M08/5/MATME/SP2/ENG/TZ1/XX+

22087304

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 8 May 2008 (morning)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

In a school with 125 girls, each student is tested to see how many sit-up exercises (sit-ups) she can do in one minute. The results are given in the table below.

Number of sit-ups	**Number of students**	**Cumulative number of students**
15	11	11
16	21	32
17	33	p
18	q	99
19	18	117
20	8	125

(This question continues on the following page)

(Question 1 continued)

(a) (i) Write down the value of p.

(ii) Find the value of q. *[3 marks]*

(b) Find the median number of sit-ups. *[2 marks]*

(c) Find the mean number of sit-ups. *[2 marks]*

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2. *[Maximum mark: 6]*

The diagram below shows triangle PQR. The length of [PQ] is 7 cm, the length of [PR] is 10 cm, and $P\hat{Q}R$ is 75°.

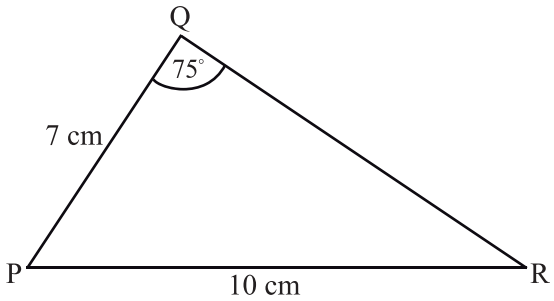

diagram not to scale

(a) Find $P\hat{R}Q$. *[3 marks]*

(b) Find the area of triangle PQR. *[3 marks]*

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3. *[Maximum mark: 6]*

The diagram below shows a circle centre O, with radius r. The length of arc ABC is 3π cm and $A\hat{O}C = \frac{2\pi}{9}$.

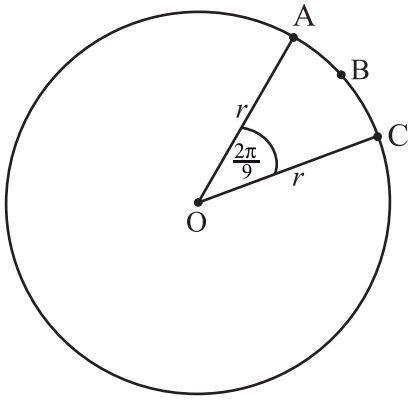

diagram not to scale

(a) Find the value of r. *[2 marks]*

(b) Find the perimeter of sector OABC. *[2 marks]*

(c) Find the area of sector OABC. *[2 marks]*

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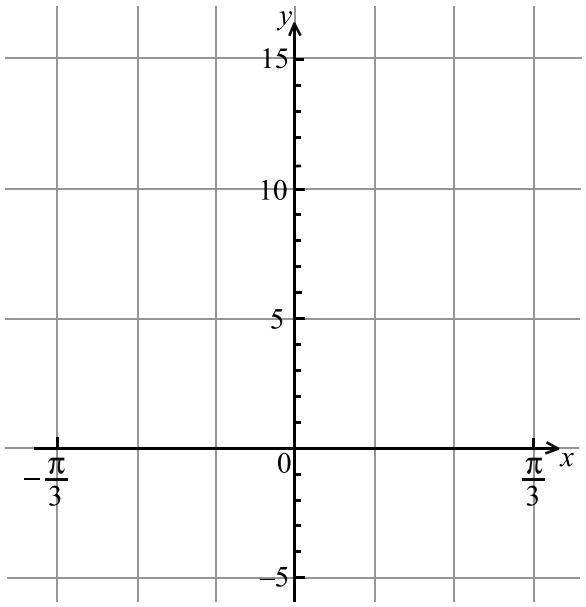
4. *[Maximum mark: 6]*

Let $f(x) = 4\tan^2 x - 4\sin x$, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$.

(a) On the grid below, sketch the graph of $y = f(x)$. *[3 marks]*



(b) Solve the equation $f(x) = 1$. *[3 marks]*

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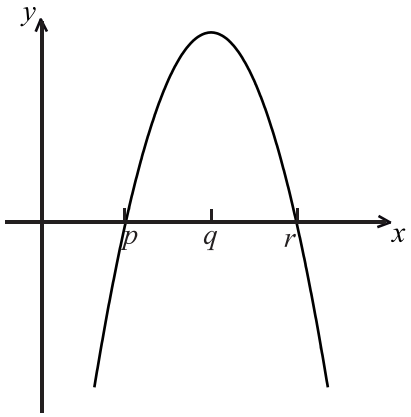
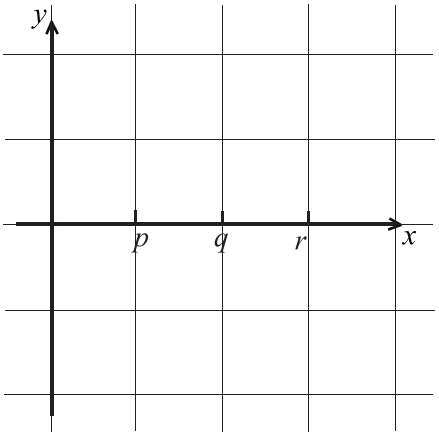
5. *[Maximum mark: 6]*

The diagram below shows part of the graph of the **gradient** function, $y = f'(x)$.



(a) On the grid below, sketch a graph of $y = f''(x)$, clearly indicating the x-intercept. *[2 marks]*



(b) Complete the table, for the graph of $y = f(x)$. *[2 marks]*

	x-coordinate
(i) Maximum point on f	
(ii) Inflexion point on f	

(c) Justify your answer to part (b)(ii). *[2 marks]*

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6. *[Maximum mark: 7]*

A factory makes switches. The probability that a switch is defective is 0.04. The factory tests a random sample of 100 switches.

(a) Find the mean number of defective switches in the sample. *[2 marks]*

(b) Find the probability that there are exactly six defective switches in the sample. *[2 marks]*

(c) Find the probability that there is at least one defective switch in the sample. *[3 marks]*

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7. *[Maximum mark: 7]*

Let $\boldsymbol{v} = 3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{w} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$. The vector $\boldsymbol{v} + p\boldsymbol{w}$ is perpendicular to $\boldsymbol{w}$. Find the value of p.

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

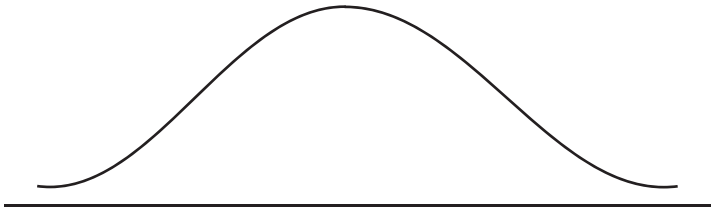
(a) One biscuit is chosen at random from the box. Find the probability that this biscuit

(i) weighs less than 8 g;

(ii) weighs between 6 g and 8 g. *[4 marks]*

(b) Five percent of the biscuits in the box weigh less than d grams.

(i) Copy and complete the following normal distribution diagram, to represent this information, by indicating d, and shading the appropriate region.



(ii) Find the value of d. *[5 marks]*

(c) The weights of biscuits in another box are normally distributed with mean μ and standard deviation 0.5 g. It is known that $20\,\%$ of the biscuits in this second box weigh less than 5 g.

Find the value of μ. *[4 marks]*

9. *[Maximum mark: 18]*

The point O has coordinates $(0, 0, 0)$, point A has coordinates $(1, -2, 3)$ and point B has coordinates $(-3, 4, 2)$.

(a) (i) Show that $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

(ii) Find $B\hat{A}O$. *[8 marks]*

(b) The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on L_1. *[2 marks]*

(c) The line L_2 passes through A and is parallel to $\overrightarrow{OB}$.

(i) Find a vector equation for L_2, giving your answer in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

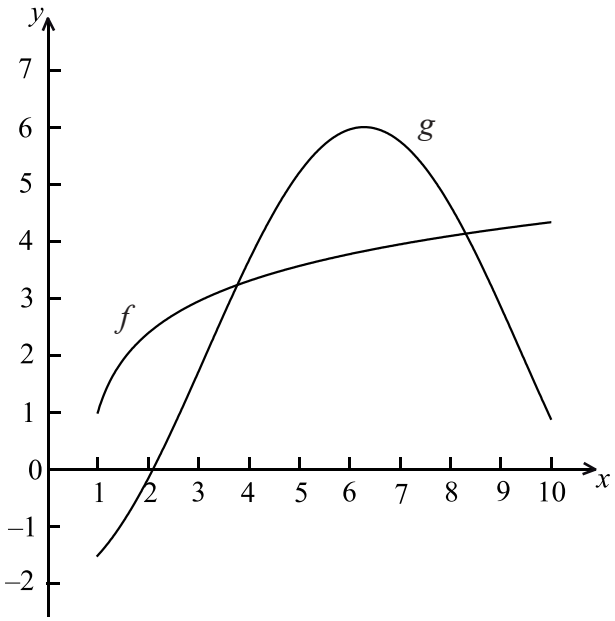
(ii) Point $C(k, -k, 5)$ is on L_2. Find the coordinates of C. *[6 marks]*

(d) The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, and passes through the point C.

Find the value of p at C. *[2 marks]*

10. *[Maximum mark: 14]*

The following diagram shows the graphs of $f(x) = \ln(3x-2)+1$ and $g(x) = -4\cos(0.5x)+2$, for $1 \le x \le 10$.



(a) Let A be the area of the region **enclosed** by the curves of f and g.

(i) Find an expression for A.

(ii) Calculate the value of A. *[6 marks]*

(b) (i) Find $f'(x)$.

(ii) Find $g'(x)$. *[4 marks]*

(c) There are two values of x for which the gradient of f is equal to the gradient of g. Find both these values of x. *[4 marks]*

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STANDARD LEVEL
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SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 6]*

Consider the infinite geometric sequence $3000, -1800, 1080, -648, \ldots$.

(a) Find the common ratio. *[2 marks]*

(b) Find the 10th term. *[2 marks]*

(c) Find the **exact** sum of the infinite sequence. *[2 marks]*

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2. *[Maximum mark: 5]*

Find the term in x^3 in the expansion of $\left(\frac{2}{3}x-3\right)^8$.

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3. *[Maximum mark: 7]*

Let $f(x) = 3x - e^{x-2} - 4$, for $-1 \le x \le 5$.

(a) Find the x-intercepts of the graph of f. *[3 marks]*

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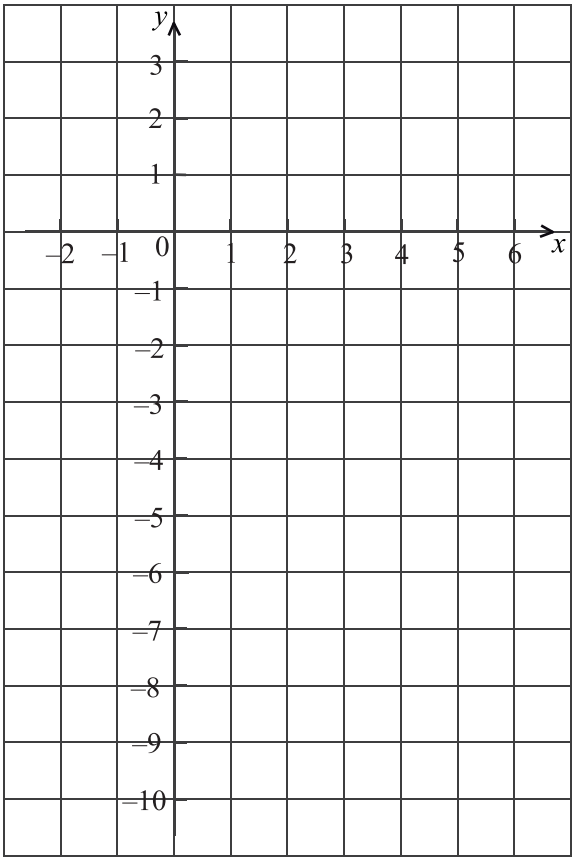
(b) On the grid below, sketch the graph of f. *[3 marks]*



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(Question 3 continued)

(c) Write down the gradient of the graph of f at $x = 2$. *[1 mark]*

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4. *[Maximum mark: 7]*

The following table shows the probability distribution of a discrete random variable X.

x	-1	0	2	3
$\mathrm{P}(X = x)$	0.2	$10k^2$	0.4	$3k$

(a) Find the value of k. *[4 marks]*

(b) Find the expected value of X. *[3 marks]*

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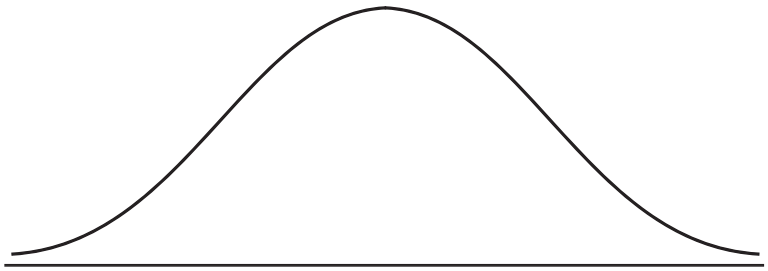
5. *[Maximum mark: 7]*

The heights of certain plants are normally distributed. The plants are classified into three categories.

The shortest 12.92 % are in category A.
The tallest 10.38 % are in category C.
All the other plants are in category B with heights between r cm and t cm.

(a) Complete the following diagram to represent this information. *[2 marks]*



(b) Given that the mean height is 6.84 cm and the standard deviation 0.25 cm, find the value of r and of t. *[5 marks]*

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6. *[Maximum mark: 7]*

Paula goes to work three days a week. On any day, the probability that she goes on a red bus is $\frac{1}{4}$.

(a) Write down the expected number of times that Paula goes to work on a red bus in one week. *[2 marks]*

In one week, find the probability that she goes to work on a red bus

(b) on exactly two days; *[2 marks]*

(c) on at least one day. *[3 marks]*

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7. *[Maximum mark: 6]*

The line L_1 is represented by $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line L_2 by $\boldsymbol{r}_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines L_1 and L_2 intersect at point T. Find the coordinates of T.

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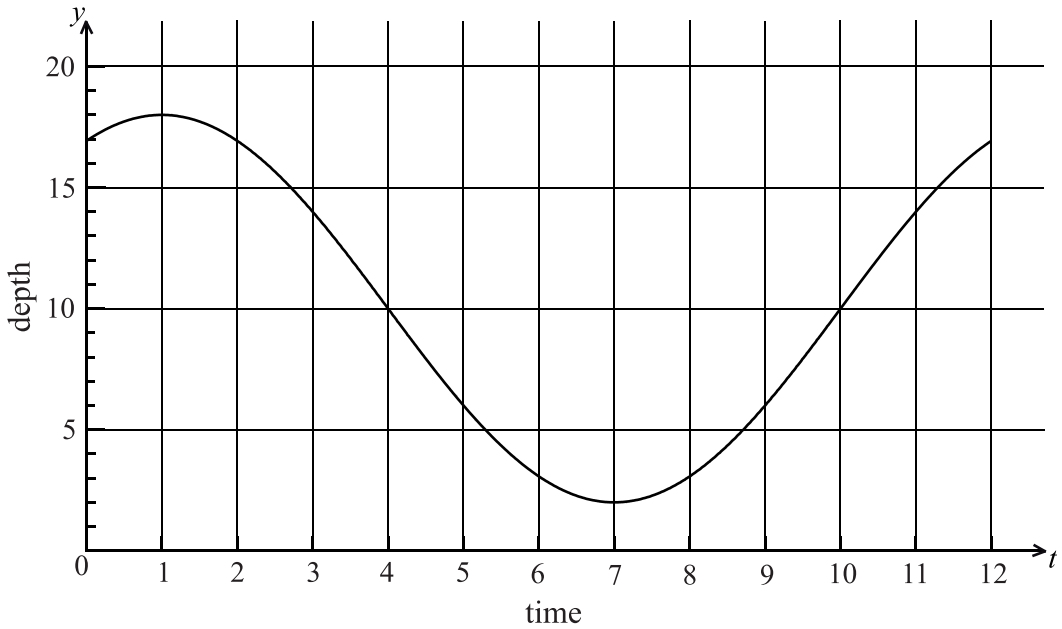
SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 11]*

The following graph shows the depth of water, y metres, at a point P, during one day. The time t is given in hours, from midnight to noon.



(a) Use the graph to write down an estimate of the value of t when

(i) the depth of water is minimum;

(ii) the depth of water is maximum;

(iii) the depth of the water is increasing most rapidly. *[3 marks]*

(b) The depth of water can be modelled by the function $y = A\cos\left(B(t-1)\right) + C$.

(i) Show that $A = 8$.

(ii) Write down the value of C.

(iii) Find the value of B. *[6 marks]*

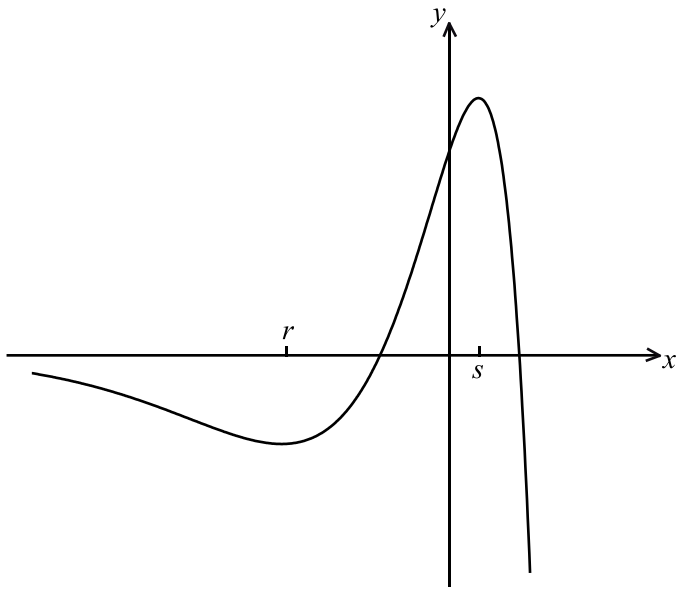
(c) A sailor knows that he cannot sail past P when the depth of the water is less than 12 m. Calculate the values of t between which he cannot sail past P. *[2 marks]*

9. *[Maximum mark: 17]*

Let $f(x) = e^x(1 - x^2)$.

(a) Show that $f'(x) = e^x(1 - 2x - x^2)$. *[3 marks]*

Part of the graph of $y = f(x)$, for $-6 \le x \le 2$, is shown below. The x-coordinates of the local minimum and maximum points are r and s respectively.



(b) Write down the **equation** of the horizontal asymptote. *[1 mark]*

(c) Write down the value of r and of s. *[4 marks]*

(d) Let L be the normal to the curve of f at P(0, 1). Show that L has equation $x + y = 1$. *[4 marks]*

(e) Let R be the region enclosed by the curve $y = f(x)$ and the line L.

(i) Find an expression for the area of R.

(ii) Calculate the area of R. *[5 marks]*

10. *[Maximum mark: 17]*

A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After n years the number of taxis, T, in the city is given by

$$T = 280 \times 1.12^n .$$

(a) (i) Find the number of taxis in the city at the end of 2005.

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000. *[6 marks]*

(b) At the end of 2000 there were 25 600 people in the city who used taxis. After n years the number of people, P, in the city who used taxis is given by

$$P = \frac{2\,560\,000}{10 + 90\mathrm{e}^{-0.1n}} .$$

(i) Find the value of P at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of P be double its value at the end of 2000? Justify your answer. *[6 marks]*

(c) Let R be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$.

(i) Find the value of R at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis? *[5 marks]*

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SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 6]*

Let $f(x) = 2x^2 + 4x - 6$.

(a) Express $f(x)$ in the form $f(x) = 2(x-h)^2 + k$. *[3 marks]*

(b) Write down the equation of the axis of symmetry of the graph of f. *[1 mark]*

(c) Express $f(x)$ in the form $f(x) = 2(x-p)(x-q)$. *[2 marks]*

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2. *[Maximum mark: 6]*

(a) Expand $(x-2)^4$ and simplify your result. *[3 marks]*

(b) Find the term in x^3 in $(3x+4)(x-2)^4$. *[3 marks]*

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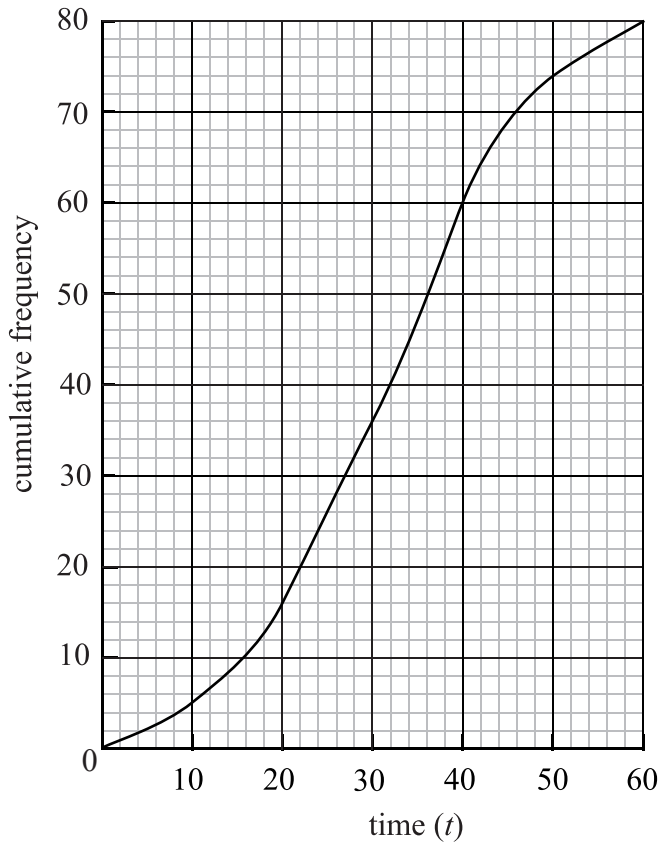
3. *[Maximum mark: 6]*

The following is a cumulative frequency diagram for the time t, in minutes, taken by 80 students to complete a task.



