Y8	UNIT 10 *Probability - Two Events* Lesson Plan 1	*Revision: Probability of One Event*

Activity | ***Notes***

1

Revising probability of one event

T: When calculating probabilities we use unbiased coins, dice or playing cards. Before learning more about this topic, let's look back over the work we have already covered.

Whole class activity.

Number of Tosses	*Number of Heads*	*Cumulative Frequency*	
50	38		
50	20		
50	11		
50	31		
50	20		
50	33		

T puts the OS with the table onto the OHP and makes Ps recall what they can remember of this topic.

Questions/answers interactively.

T: What does the OS show us? *(Someone has tested a coin for fairness)*

T: What did they do? *(They tossed the coin 300 times)*

T: What did they expect the outcome to be? *(After many tosses, the number of heads would be close to half the total number of tosses)*

T: What is the word we use for 'the number of heads'? *(Frequency)*

T: What heading would you give to the fourth column in the table, which checks the results? *(Relative frequency)*

T labels the fourth column on OS.

T: How do we calculate the relative frequency? *(The number of successful outcomes (here, the number of heads), has to be divided by the total number of trials (tosses))*

T: Come to the OHP and calculate the relative frequency after 50, 100, 150, ..., 300 trials.

P_1: The CF (cumulative frequency) is 38, so the RF (relative frequency) is:

$$\frac{38}{50} = \frac{76}{100} = 0.76 \text{ or } 76\%$$

P_2: After 100 tosses, CF = 58,

$$RF = \frac{58}{100} = 0.58\%$$

P_3: After 150 tosses, CF = 69,

$$RF = \frac{69}{150} = \frac{23}{50} = 0.46$$

P_4: After 200 tosses, CF = 100,

$$RF = 50\%$$

P_5: After 250 tosses, CF = 120,

$$RF = \frac{120}{250} = \frac{12}{25} = 0.48$$

P_6: After 300 tosses, CF = 153,

$$RF = \frac{153}{300} \times 100 \rightarrow 51\%$$

T calls Ps to count on BB and fill in the rows of the table. To show the method (row 1) and do the less straightforward calculations (rows 3 and 6) volunteer Ps should be pointed to; for the other rows, T should encourage slower Ps to do the calculations. (All Ps should be encouraged to use either fractions, decimals or percentages to express relative frequency.)

Agreement. Praising.

(continued)

Y8 | UNIT 10 *Probability - Two Events* | Lesson Plan 1 | *Revision: Probability of One Event*

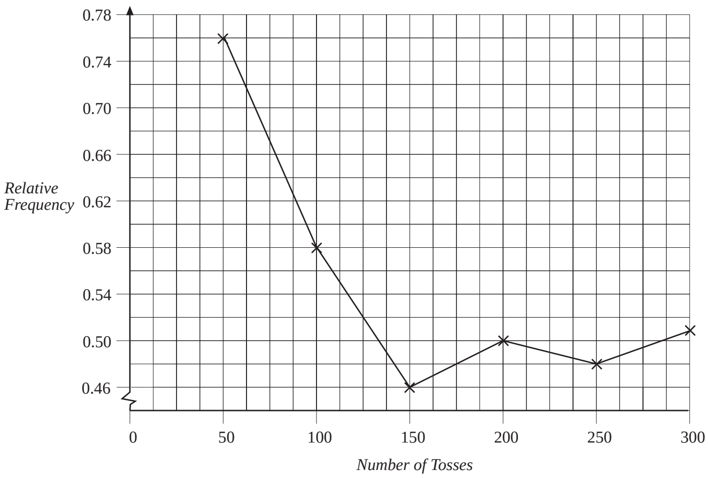
Activity | ***Notes***

1 *(continued)*

T: How could we show the fluctuation of the relative frequency?

Ps: ?

T: Let's plot the points on this relative frequency graph - we used these graphs last year ...



T sketches a grid for the relative frequency graph and calls out Ps to plot the points on it, one after another ...

... What is our conclusion? *(The coin is probably fair)*

T: What is the word we use for the relative frequencies fluctuating after many trials? *(Probability)*

T: What is the purpose of finding the probability? *(To estimate the number of outcomes of an event next time)*

T: Will we always have to do a large number of trials to find the probability? *(Not if the supplies are unbiased and the possible outcomes are equally likely)*

... then the conclusion, and further reviewing.

14 mins

2

Continuing revision

Whole class activity, revising interactively the topic already covered.

T: So, if the possible outcomes are equally likely, how do we calculate the probability of an event? *(The number of successful outcomes has to be divided by the number of possible outcomes)*

T: Let's see if that's correct ... (puts OS on OHP) ... and look at an example.

