1.) (a) $\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}, \frac{\mathrm{d}}{\mathrm{d}x}x^2 = 2x$ (seen anywhere) A1A1

attempt to substitute into the quotient rule (do **not** accept product rule) M1

e.g. $\frac{x^2\left(\frac{1}{x}\right) - 2x\ln x}{x^4}$

correct manipulation that clearly leads to result A1

e.g. $\frac{x - 2x\ln x}{x^4}, \frac{x(1-2\ln x)}{x^4}, \frac{x}{x^4} - \frac{2x\ln x}{x^4}$

$g'(x) = \frac{1-2\ln x}{x^3}$ AG N04

(b) evidence of setting the derivative equal to zero (M1)

e.g. $g'(x) = 0$, $1 - 2\ln x = 0$

$\ln x = \frac{1}{2}$ A1

$x = \mathrm{e}^{\frac{1}{2}}$ A1 N23

[7]

2.) (a) $v = 1$ A1 N1 1

(b) (i) $\frac{\mathrm{d}}{\mathrm{d}t}(2t) = 2$ A1

$\frac{\mathrm{d}}{\mathrm{d}t}(\cos 2t) = -2\sin 2t$ A1A1

Note: *Award A1 for coefficient 2 and A1 for –sin 2t.*

evidence of considering acceleration = 0 (M1)

e.g. $\frac{\mathrm{d}v}{\mathrm{d}t} = 0, 2 - 2\sin 2t = 0$

correct manipulation A1

e.g. $\sin 2k = 1$, $\sin 2t = 1$

$2k = \frac{\pi}{2}\left(\text{accept } 2t = \frac{\pi}{2}\right)$ A1

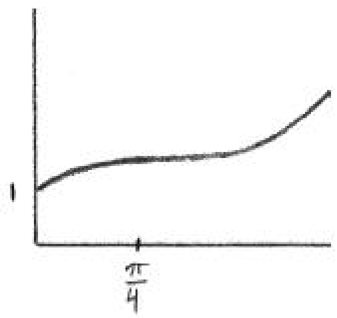
$k = \frac{\pi}{4}$ AG N0

(ii) attempt to substitute $t = \frac{\pi}{4}$ into v (M1)

e.g. $2\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{4}\right)$

$v = \frac{\text{®}}{2}$ A1 N28

(c)



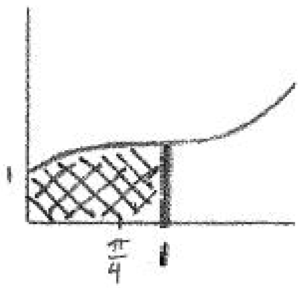
A1A1A2 N44

Notes: *Award A1 for y-intercept at (0, 1), A1 for curve having zero gradient at* $t = \frac{\pi}{4}$, *A2 for shape that is concave down to the left of* $\frac{\pi}{4}$ ***and*** *concave up to the right of* $\frac{\pi}{4}$. *If a correct curve is drawn without indicating* $t = \frac{\pi}{4}$, *do not award the second A1 for the zero gradient, but award the final A2 if appropriate. Sketch need not be drawn to scale. Only essential features need to be clear.*

(d) (i) correct expression A2

e.g. $\int_0^1 (2t + \cos 2t)\,dt, \left[t^2 + \frac{\sin 2t}{2}\right]_0^1, 1 + \frac{\sin 2}{2}, \int_0^1 v\,dt$

(ii)



A1 3

Note: *The line at t = 1 needs to be clearly after* $t = \frac{\pi}{4}$.

[16]

3.) (a) $f(1) = 2$ (A1)

$f'(x) = 4x$ A1

evidence of finding the gradient of f at $x = 1$ M1

e.g. substituting $x = 1$ into $f'(x)$

finding gradient of f at $x = 1$ A1

e.g. $f'(1) = 4$

evidence of finding equation of the line M1

e.g. $y - 2 = 4(x - 1)$, $2 = 4(1) + b$

$y = 4x - 2$ AG N05

(b) appropriate approach (M1)

e.g. $4x - 2 = 0$

$x = \frac{1}{2}$ A1 N2 2

(c) (i) bottom limit $x = 0$ (seen anywhere) (A1)

approach involving subtraction of integrals/areas (M1)

e.g. $\int f(x)$ – area of triangle, $\int f - \int l$

correct expression A2 N4

e.g. $\int_0^1 2x^2 \mathrm{d}x - \int_{0.5}^1 (4x-2)\mathrm{d}x, \int_0^1 f(x)\mathrm{d}x - \frac{1}{2}, \int_0^{0.5} 2x^2 \mathrm{d}x + \int_{0.5}^1 f(x) - (4x-2)\mathrm{d}x$

(ii) **METHOD 1 (using only integrals)**

correct integration (A1)(A1)(A1)

$\int 2x^2 \mathrm{d}x = \frac{2x^3}{3}, \int (4x-2)\mathrm{d}x = 2x^2 - 2x$

substitution of limits (M1)

e.g. $\frac{1}{12} + \frac{2}{3} - 2 + 2 - \left(\frac{1}{12} - \frac{1}{2} + 1\right)$

area = $\frac{1}{6}$ A1 N4

METHOD 2 (using integral and triangle)

area of triangle $= \frac{1}{2}$ (A1)

correct integration (A1)

$\int 2x^2 \mathrm{d}x = \frac{2x^3}{3}$

substitution of limits (M1)

e.g. $\frac{2}{3}(1)^3 - \frac{2}{3}(0)^3, \frac{2}{3} - 0$

correct simplification (A1)

e.g. $\frac{2}{3} - \frac{1}{2}$

area = $\frac{1}{6}$ A1 N4 9

[16]

4.) (a) $f(x) = -10(x + 4)(x - 6)$ A1A1 N2 2

(b) **METHOD 1**

attempting to find the x-coordinate of maximum point (M1)

e.g. averaging the x-intercepts, sketch, $y' = 0$, axis of symmetry attempting to find the y-coordinate of maximum point (M1)

e.g. $k = -10(1+4)(1-6)$

$f(x) = -10(x-1)^2 + 250$ A1A1 N44

METHOD 2

attempt to expand $f(x)$ (M1)

e.g. $-10(x^2 - 2x - 24)$

attempt to complete the square (M1)

e.g. $-10((x-1)^2 - 1 - 24)$

$f(x) = -10(x-1)^2 + 250$ A1A1 N44

(c) attempt to simplify (M1)

e.g. distributive property, $-10(x-1)(x-1) + 250$

correct simplification A1

e.g. $-10(x^2 - 6x + 4x - 24)$, $-10(x^2 - 2x + 1) + 250$

$f(x) = 240 + 20x - 10x^2$ AG N02

(d) (i) valid approach (M1)

e.g. vertex of parabola, $v'(t) = 0$

$t = 1$ A1 N2

(ii) recognizing $a(t) = v'(t)$ (M1)

$a(t) = 20 - 20t$ A1A1

speed is zero $\Rightarrow$ $t = 6$ (A1)

$a(6) = -100$ (m s^{-2}) A1 N37

[15]

5.) (a) B, D A1A1 N2 2

(b) (i) $f'(x) = -2xe^{-x^2}$ A1A1 N2

Note: *Award A1 for* e^{-x^2} *and A1 for* $-2x$.

(ii) finding the derivative of $-2x$, *i.e.* -2 (A1)

evidence of choosing the product rule (M1)

e.g. $-2e^{-x^2} - 2x \times -2xe^{-x^2}$

$-2e^{-x^2} + 4x^2e^{-x^2}$ A1

$f''(x) = (4x^2 - 2)e^{-x^2}$ AG N05

(c) valid reasoning R1

e.g. $f''(x) = 0$

attempting to solve the equation (M1)

e.g. $(4x^2 - 2) = 0$, sketch of $f''(x)$

$p = 0.707\left(=\frac{1}{\sqrt{2}}\right)$, $q = -0.707\ \left(=-\frac{1}{\sqrt{2}}\right)$ A1A1 N34

(d) evidence of using second derivative to test values on either side of POI M1

e.g. finding values, reference to graph of f', sign table

correct working A1A1

e.g. finding any two correct values either side of POI,

checking sign of f'' on either side of POI

reference to sign change of $f''(x)$ R1 N04

[15]

6.) (a) evidence of finding height, h (A1)

e.g. $\sin\theta = \frac{h}{2}$, $2\sin\theta$

evidence of finding base of triangle, b (A1)

e.g. $\cos\theta = \frac{b}{2}$, $2\cos\theta$

attempt to substitute valid values into a formula for the area of the window (M1)

e.g. two triangles plus rectangle, trapezium area formula

correct expression (must be in terms of θ) A1

e.g. $2\left(\frac{1}{2}\times 2\cos\theta \times 2\sin\theta\right) + 2\times 2\sin\theta$, $\frac{1}{2}(2\sin\theta)(2+2+4\cos\theta)$

attempt to replace $2\sin\theta\cos\theta$ by $\sin 2\theta$ M1

e.g. $4\sin\theta + 2(2\sin\theta\cos\theta)$

$y = 4\sin\theta + 2\sin 2\theta$ AG N05

(b) correct equation A1

e.g. $y = 5$, $4\sin\theta + 2\sin 2\theta = 5$

evidence of attempt to solve (M1)

e.g. a sketch, $4\sin\theta + 2\sin\theta - 5 = 0$

$\theta = 0.856\ (49.0^\circ)$, $\theta = 1.25\ (71.4^\circ)$ A1A1 N34

(c) recognition that lower area value occurs at $\theta = \frac{\pi}{2}$ (M1)

finding value of area at $\theta = \frac{\pi}{2}$ (M1)

e.g. $4\sin\left(\frac{\pi}{2}\right) + 2\sin\left(2\times\frac{\pi}{2}\right)$, draw square

$A = 4$ (A1)

recognition that maximum value of y is needed (M1)

$A = 5.19615\ldots$ (A1)

$4 < A < 5.20$ (accept $4 < A < 5.19$) A2 N57

[16]

7.) (a) $f'(x) = x^2 - 2x - 3$ A1A1A1

evidence of solving $f'(x) = 0$ (M1)

e.g. $x^2 - 2x - 3 = 0$

evidence of correct working A1

e.g. $(x + 1)(x - 3)$, $\dfrac{2 \pm \sqrt{16}}{2}$

$x = -1$ (ignore $x = 3$) (A1)

evidence of substituting **their negative** x-value into $f(x)$ (M1)

e.g. $\frac{1}{3}(-1)^3 - (-1)^2 - 3(-1)$, $-\frac{1}{3} - 1 + 3$

$y = \dfrac{5}{3}$ A1

coordinates are $\left(-1, \dfrac{5}{3}\right)$ N3

(b) (i) $(-3, -9)$ A1 N1

(ii) $(1, -4)$ A1A1N2

(iii) reflection gives $(3, 9)$ (A1)

stretch gives $\left(\dfrac{3}{2}, 9\right)$ A1A1N3

[14]

8.) (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$, $\dfrac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$ (seen anywhere) (A1)(A1)

evidence of using the quotient rule M1

correct substitution A1

e.g. $\dfrac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$, $\dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$f'(x) = \dfrac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$ A1

$f'(x) = \dfrac{-1}{\sin^2 x}$ AG N0

(b) **METHOD 1**

appropriate approach (M1)

e.g. $f'(x) = -(\sin x)^{-2}$

$f''(x) = 2(\sin^{-3} x)(\cos x)$ $\left(= \dfrac{2\cos x}{\sin^3 x}\right)$ A1A1N3

Note: *Award A1 for $2\sin^{-3} x$, A1 for $\cos x$.*

METHOD 2

derivative of $\sin^2 x = 2\sin x\cos x$ (seen anywhere) A1

evidence of choosing quotient rule (M1)

e.g. $u = -1, v = \sin^2 x$, $f'(x) = \dfrac{\sin^2 x \times 0 - (-1)2\sin x\cos x}{(\sin^2 x)^2}$

$f'(x) = \dfrac{2\sin x\cos x}{(\sin^2 x)^2}\left(= \dfrac{2\cos x}{\sin^3 x}\right)$ A1N3

(c) evidence of substituting $\dfrac{\pi}{2}$ M1

e.g. $\dfrac{-1}{\sin^2 \frac{\pi}{2}}, \dfrac{2\cos\frac{\pi}{2}}{\sin^3\frac{\pi}{2}}$

$p = -1$, $q = 0$ A1A1N1N1

(d) second derivative is zero, second derivative changes sign R1R1N2

[13]

9.) gradient of tangent = 8 (seen anywhere) (A1)

$f'(x) = 4kx^3$ (seen anywhere) A1

recognizing the gradient of the tangent is the derivative (M1)

setting the derivative equal to 8 (A1)

e.g. $4kx^3 = 8$, $kx^3 = 2$

substituting $x = 1$ (seen anywhere) (M1)

$k = 2$ A1 N4

[6]

10.) (a) substituting into the second derivative M1

e.g. $3 \times \left(-\dfrac{4}{3}\right) - 1$

$f''\left(-\dfrac{4}{3}\right) = -5$ A1

since the second derivative is negative, B is a maximum R1 N0

(b) setting $f'(x)$ equal to zero (M1)

evidence of substituting $x = 2$ $\left(\text{or } x = -\dfrac{4}{3}\right)$ (M1)

e.g. $f'(2)$

correct substitution A1

e.g. $\dfrac{3}{2}(2)^2 - 2 + p, \dfrac{3}{2}\left(-\dfrac{4}{3}\right)^2 - \left(-\dfrac{4}{3}\right) + p$

correct simplification

e.g. $6 - 2 + p = 0$, $\dfrac{8}{3} + \dfrac{4}{3} + p = 0$, $4 + p = 0$ A1

$p = -4$ AGN0

(c) evidence of integration (M1)

$f(x) = \dfrac{1}{2}x^3 - \dfrac{1}{2}x^2 - 4x + c$ A1A1A1 substituting (2, 4) or $\left(-\frac{4}{3}, \frac{358}{27}\right)$ into **their** expression (M1)

correct equation A1

e.g. $\frac{1}{2}\times 2^3 - \frac{1}{2}\times 2^2 - 4\times 2 + c = 4, \frac{1}{2}\times 8 - \frac{1}{2}\times 4 - 4\times 2 + c = 4, 4 - 2 - 8 + c = 4$

$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 10$ A1N4

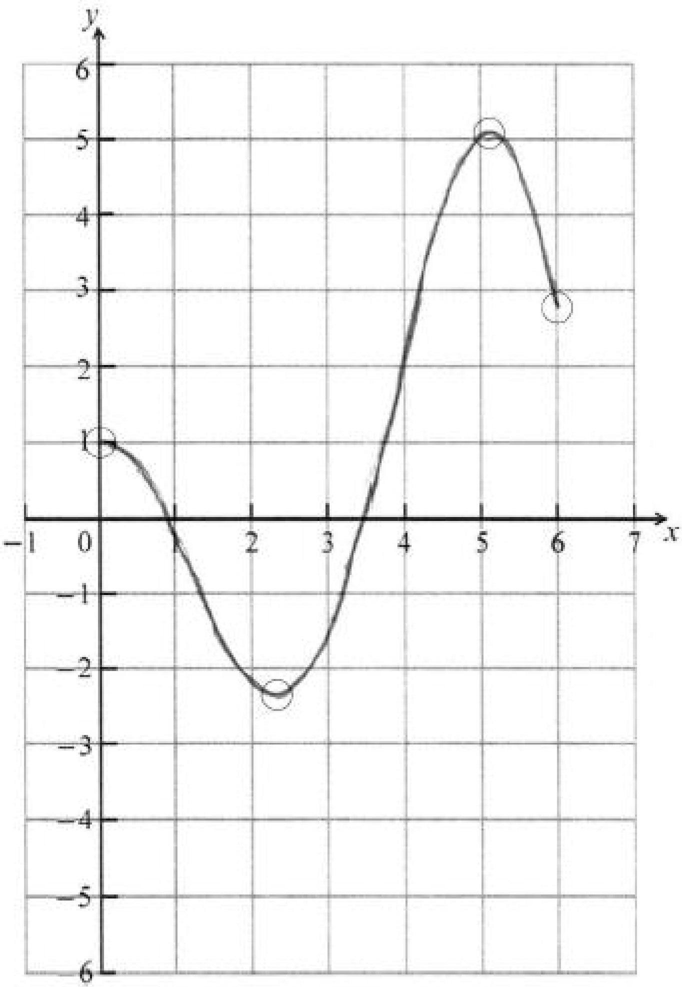
[14]

11.) (a) evidence of choosing the product rule (M1)

e.g. $x \times (-\sin x) + 1 \times \cos x$

$f'(x) = \cos x - x \sin x$ A1A1 N3

(b)



A1A1A1A1N4

Note: *Award A1 for correct domain, $0 \le x \le 6$ with endpoints in circles,*
A1 for approximately correct shape,
A1 for local minimum in circle,
A1 for local maximum in circle.

