1.) (a) $\mathrm{P}(X=2)=\frac{4}{14}\left(=\frac{2}{7}\right)$ A1 N1 1

(b) $\mathrm{P}(X = 1) = \frac{1}{14}$ (A1)

$\mathrm{P}(X = k) = \frac{k^2}{14}$ (A1)

setting the sum of probabilities = 1 M1

e.g. $\frac{1}{14}+\frac{4}{14}+\frac{k^2}{14} = 1$, $5 + k^2 = 14$

$k^2 = 9\left(\text{accept } \frac{k^2}{14}=\frac{9}{14}\right)$ A1

$k = 3$ AG N0 4

(c) correct substitution into $\mathrm{E}(X)=\sum x\mathrm{P}(X = x)$ A1

e.g. $1\left(\frac{1}{14}\right)+2\left(\frac{4}{14}\right)+3\left(\frac{9}{14}\right)$

$\mathrm{E}(X)=\frac{36}{14}\left(=\frac{18}{7}\right)$ A1 N1 2

[7]

2.) (a) (i) $s = 1$ A1 N1

(ii) evidence of appropriate approach (M1)
e.g. 21–16, $12 + 8 - q = 15$
$q = 5$ A1 N2

(iii) $p = 7$, $r = 3$ A1A1 N2 5

(b) (i) $\mathrm{P}(\text{art}|\text{music}) = \frac{5}{8}$ A2 N2

(ii) **METHOD 1**

$P(\text{art})=\frac{12}{16}\left(=\frac{3}{4}\right)$ A1

evidence of correct reasoning R1

e.g. $\frac{3}{4}\neq\frac{5}{8}$

the events are not independent AG N0

METHOD 2

$\mathrm{P}(\text{art}) \times \mathrm{P}(\text{music}) = \frac{96}{256}\left(=\frac{3}{8}\right)$ A1

evidence of correct reasoning R1

e.g. $\frac{12}{16}\times\frac{8}{16}\neq\frac{5}{16}$

the events are not independent AG N04

(c) P(first takes only music) = $\frac{3}{16}$ = (seen anywhere) A1

P(second takes only art)= $\frac{7}{15}$ (seen anywhere) A1

evidence of valid approach (M1)

e.g. $\frac{3}{16}\times\frac{7}{15}$

P(music and art)= $\frac{21}{240}\left(=\frac{7}{80}\right)$ A1 N24

[13]

3.) (a) (i) $n = 0.1$ A1 N1

(ii) $m = 0.2$, $p = 0.3$, $q = 0.4$ A1A1A1 N34

(b) appropriate approach

e.g. $\mathrm{P}(B') = 1 - \mathrm{P}(B)$, $m + q$, $1-(n + p)$ (M1)

$\mathrm{P}(B') = 0.6$ A1 N22

[6]

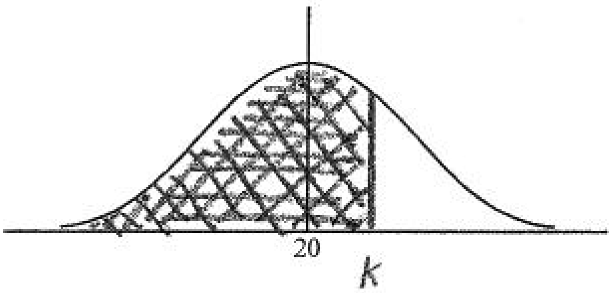
4.) (a) $\sigma = 3$ (A1)

evidence of attempt to find $\mathrm{P}(X \leq 24.5)$ (M1)

e.g. $z = 1.5$, $\frac{24.5-20}{3}$

$\mathrm{P}(X \leq 24.5) = 0.933$ A1 N33

(b) (i)



A1A1N2

Note: *Award A1 with shading that clearly extends to right of the mean, A1 for any correct label, either k, area or their value of k*

(ii) $z = 1.03(64338)$ (A1)

attempt to set up an equation (M1)

e.g. $\frac{k-20}{3} = 1.0364$, $\frac{k-20}{3} = 0.85$

$k = 23.1$ A1 N35

[8]

5.) (a) correct substitution into formula for $\mathrm{E}(X)$ (A1)

e.g. 0.05×240

E(X) =12 A1 N22

(b) evidence of recognizing binomial probability (may be seen in part (a)) (M1)

e.g. $\binom{240}{15}(0.05)^{15}(0.95)^{225}$, $X \sim B(240,0.05)$

P(X =15) = 0.0733 A1 N22

(c) P($X \le 9$) = 0.236 (A1)

evidence of valid approach (M1)

e.g. using complement, summing probabilities

P($X \ge 10$) = 0.764 A1 N33

[7]

6.) (a) evidence of valid approach involving A and B (M1)

e.g. P($A \cap$ pass) + P($B \cap$ pass), tree diagram

correct expression (A1)

e.g. P(pass) = $0.6 \times 0.8 + 0.4 \times 0.9$

P(pass) = 0.84 A1 N23

(b) evidence of recognizing complement (seen anywhere) (M1)

e.g. P(B) = x, P(A) = $1 - x$, $1 -$ P(B), $100 - x$, $x + y = 1$

evidence of valid approach (M1)

e.g. $0.8(1 - x) + 0.9x$, $0.8x + 0.9y$

correct expression A1

e.g. $0.87 = 0.8(1 - x) + 0.9x$, $0.8 \times 0.3 + 0.9 \times 0.7 = 0.87$, $0.8x + 0.9y = 0.87$

70 % from B A1 N24

[7]

7.) (a) symmetry of normal curve (M1)

e.g. P(X < 25) = 0.5

P(X > 27) = 0.2 A1 N22

(b) **METHOD 1**

finding standardized value (A1)

e.g. $\frac{27-25}{±}$

evidence of complement (M1)

e.g. $1 - p$, P(X < 27), 0.8

finding z-score (A1)

e.g. $z = 0.84\ldots$

attempt to set up equation involving the standardized value M1

e.g. $0.84 = \frac{27-25}{±}$, $0.84 = \frac{X - «}{±}$

$\sigma = 2.38$ A1 N35

METHOD 2

set up using normal CDF function and probability (M1)

e.g. $P(25 < X < 27) = 0.3$, $P(X < 27) = 0.8$

correct equation A2

e.g. $P(25 < X < 27) = 0.3$, $P(X > 27) = 0.2$

attempt to solve the equation using GDC (M1)

e.g. solver, graph, trial and error (more than two trials must be shown)

$\sigma = 2.38$ A1 N35

[7]

8.) (a) three correct pairs A1A1A1 N3 3
e.g. (2, 4), (3, 3), (4, 2), $R2G4$, $R3G3$, $R4G2$

(b) $p = \frac{1}{16}$, $q = \frac{2}{16}$, $r = \frac{2}{16}$ A1A1A1 N33

(c) let X be the number of times the sum of the dice is 5

evidence of valid approach (M1)

e.g. $X \sim B(n, p)$, tree diagram, 5 sets of outcomes produce a win

one correct parameter (A1)

e.g. $n = 4$, $p = 0.25$, $q = 0.75$

Fred wins prize is $P(X \geq 3)$ (A1)

appropriate approach to find probability M1

e.g. complement, summing probabilities, using a CDF function

correct substitution (A1)

e.g. $1 - 0.949\ldots$, $1 - \frac{243}{256}$, $0.046875 + 0.00390625$, $\frac{12}{256} + \frac{1}{256}$

probability of winning = 0.0508 $\left(\frac{13}{256}\right)$ A1 N36

[12]

9.) (a) (i) $p = 0.2$ A1 N1

(ii) $q = 0.4$ A1N1

(iii) $r = 0.1$ A1N1

(b) $P(A \mid B') = \frac{2}{3}$ A2N2

Note: *Award A1 for an unfinished answer such as* $\frac{0.2}{0.3}$.

(c) valid reason R1

e.g. $\frac{2}{3} \neq 0.5$, $0.35 \neq 0.3$

thus, A and B are not independent AGN0

[6]

10.) (a) (i) $\frac{7}{24}$ A1 N1

(ii) evidence of **multiplying** along the branches (M1)

e.g. $\frac{2}{3}\times\frac{5}{8}, \frac{1}{3}\times\frac{7}{8}$

adding probabilities of two mutually exclusive paths (M1)

e.g. $\left(\frac{1}{3}\times\frac{7}{8}\right)+\left(\frac{2}{3}\times\frac{3}{8}\right), \left(\frac{1}{3}\times\frac{1}{8}\right)+\left(\frac{2}{3}\times\frac{5}{8}\right)$

$\mathrm{P}(F) = \frac{13}{24}$ A1N2

(b) (i) $\frac{1}{3}\times\frac{1}{8}$ (A1)

$\frac{1}{24}$ A1

(ii) recognizing this is $\mathrm{P}(E \mid F)$ (M1)

e.g. $\frac{7}{24}\div\frac{13}{24}$

$\frac{168}{312}\left(=\frac{7}{13}\right)$ A2N3

(c)

X (cost in euros)	0	3	6
P (X)	$\frac{1}{9}$	$\mathbf{\frac{4}{9}}$	$\mathbf{\frac{4}{9}}$

A2A1N3

(d) correct substitution into $\mathrm{E}(X)$ formula (M1)

e.g. $0\times\frac{1}{9}+3\times\frac{4}{9}+6\times\frac{4}{9}, \frac{12}{9}+\frac{24}{9}$

$\mathrm{E}(X) = 4$ (euros) A1N2

[14]

11.) (a) evidence of recognizing binomial probability (may be seen in (b) or (c)) (M1)

e.g. probability = $\binom{7}{4}(0.9)^4(0.1)^3$, $X \sim \mathrm{B}(7, 0.9)$, complementary probabilities

probability = 0.0230 A1N2

(b) correct expression A1A1N2

e.g. $\binom{7}{4}p^4(1-p)^3$, $35p^4(1-p)^3$

Note: *Award A1 for binomial coefficient* $\left(\text{accept}\binom{7}{3}\right)$,

A1 for $p^4(1-p)^3$.

(c) evidence of attempting to solve **their** equation (M1)

e.g. $\binom{7}{4}p^4(1-p)^3 = 0.15$, sketch

$p = 0.356, 0.770$ A1A1N3

[7]

12.) (a) evidence of appropriate approach (M1)

e.g. 1 – 0.85, diagram showing values in a normal curve

$P(w \geq 82) = 0.15$ A1 N2

(b) (i) $z = -1.64$ A1 N1

(ii) evidence of appropriate approach (M1)

e.g. $-1.64 = \dfrac{x-«}{±}, \dfrac{68-76.6}{±}$

correct substitution A1

e.g. $-1.64 = \dfrac{68-76.6}{±}$

$\sigma = 5.23$ A1N1

(c) (i) $68.8 \leq$ weight ≤ 84.4A1A1A1 N3

Note: *Award A1 for 68.8, A1 for 84.4, A1 for giving answer as an interval.*

(ii) evidence of appropriate approach (M1)

e.g. $P(-1.5 \leq z \leq 1.5)$, $P(68.76 < y < 84.44)$

P(qualify) = 0.866 A1N2

(d) recognizing conditional probability (M1)

e.g. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

P(woman and qualify) = 0.25 × 0.7 (A1)

$P(\text{woman} \mid \text{qualify}) = \dfrac{0.25 \times 0.7}{0.866}$ A1

$P(\text{woman} \mid \text{qualify}) = 0.202$ A1N3

[15]

13.) (a) 36 outcomes (seen anywhere, even in denominator) (A1)

valid approach of listing ways to get sum of 5, showing at least two pairs (M1)

e.g. (1, 4)(2, 3), (1, 4)(4, 1), (1, 4)(4, 1), (2, 3)(3, 2) , lattice diagram

$P(\text{prize}) = \dfrac{4}{36}\left(=\dfrac{1}{9}\right)$ A1N3

(b) recognizing binomial probability (M1)

e.g. $B\left(8, \dfrac{1}{9}\right)$, binomial pdf, $\binom{8}{3}\left(\dfrac{1}{9}\right)^3\left(\dfrac{8}{9}\right)^5$

P(3 prizes) = 0.0426 A1N2

[5]

14.) (a) $p = \frac{4}{5}$ A1 N1

(b) multiplying along the branches (M1)

e.g. $\frac{1}{5} \times \frac{1}{4}, \frac{12}{40}$

adding products of probabilities of two mutually exclusive paths (M1)

e.g. $\frac{1}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{8}, \frac{1}{20} + \frac{12}{40}$

$\mathrm{P}(B) = \frac{14}{40}\left(= \frac{7}{20}\right)$ A1N2

(c) appropriate approach which must include A' (may be seen on diagram) (M1)

e.g. $\frac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)}\left(\text{do not accept } \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}\right)$

$\mathrm{P}(A' \mid B) = \frac{\frac{4}{5} \times \frac{3}{8}}{\frac{7}{20}}$ (A1)

$\mathrm{P}(A' \mid B) = \frac{12}{14}\left(= \frac{6}{7}\right)$ A1N2

[7]

15.) (a) (i) valid approach (M1)

e.g. np, $5 \times \frac{1}{5}$

$\mathrm{E}(X) = 1$ A1 N2

(ii) evidence of appropriate approach involving binomial (M1)

e.g. $X \sim \mathrm{B}\left(5, \frac{1}{5}\right)$

recognizing that Mark needs to answer 3 **or more** questions correctly (A1)

e.g. $\mathrm{P}(X \geq 3)$

valid approach M1

e.g. $1 - \mathrm{P}(X \leq 2)$, $\mathrm{P}(X = 3) + \mathrm{P}(X = 4) + \mathrm{P}(X = 5)$

