1.) Let $g(x) = \dfrac{\ln x}{x^2}$, for $x > 0$.

(a) Use the quotient rule to show that $g'(x) = \dfrac{1-2\ln x}{x^3}$.

(4)

(b) The graph of g has a maximum point at A. Find the x-coordinate of A.

(3)
(Total 7 marks)

2.) The velocity v m s^{-1} of a particle at time t seconds, is given by $v = 2t + \cos 2t$, for $0 \le t \le 2$.

(a) Write down the velocity of the particle when $t = 0$.

(1)

When $t = k$, the acceleration is zero.

(b) (i) Show that $k = \dfrac{\pi}{4}$.

(ii) Find the exact velocity when $t = \dfrac{\pi}{4}$.

(8)

(c) When $t < \dfrac{\pi}{4}$, $\dfrac{dv}{dt} > 0$ and when $t > \dfrac{\pi}{4}$, $\dfrac{dv}{dt} > 0$.

Sketch a graph of v against t.

(4)

(d) Let d be the distance travelled by the particle for $0 \le t \le 1$.

(i) Write down an expression for d.

(ii) Represent d on your sketch.

(3)
(Total 16 marks)

3.) Let $h(x) = \dfrac{6x}{\cos x}$. Find $h'(0)$.

(Total 6 marks)

4.) The following diagram shows part of the graph of the function $f(x) = 2x^2$.

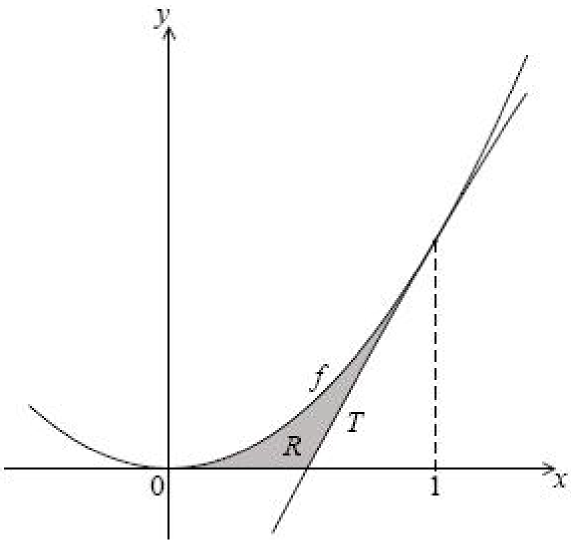


diagram not to scale

The line *T* is the tangent to the graph of *f* at $x = 1$.

(a) Show that the equation of *T* is $y = 4x - 2$.

(5)

(b) Find the x-intercept of *T*.

(2)

(c) The shaded region *R* is enclosed by the graph of *f*, the line *T*, and the x-axis.

(i) Write down an expression for the area of *R*.

(ii) Find the area of *R*.

(9)
(Total 16 marks)

5.) The following diagram shows a waterwheel with a bucket. The wheel rotates at a constant rate in an anticlockwise (counterclockwise) direction.

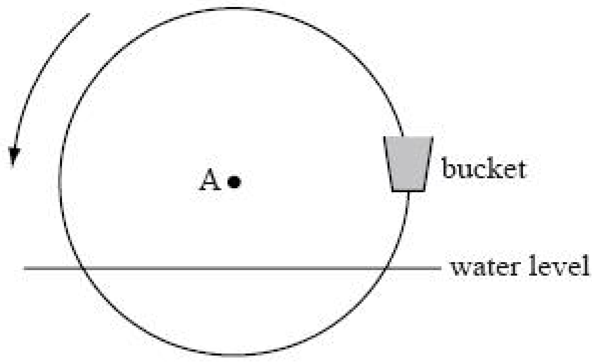


diagram not to scale

The diameter of the wheel is 8 metres. The centre of the wheel, A, is 2 metres above the water level. After *t* seconds, the height of the bucket above the water level is given by $h = a \sin bt + 2$.

(a) Show that $a = 4$.

(2)

The wheel turns at a rate of one rotation every 30 seconds.

(b) Show that $b = \frac{\pi}{15}$.

(2)

In the first rotation, there are two values of t when the bucket is **descending** at a rate of 0.5 m s^{-1}.

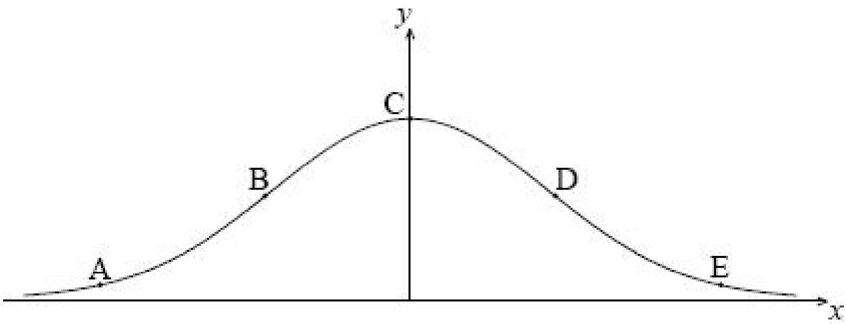
(c) Find these values of t.

(6)

(d) Determine whether the bucket is underwater at the second value of t.

(4)
(Total 14 marks)

6.) The following diagram shows the graph of $f(x) = e^{-x^2}$.



The points A, B, C, D and E lie on the graph of f. Two of these are points of inflexion.

(a) Identify the **two** points of inflexion.

(2)

(b) (i) Find $f'(x)$.

(ii) Show that $f''(x) = (4x^2 - 2)e^{-x^2}$.

(5)

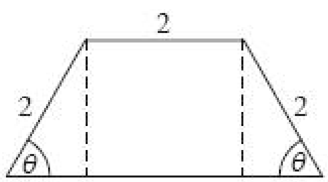
(c) Find the x-coordinate of each point of inflexion.

(4)

(d) Use the second derivative to show that one of these points is a point of inflexion.

(4)
(Total 15 marks)

7.) The diagram below shows a plan for a window in the shape of a trapezium.



Three sides of the window are 2 m long. The angle between the sloping sides of the window and the base is θ, where $0 < \theta < \frac{\pi}{2}$.

(a) Show that the area of the window is given by $y = 4 \sin \theta + 2 \sin 2\theta$.

(5)

(b) Zoe wants a window to have an area of 5 m^2. Find the two possible values of θ.

(4)

(c) John wants two windows which have the same area A but different values of θ.

Find all possible values for A.

(7)
(Total 16 marks)

8.) Let $f(x) = \dfrac{\cos x}{\sin x}$, for $\sin x \neq 0$.

(a) Use the quotient rule to show that $f'(x) = \dfrac{-1}{\sin^2 x}$.

(5)

(b) Find $f''(x)$.

(3)

In the following table, $f'\left(\frac{\pi}{2}\right) = p$ and $f''\left(\frac{\pi}{2}\right) = q$. The table also gives approximate values of $f'(x)$ and $f''(x)$ near $x = \frac{\pi}{2}$.

x	$\frac{\pi}{2} - 0.1$	$\frac{\pi}{2}$	$\frac{\pi}{2} + 0.1$
$f'(x)$	–1.01	p	–1.01
$f''(x)$	0.203	q	–0.203

(c) Find the value of p and of q.

(3)

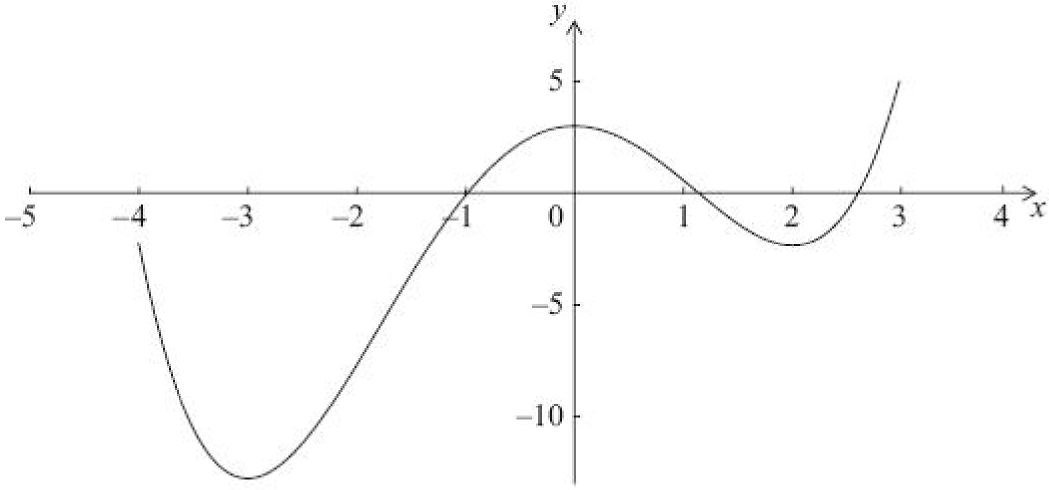
(d) Use information from the table to explain why there is a point of inflexion on the graph of f where $x = \frac{\pi}{2}$.

(2)
(Total 13 marks)

9.) Let $f(x) = kx^4$. The point P(1, k) lies on the curve of f. At P, the normal to the curve is parallel to $y = -\frac{1}{8}x$. Find the value of k.

(Total 6 marks)

10.) A function f is defined for $-4 \le x \le 3$. The graph of f is given below.



The graph has a local maximum when $x = 0$, and local minima when $x = -3$, $x = 2$.

(a) Write down the x-intercepts of the graph of the **derivative** function, f'.

(2)

(b) Write down all values of x for which $f'(x)$ is positive.

(2)

(c) At point D on the graph of f, the x-coordinate is -0.5. Explain why $f''(x) < 0$ at D.

(2)
(Total 6 marks)

11.) Consider the function f with second derivative $f''(x) = 3x - 1$. The graph of f has a minimum point at A(2, 4) and a maximum point at B$\left(-\frac{4}{3}, \frac{358}{27}\right)$.

(a) Use the second derivative to justify that B is a maximum.

(3)

(b) Given that $f' = \frac{3}{2}x^2 - x + p$, show that $p = -4$.

(4)

(c) Find $f(x)$.

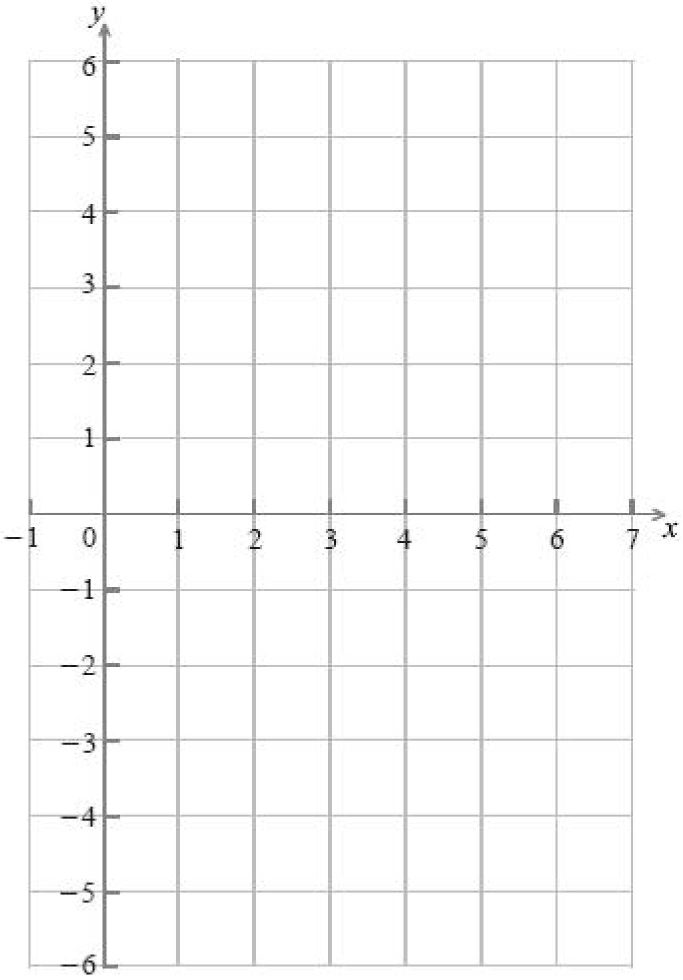
(7)
(Total 14 marks)

12.) Let $f(x) = x \cos x$, for $0 \le x \le 6$.

(a) Find $f'(x)$.

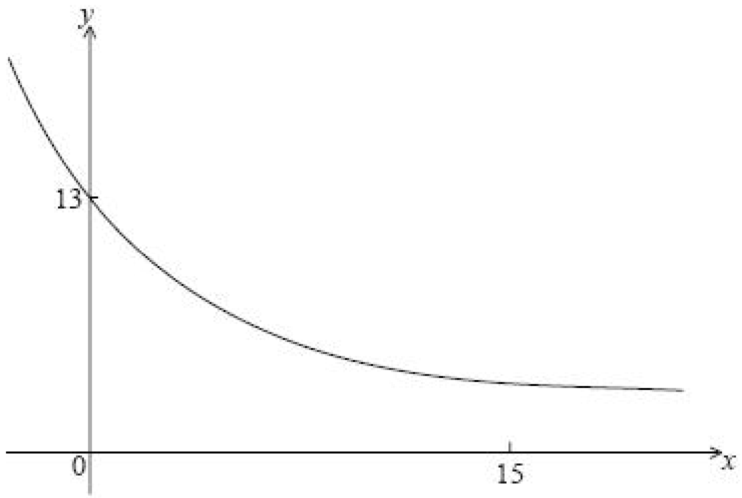
(3)

(b) On the grid below, sketch the graph of $y = f'(x)$.



(4)
(Total 7 marks)

13.) Let $f(x) = Ae^{kx} + 3$. Part of the graph of f is shown below.



The y-intercept is at (0, 13).

(a) Show that $A = 10$.

(2)

(b) Given that $f(15) = 3.49$ (correct to 3 significant figures), find the value of k.

(3)

(c) (i) Using your value of k, find $f'(x)$.

(ii) Hence, explain why f is a decreasing function.

(iii) Write down the equation of the horizontal asymptote of the graph f.

