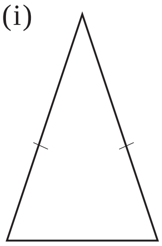
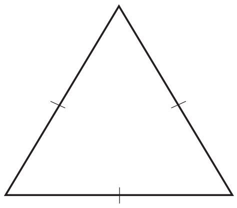
UNIT 15 *Polygons* Activities

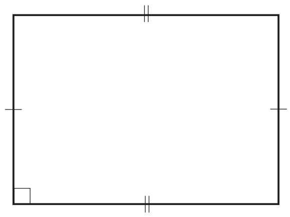
Activities

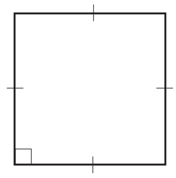
ACTIVITY 15.1 *Rotational and Line Symmetry*

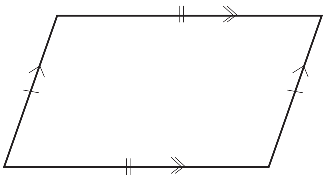
1. For each polygon below,
   (a) use dotted lines to show the lines of symmetry, if any;
   (b) check whether it has rotational symmetry and if so, state its order;
   (c) mark the centre of rotational symmetry with a cross.

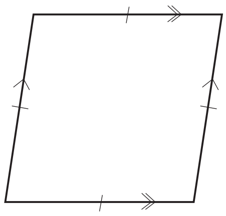
(i)

Order _____

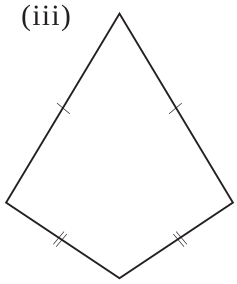
(ii)

Order _____

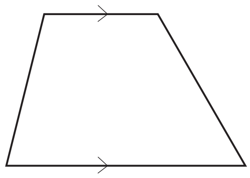
(iii)

Order _____

(iv)

Order _____

(i)

Order _____

(ii)

Order _____

(iii)

Order _____

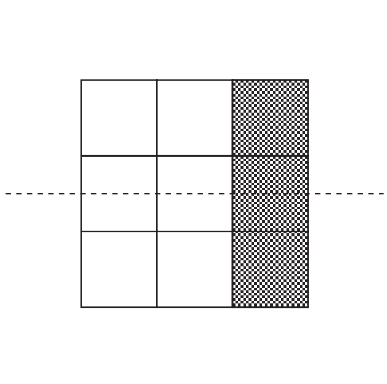
(iv)

Order _____

2. Use the results from Question 1 to complete the following table:

	Name of Polygon	*Number of Lines of Symmetry*	*Order of Rotational Symmetry*
(i)	*Isosceles triangle*		
(ii)	*Equilateral triangle*		
(iii)	*Rectangle*		
(iv)	*Square*		
(v)	*Parallelogram*		
(vi)	*Rhombus*		
(vii)	*Kite*		
(viii)	*Trapezium*		

ACTIVITY 15.2 *Lines of Symmetry*

Each of the 3×3 squares below has 3 shaded squares and one line of symmetry.



1. How many more ways can you find to shade 3 squares in a 3×3 square so that there is only one line of symmetry? Record your patterns.

2. (a) In a 3×3 square, find a pattern of 3 shaded squares which has 2 lines of symmetry.

   (b) Is it the only one? If not, try to find all such patterns.

3. Using a 3×3 square, find all the possible patterns of 4 shaded squares which have:

   (a) one line of symmetry (b) two lines of symmetry,

   (c) three lines of symmetry, (d) four lines of symmetry.