OS 10.1

T: What is happening here? *(A numbered ball is taken at random from the container)*

T: What are the possible outcomes? *(The number on the ball is 1, 2, ..., or 8)*

T: Are the outcomes equally likely? *(Yes)*

T: So what is the probability that the number is 7 ? $\left(\frac{1}{8}\right)$

T: What is the probability that the number is not 7 ? $\left(\frac{7}{8}\right)$

T: What do we call these two events? *(Complementary events)*

T: What is the probability of choosing an even number? $\left(\frac{4}{8} = \frac{1}{2}\right)$

(continued)

Y8	UNIT 10 *Probability - Two Events* Lesson Plan 1	*Revision: Probability of One Event*
Activity		***Notes***
2 *(continued)*	T: What is its complementary event? *(Not an even number ... an odd number)* T: What is its probability? $\left(\frac{4}{8}=\frac{1}{2}\right)$ T: What can you say about the probabilities of complementary events? *(They add up to 1)* T: Why? *(Since they are the only possible outcomes, together they are 100% = 1)* T: Find other pairs of complementary numbers on the OS. *((c) and (f))* T: What are their probabilities? $\left(\frac{4}{8}+\frac{4}{8}=1\right)$ T: What is the probability of event (d) or event (e) ? *(The multiples of 3 are the 3 and the 6, while the multiples of 4 are the 4 and the 8, so* $p=\frac{4}{8}$*)* T: Compare $p(\text{d})$ with $p(\text{e})$ with $p((\text{d}) \text{ or } (\text{e}))$. $p((\text{d}) \text{ or } (\text{e})) = p(\text{d}) + p(\text{e})$ T: How do we describe (d) and (e) in this case? *((d) and (e) are mutually exclusive events)* T: What would be the 'first common multiple' of 3 and 4 ? *(12)* T: Is 12 in the container? *(No)* *24 mins*	
3	**Individual work with probabilities** **PB 10.2, Q2 extended with:** **(e)** red or blue, **(f)** not green. *32 mins*	Individual work, monitored, helped. Verbal checking: T asks volunteer Ps to say the results and explain them. Discussion concerning the impossible event (part (d)) and comparing answers for questions - (a), (b) and (e) - (c) and (f) - (e) and (f). Agreement, feedback, self-correction. Praising.
4 *(continued)*	**Probabilities using playing cards** **PB 10.1, Q3** T: Can you estimate the number of times you expect to obtain a heart if you take a card from a full pack, at random, 52 times, replacing the card each time? *(The estimated number is 13)* T: Might it be far less or much more than 13 ? *(Yes)* T: Why? *(Because few trials are being carried out)* T: So if you take four cards from the pack in this way, would you expect only one of them to be a heart? *(Perhaps)* T: How many hearts would you expect when 520 cards have been drawn? *(We would expect about 130 of the cards to be hearts)*	Whole class activity. After discussing a standard 52-card pack of playing cards and the different parts of Q3, T should ask more questions. T makes Ps review how to estimate the number of outcomes. Praising wherever possible.

<table>
<tr><td>Y8</td><td>UNIT 10 Probability - Two Events Lesson Plan 1</td><td>Revision: Probability of One Event</td></tr>
<tr><td>Activity
4
(continued)</td><td>T: How do we estimate the number of times we expect an event to take place? (The probability of the event has to be multiplied by the number of trials)
T: If you take a card at random from the pack, putting the card back each time, 260 times, how many sevens would you expect to obtain? $\left(\frac{1}{13} \times 260 = 20\right)$
T: And if you take a card at random 52 times without putting it back, how many sevens would you expect to obtain? (4 sevens)
T: Are you sure? (Positive!)
40 mins</td><td>Notes
T agrees, writes on BB, Ps in Ex.Bs.
Note that, by the end of the experiment, ALL cards would have been taken!</td></tr>
<tr><td>5</td><td>Individual practice
PB 10.1, Q4 extended with:
(e) How many 5s would you expect when rolling the dice 300 times?
45 mins</td><td>Individual work, monitored, helped.
Verbal checking with explanations of results.
Agreement, feedback, self-correction. Praising.</td></tr>
<tr><td></td><td>Set homework
PB 10.1, Q6
Activity 10.1 (each P is given a copy)</td><td></td></tr>
</table>

