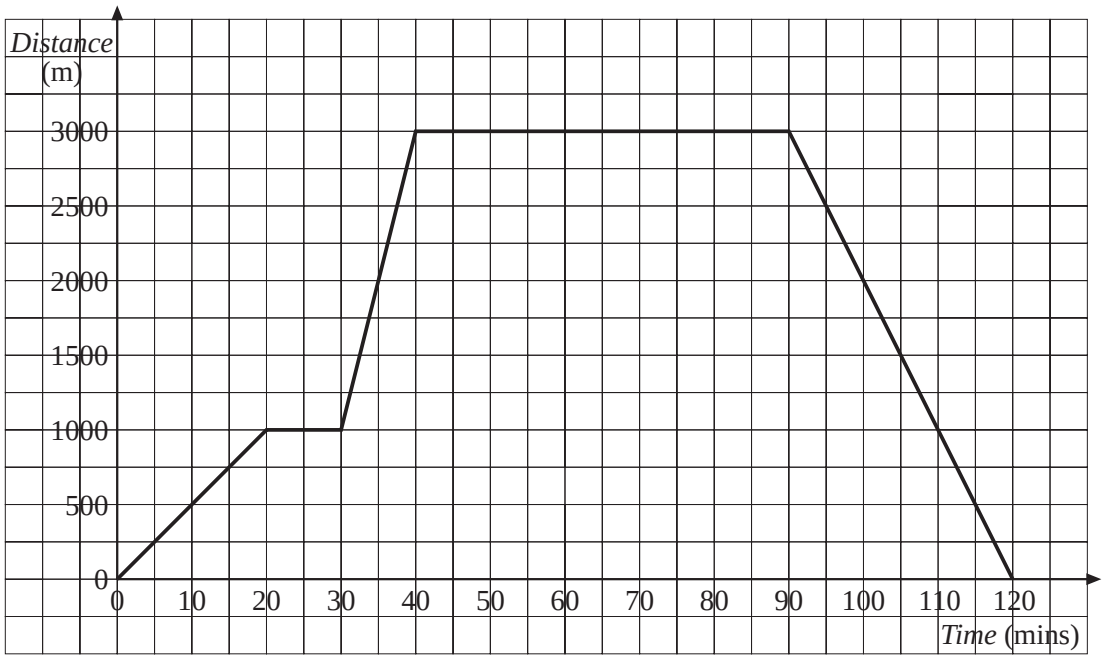
Rests, so distance does not change.

Third stage – – –

Travels 70 miles in $1\frac{1}{2}$ hours.

Example 3

The graph shows how Tom's distance from home varies with time, when he visits Ian.



(a) How long does Tom spend at Ian's?

(b) How far is it from Tom's home to Ian's?

(c) For how long does Tom stop on the way to Ian's?

(d) On which part of the journey does Tom travel the fastest?

(e) How fast does Tom walk on the way back from Ian's?

Solution

(a) The longer horizontal part of the graph represents the time that Tom is at Ian's.

Time = 90 – 40

= 50 minutes

(b) 3000 m

(c) Tom stops for 10 minutes, represented by the smaller horizontal part on the graph.

(d) He travels fastest on the second part of the journey to Ian's. This is where the graph is steepest. He travels 2000 m in 10 minutes.

$$\text{Speed} = \frac{2000}{10}$$

= 200 m/minute

$$= \frac{200 \times 60}{1000}$$

= 12 km/h

(e) Tom travels 3000 m in 30 mins.

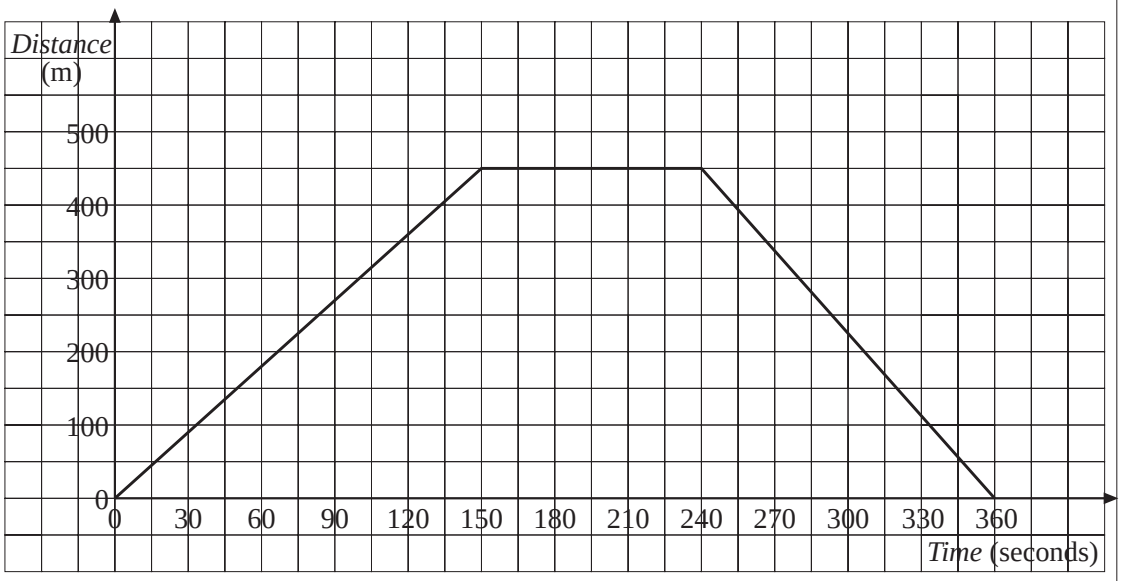
$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{3000}{30}$$