P(pass) = 0.0579 A1N3

(b) (i) evidence of summing probabilities to 1 (M1)

e.g. $0.67 + 0.05 + (a + 2b) + \ldots + 0.04 = 1$

some simplification that clearly leads to required answer

e.g. $0.76 + 4a + 2b = 1$ A1

$4a + 2b = 0.24$ AGN0

(ii) correct substitution into the formula for expected value (A1)

e.g. $0(0.67) + 1(0.05) + \ldots + 5(0.04)$

some simplification (A1)

e.g. $0.05 + 2a + 4b + \ldots + 5(0.04) = 1$

correct equation A1

e.g. $13a + 5b = 0.75$

evidence of solving (M1)

$a = 0.05$, $b = 0.02$ A1A1N4

(c) attempt to find probability Bill passes (M1)
e.g. $P(Y \geq 3)$

correct value 0.19 A1
Bill (is more likely to pass) A1N0

[17]

16.) (a) $P(A) = \frac{1}{11}$ A1 N1

(b) $P(B \mid A) = \frac{2}{10}$ A2N2

(c) recognising that $P(A \cap B) = P(A) \times P(B \mid A)$ (M1)
correct values (A1)

e.g. $P(A \cap B) = \frac{1}{11} \times \frac{2}{10}$

$P(A \cap B) = \frac{2}{110}$

A1N3

[6]

17.) (a)

3, 9	**4, 9**	**5, 9**
3, 10	**4, 10**	**5, 10**
3, 10	**4, 10**	**5, 10**

A2N2

(b) 12, 13, 14, 15 (accept 12, 13, 13, 13, 14, 14, 14, 15, 15) A2N2

(c) $P(12) = \frac{1}{9}$, $P(13) = \frac{3}{9}$, $P(14) = \frac{3}{9}$, $P(15) = \frac{2}{9}$ A2N2

(d) correct substitution into formula for $E(X)$ A1

e.g. $E(S) = 12 \times \frac{1}{9} + 13 \times \frac{3}{9} + 14 \times \frac{3}{9} + 15 \times \frac{2}{9}$

$E(S) = \frac{123}{9}$ A2N2

(e) **METHOD 1**

correct expression for expected gain $E(A)$ for 1 game (A1)

e.g. $\frac{4}{9} \times 50 - \frac{5}{9} \times 30$

$E(A) = \frac{50}{9}$

amount at end = expected gain for 1 game × 36 (M1)
= 200 (dollars) A1N2

METHOD 2

attempt to find expected number of wins and losses (M1)

e.g. $\frac{4}{5}\times 36, \frac{5}{9}\times 36$

attempt to find expected gain E(G) (M1)

e.g. $16 \times 50 - 30 \times 20$

E(G) = 200 (dollars) A1N2

[12]

18.) (a) evidence of attempt to find $P(X \le 475)$ (M1)

e.g. $P(Z \le 1.25)$

$P(X \le 475) = 0.894$ A1 N2

(b) evidence of using the complement (M1)

e.g. 0.73, $1 - p$

$z = 0.6128$ (A1)

setting up equation (M1)

e.g. $\frac{a-450}{20} = 0.6128$

$a = 462$ A1N3

[6]

19.) (a) appropriate approach (M1)

e.g. tree diagram or a table

P(win) = $P(H \cap W) + P(A \cap W)$) (M1)

= (0.65)(0.83) + (0.35)(0.26) A1

= 0.6305 (or 0.631) A1N2

(b) evidence of using complement (M1)

e.g. $1 - p$, 0.3695

choosing a formula for conditional probability (M1)

e.g. $P(H \mid W') = \frac{P(W' \cap H)}{P(W')}$

correct substitution

e.g. $\frac{(0.65)(0.17)}{0.3695}\left(=\frac{0.1105}{0.3695}\right)$ A1

P(home) = 0.299 A1N3

[8]

20.) (a) evidence of using mid-interval values (5, 15, 25, 35, 50, 67.5, 87.5) (M1)

$\sigma = 19.8$ (cm) A2 N3

(b) (i) $Q_1 = 15$, $Q_3 = 40$(A1)(A1)

$IQR = 25$ (accept any notation that suggests the interval 15 to 40) A1 N3

(ii) **METHOD 1**

60 % have a length less than k (A1)

$0.6 \times 200 = 120$ (A1)

k 30 (cm) A1N2

METHOD 2

$0.4 \times 200 = 80$ (A1)
$200 - 80 = 120$ (A1)
$k = 30$ (cm) A1N2

(c) $l < 20$ cm $\Rightarrow$ 70 fish (M1)

P(small) = $\frac{70}{200}$ (= 0.35) A1N2

(d)

Cost $\$X$	4	10	12
$P(X = x)$	**0.35**	0.565	**0.085**

A1A1N2

(e) correct substitution (of their p values) into formula for $E(X)$ (A1)
e.g. $4 \times 0.35 + 10 \times 0.565 + 12 \times 0.085$

$E(X) = 8.07$ (accept \$8.07) A1N2

[15]

21.) $A \sim N(46, 10^2)$ $B \sim N(\mu, 12^2)$

(a) $P(A > 60) = 0.0808$ A2N2

(b) correct approach (A1)

e.g. $P\left(Z < \frac{60 - \mu}{12}\right) = 0.85$, sketch

$\frac{60 - \mu}{12} = 1.036...$ (A1)

$\mu = 47.6$ A1N2

(c) (i) route A A1 N1

(ii) **METHOD 1**

$P(A < 60) = 1 - 0.0808 = 0.9192$ A1
valid reason R1
e.g. probability of A getting there on time is greater than probability of B
$0.9192 > 0.85$ N2

METHOD 2

$P(B > 60) = 1 - 0.85 = 0.15$ A1
valid reason R1
e.g. probability of A getting there late is less than probability of B
$0.0808 < 0.15$

N2

(d) (i)let X be the number of days when the van arrives before 07:00
$P(X = 5) = (0.85)^5$ (A1)
$= 0.444$ A1 N2

(ii) **METHOD 1**

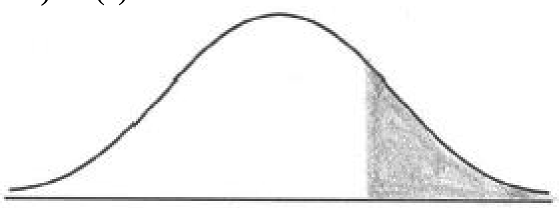
evidence of adding correct probabilities (M1)
e.g. $P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$
correct values $0.1382 + 0.3915 + 0.4437$ (A1)
$P(X \geq 3) = 0.973$ A1N3

METHOD 2

evidence of using the complement (M1)
e.g. $P(X \geq 3) = 1 - P(X \leq 2)$, $1 - p$
correct values $1 - 0.02661$ (A1)
$P(X \geq 3) = 0.973$ A1N3

[13]

22.) (a)



A1A1 N2

Note: *Award A1 for vertical line to right of mean, A1 for shading to right of* ***their*** *vertical line.*

(b) evidence of recognizing symmetry (M1)
e.g. 105 is one standard deviation above the mean so d is one standard deviation below the mean, shading the corresponding part,
$105 - 100 = 100 - d$

$d = 95$ A1N2

(c) evidence of using complement (M1)
e.g. $1 - 0.32$, $1 - p$

$P(d < X < 105) = 0.68$ A1N2

[6]

23.) (a) (i) evidence of substituting into $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ (M1)
e.g. $75 + 55 - 100$, Venn diagram

30 A1N2

(ii) 45 A1N1

(b) (i) **METHOD 1**

evidence of using complement, Venn diagram (M1)
e.g. $1 - p$, $100 - 30$

$\frac{70}{100}\left(=\frac{7}{10}\right)$ A1N2

METHOD 2

attempt to find P(only one sport), Venn diagram (M1)
e.g. $\frac{25}{100} + \frac{45}{100}$

$\frac{70}{100}\left(=\frac{7}{10}\right)$ A1N2

(ii) $\frac{45}{70}\left(=\frac{9}{14}\right)$ A2N2

(c) valid reason in words or symbols (R1)
e. g. $P(A \cap B) = 0$ if mutually exclusive, $P(A \cap B)$ if not mutually exclusive

correct statement in words or symbols A1N2
e.g. $P(A \cap B) = 0.3$, $P(A \cup B) \neq P(A) + P(B)$, $P(A) + P(B) > 1$, some students play both sports, sets intersect

(d) valid reason for independence (R1)
e.g. $P(A \cap B) = P(A) \times P(B)$, $P(B \mid A) = P(B)$

correct substitution A1A1N3
e.g. $\frac{30}{100} \neq \frac{75}{100} \times \frac{55}{100}, \frac{30}{55} \neq \frac{75}{100}$

[12]

24.) (a) $E(X) = 2$ A1 N1

(b) evidence of appropriate approach involving binomial (M1)
e.g. $\binom{10}{3}(0.2)^3$, $(0.2)^3(0.8)^7$, $X \sim B(10, 0.2)$

$P(X = 3) = 0.201$ A1N2

(c) **METHOD 1**

$P(X \leq 3) = 0.10737 + 0.26844 + 0.30199 + 0.20133\ (= 0.87912...)$ (A1)
evidence of using the complement (seen anywhere) (M1)
e.g. 1 – any probability, $P(X > 3) = 1 - P(X \leq 3)$

$P(X > 3) = 0.121$ A1N2

METHOD 2

recognizing that $P(X > 3) = P(X \geq 4)$ (M1)
e.g. summing probabilities from $X = 4$ to $X = 10$

correct expression or values (A1)
e.g. $\sum_{r=4}^{10}\binom{10}{r}(0.2)^{10-r}(0.8)^r$

$0.08808 + 0.02642 + 0.005505 + 0.000786 + 0.0000737 + 0.000004 + 0.0000001$

$P(X > 3) = 0.121$ A1N2

[6]

25.) **METHOD 1**

for independence $P(A \cap B) = P(A) \times P(B)$ (R1)
expression for $P(A \cap B)$, indicating $P(B) = 2P(A)$ (A1)
e.g. $P(A) \times 2P(A)$, $x \times 2x$

substituting into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)
correct substitution A1
e.g. $0.52 = x + 2x - 2x^2$, $0.52 = P(A) + 2P(A) - 2P(A)P(A)$
correct solutions to the equation (A2)
e.g. 0.2, 1.3 (accept the single answer 0.2)
$P(B) = 0.4$ A1 N6

METHOD 2

for independence $P(A \cap B) = P(A) \times P(B)$ (R1)

expression for $P(A \cap B)$, indicating $P(A) = \frac{1}{2}P(B)$ (A1)

e.g. $P(B) \times \frac{1}{2}P(B)$, $x \times \frac{1}{2}x$

substituting into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)

correct substitution A1

e.g. $0.52 = 0.5x + x - 0.5x^2$, $0.52 = 0.5P(B) + P(B) - 0.5P(B)P(B)$

correct solutions to the equation (A2)

e.g. 0.4, 2.6 (accept the single answer 0.4)

$P(B) = 0.4$ (accept $x = 0.4$ if x set up as $P(B)$) A1 N6

[7]

26.) (a) (i) $P(B) = \frac{3}{4}$ A1 N1

(ii) $P(R) = \frac{1}{4}$ A1 N1

(b) $p = \frac{3}{4}$ A1 N1

$s = \frac{1}{4}$, $t = \frac{3}{4}$ A1 N1

(c) (i) $P(X = 3)$

$= P$ (getting 1 and 2) $= \frac{1}{4} \times \frac{3}{4}$ A1

$= \frac{3}{16}$ AG N0

(ii) $P(X = 2) = \frac{1}{4} \times \frac{1}{4} + \frac{3}{4}\left(\text{or}\, 1 - \frac{3}{16}\right)$ (A1)

$= \frac{13}{16}$ A1 N2

(d) (i)

X	2	3
$P(X = x)$	$\frac{13}{16}$	$\frac{3}{16}$

A2 N2

(ii) evidence of using $E(X) = \sum xP(X = x)$ (M1)

$E(X) = 2\left(\frac{13}{16}\right) + 3\left(\frac{3}{16}\right)$ (A1)

$= \frac{35}{16}\left(= 2\frac{3}{16}\right)$ A1 N2

(e) win \$10 $\Rightarrow$ scores 3 one time, 2 other time (M1)

$P(3) \times P(2) = \frac{13}{16} \times \frac{3}{16}$ (seen anywhere) A1

evidence of recognizing there are different ways of winning \$10 (M1)

e.g. $P(3) \times P(2) + P(2) \times P(3)$, $2\left(\frac{13}{16} \times \frac{3}{16}\right)$,

$\frac{36}{256} + \frac{3}{256} + \frac{36}{256} + \frac{3}{256}$

$P(\text{win } \$10) = \frac{78}{256} \quad \left(= \frac{39}{128}\right)$ A1 N3

[16]

27.) (a) (i) correct calculation (A1)

e.g. $\frac{9}{20} + \frac{5}{20} - \frac{2}{20}$, $\frac{4+2+3+3}{20}$

$P(\text{male or tennis}) = \frac{12}{20} \quad \left(= \frac{3}{5}\right)$ A1 N2

(ii) correct calculation (A1)

e.g. $\frac{6}{20} \div \frac{11}{20}$, $\frac{3+3}{11}$

$P(\text{not football} \mid \text{female}) = \frac{6}{11}$ A1 N2

(b) $P(\text{first not football}) = \frac{11}{20}$, $P(\text{second not football}) = \frac{10}{19}$ A1

$P(\text{neither football}) = \frac{11}{20} \times \frac{10}{19}$ A1

$P(\text{neither football}) = \frac{110}{380} \quad \left(= \frac{11}{38}\right)$ A1 N1

[7]

28.) (a) evidence of binomial distribution (may be seen in parts (b) or (c)) (M1)

e.g. np, 100×0.04

mean = 4 A1 N2

(b) $P(X = 6) = \binom{100}{6}(0.04)^6 (0.96)^{94}$ (A1)

$= 0.105$ A1 N2

(c) for evidence of appropriate approach (M1)

e.g. complement, $1 - P(X = 0)$

$P(X = 0) = (0.96)^{100} = 0.01687...$ (A1)

