UNIT 13 *Graphs, Equations and Inequalities* Teaching Notes

Historical Background and Introduction

This unit follows on from Unit 5, *Linear Graphs and Equations*, extending this work, and also covering new topics. It is aimed primarily at those on the *Express Route*, instructing them in the higher level algebraic concepts.

One point to be stressed here is the duality between the 'equals' sign ($=$) and the 'not equal' sign ($\neq$), which itself can be replaced by the 'inequality' signs ($<$ and $>$).

It is important to stress that there can be no ambiguity here and that the 'equals' sign means precisely that 'one side of an equation *equals* the other side of the equation'. Students must be trained in this logic, and appreciate that expressions such as

$$7 \times 5 = 35 + 5 = 40$$

are not correct, and should never be tolerated, i.e. two lines are required for this calculation:

$$7 \times 5 = 35$$

and

$$35 + 5 = 40$$

Routes	**Standard**	**Academic**	**Express**
13.1 Linear Inequalities	(✓)	✓	✓
13.2 Graphs of Quadratic Expressions	×	(✓)	✓
13.3 Cubic and Reciprocal Functions	×	(✓)	✓
13.4 Solving Non-Linear Equations	×	✓	✓
13.5 Quadratic Inequalities	×	✓	✓
13.6 Equations of Perpendicular Lines	×	(✓)	✓

Language	**Standard**	**Academic**	**Express**
Linear inequalities	(✓)	✓	(✓)
Quadratic functions	×	(✓)	✓
Cubic and reciprocal functions	×	(✓)	✓
Non-linear equations	×	✓	✓
Quadratic inequalities	×	(✓)	✓

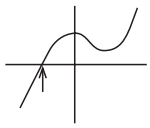
UNIT 13 Graphs, Equations and Inequalities — Teaching Notes

Misconceptions

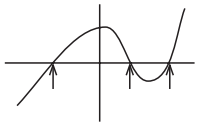
- confusion often exists when multiplying an inequality by a negative number. The sign must be changed, e.g. $x > y \Rightarrow -2x < -2y$ *not* $-2x > -2y$.

- pupils must appreciate that a cubic equation has either

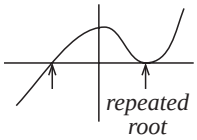
one real root



or *three* distinct real roots



or *three* real roots, with one repeated

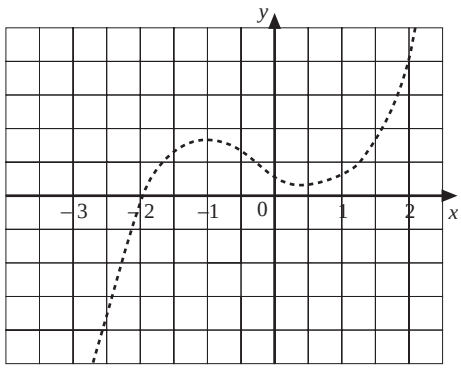


- it is easy to miss a solution when using function evaluations to find a root,

e.g. for $f(x) = 8x^3 + 10x^2 - 11x + 2$ we have

$$f(-3) = -122$$
$$f(-2) = 0$$
$$f(-1) = 15$$
$$f(0) = 2$$
$$f(1) = 9$$
$$f(2) = 84,$$

and it would be tempting to draw the graph as shown on the right, with just one real root at $x = 2$.

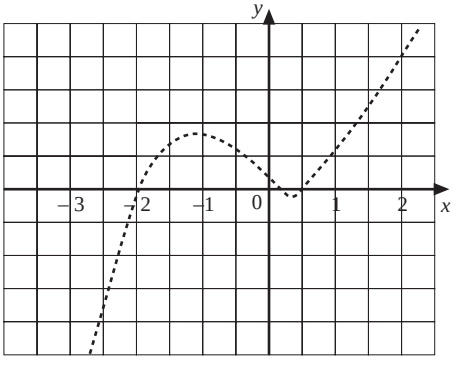


The actual graph is shown below, as, in fact,

$$f(x) = 8(x+2)\left(x - \frac{1}{4}\right)\left(x - \frac{1}{2}\right)$$

and has roots at

$$x = -2, \frac{1}{4} \text{ and } \frac{1}{2}$$



Great care must be taken with numerical/graphical approaches to solving non-linear equations.

UNIT 13 Graphs, Equations and Inequalities — Teaching Notes

Challenging Questions

The following questions are more challenging than others in the same section:

	Section	*Question No.*	*Page*
Practice Book Y9B	13.1	11	102
" "	13.2	11	107
" "	13.5	11	124