[7]

12.) evidence of integrating the acceleration function (M1) *e.g.* $\int\left(\frac{1}{t}+3\sin 2t\right)\mathrm{d}t$

correct expression $\ln t - \frac{3}{2}\cos 2t + c$ A1A1

evidence of substituting (1, 0) (M1)

e.g. $0 = \ln 1 - \frac{3}{2}\cos 2 + c$

$c = -0.624\ \left(=\frac{3}{2}\cos 2 - \ln 1 \text{ or } \frac{3}{2}\cos 2\right)$ (A1)

$v = \ln t - \frac{3}{2}\cos 2t - 0.624\ \left(=\ln t - \frac{3}{2}\cos 2t + \frac{3}{2}\cos 2 \text{ or } \ln t - \frac{3}{2}\cos 2t + \frac{3}{2}\cos 2 - \ln 1\right)$ (A1)

$v(5) = 2.24$ (accept the exact answer $\ln 5 - 1.5\cos 10 + 1.5\cos 2$) A1 N3

[7]

13.) (a) substituting (0, 13) into function M1

e.g. $13 = Ae^0 + 3$

$13 = A + 3$ A1

$A = 10$ AG N0

(b) substituting into $f(15) = 3.49$ A1

e.g. $3.49 = 10e^{15k} + 3$, $0.049 = e^{15k}$

evidence of solving equation (M1)

e.g. sketch, using ln

$k = -0.201\ \left(\text{accept } \frac{\ln 0.049}{15}\right)$ A1N2

(c) (i) $f(x) = 10e^{-0.201x} + 3$

$f'(x) = 10e^{-0.201x} \times -0.201\ (= -2.01e^{-0.201x})$ A1A1A1 N3

Note: *Award A1 for* $10e^{-0.201x}$*, A1 for* $\times -0.201$*, A1 for the derivative of 3 is zero.*

(ii) valid reason with reference to derivative R1N1

e.g. $f'(x) < 0$, derivative always negative

(iii) $y = 3$ A1N1

(d) finding limits 3.8953…, 8.6940… (seen anywhere) A1A1

evidence of integrating and subtracting functions (M1)

correct expression A1

e.g. $\int_{3.90}^{8.69} g(x) - f(x)\mathrm{d}x$, $\int_{3.90}^{8.69}[(-x^2+12x-24)-(10e^{-0.201x}+3)]\mathrm{d}x$

area = 19.5 A2N4

[16]

14.) (a) (i) substitute into gradient = $\frac{y_1 - y_2}{x_1 - x_2}$ (M1)

e.g. $\frac{f(a)-0}{a-\frac{2}{3}}$

substituting $f(a) = a^3$

e.g. $\dfrac{a^3 - 0}{a - \frac{2}{3}}$ A1

gradient = $\dfrac{a^3}{a - \frac{2}{3}}$ AGN0

(ii) correct answer A1N1

e.g. $3a^2$, $f'(a) = 3$, $f'(a) = \dfrac{a^3}{a - \frac{2}{3}}$

(iii) **METHOD 1**

evidence of approach (M1)

e.g. $f'(a)$ = gradient, $3a^2 = \dfrac{a^3}{a - \frac{2}{3}}$

simplify A1

e.g. $3a^2\left(a - \frac{2}{3}\right) = a^3$

rearrange A1

e.g. $3a^3 - 2a^2 = a^3$

evidence of solving A1

e.g. $2a^3 - 2a^2 = 2a^2(a - 1) = 0$

$a = 1$ AGN0

METHOD 2

gradient RQ = $\dfrac{-8}{-2 - \frac{2}{3}}$ A1

simplify A1

e.g. $\dfrac{-8}{-\frac{8}{3}}$, 3

evidence of approach (M1)

e.g. $f'(a)$ = gradient, $3a^2 = \dfrac{-8}{-2 - \frac{2}{3}}$, $\dfrac{a^3}{a - \frac{2}{3}} = 3$

simplify A1

e.g. $3a^2 = 3$, $a^2 = 1$

$a = 1$ AGN0

(b) approach to find area of T involving subtraction and integrals (M1)

e.g. $\int f - (3x - 2)\mathrm{d}x$, $\int_{-2}^{k} (3x - 2) - \int_{-2}^{k} x^3$, $\int (x^3 - 3x + 2)$

correct integration with correct signs A1A1A1 *e.g.* $\frac{1}{4}x^4 - \frac{3}{2}x^2 + 2x, \frac{3}{2}x^2 - 2x - \frac{1}{4}x^4$

correct limits –2 and k (seen anywhere) A1

e.g. $\int_{-2}^{k}(x^3 - 3x + 2)\mathrm{d}x, \left[\frac{1}{4}x^4 - \frac{3}{2}x^2 + 2x\right]_{-2}^{k}$

attempt to substitute k and –2 (M1)

correct substitution into **their** integral if 2 or more terms A1

e.g. $\left(\frac{1}{4}k^4 - \frac{3}{2}k^2 + 2k\right) - (4 - 6 - 4)$

setting **their** integral expression equal to $2k + 4$ (seen anywhere) (M1)

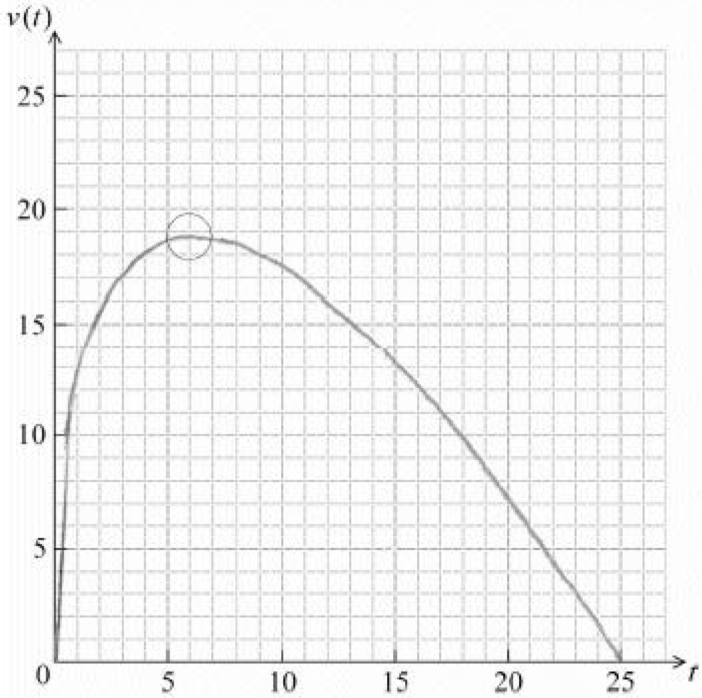
simplifying A1

e.g. $\frac{1}{4}k^4 - \frac{3}{2}k^2 + 2 = 0$

$k^4 - 6k^2 + 8 = 0$ AGN0

[16]

15.) (a)



A1A1A1 N3

Note: *Award A1 for approximately correct shape, A1 for right endpoint at (25, 0) and A1 for maximum point in circle.*

(b) (i) recognizing that d is the area under the curve (M1)

e.g. $\int v(t)$

correct expression in terms of t, with correct limits A2N3

e.g. $d = \int_0^9 (15\sqrt{t} - 3t)\mathrm{d}t, d = \int_0^9 v\mathrm{d}t$

(ii) d = 148.5 (m) (accept 149 to 3 sf) A1N1

[7]

16.) (a) valid approach R1
e.g. $f''(x) = 0$, the max and min of f' gives the points of inflexion on f

–0.114, 0.364 (accept (–0.114, 0.811) and (0.364, 2.13)) A1A1N1N1

(b) **METHOD 1**

graph of g is a quadratic function R1N1
a quadratic function does not have any points of inflexion R1N1

METHOD 2

graph of g is concave down over entire domain R1N1
therefore no change in concavity R1N1

METHOD 3

$g''(x) = -144$ R1N1
therefore no points of inflexion as $g''(x) \neq 0$ R1N1

[5]

17.) (a) evidence of valid approach (M1)
e.g. $f(x) = 0$, graph
$a = -1.73$, $b = 1.73$ $(a = -\sqrt{3}, b = \sqrt{3})$ A1A1 N3

(b) attempt to find max (M1)
e.g. setting $f'(x) = 0$, graph

$c = 1.15$ (accept (1.15, 1.13)) A1N2

(c) attempt to substitute either limits or the function into formula M1
e.g. $V = \pi\int_0^c \left[f(x)\right]^2 \mathrm{d}x, \pi\int \left[x\ln(4-x^2)\right]^2, \pi\int_0^{1.149...} y^2 \mathrm{d}x$
$V = 2.16$ A2N2

(d) valid approach recognizing 2 regions (M1)
e.g. finding 2 areas

correct working (A1)
e.g. $\int_0^{-1.73...} f(x)\mathrm{d}x + \int_0^{1.149...} f(x)\mathrm{d}x; -\int_{-1.73...}^{0} f(x)\mathrm{d}x + \int_0^{1.149...} f(x)\mathrm{d}x$

area = 2.07 (accept 2.06) A2N3

[12]

18.) evidence of choosing the product rule (M1)
$f'(x) = e^x \times (-\sin x) + \cos x \times e^x$ $(= e^x \cos x - e^x \sin x)$ A1A1

substituting π (M1)

e.g. $f'(\pi) = e^{\pi} \cos \pi - e^{\pi} \sin \pi$, $e^{\pi}(-1 - 0)$, $-e^{\pi}$

taking negative reciprocal (M1)

e.g. $-\dfrac{1}{f'(\pi)}$

gradient is $\dfrac{1}{e^{\pi}}$ A1 N3

[6]

19.) (a)

Function	Graph
displacement	A
acceleration	B

A2A2N4

(b) $t = 3$ A2N2

[6]

20.) (a) (i) $x = 3 \cos \theta$ A1 N1

(ii) $y = 3 \sin \theta$ A1N1

(b) finding area (M1)

e.g. $A = 2x \times 2y$, $A = 8 \times \dfrac{1}{2} bh$

substituting A1

e.g. $A = 4 \times 3 \sin \theta \times 3 \cos \theta$, $8 \times \dfrac{1}{2} \times 3 \cos \theta \times 3 \sin \theta$

$A = 18(2 \sin \theta \cos \theta)$ A1

$A = 18 \sin 2\theta$ AGN0

(c) (i) $\dfrac{dA}{d\_} = 36 \cos 2\theta$ A2 N2

(ii) for setting derivative equal to 0 (M1)

e.g. $36 \cos 2\theta = 0$, $\dfrac{dA}{d\_} = 0$

$2\theta = \dfrac{\pi}{2}$ (A1)

$\theta = \dfrac{\pi}{4}$ A1N2

(iii) valid reason (seen anywhere) R1

e.g. at $\dfrac{\pi}{4}$, $\dfrac{d^2A}{d\_^2} < 0$; maximum when $f''(x) < 0$

finding second derivative $\dfrac{d^2A}{d\_^2} = -72 \sin 2\theta$ A1

evidence of substituting $\dfrac{\pi}{4}$ M1 *e.g.* $-72\sin\left(2\times\frac{\pi}{4}\right), -72\sin\left(\frac{\pi}{2}\right), -72$

$\theta = \frac{\pi}{4}$ produces the maximum area AGN0

[13]

21.) (a) **METHOD 1**

evidence of substituting $-x$ for x (M1)

$f(-x) = \frac{a(-x)}{(-x)^2+1}$ A1

$f(-x) = \frac{-ax}{x^2+1}$ $(= -f(x))$ AGN0

METHOD 2

$y = -f(x)$ is reflection of $y = f(x)$ in x axis
and $y = f(-x)$ is reflection of $y = f(x)$ in y axis (M1)

sketch showing these are the same A1

$f(-x) = \frac{-ax}{x^2+1}$ $(= -f(x))$ AGN0

(b) evidence of appropriate approach (M1)
e.g. $f''(x) = 0$

to set the numerator equal to 0 (A1)
e.g. $2ax(x^2 - 3) = 0$; $(x^2 - 3) = 0$

$(0, 0)$, $\left(\sqrt{3}, \frac{a\sqrt{3}}{4}\right), \left(-\sqrt{3}, -\frac{a\sqrt{3}}{4}\right)$ (accept $x = 0$, $y = 0$ *etc.*) A1A1A1A1A1N5

(c) (i) correct expression A2

e.g. $\left[\frac{a}{2}\ln(x^2+1)\right]_3^7, \frac{a}{2}\ln 50 - \frac{a}{2}\ln 10, \frac{a}{2}(\ln 50 - \ln 10)$

area = $\frac{a}{2}\ln 5$ A1A1 N2

(ii) **METHOD 1**

recognizing that the shift does not change the area (M1)

e.g. $\int_4^8 f(x-1)dx = \int_3^7 f(x)\mathrm{d}x, \frac{a}{2}\ln 5$

recognizing that the factor of 2 doubles the area (M1)

e.g. $\int_4^8 2f(x-1)\mathrm{d}x = 2\int_4^8 f(x-1)\mathrm{d}x$ $\left(= 2\int_3^7 f(x)\mathrm{d}x\right)$

$\int_4^8 2f(x-1)\mathrm{d}x = a\ln 5$ (*i.e.* 2 × **their** answer to (c)(i)) A1N3

METHOD 2

changing variable

let $w = x - 1$, so $\frac{\mathrm{d}w}{\mathrm{d}x} = 1$

$2\int f(w)\mathrm{d}w = \frac{2a}{2}\ln(w^2+1) + c$ (M1) substituting correct limits

e.g. $\left[a\ln[(x-1)^2+1]\right]_4^8$, $\left[a\ln(w^2+1)\right]_3^7$, $a\ln 50 - a\ln 10$ (M1)

$\int_4^8 2f(x-1)\mathrm{d}x = a\ln 5$ A1N3

[16]

22.) (a) **METHOD 1**

$f'(x) = 3(x-3)^2$ A2N2

METHOD 2

attempt to expand $(x-3)^3$ (M1)

e.g. $f(x) = x^3 - 9x^2 + 27x - 27$

$f'(x) = 3x^2 - 18x + 27$ A1N2

(b) $f(3) = 0$, $f'(3) = 0$ A1N1

(c) **METHOD 1**

f' does not change sign at P R1

evidence for this R1N0

METHOD 2

f' changes sign at P so P is a maximum/minimum (*i.e.* not inflexion) R1

evidence for this R1N0

METHOD 3

finding $f(x) = \frac{1}{4}(x-3)^4 + c$ and sketching this function R1

indicating minimum at $x = 3$ R1N0

[5]

23.) **Note:** In this question, do not penalize absence of units.