Extension

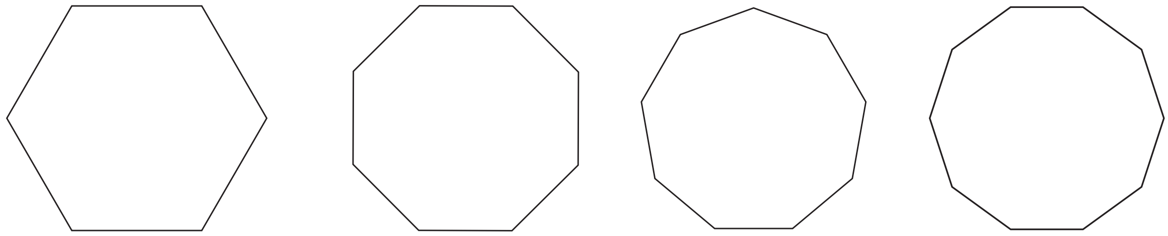
Do a similar study for a 4×4 square with different patterns of:

(a) 3 shaded squares, (b) 4 shaded squares,

(c) 5 shaded squares, (d) 6 shaded squares.

ACTIVITY 15.3 *Symmetry of Regular Polygons*

1. For each of the following regular polygons, draw in the lines of symmetry and locate the centre of rotational symmetry.



2. Use your answers to Question 1 to complete the following table:

Name of Polygon	*Number of Sides*	*Number of Lines of Symmetry*	*Order of Rotational Symmetry*
Hexagon			
Octagon			
Nonagon			
Decagon			

3. Use the completed table in Question 2 to find:

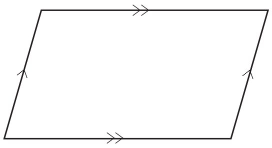
   (a) the number of lines of symmetry, and

   (b) the order of rotational symmetry for:

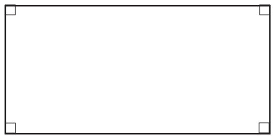
   (i) a regular 10-gon, (ii) a regular 20-gon, (iii) a regular *n*-gon.

ACTIVITY 15.4 *Special Quadrilaterals*

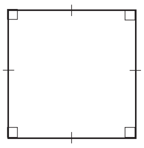
Complete the table below to identify the properties of these special quadrilaterals:



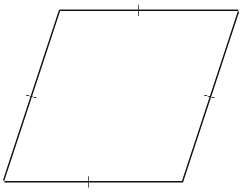
Parallelogram



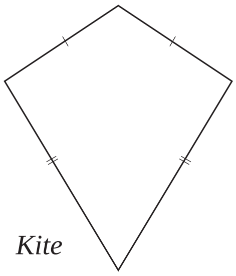
Rectangle



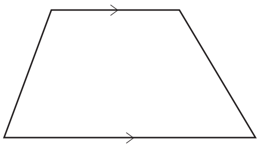
Square



Rhombus



Kite



Trapezium

Property						
All sides equal						
Opposite sides equal						
Opposite sides parallel						
Opposite angles equal						
Diagonals equal						
Diagonals bisect each other						
Diagonals intersect at right angles						
Longer diagonal bisects shorter diagonal						
Two pairs of adjacent sides equal but not all sides equal						
Only one pair of opposite sides parallel						
Only one pair of opposite angles equal						

ACTIVITY 15.5 *Transforming Polygons*

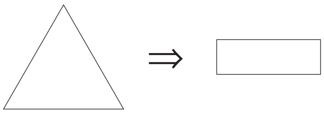
The American mathematician *David Hilbert* (1862-1942) was the first person to prove that

any polygon can be transformed into any other polygon of equal area by cutting it into a finite number of pieces and rearranging.

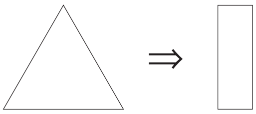
Unfortunately, the proof of this result does not tell you how to do it – just that it can be done!

We will first look at some easy examples and then show how any equilateral triangle can be made into a square of the same area.

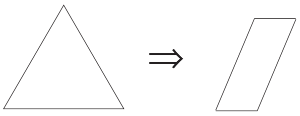
1. How can an equilateral triangle be transformed into a rectangle, which has one of its sides equal to the *height* of the triangle?



2. How can an equilateral triangle be transformed into a rectangle which has one of its sides equal to *the length of side* of the triangle?

