1.) (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln x = \dfrac{1}{x}, \dfrac{\mathrm{d}}{\mathrm{d}x}x^2 = 2x$ (seen anywhere) A1A1

attempt to substitute into the quotient rule (do **not** accept product rule) M1

e.g. $\dfrac{x^2\left(\dfrac{1}{x}\right) - 2x\ln x}{x^4}$

correct manipulation that clearly leads to result A1

e.g. $\dfrac{x - 2x\ln x}{x^4}, \dfrac{x(1-2\ln x)}{x^4}, \dfrac{x}{x^4} - \dfrac{2x\ln x}{x^4}$

$g'(x) = \dfrac{1-2\ln x}{x^3}$ AG N0

(b) evidence of setting the derivative equal to zero (M1)

e.g. $g'(x) = 0$, $1 - 2\ln x = 0$

$\ln x = \dfrac{1}{2}$ A1

$x = \mathrm{e}^{\frac{1}{2}}$ A1 N2

[7]

2.) (a) $v = 1$ A1 N1 1

(b) (i) $\dfrac{\mathrm{d}}{\mathrm{d}t}(2t) = 2$ A1

$\dfrac{\mathrm{d}}{\mathrm{d}t}(\cos 2t) = -2\sin 2t$ A1A1

Note: *Award A1 for coefficient 2 and A1 for –sin 2t.*

evidence of considering acceleration = 0 (M1)

e.g. $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 0, 2 - 2\sin 2t = 0$

correct manipulation A1

e.g. $\sin 2k = 1,\ \sin 2t = 1$

$2k = \dfrac{\pi}{2}\left(\text{accept } 2t = \dfrac{\pi}{2}\right)$ A1

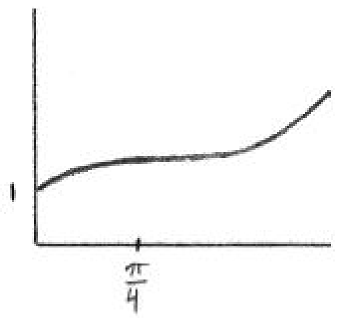
$k = \dfrac{\pi}{4}$ AG N0

(ii) attempt to substitute $t = \dfrac{\pi}{4}$ into v (M1)

e.g. $2\left(\dfrac{\pi}{4}\right) + \cos\left(\dfrac{2\pi}{4}\right)$

$v = \dfrac{\circledR}{2}$ A1 N2

(c)



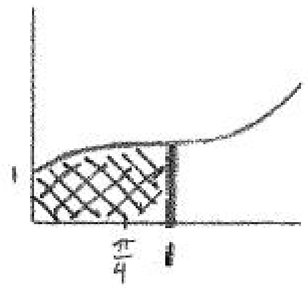
A1A1A2 N44

***Notes:** Award A1 for y-intercept at (0, 1), A1 for curve having zero gradient at $t = \frac{\pi}{4}$, A2 for shape that is concave down to the left of $\frac{\pi}{4}$ **and** concave up to the right of $\frac{\pi}{4}$. If a correct curve is drawn without indicating $t = \frac{\pi}{4}$, do not award the second A1 for the zero gradient, but award the final A2 if appropriate. Sketch need not be drawn to scale. Only essential features need to be clear.*

(d) (i) correct expression A2

e.g. $\int_0^1 (2t + \cos 2t)\,\mathrm{d}t, \left[t^2 + \frac{\sin 2t}{2}\right]_0^1, 1 + \frac{\sin 2}{2}, \int_0^1 v\,\mathrm{d}t$

(ii)



A1 3

***Note:** The line at t = 1 needs to be clearly after* $t = \frac{\pi}{4}$.

[16]

3.) **METHOD 1 (quotient)**

derivative of numerator is 6 (A1)

derivative of denominator is –sin x (A1)

attempt to substitute into quotient rule (M1)

correct substitution A1

e.g. $\frac{(\cos x)(6) - (6x)(-\sin x)}{(\cos x)^2}$

substituting $x = 0$ (A1)

e.g $\frac{(\cos 0)(6)-(6\times 0)(-\sin 0)}{(\cos 0)^2}$

$h'(0) = 6$ A1 N2

[6]

METHOD 2 (product)

$h(x) = 6x \times (\cos x)^{-1}$

derivative of $6x$ is 6 (A1)

derivative of $(\cos x)^{-1}$ is $(-(\cos x)^{-2}(-\sin x))$ (A1)

attempt to substitute into product rule (M1)

correct substitution A1

e.g. $(6x)(-(\cos x)^{-2}(-\sin x)) + (6)(\cos x)^{-1}$

substituting $x = 0$ (A1)

e.g. $(6\times 0)(-(\cos 0)^{-2}(-\sin 0)) + (6)(\cos 0)^{-1}$

$h'(0) = 6$ A1 N2

[6]

4.) (a) $f(1) = 2$ (A1)

$f'(x) = 4x$ A1

evidence of finding the gradient of f at $x = 1$ M1

e.g. substituting $x = 1$ into $f'(x)$

finding gradient of f at $x = 1$ A1

e.g. $f'(1) = 4$

evidence of finding equation of the line M1

e.g. $y - 2 = 4(x - 1)$, $2 = 4(1) + b$

$y = 4x - 2$ AG N05

(b) appropriate approach (M1)

e.g. $4x - 2 = 0$

$x = \frac{1}{2}$ A1 N22

(c) (i) bottom limit $x = 0$ (seen anywhere) (A1)

approach involving subtraction of integrals/areas (M1)

e.g. $\int f(x)$ – area of triangle, $\int f - \int l$

correct expression A2 N4

e.g. $\int_0^1 2x^2\,\mathrm{d}x - \int_{0.5}^1 (4x-2)\,\mathrm{d}x, \int_0^1 f(x)\,\mathrm{d}x - \frac{1}{2}, \int_0^{0.5} 2x^2\,\mathrm{d}x + \int_{0.5}^1 f(x) - (4x-2)\,\mathrm{d}x$

(ii) **METHOD 1 (using only integrals)**

correct integration (A1)(A1)(A1)

$\int 2x^2\,\mathrm{d}x = \frac{2x^3}{3}, \int (4x-2)\,\mathrm{d}x = 2x^2 - 2x$

substitution of limits (M1)

e.g. $\frac{1}{12}+\frac{2}{3}-2+2-\left(\frac{1}{12}-\frac{1}{2}+1\right)$

area = $\frac{1}{6}$ A1 N4

METHOD 2 (using integral and triangle)

area of triangle= $\frac{1}{2}$ (A1)

correct integration (A1)

$\int 2x^2 \mathrm{d}x = \frac{2x^3}{3}$

substitution of limits (M1)

e.g. $\frac{2}{3}(1)^3 - \frac{2}{3}(0)^3, \frac{2}{3} - 0$

correct simplification (A1)

e.g. $\frac{2}{3}-\frac{1}{2}$

area = $\frac{1}{6}$ A1 N49

[16]

5.) (a) **METHOD 1**

evidence of recognizing the amplitude is the radius (M1)

e.g. amplitude is half the diameter

$a = \frac{8}{2}$ A1

$a = 4$ AG N02

METHOD 2

evidence of recognizing the maximum height (M1)

e.g. $h = 6$, $a \sin bt + 2 = 6$

correct reasoning

e.g. $a \sin bt = 4$ and $\sin bt$ has amplitude of 1 A1

$a = 4$ AG N02

(b) **METHOD 1**

period = 30 (A1)

$b = \frac{2\pi}{30}$ A1

$b = \frac{\pi}{15}$ AG N02

METHOD 2

correct equation (A1)

e.g. $2 = 4\sin 30b + 2$, $\sin 30b = 0$

$30b = 2\pi$ A1

$b = \frac{\pi}{15}$ AG N02

(c) recognizing $h'(t) = -0.5$ (seen anywhere) R1

attempting to solve (M1)

e.g. sketch of h', finding h'

correct work involving h' A2

e.g. sketch of h' showing intersection, $-0.5 = \frac{4\pi}{15}\cos\left(\frac{\pi}{15}t\right)$

$t = 10.6$, $t = 19.4$ A1A1 N3**6**

(d) **METHOD 1**

valid reasoning for **their** conclusion (seen anywhere) R1

e.g. $h(t) < 0$ so underwater; $h(t) > 0$ so not underwater

evidence of substituting into h (M1)

e.g. $h(19.4)$, $4\sin\frac{19.4\pi}{15} + 2$

correct calculation A1

e.g. $h(19.4) = -1.19$

correct statement A1 N04

e.g. the bucket is underwater, yes

METHOD 2

valid reasoning for **their** conclusion (seen anywhere) R1

e.g. $h(t) < 0$ so underwater; $h(t) > 0$ so not underwater

evidence of valid approach (M1)

e.g. solving $h(t) = 0$, graph showing region below x-axis

correct roots A1

e.g. 17.5, 27.5

correct statement A1 N04

e.g. the bucket is underwater, yes

[14]

6.) (a) B, D A1A1 N2 2

(b) (i) $f'(x) = -2xe^{-x^2}$ A1A1N2

Note: *Award A1 for* e^{-x^2} *and A1 for* $-2x$.

(ii) finding the derivative of $-2x$, *i.e.* -2 (A1)

evidence of choosing the product rule (M1) *e.g.* $-2e^{-x^2} - 2x \times -2xe^{-x^2}$

$-2e^{-x^2} + 4x^2e^{-x^2}$ A1

$f''(x) = (4x^2 - 2)\,e^{-x^2}$ AG N05

(c) valid reasoning R1

e.g. $f''(x) = 0$

attempting to solve the equation (M1)

e.g. $(4x^2 - 2) = 0$, sketch of $f''(x)$

$p = 0.707 \left(= \frac{1}{\sqrt{2}}\right)$, $q = -0.707 \left(= -\frac{1}{\sqrt{2}}\right)$ A1A1 N34

(d) evidence of using second derivative to test values on either side of POI M1

e.g. finding values, reference to graph of f', sign table

correct working A1A1

e.g. finding any two correct values either side of POI,

checking sign of f'' on either side of POI

reference to sign change of $f''(x)$ R1 N04

[15]

7.)

(a) evidence of finding height, h (A1)

e.g. $\sin\theta = \frac{h}{2}$, $2\sin\theta$

evidence of finding base of triangle, b (A1)

e.g. $\cos\theta = \frac{b}{2}$, $2\cos\theta$

attempt to substitute valid values into a formula for the area
of the window (M1)

e.g. two triangles plus rectangle, trapezium area formula

correct expression (must be in terms of θ) A1

e.g. $2\left(\frac{1}{2} \times 2\cos\theta \times 2\sin\theta\right) + 2 \times 2\sin\theta,\ \frac{1}{2}(2\sin\theta)(2 + 2 + 4\cos\theta)$

attempt to replace $2\sin\theta\cos\theta$ by $\sin 2\theta$ M1

e.g. $4\sin\theta + 2(2\sin\theta\cos\theta)$

$y = 4\sin\theta + 2\sin 2\theta$ AG N05

(b) correct equation A1

e.g. $y = 5$, $4\sin\theta + 2\sin 2\theta = 5$

evidence of attempt to solve (M1)

e.g. a sketch, $4\sin\theta + 2\sin\theta - 5 = 0$

$\theta = 0.856$ (49.0°), $\theta = 1.25$ (71.4°) A1A1 N34

(c) recognition that lower area value occurs at $\theta = \frac{\pi}{2}$ (M1)

finding value of area at $\theta = \frac{\pi}{2}$ (M1)

e.g. $4 \sin\left(\frac{\pi}{2}\right) + 2\sin\left(2 \times \frac{\pi}{2}\right)$, draw square

$A = 4$ (A1)

recognition that maximum value of y is needed (M1)

$A = 5.19615\ldots$ (A1)

$4 < A < 5.20$ (accept $4 < A < 5.19$) A2 N57

[16]

8.) (a) $\frac{d}{dx}\sin x = \cos x, \frac{d}{dx}\cos x = -\sin x$ (seen anywhere) (A1)(A1)

evidence of using the quotient rule M1

correct substitution A1

e.g. $\frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}, \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$f'(x) = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$ A1

$f'(x) = \frac{-1}{\sin^2 x}$ AG N0

(b) **METHOD 1**

appropriate approach (M1)

e.g. $f'(x) = -(\sin x)^{-2}$

$f''(x) = 2(\sin^{-3} x)(\cos x)\left(= \frac{2\cos x}{\sin^3 x}\right)$ A1A1N3

Note: *Award A1 for 2* $\sin^{-3} x$*, A1 for* $\cos x$*.*

METHOD 2

derivative of $\sin^2 x = 2\sin x\cos x$ (seen anywhere) A1

evidence of choosing quotient rule (M1)

e.g. $u = -1, v = \sin^2 x, f''(x) = \frac{\sin^2 x \times 0 - (-1)2\sin x\cos x}{(\sin^2 x)^2}$

$f''(x) = \frac{2\sin x\cos x}{(\sin^2 x)^2}\left(= \frac{2\cos x}{\sin^3 x}\right)$ A1N3

(c) evidence of substituting $\frac{\pi}{2}$ M1

e.g. $\frac{-1}{\sin^2 \frac{\pi}{2}}, \frac{2\cos\frac{\pi}{2}}{\sin^3 \frac{\pi}{2}}$

$p = -1, q = 0$ A1A1N1N1

(d) second derivative is zero, second derivative changes sign R1R1N2

[13]

9.) gradient of tangent = 8 (seen anywhere) (A1)

$f'(x) = 4kx^3$ (seen anywhere) A1

recognizing the gradient of the tangent is the derivative (M1)

setting the derivative equal to 8 (A1)

e.g. $4kx^3 = 8$, $kx^3 = 2$

substituting $x = 1$ (seen anywhere) (M1)

$k = 2$ A1 N4

[6]

10.) (a) x-intercepts at –3, 0, 2 A2 N2

(b) $-3 < x < 0$, $2 < x < 3$ A1A1N2

(c) correct reasoning R2

e.g. the graph of f is **concave-down** (accept convex), the first derivative is decreasing

therefore the second derivative is negative AG

[6]

11.) (a) substituting into the second derivative M1

e.g. $3 \times \left(-\frac{4}{3}\right) - 1$

$f''\left(-\frac{4}{3}\right) = -5$ A1

since the second derivative is negative, B is a maximum R1 N0

(b) setting $f'(x)$ equal to zero (M1)

evidence of substituting $x = 2$ $\left(\text{or } x = -\frac{4}{3}\right)$ (M1)

e.g. $f'(2)$

correct substitution A1

e.g. $\frac{3}{2}(2)^2 - 2 + p, \frac{3}{2}\left(-\frac{4}{3}\right)^2 - \left(-\frac{4}{3}\right) + p$

correct simplification

e.g. $6 - 2 + p = 0$, $\frac{8}{3} + \frac{4}{3} + p = 0$, $4 + p = 0$ A1

$p = -4$ AGN0

(c) evidence of integration (M1)

$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + c$ A1A1A1

substituting (2, 4) or $\left(-\frac{4}{3}, \frac{358}{27}\right)$ into **their** expression (M1)

correct equation A1

e.g. $\frac{1}{2} \times 2^3 - \frac{1}{2} \times 2^2 - 4 \times 2 + c = 4, \frac{1}{2} \times 8 - \frac{1}{2} \times 4 - 4 \times 2 + c = 4, 4 - 2 - 8 + c = 4$

$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 10$ A1N4

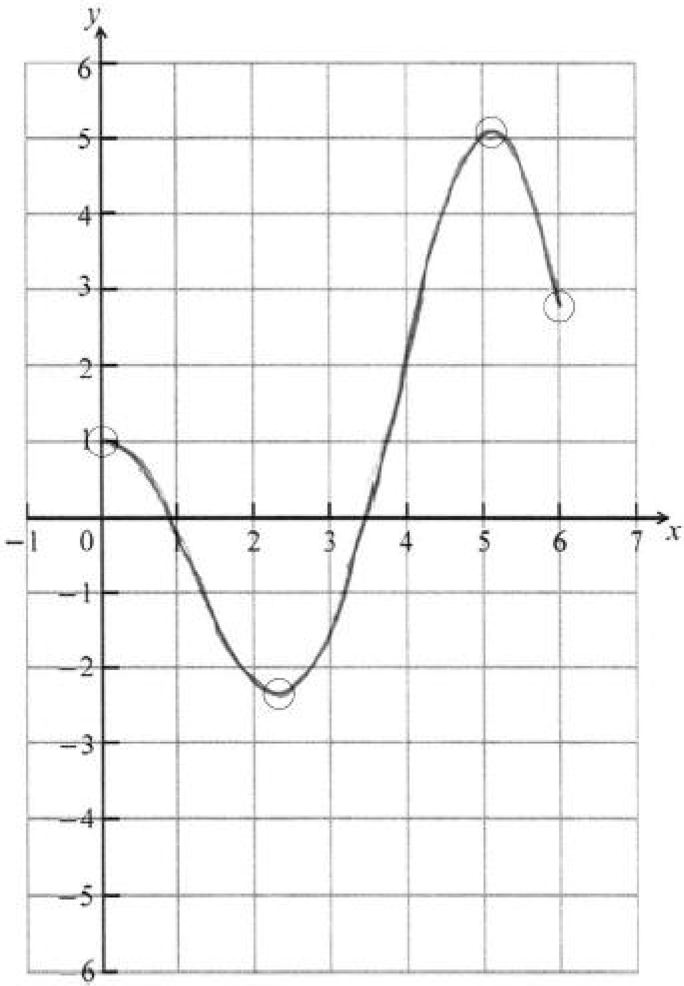
[14]

12.) (a) evidence of choosing the product rule (M1)

e.g. $x \times (-\sin x) + 1 \times \cos x$

$f'(x) = \cos x - x \sin x$ A1A1 N3

(b)



A1A1A1A1N4

Note: *Award A1 for correct domain, $0 \le x \le 6$ with endpoints in circles,*
A1 for approximately correct shape,
A1 for local minimum in circle,
A1 for local maximum in circle.