$P(X \geq 1) = 0.983$ A1 N2

[7]

29.) $X \sim N(7, 0.5^2)$

(a) (i) $z = 2$ (M1)

$P(X < 8) = P(Z < 2) = 0.977$ A1 N2

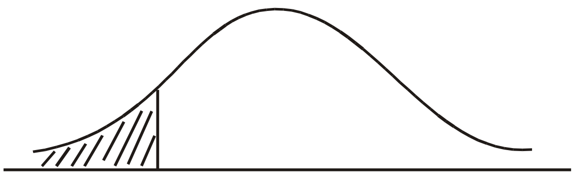
(ii) evidence of appropriate approach (M1)

e.g. symmetry, $z = -2$

$P(6 < X < 8) = 0.954$ (tables 0.955) A1 N2

Note: *Award M1A1(AP) if candidates refer to 2 standard deviations from the mean, leading to 0.95.*

(b) (i)



A1A1 N2

Note: *Award A1 for d to the left of the mean, A1 for area to the left of d shaded.*

(ii) $z = -1.645$ (A1)

$\frac{d-7}{0.5} = -1.645$ (M1)

$d = 6.18$ A1 N3

(c) $Y \sim N(\mu, 0.5^2)$

$P(Y < 5) = 0.2$ (M1)

$z = -0.84162...$ A1

$\frac{5-\mu}{0.5} = -0.8416$ (M1)

$\mu = 5.42$ A1 N3

[13]

30.) (a) evidence of using $\sum p_i = 1$ (M1)

correct substitution A1

e.g. $10k^2 + 3k + 0.6 = 1$, $10k^2 + 3k - 0.4 = 0$

$k = 0.1$ A2 N2

(b) evidence of using $E(X) = \sum p_i x_i$ (M1)

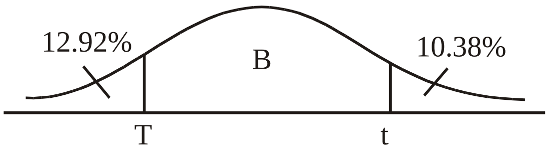
correct substitution (A1)

e.g. $-1 \times 0.2 + 2 \times 0.4 + 3 \times 0.3$

$E(X) = 1.5$ A1 N2

[7]

31.) (a)



A1A1 N2

Notes: *Award A1 for three re.g.ions, (may be shown by lines or shading) A1 for clear labelling of two re.g.ions (may be shown by percentages or cate.g.ories).*

r and t need not be labelled, but if they are, they may be interchanged.

(b) **METHOD 1**

$P(X < r) = 0.1292$ (A1)

$r = 6.56$ A1 N2

$1 - 0.1038$ (= 0.8962) (may be seen later) A1

$P(X < t) = 0.8962$ (A1)

$t = 7.16$ A1 N2

METHOD 2

finding z-values $-1.130...$, $1.260...$ A1A1

evidence of setting up one standardized equation (M1)

e.g. $\frac{r-6.84}{0.25} = -1.13\ldots,\ t = 1.260 \times 0.25 + 6.84$

$r = 6.56$, $t = 7.16$ A1A1 N2N2

[7]

32.) (a) evidence of binomial distribution (seen anywhere) (M1)

e.g. $X \sim \mathrm{B}\left(3, \frac{1}{4}\right)$

mean = $\frac{3}{4}$ $(= 0.75)$ A1 N2

(b) $P(X = 2) = \binom{3}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)$ (A1)

$P(X = 2) = 0.141$ $\left(= \frac{9}{64}\right)$ A1 N2

(c) evidence of appropriate approach M1

e.g. complement, $1 - P(X = 0)$, adding probabilities

$P(X = 0) = (0.75)^3$ $\left(= 0.422, \frac{27}{64}\right)$ (A1)

$P(X \geq 1) = 0.578 \quad \left(= \frac{37}{64}\right)$ A1 N2

[7]

33.) (a) $P(A \cap B) = P(A) \times P(B)$ $(= 0.6x)$ A1 N1

(b) (i) evidence of using $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ (M1)

correct substitution A1

e.g. $0.80 = 0.6 + x - 0.6x$, $0.2 = 0.4x$

$x = 0.5$ A1 N2

(ii) $P(A \cap B) = 0.3$ A1 N1

(c) valid reason, with reference to $P(A \cap B)$ R1 N1

e.g. $P(A \cap B) \neq 0$

[6]

34.) (a) (i) number of ways of getting $X = 6$ is 5 (A1)

$P(X = 6) = \frac{5}{36}$ A1 N2

(ii) number of ways of getting $X > 6$ is 21 (A1)

$P(X > 6) = \frac{21}{36}\left(= \frac{7}{12}\right)$ A1 N2

(iii) $P(X = 7 | X > 5) = \frac{6}{26}\left(= \frac{3}{13}\right)$ A2 N2

(b) evidence of substituting into $E(X)$ formula (M1)

finding $P(X < 6) = \frac{10}{36}$ (seen anywhere) (A2)

evidence of using $E(W) = 0$ (M1)

correct substitution A2

e.g. $3\left(\frac{5}{36}\right) + 1\left(\frac{21}{36}\right) - k\left(\frac{10}{36}\right) = 0$, $15 + 21 - 10k = 0$

$k = \frac{36}{10}$ $(= 3.6)$ A1 N4

[13]

35.) (a) evidence of using binomial probability (M1)

e.g. $P(X = 2) = \binom{7}{2}(0.18)^2(0.82)^5$

$P(X = 2) = 0.252$ A1 N2

(b) **METHOD 1**

evidence of using the complement M1

e.g. $1 - (P(X \leq 1))$

$P(X \leq 1) = 0.632$ (A1)

$P(X \geq 2) = 0.368$ A1 N2

METHOD 2

evidence of attempting to sum probabilities M1 *e.g.* P(2 heads) + P(3 heads) + ... + P(7 heads), 0.252 + 0.0923 + …

correct values for each probability (A1)
e.g. 0.252 + 0.0923 + 0.0203 + 0.00267 + 0.0002 + 0.0000061
$P(X \geq 2) = 0.368$ A1 N2

[5]

36.) (a) evidence of approach (M1)

e.g. finding 0.84…, using $\frac{23.7-21}{\pm}$

correct working (A1)

e.g. $0.84... = \frac{23.7-21}{\pm}$, graph

$\sigma = 3.21$ A1 N2

(b) (i) evidence of attempting to find $P(X < 25.4)$ (M1)
e.g. using $z = 1.37$
$P(X < 25.4) = 0.915$ A1 N2

(ii) evidence of recognizing symmetry (M1)
e.g. $b = 21 - 4.4$, using $z = -1.37$
$b = 16.6$ A1 N2

[7]

37.) **METHOD 1**

(a) $\sigma = 10$ (A1)
$1.12 \times 10 = 11.2$ A1
$11.2 + 100$ (M1)
$x = 111.2$ A1N2

(b) $100 - 11.2$ (M1)
$= 88.8$ A1N2

[6]

METHOD 2

(a) $\sigma = 10$ (A1)
Evidence of using standardisation formula (M1)

$\frac{x-100}{10} = 1.12$ A1

$x = 111.2$ A1N2

(b) $\frac{100-x}{10} = 1.12$ A1

$x = 88.8$ A1N2

[6]

38.) (a) For summing to 1 (M1)

e.g. $\frac{1}{5} + \frac{2}{5} + \frac{1}{10} + x = 1$

$x = \frac{3}{10}$ A1 N2

(b) For evidence of using $E(X) = \sum x\, f(x)$ (M1)
Correct calculation A1

e.g. $\frac{1}{5}\times 1 + 2\times\frac{2}{5} + 3\times\frac{1}{10} + 4\times\frac{3}{10}$

E(X) = $\frac{25}{10}$ (= 2.5) A1N2

(c) $\frac{1}{10}\times\frac{1}{10}$ (M1)

$\frac{1}{100}$ A1N2

[7]

39.) (a) Evidence of using the complement *e.g.* 1 – 0.06 (M1)
p = 0.94 A1 N2

(b) For evidence of using symmetry (M1)
Distance from the mean is 7 (A1)
e.g. diagram, D = mean – 7
D = 10 A1N2

(c) P(17 < H < 24) = 0.5 – 0.06 (M1)
= 0.44 A1

E(trees) = 200 × 0.44 (M1)
= 88 A1N2

[9]

40.) (a) (i) Attempt to find P(3H) = $\left(\frac{1}{3}\right)^3$ (M1)

= $\frac{1}{27}$ A1 N2

(ii) Attempt to find P(2H, 1T) (M1)
$= 3\left(\frac{1}{3}\right)^2 \frac{2}{3}$ A1
$= \frac{2}{9}$ A1N2

(b) (i) Evidence of using np $\left(\frac{1}{3}\times 12\right)$ (M1)

expected number of heads = 4 A1 N2

(ii) 4 heads, so 8 tails (A1)
E(winnings) = 4 × 10 – 8 × 6 (= 40 – 48) (M1)
= –$ 8 A1N1

[10]

41.) (a) X ~ B(100, 0.02)
E(X) = 100 × 0.02 = 2 A1 N1

(b) P(X = 3) = $\binom{100}{3}(0.02)^3\,(0.98)^{97}$ (M1)

= 0.182 A1N2

(c) **METHOD 1**

$P(X > 1) = 1 - P(X \le 1) = 1 - (P(X = 0) + P(X = 1))$ M1

$= 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99})$ (M1)

$= 0.597$ A1N2

METHOD 2

$P(X > 1) = 1 - P(X \le 1)$ (M1)

$= 1 - 0.40327$ (A1)

$= 0.597$ A1N2

Note: *Award marks as follows for finding P(X ≥ 1), if working shown.*

$P(X \ge 1)$ A0

$= 1 - P(X \le 2) = 1 - 0.67668$ M1(FT)

$= 0.323$ A1(FT)N0

[6]

42.) $X \sim N(\mu, \sigma^2)$

$P(X > 90) = 0.15$ **and** $P(X < 40) = 0.12$ (M1)

Finding standardized values 1.036, –1.175 A1A1

Setting up the equations $1.036 = \frac{90 - «}{\pm}, -1.175 = \frac{40 - «}{\pm}$ (M1)

$\mu = 66.6$, $\sigma = 22.6$ A1A1 N2N2

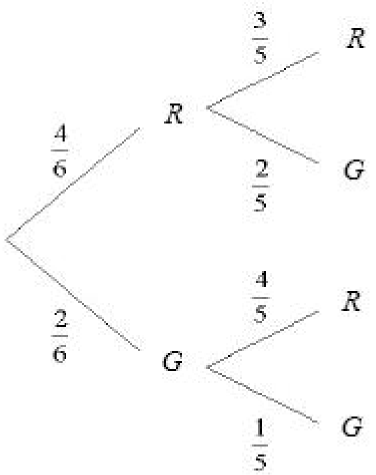
[6]

43.) (a) Using $E(X) = \sum_{0}^{2} x\,P(X = x)$ (M1)

Substituting correctly $E(X) = 0 \times \frac{3}{10} + 1 \times \frac{6}{10} + 2 \times \frac{1}{10}$ A1

$= 0.8$ A1 N2

(b) (i)



A1A1A1 N3

Note: *Award A1 for each complementary pair of probabilities,*

i.e. $\frac{4}{6}$ and $\frac{2}{6}$, $\frac{3}{5}$ and $\frac{2}{5}$, $\frac{4}{5}$ and $\frac{1}{5}$.

(ii) $P(Y = 0) = \frac{2}{5} \times \frac{1}{5} = \frac{2}{30}$ A1

$P(Y = 1) = P(RG) + P(GR)\left(= \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5}\right)$ M1

$= \frac{16}{30}$ A1

$P(Y = 2) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$ (A1)

For forming a distribution M1

y	0	1	2
$P(Y = y)$	$\frac{2}{30}$	$\frac{16}{30}$	$\frac{12}{30}$

N4

(c) $P(\text{Bag A}) = \frac{2}{6}\left(= \frac{1}{3}\right)$ (A1)

$P(\text{Bag B}) = \frac{4}{6}\left(= \frac{2}{3}\right)$ (A1)

For summing $P(A \cap RR)$ and $P(B \cap RR)$ (M1)

Substituting correctly $P(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{12}{30}$ A1

$= 0.3$ A1N3

(d) For recognising that $P(1 \text{ or } 6 \mid RR) = P(A \mid RR) = \frac{P(A \cap RR)}{P(RR)}$ (M1)

$= \frac{1}{30} \div \frac{27}{90}$ A1

$= 0.111$ A1N2

[19]

44.) (a) $\frac{3}{4}$ A1 N1

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{2}{5} + \frac{3}{4} - \frac{7}{8}$ A1

$= \frac{11}{40}$ (0.275) A1 N2

(c) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} \left(= \frac{\frac{11}{40}}{\frac{3}{4}}\right)$ A1

$= \frac{11}{30}$ (0.367) A1 N1

[6]

45.) (a) $P(H < 153) = 0.705 \Rightarrow z = 0.538(836...)$ (A1)

Standardizing $\frac{153-\mu}{5}$ (A1)

Setting up **their** equation $0.5388... = \frac{153-\mu}{5}$ M1

$\mu = 150.30\ldots$

$= 150$ (to 3sf) A1 N3

(b) $Z = \frac{153-\mu}{5} = 1.138...$ (accept 1.14 from $\mu = 150.3$, or 1.2 from $\mu = 150$) (A1)

$P(Z > 1.138) = 0.128$ (accept 0.127 from $z = 1.14$, or 0.115 from $z = 1.2$) A1 N2

[6]

