1.) The probability distribution of a discrete random variable X is given by

$$P(X = x) = \frac{x^2}{14},\ x \in \{1, 2, k\}, \text{ where } k > 0.$$

(a) Write down $P(X = 2)$.

(1)

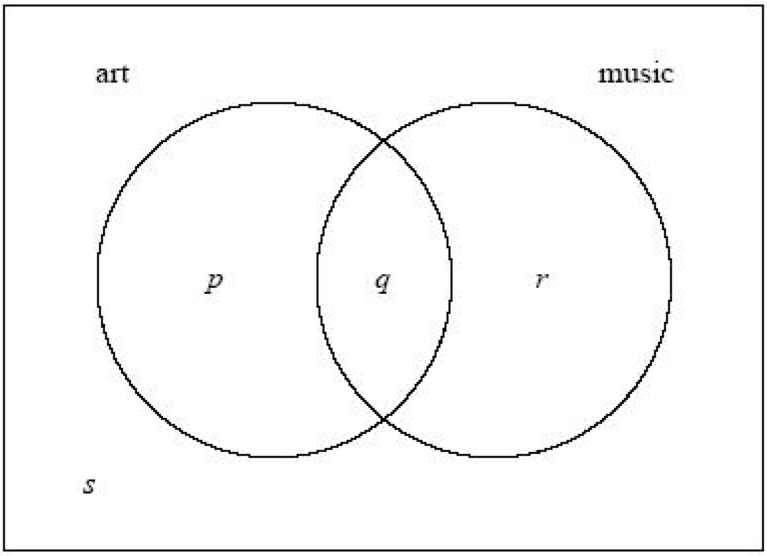
(b) Show that $k = 3$.

(4)

(c) Find $E(X)$.

(2)
(Total 7 marks)

2.) In a group of 16 students, 12 take art and 8 take music. One student takes neither art nor music. The Venn diagram below shows the events art and music. The values p, q, r and s represent numbers of students.



(a) (i) Write down the value of s.

(ii) Find the value of q.

(iii) Write down the value of p and of r.

(5)

(b) (i) A student is selected at random. Given that the student takes music, write down the probability the student takes art.

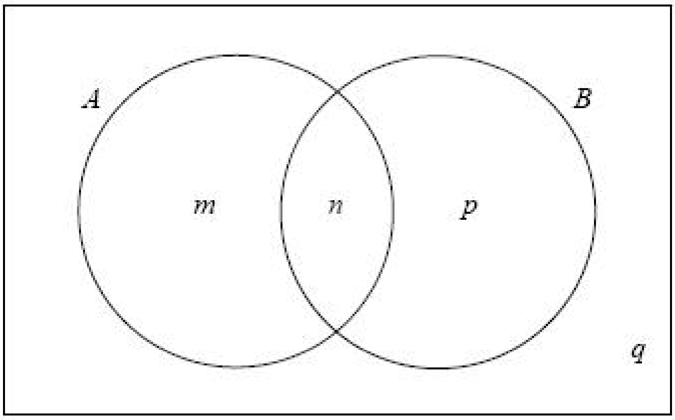
(ii) **Hence**, show that taking music and taking art are **not** independent events.

(4)

(c) Two students are selected at random, one after the other. Find the probability that the first student takes **only** music and the second student takes **only** art.

(4)
(Total 13 marks)

3.) The Venn diagram below shows events A and B where $P(A) = 0.3$, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.1$. The values m, n, p and q are probabilities.



(a) (i) Write down the value of n.

(ii) Find the value of m, of p, and of q.

(4)

(b) Find $P(B')$.

(2)

(Total 6 marks)

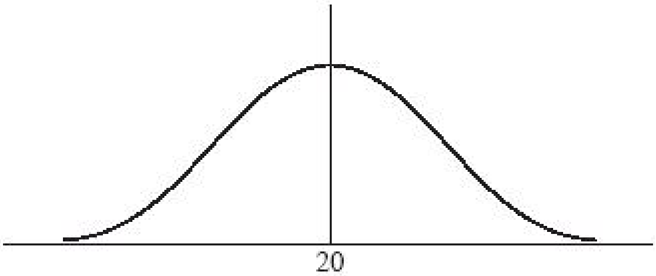
4.) A random variable X is distributed normally with a mean of 20 and variance 9.

(a) Find $P(X \leq 24.5)$.

(3)

(b) Let $P(X \leq k) = 0.85$.

(i) Represent this information on the following diagram.



(ii) Find the value of k.

(5)

(Total 8 marks)

5.) A box holds 240 eggs. The probability that an egg is brown is 0.05.

(a) Find the expected number of brown eggs in the box.

(2)

(b) Find the probability that there are 15 brown eggs in the box.

(2)

(c) Find the probability that there are at least 10 brown eggs in the box.

(3)
(Total 7 marks)

6.) A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.
90 % of the boxes made by machine B pass inspection.

A box is selected at random.

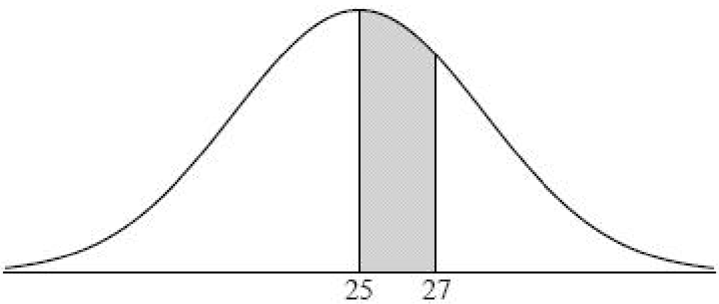
(a) Find the probability that it passes inspection.

(3)

(b) The company would like the probability that a box passes inspection to be 0.87.
Find the percentage of boxes that should be made by machine B to achieve this.

(4)
(Total 7 marks)

7.) Let the random variable X be normally distributed with mean 25, as shown in the following diagram.



