**1.** (a) On the same graph sketch the curves $y = x^2$ and $y = 3 - \frac{1}{x}$ for values of $x$ from 0 to 4 and values of $y$ from 0 to 4. Show your scales on your axes.

**(4)**

(b) Find the points of intersection of these two curves.

**(4)**

(c) (i) Find the gradient of the curve $y = 3 - \frac{1}{x}$ in terms of $x$.

(ii) Find the value of this gradient at the point (1, 2).

**(4)**

(d) Find the equation of the tangent to the curve $y = 3 - \frac{1}{x}$ at the point (1, 2).

**(3)**

**(Total 15 marks)**

**2.** The functions $f$ and $g$ are defined by

$$f : x \mapsto \frac{x+4}{x}, x \in \mathbb{R}, x \neq 0$$

$$g : x \mapsto x, x \in \mathbb{R}$$

(a) Sketch the graph of $f$ for $-10 \leq x \leq 10$.

**(4)**

(b) Write down the equations of the horizontal and vertical asymptotes of the function $f$.

**(4)**

(c) Sketch the graph of $g$ on the same axes.

**(2)**

(d) Hence, or otherwise, find the solutions of $\frac{x+4}{x} = x$.

**(4)**

(e) Write down the range of function $f$.

**(2)**
**(Total 16 marks)**

**3.** Two functions $f(x)$ and $g(x)$ are given by

$$f(x) = \frac{1}{x^2+1},$$

$$g(x) = \sqrt{x},\ x \geq 0.$$

(a) Sketch the graphs of $f(x)$ and $g(x)$ together on the same diagram using values of $x$ between –3 and 3, and values of $y$ between 0 and 2. You must label each curve.

(b) State how many solutions exist for the equation $\frac{1}{x^2+1} - \sqrt{x} = 0$.

(c) Find a solution of the equation given in part (b).

*Working:*

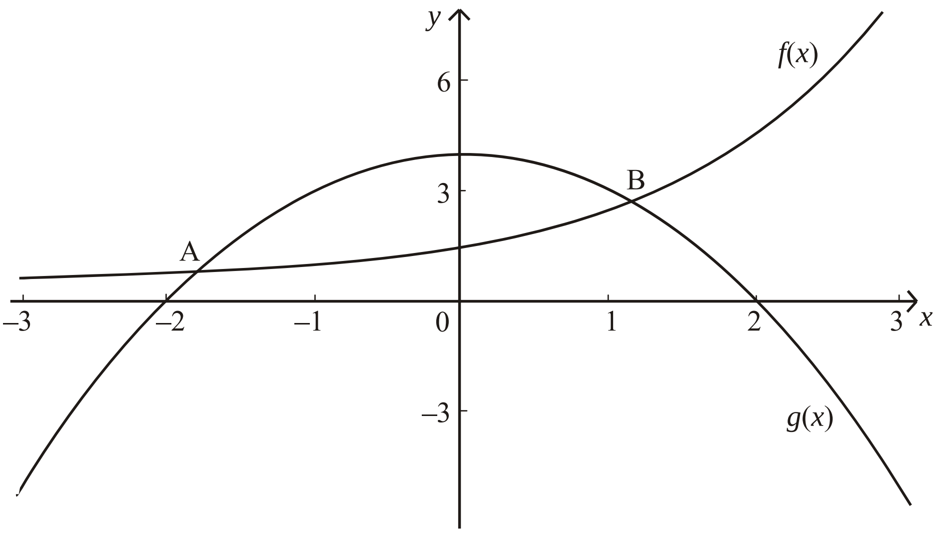
*Answers:*

(b) ..................................................

(c) ..................................................

**(Total 6 marks)**

**4.** The figure below shows the graphs of the functions $f(x) = 2^x + 0.5$ and $g(x) = 4 - x^2$ for values of $x$ between –3 and 3.




(a) Write down the coordinates of the points A and B.

(b) Write down the set of values of $x$ for which $f(x) < g(x)$.

Working:

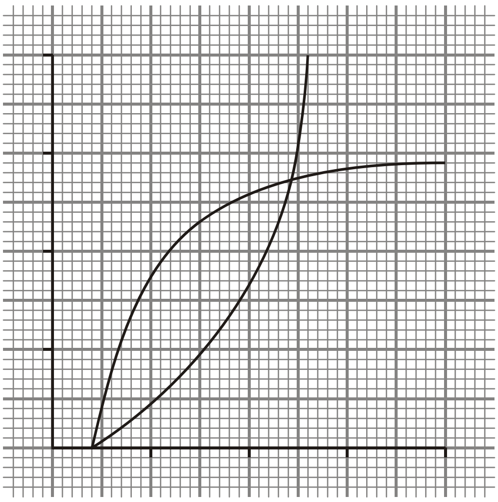
Answers:

(a) ..................................................