(a) Write down the median. *[1 mark]*

(b) Find the interquartile range. *[3 marks]*

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(Question 3 continued)

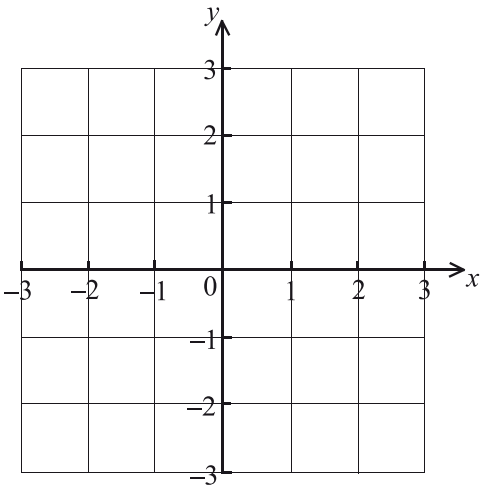
(c) Complete the frequency table below. *[2 marks]*

Time (minutes)	**Number of students**
$0 \le t < 10$	5
$10 \le t < 20$	
$20 \le t < 30$	20
$30 \le t < 40$	24
$40 \le t < 50$	
$50 \le t < 60$	6

4. *[Maximum mark: 8]*

Let $f(x) = x\cos(x - \sin x)$, $0 \le x \le 3$.

(a) Sketch the graph of f on the following set of axes. *[3 marks]*



(b) The graph of f intersects the x-axis when $x = a$, $a \ne 0$. Write down the value of a. *[1 mark]*

(c) The graph of f is revolved $360°$ about the x-axis from $x = 0$ to $x = a$. Find the volume of the solid formed. *[4 marks]*

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5. *[Maximum mark: 5]*

The probability of obtaining heads on a biased coin is 0.18. The coin is tossed seven times.

(a) Find the probability of obtaining **exactly** two heads. *[2 marks]*

(b) Find the probability of obtaining **at least** two heads. *[3 marks]*

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6. *[Maximum mark: 7]*

A ship leaves port A on a bearing of 030°. It sails a distance of 25 km to point B. At B, the ship changes direction to a bearing of 100°. It sails a distance of 40 km to reach point C. This information is shown in the diagram below.

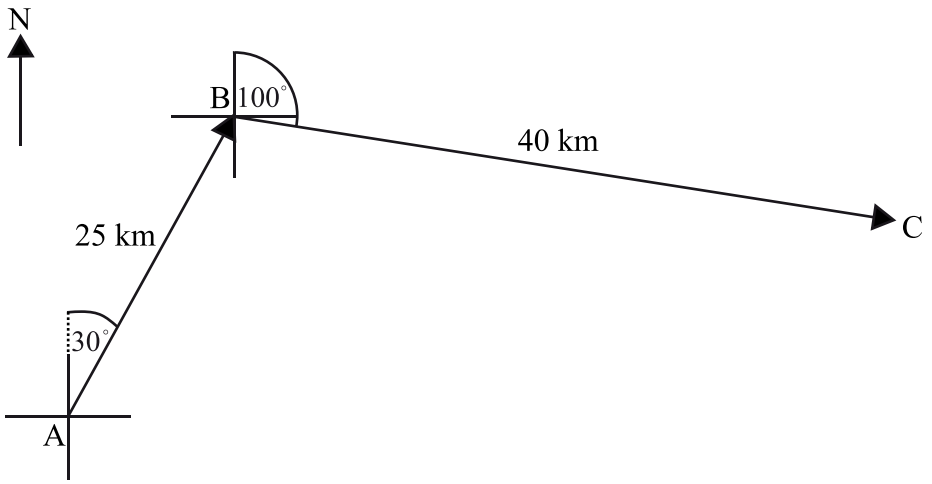

diagram not to scale

A second ship leaves port A and sails directly to C.

(a) Find the distance the second ship will travel. *[4 marks]*

(b) Find the bearing of the course taken by the second ship. *[3 marks]*

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7. *[Maximum mark: 7]*

The scores of a test given to students are normally distributed with a mean of 21. 80 % of the students have scores less than 23.7.

(a) Find the standard deviation of the scores. *[3 marks]*

A student is chosen at random. This student has the same probability of having a score less than 25.4 as having a score greater than b.

(b) (i) Find the probability the student has a score less than 25.4.

(ii) Find the value of b. *[4 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 18]*

The diagram shows a parallelogram ABCD.

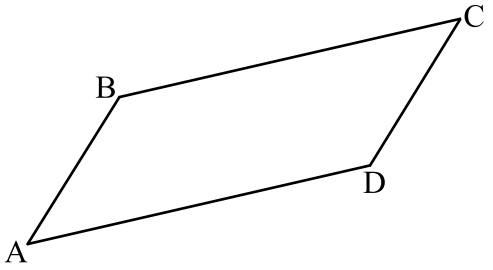

diagram not to scale

The coordinates of A, B and D are $A(1, 2, 3)$, $B(6, 4, 4)$ and $D(2, 5, 5)$.

(a) (i) Show that $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$.

(ii) Find $\overrightarrow{AD}$.

(iii) **Hence** show that $\overrightarrow{AC} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix}$. *[5 marks]*

(b) Find the coordinates of point C. *[3 marks]*

(c) (i) Find $\overrightarrow{AB} \bullet \overrightarrow{AD}$.

(ii) **Hence** find angle *A*. *[7 marks]*

(d) Hence, or otherwise, find the area of the parallelogram. *[3 marks]*

9. *[Maximum mark: 14]*

Let $f(x) = e^{2x}\cos x$, $-1 \le x \le 2$.

(a) Show that $f'(x) = e^{2x}(2\cos x - \sin x)$. *[3 marks]*

Let the line L be the normal to the curve of f at $x = 0$.

(b) Find the equation of L. *[5 marks]*

The graph of f and the line L intersect at the point $(0, 1)$ and at a second point P.

(c) (i) Find the x-coordinate of P.

(ii) Find the area of the region **enclosed** by the graph of f and the line L. *[6 marks]*

10. *[Maximum mark: 13]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 1 \\ 0 & 2 & -2 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

The matrix $\boldsymbol{B}$ satisfies the equation $\left(\boldsymbol{I} - \frac{1}{2}\boldsymbol{B}\right)^{-1} = \boldsymbol{A}$, where $\boldsymbol{I}$ is the 3×3 identity matrix.

(b) (i) Show that $\boldsymbol{B} = -2(\boldsymbol{A}^{-1} - \boldsymbol{I})$.

(ii) Find $\boldsymbol{B}$.

(iii) Write down $\det \boldsymbol{B}$.

(iv) **Hence**, explain why $\boldsymbol{B}^{-1}$ exists. *[6 marks]*

Let $\boldsymbol{BX} = \boldsymbol{C}$, where $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

(c) (i) Find $\boldsymbol{X}$.

(ii) Write down a system of equations whose solution is represented by $\boldsymbol{X}$. *[5 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 8 May 2009 (morning)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

In an arithmetic series, the first term is -7 and the sum of the first 20 terms is 620.

(a) Find the common difference. *[3 marks]*

(b) Find the value of the 78th term. *[2 marks]*

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2. *[Maximum mark: 7]*

The circle shown has centre O and radius 3.9 cm.

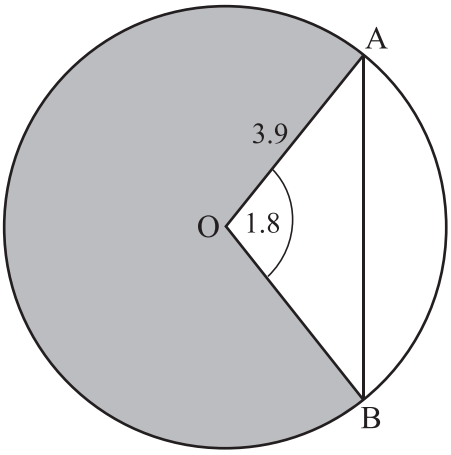

diagram not to scale

Points A and B lie on the circle and angle AOB is 1.8 radians.

(a) Find AB. *[3 marks]*

(b) Find the area of the shaded region. *[4 marks]*

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3. *[Maximum mark: 6]*

Let $f(x) = \frac{3x}{2} + 1$, $g(x) = 4\cos\left(\frac{x}{3}\right) - 1$. Let $h(x) = (g \circ f)(x)$.

(a) Find an expression for $h(x)$. *[3 marks]*

(b) Write down the period of h. *[1 mark]*

(c) Write down the range of h. *[2 marks]*

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4. *[Maximum mark: 6]*

A random variable X is distributed normally with mean 450 and standard deviation 20.

(a) Find $\mathrm{P}(X \leq 475)$. *[2 marks]*

(b) Given that $\mathrm{P}(X > a) = 0.27$, find a. *[4 marks]*

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5. *[Maximum mark: 6]*

Two lines with equations $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$ and $\boldsymbol{r}_2 = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$ intersect at the point P. Find the coordinates of P.

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6. *[Maximum mark: 7]*

In a geometric series, $u_1 = \frac{1}{81}$ and $u_4 = \frac{1}{3}$.

(a) Find the value of r. *[3 marks]*

(b) Find the smallest value of n for which $S_n > 40$. *[4 marks]*

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7. *[Maximum mark: 8]*

In any given season, a soccer team plays 65 % of their games at home.
When the team plays at home, they win 83 % of their games.
When they play away from home, they win 26 % of their games.

The team plays one game.

(a) Find the probability that the team wins the game. *[4 marks]*

(b) If the team does not win the game, find the probability that the game was played at home. *[4 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

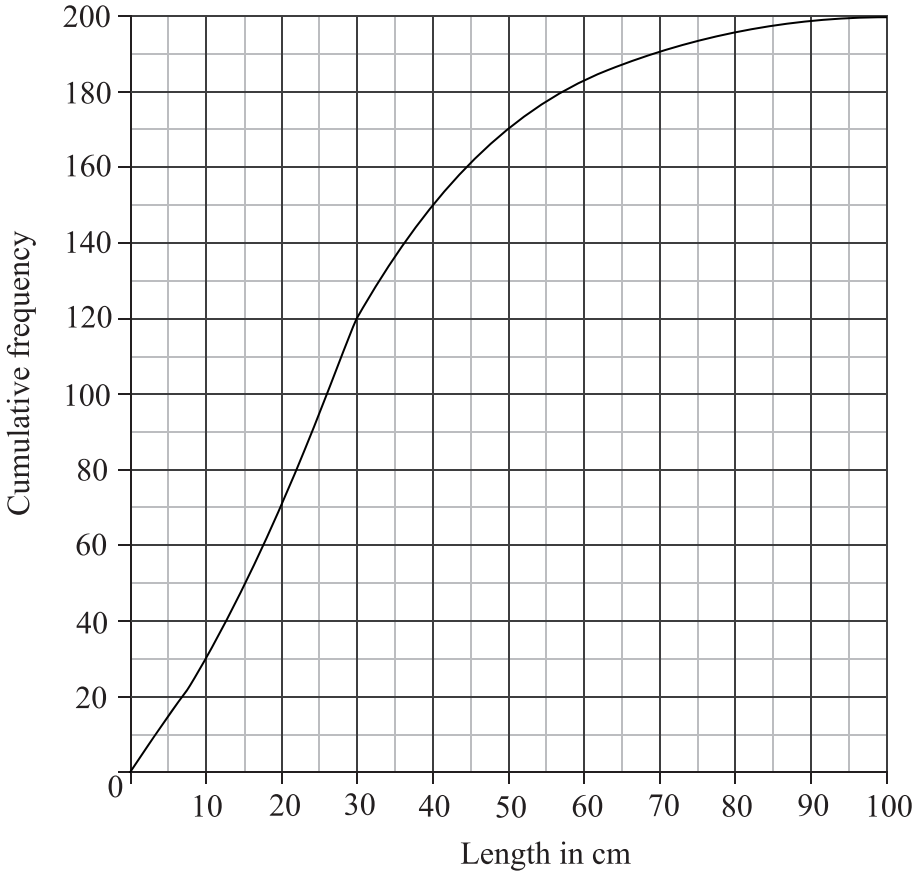
8. *[Maximum mark: 15]*

A fisherman catches 200 fish to sell. He measures the lengths, l cm of these fish, and the results are shown in the frequency table below.

Length l cm	$0 \le l < 10$	$10 \le l < 20$	$20 \le l < 30$	$30 \le l < 40$	$40 \le l < 60$	$60 \le l < 75$	$75 \le l < 100$
Frequency	30	40	50	30	33	11	6

(a) Calculate an estimate for the standard deviation of the lengths of the fish. *[3 marks]*

(b) A cumulative frequency diagram is given below for the lengths of the fish.



(This question continues on the following page)

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(Question 8 (b) continued)

Use the graph to answer the following.

(i) Estimate the interquartile range.

(ii) Given that 40 % of the fish have a length more than k cm, find the value of k. *[6 marks]*

In order to sell the fish, the fisherman classifies them as small, medium or large.

Small fish have a length less than 20 cm.
Medium fish have a length greater than or equal to 20 cm but less than 60 cm.
Large fish have a length greater than or equal to 60 cm.

(c) Write down the probability that a fish is small. *[2 marks]*

The cost of a small fish is \$4, a medium fish \$10, and a large fish \$12.

(d) Copy and complete the following table, which gives a probability distribution for the cost \$$X$. *[2 marks]*

Cost \$$X$	4	10	12
$\mathrm{P}(X = x)$		0.565	

(e) Find $\mathrm{E}(X)$. *[2 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 15]*

Let $f(x) = ax^2 + bx + c$ where a, b and c are rational numbers.

(a) The point $\mathrm{P}(-4, 3)$ lies on the curve of f. Show that $16a - 4b + c = 3$. *[2 marks]*

(b) The points $\mathrm{Q}(6, 3)$ and $\mathrm{R}(-2, -1)$ also lie on the curve of f. Write down two other linear equations in a, b and c. *[2 marks]*

(c) These three equations may be written as a matrix equation in the form $\boldsymbol{AX} = \boldsymbol{B}$, where $\boldsymbol{X} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

(i) Write down the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$.

(ii) Write down $\boldsymbol{A}^{-1}$.

(iii) **Hence** or otherwise, find $f(x)$. *[8 marks]*

(d) Write $f(x)$ in the form $f(x) = a(x-h)^2 + k$, where a, h and k are rational numbers. *[3 marks]*

10. *[Maximum mark: 15]*

Let $f(x) = x^3 - 4x + 1$.

(a) Expand $(x+h)^3$. *[2 marks]*

(b) Use the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to show that the derivative of $f(x)$ is $3x^2 - 4$. *[4 marks]*

(c) The tangent to the curve of f at the point $\mathrm{P}(1, -2)$ is parallel to the tangent at a point Q. Find the coordinates of Q. *[4 marks]*

(d) The graph of f is decreasing for $p < x < q$. Find the value of p and of q. *[3 marks]*

(e) Write down the range of values for the gradient of f. *[2 marks]*

M09/5/MATME/SP2/ENG/TZ2/XX+

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 8 May 2009 (morning)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
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- Section A: answer all of Section A in the spaces provided.
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13 pages

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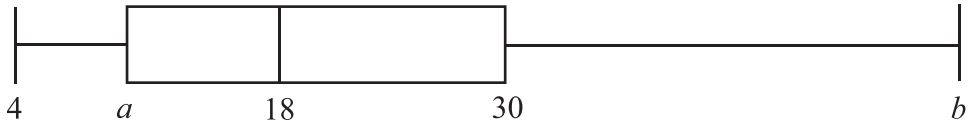
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

The following diagram is a box and whisker plot for a set of data.



The interquartile range is 20 and the range is 40.

(a) Write down the median value. *[1 mark]*

(b) Find the value of

(i) a;

(ii) b. *[4 marks]*

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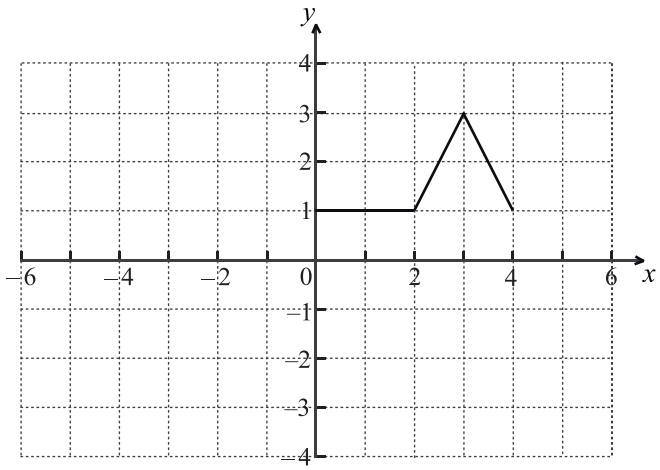
2. *[Maximum mark: 6]*

Consider the graph of f shown below.



(a) On the **same** grid sketch the graph of $y = f(-x)$. *[2 marks]*

(This question continues on the following page)

(Question 2 continued)

The following four diagrams show **images** of f under different transformations.

Diagram A

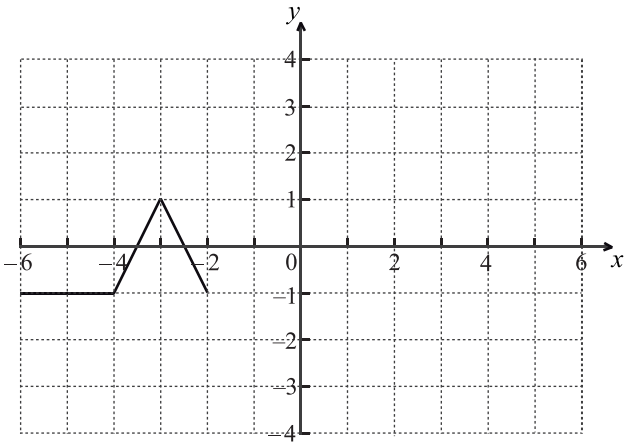


Diagram B

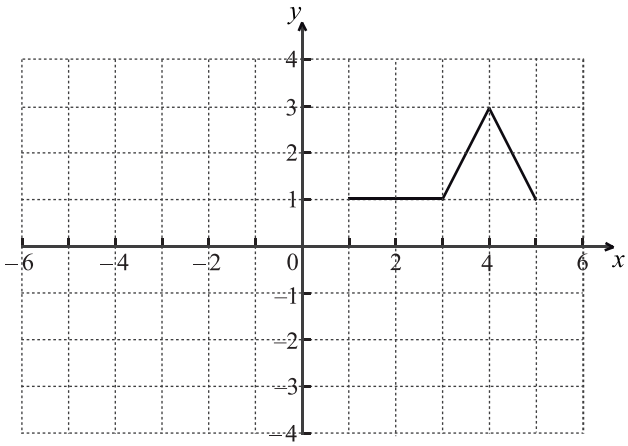


Diagram C

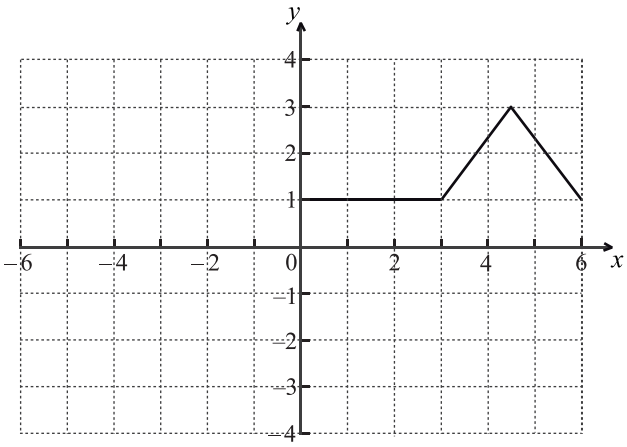
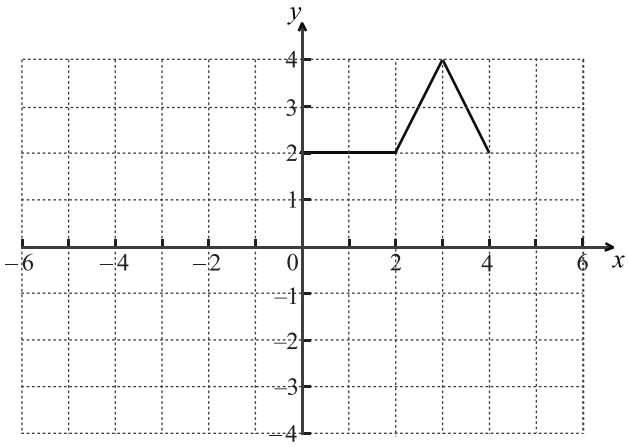


Diagram D



(b) Complete the following table. *[2 marks]*

Description of transformation	**Diagram letter**
Horizontal stretch with scale factor 1.5	
Maps f to $f(x)+1$	

(c) Give a full geometric description of the transformation that gives the image in Diagram A. *[2 marks]*

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3. *[Maximum mark: 5]*

Solve the equation $e^x = 4\sin x$, for $0 \le x \le 2\pi$.

