1.) Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

(a) Use the quotient rule to show that $g'(x) = \frac{1 - 2\ln x}{x^3}$.

(4)

(b) The graph of g has a maximum point at A. Find the x-coordinate of A.

(3)
(Total 7 marks)

2.) The velocity v m s^{-1} of a particle at time t seconds, is given by $v = 2t + \cos 2t$, for $0 \leq t \leq 2$.

(a) Write down the velocity of the particle when $t = 0$.

(1)

When $t = k$, the acceleration is zero.

(b) (i) Show that $k = \frac{\pi}{4}$.

(ii) Find the exact velocity when $t = \frac{\pi}{4}$.

(8)

(c) When $t < \frac{\pi}{4}$, $\frac{dv}{dt} > 0$ and when $t > \frac{\pi}{4}$, $\frac{dv}{dt} > 0$.

Sketch a graph of v against t.

(4)

(d) Let d be the distance travelled by the particle for $0 \leq t \leq 1$.

(i) Write down an expression for d.

(ii) Represent d on your sketch.

(3)
(Total 16 marks)

3.) The following diagram shows part of the graph of the function $f(x) = 2x^2$.

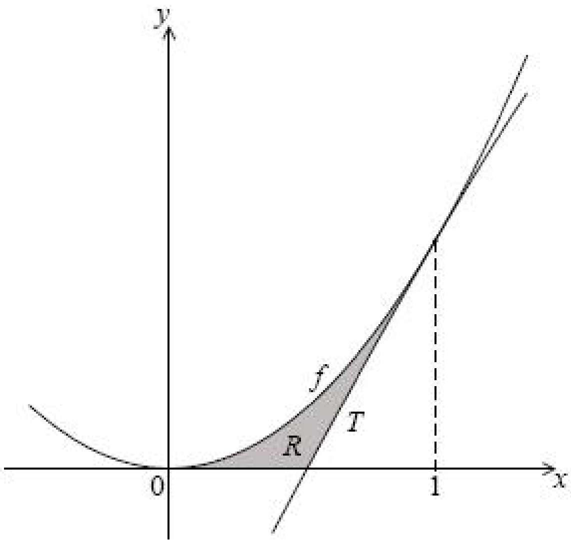


diagram not to scale

The line T is the tangent to the graph of f at $x = 1$.

(a) Show that the equation of T is $y = 4x - 2$.

(5)

(b) Find the x-intercept of T.

(2)

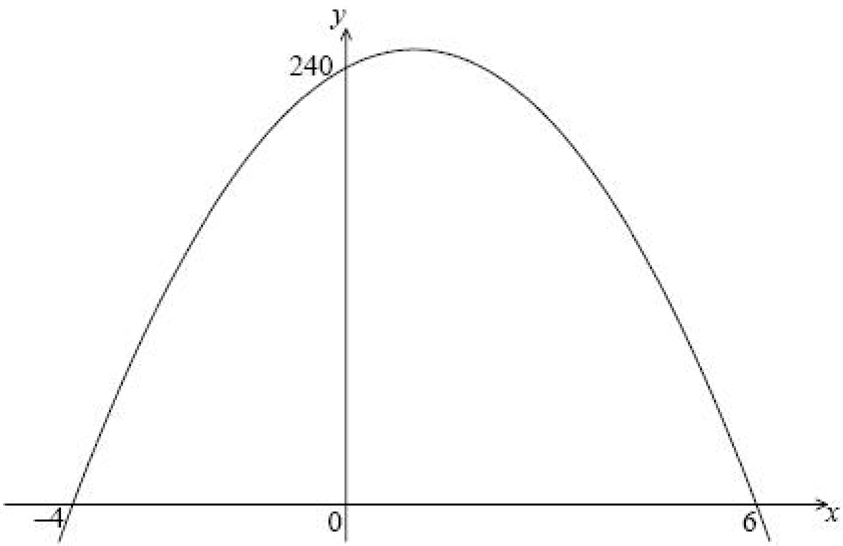
(c) The shaded region R is enclosed by the graph of f, the line T, and the x-axis.

(i) Write down an expression for the area of R.

(ii) Find the area of R.

(9)
(Total 16 marks)

4.) The following diagram shows part of the graph of a quadratic function f.



The x-intercepts are at (–4, 0) and (6, 0) and the y-intercept is at (0, 240).

(a) Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$.

(2)

(b) Find another expression for $f(x)$ in the form $f(x) = -10(x - h)^2 + k$.

(4)

(c) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$.

(2)

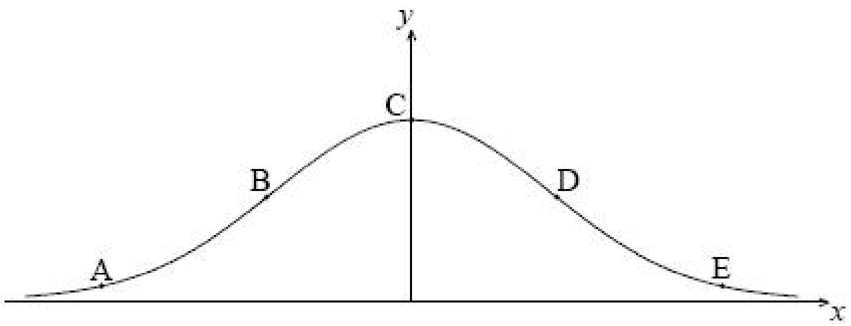
A particle moves along a straight line so that its velocity, v m s^{-1}, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \le t \le 6$.

(d) (i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero.

(7)
(Total 15 marks)

5.) The following diagram shows the graph of $f(x) = e^{-x^2}$.



The points A, B, C, D and E lie on the graph of f. Two of these are points of inflexion.

(a) Identify the **two** points of inflexion.

(2)

(b) (i) Find $f'(x)$.

(ii) Show that $f''(x) = (4x^2 - 2)e^{-x^2}$.

(5)

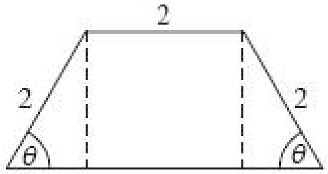
(c) Find the x-coordinate of each point of inflexion.

(4)

(d) Use the second derivative to show that one of these points is a point of inflexion.

(4)
(Total 15 marks)

6.) The diagram below shows a plan for a window in the shape of a trapezium.



Three sides of the window are 2 m long. The angle between the sloping sides of the window and the base is θ, where $0 < \theta < \frac{\pi}{2}$.

(a) Show that the area of the window is given by $y = 4 \sin \theta + 2 \sin 2\theta$.

(5)

(b) Zoe wants a window to have an area of 5 m^2. Find the two possible values of θ.

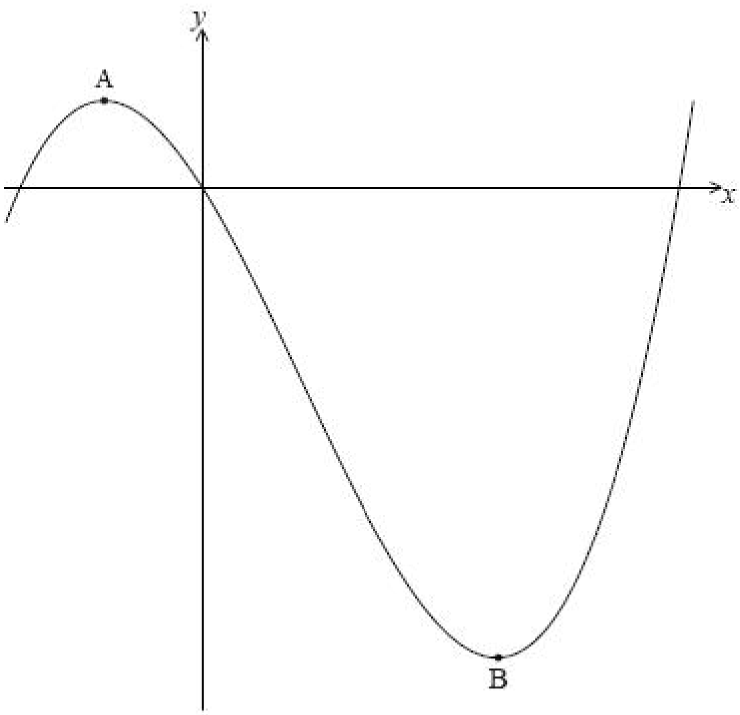
(4)

(c) John wants two windows which have the same area A but different values of θ.

Find all possible values for A.

(7)
(Total 16 marks)

7.) Let $f(x) = \frac{1}{3}x^3 - x^2 - 3x$. Part of the graph of f is shown below.



There is a maximum point at A and a minimum point at B(3, –9).

(a) Find the coordinates of A.

(8)

(b) Write down the coordinates of

(i) the image of B after reflection in the y-axis;

(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$;

(iii) the image of B after reflection in the x-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$.

(6)
(Total 14 marks)

8.) Let $f(x) = \frac{\cos x}{\sin x}$, for $\sin x \neq 0$.

(a) Use the quotient rule to show that $f'(x) = \frac{-1}{\sin^2 x}$.

(5)

(b) Find $f''(x)$.

(3)

In the following table, $f'\left(\frac{\pi}{2}\right) = p$ and $f''\left(\frac{\pi}{2}\right) = q$. The table also gives approximate values of $f'(x)$ and $f''(x)$ near $x = \frac{\pi}{2}$.

x	$\frac{\pi}{2} - 0.1$	$\frac{\pi}{2}$	$\frac{\pi}{2} + 0.1$
$f'(x)$	–1.01	p	–1.01
$f''(x)$	0.203	q	–0.203

(c) Find the value of p and of q.

(3)

(d) Use information from the table to explain why there is a point of inflexion on the graph of f where $x = \frac{\pi}{2}$.

(2)
(Total 13 marks)

9.) Let $f(x) = kx^4$. The point P(1, k) lies on the curve of f. At P, the normal to the curve is parallel to $y = -\frac{1}{8}x$. Find the value of k.

(Total 6 marks)

10.) Consider the function f with second derivative $f''(x) = 3x - 1$. The graph of f has a minimum point at A(2, 4) and a maximum point at B$\left(-\frac{4}{3}, \frac{358}{27}\right)$.

(a) Use the second derivative to justify that B is a maximum.

(3)

(b) Given that $f' = \frac{3}{2}x^2 - x + p$, show that $p = -4$.

(4)

(c) Find $f(x)$.

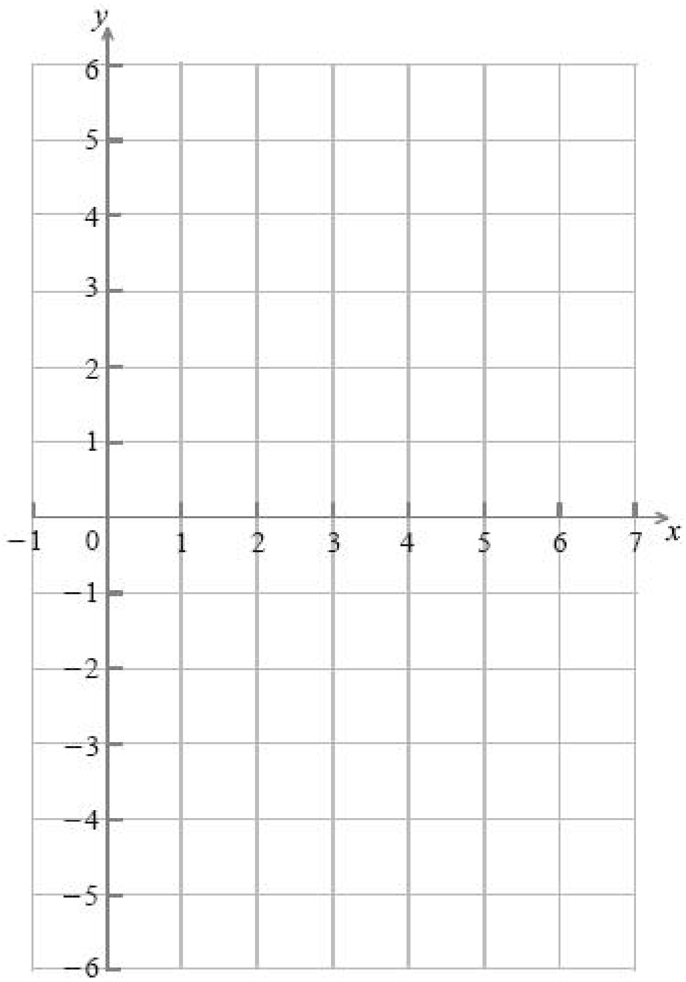
(7)
(Total 14 marks)

11.) Let $f(x) = x \cos x$, for $0 \le x \le 6$.

(a) Find $f'(x)$.

(3)

(b) On the grid below, sketch the graph of $y = f'(x)$.



