INEQUALITIES KS3 and KS4 Non-Calculator
(with some examples and solutions)

NOTE: * means "may be challenging"

Examples:

Solve the following inequalities:

1. $x + 3 \geq 7$

$x \geq 7 - 3$

$x \geq 4$

2. $3 - x < 7$

$3 - 7 < x$

$-4 < x$

or $x > -4$

3. $2x \leq 7$

$x \leq \frac{7}{2}$ $(or\ 3.5)$

Note: In Question 1 the smallest integer value that *x* may take is 4

In Question 2 the smallest integer value that *x* may take is -3

In Questions 3 the greatest integer value that *x* may take is 3

4. $2x - 3 > 7$

$2x > 7 + 3$

$2x > 10$

$x > 5$

Smallest integer value of *x* is 6

5. $3 - 2x \leq 7$

$3 - 7 \leq 2x$

$-4 \leq 2x$

$-2 \leq x$ (or $x \geq -2$)

Smallest integer value of *x* is -2

6. $6x - 3 \geq 4x - 13$

$6x - 4x \geq -13 + 3$

$2x \geq -10$

$x \geq -5$

Smallest integer value of *x* is -5

7. $4x - 3 \geq 6x - 13$

$-3 + 13 \geq 6x - 4x$

$10 \geq 2x$

$5 \geq x \text{ or } x \leq 5$

Greatest integer value of *x* is 5

*8. $-7 < 4x - 5 < 11$

$-7 + 5 < 4x < 11 + 5$ $(add\ 5\ to\ both\ sides)$

$-2 < 4x < 16$

$\frac{-2}{4} < x < \frac{16}{4}$

$-\frac{1}{2} < x < 4$ The integer values that *x* may take are: 0, 1, 2 and 3.

*9. $-6 \leq 19 - 5x < 33$

This time we split it into two inequalities and solve each one and then combine the answers.

$-6 \leq 19 - 5x$ *and* $19 - 5x < 33$

$5x \leq 19 + 6$ $19 - 33 < 5x$

$5x \leq 25$ $-4 < 5x$

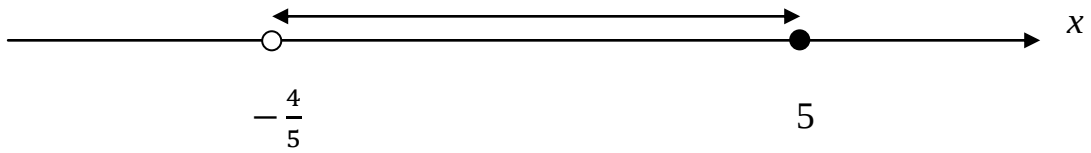
$x \leq 5$ $\frac{-4}{5} < x$ (or $-\frac{4}{5} < x$)

Now combine the two inequalities to get:

$-\frac{4}{5} < x \leq 5$

The integer values that *x* may take are: 0, 1, 2, 3 ,4 and 5.

We may now show this on a number line.



Shaded circle means 5 is included.

10. $2(x-3) \geq 16$ OR $x-3 \geq 8$

$2x-6 \geq 16$ $x \geq 11$

$2x \geq 22$

$x \geq 11$

*12. $3(x-2) \leq 11-2x$

$3x-6 \leq 11-2x$

$3x+2x \leq 11+6$

$5x \leq 17$

$x \leq \frac{17}{5}$ $(or\ 3\frac{2}{5}\ \ or\ 3.4)$

* 13. $\frac{x}{3} > 5$

$x > 5 \times 3$

$x > 15$

* 14. $\frac{x}{3} - 2 > 5$

$\frac{x}{3} > 5 + 2$

$\frac{x}{3} > 7$

$x > 7 \times 3$

$x > 21$

*15. $3 - \frac{2x}{5} \geq -1$ $3 + 1 \geq \frac{2x}{5}$ $4 \geq \frac{2x}{5}$ $20 \geq 2x$

$10 \geq x\ \ Or\ \ x \leq 10$

QUESTIONS:

Solve the following inequalities and represent each one on a number line.