The shaded region between 25 and 27 represents 30 % of the distribution.

(a) Find $P(X > 27)$.

(2)

(b) Find the standard deviation of X.

(5)
(Total 7 marks)

8.) Two fair 4-sided dice, one red and one green, are thrown. For each die, the faces are labelled 1, 2, 3, 4. The score for each die is the number which lands face down.

(a) List the pairs of scores that give a sum of 6.

(3)

The probability distribution for the sum of the scores on the two dice is shown below.

Sum	2	3	4	5	6	7	8
Probability	p	q	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	r	$\frac{1}{16}$

(b) Find the value of p, of q, and of r.

(3)

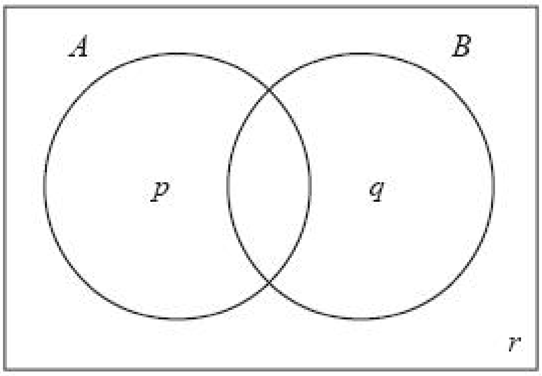
Fred plays a game. He throws two fair 4-sided dice four times. He wins a prize if the sum is 5 on three or more throws.

(c) Find the probability that Fred wins a prize.

(6)
(Total 12 marks)

9.) Consider the events A and B, where $P(A) = 0.5$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$.

The Venn diagram below shows the events A and B, and the probabilities p, q and r.



(a) Write down the value of

(i) p;

(ii) q;

(iii) r.

(3)

(b) Find the value of $P(A \mid B')$.

(2)

(c) Hence, or otherwise, show that the events A and B are **not** independent.

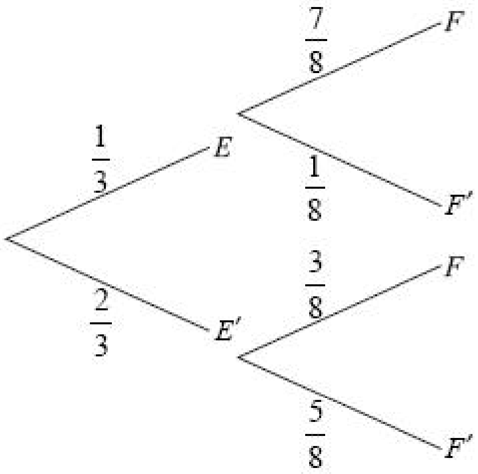
(1)
(Total 6 marks)

10.) José travels to school on a bus. On any day, the probability that José will miss the bus is $\frac{1}{3}$.

If he misses his bus, the probability that he will be late for school is $\frac{7}{8}$.

If he does not miss his bus, the probability that he will be late is $\frac{3}{8}$.

Let E be the event "he misses his bus" and F the event "he is late for school".
The information above is shown on the following tree diagram.



(a) Find

(i) $P(E \cap F)$;

(ii) $P(F)$.

(4)

(b) Find the probability that

(i) José misses his bus and is not late for school;

(ii) José missed his bus, given that he is late for school.

(5)

The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday.

(c) **Copy** and complete the probability distribution table.

***X* (cost in euros)**	0	3	6
P (*X*)	$\frac{1}{9}$		

(3)

(d) Find the expected cost for José for both days.

(2)
(Total 14 marks)

11.) Evan likes to play two games of chance, A and B.

For game A, the probability that Evan wins is 0.9. He plays game A seven times.

(a) Find the probability that he wins exactly four games.

(2)

For game B, the probability that Evan wins is p. He plays game B seven times.

(b) Write down an expression, in terms of p, for the probability that he wins exactly four games.

(2)

(c) Hence, find the values of p such that the probability that he wins exactly four games is 0.15.

(3)
(Total 7 marks)

12.) The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a) Find the probability that a player weighs more than 82 kg.

(2)

(b) (i) Write down the standardized value, z, for 68 kg.

(ii) Hence, find the standard deviation of weights.

(4)

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

(c) (i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament.

(5)

Of the players in the league, 25 % are women. Of the women, 70 % take part in the tournament.

(d) Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman.

(4)
(Total 15 marks)

13.) Jan plays a game where she tosses two fair six-sided dice. She wins a prize if the sum of her scores is 5.

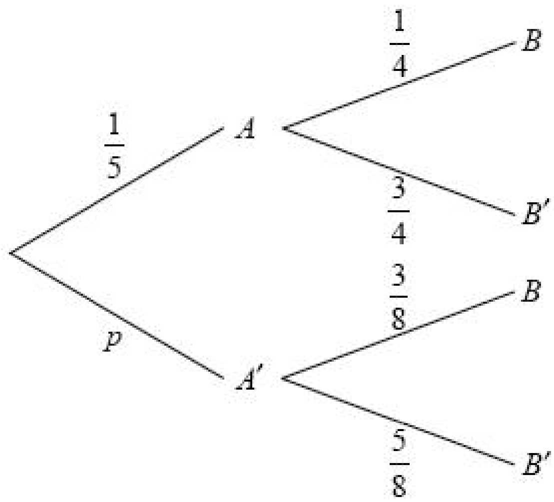
(a) Jan tosses the two dice once. Find the probability that she wins a prize.

(3)

(b) Jan tosses the two dice 8 times. Find the probability that she wins 3 prizes.

(2)
(Total 5 marks)

14.) The diagram below shows the probabilities for events A and B, with $P(A') = p$.



(a) Write down the value of p.

(1)

(b) Find $P(B)$.

(3)

(c) Find $P(A' \mid B)$.

(3)
(Total 7 marks)

15.) A test has five questions. To pass the test, at least three of the questions must be answered correctly.