(4)
(Total 7 marks)

12.) The acceleration, a m s^{-2}, of a particle at time t seconds is given by

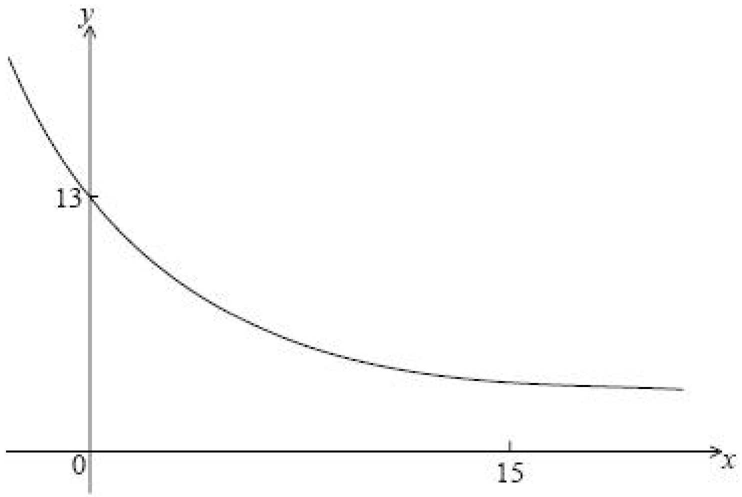
$$a = \frac{1}{t} + 3\sin 2t, \text{ for } t \geq 1.$$

The particle is at rest when $t = 1$.

Find the velocity of the particle when $t = 5$.

(Total 7 marks)

13.) Let $f(x) = Ae^{kx} + 3$. Part of the graph of f is shown below.



The y-intercept is at (0, 13).

(a) Show that A =10.

(2)

(b) Given that $f(15) = 3.49$ (correct to 3 significant figures), find the value of k.

(3)

(c) (i) Using your value of k, find $f'(x)$.

(ii) Hence, explain why f is a decreasing function.

(iii) Write down the equation of the horizontal asymptote of the graph f.

(5)

Let $g(x) = -x^2 + 12x - 24$.

(d) Find the area enclosed by the graphs of f and g.

(6)
(Total 16 marks)

14.) Let $f(x) = x^3$. The following diagram shows part of the graph of f.

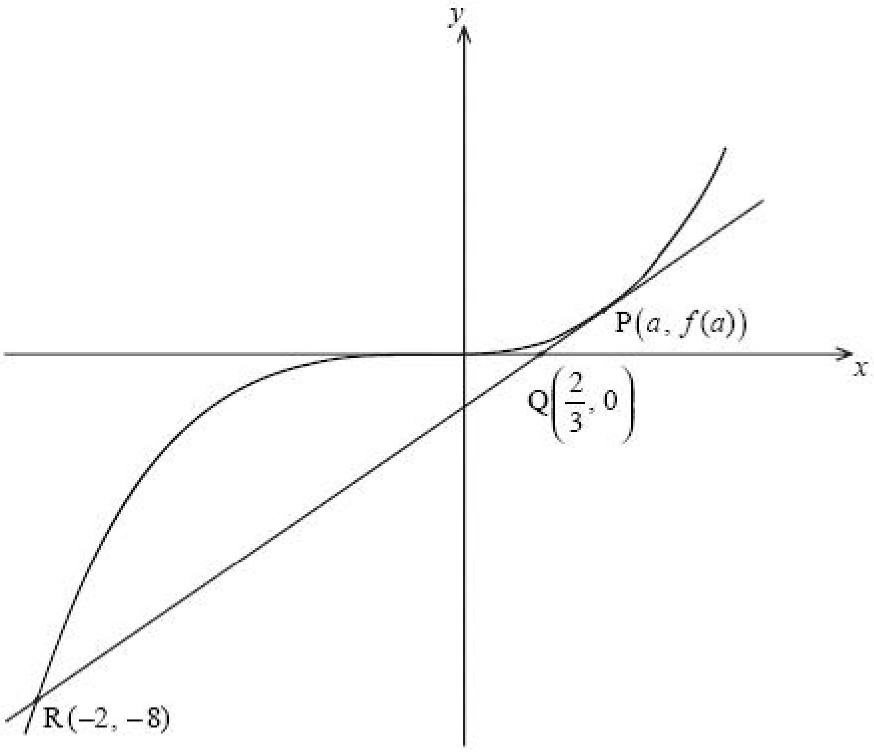


diagram not to scale

The point P (a, $f(a)$), where $a > 0$, lies on the graph of f. The tangent at P crosses the x-axis at the point Q$\left(\frac{2}{3}, 0\right)$. This tangent intersects the graph of f at the point R(–2, –8).

(a) (i) Show that the gradient of [PQ] is $\dfrac{a^3}{a - \frac{2}{3}}$.

(ii) Find $f'(a)$.

(iii) Hence show that $a = 1$.

(7)

The equation of the tangent at P is $y = 3x - 2$. Let T be the region enclosed by the graph of f, the tangent [PR] and the line $x = k$, between $x = -2$ and $x = k$ where $-2 < k < 1$. This is shown in the diagram below.

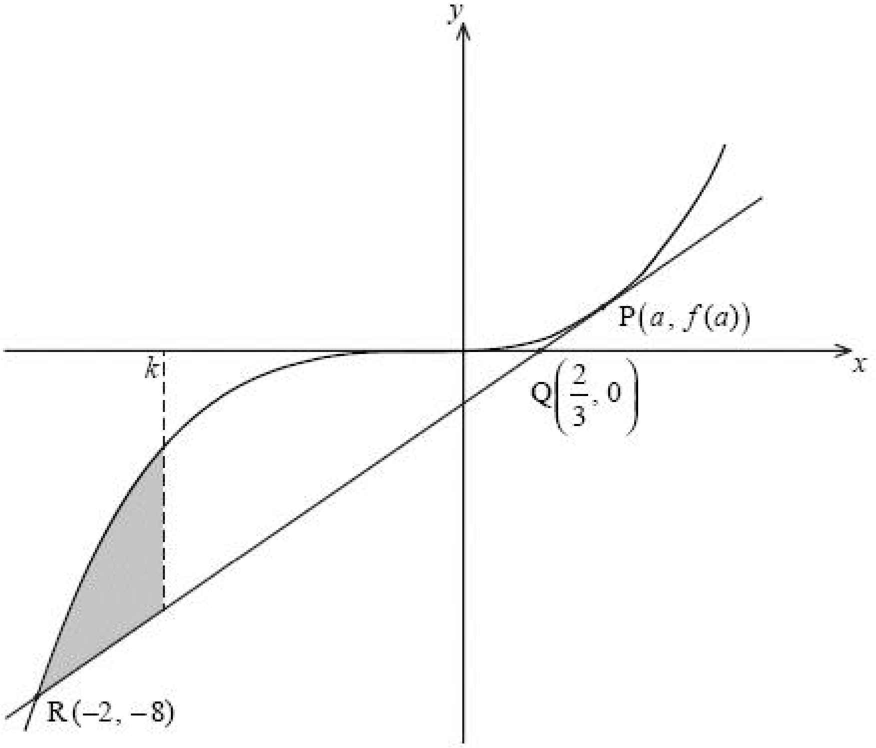


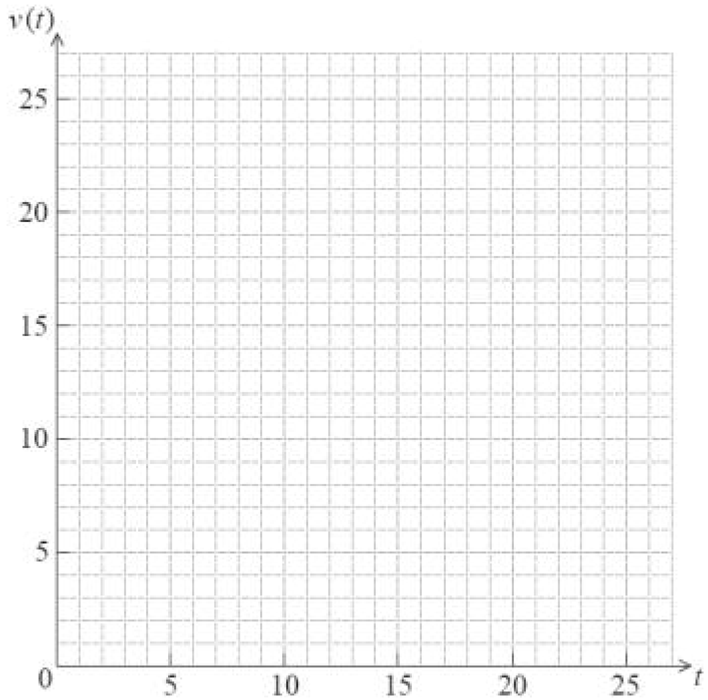
diagram not to scale

(b) Given that the area of T is $2k + 4$, show that k satisfies the equation $k^4 - 6k^2 + 8 = 0$.

(9)
(Total 16 marks)

15.) The velocity v m s^{-1} of an object after t seconds is given by $v(t) = 15\sqrt{t} - 3t$, for $0 \le t \le 25$.

(a) On the grid below, sketch the graph of v, clearly indicating the maximum point.



(3)

Let d be the distance travelled in the first nine seconds.

(b) (i) Write down an expression for d.

(ii) Hence, write down the value of d.

(4)
(Total 7 marks)

16.) Let $f'(x) = -24x^3 + 9x^2 + 3x + 1$.

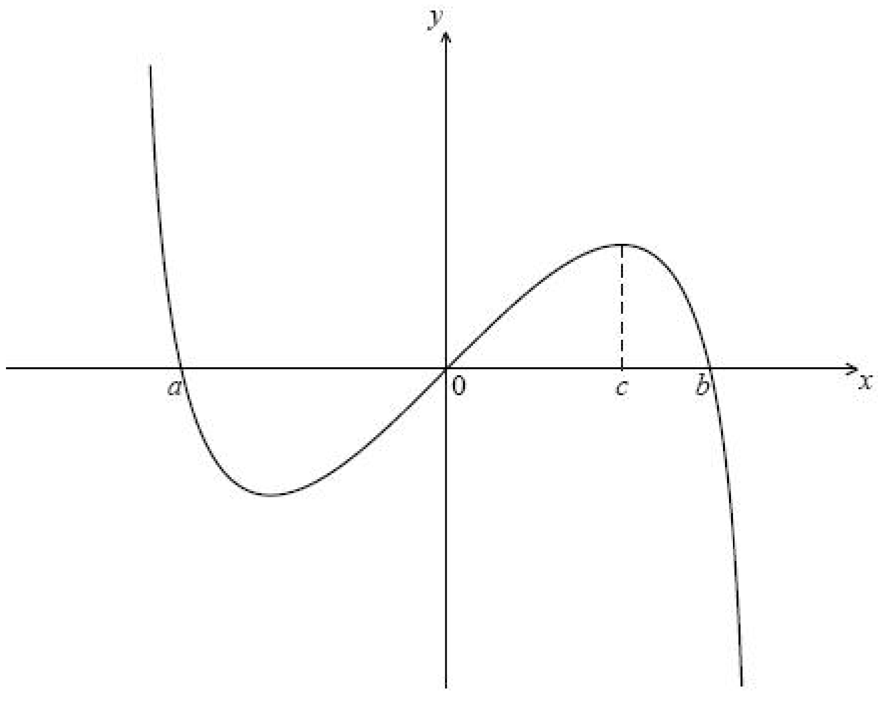
(a) There are two points of inflexion on the graph of f. Write down the x-coordinates of these points.

(3)

(b) Let $g(x) = f''(x)$. Explain why the graph of g has no points of inflexion.

(2)
(Total 5 marks)

17.) Let $f(x) = x \ln(4 - x^2)$, for $-2 < x < 2$. The graph of f is shown below.



The graph of f crosses the x-axis at $x = a$, $x = 0$ and $x = b$.

(a) Find the value of a and of b.

(3)

The graph of f has a maximum value when $x = c$.

(b) Find the value of c.

(2)

(c) The region under the graph of f from $x = 0$ to $x = c$ is rotated 360° about the x-axis. Find the volume of the solid formed.

(3)

(d) Let R be the region enclosed by the curve, the x-axis and the line $x = c$, between $x = a$ and $x = c$.

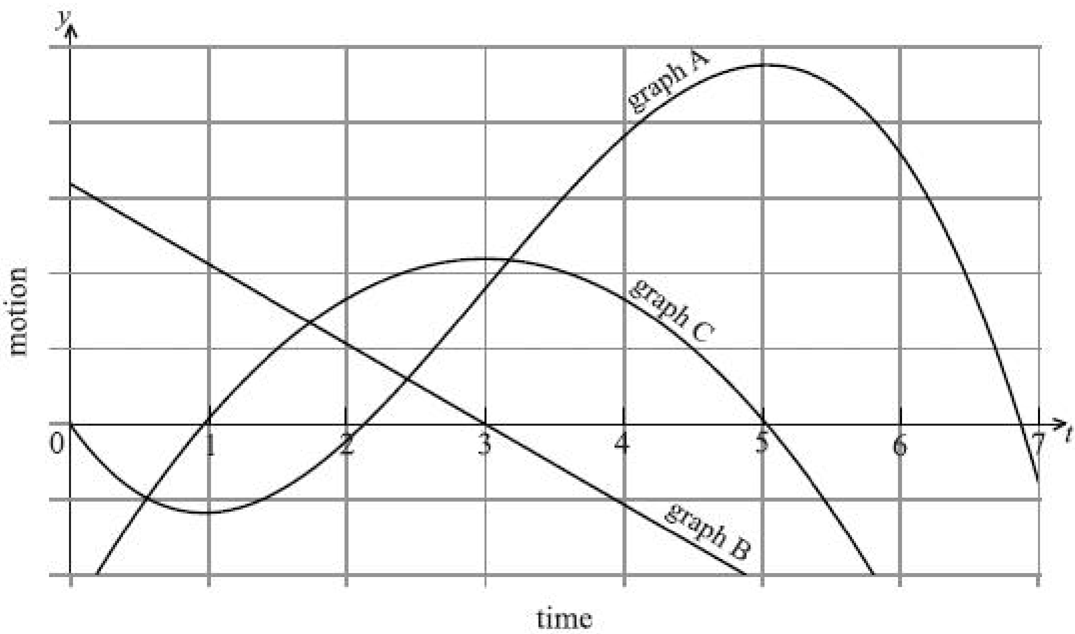
Find the area of R.