(5)

Let $g(x) = -x^2 + 12x - 24$.

(d) Find the area enclosed by the graphs of f and g.

(6)
(Total 16 marks)

14.) The number of bacteria, n, in a dish, after t minutes is given by $n = 800e^{013t}$.

(a) Find the value of n when $t = 0$.

(2)

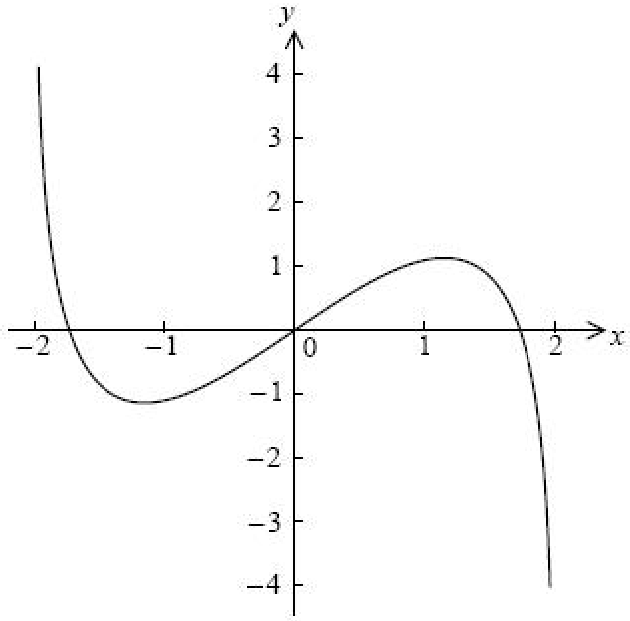
(b) Find the rate at which n is increasing when $t = 15$.

(2)

(c) After k minutes, the rate of increase in n is greater than 10 000 bacteria per minute. Find the least value of k, where $k \in \mathbb{Z}$.

(4)
(Total 8 marks)

15.) Consider $f(x) = x \ln(4 - x^2)$, for $-2 < x < 2$. The graph of f is given below.



(a) Let P and Q be points on the curve of f where the tangent to the graph of f is parallel to the x-axis.

(i) Find the x-coordinate of P and of Q.

(ii) Consider $f(x) = k$. Write down all values of k for which there are exactly two solutions.

(5)

Let $g(x) = x^3 \ln(4 - x^2)$, for $-2 < x < 2$.

(b) Show that $g'(x) = \dfrac{-2x^4}{4-x^2} + 3x^2 \ln(4 - x^2)$.

(4)

(c) Sketch the graph of g'.

(2)

(d) Consider $g'(x) = w$. Write down all values of w for which there are exactly two solutions.

(3)
(Total 14 marks)

16.) Let $g(x) = 2x \sin x$.

(a) Find $g'(x)$.

(4)

(b) Find the gradient of the graph of g at $x = \pi$.

(3)
(Total 7 marks)

17.) Let $f(x) = x^3$. The following diagram shows part of the graph of f.

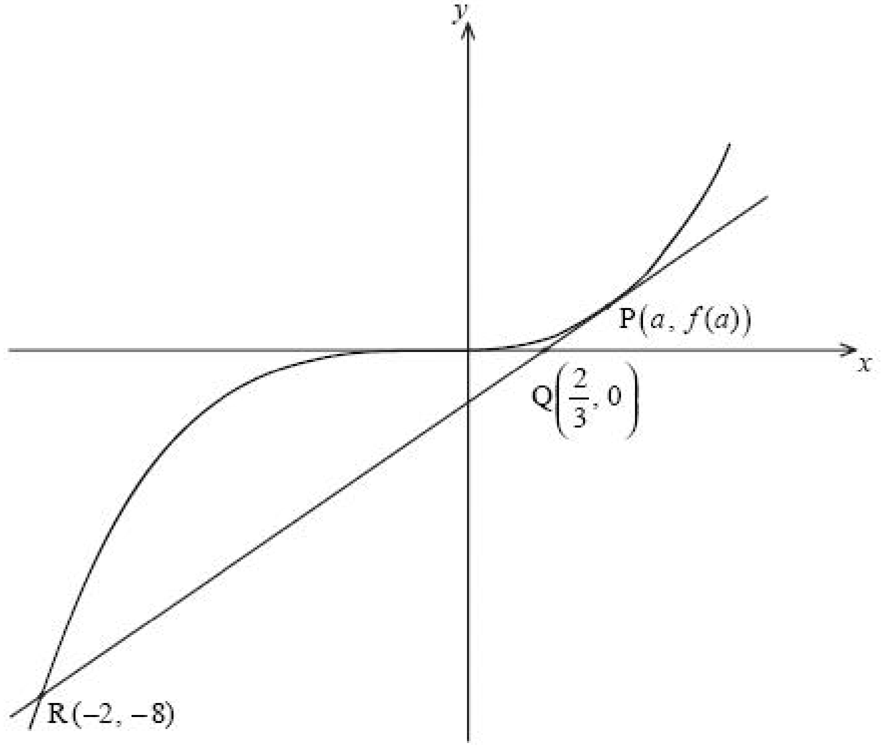


diagram not to scale

The point P (a, $f(a)$), where $a > 0$, lies on the graph of f. The tangent at P crosses the x-axis at the point $\mathrm{Q}\left(\frac{2}{3}, 0\right)$. This tangent intersects the graph of f at the point R(–2, –8).

(a) (i) Show that the gradient of [PQ] is $\dfrac{a^3}{a-\frac{2}{3}}$.

(ii) Find $f'(a)$.

(iii) Hence show that $a = 1$.

(7)

The equation of the tangent at P is $y = 3x - 2$. Let T be the region enclosed by the graph of f, the tangent [PR] and the line $x = k$, between $x = -2$ and $x = k$ where $-2 < k < 1$. This is shown in the diagram below.

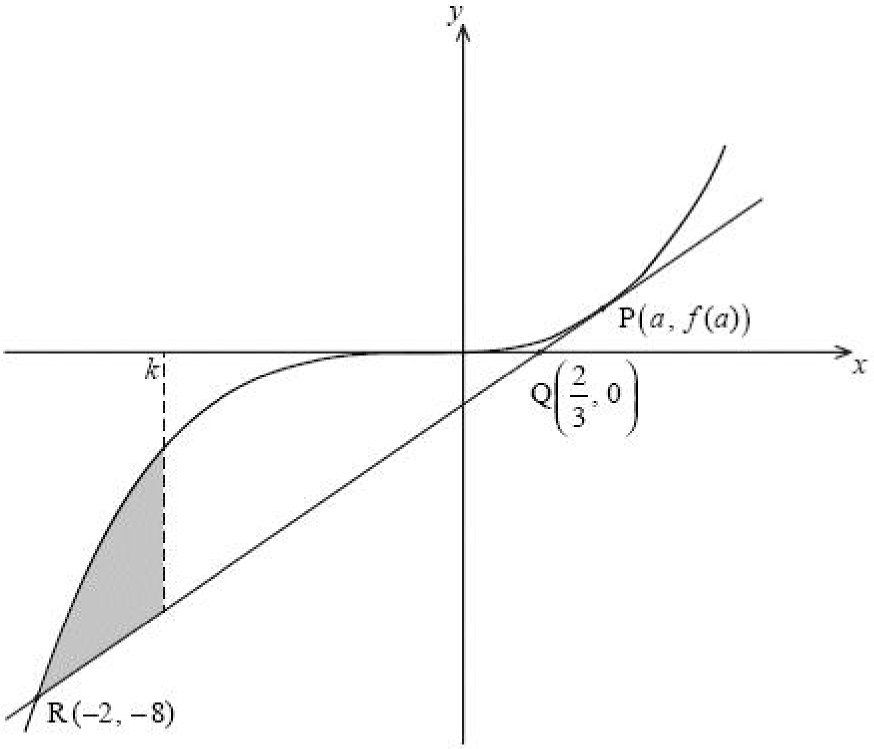


diagram not to scale

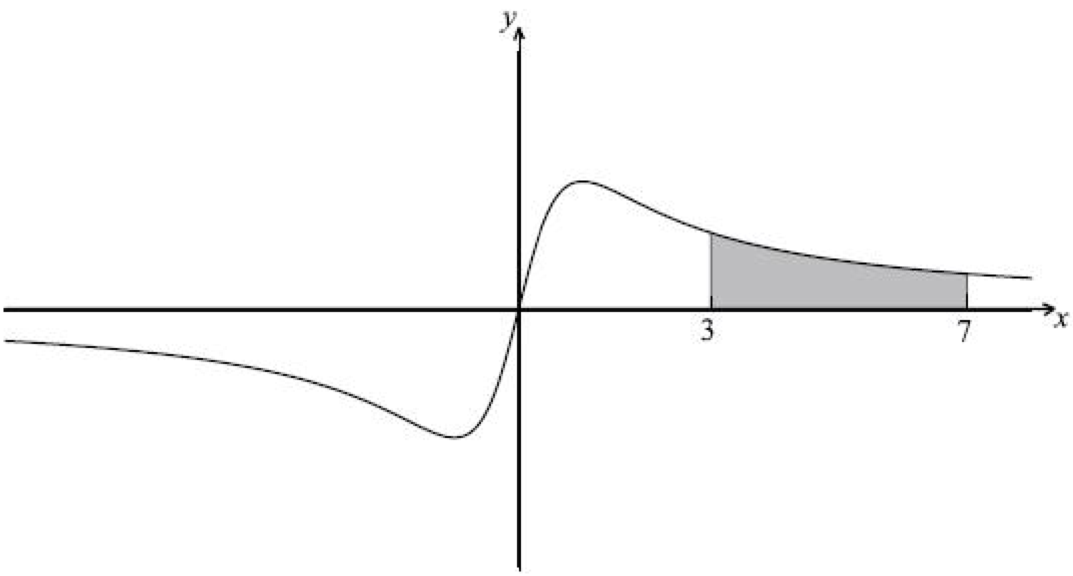
(b) Given that the area of T is $2k + 4$, show that k satisfies the equation $k^4 - 6k^2 + 8 = 0$.

(9)
(Total 16 marks)

18.) Let $f(x) = e^x \cos x$. Find the gradient of the normal to the curve of f at $x = \pi$.

(Total 6 marks)

19.) Let $f(x) = \dfrac{ax}{x^2 + 1}$, $-8 \le x \le 8$, $a \in \mathbb{R}$. The graph of f is shown below.



The region between $x = 3$ and $x = 7$ is shaded.

(a) Show that $f(-x) = -f(x)$.

(2)

(b) Given that $f''(x) = \dfrac{2ax(x^2-3)}{(x^2+1)^3}$, find the coordinates of all points of inflexion.

(7)

(c) It is given that $\int f(x)\mathrm{d}x = \dfrac{a}{2}\ln(x^2+1)+C$.

(i) Find the area of the shaded region, giving your answer in the form $p \ln q$.

(ii) Find the value of $\int_4^8 2f(x-1)\mathrm{d}x$.

(7)
(Total 16 marks)

20.) A function f has its first derivative given by $f'(x) = (x-3)^3$.

(a) Find the second derivative.

(2)

(b) Find $f'(3)$ and $f''(3)$.

(1)

(c) The point P on the graph of f has x-coordinate 3. Explain why P is not a point of inflexion.

(2)
(Total 5 marks)

21.) Let $f(x) = e^{-3x}$ and $g(x) = \sin\left(x - \dfrac{\pi}{3}\right)$.

(a) Write down

(i) $f'(x)$;

(ii) $g'(x)$.

(2)

(b) Let $h(x) = e^{-3x}\sin\left(x-\dfrac{\pi}{3}\right)$. Find the exact value of $h'\left(\dfrac{\pi}{3}\right)$.

(4)
(Total 6 marks)

22.) Let $f(x) = x^3 - 4x + 1$.

(a) Expand $(x + h)^3$.

(2)

(b) Use the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to show that the derivative of $f(x)$ is $3x^2 - 4$.

(4)

(c) The tangent to the curve of f at the point P(1, –2) is parallel to the tangent at a point Q. Find the coordinates of Q.

(4)

(d) The graph of f is decreasing for $p < x < q$. Find the value of p and of q.

(3)

(e) Write down the range of values for the gradient of f.

(2)
(Total 15 marks)

23.) Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.

(a) Sketch the graph of f.

(3)

(b) Write down

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0.

(3)

(c) Hence write $f(x)$ in the form $p \sin(qx + r)$.

(3)

(d) Write down one value of x such that $f'(x) = 0$.

(2)

(e) Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions.

(2)

(f) Let $g(x) = \ln(x + 1)$, for $0 \le x \le \pi$. There is a value of x, between 0 and 1, for which the gradient of f is equal to the gradient of g. Find this value of x.

(5)
(Total 18 marks)

24.) Consider $f(x) = x^2 + \frac{p}{x}$, $x \ne 0$, where p is a constant.

(a) Find $f'(x)$.

(2)

(b) There is a minimum value of $f(x)$ when $x = -2$. Find the value of p.

(4)
(Total 6 marks)

25.) Let $f(x) = 3 + \dfrac{20}{x^2 - 4}$, for $x \neq \pm 2$. The graph of f is given below.