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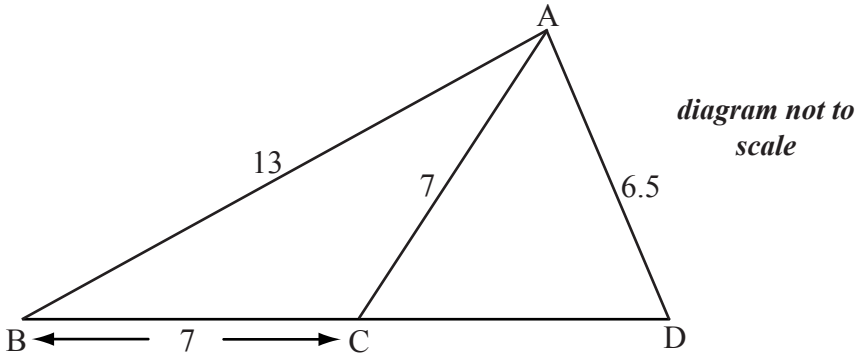
4. *[Maximum mark: 8]*

The diagram below shows a triangle ABD with $AB = 13$ cm and $AD = 6.5$ cm.
Let C be a point on the line BD such that $BC = AC = 7$ cm.



(a) Find the size of angle ACB. *[3 marks]*

(b) Find the size of angle CAD. *[5 marks]*

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5. *[Maximum mark: 7]*

(a) Expand $\sum_{r=4}^{7} 2^r$ as the sum of four terms. *[1 mark]*

(b) (i) Find the value of $\sum_{r=4}^{30} 2^r$.

(ii) Explain why $\sum_{r=4}^{\infty} 2^r$ cannot be evaluated. *[6 marks]*

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6. *[Maximum mark: 7]*

Consider the curve $y = \ln(3x - 1)$. Let P be the point on the curve where $x = 2$.

(a) Write down the gradient of the curve at P. *[2 marks]*

(b) The normal to the curve at P cuts the x-axis at R. Find the coordinates of R. *[5 marks]*

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7. *[Maximum mark: 7]*

The quadratic equation $kx^2 + (k-3)x + 1 = 0$ has two equal real roots.

(a) Find the possible values of k. *[5 marks]*

(b) **Write down** the values of k for which $x^2 + (k-3)x + k = 0$ has two equal real roots. *[2 marks]*

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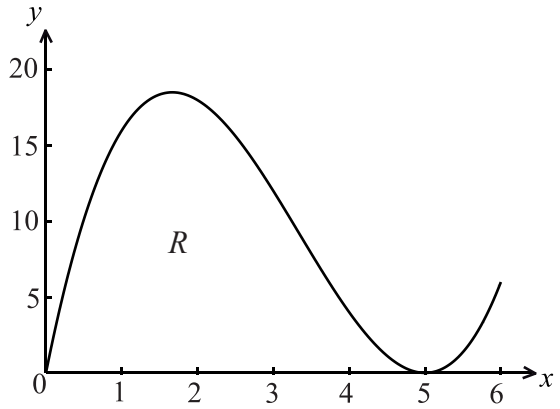
SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Let $f(x) = x(x-5)^2$, for $0 \le x \le 6$. The following diagram shows the graph of f.

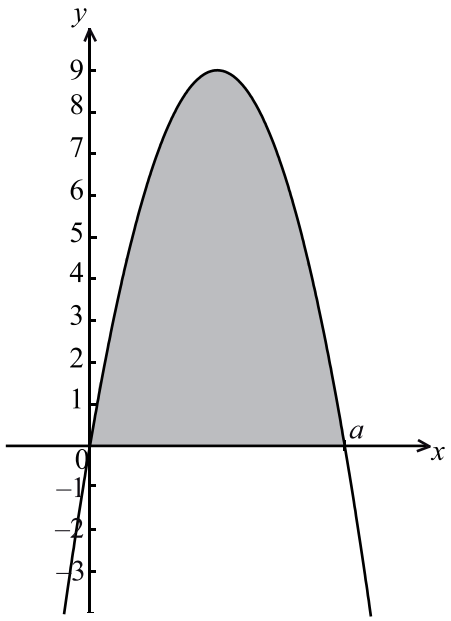


Let R be the region enclosed by the x-axis and the curve of f.

(a) Find the area of R. *[3 marks]*

(b) Find the volume of the solid formed when R is rotated through 360° about the x-axis. *[4 marks]*

(c) The diagram below shows a part of the graph of a quadratic function $g(x) = x(a-x)$. The graph of g crosses the x-axis when $x = a$.



The area of the shaded region is equal to the area of R. Find the value of a. *[7 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 13]*

A van can take either Route A or Route B for a particular journey.

If Route A is taken, the journey time may be assumed to be normally distributed with mean 46 minutes and a standard deviation 10 minutes.

If Route B is taken, the journey time may be assumed to be normally distributed with mean μ minutes and standard deviation 12 minutes.

(a) For Route A, find the probability that the journey takes **more** than 60 minutes. *[2 marks]*

(b) For Route B, the probability that the journey takes **less** than 60 minutes is 0.85. Find the value of μ. *[3 marks]*

(c) The van sets out at 06:00 and needs to arrive before 07:00.

(i) Which route should it take?

(ii) Justify your answer. *[3 marks]*

(d) On five consecutive days the van sets out at 06:00 and takes Route B. Find the probability that

(i) it arrives before 07:00 on all five days;

(ii) it arrives before 07:00 on at least three days. *[5 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 18]*

Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.

(a) Sketch the graph of f. *[3 marks]*

(b) Write down

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0. *[3 marks]*

(c) Hence write $f(x)$ in the form $p\sin(qx + r)$. *[3 marks]*

(d) Write down one value of x such that $f'(x) = 0$. *[2 marks]*

(e) Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions. *[2 marks]*

(f) Let $g(x) = \ln(x+1)$, for $0 \le x \le \pi$. There is a value of x, between 0 and 1, for which the gradient of f is equal to the gradient of g. Find this value of x. *[5 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 6 November 2009 (morning)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 6]*

In an arithmetic sequence, $S_{40} = 1900$ and $u_{40} = 106$. Find the value of u_1 and of d.

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2. *[Maximum mark: 6]*

Let $f(x) = \cos 2x$ and $g(x) = \ln(3x - 5)$.

(a) Find $f'(x)$. *[2 marks]*

(b) Find $g'(x)$. *[2 marks]*

(c) Let $h(x) = f(x) \times g(x)$. Find $h'(x)$. *[2 marks]*

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3. *[Maximum mark: 6]*

A multiple choice test consists of ten questions. Each question has five answers. Only one of the answers is correct. For each question, Jose randomly chooses one of the five answers.

(a) Find the expected number of questions Jose answers correctly. *[1 mark]*

(b) Find the probability that Jose answers exactly three questions correctly. *[2 marks]*

(c) Find the probability that Jose answers more than three questions correctly. *[3 marks]*

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4. *[Maximum mark: 6]*

Let $\boldsymbol{A} = \begin{pmatrix} 3 & 0 & 1 \\ 2 & -3 & 0 \\ 4 & -2 & 1 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Let $\boldsymbol{B}$ be a 3×3 matrix. Given that $\boldsymbol{AB} + \begin{pmatrix} -3 & 2 & 1 \\ 5 & 3 & 4 \\ -9 & 2 & 10 \end{pmatrix} = \begin{pmatrix} 7 & 6 & -7 \\ 6 & 5 & -8 \\ 1 & 7 & -5 \end{pmatrix}$, find $\boldsymbol{B}$. *[4 marks]*

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5. *[Maximum mark: 7]*

Consider the curve with equation $f(x) = px^2 + qx$, where p and q are constants. The point $\mathrm{A}(1, 3)$ lies on the curve. The tangent to the curve at A has gradient 8. Find the value of p and of q.

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6. *[Maximum mark: 7]*

Consider the independent events A and B. Given that $\mathrm{P}(B) = 2\,\mathrm{P}(A)$, and $\mathrm{P}(A \cup B) = 0.52$, find $\mathrm{P}(B)$.

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7. *[Maximum mark: 7]*

A farmer wishes to create a rectangular enclosure, ABCD, of area 525 m^2, as shown below.

The fencing used for side AB costs \$11 per metre. The fencing for the other three sides costs \$3 per metre. The farmer creates an enclosure so that the cost is a minimum. Find this minimum cost.

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*Do **NOT** write on this page.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

The following diagram shows a circle with centre O and radius 4 cm.

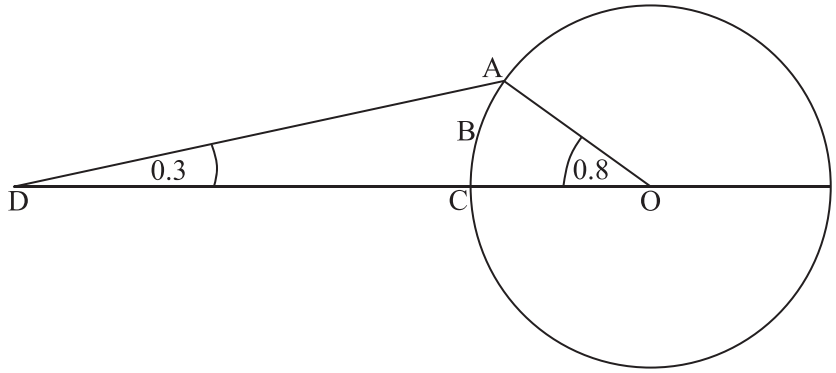

diagram not to scale

The points A, B and C lie on the circle. The point D is outside the circle, on (OC).
Angle ADC = 0.3 radians and angle AOC = 0.8 radians.

(a) Find AD. *[3 marks]*

(b) Find OD. *[4 marks]*

(c) Find the area of sector OABC. *[2 marks]*

(d) Find the area of region ABCD. *[4 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 15]*

Let $f(x) = 5\cos\frac{\pi}{4}x$ and $g(x) = -0.5x^2 + 5x - 8$, for $0 \le x \le 9$.

(a) On the same diagram, sketch the graphs of f and g. *[3 marks]*

(b) Consider the graph of f. Write down

(i) the x-intercept that lies between $x = 0$ and $x = 3$;

(ii) the period;

(iii) the amplitude. *[4 marks]*

(c) Consider the graph of g. Write down

(i) the two x-intercepts;

(ii) the equation of the axis of symmetry. *[3 marks]*

(d) Let R be the region enclosed by the graphs of f and g. Find the area of R. *[5 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 17]*

Consider the points $\mathrm{P}(2, -1, 5)$ and $\mathrm{Q}(3, -3, 8)$. Let L_1 be the line through P and Q.

(a) Show that $\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$. *[1 mark]*

(b) The line L_1 may be represented by $\boldsymbol{r} = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$.

(i) What information does the vector $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}$ give about L_1?

(ii) Write down another vector representation for L_1 using $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}$. *[3 marks]*

The point $\mathrm{T}(-1, 5, p)$ lies on L_1.

(c) Find the value of p. *[3 marks]*

The point T also lies on L_2 with equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ q \end{pmatrix}$.

(d) Show that $q = -3$. *[3 marks]*

(e) Let θ be the **obtuse** angle between L_1 and L_2. Calculate the size of θ. *[7 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 6 May 2010 (morning)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
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SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & -3 \\ -1 & -1 & 4 \\ 2 & 4 & -3 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Solve $\boldsymbol{AX} = \boldsymbol{B}$. *[3 marks]*

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2. *[Maximum mark: 6]*

Consider the arithmetic sequence $3, 9, 15, \ldots, 1353$.

(a) Write down the common difference. *[1 mark]*

(b) Find the number of terms in the sequence. *[3 marks]*

(c) Find the sum of the sequence. *[2 marks]*

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3. *[Maximum mark: 7]*

Let $f(x) = x\cos x$, for $0 \le x \le 6$.

(a) Find $f'(x)$. *[3 marks]*

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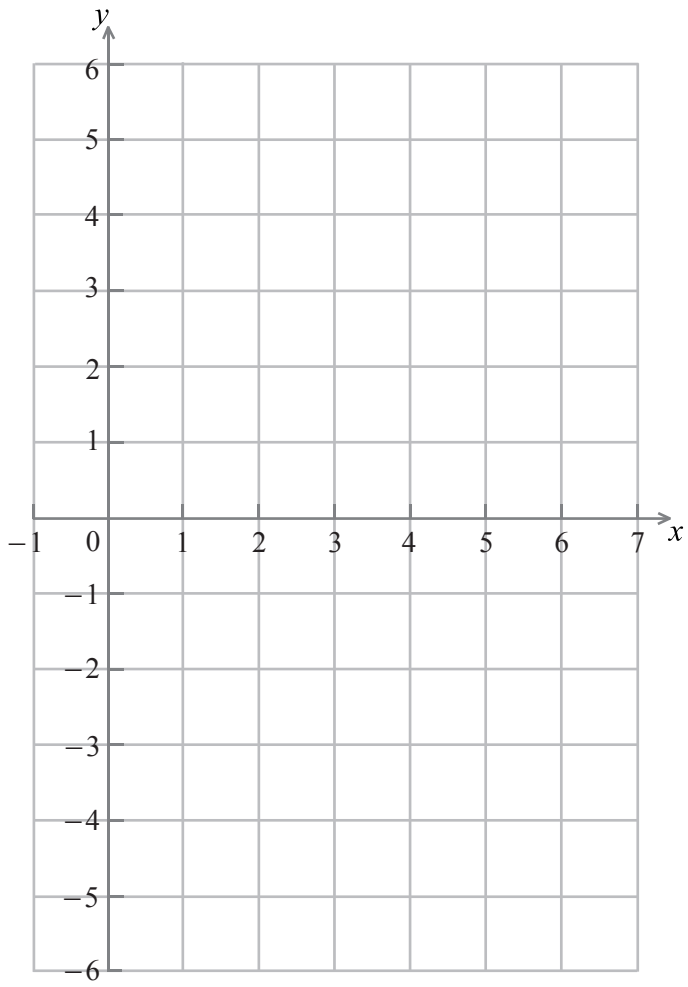
(b) On the grid below, sketch the graph of $y = f'(x)$. *[4 marks]*



4. *[Maximum mark: 6]*

The following frequency distribution of marks has mean 4.5.

Mark	1	2	3	4	5	6	7
Frequency	2	4	6	9	x	9	4

(a) Find the value of x. *[4 marks]*

(b) Write down the standard deviation. *[2 marks]*

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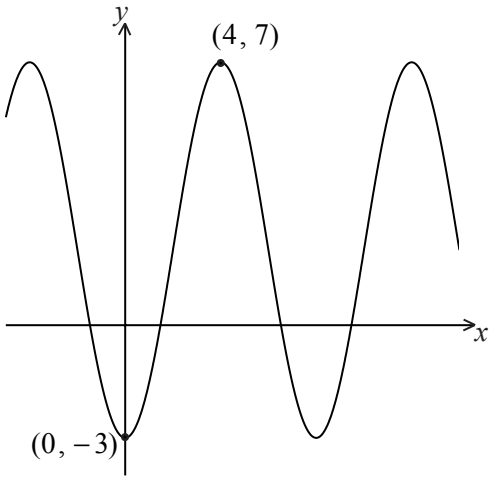
5. *[Maximum mark: 7]*

The graph of $y = p\cos qx + r$, for $-5 \le x \le 14$, is shown below.



There is a minimum point at $(0, -3)$ and a maximum point at $(4, 7)$.

(a) Find the value of

(i) p;

(ii) q;

(iii) r. *[6 marks]*

(b) The equation $y = k$ has exactly **two** solutions. Write down the value of k. *[1 mark]*

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6. *[Maximum mark: 7]*

The acceleration, $a\,\mathrm{m\,s^{-2}}$, of a particle at time t seconds is given by

$$a = \frac{1}{t} + 3\sin 2t, \text{ for } t \geq 1.$$

The particle is at rest when $t = 1$.

Find the velocity of the particle when $t = 5$.

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7. *[Maximum mark: 7]*

Evan likes to play two games of chance, A and B.

For game A, the probability that Evan wins is 0.9. He plays game A seven times.

(a) Find the probability that he wins exactly four games. *[2 marks]*

For game B, the probability that Evan wins is p. He plays game B seven times.

(b) Write down an expression, in terms of p, for the probability that he wins exactly four games. *[2 marks]*

(c) Hence, find the values of p such that the probability that he wins exactly four games is 0.15. *[3 marks]*

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*Do **NOT** write on this page.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

The diagram below shows a quadrilateral ABCD with obtuse angles $\mathrm{A\hat{B}C}$ and $\mathrm{A\hat{D}C}$.

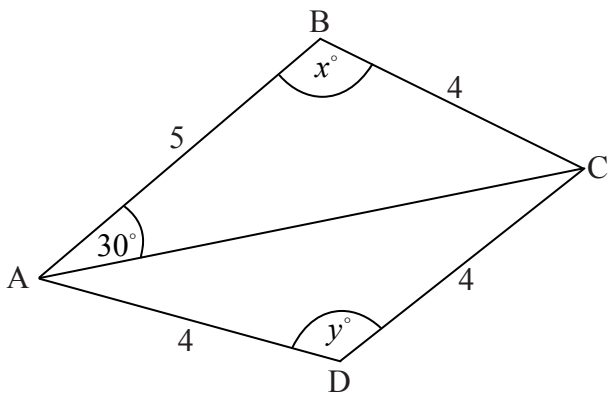


diagram not to scale

$\mathrm{AB} = 5\text{ cm}$, $\mathrm{BC} = 4\text{ cm}$, $\mathrm{CD} = 4\text{ cm}$, $\mathrm{AD} = 4\text{ cm}$, $\mathrm{B\hat{A}C} = 30^\circ$, $\mathrm{A\hat{B}C} = x^\circ$, $\mathrm{A\hat{D}C} = y^\circ$.

(a) Use the cosine rule to show that $\mathrm{AC} = \sqrt{41 - 40\cos x}$. *[1 mark]*

(b) Use the sine rule in triangle ABC to find another expression for AC. *[2 marks]*

(c) (i) Hence, find x, giving your answer to two decimal places.

(ii) Find AC. *[6 marks]*

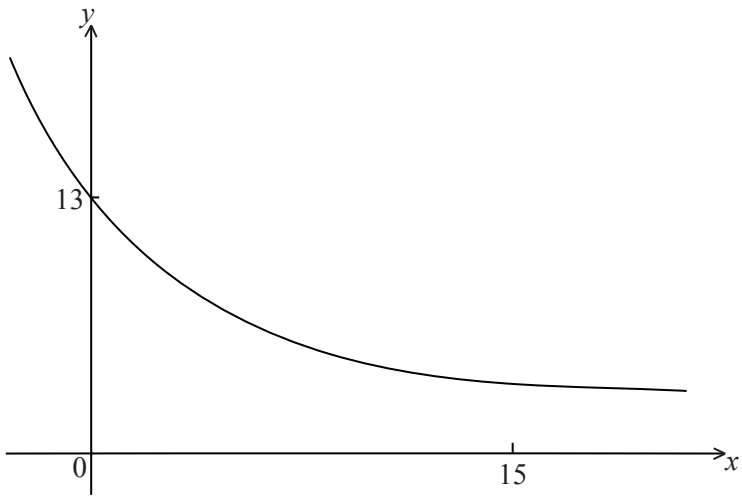
(d) (i) Find y.

(ii) Hence, or otherwise, find the area of triangle ACD. *[5 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 16]*

Let $f(x) = Ae^{kx} + 3$. Part of the graph of f is shown below.



The y-intercept is at $(0, 13)$.

(a) Show that $A = 10$. *[2 marks]*

(b) Given that $f(15) = 3.49$ (correct to 3 significant figures), find the value of k. *[3 marks]*

(c) (i) Using your value of k, find $f'(x)$.