(4)
(Total 12 marks)

18.) Let $f(x) = e^x \cos x$. Find the gradient of the normal to the curve of f at $x = \pi$.

(Total 6 marks)

19.) The following diagram shows the graphs of the **displacement**, **velocity** and **acceleration** of a moving object as functions of time, t.



(a) Complete the following table by noting which graph A, B or C corresponds to each function.

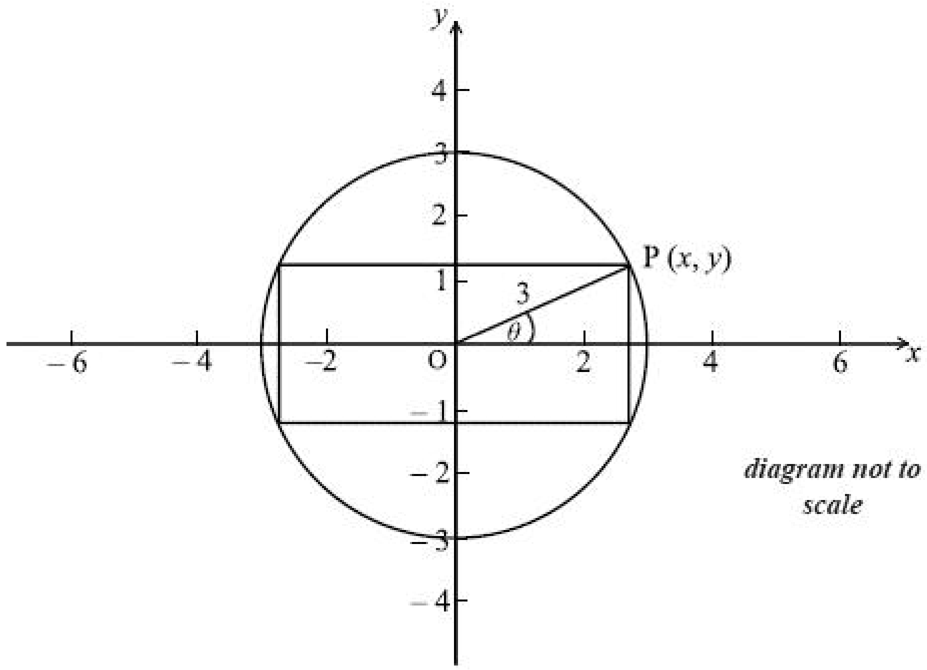
Function	Graph
displacement	
acceleration	

(4)

(b) Write down the value of t when the velocity is greatest.

(2)
(Total 6 marks)

20.) A rectangle is inscribed in a circle of radius 3 cm and centre O, as shown below.



The point P(x, y) is a vertex of the rectangle and also lies on the circle. The angle between (OP) and the x-axis is θ radians, where $0 \le \theta \le \frac{\pi}{2}$.

(a) Write down an expression in terms of θ for

(i) x;

(ii) y.

(2)

Let the area of the rectangle be A.

(b) Show that $A = 18 \sin 2\theta$.

(3)

(c) (i) Find $\frac{\mathrm{d}A}{\mathrm{d}}$.

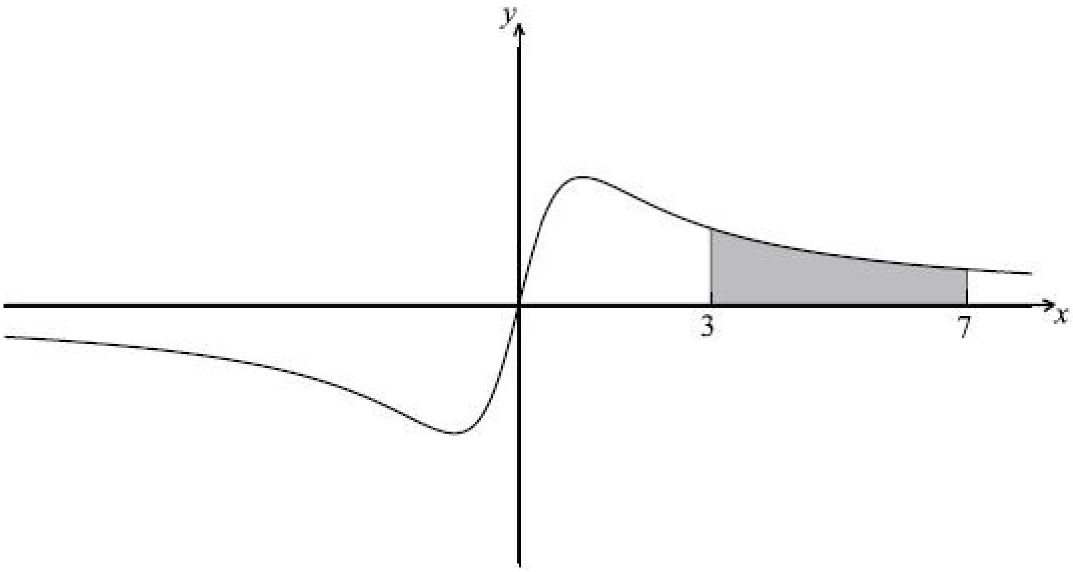
(ii) Hence, find the exact value of θ which maximizes the area of the rectangle.

(iii) Use the second derivative to justify that this value of θ does give a maximum.

(8)

(Total 13 marks)

21.) Let $f(x) = \dfrac{ax}{x^2+1}$, $-8 \le x \le 8$, $a \in \mathbb{R}$. The graph of f is shown below.



The region between $x = 3$ and $x = 7$ is shaded.

(a) Show that $f(-x) = -f(x)$.

(2)

(b) Given that $f''(x) = \dfrac{2ax(x^2-3)}{(x^2+1)^3}$, find the coordinates of all points of inflexion.

(7)

(c) It is given that $\int f(x)\mathrm{d}x = \dfrac{a}{2}\ln(x^2+1)+C$.

(i) Find the area of the shaded region, giving your answer in the form $p \ln q$.

(ii) Find the value of $\int_4^8 2f(x-1)\mathrm{d}x$.

(7)
(Total 16 marks)

22.) A function f has its first derivative given by $f'(x) = (x-3)^3$.

(a) Find the second derivative.

(2)

(b) Find $f'(3)$ and $f''(3)$.

(1)

(c) The point P on the graph of f has x-coordinate 3. Explain why P is not a point of inflexion.

(2)
(Total 5 marks)

23.) *In this question s represents displacement in metres and t represents time in seconds.*

The velocity v m s^{-1} of a moving body is given by $v = 40 - at$ where a is a non-zero constant.

(a) (i) If $s = 100$ when $t = 0$, find an expression for s in terms of a and t.

(ii) If $s = 0$ when $t = 0$, write down an expression for s in terms of a and t.

(6)

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by $v = 40 - at$, where $t = 0$ at P. The station is 500 m from P.

(b) A train M slows down so that it comes to a stop at the station.

(i) Find the time it takes train M to come to a stop, giving your answer in terms of a.

(ii) Hence show that $a = \frac{8}{5}$.

(6)

(c) For a different train N, the value of a is 4.
Show that this train will stop **before** it reaches the station.

(5)
(Total 17 marks)

24.) Let $f(x) = x^3 - 4x + 1$.

(a) Expand $(x + h)^3$.

(2)

(b) Use the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to show that the derivative of $f(x)$ is $3x^2 - 4$.

(4)

(c) The tangent to the curve of f at the point P(1, –2) is parallel to the tangent at a point Q.
Find the coordinates of Q.

(4)

(d) The graph of f is decreasing for $p < x < q$. Find the value of p and of q.

(3)

(e) Write down the range of values for the gradient of f.

(2)
(Total 15 marks)

25.) Consider the curve $y = \ln(3x - 1)$. Let P be the point on the curve where $x = 2$.

(a) Write down the gradient of the curve at P.

(2)

(b) The normal to the curve at P cuts the x-axis at R. Find the coordinates of R.

(5)
(Total 7 marks)

26.) Consider $f(x) = x^2 + \frac{p}{x}$, $x \neq 0$, where p is a constant.

(a) Find $f'(x)$.

(2)

(b) There is a minimum value of $f(x)$ when $x = -2$. Find the value of p.

(4)
(Total 6 marks)

27.) Let $f(x) = 3 + \frac{20}{x^2 - 4}$, for $x \neq \pm 2$. The graph of f is given below.

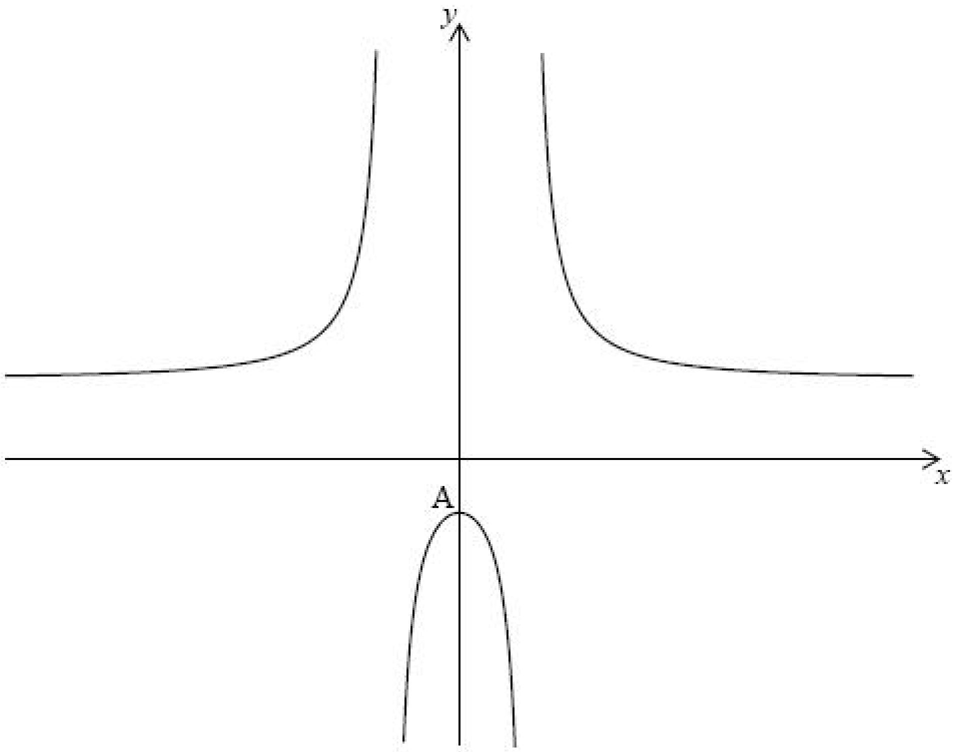


diagram not to scale

The y-intercept is at the point A.

(a) (i) Find the coordinates of A.

(ii) Show that $f'(x) = 0$ at A.

(7)

(b) The second derivative $f''(x) = \dfrac{40(3x^2+4)}{(x^2-4)^3}$. Use this to

(i) justify that the graph of f has a local maximum at A;

(ii) explain why the graph of f does **not** have a point of inflexion.

(6)

(c) Describe the behaviour of the graph of f for large $|x|$.

(1)

(d) Write down the range of f.

(2)
(Total 16 marks)

28.) Let $f(x) = \sqrt{x}$. Line L is the normal to the graph of f at the point (4, 2).

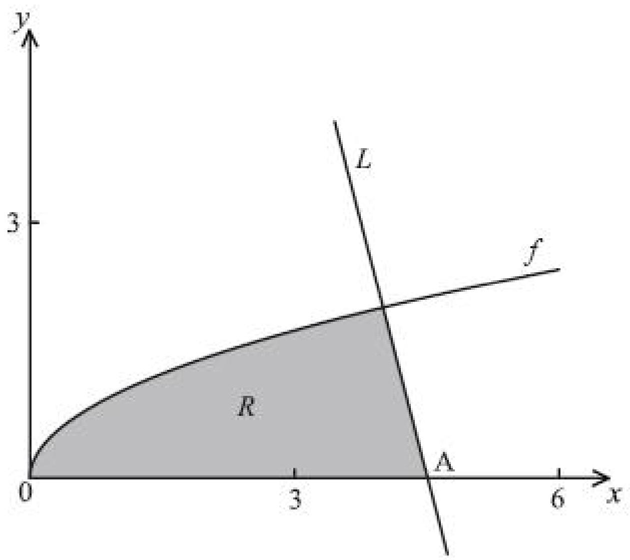
(a) Show that the equation of L is $y = -4x + 18$.

(4)

(b) Point A is the x-intercept of L. Find the x-coordinate of A.