[7]

13.) (a) substituting (0, 13) into function M1

e.g. $13 = Ae^0 + 3$

$13 = A + 3$ A1

$A = 10$ AG N0

(b) substituting into $f(15) = 3.49$ A1

e.g. $3.49 = 10e^{15k} + 3$, $0.049 = e^{15k}$

evidence of solving equation (M1)

e.g. sketch, using ln

$k = -0.201$ $\left(\text{accept } \dfrac{\ln 0.049}{15}\right)$ A1N2

(c) (i) $f(x) = 10e^{-0.201x} + 3$

$f'(x) = 10e^{-0.201x} \times -0.201$ $(= -2.01e^{-0.201x})$ A1A1A1N3

Note: *Award A1 for* $10e^{-0.201x}$, *A1 for* $\times -0.201$, *A1 for the derivative of 3 is zero.*

(ii) valid reason with reference to derivative R1N1
e.g. $f'(x) < 0$, derivative always negative

(iii) $y = 3$ A1N1

(d) finding limits 3.8953…, 8.6940… (seen anywhere) A1A1

evidence of integrating and subtracting functions (M1)
correct expression A1

e.g. $\int_{3.90}^{8.69} g(x) - f(x)\,dx$, $\int_{3.90}^{8.69} [(-x^2 + 12x - 24) - (10e^{-0.201x} + 3)]dx$

area = 19.5 A2N4

[16]

14.) (a) $n = 800e^0$ (A1)
$n = 800$ A1 N2

(b) evidence of using the derivative (M1)
$n'(15) = 731$ A1N2

(c) **METHOD 1**

setting up inequality (accept equation or reverse inequality) A1
e.g. $n'(t) > 10\,000$

evidence of appropriate approach M1
e.g. sketch, finding derivative

$k = 35.1226...$ (A1)
least value of k is 36 A1N2

METHOD 2

$n'(35) = 9842$, **and** $n'(36) = 11208$ A2
least value of k is 36 A2N2

[8]

15.) (a) (i) $-1.15, 1.15$ A1A1 N2

(ii) recognizing that it occurs at P and Q (M1)
e.g. $x = -1.15$, $x = 1.15$

$k = -1.13$, $k = 1.13$ A1A1N3

(b) evidence of choosing the product rule (M1)
e.g. $uv' + vu'$

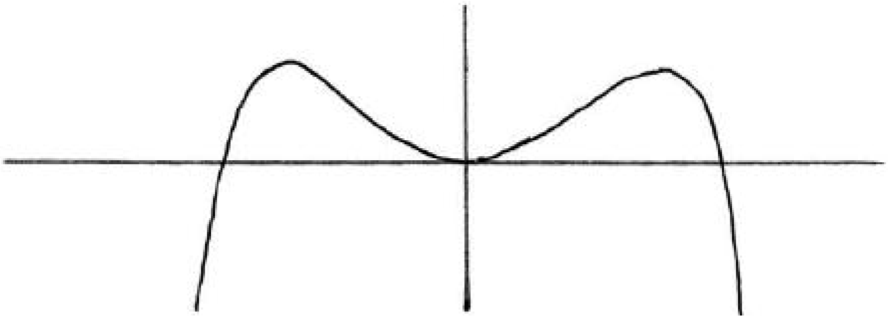
derivative of x^3 is $3x^2$ (A1)

derivative of $\ln(4 - x^2)$ is $\dfrac{-2x}{4 - x^2}$ (A1) correct substitution A1

e.g. $x^3 \times \frac{-2x}{4-x^2} + \ln(4-x^2) \times 3x^2$

$g'(x) = \frac{-2x^4}{4-x^2} + 3x^2 \ln(4-x^2)$ AGN0

(c)



A1A1N2

(d) $w = 2.69$, $w < 0$ A1A2N2

[14]

16.) (a) evidence of choosing the product rule
e.g. $uv' + vu'$ (M1)

correct derivatives $\cos x$, 2 (A1)(A1)

$g'(x) = 2x \cos x + 2 \sin x$ A1N4

(b) attempt to substitute into gradient function (M1)
e.g. $g'(\pi)$

correct substitution
e.g. $2\pi \cos \pi + 2 \sin \pi$ (A1)

gradient $= -2\pi$ A1N2

[7]

17.) (a) (i) substitute into gradient $= \frac{y_1 - y_2}{x_1 - x_2}$ (M1)

e.g. $\frac{f(a) - 0}{a - \frac{2}{3}}$

substituting $f(a) = a^3$

e.g. $\frac{a^3 - 0}{a - \frac{2}{3}}$ A1

gradient $= \frac{a^3}{a - \frac{2}{3}}$ AGN0

(ii) correct answer A1N1

e.g. $3a^2$, $f'(a) = 3$, $f'(a) = \frac{a^3}{a - \frac{2}{3}}$

(iii) **METHOD 1**

evidence of approach (M1)

e.g. $f'(a)$ = gradient, $3a^2 = \dfrac{a^3}{a-\frac{2}{3}}$

simplify A1

e.g. $3a^2\left(a-\dfrac{2}{3}\right) = a^3$

rearrange A1

e.g. $3a^3 - 2a^2 = a^3$

evidence of solving A1

e.g. $2a^3 - 2a^2 = 2a^2(a-1) = 0$

$a = 1$ AGN0

METHOD 2

gradient RQ = $\dfrac{-8}{-2-\frac{2}{3}}$ A1

simplify A1

e.g. $\dfrac{-8}{-\frac{8}{3}}$, 3

evidence of approach (M1)

e.g. $f'(a)$ = gradient, $3a^2 = \dfrac{-8}{-2-\frac{2}{3}}, \dfrac{a^3}{a-\frac{2}{3}} = 3$

simplify A1

e.g. $3a^2 = 3$, $a^2 = 1$

$a = 1$ AGN0

(b) approach to find area of T involving subtraction and integrals (M1)

e.g. $\int f-(3x-2)\mathrm{d}x, \int_{-2}^{k}(3x-2)-\int_{-2}^{k}x^3, \int(x^3-3x+2)$

correct integration with correct signs A1A1A1

e.g. $\dfrac{1}{4}x^4-\dfrac{3}{2}x^2+2x, \dfrac{3}{2}x^2-2x-\dfrac{1}{4}x^4$

correct limits –2 and k (seen anywhere) A1

e.g. $\int_{-2}^{k}(x^3-3x+2)\mathrm{d}x, \left[\dfrac{1}{4}x^4-\dfrac{3}{2}x^2+2x\right]_{-2}^{k}$

attempt to substitute k and –2 (M1)

correct substitution into **their** integral if 2 or more terms A1

e.g. $\left(\dfrac{1}{4}k^4-\dfrac{3}{2}k^2+2k\right)-(4-6-4)$

setting **their** integral expression equal to $2k + 4$ (seen anywhere) (M1)

simplifying A1

e.g. $\frac{1}{4}k^4 - \frac{3}{2}k^2 + 2 = 0$

$k^4 - 6k^2 + 8 = 0$ AGN0

[16]

18.) evidence of choosing the product rule (M1)

$f'(x) = e^x \times (-\sin x) + \cos x \times e^x$ $(= e^x \cos x - e^x \sin x)$ A1A1

substituting π (M1)

e.g. $f'(\pi) = e^{\pi} \cos \pi - e^{\pi} \sin \pi$, $e^{\pi}(-1 - 0)$, $-e^{\pi}$

taking negative reciprocal (M1)

e.g. $-\frac{1}{f'(\pi)}$

gradient is $\frac{1}{e^{\pi}}$ A1 N3

[6]

19.) (a) **METHOD 1**

evidence of substituting $-x$ for x (M1)

$f(-x) = \frac{a(-x)}{(-x)^2 + 1}$ A1

$f(-x) = \frac{-ax}{x^2 + 1}$ $(= -f(x))$ AGN0

METHOD 2

$y = -f(x)$ is reflection of $y = f(x)$ in x axis
and $y = f(-x)$ is reflection of $y = f(x)$ in y axis (M1)

sketch showing these are the same A1

$f(-x) = \frac{-ax}{x^2 + 1}$ $(= -f(x))$ AGN0

(b) evidence of appropriate approach (M1)
e.g. $f'(x) = 0$

to set the numerator equal to 0 (A1)
e.g. $2ax(x^2 - 3) = 0$; $(x^2 - 3) = 0$

$(0, 0)$, $\left(\sqrt{3}, \frac{a\sqrt{3}}{4}\right), \left(-\sqrt{3}, -\frac{a\sqrt{3}}{4}\right)$ (accept $x = 0$, $y = 0$ *etc.*) A1A1A1A1A1N5

(c) (i) correct expressionA2

e.g. $\left[\frac{a}{2}\ln(x^2 + 1)\right]_3^7, \frac{a}{2}\ln 50 - \frac{a}{2}\ln 10, \frac{a}{2}(\ln 50 - \ln 10)$

area = $\frac{a}{2}\ln 5$ A1A1N2

(ii) **METHOD 1**

recognizing that the shift does not change the area (M1) *e.g.* $\int_4^8 f(x-1)dx = \int_3^7 f(x)\mathrm{d}x, \frac{a}{2}\ln 5$

recognizing that the factor of 2 doubles the area (M1)

e.g. $\int_4^8 2f(x-1)\mathrm{d}x = 2\int_4^8 f(x-1)\mathrm{d}x \quad \left(= 2\int_3^7 f(x)\mathrm{d}x\right)$

$\int_4^8 2f(x-1)\mathrm{d}x = a\ln 5$ (*i.e.* 2 × **their** answer to (c)(i)) A1N3

METHOD 2

changing variable

let $w = x - 1$, so $\frac{\mathrm{d}w}{\mathrm{d}x} = 1$

$2\int f(w)\mathrm{d}w = \frac{2a}{2}\ln(w^2+1)+c$ (M1)

substituting correct limits

e.g. $\left[a\ln[(x-1)^2+1]\right]_4^8, \left[a\ln(w^2+1)\right]_3^7, a\ln 50 - a\ln 10$ (M1)

$\int_4^8 2f(x-1)\mathrm{d}x = a\ln 5$ A1N3

[16]

20.) (a) **METHOD 1**

$f'(x) = 3(x-3)^2$ A2N2

METHOD 2

attempt to expand $(x-3)^3$ (M1)

e.g. $f(x) = x^3 - 9x^2 + 27x - 27$

$f'(x) = 3x^2 - 18x + 27$ A1N2

(b) $f(3) = 0, f'(3) = 0$ A1N1

(c) **METHOD 1**

f' does not change sign at P R1

evidence for this R1N0

METHOD 2

f' changes sign at P so P is a maximum/minimum (*i.e.* not inflexion) R1

evidence for this R1N0

METHOD 3

finding $f(x) = \frac{1}{4}(x-3)^4 + c$ and sketching this function R1

indicating minimum at $x = 3$ R1N0

[5]

21.) (a) (i) $-3e^{-3x}$ A1 N1

(ii) $\cos\left(x - \frac{\pi}{3}\right)$ A1N1

(b) evidence of choosing product rule (M1)

e.g. $uv' + vu'$

correct expression A1

e.g. $-3e^{-3x}\sin\left(x-\frac{\pi}{3}\right)+e^{-3x}\cos\left(x-\frac{\pi}{3}\right)$

complete correct substitution of $x = \frac{\pi}{3}$ (A1)

e.g. $-3e^{-3\frac{\pi}{3}}\sin\left(\frac{\pi}{3}-\frac{\pi}{3}\right)+e^{-3\frac{\pi}{3}}\cos\left(\frac{\pi}{3}-\frac{\pi}{3}\right)$

$h'\left(\frac{\pi}{3}\right) = e^{-\pi}$ A1N3

[6]

22.) (a) attempt to expand (M1)

$(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$ A1 N2

(b) evidence of substituting x + h (M1)

correct substitution A1

e.g. $f'(x) = \lim_{h\to 0}\frac{(x+h)^3-4(x+h)+1-(x^3-4x+1)}{h}$

simplifying A1

e.g. $\frac{(x^3+3x^2h+3xh^2+h^2-4x-4h+1-x^3+4x-1)}{h}$

factoring out h A1

e.g. $\frac{h(3x^2+3xh+h^2-4)}{h}$

$f'(x) = 3x^2 - 4$ AGN0

(c) $f'(1) = -1$ (A1)

setting up an appropriate equation M1

e.g. $3x^2 - 4 = -1$

at Q, $x = -1$, $y = 4$ (Q is $(-1, 4)$) A1A1N3

(d) recognizing that f is decreasing when $f'(x) < 0$ R1

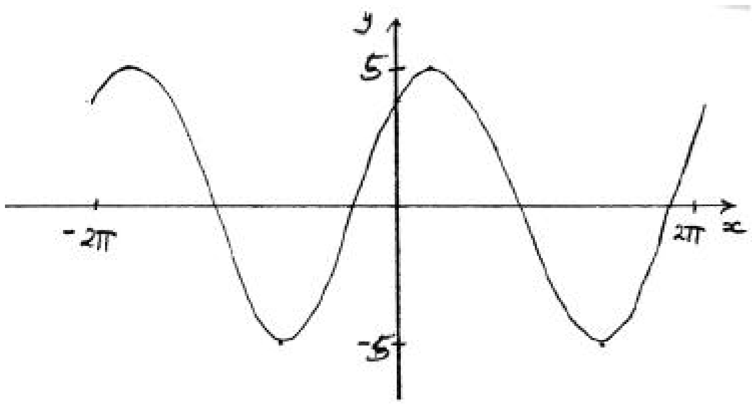
correct values for p and q (but do not accept $p = 1.15$, $q = -1.15$) A1A1N1N1

e.g. $p = -1.15$, $q = 1.15$; $\pm\frac{2}{\sqrt{3}}$; an interval such as $-1.15 \le x \le 1.15$

(e) $f'(x) \ge -4$, $y \ge -4$, $[-4, \infty[$ A2N2

[15]

23.) (a)



A1A1A1 N3

Note: *Award A1 for approximately sinusoidal shape, A1 for end points approximately correct, (–2π, 4), (2π, 4) A1 for approximately correct position of graph, (y-intercept (0, 4) maximum to right of y-axis).*

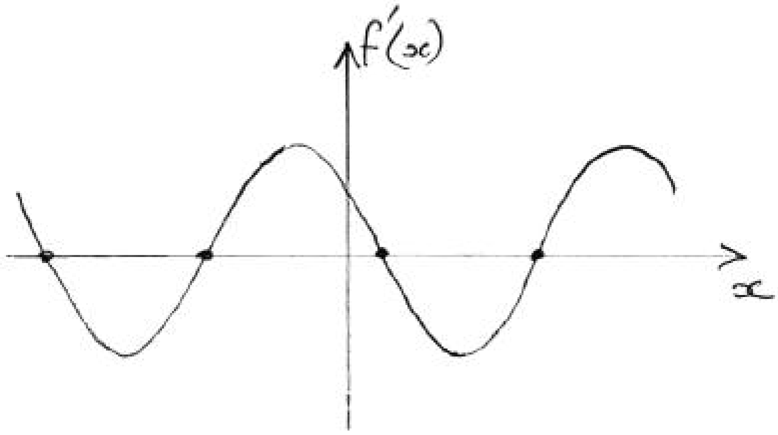
(b) (i) 5A1 N1

(ii) 2π (6.28) A1N1

(iii) –0.927 A1N1

(c) $f(x) = 5 \sin (x + 0.927)$ (accept $p = 5$, $q = 1$, $r = 0.927$) A1A1A1N3

(d) evidence of correct approach (M1)
e.g. max/min, sketch of $f'(x)$ indicating roots

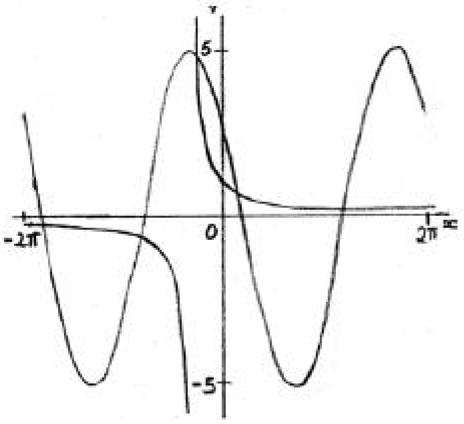


one 3 s.f. value which rounds to one of –5.6, –2.5, 0.64, 3.8 A1N2

(e) $k = –5$, $k = 5$ A1A1N2

(f) **METHOD 1**

graphical approach (but must involve derivative functions) M1
e.g.



each curve A1A1

$x = 0.511$ A2N2

METHOD 2

$g'(x) = \dfrac{1}{x+1}$ A1

$f'(x) = 3\cos x - 4\sin x \quad (5\cos(x + 0.927))$ A1

evidence of attempt to solve $g'(x) = f'(x)$ M1

$x = 0.511$ A2N2

[18]

24.) (a) $f'(x) = 2x - \dfrac{p}{x^2}$ A1A1 N2

Note: *Award A1 for 2x, A1 for* $-\dfrac{p}{x^2}$.

