1. The distances travelled to work, D km, by the employees at a large company are normally distributed with $D \sim N(30, 8^2)$.

(a) Find the probability that a randomly selected employee has a journey to work of more than 20 km.

(3)

(b) Find the upper quartile, Q_3 , of D.

(3)

(c) Write down the lower quartile, Q_1 , of D.

(1)

An outlier is defined as any value of D such that $D < h$ or $D > k$ where

$$h = Q_1 - 1.5 \times (Q_3 - Q_1) \quad \text{and} \quad k = Q_3 + 1.5 \times (Q_3 - Q_1)$$

(d) Find the value of h and the value of k.

(2)

An employee is selected at random.

(e) Find the probability that the distance travelled to work by this employee is an outlier.

(3)
(Total 12 marks)

2. The heights of a population of women are normally distributed with mean μ cm and standard deviation σ cm. It is known that 30% of the women are taller than 172 cm and 5% are shorter than 154 cm.

(a) Sketch a diagram to show the distribution of heights represented by this information.

(3)

(b) Show that $\mu = 154 + 1.6449\sigma$.

(3)

(c) Obtain a second equation and hence find the value of μ and the value of σ.
(4)

A woman is chosen at random from the population.

(d) Find the probability that she is taller than 160 cm.
(3)
(Total 13 marks)

3. The lifetimes of bulbs used in a lamp are normally distributed.
A company X sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.

(a) Find the probability of a bulb, from company X, having a lifetime of less than 830 hours.
(3)

(b) In a box of 500 bulbs, from company X, find the expected number having a lifetime of less than 830 hours.
(2)

A rival company Y sells bulbs with a mean lifetime of 860 hours and 20% of these bulbs have a lifetime of less than 818 hours.

(c) Find the standard deviation of the lifetimes of bulbs from company Y.
(4)

Both companies sell the bulbs for the same price.

(d) State which company you would recommend. Give reasons for your answer.
(2)
(Total 11 marks)

4. The random variable X has a normal distribution with mean 30 and standard deviation 5.

(a) Find $P(X < 39)$.
(2)

(b) Find the value of d such that $P(X < d) = 0.1151$

(4)

(c) Find the value of e such that $P(X > e) = 0.1151$

(2)

(d) Find $P(d < X < e)$.

(2)
(Total 10 marks)

5. A packing plant fills bags with cement. The weight X kg of a bag of cement can be modelled by a normal distribution with mean 50 kg and standard deviation 2 kg.

(a) Find $P(X > 53)$.

(3)

(b) Find the weight that is exceeded by 99% of the bags.

(5)

Three bags are selected at random.

(c) Find the probability that two weigh more than 53 kg and one weighs less than 53 kg.

(4)
(Total 12 marks)

6. The weights of bags of popcorn are normally distributed with mean of 200 g and 60% of all bags weighing between 190 g and 210 g.

(a) Write down the median weight of the bags of popcorn.

(1)

(b) Find the standard deviation of the weights of the bags of popcorn.

(5)

A shopkeeper finds that customers will complain if their bag of popcorn weighs less than 180 g.

(c) Find the probability that a customer will complain.

(3)
(Total 9 marks)

7. The random variable X has a normal distribution with mean 20 and standard deviation 4.

(a) Find $P(X > 25)$.

(3)

(b) Find the value of d such that $P(20 < X < d) = 0.4641$

(4)
(Total 7 marks)

8. The measure of intelligence, IQ, of a group of students is assumed to be Normally distributed with mean 100 and standard deviation 15.

(a) Find the probability that a student selected at random has an IQ less than 91.

(4)

The probability that a randomly selected student has an IQ of at least $100 + k$ is 0.2090.

(b) Find, to the nearest integer, the value of k.

(6)
(Total 10 marks)

9. From experience a high-jumper knows that he can clear a height of at least 1.78 m oncein 5 attempts. He also knows that he can clear a height of at least 1.65 m on 7 out of 10attempts.

Assuming that the heights the high-jumper can reach follow a Normal distribution,

(a) draw a sketch to illustrate the above information,

(3)

(b) find, to 3 decimal places, the mean and the standard deviation of the heights the high-jumper can reach,

(6)

(c) calculate the probability that he can jump at least 1.74 m.

(3)
(Total 12 marks)

10. The heights of a group of athletes are modelled by a normal distribution with mean 180 cm and standard deviation 5.2 cm. The weights of this group of athletes are modelled by a normal distribution with mean 85 kg and standard deviation 7.1 kg.

Find the probability that a randomly chosen athlete,

(a) is taller than 188 cm,

(3)

(b) weighs less than 97 kg.

(2)

(c) Assuming that for these athletes height and weight are independent, find the probability that a randomly chosen athlete is taller than 188 cm and weighs more than 97 kg.

(3)

(d) Comment on the assumption that height and weight are independent.

(1)
(Total 9 marks)

11. A scientist found that the time taken, M minutes, to carry out an experiment can be modelled by a normal random variable with mean 155 minutes and standard deviation 3.5 minutes.

Find

(a) P(M > 160).

(3)

(b) $P(150 \leq M \leq 157)$.

(4)

(c) the value of m, to 1 decimal place, such that $P(M \leq m) = 0.30$.