(2)

In the diagram below, the shaded region R is bounded by the x-axis, the graph of f and the line L.



(c) Find an expression for the area of R.

(3)

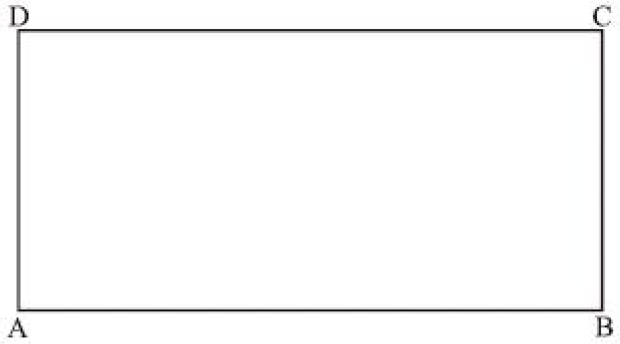
(d) The region R is rotated 360° about the x-axis. Find the volume of the solid formed, giving your answer in terms of π.

(8)
(Total 17 marks)

29.) Consider the curve with equation $f(x) = px^2 + qx$, where p and q are constants.
The point A(1, 3) lies on the curve. The tangent to the curve at A has gradient 8.
Find the value of p and of q.

(Total 7 marks)

30.) A farmer wishes to create a rectangular enclosure, ABCD, of area 525 m^2, as shown below.



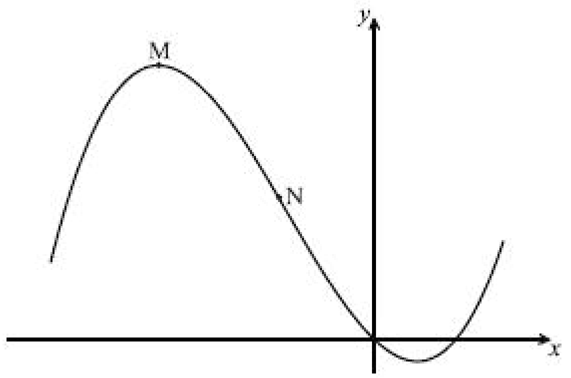
The fencing used for side AB costs \$11 per metre. The fencing for the other three sides costs \$3 per metre. The farmer creates an enclosure so that the cost is a minimum.
Find this minimum cost.

(Total 7 marks)

31.) A particle moves along a straight line so that its velocity, v m s^{-1} at time t seconds is given by $v = 6e^{3t} + 4$. When $t = 0$, the displacement, s, of the particle is 7 metres. Find an expression for s in terms of t.

(Total 7 marks)

32.) Consider $f(x) = \frac{1}{3}x^3 + 2x^2 - 5x$. Part of the graph of f is shown below. There is a maximum point at M, and a point of inflexion at N.



(a) Find $f'(x)$.

(3)

(b) Find the x-coordinate of M.

(4)

(c) Find the x-coordinate of N.

(3)

(d) The line L is the tangent to the curve of f at (3, 12). Find the equation of L in the form $y = ax + b$.

(4)
(Total 14 marks)

33.) Let $f : x \mapsto \sin^3 x$.

(a) (i) Write down the range of the function f.

(ii) Consider $f(x) = 1$, $0 \le x \le 2\pi$. Write down the number of solutions to this equation. Justify your answer.

(5)

(b) Find $f'(x)$, giving your answer in the form $a \sin^p x \cos^q x$ where $a, p, q \in \mathbb{Z}$.

(2)

(c) Let $g(x) = \sqrt{3} \sin x (\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of g is revolved through 2π about the x-axis.

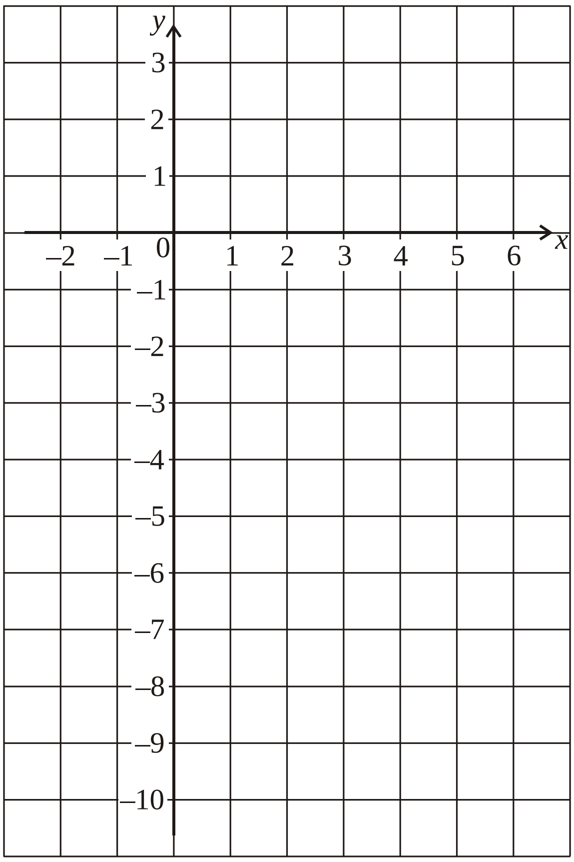
(7)
(Total 14 marks)

34.) Let $f(x) = 3x - e^{x-2} - 4$, for $-1 \le x \le 5$.

(a) Find the x-intercepts of the graph of f.

(3)

(b) On the grid below, sketch the graph of f.



(3)

(c) Write down the gradient of the graph of f at $x = 2$.

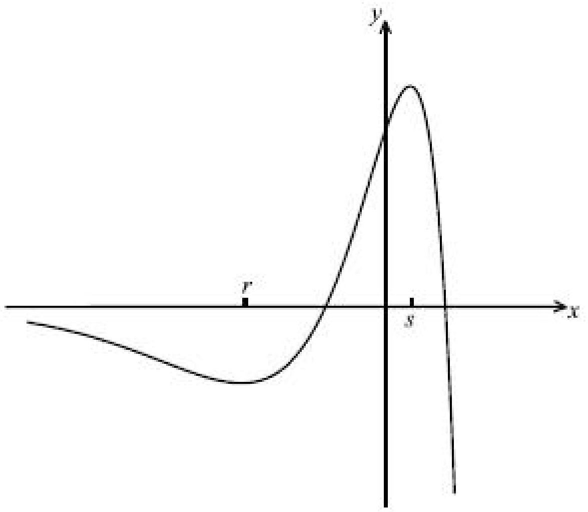
(1)
(Total 7 marks)

35.) Let $f(x) = e^x (1 - x^2)$.

(a) Show that $f'(x) = e^x (1 - 2x - x^2)$.

(3)

Part of the graph of $y = f(x)$, for $-6 \le x \le 2$, is shown below. The x-coordinates of the local minimum and maximum points are r and s respectively.



(b) Write down the **equation** of the horizontal asymptote.

(1)

(c) Write down the value of r and of s.

(4)

(d) Let L be the normal to the curve of f at P(0, 1). Show that L has equation $x + y = 1$.

(4)

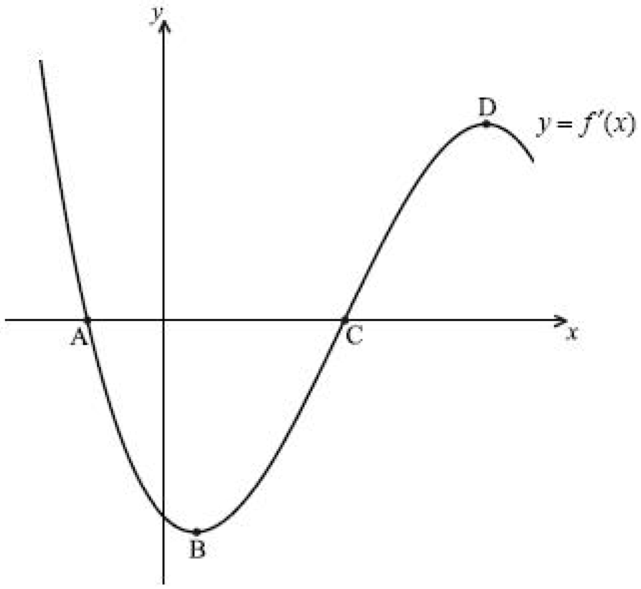
(e) Let R be the region enclosed by the curve $y = f(x)$ and the line L.

(i) Find an expression for the area of R.

(ii) Calculate the area of R.

(5)
(Total 17 marks)

36.) The diagram shows part of the graph of $y = f'(x)$. The x-intercepts are at points A and C. There is a minimum at B, and a maximum at D.



(a) (i) Write down the value of $f'(x)$ at C.

(ii) **Hence**, show that C corresponds to a minimum on the graph of f, *i.e.* it has the same x-coordinate.

(3)

(b) Which of the points A, B, D corresponds to a maximum on the graph of f?

(1)

(c) Show that B corresponds to a point of inflexion on the graph of f.

(3)
(Total 7 marks)

37.) The acceleration, a m s^{-2}, of a particle at time t seconds is given by $a = 2t + \cos t$.

(a) Find the acceleration of the particle at $t = 0$.

(2)

(b) Find the velocity, v, at time t, given that the initial velocity of the particle is 2 m s^{-1}.

(5)

(c) Find $\int_0^3 v\,dt$, giving your answer in the form $p - q \cos 3$.

(7)

(d) What information does the answer to part (c) give about the motion of the particle?

(2)
(Total 16 marks)

38.) Let $f(x) = e^{2x} \cos x$, $-1 \le x \le 2$.

(a) Show that $f'(x) = e^{2x} (2 \cos x - \sin x)$.

(3)

Let the line L be the normal to the curve of f at $x = 0$.

(b) Find the equation of L.

(5)

The graph of f and the line L intersect at the point (0, 1) and at a second point P.

(c) (i) Find the x-coordinate of P.

(ii) Find the area of the region **enclosed** by the graph of f and the line L.

(6)
(Total 14 marks)

39.) Find the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 1$. Give your answer in terms of e^2.

(Total 6 marks)

40.) The velocity v m s^{-1} of a moving body at time t seconds is given by $v = 50 - 10t$.

(a) Find its acceleration in m s^{-2}.

(2)

(b) The initial displacement s is 40 metres. Find an expression for s in terms of t.

(4)
(Total 6 marks)

41.) Let $g(x) = x^3 - 3x^2 - 9x + 5$.

(a) Find the two values of x at which the tangent to the graph of g is horizontal.

(8)

(b) For each of these values, determine whether it is a maximum or a minimum.

(6)
(Total 14 marks)

42.) The function f is defined by $f : x \mapsto -0.5x^2 + 2x + 2.5$.

Let N be the normal to the curve at the point where the graph intercepts the y-axis.

(a) Show that the equation of N may be written as $y = -0.5x + 2.5$.

(4)

(b) Find the coordinates of the other point of intersection of the normal and the curve.

(5)

(c) Let R be the region enclosed between the curve and N. Find the area of R.

(4)
(Total 13 marks)

43.) The velocity v of a particle at time t is given by $v = e^{-2t} + 12t$. The displacement of the particle at time t is s. Given that $s = 2$ when $t = 0$, express s in terms of t.

(Total 6 marks)

44.) Let $f(x) = x^3 - 3x^2 - 24x + 1$.

The tangents to the curve of f at the points P and Q are parallel to the x-axis, where P is to the left of Q.

(a) Calculate the coordinates of P and of Q.

Let N_1 and N_2 be the normals to the curve at P and Q respectively.

(b) Write down the coordinates of the points where

(i) the tangent at P intersects N_2;

(ii) the tangent at Q intersects N_1.

(Total 6 marks)

45.) The velocity, $\boldsymbol{v}$, in m s^{-1} of a particle moving in a straight line is given by $v = e^{3t-2}$, where t is the time in seconds.

(a) Find the acceleration of the particle at $t = 1$.

(b) At what value of t does the particle have a velocity of 22.3 m s^{-1}?

(c) Find the distance travelled in the first second.

(Total 6 marks)

46.) Let $f(x) = 3\cos 2x + \sin^2 x$.

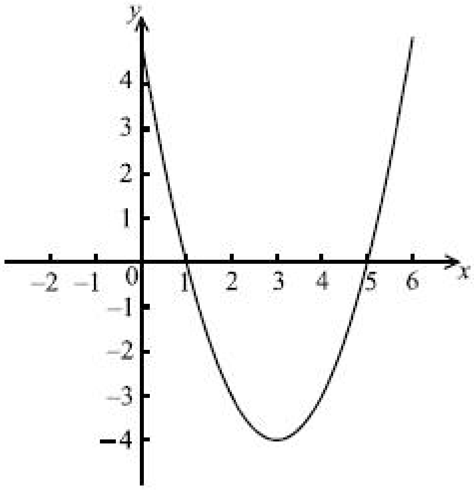
(a) Show that $f'(x) = -5\sin 2x$.

(b) In the interval $\frac{\pi}{4} \le x \le \frac{3\pi}{4}$, one normal to the graph of f has equation $x = k$.

Find the value of k.

(Total 6 marks)

47.) The following diagram shows part of the graph of a quadratic function, with equation in the form $y = (x - p)(x - q)$, where $p, q \in \mathbb{Z}$.



(a) Write down

(i) the value of p and of q;

(ii) the equation of the axis of symmetry of the curve.