46.) (a) $\frac{46}{97}\ (= 0.474)$ A1A1 N2

(b) $\frac{13}{51}\ (= 0.255)$ A1A1 N2

(c) $\frac{59}{97}\ (= 0.608)$ A2 N2

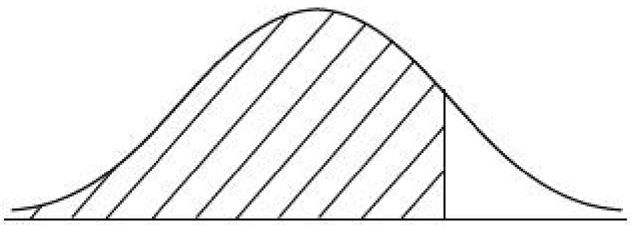
[6]

47.) (a) 0.0668 A2 N2

(b) Using the standardized value 1.645 (A1)

$k = 26.1$ kg A1 N2

(c)

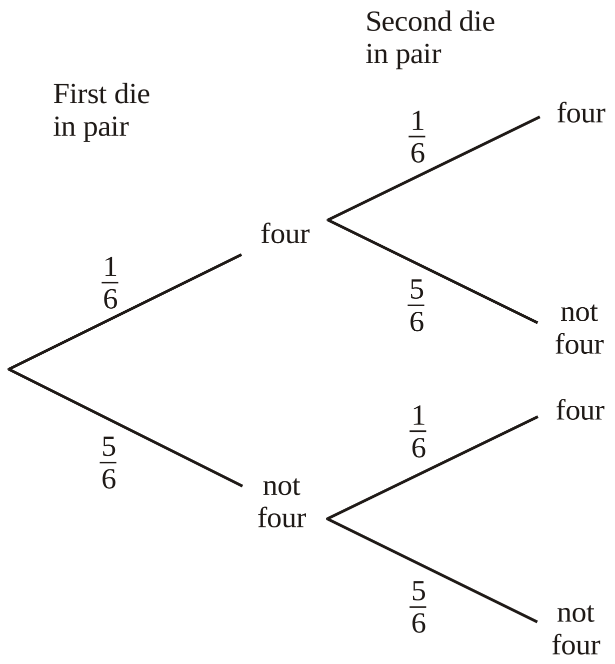


A1A1 N2

Note**:* *Award A1 for vertical line to right of the mean, A1 for shading to left of* ***their *vertical line.*

[6]

48.) (a)



A1A1A1 N3

***Note**: Award A1 for **each pair** of complementary probabilities.*

(b) $\mathrm{P}(E) = \frac{1}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{1}{6}\ \left(=\frac{5}{36}+\frac{5}{36}\right)$ (A2)

$= \frac{10}{36}\ \left(=\frac{5}{18}\text{ or } 0.278\right)$ A1 N3

(c) Evidence of recognizing the binomial distribution (M1)

eg $X \sim \mathrm{B}\left(5, \frac{5}{18}\right)$ or $p=\frac{5}{18}, q=\frac{13}{18}$

$\mathrm{P}(X = 3) = \binom{5}{3}\left(\frac{5}{18}\right)^3\left(\frac{13}{18}\right)^2$ (or other evidence of correct setup) (A1)

$= 0.112$ A1 N3

(d) **METHOD 1**

Evidence of using the complement M1

eg $\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X \leq 2)$

Correct value $1 - 0.865$ (A1)

$= 0.135$ A1 N2

METHOD 2

Evidence of adding correct probabilities M1

eg $\mathrm{P}(X \geq 3) = \mathrm{P}(X = 3) + \mathrm{P}(X = 4) + \mathrm{P}(X = 5)$

Correct values $0.1118 + 0.02150 + 0.001654$ (A1)

$= 0.135$ A1 N2

[12]

49.) (a) $P(F \cup S) = 1 - 0.14\ (= 0.86)$ (A1)

Choosing an appropriate formula (M1)

eg $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Correct substitution

eg $P(F \cap S) = 0.93 - 0.86$ A1

$P(F \cap S) = 0.07$ AG N0

Notes: *There are several valid approaches. Award (A1)(M1)A1 for relevant working using any appropriate strategy eg formula, Venn Diagram, or table.*

Award no marks for the incorrect solution

$P(F \cap S) = 1 - P(F) + P(S) = 1 - 0.93 = 0.07$

(b) Using conditional probability (M1)

eg $P(F \mid S) \left(= \frac{P(F \cap S)}{P(S)}\right)$

$P(F \mid S) = \frac{0.07}{0.62}$ (A1)

$= 0.113$ A1 N3

(c) F and S are **not** independent A1 N1

EITHER

If independent $P(F \mid S) = P(F)$, $0.113 \neq 0.31$ R1R1 N2

OR

If independent $P(F \cap S) = P(F)\,P(S)$, $0.07 \neq 0.31 \times 0.62\ (= 0.1922)$ R1R1 N2

(d) Let $P(F) = x$

$P(S) = 2P(F)\ (= 2x)$ (A1)

For independence $P(F \cap S) = P(F)P(S)\ (= 2x^2)$ (R1)

Attempt to set up a quadratic equation (M1)

eg $P(F \cup S) = P(F)P(S) - P(F)P(S)$, $0.86 = x + 2x - 2x^2$

$2x^2 - 3x + 0.86 = 0$ A2

$x = 0.386$, $x = 1.11$ (A1)

$P(F) = 0.386$ (A1) N5

[16]

50.) (a) $\frac{19}{120}\ (= 0.158)$ A1 N1

(b) $35 - (8 + 5 + 7)(= 15)$ (M1)

Probability $= \frac{15}{120} \left(= \frac{3}{24} = \frac{1}{8} = 0.125\right)$ A1 N2

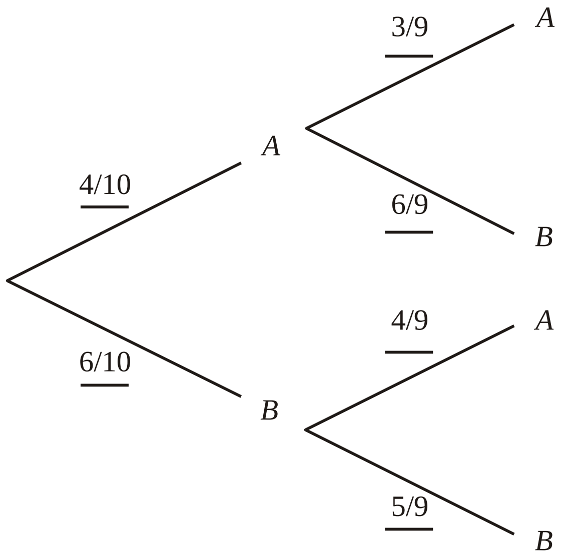
(c) Number studying = 76 (A1)

Number not studying = 120 – number studying = 44 (M1)

Probability = $\frac{44}{120}\left(=\frac{11}{30}=0.367\right)$ A1 N3

[6]

51.) (a)



A1A1A1 N3

(b) $\left(\frac{4}{10}\times\frac{6}{9}\right)+\left(\frac{6}{10}\times\frac{4}{9}\right)$ M1M1

$=\frac{48}{90}\left(\frac{8}{15}, 0.533\right)$ A1 N1

[6]

52.) (a) For summing to 1 (M1)

eg $0.1 + a + 0.3 + b = 1$

$a + b = 0.6$ A1 N2

(b) evidence of correctly using $\mathrm{E}(X)=\sum x\,f(x)$ (M1)

eg $0 \times 0.1 + 1 \times a + 2 \times 0.3 + 3 \times b$, $0.1 + a + 0.6 + 3b = 1.5$

Correct equation $0 + a + 0.6 + 3b = 1.5$ $(a + 3b = 0.9)$ (A1)

Solving simultaneously gives

$a = 0.45$ $b = 0.15$ A1A1 N3

[6]

53.) ***Note:*** *Candidates may be using tables in this question, which leads to a variety of values. Accept reasonable answers that are consistent with working shown.*

$W \sim \mathrm{N}(2.5, 0.3^2)$

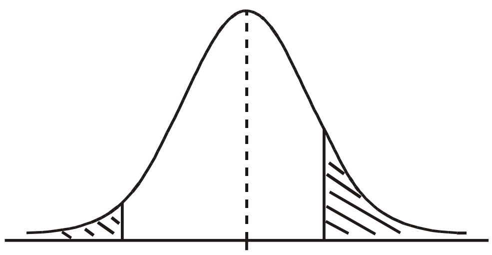
(a) (i) $z = -1.67$ (accept 1.67) (A1)

$P(W < 2) = 0.0478$ (accept answers between 0.0475 and 0.0485) A1 N2

(ii) $z = 1$ (A1)

$P(W > 2.8) = 0.159$ A1 N2

(iii)



A1A1 N2

Note: *Award A1 for a vertical line to left of mean and shading to left, A1 for vertical line to right of mean and shading to right.*

(iv) Evidence of appropriate calculation M1

eg $1 - (0.047790 + 0.15866)$, $0.8413 - 0.0478$

$P = 0.7936$ AG N0

Note: *The final value may vary depending on what level of accuracy is used.*

Accept their value in subsequent parts.

(b) (i) $X \sim B(10, 0.7935...)$

Evidence of calculation M1

eg $P(X = 10) = (0.7935...)^{10}$

$P(X = 10) = 0.0990$ (3 sf) A1 N1

(ii) **METHOD 1**

Recognizing $X \sim B(10, 0.7935...)$ (may be seen in (i)) (M1)

$P(X \le 6) = 0.1325...$ (or $P(X = 1) + ... + P(X = 6)$) (A1)

evidence of using the complement (M1)

eg $P(X \ge 7) = 1 - P(X \le 6)$, $P(X \ge 7) = 1 - P(X < 7)$

$P(X \ge 7) = 0.867$ A1 N3

METHOD 2

Recognizing $X \sim B(10, 0.7935...)$ (may be seen in (i)) (M1)

For adding terms from $P(X = 7)$ to $P(X = 10)$ (M1)

$P(X \ge 7) = 0.209235 + 0.301604 + 0.257629 + 0.099030$ (A1)

$= 0.867$ A1 N3

[13]

54.) (a) Independent $\Rightarrow P(A \cap B) = P(A) \times P(B)$ $(= 0.3 \times 0.8)$ (M1)

$= 0.24$ A1 N2

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $(= 0.3 + 0.8 - 0.24)$ M1

$= 0.86$ A1 N1

(c) No, **with** valid reason A2 N2

eg $P(A \cap B) \neq 0$ or $P(A \cup B) \neq P(A) + P(B)$ or correct numerical equivalent

[6]

55.) (a) $z = \dfrac{180 - 160}{20} = 1$ (A1)

$\Phi(1) = 0.8413$ (A1)

P(height > 180) = 1 – 0.8413

$= 0.159$ A1 N3

(b) $z = -1.1800$ (A1)

Setting up equation $-1.18 = \dfrac{d - 160}{20}$ (M1)

$d = 136$ A1 N3

[6]

56.) (a) For using $\sum p = 1$ $(0.4 + p + 0.2 + 0.07 + 0.02 = 1)$ (M1)

$p = 0.31$ A1 N2

(b) For using $E(X) = \sum x P(X = x)$ (M1)

$E(X) = 1(0.4) + 2(0.31) + 3(0.2) + 4(0.07) + 5(0.02)$ A1

$= 2$ A2 N2

[6]

57.) (a) $P(P \mid C) = \dfrac{20}{20 + 40}$ A1

$= \dfrac{1}{3}$ A1 N1

(b) $P(P \mid C') = \dfrac{30}{30 + 60}$ A1

$= \dfrac{1}{3}$ A1 N1

(c) Investigating conditions, or some relevant calculations (M1)

P is independent of *C*, **with** valid reason A1 N2

eg $P(P \mid C) = P(P \mid C')$, $P(P \mid C) = P(P)$,

$$\frac{20}{150} = \frac{50}{150} \times \frac{60}{150} \quad (ie\ \mathrm{P}(P \cap C) = \mathrm{P}(P) \times \mathrm{P}(C))$$

[6]

58.) (a) Adding probabilities (M1)

Evidence of knowing that sum = 1 for probability distribution R1

eg Sum greater than 1, sum = 1.3, sum does not equal 1 N2

(b) Equating sum to 1 ($3k + 0.7 = 1$) M1

$k = 0.1$ A1 N1

(c) (i) $\mathrm{P}(X = 0) = \dfrac{0+1}{20}$ (M1)

$= \dfrac{1}{20}$ A1 N2

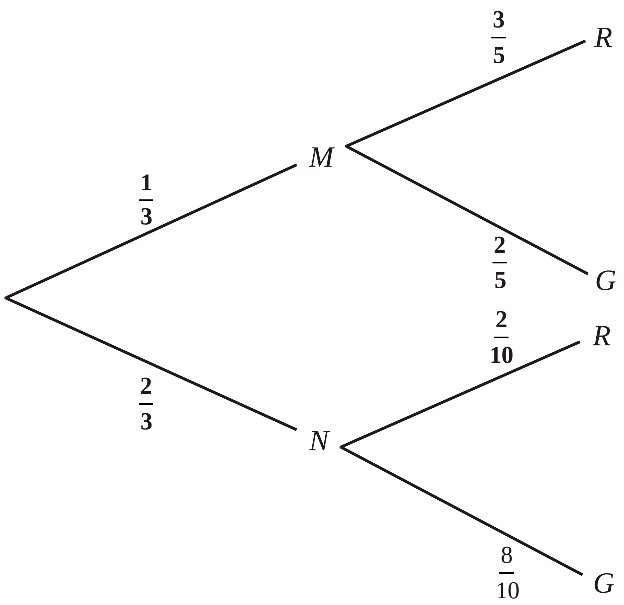
(ii) Evidence of using $\mathrm{P}(X > 0) = 1 - \mathrm{P}(X = 0)$

$\left(\text{or } \dfrac{4}{20} + \dfrac{5}{20} + \dfrac{10}{20}\right)$ (M1)

$= \dfrac{19}{20}$ A1 N2

[8]

59.) (a)



A1A1A1 N3

(b) (i) $\mathrm{P}(M \text{ and } G) = \dfrac{1}{3} \times \dfrac{2}{5} \left(= \dfrac{2}{15} = 0.133\right)$ A1 N1

(ii) $\mathrm{P}(G) = \dfrac{1}{3} \times \dfrac{2}{5} + \dfrac{2}{3} \times \dfrac{8}{10}$ (A1)(A1)

$= \frac{10}{15}\left(= \frac{2}{3} = 0.667\right)$ A1 N3

(iii) $P(M \mid G) = \frac{P(M \cap G)}{P(G)} = \frac{\frac{2}{15}}{\frac{2}{3}}$ (A1)(A1)

$= \frac{1}{5}$ or 0.2 A1 N3

(c) $P(R) = 1 - \frac{2}{3} = \frac{1}{3}$ (A1)

Evidence of using a correct formula M1

$E(\text{win}) = 2\times\frac{1}{3}+5\times\frac{2}{3}\left(\text{or } 2\times\frac{1}{3}\times\frac{3}{5}+2\times\frac{2}{3}\times\frac{2}{10}+5\times\frac{1}{3}\times\frac{2}{5}+5\times\frac{2}{3}\times\frac{8}{10}\right)$ A1

$= \$4 \left(\text{accept } \frac{12}{3}, \frac{60}{15}\right)$ A1 N2

[14]

60.)

