16 Algebra: Linear Equations

16.1 Fundamental Algebraic Skills

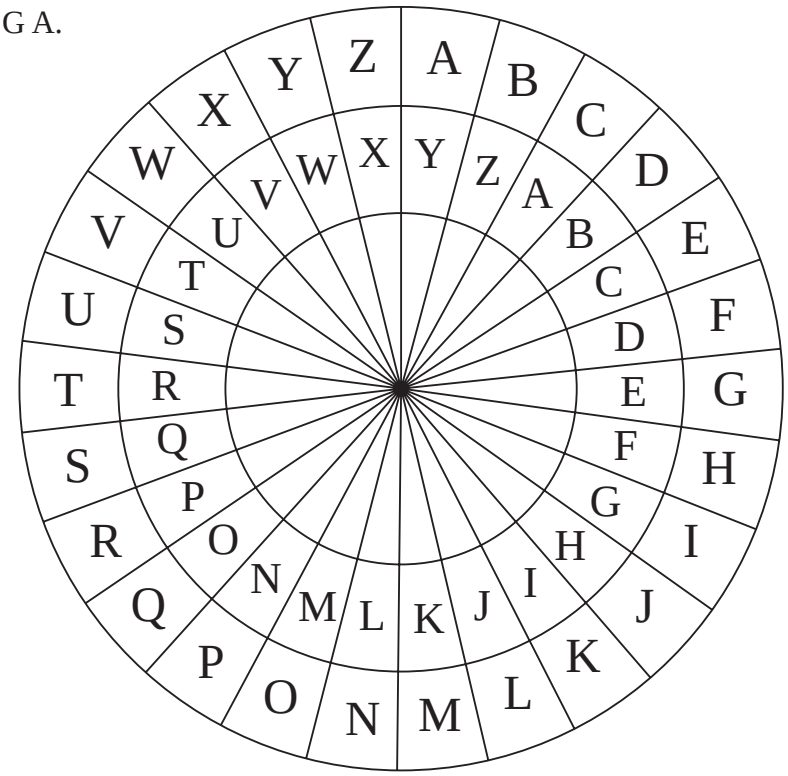
This section looks at some fundamental algebraic skills by examining codes and how to use formulae.

Example 1

Use this code wheel, which codes A on the outer ring as Y on the inner ring, to:

(a) code the word M A T H S,

(b) decode Q M L G A.



Solution

(a) Look for M on the outside circle of letters; this is coded as K which is the letter on the inside circle. Coding the other letters in the same way gives:

M	A	T	H	S
↓	↓	↓	↓	↓
K	Y	R	F	Q

(b) Look for Q on the inside circle. This decodes as S, which is the letter on the outside circle. Decoding the other letters in the same way gives:

Q	M	L	G	A
↓	↓	↓	↓	↓
S	O	N	I	C

Example 2

If $a = 4$, $b = 7$ and $c = 3$, calculate:

(a) $6 + b$ (b) $2a + b$ (c) ab (d) $a(b - c)$

Solution

(a) $6 + b = 6 + 7$
$= 13$

(b) $2a + b = 2 \times 4 + 7$ since $\boxed{2a = 2 \times a}$
$= 8 + 7$
$= 15$

(c) $ab = 4 \times 7$ since $\boxed{ab = a \times b}$
$= 28$

(d) $a(b - c) = 4 \times (7 - 3)$ since $\boxed{a(b - c) = a \times (b - c)}$
$= 4 \times 4$
$= 16$

Example 3

Simplify where possible:

(a) $2x + 4x$ (b) $5p + 7q - 3p + 2q$

(c) $y + 8y - 5y$ (d) $3t + 4s$

Solution

(a) $2x + 4x = 2 \times x + 4 \times x$
$= (x + x) + (x + x + x + x)$
$= 6 \times x$
$= 6x$

(b) $5p + 7q - 3p + 2q = 5p - 3p + 7q + 2q$
$= (5 - 3)p + (7 + 2)q$
$= 2p + 9q$

(c) $y + 8y - 5y = 1y + 8y - 5y$

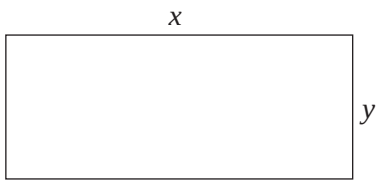
$= (1 + 8 - 5)y$

$= 4y$

(d) $3t + 4s$ cannot be simplified.