Y8	**UNIT 10** ***Probability - Two Events*** Lesson Plan 2	*Outcomes with Two Events*
Activity		***Notes***
1	**Checking homework** **(1) Activity 10.1** **(2) PB 10.1, Q6** *((a)* $\frac{1}{10}$ *(b)* $\frac{9}{10}$*)*	When checking the Activity, Ps first show their cuboctachedrons to T. Discussion about the task, problems, experiences follows, and also discussion repeating the concepts of relative frequency and probability and the connection between them Verbal checking of Q6, repeating the concept of complementary events.
	5 mins	
2A	**Introducing outcomes with two events** T: So far we've dealt only with one event and its probability. Now we'll extend the concept to the probability of two events. T: For example, let's toss two unbiased coins. What are the possible outcomes? *(2 heads, 1 head and 1 tail, 2 tails)* T: What is the probability of 2 heads? *(Perhaps* $\frac{1}{3}$*)* T: Are you sure? We'll see ... First, let's use two different objects, a dice and a coin. **OS 10.2**	Whole class activity. Introduction of outcomes with two events. Discussion of the labels of *DICE* and *COIN*, then filling in the boxes. Agreeing that the 12 possible outcomes are equally likely, so that is why it is important to know how many of them there are. Ps answer questions, slower Ps encouraged by T. Praising.

		DICE					
		1	2	3	4	5	6
COIN	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

2B	**Individual work** **PB 10.2, Q1**	Ps turn back to previous question, copying and completing the table from PB as individual work. T gives Ps two minutes for this. Then verbal checking, followed by questions.

		COIN B	
		H	*T*
COIN A	*H*	H H	H T
	T	T H	T T

	T: So how many different outcomes are there? *(Four: one '2H' two kinds of '1H, 1T' and one '2T')* T: Are they equally likely? *(Yes)* T: So what is the probability of 2 heads? *(* $\frac{1}{4}$ *)*	Praising.
	16 mins	

Y8 UNIT 10 *Probability - Two Events* Lesson Plan 2 — *Outcomes with Two Events*

Activity / *Notes*

3

Practical work

Activity 10.2 (tossing the coins only 100 times)

T: Carry out this activity, but toss the coins only 100 times as we have limited time. How many times do you think you will get 2 heads? *(25 times)*

Notes:

Ps work in pairs., each pair having two 10p coins.

T suggests that one of each pair tosses the coins and says the results while the other records them.

T monitors Ps work and helps slower ones.

When all Ps have finished, discussion takes place about how strongly the experimental results verify the predictions.

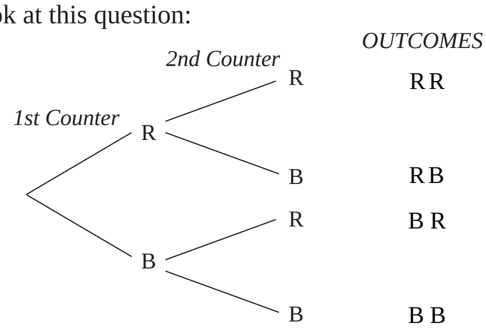
31 mins

4

Using three coins

T: You've drawn a table showing the outcomes when two coins are tossed. What about the outcomes for three coins? Where should we write the results for the third coin? We'll come to this in a minute. We'll also look at two other ways to show all the outcomes. Now look at this question:

PB 10.2, Q5 **(a)**



(b) 3

(c) 3

Notes:

Whole class activity.

T introduces the tree diagram as a possible way to list all the outcomes.

Ps read the text and look at diagram in PB. Discussion; volunteer P copies and completes the diagram on BB, others in Ex.Bs.

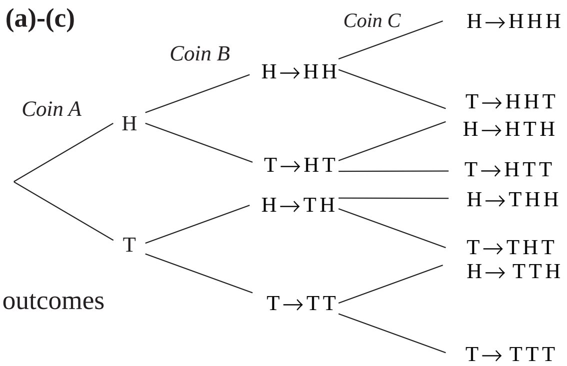
After answering parts (b) and (c) of question, T and Ps also agree that we now don't know if the outcomes are equally likely.

37 mins

5

Individual work

PB 10.2, Q6 (a)-(c)



(c) 8 outcomes

Coin A	*Coin B*	*Coin C*
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

Notes:

Individual work, monitored, helped.

When they have finished, Ps can check their solution for part (a) with solution at end of p171 of PB. For parts (b) and (c), volunteer P can draw tree diagram on BB.

Agreement, feedback, self-correction. Praising.

Then T introduces the third possible approach, systematic listing, to determine all the possible outcomes.

T explains and draws on BB, Ps in Ex.Bs.

