Practice Book   *UNIT 10   Sequences*   Answers
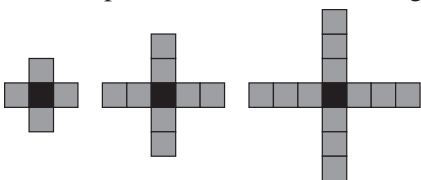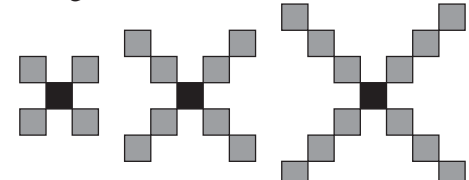
10.1 Constant Differences

1.   (a)   17, 20, 23   (b)   54, 63, 72   (c)   18, 19, 20   (d)   47, 55, 63

2.   (a)   90, 88, 86   (b)   5, 2, -1 (c)   23, 18, 13   (d) $-3, -7, -11$

3.   (a)   4, 10, 16, 22, 28   (b)   6

4.   (a)   10, 18, 26, 34, 42   (b)   8   (c)   50, 58, 66

5.   (a)   4, 11, 18, 25   (b)   7

      (c)   The first difference 7 is the multiple of *n* in the formula $u_n = 7n - 3$.

6.   (a)   11, 20, 29, 38

      (b)   The first difference 9 is the multiple of *n* in the formula $u_n = 9n + 2$.

7.   (a)   11   (b)   4, 15, 26, 37   (c)   103

8.   (a)   78, 74, 70, 66, 62   (b) -4

      (c)   The first difference -4 is the multiple of *n* in the formula $u_n = 82 - 4n$.

      (d)   2

9.   (a)   795   (b)   272   (c)   622   (d)   710

10.   (a)   B and C   (b)   D   (c)   B   (d)   C

11.   (a) -3 and 13   The numbers on this line go up in steps of 4.

      (b)   7.9, 8.0, 8.1   The numbers on this line go up in steps of 0.1.

12.   (a)   3N represents the number of grey tiles in pattern N.

      (b)   1 black and 36 grey tiles   (c)   Pattern 20   (d)   number of tiles $= 1 + 6N$

      (e)   Any suitable pattern with first three diagrams having 5, 9 and 13 tiles,
             e.g.

13.   (a)   4   (b)   24   (c)   36   (d)   10

10.2 Finding the Formula for a Linear Sequence

1.   (a)   4   (b) $u_n = 4n + 3$

2.   (a) $u_n = 4n + 2$ (b) $u_n = 2n + 9$ (c) $u_n = 7n + 2$

      (d) $u_n = 22n + 12$ (e) $u_n = 9n + 13$

3.   (a)   16   (b)   5   (c) $u_n = 5n - 4$

10.2   Answers

4. (a) $u_n = 3n - 2$ (b) $u_n = 4n - 2$ (c) $u_n = 9n - 5$

    (d) $u_n = 10n - 5$ (e) $u_n = 19n - 18$

5. (a) -2 (b) $u_n = 20 - 2n$

6. (a) $u_n = 22 - 3n$ (b) $u_n = 104 - 4n$ (c) $u_n = 48 - 7n$

    (d) $u_n = 82 - 16n$ (e) $u_n = 99 - 9n$

7. (a) -2 (b) $u_n = -2n$

8. (a) $u_n = 5 - 5n$ (b) $u_n = 2n - 20$ (c) $u_n = -2 - 3n$

    (d) $u_n = 15 - 7n$ (e) $u_n = 4n - 11$

9. (a) $u_n = 31n - 11$ (b) 20, 51, 82, 113, 144

10. (a) $u_n = 11n - 6$ (b) 5, 16, 27, 38, 49

11. (a)

pattern number	number of *grey* tiles	number of *white* tiles
5	6	10
16	17	32

    (b)

pattern number	number of *grey* tiles	number of *white* tiles
n	$n + 1$	$2n$

    (c) Total number of tiles $= (n + 1) + 2n = 3n + 1$

    (d) The number of grey tiles follows the sequence 4, 6, 8, 10, ... with constant first
         difference 2, and nth term $2n + 2$. The number of white tiles follows the sequence
         5, 8, 11, 14 ... with constant first difference 3, and nth term $3n + 2$.

         $\therefore$ total number of tiles = number of grey tiles + number of white tiles

         $$= (2n + 2) + (3n + 2) = 5n + 4$$

10.3 Second Differences and Quadratic Sequences

1. (a) 4, 9, 16, 25, 36, 49   (b) The second differences are all 2.   (c) 64, 81

2. (a) 16   (b) 6, 29, 68, 123, 194

    (c) The second differences are all 16, as expected.