***Notes**: Accept any suitable notation, as long as thecandidate's intentions are clear.*

The following symbols will be used in the markscheme.

Girls' height $G \sim N(155, 10^2)$, boys' height $B \sim N(160, 12^2)$

Height H, Female F, Male M.

(a) $P(G > 170) = 1 - P(G < 170)$ (A1)

$P(G > 170) = P\left(Z < \frac{170-155}{10}\right)$ (A1)

$P(G > 170) = 1 - \Phi(1.5) = 1 - 0.9332$

$= 0.0668$ A1 N3

(b) $z = -1.2816$ (A1)

Correct calculation (*eg* $x = 155 + -1.282 \times 10$) (A1)

$x = 142$ A1 N3

(c) Calculating one variable (A1)

eg $P(B < r) = 0.95$, $z = 1.6449$

$r = 160 + 1.645(12) = 179.74$

$= 180$ A1 N2

Any valid calculation for the second variable, including use of symmetry (A1) *eg* $P(B < q) = 0.05$, $z = -1.6449$

$q = 160 - 1.645(12) = 140.26$

$= 140$ A1 N2

***Note**: Symbols are not required in parts (d) and (e).*

(d) $P(M \cap (B > 170)) = 0.4 \times 0.2020$, $P(F \cap (G > 170)) = 0.6 \times 0.0668$ (A1)(A1)

$P(H > 170) = 0.0808 + 0.04008$ A1

$= 0.12088 = 0.121$ (3 sf) A1 N2

(e) $P(F \mid H > 170) = \dfrac{P(F \cap (H > 170))}{P(H > 170)}$ (M1)

$= \dfrac{0.60 \times 0.0668}{0.121}$ $\left(= \dfrac{0.0401}{0.121} \text{ or } \dfrac{0.04008}{0.1208}\right)$ A1

$= 0.332$ A1 N1

[17]

61.) (a) For attempting to use the formula $(P(E \cap F) = P(E)P(F))$ (M1)

Correct substitution or rearranging the formula A1

eg $\dfrac{1}{3} = \dfrac{2}{3}P(F)$, $P(F) = \dfrac{P(E \cap F)}{P(E)}$, $P(F) = \dfrac{\frac{1}{3}}{\frac{2}{3}}$

$P(F) = \dfrac{1}{2}$ A1 N2

(b) For attempting to use the formula $(P(E \cup F) = P(E) + P(F) - (P(E \cap F))$ (M1)

$P(E \cup F) = \dfrac{2}{3} + \dfrac{1}{2} - \dfrac{1}{3}$ A1

$= \dfrac{5}{6} (= 0.833)$ A1 N2

[6]

62.) **METHOD 1 Use of the GDC**

(a) Evidence of using the binomial facility, M1

that is set up with $P = \dfrac{1}{2}$ and $n = 5$.

P(X = 3) = 0.3125 $\left(0.313, \frac{5}{16}\right)$ A2 N2

(b) Evidence of set up, with 1 – P(X = 0) M1

$= 0.969 \left(= \frac{31}{32}\right)$ A2 N2

METHOD 2 Use of the formula

(a) Evidence of binomial formula (M1)

P(X = 3) = $\binom{5}{3}\left(\frac{1}{2}\right)^5$ A1

$= \frac{5}{16}$ $(= 0.313)$ A1 N2

(b) **METHOD 1**

P(at least one head) = 1 – P(X = 0) (M1)

$= 1 - \left(\frac{1}{2}\right)^5$ A1

$= \frac{31}{32}$ $(= 0.969)$ A1 N2

METHOD 2

P(at least one head) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)
+ P(X = 5) (M1)

= 0.15625 + 0.3125 + 0.3125 + 0.15625 + 0.03125 A1

= 0.969 A1 N2

[6]

63.) $X \sim N(\mu, \sigma^2)$, P(X < 3) = 0.2, P(X > 8) = 0.1

P(X < 8) = 0.9 (M1)

Attempt to set up equations (M1)

$\frac{3-\mu}{\sigma} = -0.8416, \quad \frac{8-\mu}{\sigma} = 1.282$ A1A1

$3 - \mu = -0.8416\sigma$

$8 - \mu = 1.282\sigma$

$5 = 2.1236\sigma$

$\sigma = 2.35, \quad \mu = 4.99$ A1A1 N4

[6]

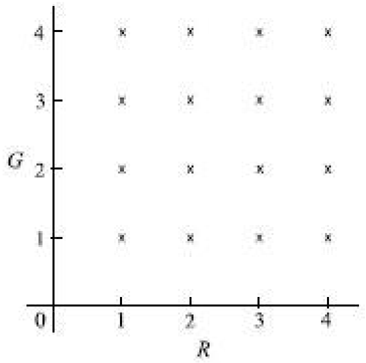
64.) (a) (i) Attempt to set up sample space, (M1)

Any **correct** representation with 16 pairs A2 N3

eg 1,1 2,1 3,1 4,1

1,2 2,2 3,2 4,2

1,3	2,3	3,3	4,3
1,4	2,4	3,4	4,4



(ii) Probability of two 4s is $\frac{1}{16}$ (= 0.0625) A1 N1

(b)

x	0	1	2
P($X = x$)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

A1A1A1 N3

(c) Evidence of selecting appropriate formula for E(X) (M1)

eg $\mathrm{E}(X) = \sum_{0}^{2} x\,\mathrm{P}(X = x)$, $\mathrm{E}(X) = np$

Correct substitution

eg $\mathrm{E}(X) = 0 \times \frac{9}{16} + 1 \times \frac{6}{16} + 2 \times \frac{1}{16}$, $\mathrm{E}(X) = 2 \times \frac{1}{4}$

$\mathrm{E}(X) = \frac{8}{16}\left(= \frac{1}{2}\right)$ A1 N2

[10]

65.) (a) $X \sim \mathrm{B}(100, 0.02)$
$E(X) = 100 \times 0.02 = 2$ A1 1

(b) $P(X = 3) = \binom{100}{3}(0.02)^3(0.98)^{97}$ (M1)

= 0.182 A1 2

(c) **METHOD 1**

$\mathrm{P}(X > 1) = 1 - \mathrm{P}(X \le 1) = 1 - (\mathrm{P}(X = 0) + \mathrm{P}(X = 1)$ M1
$= 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99})$ (M1)
= 0.597 A1 2

METHOD 2

$\mathrm{P}(X > 1) = 1 - \mathrm{P}(X \le 1)$ (M1)
= 1 – 0.40327 (A1)
= 0.597 A1 2

Note: Award marks as follows for finding P(X > 1), if working shown. (**Note** in bold)

$\mathrm{P}(X \geq 1)$	A0	
$= 1 - \mathrm{P}(X<2) = 1 - 0.67668$	M1(ft)	
$= 0.323$	A1(ft)	2

[6]

66.) $X \sim \mathrm{N}(\mu, \mathrm{s}^2)$, $\mathrm{P}(X > 90) = 0.15$ **and** $\mathrm{P}(X < 40) = 0.12$ (M1)
Finding standardized values 1.036, –1.175 A1A1

Setting up the equations $1.036 = \frac{90 - «}{±}$, $-1.175 = \frac{40 - «}{±}$ (M1)

$\mu = 66.6$, $\mathrm{s} = 22.6$ A1A1

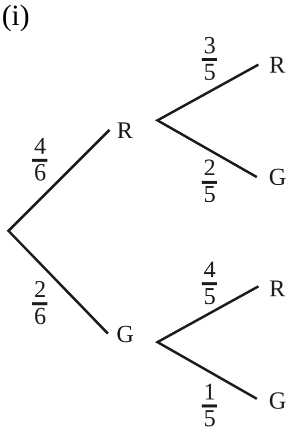
[6]

67.) (a) Using $\mathrm{E}(X) = \sum_{0}^{2} x\,\mathrm{P}(X = x)$ (M1)

Substituting correctly $E(X) = 0 \times \frac{3}{10} + 1 \times \frac{6}{10} + 2 \times \frac{1}{10}$ A1

$= \frac{8}{10}$ (0.8) A1 3

(b) (i)



A1A1A1 3

***Note**: Award (A1) for each complementary pair of probabilities, ie $\frac{4}{6}$ and $\frac{2}{6}$, $\frac{3}{5}$ and $\frac{2}{5}$, $\frac{4}{5}$ and $\frac{1}{5}$.*

(ii) $\mathrm{P}(Y = 0) = \frac{2}{5} \times \frac{1}{5} = \frac{2}{30}$ A1

$\mathrm{P}(Y = 1) = \mathrm{P}(RG) + \mathrm{P}(GR)$ $\left(= \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5}\right)$ M1

$= \frac{16}{30}$ A1

$\mathrm{P}(Y = 2) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$ (A1)

For forming a distribution M1 5

y	0	1	2

$\mathrm{P}(Y = y)$	$\frac{2}{30}$	$\frac{16}{30}$	$\frac{12}{30}$

(c) P(Bag A) = $\frac{2}{6}\left(=\frac{1}{3}\right)$ (A1)

P(BagA B) = $\frac{4}{6}\left(=\frac{2}{3}\right)$ (A1)

For summing $\mathrm{P}(A \cap RR)$ and $\mathrm{P}(B \cap RR)$ (M1)

Substituting correctly $\mathrm{P}(RR) = \frac{1}{3}\times\frac{1}{10}+\frac{2}{3}\times\frac{12}{30}$ A1

$= \frac{27}{90}\left(\frac{3}{10}, 0.3\right)$ A1 5

(d) For recognising that $\mathrm{P}(1 \text{ or } 6|RR) = \mathrm{P}(A|RR) = \frac{\mathrm{P}(A\cap RR)}{\mathrm{P}(RR)}$ (M1)

$= \frac{1}{30}\div\frac{27}{90}$ A1

$= \frac{3}{27}\left(\frac{1}{9}, 0.111\right)$ A1 3

[19]

68.) Total number of possible outcomes = 36 (may be seen anywhere) (A1)

(a) $\mathrm{P}(E) = \mathrm{P}(1,1)+\mathrm{P}(2,2)+\mathrm{P}(3,3)+\mathrm{P}(4,4)+\mathrm{P}(5,5)+\mathrm{P}(6,6)$

$= \frac{6}{36}$ (A1) (C2)

(b) $\mathrm{P}(F) = \mathrm{P}(6,4)+\mathrm{P}(5,5)+\mathrm{P}(4,6)$

$= \frac{3}{36}$ (A1) (C1)

(c) $\mathrm{P}(E\cup F) = \mathrm{P}(E) + \mathrm{P}(F) - \mathrm{P}(E \cap F)$

$\mathrm{P}(E\cap F) = \frac{1}{36}$ (A1)

$\mathrm{P}(E\cup F) = \frac{6}{36}+\frac{3}{36}-\frac{1}{36}\left(=\frac{8}{36}=\frac{2}{9}, 0.222\right)$ (M1)(A1) (C3)

[6]

69.) (a) (i) $\mathrm{P}(A) = \frac{80}{210} = \left(\frac{8}{21} = 0.381\right)$ (A1) (N1)

(ii) $\mathrm{P}(\text{year 2 art}) = \frac{35}{210} = \left(\frac{1}{6} = 0.167\right)$ (A1) (N1)

(iii) No (the events are not independent, or, they are dependent) (A1) (N1)

EITHER

$\mathrm{P}(A\cap B) = \mathrm{P}(A)\times\mathrm{P}(B)$ (to be independent) (M1)

$\mathrm{P}(B)=\frac{100}{210}\left(=\frac{10}{21}=0.476\right)$ (A1)

$\frac{1}{6}\neq\frac{8}{21}\times\frac{10}{21}$ (A1)

OR

$\mathrm{P}(A)=\mathrm{P}(A|B)$ (to be independent) (M1)

$\mathrm{P}(A|B)=\frac{35}{100}$ (A1)

$\frac{8}{21}\neq\frac{35}{100}$ (A1)

OR

$\mathrm{P}(B)=\mathrm{P}(B|A)$ (to be independent) (M1)

$\mathrm{P}(B)=\frac{100}{210}\left(=\frac{10}{21}=0.476\right),\ \mathrm{P}(B|A)=\frac{35}{80}$ (A1)

$\frac{35}{80}\neq\frac{100}{210}$ (A1) 6

***Note:** Award the first (M1) only for a **mathematical** interpretation of independence.*

(b) $n(\text{history})=85$ (A1)

$\mathrm{P}(\text{year 1}|\text{history})=\frac{50}{85}=\left(\frac{10}{17}=0.588\right)$ (A1) (N2) 2