1. $x + 5 > 9$ 2. $x - 5 < -2$ 3. $x + 9 \geq 5$

4. $5 - x \leq 5$ 5. $6 - x \geq 9$ 6. $4 - x > -2$

7. $2x > 18$ 8. $3x \leq -12$ 9. $15 > 2x$

10. $3x - 7 \geq 8$ 11. $3x + 7 \geq 1$ 12. $4x - 6 < 10$

13. $7 - 3x \geq 13$ 14. $7 - 3x < 10$ 15. $7 > 3x - 3$

16. $2x - 3 \leq x - 5$ 17. $4x - 5 > x + 16$

18. $5x > x + 2$ 19. $3(x - 2) \leq 12$

20. $4x - 5 \leq 2(3x - 7)$ $*$ 21. $-9 < 4x - 5 \leq 11$

$*$ 22. $-1 \leq 2x + 5 < 8$ $*$ 23. $11 \geq 4x - 5 \geq -9$

$*$ 24. $-1 < 5 - x \leq 7$ $*$ 25. $-1 \leq 5 - 2x < 7$

$*$ 26. $\frac{x}{6} > 4$ $*$ 27. $\frac{2x}{3} \leq 4$ $*$ 28. $\frac{2x}{3} + 5 \leq 9$

$*$ 29. $\frac{2x-1}{3} > 5$ $*$ 30. $\frac{2x-1}{3} + 1 > 6$ $*$ 31. $5 - \frac{x}{3} \geq 9$

$*$ 32. $\frac{7 - 2x}{5} > 4$ $*$ 33. $\frac{12 - 2x}{5} - 1 > 3$

Miscellaneous questions:

1. k is an integer such that $-2 \leq k < 1$

   (a) List the possible values of k.

   (b) Solve the inequality $8y \geq y + 14$

2. (a) List the possible integer values of n such that $-1 \leq n < 2$

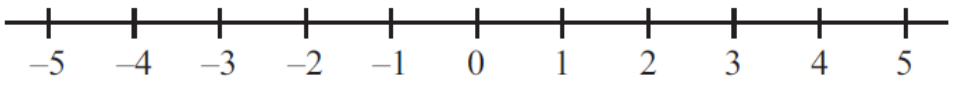
   (b) Solve the inequality $4y - 5 < 10 - y$

3. k is an integer such that $-2 < k \leq 3$.

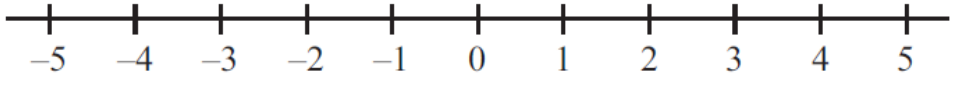
   (a) Write down all the possible values of k.

   (b) Solve the inequality $4x - 5 \geq 7$

4. $x > -2$. Show this inequality on the number line.



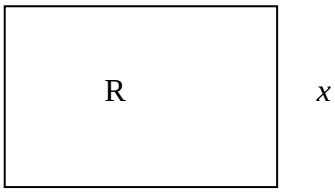
5. $-3 \leq x < 2$. Show this inequality on the number line.

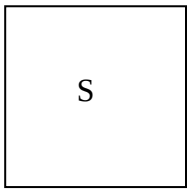


6. Solve the inequalities (a) $3x - 2 < 2x - 2$ (b) $8y + 46 \leq 14$
In each case, write down the greatest integer value of *x*.

*7. The perimeter of the rectangle, R is less than the perimeter of the Square, S.

Write down an inequality and solve it to find the range of values of *x*.

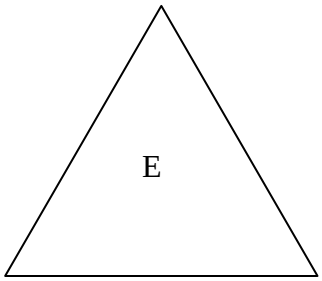


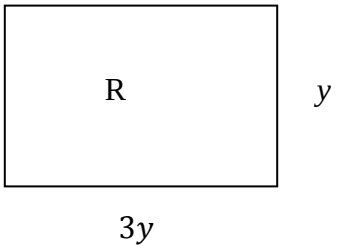


*8. The perimeter of the eqilateral triangle, E is less that the perimeter of the rectangle, R.

(a) Write down an inequality and solve it to find the range of values of *y*.

(b) What is the smallest integer value that *y* can take?

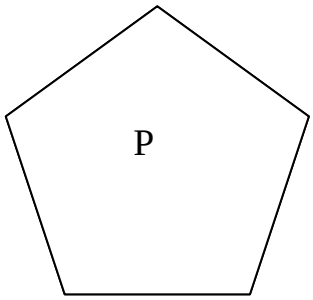




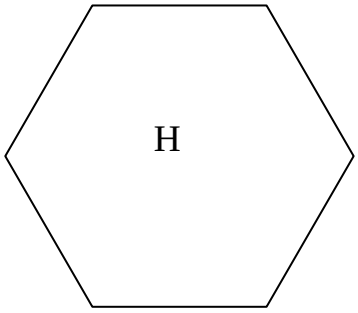
9. Solve the inequality $3x - 5 > 10 - 2x$

*10. The perimeter of the regular Hexagon, H is greater that the perimeter of the regular pentagon, P.

