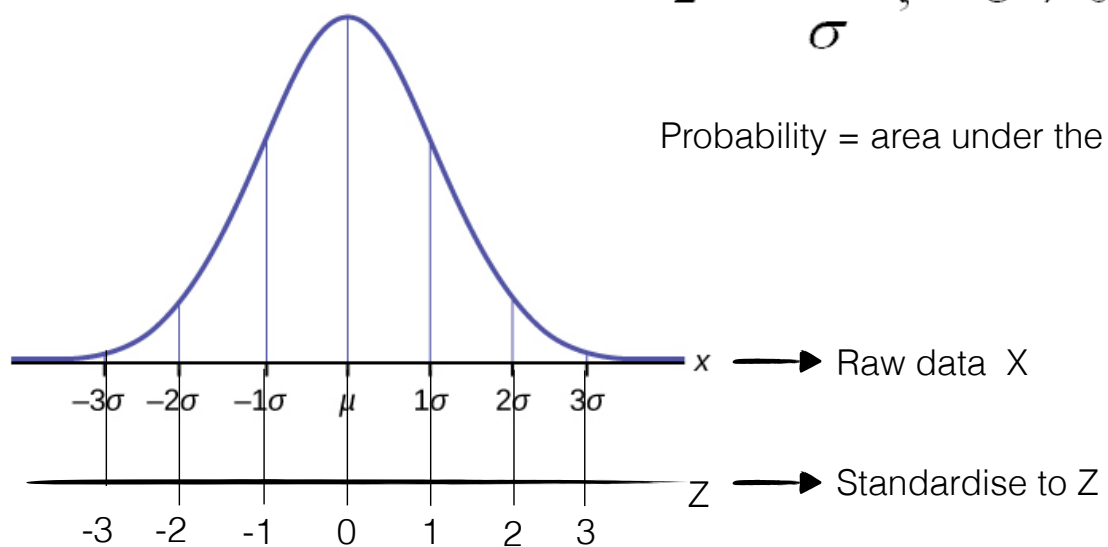


# Normal Distribution

## Normal Distribution

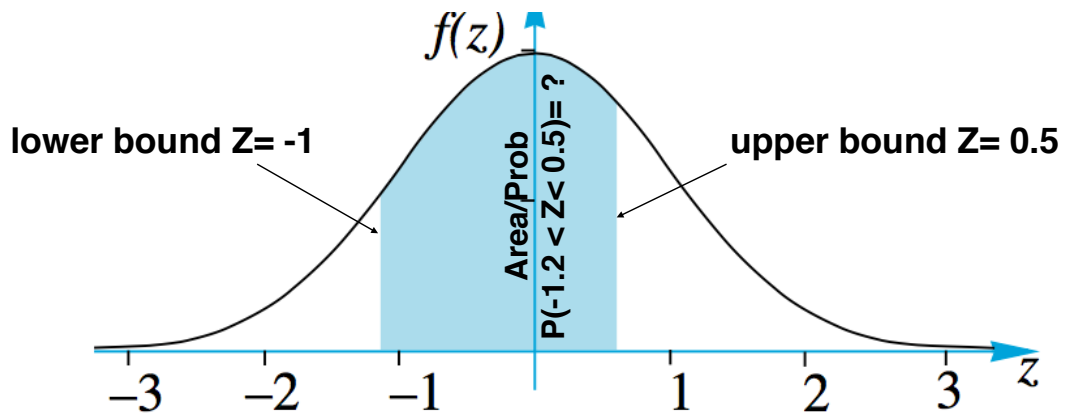
$$z = \frac{x - \mu}{\sigma}, \quad \sigma \neq 0$$