The probability that Mark answers a question correctly is $\frac{1}{5}$. Let X be the number of questions that Mark answers correctly.

(a) (i) Find $E(X)$.

(ii) Find the probability that Mark passes the test.

(6)

Bill also takes the test. Let Y be the number of questions that Bill answers correctly.
The following table is the probability distribution for Y.

y	0	1	2	3	4	5
P(*Y* = *y*)	0.67	0.05	$a + 2b$	$a - b$	$2a + b$	0.04

(b) (i) Show that $4a + 2b = 0.24$.

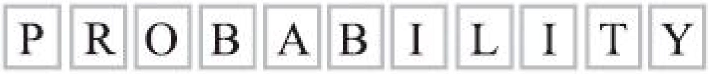
(ii) Given that $E(Y) = 1$, find a and b.

(8)

(c) Find which student is more likely to pass the test.

(3)
(Total 17 marks)

16.) The letters of the word PROBABILITY are written on 11 cards as shown below.



Two cards are drawn at random without replacement.
Let A be the event the first card drawn is the letter A.
Let B be the event the second card drawn is the letter B.

(a) Find P(A).

(1)

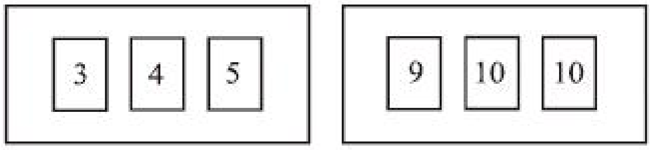
(b) Find $P(B \mid A)$.

(2)

(c) Find $P(A \cap B)$.

(3)
(Total 6 marks)

17.) Two boxes contain numbered cards as shown below.



Two cards are drawn at random, one from each box.

(a) Copy and complete the table below to show all nine equally likely outcomes.

3, 9		
3, 10		
3, 10		

(2)

Let S be the sum of the numbers on the two cards.

(b) Write down all the possible values of S.

(2)

(c) Find the probability of each value of S.

(2)

(d) Find the expected value of S.

(3)

(e) Anna plays a game where she wins \$50 if S is even and loses \$30 if S is odd.
Anna plays the game 36 times. Find the amount she expects to have at the end of the 36 games.

(3)
(Total 12 marks)

18.) A random variable X is distributed normally with mean 450 and standard deviation 20.

(a) Find $P(X \leq 475)$.

(2)

(b) Given that $P(X > a) = 0.27$, find a.

(4)
(Total 6 marks)

19.) In any given season, a soccer team plays 65 % of their games at home.
When the team plays at home, they win 83 % of their games.
When they play away from home, they win 26 % of their games.

The team plays one game.

(a) Find the probability that the team wins the game.

(4)

(b) If the team does not win the game, find the probability that the game was played at home.

(4)
(Total 8 marks)

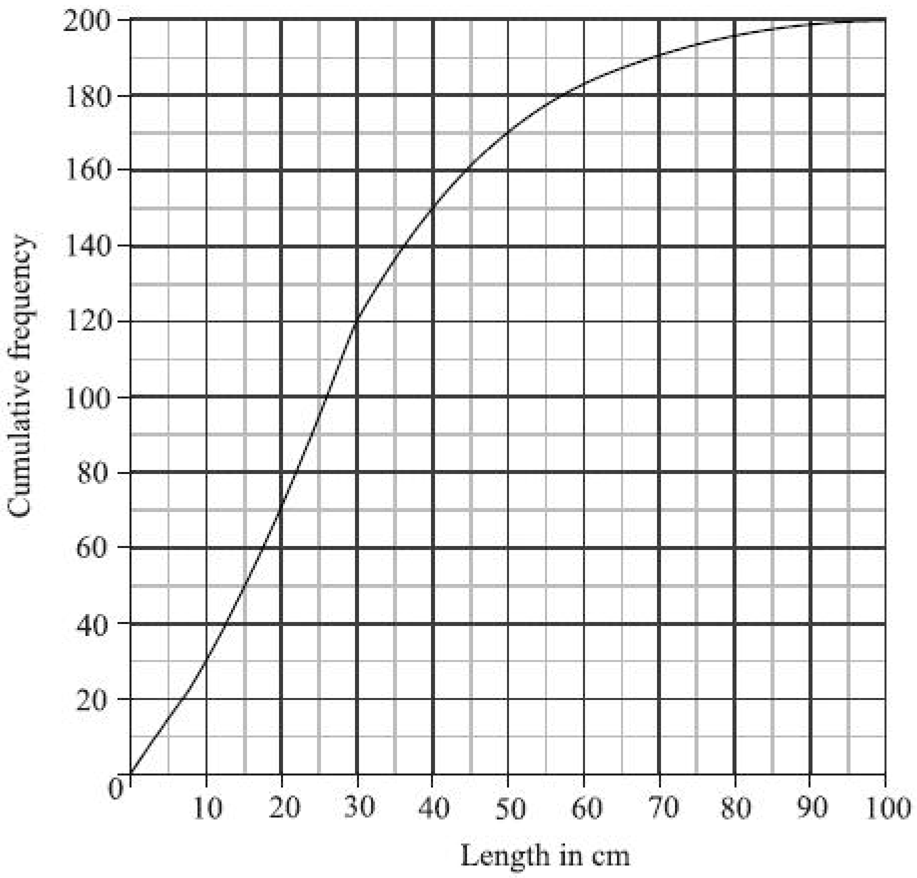
20.) A fisherman catches 200 fish to sell. He measures the lengths, l cm of these fish, and the results are shown in the frequency table below.

Length *l* cm	$0 \leq l < 10$	$10 \leq l < 20$	$20 \leq l < 30$	$30 \leq l < 40$	$40 \leq l < 60$	$60 \leq l < 75$	$75 \leq l < 100$
Frequency	30	40	50	30	33	11	6

(a) Calculate an estimate for the standard deviation of the lengths of the fish.

(3)

(b) A cumulative frequency diagram is given below for the lengths of the fish.