(b) ..................................................

**(Total 6 marks)**

1. (a)



For correct axes from 0 to 4. (A1)

For correct curve $y = x^2$. (A1)

For correct curve $y = 3 - \dfrac{1}{x}$. (A1)

For two intersections. (A1) 4

(b) (0.347, 0.121) or $x = 0.347$, $y = 0.121$ (by GDC) (G1)(G1)

(1.53, 2.35) or $x = 1.53$, $y = 2.35$. (G1)(G1)

(c) (i) $\dfrac{dy}{dx} = \dfrac{1}{x^2}$ for losing the constant. (A1)

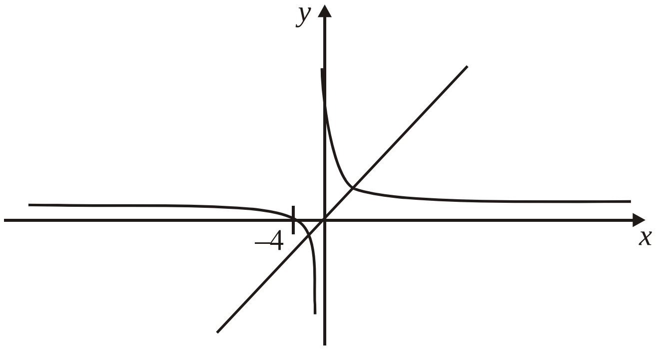
For attempting to write $\frac{1}{x}$ as a power (can be implied). (M1)

For correct answer $\frac{1}{x^2}$ or $x^{-2}$. (A1)

(ii) 1 (A1) 4

(d) For using $y = mx + c$ or equivalent with their $m$, to find $c$. (M1)
$c = 1$ (A1)
$y = x + 1$ (A1) 3
[15]

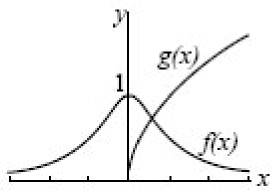
2. (a)




| | |
|---|---|
| For $x$-axis from –10 to 10. | (A1) |
| For –4 marked. | (A1) |
| For correct shape of graph. | (A1)(A1) |

| | | | |
|---|---|---|---|
| (b) | Horizontal asymptote | (A1) | |
| | $y = 1$ | (A1) | |
| | Vertical asymptote | (A1) | |
| | $x = 0$ | (A1) | 4 |
| (c) | Line drawn on sketch | (A2) | 2 |
| (d) | (2.56, 2.56) (–1.56, –1.56) | (A1)(A1)(A1)(A | |
| (e) | Range $y \in \mathbb{R}, y \neq 1$ | (A1)(A1) | |
| | | [16] | |

3. (a) With the given domain, the correct answer is




(A1)

(A1)(ft)

(A1)(ft) (C3)

***Notes:*** *Award* (A1) *for a neat window complying reasonably with the requirements.*

*The window must clearly have used x values from –3 to 3 and y values at least from 0 to 1. Axes labels are not essential. Some indication of scale must be present but this need not be a formal scale, eg tick marks, a single number on each axis or coordinates of the intersection are all adequate.*

*Award* (A1) *for each curve correct and correctly labelled with f and g or the expressions for f and g. Can follow through both curves, for example if curves are incomplete due to a poor window, and penalize only once if both curve labels are missing. Examiners should familiarize themselves with the graph of* $\frac{1}{x^2}+1$ *as this is expected to appear in error. With the correct window, this graph will not be seen at all, but with a larger y interval it might look a little like the correct graph except that it would have asymptotes at x = 0 and y = 1. Award* (A0) *for this curve.*

(b) One solution.

(A1)

(ft) (C1)

(c) Solution occurs at the point of intersection of the curves,

where $x = 0.569840$

0.570. (M1)(A1) (C2)

***Notes:*** *The* (M1) *can also be awarded for the intersection point indicated on the sketch.*

*(0.57 is an (AP))*

*If a coordinate pair is given as the answer and the x value is correct with no method mentioned, award* (C1) *or if the method is mentioned, award* (M1)(A0).

*Can follow through if curve* $\frac{1}{x^2}+1$ *is drawn, answer to (c) is then 1.75.*

**[6]**

**4.** (a) A( –1.79, 0.789) and B(1.14, 2.70) (C2) (C2)

***Notes:*** *Award* (C2) *for each pair of coordinates obtained from the GDC*

*Award* (A1)(A2)*(ft) if bracket is not used.*

(b) $-1.79 < x < 1.14$

(ft)(A1)(ft) (A1) (C2)

***Note:*** *Award* (A1) *for both numbers,* (A1) *for correct inequalities.*

**[6]**