

**ADDITIONAL
MATHEMATICS
MODULE 8**

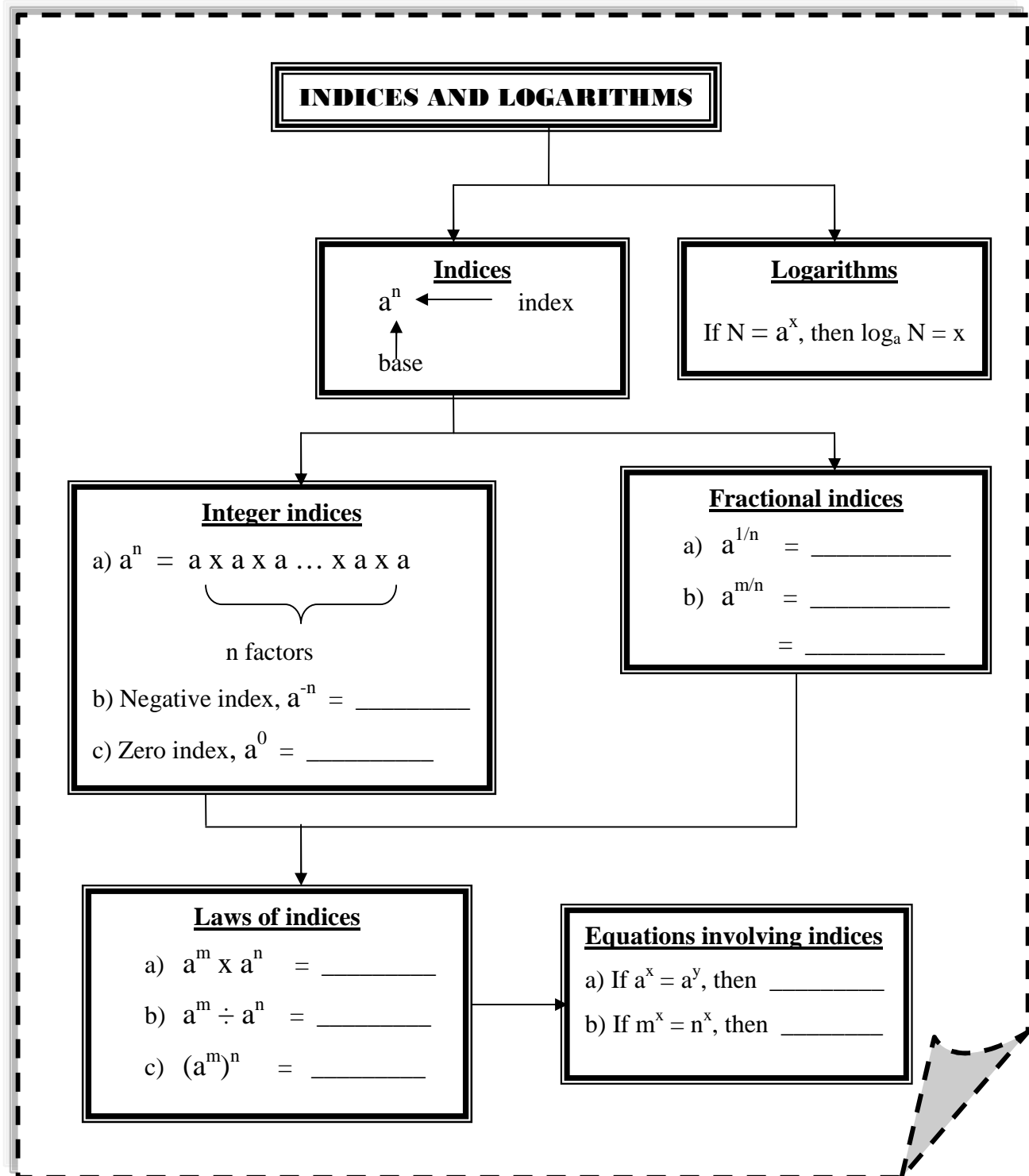
INDICES AND LOGARITHMS

CHAPTER 5 : INDICES AND LOGARITHMS

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CHAPTER 5 : INDICES AND LOGARITHMS