Write down the range of values of *x*.



$2x + 1$



$2x - 1$

*11. Solve the inequality $8 - 3x \leq 5 - 2x$

**12. Solve the following inequalities:

(a) $\frac{4x-3}{2} > \frac{x+1}{4}$

(b) $\frac{4x-3}{2} + 1 > \frac{x+5}{4}$

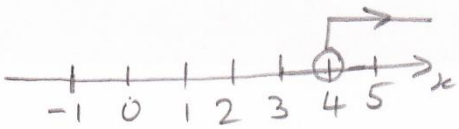
(c) $-1 < \frac{4x-5}{3} < 5$

(d) $-1 < \frac{5-4x}{3} < 5$

ANSWERS/SOLUTIONS (solutions not unique)

(1) $x+5>9$

$x>4$

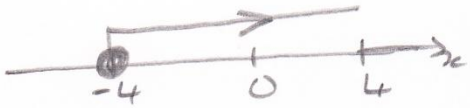


(2) $x-5<-2$

$x<3$

(3) $x+9\geq 5$

$x\geq -4$



(4) $5-x\leq 5$

$0\leq x$

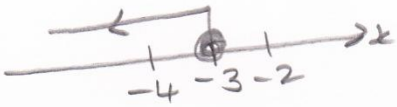
OR $x\geq 0$

(5) $6-x\geq 9$

$6-9\geq x$

$-3\geq x$

OR $x\leq -3$

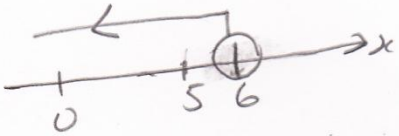


(6) $4-x>-2$

$4+2>x$

$6>x$

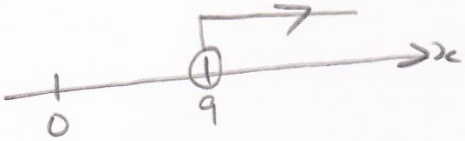
OR $x<6$



(7) $2x>18$

$x>\frac{18}{2}$

$x>9$



(8) $3x\leq -12$

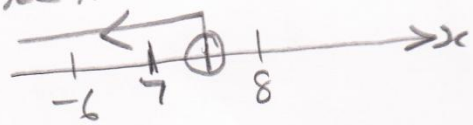
$x\leq -4$

(9) $15>2x$ OR $2x<15$

$\frac{15}{2}>x$ $x<7.5$

$7.5>x$

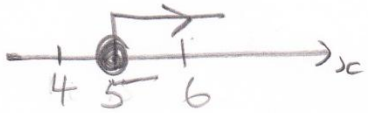
OR $x<7.5$



(10) $3x - 7 \geq 8$

$3x \geq 15$

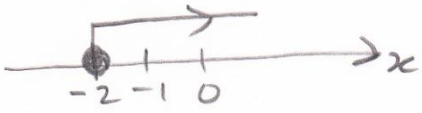
$x \geq 5$



(11) $3x + 7 \geq 1$

$3x \geq -6$

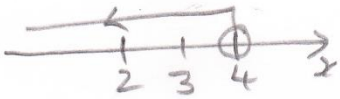
$x \geq -2$



(12) $4x - 6 < 10$

$4x < 16$

$x < 4$



(13) $7 - 3x \geq 13$

$7 - 13 \geq 3x$

$-6 \geq 3x$

$-2 \geq x$

OR $x \leq -2$

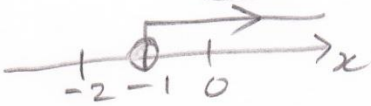
(14) $7 - 3x < 10$

$7 - 10 < 3x$

$-3 < 3x$

$-1 < x$

OR $x > -1$

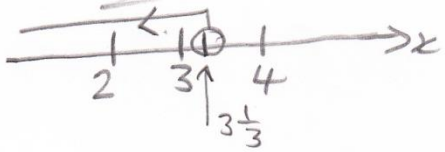


(15) $7 > 3x - 3$ OR $3x - 3 < 7$

$10 > 3x$ $\quad 3x < 10$

$\frac{10}{3} > x$ $\quad x < \frac{10}{3}$

