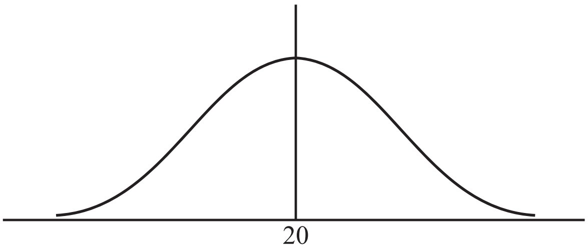
4. *[Maximum mark: 8]*

A random variable X is distributed normally with a mean of 20 and variance 9.

(a) Find $P(X \le 24.5)$. *[3 marks]*

(b) Let $P(X \le k) = 0.85$.

(i) Represent this information on the following diagram.



(ii) Find the value of k. *[5 marks]*

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5. *[Maximum mark: 7]*

A box holds 240 eggs. The probability that an egg is brown is 0.05.

(a) Find the expected number of brown eggs in the box. *[2 marks]*

(b) Find the probability that there are 15 brown eggs in the box. *[2 marks]*

(c) Find the probability that there are at least 10 brown eggs in the box. *[3 marks]*

7. *[Maximum mark: 7]*

A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.
90 % of the boxes made by machine B pass inspection.

A box is selected at random.

(a) Find the probability that it passes inspection. *[3 marks]*

(b) The company would like the probability that a box passes inspection to be 0.87. Find the percentage of boxes that should be made by machine B to achieve this. *[4 marks]*

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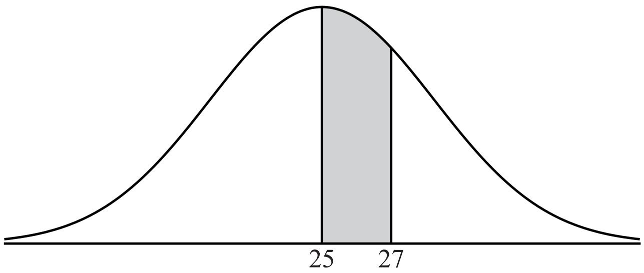
6. *[Maximum mark: 7]*

Let the random variable X be normally distributed with mean 25, as shown in the following diagram.



The shaded region between 25 and 27 represents 30 % of the distribution.

(a) Find $\mathrm{P}(X > 27)$. *[2 marks]*

(b) Find the standard deviation of X. *[5 marks]*

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7. *[Maximum mark: 7]*

The probability of obtaining "tails" when a biased coin is tossed is 0.57. The coin is tossed ten times. Find the probability of obtaining

(a) **at least** four tails; *[4 marks]*

(b) the fourth tail on the tenth toss. *[3 marks]*

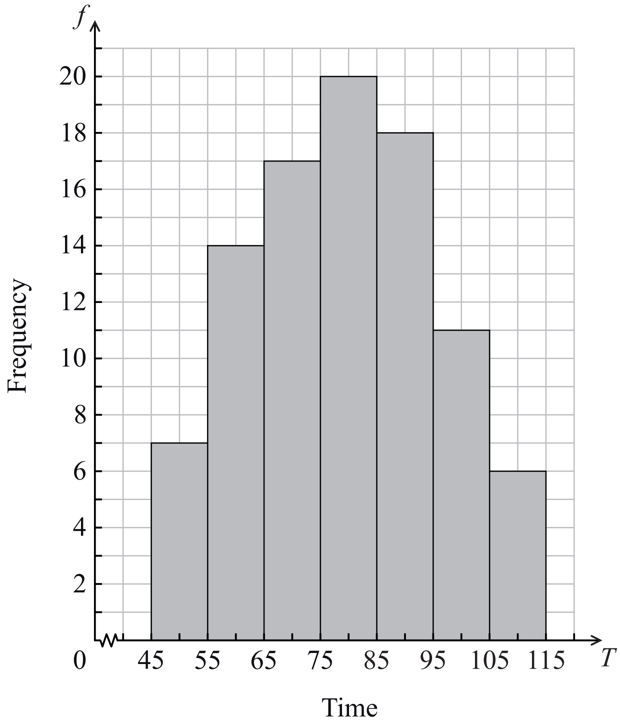
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

The histogram below shows the time T seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for T.