Use the graph to answer the following.

(i) Estimate the interquartile range.

(ii) Given that 40 % of the fish have a length more than k cm, find the value of k.

(6)

In order to sell the fish, the fisherman classifies them as small, medium or large.

Small fish have a length less than 20 cm.
Medium fish have a length greater than or equal to 20 cm but less than 60 cm.
Large fish have a length greater than or equal to 60 cm.

(c) Write down the probability that a fish is small.

(2)

The cost of a small fish is \$4, a medium fish \$10, and a large fish \$12.

(d) Copy and complete the following table, which gives a probability distribution for the cost \$$X$.

Cost \$$X$	4	10	12
$P(X = x)$		0.565	

(2)

(e) Find $E(X)$.

(2)
(Total 15 marks)

21.) A van can take either Route A or Route B for a particular journey.

If Route A is taken, the journey time may be assumed to be normally distributed with mean 46 minutes and a standard deviation 10 minutes.

If Route B is taken, the journey time may be assumed to be normally distributed with mean μ minutes and standard deviation 12 minutes.

(a) For Route A, find the probability that the journey takes **more** than 60 minutes.

(2)

(b) For Route B, the probability that the journey takes **less** than 60 minutes is 0.85. Find the value of μ.

(3)

(c) The van sets out at 06:00 and needs to arrive before 07:00.

(i) Which route should it take?

(ii) Justify your answer.

(3)

(d) On five consecutive days the van sets out at 06:00 and takes Route B. Find the probability that

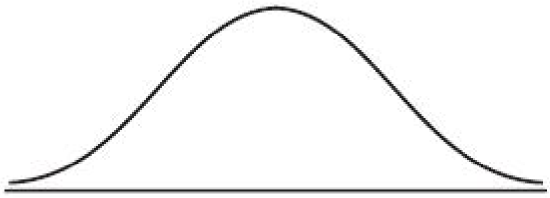
(i) it arrives before 07:00 on all five days;

(ii) it arrives before 07:00 on at least three days.

(5)
(Total 13 marks)

22.) Let X be normally distributed with mean 100 cm and standard deviation 5 cm.

(a) On the diagram below, shade the region representing $P(X > 105)$.



(2)

(b) Given that $P(X < d) = P(X > 105)$, find the value of d.

(2)

(c) Given that $P(X > 105) = 0.16$ (correct to two significant figures), find $P(d < X < 105)$.

(2)
(Total 6 marks)

23.) In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby.

(a) (i) Find the number of boys who play both sports.

(ii) Write down the number of boys who play only rugby.

(3)

(b) One boy is selected at random.

(i) Find the probability that he plays only one sport.

(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby.

(4)

Let A be the event that a boy plays football and B be the event that a boy plays rugby.

(c) Explain why A and B are **not** mutually exclusive.

(2)

(d) Show that A and B are **not** independent.

(3)
(Total 12 marks)

24.) A multiple choice test consists of ten questions. Each question has five answers. Only one of the answers is correct. For each question, Jose randomly chooses one of the five answers.

(a) Find the expected number of questions Jose answers correctly.

(1)

(b) Find the probability that Jose answers exactly three questions correctly.

(2)

(c) Find the probability that Jose answers more than three questions correctly.

(3)
(Total 6 marks)

25.) Consider the independent events A and B. Given that $P(B) = 2P(A)$, and $P(A \cup B) = 0.52$, find $P(B)$.

(Total 7 marks)

26.) A **four-sided** die has three blue faces and one red face. The die is rolled.

Let B be the event a blue face lands down, and R be the event a red face lands down.

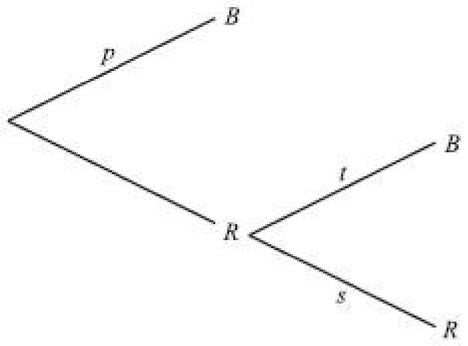
(a) Write down

(i) $P(B)$;

(ii) $P(R)$.

(2)

(b) If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the following tree diagram, where p, s, t are probabilities.



Find the value of *p*, of *s* and of *t*.

(2)

Guiseppi plays a game where he rolls the die. If a blue face lands down, he scores 2 and is finished. If the red face lands down, he scores 1 and rolls one more time. Let *X* be the total score obtained.

(c) (i) Show that P ($X = 3$) = $\frac{3}{16}$.

(ii) Find P ($X = 2$).

(3)

(d) (i) Construct a probability distribution table for *X*.

(ii) Calculate the expected value of *X*.

(5)

(e) If the total score is 3, Guiseppi wins \$10. If the total score is 2, Guiseppi gets nothing.

Guiseppi plays the game twice. Find the probability that he wins exactly \$10.

(4)
(Total 16 marks)

27.) There are 20 students in a classroom. Each student plays only one sport. The table below gives their sport and gender.

	Football	**Tennis**	**Hockey**
Female	5	3	3
Male	4	2	3

(a) One student is selected at random.

(i) Calculate the probability that the student is a male or is a tennis player.

(ii) Given that the student selected is female, calculate the probability that the student does not play football.

(4)

(b) Two students are selected at random. Calculate the probability that neither student plays football.

(3)
(Total 7 marks)

28.) A factory makes switches. The probability that a switch is defective is 0.04.

The factory tests a random sample of 100 switches.

(a) Find the mean number of defective switches in the sample.

(2)

(b) Find the probability that there are exactly six defective switches in the sample.

(2)

(c) Find the probability that there is at least one defective switch in the sample.