(ii) Hence, explain why f is a decreasing function.

(iii) Write down the equation of the horizontal asymptote of the graph f. *[5 marks]*

Let $g(x) = -x^2 + 12x - 24$.

(d) Find the area enclosed by the graphs of f and g. *[6 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 15]*

The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a) Find the probability that a player weighs more than 82 kg. *[2 marks]*

(b) (i) Write down the standardized value, z, for 68 kg.

(ii) Hence, find the standard deviation of weights. *[4 marks]*

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

(c) (i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament. *[5 marks]*

Of the players in the league, 25 % are women. Of the women, 70 % take part in the tournament.

(d) Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman. *[4 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 6 May 2010 (morning)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

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SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

The following table gives the examination grades for 120 students.

Grade	Number of students	Cumulative frequency
1	9	9
2	25	34
3	35	p
4	q	109
5	11	120

(a) Find the value of

(i) p;

(ii) q. *[4 marks]*

(b) Find the mean grade. *[2 marks]*

(c) Write down the standard deviation. *[1 mark]*

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2. *[Maximum mark: 6]*

An arithmetic sequence, $u_1, u_2, u_3 \ldots$, has $d = 11$ and $u_{27} = 263$.

(a) Find u_1. *[2 marks]*

(b) (i) Given that $u_n = 516$, find the value of n.

(ii) For this value of n, find S_n. *[4 marks]*

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3. *[Maximum mark: 5]*

Jan plays a game where she tosses two fair six-sided dice. She wins a prize if the sum of her scores is 5.

(a) Jan tosses the two dice once. Find the probability that she wins a prize. *[3 marks]*

(b) Jan tosses the two dice 8 times. Find the probability that she wins 3 prizes. *[2 marks]*

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4. *[Maximum mark: 6]*

Find the term in x^4 in the expansion of $\left(3x^2 - \frac{2}{x}\right)^5$.

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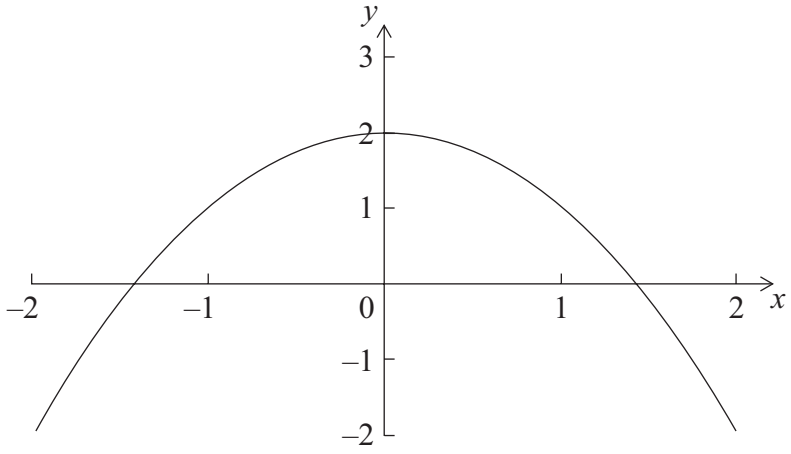
5. *[Maximum mark: 7]*

Consider $f(x) = 2 - x^2$, for $-2 \le x \le 2$ and $g(x) = \sin e^x$, for $-2 \le x \le 2$. The graph of f is given below.



(a) On the diagram above, sketch the graph of g. *[3 marks]*

(b) Solve $f(x) = g(x)$. *[2 marks]*

(c) Write down the set of values of x such that $f(x) > g(x)$. *[2 marks]*

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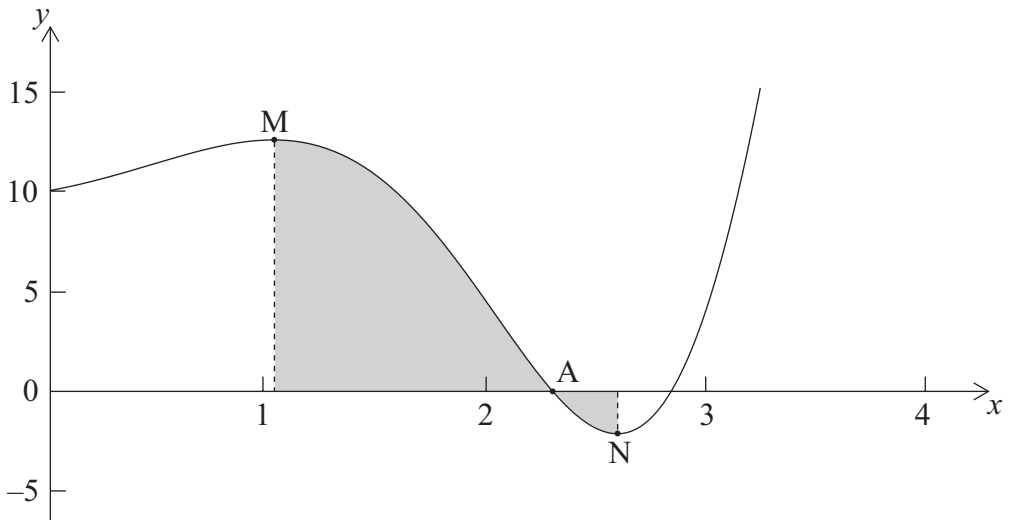
6. *[Maximum mark: 6]*

Let $f(x) = e^x \sin 2x + 10$, for $0 \le x \le 4$. Part of the graph of f is given below.



There is an x-intercept at the point A, a local maximum point at M, where $x = p$ and a local minimum point at N, where $x = q$.

(a) Write down the x-coordinate of A. *[1 mark]*

(b) Find the value of

(i) p;

(ii) q. *[2 marks]*

(c) Find $\int_p^q f(x)\,dx$. Explain why this is not the area of the shaded region. *[3 marks]*

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7. *[Maximum mark: 8]*

The number of bacteria, n, in a dish, after t minutes is given by $n = 800e^{0.13t}$.

(a) Find the value of n when $t = 0$. *[2 marks]*

(b) Find the rate at which n is increasing when $t = 15$. *[2 marks]*

(c) After k minutes, the rate of increase in n is greater than 10 000 bacteria per minute. Find the least value of k, where $k \in \mathbb{Z}$. *[4 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The diagram below shows a circle with centre O and radius 8 cm.

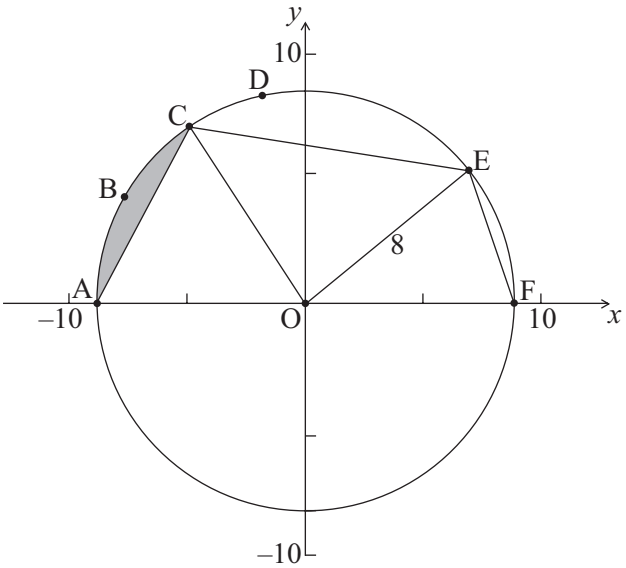


diagram not to scale

The points A, B, C, D, E and F are on the circle, and [AF] is a diameter. The length of arc ABC is 6 cm.

(a) Find the size of angle AOC. *[2 marks]*

(b) Hence find the area of the shaded region. *[6 marks]*

The area of sector OCDE is 45 cm^2.

(c) Find the size of angle COE. *[2 marks]*

(d) Find EF. *[5 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 16]*

In this question, distance is in metres.

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, p seconds after it has passed through A, is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$.

(a) (i) Write down the coordinates of A.

(ii) Find the speed of the airplane in ms^{-1}. *[4 marks]*

(b) After seven seconds the airplane passes through a point B.

(i) Find the coordinates of B.

(ii) Find the distance the airplane has travelled during the seven seconds. *[5 marks]*

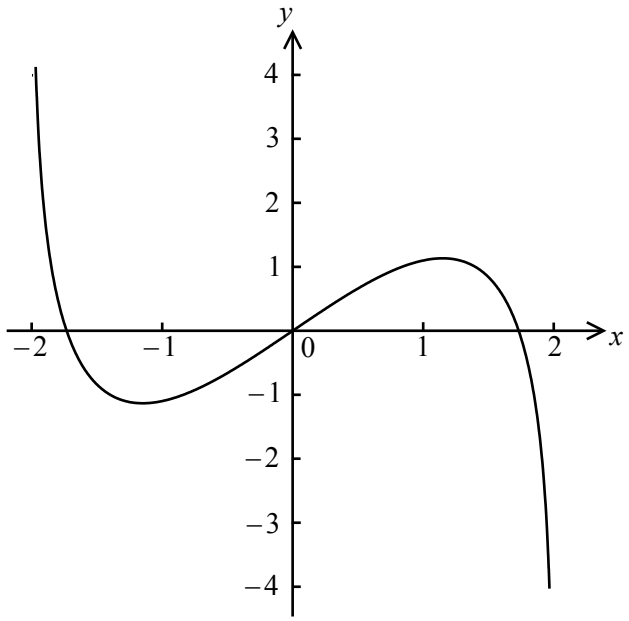
(c) Airplane 2 passes through a point C. Its position q seconds after it passes through C is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 8 \end{pmatrix} + q\begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}, a \in \mathbb{R}$.

The angle between the flight paths of Airplane 1 and Airplane 2 is $40°$. Find the two values of a. *[7 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 14]*

Consider $f(x) = x\ln(4 - x^2)$, for $-2 < x < 2$. The graph of f is given below.



(a) Let P and Q be points on the curve of f where the tangent to the graph of f is parallel to the x-axis.

(i) Find the x-coordinate of P and of Q.

(ii) Consider $f(x) = k$. Write down all values of k for which there are exactly two solutions. *[5 marks]*

Let $g(x) = x^3 \ln(4 - x^2)$, for $-2 < x < 2$.

(b) Show that $g'(x) = \dfrac{-2x^4}{4 - x^2} + 3x^2 \ln(4 - x^2)$. *[4 marks]*

(c) Sketch the graph of g'. *[2 marks]*

(d) Consider $g'(x) = w$. Write down all values of w for which there are exactly two solutions. *[3 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 5 November 2010 (morning)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 6]*

A standard die is rolled 36 times. The results are shown in the following table.

Score	1	2	3	4	5	6
Frequency	3	5	4	6	10	8

(a) Write down the standard deviation. *[2 marks]*

(b) Write down the median score. *[1 mark]*

(c) Find the interquartile range. *[3 marks]*

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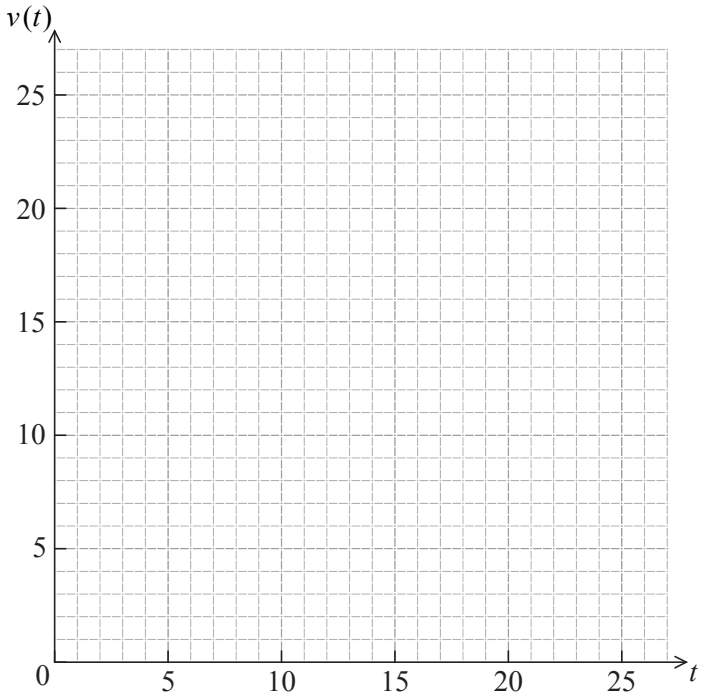
2. *[Maximum mark: 7]*

The velocity $v\ \mathrm{m\,s^{-1}}$ of an object after t seconds is given by $v(t) = 15\sqrt{t} - 3t$, for $0 \le t \le 25$.

(a) On the grid below, sketch the graph of v, clearly indicating the maximum point. *[3 marks]*



Let d be the distance travelled in the first nine seconds.

(b) (i) Write down an expression for d.

(ii) Hence, write down the value of d. *[4 marks]*

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3. *[Maximum mark: 6]*

The nth term of an arithmetic sequence is given by $u_n = 5 + 2n$.

(a) Write down the common difference. *[1 mark]*

(b) (i) Given that the nth term of this sequence is 115, find the value of n.

(ii) For this value of n, find the sum of the sequence. *[5 marks]*

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4. *[Maximum mark: 7]*

Let $\boldsymbol{v} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} k \\ -2 \\ 4 \end{pmatrix}$, for $k > 0$. The angle between $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\frac{\pi}{3}$.

Find the value of k.

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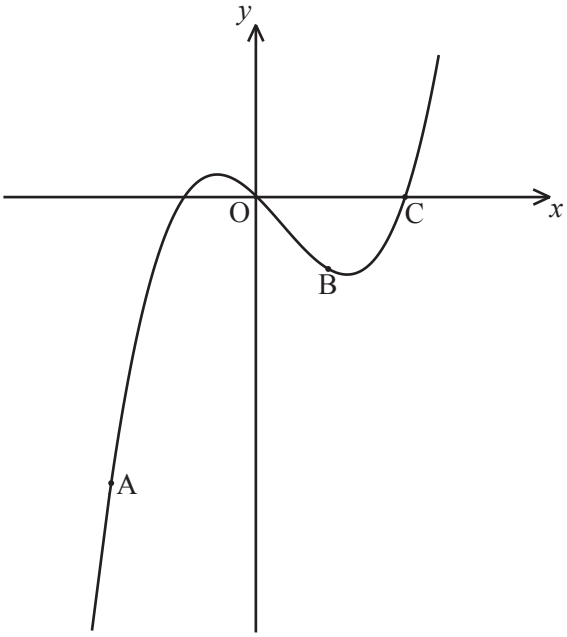
5. *[Maximum mark: 7]*

Consider the function $f(x) = px^3 + qx^2 + rx$. Part of the graph of f is shown below.



The graph passes through the origin O and the points $\mathrm{A}(-2, -8)$, $\mathrm{B}(1, -2)$ and $\mathrm{C}(2, 0)$.

(a) Find three linear equations in p, q and r. *[4 marks]*

(b) Hence find the value of p, of q and of r. *[3 marks]*

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6. *[Maximum mark: 7]*

The following diagram shows the triangle ABC.

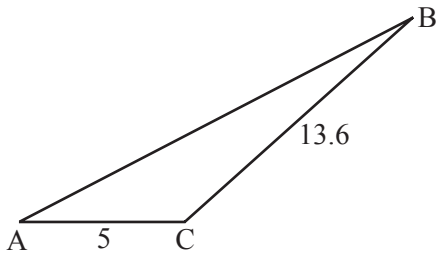

diagram not to scale

The angle at C is obtuse, $AC = 5$ cm, $BC = 13.6$ cm and the area is $20\ \text{cm}^2$.

(a) Find $A\hat{C}B$. *[4 marks]*

(b) Find AB. *[3 marks]*

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7. *[Maximum mark: 5]*

Let $f'(x) = -24x^3 + 9x^2 + 3x + 1$.

(a) There are two points of inflexion on the graph of f. Write down the x-coordinates of these points. *[3 marks]*

(b) Let $g(x) = f''(x)$. Explain why the graph of g has no points of inflexion. *[2 marks]*

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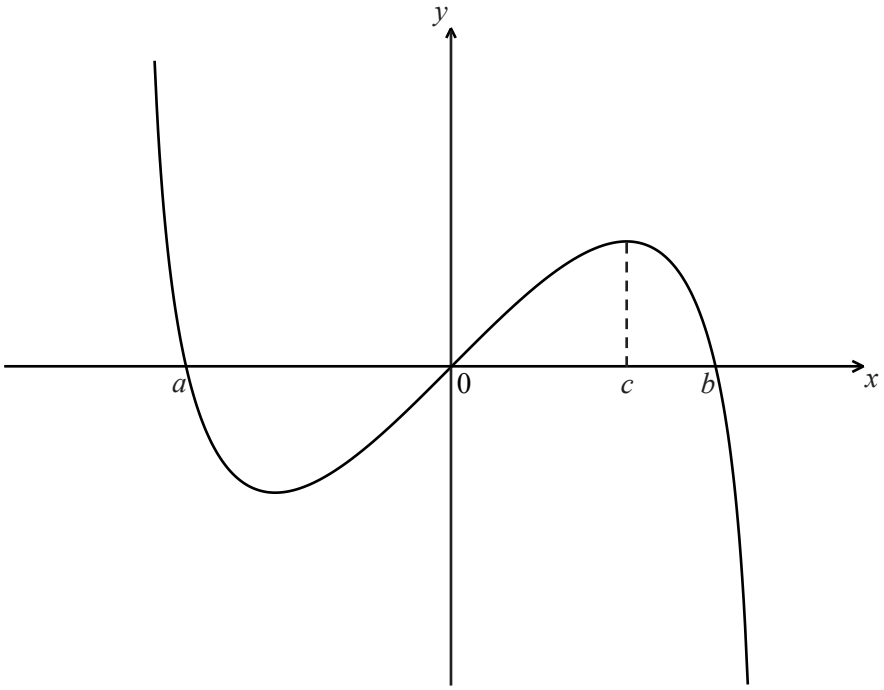
*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 12]*

Let $f(x) = x\ln(4 - x^2)$, for $-2 < x < 2$. The graph of f is shown below.



The graph of f crosses the x-axis at $x = a$, $x = 0$ and $x = b$.

(a) Find the value of a and of b. *[3 marks]*

The graph of f has a maximum value when $x = c$.

(b) Find the value of c. *[2 marks]*

(c) The region under the graph of f from $x = 0$ to $x = c$ is rotated 360° about the x-axis. Find the volume of the solid formed. *[3 marks]*

(d) Let R be the region enclosed by the curve, the x-axis and the line $x = c$, between $x = a$ and $x = c$.

Find the area of R. *[4 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9. *[Maximum mark: 17]*

A test has five questions. To pass the test, at least three of the questions must be answered correctly.

The probability that Mark answers a question correctly is $\frac{1}{5}$. Let X be the number of questions that Mark answers correctly.

(a) (i) Find $\mathrm{E}(X)$.

(ii) Find the probability that Mark passes the test. *[6 marks]*

Bill also takes the test. Let Y be the number of questions that Bill answers correctly. The following table is the probability distribution for Y.

y	0	1	2	3	4	5
$\mathbf{P}(Y = y)$	0.67	0.05	$a+2b$	$a-b$	$2a+b$	0.04

(b) (i) Show that $4a+2b=0.24$.

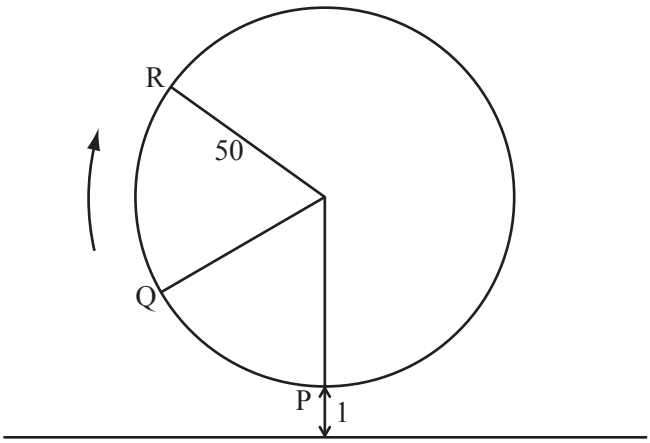
(ii) Given that $\mathrm{E}(Y)=1$, find a and b. *[8 marks]*

(c) Find which student is more likely to pass the test. *[3 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10. *[Maximum mark: 16]*

The following diagram represents a large Ferris wheel at an amusement park. The points P, Q and R represent different positions of a seat on the wheel.



The wheel has a radius of 50 metres and rotates clockwise at a rate of one revolution every 30 minutes.

A seat starts at the lowest point P, when its height is one metre above the ground.

(a) Find the height of a seat above the ground after 15 minutes. *[2 marks]*

(b) After six minutes, the seat is at point Q. Find its height above the ground at Q. *[5 marks]*

The height of the seat above ground after t minutes can be modelled by the function $h(t) = 50\sin\left(b(t-c)\right) + 51$.

