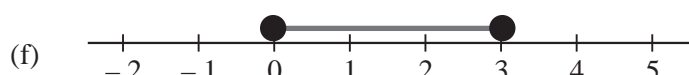
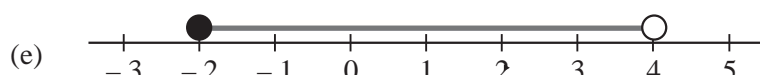
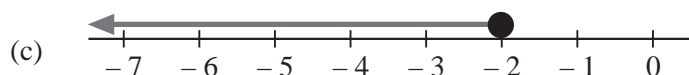
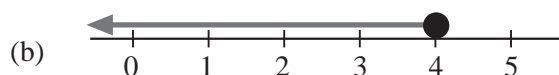
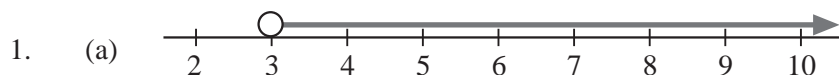


Practice Book *UNIT 13 Graphs, Equations and Inequalities*

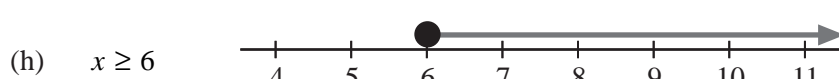
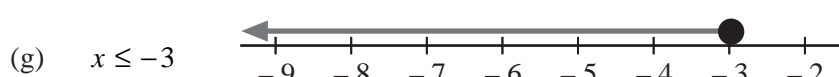
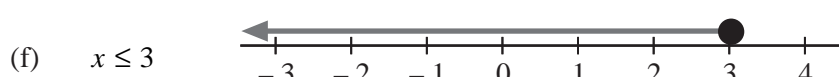
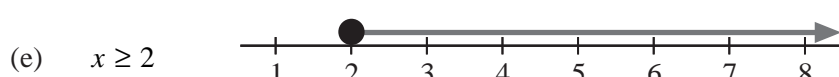
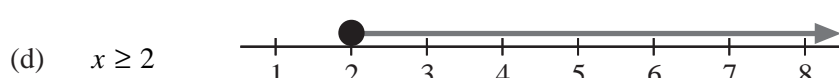
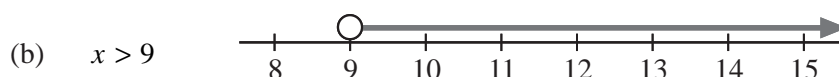
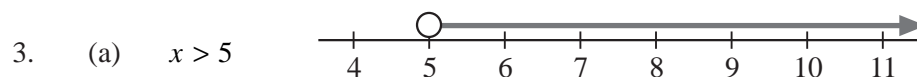
Answers

13.1 Linear Inequalities



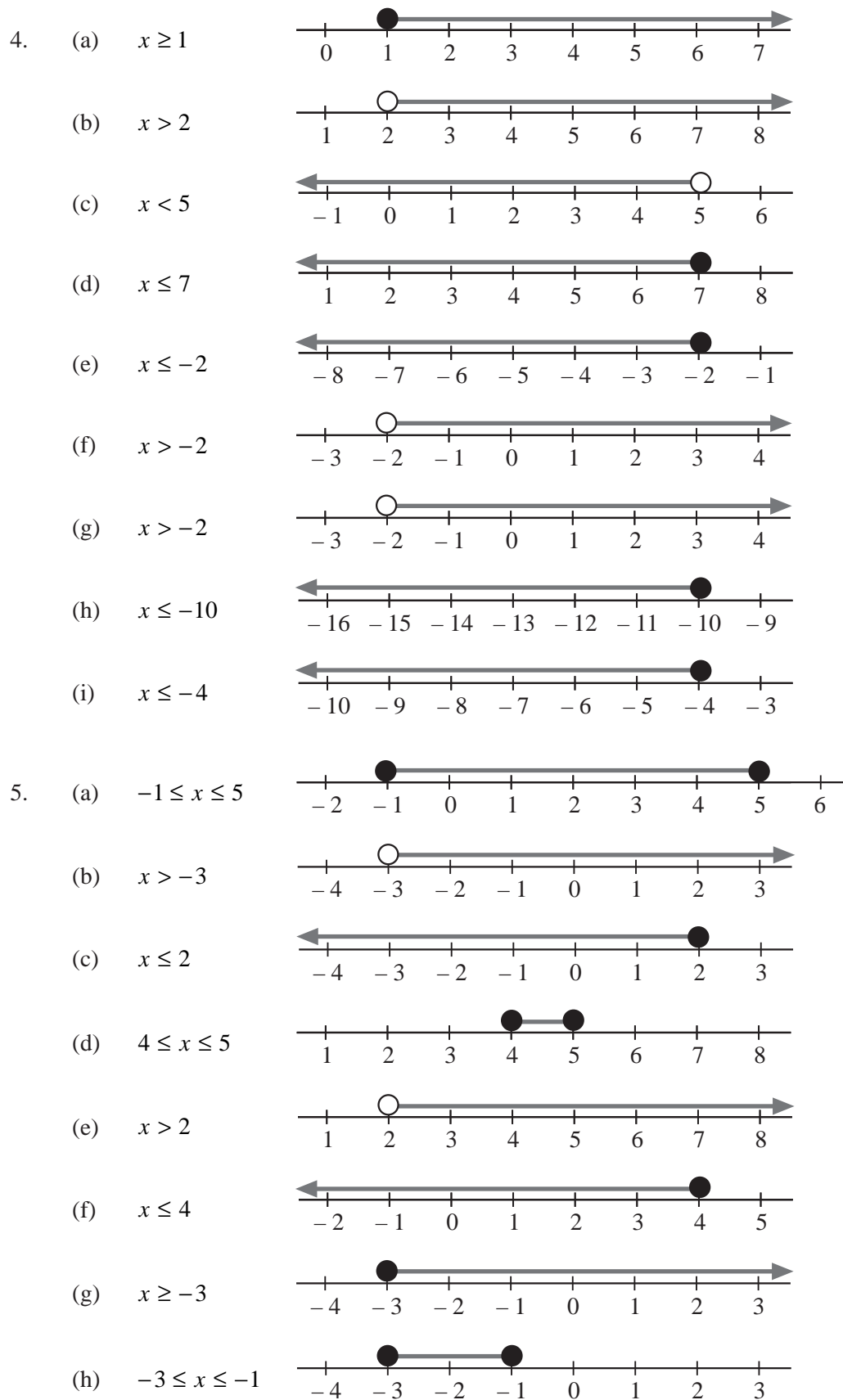
2. (a) $x \geq -2$ (b) $x < 2$ (c) $-2 < x < 3$

(d) $-3 \leq x < -1$ (e) $-4 < x \leq 2$



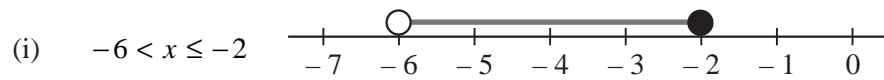
13.1

Answers



13.1

Answers



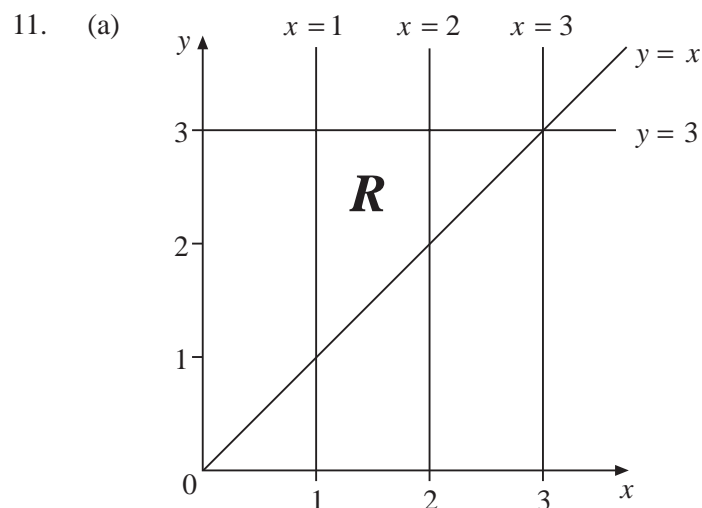
$$\begin{aligned}
 6. \quad (x+8) + x + (x+8) + x &< 44 \Rightarrow 4x + 16 < 44 \\
 &\Rightarrow 4x < 28 \\
 &\Rightarrow x < 7
 \end{aligned}$$

$$\begin{aligned}
 7. \quad 21 < (x+1) + (x+2) + (x+3) &\leq 30 \Rightarrow 21 < 3x + 6 \leq 30 \\
 &\Rightarrow 15 < 3x \leq 24 \\
 &\Rightarrow 5 < x \leq 8
 \end{aligned}$$

$$\begin{aligned}
 8. \quad 10 \leq 5(x+4) < 50 &\Rightarrow 10 \leq 5x + 20 < 50 \\
 &\Rightarrow -10 \leq 5x < 30 \\
 &\Rightarrow -2 \leq x < 6
 \end{aligned}$$

$$9. \quad 3 < \frac{30}{v} < 5 \Rightarrow 6 < v < 10$$

$$\begin{aligned}
 10. \quad 10 \leq \pi r^2 < 20 &\Rightarrow \sqrt{\frac{10}{\pi}} \leq r < \sqrt{\frac{20}{\pi}} \\
 &\Rightarrow 1.784124116 \text{ m} \leq r < 2.523132522 \text{ m} \\
 &\Rightarrow 1.78 \text{ m} < r < 2.52 \text{ m (to the nearest cm)}
 \end{aligned}$$