Example 4

Write down formulae for the area and perimeter of this rectangle:



Solution

Area $= x \times y$

$= xy$

Perimeter $= x + y + x + y$

$= 2x + 2y$

Exercises

1. Use the code wheel of Example 1 to:

   (a) code this message,

   M E E T M E A T H O M E

   (b) decode this message,

   M T C P R M W M S

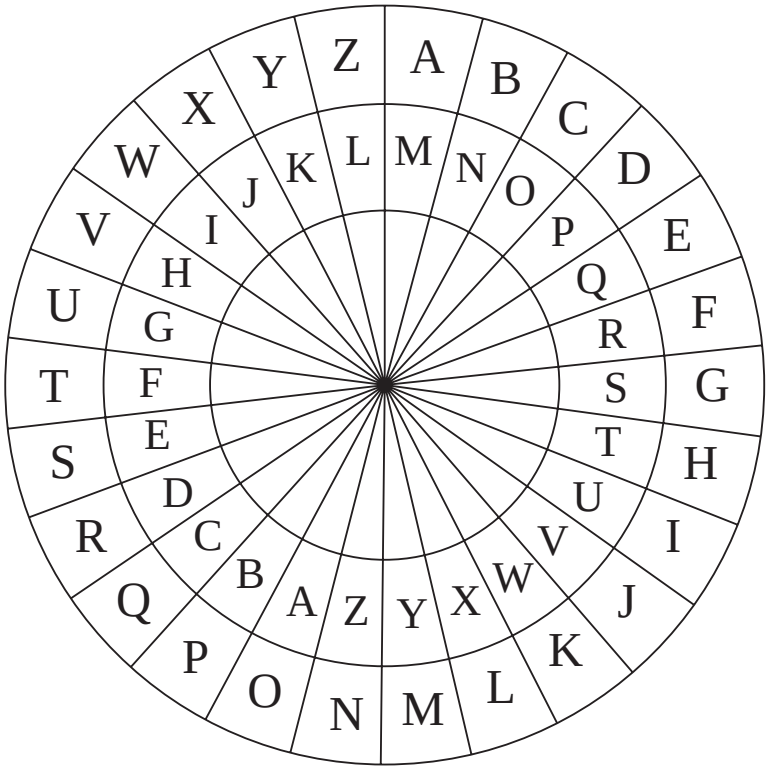
2. Use the code wheel opposite to:

   (a) code

   G O N E F I S H I N G,

   (b) decode

   T U S T R U H Q



3. Laura used a code wheel similar to the one above, but with the outer ring of letters rotated. She used her code wheel to code

	S	E	V	E	N		U	P
as	↓	↓	↓	↓	↓		↓	↓
	V	H	Y	H	Q		X	S

(a) Draw the code wheel that she used.

(b) Use the code wheel to decode:

G D Q J H U D K H D G

4. If $a = 2$, $b = 6$, $c = 10$ and $d = 3$, calculate:

(a) $a + b$ (b) $c - b$ (c) $d + 7$

(d) $3a + d$ (e) $4a$ (f) ad

(g) $3b$ (h) $2c$ (i) $3c - b$

(j) $6a + b$ (k) $3a + 2b$ (l) $4a - d$

5. If $a = 3,\ b = -1,\ c = 2$ and $d = -4$, calculate:

(a) $a - b$ (b) $a + d$ (c) $b + d$

(d) $b - d$ (e) $3d$ (f) $a + b$

(g) $c - d$ (h) $2c + d$ (i) $3a - d$

(j) $2d + 3c$ (k) $4a - 2d$ (l) $5a + 3d$

6. If $a = 7,\ b = 5,\ c = -3$ and $d = 4$, calculate:

(a) $2(a + b)$ (b) $4(a - b)$ (c) $6(a - d)$

(d) $2(a + c)$ (e) $5(b - c)$ (f) $5(d - c)$

(g) $a(b + c)$ (h) $d(b + a)$ (i) $c(b - a)$

(j) $a(2b - c)$ (k) $d(2a - 3b)$ (l) $c(d - 2)$

7. Use the formula $s = \frac{1}{2}(u + v)t$ to find s, when $u = 10,\ v = 20$ and $t = 4$.

8. Use the formula $v = u + at$ to find v, if $u = 20,\ a = -2$ and $t = 7$.

9. Simplify, where possible:

(a) $2a + 3a$ (b) $5b + 8b$

(c) $6c - 4c$ (d) $5d + 4d + 7d$

(e) $6e + 9e - 5e$ (f) $8f + 6f - 13f$

(g) $9g + 7g - 8g - 2g - 6g$ (h) $5p + 2h$

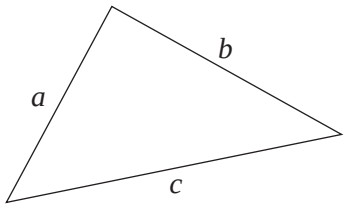
(i) $3a + 4b - 2a$ (j) $6x + 3y - 2x - y$