(c) Find the value of b and of c. *[6 marks]*

(d) Hence find the value of t the first time the seat is 96 m above the ground. *[3 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

0	0							

Thursday 5 May 2011 (morning)

Examination code

2	2	1	1	–	7	3	0	4

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
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0112

11 pages

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SECTION A

*Answer **all** questions in the boxes provided.*

1. *[Maximum mark: 6]*

The following diagram shows triangle ABC.

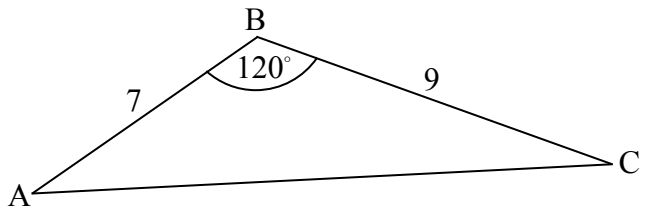

diagram not to scale

$AB = 7\,\text{cm}$, $BC = 9\,\text{cm}$ and $A\hat{B}C = 120^\circ$.

(a) Find AC. *[3 marks]*

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2. *[Maximum mark: 6]*

Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down. The graph of g is the image of the graph of f after this translation.

(a) Write down the coordinates of the vertex of the graph of g. *[2 marks]*

(b) Express g in the form $g(x) = 3(x-p)^2 + q$. *[2 marks]*

The graph of h is the reflection of the graph of g in the x-axis.

(c) Write down the coordinates of the vertex of the graph of h. *[2 marks]*

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3. *[Maximum mark: 5]*

In an arithmetic sequence $u_1 = 7$, $u_{20} = 64$ and $u_n = 3709$.

(a) Find the value of the common difference. *[3 marks]*

(b) Find the value of n. *[2 marks]*

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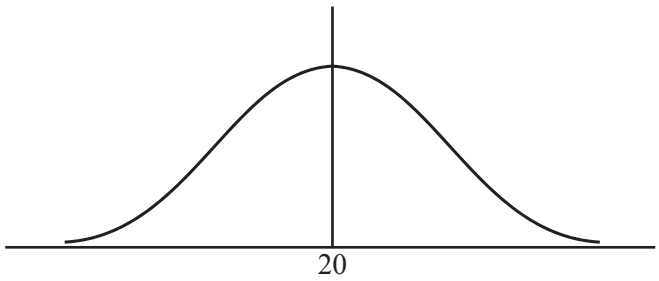
4. *[Maximum mark: 8]*

A random variable X is distributed normally with a mean of 20 and variance 9.

(a) Find $\mathrm{P}(X \le 24.5)$. *[3 marks]*

(b) Let $\mathrm{P}(X \le k) = 0.85$.

(i) Represent this information on the following diagram.



(ii) Find the value of k. *[5 marks]*

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5. *[Maximum mark: 7]*

A box holds 240 eggs. The probability that an egg is brown is 0.05.

(a) Find the expected number of brown eggs in the box. *[2 marks]*

(b) Find the probability that there are 15 brown eggs in the box. *[2 marks]*

(c) Find the probability that there are at least 10 brown eggs in the box. *[3 marks]*

6. *[Maximum mark: 6]*

Let $f(x) = \cos(x^2)$ and $g(x) = e^x$, for $-1.5 \le x \le 0.5$.

Find the area of the region enclosed by the graphs of f and g.

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7. *[Maximum mark: 7]*

A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.
90 % of the boxes made by machine B pass inspection.

A box is selected at random.

(a) Find the probability that it passes inspection. *[3 marks]*

(b) The company would like the probability that a box passes inspection to be 0.87. Find the percentage of boxes that should be made by machine B to achieve this. *[4 marks]*

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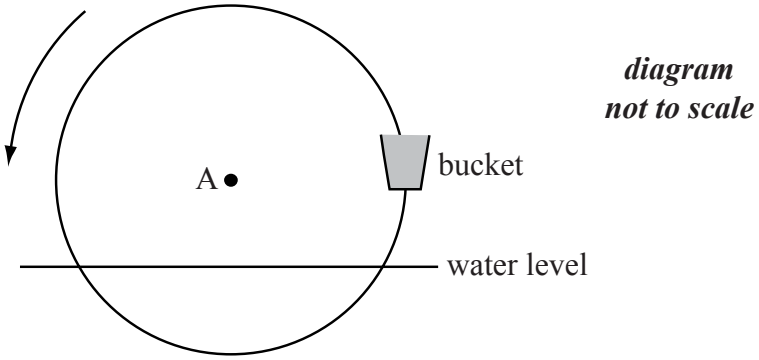
*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

The following diagram shows a waterwheel with a bucket. The wheel rotates at a constant rate in an anticlockwise (counterclockwise) direction.



The diameter of the wheel is 8 metres. The centre of the wheel, A, is 2 metres above the water level. After t seconds, the height of the bucket above the water level is given by $h = a\sin bt + 2$.

(a) Show that $a = 4$. *[2 marks]*

The wheel turns at a rate of one rotation every 30 seconds.

(b) Show that $b = \frac{\pi}{15}$. *[2 marks]*

In the first rotation, there are two values of t when the bucket is **descending** at a rate of $0.5\ \text{m s}^{-1}$.

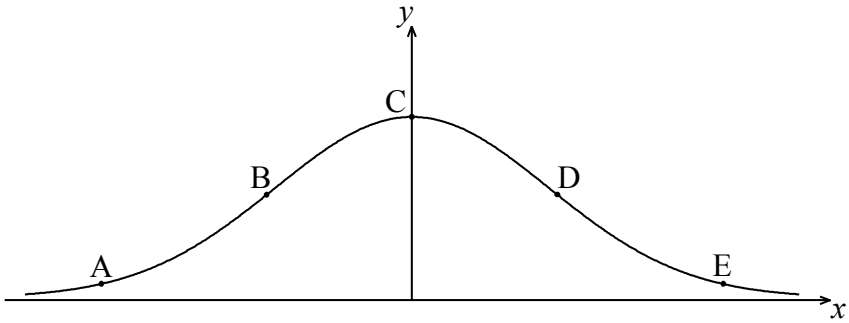
(c) Find these values of t. *[6 marks]*

(d) Determine whether the bucket is underwater at the second value of t. *[4 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9. *[Maximum mark: 15]*

The following diagram shows the graph of $f(x) = \mathrm{e}^{-x^2}$.



The points A, B, C, D and E lie on the graph of f. Two of these are points of inflexion.

(a) Identify the **two** points of inflexion. *[2 marks]*

(b) (i) Find $f'(x)$.

(ii) Show that $f''(x) = (4x^2 - 2)\mathrm{e}^{-x^2}$. *[5 marks]*

(c) Find the x-coordinate of each point of inflexion. *[4 marks]*

(d) Use the second derivative to show that one of these points is a point of inflexion. *[4 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10. *[Maximum mark: 16]*

Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

(a) Show that $f(x) = \log_3 2x$. *[2 marks]*

(b) Find the value of $f(0.5)$ and of $f(4.5)$. *[3 marks]*

The function f can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$.

(c) (i) Write down the value of a and of b.

(ii) Hence on graph paper, **sketch** the graph of f, for $-5 \le x \le 5$, $-5 \le y \le 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote. *[6 marks]*

(d) Write down the value of $f^{-1}(0)$. *[1 mark]*

The point A lies on the graph of f. At A, $x = 4.5$.

(e) On your diagram, sketch the graph of f^{-1}, noting clearly the image of point A. *[4 marks]*

Please **do not** write on this page.

Answers written on this page will not be marked.

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

0	0							

Thursday 5 May 2011 (morning)

Examination code

2	2	1	1	–	7	3	0	6

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
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0112

10 pages

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SECTION A

*Answer **all** questions in the boxes provided.*

1. *[Maximum mark: 5]*

Let $f(x) = 3x$, $g(x) = 2x - 5$ and $h(x) = (f \circ g)(x)$.

(a) Find $h(x)$. *[2 marks]*

(b) Find $h^{-1}(x)$. *[3 marks]*

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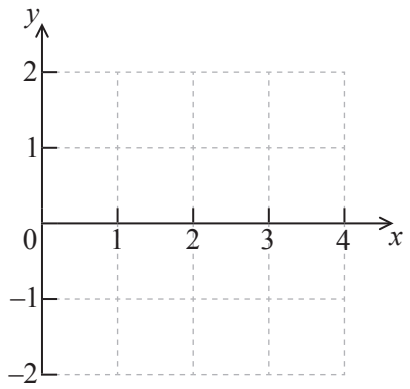
2. *[Maximum mark: 6]*

Let $g(x) = \frac{1}{2} x \sin x$, for $0 \le x \le 4$.

(a) Sketch the graph of g on the following set of axes. *[4 marks]*



(b) Hence find the value of x for which $g(x) = -1$. *[2 marks]*

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3. *[Maximum mark: 5]*

Consider the expansion of $(x+2)^{11}$.

(a) Write down the number of terms in this expansion. *[1 mark]*

(b) Find the term containing x^2. *[4 marks]*

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4. *[Maximum mark: 7]*

The system of linear equations below can be written as the matrix equation $\boldsymbol{MX} = \boldsymbol{N}$.

$$\begin{aligned} x + 6y - 3z &= -1 \\ 4x + 2y - 4z &= 12 \\ x + y + 5z &= 15 \end{aligned}$$

(a) Write down the matrices $\boldsymbol{M}$ and $\boldsymbol{N}$. *[3 marks]*

(b) Solve the **matrix** equation $\boldsymbol{MX} = \boldsymbol{N}$. *[3 marks]*

(c) Hence write down the solution of the system of linear equations. *[1 mark]*

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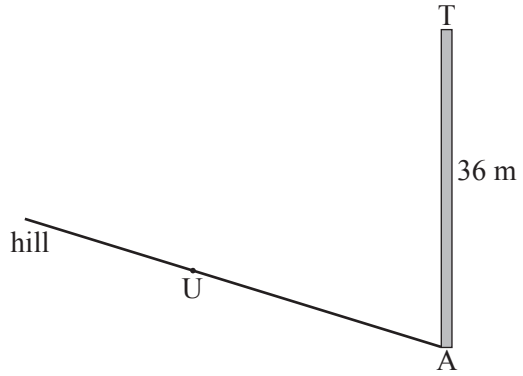
5. *[Maximum mark: 7]*

There is a vertical tower TA of height 36 m at the base A of a hill. A straight path goes up the hill from A to a point U. This information is represented by the following diagram.



The path makes a 4° angle with the horizontal.
The point U on the path is 25 m away from the base of the tower.
The top of the tower is fixed to U by a wire of length x m.

(a) Complete the diagram, showing clearly all the information above. *[3 marks]*

(b) Find x. *[4 marks]*

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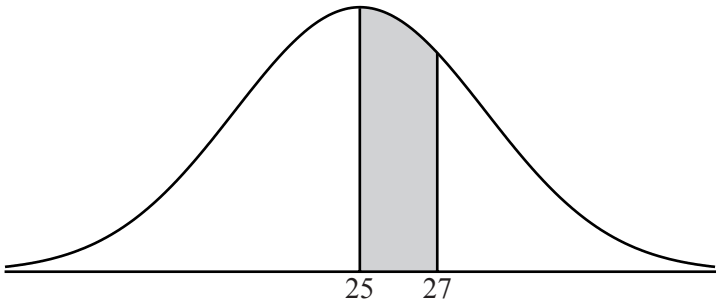
6. *[Maximum mark: 7]*

Let the random variable X be normally distributed with mean 25, as shown in the following diagram.



The shaded region between 25 and 27 represents 30 % of the distribution.

(a) Find $\mathrm{P}(X > 27)$. *[2 marks]*

(b) Find the standard deviation of X. *[5 marks]*

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7. *[Maximum mark: 8]*

A gradient function is given by $\frac{dy}{dx} = 10e^{2x} - 5$. When $x = 0$, $y = 8$. Find the value of y when $x = 1$.

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*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 17]*

Line L_1 passes through points $\mathrm{A}(1, -1, 4)$ and $\mathrm{B}(2, -2, 5)$.

(a) Find $\overrightarrow{\mathrm{AB}}$. *[2 marks]*

(b) Find an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[2 marks]*

Line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

(c) Find the angle between L_1 and L_2. *[7 marks]*

(d) The lines L_1 and L_2 intersect at point C. Find the coordinates of C. *[6 marks]*

9. *[Maximum mark: 12]*

Two fair 4-sided dice, one red and one green, are thrown. For each die, the faces are labelled 1, 2, 3, 4. The score for each die is the number which lands face down.

(a) List the pairs of scores that give a sum of 6. *[3 marks]*

The probability distribution for the sum of the scores on the two dice is shown below.

Sum	2	3	4	5	6	7	8
Probability	p	q	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	r	$\frac{1}{16}$

(b) Find the value of p, of q, and of r. *[3 marks]*

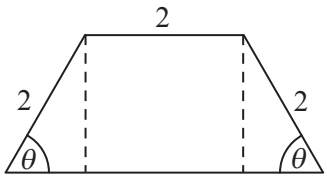
Fred plays a game. He throws two fair 4-sided dice four times. He wins a prize if the sum is 5 on three or more throws.

(c) Find the probability that Fred wins a prize. *[6 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10. *[Maximum mark: 16]*

The diagram below shows a plan for a window in the shape of a trapezium.



Three sides of the window are 2 m long. The angle between the sloping sides of the window and the base is θ, where $0 < \theta < \frac{\pi}{2}$.

(a) Show that the area of the window is given by $y = 4\sin\theta + 2\sin 2\theta$. *[5 marks]*

(b) Zoe wants a window to have an area of 5 m^2. Find the two possible values of θ. *[4 marks]*

(c) John wants two windows which have the same area A but different values of θ.

Find all possible values for A. *[7 marks]*

Please **do not** write on this page.

Answers written on this page will not be marked.

Please **do not** write on this page.

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N11/5/MATME/SP2/ENG/TZ0/XX

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

0	0							

Thursday 3 November 2011 (morning)

Examination code

8	8	1	1	–	7	3	0	2

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.

0112

11 pages

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

Let $f(x) = 2x + 4$ and $g(x) = 7x^2$.

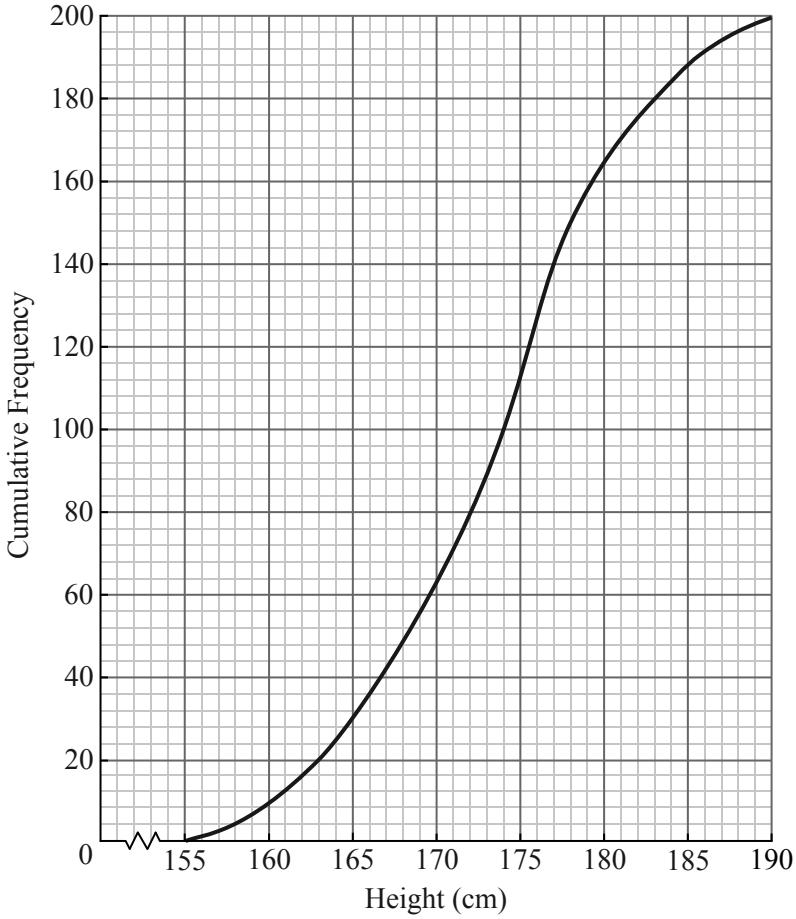
(a) Find $f^{-1}(x)$. *[3 marks]*

(b) Find $(f \circ g)(x)$. *[2 marks]*

(c) Find $(f \circ g)(3.5)$. *[2 marks]*

2. *[Maximum mark: 6]*

The cumulative frequency curve below represents the heights of 200 sixteen-year-old boys.



Use the graph to answer the following.

(a) Write down the median value. *[1 mark]*

(b) A boy is chosen at random. Find the probability that he is shorter than 161 cm. *[2 marks]*

(c) Given that 82 % of the boys are taller than h cm, find h. *[3 marks]*

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3. *[Maximum mark: 6]*

Consider the following circle with centre O and radius 6.8 cm.

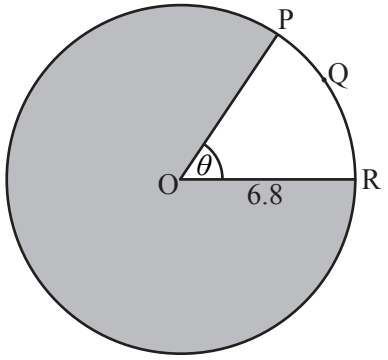

diagram not to scale

The length of the arc PQR is 8.5 cm.

(a) Find the value of θ. *[2 marks]*

(b) Find the area of the shaded region. *[4 marks]*

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4. *[Maximum mark: 6]*

Consider the triangle ABC, where $AB = 10$, $BC = 7$ and $C\hat{A}B = 30^\circ$.

(a) Find the two possible values of $A\hat{C}B$. *[4 marks]*

(b) Hence, find $A\hat{B}C$, given that it is acute. *[2 marks]*

5. *[Maximum mark: 5]*

Consider the expansion of $(3x^2 + 2)^9$.

(a) Write down the number of terms in the expansion. *[1 mark]*

(b) Find the term in x^4. *[4 marks]*

6. *[Maximum mark: 8]*

Jose takes medication. After t minutes, the concentration of medication left in his bloodstream is given by $A(t) = 10(0.5)^{0.014t}$, where A is in milligrams per litre.

(a) Write down $A(0)$. *[1 mark]*

(b) Find the concentration of medication left in his bloodstream after 50 minutes. *[2 marks]*

(c) At 13:00, when there is no medication in Jose's bloodstream, he takes his first dose of medication. He can take his medication again when the concentration of medication reaches 0.395 milligrams per litre. What time will Jose be able to take his medication again? *[5 marks]*

7. *[Maximum mark: 7]*

Let $f(t) = 2t^2 + 7$, where $t > 0$. The function v is obtained when the graph of f is transformed by

a stretch by a scale factor of $\frac{1}{3}$ parallel to the y-axis,

followed by a translation by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

(a) Find $v(t)$, giving your answer in the form $a(t-b)^2 + c$. *[4 marks]*

(b) A particle moves along a straight line so that its velocity in $\mathrm{m\,s^{-1}}$, at time t seconds, is given by v. Find the distance the particle travels between $t = 5.0$ and $t = 6.8$. *[3 marks]*

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*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

(a) Consider an infinite geometric sequence with $u_1 = 40$ and $r = \frac{1}{2}$.

(i) Find u_4.

(ii) Find the sum of the infinite sequence. *[4 marks]*

Consider an arithmetic sequence with n terms, with first term (-36) and eighth term (-8).

(b) (i) Find the common difference.

(ii) Show that $S_n = 2n^2 - 38n$. *[5 marks]*

(c) The sum of the infinite geometric sequence is equal to twice the sum of the arithmetic sequence. Find n. *[5 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9. *[Maximum mark: 16]*

A company produces a large number of water containers. Each container has two parts, a bottle and a cap. The bottles and caps are tested to check that they are not defective.

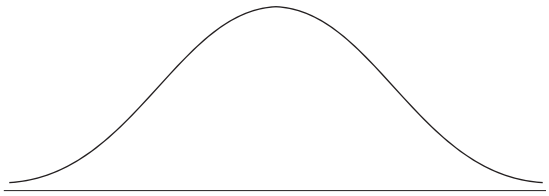
A cap has a probability of 0.012 of being defective. A random sample of 10 caps is selected for inspection.

(a) Find the probability that exactly one cap in the sample will be defective. *[2 marks]*

(b) The sample of caps passes inspection if at most one cap is defective. Find the probability that the sample passes inspection. *[2 marks]*

The heights of the bottles are normally distributed with a mean of 22 cm and a standard deviation of 0.3 cm.

