

**WHEN I WAS 14 YEARS
OLD MY FATHER WAS 42
YEARS OLD, WHICH WAS
THREE TIMES MY AGE.
NOW HE IS TWICE MY
AGE, HOW OLD AM I?**

**a small village post office is selling 1st class stamps
and 2nd class stamps in a book costing £1. if a 1st
class stamp is 9p and a 2nd class stamp is 7p. how
many ways can a £1 book be filled?**



If you were 35 years old in the year 1225 it would be a very special time mathematically, because $35^2 = 1225$. That is, the square of your age at that moment is equal to the year. This does not happen very often. Augustus de Morgan, a famous mathematician, was one of those lucky people and in 1864 he wrote:

"At some point in my life the square of my age was the same as the year."

When was he born?



**JULIE IS A CAREFUL DRIVER AND NEVER BREAKS THE SPEED
LIMIT. ONE DAY AS SHE SET OFF ON A JOURNEY IN HER CAR
SHE NOTICED THAT THE MILEOMETER WAS DISPLAYING 13931
MILES, WHICH WAS PALINDROMIC (THE SAME FORWARDS
AND BACKWARDS). AFTER COMPLETING THE JOURNEY,
WHICH TOOK TWO HOURS, JULIE COULD NOT BELIEVE THAT IT
WAS PALINDROMIC ONCE MORE. WHAT WAS HER AVERAGE**



**Three different positive integers
add to make sixteen.**

$$\square + \diamond + \nabla = 16$$

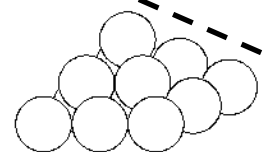
**The smallest numbers also add
to make the biggest number.**

$$\square + \diamond = \nabla$$

**Find all three possible solutions
to this problem.**

*A pile of oranges are arranged to make a square based pyramid by
having one orange on the top layer, four oranges on the second layer,
nine oranges on the third layer, and so on.*

*If there were one-thousand oranges in the pile used to make the pyramid not all of them would be needed.
How many oranges would be left over?*



IN DETAIL SOLUTIONS:

Three different positive integers add to make sixteen. The two smallest numbers add to make the biggest number. How many different solutions can you find?

Solution

Let the integers be a , b and c such that $a < b < c$.

Therefore $a + b + c = 16$.

But $a + b = c$, so $c + c = 2c = 16$, hence $c = 8$.

So solutions are: $(1,7,8)$, $(2,6,8)$ and $(3,5,8)$

A small village post office is selling 1st class stamps and 2nd class stamps in a book costing £1. If a 1st class stamp is 9p and a 2nd class stamp is 7p, how many ways can a £1 book be filled?

Solution

We are attempting to solve the Diophantine equation $9a + 7b = 100$, of which there are two positive solutions:

$$1 \times 9 + 13 \times 7 = 100$$

$$8 \times 9 + 4 \times 7 = 100$$

How many solutions are there to the equation $4a + 3b = 100$?

Try different equations and see if you can spot any patterns.

Julie is a careful driver and never breaks the speed limit. One day as she set off on a journey in her car she noticed that the odometer was displaying 13931 miles, which was palindromic (the same forwards and backwards). After completing the journey, which took two hours, Julie could not believe that it was palindromic once more.

What was her average speed during the journey?

Solution

The next palindrome after 13931 is 14041. Completing $14041 - 13931 = 110$ miles in two hours is an average speed of 55 mph. If we considered the next palindromic example, $14141 - 13931 = 210$, which means she drove at 105 mph, but we were told that she never breaks the speed limit.

If you were 35 years old in the year 1225 it would be a very special time mathematically, because $35^2 = 1225$. That is, the square of your age at that moment is equal to the year. This does not happen very often.

Augustus de Morgan, a famous mathematician, was one of those lucky people and in 1864 he wrote:

"At some point in my life the square of my age was the same as the year."

When was he born?

Solution

$$42^2 = 1764$$

If he was 42 in 1764, he would have been born in $1764 - 42 = 1722$. As he wrote it in 1864, he would have been $1864 - 1722 = 142$ years old at the time!!!

$$43^2 = 1849$$

If he was 43 in 1849, he would have been born in $1849 - 42 = 1806$, making him $1864 - 1806 = 58$ years old when he wrote the statement.

$$44^2 = 1936 \text{ (not happened by 1864)}$$

So Augustus De Morgan must have been born in 1806.

It is likely that you know somebody with the same special property with their age. When were they born and when is their special year?

When I was 14 years old my father was 42 years old, which was three times my age. Now he is twice my age, how old am I?

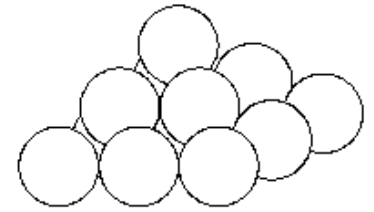
Solution

Let f be the father's age and his child's age be c .

As the father is $42 - 14 = 28$ years older than the child, $f = c + 28$.

At the moment $f = 2c$, hence $2c = c + 28$, and so $c = 28$. That is, the father's child is now 28 years old.

A pile of oranges are arranged to make a square based pyramid by having one orange on the top layer, four oranges on the second layer, nine oranges on the third layer, and so on. Such that consecutive layers will have a number of oranges equal to consecutive square numbers: 1, 4, 9, 16, 25, ...



If there were one-thousand oranges in the pile used to make the pyramid not all of them would be needed. How many oranges would be left over?

Solution

$$1^2 + 2^2 + 3^2 + ? + 13^2 = 819 \text{ and } 1000 - 819 = 181 \text{ (} 14^2 = 196 \text{)}$$

So there will be 181 oranges left over.