(3)
(Total 7 marks)

29.) A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

(a) One biscuit is chosen at random from the box. Find the probability that this biscuit

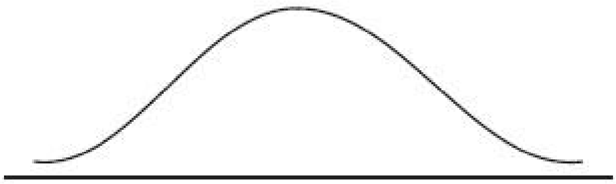
(i) weighs less than 8 g;

(ii) weighs between 6 g and 8 g.

(4)

(b) Five percent of the biscuits in the box weigh less than *d* grams.

(i) Copy and complete the following normal distribution diagram, to represent this information, by indicating *d*, and shading the appropriate region.



(ii) Find the value of *d*.

(5)

(c) The weights of biscuits in another box are normally distributed with mean m and standard deviation 0.5 g. It is known that 20% of the biscuits in this second box weigh less than 5 g.

Find the value of m

(4)
(Total 13 marks)

30.) The following table shows the probability distribution of a discrete random variable *X*.

x	–1	0	2	3
$P(X = x)$	0.2	$10k^2$	0.4	$3k$

(a) Find the value of *k*.

(4)

(b) Find the expected value of X.

(3)
(Total 7 marks)

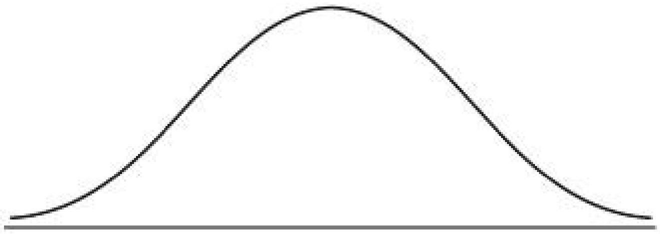
31.) The heights of certain plants are normally distributed. The plants are classified into three categories.

The shortest 12.92% are in category A.

The tallest 10.38% are in category C.

All the other plants are in category B with heights between r cm and t cm.

(a) Complete the following diagram to represent this information.



(2)

(b) Given that the mean height is 6.84 cm and the standard deviation 0.25 cm, find the value of r and of t.

(5)
(Total 7 marks)

32.) Paula goes to work three days a week. On any day, the probability that she goes on a red bus is $\frac{1}{4}$.

(a) Write down the expected number of times that Paula goes to work on a red bus in one week.

(2)

In one week, find the probability that she goes to work on a red bus

(b) on exactly two days;

(2)

(c) on at least one day.

(3)
(Total 7 marks)

33.) Let A and B be independent events, where $P(A) = 0.6$ and $P(B) = x$.

(a) Write down an expression for $P(A \cap B)$.

(1)

(b) Given that $P(A \cup B) = 0.8$,

(i) find x;

(ii) find $P(A \cap B)$.

(4)

(c) **Hence**, explain why A and B are **not** mutually exclusive.

(1)
(Total 6 marks)

34.) Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.

Score on second die

		1	2	3	4	5	6
	1	•	•	•	•	•	•
	2	•	•	•	•	•	•
Score on first die	3	•	•	•	•	•	•
	4	•	•	•	•	•	•
	5	•	•	•	•	•	•
	6	•	•	•	•	•	•

Let X be the sum of the scores on the two dice.

(a) Find

(i) $P(X = 6)$;

(ii) $P(X > 6)$;

(iii) $P(X = 7 \mid X > 5)$.

(6)

(b) Elena plays a game where she tosses two dice.

If the sum is 6, she wins 3 points.
If the sum is greater than 6, she wins 1 point.
If the sum is less than 6, she **loses** k points.

Find the value of k for which Elena's expected number of points is zero.

(7)
(Total 13 marks)

35.) The probability of obtaining heads on a biased coin is 0.18. The coin is tossed seven times.

(a) Find the probability of obtaining **exactly** two heads.

(2)

(b) Find the probability of obtaining **at least** two heads.

(3)
(Total 5 marks)

36.) The scores of a test given to students are normally distributed with a mean of 21. 80 % of the students have scores less than 23.7.

(a) Find the standard deviation of the scores.

(3)

A student is chosen at random. This student has the same probability of having a score less than 25.4 as having a score greater than *b*.

(b) (i) Find the probability the student has a score less than 25.4.

(ii) Find the value of *b*.

(4)
(Total 7 marks)

37.) A random variable X is distributed normally with a mean of 100 and a variance of 100.

(a) Find the value of X that is 1.12 standard deviations **above** the mean.

(4)

(b) Find the value of X that is 1.12 standard deviations **below** the mean.

(2)
(Total 6 marks)

38.) In a game a player rolls a biased four-faced die. The probability of each possible score is shown below.

Score	1	2	3	4
Probability	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	x

(a) Find the value of x.

(2)

(b) Find E(X).

(3)

(c) The die is rolled twice. Find the probability of obtaining two scores of 3.

(2)
(Total 7 marks)

39.) The heights of trees in a forest are normally distributed with mean height 17 metres. One tree is selected at random. The probability that a selected tree has a height greater than 24 metres is 0.06.

(a) Find the probability that the tree selected has a height less than 24 metres.

(2)

(b) The probability that the tree has a height less than D metres is 0.06.
Find the value of D.

(3)

(c) A woodcutter randomly selects 200 trees. Find the expected number of trees whose height lies between 17 metres and 24 metres.

(4)
(Total 9 marks)

40.) The probability of obtaining heads on a biased coin is $\frac{1}{3}$.

(a) Sammy tosses the coin three times. Find the probability of getting

(i) three heads;

(ii) two heads and one tail.

(5)

(b) Amir plays a game in which he tosses the coin 12 times.

(i) Find the expected number of heads.

(ii) Amir wins \$ 10 for each head obtained, and loses \$ 6 for each tail. Find his expected winnings.

(5)
(Total 10 marks)

41.) A factory makes calculators. Over a long period, 2 % of them are found to be faulty. A random sample of 100 calculators is tested.

(a) Write down the expected number of faulty calculators in the sample.