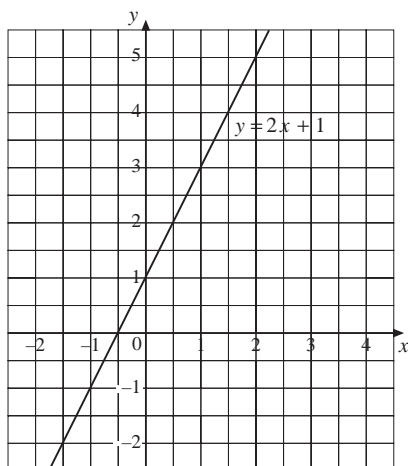
(b) $y < x$, $y > 2$ and $x < 4$

13.2

Answers

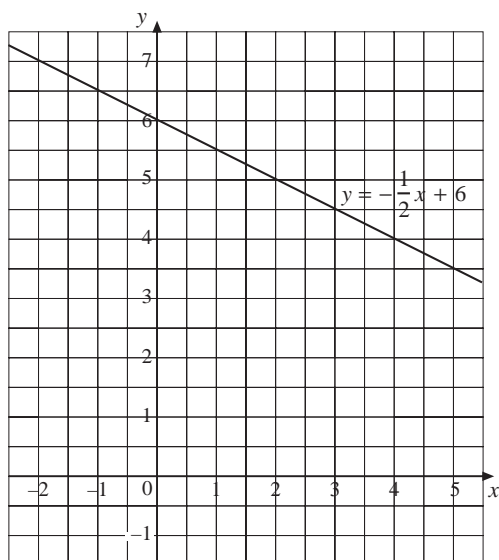
13.2 Graphs of Quadratic Functions

1. (a)

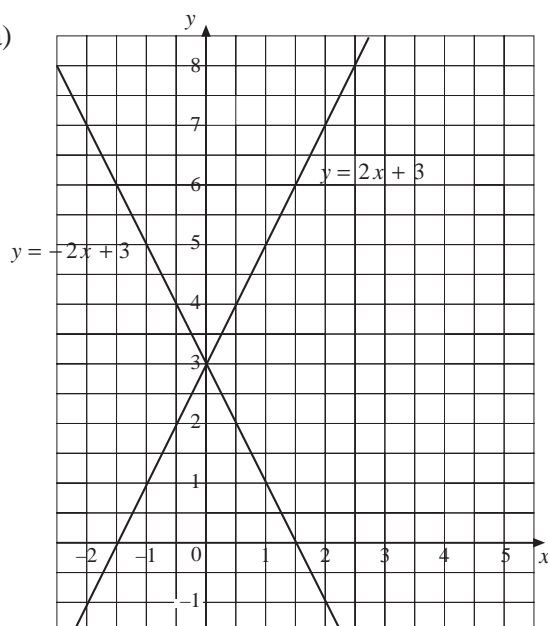


(b) Gradient = 2

2.



3. (a)

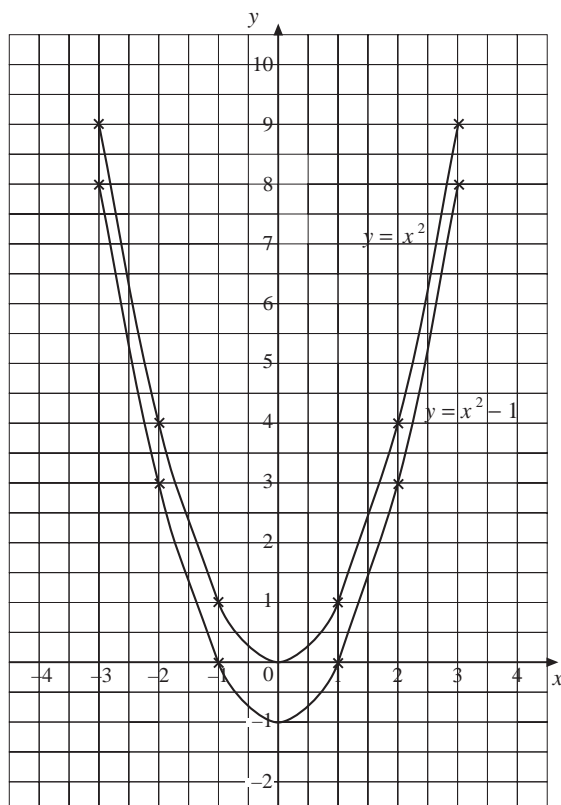


(b) The transformation is a reflection in the y-axis.

13.2

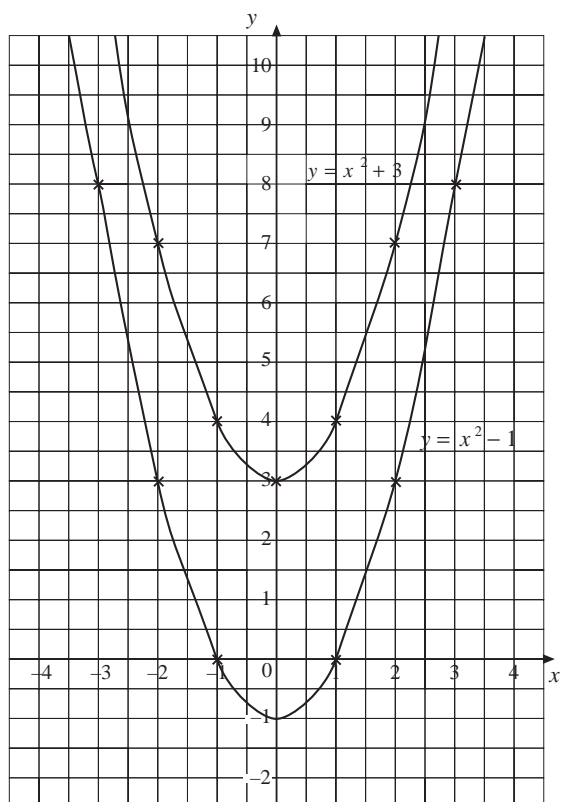
Answers

4. (a)



(b) The graph $y = x^2 - 1$ is a translation of the graph $y = x^2$ along the vector $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$.

5. (a)



(b) The graph $y = x^2 - 1$ is a translation of the graph $y = x^2 + 3$ along the vector $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$.

13.2

Answers

6. $y = x^2 + 1$ is a translation of the graph $y = x^2 - 5$ along the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

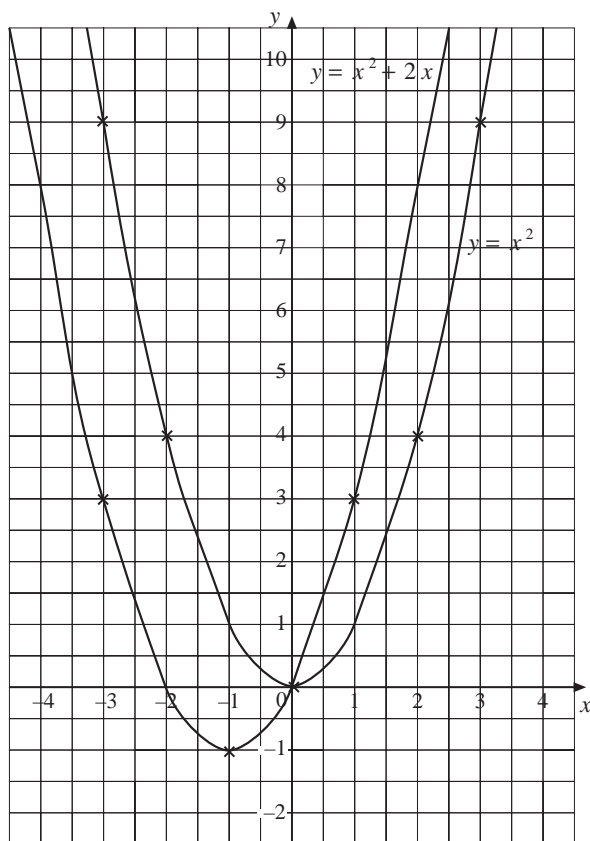
$y = x^2 + 1$ is a translation of the graph $y = x^2 + 6$ along the vector $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$.

