Trigonometry

What you need to know/be able to do:

- Definitions of $\sin\theta$, $\cos\theta$ and $\tan\theta$ (and their relation to the unit circle)
- Understand that $\sin(90^0 - x) = \cos x$
- Handle angles of depression/elevation
- Solve a triangle (i.e. find all 3 sides, all 3 angles)
- Know and use the following table:

	0^0	30^0	45^0	60^0	90^0
sin	0	1/2	√2/2	√3/2	1
cos	1	√3/2	√2/2	1/2	0
tan	0	√3/3	1	√3	∞

- Apply trigonometry to non-right triangles, via the Sine Rule and the Cosine Rule
- The Ambiguous Case of the Sine Rule
- Using trigonometry to find the area of a triangle
- Key identities: $\tan\theta = \sin\theta/\cos\theta$ and $\sin^2\theta + \cos^2\theta = 1$
- These Double angle formulae:
  - $\sin 2\theta = 2\sin\theta\cos\theta$
  - $\cos 2\theta = 2\cos^2\theta - 1 \quad = \cos^2\theta - \sin^2\theta \quad = 1 - 2\sin^2\theta$
- Domain, Range, Period and Amplitude of all 3 trig functions. This knowledge needs to be displayed algebraically and graphically
- Linking transformations of functions to the general form $y = a\sin(b(x+c)) + d$
- Solve the equations $\sin x=k$, $\cos x=k$ and $\tan x=k$, for varying domains
- The definition of the radian
- Using radian measure in circle problems (eg arc length, sector area etc) and also in the context of a given domain
- Solving quadratics which feature $\sin\theta$, $\cos\theta$ or $\tan\theta$ as the variable

The above points are specified in the IB syllabus in the section entitled ***Topic 3: Circular Functions & Trigonometry***. This isn't the end of the story though – we also need to be able to ***differentiate*** and to ***integrate*** trig functions. Trigonometry connects to ***Vectors***, via the **Scalar Product**.
There is a high likelihood that the kinds of equations you need to use technology in order to solve (in paper 2) will be trig equations – eg solve for x in: $\sin^2 3x - \sin 0.5x = 0.85$
Do you know how to use your GDC to do this?

Past Paper questions:

1. [May03 p1]

Find all solutions of the equation $\cos 3x = \cos(0.5x)$, for $0 \le x \le \pi$.

2. [Nov06 p1]

The function f is defined by $f : x \mapsto 30\sin 3x \cos 3x,\ 0 \le x \le \frac{\pi}{3}$.

(a) Write down an expression for $f(x)$ in the form $a\sin 6x$, where a is an integer.

(b) Solve $f(x) = 0$, giving your answers in terms of π.

