1.   An experiment consists of selecting a ball from a bag and spinning a coin. The bag contains 5
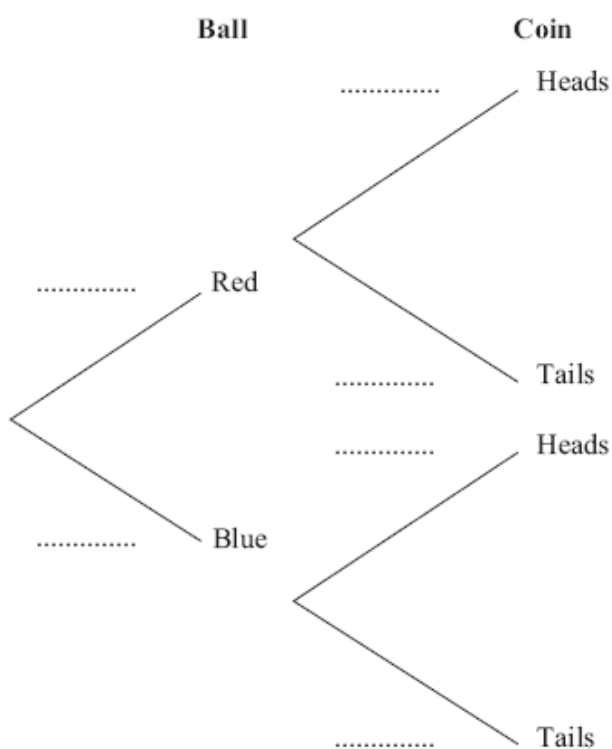   red balls and 7 blue balls. A ball is selected at random from the bag, its colour is noted and then
          the ball is returned to the bag.

          When a red ball is selected, a biased coin with probability $\dfrac{2}{3}$ of landing heads is spun.

          When a blue ball is selected a fair coin is spun.

          (a)   Complete the tree diagram below to show the possible outcomes and associated
                  probabilities.

                                                                                                        (2)

          Shivani selects a ball and spins the appropriate coin.

          (b)   Find the probability that she obtains a head.
                                                                                                        (2)

Given that Tom selected a ball at random and obtained a head when he spun the appropriate coin,

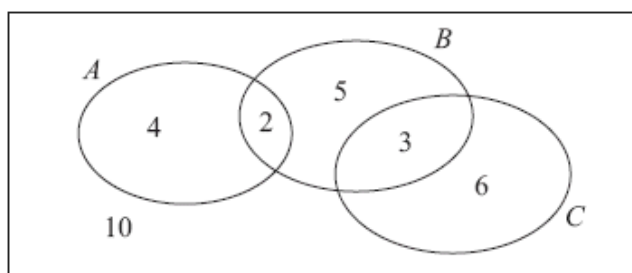(c)   find the probability that Tom selected a red ball.

(3)

Shivani and Tom each repeat this experiment.

(d)   Find the probability that the colour of the ball Shivani selects is the same as the colour of the ball Tom selects.

(3)
(Total 10 marks)

2.   The Venn diagram below shows the number of students in a class who read any of 3 popular magazines *A*, *B* and *C*.

One of these students is selected at random.

(a)   Show that the probability that the student reads more than one magazine is $\frac{1}{6}$.

(2)

(b)   Find the probability that the student reads *A* or *B* (or both).

(2)

(c)   Write down the probability that the student reads both *A* and *C*.

(1)

Given that the student reads at least one of the magazines,

(d)   find the probability that the student reads *C*.

(2)

(e)   Determine whether or not reading magazine *B* and reading magazine *C* are statistically independent.

(3)
(Total 10 marks)

3.   A jar contains 2 red, 1 blue and 1 green bead. Two beads are drawn at random from the jar without replacement.

(a)   Draw a tree diagram to illustrate all the possible outcomes and associated probabilities. State your probabilities clearly.

(3)

(b)   Find the probability that a blue bead and a green bead are drawn from the jar.

(2)
(Total 5 marks)

4.   There are 180 students at a college following a general course in computing. Students on this course can choose to take up to three extra options.

        112 take systems support,
        70 take developing software,
        81 take networking,
        35 take developing software and systems support,
        28 take networking and developing software,
        40 take systems support and networking,
        4 take all three extra options.

(a)   Draw a Venn diagram to represent this information.

(5)

A student from the course is chosen at random.

Find the probability that this student takes

(b)   none of the three extra options,

(1)

(c)   networking only.

