ACTIVITIES 13.1 - 13.2 *Notes and Solutions*

Notes and solutions given only where appropriate.

13.1 1. $100 = x^2 y$

$$xy = \frac{100}{x}$$

$$A = 2x^2 + 4xy$$

$$= 2x^2 + 4 \times \frac{100}{x}$$

$$= 2x^2 + \frac{400}{x}$$

2. $x = 4.64$ cm $y = 4.64$ cm

3. Cube

13.2 The problems become progressively more difficult. Pupils might need further practice at some of the simpler problems. You could, for example, consider different shapes, although you will need to identify *l*, *h*, *w* for each one.

1. (a) Not acceptable $(l > 600)$

   (b) Not acceptable $(l + h + w > 900)$

   (c) Acceptable

2. Taking $l = 600$, then

   $$h^2 = 610^2 - 600^2 \Rightarrow h = 110$$

   Since $l + h + w = 600 + 110 + 180 = 890 < 900$,
   the box will suffice.

   There are many possibilities as *h* could increase as *l* decreases.

   (Any volume for which $l^2 + h^2 = 610^2$ while $l + h \le 720$ is acceptable. In fact, *h* can increase up to about 122.4, by which time the $l + h$ constraint is almost violated.)

3. Using $l = 600$, $h = 0$, $w = 300$ (a *very* thin tube) means that the total length is given by

   $$d^2 = 600^2 + 0^2 + 300^2 \Rightarrow d \approx 671$$

   (The optimum answer (?))

Extension A cube of dimensions $300 \times 300 \times 300 = 27\,000\,000 \text{ cm}^3$ gives the maximum possible volume, without violating any of the constraints.

ACTIVITY 13.3 *Notes and Solutions*

13.3 One of the best ways to approach this problem is to set it as a practical exercise for group work.

1.

x	1	2	3	4	5
V	80	96	72	32	0

$x = 2$ gives maximum volume.

2. $l = 12 - 2x, \; b = 10 - 2x$ and

$$\begin{aligned} V &= x(12 - 2x)(10 - 2x) \\ &= x\left(120 - 44x + 4x^2\right) \\ &= 120x - 44x^2 + 4x^3 \end{aligned}$$

3. The graph should give a value of x about 1.8.

4.

$$\begin{aligned} V &= x(a - 2x)(a - 2x) \\ &= x\left(a^2 - 4ax + 4x^2\right) \\ &= a^2x - 4ax^2 + 4x^3 \end{aligned}$$

5. With $a = 1, \; V = x - 4x^2 + 4x^3$ and $x \approx 0.17$ gives the maximum value of V.