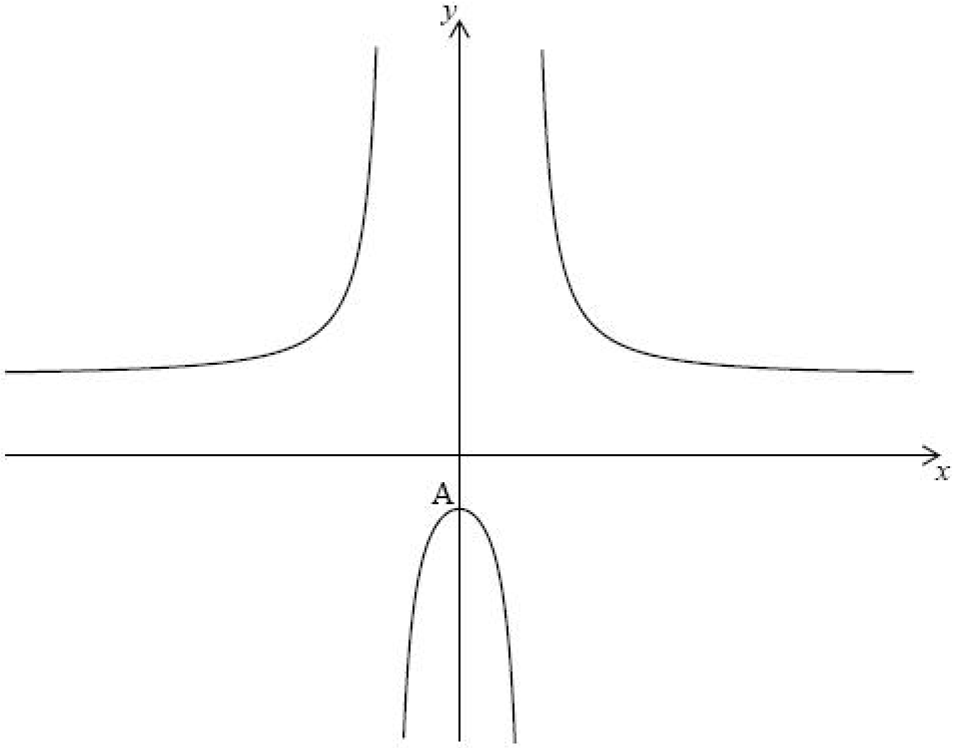


diagram not to scale

The y-intercept is at the point A.

(a) (i) Find the coordinates of A.

(ii) Show that $f'(x) = 0$ at A.

(7)

(b) The second derivative $f''(x) = \dfrac{40(3x^2 + 4)}{(x^2 - 4)^3}$. Use this to

(i) justify that the graph of f has a local maximum at A;

(ii) explain why the graph of f does **not** have a point of inflexion.

(6)

(c) Describe the behaviour of the graph of f for large $|x|$.

(1)

(d) Write down the range of f.

(2)
(Total 16 marks)

26.) Let $f(x) = \cos 2x$ and $g(x) = \ln(3x - 5)$.

(a) Find $f'(x)$.

(2)

(b) Find $g'(x)$.

(2)

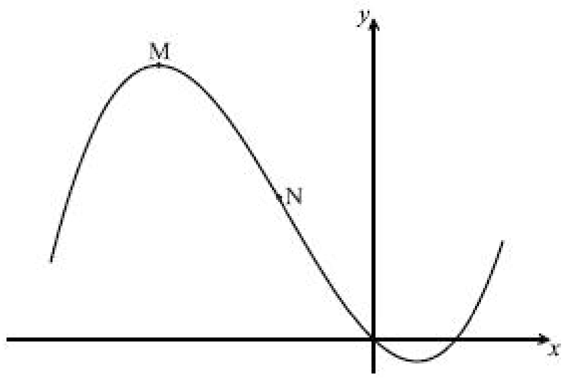
(c) Let $h(x) = f(x) \times g(x)$. Find $h'(x)$.

(2)
(Total 6 marks)

27.) Consider the curve with equation $f(x) = px^2 + qx$, where p and q are constants.
The point A(1, 3) lies on the curve. The tangent to the curve at A has gradient 8.
Find the value of p and of q.

(Total 7 marks)

28.) Consider $f(x) = \frac{1}{3}x^3 + 2x^2 - 5x$. Part of the graph of f is shown below. There is a maximum point at M, and a point of inflexion at N.



(a) Find $f'(x)$.

(3)

(b) Find the x-coordinate of M.

(4)

(c) Find the x-coordinate of N.

(3)

(d) The line L is the tangent to the curve of f at (3, 12). Find the equation of L in the form $y = ax + b$.

(4)
(Total 14 marks)

29.) Let $f : x \mapsto \sin^3 x$.

(a) (i) Write down the range of the function f.

(ii) Consider $f(x) = 1$, $0 \le x \le 2\pi$. Write down the number of solutions to this equation. Justify your answer.

(5)

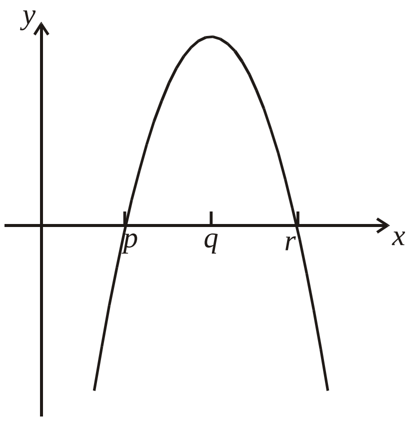
(b) Find $f'(x)$, giving your answer in the form $a \sin^p x \cos^q x$ where $a, p, q \in \mathbb{Z}$.

(2)

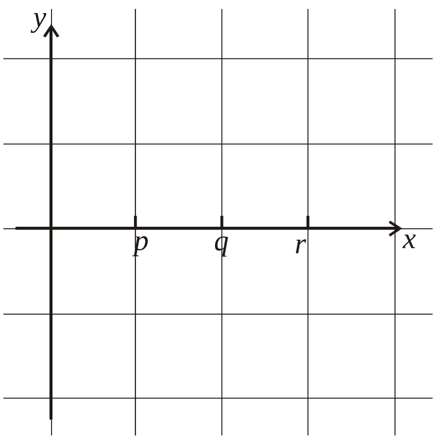
(c) Let $g(x) = \sqrt{3} \sin x (\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of g is revolved through 2π about the x-axis.

(7)
(Total 14 marks

30.) The diagram below shows part of the graph of the **gradient** function, $y = f'(x)$.



(a) On the grid below, sketch a graph of $y = f''(x)$, clearly indicating the x-intercept.



(2)

(b) Complete the table, for the graph of $y = f(x)$.

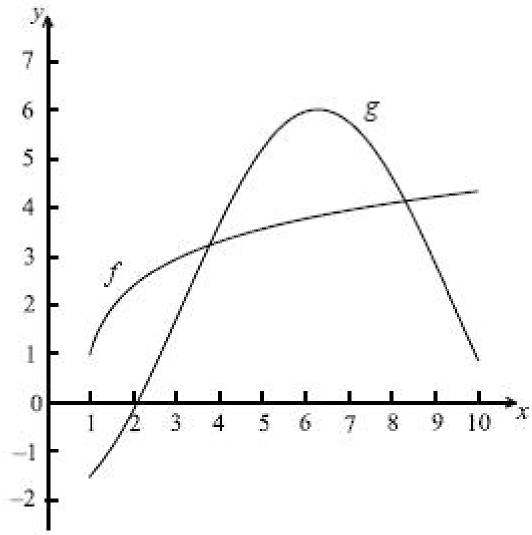
	x-coordinate
(i) Maximum point on f	
(ii) Inflexion point on f	

(2)

(c) Justify your answer to part (b) (ii).

(2)
(Total 6 marks)

31.) The following diagram shows the graphs of $f(x) = \ln(3x - 2) + 1$ and $g(x) = -4\cos(0.5x) + 2$, for $1 \le x \le 10$.



(a) Let A be the area of the region **enclosed** by the curves of f and g.

(i) Find an expression for A.

(ii) Calculate the value of A.

(6)

(b) (i) Find $f'(x)$.

(ii) Find $g'(x)$.

(4)

(c) There are two values of x for which the gradient of f is equal to the gradient of g. Find both these values of x.

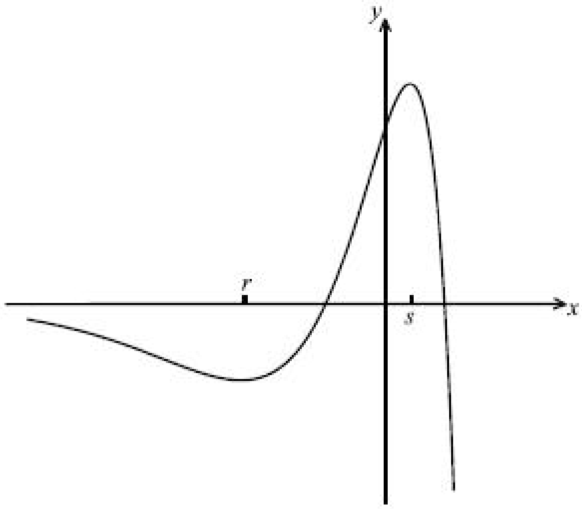
(4)
(Total 14 marks)

32.) Let $f(x) = e^{x}(1 - x^{2})$.

(a) Show that $f'(x) = e^{x}(1 - 2x - x^{2})$.

(3)

Part of the graph of $y = f(x)$, for $-6 \le x \le 2$, is shown below. The x-coordinates of the local minimum and maximum points are r and s respectively.



(b) Write down the **equation** of the horizontal asymptote. **(1)**

(c) Write down the value of r and of s. **(4)**

(d) Let L be the normal to the curve of f at P(0, 1). Show that L has equation $x + y = 1$. **(4)**

(e) Let R be the region enclosed by the curve $y = f(x)$ and the line L.

(i) Find an expression for the area of R.

(ii) Calculate the area of R.

(5)
(Total 17 marks)

33.) Let $f(x) = e^{2x} \cos x$, $-1 \le x \le 2$.

(a) Show that $f'(x) = e^{2x} (2 \cos x - \sin x)$. **(3)**

Let the line L be the normal to the curve of f at $x = 0$.

(b) Find the equation of L. **(5)**

The graph of f and the line L intersect at the point (0, 1) and at a second point P.

(c) (i) Find the x-coordinate of P.

(ii) Find the area of the region **enclosed** by the graph of f and the line L.

(6)
(Total 14 marks)

34.) Find the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 1$.
Give your answer in terms of e^2.

(Total 6 marks)

35.) The velocity v m s^{-1} of a moving body at time t seconds is given by $v = 50 - 10t$.

(a) Find its acceleration in m s^{-2}.

(2)

(b) The initial displacement s is 40 metres. Find an expression for s in terms of t.

(4)
(Total 6 marks)

36.) Let $g(x) = x^3 - 3x^2 - 9x + 5$.

(a) Find the two values of x at which the tangent to the graph of g is horizontal.

(8)

(b) For each of these values, determine whether it is a maximum or a minimum.

(6)
(Total 14 marks)

37.) The function f is given by $f(x) = 2\sin(5x - 3)$.

(a) Find $f'(x)$.

(4)

(b) Write down $\int f(x)\mathrm{d}x$.

(2)
(Total 6 marks)

38.) The function f is defined by $f : x \mapsto -0.5x^2 + 2x + 2.5$.

Let N be the normal to the curve at the point where the graph intercepts the y-axis.

(a) Show that the equation of N may be written as $y = -0.5x + 2.5$.

(4)

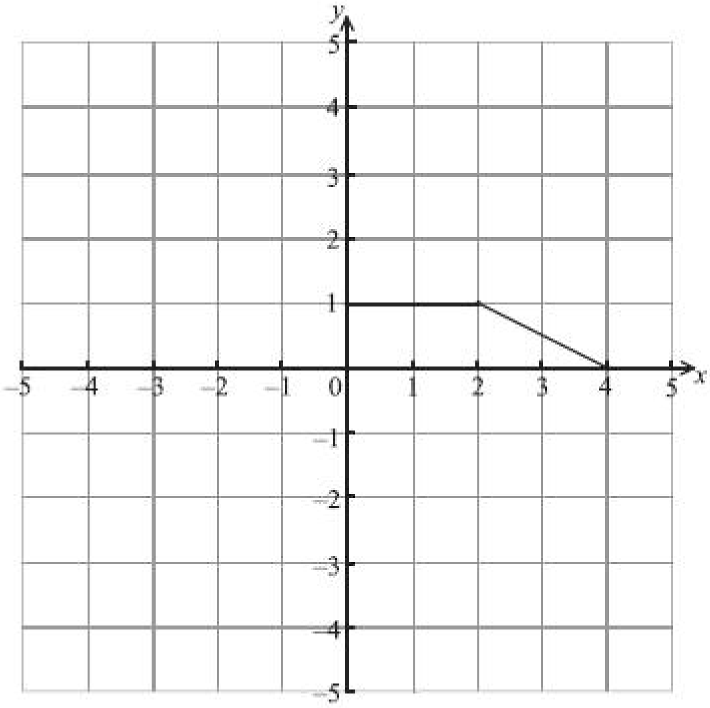
(b) Find the coordinates of the other point of intersection of the normal and the curve.

(5)

(c) Let R be the region enclosed between the curve and N. Find the area of R.

(4)
(Total 13 marks)

39.) The graph of the function $y = f(x)$, $0 \le x \le 4$, is shown below.

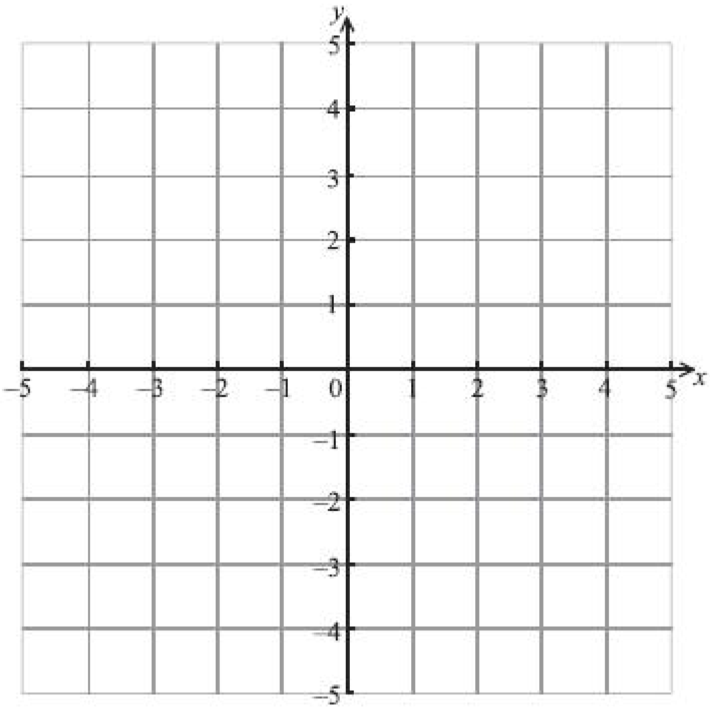


(a) Write down the value of

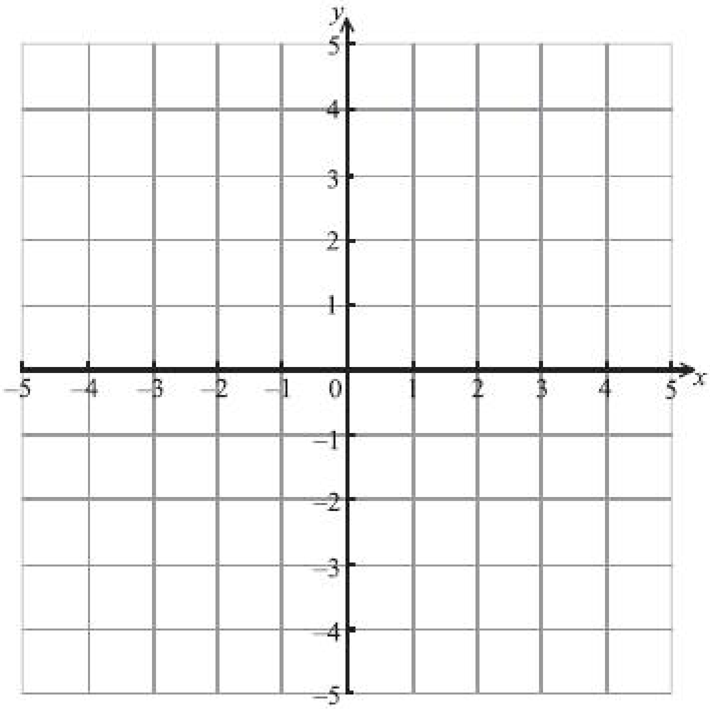
(i) $f'(1)$;

(ii) $f'(3)$.