(1)

Students who want to become technicians take systems support and networking. Given that a randomly chosen student wants to become a technician,

(d)   find the probability that this student takes all three extra options.

(2)
(Total 9 marks)

5.   On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively.

(a)   Draw a tree diagram to represent this information.

(3)

(b)   Find the probability that on a randomly chosen day

(i)   Bill travels by foot and is late,

(ii)   Bill is not late.

(4)

(c)   Given that Bill is late, find the probability that he did not travel on foot.

(4)
(Total 11 marks)

(2)

Shivani selects a ball and spins the appropriate coin.

(b)   Find the probability that she obtains a head.

(2)

Given that Tom selected a ball at random and obtained a head when he spun the appropriate coin,

(c)   find the probability that Tom selected a red ball.

(3)

Shivani and Tom each repeat this experiment.

(d)   Find the probability that the colour of the ball Shivani selects is the same as the colour of the ball Tom selects.

(3)
(Total 10 marks)

6.   (a)   Given that P(*A*) = a and P(*B*) = *b* express P(*A* $\cup$ *B*) in terms of *a* and *b* when

(i)   *A* and *B* are mutually exclusive,

(ii)   *A* and *B* are independent.

(2)

Two events *R* and *Q* are such that

P(*R* $\cap$ *Q*′) = 0.15, P(*Q*) = 0.35 and P(*R|Q*) = 0.1

Find the value of

(b)   P(*R* $\cup$ *Q*),

(1)

(c)   P($R \cap Q$),

(2)

(d)   P(R).

(2)
(Total 7 marks)

7.   A group of office workers were questioned for a health magazine and $\frac{2}{5}$ were found to take

regular exercise. When questioned about their eating habits $\frac{8\ 2}{x\ 3}$ said they always eat breakfast

and, of those who always eat breakfast $\frac{9}{25}$ also took regular exercise. Find the   probability

that a randomly selected member of the group

(a)   always eats breakfast and takes regular exercise,

(2)

(b)   does not always eat breakfast and does not take regular exercise.

(4)

(c)   Determine, giving your reason, whether or not always eating breakfast and taking regular
exercise are statistically independent.

(2)
(Total 8 marks)

8.   When Rohit plays a game, the number of points he receives is given by the discrete random
variable X with the following probability distribution.

x	0	1	2	3
P($X = x$)	0.4	0.3	0.2	0.1

(a)   Find E(X).

(2)

(b)   Find F(1.5).

(2)

(c)   Show that Var(*X*) = 1

(4)

(d)   Find Var(5 – 3*X*).

(2)

Rohit can win a prize if the total number of points he has scored after 5 games is at least 10.
After 3 games he has a total of 6 points.
You may assume that games are independent.

(e)   Find the probability that Rohit wins the prize.

(6)
(Total 16 marks)

9.   The following shows the results of a wine tasting survey of 100 people.

> 96 like wine *A*,
> 93 like wine *B*,
> 96 like wine *C*,
> 92 like *A* and *B*,
> 91 like *B* and *C*,
> 93 like *A* and *C*,
> 90 like all three wines.

(a)   Draw a Venn Diagram to represent these data.

(6)

Find the probability that a randomly selected person from the survey likes

(b)   none of the three wines,

(1)

(c)   wine *A* but not wine *B*,

(2)

(d)   any wine in the survey except wine *C*,

(2)

(e)   exactly two of the three kinds of wine.

(2)

Given that a person from the survey likes wine *A*,

(f)   find the probability that the person likes wine *C*.

(3)
(Total 16 marks)

10.   A survey of the reading habits of some students revealed that, on a regular basis, 25% read quality newspapers, 45% read tabloid newspapers and 40% do not read newspapers at all.

(a)   Find the proportion of students who read both quality and tabloid newspapers.

(3)

(b)   Draw a Venn diagram to represent this information.

(3)

A student is selected at random. Given that this student reads newspapers on a regular basis,

(c)   find the probability that this student only reads quality newspapers.

(3)
(Total 9 marks)

11.   In a factory, machines *A*, *B* and *C* are all producing metal rods of the same length. Machine *A* produces 35% of the rods, machine *B* produces 25% and the rest are produced by machine *C*. Of their production of rods, machines *A*, *B* and *C* produce 3%, 6% and 5% defective rods respectively.

(a)   Draw a tree diagram to represent this information.

