1. Consider the function $f(x) = 2x^3 - 3x^2 - 12x + 5$.

(a) (i) Find $f'(x)$.

(ii) Find the gradient of the curve $f(x)$ when $x = 3$.

(4)

(b) Find the x-coordinates of the points on the curve where the gradient is equal to -12.

(3)

(c) (i) Calculate the x-coordinates of the local maximum and minimum points.

(ii) Hence find the coordinates of the local minimum.

(6)

(d) For what values of x is the value of $f(x)$ increasing?

(2)
(Total 15 marks)

2. Consider the function $f(x) = \dfrac{3}{x^2} + x - 4$.

(a) Calculate the value of $f(x)$ when $x = 1$.

(2)

(b) Differentiate $f(x)$.

(4)

(c) Find $f'(1)$.

(2)

(d) Explain what $f'(1)$ represents.

(2)

(e) Find the equation of the tangent to the curve $f(x)$ at the point where $x = 1$.

(3)

(f) Determine the x-coordinate of the point where the gradient of the curve is zero.

(3)
(Total 16 marks)

3. The function $g(x)$ is defined by $g(x) = \dfrac{1}{8}x^4 + \dfrac{9}{4}x^2 - 5x + 7,\ x \geq 0$.

(a) Find g (2).

(2)

(b) Calculate $g'(x)$.

(3)

The graph of the function $y = g(x)$ has a tangent T_1 at the point where $x = 2$.

(c) (i) Show that the gradient of T_1 is 8.

(ii) Find the equation of T_1. Write the equation in the form $y = mx + c$.

(5)

(d) The graph has another tangent T_2 at the point $\left(1, \frac{35}{8}\right)$. T_2 has zero gradient.

Write down the equation of T_2.

(2)

(e) (i) Sketch the graph of $y = g(x)$ in the region $0 \le x \le 3$, $0 \le y \le 22$.

(ii) Add the two tangents T_1 and T_2 to your sketch, in the correct positions.

(5)
(Total 17 marks)

1. (a) (i) $f'(x) = 6x^2 - 6x - 12\ (+0) = 6x^2 - 6x - 12$ (A2)

Note: *Award (A2) for all four items correctly differentiated, (A1) for 3 correct derivatives.*

(ii) $f'(3) = 6(3)^2 - 6(3) - 12 = 24$ (M1) (A1)4

(b) $6x^2 - 6x - 12 = -12$ (M1)
$\Rightarrow 6x^2 - 6x = 0$
$\Rightarrow 6x(x - 1) = 0$
$\Rightarrow x = 0$ or $x = 1$ (A1) (A1)3

(c) (i) $f'(x) = 0 \Rightarrow 6x^2 - 6x - 12 = 0$ (M1)
$\Rightarrow 6(x^2 - x - 2) = 0$
$\Rightarrow 6(x - 2)(x + 1) = 0$ (M1)
$\Rightarrow x = 2$ or $x = -1$ (A1) (A1)

(ii) $x = 2, y = -15$ (A1)
Therefore, minimum is $(2, -15)$ (A1)6

(d) $x < -1$ and $x > 2$ (A1) (A1)2

[15]

2. (a) $f(1) = \frac{3}{1^2} + 1 - 4$ (M1)

$= 0$ (A1)

OR

$f(1) = 0$ (G2) 2

(b) $f'(x) = -\frac{6}{x^3} + 1$ (A4) 4

__Note:__ Award (A2) for $\frac{3}{x^2}$ correctly differentiated and (A1) for each other term correctly differentiated.

(c) $f'(1) = -\frac{6}{1} + 1$ for substituting $f'(x)$ (M1)

$= -5$ (A1)

OR

$f'(1) = -5$ (G2) 2

(d) The gradient of the curve where $x = 1$. (A2) 2

__Note:__ Award (A1) for gradient and (A1) for $x = 1$ or at point $(1, 0)$.

(e) $y = 0$, $x = 1$, $m = –5$ for using $y = mx + c$ with their correct values of m, x and y. (M1)

$0 = –5 \times 1 + c$

$c = 5$ (A1)

$y = –5x + 5$ (A1)

OR

$y = –5x + 5$ (G3) 3

(f) $f'(x) = 0$

$1 - \frac{6}{x^3} = 0$ (M1)(A1)

$x^3 = 6$

$x = \sqrt[3]{6}$ (1.82) (A1)

OR

1.82 (G3) 3

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3. (a) $g(2) = \frac{1}{8}(2)^4 + \frac{9}{4}(2)^2 - 5(2) + 7$ (M1)

$= 8$ (A1)

(G2) 2

(b) $g'(x) = \frac{1}{2}x^3 + \frac{9}{2}x - 5$ (A1)

(A1)(A1) 3

Notes: *Award* (A1) *for each correct term.*

An extra constant means that the –5 is incorrect.

(c) (i) $g'(2) = \frac{1}{2}(8) + \frac{9}{2}(2) - 5 = 8$

For use of their derivative function (M1)
For substitution of 2 leading to an answer of 8 only (A1)
(AG)

Note: beware that g(2) = 8 also. This receives no marks in this part.

(ii) $y = 8x + c$ (A1)

Note: *Award* (A1) *for 8x seen. Must be 8.*

Use of point (2, 8) (M1)

Note: *Follow through with y value from part (a)*

$y = 8x – 8$ (A1)(ft)(G3)

Note: *Answer must be an equation. Allow $T_1 = 8x – 8$*

OR

$8x – y = 8(2) – 8$ (A1)(M1)

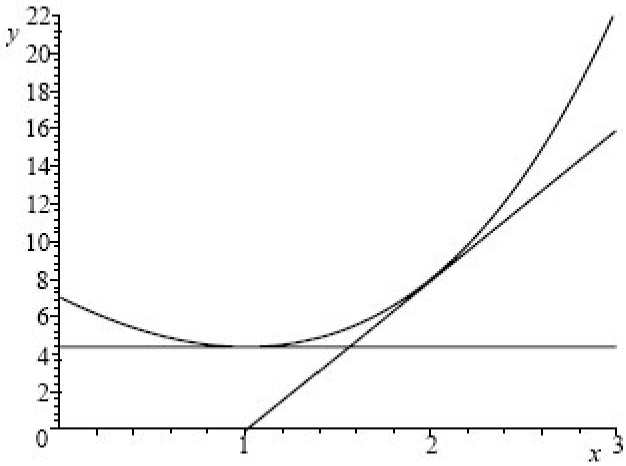
$y = 8x – 8$ (A1)(ft)(G3) 5

(d) $y = \frac{35}{8}$

For y = *Allow* T_2 = (A1)

For $\frac{35}{8}$ (4.375) (A1) 2

(e) (i)



Note: *The window must clearly have used x values from 0 to 3 and y values from 0 to 22. Axes labels are not required. Some indication of scale must be present. This need not be a formal scale. eg tick marks or single numbers on axes are adequate.* (A1)

Correct shape (A1)

Minimum in approximately correct position (A1)

(ii) Each tangent drawn at correct point.

(A1)

(ft)(A1)

Note: *Lines must be straight and must be tangents.*

5

[17]