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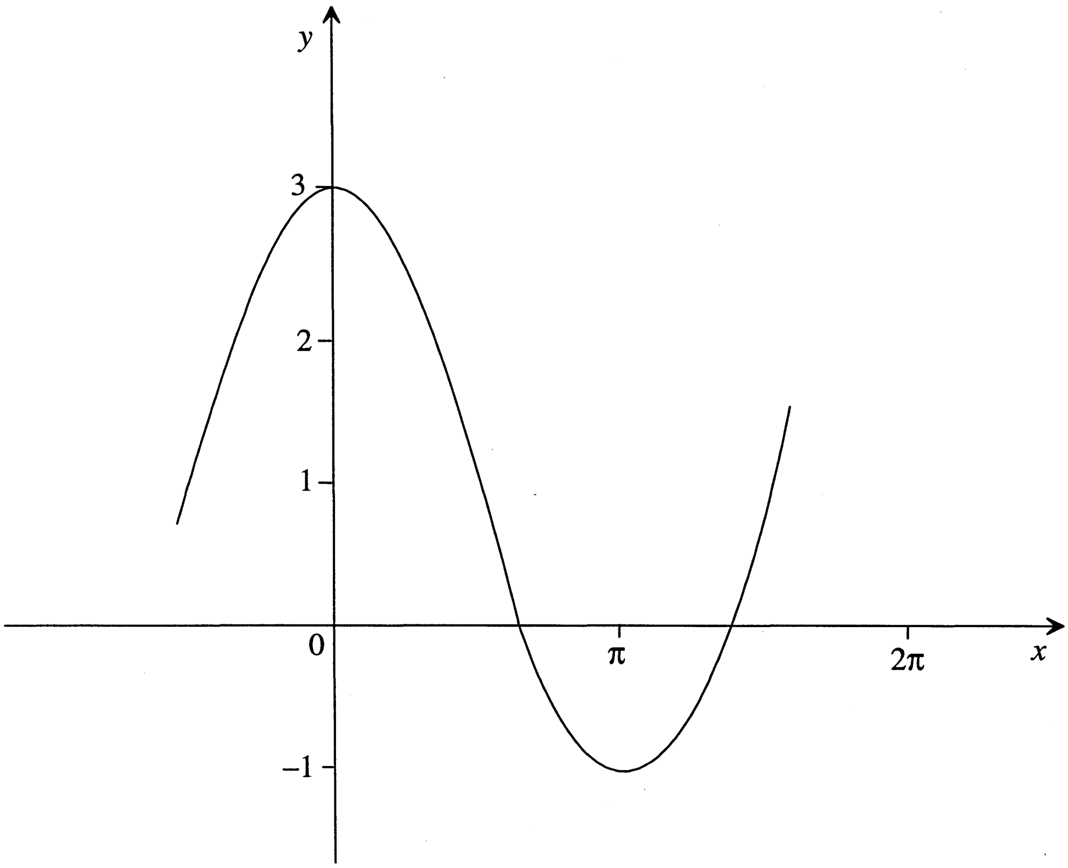
3. [May03 p1]

Part of the graph of $y = p + q\cos x$ is shown below. The graph passes through the points $(0, 3)$ and $(\pi, -1)$.



Find the value of

(a) p;

(b) q.

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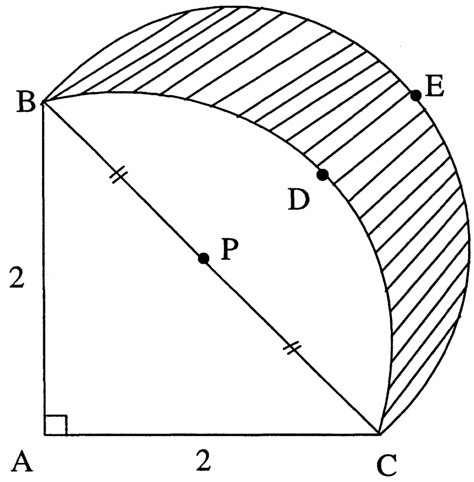
4. [May03 p1]

The diagram below shows a triangle and two arcs of circles.

The triangle ABC is a right-angled isosceles triangle, with $\mathrm{AB} = \mathrm{AC} = 2$. The point P is the midpoint of [BC].

The arc BDC is part of a circle with centre A.

The arc BEC is part of a circle with centre P.



(a) Calculate the area of the segment BDCP.

(b) Calculate the area of the shaded region BECD.

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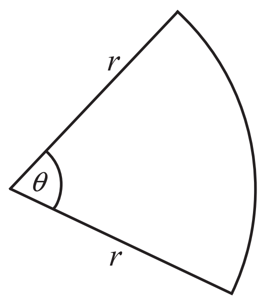
5. [May07 p1, TZ2]

The following diagram shows a sector of a circle of radius r cm, and angle θ at the centre. The perimeter of the sector is 20 cm.

