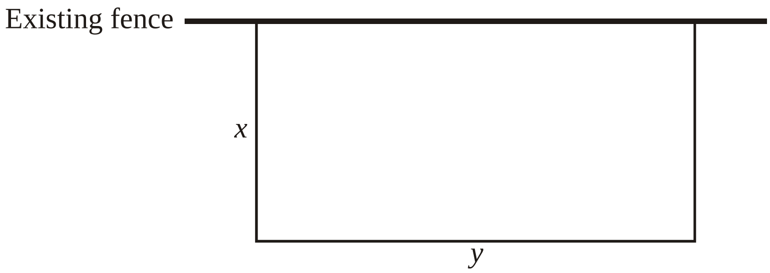
1. A farmer wishes to enclose a rectangular field using an existing fence for one of the four sides.



(a) Write an expression in terms of x and y that shows the total length of the new fence.

(1)

(b) The farmer has enough materials for 2500 metres of new fence. Show that

$$y = 2500 - 2x$$

(1)

(c) $A(x)$ represents the area of the field in terms of x.

(i) Show that

$$A(x) = 2500x - 2x^2$$

(2)

(ii) Find $A'(x)$.

(1)

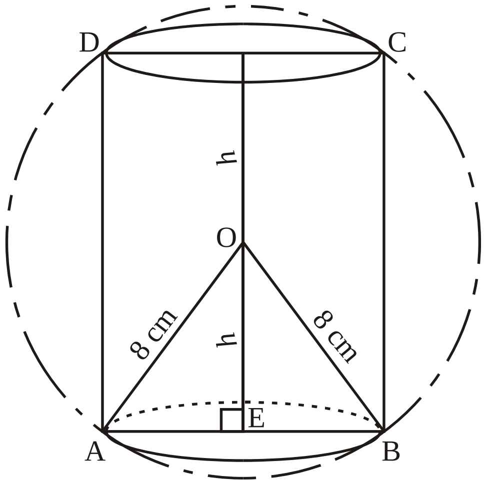
(iii) Hence or otherwise find the value of x that produces the maximum area of the field.

(3)

(iv) Find the maximum area of the field.

(3)
(Total 11 marks)

2. A cylinder is cut from a solid wooden sphere of radius 8 cm as shown in the diagram. The height of the cylinder is $2h$ cm.



(a) Find AE (the radius of the cylinder), in terms of h.

(2)

(b) Show that the volume (V) of the cylinder may be written as

$$V = 2\pi h\,(64 - h^2)\ \text{cm}^3.$$

(2)

(c) (i) Determine, correct to three significant figures, the height of the cylinder with the greatest volume that can be produced in this way.

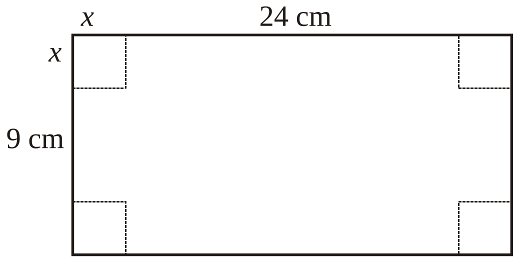
(5)

(ii) Calculate this greatest volume, giving your answer correct to the nearest cm^3.

(3)

(Total 12 marks)

3. A rectangular piece of card measures 24 cm by 9 cm. Equal squares of length x cm are cut from each corner of the card as shown in the diagram below. What is left is then folded to make an **open** box, of length l cm and width w cm.



(a) Write expressions, in terms of x, for

(i) the length, l;

(ii) the width, w.

(2)

(b) Show that the volume (B m^3) of the box is given by $B = 4x^3 - 66x^2 + 216x$.

(1)

(c) Find $\frac{\mathrm{d}B}{\mathrm{d}x}$.

(1)

(d) (i) Find the value of x which gives the maximum volume of the box.

(ii) Calculate the maximum volume of the box.

(4)

(Total 8 marks)

4. The cost of producing a mathematics textbook is \$15 (US dollars) and it is then sold for \$$x$.

(a) Find an expression for the profit made on each book sold.

(1)

A total of (100 000 – 4000x) books is sold.

(b) Show that the profit made on all the books sold is

$$P = 160\,000x - 4000x^2 - 1500\,000.$$

(3)

(c) (i) Find $\dfrac{dP}{dx}$.

(2)

(ii) Hence calculate the value of x to make a maximum profit

(2)

(d) Calculate the number of books sold to make this maximum profit.

(2)
(Total 10 marks)

5. A closed box has a square base of side x and height h.

(a) Write down an expression for the volume, V, of the box.

(1)

(b) Write down an expression for the total surface area, A, of the box.

(1)

The volume of the box is 1000 cm^3

(c) Express h in terms of x.

(2)

(d) Hence show that $A = 4000x^{-1} + 2x^2$.

(2)

(e) Find $\dfrac{dA}{dx}$.

(2)

(f) Calculate the value of x that gives a minimum surface area.

(4)

(g) Find the surface area for this value of x.