Can you find a better way to add together square numbers?

What if there was one million oranges to build a pyramid from?

By adding four consecutive integers it is possible to make different totals. For example, $16 + 17 + 18 + 19 = 70$, which is also divisible by 10.

Find two prime numbers that between them use each of the four digits 1, 2, 3 and 5 exactly once. For example, you could make 53 and 21, however, $21 = 3 \times 7$ is composite (non-prime).

Solution

As no 2-digit prime ends in 5, there can be no 2-digit solution. The only combinations would be: $21 (3 \times 7) + 53$ and $23 + 51 (3 \times 17)$, in which case at least one number in the pair is composite.

So the only possibility is that the solution comprises a single digit prime and a 3-digit prime.

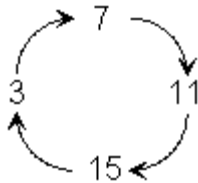
The single digit cannot be 2 or 5, as the sum of the remaining digits 135 and 123, respectively, are divisible by 3; hence any combination of them would be divisible by 3.

Therefore, the single digit prime must be 3 and the 3-digit prime could be either 251 or 521.

A chain of numbers is made by using the following rule:

Divide by 5 if the number is divisible by 5 otherwise add 4.

For example, starting with the number 7,



It can be seen that the chain has returned to 7.

Which natural number under 10 will never come back to itself?

Natural numbers are the set of whole numbers: 1, 2, 3, 4, 5, ...

Solution

Listing the chain for each starting number:

$1 \rightarrow 5 \rightarrow 1$

$2 \rightarrow 6 \rightarrow 10 \rightarrow 2$

$3 \rightarrow 7 \rightarrow 11 \rightarrow 15 \rightarrow 3$

$4 \rightarrow 8 \rightarrow 12 \rightarrow 16 \rightarrow 20 \rightarrow 4$

Having worked through the starting numbers 1-4, we can see what happens to 5-8. The only number that has not appeared is 9:

$9 \rightarrow 13 \rightarrow 17 \rightarrow 21 \rightarrow 25 \rightarrow 5 \rightarrow 1 \rightarrow 5$

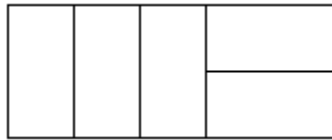
9 is unusual as it works its way into the 1,5 chain, never returning to itself.

Which numbers do not come back to themselves in general?

What happens if you change the rule from $+4/\div 5$?

Hint: Try $+3/\div 4$ and $+2/\div 3$ to start with.

A pathway measuring 5 m by 2 m is paved with stones measuring 2 m by 1 m. One way in which the pathway could be paved is as follows.



How many different ways can the path be laid?

Solution

The pathways can be paved as follows.

No horizontal blocks



1 horizontal block



2 horizontal blocks



There are 8 ways of paving a 5x2 pathway with 2x1 paving stones.

How many ways can you pave a 2 x n pathway using 2x1 stones?

What about different width pathways?

(Hint: Keep the length of the stones equal to the width of the pathway.)

Find the smallest multiple of nine containing only even digits.

Solution

If a number is divisible by nine then the sum of digits must be a multiple of nine.

But if all of the digits are even then they must sum to an even multiple of nine, and the smallest even multiple of nine is eighteen.

Hence we are looking for the most efficient way to write eighteen as the sum of even digits: $2 + 8 + 8$. That is, the smallest multiple of nine containing only even digits is 288.

Find the first five multiples of nine containing only even digits.

How many of the numbers under 100 that are divisible by 10 can you make by adding four consecutive integers?

Solution

Taking four consecutive integers, starting from n ,

$$n + (n + 1) + (n + 2) + (n + 3) = 4n + 6$$

Clearly for the total $(4n + 6)$ to be divisible by 10, $4n$ must end in a 4. So the units digit of n must be 1 or 6,

$$1 + 2 + 3 + 4 = 10$$

$$6 + 7 + 8 + 9 = 30$$

$$11 + 12 + 13 + 14 = 50$$

$$16 + 17 + 18 + 19 = 70$$

$$21 + 22 + 23 + 24 = 90$$

Giving 5 numbers under 100 that are divisible by 10 and can be made from the sum of four consecutive integers.

How many numbers under 1000, that are divisible by 5, can be made from the sum of four consecutive integers?

A palindrome is a number which reads the same forwards and backwards. For example, the number 232 is a 3-digit palindrome.

Can you find a square 3-digit palindrome, which is also palindromic when divided by 2?

Solution

If it is palindromic when divided by 2, it must be even. For a square number, n^2 , to be even, n must be even.

Listing even 3-digit squares:

$$10^2 = 100, 12^2 = 144, 14^2 = 196, 16^2 = 256,$$

$$18^2 = 324, 20^2 = 400, 22^2 = 484, 24^2 = 576,$$

$$26^2 = 676, 28^2 = 784, 30^2 = 900.$$

The two candidates are 484 and 676, but as $484/2=242$ and $676/2=338$, the palindrome we seek is 484.

Mr. and Mrs. Roberts have two daughters and three sons. At Easter time every member of the family buys one chocolate Easter egg for each other member. How many Easter eggs will be bought in total?

Solution

There are seven members of the family and each person must buy six Easter eggs.

Therefore, total number of Easter eggs is $7 \times 6 = 42$.

Use this idea to work out the following,

- (i) In a season, ten schools play home and away matches in a hockey league. How many matches that take place in a season.
 - (ii) In a netball tournament there are five teams and each team plays each other team once. How many matches will take place?
 - (iii) In a form there are thirty-two girls and two are picked at random. How many different pairs can be picked?
-