(1)

(b) Find the probability that three calculators are faulty.

(2)

(c) Find the probability that more than one calculator is faulty.

(3)
(Total 6 marks)

42.) The speeds of cars at a certain point on a straight road are normally distributed with mean μ and standard deviation σ. 15 % of the cars travelled at speeds greater than 90 km h^{-1} and 12 % of them at speeds less than 40 km h^{-1}. Find μ and σ.

(Total 6 marks)

43.) Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let X denote the number of red balls chosen. The following table shows the probability distribution for X.

X	0	1	2

$P(X = x)$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

(a) Calculate $E(X)$, the mean number of red balls chosen.

(3)

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B.

(b) (i) Draw a tree diagram to represent the above information, including the probability of each event.

(ii) Hence find the probability distribution for Y, where Y is the number of red balls chosen.

(8)

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

(c) Calculate the probability that two red balls are chosen.

(5)

(d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die.

(3)
(Total 19 marks)

44.) Consider the events A and B, where $P(A) = \frac{2}{5}$, $P(B') = \frac{1}{4}$ and $P(A \cup B) = \frac{7}{8}$.

(a) Write down $P(B)$.

(b) Find $P(A \cap B)$.

(c) Find $P(A \mid B)$.

(Total 6 marks)

45.) The heights of boys at a particular school follow a normal distribution with a standard deviation of 5 cm. The probability of a boy being shorter than 153 cm is 0.705.

(a) Calculate the mean height of the boys.

(b) Find the probability of a boy being taller than 156 cm.

(Total 6 marks)

46.) The eye colour of 97 students is recorded in the chart below.

	Brown	Blue	Green
Male	21	16	9
Female	19	19	13

One student is selected at random.

(a) Write down the probability that the student is a male.

(b) Write down the probability that the student has green eyes, given that the student is a female.

(c) Find the probability that the student has green eyes or is male.

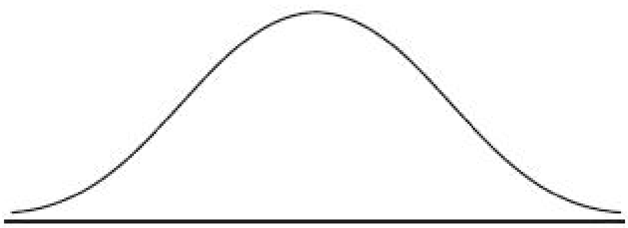
(Total 6 marks)

47.) The weights of a group of children are normally distributed with a mean of 22.5 kg and a standard deviation of 2.2 kg.

(a) Write down the probability that a child selected at random has a weight more than 25.8 kg.

(b) Of the group 95% weigh less than k kilograms. Find the value of *k*.

(c) The diagram below shows a normal curve.



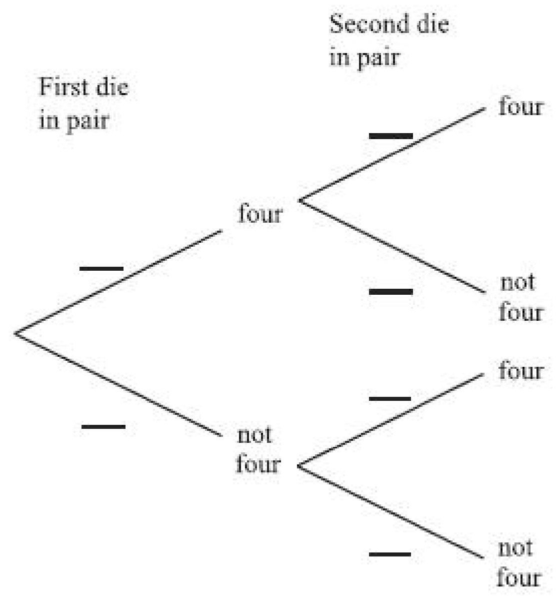
On the diagram, shade the region that represents the following information:

87% of the children weigh less than 25 kg

(Total 6 marks)

48.) A pair of fair dice is thrown.

(a) Copy and complete the tree diagram below, which shows the possible outcomes.



(3)

Let E be the event that **exactly** one four occurs when the pair of dice is thrown.

(b) Calculate P(E).

(3)

The pair of dice is now thrown five times.

(c) Calculate the probability that event E occurs **exactly** three times in the five throws.

(3)

(d) Calculate the probability that event E occurs **at least** three times in the five throws.

(3)
(Total 12 marks)

49.) Two restaurants, *Center* and *New*, sell fish rolls and salads.

Let F be the event a customer chooses a fish roll.

Let S be the event a customer chooses a salad.

Let N be the event a customer chooses neither a fish roll nor a salad.

In the *Center* restaurant P(F) = 0.31, P(S) = 0.62, P(N) = 0.14.

(a) Show that P($F \cap S$) = 0.07.

(3)

(b) Given that a customer chooses a salad, find the probability the customer also chooses a fish roll.

(3)

(c) Are F and S independent events? Justify your answer.

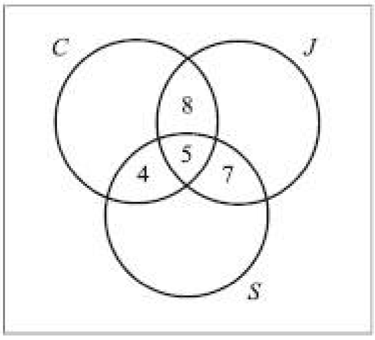
(3)

At *New* restaurant, P(N) = 0.14. Twice as many customers choose a salad as choose a fish roll. Choosing a fish roll is **independent** of choosing a salad.

(d) Find the probability that a fish roll is chosen.

(7)
(Total 16 marks)

50.) The Venn diagram below shows information about 120 students in a school. Of these, 40 study Chinese (*C*), 35 study Japanese (*J*), and 30 study Spanish (*S*).