45 mins

Y8	UNIT 10 ***Probability - Two Events*** Lesson Plan 2	*Outcomes with Two Events*
Activity		***Notes***
	Set homework **PB 10.2, Q7** (showing the possible outcomes in two different ways: using a tree diagram and also listing them systematically). **PB 10.2, Q2**	

Y8	**UNIT 10** ***Probability - Two Events*** Lesson Plan 3	*Two Dice Experiment*

Activity | ***Notes***

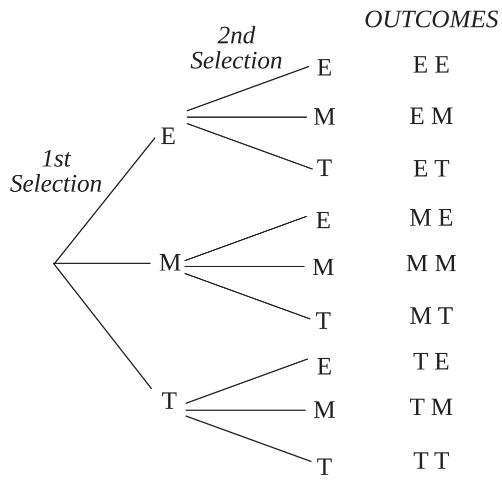
1

Checking homework

PB 10.2, Q7

(a) e.g. with systematic listing:

Sweet A	*Sweet B*
E	E
E	M
E	T
M	E
M	M
M	T
T	E
T	M
T	T



(b) 5 **(c)** 2

PB 10.2, Q2

(a)

		S (B)			
		1	2	3	4
S (A)	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

(b) 16 **(c)** 4

Notes: When checking homework, T makes Ps review the three possible approaches for listing all outcomes.
T has asked three Ps to draw a tree diagram and do a systematic listing for Q2 and draw a table for Q2 on BB as soon as Ps arrive. (Self-) correcting the tree/list/table. Then verbal checking of the answers to the questions.

Discussion, self-correction. Praising.

7 mins

2A

Mental work

M 10.1, Q2

T: In the homework you had two spinners, each numbered 1 to 4. When they are spun, which outcome is more probable:

T:	(1, 2) or (2, 1)	Ps:	Equally likely
	(1, 2) or (4, 3)		Equally likely
	Score of 2 or 8		Equally likely
	Score of 2 or 7		The score of 7

Notes: Mental warm-up preparing for this lesson's topic. T reads out questions, encourages slower Ps to answer, agrees, praises, question-by-question.
Then some extra questions referring to the table in Q2 of homework, still on BB.
Questions/answers interactively with praising where possible.

(continued)

Y8	**UNIT 10** ***Probability - Two Events*** Lesson Plan 3	*Two Dice Experiment*

Activity		***Notes***
2A *(continued)*	T: What was the total number of possible outcomes? Ps: 16 T: What do you think is the probability of a score of 2 ? Ps: $\frac{1}{16}$ T: Why? P: Because all the outcomes are equally likely and there is only one outcome which gives a score of 2.	Mental work continues, with T reading out and Ps answering. Agreement. Praising.
2B	**Further mental work** T: Let's take a fair dice and answer these questions. **M 10.1, Q1**	
	17 mins	
3	**Rolling two dice** **OS 10.3 / Activity 10.3 / OS 10.4** T: Now roll two fair dice. Do you know the gambling game played in casinos in which the score of 7 is the winner? What other scores can be obtained when two dice are thrown at the same time and their scores added up? We'll use the table to find out.	Ps work in pairs. OS appears on OHP with questions and each pair of Ps is given an Activity 10.3 sheet to work on, first dealing with only Q1 (i). T gives Ps some minutes to fill in the table, monitors Ps' work, helping slower pairs. After stopping the work, T puts only the table from OS 10.4 on OHP and Ps can check and correct their work. Feedback. Praising.
3A	**Completing the Two Dice Table**	

		2nd DICE					
		1	*2*	*3*	*4*	*5*	*6*
1st DICE	*1*	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

3B	**Questions on OS 10.3 and OS 10.4** e.g: T: How many outcomes are there is total? *(36)* T: How many outcomes give a '2' and a '5' ? *(Two outcomes: (2, 5) and (5, 2))* T: Which outcome is more likely, (6, 6) or (2, 5) ? *(They are equally likely)* T: What can you say when comparing the outcomes? *(All 36 outcomes are equally likely)* T: So what is the probability of one particular outcome? $(\frac{1}{36})$ T: How many outcomes give a score of 9 ? *(Four)* T: List them. *(The outcomes (3, 6), (4, 5), (5, 4) and (6, 3)* T: So what is the probability of getting a score of 9 ? $p(9) = \frac{4}{36} = \frac{1}{9}$	Whole class activity, discussing and answering questions on OS 10.3 and OS 10.4. T asks (reads out) questions from the OSs (and also other questions) in turn. Questions/answers interactively with all Ps contributing.
(continued)	etc.	Praising wherever possible.