(b) evidence of equating derivative to 0 (seen anywhere) (M1)

evidence of finding $f'(-2)$ (seen anywhere) (M1)

correct equation A1

e.g. $-4 - \dfrac{p}{4} = 0, -16 - p = 0$

$p = -16$ A1N3

[6]

25.) (a) (i) coordinates of A are $(0, -2)$ A1A1 N2

(ii) derivative of $x^2 - 4 = 2x$ (seen anywhere) (A1)

evidence of correct approach (M1)

e.g. quotient rule, chain rule

finding $f'(x)$ A2

e.g. $f'(x) = 20 \times (-1) \times (x^2 - 4)^{-2} \times (2x)$, $\dfrac{(x^2-4)(0)-(20)(2x)}{(x^2-4)^2}$

substituting $x = 0$ into $f'(x)$ (do **not** accept solving $f'(x) = 0$) M1

at A $f'(x) = 0$ AGN0

(b) (i) reference to $f'(x) = 0$ (seen anywhere)(R1)

reference to $f''(0)$ is negative (seen anywhere) R1

evidence of substituting $x = 0$ into $f''(x)$ M1 finding $f''(0) = \frac{40 \times 4}{(-4)^3}\left(= \frac{-5}{2}\right)$ A1

then the graph must have a local maximum AG

(ii) reference to $f''(x) = 0$ at point of inflexion, (R1)

recognizing that the second derivative is never 0 A1N2

e.g. $40(3x^2 + 4) \neq 0$, $3x^2 + 4 \neq 0$, $x^2 \neq -\frac{4}{3}$, the numerator is always positive

Note: *Do **not** accept the use of the first derivative in part (b).*

(c) correct (informal) statement, including reference to approaching $y = 3$ A1N1

e.g. getting closer to the line $y = 3$, horizontal asymptote at $y = 3$

(d) **correct** inequalities, $y \leq -2$, $y > 3$, ***FT*** from (a)(i) and (c) A1A1N2

[16]

26.) (a) $f'(x) = -\sin 2x \times 2$ ($= -2 \sin 2x$) A1A1 N2

Note: *Award A1 for 2, A1 for sin 2x.*

(b) $g'(x) = 3 \times \frac{1}{3x-5}\left(= \frac{3}{3x-5}\right)$ A1A1N2

Note: *Award A1 for 3, A1 for* $\frac{1}{3x-5}$.

(c) evidence of using product rule (M1)

$h'(x) = (\cos 2x)\left(\frac{3}{3x-5}\right) + \ln(3x-5)(-2 \sin 2x)$ A1N2

[6]

27.) substituting $x = 1$, $y = 3$ into $f(x)$(M1)

$3 = p + q$ A1

finding derivative (M1)

$f'(x) = 2px + q$ A1

correct substitution, $2p + q = 8$ A1

$p = 5$, $q = -2$ A1A1 N2N2

[7]

28.) (a) $f'(x) = x^2 + 4x - 5$ A1A1A1 N3

(b) evidence of attempting to solve $f'(x) = 0$ (M1)

evidence of correct working A1

e.g. $(x + 5)(x - 1)$, $\frac{-4 \pm \sqrt{16+20}}{2}$, sketch

$x = -5$, $x = 1$ (A1)

so $x = -5$ A1 N2

(c) **METHOD 1**

$f''(x) = 2x + 4$ (may be seen later) A1

evidence of setting second derivative = 0 (M1)

e.g. $2x + 4 = 0$

$x = -2$ A1 N2

METHOD 2

evidence of use of symmetry (M1)

e.g. midpoint of max/min, reference to shape of cubic

correct calculation A1

e.g. $\frac{-5+1}{2}$,

$x = -2$ A1 N2

(d) attempting to find the value of the derivative when $x = 3$ (M1)

$f'(3) = 16$ A1

valid approach to finding the equation of a line M1

e.g. $y - 12 = 16(x - 3)$, $12 = 16 \times 3 + b$

$y = 16x - 36$ A1 N2

[14]

29.) (a) (i) range of f is $[-1, 1]$, $(-1 \le f(x) \le 1)$ A2 N2

(ii) $\sin^3 x = 1 \Rightarrow \sin x = 1$ A1

justification for one solution on $[0, 2\pi]$ R1

e.g. $x = \frac{\pi}{2}$, unit circle, sketch of $\sin x$

1 solution (seen anywhere) A1 N1

(b) $f'(x) = 3\sin^2 x \cos x$ A2 N2

(c) using $V = \int_a^b \pi y^2 \mathrm{d}x$ (M1)

$$V = \int_0^{\frac{\pi}{2}} \pi \left(\sqrt{3} \sin x \cos^{\frac{1}{2}} x \right)^2 \mathrm{d}x$$ (A1)

$$= \pi \int_0^{\frac{\pi}{2}} 3 \sin^2 x \cos x \, \mathrm{d}x$$ A1

$$V = \pi \left[\sin^3 x \right]_0^{\frac{\pi}{2}} \left(= \pi \left(\sin^3 \left(\frac{\pi}{2} \right) - \sin^3 0 \right) \right)$$ A2

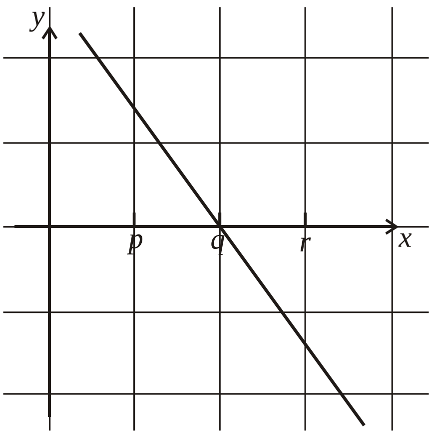
evidence of using $\sin \frac{\pi}{2} = 1$ and $\sin 0 = 0$ (A1)

e.g. $\pi(1 - 0)$

$V = \pi$ A1 N1

[14]

30.) (a)



A1A1 N2

Note: *Award A1 for ne.g.ative gradient throughout, A1 for x-intercept of q. It need not be linear.*

(b)

		x-coordinate
(i)	Maximum point on f	r
(ii)	Inflexion point on f	q

A1 N1

A1 N1

(c) **METHOD 1**

Second derivative is zero, second derivative changes sign. R1R1 N2

METHOD 2

There is a maximum on the graph of the first derivative. R2 N2

[6]

31.) (a) (i) intersection points x =3.77, x = 8.30 (may be seen as the limits) (A1)(A1)

approach involving subtraction and integrals (M1)

fully correct expression A2

e.g. $\int_{3.77}^{8.30}((-4\cos(0.5x)+2)-(\ln(3x-2)+1))\mathrm{d}x$,

$\int_{3.77}^{8.30} g(x)\mathrm{d}x - \int_{3.77}^{8.30} f(x)\mathrm{d}x$ N5

(ii) $A = 6.46$ A1 N1

(b) (i) $f'(x) = \dfrac{3}{3x-2}$ A1A1N2

Note: *Award A1 for numerator (3), A1 for denominator (3x - 2), but penalize 1 mark for additional terms.*

(ii) $g'(x) = 2 \sin (0.5x)$ A1A1 N2

Note: *Award A1 for 2, A1 for sin (0.5x), but penalize 1 mark for additional terms.*

(c) evidence of using derivatives for gradients (M1)

correct approach (A1)

e.g. $f'(x) = g'(x)$, points of intersection

$x = 1.43$, $x = 6.10$ A1A1 N2N2

[14]

32.) (a) evidence of using the product rule M1

$f'(x) = e^x(1 - x^2) + e^x(-2x)$ A1A1

Note: *Award A1 for* $e^x(1- x^2)$, *A1 for* $e^x(- 2x)$.

$f'(x) = e^x(1 - 2x - x^2)$ AG N0

(b) $y = 0$ A1 N1

(c) at the local maximum or minimum point

$f'(x) = 0$ $(e^x(1 - 2x - x^2) = 0)$ (M1)

$\Rightarrow 1 - 2x - x^2 = 0$ (M1)

$r = -2.41$ $s = 0.414$ A1A1 N2N2

(d) $f'(0) = 1$ A1

gradient of the normal $= -1$ A1

evidence of substituting into an equation for a straight line (M1)

correct substitution A1

e.g. $y - 1 = -1(x - 0)$, $y - 1 = -x$, $y = -x + 1$

$x + y = 1$ AG N0

(e) (i) intersection points at $x = 0$ and $x = 1$ (may be seen as the limits) (A1)

approach involving subtraction and integrals (M1)

fully correct expression A2 N4

e.g. $\int_0^1 \left(e^x\left(1-x^2\right)-\left(1-x\right)\right)dx, \int_0^1 f\left(x\right)dx - \int_0^1 \left(1-x\right)dx$

(ii) area $R = 0.5$ A1 N1

[17]

33.) (a) correctly finding the derivative of e^{2x}, *i.e.* $2e^{2x}$ A1

correctly finding the derivative of $\cos x$, *i.e.* $-\sin x$ A1

evidence of using the product rule, seen anywhere M1

e.g. $f'(x) = 2e^{2x} \cos x - e^{2x} \sin x$

$f'(x) = e^{2x}(2 \cos x - \sin x)$ AG N0

(b) evidence of finding $f(0) = 1$, seen anywhere A1

attempt to find the gradient of f (M1)

e.g. substituting $x = 0$ into $f'(x)$

value of the gradient of f A1

e.g. $f'(0) = 2$, equation of tangent is $y = 2x + 1$

gradient of normal $= -\frac{1}{2}$ (A1)

$y - 1 = -\frac{1}{2}x$ $\left(y = -\frac{1}{2}x + 1\right)$ A1 N3

(c) (i) evidence of equating correct functions M1

e.g. $e^{2x}\cos x = -\frac{1}{2}x + 1$, sketch showing intersection of graphs

$x = 1.56$ A1 N1

(ii) evidence of approach involving subtraction of integrals/areas (M1)

e.g. $\int [f(x) - g(x)]\mathrm{d}x$, $\int f(x)\mathrm{d}x$ – area under trapezium

fully correct integral expression A2

e.g. $\int_0^{1.56}\left[e^{2x}\cos x - \left(-\frac{1}{2}x + 1\right)\right]\mathrm{d}x$, $\int_0^{1.56} e^{2x}\cos x\,\mathrm{d}x - 0.951...$

area = 3.28 A1 N2

[14]

34.) Attempt to differentiate (M1)

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2e^{2x}$ A1

At $x = 1$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 2e^2$ A1

$y = e^2$ A1

Equation of tangent is $y - e^2 = 2e^2(x - 1)$ $(y = 2e^2x - e^2)$M1A1 N2

[6]

35.) (a) $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ (M1)

$= -10$ (m s^{-2}) A1 N2

(b) $s = \int v\,\mathrm{d}t$ (M1)

$= 50t - 5t^2 + c$ A1

$40 = 50(0) - 5(0) + c \Rightarrow c = 40$ A1

$s = 50t - 5t^2 + 40$ A1N2

Note: *Award **(M1)** and the first **A1** in part (b) if c is missing, but do **not** award the final 2 marks.*

[6]

36.) (a) Attempt to differentiate (M1)

$g'(x) = 3x^2 - 6x - 9$ A1A1A1

for setting derivative equal to zero M1

$3x^2 - 6x - 9 = 0$

Solving A1

e.g. $3(x - 3)(x + 1) = 0$
$x = 3 \quad x = -1$ A1A1N3

(b) **METHOD 1**
$g'(x < -1)$ is positive, $g'(x > -1)$ is negative A1A1
$g'(x < 3)$ is negative, $g'(x > 3)$ is positive A1A1

min when $x = 3$, max when $x = -1$ A1A1N2

METHOD 2
Evidence of using second derivative (M1)
$g''(x) = 6x - 6$ A1
$g''(3) = 12$ (or positive), $g''(-1) = -12$ (or negative) A1A1

min when $x = 3$, max when $x = -1$ A1A1N2

[14]

37.) (a) Using the chain rule (M1)
$f'(x) = (2 \cos(5x - 3))5$ $(= 10 \cos(5x - 3))$ A1
$f''(x) = -(10 \sin(5x - 3))5$
$= -50\sin(5x - 3)$ A1A1 N2

Note: *Award A1 for sin (5x – 3), A1 for –50.*

(b) $\int f(x)\mathrm{d}x = -\frac{2}{5}\cos(5x - 3) + c$ A1A1N2

Note: *Award A1 for cos(5x – 3), A1 for* $-\frac{2}{5}$.

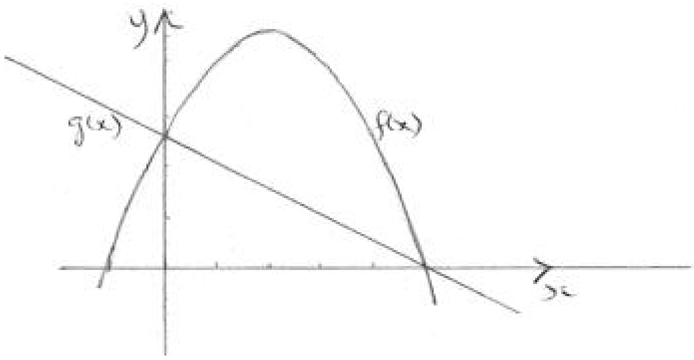
[6]

38.) (a) Curve intersects y-axis when $x = 0$ (A1)
Gradient of tangent at y-intercept = 2 A1
$\Rightarrow$ gradient of $N = -\frac{1}{2}(= -0.5)$ A1
Finding y-intercept, 2.5 A1
Therefore, equation of N is $y = -0.5x + 2.5$ AG N0

(b) N intersects curve when $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$ A1
Solving equation (M1)
e.g. sketch, factorising
$\Rightarrow x = 0$ or $x = 5$ A1

Other point when $x = 5$ (R1)
$x = 5 \Rightarrow y = 0$ (so other point $(5, 0)$) A1N2

(c)



Using appropriate method, with subtraction/correct expression, **correct** limitsM1A1 *e.g.* $\int_0^5 f(x)\mathrm{d}x - \int_0^5 g(x)\mathrm{d}x, \int_0^5 (-0.5x^2 + 2.5x)\mathrm{d}x$

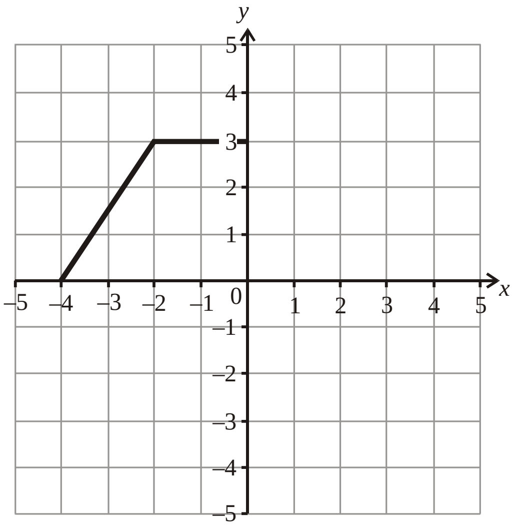
Area = 10.4 A2N2

[13]