Time	$45 \le T < 55$	$55 \le T < 65$	$65 \le T < 75$	$75 \le T < 85$	$85 \le T < 95$	$95 \le T < 105$	$105 \le T < 115$
Frequency	7	14	p	20	18	q	6

(a) (i) Write down the value of p and of q.

(ii) Write down the median class. *[3 marks]*

(b) A child is selected at random. Find the probability that the child takes less than 95 seconds to solve the puzzle. *[2 marks]*

(This question continues on the following page)

*Do **NOT** write solutions on this page.*

(Question 8 continued)

Consider the class interval $45 \le T < 55$.

(c) (i) Write down the interval width.

(ii) Write down the mid-interval value. *[2 marks]*

(d) Hence find an estimate for the

(i) mean;

(ii) standard deviation. *[4 marks]*

John assumes that T is normally distributed and uses this to estimate the probability that a child takes less than 95 seconds to solve the puzzle.

(e) Find John's estimate. *[2 marks]*

4. *[Maximum mark: 6]*

The heights of a group of seven-year-old children are normally distributed with mean 117 cm and standard deviation 5 cm. A child is chosen at random from the group.

(a) Find the probability that this child is taller than 122.5 cm. *[3 marks]*

(b) The probability that this child is shorter than k cm is 0.65. Find the value of k. *[3 marks]*

7. *[Maximum mark: 8]*

A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested.

(a) Find the probability that there is at least one defective lamp in the sample. *[4 marks]*

(b) Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps. *[4 marks]*

2. *[Maximum mark: 6]*

The following table shows the Diploma score x and university entrance mark y for seven IB Diploma students.

Diploma score (x)	28	30	27	31	32	25	27
University entrance mark (y)	73.9	78.1	70.2	82.2	85.5	62.7	69.4

(a) Find the correlation coefficient. *[2]*

The relationship can be modelled by the regression line with equation $y = ax + b$.

(b) Write down the value of a and of b. *[2]*

Rita scored a total of 26 in her IB Diploma.

(c) Use your regression line to estimate Rita's university entrance mark. *[2]*

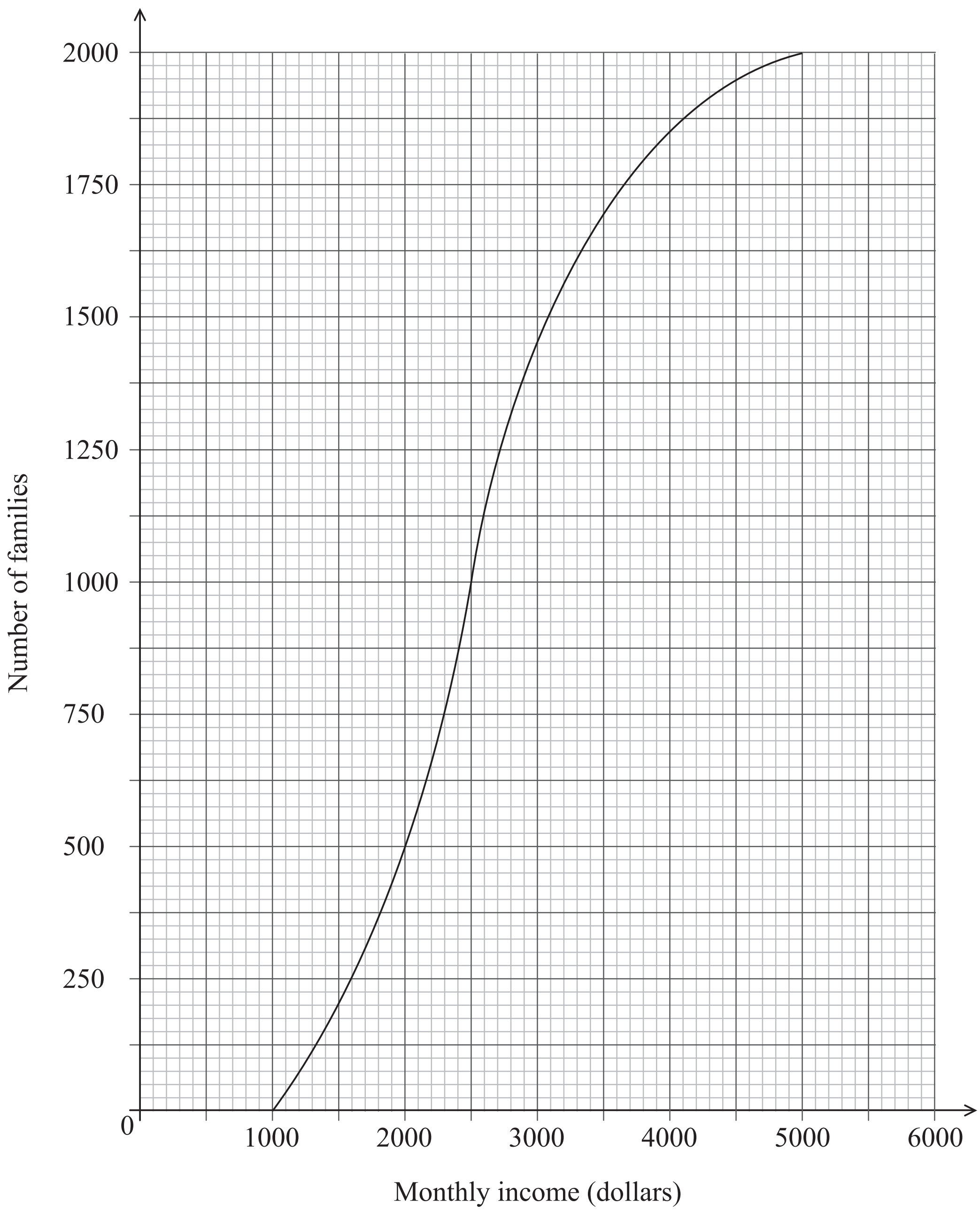
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The following cumulative frequency graph shows the monthly income, I dollars, of 2000 families.



