UNIT 16 Inequalities *NC: Algebra 3c and 3d*

TOPICS (Text and Practice Books)		**St**	**Ac**	**Ex**	**Sp**
16.1	*Inequalities on a Number Line*	✓	✓	-	-
16.2	*Solution of Linear Inequalities*	✓	✓	✓	✓
16.3	*Inequalities Involving Quadradic Terms*	✓	✓	✓	✓
16.4	*Graphical Approach to Inequalities*	✗	✓	✓	✓
16.5	*Dealing with More than One Inequality*	✗	✗	✓	✓

Activities

16.1	*Archimedes' Inequality for π*	✗	✓	✓	✓
16.2	*Isoperimetric Inequalities*	✓	✓	✓	✓
16.3	*Inequalities for Mean Values*	✗	✓	✓	✓
16.4	*Linear Programming*	✗	✗	✗	✓

OH Slides

16.1	*Inequalities on a Number Line*	✓	✓	-	-
16.2	*Solving Linear Inequalities*	✓	✓	✓	✓
16.3	*Inequalities Involving Quadratic Terms*	✓	✓	✓	✓
16.4	*Graphical Approach*	✗	✗	✓	✓

Mental Tests

16.1	✓	✓	-	-
16.2	✓	✓	-	-
16.3	✗	✗	✓	✓
16.4	✗	✗	✓	✓

Revision Tests

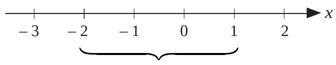
16.1	✓	-	-	-
16.2	✗	✓	-	-
16.3	✗	✗	✓	✓

UNIT 16 *Inequalities* Teaching Notes

Background and Preparatory Work

It may seem strange to be dealing with inequalities towards the end of this course, since the idea of an inequality is fundamental to the whole of mathematics, i.e. $1 > 0$, $2 > 1$, etc.

On the other hand, students will have met inequalities in a variety of forms earlier in this course but it is this Unit which picks up the threads. Apart from the obvious trivial inequalities (which are seen clearly on a number line) the first important inequality was derived by *Archimedes of Syracuse* (287–212BC), who used 96 -sided regular polygons (one inside and one outside a circle) to obtain



$-2 < 1$ or $1 > -2$

See internet address: http://www-groups.dcs.st-and.ac.uk:80/history/

A 16.1

$$3\frac{10}{71} < \pi < 3\frac{1}{7}.$$

Of course, with calculators and computers we can dramatically narrow this inequality – indeed, to any desired degree of accuracy – but remember that *Archimedes* had none of this technology to help him!

In addition to his work on estimating π, Archimedes' great claim to fame arises from his theorem which gives the weight of a body immersed in a liquid (*Archimedes' Principal*). Unfortunately he was killed during the capture of Syracuse by the Romans; his death is recounted by *Plutarch*:

> *As fate would have it, Archimedes was intent on working out some problem by a diagram, and having fixed both his mind and eyes upon the subject of his speculation, he did not notice the entry of the Romans nor that the city was taken. In this transport of study a soldier unexpectedly came up to him and commanded that he accompany him. When he declined to do this before he had finished his problem, the enraged soldier drew his sword and ran him through.*

Inequalities are also used in scientific notation, where numbers are expressed in the form

UNIT 1.7

$$x \times 10^{a}$$

where a is an integer and $1 \le x < 10$. This leads to space and time scales.

OS 1.2 and 1.3

Another use of inequalities is in quantifying the accuracy of a measurement; for example, a height measured as 176 cm to the nearest cm really means that the height is somewhere in the range

UNIT 6.7

$$175.5 \le \text{height} < 176.5.$$

There are also some classic inequalities which have been around for some time. The idea of isoperimetric inequalities has been derived from the Greek legend of *Princess Dido*; and the inequality A 16.2

$$A \geq G \geq H$$

A 16.3

where the A is the arithmetic mean of a set of positive numbers, G the geometric mean and H the harmonic mean, has been known for some time although it is difficult to prove in general.

A more recent application of solving sets of inequalities is that of *linear programming*. This technique was developed during the 2nd World War by the mathematicians, *John von Neuman* (1905–1957) and *George Dantzig* (1914–), in order to solve problems of optimising convoys across the Atlantic. Since then, the technique has been expanded and used in a variety of contexts – essentially when you are trying to maximise or minimise a quantity (e.g. profit) subject to a number of inequality constraints. It is just beyond the scope of the *GCSE Mathematics Higher Tier* syllabus but we have included an activity to show how it works. A 16.4

In summary, inequalities are a fundamental building block in mathematics and although this chapter deals with particular applications, it should be stressed that this *is* mainstream mathematics.

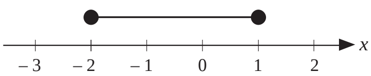
Teaching Points

Introduction

This unit is based on inequalities and their applications. It is important to stress the idea of a *number line* in the first sections, as this is fundamental to complete understanding of the topic.

For example, the inequality

$$-2 \leq x \leq 1$$



can be illustrated with reference to a number line and, for integer solutions, we can see that

$$x \in \{-2, -1, 0, 1\}.$$

Language/Notation

Pupils must be familiar with the usual notation for inequalities:

$$<, \ \leq, \ >, \ \geq.$$

It is also sometimes useful to use $\nless$ and $\ngtr$ which mean 'not less than' and 'not more than', although this is not fundamental to the development here.

Language used includes

- linear inequalities
- quadratic inequalities

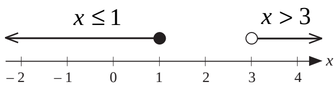
whereas *feasible region* and *linear programming* are beyond the GCSE syllabus but are included here as an activity. A 16.2

Key Points

- You must be careful to differentiate between $<$ and $\leq$.
- Representing $<$ and $\leq$ on a number line must be made clear,

  i.e. ○ for the end point of $<$

  ● for the end point of $\leq$.

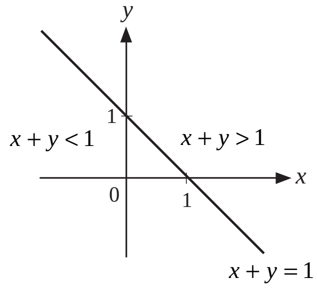


- A set of inequalities is solved by finding the values of x and y which satisfy all the inequalities.
- The *graphical approach* for solving sets of inequalities is the preferred method.

Misconceptions

Note that:

- multiplying an inequality by a negative number changes the inequality sign around, e.g. $x \geq 4 \Rightarrow -x \leq -4$;
- care must taken in identifying the region satisfied by an inequality of the form $ax + by \leq c$, as it is easy to choose the wrong side of the line;
- shading *in* or *out* can be confusing.



Key Concept

- The equation $ax + by = c$ defines a line; one side of the line satisfies $ax + by > c$ and the other side satisfies $ax + by < c$ (see example above).