A student is chosen at random from the group. Find the probability that the student

(a) studies exactly two of these languages;

(1)

(b) studies only Japanese;

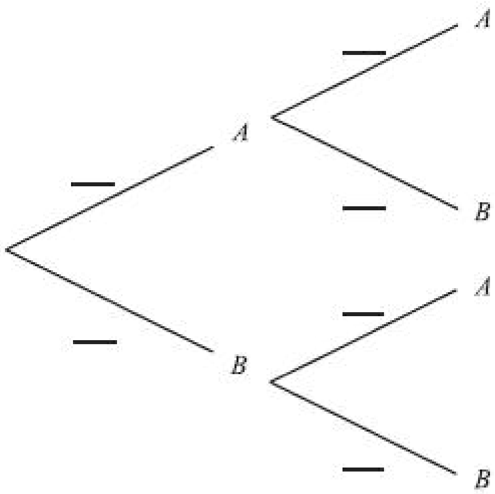
(2)

(c) does not study any of these languages.

(3)
(Total 6 marks)

51.) A bag contains four apples (*A*) and six bananas (*B*). A fruit is taken from the bag and eaten. Then a second fruit is taken and eaten.

(a) Complete the tree diagram below by writing probabilities in the spaces provided.



(3)

(b) Find the probability that one of each type of fruit was eaten.

(3)

(Total 6 marks)

52.) A discrete random variable X has a probability distribution as shown in the table below.

x	0	1	2	3
$P(X = x)$	0.1	a	0.3	b

(a) Find the value of $a + b$.

(2)

(b) Given that $E(X) = 1.5$, find the value of a and of b.

(4)
(Total 6 marks)

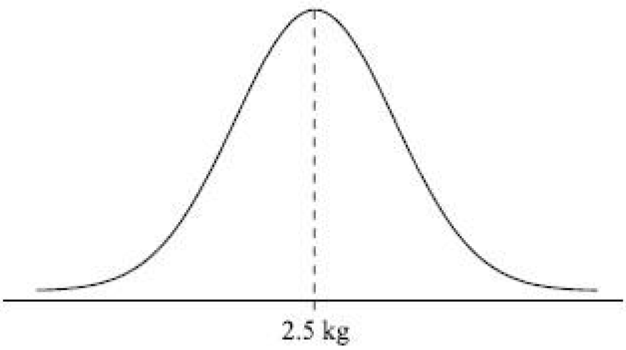
53.) The weights of chickens for sale in a shop are normally distributed with mean 2.5 kg and standard deviation 0.3 kg.

(a) A chicken is chosen at random.

(i) Find the probability that it weighs less than 2 kg.

(ii) Find the probability that it weighs more than 2.8 kg.

(iii) Copy the diagram below. Shade the areas that represent the probabilities from parts (i) and (ii).



(iv) **Hence** show that the probability that it weighs between 2 kg and 2.8 kg is 0.7936 (to four significant figures).

(7)

(b) A customer buys 10 chickens.

(i) Find the probability that all 10 chickens weigh between 2 kg and 2.8 kg.

(ii) Find the probability that at least 7 of the chickens weigh between 2 kg and 2.8 kg.

(6)
(Total 13 marks)

54.) Let A and B be independent events such that P(A) = 0.3 and P(B) = 0.8.

(a) Find P(*A* ą *B*).

(b) Find P(*A* Ć *B*).

(c) Are A and B mutually exclusive? Justify your answer.

(Total 6 marks)

55.) The heights of a group of students are normally distributed with a mean of 160 cm and a standard deviation of 20 cm.

(a) A student is chosen at random. Find the probability that the student's height is greater than 180 cm.

(b) In this group of students, 11.9% have heights less than d cm. Find the value of d.

(Total 6 marks)

56.) The probability distribution of the discrete random variable X is given by the following table.

x	1	2	3	4	5
P(X = x)	0.4	p	0.2	0.07	0.02

(a) Find the value of p.

(b) Calculate the expected value of X.

(Total 6 marks)

57.) In a class, 40 students take chemistry only, 30 take physics only, 20 take both chemistry and physics, and 60 take neither.

(a) Find the probability that a student takes physics given that the student takes chemistry.

(b) Find the probability that a student takes physics given that the student does **not** take chemistry.

(c) State whether the events "taking chemistry" and "taking physics" are mutually exclusive, independent, or neither. Justify your answer.

(Total 6 marks)

58.) Three students, Kim, Ching Li and Jonathan each have a pack of cards, from which they select a card at random. Each card has a 0, 3, 4, or 9 printed on it.

(a) Kim states that the probability distribution for her pack of cards is as follows.

x	0	3	4	9
P(X = x)	0.3	0.45	0.2	0.35

Explain why Kim is incorrect.

(2)

(b) Ching Li correctly states that the probability distribution for her pack of cards is as follows.

x	0	3	4	9
$P(X = x)$	0.4	k	$2k$	0.3

Find the value of k.

(2)

(c) Jonathan correctly states that the probability distribution for his pack of cards is given by $P(X = x) = \frac{x+1}{20}$. One card is drawn at random from his pack.

(i) Calculate the probability that the number on the card drawn is 0.

(ii) Calculate the probability that the number on the card drawn is greater than 0.

(4)
(Total 8 marks)

59.) A game is played, where a die is tossed and a marble selected from a bag.

Bag M contains 3 red marbles (R) and 2 green marbles (G).

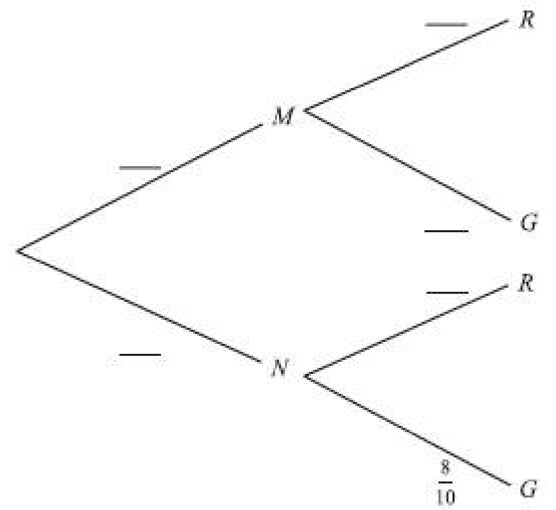
Bag N contains 2 red marbles and 8 green marbles.