39.) (a) (i) 0 A1 N1

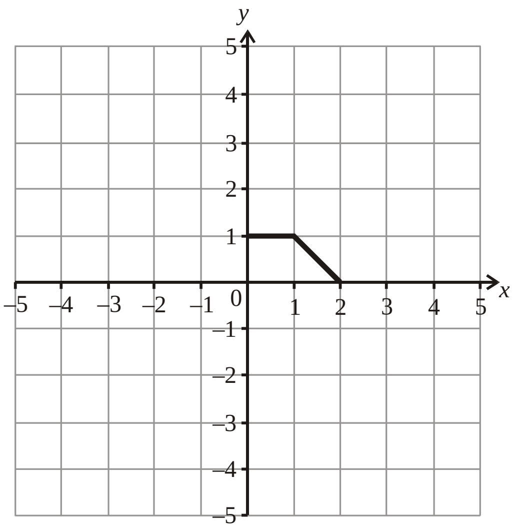
(ii) $-\frac{1}{2}$ A1 N1

(b)



A2 N2

(c)



A2 N2

[6]

40.) (a) **METHOD 1**

$f'(x) = -6 \sin 2x + 2 \sin x \cos x$ A1A1A1

$= -6 \sin 2x + \sin 2x$ A1

$= -5 \sin 2x$ AG N0

METHOD 2

$\sin^2 x = \dfrac{1-\cos 2x}{2}$ (A1)

$f(x) = 3\cos 2x + \dfrac{1}{2} - \dfrac{1}{2}\cos 2x$ A1

$f(x) = \dfrac{5}{2}\cos 2x + \dfrac{1}{2}$ A1

$f'(x) = 2\left(\dfrac{5}{2}\right)(-\sin 2x)$ A1

$f'(x) = -5\sin 2x$ AG N0

(b) $k = \dfrac{\pi}{2}$ $(=1.57)$ A2 N2

[6]

41.) (a) (i) $p = 1$, $q = 5$ (or $p = 5$, $q = 1$) A1A1 N2

(ii) $x = 3$ (must be an equation) A1 N1

(b) $y = (x - 1)(x - 5)$

$= x^2 - 6x + 5$ (A1)

$= (x - 3)^2 - 4$ (accept $h = 3$, $k = -4$) A1A1 N3

(c) $\dfrac{dy}{dx} = 2(x-3)$ $(=2x-6)$ A1A1 N2

(d) When $x = 0$, $\dfrac{dy}{dx} = -6$ (A1)

$y - 5 = -6(x - 0)$ ($y = -6x + 5$ or equivalent) A1 N2

[10]

42.) (a) π (3.14) (accept (π, 0), (3.14, 0)) A1 N1

(b) (i) For using the product rule (M1)

$f'(x) = e^x \cos x + e^x \sin x = e^x(\cos x + \sin x)$ A1A1 N3

(ii) At B, $f'(x) = 0$ A1 N1

(c) $f''(x) = e^x \cos x - e^x \sin x + e^x \sin x + e^x \cos x$ A1A1

$= 2e^x \cos x$ AG N0

(d) (i) At A, $f''(x) = 0$ A1N1

(ii) Evidence of setting up **their** equation (may be seen in part (d)(i)) A1

eg $2e^x \cos x = 0$, $\cos x = 0$

$x = \frac{\pi}{2}\ (=1.57)$, $y = e^{\frac{\pi}{2}}\ (=4.81)$ A1A1

Coordinates are $\left(\frac{\pi}{2}, e^{\frac{\pi}{2}}\right)(1.57, 4.81)$ N2

(e) (i) $\int_0^{\pi} e^x \sin x\,dx$ or $\int_0^{\pi} f(x)\,dx$ A2N2

(ii) Area = 12.1 A2 N2

[15]

43.) (a) (i) $p = 2$ A1 N1

(ii) $q = 1$ A1 N1

(b) (i) $f(x) = 0$ (M1)

$2 - \frac{3x}{x^2-1} = 0$ $(2x^2 - 3x - 2 = 0)$ A1

$x = -\frac{1}{2}$ $x = 2$

$\left(-\frac{1}{2}, 0\right)$ A1 N2

(ii) Using $V = \int_a^b \pi y^2 dx$ (limits not required) (M1)

$V = \int_{-\frac{1}{2}}^{0} \pi\left(2 - \frac{3x}{x^2-1}\right)^2 dx$ A2

$V = 2.52$ A1 N2

(c) (i) Evidence of appropriate method M1

eg Product or quotient rule

Correct derivatives of $3x$ and $x^2 - 1$ A1A1

Correct substitution A1

eg $\frac{-3(x^2-1)-(-3x)(2x)}{(x^2-1)^2}$

$f'(x) = \frac{-3x^2+3+6x^2}{(x^2-1)^2}$ A1

$f'(x) = \frac{3x^2+3}{(x^2-1)^2} = \frac{3(x^2+1)}{(x^2-1)^2}$ AG N0

(ii) **METHOD 1**

Evidence of using $f'(x) = 0$ at max/min (M1)

$3(x^2 + 1) = 0$ $(3x^2 + 3 = 0)$ A1

no (real) solution R1

Therefore, no maximum or minimum. AG N0

METHOD 2

Evidence of using $f'(x) = 0$ at max/min (M1)

Sketch of $f'(x)$ with good asymptotic behaviour A1

Never crosses the x-axis R1

Therefore, no maximum or minimum. AG N0

METHOD 3

Evidence of using $f'(x) = 0$ at max/min (M1)

Evidence of considering the sign of $f'(x)$ A1

$f'(x)$ is an increasing function ($f'(x) > 0$, always) R1

Therefore, no maximum or minimum. AG N0

(d) For using integral (M1)

Area = $\int_0^a g(x)\,\mathrm{d}x \left(\text{or} \int_0^a f'(x)\,\mathrm{d}x \text{ or} \int_0^a \frac{3x^2+3}{(x^2-1)^2}\,\mathrm{d}x\right)$ A1

Recognizing that $\int_0^a g(x)\,\mathrm{d}x = f(x)\Big|_0^a$ A2

Setting up equation (seen anywhere) (M1)

Correct equation A1

eg $\int_0^a \frac{3x^2+3}{(x^2-1)^2}\,\mathrm{d}x = 2$, $\left[2 - \frac{3a}{a^2-1}\right] - [2-0] = 2$, $2a^2 + 3a - 2 = 0$

$a = \frac{1}{2}$ $\quad a = -2$

$a = \frac{1}{2}$ A1 N2

[24]

44.) (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = 3\cos 3x$ A1 N1

(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{\cos^2 x} + \tan x$ accept $x\sec^2 x + \tan x$ A1A1 N2

(c) **METHOD 1**

Evidence of using the quotient rule (M1)

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x \times \frac{1}{x} - \ln x}{x^2}$ A1A1

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1-\ln x}{x^2}$ N3

METHOD 2

$y = x^{-1} \text{In } x$

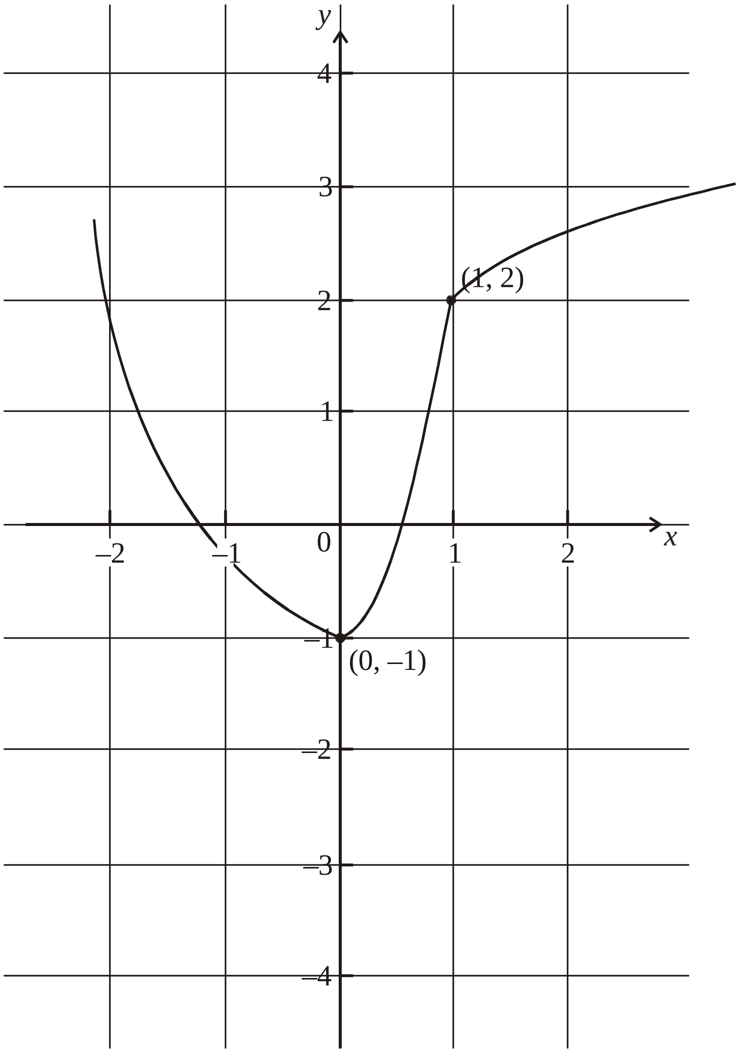
Evidence of using the product rule (M1)

$$\frac{dy}{dx} = x^{-1} \times \frac{1}{x} + \ln x(-1)(x^{-2})$$ A1A1

$$\frac{dy}{dx} = \frac{1}{x^2} - \frac{\ln x}{x^2}$$ N3

[6]

45.)



A1A1A1A1A1A1 N6

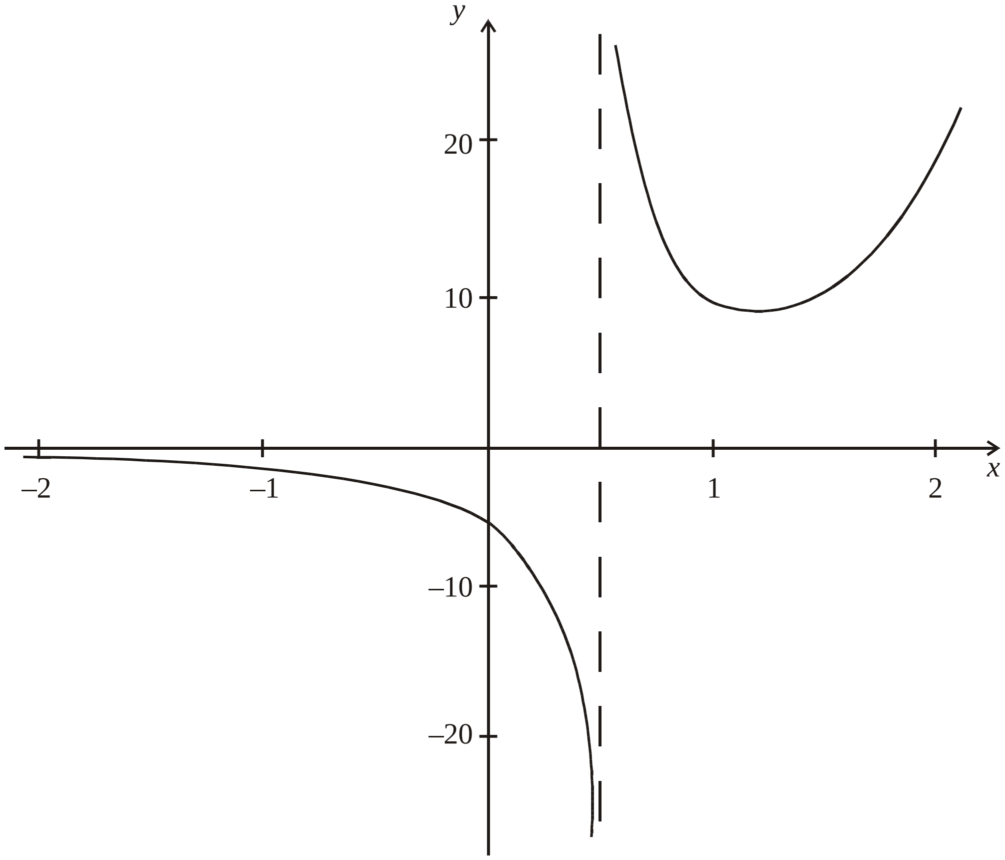
Notes: *On interval [- 2,0], award A1 for decreasing, A1 for concave up.*

On interval [0,1], award A1 for increasing, A1 for concave up.

On interval [1,2], award A1 for change of concavity, A1 for concave down.

[6]

46.) (a)



A1A1A1 N3

Note: *Award A1 for the left branch asymptotic to the x-axis and crossing the y-axis,*
A1 for the right branch approximately the correct shape,
A1 for a vertical asymptote at approximately $x = \frac{1}{2}$.

(b) (i) $x = \frac{1}{2}$ (must be an equation) A1N1

(ii) $\int_0^2 f(x)\,\mathrm{d}x$ A1 N1

(iii) Valid reason R1 N1

eg reference to area undefined or discontinuity

Note: *GDC reason* ***not*** *acceptable.*

(c) (i) $V = \pi \int_1^{1.5} f(x)^2\,\mathrm{d}x$ A2N2

(ii) $V = 105$ (accept $33.3\,\pi$) A2 N2

(d) $f'(x) = 2e^{2x-1} - 10(2x-1)^{-2}$ A1A1A1A1 N4

(e) (i) $x = 1.11$ (accept (1.11, 7.49)) A1N1

(ii) $p = 0$, $q = 7.49$ (accept $0 \le k < 7.49$) A1A1 N2

[17]

47.) ***Note:*** *Accept **exact** answers given in terms of* p.

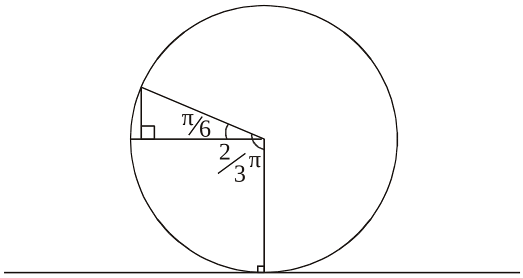
(a) Evidence of using *l* = ***r***q (M1)

arc AB = 7.85 (m) A1 N2

(b) Evidence of using $A = \frac{1}{2} r^2 \theta$ (M1)

Area of sector AOB = 58.9 (m^2) A1 N2

(c) **METHOD 1**



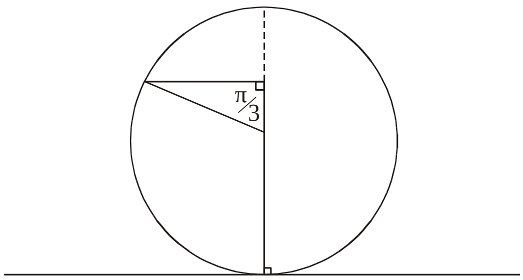
angle = $\frac{\pi}{6}$ $(30°)$ (A1)

attempt to find 15 sin $\frac{\pi}{6}$ M1

height = 15 + 15 sin $\frac{\pi}{6}$

= 22.5 (m) A1 N2

METHOD 2



angle = $\frac{\pi}{3}$ $(60°)$ (A1)

attempt to find 15 cos $\frac{\pi}{3}$ M1

height = 15 + 15 cos $\frac{\pi}{3}$

= 22.5 (m) A1 N2

(d) (i) $h\left(\frac{\pi}{4}\right) = 15 - 15\cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$ (M1)

$= 25.6$ (m) A1 N2

(ii) $h(0) = 15 - 15\cos\left(0 + \frac{\pi}{4}\right)$ (M1)

$= 4.39$(m) A1 N2

(iii) **METHOD 1**

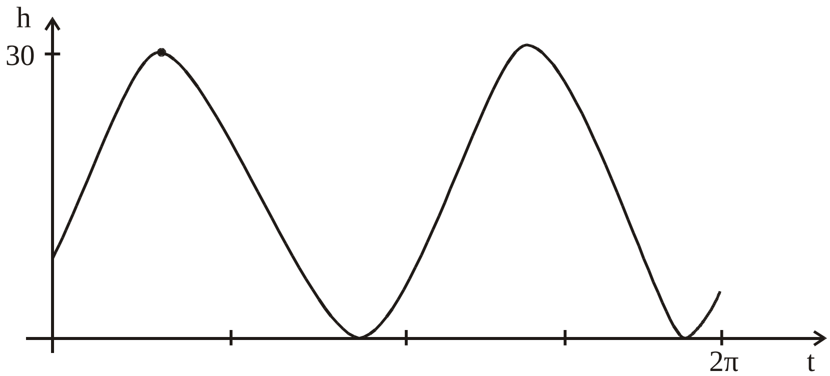
Highest point when $h = 30$ R1

$30 = 15 - 15\cos\left(2t + \frac{\pi}{4}\right)$ M1

$\cos\left(2t + \frac{\pi}{4}\right) = -1$ (A1)