Y8	UNIT 10 *Probability - Two Events* Lesson Plan 3	*Two Dice Experiment*

Activity

3C *(continued)*

Activity 10.3, Q1 (ii)

T: If we are sitting in a casino watching people gambling and we know the probabilities, what are we able to do?

(We can estimate the number of any event from a large number of throws)

T: For example, what is the probability of getting a score of 3 ?

$$p(3) = \frac{2}{36} = \frac{1}{18}$$

T: How many 3s do you expect when throwing a pair of fair dice 180 times? $\left(\frac{1}{18} \text{ of } 180 = 10\right)$

T: Complete the second table on your sheet.

Score	*Probability*	*Tallies (for Q2)*	*Number Expected in 180 Throws*
2	$\frac{1}{36}$		5
3	$\frac{2}{36} = \frac{1}{18}$		10
4	$\frac{3}{36} = \frac{1}{12}$		15
5	$\frac{4}{36} = \frac{1}{9}$		20
6	$\frac{5}{36}$		25
7	$\frac{6}{36} = \frac{1}{6}$		30
8	$\frac{5}{36}$		25
9	$\frac{4}{36} = \frac{1}{9}$		20
10	$\frac{3}{36} = \frac{1}{12}$		15
11	$\frac{2}{36} = \frac{1}{18}$		10
12	$\frac{1}{36}$		5

3D **Activity 10.3, Q2, Q3**

45 mins (or more)

Set homework

PB 10.3, Q1

PB 10.3, Q7

Notes

Ps work in pairs after a short discussion.

(Before starting, T asks Ps to divide the third column into two columns, to give space for tallies in Q2.)

T monitors and helps Ps work, stopping them when almost all have finished.

T puts Activity 10.3 sheet on OHP, waits for the answers for each row, agrees/waits for correction, praises, fills in the rows at OHP. Ps check their answers and correct their work where necessary. Those pairs who did not finish copy the missing answers from OS. Feedback. Praising.

Ps work in pairs, monitored, helped if necessary. Each pair is given two dice. One member of the pair rolls the dice and says the sums, the other records them.

T should wait for all pairs to finish their 180 throwings and add up the tallies so that they can compare their results with the expected ones.

T and Ps discuss the work and then combine the results for the whole class.

If there is not enough time, the table can be completed at home and the results combined at the beginning of the next lesson. Praising.

Y8	**UNIT 10** ***Probability - Two Events*** Lesson Plan 4	*Probabilities Using Listings*
Activity		***Notes***
1	**Checking homework** **PB 10.3, Q1** (a) $p(9) = \frac{1}{9}$ (b) $p(\text{odd}) = \frac{1}{2}$ (c) $p(> 10) = \frac{1}{12}$ (d) $p(< 8) = \frac{7}{12}$ **PB 10.3, Q7** (a) $p(\text{at least 1 head}) = \frac{3}{4}$ (b) $p(\text{no head}) = \frac{1}{4}$ e.g. **PB 10.3, Q1 (c)** P (shows on OS): There are 3 outcomes that give a score greater than 10, so the probability of this event is $\frac{3}{36} = \frac{1}{12}$. *6 mins*	Firstly, T puts the table (e.g. OS 10.4) which shows the possible outcomes when rolling two fair dice on OHP, and asks Ps to answer the parts of Q1, showing and explaining at OHP. In the meantime, T and Ps discuss the fact that they can find the probabilities in each case because the outcomes for the two events are equally likely. They also recall that this has been shown by experiments in the last lesson. Then T puts the table showing the possible outcomes when tossing two unbiased coins on OHP and encourages slower Ps to answer the questions. Agreement, feedback, self-correction, praising, for both tasks.
2	**Outcomes of pairs of events** T: Today we're going to see more pairs of events, find the outcomes that are equally likely and find the probability of any event. Let's combine first tossing a coin and rolling a dice. **PB 10.3, Q3** T: How many outcomes are there? *(12)* T: What can you say about their probabilities? *(They are equally likely)* T: What is the probability for each one? $\left(\frac{1}{12}\right)$ T: For example, what is the probability of obtaining a head and a 3 ? $p(\text{H3}) = \frac{1}{12}$ T: Which events give a tail and an even number? *(T2, T4, T6)* T: So what is the probability of obtaining a tail and an even number? $\left(p(\text{T and even}) = \frac{3}{12} = \frac{1}{4}\right)$ T: A head and a prime number? $(\text{H2, H3, H5, so } p = \frac{1}{4}$ *as well)* *14 mins*	Whole class activity. Discussion recalling the tables Ps used two lessons ago. Then T sketches the table on BB and a volunteer slower P comes to label the rows and columns; another P fills in the boxes. T agrees, praises, Ps draw the table in Ex.Bs. Then questions/answers interactively. T agrees, praises, writes results (answers) on BB below the table, Ps in Ex.Bs.