Probability = area under the curve



Probability = area under the curve

**Normal Cdf**    **Z system ,  $\mu = 0$  ,  $\sigma = 1$  , put lower bound and upper bound**



## **EXERCISES 17.1**

**Normal Cdf    Z system ,  $\mu = 0$  ,  $\sigma = 1$**

**1.** For the standard normal variable Z, find:

- |                    |                     |                     |                       |
|--------------------|---------------------|---------------------|-----------------------|
| (i) $p(Z < 0.5)$   | (ii) $p(Z < 1.84)$  | (iii) $p(Z < 1.62)$ | (iv) $p(-2.7 < Z)$    |
| (v) $p(-1.97 < Z)$ | (vi) $p(Z < -2.55)$ | (vii) $p(1.9 < Z)$  | (viii) $p(Z < -1.56)$ |
| (ix) $p(2.44 < Z)$ | (x) $p(-0.95 < Z)$  | (xi) $p(Z < 0.37)$  | (xii) $p(1.39 < Z)$   |

2. For the standard normal variable  $z$ , find:

**Normal Cdf Z system ,  $\mu = 0$  ,  $\sigma = 1$**

- |                              |                              |                              |
|------------------------------|------------------------------|------------------------------|
| (i) $p(1.75 < Z < 2.65)$     | (ii) $p(0.3 < Z < 2.5)$      | (iii) $p(1.35 < Z < 1.94)$   |
| (iv) $p(-1.92 < Z < -1.38)$  | (v) $p(2.23 < Z < 2.92)$     | (vi) $p(-1.51 < Z < -0.37)$  |
| (vii) $p(-2.17 < Z < 0.76)$  | (viii) $p(1.67 < Z < 2.22)$  | (ix) $p(-0.89 < Z < 0.8)$    |
| (x) $p(-2.64 < Z < -1.04)$   | (xi) $p(-1.43 < Z < 2.74)$   | (xii) $p(-1.59 < Z < -0.46)$ |
| (xiii) $p(-2.12 < Z < 0.58)$ | (xiv) $p(-2.61 < Z < 1.39)$  | (xv) $p(-1.86 < Z < 0.13)$   |
| (xvi) $p(-2.56 < Z < 0.92)$  | (xvii) $p(-1.75 < Z < 2.03)$ | (xviii) $p(-0.9 < Z < 1.34)$ |

**Normal Cdf**

**X system , get value of  $\mu$  ,  $\sigma$**

**Z system ,  $\mu = 0$  ,  $\sigma = 1$**

### EXAMPLE 17.5

find  $X$  is a normal random variable with mean  $\mu = 80$  and variance  $\sigma^2 = 16$  ,

- (a)  $p(X \leq 78)$     (b)  $p(76 \leq X \leq 84)$     (c)  $p(X \geq 86)$

Probability = area under the curve

**InvNormalCdf** Z system ,  $\mu = 0$  ,  $\sigma = 1$  , put the value of area

**put area -> get z value**

**EXAMPLE 17.6**

Find the values of  $a$  in each of these statements that refer to the standard normal variable,  $z$ .

- (a)  $p(Z < a) = 0.5478$       (b)  $p(Z > a) = 0.6$       (c)  $p(Z < a) = 0.05$

- 11.** If  $X$  is a normal random variable with a mean of 8 and a standard deviation of 1, find the value of  $c$ , such that
- (a)  $p(X > c) = 0.90$       (b)  $p(X \leq c) = 0.60$

- 12.** If  $X$  is a normal random variable with a mean of 50 and a standard deviation of 5, find the value of  $c$ , such that
- (a)  $p(X \leq c) = 0.95$       (b)  $p(X \geq c) = 0.95$       (c)  $p(-c \leq X \leq c) = 0.95$

- 28.** A normally distributed variable,  $X$ , has a standard deviation of 2.6.  
 $p(X < 322.68) = 0.6032$ . Find the mean of  $X$ .

- 35.** (a) Find the mean and standard deviation of the normal random variable  $X$ , given that  $P(X < 50) = 0.05$  and  $P(X > 80) = 0.1$ .

**EXAMPLE 17.8**

The Board of Examiners have decided that 85% of all candidates sitting Mathematical Methods will obtain a pass grade in the examination. The actual examination marks are found to be normally distributed with a mean of 55 and a variance of 16. What is the lowest score a student can get on the exam to be awarded a pass grade?