(3)

(b) Find the equation of the function in the form $y = (x - h)^2 + k$, where $h, k \in \mathbb{Z}$.

(3)

(c) Find $\frac{dy}{dx}$.

(2)

(d) Let T be the tangent to the curve at the point (0, 5). Find the equation of T.

(2)
(Total 10 marks)

48.) The function $f(x)$ is defined as $f(x) = 3 + \frac{1}{2x-5}$, $x \neq \frac{5}{2}$.

(a) Sketch the curve of f for $-5 \leq x \leq 5$, showing the asymptotes.

(3)

(b) Using your sketch, write down

(i) the equation of each asymptote;

(ii) the value of the x-intercept;

(iii) the value of the y-intercept.

(4)

(c) The region enclosed by the curve of f, the x-axis, and the lines $x = 3$ and $x = a$, is revolved through 360° about the x-axis. Let V be the volume of the solid formed.

(i) Find $\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx$.

(ii) Hence, given that $V = \pi\left(\frac{28}{3} + 3\ln 3\right)$, find the value of a.

(10)
(Total 17 marks)

49.) Consider the function $f(x) = 4x^3 + 2x$. Find the equation of the normal to the curve of f at the point where $x = 1$.

(Total 6 marks)

50.) Consider the function $f(x)\ e^{(2x-1)} + \left(\frac{5}{(2x-1)}\right)$, $x \neq \frac{1}{2}$.

(a) Sketch the curve of f for $-2 \leq x \leq 2$, including any asymptotes.

(3)

(b) (i) Write down the equation of the vertical asymptote of f.

(ii) Write down which one of the following expressions does **not** represent an area between the curve of f and the x-axis.

$$\int_1^2 f(x)\mathrm{d}x$$

$$\int_0^2 f(x)\mathrm{d}x$$

(iii) Justify your answer.

(3)

(c) The region between the curve and the x-axis between $x = 1$ and $x = 1.5$ is rotated through 360° about the x-axis. Let V be the volume formed.

(i) Write down an expression to represent V.

(ii) Hence write down the value of V.

(4)

(d) Find $f'(x)$.

(4)

(e) (i) Write down the value of x at the minimum point on the curve of f.

(ii) The equation $f(x) = k$ has no solutions for $p \leq k < q$. Write down the value of p and of q.

(3)
(Total 17 marks)

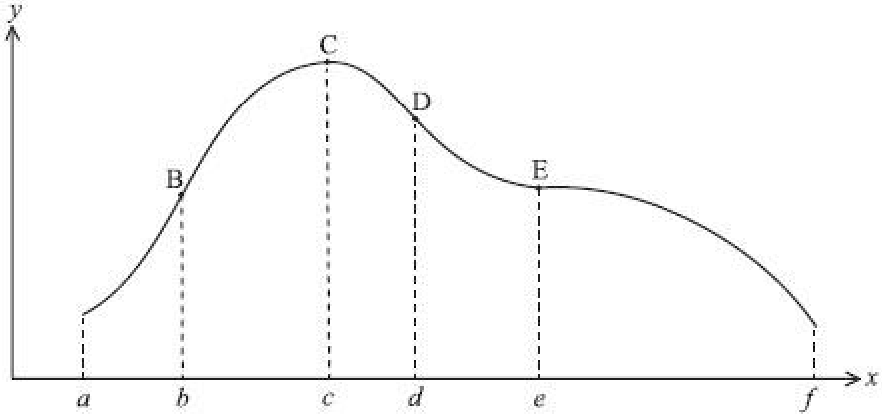
51.) The velocity, v m s^{-1}, of a moving object at time t seconds is given by $v = 4t^3 - 2t$.

When $t = 2$, the displacement, s, of the object is 8 metres.

Find an expression for s in terms of t.

(Total 6 marks)

52.) The graph of a function g is given in the diagram below.



The gradient of the curve has its maximum value at point B and its minimum value at point D. The tangent is horizontal at points C and E.

(a) Complete the table below, by stating whether the first derivative g' is positive or negative, and whether the second derivative g'' is positive or negative.

Interval	g'	g''
$a < x < b$		
$e < x < f$		

(b) Complete the table below by noting the points on the graph described by the following conditions.

Conditions	Point
$g'(x) = 0, g''(x) < 0$	
$g'(x) < 0, g''(x) = 0$	

(Total 6 marks)

53.) Consider the function $f : x \mapsto 3x^2 - 5x + k$.

(a) Write down $f'(x)$.

The equation of the tangent to the graph of f at $x = p$ is $y = 7x - 9$. Find the value of

(b) p;

(c) k.

(Total 6 marks)

54.) The displacement s metres at time t seconds is given by

$$s = 5 \cos 3t + t^2 + 10, \text{ for } t \geq 0.$$

(a) Write down the minimum value of s.

(b) Find the acceleration, a, at time t.

(c) Find the value of t when the **maximum** value of a first occurs.

(Total 6 marks)

55.) Let $f(x) = -\frac{3}{4}x^2 + x + 4$.

(a) (i) Write down $f'(x)$.

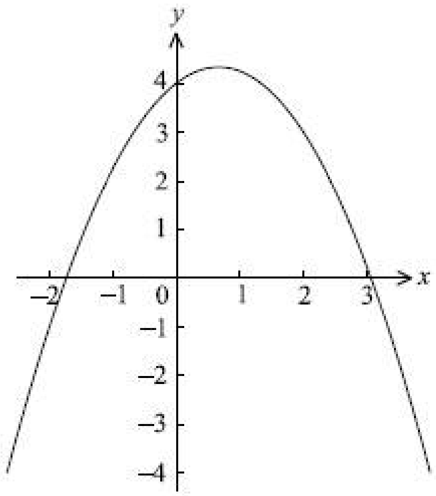
(ii) Find the equation of the normal to the curve of f at (2, 3).

(iii) This normal intersects the curve of f at (2, 3) and at one other point P.

Find the x-coordinate of P.

(9)

Part of the graph of f is given below.



(b) Let R be the region under the curve of f from $x = -1$ to $x = 2$.

(i) Write down an expression for the area of R.

(ii) Calculate this area.

(iii) The region R is revolved through 360° about the x-axis. Write down an expression for the volume of the solid formed.

(6)

(c) Find $\int_1^k f(x)\,\mathrm{d}x$, giving your answer in terms of k.

(6)
(Total 21 marks)

56.) The velocity v in m s^{-1} of a moving body at time t seconds is given by $v = e^{2t-1}$. When t = 0 5. the displacement of the body is 10 m. Find the displacement when t =1.

(Total 6 marks)

57.) The velocity v m s^{-1} of a moving body at time t seconds is given by $v = 50 - 10t$.

(a) Find its acceleration in m s^{-2}.

(b) The initial displacement s is 40 metres. Find an expression for s in terms of t.

(Total 6 marks)

58.) The function f is defined by $f : x \mapsto -0.5x^2 + 2x + 2.5$.

(a) Write down

(i) $f'(x)$;

(ii) $f'(0)$.

(2)

(b) Let N be the normal to the curve at the point where the graph intercepts the y-axis. Show that the equation of N may be written as $y = -0.5x + 2.5$.

(3)

Let $g : x \mapsto -0.5x + 2.5$

(c) (i) Find the solutions of $f(x) = g(x)$.

(ii) Hence find the coordinates of the other point of intersection of the normal and the curve.

(6)

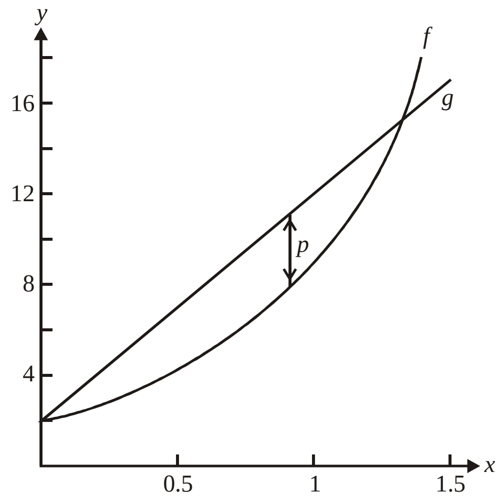
(d) Let R be the region enclosed between the curve and N.

(i) Write down an expression for the area of R.

(ii) Hence write down the area of R.

(5)
(Total 16 marks)

59.) The diagram below shows the graphs of $f(x) = 1 + e^{2x}$, $g(x) = 10x + 2$, $0 \le x \le 1.5$.

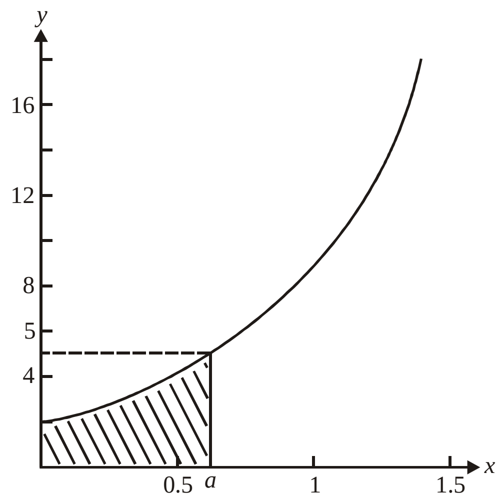


(a) (i) Write down an expression for the vertical distance p between the graphs of f and g.

(ii) Given that p has a maximum value for $0 \le x \le 1.5$, find the value of x at which this occurs.

(6)

The graph of $y = f(x)$ only is shown in the diagram below. When $x = a$, $y = 5$.



(b) (i) Find $f^{-1}(x)$.

(ii) **Hence** show that $a = \ln 2$.

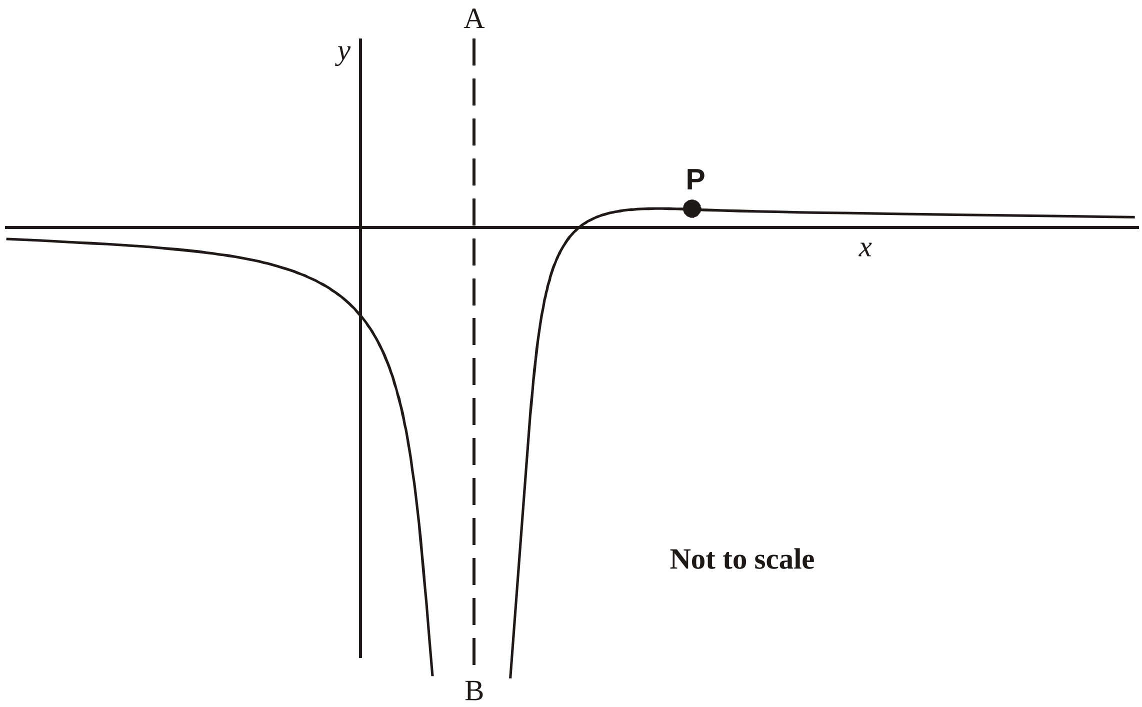
(5)

(c) The region shaded in the diagram is rotated through 360° about the x-axis. Write down an expression for the volume obtained.

(3)
(Total 14 marks)

60.) Consider the function $h : x \mapsto \dfrac{x-2}{(x-1)^2}, x \neq 1.$

A sketch of part of the graph of h is given below.



The line (AB) is a vertical asymptote. The point P is a point of inflexion.

(a) Write down the **equation** of the vertical asymptote.

(1)

(b) Find $h'(x)$, writing your answer in the form

$$\frac{a-x}{(x-1)^n}$$

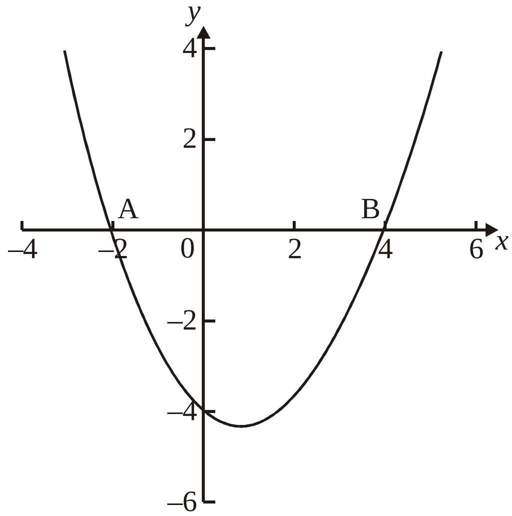
where a and n are constants to be determined.