(k) $8t - 6t + 7s - 2s$

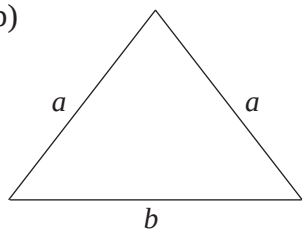
(l) $11m + 3n - 5p + 2q - 2n + 9q - 8m + 14p$

10. Write down formulae for the perimeter of each of these shapes:

(a)



(b)



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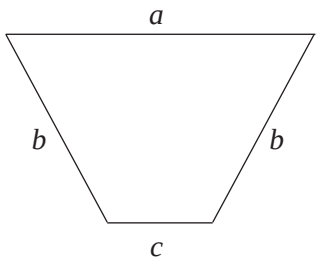

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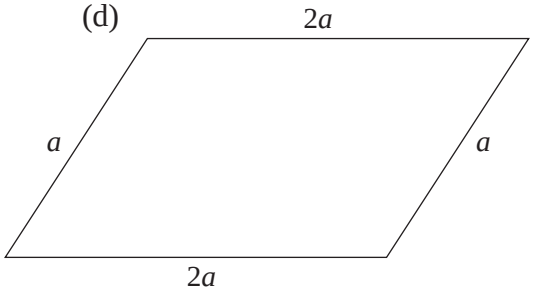
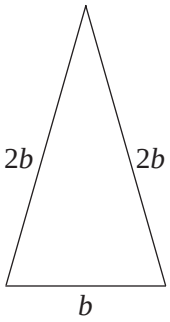
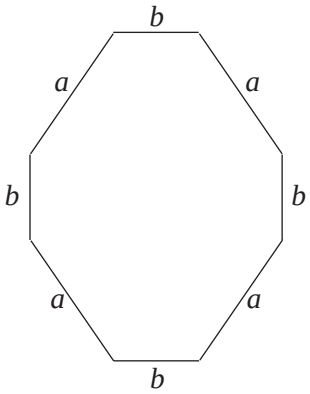


(e)



(f)



11. Sam asks her friend to think of a number, multiply it by 2 and then add 5. If the number her friend starts with is x, write down a formula for the number her friend gets.

12. A removal firm makes a fixed charge of £50, plus £2 for every mile travelled. Write down the formula for the cost of a removal when travelling x miles.

13. A taxi driver charges passengers £1, plus 50p per mile. Write down a formula for the cost of travelling x miles.

16.2 Function Machines

In this section we look at how to find the input and output of function machines, building on the work on number machines in Book Y7A.

INPUT → [FUNCTION MACHINE] → OUTPUT

Example 1

Calculate the output of each of these function machines:

(a) 4 → [× 5] → ?

(b) 5 → [× 2] → [− 1] → ?

(c) $-3 \longrightarrow \boxed{+\ 8} \longrightarrow \boxed{\times\ 7} \longrightarrow ?$

Solution

(a) The input is simply multiplied by 5 to give 20:

$4 \longrightarrow \boxed{\times\ 5} \longrightarrow 20$

(b) The input is multiplied by 2 to give 10, and then 1 is subtracted from this to give 9:

$5 \longrightarrow \boxed{\times\ 2} \xrightarrow{10} \boxed{-\ 1} \longrightarrow 9$

(c) Firstly, 8 is added to the input to give 5, and this is then multiplied by 7 to give 35:

$-3 \longrightarrow \boxed{+\ 8} \xrightarrow{5} \boxed{\times\ 7} \longrightarrow 35$

Example 2

Calculate the input for each of these function machines:

(a) $? \longrightarrow \boxed{\times\ 4} \longrightarrow 8$

(b) $? \longrightarrow \boxed{+\ 2} \longrightarrow \boxed{\times\ 5} \longrightarrow 25$