(b) On the diagram below, draw the graph of $y = 3f(-x)$.



(c) On the diagram below, draw the graph of $y = f(2x)$.



(Total 6 marks)

40.) Let $f(x) = 3 \cos 2x + \sin^2 x$.

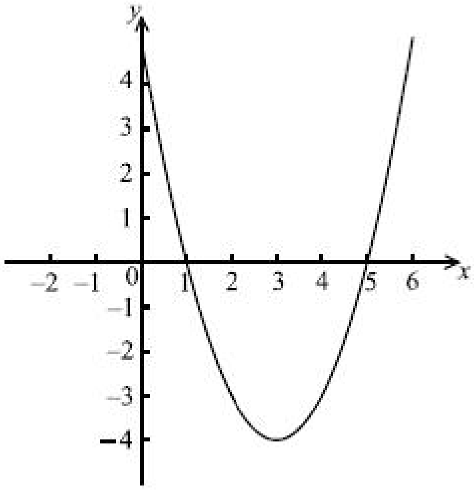
(a) Show that $f'(x) = -5 \sin 2x$.

(b) In the interval $\frac{\pi}{4} \le x \le \frac{3\pi}{4}$, one normal to the graph of f has equation $x = k$.

Find the value of k.

(Total 6 marks)

41.) The following diagram shows part of the graph of a quadratic function, with equation in the form $y = (x - p)(x - q)$, where $p, q \in \mathbb{Z}$.



(a) Write down

(i) the value of p and of q;

(ii) the equation of the axis of symmetry of the curve.

(3)

(b) Find the equation of the function in the form $y = (x - h)^2 + k$, where $h, k \in \mathbb{Z}$.

(3)

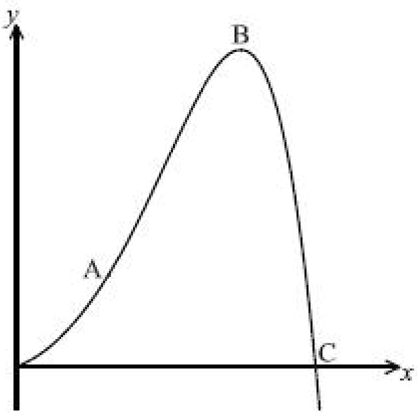
(c) Find $\frac{dy}{dx}$.

(2)

(d) Let T be the tangent to the curve at the point (0, 5). Find the equation of T.

(2)
(Total 10 marks)

42.) The function f is defined as $f(x) = e^x \sin x$, where x is in radians. Part of the curve of f is shown below.



There is a point of inflexion at A, and a local maximum point at B. The curve of f intersects the x-axis at the point C.

(a) Write down the x-coordinate of the point C.

(1)

(b) (i) Find $f'(x)$.

(ii) Write down the value of $f'(x)$ at the point B.

(4)

(c) Show that $f''(x) = 2e^x \cos x$.

(2)

(d) (i) Write down the value of $f''(x)$ at A, the point of inflexion.

(ii) Hence, calculate the coordinates of A.

(4)

(e) Let R be the region enclosed by the curve and the x-axis, between the origin and C.

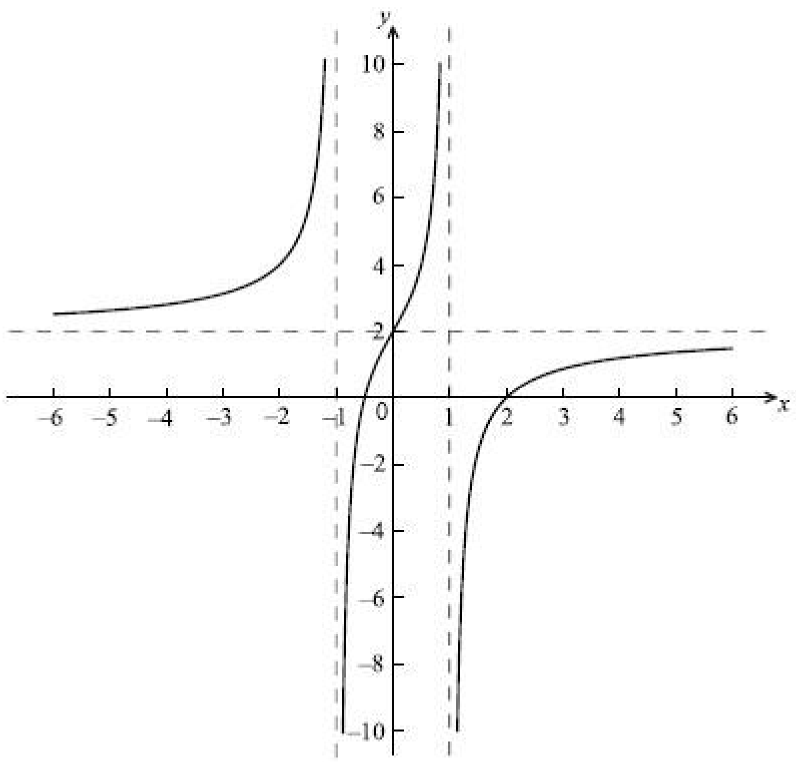
(i) Write down an expression for the area of R.

(ii) Find the area of R.

(4)
(Total 15 marks)

43.) Let $f(x) = p - \dfrac{3x}{x^2 - q^2}$, where $p, q \in \mathbb{R}^+$.

Part of the graph of f, including the asymptotes, is shown below.



(a) The equations of the asymptotes are $x = 1$, $x = -1$, $y = 2$. Write down the value of

(i) p;

(ii) q.

(2)

(b) Let R be the region bounded by the graph of f, the x-axis, and the y-axis.

(i) Find the negative x-intercept of f.

(ii) Hence find the volume obtained when R is revolved through 360° about the x-axis.

(7)

(c) (i) Show that $f'(x) = \dfrac{3(x^2+1)}{(x^2-1)^2}$.

(ii) Hence, show that there are no maximum or minimum points on the graph of f.

(8)

(d) Let $g(x) = f'(x)$. Let A be the area of the region enclosed by the graph of g and the x-axis, between $x = 0$ and $x = a$, where $a > 0$. Given that $A = 2$, find the value of a.

(7)
(Total 24 marks)

44.) Differentiate each of the following with respect to x.

(a) $y = \sin 3x$

(1)

(b) $y = x \tan x$

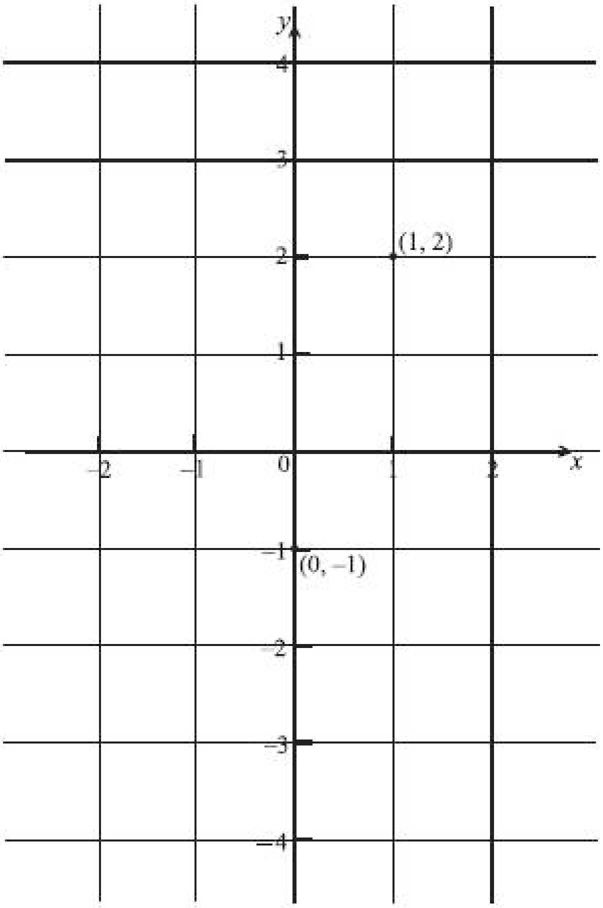
(2)

(c) $y = \dfrac{\ln x}{x}$

(3)
(Total 6 marks)

45.) On the axes below, sketch a curve $y = f(x)$ which satisfies the following conditions.

x	$f(x)$	$f'(x)$	$f''(x)$
$-2 \le x < 0$		negative	positive
0	–1	0	positive
$0 < x < 1$		positive	positive
1	2	positive	0
$1 < x \le 2$		positive	negative



(Total 6 marks)

46.) Consider the function $f(x)\ \mathrm{e}^{(2x-1)} + \left(\dfrac{5}{(2x-1)}\right)$, $x \neq \dfrac{1}{2}$.

(a) Sketch the curve of f for $-2 \le x \le 2$, including any asymptotes.

(3)

(b) (i) Write down the equation of the vertical asymptote of f.

(ii) Write down which one of the following expressions does **not** represent an area between the curve of f and the x-axis.

$$\int_1^2 f(x)\mathrm{d}x$$

$$\int_0^2 f(x)\mathrm{d}x$$

(iii) Justify your answer.

(3)

(c) The region between the curve and the x-axis between $x = 1$ and $x = 1.5$ is rotated through 360° about the x-axis. Let V be the volume formed.

(i) Write down an expression to represent V.

(ii) Hence write down the value of *V*.

(4)

(d) Find $f'(x)$.

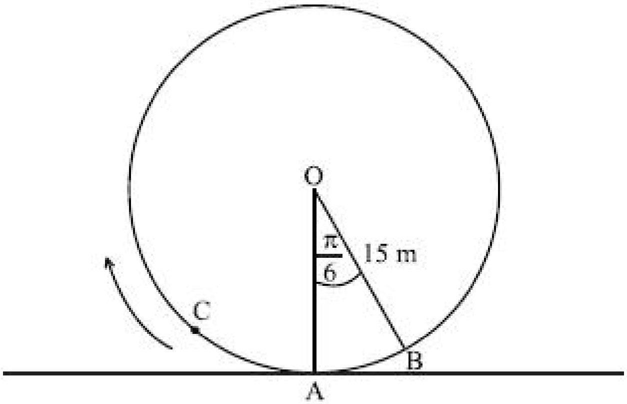
(4)

(e) (i) Write down the value of x at the minimum point on the curve of f.

(ii) The equation $f(x) = k$ has no solutions for $p \le k < q$. Write down the value of p and of q.

(3)
(Total 17 marks)

47.) A Ferris wheel with centre O and a radius of 15 metres is represented in the diagram below. Initially seat A is at ground level. The next seat is B, where $\mathrm{A\hat{O}B} = \frac{\pi}{6}$.



(a) Find the length of the arc AB.

(2)

(b) Find the area of the sector AOB.

(2)

(c) The wheel turns clockwise through an angle of $\frac{2\pi}{3}$. Find the height of A above the ground.

(3)

The height, *h* metres, of seat C above the ground after *t* minutes, can be modelled by the function

$$h(t) = 15 - 15\cos\left(2t + \frac{\pi}{4}\right).$$

(d) (i) Find the height of seat C when $t = \frac{\pi}{4}$.

(ii) Find the initial height of seat C.

(iii) Find the time at which seat C first reaches its highest point.

(8)

(e) Find $h'(t)$.

(2)

(f) For $0 \le t \le \pi$,

(i) sketch the graph of h';

(ii) find the time at which the height is changing most rapidly.

(5)
(Total 22 marks)

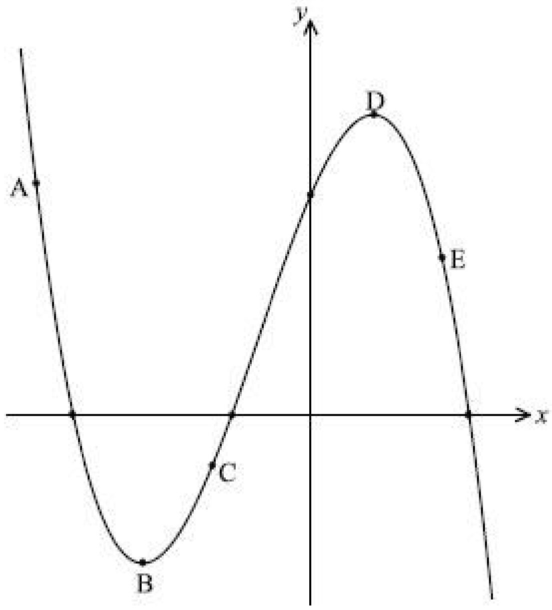
48.) (a) Let $f(x) = e^{5x}$. Write down $f'(x)$.

(b) Let $g(x) = \sin 2x$. Write down $g'(x)$.