$$= 100 \text{ m/minute}$$

Exercises

1. The following graph illustrates how Jamil moves as he goes to the paper shop:



(a) How long does it take Jamil to cycle to the shop?

(b) What distance does Jamil cycle to get to the shop?

(c) Calculate the speed at which Jamil cycles to the shop.

(d) How long does Jamil spend at the shop?

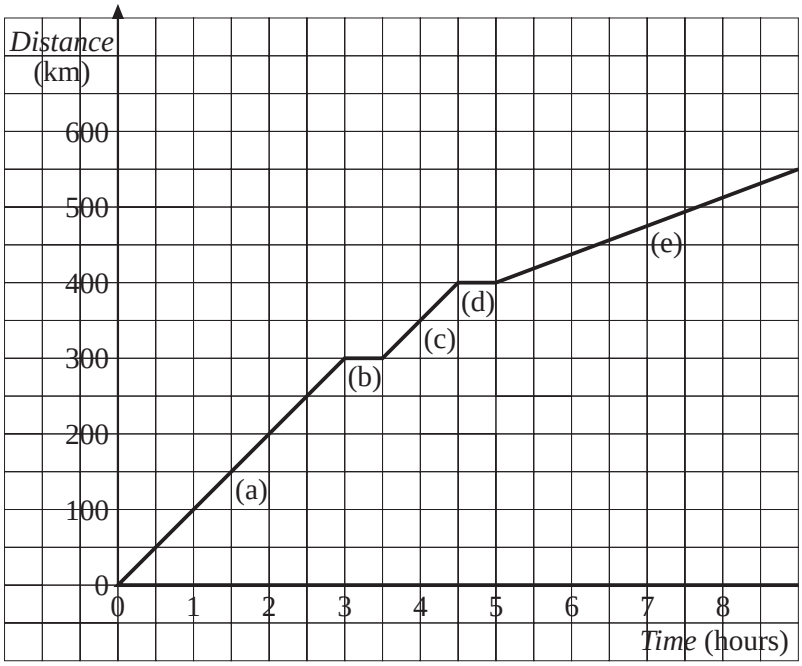
(e) Calculate the speed at which Jamil cycles on his way home.

2. On a journey, Vera

- drives 200 miles in 4 hours
- rests for 1 hour
- drives another 100 miles in 2 hours.

Draw a distance-time graph for Vera's journey.

3. Describe the 5 parts of the journey (labelled (a), (b), (c), (d) and (e)) represented by the following distance-time graph:



4. Ray walks 420 m from his house to a shop in 7 minutes. He spends 5 minutes at the shop and then walks home in 6 minutes.

   (a) Draw a distance-time graph for Ray's shopping trip.

   (b) Calculate the speed at which Ray walks on each part of the journey.

5. Mary sprints 200 m in 30 seconds, rests for 45 seconds and then walks back in $1\frac{1}{2}$ minutes to where she started the race.

   (a) Draw a distance-time graph for Mary.

   (b) Calculate the speed at which Mary runs.

   (c) Calculate the speed at which Mary walks.

6. After morning school, Mike walks home from school to have his lunch. The distance-time graph on the next page describes his journey on one day, showing his distance from home.