Y8	UNIT 10 ***Probability - Two Events*** Lesson Plan 4	*Probabilities Using Listings*

Activity | ***Notes***

3

Whole class practice

T: Now you're on your own! You have to interpret the following problem and my role is only to agree with you.

PB 10.3, Q9

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

(a) $p(12) = \frac{4}{36} = \frac{1}{9}$

(b) $p(20) = \frac{2}{36} = \frac{1}{18}$

(c) $p(>25) = \frac{3}{36} = \frac{1}{12}$

(d) $p(<30) = 1 - p(>25) = \frac{11}{12}$

(e) $p(\text{even}) = 1 - p(\text{odd}) = 1 - \frac{3 \times 3}{36} = \frac{3}{4}$

Whole class activity.

Ps read the task in PB, volunteer, suggest what should be done. T agrees and puts empty table on BB (e.g. OS 10.3, with everything else covered) and allows Ps to do what they say.

When answering the questions, T asks Ps to give the simplest solution (without counting the number of outcomes) for tasks (d) and (e) $\rightarrow$ T making Ps discover that (d) is the complementary event of (c). Similarly for (e), where it's much easier to count the number of outcomes that give a total score that is an *odd* number.

Agreement. Praising.

24 mins

4

Individual work

PB 10.3, Q2

	1	2	3	4
1	2	3	4	5
3	4	5	6	7
5	6	7	8	9
7	8	9	10	11

$p(\text{a}): p(7) = \frac{2}{16} = \frac{1}{8}$

$p(\text{b}): p(6) = \frac{1}{8}$

$p(\text{c}): p(>10) = \frac{1}{16}$

$p(\text{d}): p(<5) = \frac{4}{16} = \frac{1}{4}$

Individual work, monitored, helped.

Checking: T has prepared an OS with the completed table on it (or sketches it quickly on BB just before stopping the work). Ps check and correct their work, then volunteer to answer the questions.

Agreement, feedback, self-correction. Praising.

32 mins

Y8	UNIT 10 ***Probability - Two Events*** Lesson Plan 4	*Probabilities Using Listings*
Activity		***Notes***
5A	**Introducing multiplication law** T: Let's turn back to fair dice and unbiased coins in PB 10.3, Q3 and compare the probabilities of combined events with the probabilities of single ones. T: Toss an unbiased coin. What is the probability that you obtain a head? $\left(\frac{1}{2}\right)$ T: Toss another coin. What is the probability that you obtain a head? $\left(\frac{1}{2}\right)$ T: Toss them together. Give the probability of two heads. $\left(\frac{1}{4}\right)$ T: What do you notice? $\left(\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}\right)$ T: Roll a blue fair dice. What is the probability that you obtain a 6 ? $\left(\frac{1}{6}\right)$ T: Roll a red one, $p(6) = ?$ $\left(\frac{1}{6}\right)$ T: Roll them together. What is the probability of obtaining two 6s? $\left(\frac{1}{36} \text{ which is } \frac{1}{6} \times \frac{1}{6}\right)$ T: Look at the first task in this lesson and then try to explain the answers to the questions. P_1: We've found that $p(\text{H3}) = \frac{1}{12}$ and that is $\frac{1}{2} \times \frac{1}{6}$, the product of the probability of obtaining a head and the probability of obtaining a 3. P_2: $p(\text{even}) = \frac{3}{6}$ and $p(\text{tail}) = \frac{1}{2}$ and here again $p(\text{T and even}) = p(\text{T}) \times p(\text{even})$ P_3: The situation is the same in question Q3 (c): $p(\text{prime}) = \frac{1}{2}$, $p(\text{head}) = \frac{1}{2}$ and $p(\text{prime and head}) = p(\text{prime}) \times p(\text{head})$ T: What have you discovered? Ps: The probability of a combined event is the product of the single events. T: That's true here, but we'll see some different problems later.	Introducing multiplication law. Firstly mental work (answers/ questions) looking at 'two coin' and 'two dice' problems, with T writing on BB ... (T writes on BB) (T writes on BB) ... then whole class activity, trying to calculate the probabilities of the outcome of the 'one coin - one dice' problem. Ps can calculate the answers for the first question in their Ex.Bs, volunteer, come to BB, write and explain their ideas. T agrees, praises.
5B	T: Finally, let's see how we can illustrate these solutions. **OS 10.5** *45 mins*	Whole class activity. Task appears on OHP. Start with a short discussion regarding the type of tree diagram that is needed (here: six, not six), then volunteer Ps complete the diagram and determine the probabilities asked for on the OS. Agreement. Praising.
	Set homework **PB 10.3, Q5** **PB 10.4, Q3**	