$y = x^2 - 5$ is a translation of the graph $y = x^2 + 6$ along the vector $\begin{pmatrix} 0 \\ -11 \end{pmatrix}$.

7. (a)

x	-4	-3	-2	-1	0	1	2	3	4
$x^2 + 2x$	8	3	0	-1	0	3	8	15	24

(b)

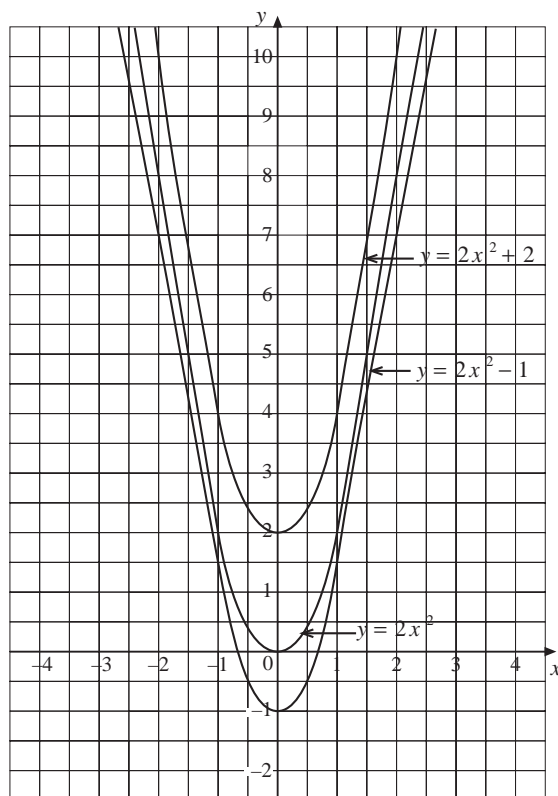


(c) Graph $y = x^2 + 2x$ is a translation of the graph $y = x^2$ along the vector $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$.

13.2

Answers

8. (a) and (b)

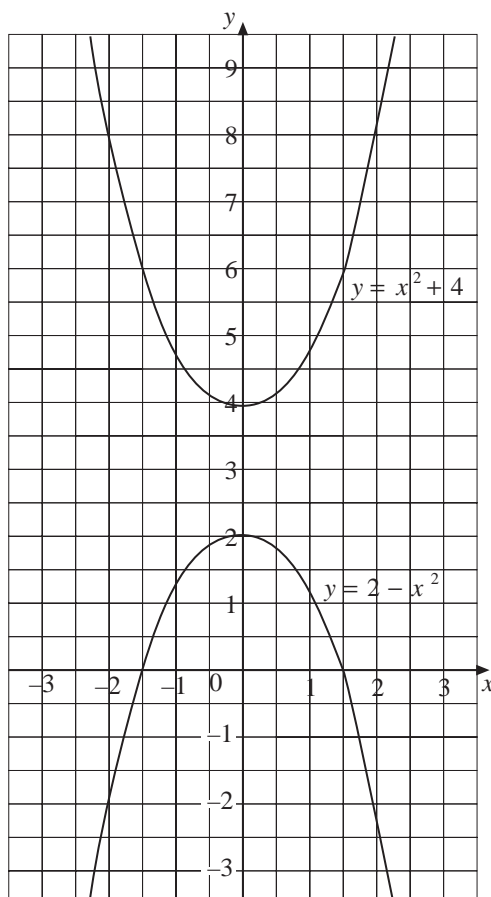


(c) $y = 2x^2 - 1$ is a translation of the graph $y = 2x^2$ along the vector $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$.

$y = 2x^2 + 2$ is a translation of the graph $y = 2x^2$ along the vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

$y = 2x^2 - 1$ is a translation of the graph $y = 2x^2 + 2$ along the vector $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$.

9. (a)

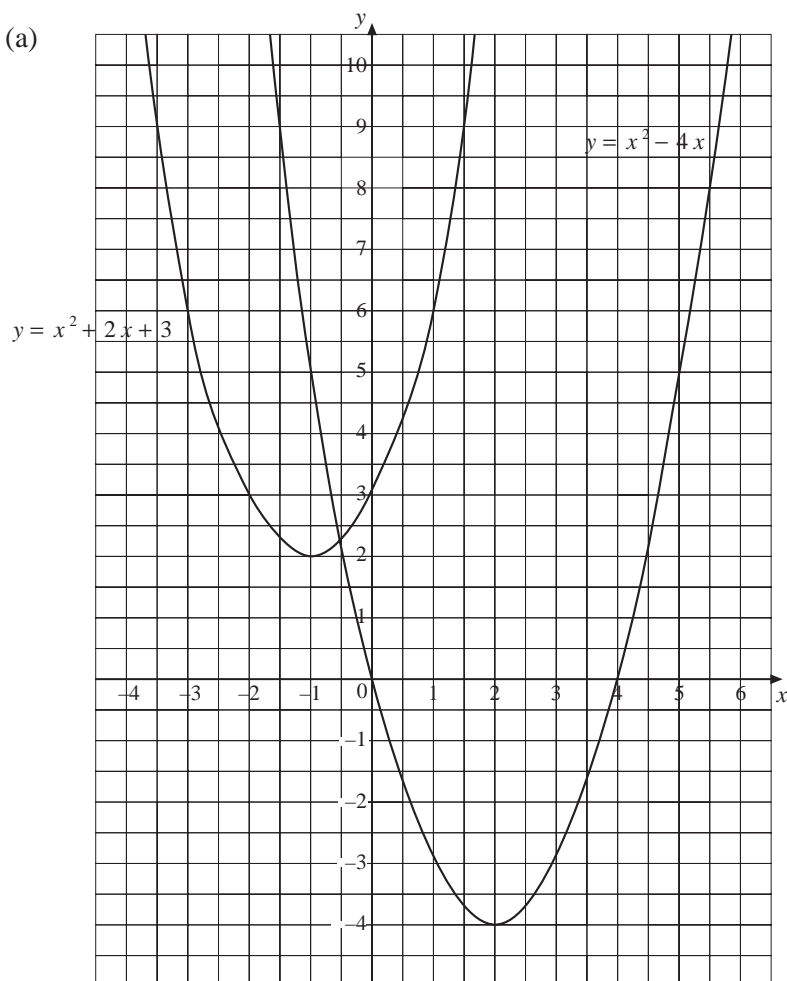


(b) The graph $y = 2 - x^2$ is a reflection of the curve of $y = x^2 + 4$ in the line $y = 3$.