(c) $\left(\frac{110}{210}\times\frac{100}{209}\right)+\left(\frac{100}{210}\times\frac{110}{209}\right)\left(=2\times\frac{110}{210}\times\frac{100}{209}\right)$ (M1)(A1)(A1)

$=\frac{200}{399}(=0.501)$ (A1) (N2) 4

[12]

70.) (i) $\mathrm{P}(X>3\,200)=\mathrm{P}(Z>0.4)$ (M1)

$=1-0.6554=34.5\,\%\ (=0.345)$ (A1) (N2)

(ii) $\mathrm{P}(2\,300<X<3\,300)=\mathrm{P}(-1.4<Z<0.6)$ (M1)

$=0.4192+0.2257$

$=0.645$ (A1)

$\mathrm{P}(\text{both})=(0.645)^2=0.416$ (A1) (N2)

(iii) $0.7422=\mathrm{P}(Z<0.65)$ (A1)

$\frac{d-3\,000}{500}=0.65$ (A1)

$d=\$\,3\,325\ (=\$\,3\,330\text{ to 3 s.f.})$ (Accept \$3325.07) (A1) (N3)

[8]

71.) Correct probabilities $\left(\frac{13}{24}\right),\left(\frac{12}{23}\right),\left(\frac{11}{22}\right),\left(\frac{10}{21}\right)$ (A1)(A1)(A1)(A1)

Multiplying $\left(\frac{13}{24}\times\frac{12}{23}\times\frac{11}{22}\times\frac{10}{21}\right)$ (M1)

P(4 girls) = $\frac{17160}{255024}\left(=\frac{65}{966}=0.0673\right)$ (A1) (C6)

[6]

72.) For using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)

Let $P(A) = x$ then $P(B) = 3x$

$P(A \cap B) = P(A) \times 3P(A)$ $(= 3x^2)$ (A1)

$0.68 = x + 3x - 3x^2$ (A1)

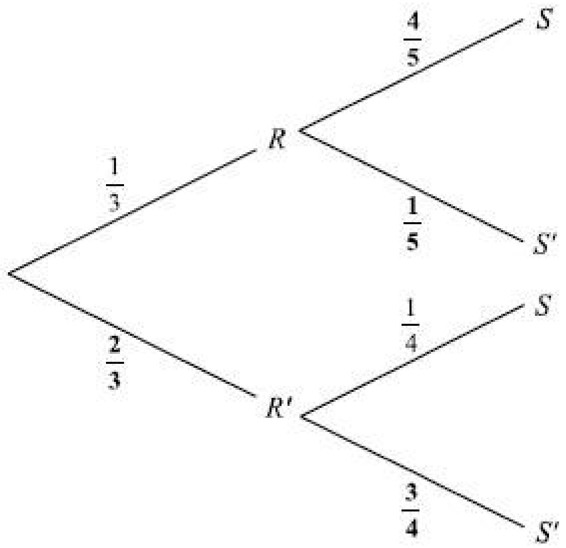
$3x^2 - 4x + 0.68 = 0$

$x = 0.2$ ($x = 1.133$, not possible) (A2)

$P(B) = 3x = 0.6$ (A1) (C6)

[6]

73.) (a)



(A1)(A1)(A1)

(b) (i) P(R ą S) = $\frac{1}{3}\times\frac{4}{5}\left(=\frac{4}{15}=0.267\right)$ (A1) (N1)

(ii) $P(S) = \frac{1}{3}\times\frac{4}{5}+\frac{2}{3}\times\frac{1}{4}$ (A1)(A1)

$= \frac{13}{30}$ $(= 0.433)$ (A1) (N3)

(iii) $\mathrm{P}(R \mid S) = \dfrac{\frac{4}{15}}{\frac{13}{30}}$ (A1)(A1)

$= \dfrac{8}{13}$ (= 0.615) (A1) (N3)

[10]

74.) (a) $z = \dfrac{185-170}{20} = 0.75$ (M1)(A1)

$\mathrm{P}(Z < 0.75) = 0.773$ (A1) (N3)

(b) $z = -0.47$ (may be implied) (A1)

$-0.47 = \dfrac{d-170}{20}$ (M1)

$d = 161$ (A1) (N3)

[6]

75.) (a) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (M1)

$\mathrm{P}(A \cap B) = \dfrac{1}{2} + \dfrac{3}{4} - \dfrac{7}{8}$

$= \dfrac{3}{8}$ (A1) (C2)

(b) $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} \left(= \dfrac{\frac{3}{8}}{\frac{3}{4}} \right)$ (M1)

$= \dfrac{1}{2}$ (A1) (C2)

(c) Yes, the events are independent (A1) (C1)

EITHER

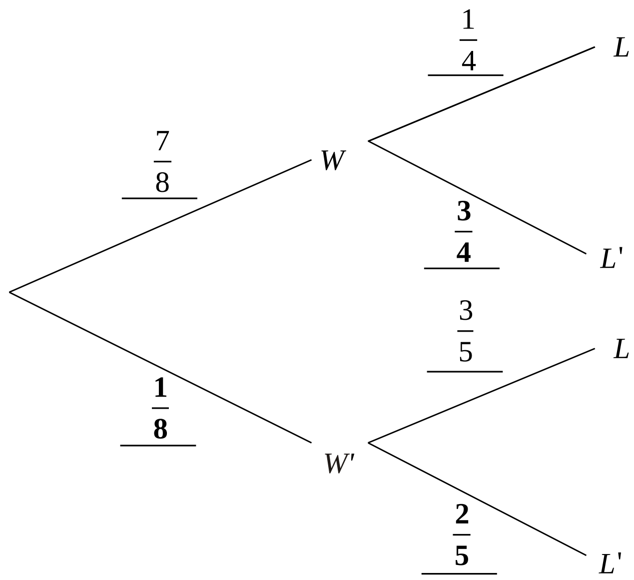
$\mathrm{P}(A \mid B) = \mathrm{P}(A)$ (R1) (C1)

OR

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ (R1) (C1)

[6]

76.) (a)



(A1)(A1)(A1)(A1) 4

Note: *Award (A1) for the given probabilities* $\left(\frac{7}{8}, \frac{1}{4}, \frac{3}{5}\right)$ *in the correct positions, and (A1) for each* ***bold*** *value.*

(b) Probability that Dumisani will be late is $\frac{7}{8} \times \frac{1}{4} + \frac{1}{8} \times \frac{3}{5}$ (A1)(A1)

$= \frac{47}{160}$ (0.294) (A1) (N2) 3

(c) $\mathrm{P}(W | L) = \frac{\mathrm{P}(W \cap L)}{\mathrm{P}(L)}$

$\mathrm{P}(W \cap L) = \frac{7}{8} \times \frac{1}{4}$ (A1)

$\mathrm{P}(L) = \frac{47}{160}$ (A1)

$\mathrm{P}(W | L) = \frac{\frac{7}{32}}{\frac{47}{160}}$ (M1)

$= \frac{35}{47}$ (= 0.745) (A1) (N3)4

[11]

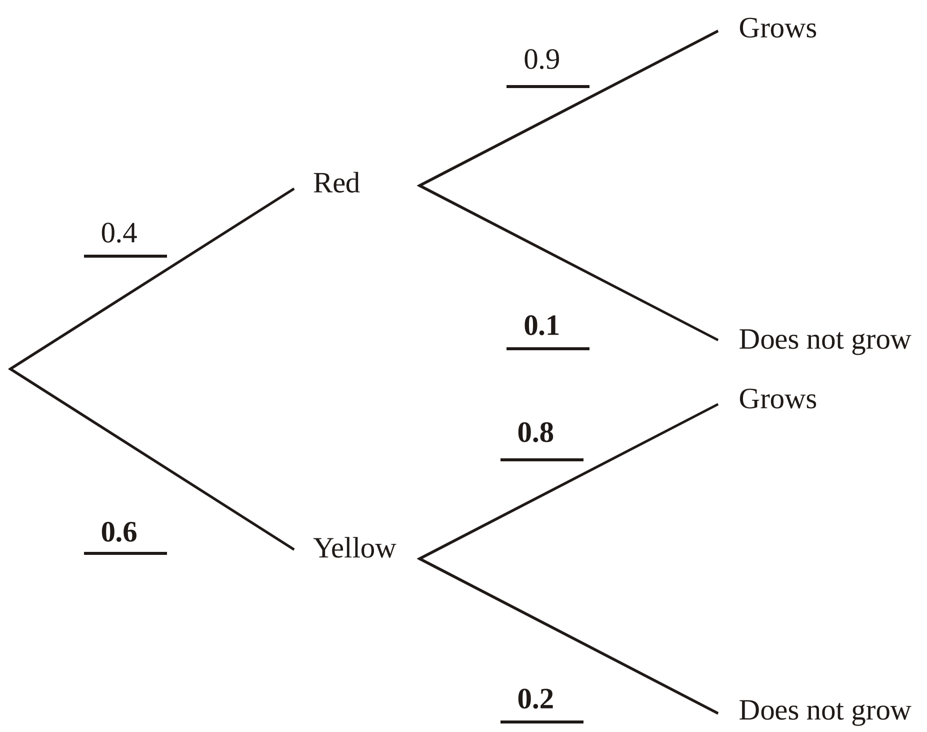
77.) (a) $\frac{120}{360}\left(= \frac{1}{3} = 0.333\right)$ (A1)(A1) (C2)

(b) $\frac{90+120}{360}\left(= \frac{210}{360} = \frac{7}{12} = 0.583\right)$ (A2) (C2)

(c) $\frac{90}{210}\left(= \frac{3}{7} = 0.429\right)$ $\left(\text{Accept } \frac{\frac{1}{4}}{\frac{7}{12}}\right)$ (A1)(A1) (C2)

[6]

78.) (a)



(A3) (N3) 3

(b) (i) 0.4×0.9 (A1)

$= 0.36$ (A1) (N2)

(ii) $0.36 + 0.6 \times 0.8$ $(= 0.36 + 0.48)$ (A1)

$= 0.84$ (A1) (N1)

(iii) $\frac{P(\text{red} \cap \text{grows})}{P(\text{grows})}$ (may be implied) (M1)

$= \frac{0.36}{0.84}$ (A1)

$= 0.429 \left(\frac{3}{7}\right)$ (A1)(N2) 7

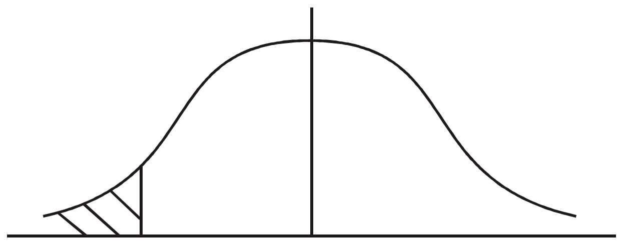
[10]

79.) (a) (i) $a = -1$ (A1)
$b = 0.5$ (A1)

(ii) (a) 0.841 (A2)

(b) $0.6915 - 0.1587$ (or $0.8413 - 0.3085$) (M1)
$= 0.533$ (3 sf) (A1) (N2) 6

(b) (i) Sketch of normal curve (A1)(A1)



(ii) $c = 0.647$ (A2) 4

[10]

80.) (a) Independent (I) (C2)

(b) Mutually exclusive (M) (C2)

(c) Neither (N) (C2)

Note: *Award part marks if the candidate shows understanding of I and/or M*

eg I P(A ą B) = P(A)P(B) (M1)

M P(A Ć B) = P(A) + P(B) (M1)

[6]

81.) **Method 1**

$b^2 - 4ac = 9 - 4k$ (M1)

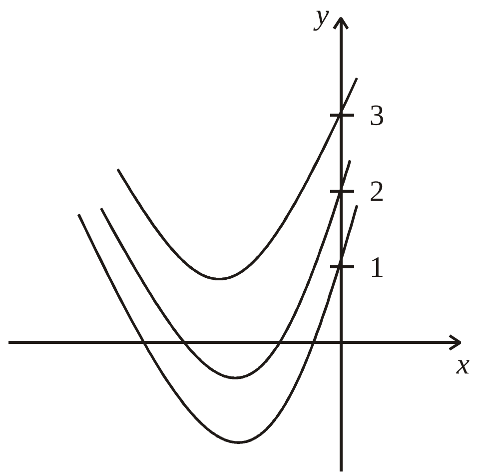
$9 - 4k > 0$ (M1)

$2.25 > k$ (A1)

crosses the x-axis if $k = 1$ **or** $k = 2$ (A1)(A1)

probability $= \frac{2}{7}$ (A1) (C6)

Method 2



(M2)(M1)

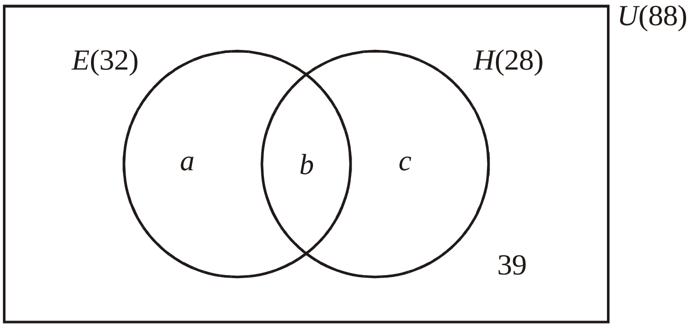
Note: *Award (M2) for one (relevant) curve; (M1) for a second one.*

$k = 1$ **or** $k = 2$ (G1)(G1)

probability $= \frac{2}{7}$ (A1) (C6)

[6]

82.) (a)



$n(E \cup H) = a + b + c = 88 - 39 = 49$ (M1)

$n(E \cup H) = 32 + 28 - b = 49$

$60 - 49 = b = 11$ (A1)

$a = 32 - 11 = 21$ (A1)

$c = 28 - 11 = 17$ (A1) 4

Note: *Award (A3) for correct answers with no working.*

(b) (i) $P(E \cap H) = \dfrac{11}{88} = \dfrac{1}{8}$ (A1)

(ii) $P(H'|E) = \dfrac{P(H' \cap E)}{P(E)} = \dfrac{\frac{21}{88}}{\frac{32}{88}}$ (M1)

$= \dfrac{21}{32}$ (= 0.656) (A1)

OR

Required probability = $\dfrac{21}{32}$ (A1)(A1) 3

(c) (i) P(none in economics) = $\dfrac{56 \times 55 \times 54}{88 \times 87 \times 86}$ (M1)(A1)

= 0.253 (A1)

Notes: *Award (M0)(A0)(A1)(ft) for* $\left(\dfrac{56}{88}\right)^3$ *= 0.258.*

Award no marks for $\dfrac{56 \times 55 \times 54}{88 \times 88 \times 88}$.