Y8	**UNIT 10** ***Probability - Two Events*** Lesson Plan 5	*Probabilities Using Tree Diagrams*
Activity		***Notes***
1	**Reviewing work covered** T: Let's go over what we've covered in this unit.	Reviewing the work covered in this unit, first with mental work, then checking and explaining the homework.
1A	**Mental Test** **M 10.2**	The questions of M 10.2 are read out by T. To help slower Ps, - T sketches a spinner with numbers 1 to 7 before asking Q1 - Ps are allowed to use Ex.Bs to draw up a table for Q2 (and Q3?) - T suggests that they look back to the table drawn for the two dice problem in their Ex.Bs - T encourages Ps! Questions/answers with all slower Ps contributing. Agreement. Praising.
1B	**Checking homework** **PB 10.3, Q5** T: Let's see how we've tackled this slightly more difficult problem.	

	1	2	3	4
–1	0	1	2	3
0	1	2	3	4
1	2	3	4	5
2	3	4	5	6

P (a): $p(6) = \frac{1}{16}$

P (b): $p(0) = \frac{1}{16}$

P (c): $p(1) = \frac{2}{16} = \frac{1}{8}$

p (d): $p(3) = \frac{4}{16} = \frac{1}{4}$

Notes (1B): T asks a volunteer P to draw and explain the table they prepared at home.

After agreeing, feedback, self-correction, T praises.

Then T chooses Ps who had difficulties with the table to answer the questions, giving explanations.

Agreement. Praising. (Feedback)

1C **PB 10.4, Q3**

T: Can you remember the last problem in the previous lesson? The problem was to obtain two 6s, one 6 or no 6, and that can be done by drawing a tree diagram. Let's see what kind of tree diagram we should have drawn for this homework question.

Notes (1C): The new topic (multiplication law for independent events) was only touched on in the previous lesson, so the preparation of a suitable tree diagram will have been difficult for many Ps.

T (walking among Ps) looks at the volunteer (probably stronger) Ps' Ex.Bs and chooses a successful one to explain solution at BB.

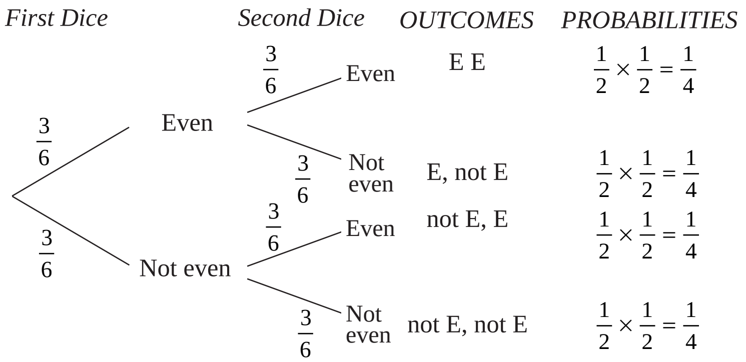
(continued)

Y8 UNIT 10 *Probability - Two Events* Lesson Plan 5

Probabilities Using Tree Diagrams

Activity		***Notes***

1C *(continued)*



So $p(\text{two even}) = \frac{1}{4}$

$p(\text{at least one even}) = \frac{3}{4}$

$p(\text{no even}) = \frac{1}{4}$

Notes: Agreement. Praising.

Ps (who failed) correct their tree diagrams.

1D

T: Let's look again at the previous problem as an example of multiplication law (figure at top of p178 in PB).

Notes: T puts the figure at the top of p178 of PB on OHP (prepared in advance) and T and Ps discuss when multiplication law is true (see 'Note' under figure in PB).

10 mins

2

Multiplication law
OS 10.6

P_2: $\text{Y Y} \rightarrow \frac{5}{9} \times \frac{5}{9}$

$\text{Y G} \rightarrow \frac{5}{9} \times \frac{4}{9}$

$\text{G Y} \rightarrow \frac{4}{9} \times \frac{5}{9}$

$\text{G G} \rightarrow \frac{4}{9} \times \frac{4}{9}$

P_3: $p(2\,\text{Y}) = \frac{25}{81}$

$p(2\,\text{G}) = \frac{16}{81}$

$p(1\text{G}\ 1\text{Y}) = \frac{20}{81} + \frac{20}{81} = \frac{40}{81}$

T: Have we listed all the possible outcomes?