(4)
(Total 11 marks)

12. The random variable X is normally distributed with mean 79 and variance 144.

Find

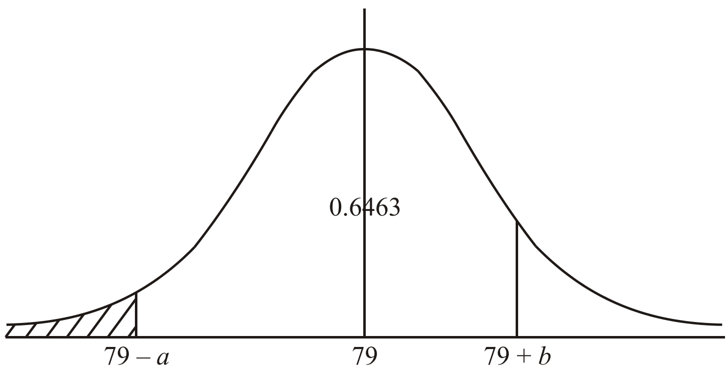
(a) $P(X < 70)$,

(3)

(b) $P(64 < X < 96)$.

(3)

It is known that $P(79 - a \leq X \leq 79 + b) = 0.6463$. This information is shown in the figure below.



Given that $P(X \geq 79 + b) = 2P(X \leq 79 - a)$,

(c) show that the area of the shaded region is 0.1179.

(3)

(d) Find the value of b.

(4)
(Total 13 marks)

13. The random variable X ~ N(μ, σ^2).

It is known that

$P(X \leq 66) = 0.0359$ and $P(X \geq 81) = 0.1151$.

(a) In the space below, give a clearly labelled sketch to represent these probabilities on a Normal curve.

(1)

(b) (i) Show that the value of σ is 5.

(ii) Find the value of μ.

(8)

(c) Find $P(69 \leq X \leq 83)$.

(3)
(Total 12 marks)

14. Students in Mr Brawn's exercise class have to do press-ups and sit-ups. The number of press-ups x and the number of sit-ups y done by a random sample of 8 students are summarised below.

$$\Sigma x = 272, \quad \Sigma x^2 = 10\ 164, \quad \Sigma xy = 11\ 222,$$

$$\Sigma y = 320, \quad \Sigma y^2 = 13\ 464.$$

(a) Evaluate S_{xx}, S_{yy} and S_{xy}.
(4)

(b) Calculate, to 3 decimal places, the product moment correlation coefficient between x and y.
(3)

(c) Give an interpretation of your coefficient.
(2)

(d) Calculate the mean and the standard deviation of the number of press-ups done by these students.
(4)

Mr Brawn assumes that the number of press-ups that can be done by any student can be modelled by a normal distribution with mean μ and standard deviation σ. Assuming that μ and σ. take the same values as those calculated in part (d),

(e) find the value of a such that $P(\mu - a < X < \mu + a) = 0.95$.
(3)

(f) Comment on Mr Brawn's assumption of normality.
(2)
(Total 18 marks)

15. A health club lets members use, on each visit, its facilities for as long as they wish. The club's records suggest that the length of a visit can be modelled by a normal distribution with mean 90 minutes. Only 20% of members stay for more than 125 minutes.

(a) Find the standard deviation of the normal distribution.
(4)

(b) Find the probability that a visit lasts less than 25 minutes.

(3)

The club introduces a closing time of 10:00 pm. Tara arrives at the club at 8:00 pm.

(c) Explain whether or not this normal distribution is still a suitable model for the length of her visit.

(2)
(Total 9 marks)

16. The random variable X is normally distributed with mean μ and variance σ^2.

(*a*) Write down 3 properties of the distribution of X.

(3)

Given that $\mu = 27$ and $\sigma = 10$

(b) find $P(26 < X < 28)$.

(4)
(Total 7 marks)

17. Cooking sauces are sold in jars containing a stated weight of 500 g of sauce. The jars are filled by a machine. The actual weight of sauce in each jar is normally distributed with mean 505 g and standard deviation 10 g.

(a) (i) Find the probability of a jar containing less than the stated weight.

(ii) In a box of 30 jars, find the expected number of jars containing less than the stated weight.

(5)

The mean weight of sauce is changed so that 1% of the jars contain less than the stated weight. The standard deviation stays the same.

(b) Find the new mean weight of sauce.

(4)
(Total 9 marks)

18. The lifetimes of batteries used for a computer game have a mean of 12 hours and a standard deviation of 3 hours. Battery lifetimes may be assumed to be normally distributed.

Find the lifetime, t hours, of a battery such that 1 battery in 5 will have a lifetime longer than t.

(Total 6 marks)

19. A drinks machine dispenses coffee into cups. A sign on the machine indicates that each cup contains 50 ml of coffee. The machine actually dispenses a mean amount of 55 ml per cup and 10% of the cups contain less than the amount stated on the sign. Assuming that the amount of coffee dispensed into each cup is normally distributed find

(a) the standard deviation of the amount of coffee dispensed per cup in ml,

(4)

(b) the percentage of cups that contain more than 61 ml.

(3)

Following complaints, the owners of the machine make adjustments. Only 2.5% of cups now contain less than 50 ml. The standard deviation of the amount dispensed is reduced to 3 ml.

Assuming that the amount of coffee dispensed is still normally distributed,

(c) find the new mean amount of coffee per cup.

(4)
(Total 11 marks)

20. The weight of coffee in glass jars labelled 100 g is normally distributed with mean 101.80 g and standard deviation 0.72 g. The weight of an empty glass jar is normally distributed with mean 260.00 g and standard deviation 5.45 g. The weight of a glass jar is independent of the weight of the coffee it contains.

Find the probability that a randomly selected jar weighs less than 266 g and contains less than 100 g of coffee. Give your answer to 2 significant figures.

(Total 8 marks)