(a) (i) $s = \int(40 - at)\mathrm{d}t$ (M1)

$s = 40t - \frac{1}{2}at^2 + c$ (A1)(A1)

substituting $s = 100$ when $t = 0$ ($c = 100$) (M1)

$s = 40t - \frac{1}{2}at^2 + 100$ A1 N5

(ii) $s = 40t - \frac{1}{2}at^2$ A1 N1

(b) (i) stops at station, so $v = 0$ (M1)

$t = \frac{40}{a}$ (seconds) A1 N2

(ii) evidence of choosing formula for s from (a) (ii) (M1)

substituting $t = \frac{40}{a}$ (M1)

e.g. $40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}$

setting up equation M1

e.g. $500 = s$, $500 = 40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}, 500 = \frac{1600}{a} - \frac{800}{a}$

evidence of simplification to an expression which obviously

leads to $a = \frac{8}{5}$ A1

e.g. $500a = 800$, $5 = \frac{8}{a}$, $1000a = 3200 - 1600$

$a = \frac{8}{5}$ AGN0

(c) **METHOD 1**

$v = 40 - 4t$, stops when $v = 0$

$40 - 4t = 0$ (A1)

$t = 10$ A1

substituting into expression for s M1

$s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$

$s = 200$ A1

since $200 < 500$ (allow ***FT*** on their s, if $s < 500$) R1

train stops before the station AGN0

METHOD 2

from (b) $t = \frac{40}{4} = 10$ A2

substituting into expression for s

e.g. $s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$ M1

$s = 200$ A1

since $200 < 500$, R1

train stops before the station AGN0

METHOD 3

a is deceleration A2

$4 > \frac{8}{5}$ A1

so stops in shorter time (A1)

so less distance travelled R1

so stops before station AGN0

[17]

24.) (a) attempt to expand (M1)

$(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$ A1 N2

(b) evidence of substituting x + h (M1)

correct substitution A1

e.g. $f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - 4(x+h) + 1 - (x^3 - 4x + 1)}{h}$

simplifying A1

e.g. $\dfrac{(x^3+3x^2h+3xh^2+h^2-4x-4h+1-x^3+4x-1)}{h}$

factoring out h A1

e.g. $\dfrac{h(3x^2+3xh+h^2-4)}{h}$

$f'(x) = 3x^2 - 4$ AGN0

(c) $f'(1) = -1$ (A1)

setting up an appropriate equation M1

e.g. $3x^2 - 4 = -1$

at Q, $x = -1$, $y = 4$ (Q is $(-1, 4)$) A1A1N3

(d) recognizing that f is decreasing when $f'(x) < 0$ R1

correct values for p and q (but do not accept $p = 1.15$, $q = -1.15$) A1A1N1N1

e.g. $p = -1.15$, $q = 1.15$; $\pm\dfrac{2}{\sqrt{3}}$; an interval such as $-1.15 \le x \le 1.15$

(e) $f'(x) \ge -4$, $y \ge -4$, $[-4, \infty[$ A2N2

[15]

25.) (a) gradient is 0.6 A2 N2

(b) at R, $y = 0$ (seen anywhere) A1

at $x = 2$, $y = \ln 5$ (= 1.609...) (A1)

gradient of normal = –1.6666... (A1)

evidence of finding correct equation of normal A1

e.g. $y - \ln 5 = -\dfrac{5}{3}(x-2)$, $y = -1.67x + c$

$x = 2.97$ (accept 2.96) A1

coordinates of R are (2.97, 0) N3

[7]

26.) (a) $f'(x) = 2x - \dfrac{p}{x^2}$ A1A1 N2

Note: *Award A1 for 2x, A1 for* $-\dfrac{p}{x^2}$.

(b) evidence of equating derivative to 0 (seen anywhere) (M1)

evidence of finding $f'(-2)$ (seen anywhere) (M1)

correct equation A1

e.g. $-4 - \dfrac{p}{4} = 0$, $-16 - p = 0$

$p = -16$ A1N3

[6]

27.) (a) (i) coordinates of A are (0, –2) A1A1 N2

(ii) derivative of $x^2 - 4 = 2x$ (seen anywhere) (A1)
evidence of correct approach (M1)
e.g. quotient rule, chain rule

finding $f'(x)$ A2

e.g. $f'(x) = 20 \times (-1) \times (x^2 - 4)^{-2} \times (2x)$, $\frac{(x^2-4)(0)-(20)(2x)}{(x^2-4)^2}$

substituting $x = 0$ into $f'(x)$ (do **not** accept solving $f'(x) = 0$) M1
at A $f'(x) = 0$ AGN0

(b) (i) reference to $f'(x) = 0$ (seen anywhere) (R1)
reference to $f''(0)$ is negative (seen anywhere) R1
evidence of substituting $x = 0$ into $f''(x)$ M1

finding $f''(0) = \frac{40\times 4}{(-4)^3}\left(=\frac{-5}{2}\right)$ A1

then the graph must have a local maximum AG

(ii) reference to $f''(x) = 0$ at point of inflexion, (R1)
recognizing that the second derivative is never 0 A1N2

e.g. $40(3x^2 + 4) \neq 0$, $3x^2 + 4 \neq 0$, $x^2 \neq -\frac{4}{3}$, the numerator is always positive

Note: *Do* ***not*** *accept the use of the first derivative in part (b).*

(c) correct (informal) statement, including reference to approaching $y = 3$ A1N1
e.g. getting closer to the line $y = 3$, horizontal asymptote at $y = 3$

(d) **correct** inequalities, $y \le -2$, $y > 3$, ***FT*** from (a)(i) and (c) A1A1N2

[16]

28.) (a) finding derivative (A1)

e.g. $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}, \frac{1}{2\sqrt{x}}$

correct value of derivative or its negative reciprocal (seen anywhere) A1

e.g. $\frac{1}{2\sqrt{4}}, \frac{1}{4}$

gradient of normal = $-\frac{1}{\text{gradient of tangent}}$ (seen anywhere) A1

e.g. $-\frac{1}{f'(4)} = -4,\ -2\sqrt{x}$

substituting into equation of line (for normal) M1
e.g. $y - 2 = -4(x - 4)$

$y = -4x + 18$ AGN0

(b) recognition that $y = 0$ at A (M1)
e.g. $-4x + 18 = 0$

$x = \frac{18}{4}\left(= \frac{9}{2}\right)$ A1N2

(c) splitting into two appropriate parts (areas and/or integrals) (M1)
correct expression for area of R A2N3

e.g. area of $R = \int_0^4 \sqrt{x}\mathrm{d}x + \int_4^{4.5}(-4x+18)\mathrm{d}x, \int_0^4 \sqrt{x}\mathrm{d}x + \frac{1}{2}\times 0.5 \times 2$ (triangle)

Note: *Award A1 if dx is missing.*

(d) correct expression for the volume from $x = 0$ to $x = 4$ (A1)

e.g. $V = \int_0^4 \pi\left[f(x)^2\right]\mathrm{d}x, \int_0^4 \pi\sqrt{x}^2\mathrm{d}x, \int_0^4 \pi x \mathrm{d}x$

$V = \left[\frac{1}{2}\pi x^2\right]_0^4$ A1

$V = \pi\left(\frac{1}{2}\times 16 - \frac{1}{2}\times 0\right)$ (A1)

$V = 8\pi$ A1

finding the volume from $x = 4$ to $x = 4.5$

EITHER

recognizing a cone (M1)

e.g. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(2)^2 \times \frac{1}{2}$ (A1)

$= \frac{2\pi}{3}$ A1

total volume is $8\pi + \frac{2}{3}\pi \quad \left(= \frac{26}{3}\pi\right)$ A1N4

OR

$V = \pi\int_4^{4.5}(-4x+18)^2\mathrm{d}x$ (M1)

$= \int_4^{4.5}\pi(16x^2 - 144x + 324)\mathrm{d}x$

$= \pi\left[\frac{16}{3}x^3 - 72x^2 + 324x\right]_4^{4.5}$ A1

$= \frac{2\pi}{3}$ A1

total volume is $8\pi + \frac{2}{3}\pi \quad \left(= \frac{26}{3}\pi\right)$ A1N4

[17]

29.) substituting $x = 1$, $y = 3$ into $f(x)$(M1)
$3 = p + q$ A1

finding derivative (M1)
$f'(x) = 2px + q$ A1
correct substitution, $2p + q = 8$ A1

$p = 5$, $q = -2$ A1A1 N2N2

[7]

30.) **METHOD 1**

correct expression for **second** side, using area = 525 (A1)

e.g. let AB = x, AD = $\frac{525}{x}$

attempt to set up cost function using \$3 for three sides and \$11 for one side(M1)
e.g. 3(AD + BC + CD) + 11AB

correct expression for cost A2

e.g. $\frac{525}{x}\times 3+\frac{525}{x}\times 3+11x+3x, \frac{525}{\text{AB}}\times 3+\frac{525}{\text{AB}}\times 3+11\text{AB}+3\text{AB}, \frac{3150}{x}+14x$

EITHER

sketch of cost function (M1)
identifying minimum point (A1)
e.g. marking point on graph, $x = 15$

minimum cost is 420 (dollars) A1 N4

OR

correct derivative (may be seen in equation below) (A1)

e.g. $C'(x) = \frac{-1575}{x^2}+\frac{-1575}{x^2}+14$

setting their derivative equal to 0 (seen anywhere) (M1)

e.g. $\frac{-3150}{x^2}+14=0$

minimum cost is 420 (dollars) A1 N4

METHOD 2

correct expression for **second** side, using area = 525 (A1)

e.g. let AD = x, AB = $\frac{525}{x}$

attempt to set up cost function using \$3 for three sides and \$11 for one side(M1)
e.g. 3(AD + BC + CD) + 11AB

correct expression for cost A2

e.g. $3\left(x+x+\frac{525}{x}\right)+\frac{525}{x}\times 11, 3\left(\text{AD}+\text{AD}+\frac{525}{\text{AD}}\right)+\frac{525}{\text{AD}}\times 11, 6x+\frac{7350}{x}$

EITHER

sketch of cost function (M1)
identifying minimum point (A1)
e.g. marking point on graph, $x = 35$

minimum cost is 420 (dollars) A1 N4

OR

correct derivative (may be seen in equation below) (A1)

e.g. $C'(x) = 6 - \frac{7350}{x^2}$

setting their derivative equal to 0 (seen anywhere) (M1)

e.g. $6 - \frac{7350}{x^2} = 0$

minimum cost is 420 (dollars) A1 N4

[7]

31.) evidence of anti-differentiation (M1)

e.g. $s = \int \left(6e^{3x} + 4\right) dx$

$s = 2e^{3t} + 4t + C$ A2A1

substituting $t = 0$, (M1)

$7 = 2 + C$ A1

$C = 5$

$s = 2e^{3t} + 4t + 5$ A1 N3

[7]

32.) (a) $f'(x) = x^2 + 4x - 5$ A1A1A1 N3

(b) evidence of attempting to solve $f'(x) = 0$ (M1)

evidence of correct working A1

e.g. $(x + 5)(x - 1)$, $\frac{-4 \pm \sqrt{16+20}}{2}$, sketch

$x = -5$, $x = 1$ (A1)

so $x = -5$ A1 N2

(c) **METHOD 1**

$f''(x) = 2x + 4$ (may be seen later) A1

evidence of setting second derivative = 0 (M1)

e.g. $2x + 4 = 0$

$x = -2$ A1 N2

METHOD 2

evidence of use of symmetry (M1)

e.g. midpoint of max/min, reference to shape of cubic

correct calculation A1

e.g. $\frac{-5+1}{2}$,

$x = -2$ A1 N2

(d) attempting to find the value of the derivative when $x = 3$ (M1)

$f'(3) = 16$ A1

valid approach to finding the equation of a line M1

e.g. $y - 12 = 16(x - 3)$, $12 = 16 \times 3 + b$

$y = 16x - 36$ A1 N2

[14]

33.) (a) (i) range of f is $[-1, 1]$, $(-1 \le f(x) \le 1)$ A2 N2

(ii) $\sin^3 x = 1 \Rightarrow \sin x = 1$ A1

justification for one solution on $[0, 2\pi]$ R1

e.g. $x = \frac{\pi}{2}$, unit circle, sketch of $\sin x$

1 solution (seen anywhere) A1 N1

(b) $f'(x) = 3\sin^2 x \cos x$ A2 N2

(c) using $V = \int_a^b \pi y^2 \mathrm{d}x$ (M1)

$$V = \int_0^{\frac{\pi}{2}} \pi \left(\sqrt{3} \sin x \cos^{\frac{1}{2}} x \right)^2 \mathrm{d}x$$ (A1)

$$= \pi \int_0^{\frac{\pi}{2}} 3 \sin^2 x \cos x \,\mathrm{d}x$$ A1

$$V = \pi \left[\sin^3 x\right]_0^{\frac{\pi}{2}} \left(= \pi \left(\sin^3 \left(\frac{\pi}{2} \right) - \sin^3 0 \right) \right)$$ A2

evidence of using $\sin \frac{\pi}{2} = 1$ and $\sin 0 = 0$ (A1)

e.g. $\pi(1 - 0)$

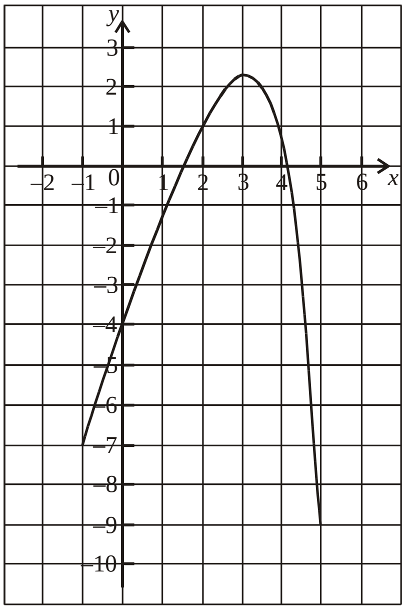
$V = \pi$ A1 N1

[14]

34.) (a) intercepts when $f(x) = 0$ (M1)

(1.54, 0) (4.13, 0) (accept $x = 1.54$ $x = 4.13$) A1A1 N3

(b)



A1A1A1 N3

Note: *Award A1 for passing through approximately (0, - 4), A1 for correct shape, A1 for a range of approximately - 9 to 2.3.*

(c) gradient is 2 A1 N1

[7]

35.) (a) evidence of using the product rule M1

$f'(x) = e^x(1 - x^2) + e^x(-2x)$ A1A1

Note: *Award A1 for $e^x(1 - x^2)$, A1 for $e^x(-2x)$.*

$f'(x) = e^x(1 - 2x - x^2)$ AG N0

(b) $y = 0$ A1 N1

(c) at the local maximum or minimum point

$f'(x) = 0$ $(e^x(1 - 2x - x^2) = 0)$ (M1)

$\Rightarrow 1 - 2x - x^2 = 0$ (M1)

$r = -2.41$ $s = 0.414$ A1A1 N2N2

(d) $f'(0) = 1$ A1

gradient of the normal $= -1$ A1

evidence of substituting into an equation for a straight line (M1)

correct substitution A1

e.g. $y - 1 = -1(x - 0)$, $y - 1 = -x$, $y = -x + 1$

$x + y = 1$ AG N0

(e) (i)intersection points at $x = 0$ and $x = 1$ (may be seen as the limits) (A1)

approach involving subtraction and integrals (M1)

fully correct expression A2 N4

e.g. $\int_0^1 \left(e^x\left(1-x^2\right)-\left(1-x\right)\right)dx, \int_0^1 f\left(x\right)dx - \int_0^1 \left(1-x\right)dx$

(ii) area $R = 0.5$ A1 N1

[17]

36.) (a) (i) $f'(x) = 0$ A1 N1

(ii) **METHOD 1**

$f'(x) < 0$ to the left of C, $f'(x) > 0$ to the right of C R1R1 N2

METHOD 2

$f''(x) > 0$ R2 N2

(b) A A1 N1

(c) **METHOD 1**

$f''(x) = 0$ R2

Discussion of sign change of $f''(x)$ R1
e.g. $f''(x) < 0$ to the left of B and $f''(x) > 0$ to the right of B; $f''(x)$ changes sign either side of B
B is a point of inflexion AG N0

METHOD 2

B is a minimum on the graph of the derivative f' R2
Discussion of sign change of $f''(x)$ R1
e.g. $f''(x) < 0$ to the left of B and $f''(x) > 0$ to the right of B; $f''(x)$ changes sign either side of B
B is a point of inflexion AG N0

[7]

37.) (a) substituting $t = 0$ (M1)
e.g. $a(0) = 0 + \cos 0$
$a(0) = 1$ A1 N2

(b) evidence of integrating the acceleration function (M1)
e.g. $\int (2t + \cos t)dt$

correct expression $t^2 + \sin t + c$ A1A1

Note: *If "+c" is omitted, award no further marks.*

evidence of substituting (0, 2) into indefinite integral (M1)
e.g. $2 = 0 + \sin 0 + c$, $c = 2$
$v(t) = t^2 + \sin t + 2$ A1 N3

(c) $\int (t^2 + \sin t + 2)dt = \frac{t^3}{3} - \cos t + 2t$ A1A1A1

Note: *Award A1 for each correct term.*

evidence of using $v(3) – v(0)$ (M1)
correct substitution A1
e.g. $(9 – \cos 3 + 6) – (0 – \cos 0 + 0)$, $(15 – \cos 3) – (–1)$
$16 – \cos 3$ (accept $p = 16$, $q = –1$) A1A1 N3

(d) reference to motion, reference to first 3 seconds R1R1 N2
e.g. displacement in 3 seconds, distance travelled in 3 seconds

[16]

38.) (a) correctly finding the derivative of e^{2x}, *i.e.* $2e^{2x}$ A1
correctly finding the derivative of $\cos x$, *i.e.* $-\sin x$ A1
evidence of using the product rule, seen anywhere M1
e.g. $f'(x) = 2e^{2x}\cos x - e^{2x}\sin x$
$f'(x) = e^{2x}(2\cos x - \sin x)$ AG N0