(3)
(Total 15 marks)

1. (a) $2x + y$ (A1) 1

(b) $2500 = 2x + y$ (M1)
$2500 - 2x = y$ (AG) 1

(c) (i) Area $A(x) = xy$ (M1)
$= x(2500 - 2x)$ (M1)
$= 2500x - 2x^2$ (AG) 2

(ii) $A'(x) = 2500 - 4x$ (A1) 1

(iii) $A'(x) = 0$
$0 = 2500 - 4x$ (M1)
$4x = 2500$ (M1)
$x = 625$ (A1) 3

(iv) $A(x) = 2500x - 2x^2$
$A(625) = 2500 \times 625 - 2(625)^2$ (M2)
$= 781250$
$= 781000 \text{ m}^2$ (A1) 3

[11]

2. (a) $AE^2 + OE^2 = OA^2$ .... (3)
$\Rightarrow AE^2 + k^2 = 8^2$ (M1)

Note: *Award (M1) for using and substituting correctly in equation (3).*

$AE^2 = \sqrt{64 - h^2}$ (A1) 2

(b) Volume (V) $= 2h\pi r^2$ (M1)

$= 2\pi h(AE^2)$	(M1)	
$= 2\pi h(64 - h^2)$ cm^3 .... (4)	(AG)	2

(c) (i) From (b) $V = 128\pi h - 2\pi h^3$ (M1)

__Note:__ Award (M1) for using equation (4) or any other correct approach.

$\frac{dV}{dh} = 128\pi - 6\pi h^2 = 0$ at maximum/minimum points (M2)

__Note:__ Award (M2) for correctly differentiating V w.r.t. x.

$\Rightarrow h = \sqrt{\frac{64}{3}} = \pm 4.62$ cm (3 s.f.) (A1)

Test to show that V is maximum when $h = 4.62$ (R1) 5

__Note:__ Award (R1) for testing to confirm V is indeed maximum.

(ii) $AE^2 = 64 - h^2$

$= 64 - \frac{64}{3} = \frac{128}{3}$ (M1)

__Notes:__ Follow through with candidate's AE from part (a) (M1) is for correctly obtaining candidate's AE^2.

Therefore maximum volume $= \pi r^2(2h) = \pi\left(\frac{128}{3}\right)\left(2\left(\sqrt{\frac{64}{3}}\right)\right)$ (M1)

__Note:__ Follow through with candidate's AE^2

$= 1238.7187... = 1239$ cm^3 (nearest cm^3) (A1) 3

__Notes:__ Correct answer only.
Accept 1238 cm^3 if and only if candidate uses $\pi = 3.14$

[12]

3. (a) (i) $l = 24 - 2x$ (A1)

(ii) $w = 9 - 2x$ (A1) 2

(b) $B = x(24 - 2x)(9 - 2x)$ (M1)
$= 4x^3 - 66x^2 + 216x$ (AG) 1

(c) $\frac{dB}{dx} = 12x^2 - 132x + 216$ (A1) 1

(d) (i) $\frac{dB}{dx} = 0 \Rightarrow x^2 - 11x + 18 = 0$

$(x - 2)(x - 9) = 0$ (M1)

$\Rightarrow x = 2$ or $x = 9$ (not possible)

Therefore, $x = 2$ cm. (A1)

(ii) $B = 4(2)^3 - 66(2)^2 + 216(2)$ (or $2 \times 20 \times 5$) (M1)

$= 200$ cm^3 (A1) 4

[8]

4. (a) $x - 15$ (A1) 1

(b) Profit = $(x - 15)(100\,000 - 4000x)$ (M1)

$= 100000x - 4000x^2 - 1500\,000 + 60\,000x$ (A2)

Note: *Award (A1) for one error, (A0) for 2 or more errors.*

$= 160\,000x - 4000x^2 - 1500\,000$ (AG) 3

(c) (i) $\frac{dP}{dx} = 160000 - 8000x$ (A1)(A1)

(ii) $0 = 160000 - 8000x$ (M1)

$x = \frac{160000}{8000}$

$x = 20$ (A1) 4

(d) Books sold = $100\,000 - 4000 \times 20$ (M1)

$= 20000$ (A1)

OR

Books = 20 000 (A2) 2

[10]

5. (a) $V = x^2h$ (A1) 1

(b) $A = 2x^2 + 4xh$ (A1) 1

(c) $1000 = x^2h$ (M1)

$h = \dfrac{1000}{x^2}$ (A1) 2

(d) $A = 2x^2 + 4x\left(\dfrac{1000}{x^2}\right)$ (M1)

$A = 2x^2 + \dfrac{4000}{x}$ (A1)

$= 2x^2 + 4000x^{-1}$ (AG) 2

(e) $\dfrac{\mathrm{d}A}{\mathrm{d}x} = 4x - 4000x^{-2}$ (A2) 2

(f) $4x - 4000x^{-2} = 0$ (M1)

$4x^3 - 4000 = 0$ (M1)

$4x^3 = 4000$

$x^3 = 1000$ (A1)

$x = 10$ (A1)

OR

$x = 10$ (G4) 4

(g) $h = \dfrac{1000}{100} = 10$ (A1)

$A = 2(100) + 4(10)(10)$ (M1)

$= 200 + 400 = 600$ (A1)

OR

$A = 600$ (G3) 3

[15]