(c) (i) **Copy** and complete the following diagram, shading the region representing where the heights are less than 22.63 cm.



(ii) Find the probability that the height of a bottle is less than 22.63 cm. *[5 marks]*

(d) (i) A bottle is accepted if its height lies between 21.37 cm and 22.63 cm. Find the probability that a bottle selected at random is accepted.

(ii) A sample of 10 bottles passes inspection if all of the bottles in the sample are accepted. Find the probability that the sample passes inspection. *[5 marks]*

(e) The bottles and caps are manufactured separately. A sample of 10 bottles and a sample of 10 caps are randomly selected for testing. Find the probability that both samples pass inspection. *[2 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10. *[Maximum mark: 15]*

Let $f(x) = \frac{20x}{e^{0.3x}}$, for $0 \le x \le 20$.

(a) Sketch the graph of f. *[3 marks]*

(b) (i) Write down the x-coordinate of the maximum point on the graph of f.

(ii) Write down the interval where f is increasing. *[3 marks]*

(c) Show that $f'(x) = \frac{20 - 6x}{e^{0.3x}}$. *[5 marks]*

(d) Find the interval where the rate of change of f is increasing. *[4 marks]*

Please **do not** write on this page.

Answers written on this page will not be marked.

M12/5/MATME/SP2/ENG/TZ1/XX

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

0	0							

Friday 4 May 2012 (morning)

Examination code

2	2	1	2	–	7	3	0	4

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0116

13 pages

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Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The first three terms of an arithmetic sequence are 36, 40, 44, ….

(a) (i) Write down the value of d.

(ii) Find u_8. *[3 marks]*

(b) (i) Show that $S_n = 2n^2 + 34n$.

(ii) Hence, write down the value of S_{14}. *[3 marks]*

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2. *[Maximum mark: 7]*

Let $f(x) = 2x^2 - 8x - 9$.

(a) (i) Write down the coordinates of the vertex.

(ii) Hence or otherwise, express the function in the form $f(x) = 2(x-h)^2 + k$. *[4 marks]*

(b) Solve the equation $f(x) = 0$. *[3 marks]*

3. *[Maximum mark: 6]*

Let $\boldsymbol{M} = \begin{pmatrix} x & 2x \\ x^2 & 1 \end{pmatrix}$ and $\boldsymbol{N} = \begin{pmatrix} 2 & 3 & -1 \\ 4 & 2 & 0 \\ 1 & 5 & 1 \end{pmatrix}$.

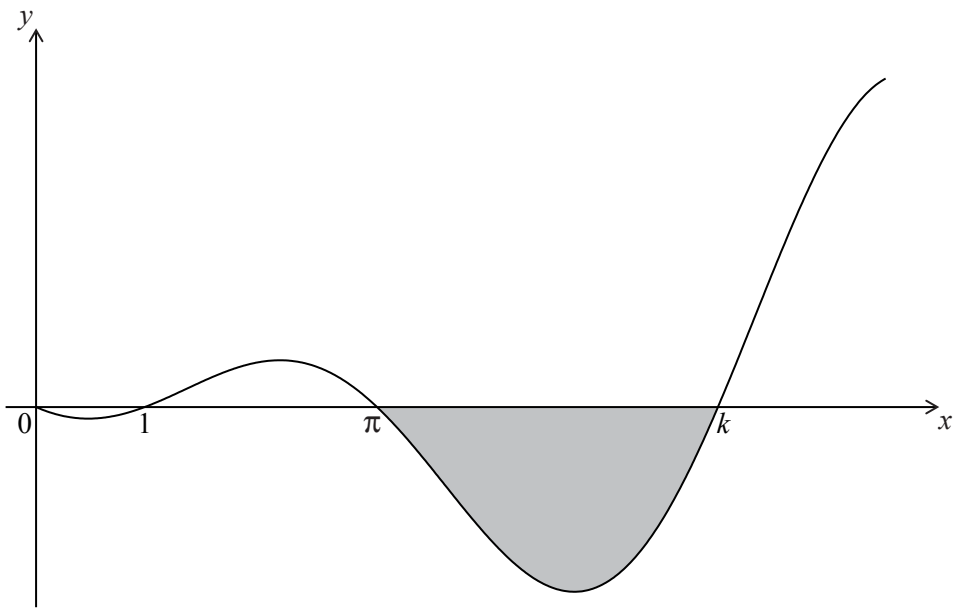
(a) Find $\det \boldsymbol{M}$. *[2 marks]*

(b) Write down $\det \boldsymbol{N}$. *[1 mark]*

(c) Find the value of x for which $\det \boldsymbol{M} = \det \boldsymbol{N}$. *[3 marks]*

4. *[Maximum mark: 7]*

The graph of $y = (x-1)\sin x$, for $0 \le x \le \frac{5\pi}{2}$, is shown below.



The graph has x-intercepts at 0, 1, π and k.

(a) Find k. *[2 marks]*

The shaded region is rotated 360° about the x-axis. Let V be the volume of the solid formed.

(b) Write down an expression for V. *[3 marks]*

(c) Find V. *[2 marks]*

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5. *[Maximum mark: 6]*

Let $\boldsymbol{M} = \begin{pmatrix} p & -1 & -2 \\ 1 & 1 & -2 \\ 1 & q & -1 \end{pmatrix}$ and $\boldsymbol{M}^{-1} = \frac{1}{2}\begin{pmatrix} 3 & -5 & 4 \\ -1 & 1 & 0 \\ 1 & -3 & 2 \end{pmatrix}$.

(a) Find the value of p and of q. *[3 marks]*

(b) Solve the system of linear equations.

$$\begin{aligned} px - y - 2z &= 7 \\ x + y - 2z &= 2 \\ x + qy - z &= -3 \end{aligned}$$

[3 marks]

6. *[Maximum mark: 6]*

Consider the expansion of $\left(2x^3 + \frac{b}{x}\right)^8 = 256x^{24} + 3072x^{20} + \ldots + kx^0 + \ldots$.

(a) Find b. *[3 marks]*

(b) Find k. *[3 marks]*

7. *[Maximum mark: 7]*

The probability of obtaining "tails" when a biased coin is tossed is 0.57. The coin is tossed ten times. Find the probability of obtaining

(a) **at least** four tails; *[4 marks]*

(b) the fourth tail on the tenth toss. *[3 marks]*

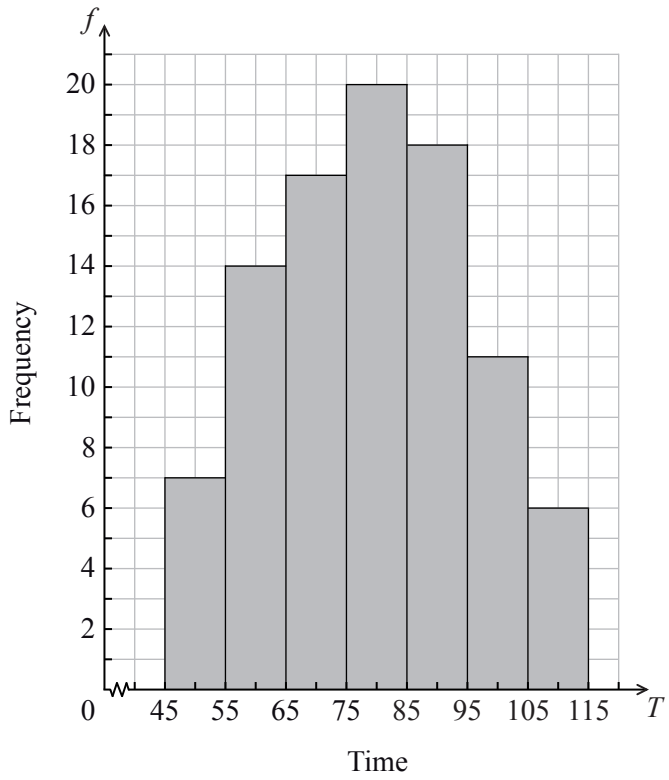
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

The histogram below shows the time T seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for T.

Time	$45 \le T < 55$	$55 \le T < 65$	$65 \le T < 75$	$75 \le T < 85$	$85 \le T < 95$	$95 \le T < 105$	$105 \le T < 115$
Frequency	7	14	p	20	18	q	6

(a) (i) Write down the value of p and of q.

(ii) Write down the median class. *[3 marks]*

(b) A child is selected at random. Find the probability that the child takes less than 95 seconds to solve the puzzle. *[2 marks]*

(This question continues on the following page)

*Do **NOT** write solutions on this page.*

(Question 8 continued)

Consider the class interval $45 \le T < 55$.

(c) (i) Write down the interval width.

(ii) Write down the mid-interval value. *[2 marks]*

(d) Hence find an estimate for the

(i) mean;

(ii) standard deviation. *[4 marks]*

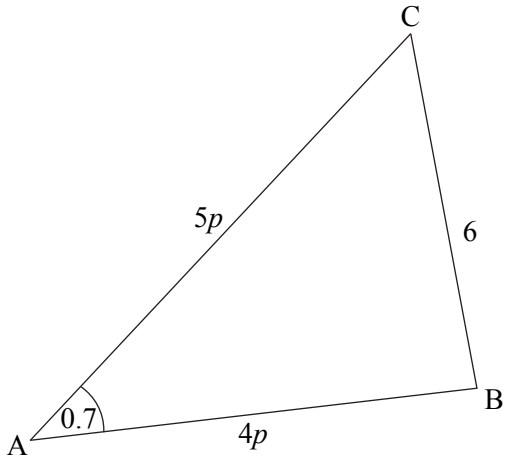
John assumes that T is normally distributed and uses this to estimate the probability that a child takes less than 95 seconds to solve the puzzle.

(e) Find John's estimate. *[2 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

The following diagram shows a triangle ABC.

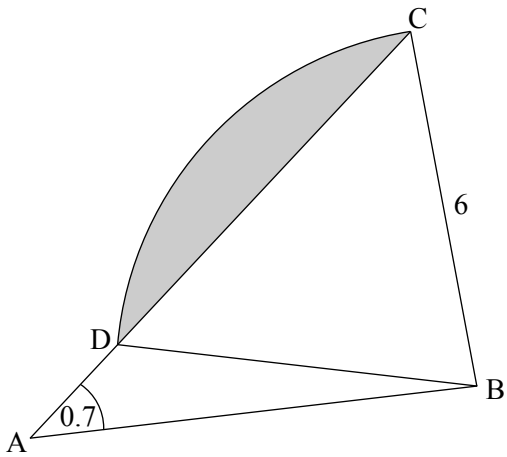


$\mathrm{BC} = 6$, $\mathrm{C\hat{A}B} = 0.7$ radians, $\mathrm{AB} = 4p$, $\mathrm{AC} = 5p$, where $p > 0$.

(a) (i) Show that $p^2(41 - 40\cos 0.7) = 36$.

(ii) Find p. *[4 marks]*

Consider the circle with centre B that passes through the point C. The circle cuts the line CA at D, and $\mathrm{A\hat{D}B}$ is obtuse. Part of the circle is shown in the following diagram.



(b) Write down the length of BD. *[1 mark]*

(c) Find $\mathrm{A\hat{D}B}$. *[4 marks]*

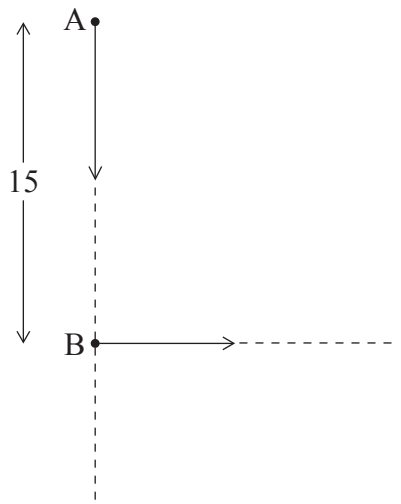
(d) (i) Show that $\mathrm{C\hat{B}D} = 1.29$ radians, correct to 2 decimal places.

(ii) Hence, find the area of the shaded region. *[6 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 17]*

The following diagram shows two ships A and B. At noon, ship A was 15 km due north of ship B. Ship A was moving south at 15 km h^{-1} and ship B was moving east at 11 km h^{-1}.



(a) Find the distance between the ships

(i) at 13:00;

(ii) at 14:00. *[5 marks]*

Let $s(t)$ be the distance between the ships t hours after noon, for $0 \le t \le 4$.

(b) Show that $s(t) = \sqrt{346t^2 - 450t + 225}$. *[6 marks]*

(c) Sketch the graph of $s(t)$. *[3 marks]*

(d) Due to poor weather, the captain of ship A can only see another ship if they are less than 8 km apart. Explain why the captain cannot see ship B between noon and 16:00. *[3 marks]*

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M12/5/MATME/SP2/ENG/TZ2/XX

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

0	0							

Friday 4 May 2012 (morning)

Examination code

2	2	1	2	–	7	3	0	6

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0112

11 pages

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The following diagram shows ΔPQR, where $RQ = 9$ cm, $P\hat{R}Q = 70^\circ$ and $P\hat{Q}R = 45^\circ$.

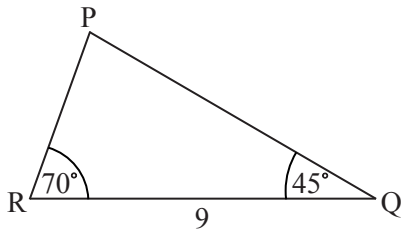

diagram not to scale

(a) Find $R\hat{P}Q$. *[1 mark]*

(b) Find PR. *[3 marks]*

(c) Find the area of ΔPQR. *[2 marks]*

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2. *[Maximum mark: 6]*

Let $f(x) = \cos(e^x)$, for $-2 \le x \le 2$.

(a) Find $f'(x)$. *[2 marks]*

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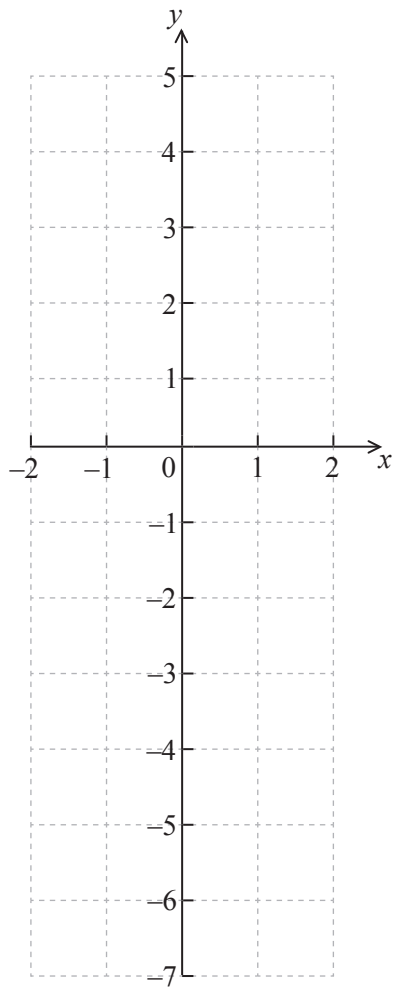
(b) On the grid below, sketch the graph of $f'(x)$. *[4 marks]*



3. *[Maximum mark: 6]*

The first term of a geometric sequence is 200 and the sum of the first four terms is 324.8.

(a) Find the common ratio. *[4 marks]*

(b) Find the tenth term. *[2 marks]*

4. *[Maximum mark: 6]*

The heights of a group of seven-year-old children are normally distributed with mean 117 cm and standard deviation 5 cm. A child is chosen at random from the group.

(a) Find the probability that this child is taller than 122.5 cm. *[3 marks]*

(b) The probability that this child is shorter than k cm is 0.65. Find the value of k. *[3 marks]*

5. *[Maximum mark: 6]*

A particle moves in a straight line with velocity $v = 12t - 2t^3 - 1$, for $t \geq 0$, where v is in centimetres per second and t is in seconds.

(a) Find the acceleration of the particle after 2.7 seconds. *[3 marks]*

(b) Find the displacement of the particle after 1.3 seconds. *[3 marks]*

6. *[Maximum mark: 7]*

Let $\boldsymbol{A} = \begin{pmatrix} -1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & -2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 3 & 1 & 2 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Let $\boldsymbol{C}$ be a 3×3 matrix such that $\boldsymbol{ACA}^{-1} = \boldsymbol{B}$. Find $\boldsymbol{C}$. *[5 marks]*

7. *[Maximum mark: 8]*

A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested.

(a) Find the probability that there is at least one defective lamp in the sample. *[4 marks]*

(b) Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps. *[4 marks]*

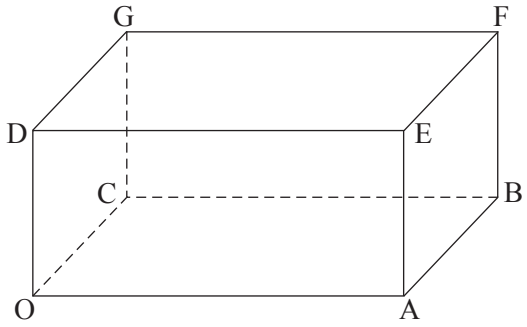
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 16]*

The following diagram shows the cuboid (rectangular solid) OABCDEFG, where O is the origin, and $\overrightarrow{OA} = 4\boldsymbol{i}$, $\overrightarrow{OC} = 3\boldsymbol{j}$, $\overrightarrow{OD} = 2\boldsymbol{k}$.



(a) (i) Find $\overrightarrow{OB}$.

(ii) Find $\overrightarrow{OF}$.

(iii) Show that $\overrightarrow{AG} = -4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$. *[5 marks]*

(b) Write down a vector equation for

(i) the line OF;

(ii) the line AG. *[4 marks]*

(c) Find the obtuse angle between the lines OF and AG. *[7 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 13]*

Let $f(x) = ax^3 + bx^2 + c$, where a, b and c are real numbers. The graph of f passes through the point $(2, 9)$.

(a) Show that $8a + 4b + c = 9$. *[2 marks]*

The graph of f has a local minimum at $(1, 4)$.

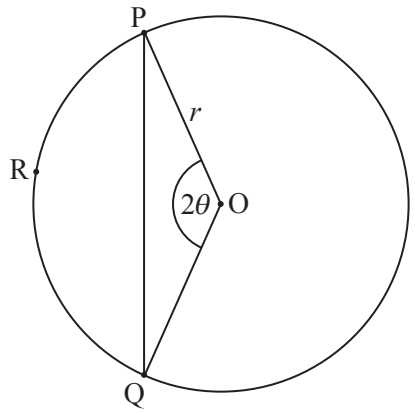
(b) Find two other equations in a, b and c, giving your answers in a similar form to part (a). *[7 marks]*

(c) Find the value of a, of b and of c. *[4 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

Consider the following circle with centre O and radius r.



The points P, R and Q are on the circumference, $\mathrm{P\hat{O}Q} = 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

(a) Use the cosine rule to show that $\mathrm{PQ} = 2r\sin\theta$. *[4 marks]*

Let l be the length of the arc PRQ.

(b) Given that $1.3\,\mathrm{PQ} - l = 0$, find the value of θ. *[5 marks]*

Consider the function $f(\theta) = 2.6\sin\theta - 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

(c) (i) Sketch the graph of f.

(ii) Write down the root of $f(\theta) = 0$. *[4 marks]*

(d) Use the graph of f to find the values of θ for which $l < 1.3\,\mathrm{PQ}$. *[3 marks]*

Please **do not** write on this page.

Answers written on this page
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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

0	0							

Wednesday 7 November 2012 (morning)

Examination code

8	8	1	2	–	7	3	0	2

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0112

12 pages

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The first three terms of an arithmetic sequence are 5, 6.7, 8.4.

(a) Find the common difference. *[2 marks]*

(b) Find the 28th term of the sequence. *[2 marks]*

(c) Find the sum of the first 28 terms. *[2 marks]*

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2. *[Maximum mark: 5]*

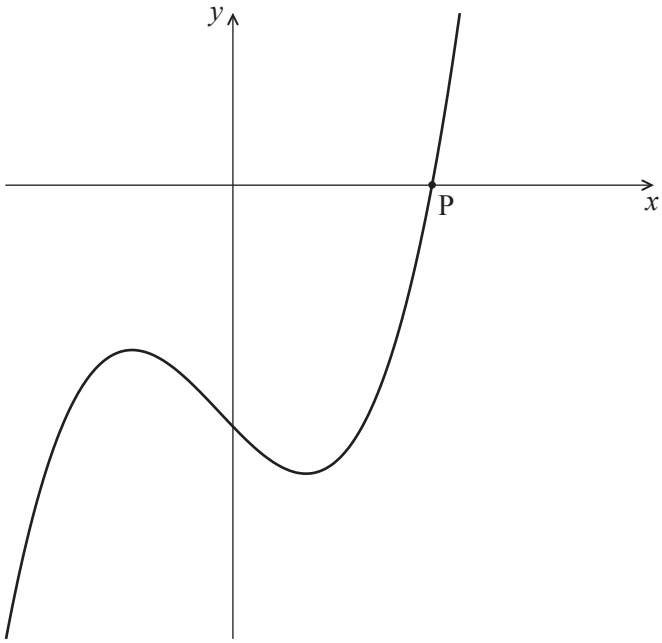
Let $\boldsymbol{A} = \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Hence or otherwise, find $\boldsymbol{B}$, given that $\boldsymbol{AB} = \begin{pmatrix} -1 & 6 & -1 \\ 5 & -1 & 3 \\ 5 & 2 & 7 \end{pmatrix}$. *[3 marks]*

3. *[Maximum mark: 6]*

Let $f(x) = x^3 - 2x - 4$. The following diagram shows part of the curve of f.