(b) evidence of finding $f(0) = 1$, seen anywhere A1

attempt to find the gradient of f (M1)
e.g. substituting $x = 0$ into $f'(x)$

value of the gradient of f A1
e.g. $f'(0) = 2$, equation of tangent is $y = 2x + 1$

gradient of normal = $-\frac{1}{2}$ (A1)

$y - 1 = -\frac{1}{2}x$ $\left(y = -\frac{1}{2}x + 1\right)$ A1 N3

(c) (i) evidence of equating correct functions M1
e.g. $e^{2x}\cos x = -\frac{1}{2}x + 1$, sketch showing intersection of graphs
$x = 1.56$ A1 N1

(ii) evidence of approach involving subtraction of integrals/areas (M1)
e.g. $\int [f(x) - g(x)]dx, \int f(x)dx$ – area under trapezium
fully correct integral expression A2
e.g. $\int_0^{1.56}\left[e^{2x}\cos x - \left(-\frac{1}{2}x + 1\right)\right]dx, \int_0^{1.56} e^{2x}\cos x dx - 0.951...$
area = 3.28 A1 N2

[14]

39.) Attempt to differentiate (M1)
$\frac{dy}{dx} = 2e^{2x}$ A1
At $x = 1$ $\frac{dy}{dx} = 2e^2$ A1
$y = e^2$ A1
Equation of tangent is $y - e^2 = 2e^2(x - 1)$ ($y = 2e^2x - e^2$)M1A1 N2

[6]

40.) (a) $a = \frac{dv}{dt}$ (M1)
$= -10$ (m s^{-2}) A1 N2

(b) $s = \int v\, dt$ (M1)
$= 50t - 5t^2 + c$ A1
$40 = 50(0) - 5(0) + c \Rightarrow c = 40$ A1
$s = 50t - 5t^2 + 40$ A1N2

Note: *Award* ***(M1)*** *and the first* ***A1*** *in part (b) if c is missing, but do* ***not*** *award the final 2 marks.*

[6]

41.) (a) Attempt to differentiate (M1)
$g'(x) = 3x^2 - 6x - 9$ A1A1A1

for setting derivative equal to zero M1
$3x^2 - 6x - 9 = 0$

Solving A1
e.g. $3(x - 3)(x + 1) = 0$
$x = 3$ $x = -1$ A1A1N3

(b) **METHOD 1**
$g'(x < -1)$ is positive, $g'(x > -1)$ is negative A1A1
$g'(x < 3)$ is negative, $g'(x > 3)$ is positive A1A1

min when $x = 3$, max when $x = -1$ A1A1N2

METHOD 2
Evidence of using second derivative (M1)
$g''(x) = 6x - 6$ A1
$g''(3) = 12$ (or positive), $g''(-1) = -12$ (or negative) A1A1

min when $x = 3$, max when $x = -1$ A1A1N2

[14]

42.) (a) Curve intersects y-axis when $x = 0$ (A1)
Gradient of tangent at y-intercept = 2 A1

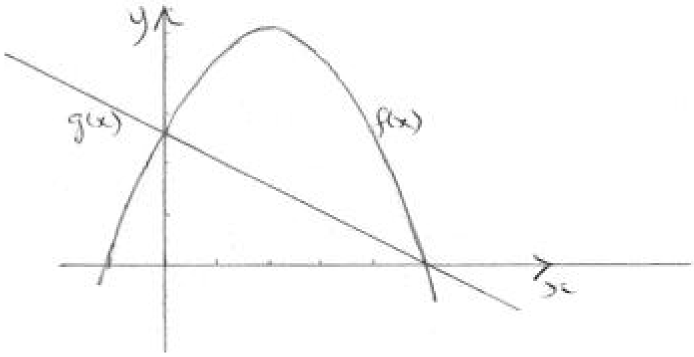
$\Rightarrow$ gradient of $N = -\frac{1}{2}$ $(= -0.5)$ A1

Finding y-intercept, 2.5 A1
Therefore, equation of N is $y = -0.5x + 2.5$ AG N0

(b) N intersects curve when $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$ A1
Solving equation (M1)
e.g. sketch, factorising
$\Rightarrow$ $x = 0$ or $x = 5$ A1

Other point when $x = 5$ (R1)
$x = 5 \Rightarrow y = 0$ (so other point (5, 0)) A1N2

(c)



Using appropriate method, with subtraction/correct expression, **correct** limits M1A1

e.g. $\int_0^5 f(x)\mathrm{d}x - \int_0^5 g(x)\mathrm{d}x, \int_0^5 (-0.5x^2 + 2.5x)\mathrm{d}x$

Area = 10.4 A2N2

[13]

43.) Evidence of integration (M1)

$s = -0.5\,\mathrm{e}^{-2t} + 6t^2 + c$ A1A1

Substituting $t = 0$, $s = 2$ (M1)

eg $2 = -0.5 + c$

$c = 2.5$ (A1)

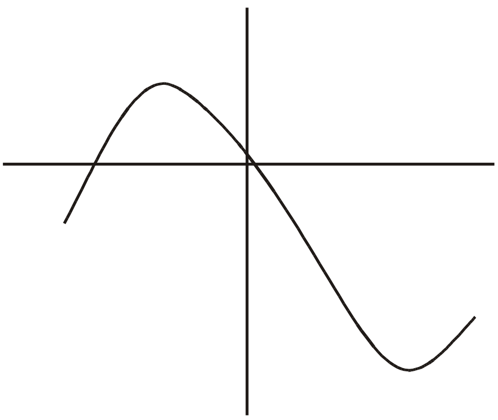
$s = -0.5\,\mathrm{e}^{-2t} + 6t^2 + 2.5$ A1 N4

[6]

44.) (a) **EITHER**

Recognizing that tangents parallel to the x-axis mean maximum and minimum (may be seen on sketch) R1

Sketch of graph of f M1



OR

Evidence of using $f'(x) = 0$ M1

Finding $f'(x) = 3x^2 - 6x - 24$ A1

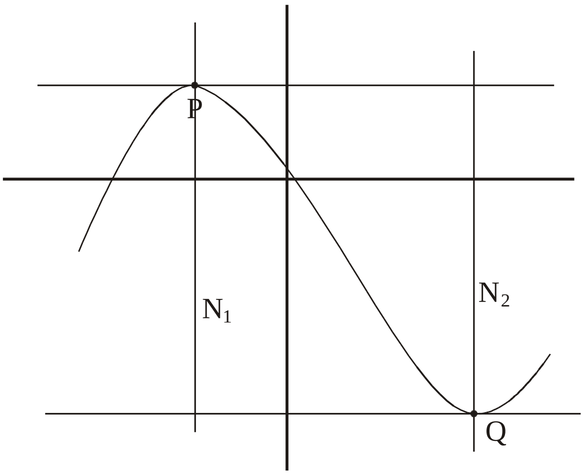
$3x^2 - 6x - 24 = 0$

Solutions $x = -2$ or $x = 4$

THEN

Coordinates are P(−2, 29) and Q(4, −79) A1A1 N1N1

(b)



(i) (4, 29) A1 N1

(ii) (–2, –79) A1 N1

[6]

45.) (a) Evidence of using $a = \frac{dv}{dt}$ (M1)

eg $3e^{3t-2}$

$a(1) = 3e$ (= 8.15) A1 N2

(b) Attempt to solve $e^{3t-2} = 22.3$ (M1)

eg $(3t – 2)$ (ln e) = ln 22.3, sketch

$t = 1.70$ A1 N2

(c) Evidence of using $s = \int v\,dt$ (limits not required) M1

$e.g.\ \int e^{3t-2}\,dt, \frac{1}{3}\left[e^{3t-2}\right]_0^1$

$\frac{1}{3}\left(e^1 - e^{-2}\right)\ \left[= \frac{1}{3}\left(e - e^{-2}\right) = 0.861\right]$ A1 N1

[6]

46.) (a) **METHOD 1**

$f'(x) = –6 \sin 2x + 2 \sin x \cos x$ A1A1A1

$= –6 \sin 2x + \sin 2x$ A1

$= –5 \sin 2x$ AG N0

METHOD 2

$\sin^2 x = \frac{1 - \cos 2x}{2}$ (A1)

$f(x) = 3\cos 2x + \frac{1}{2} - \frac{1}{2}\cos 2x$ A1

$f(x) = \frac{5}{2}\cos 2x + \frac{1}{2}$ A1

$f'(x) = 2\left(\frac{5}{2}\right)(-\sin 2x)$ A1

$f'(x) = -5\sin 2x$ AG N0

(b) $k = \frac{\pi}{2}$ $(=1.57)$ A2 N2

[6]

47.) (a) (i) $p = 1$, $q = 5$ (or $p = 5$, $q = 1$) A1A1 N2

(ii) $x = 3$ (must be an equation) A1 N1

(b) $y = (x - 1)(x - 5)$

$= x^2 - 6x + 5$ (A1)

$= (x - 3)^2 - 4$ (accept $h = 3$, $k = -4$) A1A1 N3

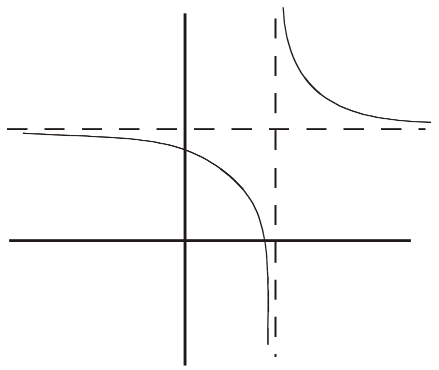
(c) $\frac{dy}{dx} = 2(x-3)$ $(= 2x - 6)$ A1A1 N2

(d) When $x = 0$, $\frac{dy}{dx} = -6$ (A1)

$y - 5 = -6(x - 0)$ ($y = -6x + 5$ or equivalent) A1 N2

[10]

48.) (a)



A1A1A1 N3

Notes**:* *Award A1 for* ***both *asymptotes shown.*
The asymptotes need not be labelled.
Award A1 for the left branch in ***approximately*** *correct position,*
A1 for the right branch in ***approximately*** *correct position.*

(b) (i) $y = 3, x = \frac{5}{2}$ (must be equations) A1A1 N2

(ii) $x = \frac{14}{6}$ $\left(\frac{7}{3} \text{ or } 2.33, \text{ also accept} \left(\frac{14}{6}, 0\right)\right)$ A1 N1

(iii) $y = \frac{14}{6}$ $(y = 2.8)$ $\left(\text{accept} \left(0, \frac{14}{5}\right) \text{or} (0, 2.8)\right)$ A1 N1

(c) (i) $\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx = 9x +$

$3\ln(2x-5) - \frac{1}{2(2x-5)} + C$ A1A1A1

A1A1 N5

(ii) Evidence of using $V = \int_a^b \pi y^2 dx$ (M1)

Correct expression A1

eg $\int_3^a \pi \left(3 + \frac{1}{2x-5}\right)^2 dx, \pi \int_3^a \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx,$

$\left[9x + 3\ln(2x-5) - \frac{1}{2(2x-5)}\right]_3^a$

Substituting $\left(9a + 3\ln(2a-5) - \frac{1}{2(2a-5)}\right) - \left(27 + 3\ln 1 - \frac{1}{2}\right)$ A1

Setting up an equation (M1)

$9a - \frac{1}{2(2a-5)} - 27 + \frac{1}{2} + 3\ln(2a-5) - 3\ln 1 = \left(\frac{28}{3} + 3\ln 3\right)$

Solving gives $a = 4$ A1 N2

[17]

49.) $f'(x) = 12x^2 + 2$ A1A1

When $x = 1$, $f(1) = 6$ (seen anywhere) (A1)

When $x = 1$, f ȧ $(1) = 14$ (A1)

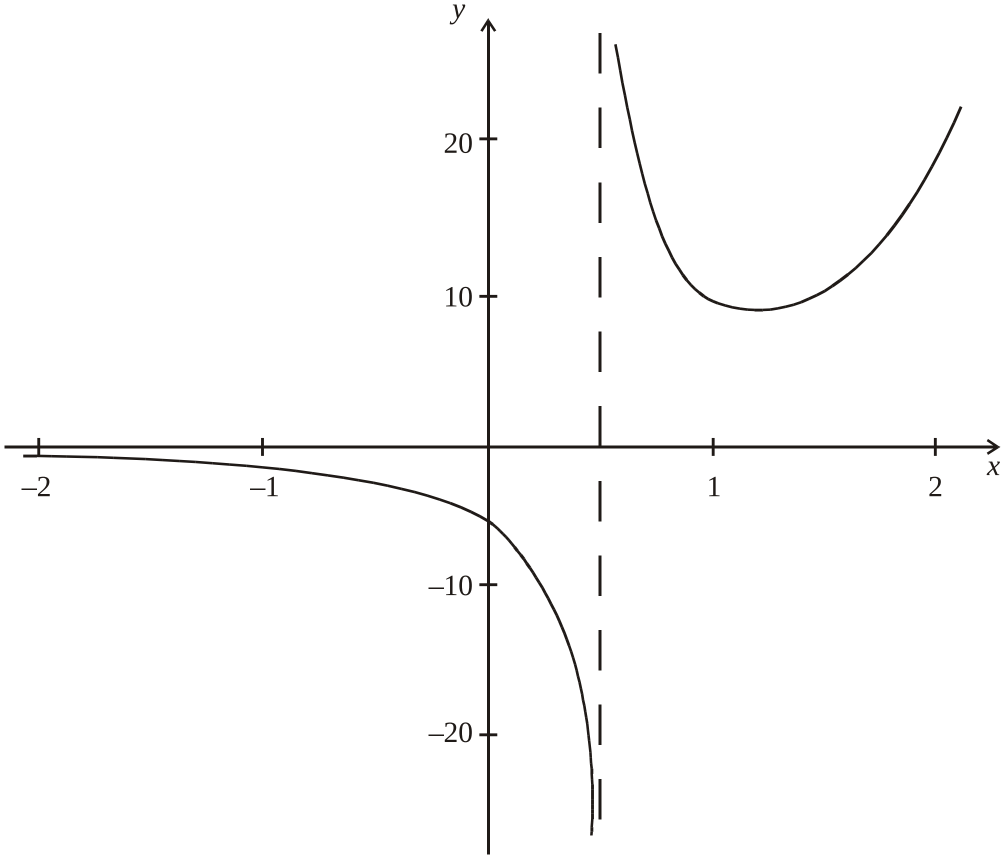
Evidence of taking the negative reciprocal (M1)

eg $\frac{-1}{14}x, \frac{1}{-14}, -0.0714$

Equation is $y - 6 = -\frac{1}{14}(x-1)$ $\left(y = -\frac{1}{14}x + \frac{85}{14}, y = -0.0714x + 6.07\right)$ A1 N4

[6]

50.) (a)



A1A1A1 N3

Note: *Award A1 for the left branch asymptotic to the x-axis and crossing the y-axis,*

A1 for the right branch approximately the correct shape,

A1 for a vertical asymptote at approximately $x = \frac{1}{2}$.