5.1 CONCEPTUAL MAP



5.2 INDICES AND LAWS OF INDICES

Examples	Hit the Buttons
Find the values of the following. 1. $6^3 = 216$	<div>6</div> <div>^</div> <div>3</div> <div>=</div>
2. $4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	<div>1</div> <div>a b / c</div> <div>4</div> <div>x²</div> <div>=</div>
3. $8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2$ $= 4$	<div>(</div> <div>3</div> <div>shift</div> <div>^</div> <div>8</div> <div>)</div> <div>x²</div> <div>=</div>

EXERCISE 5.2

a) Evaluate the following

1.	0.4^4 <u>Solution</u>	2.	$\left(\frac{-27}{8}\right)^{\frac{2}{3}}$ <u>Solution</u>
3.	$\left(\frac{1}{8}\right)^{-3}$ <u>Solution</u>	4.	$\frac{1}{4^{-\frac{1}{2}}}$ <u>Solution</u>

5.3 SIMPLIFY ALGEBRAIC EXPRESSIONS

Examples	Solution
<p>Simplify</p> <p>1. $2a^2 \times 3a^3$ $= (2 \times 3) \times (a^2 \times a^3)$ $= 6 \times a^{2+3}$</p>	<p>Sign : $(+) \times (+) = (+)$ Number : $2 \times 3 = 6$ Unknown/ base : $a^2 \times a^3 = a^5$</p>
<p>2. $3^{3n-2} \div 3^{n-3}$ $= 3^{3n-2-(n-3)}$ ← $= 3^{3n-2-n+3}$ $= 3^{2n+1}$</p>	<p>$a^m \div a^n = a^{m-n}$ Use the laws of indices to simplify the division</p>

EXERCISE 5.3

Simplify the following

<p>1. $(3^4)^{2n} \times 3^n$</p> <p><u>Solution</u></p>	<p>2. $3^{1-2n} \times 3^4 \times 3^{3n+2}$</p> <p><u>Solution</u></p>
<p>3. $8^{n+1} \div 4^{2n-1}$</p> <p><u>Solution</u></p>	<p>4. $a^2 \times 2a^3 \div (4a)^2$</p> <p><u>Solution</u></p>

5.	$\frac{p^{n+2} \times p^{3-2n}}{p^{5+3n}}$ <p><u>Solution</u></p>	6.	$\frac{18a^7b^2}{6a^2b^2}$ <p><u>Solution</u></p>
7.	$3^x - 3^{x-1}$ <p><u>Solution</u></p>	8.	$2^{n+2} - 2^n + 12(2^{n-1})$ <p><u>Solution</u></p>

5.4 EQUATIONS INVOLVING INDICES

Examples	Steps / Method
<p>Solve each of the following equations</p> <p>1. $3^{3x} = 81$ $3^{3x} = 3^4$ ← $3x = 4$ ← $x = \frac{4}{3}$</p>	<p>Express in the _____ and compare the indices</p> <p>_____ the indices</p>
<p>2. $2^x \cdot 4^{x+1} = 64$ $2^x \cdot 2^{2(x+1)} = 2^6$ ← $x + 2x + 2 = 6$ ← $3x = 4$ $x = \frac{4}{3}$</p>	<p>Use _____ as the same base throughout</p> <p>_____ the indices using the laws of indices</p>

<p>3. $8^{x^2} - 16^{x+1} = 0$</p> <p>$(2^3)^{x^2} - (2^4)^{x+1} = 0$ ←</p> <p>$(2^3)^{x^2} = (2^4)^{x+1}$</p> <p>$2^{3x^2} = 2^{4x+4}$</p> <p>$3x^2 = 4x + 4$ ←</p> <p>$3x^2 - 4x - 4 = 0$ ←</p> <p>$(3x + 2)(x - 2) = 0$ ←</p> <p>$x = \frac{-2}{3}, \quad x = 2$</p>	<p>Use 2 as the same base throughout</p> <p>_____ of a quadratic equation</p> <p>_____</p>
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EXERCISE 5.4

1.	<p>Solve $8x^3 = 27$</p> <p><u>Solution</u></p>	2.	<p>Solve $8^x = \frac{1}{4}$</p> <p><u>Solution</u></p>
3.	<p>Solve $2^x \cdot 3^x = 36$</p> <p><u>Solution</u></p>	4.	<p>Solve $4x^{\frac{1}{2}} = 10$</p> <p><u>Solution</u></p>