(c) $? \longrightarrow \boxed{-\ 5} \longrightarrow \boxed{\times\ 3} \longrightarrow 6$

Solution

The missing inputs can be found by reversing the machines and using the inverse (i.e. opposite) operations in each machine:

(a) $? \longrightarrow \boxed{\times\ 4} \longrightarrow 8$

$2 \longleftarrow \boxed{\div\ 4} \longleftarrow 8$

(b) $? \longrightarrow \boxed{+\ 2} \longrightarrow \boxed{\times\ 5} \longrightarrow 25$

$3 \longleftarrow \boxed{-\ 2} \xleftarrow{5} \boxed{\div\ 5} \longleftarrow 25$

(c) $? \longrightarrow \boxed{-\ 5} \longrightarrow \boxed{\times\ 3} \longrightarrow 6$

$7 \longleftarrow \boxed{+\ 5} \xleftarrow{2} \boxed{\div\ 3} \longleftarrow 6$

Note that:

Operation	*Inverse Operation*
+	–
–	+
×	÷
÷	×

Exercises

1. What is the output of each of these function machines:

(a) 4 → [+ 6] → ?

(b) 3 → [× 10] → ?

(c) 10 → [– 7] → ?

(d) 14 → [÷ 2] → ?

(e) 21 → [÷ 3] → ?

(f) 100 → [× 5] → ?

2. What is the output of each of these function machines:

(a) 3 → [× 4] → [– 7] → ?

(b) 10 → [– 8] → [× 7] → ?

(c) 8 → [– 5] → [× 5] → ?

(d) –2 → [× 6] → [+ 20] → ?

(e) 7 → [+ 2] → [÷ 3] → ?

(f) –5 → [+ 8] → [× 9] → ?

3. What is the output of each of these function machines:

(a) ? → [× 5] → 30 (b) ? → [+ 8] → 12

(c) ? → [− 9] → 11 (d) ? → [÷ 4] → 5

(e) ? → [+ 12] → 21 (f) ? → [× 7] → 42

4. What is the input of each of these *double function* machines:

(a) ? → [+ 1] → [× 4] → 12

(b) ? → [+ 7] → [÷ 6] → 4

(c) ? → [× 4] → [+ 9] → 37

(d) ? → [× 9] → [− 20] → 34

(e) ? → [÷ 6] → [− 1] → 7

(f) ? → [− 6] → [÷ 7] → 9

(g) ? → [+ 8] → [× 4] → 24

(h) ? → [× 2] → [+ 7] → −3

5. Here is a *triple function* machine:

Input → [× 7] → [− 5] → [÷ 2] → Output

(a) What is the *output* if the input is 8.

(b) What is the *input* if the output is 22.

(c) What is the *input* if the output is -13.

6. A number is multiplied by 10, and then 6 is added to get 36.
What was the number?

7. Karen asks her teacher, Miss Sharp, how old she is. Miss Sharp replies that if you double her age, add 7 and then divide by 3, you get 21. How old is Miss Sharp?

8. Sally is given her pocket money. She puts half in the bank and then spends £3 in one shop and £2.50 in another shop. She goes home with £1.25. How much pocket money was she given?

9. A bus has its maximum number of passengers when it leaves the bus station. At the first stop, half of the passengers get off. At the next stop 7 people get on and at the next stop 16 people get off. There are now 17 people on the bus. How many passengers were on the bus when it left the bus station?

10. Prakesh buys a tomato plant. In the first week it doubles its height. In the second week it grows 8 cm. In the third week it grows 5 cm. What was the height of the plant when Prakesh bought it if it is now 35 cm in height?

16.3 Linear Equations

An *equation* is a statement, such as $3x + 2 = 17$, which contains an unknown number, in this case, *x*. The aim of this section is to show how to find the unknown number, *x*.

All equations contain an 'equals' sign.