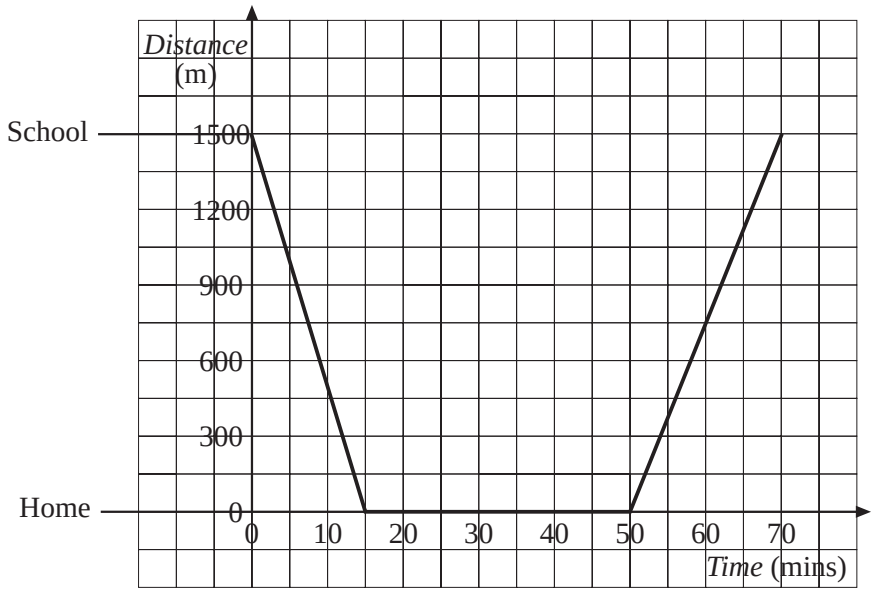
   (a) How far is Mike's home from school?

   (b) How long does it take Mike to walk home?

   (c) At what speed does he walk on the way home? Give your answer in m/s.

   (d) How long does Mike spend at home?

(e) At what speed does he walk back to school? Give your answer in m/s.

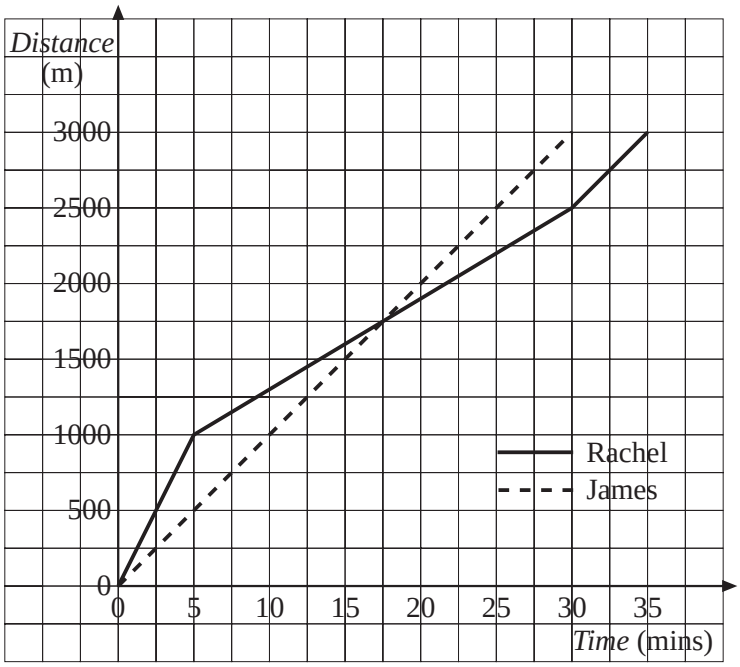


7. Helen cycles for 20 minutes at 5 m/s and then for a further 10 minutes at 4 m/s.

(a) How far does she cycle altogether?

(b) Draw a distance-time graph for her ride.

8. The distance-time graph shown is for a 3000 m cross-country race, run by Rachel and James.



(a) Describe how James runs the race.

(b) Describe how Rachel runs the race.

(c) When, and how far from the start, does James catch up with Rachel?

(d) Calculate the speed at which James runs.

(e) Calculate the different speeds at which Rachel runs.

(f) Who wins the race?

9. Josh completes a 10 000 m race. He runs the first 2000 m at 5 m/s, the next 7400 m at 4 m/s and the last 600 m at 6 m/s.

(a) Draw a distance-time graph for Josh's race.

(b) How long does he take to complete the race?

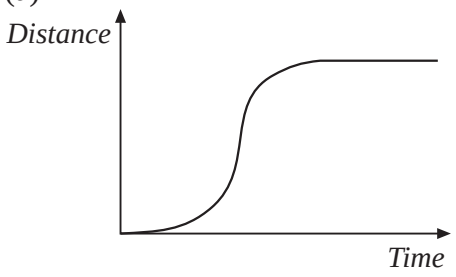
10. Emma runs a 2000 m race. She runs at 5 m/s for the first part of the race and at 4 m/s for the rest of the race. She complete the race in 440 seconds.

(a) Draw a distance-time graph for Emma's race.