13.2

Answers

10. (a)



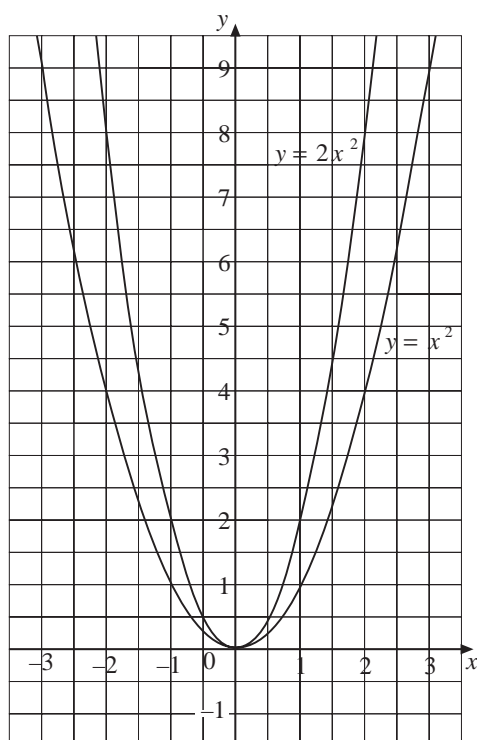
(b)

The graph

$y = x^2 + 2x + 3$ is a translation of the graph $y = x^2 - 4x$ along the

vector $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

11. (a)



(b) $y = -x^2$

(c) $y = x^2 + 1$

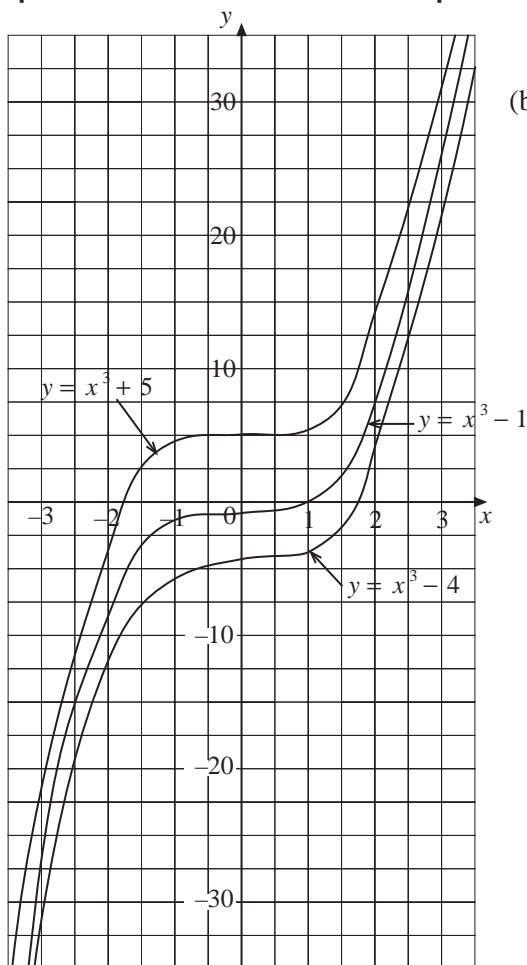
(d) $y > x^2$ and $y < 2$

13.3

Answers

13.3 Graphs of Cubic and Reciprocal Functions

1. (a)

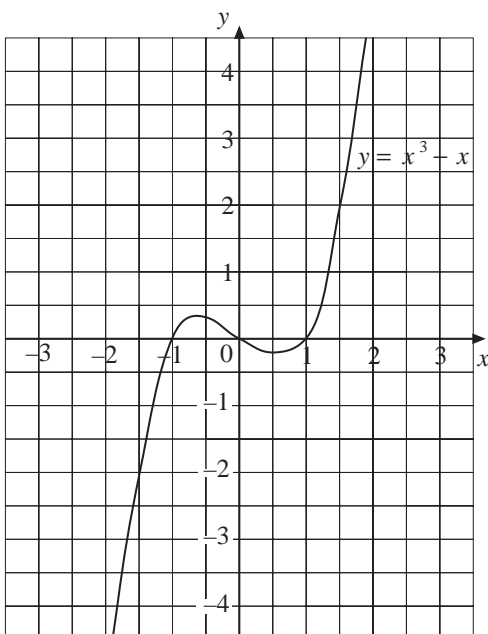


(b) $y = x^3 - 1$ is a translation of the graph $y = x^3 + 5$ along the vector $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$.

$y = x^3 - 4$ is a translation of the graph $y = x^3 + 5$ along the vector $\begin{pmatrix} 0 \\ -9 \end{pmatrix}$.

$y = x^3 - 1$ is a translation of the graph $y = x^3 - 4$ along the vector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$.

2. (a)

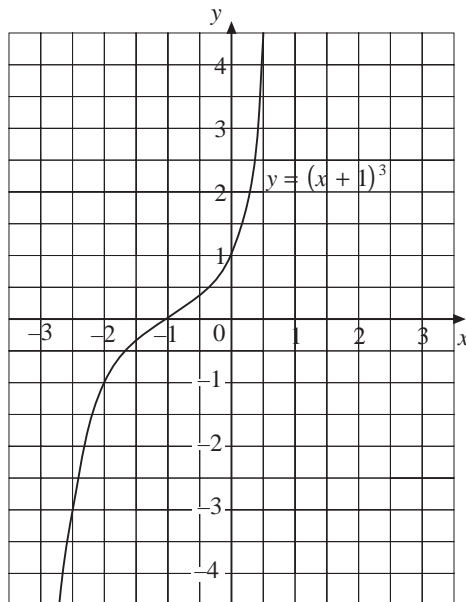


(b) The curve has rotational symmetry of order 2 about the origin.

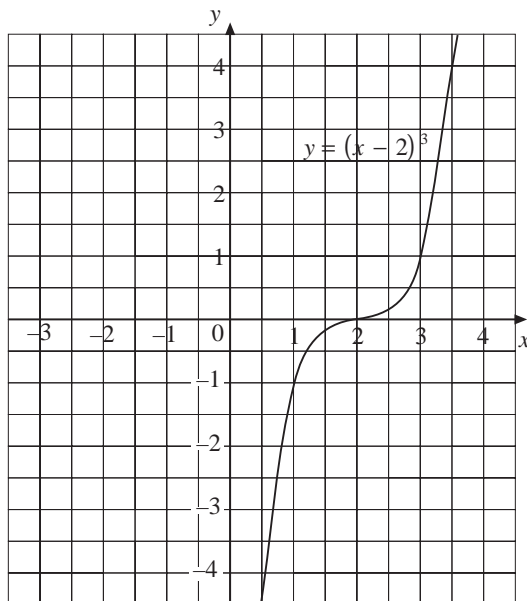
13.3

Answers

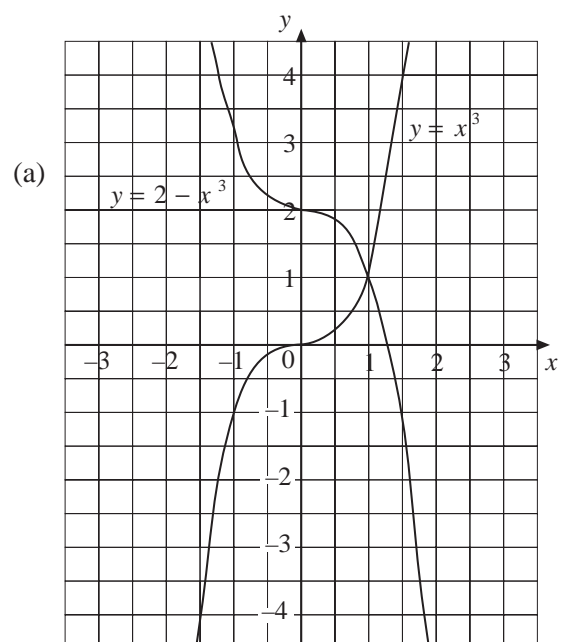
3. (a)

(b) $y = (x + 1)^3$ is a translation of the graph $y = x^3$ along the vector $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ (c) The curve has rotational symmetry of order 2 about $(-1, 0)$.

4. (a)

(b) The curve has rotational symmetry of order 2 about $(2, 0)$.

5.

(b) $y = 2 - x^3$ is a reflection of the graph $y = x^3$ in the line $y = 1$, and vice versa.