5.	Solve $3(9^{x+4}) = 27^{x+1}$ <u>Solution</u>	6.	Solve $5^{3x} \div 25^{x+1} = \frac{1}{125}$ <u>Solution</u>
7.	Solve $\sqrt{27^{x+4}} = \frac{1}{3^{x+3}9^x}$ <u>Solution</u>	8.	Solve $125^{x-1} = \frac{25}{\sqrt{5^{x+3}}}$ <u>Solution</u>

5.5 PAST YEAR QUESTIONS

1.	<p>Solve the equation $2^{x+4} - 2^{x+3} = 1$ (SPM 05)</p> <p><u>Solution</u></p>
2.	<p>Solve the equation $32^{4x} = 4^{8x+6}$ (SPM 04)</p> <p><u>Solution</u></p>

5.6 ASSESSMENT

Answer All the questions

Time : 30 minutes

1.	<p>Solve the equation $16^{3x} = 8^{6x+1}$</p> <p><u>Solution</u></p>
2.	<p>Solve the equation $3^{2x} = \frac{1}{243}$</p> <p><u>Solution</u></p>
3.	<p>Solve the equation $8^x \cdot 2^x = 32$</p> <p><u>Solution</u></p>

4.	<p>Solve the equation $3^{x^2} - 9^{6-2x} = 0$</p> <p><u>Solution</u></p>
5.	<p>Solve the equation $\sqrt{8^{x-2}} = \frac{1}{4^{1-x} 2^x}$</p> <p><u>Solution</u></p>
6.	<p>Solve the equation $5^{2x} - 5^{2x-1} = 100$</p> <p><u>Solution</u></p>

ANSWERS

EXERCISE 1.1

1. 0.0256
2. 2.25
3. 512
4. 2

EXERCISE 2.1

- | | |
|-----------------------|----------------------------------|
| 1. 3^{9n} | 5. 2^{n+1} |
| 2. 3^{n+7} | 6. $2a^5$ |
| 3. $\frac{1}{p^{4n}}$ | 7. $3^x\left(\frac{2}{3}\right)$ |
| 4. $\frac{1}{8}a^3$ | 8. $2^n(9)$ |

EQUATION INVOLVING INDICES

1. same base, compare
2. 2, Add
3. General form, solving by factorization

EXERCISE 3.1

- | | |
|-----------------------|-------------|
| 1. $x = \frac{3}{2}$ | 5. $x = 6$ |
| 2. $x = \frac{-2}{3}$ | 6. $x = -1$ |
| 3. $x = 2$ | 7. $x = -2$ |
| 4. $x = 6.25$ | 8. $x = 1$ |

PAST YEAR QUESTIONS

1. $x = -1$
2. $x = 3$

ASSESSMENT

- | | |
|-----------------------|----------------------|
| 1. $x = \frac{-5}{2}$ | 4. $x = 6, -2$ |
| 2. $x = \frac{-1}{2}$ | 5. $x = 2$ |
| 3. $x = \frac{5}{4}$ | 6. $x = \frac{3}{2}$ |