$t = 1.18$ $\left(\text{accept } \frac{3\pi}{8}\right)$ A1 N2

METHOD 2



Sketch of graph of h M2

Correct maximum indicated (A1)

$t = 1.18$ A1 N2

METHOD 3

Evidence of setting $h'(t) = 0$ M1

$\sin\left(2t + \frac{\pi}{4}\right) = 0$ (A1)

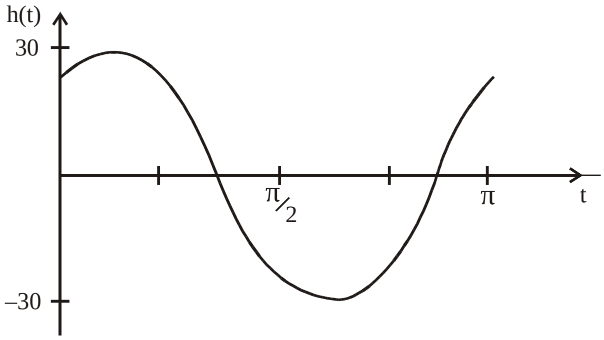
Justification of maximum R1

eg reasoning from diagram, first derivative test, second derivative test

$t = 1.18$ $\left(\text{accept } \frac{3\pi}{8}\right)$ A1 N2

(e) $h'(t) = 30\sin\left(2t + \frac{\pi}{4}\right)$ (may be seen in part (d)) A1A1 N2

(f) (i)



A1A1A1 N3

Notes: *Award A1 for range - 30 to 30, A1 for two zeros.*

Award A1 for approximate correct ***sinusoidal*** *shape.*

(ii) **METHOD 1**

Maximum on graph of h' (M1)

$t = 0.393$ A1 N2

METHOD 2

Minimum on graph of h' (M1)

$t = 1.96$ A1 N2

METHOD 3

Solving $h''(t) = 0$ (M1)

One or both correct answers A1

$t = 0.393$, $t = 1.96$ N2

[22]

48.) (a) $f'(x) = 5e^{5x}$ A1A1 N2

(b) $g'(x) = 2 \cos 2x$ A1A1 N2

(c) $h' = fg' + gf'$ (M1)

$= e^{5x}(2 \cos 2x) + \sin 2x\,(5e^{5x})$ A1 N2

[6]

49.) (a)

	A	B	E
$f'(x)$	negative	0	negative

A1A1A1 N3

(b)

	A	B	E
$f''(x)$	positive	positive	negative

A1A1A1 N3

[6]

50.) (a)

Interval	g'	g''
$a < x < b$	**positive**	**positive**
$e < x < f$	**negative**	**negative**

A1A1

A1A1 N4

(b)

Conditions	Point
$g'(x) = 0$, $g''(x) < 0$	**C**
$g'(x) < 0$, $g''(x) = 0$	**D**

A1 N1

A1 N1

[6]

51.) (a) $f'(x) = 6x - 5$ A1 N1

(b) $f'(p) = 7$ (or $6p - 5 = 7$) M1

$p = 2$ A1 N1

(c) Setting $y(2) = f(2)$ (M1)

Substituting $y(2) = 7 \times 2 - 9$ $(= 5)$, **and** $f(2) = 3 \times 2^2 - 5 \times 2 + k$ $(= k + 2)$ A1

$k + 2 = 5$

$k = 3$ A1 N2

[6]

52.) (a) $f'(x) = 2xe^{-x} - x^2e^{-x}$ $(= (2x - x^2)e^{-x} = x(2 - x)e^{-x})$ A1A1 N2

(b) Maximum occurs at $x = 2$ (A1)

Exact maximum value $= 4e^{-2}$ A1 N2

(c) For inflexion, $f''(x) = 0$ $\left((x^2 - 4x + 2) = 0,\ x = \frac{4 \pm \sqrt{16 - 8}}{2}, \text{etc.} \right)$ M1

$x = \frac{4 + \sqrt{8}}{2}$ $(= 2 + \sqrt{2})$ A1 N1

[6]

53.) (a) (i) $f'(x) = -\frac{3}{2}x+1$ A1A1 N2

(ii) For using the derivative to find the gradient of the tangent (M1)

$f'(2) = -2$ (A1)

Using negative reciprocal to find the gradient of the normal $\left(\frac{1}{2}\right)$ M1

$y-3=\frac{1}{2}(x-2)$ $\left(\text{or } y=\frac{1}{2}x+2\right)$ A1 N3

(iii) Equating $-\frac{3}{4}x^2+x+4=\frac{1}{2}x+2$ (or sketch of graph) M1

$3x^2 - 2x - 8 = 0$ (A1)

$(3x + 4)(x - 2) = 0$

$x = -\frac{4}{3}\ (=-1.33)$ (accept $\left(-\frac{4}{3},\frac{4}{3}\right)$ or $x=-\frac{4}{3}, x=2$) A1 N2

(b) (i) Any **completely** correct expression (accept absence of dx) A2

eg $\int_{-1}^{2}\left(-\frac{3}{4}x^2+x+4\right)\mathrm{d}x, \left[-\frac{1}{4}x^3+\frac{1}{2}x^2+4x\right]_{-1}^{2}$ N2

(ii) Area = $\frac{45}{4}\ (=11.25)$ (accept 11.3) A1 N1

(iii) Attempting to **use** the formula for the volume (M1)

eg $\int_{-1}^{2}\pi\left(-\frac{3}{4}x^2+x+4\right)\mathrm{d}x, \pi\int_{-1}^{2}\left(-\frac{3}{4}x^2+x+4\right)^2\mathrm{d}x$ A2 N3

(c) $\int_{1}^{k} f(x)\,\mathrm{d}x=\left[-\frac{1}{4}x^3+\frac{1}{2}x^2+4x\right]_{1}^{k}$ A1A1A1

Note: *Award A1 for $-\frac{1}{4}x^3$, A1 for $\frac{1}{2}x^2$, A1 for 4x.*

Substituting $\left(-\frac{1}{4}k^3+\frac{1}{2}k^2+4k\right)-\left(-\frac{1}{4}+\frac{1}{2}+4\right)$ (M1)(A1)

$= -\frac{1}{4}k^3+\frac{1}{2}k^2+4k-4.25$ A1 N3

[21]

54.)

	Graph	Diagram

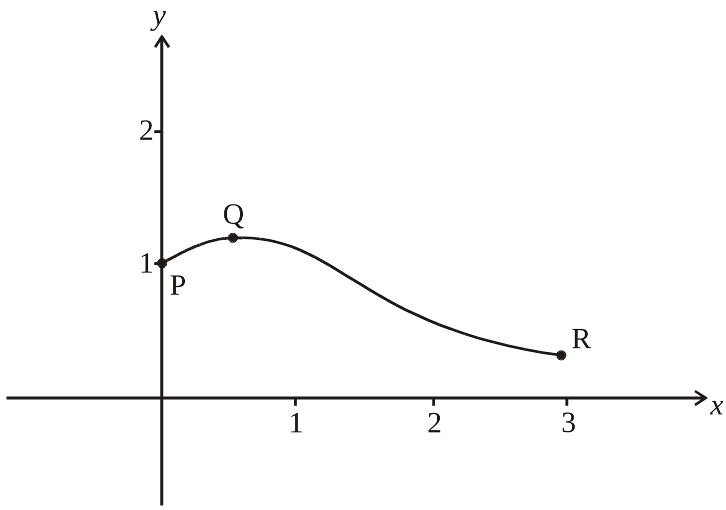
(a)	fâ (x)	I
(b)	f ð (x)	IV

A3 N3

A3 N3

[6]

55.) (a)



A1A1A1 N3

***Note**: Award A1 for the shape of the curve,*
A1 for correct domain,
*A1 for labelling **both** points P and Q in approximately correct positions.*

(b) (i) Correctly finding derivative of $2x + 1$ *ie* 2 (A1)

Correctly finding derivative of e^{-x} *ie* $-e^{-x}$ (A1)

Evidence of using the product rule (M1)

$f'(x) = 2e^{-x} + (2x + 1)(-e^{-x})$ A1

$= (1 - 2x)e^{-x}$ AG N0

(ii) At **Q**, $f'(x) = 0$ (M1)

$x = 0.5$, $y = 2e^{-0.5}$ A1A1

Q is $(0.5, 2e^{-0.5})$ N3

(c) $1 \leq k < 2e^{-0.5}$ A2 N2

(d) Using f ð (x) = 0 at the point of inflexion M1

$e^{-x}(-3 + 2x) = 0$

This equation has only one root. R1

So f has only one point of inflexion. AG N0

(e) At R, $y = 7e^{-3}$ (= 0.34850 ...) (A1)

Gradient of (PR) is $\dfrac{7e^{-3} - 1}{3}$ $(= -0.2172)$ (A1)

Equation of (PR) is $g\,(x) = \left(\frac{7e^{-3}-1}{3}\right)x+1\left(=-0.2172x+1\right)$ A1

Evidence of appropriate method, involving subtraction of integrals or areas M2

Correct limits/endpoints A1

eg $\int_0^3 (f(x)-g(x))\,dx$, area under curve − area under PR

Shaded area is $\int_0^3 \left((2x+1)e^{-x} - \left(\frac{7e^{-3}-1}{3}x+1\right)\right)dx$

= 0.529 A1 N4

[21]

56.) (a) Using the chain rule (M1)
$f'(x)$ =(2 cos (5x–3)) 5 (= 10 cos (5x – 3)) A1
$f''(x)$ = –(10 sin (5x–3)) 5
= –50 sin (5x – 3) A1A1 4

***Note**: Award (A1) for sin (5x – 3), (A1) for –50.*

(b) $\int f(x)dx = \frac{2}{5}\cos(5x-3)+c$ A1A1 2

***Note**: Award (A1) for cos (5x–3), (A1) for* $-\frac{2}{5}$.

[6]

57.) (a) (i) $f'(x) = -x + 2$ A1

(ii) $f'(0) = 2$ A1 2

(b) Gradient of tangent at y-intercept = $f'(0)$ = 2

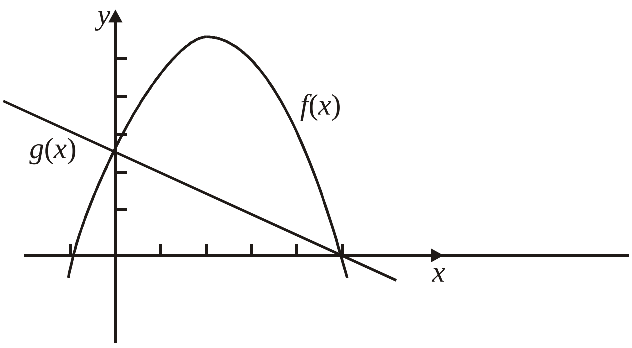
$\Rightarrow$ gradient of normal = $\frac{1}{2}$ (= –0.5) A1

Finding y-intercept is 2.5 A1
Therefore, equation of the normal is

y – 2.5 = ~(x – 0) (y – 2.5 = –0.5x) M1
(y = –0.5x + 2.5 (AG) 3

(c) (i) **EITHER**
solving $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$ (M1)A1
$\Rightarrow x = 0$ or $x = 5$ A1 2

OR



M1

Curves intersect at $x = 0$, $x = 5$ (A1)

So solutions to $f(x) = g(x)$ are $x = 0$, $x = 5$ A1 2

OR

$\Rightarrow 0.5x^2 - 2.5x = 0$ (A1)

$\Rightarrow -0.5x(x - 5) = 0$ M1

$\Rightarrow x = 0$ or $x = 5$ A1 2

(ii) Curve and normal intersect when $x = 0$ or $x = 5$ (M2)

Other point is when $x = 5$

$\Rightarrow y = -0.5(5) + 2.5 = 0$ (so other point (5, 0)) A1 2

(d) (i)Area = $\int_0^5 (f(x) - g(x))\mathrm{d}x \left(\text{or } \int_0^5 (-0.5x^2 + 2x + 2.5)\mathrm{d}x - \frac{1}{2} \times 5 \times 2.5\right)$

A1A1A1 3

Note*: Award (A1) for the integral, (A1) for both correct limits on the integral, and (A1) for the difference.*

(ii) Area = Area under curve – area under line ($A = A_1 - A_2$) (M1)

(A1) = $\frac{50}{3}, A_2 = \frac{25}{4}$

Area = $\frac{50}{3} - \frac{25}{4} = \frac{125}{12}$ (or 10.4 (3sf) A1 2

[16]

58.) (a) $x = 1$ (A1) 1

(b) Using quotient rule (M1)

Substituting correctly $g'(x) = \dfrac{(x-1)^2(1) - (x-2)[2(x-1)]}{(x-1)^4}$ A1

$= \dfrac{(x-1) - (2x-4)}{(x-1)^3}$ (A1)

$= \dfrac{3-x}{(x-1)^3}$ (Accept $a = 3$, $n = 3$) A1 4

(c) Recognizing at point of inflexion $g''(x) = 0$ M1

$x = 4$ A1

Finding corresponding y-value = $\frac{2}{9}$ = 0.222 *ie* $\mathrm{P}\left(4, \frac{2}{9}\right)$ A1 3

[8]

59.) (a) $f'(x)=5(3x+4)^4\times 3$ $\left(=15(3x+4)^4\right)$ (A1)(A1)(A1) (C3)

(b) $\int(3x+4)^5\,\mathrm{d}x=\frac{1}{3}\times\frac{1}{6}(3x+4)^6+c$ $\left(=\frac{(3x+4)^6}{18}+c\right)$ (A1)(A1)(A1) (C3)

[6]

60.) (a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)+g(x)\right)=f'(x)+g'(x)$ $\left(=f'(4)+g'(4)\right)$ (M1)

$=7+4$

$=11$ (A1) (C2)

(b) $\int_1^3\left(g'(x)+6\right)\mathrm{d}x=\left[g(x)\right]_1^3+\left[6x\right]_1^3$ (A1)(A1)

$=\left(g(3)-g(1)\right)+\left(18-6\right)$ $\left(=(2-1)+12\right)$ (A1)

$=13$ (A1) (C4)

[6]

61.) (a) (i) $p=-2$, $q=4$ (or $p=4, q=-2$) (A1)(A1) (N1)(N1)

(ii) $y=a(x+2)(x-4)$

$8=a(6+2)(6-4)$ (M1)

$8=16a$

$a=\frac{1}{2}$ (A1) (N1)

(iii) $y=\frac{1}{2}(x+2)(x-4)$

$y=\frac{1}{2}(x^2-2x-8)$

$y=\frac{1}{2}x^2-x-4$ (A1) (N1)5

(b) (i) $\frac{\mathrm{d}y}{\mathrm{d}x}=x-1$ (A1)(N1)

(ii) $x-1=7$ (M1)

$x=8,\ y=20$ $\left(\text{P is }(8, 20)\right)$ (A1)(A1) (N2)4

(c) (i) when x = 4, gradient of tangent is 4 – 1 = 3 (may be implied) (A1)

gradient of normal is $-\frac{1}{3}$ (A1)

$y-0=-\frac{1}{3}(x-4)$ $\left(y=-\frac{1}{3}x+\frac{4}{3}\right)$ (A1) (N3)

(ii) $\frac{1}{2}x^2-x-4=-\frac{1}{3}x+\frac{4}{3}$ (or sketch/graph) (M1)

$\frac{1}{2}x^2-\frac{2}{3}x-\frac{16}{3}=0$

$3x^2-4x-32=0$ (may be implied) (A1)

$(3x+8)(x-4)=0$

$x=-\frac{8}{3}$ or $x=4$

$x=-\frac{8}{3}$ (-2.67) (A1) (N2)6

[15]

62.) (a) $x=\frac{1}{5}$ or $5x-1=0$ (A1) (N1) 1

(b) $f'(x)=\frac{(5x-1)(6x)-(3x^2)(5)}{(5x-1)^2}$ (M1)(A1)

$=\frac{30x^2-6x-15x^2}{(5x-1)^2}$ (may be implied) (A1)

$=\frac{15x^2-6x}{(5x-1)^2}$ (accept a = 15, b = –6) (A1) (N2)4

[5]

63.) (a) $x = 1$ (A1)

EITHER

The gradient of $g(x)$ goes from positive to negative (R1)

OR

$g(x)$ goes from increasing to decreasing (R1)

OR

when $x=1$, $g'(x)$ is negative (R1) 2

(b) $-3<x<-2$ **and** $1<x<3$ (A1)

$g'(x)$ is negative (R1) 2

(c) $x=-\frac{1}{2}$ (A1)