13.3

Answers

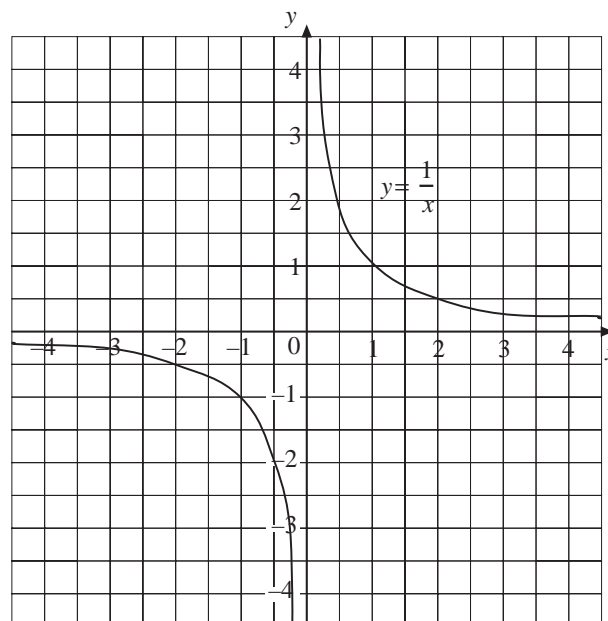
6. (a)

x	-4	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	4
$\frac{1}{x}$	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	-2	2	1	$\frac{1}{2}$	$\frac{1}{4}$

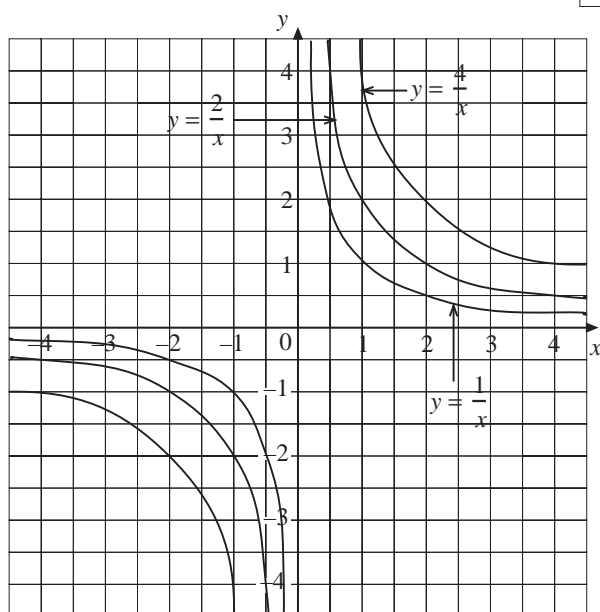
(c) The curve is symmetrical about the lines $y = x$ and $y = -x$.

The curve has rotational symmetry of order 2 about the origin.

(b)



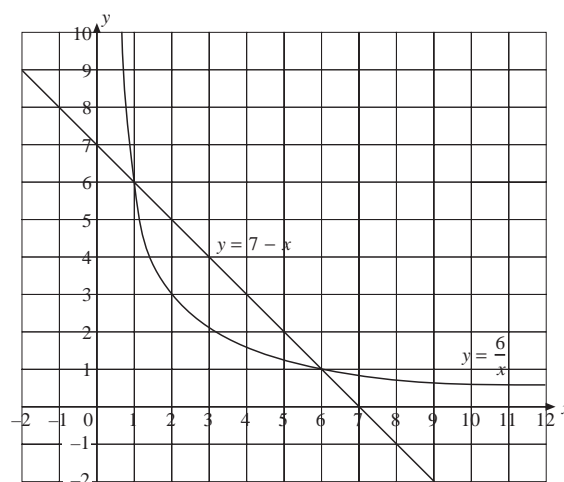
7.



8.

(b) The line $y = 7 - x$ and the curve $y = \frac{6}{x}$ meet at (6, 1) and (1, 6).

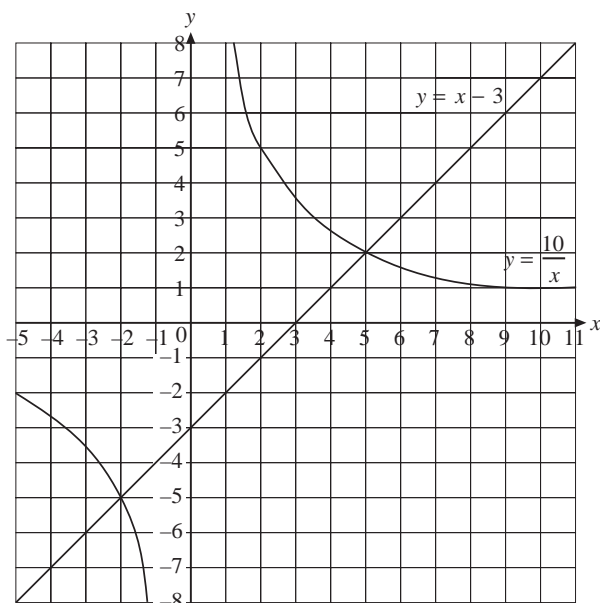
(a)



13.3

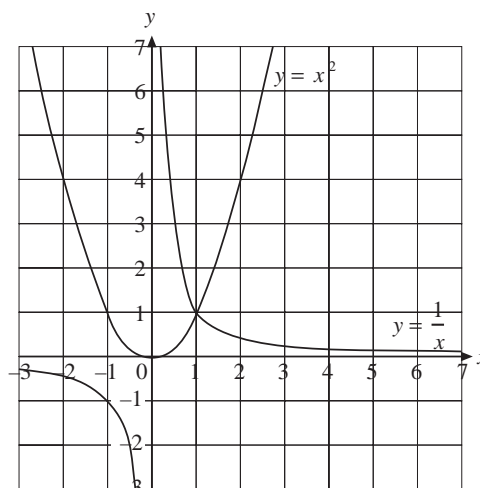
Answers

9. (a)

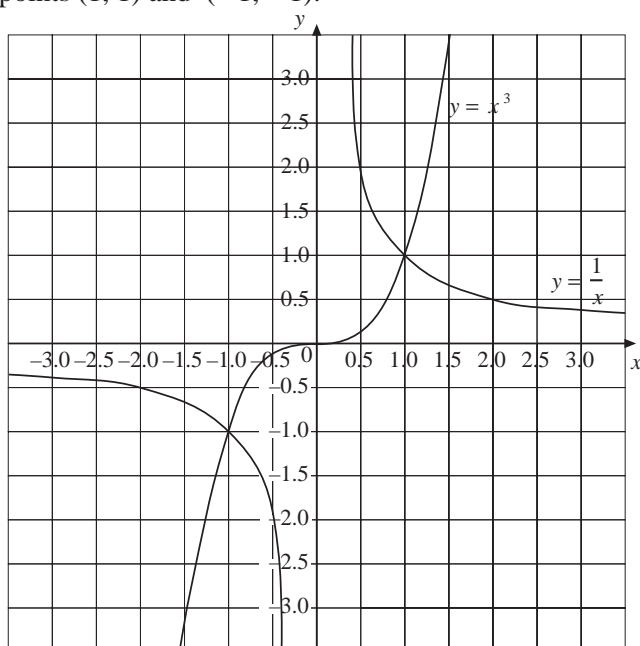


(b) The line $y = x - 3$ meets the curve $y = \frac{10}{x}$ at the points $(5, 2)$ and $(-2, -5)$.

10. (a) The curve $y = x^2$ meets the curve $y = \frac{1}{x}$ at the point $(1, 1)$.



(b) The curve $y = x^3$ meets the curve $y = \frac{1}{x}$ at the points $(1, 1)$ and $(-1, -1)$.

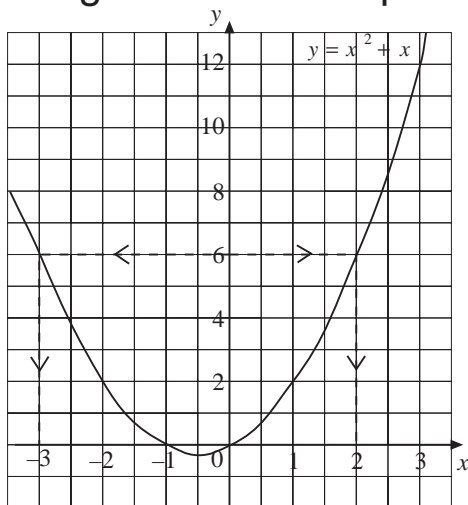


13.4

Answers

13.4 Solving Non-Linear Equations

1. (a)

(b) The solutions are $x = -3$ and $x = 2$.

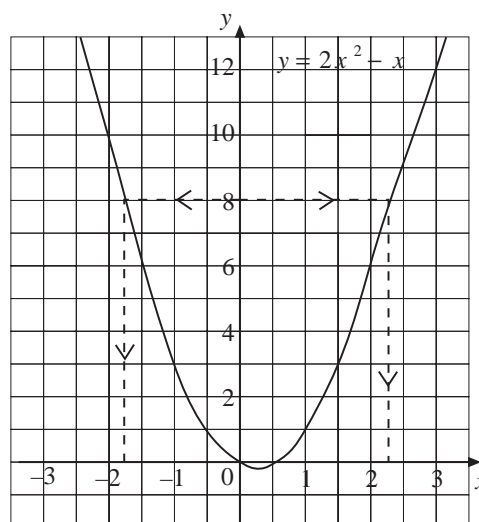
2.