**ADDITIONAL
MATHEMATICS
MODULE 9**

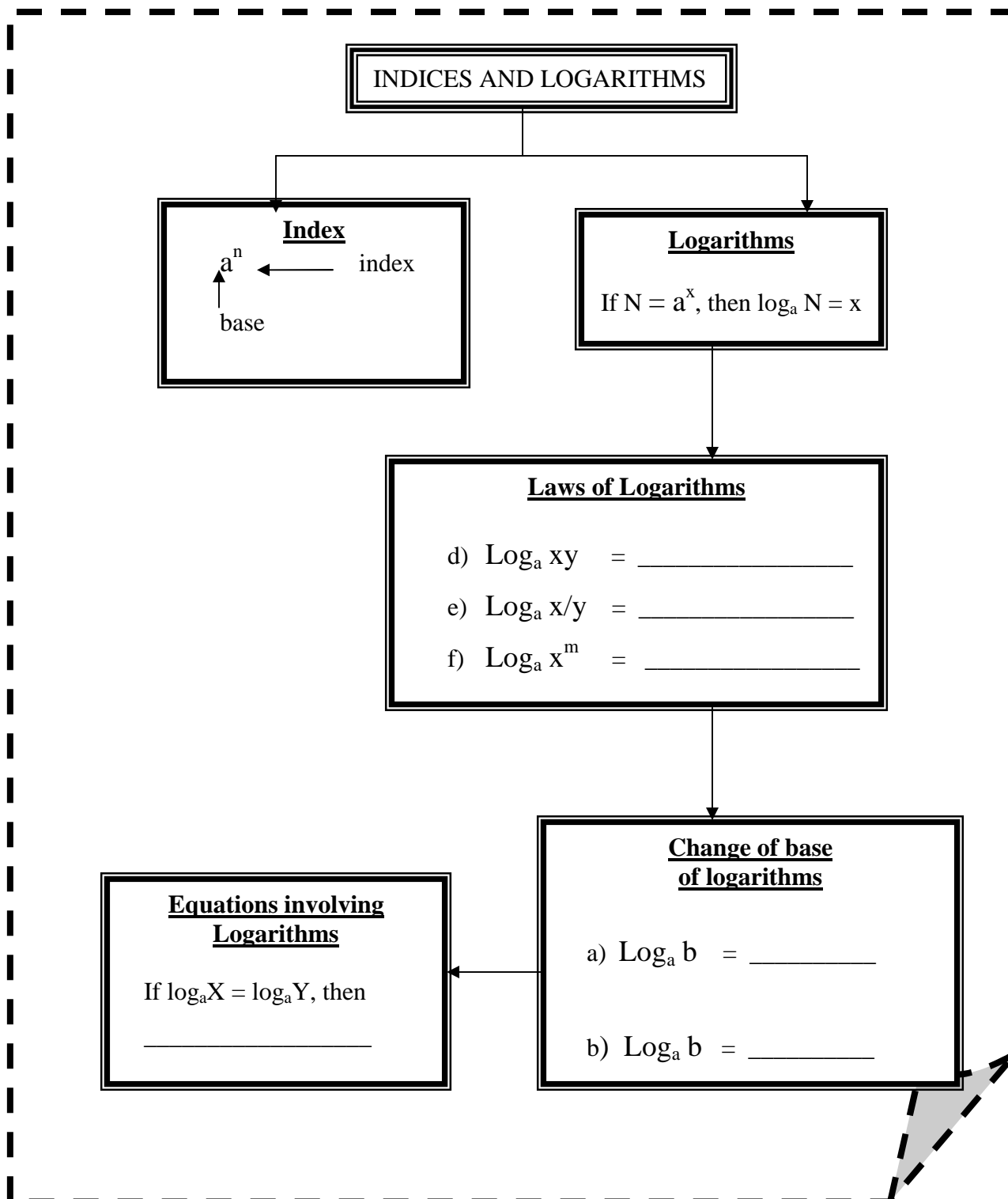
INDICES AND LOGARITHMS

CHAPTER 5 : INDICES AND LOGARITHMS

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CHAPTER 5 : INDICES AND LOGARITHMS

5.1 CONCEPTUAL MAP



5.2 LOGARITHM AND LAWS OF LOGARITHM

Find the value of the following

Examples	Solution
1. $\log_4 \frac{1}{64}$	Let . $\log_4 \frac{1}{64} = x$ Then $4^x = 4^{-3}$ $x = -3$
2. $\log_5 \sqrt{5}$	Let . $\log_5 \sqrt{5} = x$ Then $5^x = 5^{\frac{1}{2}}$ $x = \frac{1}{2}$
3. $\log_2 x = -5$	Given $\log_2 x = -5$ Then $x = 2^{-5}$ $= \frac{1}{2^5} = \frac{1}{32}$

EXERCISE 5.2

Find the value of each of the following

1. $\log_2 128$	2. $\log_5 \sqrt{125}$
3. $\log_8 \frac{1}{64}$	4. $\log_{10} \left(\frac{2}{5}\right)^3$

5. $\log_{100} 10$	6. $\log_{\sqrt{4}} 64$
7. $\log_9 x = -1$	8. $\log_x \frac{1}{25} = -2$