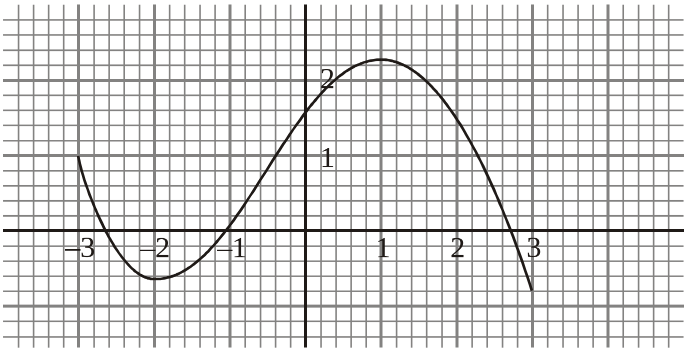
EITHER

$g'(x)$ changes from positive to negative (R1)

OR

concavity changes (R1) 2

(d)



(A3) 3

[9]

64.) (a) $f'(x) = 3(2x + 7)^2 \times 2$ (A1)(A1)

$= 6(2x + 7)^2$ $(= 24x^2 + 168x + 294)$ (C2)

(b) $g'(x) = 2\cos(4x)(-\sin(4x))(4)$ (A1)(A1)(A1)(A1)

$= -8\cos(4x)\sin(4x)$ $(= -4\sin(8x))$ (C4)

[6]

65.) (a) $f'(x) = 3x^2 - 4x - 0$ (A1)(A1)(A1)

$= 3x^2 - 4x$ (C3)

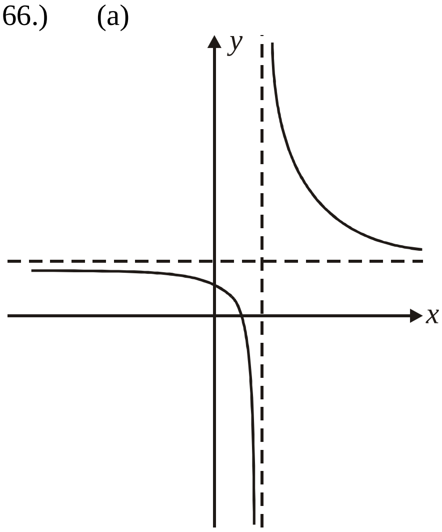
(b) Gradient $= f'(2)$ (M1)

$= 3 \times 4 - 4 \times 2$ (A1)

$= 4$ (A1) (C3)

[6]

66.) (a)



(A1)(A1) 2

Note: *Award (A1) for a second branch in approximately the correct position, and (A1) for the second branch having positive x and y intercepts. Asymptotes need not be drawn.*

(b) (i) x-intercept $= \frac{1}{2}\left(\text{Accept}\left(\frac{1}{2}, 0\right), x = \frac{1}{2}\right)$ (A1)

y-intercept = 1 (Accept (0, 1), $y = 1$) (A1)

(ii) horizontal asymptote $y = 2$ (A1)
vertical asymptote $x = 1$ (A1) 4

(c) (i) $f'(x) = 0 - (x-1)^{-2} \left(= \frac{-1}{(x-1)^2}\right)$ (A2)

(ii) no maximum / minimum points.
since $\frac{-1}{(x-1)^2} \neq 0$ (R1) 3

(d) (i) $2x + \ln(x-1) + c$ (accept $\ln|x-1|$)(A1)(A1)
(A1)

(ii) $A = \int_2^4 f(x)\mathrm{d}x \left(\text{Accept} \int_2^4 \left(2 + \frac{1}{x-1}\right)\mathrm{d}x, \left[2x + \ln(x-1)\right]_2^4\right)$ (M1)(A1)

Notes: *Award (A1) for* ***both*** *correct limits.*
Award (M0)(A0) for an incorrect function.

(iii) $A = \left[2x + \ln(x-1)\right]_2^4$
$= (8 + \ln 3) - (4 + \ln 1)$ (M1)
$= 4 + \ln 3 (= 5.10$, to 3 sf) (A1) (N2) 7

[16]

67.) **METHOD 1**

$f(x) = 6x^{\frac{2}{3}}$ (A2)

$f'(x) = 4x^{-\frac{1}{3}} \left(= \frac{4}{x^{\frac{1}{3}}} = \frac{4}{\sqrt[3]{x}}\right)$ (A2)(A2) (C6)

METHOD 2

$f(x) = 6(x^2)^{\frac{1}{3}}$ (A1)

$f'(x) = 6 \times \frac{1}{3}(x^2)^{-\frac{2}{3}} \times 2x$ (A2)(A2)

$f'(x) = 4x^{-\frac{1}{3}}$ (A1) (C6)

[6]

68.) (a) $p = 100\mathrm{e}^0$ (M1)
$= 100$ (A1) (C2)

(b) Rate of increase is $\frac{\mathrm{d}p}{\mathrm{d}t}$ (M1)

$\frac{\mathrm{d}p}{\mathrm{d}t} = 0.05 \times 100\mathrm{e}^{0.05t} \quad = 5\mathrm{e}^{0.05t}$ (A1)(A1)

When $t = 10$

$\frac{\mathrm{d}p}{\mathrm{d}t} = 5\mathrm{e}^{0.05(10)}$

$= 5e^{0.5} \quad \left(= 8.24 = 5\sqrt{e}\right)$ (A1) (C4)

[6]

69.) (a) (i) 1 (A1) (C1)

(ii) 2 (A1) (C1)

(iii) $f'(14) = f'(2)$ (or $f'(5)$ or $f'(8)$) (M1)

$= -1$ (A1) (C2)

(b) There are five repeated periods of the graph, each with two solutions, (*ie* number of solutions is 5 × 2) (R1)

$= 10$ (A1) (C2)

[6]

70.) (a) (i) $f'(x) = -6\sin 2x$ (A1)(A1)

(ii) **EITHER**

$f'(x) = -12\sin x\cos x = 0$

$\Rightarrow \sin x = 0$ or $\cos x = 0$ (M1)

OR

$\sin 2x = 0$,

for $0 \le 2x \le 2$ (M1)

THEN

$x = 0, \frac{\pi}{2}, \pi$ (A1)(A1)(A1) (N4) 6

(b) (i) translation (A1)

in the y-direction of –1 (A1)

(ii) 1.11 (1.10 from TRACE is subject to **AP**) (A2) 4

[10]

71.) (a) $y = 0$ (A1) 1

(b) $f'(x) = \frac{-2x}{(1+x^2)^2}$ (A1)(A1)(A1) 3

(c) $\frac{6x^2 - 2}{(1+x^2)^3} = 0$ (or sketch of $f'(x)$ showing the maximum) (M1)

$6x^2 - 2 = 0$ (A1)

$x = \pm\sqrt{\frac{1}{3}}$ (A1)

$x = \frac{-1}{\sqrt{3}} (= -0.577)$ (A1) (N4) 4

(d) $\int_{-0.5}^{0.5} \frac{1}{1+x^2}\,\mathrm{d}x \left(= 2 \int_{0}^{0.5} \frac{1}{1+x^2}\,\mathrm{d}x = 2 \int_{-0.5}^{0} \frac{1}{1+x^2}\,\mathrm{d}x \right)$ (A1)(A1) 2

[10]

72.) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{\frac{x}{3}}\right) = \frac{1}{3}\mathrm{e}^{\frac{x}{3}}$ (A1)(A1)

$\frac{\mathrm{d}}{\mathrm{d}x}(5 \cos^2 x) = -10 \cos x \sin x$ (A1)(A1)(A1)

$f'(x) = \frac{1}{3}\mathrm{e}^{\frac{x}{3}} - 10 \cos x \sin x$ (A1) (C6)

[6]

73.) (a) (i) $\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ (A1)

therefore $\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right) = 0$ (AG)

(ii) $\cos x + \sin x = 0 \Rightarrow 1 + \tan x = 0$

$\Rightarrow \tan x = -1$ (M1)

$x = \frac{3\pi}{4}$ (A1)

Note: *Award (A0) for 2.36.*

OR

$x = \frac{3\pi}{4}$ (G2) 3

(b) $y = \mathrm{e}^x(\cos x + \sin x)$

$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x(\cos x + \sin x) + \mathrm{e}^x(-\sin x + \cos x)$ (M1)(A1)(A1) 3

$= 2\mathrm{e}^x \cos x$

(c) $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ for a turning point $\Rightarrow 2\mathrm{e}^x \cos x = 0$ (M1)

$\Rightarrow \cos x = 0$ (A1)

$\Rightarrow x = \frac{\pi}{2} \Rightarrow a = \frac{\pi}{2}$ (A1)

$y = \mathrm{e}^{\frac{\pi}{2}}\left(\cos \frac{\pi}{2} + \sin \frac{\pi}{2}\right) = \mathrm{e}^{\frac{\pi}{2}}$

$b = \mathrm{e}^{\frac{\pi}{2}}$ (A1) 4

Note: *Award (M1)(A1)(A0)(A0) for $a = 1.57$, $b = 4.81$.*

(d) At D, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$ (M1)

$2\mathrm{e}^x \cos x - 2\mathrm{e}^x \sin x = 0$ (A1)

$2\mathrm{e}^x (\cos x - \sin x) = 0$

$\Rightarrow \cos x - \sin x = 0$ (A1)

$\Rightarrow x = \dfrac{\pi}{4}$ (A1)

$\Rightarrow y = e^{\frac{\pi}{4}}(\cos \dfrac{\pi}{4} + \sin \dfrac{\pi}{4})$ (A1)

$= \sqrt{2}\, e^{\frac{\pi}{4}}$ (AG) 5

(e) Required area = $\int_0^{\frac{3}{4}} e^x (\cos x + \sin x)dx$ (M1)

$= 7.46$ sq units (G1)

OR

Area = 7.46 sq units (G2) 2

***Note**: Award (M1)(G0) for the answer 9.81 obtained if the calculator is in degree mode.*

[17]

74.) (a) (i) $f'(x) = -2e^{-2x}$ (A1)

(ii) $f'(x)$ is always negative (R1) 2

(b) (i) $y = 1 + e^{-2\times-\frac{1}{2}}$ (= 1 + e) (A1)

(ii) $f'\left(-\dfrac{1}{2}\right) = -2e^{-2\times-\frac{1}{2}}$ (= –2e) (A1) 2

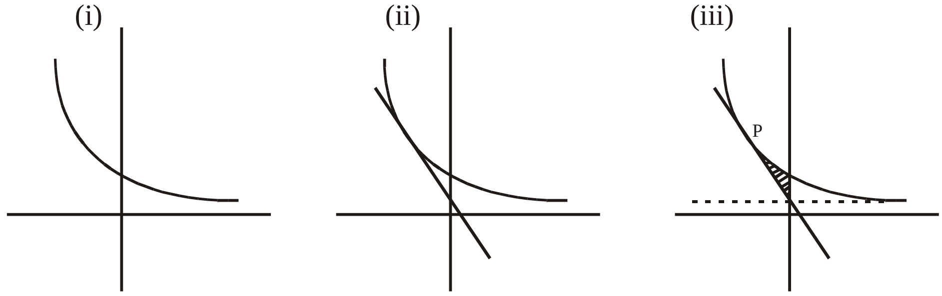
***Note:** In part (b) the answers do not need to be simplified.*

(c) $y - (1 + e) = -2e\left(x + \dfrac{1}{2}\right)$ (M1)

$y = -2ex + 1$ ($y = -5.44\,x + 1$) (A1)(A1) 3

(d)

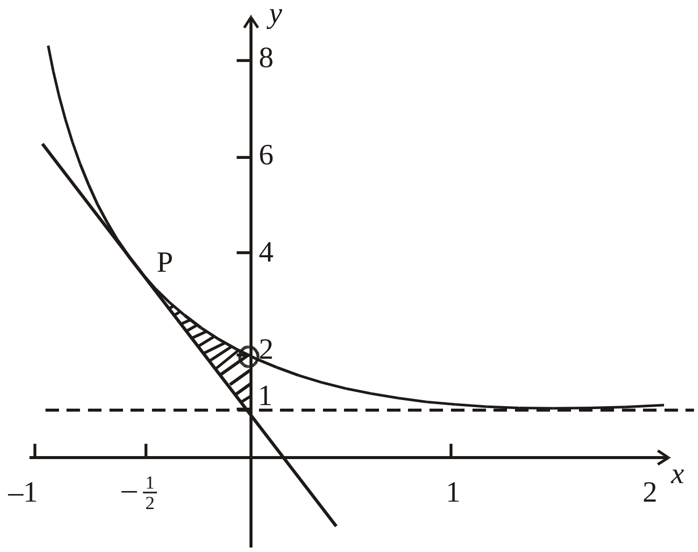
(i) (ii) (iii)



(A1)(A1)(A1)

***Notes:** Award (A1) for each correct answer. Do **not** allow **(ft)** on an incorrect answer to part (i). The correct final diagram is shown below. Do not penalize if the horizontal asymptote is missing. Axes do not need to be labelled.*

(i)(ii)(iii)



(iv) Area = $\int_{-\frac{1}{2}}^{0} [(1+e^{-2x}) - (-2ex+1)]dx$ (or equivalent) (M1)(M1)

Notes: *Award (M1) for the limits, (M1) for the function.*
Accept difference of integrals as well as integral of difference.
Area below line may be calculated geometrically.

Area = $\int_{-\frac{1}{2}}^{0} [(e^{-2x} + 2ex)dx$

$= \left[-\frac{1}{2}e^{-2x} + ex^2\right]_{-\frac{1}{2}}^{0}$ (A1)

= 0.1795 …= 0.180 (3 sf) (A1)

OR

Area = 0.180 (G2) 7

[14]

75.) ***Note***: *Do not penalize missing units in this question.*

(a) (i) At release(P), $t = 0$ (M1)
$s = 48 + 10 \cos 0$
$= 58$ cm below ceiling (A1)

(ii) $58 = 48 + 10 \cos 2\pi t$ (M1)
$\cos 2\pi t = 1$ (A1)
$t = 1$sec (A1)

OR

$t = 1$sec (G3) 5

(b) (i) $\frac{ds}{dt} = -20\pi \sin 2\pi t$(A1)(A1)

Note: *Award (A1) for –20π, and (A1) for sin 2pt.*

(ii) $v = \frac{ds}{dt} = -20\pi \sin 2\pi t = 0$ (M1)
$\sin 2\pi t = 0$
$t = 0, \frac{1}{2}$ ... (at least 2 values) (A1)

$s = 48 + 10 \cos 0$ **or** $s = 48 + 10 \cos \pi$ (M1)

$= 58$ cm (at P) $= 38$ cm (20 cm above P) (A1)(A1) 7

Note: *Accept these answers without working for full marks. May be deduced from recognizing that amplitude is 10.*

(c) $48 + 10 \cos 2\pi t = 60 + 15 \cos 4\pi t$ (M1)

$t = 0.162$ secs(A1)

OR

$t = 0.162$ secs (G2) 2

(d) 12 times (G2) 2

Note: *If either of the correct answers to parts (c) and (d) are missing and suitable graphs have been sketched, award (G2) for sketch of suitable graph(s); (A1) for t = 0.162; (A1) for 12.*

[16]

76.) (a) $x = 1$ (A1) 1

(b) (i) $f(-1000) = 2.01$ (A1)

(ii) $y = 2$ (A1) 2

(c) $f'(x) = \dfrac{(x-1)^2(4x-13) - 2(x-1)(2x^2 - 13x + 20)}{(x-1)^4}$ (A1)(A1)

$= \dfrac{(4x^2 - 17x + 13) - (4x^2 - 26x + 40)}{(x-1)^3}$ (A1)

$= \dfrac{9x - 27}{(x-1)^3}$ (AG) 3

Notes: *Award (M1) for the* ***correct*** *use of the quotient rule, the first (A1) for the placement of the correct expressions into the quotient rule.*
Award the second (A1) for doing sufficient simplification to make the given answer reasonably obvious.

(d) $f'(3) = 0$ $\Rightarrow$ stationary (or turning) point (R1)

$f''(3) = \dfrac{18}{16} > 0$ $\Rightarrow$ minimum (R1) 2

(e) Point of inflexion $\Rightarrow f''(x) = 0 \Rightarrow x = 4$ (A1)

$x = 4 \Rightarrow y = 0$ $\Rightarrow$ Point of inflexion = (4, 0) (A1)

OR

Point of inflexion = (4, 0) (G2) 2

[10]

77.)