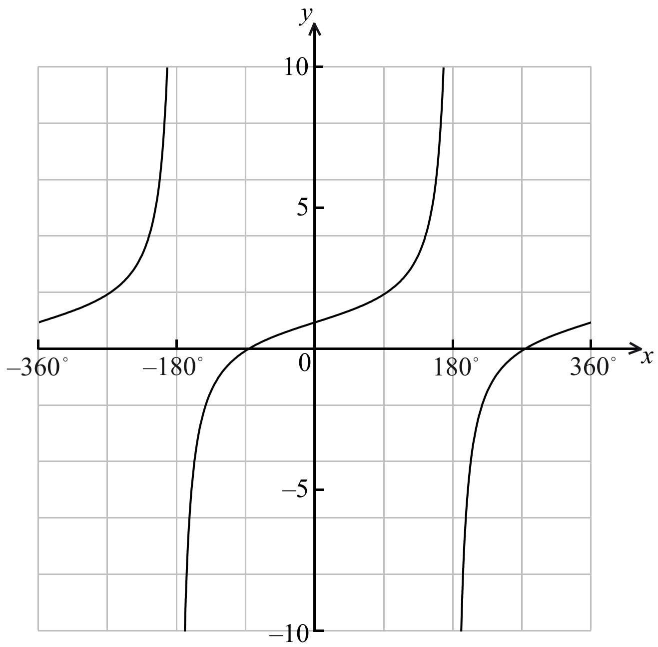


(a) Show that $\theta = \dfrac{20 - 2r}{r}$.

(b) The area of the sector is $25\ \text{cm}^2$. Find the value of r.

6. [Nov07, p1]

The diagram below shows the graph of $f(x)=1+\tan\left(\frac{x}{2}\right)$ for $-360^{\circ}\le x\le 360^{\circ}$.



(a) On the same diagram, draw the asymptotes. *[2 marks]*

(b) Write down

(i) the period of the function;

(ii) the value of $f(90^{\circ})$. *[2 marks]*

(c) Solve $f(x)=0$ for $-360^{\circ}\le x\le 360^{\circ}$. *[2 marks]*

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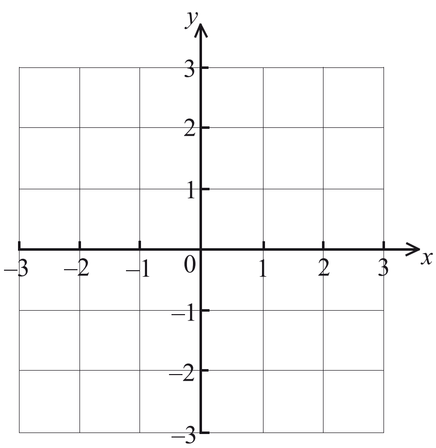
7. [Nov08, p2]

Let $f(x)=x\cos(x-\sin x)$, $0\le x\le 3$.

(a) Sketch the graph of f on the following set of axes. *[3 marks]*



(b) The graph of f intersects the x-axis when $x=a$, $a\neq 0$. Write down the value of a. *[1 mark]*

8. [May09, p2, TZ2]

The diagram below shows a triangle ABD with $\text{AB} = 13$ cm and $\text{AD} = 6.5$ cm.
Let C be a point on the line BD such that $\text{BC} = \text{AC} = 7$ cm.

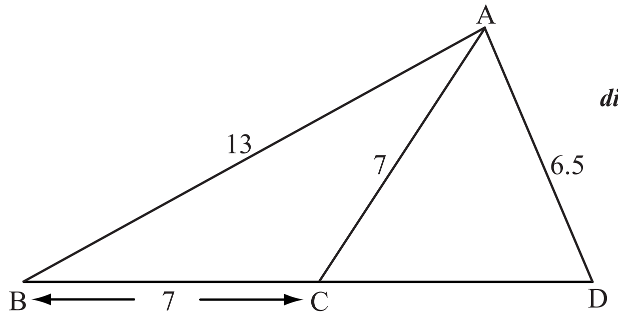


diagram not to scale

(a) Find the size of angle ACB. *[3 marks]*

(b) Find the size of angle CAD. *[5 marks]*

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9. [May09, p2, TZ2]

Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.

(a) Sketch the graph of f. *[3 marks]*

(b) Write down

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0. *[3 marks]*

(c) Hence write $f(x)$ in the form $p\sin(qx+r)$. *[3 marks]*

(d) Write down one value of x such that $f'(x) = 0$. *[2 marks]*

(e) Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions. *[2 marks]*

10. [May01, p1]

(a) Write the expression $3\sin^2 x + 4\cos x$ in the form $a\cos^2 x + b\cos x + c$.

(b) Hence or otherwise, solve the equation

$$3\sin^2 x + 4\cos x - 4 = 0, \qquad 0° \le x \le 90°.$$