The curve crosses the x-axis at the point P.

(a) Write down the x-coordinate of P. *[1 mark]*

(b) Write down the gradient of the curve at P. *[2 marks]*

(c) Find the equation of the normal to the curve at P, giving your equation in the form $y = ax + b$. *[3 marks]*

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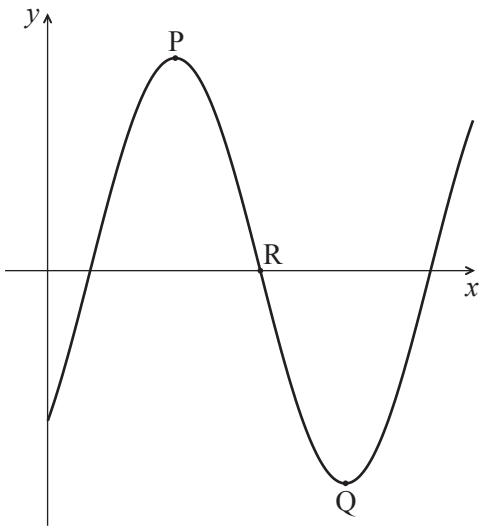
4. *[Maximum mark: 7]*

The third term in the expansion of $(2x + p)^6$ is $60x^4$. Find the possible values of p.

5. *[Maximum mark: 6]*

Let $f(x) = a\cos\big(b(x-c)\big)$. The diagram below shows part of the graph of f, for $0 \le x \le 10$.



The graph has a local maximum at $\mathrm{P}(3, 5)$, a local minimum at $\mathrm{Q}(7, -5)$, and crosses the x-axis at R.

(a) Write down the value of

(i) a;

(ii) c. *[2 marks]*

(b) Find the value of b. *[2 marks]*

(c) Find the x-coordinate of R. *[2 marks]*

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6. *[Maximum mark: 8]*

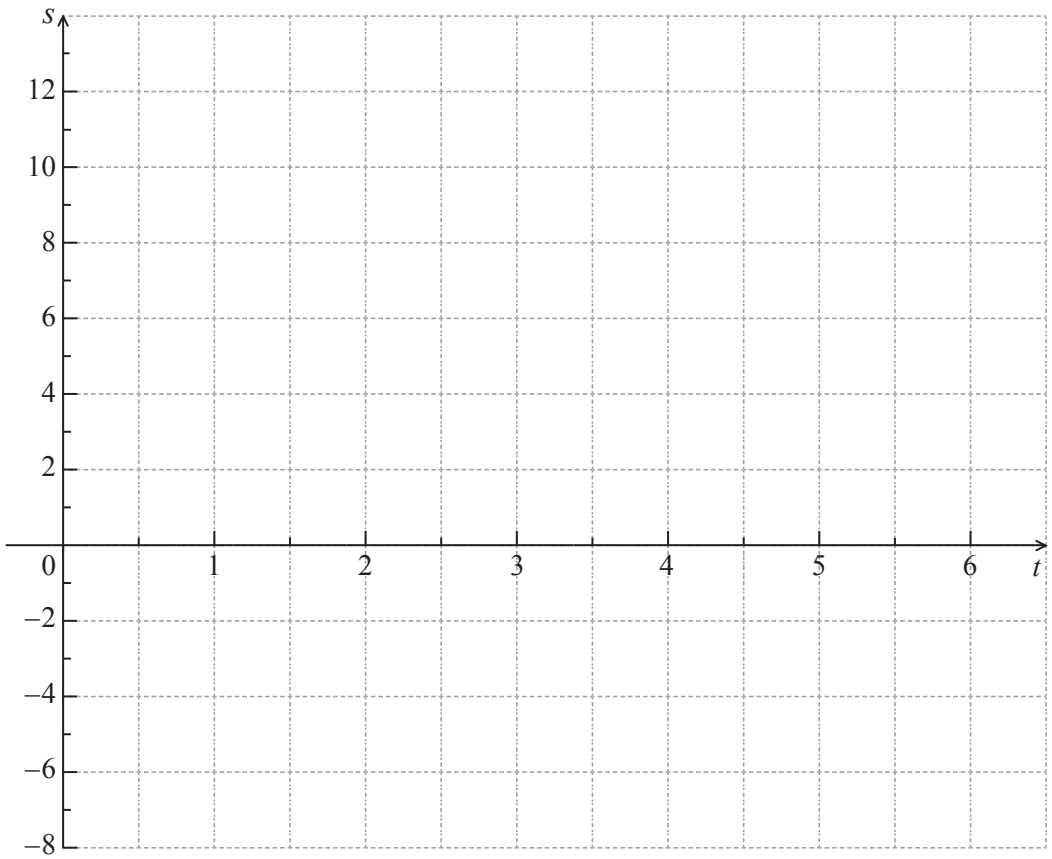
In a large city, the time taken to travel to work is normally distributed with mean μ and standard deviation σ. It is found that 4 % of the population take less than 5 minutes to get to work, and 70 % take less than 25 minutes.

Find the value of μ and of σ.

7. *[Maximum mark: 7]*

A particle's displacement, in metres, is given by $s(t) = 2t\cos t$, for $0 \le t \le 6$, where t is the time in seconds.

(a) On the grid below, sketch the graph of s. *[4 marks]*



(This question continues on the following page)

(Question 7 continued)

(b) Find the maximum velocity of the particle. *[3 marks]*

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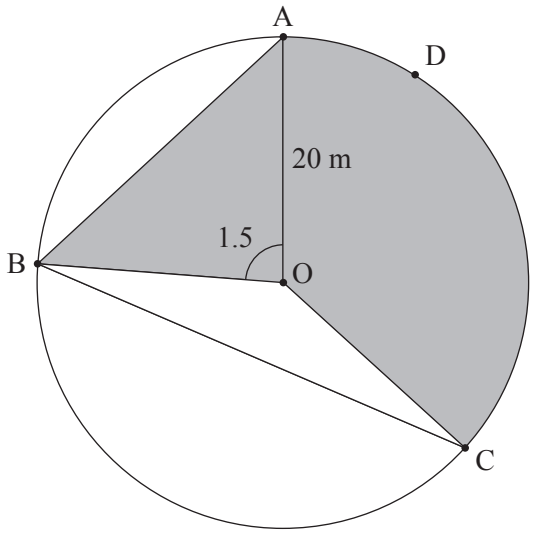
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The following diagram shows a circular play area for children.



The circle has centre O and a radius of 20 m, and the points A, B, C and D lie on the circle. Angle AOB is 1.5 radians.

(a) Find the length of the chord [AB]. *[3 marks]*

(b) Find the area of triangle AOB. *[2 marks]*

Angle BOC is 2.4 radians.

(c) Find the length of arc ADC. *[3 marks]*

(d) Find the area of the shaded region. *[3 marks]*

(e) The shaded region is to be painted red. Red paint is sold in cans which cost \$32 each. One can covers 140 m^2. How much does it cost to buy the paint? *[4 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

Consider the function $f(x) = x^2 - 4x + 1$.

(a) Sketch the graph of f, for $-1 \le x \le 5$. *[4 marks]*

This function can also be written as $f(x) = (x - p)^2 - 3$.

(b) Write down the value of p. *[1 mark]*

The graph of g is obtained by reflecting the graph of f in the x-axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

(c) Show that $g(x) = -x^2 + 4x + 5$. *[4 marks]*

The graphs of f and g intersect at two points.

(d) Write down the x-coordinates of these two points. *[3 marks]*

Let R be the region enclosed by the graphs of f and g.

(e) Find the area of R. *[3 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

At a large school, students are required to learn at least one language, Spanish or French. It is known that 75 % of the students learn Spanish, and 40 % learn French.

(a) Find the percentage of students who learn **both** Spanish and French. *[2 marks]*

(b) Find the percentage of students who learn Spanish, but not French. *[2 marks]*

At this school, 52 % of the students are girls, and 85 % of the girls learn Spanish.

(c) A student is chosen at random. Let G be the event that the student is a girl, and let S be the event that the student learns Spanish.

(i) Find $\mathrm{P}(G \cap S)$.

(ii) Show that G and S are **not** independent. *[5 marks]*

(d) A boy is chosen at random. Find the probability that he learns Spanish. *[6 marks]*

M13/5/MATME/SP2/ENG/TZ1/XX

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

0	0							

Friday 10 May 2013 (morning)

Examination code

2	2	1	3	–	7	3	0	4

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
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- The maximum mark for this examination paper is *[90 marks]*.

0112

11 pages

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

An arithmetic sequence is given by $5, 8, 11, \ldots$.

(a) Write down the value of d. *[1 mark]*

(b) Find

(i) u_{100};

(ii) S_{100}. *[4 marks]*

(c) Given that $u_n = 1502$, find the value of n. *[2 marks]*

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2. *[Maximum mark: 6]*

Consider the following cumulative frequency table.

x	**Frequency**	**Cumulative frequency**
5	2	2
15	10	12
25	14	26
35	p	35
45	6	41

(a) Find the value of p. *[2 marks]*

(b) Find

(i) the mean;

(ii) the variance. *[4 marks]*

3. *[Maximum mark: 5]*

In the expansion of $(3x-2)^{12}$, the term in x^5 can be expressed as $\binom{12}{r}\times(3x)^p\times(-2)^q$.

(a) Write down the value of p, of q and of r. *[3 marks]*

(b) Find the coefficient of the term in x^5. *[2 marks]*

4. *[Maximum mark: 6]*

Consider the system of equations
$$\begin{aligned} -x - y + z &= 2.5 \\ x + y &= 1 \\ -2x - y + 2z &= -3 \end{aligned}$$

This system can be represented by the matrix equation $\boldsymbol{AX} = \boldsymbol{B}$, where $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

(a) (i) Write down the matrix $\boldsymbol{A}$.

(ii) Write down the matrix $\boldsymbol{A}^{-1}$. *[3 marks]*

(b) Hence, find $\boldsymbol{X}$. *[3 marks]*

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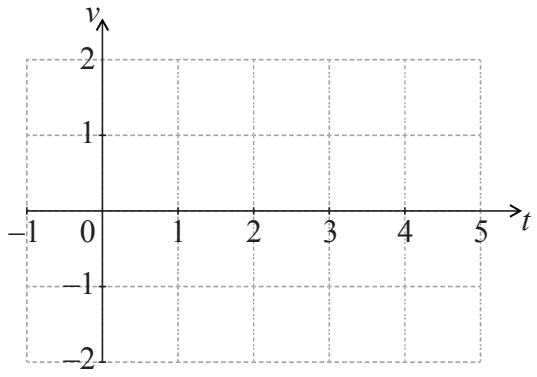
5. *[Maximum mark: 8]*

The velocity of a particle in ms^{-1} is given by $v = e^{\sin t} - 1$, for $0 \le t \le 5$.

(a) On the grid below, sketch the graph of v. *[3 marks]*



(b) (i) Write down the positive t-intercept.

(ii) Find the total distance travelled by the particle in the first five seconds. *[5 marks]*

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6. *[Maximum mark: 6]*

Let f and g be functions such that $g(x) = 2f(x+1) + 5$.

(a) The graph of f is mapped to the graph of g under the following transformations:

vertical stretch by a factor of k, followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k;

(ii) p;

(iii) q. *[3 marks]*

(b) Let $h(x) = -g(3x)$. The point $\mathrm{A}(6, 5)$ on the graph of g is mapped to the point A' on the graph of h. Find A'. *[3 marks]*

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7. *[Maximum mark: 7]*

A random variable X is normally distributed with $\mu = 150$ and $\sigma = 10$.

Find the interquartile range of X.

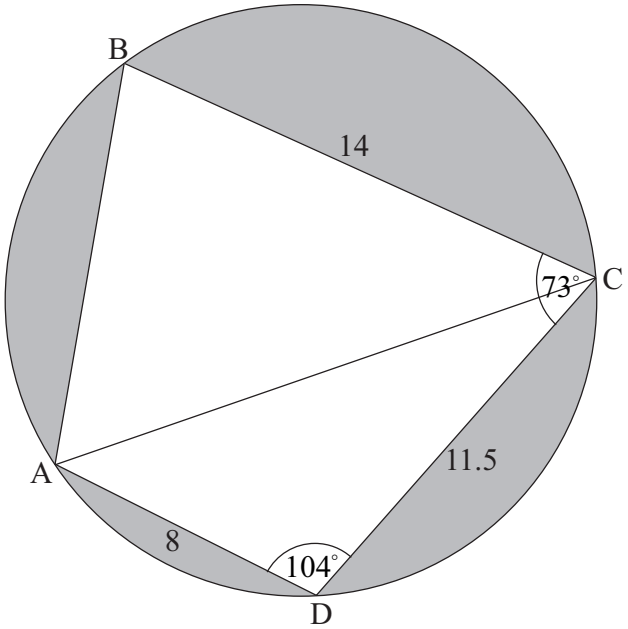
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

The diagram shows a circle of radius 8 metres. The points ABCD lie on the circumference of the circle.



BC = 14 m, CD = 11.5 m, AD = 8 m, $A\hat{D}C = 104^\circ$, and $B\hat{C}D = 73^\circ$

(a) Find AC. *[3 marks]*

(b) (i) Find $A\hat{C}D$.

(ii) Hence, find $A\hat{C}B$. *[5 marks]*

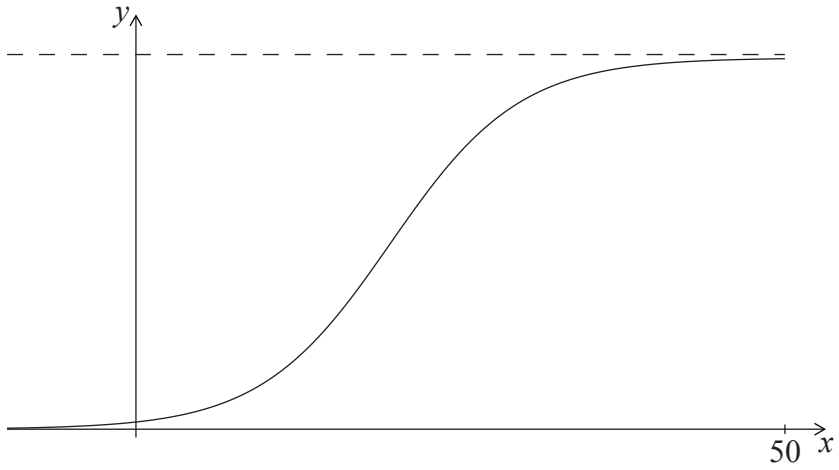
(c) Find the area of triangle ADC. *[2 marks]*

(d) Hence or otherwise, find the total area of the shaded regions. *[4 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

Let $f(x) = \dfrac{100}{\left(1 + 50\mathrm{e}^{-0.2x}\right)}$. Part of the graph of f is shown below.



(a) Write down $f(0)$. *[1 mark]*

(b) Solve $f(x) = 95$. *[2 marks]*

(c) Find the range of f. *[3 marks]*

(d) Show that $f'(x) = \dfrac{1000\mathrm{e}^{-0.2x}}{\left(1 + 50\mathrm{e}^{-0.2x}\right)^2}$. *[5 marks]*

(e) Find the maximum rate of change of f. *[4 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

A Ferris wheel with diameter 122 metres rotates clockwise at a constant speed. The wheel completes 2.4 rotations every hour. The bottom of the wheel is 13 metres above the ground.

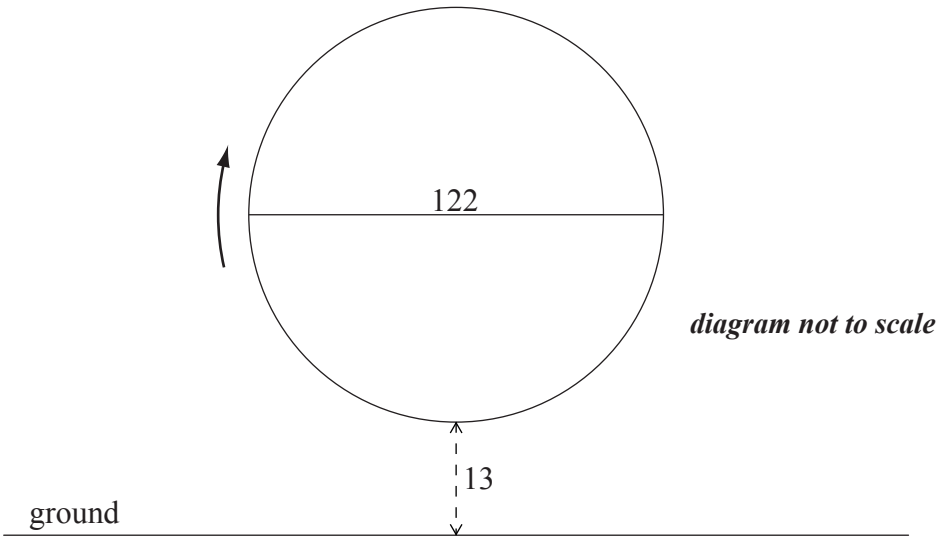


diagram not to scale

A seat starts at the bottom of the wheel.

(a) Find the maximum height above the ground of the seat. *[2 marks]*

After t minutes, the height h metres above the ground of the seat is given by

$$h = 74 + a\cos bt\ .$$

(b) (i) Show that the period of h is 25 minutes.

(ii) Write down the **exact** value of b. *[2 marks]*

(c) Find the value of a. *[3 marks]*

(d) Sketch the graph of h, for $0 \le t \le 50$. *[4 marks]*

(e) In one rotation of the wheel, find the probability that a randomly selected seat is at least 105 metres above the ground. *[5 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

M13/5/MATME/SP2/ENG/TZ2/XX

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

0	0							

Friday 10 May 2013 (morning)

Examination code

2	2	1	3	–	7	3	0	6

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0112

11 pages

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Solve $\boldsymbol{AX} = \boldsymbol{B}$. *[3 marks]*

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2. *[Maximum mark: 6]*

The random variable X is normally distributed with mean 20 and standard deviation 5.

(a) Find $\mathrm{P}(X \leq 22.9)$. *[3 marks]*

(b) Given that $\mathrm{P}(X < k) = 0.55$, find the value of k. *[3 marks]*

3. *[Maximum mark: 6]*

The following diagram shows a triangle ABC.

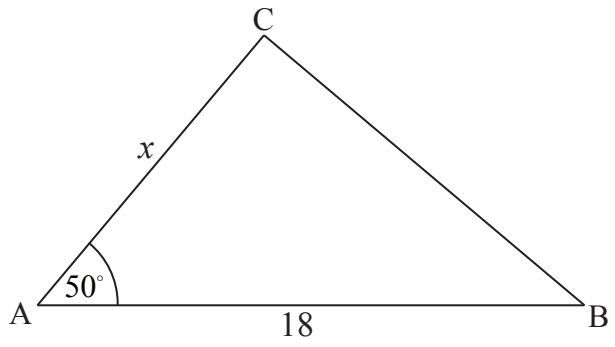

diagram not to scale

The area of triangle ABC is $80\ \text{cm}^2$, $\text{AB} = 18\ \text{cm}$, $\text{AC} = x\ \text{cm}$ and $\text{B}\hat{\text{A}}\text{C} = 50^\circ$.

(a) Find x. *[3 marks]*

(b) Find BC. *[3 marks]*

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4. *[Maximum mark: 7]*

Line L_1 has equation $\boldsymbol{r}_1 = \begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix}$ and line L_2 has equation $\boldsymbol{r}_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$.

Lines L_1 and L_2 intersect at point A. Find the coordinates of A.

5. *[Maximum mark: 6]*

The sum of the first three terms of a geometric sequence is 62.755, and the sum of the infinite sequence is 440. Find the common ratio.

6. *[Maximum mark: 7]*

The constant term in the expansion of $\left(\frac{x}{a}+\frac{a^2}{x}\right)^6$, where $a \in \mathbb{Z}$, is 1280. Find a.

7. *[Maximum mark: 8]*

The following diagram shows a circle with centre O and radius r cm.

