Double Angle Exercises

1. Given that $\sin\theta = -\frac{5}{6}, \frac{3\pi}{2} \le \theta \le 2\pi$, evaluate

(a) $\sin 2\theta$ (b) $\cos 2\theta$ (c) $\tan 2\theta$ (d) $\sin 4\theta$

2. Given that $\tan x = -3, \frac{\pi}{2} \le x \le \pi$, evaluate

(a) $\sin 2x$ (b) $\cos 2x$ (c) $\tan 2x$ (d) $\tan 4x$

3. Find the exact value of $\sin\frac{\pi}{12}$

4. Given that $\tan x = \frac{a}{b}, \pi \le x \le \frac{3\pi}{2}$, evaluate

(a) $\sin 2x$ (b) $\operatorname{cosec} 2x$ (c) $\cos 4x$ (d) $\tan 2x$

5. Prove the following identities:

(a) $\tan(\theta + \phi) + \tan(\theta - \phi) = \dfrac{2\sin 2\theta}{\cos 2\theta + \cos 2\phi}$ (b) $\dfrac{1 + \cos 2y}{\sin 2y} = \dfrac{\sin 2y}{1 - \cos 2y}$

(c) $\cos^4\alpha - \sin^4\alpha = 1 - 2\sin^2\alpha$ (d) $\dfrac{1}{\sin y\cos y} - \dfrac{\cos y}{\sin y} = \tan y$

(e) $\dfrac{1 + \sin 2\theta}{\cos 2\theta} = \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$ (f) $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$

(g) $\cos\beta + \sin\beta = \dfrac{\cos 2\beta}{\cos\beta - \sin\beta}$ (h) $\sin^2\dfrac{\theta}{2} = \dfrac{1 - \cos\theta}{2}$
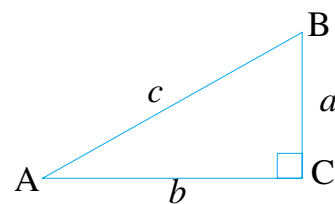
(i) $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \dfrac{1}{2}\sin 2x$ (j) $\cos 4x = 8\cos^4 x - 8\cos^2 x + 1$

6. For the right-angled triangle shown, prove that

(a) $\sin 2\alpha = \dfrac{2ab}{c^2}$ (b) $\cos 2\alpha = \dfrac{b^2 - a^2}{c^2}$

(c) $\sin\dfrac{1}{2}\alpha = \sqrt{\dfrac{c - b}{2c}}$ (d) $\cos\dfrac{1}{2}\alpha = \sqrt{\dfrac{c + b}{2c}}$

7. Solve the following for $0 \le x \le 2\pi$

(a) $\sin x = \sin 2x$ (b) $\sin x = \cos 2x$ (c) $\tan 2x = 4\tan x$

Answers

1. (a) $-\dfrac{5\sqrt{11}}{18}$ (b) $-\dfrac{7}{18}$ (c) $\dfrac{5\sqrt{11}}{7}$ (d) $\dfrac{35\sqrt{11}}{162}$ **2.** (a) $-\dfrac{3}{5}$ (b) $-\dfrac{4}{5}$ (c) $\dfrac{3}{4}$ (d) $\dfrac{24}{7}$ **3.** $\dfrac{(\sqrt{3}-1)\sqrt{2}}{4}$

4. (a) $\dfrac{2ab}{a^2+b^2}$ (b) $\dfrac{a^2+b^2}{2ab}$ (c) $\dfrac{a^4-6a^2b^2+b^4}{(a^2+b^2)^2}$ (d) $\dfrac{2ab}{b^2-a^2}$ **7.** (a) $0, \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}, 2\pi$ (b) $\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$

(c) $0, \pi, 2\pi, \alpha, \pi \pm \alpha, 2\pi - \alpha, \alpha = \tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$