(b) (i) $x = \frac{1}{2}$ (must be an equation) A1 N1

(ii) $\int_0^2 f(x)\,\mathrm{d}x$ A1 N1

(iii) Valid reason R1 N1

eg reference to area undefined or discontinuity

Note: *GDC reason* ***not*** *acceptable.*

(c) (i) $V = \pi \int_1^{1.5} f(x)^2\,\mathrm{d}x$ A2 N2

(ii) $V = 105$ (accept 33.3π) A2 N2

(d) $f'(x) = 2e^{2x-1} - 10(2x-1)^{-2}$ A1A1A1A1 N4

(e) (i) $x = 1.11$ (accept (1.11, 7.49)) A1 N1

(ii) $p = 0$, $q = 7.49$ (accept $0 \le k < 7.49$) A1A1 N2

[17]

51.) Finding anti-derivative of $4t^3 - 2t$ (M1)

$s = t^4 - t^2 + c$ A1A1

Substituting correctly $8 = 2^4 - 2^2 + c$ A1

Note: *Exception to the* ***FT*** *rule. Allow full* ***FT*** *on incorrect integration.*

$c = -4$ (A1)

$s = t^4 - t^2 - 4$ A1 N3

[6]

52.) (a)

Interval	g'	g''
$a < x < b$	**positive**	**positive**
$e < x < f$	**negative**	**negative**

A1A1

A1A1 N4

(b)

Conditions	Point
$g'(x) = 0$, $g''(x) < 0$	**C**
$g'(x) < 0$, $g''(x) = 0$	**D**

A1 N1

A1 N1

[6]

53.) (a) $f'(x) = 6x - 5$ A1 N1

(b) $f'(p) = 7$ (or $6p - 5 = 7$) M1

$p = 2$ A1 N1

(c) Setting $y(2) = f(2)$ (M1)

Substituting $y(2) = 7 \times 2 - 9$ ($= 5$), **and** $f(2) = 3 \times 2^2 - 5 \times 2 + k$ ($= k + 2$) A1

$k + 2 = 5$

$k = 3$ A1 N2

[6]

54.) (a) $S_{min} = 6.05$ (accept $(1, 6.05)$) A1 N1

(b) $\frac{ds}{dt} = -15 \sin 3t + 2t$ A1

$a = \frac{d^2 s}{dt^2}$ (M1)

$a = \frac{d^2 s}{dt^2} = -45 \cos 3t + 2$ (Exception to ***FT*** rule : allow ***FT*** from $\frac{ds}{dt}$) A1 N2

(c) **EITHER**

Maximum value of a when $\cos 3t$ is minimum *ie* $\cos 3t = -1$ (A1)

OR

At maximum $\frac{da}{dt} = 0$ $(135 \sin 3t = 0)$ (A1)

THEN

$t = \frac{\pi}{3}$ (accept 1.05; do **not** accept 60°) A1 N2

[6]

55.) (a) (i) $f'(x) = -\frac{3}{2}x + 1$ A1A1N2

(ii) For using the derivative to find the gradient of the tangent (M1)

$f'(2) = -2$ (A1)

Using negative reciprocal to find the gradient of the normal $\left(\frac{1}{2}\right)$ M1

$y - 3 = \frac{1}{2}(x - 2)$ $\left(\text{or } y = \frac{1}{2}x + 2\right)$ A1 N3

(iii) Equating $-\frac{3}{4}x^2 + x + 4 = \frac{1}{2}x + 2$ (or sketch of graph) M1

$3x^2 - 2x - 8 = 0$ (A1)

$(3x + 4)(x - 2) = 0$

$x = -\frac{4}{3}$ $(= -1.33)$ (accept $\left(-\frac{4}{3}, \frac{4}{3}\right)$ or $x = -\frac{4}{3}, x = 2$) A1 N2

(b) (i) Any **completely** correct expression (accept absence of dx) A2

eg $\int_{-1}^{2}\left(-\frac{3}{4}x^2 + x + 4\right)dx, \left[-\frac{1}{4}x^3 + \frac{1}{2}x^2 + 4x\right]_{-1}^{2}$ N2

(ii) Area = $\frac{45}{4}$ $(= 11.25)$ (accept 11.3) A1 N1

(iii) Attempting to **use** the formula for the volume (M1)

$$eg\ \int_{-1}^{2}\pi\left(-\frac{3}{4}x^2+x+4\right)\mathrm{d}x,\ \pi\int_{-1}^{2}\left(-\frac{3}{4}x^2+x+4\right)^2\mathrm{d}x$$ A2 N3

(c) $\int_1^k f(x)\,\mathrm{d}x=\left[-\frac{1}{4}x^3+\frac{1}{2}x^2+4x\right]_1^k$ A1A1A1

Note: *Award A1 for* $-\frac{1}{4}x^3$ *, A1 for* $\frac{1}{2}x^2$ *, A1 for 4x.*

Substituting $\left(-\frac{1}{4}k^3+\frac{1}{2}k^2+4k\right)-\left(-\frac{1}{4}+\frac{1}{2}+4\right)$ (M1)(A1)

$= -\frac{1}{4}k^3+\frac{1}{2}k^2+4k-4.25$ A1 N3

[21]

56.) $s=\int v\,\mathrm{d}t$ (M1)

$s=\frac{1}{2}\mathrm{e}^{2t-1}+c$ A1A1

Substituting $t = 0.5$

$\frac{1}{2}+c=10$

$c = 9.5$ (A1)

Substituting $t = 1$ M1

$s = \frac{1}{2}\mathrm{e}+9.5\,(=10.9 \text{ to } 3\, s.f.)$ A1 N3

[6]

57.) (a) $a=\frac{\mathrm{d}v}{\mathrm{d}t}$ (M1)

$= -10$ A1 3

(b) $s = \int vdt$ (M1)

$= 50t - 5t^2 + c$ A1

$40 = 50(0) - 5(0) + c \Rightarrow c = 40$ A1

$s = 50t - 5t^2 + 40$ A1 3

Note*: Award (M1) and the first (A1) in part (b) if c is missing, but do not award the final 2 marks.*

[6]

58.) (a) (i) $f'(x) = -x + 2$ A1

(ii) $f'(0) = 2$ A1 2

(b) Gradient of tangent at y-intercept $= f'(0) = 2$

$\Rightarrow$ gradient of normal $= \frac{1}{2}$ $(= -0.5)$ A1

Finding y-intercept is 2.5 A1

Therefore, equation of the normal is

$y - 2.5 = \sim(x - 0)$ $(y - 2.5 = -0.5x)$ M1

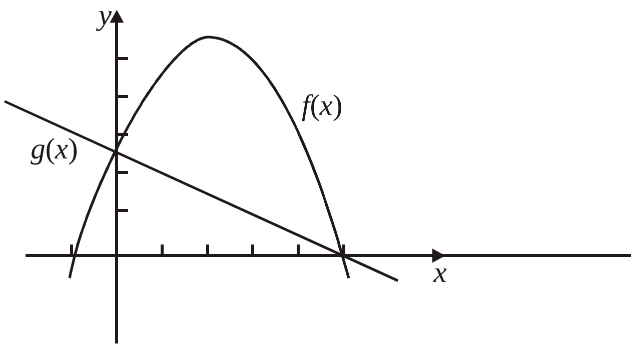
$(y = -0.5x + 2.5$ (AG) 3

(c) (i) **EITHER**

solving $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$ (M1)A1

$\Rightarrow x = 0$ or $x = 5$ A1 2

OR



M1

Curves intersect at $x = 0$, $x = 5$ (A1)

So solutions to $f(x) = g(x)$ are $x = 0$, $x = 5$ A1 2

OR

$\Rightarrow 0.5x^2 - 2.5x = 0$ (A1)

$\Rightarrow -0.5x(x - 5) = 0$ M1

$\Rightarrow x = 0$ or $x = 5$ A1 2

(ii) Curve and normal intersect when $x = 0$ or $x = 5$ (M2)

Other point is when $x = 5$

$\Rightarrow y = -0.5(5) + 2.5 = 0$ (so other point (5, 0) A1 2

(d) (i) Area $= \int_0^5 (f(x) - g(x))\mathrm{d}x \left(\text{or } \int_0^5 (-0.5x^2 + 2x + 2.5)\mathrm{d}x - \frac{1}{2} \times 5 \times 2.5\right)$

A1A1A1 3

***Note**: Award (A1) for the integral, (A1) for both correct limits on the integral, and (A1) for the difference.*

(ii) Area = Area under curve – area under line $(A = A_1 - A_2)$ (M1)

(A1) $= \frac{50}{3}, A_2 = \frac{25}{4}$

Area $= \frac{50}{3} - \frac{25}{4} = \frac{125}{12}$ (or 10.4 (3sf) A1 2

[16]

59.) (a) (i) $p = (10x + 2) - (1 + e^{2x})$ A2 2

***Note**: Award (A1) for $(l + e^{2x}) - (10x + 2)$.*

(ii) $\frac{dp}{dx} = 10 - 2e^{2x}$ A1A1

$\frac{dp}{dx} = 0 \quad (10 - 2e^{2x} = 0)$ M1

$x = \frac{\ln 5}{2} \quad (= 0.805)$ A1 4

(b) (i) **METHOD 1**

$x = 1 + e^{2y}$ M1

$\ln(x - 1) = 2y$ A1

$f^{-1}(x) = \frac{\ln(x-1)}{2} \left(\text{Allow } y = \frac{\ln(x-1)}{2}\right)$ A1 3

METHOD 2

$y - 1 = e^{2x}$ A1

$\frac{\ln(y-1)}{2} = x$ M1

$f^{-1}(x) = \frac{\ln(x-1)}{2} \left(\text{Allow } y = \frac{\ln(x-1)}{2}\right)$ A1 3

(ii) $a = \frac{\ln(5-1)}{2} \left(= \frac{1}{2}\ln 2^2\right)$ M1

$= \frac{1}{2} \times 2\ln 2$ A1

$= \ln 2$ AG 2

(c) Using $V = \int_a^b \pi y^2 \, dx$ (M1)

Volume $= \int_0^{\ln 2} \pi(1 + e^{2x})^2 \, dx \quad \left(\text{or} \int_0^{0.805} \pi(1 + e^{2x})^2 \, dx\right)$ A2 3

[14]

60.) (a) $x = 1$ (A1) 1

(b) Using quotient rule (M1)

Substituting correctly $g'(x) = \frac{(x-1)^2(1) - (x-2)[2(x-1)]}{(x-1)^4}$ A1

$= \frac{(x-1) - (2x-4)}{(x-1)^3}$ (A1)

$= \frac{3-x}{(x-1)^3}$ (Accept $a = 3$, $n = 3$) A1 4

(c) Recognizing at point of inflexion $g''(x) = 0$ M1

$x = 4$ A1

Finding corresponding y-value = $\frac{2}{9}$ = 0.222 *ie* $P\left(4, \frac{2}{9}\right)$ A1 3

[8]

61.) (a) (i)
$p = -2 \quad q = 4$ (or $p = 4, q = -2$) (A1)(A1)(N1)
(N1)

(ii) $y = a(x+2)(x-4)$

$8 = a(6+2)(6-4)$ (M1)

$8 = 16a$

$a = \frac{1}{2}$ (A1) (N1)

(iii) $y = \frac{1}{2}(x+2)(x-4)$

$y = \frac{1}{2}(x^2 - 2x - 8)$

$y = \frac{1}{2}x^2 - x - 4$ (A1) (N1)5

(b) (i) $\frac{\mathrm{d}y}{\mathrm{d}x} = x - 1$ (A1) (N1)

(ii) $x - 1 = 7$ (M1)

$x = 8,\ y = 20$ (P is (8, 20)) (A1)(A1) (N2)4

(c) (i)when x = 4, gradient of tangent is 4 – 1 = 3 (may be implied)(A1)

gradient of normal is $-\frac{1}{3}$ (A1)

$y - 0 = -\frac{1}{3}(x - 4)$ $\left(y = -\frac{1}{3}x + \frac{4}{3}\right)$ (A1) (N3)

(ii) $\frac{1}{2}x^2 - x - 4 = -\frac{1}{3}x + \frac{4}{3}$ (or sketch/graph) (M1)

$\frac{1}{2}x^2 - \frac{2}{3}x - \frac{16}{3} = 0$

$3x^2 - 4x - 32 = 0$ (may be implied) (A1)

$(3x+8)(x-4) = 0$

$x = -\frac{8}{3}$ or $x = 4$

$x = -\frac{8}{3}$ (-2.67) (A1) (N2)6

[15]

62.) (a) x = 1(A1)

EITHER

The gradient of $g(x)$ goes from positive to negative (R1)

OR

$g(x)$ goes from increasing to decreasing (R1)

OR

when $x = 1$, $g'(x)$ is negative (R1) 2

(b) $-3 < x < -2$ **and** $1 < x < 3$ (A1)

$g'(x)$ is negative (R1) 2

(c) $x = -\frac{1}{2}$ (A1)

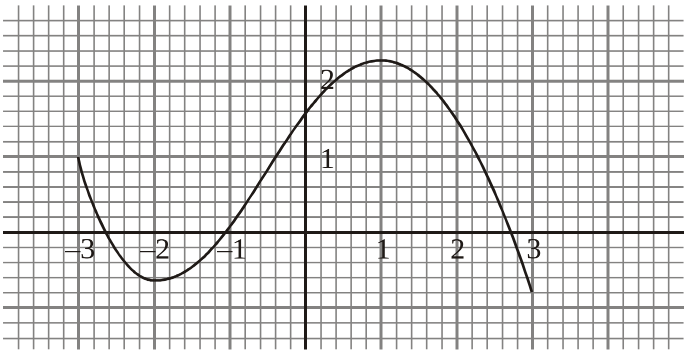
EITHER

$g'(x)$ changes from positive to negative (R1)

OR

concavity changes (R1) 2

(d)



(A3) 3

[9]

63.) **METHOD 1**

$l + 2w = 60$ (M1)

$l = 60 - 2w$ (A1)

$A = w(60 - 2w)$ $\quad$ $(= 60w - 2w^2)$ (A1)

$\frac{dA}{dw} = 60 - 4w$ (A1)

Using $\frac{dA}{dw} = 0$ $\quad$ $(60 - 4w = 0)$ (M1)

$w = 15$ (A1) (C6)

METHOD 2

$w + 2l = 60$ (A1)

$w = 60 - 2l$ (A1)

$A = l(60 - 2l)$ $\quad$ $(= 60l - 2l^2)$ (A1)

$\frac{dA}{dl} = 60 - 4l$ (A1)

Using $\frac{dA}{dl} = 0$ $(60 - 4l = 0)$ (M1)

$l = 15$

$w = 30$ (A1) (C6)

[6]

64.) (a) $s = 25t - \frac{4}{3}t^3 + c$ (M1)

(A1)(A1)

Note: Award no further marks if "c" is missing.

Substituting $s = 10$ and $t = 3$ (M1)

$10 = 25 \times 3 - \frac{4}{3}(3)^3 + c$

$10 = 75 - 36 + c$

$c = -29$ (A1)

$s = 25t - \frac{4}{3}t^3 - 29$ (A1) (N3)

(b) **METHOD 1**

s is a maximum when $v = \frac{ds}{dt} = 0$ (may be implied) (M1)

$25 - 4t^2 = 0$ (A1)

$t^2 = \frac{25}{4}$

$t = \frac{5}{2}$ (A1) (N2)

METHOD 2

Using maximum of s ($12\frac{2}{3}$, may be implied) (M1)

$25t - \frac{4}{3}t^3 - 29 = 12\frac{2}{3}$ (A1)

t = 2.5 (A1) (N2)

(c) $25t - \frac{4}{3}t^3 - 29 > 0$ (accept equation) (M1)

$m = 1.27$, $n = 3.55$ (A1)(A1) (N3)

[12]

65.) (a) $d = \int_0^4 (4t + 5 - 5e^{-t})\,dt$ (M1)(A1)(A1)(C3)

Note: *Award (M1) for* ∫, *(A1) for **both** limits, (A1) for* $4t + 5 - 5e^{-t}$

(b) $d = [2t^2 + 5t + 5e^{-t}]_0^4$ (A1)(A1)

Note: *Award (A1) for* $2t^2 + 5t$, *(A1) for* $5e^{-t}$.