(4)

(c) Given that $h''(x) = \dfrac{2x-8}{(x-1)^4}$, calculate the coordinates of P.

(3)
(Total 8 marks)

61.) The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve intersects the x-axis at A(–2, 0) and B(4, 0). The curve of $y = f(x)$ is shown in the diagram below.



(a) (i) Write down the value of p and of q.

(ii) Given that the point (6, 8) is on the curve, find the value of a.

(iii) Write the equation of the curve in the form $y = ax^2 + bx + c$.

(5)

(b) (i) Find $\frac{dy}{dx}$.

(ii) A tangent is drawn to the curve at a point P. The gradient of this tangent is 7. Find the coordinates of P.

(4)

(c) The line L passes through B(4, 0), and is perpendicular to the tangent to the curve at point B.

(i) Find the equation of L.

(ii) Find the x-coordinate of the point where L intersects the curve again.

(6)
(Total 15 marks)

62.) The function $g(x)$ is defined for $-3 \le x \le 3$. The behaviour of $g'(x)$ and $g''(x)$ is given in the tables below.

x	$-3 < x < -2$	-2	$-2 < x < 1$	1	$1 < x < 3$
$g'(x)$	negative	0	positive	0	negative

x	$-3 < x < -\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2} < x < 3$
$g''(x)$	positive	0	negative

Use the information above to answer the following. In each case, justify your answer.

(a) Write down the value of x for which g has a maximum.

(2)

(b) On which intervals is the value of g decreasing?

(2)

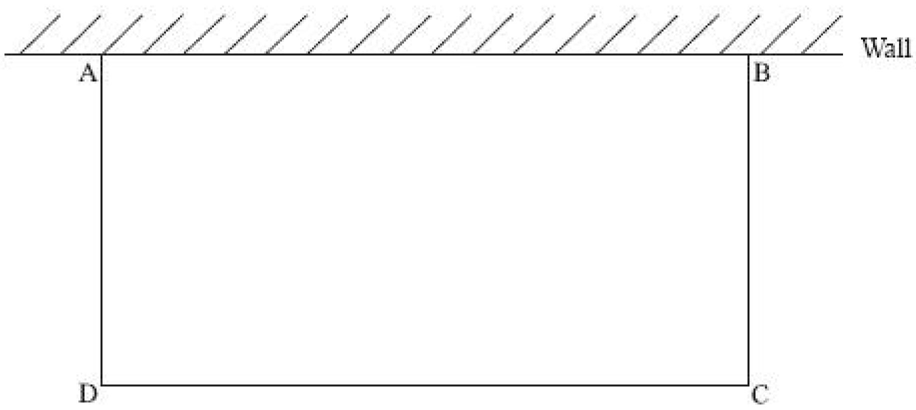
(c) Write down the value of x for which the graph of g has a point of inflexion.

(2)

(d) Given that $g(-3) = 1$, sketch the graph of g. On the sketch, clearly indicate the position of the maximum point, the minimum point, and the point of inflexion.

(3)
(Total 9 marks)

63.) The following diagram shows a rectangular area ABCD enclosed on three sides by 60 m of fencing, and on the fourth by a wall AB.



Find the width of the rectangle that gives its maximum area.

Working:

Answers:

....

(Total 6 marks)

64.) A particle moves with a velocity v m s^{-1} given by $v = 25 - 4t^2$ where $t \geq 0$.

(a) The displacement, s metres, is 10 when t is 3. Find an expression for s in terms of t.

(6)

(b) Find t when s reaches its maximum value.

(3)

(c) The particle has a positive displacement for $m \leq t \leq n$. Find the value of m and the value of n.

(3)
(Total 12 marks)

65.) A car starts by moving from a fixed point A. Its velocity, v m s^{-1} after t seconds is given by $v = 4t + 5 - 5e^{-t}$. Let d be the displacement from A when $t = 4$.

(a) Write down an integral which represents d.

(b) Calculate the value of d.

Working:

Answers:

(a) ……………………………………………..

(b) ……………………………………………..

(Total 6 marks)

66.) The displacement s metres of a car, t seconds after leaving a fixed point A, is given by

$$s = 10t - 0.5t^2.$$

(a) Calculate the velocity when $t = 0$.

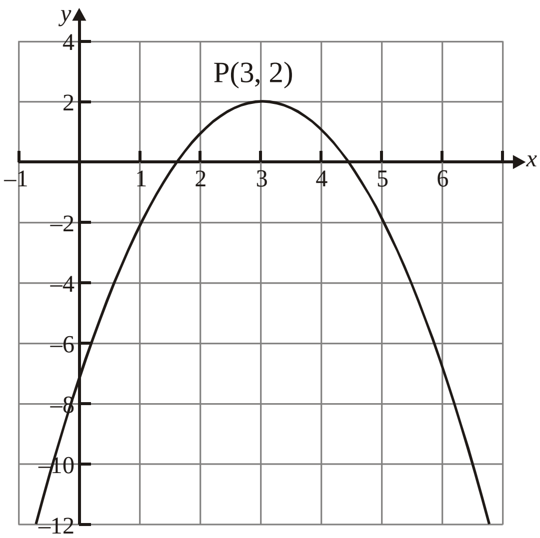
(b) Calculate the value of t when the velocity is zero.

(c) Calculate the displacement of the car from A when the velocity is zero.

Working:	
	Answers:
	(a) …………………………………………..
	(b) …………………………………………..
	(c) …………………………………………..

(Total 6 marks)

67.) The function $f(x)$ is defined as $f(x) = -(x - h)^2 + k$. The diagram below shows part of the graph of $f(x)$. The maximum point on the curve is P (3, 2).



(a) Write down the value of

(i) h;

(ii) k.

(2)

(b) Show that $f(x)$ can be written as $f(x) = -x^2 + 6x - 7$.

(1)

(c) Find $f'(x)$.

(2)

The point Q lies on the curve and has coordinates (4, 1). A straight line L, through Q, is perpendicular to the tangent at Q.

(d) (i) Calculate the gradient of L.

(ii) Find the equation of L.

(iii) The line L intersects the curve again at R. Find the x-coordinate of R.

(8)
(Total 13 marks)

68.) Let $f(x) = \dfrac{1}{1+x^2}$.

(a) Write down the equation of the horizontal asymptote of the graph of f.

(1)

(b) Find $f'(x)$.

(3)

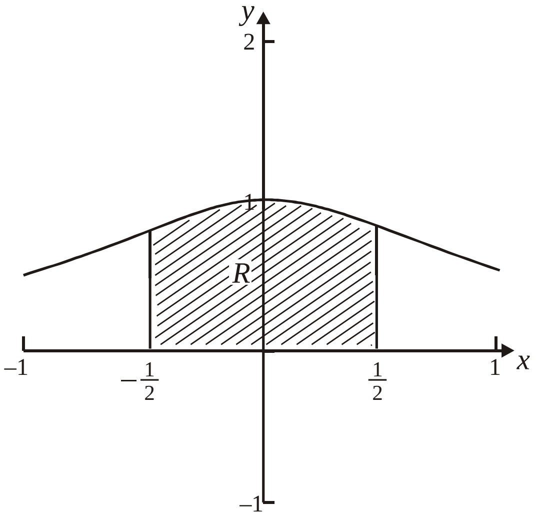
(c) The second derivative is given by $f''(x) = \dfrac{6x^2-2}{(1+x^2)^3}$.

Let A be the point on the curve of f where the gradient of the tangent is a maximum. Find the x-coordinate of A.

(4)

(d) Let R be the region under the graph of f, between $x = -\dfrac{1}{2}$ and $x = \dfrac{1}{2}$,

as shaded in the diagram below

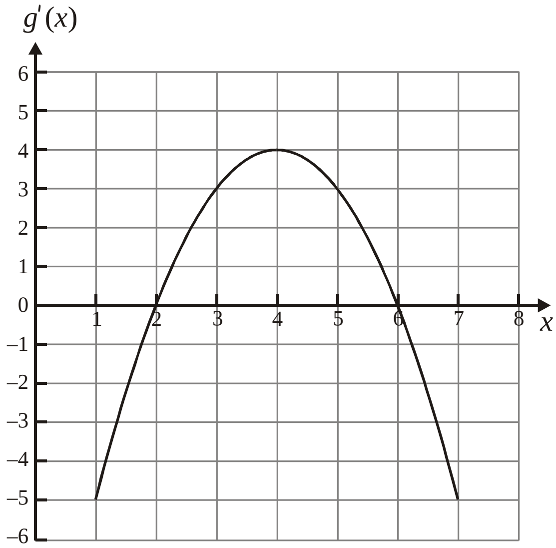


Write down the definite integral which represents the area of R.

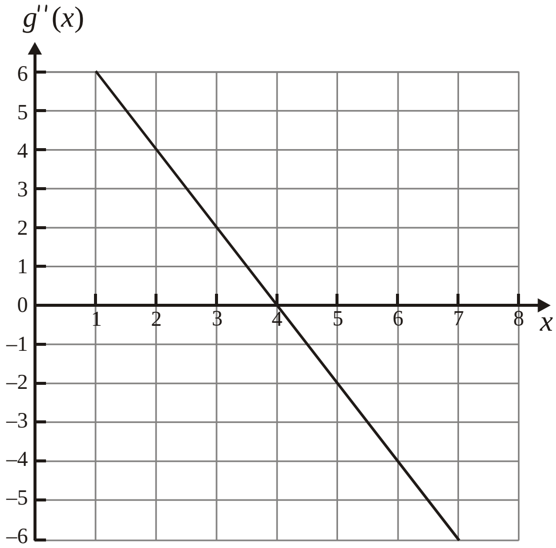
(2)
(Total 10 marks)

69.) Let $y = g(x)$ be a function of x for $1 \le x \le 7$. The graph of g has an inflexion point at P, and a minimum point at M.

Partial sketches of the curves of g' and g'' are shown below.



$y = g'(x)$



$y = g''(x)$

Use the above information to answer the following.

(a) Write down the x-coordinate of P, **and** justify your answer.

(2)

(b) Write down the x-coordinate of M, **and** justify your answer.

(2)

(c) Given that $g(4) = 0$, sketch the graph of g. On the sketch, mark the points P and M.

(4)
(Total 8 marks)

70.) An aircraft lands on a runway. Its velocity v m s^{-1} at time t seconds after landing is given by the equation $v = 50 + 50e^{-0.5t}$, where $0 \le t \le 4$.

(a) Find the velocity of the aircraft

(i) when it lands;

(ii) when $t = 4$.

(4)

(b) Write down an integral which represents the distance travelled in the first four seconds.

(3)

(c) Calculate the distance travelled in the first four seconds.

(2)

After four seconds, the aircraft slows down (decelerates) **at a constant rate** and comes to rest when $t = 11$.

(d) **Sketch** a graph of velocity against time for $0 \le t \le 11$. Clearly label the axes and mark on the graph the point where $t = 4$.

(5)

(e) Find the constant rate at which the aircraft is slowing down (decelerating) between $t = 4$ and $t = 11$.

(2)

(f) Calculate the distance travelled by the aircraft between $t = 4$ and $t = 11$.

(2)
(Total 18 marks)

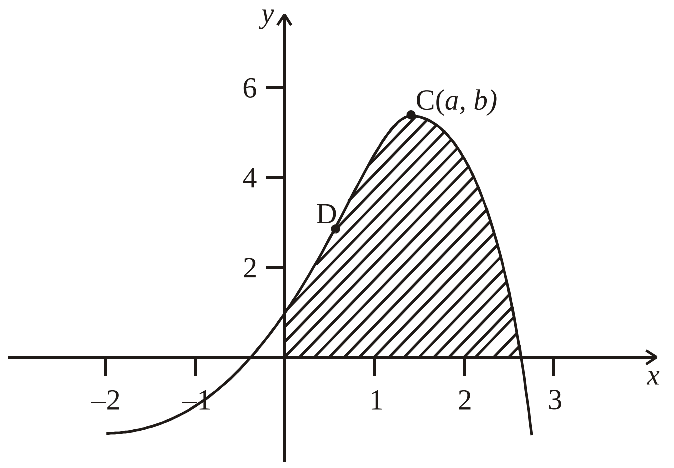
71.) Consider the function $f(x) = \cos x + \sin x$.

(a) (i) Show that $f(-\frac{\pi}{4}) = 0$.

(ii) Find in terms of π, the smallest **positive** value of x which satisfies $f(x) = 0$.

(3)

The diagram shows the graph of $y = e^x(\cos x + \sin x)$, $-2 \le x \le 3$. The graph has a maximum turning point at C(a, b) and a point of inflexion at D.



(b) Find $\frac{dy}{dx}$.

(3)

(c) Find the **exact** value of a and of b.