5.3 FINDING LOGARITHM OF NUMBERS

Examples	Solution
Evaluate the following 1. $\log_3 18 - \log_3 6$ $= \log_3 \frac{18}{6}$ $= \log_3 3$ $= 1$	2. $\log_4 2 + \log_4 8$ $= \log_4 (2 \times 8)$ $= \log_4 4^2$ $= 2 \log_4 4$ $= 2(1)$ $= 2$
3. Given that $\log_a 2 = 0.123$ and $\log_a 3 = 0.256$, calculate the following a) $\log_a 12 = \log_a (2 \times 2 \times 3)$ $= \log_a 2^2 + \log_a 3$ $= 2\log_a 2 + \log_a 3$ $= 2(0.123) + 0.256$ $= 0.502$	

$$\begin{aligned}
 \text{b) } \log_a \sqrt{6} &= \log_a (6)^{\frac{1}{2}} \\
 &= \frac{1}{2} \log_a (2 \times 3) \\
 &= \frac{1}{2} [\log_a 2 + \log_a 3] \\
 &= \frac{1}{2} [0.123 + 0.256] \\
 &= 0.1895
 \end{aligned}$$

4. Given that $\log_x 2 = a$ and $\log_x 3 = b$, write $\log_x \frac{4}{9x^2}$ in term of a and b.

$$\begin{aligned}
 \log_x \frac{4}{9x^2} &= \log_x 4 - \log_x 9x^2 \\
 &= \log_x 2^2 - [\log_x 9 + \log_x x^2] \\
 &= 2 \log_x 2 - \log_x 3^2 - 2 \log_x x \\
 &= 2a - 2 \log_x 3 - 2(1) \\
 &= 2a - 2b - 2
 \end{aligned}$$

EXERCISE 5.3

Simplify each of the following logarithms expression

1. $\log_4 2 + \log_4 32$	2. $\log_2 48 + \log_2 3 - \log_2 9$
3. $2 \log_5 10 + 3 \log_5 2 - \log_5 32$	4. $4 \log_x 2 - 3 \log_x 4 + \log_x 6$

<p>5. Express $\log_a \frac{b^2}{c}$ in terms of $\log_a b$ and $\log_a c$</p> <p><u>Solution</u></p>	<p>6. Given that $\log_2 6 = 2.59$ and $\log_2 5 = 2.32$ find the value of $\log_{10} 30$</p> <p><u>Solution</u></p>
<p>7. Given $\log_3 2 = 0.631$ and $\log_3 5 = 1.465$, find the values of</p> <p>a) $\log_3 20$</p> <p>b) $\log_3 \frac{4}{5}$</p> <p>c) $\log_3 7.5$</p> <p><u>Solution</u></p>	

5.4 CHANGING THE BASE OF LOGARITHM

Examples	
Find the value of the logarithm	
1.	$\log_3 5 = \frac{\log_{10} 5}{\log_{10} 3}$ $= \frac{0.699}{0.4771}$ $= 1.465$

EXERCISE 5.4.1

Find the value of the logarithm by changing the base of the logarithm

1.	$\text{Log}_4 3$	2.	$\text{Log}_3 10$
3.	$\text{Log}_2 0.1$	4.	$\text{Log}_5 0.048$
5.	$\text{Log}_4 \frac{1}{5}$	6.	$\text{Log}_2 \left(\frac{3}{4} \right)$

Examples	
Find the value of the logarithm	
1. $\log_4 5$	$= \frac{\log_2 5}{\log_2 4}$ $= \frac{2.322}{2\log_2 2}$ $= \frac{2.322}{2}$ $= 1.161$

EXERCISE 5.4.2

Given that $\log_2 5 = 2.322$ and $\log_2 3 = 1.585$, find the value of the logarithm

1.	$\text{Log}_3 5$	2.	$\text{Log}_8 15$
3.	$\text{Log}_9 125$	4.	$\text{Log}_8 25$

Examples
Find the value of the logarithm
$1. \log_{10} p^2 = \frac{\log_4 p^2}{\log_4 16}$ $= \frac{2\log_4 p}{2\log_4 4}$ $= \frac{2m}{2}$ $= m$

EXERCISE 5.4.3

Given that $\log_4 p = m$. Express the following in terms of m

1.	$\text{Log}_p 4$	2.	$\text{Log}_8 p$
3.	$\text{Log}_{4p} 64$	4.	$\text{Log}_p \frac{1}{256}$