(b) (i) $x = 2.25$ or -1.75 (ii) $x = 1.85$ or -1.35

3. The following are the correct answers to the given equations to 2 decimal places; the answers obtained visually from the graph are in brackets.

(a) $x = 3.43$ (3.6)(b) $x = -2.66$ (-2.7)(c) $x = -1.76, -0.36$ or 3.12 (-1.75, -0.35 or 3.2)(d) $x = -2.32, 0.64$ or 2.68 (-2.3, 0.65 or 2.7)

(a)



4.

<i>Trial x</i>	$x^3 + x$	<i>Comment</i>
9	738	$9 < x$
10	1010	$9 < x < 10$
9.8	950.992	$9.8 < x < 10$
9.9	980.199	$9.9 < x < 10$
9.94	992.047784	$9.94 < x < 10$
9.95	995.024875	$9.95 < x < 10$
9.96	998.007936	$9.96 < x < 10$
9.97	1000.99697	$9.96 < x < 9.97$
9.965	999.50170713	$9.965 < x < 9.97$

So the solution is $x = 9.97$ (to 2 decimal places)

13.4

Answers

5.

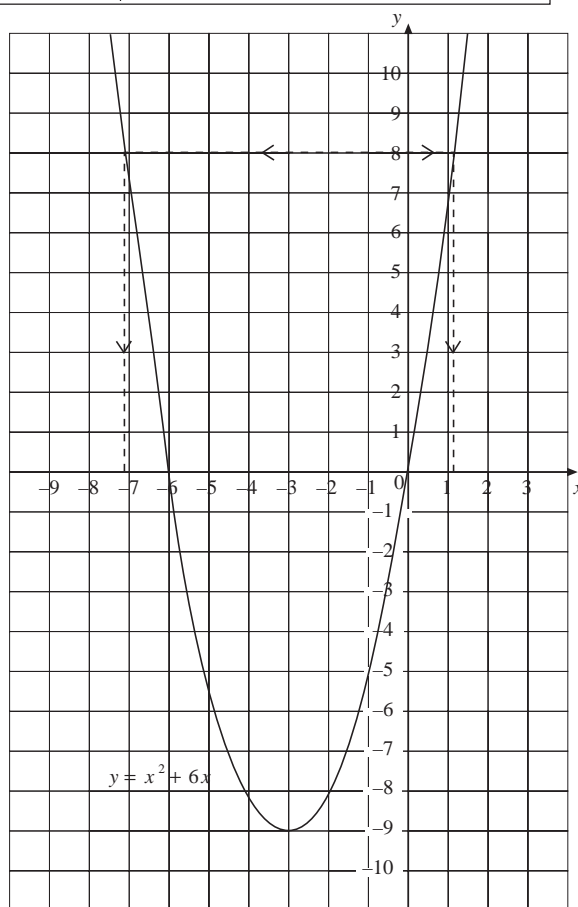
<i>Trial x</i>	$x + \sqrt{x}$	<i>Comment</i>
3.1	4.86068169	3.1 < x
3.2	4.98885438	3.2 < x
3.3	5.11659021	3.2 < x < 3.3
3.21	5.00164729	3.20 < x < 3.21
3.205	4.99525138	3.205 < x < 3.21

So the solution is $x = 3.21$ (to 2 d.p.).

6.

From the graph,

$x = 1.1$ or -7.1



<i>Trial x</i>	$x^2 + 6x$	<i>Comment</i>
1	7	1 < x
2	16	1 < x < 2
1.1	7.81	1.1 < x < 2
1.2	8.64	1.1 < x < 1.2
1.11	7.8921	1.11 < x < 1.2
1.12	7.9744	1.12 < x < 1.2
1.13	8.0569	1.12 < x < 1.13
1.125	8.015625	1.12 < x < 1.125

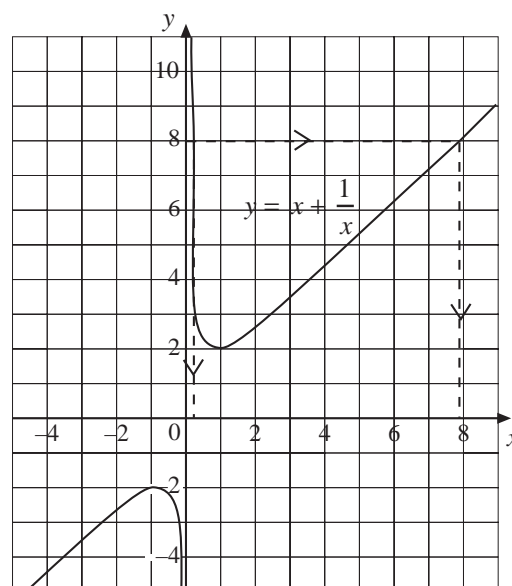
13.4

Answers

<i>Trial x</i>	$x^2 + 6x$	<i>Comment</i>
-7	7.00000000	$x < -7$
-8	16.00000000	$-8 < x < -7$
-7.1	7.81000000	$-8 < x < -7.1$
-7.2	8.64000000	$-7.2 < x < -7.1$
-7.11	7.89210000	$-7.2 < x < -7.11$
-7.12	7.97440000	$-7.2 < x < -7.12$
-7.13	8.05690000	$-7.13 < x < -7.12$
-7.125	8.01562500	$-7.125 < x < -7.12$

So the solution is $x = 1.12$ or -7.12 (to 2 decimal places).

7. (a) From the graph the solutions are
 $x = 0.2$ and 7.8



(b)

<i>Trial x</i>	$x + \frac{1}{x}$	<i>Comment</i>
0.1	10.1	$0.1 < x$
0.2	5.2	$0.1 < x < 0.2$
0.15	6.81666667	$0.1 < x < 0.15$
0.14	7.28285714	$0.1 < x < 0.14$
0.13	7.82230769	$0.1 < x < 0.13$
0.12	8.45333333	$0.12 < x < 0.13$
0.125	8.125	$0.125 < x < 0.13$

13.4

Answers

<i>Trial x</i>	$x + \frac{1}{x}$	<i>Comment</i>
7.8	7.92820513	$7.8 < x$
7.9	8.02658228	$7.8 < x < 7.9$
7.85	7.97738854	$7.85 < x < 7.9$
7.86	7.98722646	$7.86 < x < 7.9$
7.87	7.99706480	$7.87 < x < 7.9$
7.88	8.00690355	$7.87 < x < 7.88$
7.875	8.00198413	$7.87 < x < 7.875$

So the solution to $x + \frac{1}{x}$ is $x = 0.13$ or 7.87 (to 2 decimal places).

8.

<i>Trial x</i>	$8x^2 - x^3$	<i>Comment</i>
0	0	$0 < x$
1	7	$0 < x < 1$
0.7	3.577	$0.7 < x < 1$
0.8	4.608	$0.8 < x < 1$
0.9	5.751	$0.8 < x < 0.9$
0.85	5.165875	$0.8 < x < 0.85$

<i>Trial x</i>	$8x^2 - x^3$	<i>Comment</i>
0	0	$x < 0$
-1	9	$-1 < x < 0$
-0.6	3.096	$-0.6 < x < 0$
-0.7	4.263	$-0.7 < x < 0$
-0.8	5.632	$-0.8 < x < -0.7$
-0.75	4.921875	$-0.8 < x < -0.75$

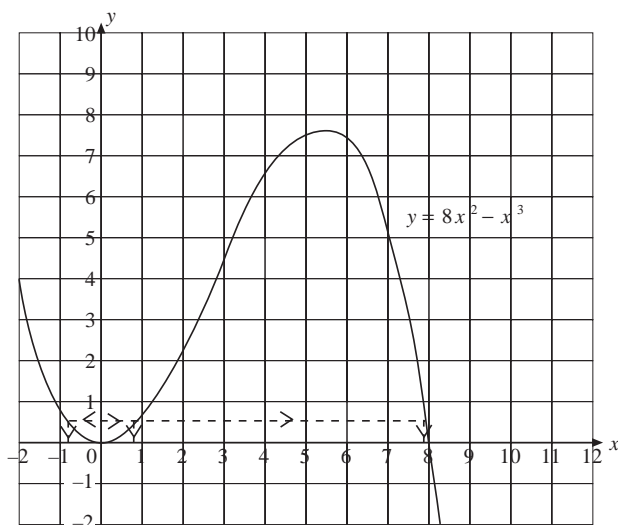
<i>Trial x</i>	$8x^2 - x^3$	<i>Comment</i>
7.7	17.787	$7.7 < x$
7.8	12.168	$7.8 < x$
7.9	6.241	$7.9 < x$
8	0	$7.9 < x < 8$
7.95	3.160125	$7.9 < x < 7.95$

Graph on next page.