Function	Derivative diagram	
f_1	(d)	(AG)
f_2	(e)	(A2)
f_3	(b)	(A2)

f_4	(a)

(A2)

(C6)

[6]

78.) (a) $\int_0^1 e^{-kx}\,dx = \left[-\frac{1}{k}e^{-kx}\right]_0^1$ (A1)

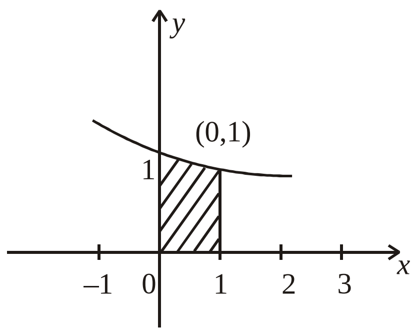
$= -\frac{1}{k}(e^{-k} - e^{0})$ (A1)

$= -\frac{1}{k}(e^{-k} - 1)$ (A1)

$= -\frac{1}{k}(1 - e^{-k})$ (AG) 3

(b) $k = 0.5$

(i)



(A2)

Note: *Award (A1) for shape, and (A1) for the point (0,1).*

(ii) Shading (see graph) (A1)

(iii) Area = $\int_0^1 e^{-kx}\,dx$ for k = 0.5 (M1)

$= \frac{1}{0.5}(1 - e^{0.5})$

= 0.787 (3 sf) (A1)

OR

Area = 0.787 (3 sf) (G2) 5

(c) (i) $\frac{dy}{dx} = -ke^{-kx}$ (A1)

(ii) $x = 1$ $y = 0.8$ $\Rightarrow 0.8 = e^{-k}$ (A1)

$\ln 0.8 = -k$

$k = 0.223$ (A1)

(iii) At $x = 1$ $\frac{dy}{dx} = -0.223e^{-0.223}$ (M1)

= –0.179 (accept –0.178) (A1)

OR

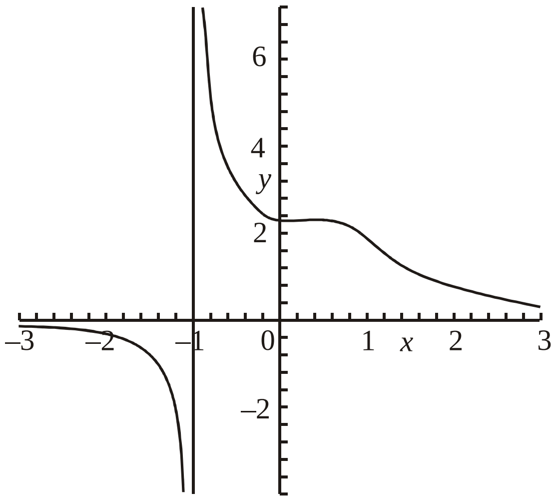
$\frac{dy}{dx} = -0.178$ or -0.179 (G2) 5

[13]

79.) (a) (i) Vertical asymptote $x = -1$ (A1)

(ii) Horizontal asymptote $y = 0$ (A1)

(iii)



(A1)(A1)

Note: *Award (A1) for each branch.*

4

(b) (i) $f'(x) = \dfrac{-6x^2}{\left(1+x^3\right)^2}$

$f''(x) = \dfrac{\left(1+x^3\right)^2(-12x) + 6x^2(2)(1+x^3)^1\left(3x^2\right)}{\left(1+x^3\right)^4}$ (M1)

$= \dfrac{\left(1+x^3\right)(-12x) + 36x^4}{\left(1+x^3\right)^3}$ (A1)

$= \dfrac{-12 - 12x^4 + 36x^4}{\left(1+x^3\right)^3}$ (A1)

$= \dfrac{12x\left(2x^3 - 1\right)}{\left(1+x^3\right)^3}$ (AG)

(ii) Point of inflexion => $f''(x) = 0$ (M1)

=> $x = 0$ or $x = \sqrt[3]{\dfrac{1}{2}}$

$x = 0$ or $x = 0.794$ (3 sf) (A1)(A1)

OR

$x = 0$, $x = 0.794$ (G1)(G2) 6

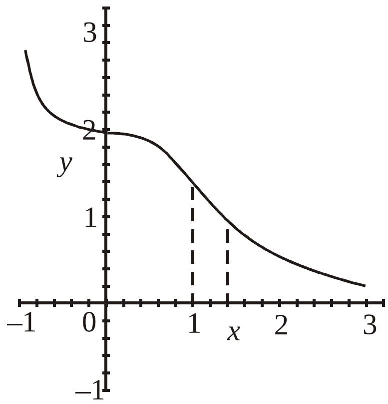
(c) (i) Approximate value of $\int_1^3 f(x)\,dx$, $h = \dfrac{b-a}{n} = \dfrac{2}{5}$ (A1)

$= \dfrac{1}{5}$ [1 + 1.068377 + ... + 0.215332 + 0.071429] (A1)

$= \dfrac{1}{5}$ (3.284025)

$= 0.656805$ (A1)

(ii) $\int_1^3 f(x)\,\mathrm{d}x = 0.637599$



(A1)

Between 1 and 3, the graph is 'concave up', so that the straight lines forming the trapezia are all **above** the graph. (R1) 5

[15]

80.) $f(x) = x^{\frac{3}{2}}$ (M1)

(a) $f'(x) = \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}x^{\frac{1}{2}}$ (or $\frac{3}{2}\sqrt{x}$) (M1)(A1) (C3)

(b) $\int x^{\frac{3}{2}}\mathrm{d}x = \frac{1}{\frac{3}{2}+1}x^{\frac{3}{2}+1} + c$ (M1)

$= \frac{2}{5}x^{\frac{5}{2}} + c$ (or $\frac{2}{5}\sqrt{x^5} + c$) (A1)(A1) (C3)

Notes: *Do not penalize the absence of c.*

Award (A1) for $\frac{5}{2}$ *and (A1) for* $x^{\frac{5}{2}}$.

[6]

81.) (a) At A, $x = 0$ => $y = \sin(e^0) = \sin(1)$ (M1)
=> coordinates of A = (0,0.841) (A1)
OR
A(0, 0.841) (G2) 2

(b) $\sin(e^x) = 0$ => $e^x = \pi$ (M1)
=> $x = \ln \pi$ (or $k = \pi$) (A1)
OR
$x = \ln \pi$ (or $k = \pi$) (A2) 2

(c) (i) Maximum value of sin function = 1 (A1)

(ii) $\frac{\mathrm{d}y}{\mathrm{d}x} = e^x \cos(e^x)$ (A1)(A1)

Note: *Award (A1) for cos* (e^x) *and (A1) for* e^x.

(iii) $\frac{dy}{dx} = 0$ at a maximum (R1)

$e^x \cos(e^x) = 0$

$\Rightarrow e^x = 0$ (impossible) or $\cos(e^x) = 0$ (M1)

$\Rightarrow e^x = \frac{\pi}{2} \Rightarrow x = \ln \frac{\pi}{2}$ (A1)(AG) 6

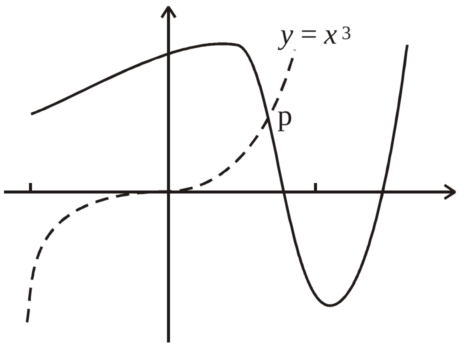
(d) (i) Area = $\int_0^{\ln \pi} \sin(e^x)\,dx$ (A1)(A1)

(A1)

Note: *Award (A1) for 0, (A1) for ln π, (A1) for sin* (e^x).

(ii) Integral = 0.90585 = 0.906 (3 sf) (G2) 5

(e)



(M1)

At P, x = 0.87656 = 0.877 (3 sf) (G2) 3

[18]

82.) (a) $\frac{dy}{dx} = (2x)\left[\frac{1}{2}(1+x^2)^{-\frac{1}{2}}(2x)\right] + (2)(1+x^2)^{\frac{1}{2}}$ (M1)(M1)

Note: *Award (M1) for correct use of product rule, (M1) for correct use of chain rule.*

$\frac{dy}{dx} = \frac{2x^2}{\sqrt{1+x^2}} + 2\sqrt{1+x^2}$ (A1) 3

(b) $u = 1 + x^2 \Rightarrow \frac{du}{dx} = 2x$ (or $du = 2x\,dx$) (M1)

$\Rightarrow \int 2x\sqrt{1+x^2}\,dx = \int u^{\frac{1}{2}}\left(\frac{du}{dx}\right)dx = \int u^{\frac{1}{2}}\,du$ (M1)

$= \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{3}(1 + x^2)^{\frac{3}{2}} + C$ (A1)(AG) 3

Note: *Accept proof by differentiation.*

(c) $R = \int_0^k 2x\sqrt{1+x^2}\,\mathrm{d}x = \left[\frac{2}{3}\left(1+x^2\right)^{3/2}\right]_0^k = 1$ (M1)

$\Rightarrow \frac{2}{3}\left(1+k^2\right)^{\frac{3}{2}} - \frac{2}{3}(1) = 1 \Rightarrow (1+k^2)^{\frac{3}{2}} = \frac{5}{2}$ (M1)

$\Rightarrow k = 0.9176 = 0.918$ (A1)

OR

$k = 0.918$ (G3) 3

[9]

83.) (a) (i) $f'(x) = \dfrac{\left(x \times \frac{1}{2x} \times 2\right) - (\ln 2x \times 1)}{x^2}$ (M1)(M1)

Note: *Award (M1) for the correct use of the quotient rule and (M1) for correct substitution.*

$= \dfrac{1 - \ln 2x}{x^2}$ (AG)

(ii) $f'(x) = 0$ for max/min. (R1)

$\dfrac{1 - \ln 2x}{x^2} = 0$ only at 1 point, when $x = \dfrac{\mathrm{e}}{2}$ (R1)

Note: *Award no marks if the reason given is of the sort "by looking at the graph".*

(iii) Maximum point when $f'(x) = 0$.

$f'(x) = 0$ for $x = \dfrac{\mathrm{e}}{2}$ $(= 1.36)$ (A1)

$y = f\left(\dfrac{\mathrm{e}}{2}\right) = \dfrac{2}{\mathrm{e}}$ $(= 0.736)$ (A1) 6

Note: *Award (A1) per correct coordinate if the answer is found using the GDC, regardless of method. If one or both coordinates are wrong, you may award up to 1 mark for method.*

(b) $f''(x) = \dfrac{-\frac{1}{2x} \times 2 \times x^2 - (1 - \ln 2x)2x}{x^4}$ (M1)(M1)

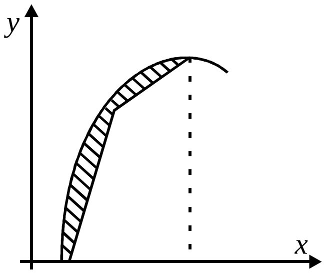
$= \dfrac{2\ln 2x - 3}{x^3}$ (AG)

Inflexion point $\Rightarrow f''(x) = 0$ (M1)

$\Rightarrow 2\ln 2x = 3$ (M1)

$x = \dfrac{\mathrm{e}^{1.5}}{2}$ $(= 2.24)$ (A1) 6

(c) (i) The trapezium rule would underestimate the area of *S*. (A1)



Shaded area not included when using the trapezium rule
(or similar reasonable explanation). (R2) 3

(ii) $u = \ln 2x$; $\mathrm{d}u = \dfrac{1}{2x} \times 2\mathrm{d}x = \dfrac{1}{x}\,\mathrm{d}x$ (M1)

$\int \dfrac{\ln 2}{x}\mathrm{d}x = \int u\mathrm{d}u$ (M1)

$= \dfrac{u^2}{2} + C$ (A1)

$= \dfrac{(\ln 2x)^2}{2} + C$ (A1) 4

(iii) Area of $S = \int_{0.5}^{\frac{e}{2}} \dfrac{\ln 2x}{x}\,\mathrm{d}x$ (M1)(A1)

Note: ***Award (M1) for the integral expression,
and (A1) for the limits.*** *(M1)*

$= \dfrac{\left(\ln 2\left(\frac{e}{2}\right)\right)^2}{2} - \dfrac{(\ln(2 \times 0.5))^2}{2}$ (M1)

$= \dfrac{1}{2} - 0 = \dfrac{1}{2}$ (A1) 4

Note: *Award only (A1)(M0)(M0)(A1) if the area (to 3 sf or
exactly) is found on the GDC.*

(d) (i) If $x_1 = 1$, then $x_2 = -1.26$ (M1)
$f(x_2) = f(-1.26)$ does not exist, so x_3 cannot be calculated. (R2) 3

(ii) $x_2 = 0.4 - \dfrac{f(0.4)}{f'(0.4)} = 0.47297$ (A1)

Absolute error = $|0.5 - 0.47297| = 0.02703$ (A1)

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = 0.49787$ (A1)

Absolute error = $|0.5 - 0.49787| = 0.00213$ (A1) 4
which is less than 0.01.

Notes: *Absolute errors need not be explicitly given.
Award (A3) if further terms are listed, without stating that they
are unnecessary.*

[30]

84.) (a) $f'(x) = k\cos x + 3$ (A1)(A1) (C2)

(b) $k\cos\left(\dfrac{\pi}{3}\right) + 3 = 8$ (M1)

$\Rightarrow k\left(\frac{1}{2}\right) + 3 = 8$

$\Rightarrow k = 10$ (A1) (C2)

[4]

85.) (a) (i) $v(0) = 50 - 50e^0 = 0$ (A1)

(ii) $v(10) = 50 - 50e^{-2} = 43.2$ (A1) 2

(b) (i) $a = \frac{dv}{dt} = -50(-0.2e^{-0.2t})$ (M1)

$= 10e^{-0.2t}$ (A1)

(ii) $a(0) = 10e^0 = 10$ (A1) 3

(c) (i) $t \to \infty \Rightarrow v \to 50$ (A1)

(ii) $t \to \infty \Rightarrow a \to 0$ (A1)

(iii) when $a = 0$, v is constant at 50 (R1) 3

(d) (i) $y = \int v\,dt$ (M1)

$= 50t - \frac{e^{-0.2t}}{-0.2} + k$ (A1)

$= 50t + 250e^{-0.2t} + k$ (AG)

(ii) $0 = 50(0) + 250e^0 + k = 250 + k$ (M1)

$\Rightarrow k = -250$ (A1)

(iii) Solve $250 = 50t + 250e^{-0.2t} - 250$ (M1)

$\Rightarrow 50t + 250e^{-0.2t} - 500 = 0$

$\Rightarrow t + 5e^{-0.2t} - 10 = 0$

$\Rightarrow t = 9.207\ s$ (G2) 7

[15]

86.) (a) (i) $x = -\frac{5}{2}$ (A1)

(ii) $y = \frac{3}{2}$ (A1) 2

(b) By quotient rule (M1)

$\frac{dy}{dx} = \frac{(2x+5)(3)-(3x-2)(2)}{(2x+5)^2}$ (A1)

$= \frac{19}{(2x+5)^2}$ (A1) 3

(c) There are no points of inflexion. (A1) 1

[6]

87.) $f(1) = 1^2 - 3b + c + 2 = 0$ (M1)

$f'(x) = 2x - 3\text{b}$,
$f'(3) = 6 - 3b = 0$ (M1)

$3b = 6$, $b = 2$ (A1)

$1 - 3(2) + c + 2 = 0$, $c = 3$ (A1)

Note: *In the event of no working shown, award (C2) for 1 correct answer.*

[4]

88.) (a) (i) ***Note:*** *Range of f = {y : 0 á y á 2} (graphic display calculator)*
So let a = 0 – Č, b = 2 + Č, with 0 < Č < 2

For example, $a = -1$ $b = 3$ *etc*. (A1)(A1)

(ii) As $x \to \infty$, $\dfrac{2x}{1+x^2} \to 0$, $f(x) \to 1$; $y = 1$ (A1) 3

(b) $f'(x) = \dfrac{\text{d}}{\text{d}x}\left(1 - \dfrac{2x}{1+x^2}\right)$

$= 0 - \left(\dfrac{(1+x^2)\times(2) - (2x)(2x)}{(1+x^2)^2}\right)$ (A1)(A1)(A1)

$= \dfrac{4x^2 - 2(1+x^2)}{(1+x^2)^2}$ (A1)

$= \dfrac{2x^2 - 2}{(1+x^2)^2}$ (AG) 4

(c) $\left.\begin{aligned} f'(x) = 0 &\Leftrightarrow 2x^2 - 2 = 0 \\ &\Leftrightarrow x = \pm 1 \end{aligned}\right\}$ (M1)

From graphic display calculator inspection, or $f'(x)$ on each side of -1, max when $x = -1$ (M1)

$f(-1) = 1 - \dfrac{-2}{1+1} = 1 + 1 = 2$
$(-1, 2)$ (A1) 3

(d) (i) $\displaystyle\int f(x)\text{d}x = \int\left(1 - \frac{2x}{1+x^2}\right)\text{d}x$

$= x - \displaystyle\int \frac{1}{u}\,\text{d}u$ (A1)(M1)

Note: *Award (A1) for x, and (M1) for* $\displaystyle\int \frac{1}{u}$ *du.*

$= x - \ln u + C$ (M1)

