Mathematics Standard Level

for the IB Diploma

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Geometry

(Topic 3: Circular functions & trigonometry & Topic 4: Vectors)

Calculus

(Topic 6: Calculus)

Statistics and probability

(Topic 5: Statistics & probability)

Introduction

Structure of the book

The book is split roughly into four blocks. Chapters 1 to 7 cover algebra and functions, chapters 8 to 11 cover geometry, chapters 12 to 15 cover calculus, and chapters 16 to 18 cover probability and statistics. Chapter 19 contains questions that mix together different parts of the course – a favourite trick in International Baccalaureate® (IB) examinations.

You do not have to work through the book in the order presented, but given how much the IB likes to mix up topics, you will find that some questions refer to material in previous chapters. In such cases, a 'rewind panel' will tell you that the material has been covered earlier, so that you can decide whether to remind yourself or move on.

In the book we have tried to include only material that will be examinable. There are many proofs and ideas that are useful and interesting but which are not included in the main text; these can be found on the CD-ROM should you wish to explore them.

Each chapter starts with a list of learning objectives which give you an idea about what the chapter contains. There is also an introductory problem that illustrates what you will be able to do after you have completed the chapter. Some introductory problems relate to 'real life' situations, while others are purely mathematical. You should not expect to be able to solve the problem at the start, but you may want to think about possible strategies and what sort of new facts and methods would help you. The solution to the introductory problem is provided at the end of the chapter, after the summary of the chapter contents.

Key point boxes

The most important ideas and formulae are emphasised in the Key point boxes. They also highlight which formulae are given in the Formula booklet. 

Worked examples

Each worked example is split into two columns. On the right is what you should write down in your solution. Sometimes examples may go into more detail than you strictly need, but they are designed to give you an idea of what is required to score full method marks in examinations. Mathematics, however, is about much more than remembering methods and preparing for examinations. So, on the left of each worked example are notes that describe the thought processes and suggest which approach you could use to tackle the question. We hope that these will

help you learn how to solve problems that differ from the worked examples. It is very deliberate that some of the exercise questions require you to do more than just repeat the methods in the worked examples – mathematics is about thinking!

Signposts

There are several kinds of boxes that appear throughout the book.

Theory of knowledge issues

Every lesson is really a 'theory of knowledge' lesson, but sometimes the connections may not be obvious. Although mathematics is frequently cited as an example of certainty and truth, things are often not so clear-cut. In these boxes we will try to highlight some of the weaknesses and ambiguities in mathematics, as well as showing how mathematics links to other areas of knowledge.

From another perspective

Mathematics is often described as a unified international language, but the International Baccalaureate encourages looking at things in various ways. As well as highlighting some differences between mathematicians from different parts of the world, these boxes also discuss other perspectives on the mathematics we are covering – historical, pragmatic and cultural.

Research explorer

As part of your course, you will be asked to write a report on an area of mathematics beyond the syllabus, related to a topic that changes from year to year. It is sometimes difficult to know which topics are suitable as a basis for such reports, so we have tried to show where a topic can act as a jumping-off point for further work. These can also give you ideas for the extended essay. There is a lot of great mathematics out there!

Exam hints

Although we encourage you to think of mathematics as more than just a subject to be studied in order to pass an examination, it is useful to be aware of some common errors so that you can try to avoid making them yourself. In these boxes we highlight common pitfalls; we also point out where graphical calculators can be used effectively to simplify a question or speed up your work, often referring to the relevant calculator skills sheet on the CD-ROM.

Fast forward / Rewind

Mathematics is all about making links. You might be interested in seeing how something you have just learned will be used elsewhere in the course, or you may need to go back and remind yourself of a previous topic. These boxes indicate connections with other sections of the book to help you find your way around.

How to use the questions

The colour coding

The questions are colour-coded to distinguish between different levels.

Black questions are drill questions. They are meant to help you practise the methods described in the book, but they are usually not structured like typical questions that appear in the examination. This does not mean they are easy – in fact, some of them are quite tough – but they are generally similar in style to the worked examples.

Each differently numbered drill question tests a different skill. Lettered subparts (a), (b), (c), … of a question are of increasing difficulty. Within each lettered part there may be multiple roman-numeral parts (i), (ii), (iii), …, which are all of similar difficulty. Unless you want to get lots of practice, we recommend that you do only one roman-numeral part and then check your answer. If you have made a mistake, you may want to think about what went wrong and then attempt another of the roman-numeral parts.

Green questions are examination-style questions which should be accessible to students on the way to achieving a grade 3 or 4.

Blue questions are harder examination-style questions. If you are aiming for a grade 5 or 6, you should be able to make significant progress through most of these.

Red questions are at the very top end of difficulty among examination-style questions. If you can do these, then you are likely to be on course for a grade 7.

Gold questions are those that are *not* typically set in the examination but which are designed to provoke thinking and discussion, in order to help you gain a better understanding of a particular concept.

At the end of each chapter you will see longer questions typical of the second section of the IB examination. The parts (a), (b), (c), … of these follow the same colour-coding scheme.

Of course, these are just **guidelines**. If you are aiming for a grade 6, do not be surprised if occasionally you find a green question you cannot do; people are rarely equally good at all areas of the syllabus. Similarly, even if you are able to do all the red questions, that does not guarantee you will get a grade 7 – after all, in the examination you will have to deal with time pressure and examination stress! It is also worth remembering that these questions are graded according to our experience of the final examination. When you first start the course, you may well find the questions harder than you would do by the end of the course, so try not to get discouraged.

Calculator versus non-calculator questions

In the final examination there will be one paper in which calculators are not allowed. Some questions require a calculator, but most could appear in either the calculator or the non-calculator paper.

Certain types of question are particularly common in the non-calculator paper, and you need to know how to deal with them. They are indicated by the non-calculator symbol.

On the other hand, some questions can be done in a clever way using a calculator, or cannot realistically be done without using a calculator. These are marked with a calculator symbol.

Note, however, that in the final examination you will not get any calculator/non-calculator indications, so you must make sure to learn which types of questions have an easy calculator method. The calculator skills sheets on the CD-ROM can help with this.

With questions that are not labelled with either the calculator or the non-calculator symbol, you could mix up practising with and without a calculator. Be careful not to become too reliant on your calculator – half of the core examination needs to be done without one!

On the CD-ROM

On the CD-ROM there are various materials that you might find useful.

Prior learning

The International Baccalaureate syllabus lists what candidates are expected to know before taking the examination. Not all the topics on the list are explicitly covered in the course, but knowledge of them may be needed to answer examination questions. Don't worry, you do not have to be familiar with all the 'prior learning' topics before starting the course: we have indicated in the rewind panels where a particular concept or skill is required, and on the CD-ROM you can find a self-assessment test for checking your knowledge, as well as some worksheets to help you learn any skills that you might be missing.

Coursebook support

Supporting worksheets include:

- calculator skills sheets that give instructions for making optimal use of some of the recommended graphical calculators
- fill-in proof sheets to allow you to re-create proofs that are not required in the examination
- self-discovery sheets to encourage you to investigate new results for yourself in the examination
- supplementary sheets exploring some applications, international and historical perspectives of the mathematics covered in the syllabus.

e-version

A flat pdf of the whole coursebook (for days when you don't want to carry the paperback!)

We hope that you will find Standard Level Mathematics for the IB diploma an interesting and enriching course. You may also find it quite challenging, but do not get intimidated – frequently, topics start to make sense only after lots of revision and practice. Persevere and you will succeed!

The author team.