(c) Let $h(x) = e^{5x} \sin 2x$. Find $h'(x)$.

(Total 6 marks)

49.) The following diagram shows part of the curve of a function f. The points A, B, C, D and E lie on the curve, where B is a minimum point and D is a maximum point.



(a) Complete the following table, noting whether $f'(x)$ is positive, negative or zero at the given points.

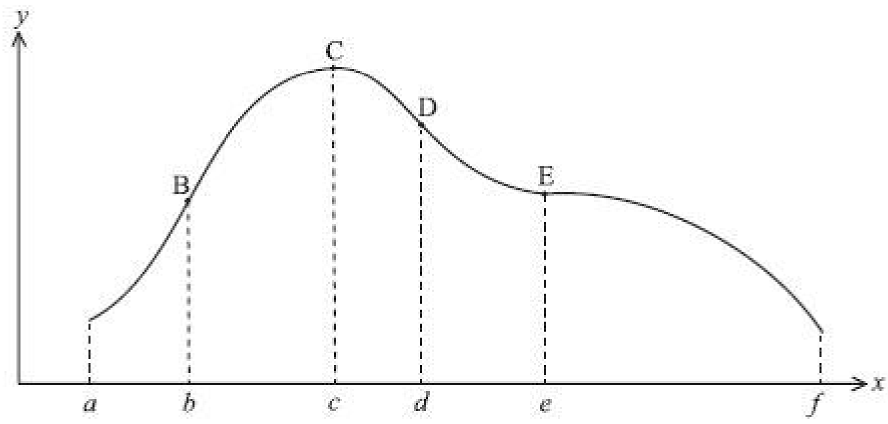
	A	B	E
$f'(x)$			

(b) Complete the following table, noting whether $f''(x)$ is positive, negative or zero at the given points.

	A	C	E
$f''(x)$			

(Total 6 marks)

50.) The graph of a function g is given in the diagram below.



The gradient of the curve has its maximum value at point B and its minimum value at point D. The tangent is horizontal at points C and E.

(a) Complete the table below, by stating whether the first derivative g' is positive or negative, and whether the second derivative g'' is positive or negative.

Interval	g'	g''
$a < x < b$		
$e < x < f$		

(b) Complete the table below by noting the points on the graph described by the following conditions.

Conditions	Point
$g'(x) = 0, g''(x) < 0$	
$g'(x) < 0, g''(x) = 0$	

(Total 6 marks)

51.) Consider the function $f : x \mapsto 3x^2 - 5x + k$.

(a) Write down $f'(x)$.

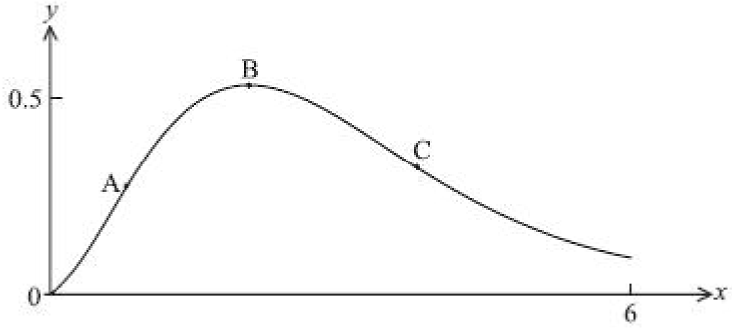
The equation of the tangent to the graph of f at $x = p$ is $y = 7x - 9$. Find the value of

(b) p;

(c) k.

(Total 6 marks)

52.) The diagram below shows the graph of $f(x) = x^2 e^{-x}$ for $0 \le x \le 6$. There are points of inflexion at A and C and there is a maximum at B.



(a) Using the product rule for differentiation, find $f'(x)$.

(b) Find the **exact** value of the ***y*-coordinate** of B.

(c) The second derivative of f is $f''(x) = (x^2 - 4x + 2)\,e^{-x}$. Use this result to find the **exact** value of the x-coordinate of C.

(Total 6 marks)

53.) Let $f(x) = -\frac{3}{4}x^2 + x + 4$.

(a) (i) Write down $f'(x)$.

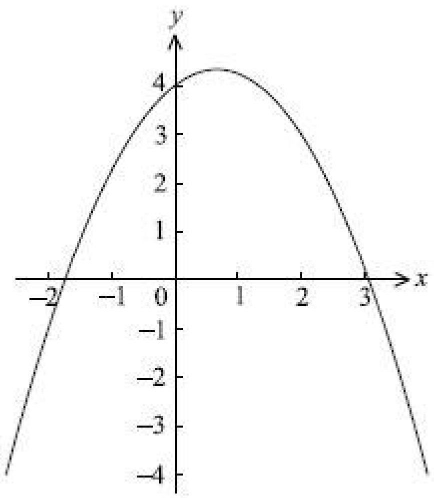
(ii) Find the equation of the normal to the curve of f at (2, 3).

(iii) This normal intersects the curve of f at (2, 3) and at one other point P.

Find the x-coordinate of P.

(9)

Part of the graph of f is given below.



(b) Let R be the region under the curve of f from $x = -1$ to $x = 2$.

(i) Write down an expression for the area of R.

(ii) Calculate this area.

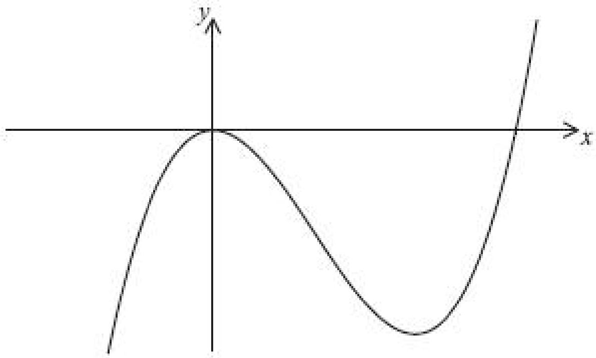
(iii) The region R is revolved through 360° about the x-axis. Write down an expression for the volume of the solid formed.

(6)

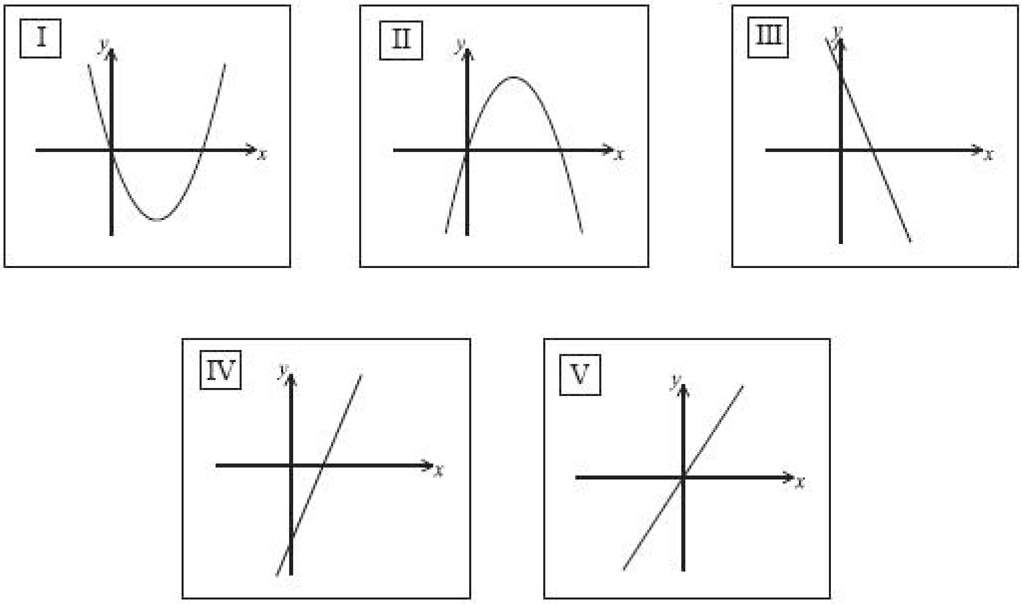
(c) Find $\int_1^k f(x)\,dx$, giving your answer in terms of k.

(6)
(Total 21 marks)

54.) The following diagram shows the graph of a function f.



Consider the following diagrams.



Complete the table below, noting which one of the diagrams above represents the graph of

(a) $f'(x)$;

(b) $f''(x)$.

	Graph	Diagram

(a)	$f'(x)$	
(b)	$f''(x)$	

(Total 6 marks)

55.) The function f is defined as $f(x) = (2x + 1)\,e^{-x}$, $0 \le x \le 3$. The point P(0, 1) lies on the graph of $f(x)$, and there is a maximum point at Q.

(a) Sketch the graph of $y = f(x)$, labelling the points P and Q.

(3)

(b) (i) Show that $f'(x) = (1 - 2x)\,e^{-x}$.

(ii) Find the **exact** coordinates of Q.

(7)

(c) The equation $f(x) = k$, where $k \in \mathbb{R}$, has two solutions. Write down the range of values of k.

(2)

(d) Given that $f''(x) = e^{-x}(-3 + 2x)$, show that the curve of f has only one point of inflexion.

(2)

(e) Let R be the point on the curve of f with x-coordinate 3. Find the area of the region enclosed by the curve and the line (PR).

(7)
(Total 21 marks)

56.) The function f is given by $f(x) = 2\sin(5x - 3)$.

(a) Find $f''(x)$.

(b) Write down $\int f(x)\mathrm{d}x$.

(Total 6 marks)

57.) The function f is defined by $f : x \mapsto -0.5x^2 + 2x + 2.5$.

(a) Write down

(i) $f'(x)$;

(ii) $f'(0)$.

(2)

(b) Let N be the normal to the curve at the point where the graph intercepts the y-axis. Show that the equation of N may be written as $y = -0.5x + 2.5$.

(3)

Let $g : x \mapsto -0.5x + 2.5$

(c) (i) Find the solutions of $f(x) = g(x)$.

(ii) Hence find the coordinates of the other point of intersection of the normal and the curve.

(6)

(d) Let R be the region enclosed between the curve and N.

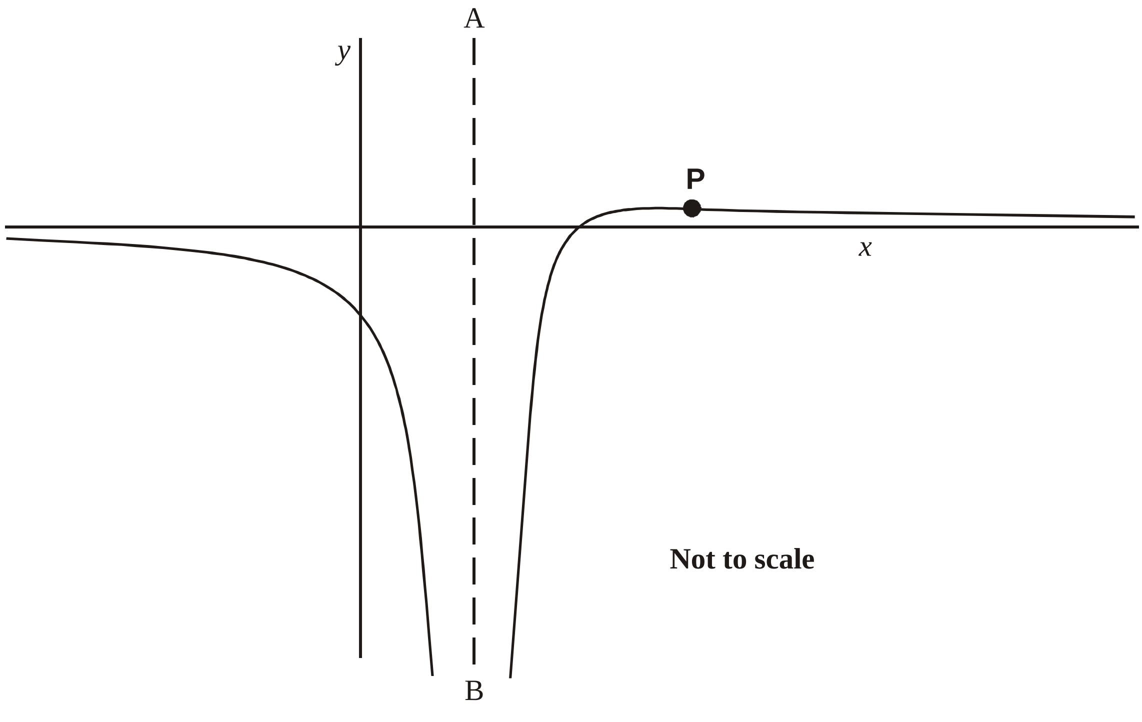
(i) Write down an expression for the area of R.

(ii) Hence write down the area of R.

(5)
(Total 16 marks)

58.) Consider the function $h : x \mapsto \dfrac{x-2}{(x-1)^2}, x \neq 1$.

A sketch of part of the graph of h is given below.



The line (AB) is a vertical asymptote. The point P is a point of inflexion.

(a) Write down the **equation** of the vertical asymptote.

(1)

(b) Find $h'(x)$, writing your answer in the form

$$\frac{a-x}{(x-1)^n}$$

where a and n are constants to be determined.

(4)

(c) Given that $h''(x) = \dfrac{2x-8}{(x-1)^4}$, calculate the coordinates of P.

(3)
(Total 8 marks)

59.) Let $f(x) = (3x + 4)^5$. Find

(a) $f'(x)$;

(b) $\int f(x)\mathrm{d}x$.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

60.) The table below shows some values of two functions, f and g, and of their derivatives f' and g'.

x	1	2	3	4
$f(x)$	5	4	–1	3
$g(x)$	1	–2	2	–5
$f'(x)$	5	6	0	7
$g'(x)$	–6	–4	–3	4

Calculate the following.