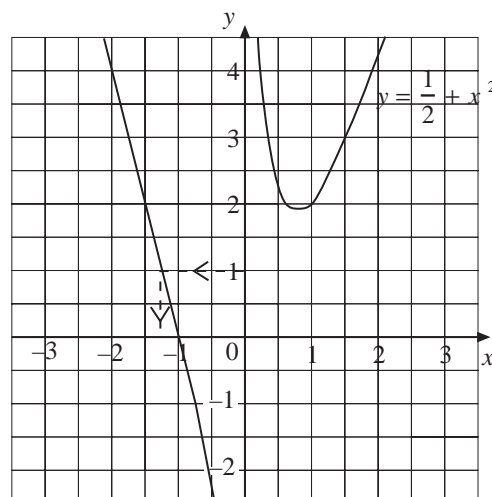
So the complete solution to $8x^2 - x^3 = 5$ is $x = -0.8, 0.8$ or 7.9 (to 1 decimal place).

13.4

Answers



9. The graph shows that the solution is between $x = -1$ and -2 , and is approximately -1.3 .



<i>Trial x</i>	$\frac{1}{x} + x^2$	<i>Comment</i>
-1	0	$x < -1$
-2	3.5	$-2 < x < -1$
-1.3	0.92076923	$-2 < x < -1.3$
-1.4	1.24571429	$-1.4 < x < -1.3$
1.32	0.98482424	$-1.4 < x < -1.32$
1.33	1.0170203	$-1.33 < x < -1.32$
1.325	1.00090802	$-1.325 < x < -1.32$

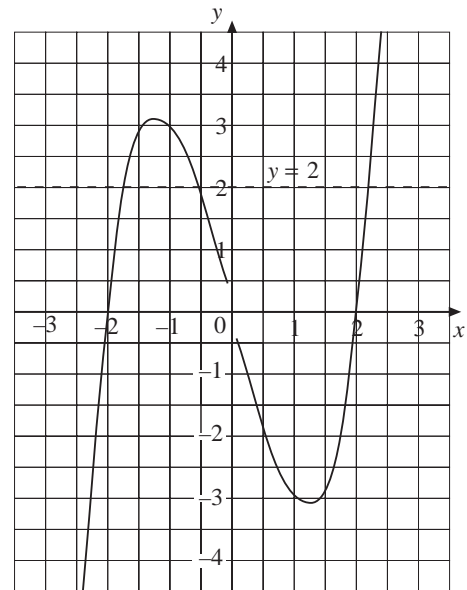
The solution to $\frac{1}{x} + x^2 = 1$ is $x = -1.32$ (to 2 decimal places).

13.4

Answers

10. The graph shows that one solution is between $x = -1$ and -2 , another between $x = -1$ and 0 , and the other between $x = 2$ and 3 .

By trial and improvement, the complete solution to $x^3 - 4x = 2$ is $x = -1.68, -0.54$ or 2.21 (to 2 decimal places).



<i>Trial x</i>	$x^3 - 4x$	<i>Comment</i>
-1	3	$x < -1$
-2	0	$-2 < x < -1$
-1.6	2.304	$-2 < x < -1.6$
-1.7	1.887	$-1.7 < x < -1.6$
-1.67	2.022537	$-1.7 < x < -1.67$
-1.68	1.978368	$-1.68 < x < -1.67$
-1.675	2.00057813	$-1.68 < x < -1.675$

<i>Trial x</i>	$x^3 - 4x$	<i>Comment</i>
0	0	$x < 0$
-1	3	$-1 < x < 0$
-0.6	2.184	$-0.6 < x < 0$
-0.5	1.875	$-0.6 < x < -0.5$
-0.54	2.002536	$-0.54 < x < -0.5$
-0.53	1.971123	$-0.54 < x < -0.53$
-0.535	1.98686963	$-0.54 < x < -0.535$

<i>Trial x</i>	$x^3 - 4x$	<i>Comment</i>
2	0	$2 < x$
3	15	$2 < x < 3$
2.2	1.848	$2.2 < x < 3$
2.3	2.967	$2.2 < x < 2.3$
2.21	1.953861	$2.21 < x < 2.3$
2.22	2.061048	$2.21 < x < 2.22$
2.215	2.00728838	$2.21 < x < 2.215$

13.4

Answers

11. (a)

x	-2	-1	0	1	2	3
$y = x^2 + x - 5$	-3	-5	-5	-3	1	7

(b)

<i>Trial x</i>	$x^2 + x - 5$	<i>Comment</i>
1	-3	$1 < x$
2	1	$1 < x < 2$
1.6	-0.84	$1.6 < x < 2$
1.7	-0.41	$1.7 < x < 2$
1.8	0.04	$1.7 < x < 1.8$
1.75	-0.1875	$1.75 < x < 1.8$

So the solution to $x^2 + x - 5 = 0$ is $x = 1.8$ (to 1 d.p.).

12. (a) There is a root between $x = 0$ and $x = 1$ because the difference is positive when $x = 0$ and it is negative when $x = 1$.

(b) The root lies between $x = 0.5$ and $x = 0.6$.

(c)

x	$2x^2$	$10x - 5$	<i>Difference</i>
0.54	0.5832	0.4	0.1832
0.55	0.605	0.5	0.105
0.56	0.6272	0.6	0.0272
0.57	0.6498	0.7	-0.0502
0.58	0.6728	0.8	-0.1272

So the solution to $2x^2 = 10x - 5$ lies between $x = 0.56$ and $x = 0.57$.

13. (a) The solutions to $x^2 + 4x - 2 = 0$ are $x = -4.5$ and $x = 0.5$.

(b) Any equation of the form $x^2 + 4x + k = 0$ where $k \leq 4$.

(c) $x^2 + 4x + 5 = 0$ is equivalent to the equation $x^2 + 4x = -5$, which has no solution because the curve $y = x^2 + 4x$ does not meet the line $y = -5$.

(d) $x^2 + 4x = 2 \Rightarrow x(x + 4) = 2 \Rightarrow x = \frac{2}{x + 4}$

\therefore the values of the iterative equation $x_{n+1} = \frac{2}{x_n + 4}$ should approach a solution of the equation $x^2 + 4x = 2$.

(e) $x_2 = -2 \quad x_3 = -5 \quad x_4 = -4.4 \quad x_5 = -4.45454545$

13.5

Answers

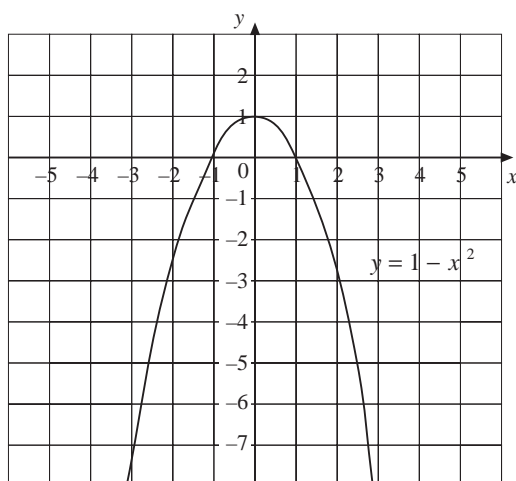
13.5 Quadratic Inequalities

1.

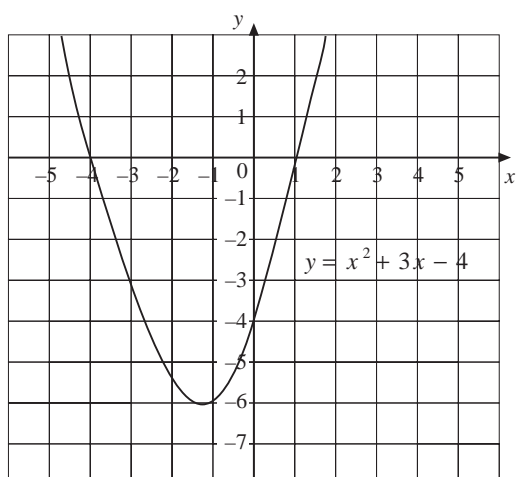
(b) $x \leq -3$ or $x \geq 1$

(c) $-3 < x < 1$

2.



3.