(3)

(b) Find the probability that a randomly selected rod is

(i) produced by machine *A* and is defective,

(ii) is defective.

(5)

(c) Given that a randomly selected rod is defective, find the probability that it was produced by machine *C*.

(3)
(Total 11 marks)

12. The random variable *X* has probability function

$$P(X = x) = \frac{(2x - 1)}{36} \qquad x = 1, 2, 3, 4, 5, 6.$$

(a) Construct a table giving the probability distribution of *X*.

(3)

Find

(b) $P(2 < X \leq 5)$,

(2)

(c) the exact value of $E(X)$.

(2)

(d) Show that $Var(X) = 1.97$ to 3 significant figures.

(4)

(e) Find $Var(2 - 3X)$.

(2)
(Total 13 marks)

13.   A group of 100 people produced the following information relating to three attributes.
The attributes were wearing glasses, being left handed and having dark hair.
Glasses were worn by 36 people, 28 were left handed and 36 had dark hair.
There were 17 who wore glasses and were left handed, 19 who wore glasses and had
dark hair and 15 who were left handed and had dark hair. Only 10 people wore glasses,
were left handed and had dark hair.

(a)   Represent these data on a Venn diagram.

(6)

A person was selected at random from this group.

Find the probability that this person

(b)   wore glasses but was not left handed and did not have dark hair,

(1)

(c)   did not wear glasses, was not left handed and did not have dark hair,

(1)

(d)   had only two of the attributes,

(2)

(e)   wore glasses given that they were left handed and had dark hair.

....

....

....

....

(3)
(Total 13 marks)

14.   A bag contains 9 blue balls and 3 red balls. A ball is selected at random from the bag and its
colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is
recorded.

(a)   Draw a tree diagram to represent this information.

(3)

Find the probability that

(b)   the second ball selected is red,

(2)

(c)   both balls selected are red, given that the second ball selected is red.

(2)
(Total 7 marks)

15.   For the events A and B,

$$P (A \cap B') = 0.32, \qquad P(A' \cap B) = 0.11 \qquad \text{and} \qquad P(A \cup B) = 0.65$$

(a)   Draw a Venn diagram to illustrate the complete sample space for the events A and B.

(3)

(b)   Write down the value of P(A) and the value of P(B).

(3)

(c)   Find P($A \mid B'$).

(2)

(d)   Determine whether or not A and B are independent.

(3)
(Total 11 marks)

16.   In a school there are 148 students in Years 12 and 13 studying Science, Humanities or Arts subjects. Of these students, 89 wear glasses and the others do not. There are 30 Science students of whom 18 wear glasses. The corresponding figures for the Humanities students are 68 and 44 respectively.

A student is chosen at random.

Find the probability that this student

(a)   is studying Arts subjects,

(4)

(b)   does not wear glasses, given that the student is studying Arts subjects.

(2)

Amongst the Science students, 80% are right-handed. Corresponding percentages for Humanities and Arts students are 75% and 70% respectively.

A student is again chosen at random.

(c)   Find the probability that this student is right-handed.

(3)

(d)   Given that this student is right-handed, find the probability that the student is studying Science subjects.

(3)
(Total 12 marks)

17. A company assembles drills using components from two sources. Goodbuy supplies 85% of the components and Amart supplies the rest. It is known that 3% of the components supplied by Goodbuy are faulty and 6% of those supplied by Amart are faulty.

(a) Represent this information on a tree diagram.

(3)

An assembled drill is selected at random.

(b) Find the probability that it is not faulty.

(3)
(Total 6 marks)

18. Articles made on a lathe are subject to three kinds of defect, *A*, *B* or *C*. A sample of 1000 articles was inspected and the following results were obtained.

31 had a type *A* defect
37 had a type *B* defect
42 had a type *C* defect
11 had both type *A* and type *B* defects
13 had both type *B* and type *C* defects
10 had both type *A* and type *C* defects
6 had all three types of defect.

(a) Draw a Venn diagram to represent these data.

(6)

Find the probability that a randomly selected article from this sample had

(b) no defects,

(1)

(c) no more than one of these defects.

(2)

An article selected at random from this sample had only one defect.

(d) Find the probability that it was a type *B* defect.

(2)

Two different articles were selected at random from this sample.

(e)   Find the probability that both had type *B* defects.

(2)
(Total 13 marks)

19.   The events *A* and *B* are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$.

(a)   Using the space below, represent these probabilities in a Venn diagram.

