UNIT 10 *Sequences* Activities

Activities

ACTIVITY 10.1 *Finding the Limit of a Sequence*

1. (a) Complete the table below. (Use a spreadsheet or a calculator.)

n	$\dfrac{2n+1}{n-1}$	$\dfrac{3n+4}{2n+1}$	$\dfrac{5n+2}{n+1}$	$\dfrac{6n-3}{2n+1}$
2				
5				
10				
20				
50				
100				
1000				
10 000				
100 000				
1 000 000				

(b) What is the limit of each sequence as n becomes large?

2. Predict what will happen to $\dfrac{an+b}{cn+d}$ as n becomes large.

ACTIVITY 10.2 *Ulam's Sequence*

A sequence can be made for many reasons and it can be very interesting to try to invent the rules for one and investigate what happens. Here is a sequence the rules for which were invented by *Stanislaw Ulam*, an American mathematician.

Step 1 Start with the two numbers, 1 and 2.

Step 2 Look at all other numbers in turn, starting with 3.

Step 3 If a new number can be made by:

(a) adding two different numbers which are already in the sequence, and

(b) this can be done in *only one way*,

then that new number belongs to the sequence.

For example:

$\boxed{3}$ is in *Ulam's Sequence* because $3 = 1 + 2$, and both these numbers are already in the sequence. Also, 3 cannot be made in any other way by adding two *different* numbers.

$\boxed{5}$ is *NOT* in *Ulam's Sequence* because although $5 = 2 + 3$, (both of which are already in the sequence), 5 also equals $1 + 4$, so there is *more than one* way of satisfying part (a) of *Step 3* above.

1. Write down the first 10 terms of *Ulam's Sequence.*
2. What are the next two numbers (after 5) which must be left out? Why?
3. What is the first number left out which can be made in three ways?
4. Continue the sequence for another four terms.
5. What is the next consecutive pair of numbers after (3, 4) ?

Extension

Investigate what would happen if:

(a) you used another pair of numbers as a starting point
(e.g. change the order, use a zero as one of the pair or use a negative number),

(b) the sequence required *three* numbers to be added.

ACTIVITY 10.3 *General Formula for Generating Quadratic Equations*

Given a sequence of the type

$$2, \quad 3, \quad 5, \quad 8, \quad 12, \quad 17, \quad \ldots, \quad \ldots$$

it is soon clear that it could have been generated by a quadratic sequence, as its second differences are constant and non-zero. There is though a more general method of identifying the constants *a*, *b* and *c* in a general quadratic expression of the form

$$u_n = an^2 + bn + c$$

that fits with given data.

1. Given that a sequence is generated by the formula $u_n = an^2 + bn + c$, for constants a, b, c write down (along a line) the first 5 terms.

2. Calculate the first and second differences for this general quadratic sequence.

3. Calculate the first and second differences for the sequence

$$2, \quad 3, \quad 5, \quad 8, \quad 12, \quad 17, \quad \ldots, \quad \ldots$$

4. Using the first term of the

   (a) second difference, (b) first difference, (c) sequence,

   determine the values of *a*, *b* and *c*.

5. Repeat the procedure for the sequence

$$11, \quad 16, \quad 25, \quad 38, \quad 55, \quad \ldots, \quad \ldots$$

Extension

Analyse sequences generated by a general cubic in the same way, and use your results to determine the cubic formula for the sequence

$$1, \quad 6, \quad 23, \quad 58, \quad 117, \quad \ldots, \quad \ldots$$