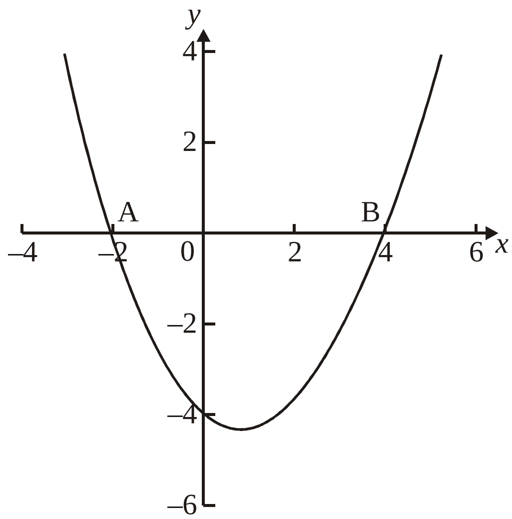
(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}(f(x) + g(x))$, when $x = 4$;

(b) $\int_1^3 (g'(x) + 6)\mathrm{d}x$.

Working:	
	Answers:
	(a) ....
	(b) ....

(Total 6 marks)

61.) The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve intersects the x-axis at A(–2, 0) and B(4, 0). The curve of $y = f(x)$ is shown in the diagram below.



(a) (i) Write down the value of p and of q.

(ii) Given that the point (6, 8) is on the curve, find the value of a.

(iii) Write the equation of the curve in the form $y = ax^2 + bx + c$.

(5)

(b) (i) Find $\frac{dy}{dx}$.

(ii) A tangent is drawn to the curve at a point P. The gradient of this tangent is 7. Find the coordinates of P.

(4)

(c) The line L passes through B(4, 0), and is perpendicular to the tangent to the curve at point B.

(i) Find the equation of L.

(ii) Find the x-coordinate of the point where L intersects the curve again.

(6)
(Total 15 marks)

62.) Let $f(x) = \dfrac{3x^2}{5x-1}$.

(a) Write down the **equation** of the vertical asymptote of $y = f(x)$.

(1)

(b) Find $f'(x)$. Give your answer in the form $\dfrac{ax^2+bx}{(5x-1)^2}$ where a and $b \in \mathbb{Z}$.

(4)
(Total 5 marks)

63.) The function $g(x)$ is defined for $-3 \le x \le 3$. The behaviour of $g'(x)$ and $g''(x)$ is given in the tables below.

x	$-3 < x < -2$	-2	$-2 < x < 1$	1	$1 < x < 3$
$g'(x)$	negative	0	positive	0	negative

x	$-3 < x < -\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2} < x < 3$
$g''(x)$	positive	0	negative

Use the information above to answer the following. In each case, justify your answer.

(a) Write down the value of x for which g has a maximum.

(2)

(b) On which intervals is the value of g decreasing?

(2)

(c) Write down the value of x for which the graph of g has a point of inflexion.

(2)

(d) Given that $g(-3) = 1$, sketch the graph of g. On the sketch, clearly indicate the position of the maximum point, the minimum point, and the point of inflexion.

(3)
(Total 9 marks)

64.) Let $f(x) = (2x + 7)^3$ and $g(x) \cos^2(4x)$. Find

(a) $f'(x)$;

(b) $g'(x)$.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

65.) Let $f(x) = x^3 - 2x^2 - 1$.

(a) Find $f'(x)$.

(b) Find the gradient of the curve of $f(x)$ at the point $(2, -1)$.

Working:

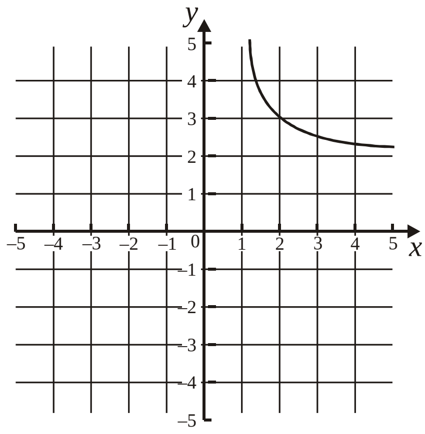
Answers:

(a) ....

(b) ....

(Total 6 marks)

66.) (a) Consider the function $f(x) = 2 + \dfrac{1}{x-1}$. The diagram below is a sketch of part of the graph of $y = f(x)$.



Copy and complete the sketch of $f(x)$.

(2)

(b) (i) Write down the x-intercepts and y-intercepts of $f(x)$.

(ii) Write down the equations of the asymptotes of $f(x)$.

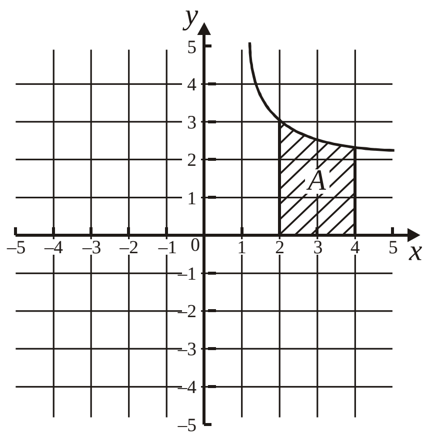
(4)

(c) (i) Find $f'(x)$.

(ii) There are no maximum or minimum points on the graph of $f(x)$.
Use your expression for $f'(x)$ to explain why.

(3)

The region enclosed by the graph of $f(x)$, the x-axis and the lines $x = 2$ and $x = 4$, is labelled A, as shown below.



(d) (i) Find $\int f(x)\,dx$.

(ii) Write down an expression that represents the area labelled A.

(iii) Find the area of A.

(7)
(Total 16 marks)

67.) Let $f(x) = 6\sqrt[3]{x^2}$. Find $f'(x)$.

Working:

Answer:

…………………………………………..……

(Total 6 marks)

68.) The population p of bacteria at time t is given by $p = 100e^{0.05t}$.

Calculate

(a) the value of p when $t = 0$;

(b) the rate of increase of the population when $t = 10$.

Working:

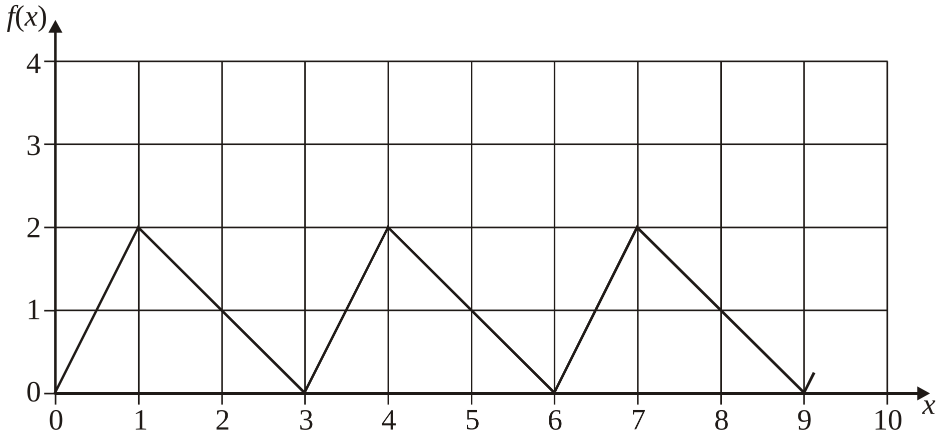
Answers:

(a) ……………………………………………..

(b) ……………………………………………..

(Total 6 marks)

69.) Part of the graph of the periodic function f is shown below. The domain of f is $0 \le x \le 15$ and the period is 3.



(a) Find

(i) $f(2)$;

(ii) $f'(6.5)$;

(iii) $f'(14)$.

(b) How many solutions are there to the equation $f(x) = 1$ over the given domain?

Working:

Answers:

(a) (i) ……………………………………………

(ii) ……………………………………………

(iii) ……………………………………………

(b) ……………………………………………

(Total 6 marks)

70.) Let $f(x) = 1 + 3\cos(2x)$ for $0 \le x \le \pi$, and x is in radians.

(a) (i) Find $f'(x)$.

(ii) Find the values for x for which $f'(x) = 0$, giving your answers in terms of π.

(6)

The function $g(x)$ is defined as $g(x) = f(2x) - 1$, $0 \le x \le \dfrac{\pi}{2}$.

(b) (i) The graph of f may be transformed to the graph of g by a stretch in the x-

direction with scale factor $\frac{1}{2}$ followed by another transformation. Describe fully this other transformation.

(ii) Find the solution to the equation $g(x) = f(x)$

(4)
(Total 10 marks)

71.) Let $f(x) = \dfrac{1}{1+x^2}$.

(a) Write down the equation of the horizontal asymptote of the graph of f.

(1)

(b) Find $f'(x)$.

(3)

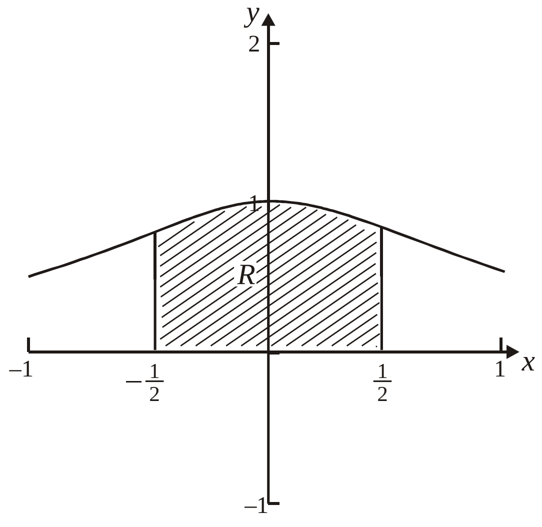
(c) The second derivative is given by $f''(x) = \dfrac{6x^2-2}{(1+x^2)^3}$.

Let A be the point on the curve of f where the gradient of the tangent is a maximum. Find the x-coordinate of A.

(4)

(d) Let R be the region under the graph of f, between $x = -\dfrac{1}{2}$ and $x = \dfrac{1}{2}$,

as shaded in the diagram below



Write down the definite integral which represents the area of R.

(2)
(Total 10 marks)

72.) Let $f(x) = e^{\frac{x}{3}} + 5\cos^2 x$. Find $f'(x)$.

Working:

Answer:

....

(Total 6 marks)

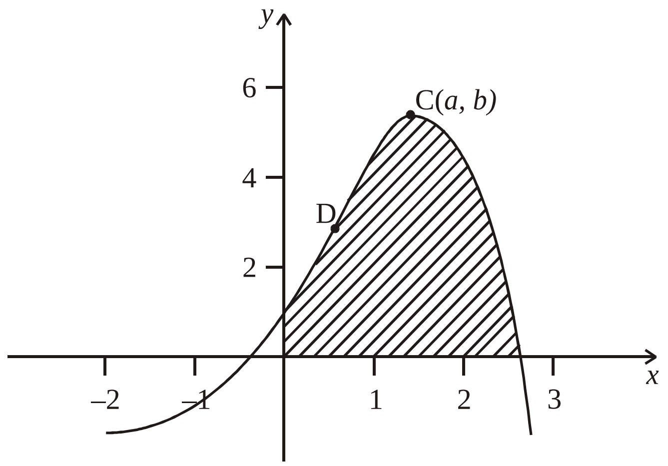
73.) Consider the function $f(x) = \cos x + \sin x$.

(a) (i) Show that $f\left(-\frac{\pi}{4}\right) = 0$.

(ii) Find in terms of π, the smallest **positive** value of x which satisfies $f(x) = 0$.

(3)

The diagram shows the graph of $y = e^x(\cos x + \sin x)$, $-2 \le x \le 3$. The graph has a maximum turning point at $C(a, b)$ and a point of inflexion at D.



(b) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(3)

(c) Find the **exact** value of a and of b.

(4)

(d) Show that at D, $y = \sqrt{2}e^{\frac{\pi}{4}}$.

(5)

(e) Find the area of the shaded region.

(2)
(Total 17 marks)

74.) Consider the function $f(x) = 1 + e^{-2x}$.

(a) (i) Find $f'(x)$.

(ii) Explain briefly how this shows that $f(x)$ is a decreasing function for all values of x (*ie* that $f(x)$ always decreases in value as x increases).

(2)

Let P be the point on the graph of f where $x = -\frac{1}{2}$.

(b) Find an expression in terms of e for

(i) the y-coordinate of P;

(ii) the gradient of the tangent to the curve at P.

(2)

(c) Find the equation of the tangent to the curve at P, giving your answer in the form $y = ax + b$.

(3)

(d) (i) Sketch the curve of f for $-1 \le x \le 2$.

(ii) Draw the tangent at $x = -\frac{1}{2}$.

(iii) Shade the area enclosed by the curve, the tangent and the y-axis.

(iv) Find this area.

(7)
(Total 14 marks)

75.) ***Note: Radians are used throughout this question.***

A mass is suspended from the ceiling on a spring. It is pulled down to point P and then released. It oscillates up and down.

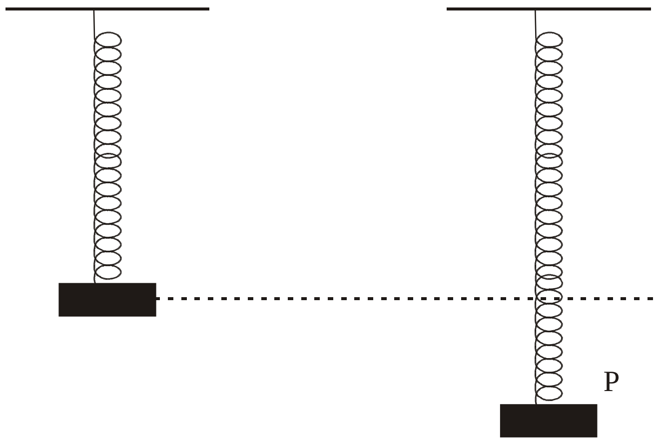


diagram not to scale

Its distance, s cm, from the ceiling, is modelled by the function $s = 48 + 10 \cos 2\pi t$ where t is the time in seconds from release.

(a) (i) What is the distance of the point P from the ceiling?

(ii) How long is it until the mass is next at P?

(5)

(b) (i) Find $\frac{\mathrm{d}s}{\mathrm{d}t}$.

(ii) Where is the mass when the velocity is zero?

(7)

A second mass is suspended on another spring. Its distance r cm from the ceiling is modelled by the function $r = 60 + 15 \cos 4\pi t$. The two masses are released at the same instant.

(c) Find the value of t when they are first at the same distance below the ceiling.