(4)

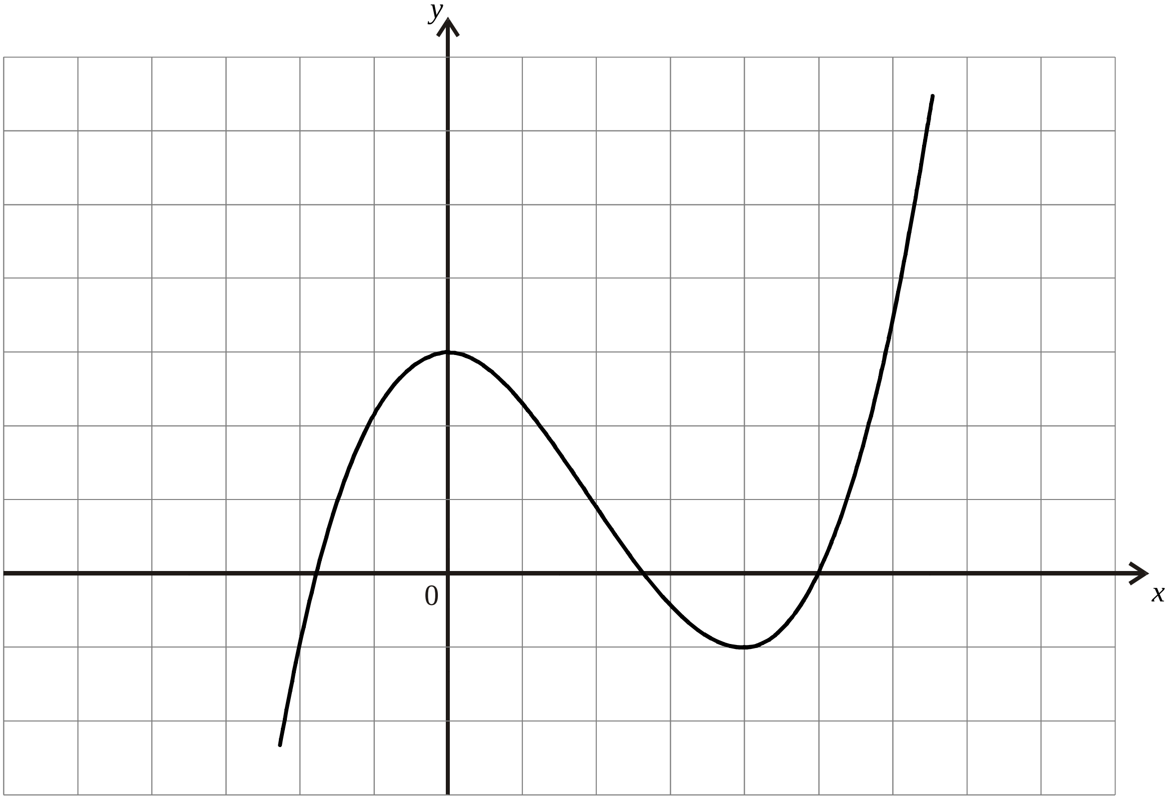
(d) Show that at D, $y = \sqrt{2}e^{\frac{\pi}{4}}$.

(5)

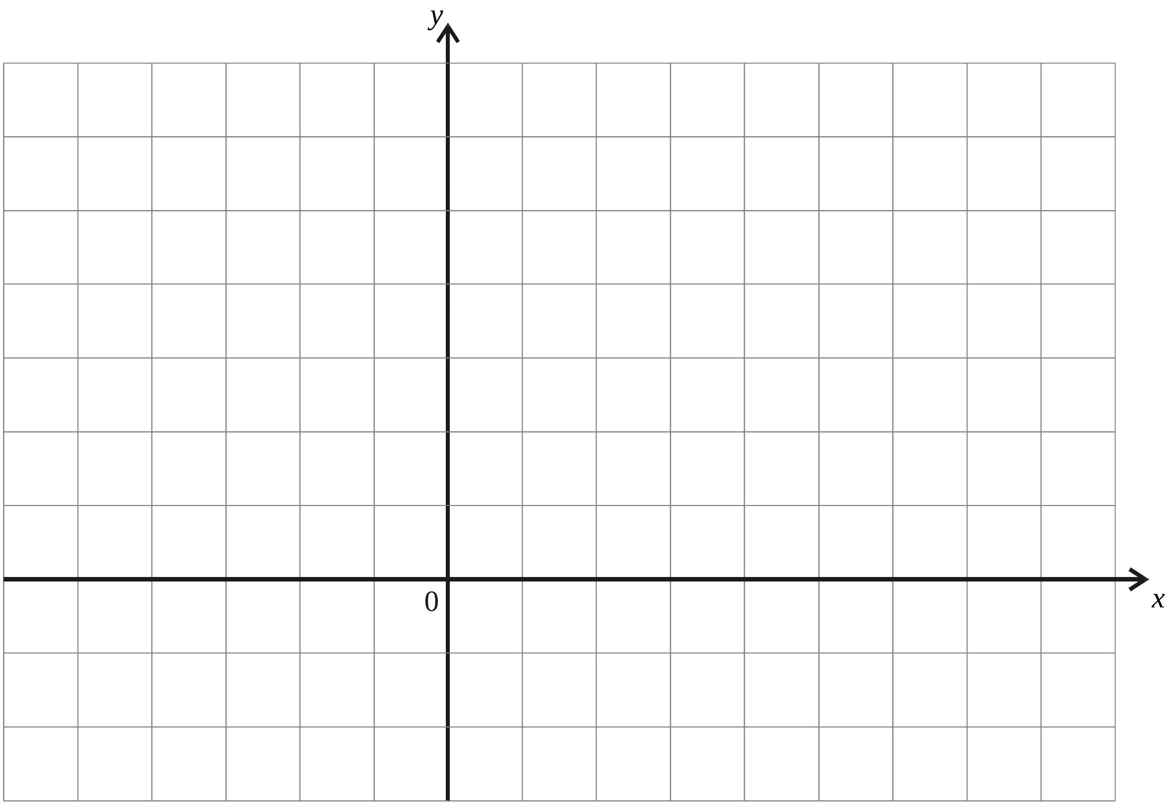
(e) Find the area of the shaded region.

(2)
(Total 17 marks)

72.) The diagram shows the graph of $y = f(x)$.



On the grid below sketch the graph of $y = f'(x)$.



(Total 6 marks)

73.) Consider the function $f(x) = 1 + e^{-2x}$.

(a) (i) Find $f'(x)$.

(ii) Explain briefly how this shows that $f(x)$ is a decreasing function for all values of x (*ie* that $f(x)$ always decreases in value as x increases).

(2)

Let P be the point on the graph of f where $x = -\frac{1}{2}$.

(b) Find an expression in terms of e for

(i) the y-coordinate of P;

(ii) the gradient of the tangent to the curve at P.

(2)

(c) Find the equation of the tangent to the curve at P, giving your answer in the form $y = ax + b$.

(3)

(d) (i) Sketch the curve of f for $-1 \le x \le 2$.

(ii) Draw the tangent at $x = -\frac{1}{2}$.

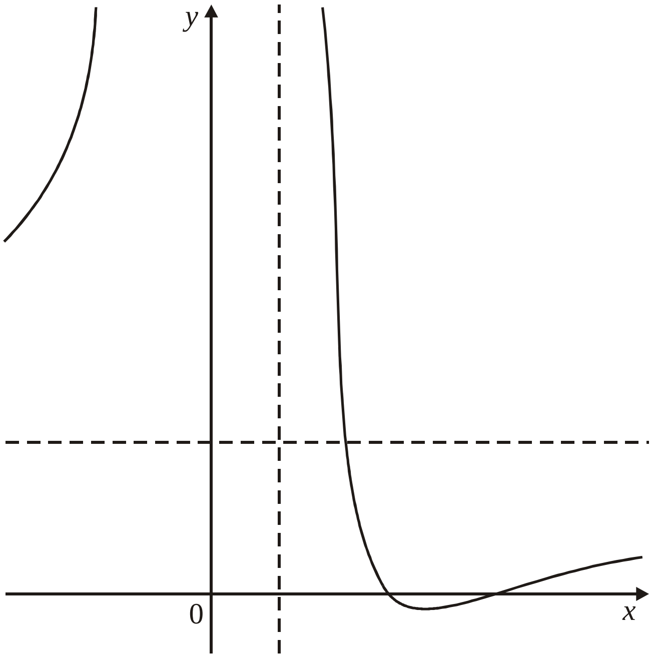
(iii) Shade the area enclosed by the curve, the tangent and the y-axis.

(iv) Find this area.

(7)
(Total 14 marks)

74.) Consider the function f given by $f(x) = \dfrac{2x^2 - 13x + 20}{(x-1)^2}$, $x \neq 1$.

A part of the graph of f is given below.



The graph has a vertical asymptote and a horizontal asymptote, as shown.

(a) Write down the **equation** of the vertical asymptote.

(1)

(b) $f(100) = 1.91$ $f(-100) = 2.09$ $f(1000) = 1.99$

(i) Evaluate $f(-1000)$.

(ii) Write down the **equation** of the horizontal asymptote.

(2)

(c) Show that $f'(x) = \dfrac{9x - 27}{(x-1)^3}$, $x \neq 1$.

(3)

The second derivative is given by $f''(x) = \dfrac{72 - 18x}{(x-1)^4}$, $x \neq 1$.

(d) Using values of $f'(x)$ and $f''(x)$ explain why a minimum must occur at $x = 3$.

(2)

(e) There is a point of inflexion on the graph of f. Write down the coordinates of this point.

(2)
(Total 10 marks)

75.) The graph of $y = x^3 - 10x^2 + 12x + 23$ has a maximum point between $x = -1$ and $x = 3$. Find the coordinates of this maximum point.

Working:

Answer:

....

(Total 6 marks)

76.) In this question, s represents displacement in metres, and t represents time in seconds.

(a) The velocity v m s^{-1} of a moving body may be written as $v = \frac{\mathrm{d}s}{\mathrm{d}t} = 30 - at$, where a is a constant. Given that $s = 0$ when $t = 0$, find an expression for s in terms of a and t.

(5)

Trains approaching a station start to slow down when they pass a signal which is 200 m from the station.

(b) The velocity of Train 1 t seconds after passing the signal is given by $v = 30 - 5t$.

(i) Write down its velocity as it passes the signal.

(ii) Show that it will stop before reaching the station.

(5)

(c) Train 2 slows down so that it stops at the station. Its velocity is given by $v = \frac{\mathrm{d}s}{\mathrm{d}t} = 30 - at$, where a is a constant.

(i) Find, in terms of a, the time taken to stop.

(ii) Use your solutions to parts (a) and (c)(i) to find the value of a.

(5)

(Total 15 marks)

77.) Consider the function $h(x) = x^{\frac{1}{5}}$.

(i) Find the equation of the tangent to the graph of h at the point where $x = a$, ($a \neq 0$). Write the equation in the form $y = mx + c$.

(ii) Show that this tangent intersects the x-axis at the point ($-4a$, 0).

(Total 5 marks)

78.) The point P ($\frac{1}{2}$, 0) lies on the graph of the curve of $y = \sin(2x - 1)$.

Find the gradient of the tangent to the curve at P.

Working:

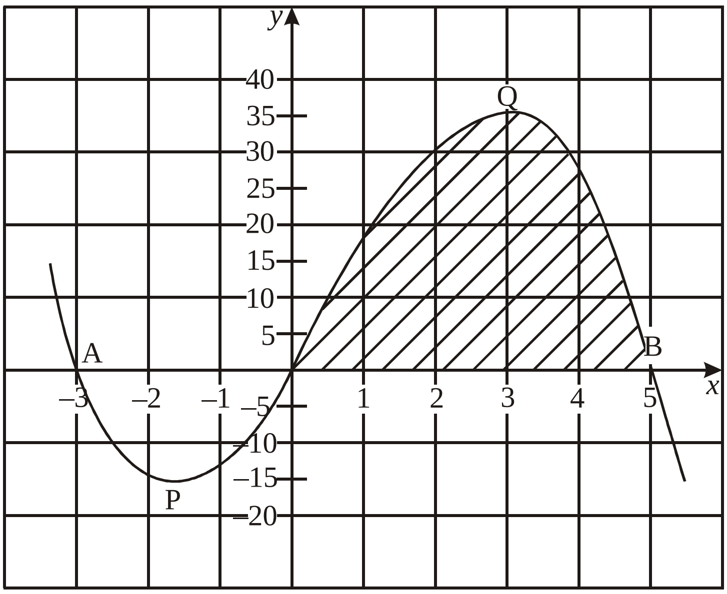
Answer:

....

(Total 4 marks)

79.) The diagram below shows part of the graph of the function

$$f : x \mapsto -x^3 + 2x^2 + 15x.$$



The graph intercepts the x-axis at A(–3, 0), B(5, 0) and the origin, O. There is a minimum point at P and a maximum point at Q.

(a) The function may also be written in the form $f : x \mapsto -x(x-a)(x-b)$, where $a < b$. Write down the value of

(i) a;

(ii) b.

(2)

(b) Find

(i) $f'(x)$;

(ii) the **exact** values of x at which $f'(x) = 0$;

(iii) the value of the function at Q.

(7)

(c) (i) Find the equation of the tangent to the graph of f at O.

(ii) This tangent cuts the graph of f at another point. Give the x-coordinate of this point.

(4)

(d) Determine the area of the shaded region.