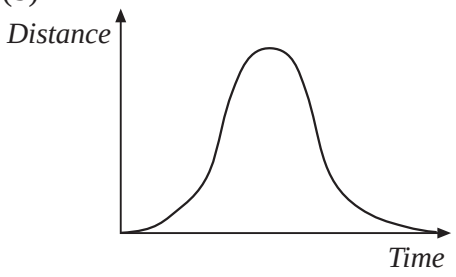
(b) How far does she run at each speed?

11. Describe the journey shown in each of the following graphs:

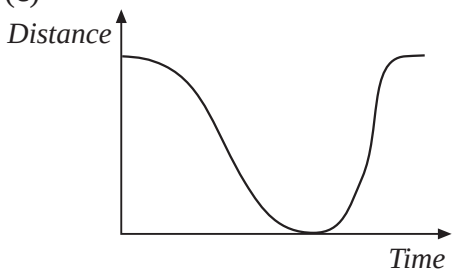
(a)



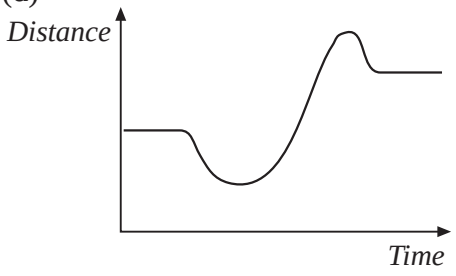
(b)



(c)



(d)



18.5 Other Compound Measures

In the section so far we have considered speed in several different contexts. We will now look at other things such as goals per game, and postage rates.

Example 1

In a football season, Ivor Boot scores 27 goals in 40 matches. Calculate his average scoring rate in goals per match, goals per minute and goals per hour.

Solution

$$\text{Scoring rate} = \frac{27}{40}$$

$$= 0.675 \text{ goals per match}$$

$$\text{Scoring rate} = \frac{27}{40 \times 90} \quad \text{(there are 90 minutes per match)}$$

$$= 0.0075 \text{ goals per minute}$$

$$\text{Scoring rate} = \frac{27}{40 \times 1\frac{1}{2}}$$

$$= 0.45 \text{ goals per hour}$$

Example 2

A package has a mass of 200 grams. It can be posted first class for 60p, or second class for 47p. Calculate the cost per gram for first and second class post.

Solution

First Class Cost per gram $= \frac{60}{200}$

$= 0.3\text{p}$

Second Class Cost per gram $= \frac{47}{200}$

$= 0.235\text{p}$

Exercises

1. Three boys play football for a school team. The numbers of goals scored and matches played are listed below:

	Number of Goals Scored	*Number of Matches Played*
Ian	16	32
Ben	22	40
Sergio	9	20

   (a) Who scores the most goals per match?

   (b) Who scores the least goals per match?

2. Alison plays 20 games for her school hockey team and scores 18 goals. Each match lasts 90 minutes.

   Calculate her scoring rate in:

   (a) goals per hour,

   (b) goals per minute,

   (c) goals per match.

3. When playing football, Jai claims to be able, on average, to score a goal every 40 minutes. How many goals would you expect him to score in:

   (a) 90 minutes,

   (b) 1 hour,

   (c) 5 matches,

   (d) 40 matches ?

4. It costs 96p to send an air mail letter of mass 40 grams to Africa, and 107p to send it to China.

   (a) Calculate the cost per gram for each destination.

   (b) If the same rates apply to a 50 gram letter, calculate the cost for each destination.

5. A package of mass 80 grams costs 39p to post first class and 31p to post second class. Calculate the cost per gram for first and second class post.

6. A taxi driver charges £3.20 for a 4 km journey. How much does he charge:

   (a) per km,

   (b) per metre ?

7. A taxi service makes a fixed charge of £1.20 and then 78p per km. Calculate the cost for journeys of the following lengths:

   (a) 1 km (b) 2 km

   (c) 4.5 km (d) 10.5 km

8. Alexi buys a 20 m length of fabric for £18.60.

   (a) What is the cost per m of the fabric?

   (b) What would be the cost of 9.2 m of the fabric?

9. Five people work in a shop. The following table lists the hours worked and the total paid in one week:

	Hours Worked	*Total Paid*
Dee	8	£28.64
Nadina	12	£43.44
Lisa	42	£302.40
Mary	38	£136.80
Clare	35	£134.40

   (a) Who is paid the *most* per hour?

   (b) Who is paid the *least* per hour?

10. A 5 litre tin of paint is used to paint a wall that measures 6.25 m by 4 m. Calculate the rate at which paint is applied to the wall, in:

    (a) litres per m^2,

    (b) cm^3 per m^2,

    (c) ml per cm^2.