(2)

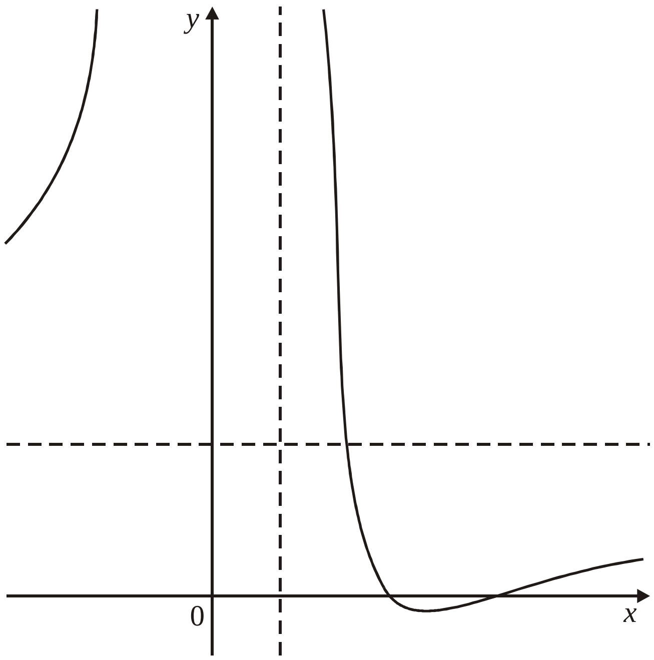
(d) In the first three seconds, how many times are the two masses at the same height?

(2)

(Total 16 marks)

76.) Consider the function f given by $f(x) = \dfrac{2x^2 - 13x + 20}{(x-1)^2}$, $x \neq 1$.

A part of the graph of f is given below.



The graph has a vertical asymptote and a horizontal asymptote, as shown.

(a) Write down the **equation** of the vertical asymptote.

(1)

(b) $f(100) = 1.91$ $f(-100) = 2.09$ $f(1000) = 1.99$

(i) Evaluate $f(-1000)$.

(ii) Write down the **equation** of the horizontal asymptote.

(2)

(c) Show that $f'(x) = \dfrac{9x - 27}{(x-1)^3}$, $x \neq 1$.

(3)

The second derivative is given by $f''(x) = \dfrac{72 - 18x}{(x-1)^4}$, $x \neq 1$.

(d) Using values of $f'(x)$ and $f''(x)$ explain why a minimum must occur at $x = 3$.

(2)

(e) There is a point of inflexion on the graph of f. Write down the coordinates of this point.

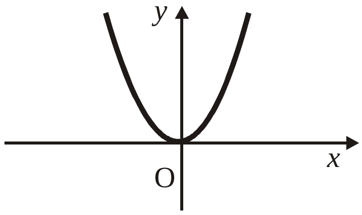
(2)
(Total 10 marks)

77.) **Figure 1** shows the graphs of the functions f_1, f_2, f_3, f_4.

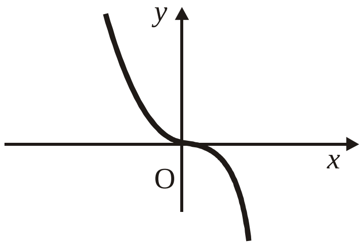
Figure 2 includes the graphs of the derivatives of the functions shown in **Figure 1**, *eg* the derivative of f_1 is shown in diagram (d).

Figure 1 **Figure 2**

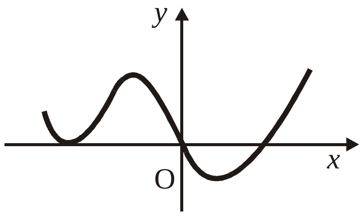
f_1



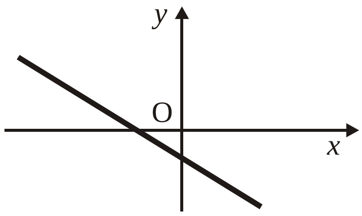
f_2



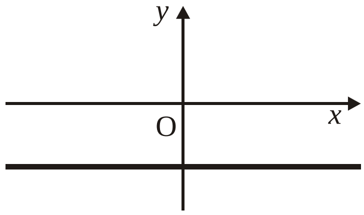
f_3



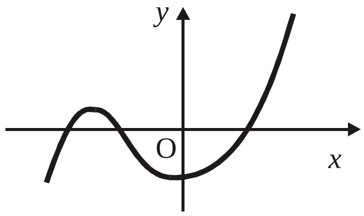
f_4



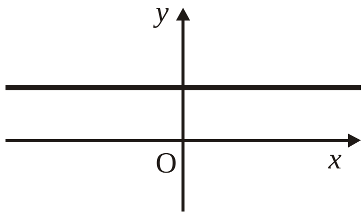
(a)



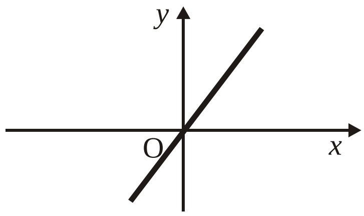
(b)



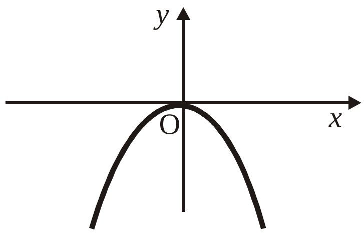
(c)



(d)



(e)



Complete the table below by matching each function with its derivative.

Function	Derivative diagram
f_1	(d)
f_2	
f_3	
f_4	

Working:

(Total 6 marks)

78.) Consider functions of the form $y = e^{-kx}$

(a) Show that $\int_0^1 e^{-kx}dx = \frac{1}{k}(1 - e^{-k})$.

(3)

(b) Let $k = 0.5$

(i) Sketch the graph of $y = e^{-0.5x}$, for $-1 \le x \le 3$, indicating the coordinates of the y-intercept.

(ii) Shade the region enclosed by this graph, the x-axis, y-axis and the line $x = 1$.

(iii) Find the area of this region.

(5)

(c) (i) Find $\frac{dy}{dx}$ in terms of k, where $y = e^{-kx}$.

The point P(1, 0.8) lies on the graph of the function $y = e^{-kx}$.

(ii) Find the value of k in this case.

(iii) Find the gradient of the tangent to the curve at P.

(5)
(Total 13 marks)

79.) Let the function f be defined by $f(x) = \frac{2}{1+x^3}$, $x \ne -1$.

(a) (i) Write down the equation of the vertical asymptote of the graph of f.

(ii) Write down the equation of the horizontal asymptote of the graph of f.

(iii) Sketch the graph of f in the domain $-3 \le x \le 3$.

(4)

(b) (i) Using the fact that $f'(x) = \dfrac{-6x^2}{(1+x^3)^2}$, show that the second derivative

$f''(x) = \dfrac{12x(2x^3-1)}{(1+x^3)^3}$.

(ii) Find the x-coordinates of the points of inflexion of the graph of f.

(6)

(c) The table below gives some values of $f(x)$ and $2f(x)$.

x	$f(x)$	$2f(x)$
1	1	2
1.4	0.534188	1.068376
1.8	0.292740	0.585480
2.2	0.171703	0.343407
2.6	0.107666	0.215332
3	0.071429	0.142857

(i) Use the trapezium rule with five sub-intervals to approximate the integral $\int_1^3 f(x)\,dx$.

(ii) Given that $\int_1^3 f(x)\,dx = 0.637599$, use a diagram to explain why your answer is greater than this.

(5)
(Total 15 marks)

80.) Let $f(x) = \sqrt{x^3}$. Find

(a) $f'(x)$;

(b) $\int f(x)\,dx$.

Working:

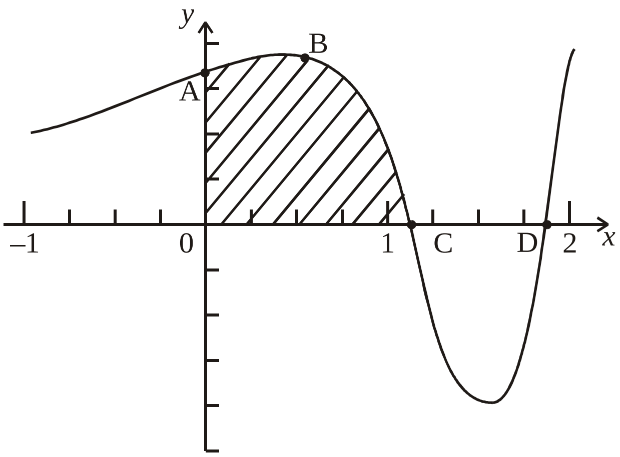
Answers:

(a) ....

(b) ....

(Total 6 marks)

81.) The diagram below shows a sketch of the graph of the function $y = \sin(e^x)$ where $-1 \le x \le 2$, and x is in **radians**. The graph cuts the y-axis at A, and the x-axis at C and D. It has a maximum point at B.



(a) Find the coordinates of A.

(2)

(b) The coordinates of C may be written as $(\ln k, 0)$. Find the **exact** value of k.

(2)

(c) (i) Write down the y-coordinate of B.

(ii) Find $\frac{dy}{dx}$.

(iii) Hence, show that at B, $x = \ln \frac{\pi}{2}$.

(6)

(d) (i) Write down the integral which represents the shaded area.

(ii) Evaluate this integral.

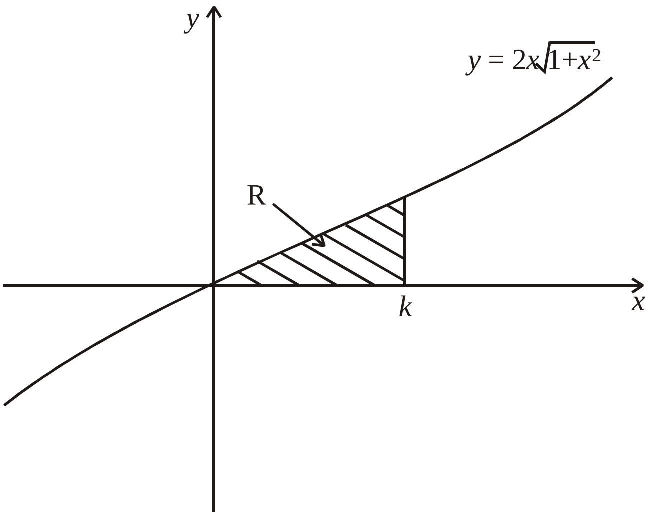
(5)

(e) (i) Copy the above diagram into your answer booklet. (There is no need to copy the shading.) On your diagram, sketch the graph of $y = x^3$.

(ii) The two graphs intersect at the point P. Find the x-coordinate of P.

(3)
(Total 18 marks)

82.) The diagram below shows the shaded region R enclosed by the graph of $y = 2x\sqrt{1+x^2}$, the x-axis, and the vertical line $x = k$.



(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(3)

(b) Using the substitution $u = 1 + x^2$ or otherwise, show that

$$\int 2x\sqrt{1+x^2}\,\mathrm{d}x = \frac{2}{3}(1+x^2)^{\frac{3}{2}} + c.$$

(3)

(c) Given that the area of R equals 1, find the value of k.

(3)
(Total 9 marks)

83.) The function f is given by $f(x) = \frac{\ln 2x}{x}$, $x > 0$.

(a) (i) Show that $f'(x) = \frac{1 - \ln 2x}{x^2}$.

Hence

(ii) prove that the graph of f can have only one local maximum or minimum point;

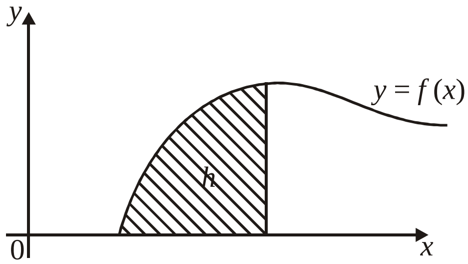
(iii) find the coordinates of the maximum point on the graph of f.

(6)

(b) By showing that the second derivative $f''(x) = \frac{21\text{n } 2x - 3}{x^3}$ or otherwise, find the coordinates of the point of inflexion on the graph of f.

(6)

(c) The region S is enclosed by the graph of f, the x-axis, and the vertical line through the maximum point of f, as shown in the diagram below.



(i) Would the trapezium rule overestimate or underestimate the area of S? Justify your answer by drawing a diagram or otherwise.

(3)

(ii) Find $\int f(x)\,dx$, by using the substitution $u = \ln 2x$, or otherwise.

(4)

(iii) Using $\int f(x)\,dx$, find the area of S.

(4)

(d) The Newton–Raphson method is to be used to solve the equation $f(x) = 0$.

(i) Show that it is not possible to find a solution using a starting value of $x_1 = 1$.

(3)

(ii) Starting with $x_1 = 0.4$, calculate successive approximations $x_2, x_3, \ldots$ for the root of the equation until the absolute error is less than 0.01. Give all answers correct to **five** decimal places.

(4)
(Total 30 marks)

84.) Consider the function $f(x) = k \sin x + 3x$, where k is a constant.

(a) Find $f'(x)$.

(b) When $x = \frac{\text{®}}{3}$, the gradient of the curve of $f(x)$ is 8. Find the value of k.

Working:	
	Answers:
	(a) ....
	(b) ....

(Total 4 marks)

85.) A ball is dropped vertically from a great height. Its velocity v is given by

$$v = 50 - 50e^{-0.2t}, \ t \geq 0$$

where v is in metres per second and t is in seconds.

(a) Find the value of v when

(i) $t = 0$;

(ii) $t = 10$.

(2)

(b) (i) Find an expression for the acceleration, a, as a function of t.

(ii) What is the value of a when $t = 0$?

(3)

(c) (i) As t becomes large, what value does v approach?

(ii) As t becomes large, what value does a approach?

(iii) Explain the relationship between the answers to parts (i) and (ii).

(3)

(d) Let y metres be the distance fallen after t seconds.

(i) Show that $y = 50t + 250e^{-0.2t} + k$, where k is a constant.

(ii) Given that $y = 0$ when $t = 0$, find the value of k.

(iii) Find the time required to fall 250 m, giving your answer correct to **four** significant figures.

(7)
(Total 15 marks)

86.) *Radian measure is used, where appropriate, throughout the question.*

Consider the function $y = \frac{3x-2}{2x-5}$.

The graph of this function has a vertical and a horizontal asymptote.