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*Do **NOT** write solutions on this page.*

(Question 8 continued)

(a) Find the median monthly income. *[2]*

(b) (i) Write down the number of families who have a monthly income of 2000 dollars or less.

(ii) Find the number of families who have a monthly income of more than 4000 dollars. *[4]*

The 2000 families live in two different types of housing. The following table gives information about the number of families living in each type of housing and their monthly income I.

	$1000 < I \le 2000$	$2000 < I \le 4000$	$4000 < I \le 5000$
Apartment	436	765	28
Villa	64	p	122

(c) Find the value of p. *[2]*

(d) A family is chosen at random.

(i) Find the probability that this family lives in an apartment.

(ii) Find the probability that this family lives in an apartment, given that its monthly income is greater than 4000 dollars. *[4]*

(e) Estimate the mean monthly income for families living in a villa. *[3]*

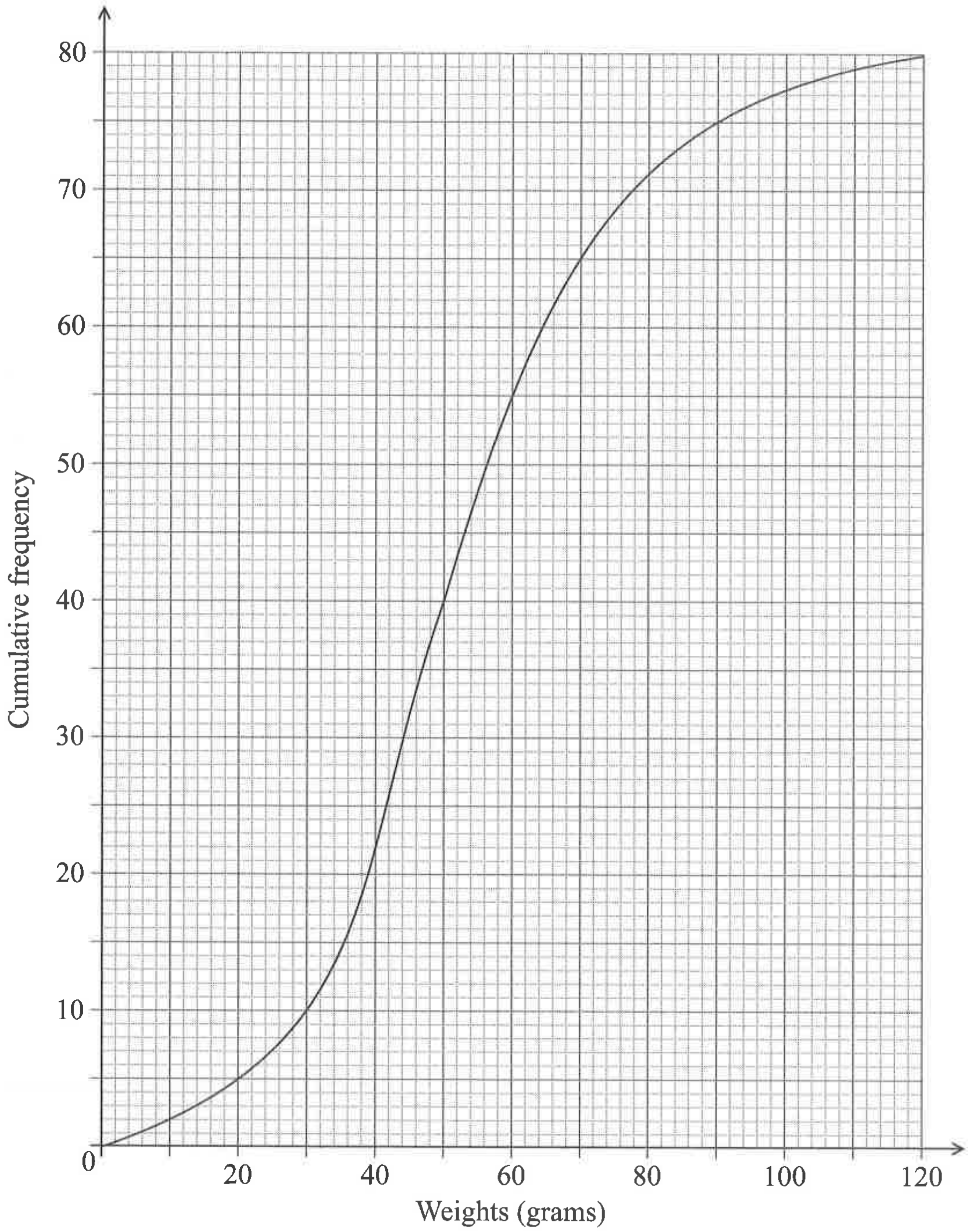
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 16]*

The weights in grams of 80 rats are shown in the following cumulative frequency diagram.



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*Do **NOT** write solutions on this page.*

(Question 8 continued)

(a) (i) Write down the median weight of the rats.

(ii) Find the percentage of rats that weigh 70 grams or less. *[4]*

The same data is presented in the following table.

Weights w grams	$0 \le w \le 30$	$30 < w \le 60$	$60 < w \le 90$	$90 < w \le 120$
