UNIT 17 Using Graphs

NC : Further Material 4
Algebra b, c, d

		St	Ac	Ex	Sp
TOPICS (Text and Practice Book)					
17.1	*Transformations of Graphs*	✗	✗	✓	✓
17.2	*Areas Under Graphs*	✗	✗	✓	✓
17.3	*Tangents to Curves*	✗	✗	✓	✓
17.4	*Finding Coefficients*	✗	✗	✓	✓
Activities					
17.1	*Pursuit Curves*	✗	✗	✓	✓
17.2	*Relationship between Distance-Time and Speed-Time Graphs*	✗	✗	✓	✓
17.3	*Race Commentary*	✗	✗	✓	✓
17.4	*Estimating Distances Travelled*	✗	✗	✓	✓
OH Slides					
17.1	*Graph Transforms 1*	✗	✗	✓	✓
17.2	*Graph Transforms 2*	✗	✗	✓	✓
17.3	*Distance-Time Graphs*	✗	✗	✓	✓
17.4	*Speed-Time Graphs*	✗	✗	✓	✓
Mental Tests					
17.1		✗	✗	✓	✓
17.2		✗	✗	✓	✓
Revision Test					
17.1		✗	✗	✓	✓

UNIT 17 *Using Graphs* Teaching Notes

Background and Preparatory Work

This unit starts with the topic of transformation of graphs, which relates to the earlier work on transformation. In the early work, we were transforming shapes with

UNIT 14

- reflection
- rotation
- translation

and combinations of these. Here we are transforming graphs of functions, which in some ways is the algebraic equivalent of the earlier work. Here, though, we will be considering four types of transformation:

$$\left.\begin{array}{l} y = f(x) + a \\ y = f(x + a) \end{array}\right\} \text{corresponding to horizontal or vertical translations}$$

$$\left.\begin{array}{l} y = f(kx) \\ y = kf(x) \end{array}\right\} \text{ these are horizontal or vertical stretches/compressions.}$$

There are special cases which can be related back to work in Unit 14, namely:

$$k = -1 \text{ in } y = f(kx) \Rightarrow y = f(-x),$$

which is a reflection in the y-axis; and

$$k = -1 \text{ in } y = kf(x) \Rightarrow y = -f(x)$$

which is a reflection in the *x*-axis. It may be helpful for students to appreciate that these geometric and algebraic approaches are essentially equivalent.

The latter part of this unit deals with the use of graphs in practical situations, namely speed-time and distance-time graphs. This also relates back to work in an earlier unit but takes the analysis a stage further using:

UNIT 13.6

- the area under a speed-time graph to find the distance,
- the gradient of a distance-time graph as the speed,
- the gradient of a speed-time graph as the acceleration.

Of course, these topics relate back to the beginnings of kinematics and the development of Calculus by *Newton* (1643–1727) and *Leibnitz* (1646–1716), which were the starting points for the modern and highly relevant topics of Mechanics.

Teaching Points

Introduction

This unit, which is only for the *Express/Special* routes, links together the transformation of functions (and hence their graphs) and uses graphs to illustrate physical situations. It builds on the work of transformation geometry (Unit 14) and travel graphs (Unit 13, section 6).

Language/Notation

The language needed for the transformation section includes:

- translation
- compression
- stretches
- reflection

while the other sections require a knowledge of

- tangents - gradient of curves
- trapezium rule (for estimating the area under a curve).

Key Points

- Speed (velocity) is the gradient of distance-time graphs.
- Acceleration is the gradient of speed-time graphs.
- The area under a speed-time graph is the distance travelled.

Misconceptions

Note that:

- if data points (x_i, y_i) follow a relationship of the form $y = k\sqrt{x}$, then plotting points y_i against $\sqrt{x_i}$ should approximate to a straight line;
- care must taken with units, particularly when finding the distance as the area under a speed-time graph. e.g. distance in metres is found when speed is in metres per second and time is in seconds;
- $y = f(x + a)$ moves the graph $y = f(x)$ distance *a* to the left, not the right.
- when $y = f(x)$ is defined as $0 \le x \le a$, then $y = f(2x)$ is defined as $0 \le x \le \frac{a}{2}$, not $0 \le x \le 2a$.

Key Concepts

For transformations:

- $y = f(x) + a$ moves the graph of $y = f(x)$ up by *a* units,
- $y = f(x + a)$ moves the graph of $y = f(x)$ to the left by *a* units
- $y = f(kx)$ compresses the graph of $y = f(x)$ by a factor *k* in the *x*-direction
- $y = kf(x)$ stretches the graph of $y = f(x)$ by a factor *k* in the *y*-direction.