Ps: Yes.

T: Is there any outcome we have counted more than once?

Ps: No.

T: How can we check this?

(continued)

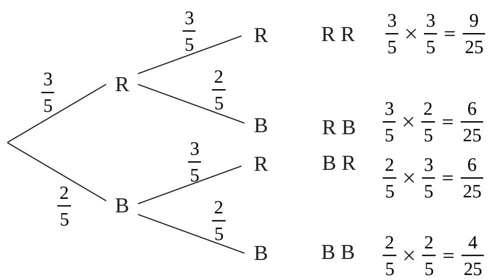
Notes: Task appears on OHP.

Whole class activity with slower Ps contributing:

- one writing probabilities on the tree diagram
- one completing the columns for 'Outcome' and 'Probability'
- one answering the questions
- one checking results.

T lets encouraged slower Ps explain and work at OHP and asks the other Ps to help them when necessary.

Y8	UNIT 10 *Probability - Two Events* Lesson Plan 5	*Probabilities Using Tree Diagrams*

Activity		***Notes***
2 *(continued)*	Ps: ? T (to a slower P): Please come to front and add up the probabilities of all the outcomes on BB. P_4 (writes on BB): $\frac{25}{81}+\frac{20}{81}+\frac{20}{81}+\frac{16}{81}=\frac{81}{81}=1$ T: Why is this correct? *(Because the four possible outcomes give the certain event and they have no intersection)* T: Ensuring that the probabilities add up to 1 is a useful means of checking our working. *28 mins*	Agreement. Praising. All Ps draw the tree diagram and write the correct answers and checking in their Ex.Bs.
3	**Practice using tree diagrams** **PB 10.4, Q1** T: How does the tree diagram you have drawn here compare with the one in the previous question? P: It is similar. T: What should we write on it? P_1: For the first ball we can choose red with the probability of $\frac{3}{5}$ or blue with $p=\frac{2}{5}$. T: And then? P_2: The process is repeated when taking the second ball. T: What were the possible outcomes and what were their probabilities? P_3: $p(\text{R R})=\frac{3}{5}\times\frac{3}{5}=\frac{9}{25}$ P_4: $p(\text{R B})=\frac{6}{25}$ P_5: $p(\text{B R})=\frac{6}{25}$ P_6: $p(\text{B B})=\frac{4}{25}$ T: Have you checked your working? Ps: Yes, the probabilities add up to 1. T: Let's look at parts (b) and (c). P_7: Both balls are the same colour when we obtain RR or BB, so the probability is $\frac{9}{25}+\frac{4}{25}=\frac{13}{25}$. P_8: At least one of the balls is red when obtaining RR, RB or BR Their probabilities add up to $\frac{21}{25}$. T: Did anyone answer part (c) in a different way? P_9: The answer to part (c) includes everything except the outcome BB. So we can determine its probability by subtracting $p(\text{B B})$ from 1. *38 mins*	Individual work, monitored, helped. Verbal checking in detail, with as many Ps as possible contributing: T asks, Ps answer, T agrees or waits for correction, writes results in BB (Ps self-correct). T sketches an empty tree diagram in BB. T writes on BB. T completes the diagram.  T writes on BB. Feedback. Praising.

Y8	UNIT 10 ***Probability - Two Events*** Lesson Plan 5	*Probabilities Using Tree Diagrams*
Activity		***Notes***
4	**Tossing three coins** T: How do we determine the probabilities if we have to take three balls or roll three dice or toss three coins? P: We can extend the tree diagram. T: Let's look at tossing three coins. **Activity 10.4** (completing the tree diagram and answering Q1 and Q2) e.g. **1. (a)** P: There is only one outcome which gives three heads, so $p(3\text{H}) = \frac{1}{8}$ etc. e.g. **2. (a)** P: Looking at the probabilities, the expected frequency for obtaining 3 heads from carrying out the experiment 40 times is $\frac{1}{8} \times 40 = 5$ etc. *45 mins*	Task appears on OHP and each P is given a copy of Activity 10.4. T asks volunteer Ps to come out and complete the diagram on OS (agrees, praises). All Ps complete it on their copy. Then Q1 questions are answered with a slower P finding the outcomes and adding up the probabilities mentally for each question. Agreement, praising, Ps write correct results on their copies. Then volunteer Ps are pointed to, to answer the different parts of Q2 by counting mentally. Agreement, praising, Ps write correct answers on their copies.
	Set homework **(1) Complete Activity 10.4 (carrying out the experiment 40 times, completing the table and answering Q3).** **(2) PB 10.4, Q2** **PB 10.4, Q4**	