5.5 EQUATION INVOLVING LOGARITHMS

<p>1. Solve the equation : $\text{Log}_{10}(x - 5) = \log_{10}(x - 1) + 2$</p> <p><u>Solution</u> $\text{Log}_{10}(x - 5) = \log_{10}(x - 1) + 2$ $\text{Log}_{10}(x - 5) - \log_{10}(x - 1) = -2$ $\text{Log}_{10} \frac{x-5}{x-1} = 2$ $\frac{x-5}{x-1} = 10^2$ $x - 5 = 100(x - 1)$ $x - 5 = 100x - 100$ $99x = 95$ $x = \frac{95}{99}$</p>	<p>2. Solve the equation: $3^x = 5^{2x+1}$</p> <p><u>Solution</u> $x \log_{10} 3 = (2x + 1) \log_{10} 5$ $x \log_{10} 3 = 2x \log_{10} 5 + \log_{10} 5$ $x \log_{10} 3 - 2x \log_{10} 5 = \log_{10} 5$ $x(\log_{10} 3 - 2 \log_{10} 5) = \log_{10} 5$ $x = \frac{\log_{10} 5}{\log_{10} 3 - 2 \log_{10} 5}$ $x = -0.7591$</p>
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EXERCISE 5.5

Solve each of the equation

1.	$5^x = 27$	2.	$5^x = 0.8$
3.	$3^{x+1} = 18$	4.	$5^{x-2} = 4^{x+1}$

5.	$2^x 3^x = 5^{x+1}$	6.	$3^x \cdot 4^{x+1} = 6$
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5.6 SPM QUESTIONS

SPM 2005(Paper 1)

1. Solve the equation $\log_3 4x - \log_3 (2x - 1) = 1$

[3 marks]

SPM 2005(Paper 1)

2. Given that $\log_m = p$ and $\log_m 3 = r$, express $\log_m \left(\frac{27m}{4} \right)$ in term of p and r

[4 marks]

SPM 2004(Paper I)

3. Given that $\log_5 2 = m$ and $\log_5 = p$, express $\log_5 4.9$ in term of m and p

[4 marks]

SPM 2003(Paper I)

4. Given that $\log_2 T - \log_4 V = 3$, express T in term of V . Express T in term of V

[4 marks]

SPM 2003(Paper I)

5. Solve the equation $4^{2x-1} = 7^x$

[4 marks]

5.7 ASSESSMENT

Answer All the questions

Time : 30 minutes

1.	Solve the equation $\log_5(8x - 4) = 2\log_5 3 + \log_5 4$
2.	Given $x = \log_3 k = m$ and $\log_2 k = n$. Find the value of $\log_k 24$ in term of m and n
3.	Given that $\log_3 T + \log_9 V = 4$, express T in terms of V
4.	Solve the equation $5^{2x+1} = 9^x$

ANSWERS

EXERCISE 5.2

- | | | |
|------------------|------------------|------------------|
| 1. 7 | 4. -1.1938 | 7. $\frac{1}{9}$ |
| 2. $\frac{3}{2}$ | 5. $\frac{1}{2}$ | 8. 5 |
| 3. -2 | 6. 6 | |

EXERCISE 5.3

- | | | |
|-------------------------|----------------------------|----------|
| 1. 3 | 5. $2 \log_a b - \log_a c$ | |
| 2. 4 | 6. 4.91 | |
| 3. 2 | 7. a) 2.727 | c) 1.834 |
| 4. $\log_x \frac{3}{2}$ | b) -0.203 | |

EXERCISE 5.4.1

- | | |
|-----------|-----------|
| 1. 0.7924 | 3. -3.322 |
| 2. 2.096 | 4. -1.887 |

EXERCISE 5.4.2

- | | |
|----------|-----------|
| 1. 1.465 | 3. 2.1975 |
| 2. 1.302 | 4. -1.548 |

EXERCISE 5.4.3

- | | |
|-------------------|--------------------|
| 1. $\frac{1}{m}$ | 3. $\frac{3}{1+m}$ |
| 2. $\frac{2}{3}m$ | 4. $\frac{-4}{m}$ |

EXERCISE 5.5

- | | |
|------------|-----------|
| 1. 2.048 | 4. 20.64 |
| 2. -0.1386 | 5. 8.827 |
| 3. 1.630 | 6. 0.1632 |

SPM QUESTIONS

- | | |
|----------------------|--------------------|
| 1. $x = \frac{3}{2}$ | 4. $T = 8\sqrt{v}$ |
| 2. $3r - 2p + 1$ | 5. 1.677 |
| 3. $2p - m - 1$ | |

ASSESSMENT

- | | |
|--------------------------------|------------------------------|
| 1. 5 | 3. $T = \frac{81}{\sqrt{v}}$ |
| 2. $\frac{1}{m} + \frac{3}{n}$ | 4. -1.575 |