UNIT 10 *Sequences* Extra Exercises 10.1

1. Write down the next 3 terms of each of the following sequences:
   (a) 4, 7, 10, 13, 16, ...
   (b) 8, 20, 32, 44, 56, ...
   (c) 13, 24, 35, 46, 57, ...
   (d) 30, 28, 26, 24, 22, ...
   (e) 10, 6, 2, -2, -6, ...

2. A sequence is defined by the formula $u_n = 5n + 2$.
   (a) Calculate the first 5 terms of the sequence.
   (b) What is the 6th term of the sequence?
   (c) What is the 20th term of the sequence?

3. A sequence is defined by the formula $u_n = 17 - 2n$.
   (a) Calculate the first 4 terms of the sequence.
   (b) What is the 8th term of the sequence?
   (c) What is the 10th term of the sequence?

4. (a) Calculate the 5th term of the sequence $u_n = 18 + 5n$.
   (b) Calculate the 20th term of the sequence $u_n = 7 + 4n$.
   (c) Calculate the 10th term of the sequence $u_n = 8 - 2n$.
   (d) Calculate the 100th term of the sequence $u_n = 3 + 8n$.
   (e) Calculate the 99th term of the sequence $u_n = 7 + 6n$.

UNIT 10 *Sequences* Extra Exercises 10.2

1. For the sequence

   3, 16, 29, 42, 55, ...

   (a) calculate the difference between each term,

   (b) determine the formula that generates the sequence.

2. (a) Determine the formula that generates the sequence

   2, 9, 16, 23, 30, ...

   (b) Use the formula to calculate the 45th term of the sequence.

3. For each of the following sequences, determine the general formula and use it to calculate the 40th term:

   (a) 4, 7, 10, 13, 16, ...

   (b) 11, 16, 21, 26, 31, ...

   (c) 15, 22, 29, 36, 43, ...

   (d) 22, 26, 30, 34, 38, ...

   (e) 1, 15, 29, 43, 57, ...

4. Determine the 50th term of the sequence

   100, 96, 92, 88, 84, ...

5. A linear sequence has a first term of 17 and a difference between terms of 8. Calculate the 18th term of the sequence.

6. Write down the general formula for the sequence

   10, 6, 2, – 2, ...

UNIT 10 *Sequences* Extra Exercises 10.3

1. (a) Calculate the first 5 terms of the sequence $u_n = n^2 + n - 2$.

   (b) Calculate the second differences for the sequence.

   (c) Comment on the values you obtain.

2. What would you expect the second differences to be for each of these sequences?

   (a) $u_n = n^2 + 3$

   (b) $u_n = 5n^2 - n + 2$

   (c) $u_n = \frac{1}{2}n^2 + 2n + 3$

   (d) $u_n = 4n^2 - 3n$

3. (a) Explain why the following sequence has a formula of the form $u_n = n^2 + an + b$.

   1, 6, 13, 22, 33, 46

   (b) Determine the formula for the sequence

   0, 2, 4, 6, 8, 10, ...

   (c) Hence state the formula for the sequence in part (a).

4. Determine the formula for each of the following sequences:

   (a) -1, 3, 9, 17, 27, 39, ...

   (b) -2, -2, 0, 4, 10, 18, ...

   (c) 1, 9, 23, 43, 69, 101, ...

   (d) 5, 18, 41, 74, 117, 170, ...

   (e) 7, 14, 25, 40, 59, 82, ...

   (f) 6.5, 9, 12.5, 17, 22.5, 29, ...

UNIT 10 *Sequences* Extra Exercises 10.4

1. Calculate the next 3 terms of each of the following sequences:

   (a) 4, 5, 7, 10, 14, ...

   (b) 1, 3, 7, 15, 31, ...

   (c) 0, 5, 15, 30, 50, ...

   (d) 100, 90, 81, 73, 66, ...

   (e) 7, 8, 6, 9, 5, 10, ...

2. Write down the next 3 fractions in each of the following sequences:

   (a) $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \ldots$

   (b) $\frac{3}{7}, \frac{5}{10}, \frac{7}{13}, \frac{9}{16}, \ldots$

3. Determine the general formula for the sequence

   $$\frac{3}{8}, \frac{5}{11}, \frac{7}{14}, \frac{9}{17}, \frac{11}{20}, \ldots$$

4. What happens to the sequence $u_n = \frac{3n}{n+1}$ as n becomes large? Answer the question by completing a copy of the following table:

n	1	10	100	1000	5000
u_n					

5. What happens to each of the following sequences as n becomes large:

   (a) $u_n = \frac{n}{n+1}$ (b) $u_n = \frac{7n-1}{2n}$

   (c) $u_n = \frac{3n}{2n+6}$ (d) $u_n = \frac{8n-3}{4n}$

Extra Exercises 10.1 Answers

1. (a) ... , 19, 22, 25, ... (b) ... , 68, 80, 92, ...
   (c) ... , 68, 79, 90, ... (d) ... , 20, 18, 16, ...
   (e) ... , – 10, – 14, – 18, ...
2. (a) 7, 12, 17, 22, 27 (b) 32 (c) 102
3. (a) 15, 13, 11, 9 (b) 1 (c) – 3
4. (a) 43 (b) 87 (c) – 12 (d) 803 (e) 601

Extra Exercises 10.2 Answers

1. (a) 13 (b) $u_n = 13n - 10$
2. (a) $u_n = 7n - 5$ (b) 310
3. (a) $u_n = 3n + 1$, 121 (b) $u_n = 5n + 6$, 206
   (c) $u_n = 7n + 8$, 288 (d) $u_n = 4n + 18$, 178
   (e) $u_n = 14n - 13$, 547
4. – 96 5. 153 6. $u_n = 14 - 4n$

Extra Exercises 10.3 Answers

1. (a) 0, 4, 10, 18, 28 (b) 2, 2, 2, ... (c) They are all 2.
2. (a) 2 (b) 10 (c) 1 (d) 8
3. (a) Second difference is 2. (b) $u_n = 2n - 2$ (c) $u_n = n^2 + 2n - 2$
4. (a) $u_n = n^2 + n - 3$ (b) $u_n = n^2 - 3n$ (c) $u_n = 3n^2 - n - 1$
   (d) $u_n = 5n^2 - 2n + 2$ (e) $u_n = 2n^2 + n + 4$ (f) $u_n = \frac{1}{2}n^2 + n + 5$

Extra Exercises 10.4 Answers

1. (a) ... , 19, 25, 32, ... (b) ... , 63, 127, 255, ...
   (c) ... , 75, 105, 140, ... (d) ... , 60, 55, 51, ...
   (e) ... , 4, 11, 3, ...
2. (a) ..., $\frac{9}{32}$, $\frac{11}{64}$, $\frac{13}{128}$, ... (b) ..., $\frac{11}{19}$, $\frac{13}{22}$, $\frac{15}{25}$, ...
3. $u_n = \frac{1 + 2n}{5 + 3n}$
4.

n	1	10	100	1000	5000
u_n	1.5	2.727	2.970	2.997	2.999

   (u_n to 4 s.f.)
   u_n becomes closer to 3 as n becomes large.
5. (a) $u_n \to 1$ (b) $u_n \to 3.5$ (c) $u_n \to 1.5$ (d) $u_n \to 2$