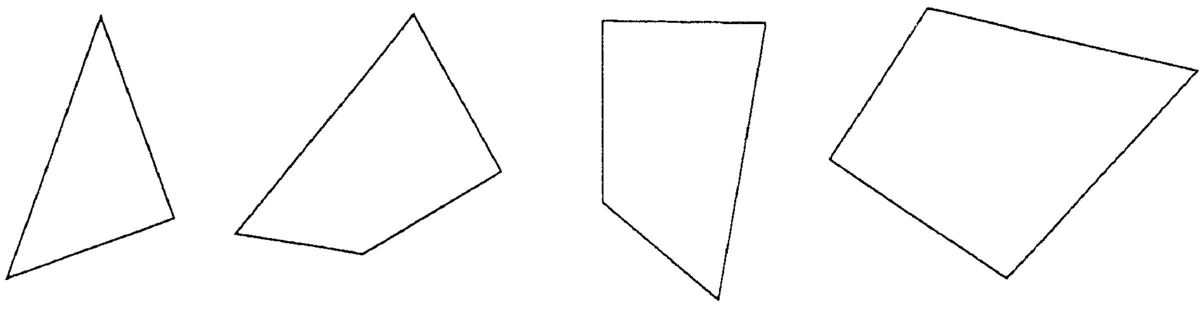


3. How can an equilateral triangle be transformed into a parallelogram with a height equal to *half* the length of a side of the triangle?



These problems are all quite straightforward in their construction. A much more difficult problem is to transform an equilateral triangle into a *square* of the same area.

4. Cut out the pieces shown below and check that you can make both an *equilateral triangle* and a s*quare* from them. The pieces must all be kept the same way up.



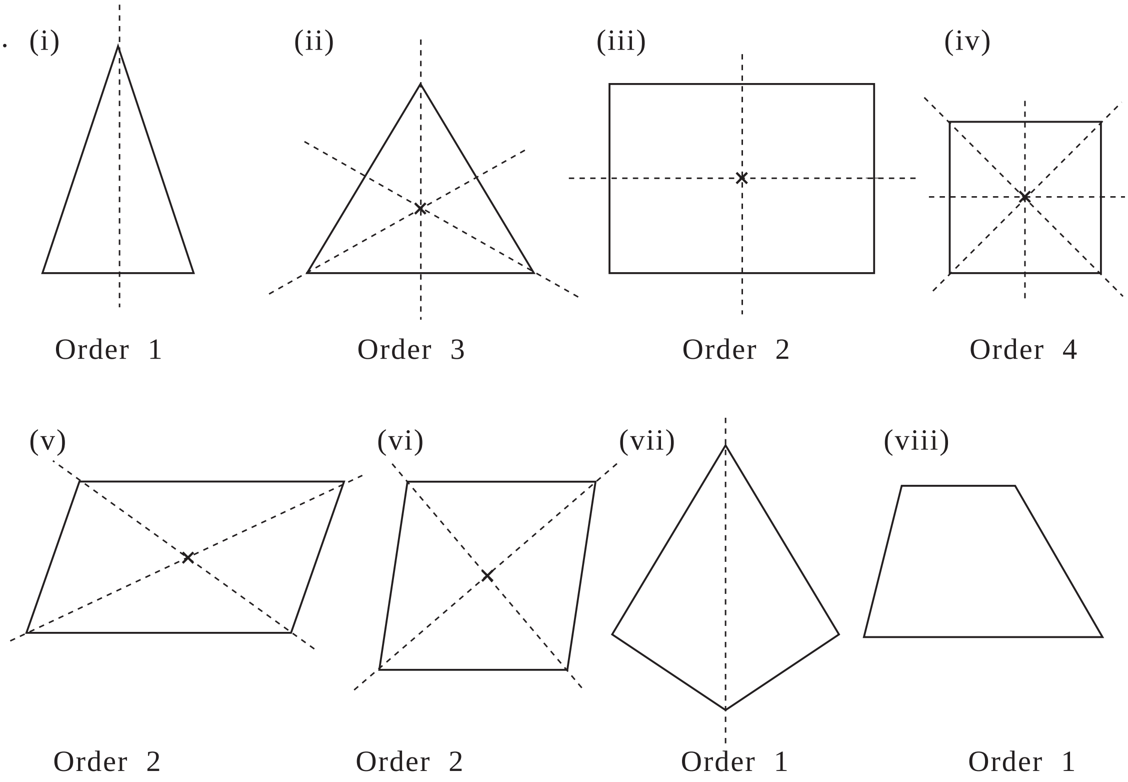
Extension

Starting with an equilateral triangle, and by making suitable cuttings, see what shapes you can make.