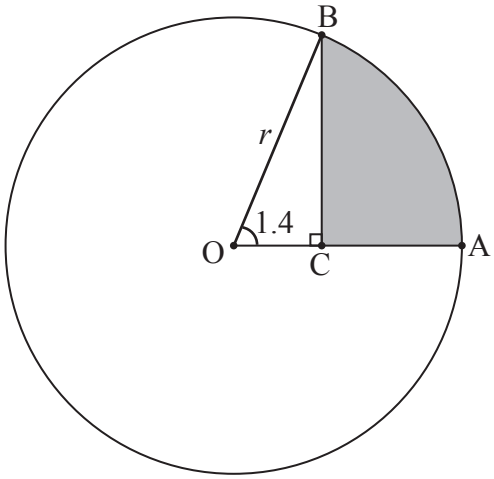

diagram not to scale

Points A and B are on the circumference of the circle and $\mathrm{A\hat{O}B} = 1.4$ radians.
The point C is on [OA] such that $\mathrm{B\hat{C}O} = \frac{\pi}{2}$ radians.

(a) Show that $\mathrm{OC} = r\cos 1.4$. *[1 mark]*

(b) The area of the shaded region is $25\ \mathrm{cm}^2$. Find the value of r. *[7 marks]*

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SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

Consider the points $A(5, 2, 1)$, $B(6, 5, 3)$, and $C(7, 6, a+1)$, where $a \in \mathbb{R}$.

(a) Find

(i) $\overrightarrow{AB}$;

(ii) $\overrightarrow{AC}$. *[3 marks]*

Let α be the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(b) Find the value of a for which $\alpha = \frac{\pi}{2}$. *[4 marks]*

(c) (i) Show that $\cos\alpha = \frac{2a+14}{\sqrt{14a^2+280}}$.

(ii) Hence, find the value of a for which $\alpha = 1.2$. *[8 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

A bag contains four gold balls and six silver balls.

(a) Two balls are drawn at random from the bag, with replacement. Let X be the number of gold balls drawn from the bag.

(i) Find $\mathrm{P}(X = 0)$.

(ii) Find $\mathrm{P}(X = 1)$.

(iii) Hence, find $\mathrm{E}(X)$. *[8 marks]*

Fourteen balls are drawn from the bag, with replacement.

(b) Find the probability that exactly five of the balls are gold. *[2 marks]*

(c) Find the probability that at most five of the balls are gold. *[2 marks]*

(d) Given that at most five of the balls are gold, find the probability that exactly five of the balls are gold. Give the answer correct to two decimal places. *[3 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

Let $f(x) = e^{\frac{x}{4}}$ and $g(x) = mx$, where $m \geq 0$, and $-5 \leq x \leq 5$. Let R be the region enclosed by the y-axis, the graph of f, and the graph of g.

(a) Let $m = 1$.

(i) Sketch the graphs of f and g on the same axes.

(ii) Find the area of R. *[7 marks]*

(b) Consider all values of m such that the graphs of f and g intersect. Find the value of m that gives the greatest value for the area of R. *[8 marks]*

Please **do not** write on this page.

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

0	0							

Tuesday 12 November 2013 (morning)

Examination code

8	8	1	3	–	7	3	0	2

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

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11 pages

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Let $\boldsymbol{A} = \begin{pmatrix} 0 & 5 & 4 \\ 1 & 2 & 1 \\ 2 & 2 & 0 \end{pmatrix}$, and $\boldsymbol{B} = \begin{pmatrix} 11 \\ 7 \\ 10 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2]*

(b) Hence or otherwise, solve the equation $\boldsymbol{AX} = \boldsymbol{B}$. *[3]*

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2. *[Maximum mark: 6]*

Let $f(x) = (x-1)(x-4)$.

(a) Find the x-intercepts of the graph of f. *[3]*

(b) The region enclosed by the graph of f and the x-axis is rotated 360° about the x-axis. Find the volume of the solid formed. *[3]*

3. *[Maximum mark: 6]*

Let $f(x) = \sqrt[3]{x^4} - \frac{1}{2}$.

(a) Find $f'(x)$. *[2]*

(b) Find $\int f(x)\mathrm{d}x$. *[4]*

4. *[Maximum mark: 6]*

Two events A and B are such that $\mathrm{P}(A) = 0.2$ and $\mathrm{P}(A \cup B) = 0.5$.

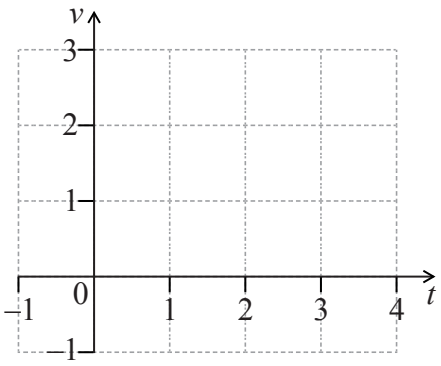
(a) Given that A and B are mutually exclusive, find $\mathrm{P}(B)$. *[2]*

(b) Given that A and B are independent, find $\mathrm{P}(B)$. *[4]*

5. *[Maximum mark: 8]*

A particle moves along a straight line such that its velocity, $v\,\mathrm{m\,s^{-1}}$, is given by $v(t) = 10t\mathrm{e}^{-1.7t}$, for $t \geq 0$.

(a) On the grid below, sketch the graph of v, for $0 \leq t \leq 4$. *[3]*



(b) Find the distance travelled by the particle in the first three seconds. *[2]*

(c) Find the velocity of the particle when its acceleration is zero. *[3]*

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6. *[Maximum mark: 7]*

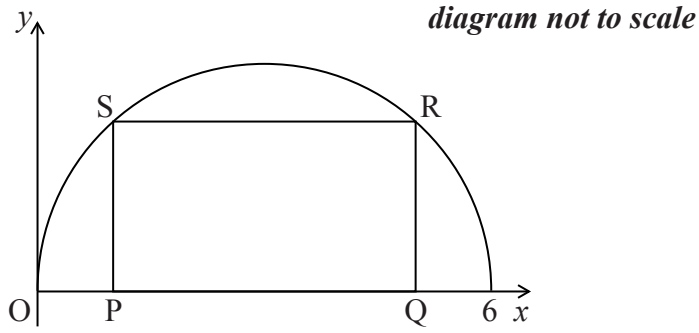
The time taken for a student to complete a task is normally distributed with a mean of 20 minutes and a standard deviation of 1.25 minutes.

(a) A student is selected at random. Find the probability that the student completes the task in less than 21.8 minutes. *[2]*

(b) The probability that a student takes between k and 21.8 minutes is 0.3. Find the value of k. *[5]*

7. *[Maximum mark: 7]*

Consider the graph of the semicircle given by $f(x)=\sqrt{6x-x^2}$, for $0 \le x \le 6$. A rectangle PQRS is drawn with upper vertices R and S on the graph of f, and PQ on the x-axis, as shown in the following diagram.



(a) Let $\text{OP} = x$.

(i) Find PQ, giving your answer in terms of x.

(ii) Hence, write down an expression for the area of the rectangle, giving your answer in terms of x. *[3]*

(b) (i) Find the rate of change of area when $x = 2$.

(ii) The area is decreasing for $a < x < b$. Find the value of a and of b. *[4]*

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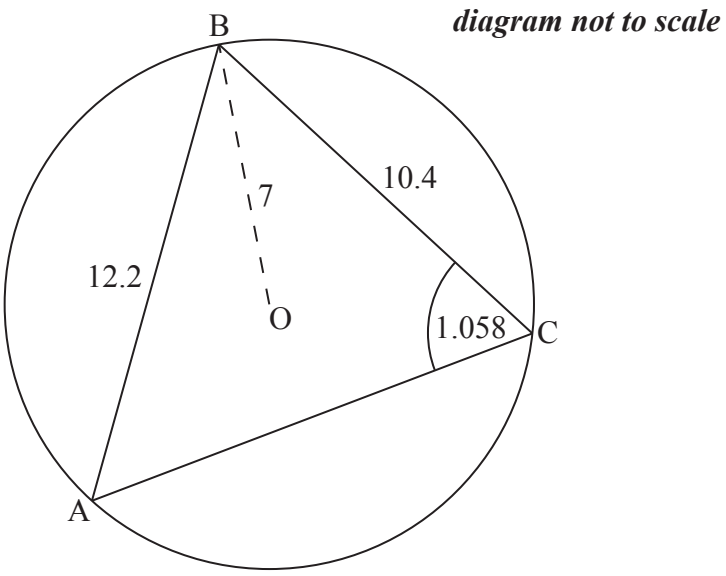
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Consider a circle with centre O and radius 7 cm. Triangle ABC is drawn such that its vertices are on the circumference of the circle.



$AB = 12.2\,\text{cm}$, $BC = 10.4\,\text{cm}$ and $A\hat{C}B = 1.058$ radians.

(a) Find $B\hat{A}C$. *[3]*

(b) Find AC. *[5]*

(c) Hence or otherwise, find the length of arc ABC. *[6]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 17]*

Consider the lines L_1 and L_2 with equations $L_1 : \boldsymbol{r} = \begin{pmatrix} 11 \\ 8 \\ 2 \end{pmatrix} + s \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$ and $L_2 : \boldsymbol{r} = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$.

The lines intersect at point P.

(a) Find the coordinates of P. *[6]*

(b) Show that the lines are perpendicular. *[5]*

(c) The point $\mathrm{Q}(7, 5, 3)$ lies on L_1. The point R is the reflection of Q in the line L_2. Find the coordinates of R. *[6]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 14]*

Samantha goes to school five days a week. When it rains, the probability that she goes to school by bus is 0.5. When it does not rain, the probability that she goes to school by bus is 0.3. The probability that it rains on any given day is 0.2.

(a) On a randomly selected school day, find the probability that Samantha goes to school by bus. *[4]*

(b) Given that Samantha went to school by bus on Monday, find the probability that it was raining. *[3]*

(c) In a randomly chosen school week, find the probability that Samantha goes to school by bus on exactly three days. *[2]*

(d) After n school days, the probability that Samantha goes to school by bus at least once is greater than 0.95. Find the smallest value of n. *[5]*

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

Wednesday 14 May 2014 (morning)

1 hour 30 minutes

Examination code

2	2	1	4	–	7	3	0	4

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

12 pages

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The following diagram shows triangle ABC.

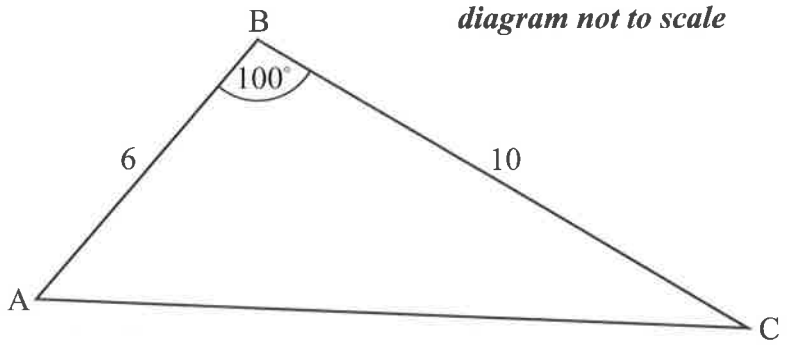

diagram not to scale

$AB = 6\,\text{cm}$, $BC = 10\,\text{cm}$, and $A\hat{B}C = 100^\circ$.

(a) Find AC. *[3]*

(b) Find $B\hat{C}A$. *[3]*

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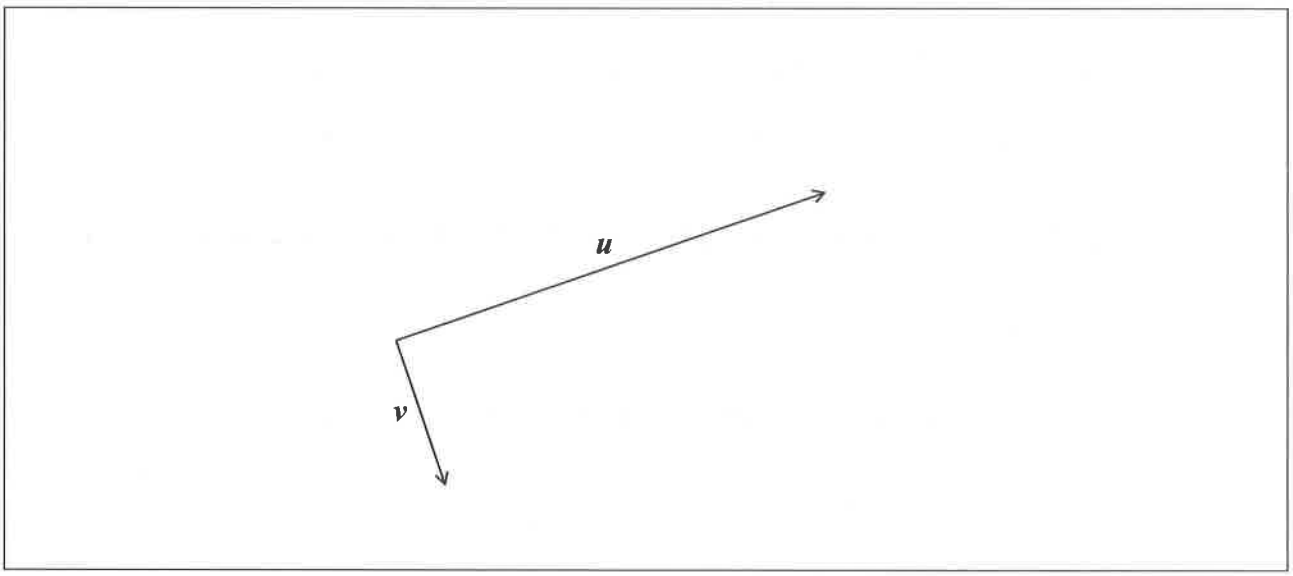
4. *[Maximum mark: 6]*

The following diagram shows two perpendicular vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.



(a) Let $\boldsymbol{w} = \boldsymbol{u} - \boldsymbol{v}$. Represent $\boldsymbol{w}$ on the diagram above. *[2]*

(b) Given that $\boldsymbol{u} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 5 \\ n \\ 3 \end{pmatrix}$, where $n \in \mathbb{Z}$, find n. *[4]*

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5. *[Maximum mark: 6]*

The population of deer in an enclosed game reserve is modelled by the function $P(t) = 210\sin(0.5t - 2.6) + 990$, where t is in months, and $t = 1$ corresponds to 1 January 2014.

(a) Find the number of deer in the reserve on 1 May 2014. *[3]*

(b) (i) Find the rate of change of the deer population on 1 May 2014.

(ii) Interpret the answer to part (i) with reference to the deer population size on 1 May 2014. *[3]*

6. *[Maximum mark: 8]*

Ramiro and Lautaro are travelling from Buenos Aires to El Moro.

Ramiro travels in a vehicle whose velocity in m s^{-1} is given by $V_R = 40 - t^2$, where t is in seconds.

Lautaro travels in a vehicle whose displacement from Buenos Aires in metres is given by $S_L = 2t^2 + 60$.

When $t = 0$, both vehicles are at the same point.

Find Ramiro's displacement from Buenos Aires when $t = 10$.

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7. *[Maximum mark: 7]*

Let $f(x) = \dfrac{g(x)}{h(x)}$, where $g(2) = 18$, $h(2) = 6$, $g'(2) = 5$, and $h'(2) = 2$. Find the equation of the normal to the graph of f at $x = 2$.

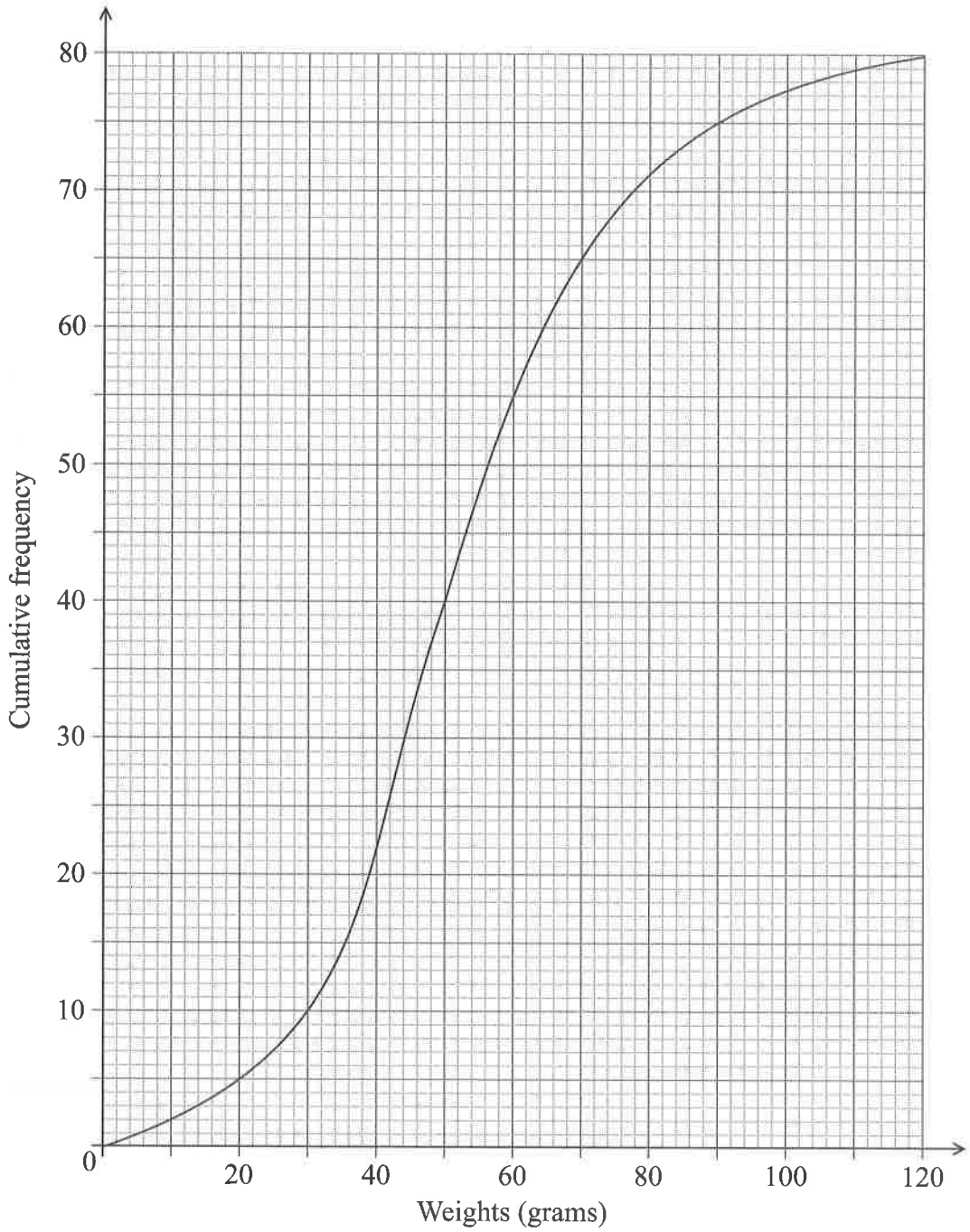
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 16]*

The weights in grams of 80 rats are shown in the following cumulative frequency diagram.



(This question continues on the following page)

*Do **NOT** write solutions on this page.*

(Question 8 continued)

(a) (i) Write down the median weight of the rats.

(ii) Find the percentage of rats that weigh 70 grams or less. *[4]*

The same data is presented in the following table.

Weights w grams	$0 \le w \le 30$	$30 < w \le 60$	$60 < w \le 90$	$90 < w \le 120$
Frequency	p	45	q	5

(b) (i) Write down the value of p.

(ii) Find the value of q. *[4]*

(c) Use the values from the table to estimate the mean and standard deviation of the weights. *[3]*

Assume that the weights of these rats are normally distributed with the mean and standard deviation estimated in part (c).

(d) Find the percentage of rats that weigh 70 grams or less. *[2]*

(e) A sample of five rats is chosen at random. Find the probability that at most three rats weigh 70 grams or less. *[3]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

Let $f(x) = \cos\left(\frac{\pi}{4}x\right) + \sin\left(\frac{\pi}{4}x\right)$, for $-4 \le x \le 4$.

(a) Sketch the graph of f. *[3]*

(b) Find the values of x where the function is decreasing. *[5]*

(c) The function f can also be written in the form $f(x) = a\sin\left(\frac{\pi}{4}(x+c)\right)$, where $a \in \mathbb{R}$, and $0 \le c \le 2$. Find the value of

(i) a;

(ii) c. *[7]*

10. *[Maximum mark: 14]*

Let $f(x) = \frac{3x}{x-q}$, where $x \ne q$.

(a) Write down the equations of the vertical and horizontal asymptotes of the graph of f. *[2]*

The vertical and horizontal asymptotes to the graph of f intersect at the point $\mathrm{Q}(1, 3)$.

(b) Find the value of q. *[2]*

(c) The point $\mathrm{P}(x, y)$ lies on the graph of f. Show that $\mathrm{PQ} = \sqrt{(x-1)^2 + \left(\frac{3}{x-1}\right)^2}$. *[4]*

(d) Hence find the coordinates of the points on the graph of f that are closest to $(1, 3)$. *[6]*