3. (a) The second differences are all 2.   (b) $u_n = n^2 + 5$

4. (a) $u_n = 2n^2 - 1$ (b) $u_n = 4n^2 + 2$

    (c) $u_n = 5n^2 - 10$ (d) $u_n = 3n^2 - 2$

5. (a) The second differences are all 4.   (b) 0, 1, 2, 3, 4, 5

    (c) Simpler sequence $= n - 1$ (d) $u_n = 2n^2 + n - 1$

10.3

Answers

6. (a) The second differences are all 8. (b) $u_n = 4n^2 - n + 3$

7. (a) $u_n = 4n^2 + 2n - 3$ (b) $u_n = 3n^2 + 4n - 2$

(c) $u_n = 4n^2 + 2n + 3$ (d) $u_n = 5n^2 + n - 10$

8. (a) The second differences are all -4. (b) $u_n = -2n^2 + n + 10$

(c) 20th term $= -2 \times 20^2 + 20 + 10 = -800 + 30 = -770$

9. $u_n = -n^2 + 7n$

10. (a) The first differences are 7, 19, 37, 61, 91.
The second differences are 12, 18, 24, 30.
The third differences are all 6.

(b) $u_n = n^3 + 5$

11. (a) 4 black tiles and 64 grey tiles (b) 4 black tiles and 256 grey tiles

(c) 4 black tiles and P^2 grey tiles (d) $T = P^2 + 4$

10.4 Special Sequences

1. (a) 29, 47, 76 (b) 57, 92, 149 (c) $\dfrac{6}{17}, \dfrac{7}{20}, \dfrac{8}{23}$

(d) 1215, 3645, 10 935

2. (a) 0, 3, 3, 6, 9, 15 (b) 3, 4, 11, 26, 63, 152
(c) 6, 10, 24, 62, 162, 424 (d) 1, 2, 2, 4, 8, 32

3. (a) 1024, 4096, 16 384 (b) $u_n = 4^{n-1}$ or $u_n = \dfrac{1}{4} \times 4^n$

4. (a) $u_n = 3 \times 5^n$ (b) $u_n = 3^{n-1}$ or $u_n = \dfrac{1}{3} \times 3^n$

(c) $u_n = 2 \times 10^n$ (d) $u_n = 4 \times 7^{n-1}$ or $\dfrac{4}{7} \times 7^n$

5. (a) $u_n = 6n - 5$ (b) $u_n = 8n - 6$ (c) $u_n = \dfrac{6n - 5}{8n - 6}$

6. (a) $u_n = \dfrac{n}{n + 3}$ (b) $u_n = \dfrac{2n - 1}{8n - 5}$ (c) $u_n = \dfrac{9n - 4}{5n + 2}$

(d) $u_n = \dfrac{6n - 5}{5 \times 2^n}$

10.4

Answers

7. (a) $u_n = \dfrac{3n}{4n-1}$ 10th term $= \dfrac{30}{39} = \dfrac{10}{13}$

    (b) $u_n = \dfrac{5n-4}{3n+2}$ 10th term $= \dfrac{46}{32} = \dfrac{23}{16}$

8. (a)

n	u_n	u_n to 3 decimal places
1	1	1.000
5	$1\frac{2}{3}$	1.667
10	$1\frac{9}{11}$	1.818
50	$1\frac{49}{51}$	1.961
100	$1\frac{99}{101}$	1.980
500	$1\frac{499}{501}$	1.996
1000	$1\frac{999}{1001}$	1.998
2000	$1\frac{1999}{2001}$	1.999

    (b) As n becomes larger and larger, u_n gets closer and closer to 2.

9. (a)

n	u_n to 3 decimal places
1	4.000
5	4.800
10	4.900
50	4.980
100	4.990
500	4.998
1000	4.999
2000	5.000

As n becomes larger and larger, u_n gets closer and closer to 5.

    (b)

n	u_n to 3 decimal places
1	7.000
5	6.200
10	6.100
50	6.020
100	6.010
500	6.002
1000	6.001
2000	6.001

As n becomes larger and larger, u_n gets closer and closer to 6.

10.4

Answers

(c)

n	u_n to 3 decimal places
1	2.000
5	2.667
10	2.818
50	2.961
100	2.980
500	2.996
1000	2.998
2000	2.999

As n becomes larger and larger, u_n gets closer and closer to 3.

(d)

n	u_n to 3 decimal places
1	1.000
5	0.600
10	0.550
50	0.510
100	0.505
500	0.501
1000	0.501
2000	0.500

As n becomes larger and larger, u_n gets closer and closer to $\frac{1}{2}$.

10. (a) $u_n \to 4$ as n increases   (b) $u_n \to 7$ as n increases

 (c) $u_n \to 0.5$ as n increases   (d) $u_n \to 2$ as n increases

11. (a) Multiply by 2 each time.

 (b) For example,

 $1 \to 5 \to 9 \to 13 \to 17$ $+4$ each time

 $1 \to 5 \to 25 \to 125 \to 625$ $\times 5$ each time

 $1 \to 5 \to 13 \to 29 \to 61$ $\times 2 + 3$ each time

 (There are *many* alternatives.)

12. (a) $u_n = \dfrac{n}{2n + 1}$ (b) Graph 2   (c) $\dfrac{2}{5}, \dfrac{3}{10}, \dfrac{4}{17}$ (d) Graph 1