Notes: *Award (M1) for ln u or award (A2) by inspection.*

$= x - \ln(1 + x^2) + C$ (A1) 4

Note: *Award (A1) for ln(1 +* x^2*).*

(ii) Area $= \displaystyle\int_0^1 f(x)\text{d}x$ (A1)

Note: *Award (A1) for upper and lower limits.*

$= [x - \ln(1 + x^2)]_0^1$ (M1)

$= (1 - 0) - (\ln 2 - \ln 1)$ (A1)

$= 1 - \ln 2$ (A1) 4

Note: *Award (A0) for 0.307*

[18]

89.) (a) $\frac{f(5+h) - f(5)}{h} = \frac{(5.1)^3 - 5^3}{0.1}$

$= 76.51$ (or 76.5 to 3 sf) (A1) (C1)

(b) $\lim_{h \to 0} \frac{f(5+h) - f(5)}{h} = f'(5)$ (M1)

$= 3(5)^2$ (A1)

$= 75$ (A1) (C3)

[4]

90.) (a) (i) $t = 0$ $s = 800$

$t = 5$ $s = 800 + 500 - 100 = 1200$ (M1)

distance in first 5 seconds $= 1200 - 800$

$= 400$ m (A1) 2

(ii) $v = \frac{\mathrm{d}s}{\mathrm{d}t} = 100 - 8t$ (A1)

At $t = 5$, velocity $= 100 - 40$ (M1)

$= 60$ m s^{-1} (A1) 3

(iii) Velocity $= 36$ m s^{-1} $\Rightarrow 100 - 8t = 36$ (M1)

$t = 8$ seconds after touchdown. (A1) 2

(iv) When $t = 8$, $s = 800 + 100(8) - 4(8)^2$ (M1)

$= 800 + 800 - 256$ (A1)

$= 1344$ m (A1) 3

(b) If it touches down at P, it has $2000 - 1344 = 656$ m to stop. (M1)

To come to rest, $100 - 8t = 0 \Rightarrow t = 12.5$ s (M1)

Distance covered in 12.5 $s = 100(12.5) - 4(12.5)^2$ (M1)

$= 1250 - 625$

$= 625$ (A1)

Since $625 < 656$, it can stop safely. (R1) 5

[15]

91.) (a) $y = \mathrm{e}^{2x} \cos x$

$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{2x}(-\sin x) + \cos x\,(2\mathrm{e}^{2x})$ (A1)(M1)

$= \mathrm{e}^{2x}(2\cos x - \sin x)$ (AG) 2

(b) $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2\mathrm{e}^{2x}(2\cos x - \sin x) + \mathrm{e}^{2x}(-2\sin x - \cos x)$ (A1)(A1)

$= \mathrm{e}^{2x}(4\cos x - 2\sin x - 2\sin x - \cos x)$ (A1)

$= \mathrm{e}^{2x}(3\cos x - 4\sin x)$ (A1) 4

(c) (i) At P, $\frac{d^2y}{dx^2} = 0$ (R1)

$\Rightarrow 3\cos x = 4\sin x$ (M1)

$\Rightarrow \tan x = \frac{3}{4}$

At P, $x = a$, *ie* $\tan a = \frac{3}{4}$ (A1)

(ii) The gradient at any point $e^{2x}(2\cos x - \sin x)$ (M1)

Therefore, the gradient at P = $e^{2a}(2\cos a - \sin a)$

When $\tan a = \frac{3}{4}$, $\cos a = \frac{4}{5}$, $\sin a = \frac{3}{5}$ (A1)(A1)

(by drawing a right triangle, or by calculator)

Therefore, the gradient at P = $e^{2a}\left(\frac{8}{5} - \frac{3}{5}\right)$ (A1)

$= e^{2a}$ (A1) 8

[14]

92.) (a) $f'(x) = 3(2x + 5)^2 \times 2$ (M1)(A1)

Note: *Award (M1) for an attempt to use the chain rule.*

$= 6(2x + 5)^2$ (C2)

(b) $\int f(x)dx = \frac{(2x+5)^4}{4 \times 2} + c$ (A2) (C2)

Note: *Award (A1) for* $(2x + 5)^4$ *and (A1) for /8.*

[4]

93.) (a) $t = 2 \Rightarrow h = 50 - 5(2^2) = 50 - 20$

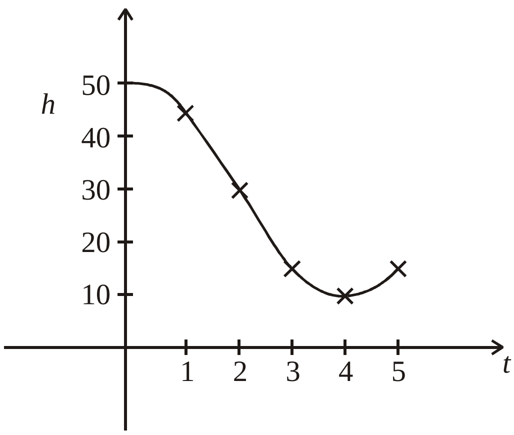
$= 30$ (A1)

OR

$h = 90 - 40(2) + 5(2^2)$

$= 30$ (A1) 1

(b)



(A4) 4

Note: *Award (A1) for marked scales on each axis, (A1) for each* *section of the curve.*

(c) (i) $\frac{dh}{dt} = \frac{d}{dt}(50 - 5t^2)$

$= 0 - 10t = -10t$ (A1)

(ii) $\frac{dh}{dt} = \frac{d}{dt}(90 - 40t + 5t^2)$

$= 0 - 40 + 10t = -40 + 10t$ (A1) 2

(d) When $t = 2$ (i) $\frac{dh}{dt} = -10(2)$ **or** $\frac{dh}{dt} = -40 + 10 \times 2$ (M1)

$= -20$ $= -20$ (A1) 2

(e) $\frac{dh}{dt} = 0 \Rightarrow -10t = 0 (0 \le t \le 2)$ **or** $-40 + 10t = 0 (2 \le t \le 5)$ (M1)

$t = 0$ **or** $t = 4$ (A1)(A1) 3

(f) When $t = 4$ (M1)

$h = 90 - 40(4) + 5(4^2)$ (M1)

$= 90 - 160 + 80$

$= 10$ (A1) 3

[15]

94.) (a) From graph, period = 2π (A1) 1

(b) Range = $\{y \mid -0.4 < y < 0.4\}$ (A1) 1

(c) (i) $f'(x) = \frac{d}{dx}\{\cos x (\sin x)^2\}$

$= \cos x (2 \sin x \cos x) - \sin x (\sin x)^2$ **or** $-3 \sin^3 x + 2 \sin x$ (M1)(A1)(A1)

Note: *Award (M1) for using the product rule and (A1) for each part.*

(ii) $f'(x) = 0$ (M1)

$\Rightarrow \sin x\{2 \cos x - \sin^2 x\} = 0$ **or** $\sin x\{3 \cos x - 1\} = 0$ (A1)

$\Rightarrow 3 \cos^2 x - 1 = 0$

$\Rightarrow \cos x = \pm\sqrt{\left(\frac{1}{3}\right)}$ (A1)

At A, $f(x) > 0$, hence $\cos x = \sqrt{\left(\frac{1}{3}\right)}$ (R1)(AG)

(iii) $f(x) = \sqrt{\left(\frac{1}{3}\right)\left(1 - \left(\sqrt{\left(\frac{1}{3}\right)}\right)^2\right)}$ (M1)

$= \frac{2}{3} \times \frac{1}{\sqrt{3}} = \frac{2}{9}\sqrt{3}$ (A1) 9

(d) $x = \frac{\pi}{2}$ (A1) 1

(e) (i) $\int (\cos x)(\sin x)^2 \, dx = \frac{1}{3}\sin^3 x + c$ (M1)(A1)

(ii) Area = $\int_0^{\pi/2}(\cos x)(\sin x)^2\,dx = \frac{1}{3}\left\{\left(\sin\frac{\pi}{2}\right)^3 - (\sin 0)^3\right\}$ (M1)

$= \frac{1}{3}$ (A1) 4

(f) At C $f''(x) = 0$ (M1)

$\Leftrightarrow 9\cos^3 x - 7\cos x = 0$

$\Leftrightarrow \cos x(9\cos^2 x - 7) = 0$ (M1)

$\Rightarrow x = \frac{\pi}{2}$ (reject) ***or*** $x = \arccos\frac{\sqrt{7}}{3} = 0.491$ (3 sf) (A1)(A1) 4

[20]

95.) (a) $y = \sqrt{3-4x} = (3-4x)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(3-4x)^{-\frac{1}{2}}(-4)$ (A1)(A1) (C2)

Note: *Award (A1) for each element, to a maximum of [2 marks].*

(b) $y = e^{\sin x}$

$\frac{dy}{dx} = (\cos x)(e^{\sin x})$ (A1)(A1) (C2)

Note: *Award (A1) for each element.*

[4]

96.) (a) $f''(x) = 2x - 2$

$\Rightarrow$ $f'(x) = x^2 - 2x + c$ (M1)(M1)

$= 0$ when $x = 3$

$\Rightarrow$ $0 = 9 - 6 + c$

$c = -3$ (A1)

$f'(x) = x^2 - 2x - 3$ (AG)

$f(x) = \frac{x^3}{3} - x^2 - 3x + d$ (M1)

When $x = 3$, $f(x) = -7$

$\Rightarrow$ $-7 = 9 - 9 - 9 + d$ (M1)

$\Rightarrow$ $d = 2$ (A1) 6

$\Rightarrow$ $f(x) = \frac{x^3}{3} - x^2 - 3x + 2$

(b) $f(0) = 2$ (A1)

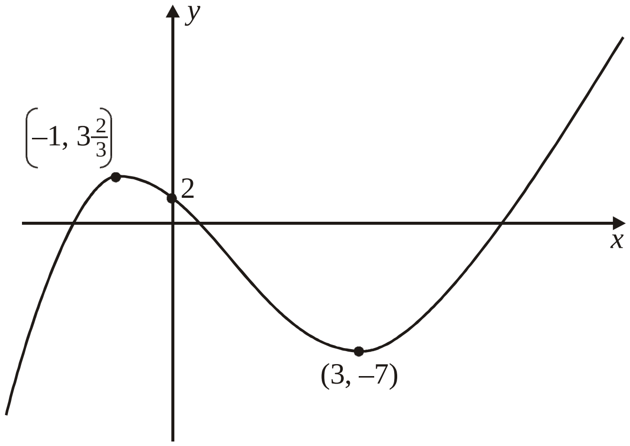
$f(-1) = -\frac{1}{3} - 1 + 3 + 2$

$= 3\frac{2}{3}$ (A1)

$f'(-1) = 1 + 2 - 3$

$= 0$ (A1) 3

(c) $f'(-1) = 0 \Rightarrow \left(-1,\ 3\frac{2}{3}\right)$ is a stationary point



(A4) 4

Note: *Award (A1) for maximum, (A1) for (0, 2) (A1) for (3, –7), (A1) for cubic.*

[13]

97.) (a) $\frac{\mathrm{d}}{\mathrm{d}x}(x^2 + 1)^2$

$= 2(x^2 + 1) \times (2x)$ (M1)(M1) (C2)

$= 4x(x^2 + 1)$

(b) $\frac{\mathrm{d}}{\mathrm{d}x}(\ln(3x - 1))$

$= \frac{1}{3x-1} \times (3)$ (M1)(M1) (C2)

$= \frac{3}{3x-1}$

[4]

98.) (a) $f(1) = 3$ $f(5) = 3$ (A1)(A1) 2

(b) **EITHER** distance between successive maxima = period (M1)

= 5 – 1 (A1)

= 4 (AG)

OR Period of $\sin kx = \frac{2\pi}{k}$; (M1)

so period $= \frac{2\pi}{\frac{\pi}{2}}$ (A1)

= 4 (AG) 2

(c) **EITHER** $A \sin\left(\frac{\pi}{2}\right) + B = 3$ and $A \sin\left(\frac{3\pi}{2}\right) + B = -1$ (M1) (M1)

$\Leftrightarrow A + B = 3, -A + B = -1$ (A1)(A1)

$\Leftrightarrow A = 2, B = 1$ (AG)(A1)

OR Amplitude = A (M1)

$A = \dfrac{3-(-1)}{2} = \dfrac{4}{2}$ (M1)

$A = 2$ (AG)

Midpoint value $= B$ (M1)

$B = \dfrac{3+(-1)}{2} = \dfrac{2}{2}$ (M1)

$B = 1$ (A1) 5

***Note:** As the values of $A = 2$ and $B = 1$ are likely to be quite obvious to a bright student, do not insist on too detailed a proof.*

(d) $f(x) = 2\sin\left(\dfrac{\pi}{2}x\right) + 1$

$f'(x) = \left(\dfrac{\pi}{2}\right)2\cos\left(\dfrac{\pi}{2}x\right) + 0$ (M1)(A2)

***Note:** Award (M1) for the chain rule, (A1) for $\left(\dfrac{\pi}{2}\right)$, (A1) for $2\cos\left(\dfrac{\pi}{2}x\right)$.*

$= \pi\cos\left(\dfrac{\pi}{2}x\right)$ (A1) 4

***Notes:** Since the result is given, make sure that reasoning is valid. In particular, the final (A1) is for simplifying the result of the chain rule calculation. If the preceding steps are not valid, this final mark should not be given. Beware of "fudged" results.*

(e) (i) $y = k - \pi x$ is a tangent $\Rightarrow -\pi = \pi\cos\left(\dfrac{\pi}{2}x\right)$ (M1)

$\Rightarrow -1 = \cos\left(\dfrac{\pi}{2}x\right)$ (A1)

$\Rightarrow \dfrac{\pi}{2}x = \pi$ or 3π or ...

$\Rightarrow x = 2$ or 6 ... (A1)

Since $0 \le x \le 5$, we take $x = 2$, so the point is $(2, 1)$ (A1)

(ii) Tangent line is: $y = -\pi(x-2) + 1$ (M1)

$y = (2\pi + 1) - \pi x$

$k = 2\pi + 1$ (A1) 6

(f) $f(x) = 2 \Rightarrow 2\sin\left(\dfrac{\pi}{2}x\right) + 1 = 2$ (A1)

$\Rightarrow \sin\left(\dfrac{\pi}{2}x\right) = \dfrac{1}{2}$ (A1)

$\Rightarrow \dfrac{\pi}{2}x = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$ or $\dfrac{13\pi}{6}$

$x = \dfrac{1}{3}$ or $\dfrac{5}{3}$ or $\dfrac{13}{3}$ (A1)(A1)(A1) 5

[24]

99.) (a) $y = \ln x \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{x}$ (A1)

when $x = \mathrm{e}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\mathrm{e}}$

tangent line: $y = \left(\dfrac{1}{\mathrm{e}}\right)(x - \mathrm{e}) + 1$ (M1)

$y = \dfrac{1}{\mathrm{e}}(x) - 1 + 1 = \dfrac{x}{\mathrm{e}}$ (A1)

$x = 0 \Rightarrow y = \dfrac{0}{\mathrm{e}} = 0$ (M1)

(0, 0) is on line (AG) 4

(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}(x \ln x - x) = (1) \times \ln x + x \times \left(\dfrac{1}{x}\right) - 1 = \ln x$ (M1)(A1)(AG) 2

Note: *Award (M1) for applying the product rule, and (A1) for* $(1) \times \ln x + x \times \left(\dfrac{1}{x}\right)$.

(c) Area = area of triangle – area under curve (M1)

$= \left(\dfrac{1}{2} \times \mathrm{e} \times 1\right) - \int_1^{\mathrm{e}} \ln x \mathrm{d}x$ (A1)

$= \dfrac{\mathrm{e}}{2} - [x \ln x - x]_1^{\mathrm{e}}$ (A1)

$= \dfrac{\mathrm{e}}{2} - \{(\mathrm{e} \ln \mathrm{e} - 1 \ln 1) - (\mathrm{e} - 1)\}$ (A1)

$= \dfrac{\mathrm{e}}{2} - \{\mathrm{e} - 0 - \mathrm{e} + 1\}$

$= \dfrac{1}{2}\mathrm{e} - 1.$ (AG) 4

[10]

100.) A curve has equation $y = x(x - 4)^2$.

(a) For this curve find

(i) the x-intercepts;

(ii) the coordinates of the maximum point;

(iii) the coordinates of the point of inflexion.

(9)

(b) Use your answers to part (a) to sketch a graph of the curve for $0 \le x \le 4$, clearly indicating the features you have found in part (a).

(3)

(c) (i) On your sketch indicate by shading the region whose area is given by the following integral:

$$\int_0^4 x(x-4)^2 \, \mathrm{d}x.$$

(ii) Explain, using your answer to part (a), why the value of this integral is greater than 0 but less than 40.

(3)

(Total 15 marks)