(a) Write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote.

(2)

(b) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$, simplifying the answer as much as possible.

(3)

(c) How many points of inflexion does the graph of this function have?

(1)
(Total 6 marks)

87.) Given the function $f(x) = x^2 - 3bx + (c + 2)$, determine the values of b and c such that $f(1) = 0$ and *f*â (3) = 0.

Working:

Answer:

....

(Total 4 marks)

88.) The function f is given by

$$f(x) = 1 - \frac{2x}{1+x^2}$$

(a) (i) To display the graph of $y = f(x)$ for $-10 \le x \le 10$, a suitable interval for y, $a \le$

$y \le b$ must be chosen. Suggest appropriate values for a and b .

(ii) Give the equation of the asymptote of the graph.

(3)

(b) Show that $f'(x) = \dfrac{2x^2 - 2}{(1 + x^2)^2}$.

(4)

(c) **Use your answer to part (b)** to find the coordinates of the maximum point of the graph.

(3)

(d) (i) **Either** by inspection or by using an appropriate substitution, find

$$\int f(x)\,dx$$

(ii) **Hence** find the exact area of the region enclosed by the graph of f, the x-axis and the y-axis.

(8)
(Total 18 marks)

89.) Let $f(x) = x^3$.

(a) Evaluate $\dfrac{f(5+h) - f(5)}{h}$ for $h = 0.1$.

(b) What number does $\dfrac{f(5+h) - f(5)}{h}$ approach as h approaches zero?

Working:

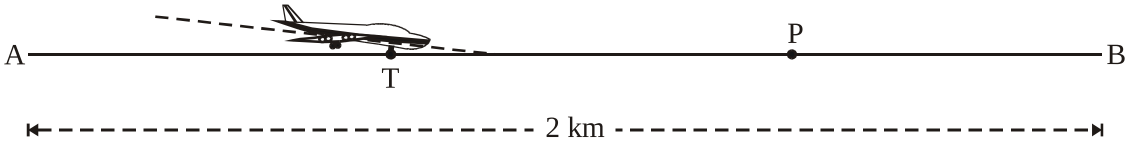
Answers:

(a) ....

(b) ....

(Total 4 marks)

90.) The main runway at *Concordville* airport is 2 km long. An airplane, landing at *Concordville*, touches down at point T, and immediately starts to slow down. The point A is at the southern end of the runway. A marker is located at point P on the runway.



Not to scale

As the airplane slows down, its distance, s, from A, is given by

$$s = c + 100t - 4t^2,$$

where t is the time in seconds after touchdown, and c metres is the distance of T from A.

(a) The airplane touches down 800 m from A, (*ie* $c = 800$).

(i) Find the distance travelled by the airplane in the first 5 seconds after touchdown.

(2)

(ii) Write down an expression for the velocity of the airplane at time t seconds after touchdown, and hence find the velocity after 5 seconds.

(3)

The airplane passes the marker at P with a velocity of 36 m s^{-1}. Find

(iii) how many seconds after touchdown it passes the marker;

(2)

(iv) the distance from P to A.

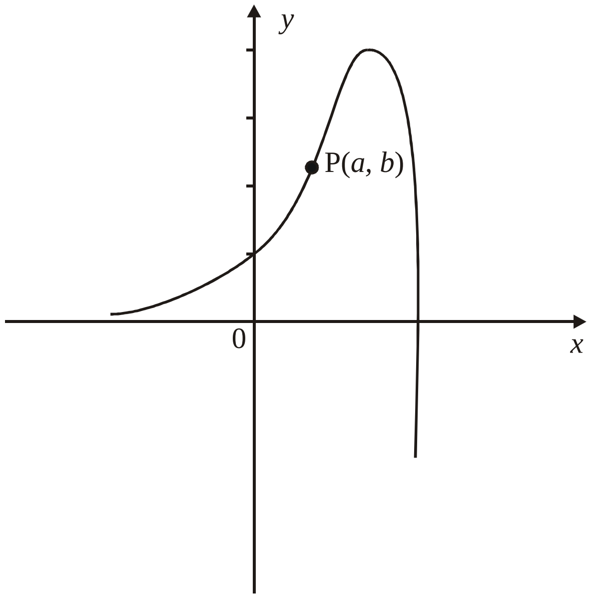
(3)

(b) Show that if the airplane touches down before reaching the point P, it can stop before reaching the northern end, B, of the runway.

(5)
(Total 15 marks)

91.) The diagram shows part of the graph of the curve with equation

$$y = e^{2x} \cos x.$$



(a) Show that $\dfrac{dy}{dx} = e^{2x}(2\cos x - \sin x)$.

(2)

(b) Find $\dfrac{d^2y}{dx^2}$.

(4)

There is an inflexion point at P (a, b).

(c) Use the results from parts (a) and (b) to prove that:

(i) $\tan a = \dfrac{3}{4}$;

(3)

(ii) the gradient of the curve at P is e^{2a}.

(5)
(Total 14 marks)

92.) Given that $f(x) = (2x + 5)^3$ find

(a) $f'(x)$;

(b) $\int f(x)\mathrm{d}x$.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

93.) A rock-climber slips off a rock-face and falls vertically. At first he falls freely, but after 2 seconds a safety rope slows him down. The height h metres of the rock-climber after t seconds of the fall is given by:

$$h = 50 - 5t^2, \qquad 0 \le t \le 2$$

$$h = 90 - 40t + 5t^2, \qquad 2 \le t \le 5$$

(a) Find the height of the rock-climber when $t = 2$.

(1)

(b) Sketch a graph of h against t for $0 \le t \le 5$.

(4)

(c) Find $\frac{\mathrm{d}h}{\mathrm{d}t}$ for:

(i) $0 \le t \le 2$

(ii) $2 \le t \le 5$

(2)

(d) Find the velocity of the rock-climber when $t = 2$.

(2)

(e) Find the times when the velocity of the rock-climber is zero.

(3)

(f) Find the minimum height of the rock-climber for $0 \le t \le 5$.

(3)

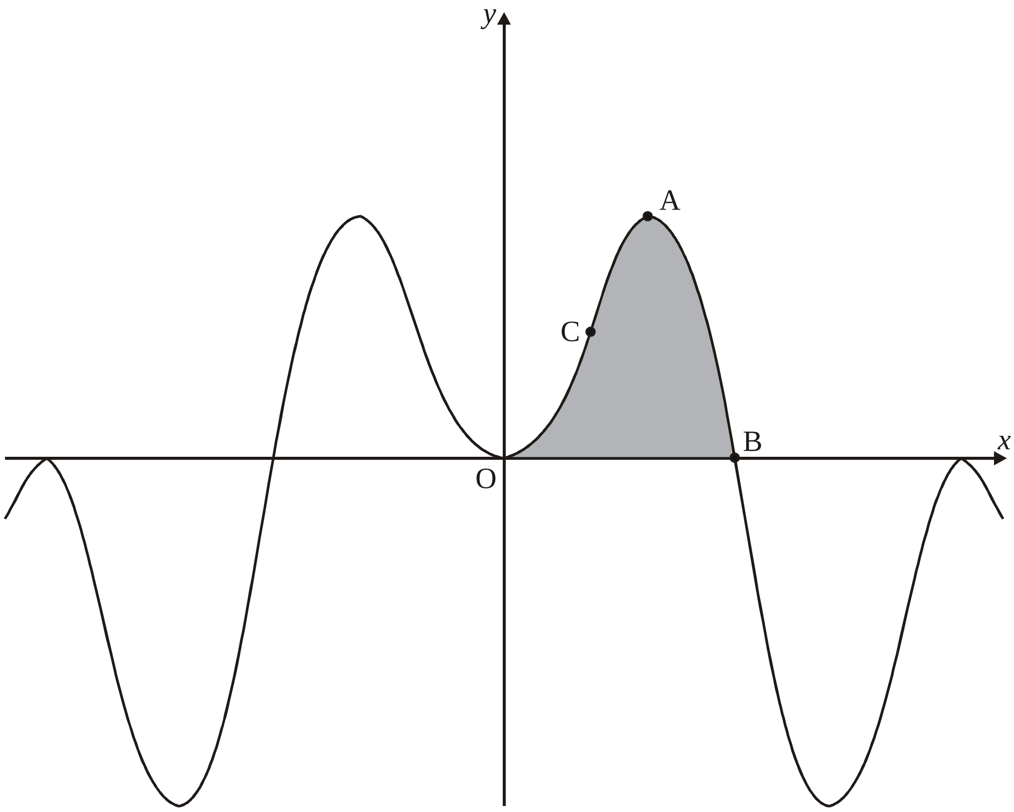
(Total 15 marks)

94.) **In this part of the question, radians are used throughout.**

The function f is given by

$$f(x) = (\sin x)^2 \cos x.$$

The following diagram shows part of the graph of $y = f(x)$.



The point A is a maximum point, the point B lies on the x-axis, and the point C is a point of inflexion.

(a) Give the period of f.

(1)

(b) From consideration of the graph of $y = f(x)$, find **to an accuracy of one significant figure** the range of f.

(1)

(c) (i) Find $f'(x)$.

(ii) Hence show that at the point A, $\cos x = \sqrt{\frac{1}{3}}$.

(iii) Find the exact maximum value.

(9)

(d) Find the exact value of the x-coordinate at the point B.

(1)

(e) (i) Find $\int f(x)\,dx$.

(ii) Find the area of the shaded region in the diagram.

(4)

(f) Given that $f''(x) = 9(\cos x)^3 - 7\cos x$, find the x-coordinate at the point C.

(4)
(Total 20 marks)

95.) Differentiate with respect to x

(a) $\sqrt{3-4x}$

(b) $e^{\sin x}$

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

96.) The function f is such that $f''(x) = 2x - 2$.

When the graph of f is drawn, it has a minimum point at (3, –7).

(a) Show that $f'(x) = x^2 - 2x - 3$ and hence find $f(x)$.

(6)

(b) Find $f(0)$, $f(-1)$ and $f'(-1)$.

(3)

(c) Hence sketch the graph of f, labelling it with the information obtained in part (b).

(4)

(**Note:** It is **not** necessary to find the coordinates of the points where the graph cuts the x-axis.)

(Total 13 marks)

97.) Differentiate with respect to x:

(a) $(x^2 + 1)^2$.

(b) 1n(3x – 1).

Working:

Answers:

(a) ....

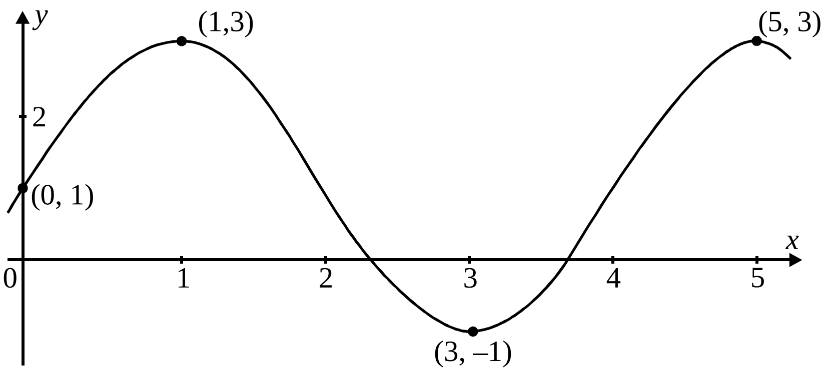
(b) ....

(Total 4 marks)

98.) The diagram shows the graph of the function f given by

$$f(x) = A \sin\left(\frac{\pi}{2}x\right) + B,$$

for $0 \le x \le 5$, where A and B are constants, and x is measured in radians.



The graph includes the points (1, 3) and (5, 3), which are maximum points of the graph.

(a) Write down the values of $f(1)$ and $f(5)$.

(2)

(b) Show that the period of f is 4.

(2)

The point (3, –1) is a minimum point of the graph.

(c) Show that A = 2, and find the value of B.

(5)

(d) Show that $f'(x) = \pi \cos\left(\frac{\pi}{2}x\right)$.

(4)

The line $y = k - \pi x$ is a tangent line to the graph for $0 \le x \le 5$.

(e) Find

(i) the point where this tangent meets the curve;

(ii) the value of k.

(6)

(f) Solve the equation $f(x) = 2$ for $0 \leq x \leq 5$.

(5)
(Total 24 marks)

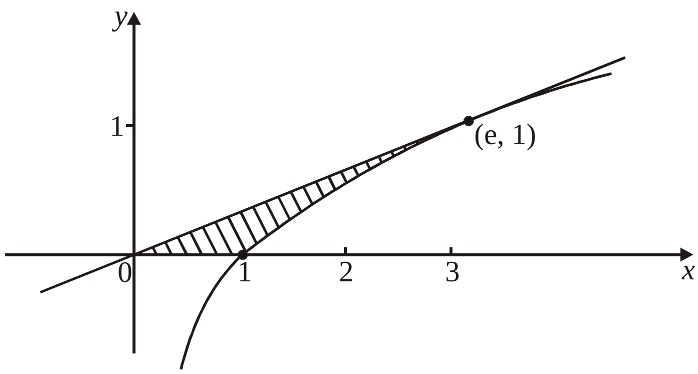
99.) (a) Find the equation of the tangent line to the curve $y = \ln x$ at the point (e, 1), and verify that the origin is on this line.

(4)

(b) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x \ln x - x) = \ln x$.

(2)

(c) The diagram shows the region enclosed by the curve $y = \ln x$, the tangent line in part (a), and the line $y = 0$.



Use the result of part (b) to show that the area of this region is $\frac{1}{2}e - 1$.

(4)
(Total 10 marks)

100.) A curve has equation $y = x(x - 4)^2$.

(a) For this curve find

(i) the x-intercepts;

(ii) the coordinates of the maximum point;

(iii) the coordinates of the point of inflexion.

(9)

(b) Use your answers to part (a) to sketch a graph of the curve for $0 \leq x \leq 4$, clearly indicating the features you have found in part (a).

(3)

(c) (i) On your sketch indicate by shading the region whose area is given by the following integral:

$$\int_0^4 x(x-4)^2 \, \mathrm{d}x.$$

(ii) Explain, using your answer to part (a), why the value of this integral is greater than 0 but less than 40.

(3)
(Total 15 marks)