(ii) P(at least one) = 1 – 0.253 (M1)

= 0.747 (A1)

OR

$3\left(\dfrac{32}{88} \times \dfrac{56}{87} \times \dfrac{55}{86}\right) + 3\left(\dfrac{32}{88} \times \dfrac{31}{87} \times \dfrac{56}{86}\right) + \dfrac{32}{88} \times \dfrac{31}{87} \times \dfrac{30}{86}$ (M1)

= 0.747 (A1) 5

[12]

83.) $X \sim N(80, 8^2)$

(a) $P(X < 72) = P(Z < -1)$ (M1)

$= 1 - 0.8413$

$= 0.159$ (A1)

OR

$P(X < 72) = 0.159$ (G2) 2

(b) (i) $P(72 < X < 90) = P(-1 < Z < 1.25)$ (M1)

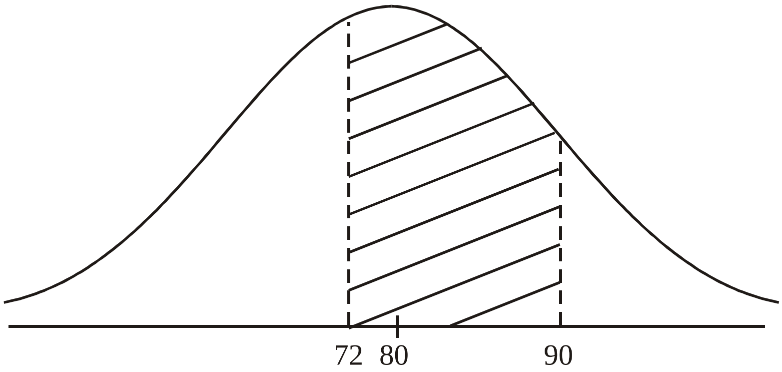
$= 0.3413 + 0.3944$ (A1)

$= 0.736$ (A1)

OR

$P(72 < X < 90) = 0.736$ (G3)

(ii)



(A1)(A1) 5

Note: *Award (A1) for a normal curve and (A1) for the shaded area, which should not be symmetrical.*

(c) 4% fail in less than x months

$\Rightarrow x = 80 - 8 \times \Phi^{-1}(0.96)$ (M1)

$= 80 - 8 \times 1.751$ (A1)

$= 66.0$ months (A1)

OR

$x = 66.0$ months (G3) 3

[10]

84.) $P(RR) = \frac{7}{12} \times \frac{6}{11} \left(= \frac{7}{22}\right)$ (M1)(A1)

$P(YY) = \frac{5}{12} \times \frac{4}{11} \left(= \frac{5}{33}\right)$ (M1)(A1)

P (same colour) = P(RR) + P(YY) (M1)

$= \frac{31}{66}$ (= 0.470, 3 sf) (A1) (C6)

Note: *Award C2 for* $\left(\frac{7}{12}\right)^2 + \left(\frac{5}{12}\right)^2 = \frac{74}{144}$.

[6]

85.) (a) $P(M \geq 350) = 1 - P(M < 350) = 1 - P\left(Z < \frac{350 - 310}{30}\right)$ (M1)

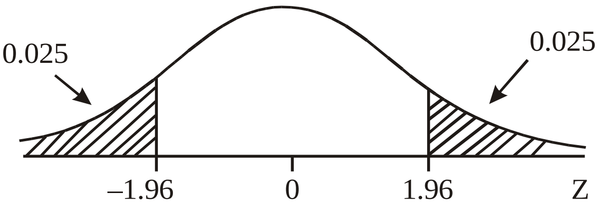
$= 1 - P(Z < 1.333) = 1 - 0.9088$

$= 0.0912$ (accept 0.0910 to 0.0920) (A1)

OR

$P(M \geq 350) = 0.0912$ (G2)

(b)



$P(Z < 1.96) = 1 – 0.025 = 0.975$ (A1)

$1.96\ (30) = 58.8$ (M1)

$310 – 58.8 < M < 310 + 58.8 \Rightarrow a = 251, b = 369$ (A1)

OR

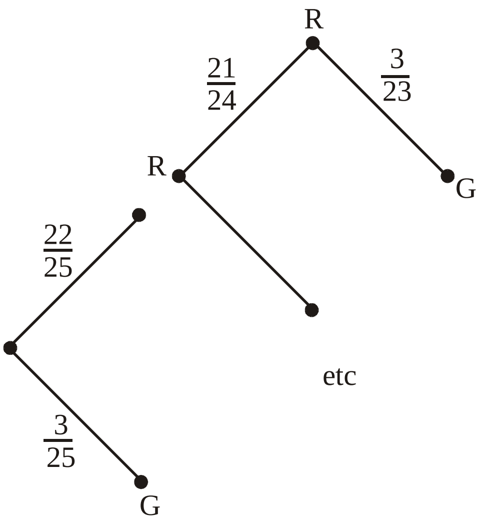
$251 < M < 369$ (G3)

Note: *Award (G1) if only one of the end points is correct.*

[5]

86.) (a) $P = \frac{22}{23}$ (= 0.957 (3 sf))(A2) (C2)

(b)



(M1)

OR

P = P (RRG) + P (RGR) + P (GRR) (M1)

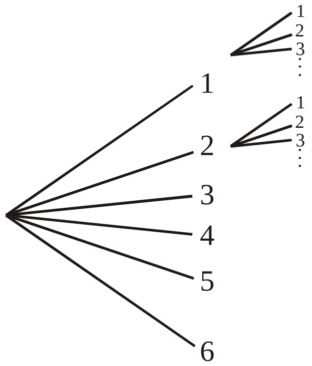
$$\frac{22}{25}\times\frac{21}{24}\times\frac{3}{23}+\frac{22}{25}\times\frac{3}{24}\times\frac{21}{23}+\frac{3}{25}\times\frac{22}{24}\times\frac{21}{23}$$ (M1)(A1)

$= \frac{693}{2300}$ (= 0.301 (3 sf)) (A1) (C4)

[6]

87.) Sample space ={(1, 1), (1, 2) ... (6, 5), (6, 6)}

(This may be indicated in other ways, for example, a grid or a tree diagram, partly or fully completed)



(a) $\quad$ P (S < 8) = $\dfrac{6+5+4+3+2+1}{36}$ (M1)

$= \dfrac{7}{12}$ (A1)

OR

P (S < 8) = $\dfrac{7}{12}$ (A2)

(b) $\quad$ P (at least one 3) = $\dfrac{1+1+6+1+1+1}{36}$ (M1)

$= \dfrac{11}{36}$ (A1)

OR

P (at least one 3) = $\dfrac{11}{36}$ (A2)

(c) $\quad$ P (at least one 3 | S < 8) = $\dfrac{\text{P(at least one } 3 \cap S < 8)}{\text{P}(S < 8)}$ (M1)

$= \dfrac{7/36}{7/12}$ (A1)

$= \dfrac{1}{3}$ (A1)

[7]

88.) (a) (These answers may be obtained from a calculator or by finding z in each case and the corresponding area.)

$M \sim N$ (750, 625)

(i) P (M < 740 g) = 0.345 (G2)

OR

$z = –0.4$ $P(z < –0.4) = 0.345$ (A1)(A1)

(ii) $P(M > 780 \text{ g}) = 0.115$ (G2)

OR

$z = 1.2$ $P(z > 1.2) = 1 – 0.885 = 0.115$ (A1)(A1)

(iii) $P(740 < M < 780) = 0.540$ (G1)

OR

$1 – (0.345 + 0.115) = 0.540$ (A1) 5

(b) Independent events

Therefore, P (both < 740) = 0.345^2 (M1)
$= 0.119$ (A1) 2

(c) 70% have mass < 763 g (G1)
Therefore, 70% have mass of at least 750 – 13
$x = 737$ g (A1) 2

[9]

89.) (a) $P(A \cup B) = P(A) + P(B) – P(A \cap B) \Rightarrow P(A \cap B) = P(A) + P(B) – P(A \cup B)$ (M1)

$= \frac{3}{11} + \frac{4}{11} - \frac{6}{11}$ (M1)

$= \frac{1}{11}$ (0.0909) (A1) (C3)

(b) For independent events, $P(A \cap B) = P(A) \times P(B)$ (M1)

$= \frac{3}{11} \times \frac{4}{11}$ (A1)

$= \frac{12}{121}$ (0.0992) (A1) (C3)

[6]

90.) ***Note:*** *Where accuracy is not specified, accept answers with greater than 3 sf accuracy, provided they are correct as far as 3 sf*

(a) $z = \frac{197 - 187.5}{9.5} = 1.00$ (M1)

$P(Z > 1) = 1 – \Phi(1) = 1 – 0.8413 = 0.1587$
$= 0.159$ (3 sf) (A1)
$= 15.9\%$ (A1)

OR

$P(H > 197) = 0.159$ (G2)
$= 15.9\%$ (A1) 3

(b) Finding the 99th percentile

$\Phi(a) = 0.99 \Rightarrow a = 2.327$ (accept 2.33) (A1)
=> 99% of heights under $187.5 + 2.327(9.5) = 209.6065$ (M1)
$= 210$ (3 sf) (A1)

OR

99% of heights under 209.6 = 210 cm (3 sf) (G3)

Height of standard doorway = 210 + 17 = 227 cm (A1) 4

[7]

91.) P(different colours) = 1 – [P(GG) + P(RR) + P(WW)] (M1)

$= 1 - \left(\frac{10}{6}\times\frac{9}{25}+\frac{10}{26}\times\frac{9}{25}+\frac{6}{26}\times\frac{5}{25}\right)$ (A1)

$= 1 - \left(\frac{210}{650}\right)$ (A1)

$= \frac{44}{65}$ (= 0.677, to 3 sf) (A1) (C4)

OR

P(different colours) = P(GR) + P(RG) + P(GW) + P(WG) + P(RW) + P(WR)(A1)

$= 4\left(\frac{10}{26}\times\frac{6}{25}\right)+2\left(\frac{10}{26}\times\frac{10}{25}\right)$ (A1)(A1)

$= \frac{44}{65}$ (= 0.677, to 3 sf) (A1) (C4)

[4]

92.) (a) $s = 7.41$(3 sf) (G3) 3

(b)

Weight (W)	$W \le 85$	$W \le 90$	$W \le 95$	$W \le 100$	$W \le 105$	$W \le 110$	$W \le 115$
Number of packets	5	15	**30**	**56**	**69**	**76**	80

(A1) 1

(c) (i)From the graph, the median is approximately 96.8.
Answer: 97 (nearest gram). (A2)

(ii) From the graph, the upper or third quartile is approximately 101.2.
Answer: 101 (nearest gram). (A2) 4

(d) Sum = 0, since the sum of the deviations from the mean is zero. (A2)
OR

$\sum (W_i - \overline{W}) = \sum W_i - \left(80\frac{\sum W_i}{80}\right) = 0$ (M1)(A1) 2

(e) Let A be the event: $W > 100$, and B the event: $85 < W \le 110$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ (M1)

$P(A \cap B) = \frac{20}{80}$ (A1)

$P(B) = \frac{71}{80}$ (A1)

$P(A|B) = 0.282$ (A1)

OR

71 packets with weight $85 < W \le 110$. (M1)
Of these, 20 packets have weight $W > 100$. (M1)

Required probability = $\frac{20}{71}$ (A1)

= 0.282 (A1) 4

Notes: *Award (A2) for a correct final answer with no reasoning.*
Award up to (M2) for correct reasoning or method.