To solve the equation, you need to reorganise it so that the unknown value is by itself on one side of the equation. This is done by performing operations on the equation. When you do this, in order to keep the equality of the sides, you must remember that

> *whatever you do to one side of an equation, you must also do the same to the other side*

Example 1

Solve these equations:

(a) $x + 2 = 8$ (b) $x - 4 = 3$ (c) $3x = 12$

(d) $\frac{x}{2} = 7$ (e) $2x + 5 = 11$ (f) $3 - 2x = 7$

Solution

(a) To solve this equation, subtract 2 from each side of the equation:

$$x + 2 = 8$$

$$x + 2 - 2 = 8 - 2$$

$$x = 6$$

(b) To solve this equation, add 4 to both sides of the equation:

$$x - 4 = 3$$
$$x - 4 + 4 = 3 + 4$$
$$x = 7$$

(c) To solve this equation, divide both sides of the equation by 3:

$$3x = 12$$
$$\frac{3x}{3} = \frac{12}{3}$$
$$x = 4$$

(d) To solve this equation, multiply both sides of the equation by 2:

$$\frac{x}{2} = 7$$
$$2 \times \frac{x}{2} = 2 \times 7$$
$$x = 14$$

(e) This equation must be solved in 2 stages.
First, subtract 5 from both sides:

$$2x + 5 = 11$$
$$2x + 5 - 5 = 11 - 5$$
$$2x = 6$$

Then, divide both sides of the equation by 2:

$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

(f) First, subtract 3 from both sides:

$$3 - 2x = 7$$
$$3 - 2x - 3 = 7 - 3$$
$$-2x = 4$$

Then divide both sides by (-2):

$$\frac{-2x}{-2} = \frac{4}{-2}$$
$$x = -2$$

Example 2

Solve these equations:

(a) $3x + 2 = 4x - 3$

(b) $2x + 7 = 8x - 11$

Solution

These equations contain x on both sides. The first step is to change them so that x is on only *one* side of the equation. Choose the side which has the most x; here, the right hand side.

(a) Subtract $3x$ from both sides of the equation:

$$3x + 2 = 4x - 3$$
$$3x + 2 - 3x = 4x - 3 - 3x$$
$$2 = x - 3$$

Then add 3 to both sides of the equation:

$$2 = x - 3$$
$$2 + 3 = x - 3 + 3$$
$$5 = x$$
$$\text{so} \quad x = 5$$

> Note: *it is conventional to give the answer with the unknown value, x, on the left hand side, and its value on the right hand side*.

(b) First, subtract $2x$ from both sides of the equation:

$$2x + 7 = 8x - 11$$
$$2x + 7 - 2x = 8x - 11 - 2x$$
$$7 = 6x - 11$$

Next, add 11 to both sides of the equation:

$$7 + 11 = 6x - 11 + 11$$
$$18 = 6x$$

Then divide both sides by 6:

$$\frac{18}{6} = \frac{6x}{6}$$
$$3 = x$$
$$\text{so} \quad x = 3$$

Example 3

You ask a friend to think of a number. He then multiplies it by 5 and subtracts 7. He gets the answer 43.

(a) Use this information to write down an equation for x, the unknown number.

(b) Solve your equation for x.

Solution

(a) As x = number your friend thought of, then

$$x \longrightarrow \boxed{\times 5} \xrightarrow{5x} \boxed{-7} \longrightarrow 5x - 7$$

So $5x - 7 = 43$

(b) First, add 7 to both sides of the equation to give

$$5x = 50$$

Then divide both sides by 5 to give

$$x = 10$$

and this is the number that your friend thought of.

Exercises

1. Solve these equations:

(a)	$x + 2 = 8$	(b)	$x + 5 = 11$	(c)	$x - 6 = 2$
(d)	$x - 4 = 3$	(e)	$2x = 18$	(f)	$3x = 24$
(g)	$\frac{x}{6} = 4$	(h)	$\frac{x}{5} = 9$	(i)	$6x = 54$
(j)	$x + 12 = 10$	(k)	$x + 5 = 3$	(l)	$x - 22 = -4$
(m)	$\frac{x}{7} = -2$	(n)	$10x = 0$	(o)	$\frac{x}{2} + 4 = 5$

2. Solve these equations:

(a)	$2x + 4 = 14$	(b)	$3x + 7 = 25$	(c)	$4x + 2 = 22$
(d)	$6x - 4 = 26$	(e)	$5x - 3 = 32$	(f)	$11x - 4 = 29$
(g)	$3x + 4 = 25$	(h)	$5x - 8 = 37$	(i)	$6x + 7 = 31$
(j)	$3x + 11 = 5$	(k)	$6x + 2 = -10$	(l)	$7x + 44 = 2$