(2)
(Total 15 marks)

80.) A ball is dropped vertically from a great height. Its velocity v is given by

$$v = 50 - 50e^{-0.2t}, \ t \geq 0$$

where v is in metres per second and t is in seconds.

(a) Find the value of v when

(i) $t = 0$;

(ii) $t = 10$.

(2)

(b) (i) Find an expression for the acceleration, a, as a function of t.

(ii) What is the value of a when $t = 0$?

(3)

(c) (i) As t becomes large, what value does v approach?

(ii) As t becomes large, what value does a approach?

(iii) Explain the relationship between the answers to parts (i) and (ii).

(3)

(d) Let y metres be the distance fallen after t seconds.

(i) Show that $y = 50t + 250e^{-0.2t} + k$, where k is a constant.

(ii) Given that $y = 0$ when $t = 0$, find the value of k.

(iii) Find the time required to fall 250 m, giving your answer correct to **four** significant figures.

(7)
(Total 15 marks)

81.) *Radian measure is used, where appropriate, throughout the question.*

Consider the function $y = \dfrac{3x-2}{2x-5}$.

The graph of this function has a vertical and a horizontal asymptote.

(a) Write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote.

(2)

(b) Find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$, simplifying the answer as much as possible.

(3)

(c) How many points of inflexion does the graph of this function have?

(1)
(Total 6 marks)

82.) A ball is thrown vertically upwards into the air. The height, h metres, of the ball above the ground after t seconds is given by

$$h = 2 + 20t - 5t^2, \; t \geq 0$$

(a) Find the **initial** height above the ground of the ball (that is, its height at the instant when it is released).

(2)

(b) Show that the height of the ball after one second is 17 metres.

(2)

(c) At a later time the ball is **again** at a height of 17 metres.

(i) Write down an equation that t must satisfy when the ball is at a height of 17 metres.

(ii) Solve the equation **algebraically**.

(4)

(d) (i) Find $\dfrac{\mathrm{d}h}{\mathrm{d}t}$.

(ii) Find the **initial** velocity of the ball (that is, its velocity at the instant when it is released).

(iii) Find **when** the ball reaches its maximum height.

(iv) Find the maximum height of the ball.

(7)
(Total 15 marks)

83.) Find the coordinates of the point on the graph of $y = x^2 - x$ at which the tangent is parallel to the line $y = 5x$.

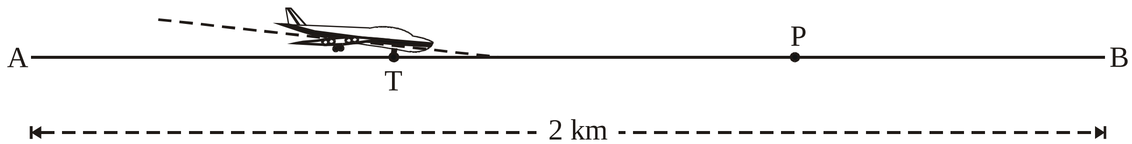
Working:

Answer:

....

(Total 4 marks)

84.) The main runway at *Concordville* airport is 2 km long. An airplane, landing at *Concordville*, touches down at point T, and immediately starts to slow down. The point A is at the southern end of the runway. A marker is located at point P on the runway.



Not to scale

As the airplane slows down, its distance, s, from A, is given by

$$s = c + 100t - 4t^2,$$

where t is the time in seconds after touchdown, and c metres is the distance of T from A.

(a) The airplane touches down 800 m from A, (*ie* $c = 800$).

(i) Find the distance travelled by the airplane in the first 5 seconds after touchdown.

(2)

(ii) Write down an expression for the velocity of the airplane at time t seconds after touchdown, and hence find the velocity after 5 seconds.

(3)

The airplane passes the marker at P with a velocity of 36 m s^{-1}. Find

(iii) how many seconds after touchdown it passes the marker;

(2)

(iv) the distance from P to A.

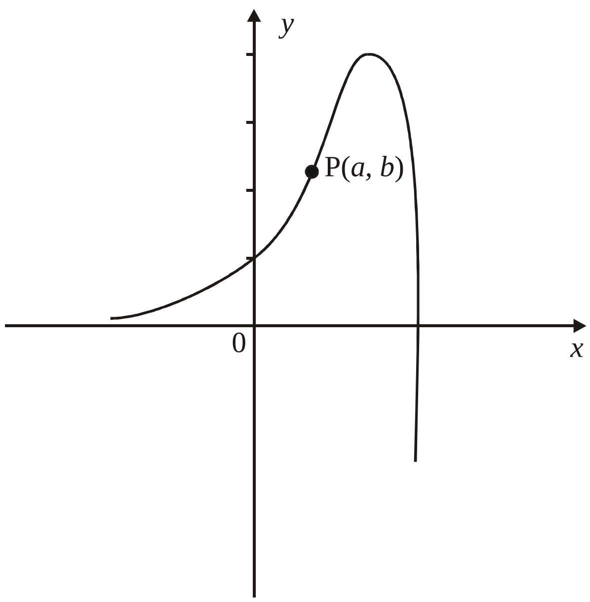
(3)

(b) Show that if the airplane touches down before reaching the point P, it can stop before reaching the northern end, B, of the runway.

(5)
(Total 15 marks)

85.) The diagram shows part of the graph of the curve with equation

$$y = e^{2x} \cos x.$$



(a) Show that $\frac{dy}{dx} = e^{2x}(2 \cos x - \sin x)$.

(2)

(b) Find $\frac{d^2y}{dx^2}$.

(4)

There is an inflexion point at P (a, b).

(c) Use the results from parts (a) and (b) to prove that:

(i) $\tan a = \frac{3}{4}$;

(3)

(ii) the gradient of the curve at P is e^{2a}.

(5)
(Total 14 marks)

86.) A rock-climber slips off a rock-face and falls vertically. At first he falls freely, but after 2 seconds a safety rope slows him down. The height *h* metres of the rock-climber after *t* seconds of the fall is given by:

$$h = 50 - 5t^2, \qquad 0 \leq t \leq 2$$

$$h = 90 - 40t + 5t^2, \qquad 2 \leq t \leq 5$$

(a) Find the height of the rock-climber when $t = 2$.

(1)

(b) Sketch a graph of h against t for $0 \leq t$ á 5.

(4)

(c) Find $\frac{dh}{dt}$ for:

(i) $0 \leq t \leq 2$

(ii) $2 \leq t \leq 5$

(2)

(d) Find the velocity of the rock-climber when $t = 2$.

(2)

(e) Find the times when the velocity of the rock-climber is zero.

(3)

(f) Find the minimum height of the rock-climber for $0 \leq t \leq 5$.

(3)
(Total 15 marks)

87.) Find the equation of the normal to the curve with equation

$$y = x^3 + 1$$

at the point (1, 2).

Working:

Answer:

....

(Total 4 marks)

88.) The function f is given by

$$f(x) = \frac{2x+1}{x-3}, \; x \in \mathbb{R}, x \neq 3.$$

(a) (i) Show that $y = 2$ is an asymptote of the graph of $y = f(x)$.

(2)

(ii) Find the vertical asymptote of the graph.

(1)

(iii) Write down the coordinates of the point P at which the asymptotes intersect.

(1)

(b) Find the points of intersection of the graph and the axes.

(4)

(c) Hence sketch the graph of $y = f(x)$, showing the asymptotes by dotted lines.

(4)

(d) Show that $f'(x) = \frac{-7}{(x-3)^2}$ and hence find the equation of the tangent at the point S where $x = 4$.

(6)

(e) The tangent at the point T on the graph is parallel to the tangent at S.

Find the coordinates of T.

(5)

(f) Show that P is the midpoint of [ST].

(l)
(Total 24 marks)

89.) The function f is such that $f''(x) = 2x - 2$.

When the graph of f is drawn, it has a minimum point at (3, –7).

(a) Show that $f'(x) = x^2 - 2x - 3$ and hence find $f(x)$.

(6)

(b) Find $f(0)$, $f(-1)$ and $f'(-1)$.

(3)

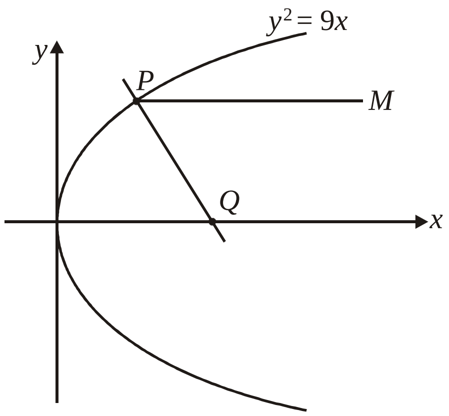
(c) Hence sketch the graph of f, labelling it with the information obtained in part (b).

(4)

(**Note:** It is **not** necessary to find the coordinates of the points where the graph cuts the x-axis.)

(Total 13 marks)

90.) The parabola shown has equation $y^2 = 9x$.



(a) Verify that the point P (4, 6) is on the parabola.

(2)

The line (PQ) is the normal to the parabola at the point P, and cuts the x-axis at Q.

(b) (i) Find the equation of (PQ) in the form $ax + by + c = 0$.

(5)

(ii) Find the coordinates of Q.

(2)

S is the point $\left(\frac{9}{4}, 0\right)$.

(c) Verify that $SP = SQ$.

(4)

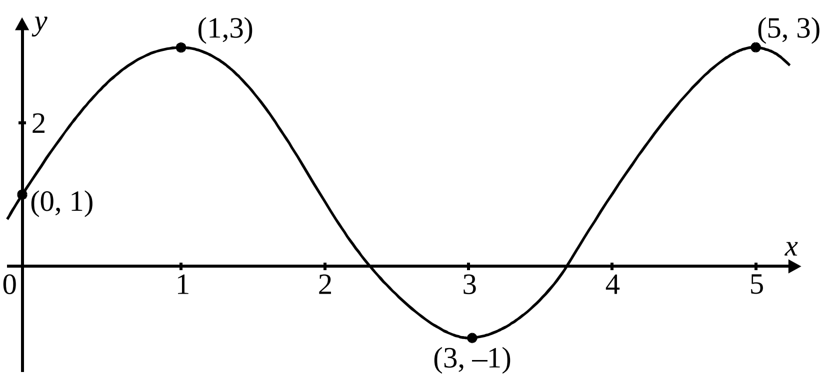
(d) The line (PM) is parallel to the x-axis. From part (c), explain why (QP) bisects the angle $\mathrm{S\hat{P}M}$.

(3)
(Total 16 marks)

91.) The diagram shows the graph of the function f given by

$$f(x) = A \sin\left(\frac{\text{®}}{2}x\right) + B,$$

for $0 \le x \le 5$, where A and B are constants, and x is measured in radians.



The graph includes the points (1, 3) and (5, 3), which are maximum points of the graph.

(a) Write down the values of $f(1)$ and $f(5)$.

(2)

(b) Show that the period of f is 4.

(2)

The point (3, –1) is a minimum point of the graph.

(c) Show that $A = 2$, and find the value of B.

(5)

(d) Show that fâ (x) = p cos $\left(\frac{®}{2}x\right)$.

(4)

The line $y = k$ – px is a tangent line to the graph for $0 \le x \le 5$.

(e) Find

(i) the point where this tangent meets the curve;

(ii) the value of k.

(6)

(f) Solve the equation $f(x) = 2$ for $0 \le x \le 5$.

(5)
(Total 24 marks)

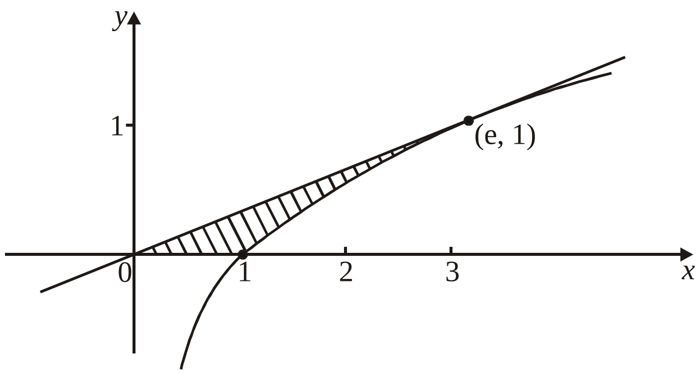
92.) (a) Find the equation of the tangent line to the curve $y = \ln x$ at the point (e, 1), and verify that the origin is on this line.

(4)

(b) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x \ln x - x) = \ln x$.

(2)

(c) The diagram shows the region enclosed by the curve $y = \ln x$, the tangent line in part (a), and the line $y = 0$.



Use the result of part (b) to show that the area of this region is $\frac{1}{2}$e – 1.

(4)
(Total 10 marks)

93.) A curve has equation $y = x(x-4)^2$.

(a) For this curve find

(i) the x-intercepts;

(ii) the coordinates of the maximum point;

(iii) the coordinates of the point of inflexion.

(9)

(b) Use your answers to part (a) to sketch a graph of the curve for $0 \le x \le 4$, clearly indicating the features you have found in part (a).

(3)

(c) (i) On your sketch indicate by shading the region whose area is given by the following integral:

$$\int_0^4 x(x-4)^2\,dx.$$

(ii) Explain, using your answer to part (a), why the value of this integral is greater than 0 but less than 40.

(3)

(Total 15 marks)