[14]

93.) (a) Let X be the random variable for the IQ.
$X \sim N(100, 225)$
$P(90 < X < 125) = P(-0.67 < Z < 1.67)$ (M1)
$= 0.701$
70.1 percent of the population (accept 70 percent). (A1)

OR

$P(90 < X < 125) = 70.1\%$ (G2) 2

(b) $P(X \geq 125) = 0.0475$ (or 0.0478) (M1)
P(both persons having IQ ≥ 125) $= (0.0475)^2$ (or $(0.0478)^2$) (M1)
$= 0.00226$ (or 0.00228) (A1) 3

(c) Null hypothesis (H_0): mean IQ of people with disorder is 100 (M1)
Alternative hypothesis (H_1): mean IQ of people with disorder is less than 100 (M1)

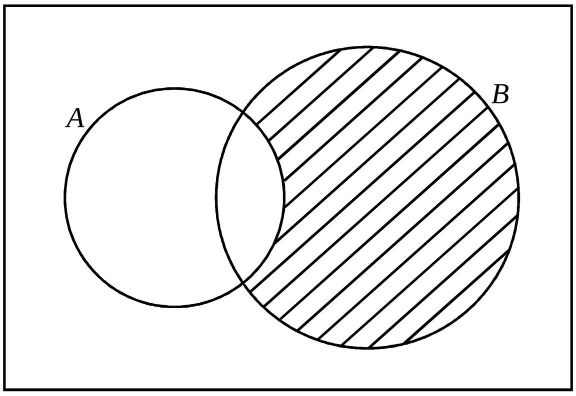
$$P(\overline{X} < 95.2) = P\left(Z < \left(\frac{95.2 - 100}{\frac{15}{\sqrt{25}}}\right)\right) = P(Z < -1.6) = 1 - 0.9452$$

$= 0.0548$ (A1)

The probability that the sample mean is 95.2 and the null hypothesis true is $0.0548 > 0.05$. Hence the evidence is not sufficient. (R1) 4

[9]

94.) (a) U



(A1) (C1)

(b) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$65 = 30 + 50 - n(A \cap B)$

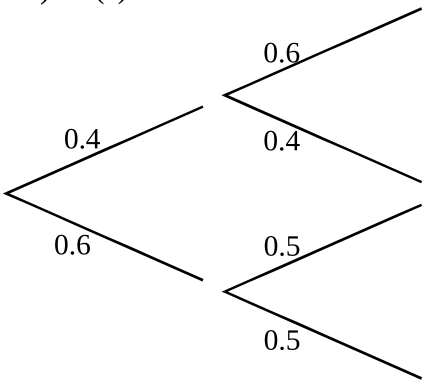
$\Rightarrow n(A \cap B) = 15$ (may be on the diagram) (M1)

$n(B \cap A') = 50 - 15 = 35$ (A1) (C2)

(c) $P(B \cap A') = \dfrac{n(B \cap A')}{n(U)} = \dfrac{35}{100} = 0.35$ (A1) (C1)

[4]

95.) (a)



(A1) (C1)

(b) $P(B) = 0.4(0.6) + 0.6\,(0.5) = 0.24 + 0.30$ (M1)

$= 0.54$ (A1) (C2)

(c) $P(C \mid B) = \dfrac{P(B \cap C)}{P(B)} = \dfrac{0.24}{0.54} = \dfrac{4}{9}$ (= 0.444, 3 sf) (A1) (C1)

[4]

96.) (a) $Z = \dfrac{25 - 25.7}{0.50} = -1.4$ (M1)

$P(Z < -1.4) = 1 - P(Z < 1.4)$

$= 1 - 0.9192$

$= 0.0808$ (A1)

OR

$P(W < 25) = 0.0808$ (G2) 2

(b) $P(Z < -a) = 0.025 \Rightarrow P(Z < a) = 0.975$

$\Rightarrow a = 1.960$ (A1)

$\frac{25 - «}{0.50}$ = –1.96 ⇒ μ = 25 + 1.96 (0.50) (M1)

= 25 + 0.98 = 25.98 (A1)

= 26.0 (3 sf) (AG)

OR

$\frac{25.0 - 26.0}{0.50}$ = –2.00 (M1)

P(Z < –2.00) = 1 – P(Z < 2.00)

= 1 – 0.9772 = 0.0228 (A1)

≈ 0.025 (A1)

OR

μ = 25.98 (G2)

⇒ mean = 26.0 (3 sf) (A1)(AG) 3

(c) Clearly, by symmetry μ = 25.5 (A1)

$Z = \frac{25.0 - 25.5}{\pm}$ = –1.96 ⇒ 0.5 = 1.96σ (M1)

⇒ σ = 0.255 kg (A1) 3

(d) On average, $\frac{\text{cement saving}}{\text{bag}}$ = 0.5 kg (A1)

$\frac{\text{cost saving}}{\text{bag}}$ = 0.5(0.80) = $0.40 (M1)

To save $5000 takes $\frac{5000}{0.40}$ = 12500 bags (A1) 3

[11]

97.) (a)

	Males	Females	Totals
Unemployed	**20**	**40**	**60**
Employed	**90**	**50**	**140**
Totals	**110**	**90**	200

Note: *Award (A1) if at least 4 entries are correct.*
Award (A2) if all 8 entries are correct.

(b) (i) P(unemployed female) = $\frac{40}{200} = \frac{1}{5}$ (A1)

(ii) P(male I employed person) = $\frac{90}{140} = \frac{9}{14}$ (A1)

[4]

98.) (a)

	Boy	Girl	Total
TV	**13**	**25**	**38**
Sport	33	29	**62**
Total	46	**54**	100

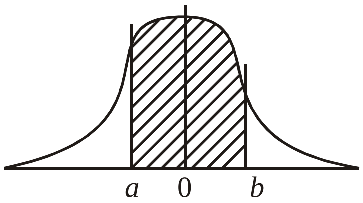
P(TV) = $\frac{38}{100}$ (A1) (C2)

(b) P(TV | Boy) = $\frac{13}{46}$ (= 0.283 to 3 sf) (A2) (C2)

Notes: *Award (A1) for numerator and (A1) for denominator. Accept equivalent answers.*

[4]

99.) (a) Let X be the lifespan in hours
$X \sim N(57, 4.4^2)$



(i) $a = -0.455$ (3 sf) (A1)
$b = 0.682$ (3 sf) (A1)

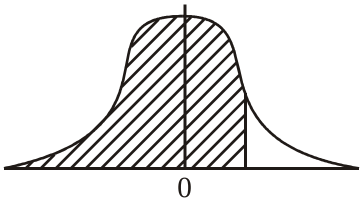
(ii) (a) $P(X > 55) = P(Z > -0.455)$
$= 0.675$ (A1)

(b) $P(55 \le X \le 60) = P\left(\frac{2}{4.4} \le Z \le \frac{3}{4.4}\right)$
$\approx P(0.455 \le Z \le 0.682)$
$\approx 0.6754 + 0.752 - 1$ (A1)
$= 0.428$ (3sf) (A1)

OR

$P(55 \le X \le 60) = 0.428$ (3 sf) (G2) 5

(b) 90% have died $\Rightarrow$ shaded area = 0.9 (M1)



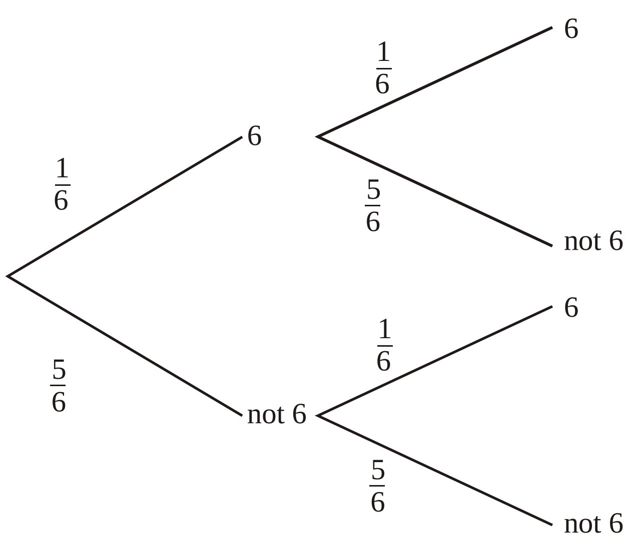
(A1)

Hence $t = 57 + (4.4 \times 1.282)$ (M1)
$= 57 + 5.64$ (A1)
$= 62.6$ hours (A1)

OR $t = 62.6$ hours (G3) 5

[10]

100.) (a)



6, 6	$\frac{1}{36}$
6, not 6	$\frac{5}{36}$
not 6, 6	$\frac{5}{36}$
not 6, not 6	$\frac{25}{36}$

(M2) (C2)

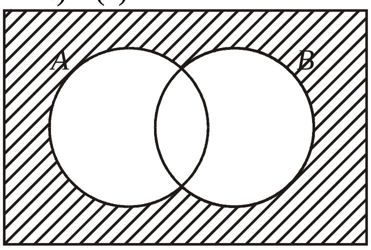
Notes: *Award (M1) for probabilities* $\frac{1}{6}, \frac{5}{6}$ *correctly entered on diagram.*
Award (M1) for correctly listing the outcomes 6, 6; 6 not 6; not 6, 6; not 6, not 6, or the corresponding probabilities.

(b) P(one or more sixes) = $\frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6}$ **or** $\left(1 - \frac{5}{6} \times \frac{5}{6}\right)$ (M1)

$= \frac{11}{36}$ (A1) (C2)

[4]

101.) (a)



(A1) (C1)

(b) (i) $n(A \cap B) = 2$ (A1) (C1)

(ii) $P(A \cap B) = \frac{2}{36}\left(\text{or } \frac{1}{18}\right)$ (allow **ft** from (b)(i)) (A1) (C1)

(c) $n(A \cap B) \neq 0$ (or equivalent) (R1) (C1)

[4]

102.) (a) ***Note:*** *Candidates using tables may get slightly different answers, especially if they do not interpolate. Accept these answers.*

$P(\text{speed} > 50) = 0.3 = 1 - \Phi\left(\frac{50 - «}{10}\right)$ (A1)

Hence, $\frac{50 - «}{10} = \Phi^{-1}(0.7)$ (M1)

$\mu = 50 - 10\Phi^{-1}(0.7)$ (M1)
$= 44.75599 \ldots\ldots = 44.8$ km/h (3 sf) (accept 44.7) (AG) 3

(b) H_1: “the mean speed has been reduced by the campaign”. (A1) 1

(c) One-tailed; because H_1 involves only “<”. (A2) 2

(d) For a one-tailed test at 5% level, critical region is
$Z < \mu_m - 1.64\sigma_m$ *(accept –1.65s$_m$)* (M1)

Now, $\mu_m = \mu = 44.75\ldots$; $\sigma_m = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2$ *(allow ft)* (A1)

So test statistic is $44.75\ldots - 1.64 \times 2 = 41.47$ (A1)

Now $41.3 < 41.47$ so reject H_0, yes. (A1) 4

[10]

103.) $p(\text{Red}) = \frac{35}{40} = \frac{7}{8}$ $p(\text{Black}) = \frac{5}{40} = \frac{1}{8}$

(a) (i) $p(\text{one black}) = \binom{8}{1}\left(\frac{1}{8}\right)^1\left(\frac{7}{8}\right)^7$ (M1)(A1)
$= 0.393$ to 3 sf (A1) 3

(ii) $p(\text{at least one black}) = 1 - p(\text{none})$ (M1)
$= 1 - \binom{8}{0}\left(\frac{1}{8}\right)^0\left(\frac{7}{8}\right)^8$ (A1)
$= 1 - 0.344$
$= 0.656$ (A1) 3

(b) 400 draws: expected number of blacks $= \frac{400}{8}$ (M1)
$= 50$ (A1) 2

[8]

104.) (a) $p(A \cap B) = 0.6 + 0.8 - 1$ (M1)
$= 0.4$ (A1) (C2)

(b) $p(\complement A \cup \complement B) = p(\complement(A \cap B)) = 1 - 0.4$ (M1)
$= 0.6$ (A1) (C2)

[4]

105.) (a) Area $A = 0.1$ (A1) 1

(b) **EITHER** Since $p(X \geq 12) = p(X \leq 8)$, (M1)
then 8 and 12 are symmetrically disposed around the (M1)(R1)
mean.

Thus mean = $\dfrac{8+12}{2}$ (M1)

= 10 (A1)

Notes: *If a candidate says simply "by symmetry m= 10" with no further explanation award [3 marks] (M1, A1, R1). As a full explanation is requested award an additional (A1) for saying since p(X < 8) = p(X > 12) and another (A1) for saying that the normal curve is symmetric.*

OR $p\ (X \geq 12) = 0.1 \quad \Rightarrow p\left(Z \geq \dfrac{12-«}{±}\right) = 0.1$ (M1)

$\Rightarrow p\left(Z \leq \dfrac{12-«}{±}\right) = 0.9$

$p\ (X \leq 8) = 0.1 \quad \Rightarrow p\left(Z \leq \dfrac{8-«}{±}\right) = 0.1$

$\Rightarrow p\left(Z \leq \dfrac{«-8}{±}\right) = 0.9$ (A1)

So $\dfrac{12-«}{±} = \dfrac{«-8}{±}$ (M1)

$\Rightarrow$ 12 – m= m– 8 (M1)

$\Rightarrow$ m= 10 (A1) 5

(c) $\Phi\left(\dfrac{12-10}{±}\right) = 0.9$ (A1)(M1)(A1)

Note: *Award (A1) for* $\left(\dfrac{12-10}{±}\right)$*, (M1) for standardizing, and (A1) for 0.9.*

$\Rightarrow \dfrac{2}{±} = 1.282$ (or 1.28) (A1)

s $= \dfrac{2}{1.282}\left(\text{or } \dfrac{2}{1.28}\right)$ (A1)

= 1.56 (3 sf) (AG) 5

Note: *Working backwards from* s *= 1.56 to show it leads the given data should receive a maximum of [3 marks] if done correctly.*

(d) $p\ (X \leq 11) = p\left(Z \leq \dfrac{11-10}{1.561}\right)$ (or 1.56) (M1)(A1)

Note: *Award (M1) for standardizing and (A1) for* $\left(\dfrac{11-10}{1.561}\right)$.

= p ($Z \leq 0.6407$) (or 0.641 or 0.64) (A1)

= $\Phi(0.6407)$ (M1)

= 0.739 (3 sf) (A1) 5

[16]

106.) (a) p (4 heads) = $\dbinom{8}{4}\left(\dfrac{1}{2}\right)^{4}\left(\dfrac{1}{2}\right)^{8-4}$ (M1)

$= \dfrac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} \times \left(\dfrac{1}{2}\right)^8$

$= \dfrac{70}{256} \cong 0.273$ (3 sf) (A1) 2

(b) p (3 heads) = $\dbinom{8}{3}\left(\dfrac{1}{2}\right)^3\left(\dfrac{1}{2}\right)^{8-3} = \dfrac{8 \times 7 \times 6}{1 \times 2 \times 3} \times \left(\dfrac{1}{2}\right)^8$

$= \dfrac{56}{256} \cong 0.219$ (3 sf) (A1) 1

(c) p (5 heads) = p (3 heads) (by symmetry) (M1)

p (3 or 4 or 5 heads) = p (4) + 2p (3) (M1)

$= \dfrac{70 + 2 \times 56}{256} = \dfrac{182}{256}$

≈ 0.711 (3 sf) (A1) 3

[6]