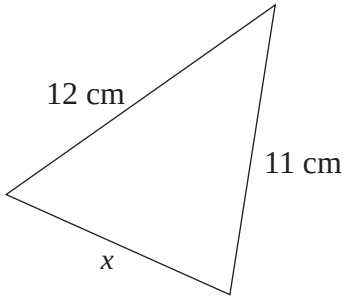
3. Solve these equations, giving your answers as fractions or mixed numbers:

(a) $3x = 4$ (b) $5x = 7$ (c) $2x + 8 = 13$

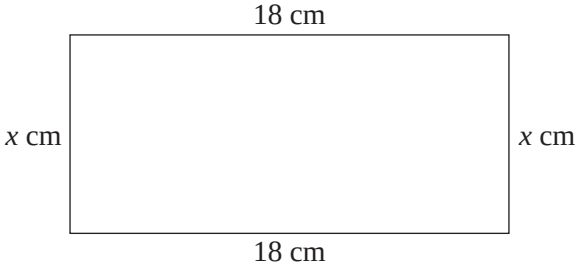
(d) $8x + 2 = 5$ (e) $2x + 6 = 9$ (f) $4x - 7 = 10$

4. The perimeter of this triangle is 31 cm.

Use this information to write down an equation for x and solve it to find x.



5. (a) Write down an expression for the length of the perimeter of this rectangle:



(b) Find x if the perimeter length is 48 cm.

(c) Find x if the perimeter length is 45 cm.

6. Tom asks each of his friends to think of their age, double it and then take away 10.

Here are the answers he is given:

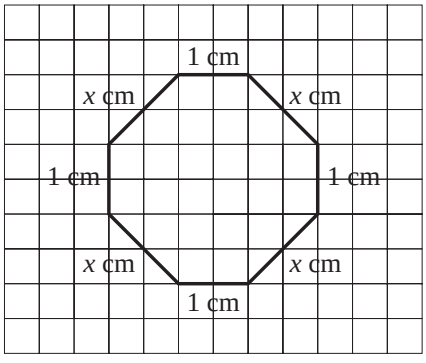
Ben	*Ian*	*Adam*	*Sergio*
8	10	14	11

(a) Using x to represent Ben's age, write down an equation for x and solve it to find Ben's age.

(b) Write down and solve equations to find the ages of Ian, Adam and Sergio.

7. The perimeter of this octagon is 9.6 cm.

Write down an equation and solve it to find x.



8. Solve these equations:

(a) $x + 2 = 2x - 1$ (b) $8x - 1 = 4x + 11$

(c) $5x + 2 = 6x - 4$ (d) $11x - 4 = 2x + 23$

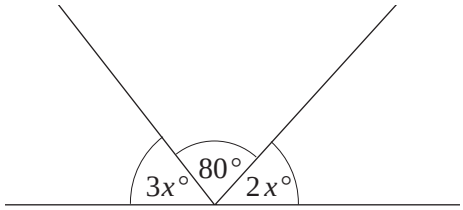
(e) $5x + 1 = 6x - 8$ (f) $3x + 2 + 5x = x + 44$

(g) $6x + 2 - 2x = x + 23$ (h) $2x - 3 = 6x + x - 58$

(i) $3x + 2 = x - 8$ (j) $4x - 2 = 2x - 8$

(k) $3x + 82 = 10x + 12$ (l) $6x - 10 = 2x - 14$

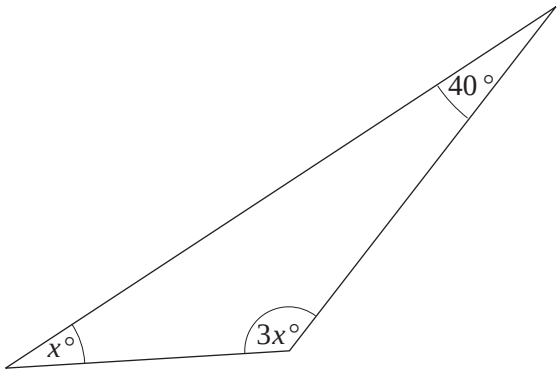
9. The diagram below shows three angles on a straight line:



(a) Write down an equation and use it to find x.

(b) Write down the sizes of the two unknown angles and check that the three angles shown add up to 180 °.

10. Use an equation to find the sizes of the unknown angles in this triangle:



11. Karen thinks of a number, multiplies it by 3 and then adds 10. Her answer is 11 more than the number she thought of. If x is her original number, write down an equation and solve it to find x.