ACTIVITY 15.1 *Notes for Solutions*

Notes and solutions given only where appropriate.

15.1 1.

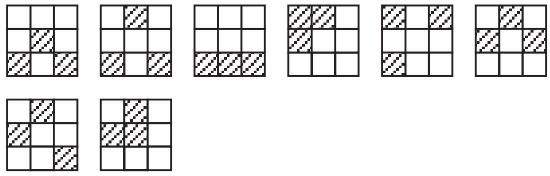


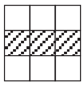
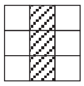
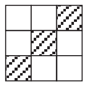
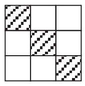
2.

	Name of Polygon	Number of Lines of Symmetry	Order of Rotational Symmetry
(i)	*Isosceles triangle*	1	1
(ii)	*Equilateral triangle*	3	3
(iii)	*Rectangle*	2	2
(iv)	*Square*	4	4
(v)	*Parallelogram*	0	2
(vi)	*Rhombus*	2	2
(vii)	*Kite*	1	1
(viii)	*Trapezium*	0	1

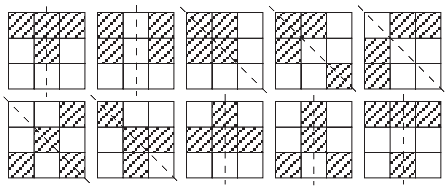
ACTIVITIES 15.2 - 15.4 *Notes for Solutions*

15.2 1. 32 Each of these patterns can be be rotated to give 4 different squares with only one line of symmetry.



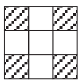
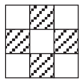
2. 4 possibilities:  or  ,  or 

3. (a) 10 basic designs:



(b) none

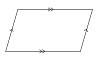
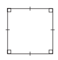
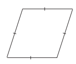
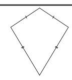
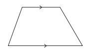
(c) none

(d)  or 

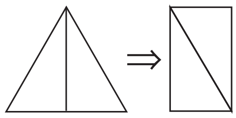
15.3

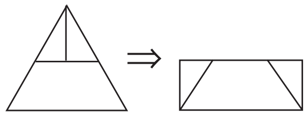
Name of Polygon	*Number of Sides*	*Number of Lines of Symmetry*	*Order of Rotational Symmetry*
Hexagon	6	6	6
Octagon	8	8	8
Nonagon	9	9	9
Decagon	10	10	10

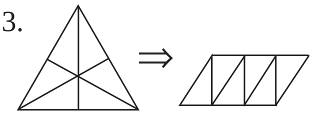
15.4

Property						
All sides equal	✗	✗	✓	✓	✗	✗
Opposite sides equal	✓	✓	✓	✓	✗	✗
Opposite sides parallel	✓	✓	✓	✓	✗	✗
Opposite angles equal	✓	✓	✓	✓	✗	✗
Diagonals equal	✗	✓	✓	✗	✗	✗
Diagonals bisect each other	✓	✓	✓	✓	✗	✗
Diagonals intersect at right angles	✗	✗	✓	✓	✓	✗
Longer diagonal bisects shorter diagonal	✓	✓	✓	✓	✓	✗
Two pairs of adjacent sides equal but not all sides equal	✗	✗	✗	✗	✓	✗
Only one pair of opposite sides parallel	✗	✗	✗	✗	✗	✓
Only one pair of opposite angles equal	✗	✗	✗	✗	✓	✗

ACTIVITY 15.5 *Notes for Solutions*

15.5 1. 

2. 

3. 

4.