IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MATHEMATICAL METHODS
STANDARD LEVEL
PAPER 1

Thursday **3** November 2005 **(afternoon)**

Candidate session number

0	0								

1 hour

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. In triangle PQR, PQ is 10 cm, QR is 8 cm and angle PQR is acute. The area of the triangle is 20 cm^2. Find the size of angle PQR.

Working:

Answer:

2. A class contains 13 girls and 11 boys. The teacher randomly selects four students. Determine the probability that all four students selected are girls.

Working:

Answer:

3.   Given that $\left(3+\sqrt{7}\right)^3 = p + q\sqrt{7}$ where p and q are integers, find

(a) p;

(b) q.

Working:

Answers:

(a) _____

(b) _____

4. The following table shows the mathematics marks scored by students.

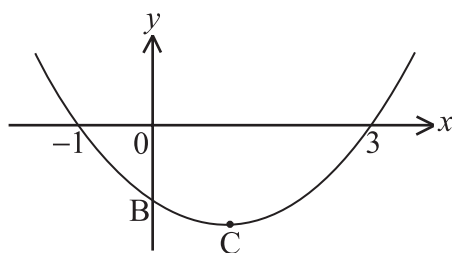| Mark	1	2	3	4	5	6	7
Frequency	0	4	6	k	8	6	6

The mean mark is 4.6.

(a) Find the value of k.

(b) Write down the mode.

Working:

Answers:

(a) _____

(b) _____

0413

5. Part of the graph of $f(x) = (x - p)(x - q)$ is shown below.

The vertex is at C. The graph crosses the y-axis at B.

(a) Write down the value of p and of q.

(b) Find the coordinates of C.

(c) Write down the y-coordinate of B.

Working:

Answers:

(a) _____

(b) _____

(c) _____

6. Consider the functions $f(x) = 2x$ and $g(x) = \dfrac{1}{x-3}$, $x \neq 3$.

  (a) Calculate $(f \circ g)(4)$.

  (b) Find $g^{-1}(x)$.

  (c) Write down the domain of g^{-1}.

Working:

Answers:

(a) _____

(b) _____

(c) _____

7. The first term of an infinite geometric sequence is 18, while the third term is 8. There are two possible sequences. Find the sum of each sequence.

Working:

Answers:

_____

_____

8. Given $\displaystyle\int_3^k \frac{1}{x-2}\,\mathrm{d}x = \ln 7$, find the value of k.

Working:

Answer:

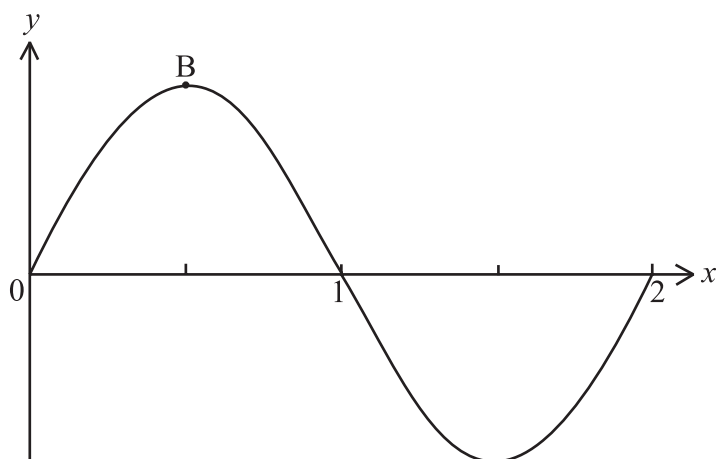...

0713

9. A machine was purchased for $\$10\,000$. Its value V after t years is given by $V = 10\,000\,e^{-0.3t}$. The machine must be replaced at the end of the year in which its value drops below $\$1500$. Determine in how many years the machine will need to be replaced.

Working:

Answer:

10. Let $f(x) = 6\sin \pi x$, and $g(x) = 6e^{-x} - 3$, for $0 \leq x \leq 2$. The graph of f is shown on the diagram below. There is a maximum value at $B(0.5, b)$.

(a)   Write down the value of b.

(b)   On the same diagram, sketch the graph of g.

(c)   Solve $f(x) = g(x)$, for $0.5 \leq x \leq 1.5$.

Working:

Answers:

(a)   _____

(c)   _____

11. Let $f(x) = (2x+7)^3$ and $g(x) = \cos^2(4x)$. Find

(a) $f'(x)$;

(b) $g'(x)$.

Working:

Answers:

(a) _____

(b) _____

12. The events A and B are independent such that $P(B) = 3P(A)$ and $P(A \cup B) = 0.68$. Find $P(B)$.

Working:

Answer:

_____

1013

13. A boat B moves with constant velocity along a straight line. Its velocity vector is given by $v = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.
At time $t = 0$ it is at the point $(-2, 1)$.

(a) Find the magnitude of v.

(b) Find the coordinates of B when $t = 2$.

(c) Write down a vector equation representing the position of B, giving your answer in the form $r = a + tb$.

Working:

Answers:

(a) _____

(b) _____

(c) _____

1113

14. Consider the equation $3\cos 2x + \sin x = 1$.
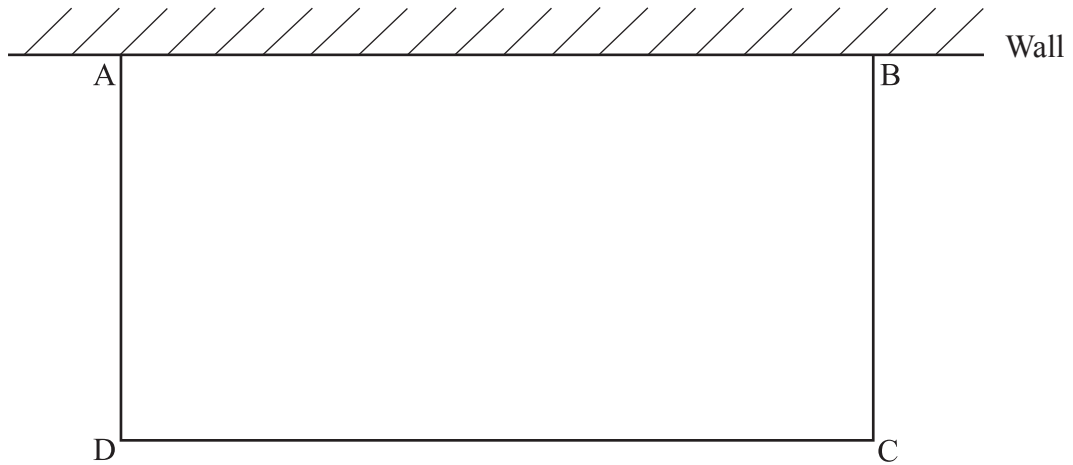
(a) Write this equation in the form $f(x) = 0$, where $f(x) = p\sin^2 x + q\sin x + r$, and $p, q, r \in \mathbb{Z}$.

(b) Factorize $f(x)$.

(c) Write down the number of solutions of $f(x) = 0$, for $0 \le x < 2\pi$.

Working:

Answers:

(a) _____

(b) _____

(c) _____

15. The following diagram shows a rectangular area ABCD enclosed on three sides by 60 m of fencing, and on the fourth by a wall AB.

Find the width of the rectangle that gives its maximum area.

Working:

Answer: