22127306

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 4 May 2012 (morning)

1 hour 30 minutes

Candidate session number

0	0							

Examination code

2	2	1	2	–	7	3	0	6

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The following diagram shows ΔPQR, where $RQ = 9$ cm, $P\hat{R}Q = 70^\circ$ and $P\hat{Q}R = 45^\circ$.

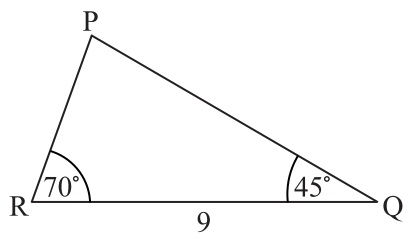


diagram not to scale

(a) Find $R\hat{P}Q$. *[1 mark]*

(b) Find PR. *[3 marks]*

(c) Find the area of ΔPQR. *[2 marks]*

2. *[Maximum mark: 6]*

Let $f(x) = \cos(e^x)$, for $-2 \le x \le 2$.

(a) Find $f'(x)$. *[2 marks]*

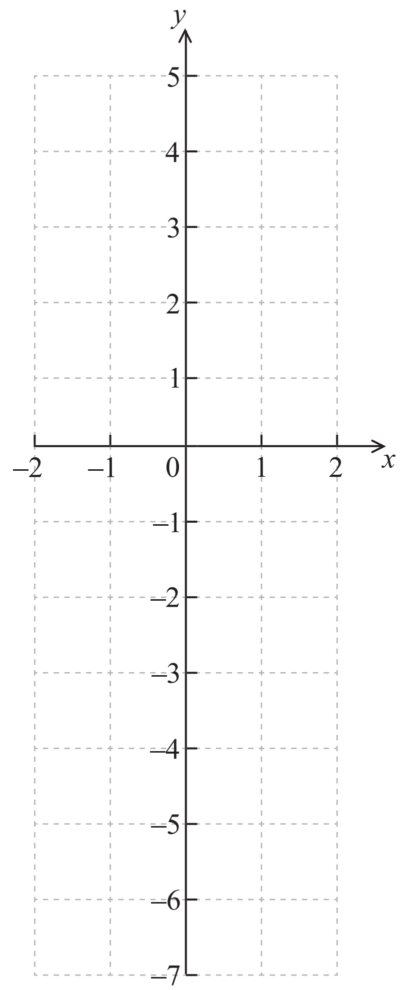
. . . .

. . . .

. . . .

. . . .

(b) On the grid below, sketch the graph of $f'(x)$. *[4 marks]*



3. *[Maximum mark: 6]*

The first term of a geometric sequence is 200 and the sum of the first four terms is 324.8.

(a) Find the common ratio. *[4 marks]*

(b) Find the tenth term. *[2 marks]*

4. *[Maximum mark: 6]*

The heights of a group of seven-year-old children are normally distributed with mean 117 cm and standard deviation 5 cm. A child is chosen at random from the group.

(a) Find the probability that this child is taller than 122.5 cm. *[3 marks]*

(b) The probability that this child is shorter than k cm is 0.65. Find the value of k. *[3 marks]*

5. *[Maximum mark: 6]*

A particle moves in a straight line with velocity $v = 12t - 2t^3 - 1$, for $t \geq 0$, where v is in centimetres per second and t is in seconds.

(a) Find the acceleration of the particle after 2.7 seconds. *[3 marks]*

(b) Find the displacement of the particle after 1.3 seconds. *[3 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

6. *[Maximum mark: 7]*

Let $\boldsymbol{A} = \begin{pmatrix} -1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & -2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 3 & 1 & 2 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Let $\boldsymbol{C}$ be a 3×3 matrix such that $\boldsymbol{ACA}^{-1} = \boldsymbol{B}$. Find $\boldsymbol{C}$. *[5 marks]*

7. *[Maximum mark: 8]*

A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested.

(a) Find the probability that there is at least one defective lamp in the sample. *[4 marks]*

(b) Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps. *[4 marks]*

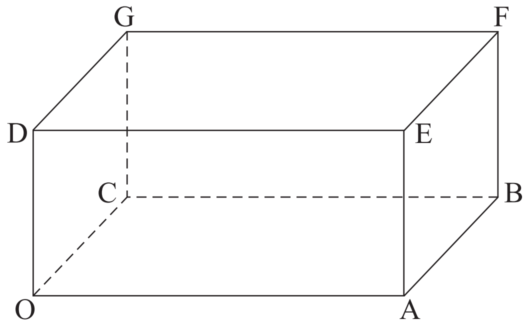
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 16]*

The following diagram shows the cuboid (rectangular solid) OABCDEFG, where O is the origin, and $\overrightarrow{\text{OA}} = 4\boldsymbol{i}$, $\overrightarrow{\text{OC}} = 3\boldsymbol{j}$, $\overrightarrow{\text{OD}} = 2\boldsymbol{k}$.



(a) (i) Find $\overrightarrow{\text{OB}}$.

(ii) Find $\overrightarrow{\text{OF}}$.

(iii) Show that $\overrightarrow{\text{AG}} = -4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$. *[5 marks]*

(b) Write down a vector equation for

(i) the line OF;

(ii) the line AG. *[4 marks]*

(c) Find the obtuse angle between the lines OF and AG. *[7 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 13]*

Let $f(x) = ax^3 + bx^2 + c$, where a, b and c are real numbers. The graph of f passes through the point $(2, 9)$.

(a) Show that $8a + 4b + c = 9$. *[2 marks]*

The graph of f has a local minimum at $(1, 4)$.

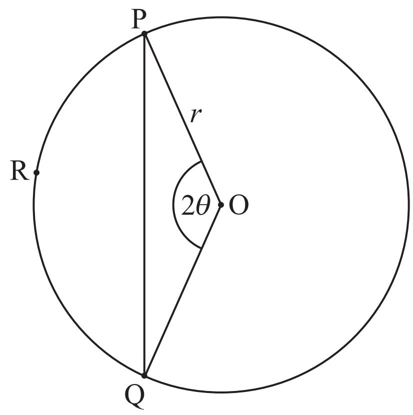
(b) Find two other equations in a, b and c, giving your answers in a similar form to part (a). *[7 marks]*

(c) Find the value of a, of b and of c. *[4 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

Consider the following circle with centre O and radius r.



The points P, R and Q are on the circumference, $\hat{POQ} = 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

(a) Use the cosine rule to show that $PQ = 2r\sin\theta$. *[4 marks]*

Let l be the length of the arc PRQ.

(b) Given that $1.3PQ - l = 0$, find the value of θ. *[5 marks]*

Consider the function $f(\theta) = 2.6\sin\theta - 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

(c) (i) Sketch the graph of f.

(ii) Write down the root of $f(\theta) = 0$. *[4 marks]*

(d) Use the graph of f to find the values of θ for which $l < 1.3PQ$. *[3 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.