(4)

Hence, or otherwise, find

(b) $P(A \cup B)$,

(1)

(c) $P(A \mid B')$

(2)
(Total 7 marks)

20.   A fair die has six faces numbered 1, 2, 2, 3, 3 and 3. The die is rolled twice and the number showing on the uppermost face is recorded each time.

Find the probability that the sum of the two numbers recorded is at least 5.

(Total 5 marks)

21.   Three events *A*, *B* and *C* are defined in the sample space *S*. The events *A* and *B* are mutually exclusive and *A* and *C* are independent.

(a)   Draw a Venn diagram to illustrate the relationships between the 3 events and the sample space.

(3)

Given that P(*A*) = 0.2, P(*B*) = 0.4 and P(*A* ∪ *C*) = 0.7, find

(b)   P(*A*│*C*),

(2)

(c)   P(*A* ∪ *B*),

(2)

(d)   P(*C*).

(4)
(Total 11 marks)

22.   The events *A* and *B* are such that P(*A*) = $\dfrac{2}{5}$, P(*B*) = $\dfrac{1}{2}$ and P(*A*│*B*′) = $\dfrac{4}{5}$.

(a)   Find

(i)   P(*A* ∩ *B*′),

(ii)   P(*A* ∩ *B*),

(iii) P($A \cup B$),

(iv) P($A | B$).

(7)

(b) State, with a reason, whether or not A and B are

(i) mutually exclusive,

(2)

(ii) independent.

(2)
(Total 11 marks)

23. One of the objectives of a computer game is to collect keys. There are three stages to the game. The probability of collecting a key at the first stage is $\dfrac{2}{3}$, at the second stage is $\dfrac{1}{2}$, and at the third stage is $\dfrac{1}{4}$.

(a) Draw a tree diagram to represent the 3 stages of the game.

(4)

(b) Find the probability of collecting all 3 keys.

(2)

(c) Find the probability of collecting exactly one key in a game.

(5)

(d)   Calculate the probability that keys are not collected on at least 2 successive stages in a
        game.

(5)
(Total 16 marks)

24.   A fairground game involves trying to hit a moving target with a gunshot. A round consists of up
        to 3 shots. Ten points are scored if a player hits the target, but the round is over if the player
        misses. Linda has a constant probability of 0.6 of hitting the target and shots are independent of
        one another.

(a)   Find the probability that Linda scores 30 points in a round.

(2)

The random variable X is the number of points Linda scores in a round.

(b)   Find the probability distribution of X.

(5)

(c)   Find the mean and the standard deviation of X.

(5)

A game consists of 2 rounds.

(d)   Find the probability that Linda scores more points in round 2 than in round 1.

(6)
(Total 18 marks)

25.   Explain what you understand by

(a)   a sample space,

(1)

(b) an event.

(1)

Two events A and B are independent, such that $P(A) = \dfrac{1}{3}$ and $P(B) = \dfrac{1}{4}$.

Find

(c) $P(A \cap B)$,

(1)

(d) $P(A \mid B)$,

(2)

(e) $P(A \cup B)$.

(2)
(Total 7 marks)

26. A car dealer offers purchasers a three year warranty on a new car. He sells two models, the Zippy and the Nifty. For the first 50 cars sold of each model the number of claims under the warranty is shown in the table below.

	Claim	No claim
Zippy	35	15
Nifty	40	10

One of these purchasers is chosen at random. Let A be the event that no claim is made by the purchaser under the warranty and B the event that the car purchased is a Nifty.

(a) Find $P(A \cap B)$.

(2)

(b) Find $P(A')$.

(2)

Given that the purchaser chosen does not make a claim under the warranty,

(c) find the probability that the car purchased is a Zippy.

(2)

(d) Show that making a claim is not independent of the make of the car purchased.

Comment on this result.

(3)
(Total 9 marks)

27. A keep-fit enthusiast swims, runs or cycles each day with probabilities 0.2, 0.3 and 0.5 respectively. If he swims he then spends time in the sauna with probability 0.35. The probabilities that he spends time in the sauna after running or cycling are 0.2 and 0.45 respectively.

(a) Represent this information on a tree diagram.

(3)

(b) Find the probability that on any particular day he uses the sauna.

(3)

(c) Given that he uses the sauna one day, find the probability that he had been swimming.

(3)

(d) Given that he did not use the sauna one day, find the probability that he had been swimming.

(6)
(Total 15 marks)