IBDP Mathematics SL

Past Paper Problems — Topic: Exponents and Log

Student Name: — Date:

1. Solve the equation $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$.

(Total 4 marks)

2. Solve the equation $4^{3x-1} = 1.5625 \times 10^{-2}$.

(Total 4 marks)

3. If $\log_a 2 = x$ and $\log_a 5 = y$, find in terms of x and y, expressions for

(a) $\log_2 5$;

(b) $\log_a 20$.

(Total 4 marks)

4. Let $\log_{10} P = x$, $\log_{10} Q = y$ and $\log_{10} R = z$. Express $\log_{10}\left(\frac{P}{QR^3}\right)^2$ in terms of x , y and z.

(Total 4 marks)

5. Consider the following statements

A: $\log_{10}(10^x) > 0$.

B: $-0.5 \le \cos(0.5x) \le 0.5$.

C: $-\frac{\pi}{2} \le \arctan x \le \frac{\pi}{2}$.

(a) Determine which statements are true for all real numbers x. Write your answers (yes or no) in the table below.

Statement	(a) Is the statement true for all real numbers x? (Yes/No)	(b) If not true, example
A		
B		
C		

(b) If a statement is not true for all x, complete the last column by giving an example of one value of x for which the statement is false.

(Total 6 marks)

6. Solve the equation $\log_9 81 + \log_9 \frac{1}{9} + \log_9 3 = \log_9 x$.

(Total 4 marks)

7. Solve the equation $\log_{27} x = 1 - \log_{27} (x - 0.4)$.

(Total 6 marks)

8. Given that $\log_5 x = y$, express each of the following in terms of y.

(a) $\log_5 x^2$

(b) $\log_5\left(\frac{1}{x}\right)$

(c) $\log_{25} x$

(Total 6 marks)

9. Let $p = \log_{10} x$, $q = \log_{10} y$ and $r = \log_{10} z$.

Write the expression $\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right)$ in terms of p, q and r.

(Total 6 marks)

10. Let $a = \log x$, $b = \log y$, and $c = \log z$.

Write $\log\left(\frac{x^2\sqrt{y}}{z^3}\right)$ in terms of a, b and c.

(Total 6 marks)

11. Find the **exact** solution of the equation $9^{2x} = 27^{(1-x)}$.

(Total 6 marks)

12. (a) Given that $\log_3 x - \log_3 (x - 5) = \log_3 A$, express A in terms of x.

(b) Hence or otherwise, solve the equation $\log_3 x - \log_3 (x - 5) = 1$.

(Total 6 marks)

13. (a) Let $\log_c 3 = p$ and $\log_c 5 = q$. Find an expression in terms of p and q for

(i) $\log_c 15$;

(ii) $\log_c 25$.

(b) Find the value of d if $\log_d 6 = \frac{1}{2}$.

(Total 6 marks)

14. Let $\ln a = p$, $\ln b = q$. Write the following expressions in terms of p and q.

(a) $\ln a^3 b$ (b) $\ln\left(\frac{\sqrt{a}}{b}\right)$

(Total 6 marks)

15. Given that $p = \log_a 5$, $q = \log_a 2$, express the following in terms of p and/or q.

(a) $\log_a 10$

(b) $\log_a 8$

(c) $\log_a 2.5$

(Total 6 marks)

16. (a) Expand $\left(e+\frac{1}{e}\right)^4$ in terms of e.

(4)

(b) Express $\left(e+\frac{1}{e}\right)^4 + \left(e-\frac{1}{e}\right)^4$ as the sum of three terms.

(2)
(Total 6 marks)