$= (32 + 20 + 5e^{-4}) - (5)$

$= 47 + 5e^{-4}$ (47.1, 3sf) (A1) (C3)

[6]

66.) (a) Velocity is $\frac{ds}{dt}$. (M1)

$\frac{ds}{dt} = 10 - t$ (A1)

10 (m s^{-1}) (A1) (C3)

(b) The velocity is zero when $\frac{ds}{dt} = 0$ (M1)

$10 - t = 0$

$t = 10$ (secs) (A1) (C2)

(c) $s = 50$ (metres) (A1) (C1)

Note: *Do not penalize absence of units.*

[6]

67.) (a) $h = 3$ (A1)

$k = 2$ (A1) 2

(b) $f(x) = -(x - 3)^2 + 2$

$= -x^2 + 6x - 9 + 2$ (must be a correct expression) (A1)

$= -x^2 + 6x - 7$ (AG) 1

(c) $f'(x) = -2x + 6$ (A2) 2

(d) (i) tangent gradient $= -2$ (A1)

gradient of $L = \frac{1}{2}$ (A1)(N2) 2

(ii) **EITHER**

equation of L is $y = \frac{1}{2}x + c$ (M1)

$c = -1$. (A1)

$y = \frac{1}{2}x - 1$

OR

$y - 1 = \frac{1}{2}(x - 4)$ (A2) (N2) 2

(iii) **EITHER**

$-x^2 + 6x - 7 = \frac{1}{2}x - 1$ (M1)

$2x^2 - 11x + 12 = 0$ (may be implied) (A1)

$(2x-3)(x-4) = 0$ (may be implied) (A1)

$x = 1.5$ (A1) (N3) 4

OR

$-x^2 + 6x - 7 = \frac{1}{2}x - 1$ (or a sketch) (M1)

$x = 1.5$ (A3)(N3) 8

[13]

68.) (a) $y = 0$ (A1)1

(b) $f'(x) = \frac{-2x}{(1+x^2)^2}$ (A1)(A1)(A1) 3

(c) $\frac{6x^2 - 2}{(1+x^2)^3} = 0$ (or sketch of $f'(x)$ showing the maximum) (M1)

$6x^2 - 2 = 0$ (A1)

$x = \pm\sqrt{\frac{1}{3}}$ (A1)

$x = \frac{-1}{\sqrt{3}} (= -0.577)$ (A1) (N4) 4

(d) $\int_{-0.5}^{0.5} \frac{1}{1+x^2} \mathrm{d}x \left(= 2 \int_{0}^{0.5} \frac{1}{1+x^2} \mathrm{d}x = 2 \int_{-0.5}^{0} \frac{1}{1+x^2} \mathrm{d}x \right)$ (A1)(A1)2

[10]

69.) (a) $x = 4$ (A1)

g'' changes sign at $x = 4$ or concavity changes (R1) 2

(b) $x = 2$ (A1)

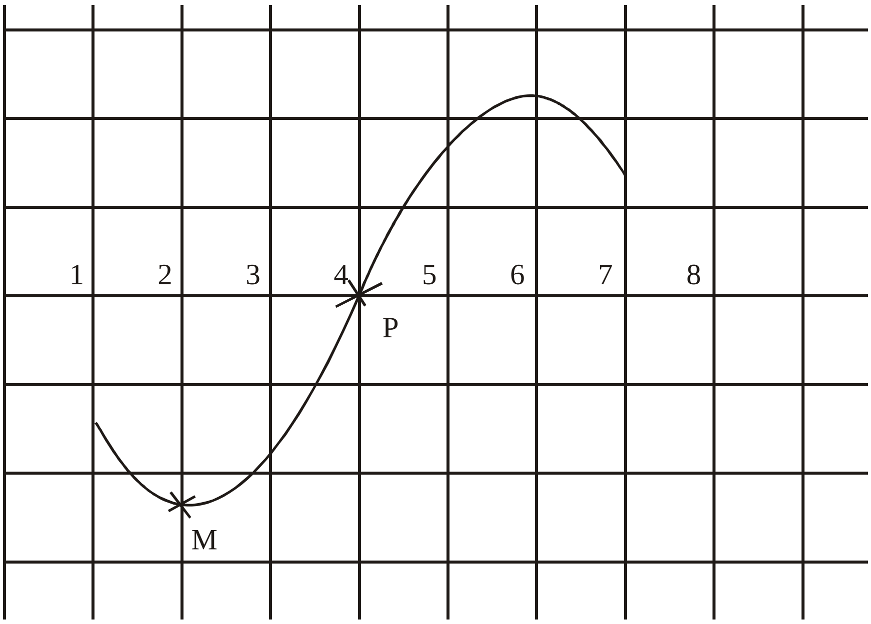
EITHER

g' goes from negative to positive (R1)

OR

$g'(2) = 0$ **and** $g''(2)$ is positive (R1) 2

(c)



(A2)(A1)(A1) 4

Note: *Award (A2) for a suitable cubic curve through (4, 0), (A1) for M at x = 2, (A1) for P at (4, 0).*

[8]

70.) (a) (i) When $t = 0$,

$v = 50 + 50e^0$ (A1)

$= 100 \text{ m s}^{-1}$ (A1)

(ii) When $t = 4$, $v = 50 + 50e^{-2}$ (A1)

$= 56.8 \text{ m s}^{-1}$ (A1) 4

(b) $v = \dfrac{ds}{dt} \Rightarrow s = \int v\, dt$

$\int_0^4 \left(50 + 50e^{-0.5t}\right) dt$ (A1)(A1)(A1) 3

Note: *Award (A1) for each limit in the correct position and (A1) for the function.*

(c) Distance travelled in 4 seconds $= \int_0^4 \left(50 + 50e^{-0.5t}\right) dt$

$= [50t - 100e^{-0.5t}]_0^4$ (A1)

$= (200 - 100e^{-2}) - (0 - 100e^0)$

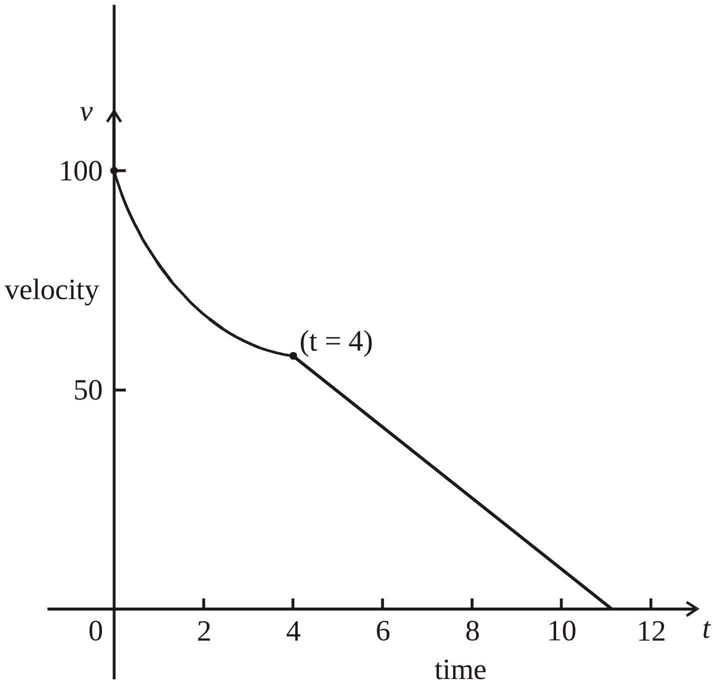
$= 286$ m (3 sf) (A1)

Note: *Award first (A1) for $[50t - 100e^{-0.5t}]$, ie limits not required.*

OR

Distance travelled in 4 seconds = 286 m (3 sf) (G2) 2

(d)



Notes: *Award (A1) for the exponential part, (A1) for the straight line through (11, 0),*
Award (A1) for indication of time on x-axis ***and*** *velocity on y-axis,*
(A1) for scale on x-axis ***and*** *y-axis.*
Award (A1) for marking the point where t = 4.

5

(e) Constant rate = $\frac{56.8}{7}$ (M1)

$= 8.11 \text{ m s}^{-2}$ (A1) 2

Note: *Award (M1)(A0) for –8.11.*

(f) distance = $\frac{1}{2}(7)(56.8)$ (M1)

= 199 m (A1) 2

Note: *Do not award* ***ft*** *in parts (e) and (f) if candidate has not used a straight line for t = 4 to t = 11 or if they continue the exponential beyond t = 4.*

[18]

71.) (a) (i) cos $\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, sin $\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ (A1)

therefore $\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right) = 0$ (AG)

(ii) $\cos x + \sin x = 0 \Rightarrow 1 + \tan x = 0$

$\Rightarrow \tan x = -1$ (M1)

$x = \frac{3\pi}{4}$ (A1)

Note: *Award (A0) for 2.36.*

OR

$x = \frac{3\pi}{4}$ (G2) 3

(b) $y = e^x(\cos x + \sin x)$

$\frac{dy}{dx} = e^x(\cos x + \sin x) + e^x(-\sin x + \cos x)$ (M1)(A1)(A1) 3

$= 2e^x \cos x$

(c) $\frac{dy}{dx} = 0$ for a turning point $\Rightarrow 2e^x \cos x = 0$ (M1)

$\Rightarrow \cos x = 0$ (A1)

$\Rightarrow x = \frac{\pi}{2} \Rightarrow a = \frac{\pi}{2}$ (A1)

$y = e^{\frac{\pi}{2}}(\cos \frac{\pi}{2} + \sin \frac{\pi}{2}) = e^{\frac{\pi}{2}}$

$b = e^{\frac{\pi}{2}}$ (A1) 4

Note: *Award (M1)(A1)(A0)(A0) for $a = 1.57$, $b = 4.81$.*

(d) At D, $\frac{d^2y}{dx^2} = 0$ (M1)

$2e^x \cos x - 2e^x \sin x = 0$ (A1)

$2e^x (\cos x - \sin x) = 0$

$\Rightarrow \cos x - \sin x = 0$ (A1)

$\Rightarrow x = \frac{\pi}{4}$ (A1)

$\Rightarrow y = e^{\frac{\pi}{4}}(\cos \frac{\pi}{4} + \sin \frac{\pi}{4})$ (A1)

$= \sqrt{2}\, e^{\frac{\pi}{4}}$ (AG) 5

(e) Required area $= \int_0^{\frac{3}{4}} e^x (\cos x + \sin x)dx$ (M1)

$= 7.46$ sq units (G1)

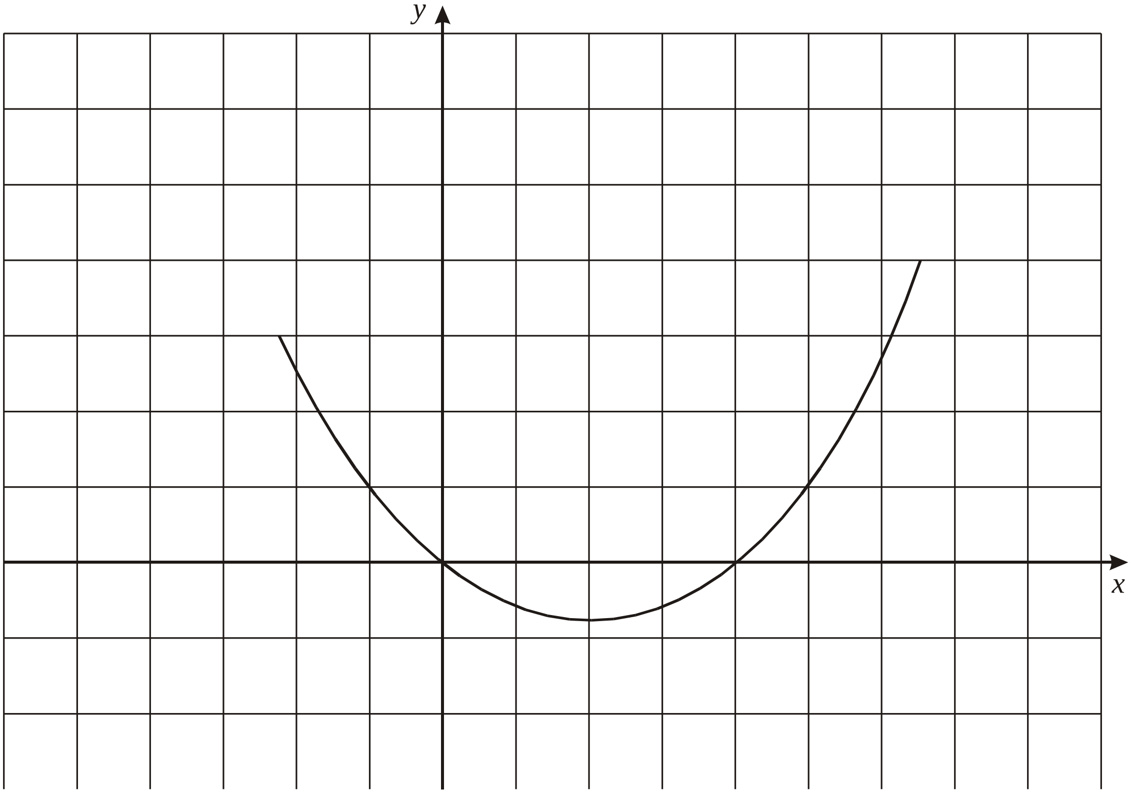
OR

Area = 7.46 sq units (G2) 2

Note: *Award (M1)(G0) for the answer 9.81 obtained if the calculator is in degree mode.*

[17]

72.)



(A2)(A1)(A1)(A2) (C6)

Note: *Award A2 for correct shape (approximately parabolic), A1 A1 for intercepts at 0 and 4, A2 for minimum between x = 1.5 and x = 2.5.*

[6]

73.) (a) (i) $f'(x) = -2e^{-2x}$ (A1)

(ii) $f'(x)$ is always negative (R1) 2

(b) (i) $y = 1 + e^{-2\times -\frac{1}{2}}$ $(= 1 + e)$ (A1)

(ii) $f'\left(-\frac{1}{2}\right) = -2e^{-2\times -\frac{1}{2}}$ $(= -2e)$ (A1) 2

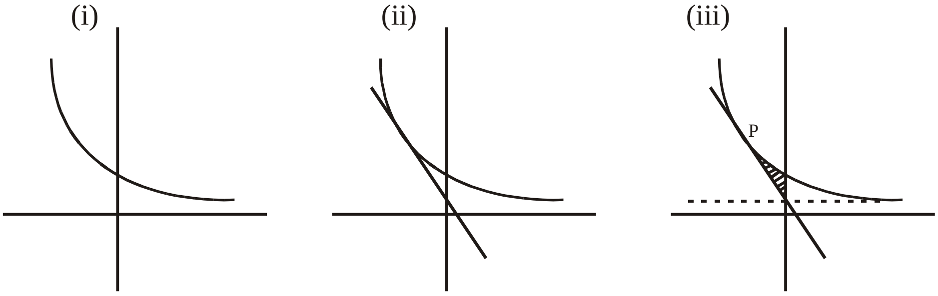
Note: *In part (b) the answers do not need to be simplified.*

(c) $y - (1 + e) = -2e\left(x + \frac{1}{2}\right)$ (M1)

$y = -2ex + 1$ $(y = -5.44x + 1)$ (A1)(A1) 3

(d)

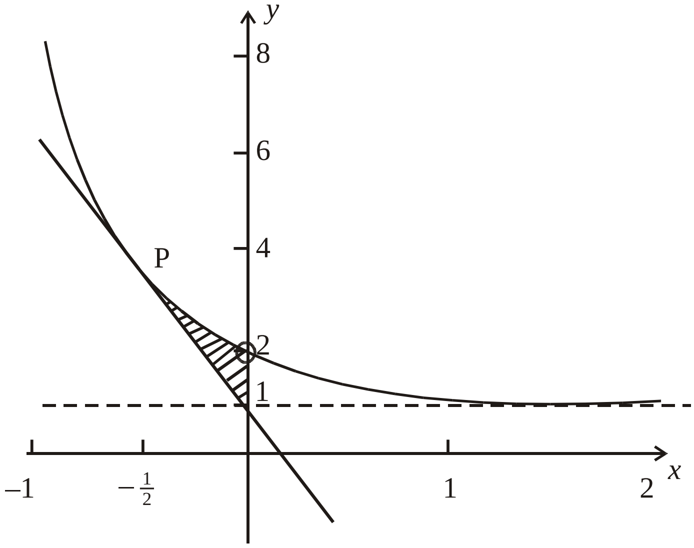
(i) (ii) (iii)



(A1)(A1)(A1)

Notes: *Award (A1) for each correct answer. Do* ***not*** *allow* ***(ft)*** *on an incorrect answer to part (i). The correct final diagram is shown below. Do not penalize if the horizontal asymptote is missing. Axes do not need to be labelled.*

(i)(ii)(iii)



(iv) Area $= \int_{-\frac{1}{2}}^{0} [(1+\mathrm{e}^{-2x}) - (-2\mathrm{e}x+1)]\mathrm{d}x$ (or equivalent) (M1)(M1)

Notes: *Award (M1) for the limits, (M1) for the function. Accept difference of integrals as well as integral of difference. Area below line may be calculated geometrically.*

Area $= \int_{-\frac{1}{2}}^{0} [(\mathrm{e}^{-2x} + 2\mathrm{e}x)\mathrm{d}x$

$= \left[-\frac{1}{2}\mathrm{e}^{-2x} + \mathrm{e}x^2\right]_{-\frac{1}{2}}^{0}$ (A1)

$= 0.1795 \ldots = 0.180$ (3 sf) (A1)

OR

Area = 0.180 (G2) 7

[14]

74.) (a) $x = 1$ (A1) 1

(b) (i) $f(-1000) = 2.01$ (A1)

(ii) $y = 2$ (A1) 2

(c) $f'(x) = \dfrac{(x-1)^2(4x-13) - 2(x-1)(2x^2 - 13x + 20)}{(x-1)^4}$ (A1)(A1)

$= \dfrac{(4x^2 - 17x + 13) - (4x^2 - 26x + 40)}{(x-1)^3}$ (A1)

$= \dfrac{9x - 27}{(x-1)^3}$ (AG) 3

Notes: *Award (M1) for the* ***correct*** *use of the quotient rule, the first (A1) for the placement of the correct expressions into the quotient rule.*
Award the second (A1) for doing sufficient simplification to make the given answer reasonably obvious.

(d) $f'(3) = 0$ $\Rightarrow$ stationary (or turning) point (R1)

$f''(3) = \dfrac{18}{16} > 0$ $\Rightarrow$ minimum (R1) 2

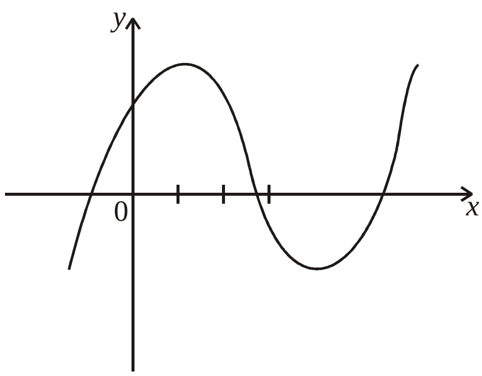
(e) Point of inflexion $\Rightarrow f''(x) = 0 \Rightarrow x = 4$ (A1)
$x = 4 \Rightarrow y = 0$ $\Rightarrow$ Point of inflexion = (4, 0) (A1)

OR

Point of inflexion = (4, 0) (G2) 2

[10]

75.) **METHOD 1**



Using gdc coordinates of maximum are
(0.667, 26.9) (G3)(G3) (C6)

METHOD 2

At maximum $\dfrac{dy}{dx} = 3x^2 - 20x + 12 = 0 = (3x - 2)(x - 6)$ (M1)(A1)(M1)

=> $x = \dfrac{2}{3}$ must be where maximum occurs (A1)

$x = \dfrac{2}{3}$ => $y = \left(\dfrac{2}{3}\right)^3 - 10\left(\dfrac{2}{3}\right)^2 + 12\left(\dfrac{2}{3}\right) + 23 = \dfrac{725}{27}$ (= 26.9, 3 sf)(M1)(A1)

Maximum at $\left(\frac{2}{3}, \frac{725}{27}\right)$ (C4)(C2)

[6]

76.) (a) $\frac{ds}{dt} = 30 - at \Rightarrow s = 30t - a\frac{t^2}{2} + C$ (A1)(A1)(A1)

Note: *Award (A1) for 30t, (A1) for* $a\frac{t^2}{2}$*, (A1) for C.*

$t = 0 \Rightarrow s = 30(0) - a\frac{(0^2)}{2} + C = 0 + C \Rightarrow C = 0$ (M1)

$\Rightarrow s = 30t - \frac{1}{2}at^2$ (A1) 5

(b) (i) vel = 30 – 5(0) = 30 m s^{-1} (A1)

(ii) Train will stop when $0 = 30 - 5t \Rightarrow t = 6$ (M1)

Distance travelled $= 30t - \frac{1}{2}at^2$

$= 30(6) - \frac{1}{2}(5)(6^2)$ (M1)

= 90m (A1)

90 < 200 => train stops before station. (R1)(AG) 5

(c) (i) $0 = 30 - at \Rightarrow t = \frac{30}{a}$ (A1)

(ii) $30\left(\frac{30}{a}\right) - \frac{1}{2}(a)\left(\frac{30}{a}\right)^2 = 200$ (M1)(M1)

Note: *Award (M1) for substituting* $\frac{30}{a}$*, (Ml) for setting equal to 200.*

$\Rightarrow \frac{900}{a} - \frac{450}{a} = \frac{450}{a} = 200$ (A1)

$\Rightarrow a = \frac{450}{200} = \frac{9}{4} = 2.25$ m s^{-2} (A1) 5

Note: *Do not penalize lack of units in answers.*

[15]

77.) (i) At $x = a$, $h(x) = a^{\frac{1}{5}}$

$h'(x) = \frac{1}{5}x^{-\frac{4}{5}} \Rightarrow h'(a) = \frac{1}{5a^{\frac{4}{5}}}$ = gradient of tangent (A1)

$\Rightarrow\ y - a^{\frac{1}{5}} = \dfrac{1}{5a^{\frac{4}{5}}}(x - a) = \dfrac{1}{5a^{\frac{4}{5}}}x - \dfrac{1}{5}a^{\frac{1}{5}}$ (M1)

$\Rightarrow\ y = \dfrac{1}{5a^{\frac{4}{5}}}x + \dfrac{4}{5}a^{\frac{1}{5}}$ (A1)

(ii) tangent intersects x-axis => $y = 0$

$\Rightarrow \dfrac{1}{5a^{\frac{4}{5}}}x = -\dfrac{4}{5}a^{\frac{1}{5}}$ (M1)

$\Rightarrow x = 5a^{\frac{4}{5}}\left(-\dfrac{4}{5}a^{\frac{1}{5}}\right) = -4a$ (M1)(AG) 5

[5]

78.) $y = \sin(2x - 1)$

$\dfrac{dy}{dx} = 2\cos(2x - 1)$ (A1)(A1)

At $\left(\dfrac{1}{2}, 0\right)$, the gradient of the tangent = 2 cos 0 (A1)

= 2 (A1) (C4)

[4]

79.) (a) (i) $a = -3$ (A1)

(ii) $b = 5$ (A1) 2

(b) (i) $f'(x) = -3x^2 + 4x + 15$ (A2)

(ii) $-3x^2 + 4x + 15 = 0$

$-(3x + 5)(x - 3) = 0$ (M1)

$x = -\dfrac{5}{3}$ or $x = 3$ (A1)(A1)

OR

$x = -\dfrac{5}{3}$ or $x = 3$ (G3)

(iii) $x = 3 \Rightarrow f(3) = -3^3 + 2(3^2) + 15(3)$ (M1)

$= -27 + 18 + 45 = 36$ (A1)

OR

$f(3) = 36$ (G2) 7

(c) (i) $f'(x) = 15$ at $x = 0$ (M1)

Line through (0, 0) of gradient 15

$\Rightarrow y = 15x$ (A1)

OR

$y = 15x$ (G2)

(ii) $-x^3 + 2x^2 + 15x = 15x$ (M1)

$\Rightarrow -x^3 + 2x^2 = 0$

$\Rightarrow -x^2(x - 2) = 0$

$\Rightarrow x = 2$ (A1)

OR
$x = 2$ (G2) 4

(d) Area = 115 (3 sf) (G2)
OR

Area = $\int_0^6 (-x^3 + 2x^2 + 15x)\mathrm{d}x = \left[-\frac{x^4}{4} + 2\frac{x^3}{3} + 15\frac{x^2}{2}\right]_0^5$ (M1)

$= \frac{1375}{12}$ = 115 (3 sf) (A1) 2

[15]

80.) (a) (i) $v(0) = 50 - 50\mathrm{e}^0 = 0$ (A1)

(ii) $v(10) = 50 - 50\mathrm{e}^{-2} = 43.2$ (A1) 2

(b) (i) $a = \frac{\mathrm{d}v}{\mathrm{d}t} = -50(-0.2\mathrm{e}^{-0.2t})$ (M1)

$= 10\mathrm{e}^{-0.2t}$ (A1)

(ii) $a(0) = 10\mathrm{e}^0 = 10$ (A1) 3

(c) (i) $t \to \infty \Rightarrow v \to 50$ (A1)

(ii) $t \to \infty \Rightarrow a \to 0$ (A1)

(iii) when $a = 0$, v is constant at 50 (R1) 3

(d) (i) $y = \int v\mathrm{d}t$ (M1)

$= 50t - \frac{\mathrm{e}^{-0.2t}}{-0.2} + k$ (A1)

$= 50t + 250\mathrm{e}^{-0.2t} + k$ (AG)

(ii) $0 = 50(0) + 250\mathrm{e}^0 + k = 250 + k$ (M1)
$\Rightarrow k = -250$ (A1)

(iii) Solve $250 = 50t + 250\mathrm{e}^{-0.2t} - 250$ (M1)
$\Rightarrow 50t + 250\mathrm{e}^{-0.2t} - 500 = 0$
$\Rightarrow t + 5\mathrm{e}^{-0.2t} - 10 = 0$
$\Rightarrow t = 9.207\ s$ (G2) 7

[15]

81.) (a) (i) $x = -\frac{5}{2}$ (A1)

(ii) $y = \frac{3}{2}$ (A1) 2

(b) By quotient rule (M1)

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2x+5)(3) - (3x-2)(2)}{(2x+5)^2}$ (A1)

$= \dfrac{19}{(2x+5)^2}$ (A1) 3

(c) There are no points of inflexion. (A1) 1

[6]

82.) (a) When $t = 0$, (M1)

$h = 2 + 20 \times 0 - 5 \times 0^2 = 2$ $h = 2$ (A1) 2

(b) When $t = 1$, (M1)

$h = 2 + 20 \times 1 - 5 \times 1^2$ (A1)

$= 17$ (AG) 2

(c) (i) $h = 17 \Rightarrow 17 = 2 + 20t - 5t^2$ (M1)

(ii) $5t^2 - 20t + 15 = 0$ (M1)

$\Leftrightarrow 5(t^2 - 4t + 3) = 0$

$\Leftrightarrow (t - 3)(t - 1) = 0$ (M1)

Note: *Award (M1) for factorizing or using the formula*

$\Leftrightarrow t = 3$ or 1 (A1) 4

Note: *Award (A1) for $t = 3$*

(d) (i) $h = 2 + 20t - 5t^2$

$\Rightarrow \dfrac{dh}{dt} = 0 + 20 - 10t$

$= 20 - 10t$ (A1)(A1)

(ii) $t = 0$ (M0)

$\Rightarrow \dfrac{dh}{dt} = 20 - 10 \times 0 = 20$ (A1)

(iii) $\dfrac{dh}{dt} = 0$ (M1)

$\Leftrightarrow 20 - 10t = 0 \Leftrightarrow t = 2$ (A1)

(iv) $t = 2$ (M1)

$\Rightarrow h = 2 + 20 \times 2 - 5 \times 2^2 = 22 \Rightarrow h = 22$ (A1) 7

[15]

83.) $y = x^2 - x$

$\dfrac{dy}{dx} = 2x - 1$ = gradient at any point. (M1)

Line parallel to $y = 5x$

$\Rightarrow 2x - 1 = 5$ (M1)

$x = 3$ (A1)

$y = 6$ (A1)

Point (3, 6) (C2)(C2)

[4]

84.) (a) (i) $t = 0$ $s = 800$
$t = 5$ $s = 800 + 500 - 100 = 1200$ (M1)
distance in first 5 seconds = 1200 – 800
= 400 m (A1) 2

(ii) $v = \frac{ds}{dt} = 100 - 8t$ (A1)

At $t = 5$, velocity = 100 – 40 (M1)
= 60 m s^{-1} (A1) 3

(iii) Velocity = 36 m $s^{-1} \Rightarrow 100 - 8t = 36$ (M1)
t = 8 seconds after touchdown. (A1) 2

(iv) When $t = 8$, $s = 800 + 100(8) - 4(8)^2$ (M1)
$= 800 + 800 - 256$ (A1)
= 1344 m (A1) 3

(b) If it touches down at P, it has 2000 – 1344 = 656 m to stop. (M1)
To come to rest, $100 - 8t = 0 \Rightarrow t = 12.5$ s (M1)

Distance covered in 12.5 $s = 100(12.5) - 4(12.5)^2$ (M1)
$= 1250 - 625$
$= 625$ (A1)
Since 625 < 656, it can stop safely. (R1) 5

[15]

85.) (a) $y = e^{2x} \cos x$
$\frac{dy}{dx} = e^{2x}(-\sin x) + \cos x\,(2e^{2x})$ (A1)(M1)
$= e^{2x}(2 \cos x - \sin x)$ (AG) 2

(b) $\frac{d^2y}{dx^2} = 2e^{2x}(2 \cos x - \sin x) + e^{2x}(-2 \sin x - \cos x)$ (A1)(A1)
$= e^{2x}(4 \cos x - 2 \sin x - 2 \sin x - \cos x)$ (A1)
$= e^{2x}(3 \cos x - 4 \sin x)$ (A1) 4

(c) (i) At P, $\frac{d^2y}{dx^2} = 0$ (R1)

$\Rightarrow 3 \cos x = 4 \sin x$ (M1)
$\Rightarrow \tan x = \frac{3}{4}$
At P, $x = a$, *ie* $\tan a = \frac{3}{4}$ (A1)

(ii) The gradient at any point $e^{2x}(2 \cos x - \sin x)$ (M1)
Therefore, the gradient at P = $e^{2a}(2 \cos a - \sin a)$
When $\tan a = \frac{3}{4}$, $\cos a = \frac{4}{5}$, $\sin a = \frac{3}{5}$ (A1)(A1)
(by drawing a right triangle, or by calculator)
Therefore, the gradient at P = $e^{2a}\left(\frac{8}{5} - \frac{3}{5}\right)$ (A1)
$= e^{2a}$ (A1) 8

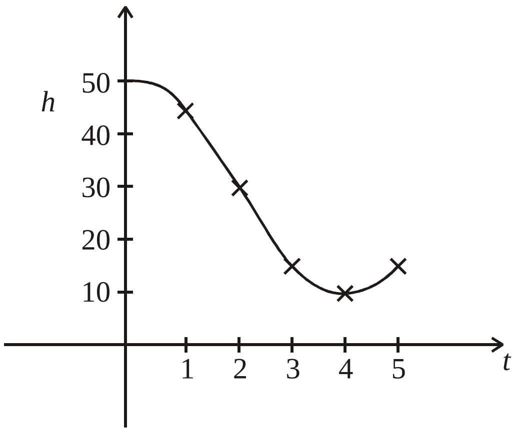
[14]

86.) (a) $t = 2 \Rightarrow h = 50 - 5(2^2) = 50 - 20$
$= 30$ (A1)

OR

$h = 90 - 40(2) + 5(2^2)$
$= 30$ (A1) 1

(b)



(A4) 4

Note: *Award (A1) for marked scales on each axis, (A1) for each section of the curve.*

(c) (i) $\dfrac{dh}{dt} = \dfrac{d}{dt}(50 - 5t^2)$
$= 0 - 10t = -10t$ (A1)

(ii) $\dfrac{dh}{dt} = \dfrac{d}{dt}(90 - 40t + 5t^2)$
$= 0 - 40 + 10t = -40 + 10t$ (A1) 2

(d) When $t = 2$ (i) $\dfrac{dh}{dt} = -10(2)$ **or** $\dfrac{dh}{dt} = -40 + 10 \times 2$ (M1)
$= -20$ $= -20$ (A1) 2

(e) $\dfrac{dh}{dt} = 0 \Rightarrow -10t = 0 (0 \le t \le 2)$ **or** $-40 + 10t = 0 (2 \le t \le 5)$ (M1)
$t = 0$ **or** $t = 4$ (A1)(A1) 3

(f) When $t = 4$ (M1)
$h = 90 - 40(4) + 5(4^2)$ (M1)
$= 90 - 160 + 80$
$= 10$ (A1) 3

[15]

87.) $y = x^3 + 1$

$\dfrac{dy}{dx} = 3x^2$

= Slope of tangent at any point

Therefore at point where $x = 1$, slope = 3 (M1)

$\Rightarrow$ Slope of normal = $-\frac{1}{3}$ (M1)(A1)

$\Rightarrow$ Equation of normal: $y - 2 = -\frac{1}{3}(x - 1)$

$3y - 6 = -x + 1$

$x + 3y - 7 = 0$ (A1) (C4)

Note: *Accept equivalent forms eg* $y = -\frac{1}{3}x + 2\frac{1}{3}$

[4]

88.) (a) (i) $f(x) = \frac{2x+1}{x-3}$

$= 2 + \frac{7}{x-3}$ by division or otherwise (M1)

Therefore as $|x| \to \infty$ $f(x) \to 2$ (A1)

$\Rightarrow$ *y = 2* is an asymptote (AG)

OR $\lim\limits_{x\to\infty} \frac{2x+1}{x-3} = 2$ (M1)(A1)

$\Rightarrow$ $y = 2$ is an asymptote (AG)

OR make x the subject

$yx - 3y = 2x + 1$

$x(y - 2) = 1 + 3y$ (M1)

$x = \frac{1+3y}{y-2}$ (A1)

$\Rightarrow$ $y = 2$ is an asymptote (AG)

Note: *Accept inexact methods based on the ratio of the coefficients of x.*

(ii) Asymptote at $x = 3$ (A1)

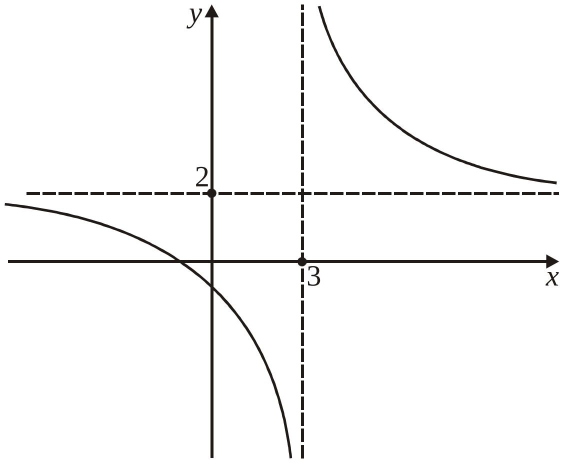
(iii) $P(3, 2)$ (A1) 4

(b) $f(x) = 0 \Rightarrow x = -\frac{1}{2}$ $\left(-\frac{1}{2}, 0\right)$ (M1)(A1)

$x = 0 \Rightarrow f(x) = -\frac{1}{3}$ $\left(0, -\frac{1}{3}\right)$ (M1)(A1) 4

Note: *These do not have to be in coordinate form.*

(c)



(A4) 4

Note: *Asymptotes (A1)*
Intercepts (A1)
"Shape" (A2).

(d) $f'(x) = \dfrac{(x-3)(2)-(2x+1)}{(x-3)^2}$ (M1)

$= \dfrac{-7}{(x-3)^2}$ (A1)

= Slope at any point

Therefore slope when $x = 4$ is -7 (A1)
And $f(4) = 9$ ie $S(4, 9)$ (A1)
$\Rightarrow$ Equation of tangent: $y - 9 = -7(x - 4)$ (M1)
$7x + y - 37 = 0$ (A1) 6

(e) at T, $\dfrac{-7}{(x-3)^2} = -7$ (M1)

$\Rightarrow (x-3)^2 = 1$ (A1)
$x - 3 = \pm 1$ (A1)

$\left.\begin{array}{l} x = 4 \text{ or } 2 \\ y = 9 \text{ or } -5 \end{array}\right\} \begin{array}{l} S(4,9) \\ T(2,-5) \end{array}$ (A1)(A1) 5

(f) Midpoint $[ST] = \left(\dfrac{4+2}{2}, \dfrac{9-5}{2}\right)$

$= (3, 2)$
= point P (A1) 1

[24]

89.) (a) (i) $f(x) = \dfrac{2x+1}{x-3}$

$= 2 + \dfrac{7}{x-3}$ by division or otherwise (M1)

Therefore as $|x| \to \infty$ $f(x) \to 2$ (A1)

$\Rightarrow$ *y = 2* is an asymptote (AG)

OR $\lim\limits_{x\to\infty} \dfrac{2x+1}{x-3} = 2$ (M1)(A1)

$\Rightarrow y = 2$ is an asymptote (AG)

OR make *x* the subject

$yx - 3y = 2x + 1$

$x(y - 2) = 1 + 3y$ (M1)

$x = \dfrac{1+3y}{y-2}$ (A1)

$\Rightarrow y = 2$ is an asymptote (AG)

Note: *Accept inexact methods based on the ratio of the coefficients of x.*

(ii) Asymptote at $x = 3$ (A1)

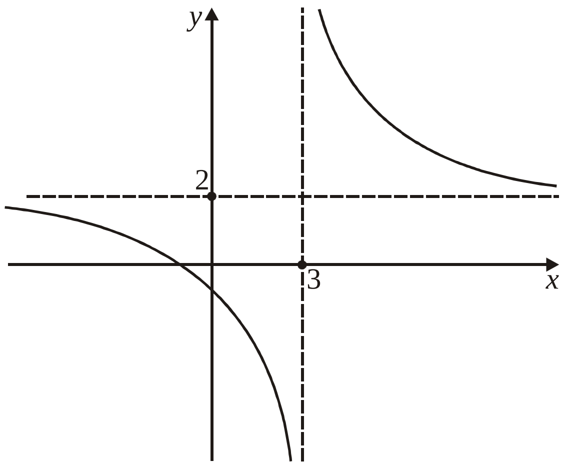
(iii) $P(3, 2)$ (A1) 4

(b) $f(x) = 0 \Rightarrow x = -\dfrac{1}{2}$ $\left(-\dfrac{1}{2}, 0\right)$ (M1)(A1)

$x = 0 \Rightarrow f(x) = -\dfrac{1}{3}$ $\left(0, -\dfrac{1}{3}\right)$ (M1)(A1) 4

Note: *These do not have to be in coordinate form.*

(c)



(A4) 4

Note: *Asymptotes (A1)*
Intercepts (A1)
“Shape” (A2).

(d) $f'(x) = \dfrac{(x-3)(2)-(2x+1)}{(x-3)^2}$ (M1)

$= \dfrac{-7}{(x-3)^2}$ (A1)

= Slope at any point

Therefore slope when $x = 4$ is -7 (A1)

And $f(4) = 9$ ie $S(4, 9)$ (A1)

$\Rightarrow$ Equation of tangent: $y - 9 = -7(x - 4)$ (M1)

$7x + y - 37 = 0$ (A1) 6

(e) at T, $\dfrac{-7}{(x-3)^2} = -7$ (M1)

$\Rightarrow (x-3)^2 = 1$ (A1)

$x - 3 = \pm 1$ (A1)

$\left.\begin{array}{l} x = 4 \text{ or } 2 \\ y = 9 \text{ or } -5 \end{array}\right\}$ $\begin{array}{l} S(4, 9) \\ T(2, -5) \end{array}$ (A1)(A1) 5

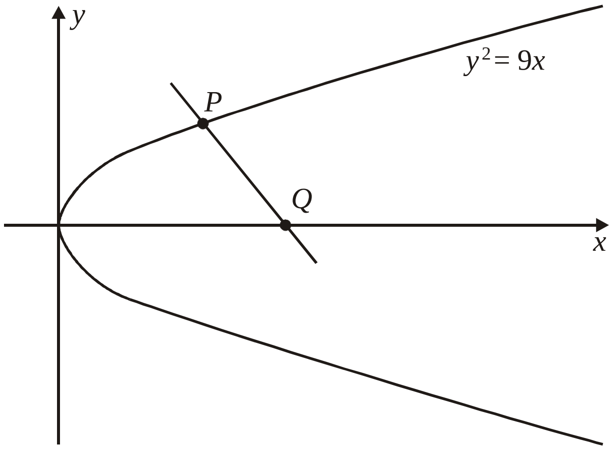
(f) Midpoint $[ST] = \left(\dfrac{4+2}{2}, \dfrac{9-5}{2}\right)$

$= (3, 2)$

$=$ point P (A1) 1

[24]

90.) (a)



$y^2 = 9x$

$6^2 = 9(4)$ (M1)

$36 = 36$ (A1) 2

$\Rightarrow (4, 6)$ on parabola

(b) (i) $y = 3\sqrt{x}$

$\dfrac{dy}{dx} = \dfrac{3}{2\sqrt{x}}$ (M1)

= Slope at any point

Therefore at (4, 6), slope of tangent $= \dfrac{3}{4}$ (A1)

$\Rightarrow$ Slope of normal $= -\dfrac{4}{3}$ (A1)

Therefore equation of normal is $y - 6 = -\dfrac{4}{3}(x - 4)$(M1)

$3y - 18 = -4x + 16$

$4x + 3y - 34 = 0$ (A1) 5

Notes: Candidates may differentiate implicitly to obtain
$\dfrac{dy}{dx} = \dfrac{9}{2y}$.

Answer must be given in the form $ax + by + c = 0$.

(ii) Coordinates of Q:

$y = 0$, $4x = 34$

$x = \dfrac{17}{2}$ (A1)

$Q\left(\dfrac{17}{2}, 0\right)$ (A1) 2

(c) $SP = \sqrt{\left(\dfrac{9}{4} - 4\right)^2 + (0-6)^2}$ (M1)

$= \sqrt{\dfrac{49}{16} + 36}$

$= \dfrac{25}{4}$ (A1)

$SQ = \dfrac{17}{2} - \dfrac{9}{4}$ (M1)

$= \dfrac{34}{4} - \dfrac{9}{4}$

$= \dfrac{25}{4}$ (A1) 4

(d) $|SP| = |SQ| \Rightarrow S\hat{P}Q = S\hat{Q}P$ (M1)

But $S\hat{Q}P = M\hat{P}Q$ (alternate angles) (A1)

$\Rightarrow M\hat{P}Q = S\hat{P}Q$ (A1) 3

[16]

91.) (a) $f(1) = 3$ $f(5) = 3$ (A1)(A1) 2

(b) **EITHER** distance between successive maxima = period (M1)
= 5 – 1 (A1)
= 4 (AG)

OR Period of $\sin kx = \dfrac{2\pi}{k}$; (M1)

so period = $\dfrac{2\pi}{\frac{\pi}{2}}$ (A1)

= 4 (AG) 2

(c) **EITHER** $A \sin\left(\dfrac{\pi}{2}\right) + B = 3$ and $A \sin\left(\dfrac{3\pi}{2}\right) + B = -1$ (M1) (M1)

$\Leftrightarrow A + B = 3, -A + B = -1$ (A1)(A1)

$\Leftrightarrow A = 2, B = 1$ (AG)(A1)

OR Amplitude = A (M1)

$A = \dfrac{3-(-1)}{2} = \dfrac{4}{2}$ (M1)

$A = 2$ (AG)

Midpoint value = B (M1)

$B = \dfrac{3+(-1)}{2} = \dfrac{2}{2}$ (M1)

$B = 1$ (A1) 5

Note: *As the values of $A = 2$ and $B = 1$ are likely to be quite obvious to a bright student, do not insist on too detailed a proof.*

(d) $f(x) = 2\sin\left(\frac{\pi}{2}x\right) + 1$

$f'(x) = \left(\frac{\pi}{2}\right)2\cos\left(\frac{\pi}{2}x\right) + 0$ (M1)(A2)

Note: *Award (M1) for the chain rule, (A1) for* $\left(\frac{\pi}{2}\right)$*, (A1) for* *2 cos*$\left(\frac{\pi}{2}x\right)$.

$= \pi\cos\left(\frac{\pi}{2}x\right)$ (A1) 4

Notes: *Since the result is given, make sure that reasoning is valid. In particular, the final (A1) is for simplifying the result of the chain rule calculation. If the preceding steps are not valid, this final mark should not be given. Beware of "fudged" results.*

(e) (i) $y = k - \pi x$ is a tangent $\Rightarrow -\pi = \pi\cos\left(\frac{\pi}{2}x\right)$ (M1)

$\Rightarrow -1 = \cos\left(\frac{\pi}{2}x\right)$ (A1)

$\Rightarrow \frac{\pi}{2}x = \pi$ or 3π or ...

$\Rightarrow x = 2$ or 6 ... (A1)

Since $0 \le x \le 5$, we take $x = 2$, so the point is $(2, 1)$ (A1)

(ii) Tangent line is: $y = -\pi(x - 2) + 1$ (M1)

$y = (2\pi + 1) - \pi x$

$k = 2\pi + 1$ (A1) 6

(f) $f(x) = 2 \Rightarrow 2\sin\left(\frac{\pi}{2}x\right) + 1 = 2$ (A1)

$\Rightarrow \sin\left(\frac{\pi}{2}x\right) = \frac{1}{2}$ (A1)

$\Rightarrow \frac{\pi}{2}x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ or $\frac{13\pi}{6}$

$x = \frac{1}{3}$ or $\frac{5}{3}$ or $\frac{13}{3}$ (A1)(A1)(A1) 5

[24]

92.) (a) $y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$ (A1)

when $x = e$, $\frac{dy}{dx} = \frac{1}{e}$

tangent line: $y = \left(\frac{1}{e}\right)(x - e) + 1$ (M1)

$y = \frac{1}{e}(x) - 1 + 1 = \frac{x}{e}$ (A1)

$x = 0 \Rightarrow y = \dfrac{0}{e} = 0$ (M1)

(0, 0) is on line (AG) 4

(b) $\dfrac{d}{dx}(x \ln x - x) = (1) \times \ln x + x \times \left(\dfrac{1}{x}\right) - 1 = \ln x$ (M1)(A1)(AG) 2

Note: *Award (M1) for applying the product rule, and (A1) for $(1) \times \ln x + x \times \left(\dfrac{1}{x}\right)$.*

(c) Area = area of triangle – area under curve (M1)

$= \left(\dfrac{1}{2} \times e \times 1\right) - \int_1^e \ln x\, dx$ (A1)

$= \dfrac{e}{2} - [x \ln x - x]_1^e$ (A1)

$= \dfrac{e}{2} - \{(e \ln e - 1 \ln 1) - (e - 1)\}$ (A1)

$= \dfrac{e}{2} - \{e - 0 - e + 1\}$

$= \dfrac{1}{2}e - 1.$ (AG) 4

[10]

93.) (a) $y = x(x-4)^2$

(i) $y = 0 \Leftrightarrow x = 0$ or $x = 4$ (A1)

(ii) $\dfrac{dy}{dx} = 1(x-4)^2 + x \times 2(x-4) = (x-4)(x-4+2x)$

$= (x-4)(3x-4)$ (A1)

$\dfrac{dy}{dx} = 0 \Rightarrow x = 4$ or $x = \dfrac{4}{3}$ (A1)

$$\left.\begin{aligned} x = 1 &\Rightarrow \frac{dy}{dx} = (-3)(-1) = 3 > 0 \\ x = 2 &\Rightarrow \frac{dy}{dx} = (-2)(2) = -4 < 0 \end{aligned}\right\} \Rightarrow \frac{4}{3} \text{ is a } \textbf{maximum}$$ (R1)

Note: *A second derivative test may be used.*

$x = \dfrac{4}{3} \Rightarrow y = \dfrac{4}{3} \times \left(\dfrac{4}{3} - 4\right)^2 = \dfrac{4}{3} \times \left(\dfrac{-8}{3}\right)^2 = \dfrac{4}{3} \times \dfrac{64}{9} = \dfrac{256}{27}$

$\left(\dfrac{4}{3}, \dfrac{256}{27}\right)$ (A1)

Note: *Proving that $\left(\dfrac{4}{3}, \dfrac{256}{27}\right)$ is a maximum is not necessary to receive full credit of [4 marks] for this part.*

(iii) $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\big((x-4)(3x-4)\big) = \dfrac{d}{dx}(3x^2 - 16x + 16) = 6x - 16$ (A1)

$\dfrac{d^2y}{dx^2} = 0 \Leftrightarrow 6x - 16 = 0$ (M1)

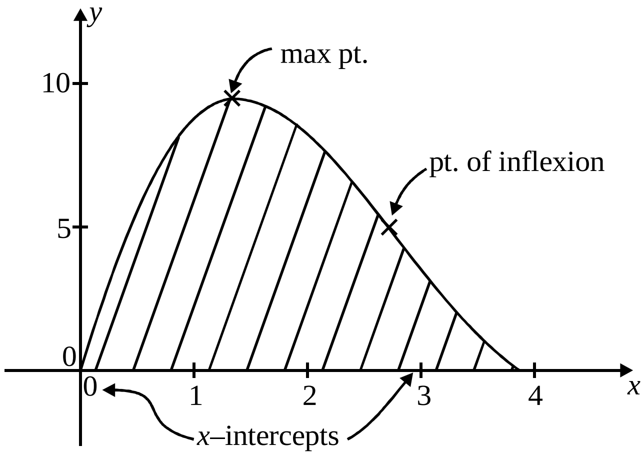
$\Leftrightarrow x = \dfrac{8}{3}$ (A1)

$$x = \frac{8}{3} \Rightarrow y = \frac{8}{3}\left(\frac{8}{3} - 4\right)^2 = \frac{8}{3}\left(\frac{-4}{3}\right)^2 = \frac{8}{3} \times \frac{16}{9} = \frac{128}{27}$$

$\left(\dfrac{8}{3}, \dfrac{128}{27}\right)$ (A1) 9

Note: *GDC use is likely to give the answer (1.33, 9.48). If this answer is given with no explanation, award (A2), If the answer is given with the explanation "used GDC" or equivalent, award full credit.*

(b)



(A3) 3

Note: *Award (A1) for intercepts, (A1) for maximum and (A1) for point of inflexion.*

(c) (i) See diagram above (A1)

(ii) $0 < y < 10$ for $0 \le x \le 4$ (R1)

So $\int_0^4 0\,\mathrm{d}x < \int_0^4 y\,\mathrm{d}x < \int_0^4 10\,\mathrm{d}x \Rightarrow 0 < \int_0^4 y\,\mathrm{d}x < 40$ (R1) 3

[15]