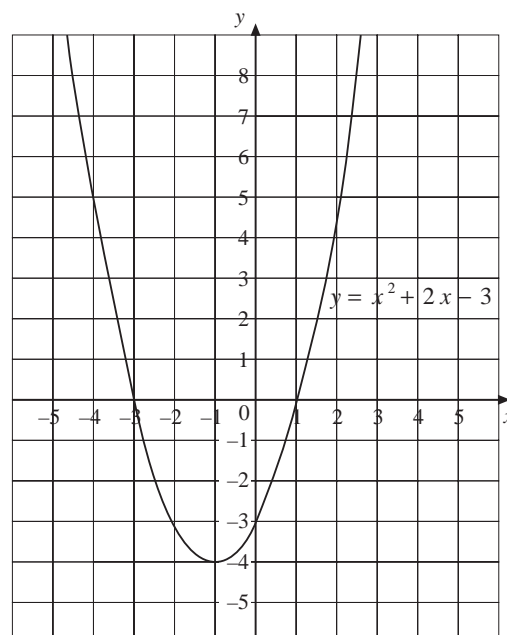
4. (a) $x^2 - 5x = x(x - 5)$

(c) The solution is $0 < x < 5$.

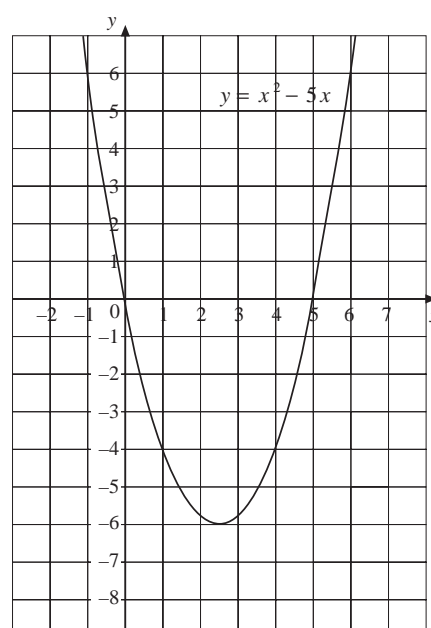
5. (a) $x < -5$ or $x > 0$ (b) $0 \leq x \leq 3$

(c) $x < 0$ or $x > 1$ (d) $0 \leq x \leq 2$

(a)

The solution is $x < -1$ or $x > 1$.The solution is $-4 \leq x \leq 1$.

(b)



13.5

Answers

6. (a) $x^2 - 49 = (x - 7)(x + 7)$

(c) $x < -7$ or $x > 7$

7. (a) $-6 < x < 6$

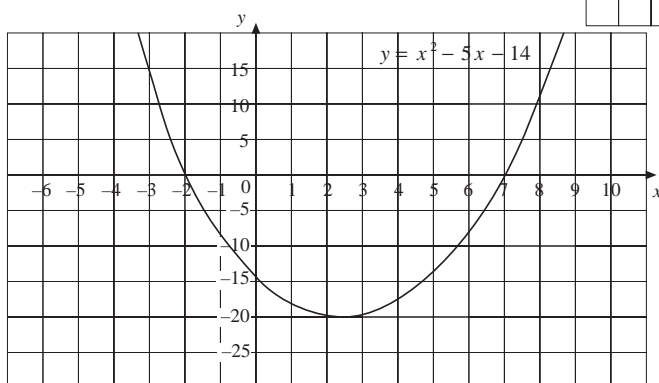
(b) $-10 \leq x \leq 10$

(c) $-4 \leq x \leq 4$

(d) $x < -9$ or $x > 9$

8. (a) $x^2 - 5x - 14 = (x - 7)(x + 2)$

(b)



(c) $x \leq -2$ or $x \geq 7$

9. (a) $-3 < x < 9$ (b) $-4 \leq x \leq -3$

(c) $x < 5$ or $x > 8$ (d) $x \leq -2$ or $x \geq 9$

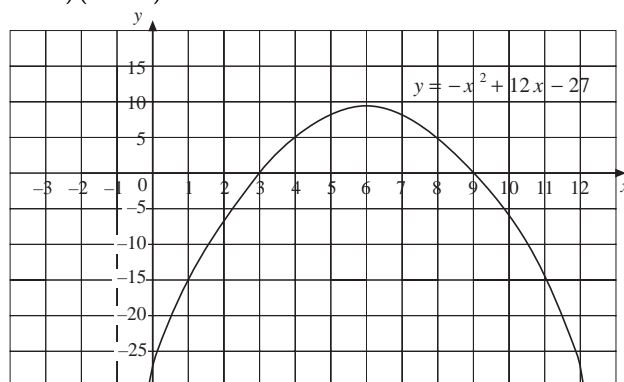
10. (a) $-x^2 + 12x - 27 = (-x + 3)(x - 9) = -(x - 3)(x - 9)$

(c) $x < 3$ or $x > 9$

11. (a)

n	$\frac{n(n+1)}{2}$
22	253
23	276
24	300
25	325
26	351
27	378
28	406

(b)



So $300 < \frac{n(n+1)}{2} < 360$ for $n = 25$ or $n = 26$, giving triangular numbers 325 and 351.

13.5

Answers

(b)

n	$\frac{n(n+1)}{2}$
70	2485
71	2556
72	2628
73	2701
74	2775
75	2850
76	2926

So $2700 < \frac{n(n+1)}{2}$ for $n \geq 73$, giving the two smallest triangular numbers over 2700 as 2701 and 2775.

13.6 Equations of Perpendicular Lines

1. (a) and (c)

(b) $y = -\frac{1}{2}x$

2. (a) Lines C and E

- (b) Line D

3. Gradient of AB = $\frac{1-3}{6-1} = \frac{-2}{5}$

Gradient of CD = $\frac{6-1}{5-3} = \frac{5}{2}$

As $\frac{-1}{\left(\frac{-2}{5}\right)} = \frac{5}{2}$, it follows that AB is perpendicular to CD.

4. Gradient of QR = $\frac{5-3}{6-7} = \frac{2}{-1} = -2$

Gradient of RS = $\frac{3-5}{2-6} = \frac{-2}{-4} = \frac{1}{2}$

Gradient of ST = $\frac{1-3}{3-2} = \frac{-2}{1} = -2$

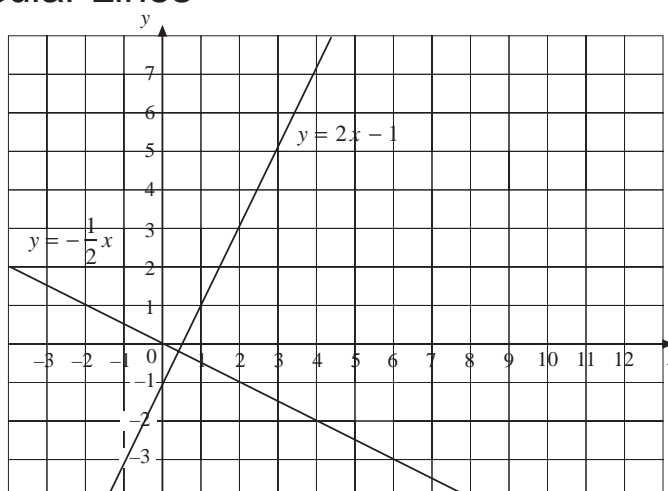
Gradient of TQ = $\frac{1-3}{3-7} = \frac{-2}{-4} = \frac{1}{2}$

$\therefore QR \perp RS \perp ST \perp TQ$ and $QR \parallel ST$ and $RS \parallel TQ$, so QRST is a rectangle.

5. $y = x - 1$

6. Yes the lines are perpendicular because the first line has gradient = $\frac{1}{3}$, the second has gradient -3 ,

and $\frac{-1}{\left(\frac{1}{3}\right)} = -3$



13.6

Answers

7. (a) $y = \frac{-4}{7}x$ or $7y + 4x = 0$

(b) $y = \frac{-4}{7}x + 9\frac{2}{7}$ or $7y + 4x = 65$

8. (a) $y = 9 - 2x$ (b) $(2, 5)$

9. $y = 22 - 4x$ and $y = \frac{1}{4}x + 5$

10. The line through the origin has gradient $\frac{5}{6}$ so the perpendicular line has gradient $\frac{-6}{5}$ and equation

$y = \frac{-6}{5}x + 12\frac{1}{5}$, or $5y + 6x = 61$. This second line crosses the x -axis at the point $\left(10\frac{1}{6}, 0\right)$.