Frequency	p	45	q	5

(b) (i) Write down the value of p.

(ii) Find the value of q. *[4]*

(c) Use the values from the table to estimate the mean and standard deviation of the weights. *[3]*

Assume that the weights of these rats are normally distributed with the mean and standard deviation estimated in part (c).

(d) Find the percentage of rats that weigh 70 grams or less. *[2]*

(e) A sample of five rats is chosen at random. Find the probability that at most three rats weigh 70 grams or less. *[3]*

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

The following table shows the average number of hours per day spent watching television by seven mothers and each mother's youngest child.

Hours per day that a mother watches television (x)	2.5	3.0	3.2	3.3	4.0	4.5	5.8
Hours per day that her child watches television (y)	1.8	2.2	2.6	2.5	3.0	3.2	3.5

The relationship can be modelled by the regression line with equation $y = ax + b$.

(a) (i) Find the correlation coefficient.

(ii) Write down the value of a and of b. [4]

Elizabeth watches television for an average of 3.7 hours per day.

(b) Use your regression line to predict the average number of hours of television watched per day by Elizabeth's youngest child. Give your answer correct to one decimal place. [3]

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9. [Maximum mark: 16]

A company makes containers of yogurt. The volume of yogurt in the containers is normally distributed with a mean of $260\,\mathrm{ml}$ and standard deviation of $6\,\mathrm{ml}$.

A container which contains less than $250\,\mathrm{ml}$ of yogurt is **underfilled**.

(a) A container is chosen at random. Find the probability that it is underfilled. [2]

The company decides that the probability of a container being underfilled should be reduced to 0.02. It decreases the standard deviation to σ and leaves the mean unchanged.

(b) Find σ. [4]

The company changes to the new standard deviation, σ, and leaves the mean unchanged. A container is chosen at random for inspection. It passes inspection if its volume of yogurt is between 250 and $271\,\mathrm{ml}$.

(c) (i) Find the probability that it passes inspection.

(ii) Given that the container is **not** underfilled, find the probability that it passes inspection. [6]

(d) A sample of 50 containers is chosen at random. Find the probability that 48 or more of the containers pass inspection. [4]

3. [Maximum mark: 6]

The following table shows the sales, y millions of dollars, of a company, x years after it opened.

Time after opening (x years)	2	4	6	8	10
Sales (y millions of dollars)	12	20	30	36	52

The relationship between the variables is modelled by the regression line with equation $y = ax + b$.

(a) (i) Find the value of a and of b.

(ii) Write down the value of r. [4]

(b) Hence estimate the sales in millions of dollars after seven years. [2]

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6. [Maximum mark: 7]

Ramiro walks to work each morning. During the first minute he walks 80 metres. In each subsequent minute he walks $90\,\%$ of the distance walked during the previous minute. The distance between his house and work is 660 metres. Ramiro leaves his house at $08{:}00$ and has to be at work by $08{:}15$.

Explain why he will not be at work on time.

Do **not** write solutions on this page.

9. [Maximum mark: 16]

A machine manufactures a large number of nails. The length, L mm, of a nail is normally distributed, where $L \sim \mathrm{N}\left(50, \sigma^2\right)$.

(a) Find $\mathrm{P}(50-\sigma < L < 50+2\sigma)$. [3]

(b) The probability that the length of a nail is less than $53.92\,\mathrm{mm}$ is 0.975.
Show that $\sigma = 2.00$ (correct to three significant figures). [2]

All nails with length at least t mm are classified as large nails.

(c) A nail is chosen at random. The probability that it is a large nail is 0.75.
Find the value of t. [3]

(d) (i) A nail is chosen at random from the large nails. Find the probability that the length of this nail is less than $50.1\,\mathrm{mm}$.

(ii) Ten nails are chosen at random from the large nails. Find the probability that at least two nails have a length that is less than $50.1\,\mathrm{mm}$. [8]

2. *[Maximum mark: 6]*

Consider the following cumulative frequency table.

x	**Frequency**	**Cumulative frequency**
5	2	2
15	10	12
25	14	26
35	p	35
45	6	41

(a) Find the value of p. *[2 marks]*

(b) Find

(i) the mean;

(ii) the variance. *[4 marks]*

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7. *[Maximum mark: 7]*

A random variable X is normally distributed with $\mu = 150$ and $\sigma = 10$.

Find the interquartile range of X.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

The following table gives the examination grades for 120 students.

Grade	Number of students	Cumulative frequency
1	9	9
2	25	34
3	35	p
4	q	109
5	11	120

(a) Find the value of

(i) p;

(ii) q. *[4 marks]*

(b) Find the mean grade. *[2 marks]*

(c) Write down the standard deviation. *[1 mark]*

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3. *[Maximum mark: 5]*

Jan plays a game where she tosses two fair six-sided dice. She wins a prize if the sum of her scores is 5.

(a) Jan tosses the two dice once. Find the probability that she wins a prize. *[3 marks]*

(b) Jan tosses the two dice 8 times. Find the probability that she wins 3 prizes. *[2 marks]*

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10. *[Maximum mark: 15]*

The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a) Find the probability that a player weighs more than 82 kg. *[2 marks]*

(b) (i) Write down the standardized value, z, for 68 kg.

(ii) Hence, find the standard deviation of weights. *[4 marks]*

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

(c) (i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament. *[5 marks]*

Of the players in the league, 25 % are women. Of the women, 70 % take part in the tournament.

(d) Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman. *[4 marks]*