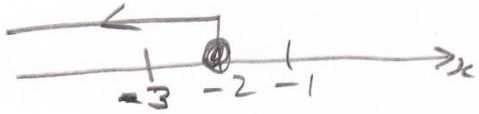
OR $x < 3\frac{1}{3}$



(16) $2x - 3 \leq x - 5$

$2x - x \leq -5 + 3$

$x \leq -2$

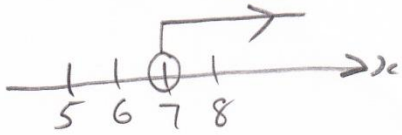


(17) $4x - 5 > x + 16$

$4x - x > 16 + 5$

$3x > 21$

$x > 7$

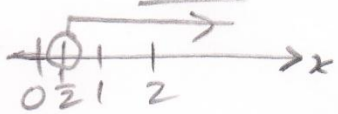


(18) $5x > x + 2$

$4x > 2$

$x > \frac{2}{4}$

or $x > \frac{1}{2}$



(19) $3(x-2) \leq 12$ OR $x - 2 \leq 4$

$3x - 6 \leq 12$

$3x \leq 18$

$x \leq 6$

$x \leq 6$

(20) $4x - 5 \leq 2(3x - 7)$

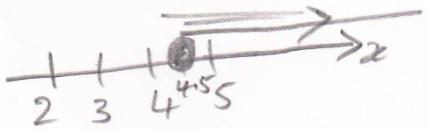
$4x - 5 \leq 6x - 14$

$-5 + 14 \leq 6x - 4x$

$9 \leq 2x$

$4.5 \leq x$

OR $x \geq 4.5$



(21) $-9 < 4x - 5 \leq 11$

$-9 + 5 < 4x \leq 11 + 5$

$-4 < 4x \leq 16$

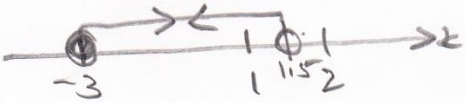
$-1 < x \leq 4$

(22) $-1 \leq 2x + 5 < 8$

$-1 - 5 \leq 2x < 8 - 5$

$-6 \leq 2x < 3$

$-3 \leq x < 1.5$



(23) $11 \geq 4x - 5 \geq -9$

$16 \geq 4x \geq -4$

$4 \geq x \geq -1$

OR $-1 \leq x \leq 4$

(24) $-1 < 5 - x \leq 7$

Time to split

$-1 < 5 - x$ and $5 - x \leq 7$

$x < 5 + 1$ $\quad 5 - 7 \leq x$

$x < 6$ $\quad -2 \leq x$

combine

$-2 \leq x < 6$

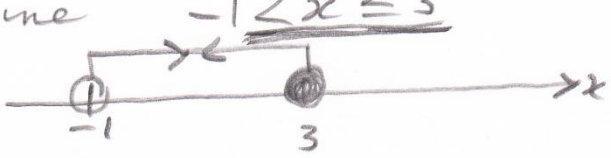
(25) $-1 \leq 5-2x < 7$

Split $-1 \leq 5-2x$ and $5-2x < 7$

$2x \leq 6$ $\quad -2 < 2x$

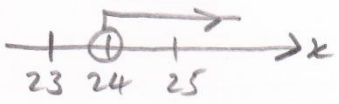
$x \leq 3$ $\quad -1 < x$

Combine $-1 < x \leq 3$



(26) $\frac{x}{6} > 4$

$x > 24$



(27) $\frac{2x}{3} \leq 4$

$2x \leq 12$

$x \leq 6$

(28) $\frac{2x}{3} + 5 \leq 9$

$\frac{2x}{3} \leq 4$

$2x \leq 12$

$x \leq 6$

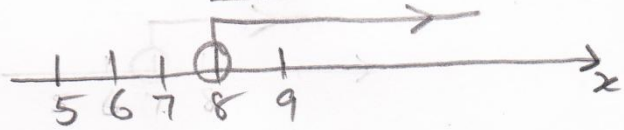
Same diagram.

(29) $\frac{2x-1}{3} > 5$

$2x-1 > 15$

$2x > 16$

$x > 8$



(30) $\frac{2x-1}{3} + 1 > 6$

$\frac{2x-1}{3} > 5$

$2x-1 > 15$

$2x > 16$

$x > 8$

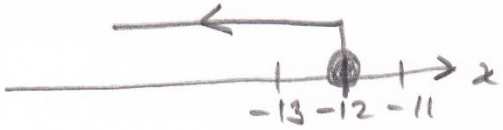
(31) $5 - \frac{x}{3} \geq 9$ When the subject (x) is negative, "take it to the otherside"

$5 - 9 \geq \frac{x}{3}$

$-4 \geq \frac{x}{3}$

$-12 \geq x$

OR $x \leq -12$



(32) $\frac{7-2x}{5} > 4$

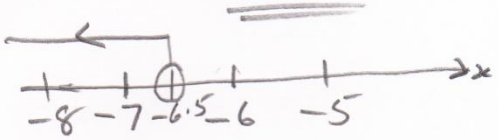
$7 - 2x > 20$

$7 - 20 > 2x$

$-13 > 2x$

$-6.5 > x$

OR $x < -6.5$



(33) $\frac{12-2x}{5} - 1 > 3$

$\frac{12-2x}{5} > 4$

$12 - 2x > 20$

$12 - 20 > 2x$

$-8 > 2x$

$-4 > x$

OR $x < -4$

Miscellaneous

(1) @ $-2, -1, 0$

(b) $8y \geq y + 14$

$7y \geq 14$

$y \geq 2$

(2) @ $-1, 0, 1$

(b) $4y - 5 < 10 - y$

$4y + y < 10 + 5$

$5y < 15$

$y < 3$

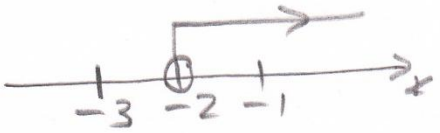
③ (a) $-1, 0, 1, 2, 3$

(b) $4x - 5 \geq 7$

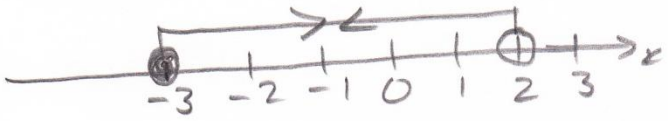
$4x \geq 12$

$x \geq 3$

④ $x > -2$



⑤ $-3 \leq x < 2$



⑥ (a) $3x - 2 < 2x - 2$

$x < 0$ greatest value $= -1$

(b) $8y + 46 \leq 14$

$8y \leq 14 - 46$

$8y \leq -32$

$y \leq -4$

greatest value $= -4$

⑦ Perimeter of $R = 2(3x + x)$

$= 2 \times 4x$

$= 8x$

Perimeter of $S = 4(x + 1)$

$= 4x + 4$

$8x < 4x + 4$

$4x < 4$

$x < 1$

⑧ (a) $3(2y + 1) < 2(4y)$

$6y + 3 < 8y$

$3 < 2y$

$1.5 < y$

or $y > 1.5$

(b) 2

(9) $3x - 5 > 10 - 2x$

$3x + 2x > 10 + 5$

$5x > 15$

$x > 3$

(10) $6(2x - 1) > 5(2x + 1)$

$12x - 6 > 10x + 5$

$2x > 11$

$x > 5.5$

(11) $8 - 3x \leq 5 - 2x$

$8 - 5 \leq -2x + 3x$

$3 \leq x$

or $x \geq 3$

(12) (a) $\frac{4x - 3}{2} > \frac{x + 1}{4}$

$\times 4 \Rightarrow 2(4x - 3) > x + 1$

$8x - 6 > x + 1$

$7x > 7$

$x > 1$

Continued on the next page.

(12) (b) $\frac{4x-3}{2} + 1 > \frac{x+5}{4}$

$\times 4 \Rightarrow \quad 2(4x-3) + 4 > x+5$

$8x - 6 + 4 > x + 5$

$8x - x > 5 + 6 - 4$

$7x > 7$

$x > 1$

(c) $-1 < \frac{4x-5}{3} < 5$

$\times 3 \Rightarrow \quad -3 < 4x - 5 < 15$

$-3 + 5 < 4x < 15 + 5$

$2 < 4x < 20$

$0.5 < x < 5$

(d) $-1 < \frac{5-4x}{3} < 5$

$\times 3 \Rightarrow \quad -3 < 5 - 4x < 15$

Split $-3 < 5 - 4x$ and $5 - 4x < 15$

$4x < 8$ $\qquad$ $5 - 15 < 4x$

$x < 2$ $\qquad$ $-10 < 4x$

$-2.5 < x$

Combine

$-2.5 < x < 2$

I hope you find this useful. Apologies for the hand written solutions. Time is to blame!

If you find any errors, please let me know. Thank you.