A fair six-sided die is tossed. If a 3 or 5 appears on the die, bag M is selected (M).

If any other number appears, bag N is selected (N).

A single marble is then drawn at random from the selected bag.

(a) **Copy** and **complete** the probability tree diagram on **your answer sheet.**



(3)

(b) (i) Write down the probability that bag M is selected and a green marble drawn from it.

(ii) Find the probability that a green marble is drawn from either bag.

(iii) Given that the marble is green, calculate the probability that it came from Bag M.

(7)

(c) A player wins \$2 for a red marble and \$5 for a green marble. What are his expected winnings?

(4)
(Total 14 marks)

60.) In a large school, the heights of all fourteen-year-old students are measured.

The heights of the girls are normally distributed with mean 155 cm and standard deviation 10 cm.

The heights of the boys are normally distributed with mean 160 cm and standard deviation 12 cm.

(a) Find the probability that a girl is taller than 170 cm.

(3)

(b) Given that 10% of the girls are shorter than x cm, find x.

(3)

(c) Given that 90% of the boys have heights between q cm and r cm where q and r are symmetrical about 160 cm, and $q < r$, find the value of q and of r.

(4)

In the group of fourteen-year-old students, 60% are girls and 40% are boys.

The probability that a girl is taller than 170 cm was found in part (a).

The probability that a boy is taller than 170 cm is 0.202.

A fourteen-year-old student is selected at random.

(d) Calculate the probability that the student is taller than 170 cm.

(4)

(e) Given that the student is taller than 170 cm, what is the probability the student is a girl?

(3)
(Total 17 marks)

61.) Events E and F are independent, with $\mathrm{P}(E) = \frac{2}{3}$ and $\mathrm{P}(E \cap F) = \frac{1}{3}$. Calculate

(a) $\mathrm{P}(F)$;

(b) $\mathrm{P}(E \cup F)$.

(Total 6 marks)

62.) A fair coin is tossed five times. Calculate the probability of obtaining

(a) exactly three heads;

(b) at least one head.

(Total 6 marks)

63.) The heights of certain flowers follow a normal distribution. It is known that 20% of these flowers have a height less than 3 cm and 10% have a height greater than 8 cm.

Find the value of the mean μ and the standard deviation σ.

(Total 6 marks)

64.) Two fair **four**-sided dice, one red and one green, are thrown. For each die, the faces are labelled 1, 2, 3, 4. The score for each die is the number which lands face down.

(a) Write down

(i) the sample space;

(ii) the probability that two scores of 4 are obtained.

(4)

Let X be the number of 4s that land face down.

(b) **Copy** and complete the following probability distribution for X.

x	0	1	2
$P(X = x)$			

(3)

(c) Find $E(X)$.

(3)
(Total 10 marks)

65.) A factory makes calculators. Over a long period, 2% of them are found to be faulty. A random sample of 100 calculators is tested.

(a) Write down the expected number of faulty calculators in the sample.

(b) Find the probability that three calculators are faulty.

(c) Find the probability that more than one calculator is faulty.

(Total 6 marks)

66.) The speeds of cars at a certain point on a straight road are normally distributed with mean μ and standard deviation σ. 15% of the cars travelled at speeds greater than 90 km h^{-1} and 12% of them at speeds less than 40 km h^{-1}. Find μ and σ.

(Total 6 marks)

67.) Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let *X* denote the number of red balls chosen. The following table shows the probability distribution for *X*

X	0	1	2
P(*X* = *x*)	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

(a) Calculate E(*X*), the mean number of red balls chosen.

(3)

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B.

(b) (i) Draw a tree diagram to represent the above information, including the probability of each event.

(ii) Hence find the probability distribution for *Y*, where *Y* is the number of red balls chosen.

(8)

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

(c) Calculate the probability that two red balls are chosen.

(5)

(d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die.

(3)

(Total 19 marks)

68.) Two unbiased 6-sided dice are rolled, a red one and a black one. Let *E* and *F* be the events

E : the same number appears on both dice;

F : the sum of the numbers is 10.

Find

(a) P(*E*);

(b) P(*F*);

(c) P($E \cup F$).

Working:

Answers:

(a) ....

(b) ....

(c) ....

(Total 6 marks)

69.) The table below shows the subjects studied by 210 students at a college.

	Year 1	Year 2	Totals
History	50	35	85
Science	15	30	45
Art	45	35	80
Totals	110	100	210

(a) A student from the college is selected at random.

Let A be the event the student studies Art.
Let B be the event the student is in Year 2.

(i) Find $P(A)$.

(ii) Find the probability that the student is a Year 2 Art student.

(iii) Are the events A and B independent? Justify your answer.

(6)

(b) Given that a History student is selected at random, calculate the probability that the student is in Year 1.

(2)

(c) Two students are selected at random from the college. Calculate the probability that one student is in Year 1, and the other in Year 2.

(4)
(Total 12 marks)

70.) Residents of a small town have savings which are normally distributed with a mean of \$3000 and a standard deviation of \$500.

(i) What percentage of townspeople have savings greater than \$3200?

(ii) Two townspeople are chosen at random. What is the probability that **both** of them have savings between \$2300 and \$3300?

(iii) The percentage of townspeople with savings less than d dollars is 74.22%. Find the value of d.

(Total 8 marks)

71.) A class contains 13 girls and 11 boys. The teacher randomly selects four students. Determine the probability that all four students selected are girls.

Working:

Answers:

....

(Total 6 marks)

72.) The events A and B are independent such that $P(B) = 3P(A)$ and $P(A \cup B) = 0.68$. Find $P(B)$

Working:

Answers:

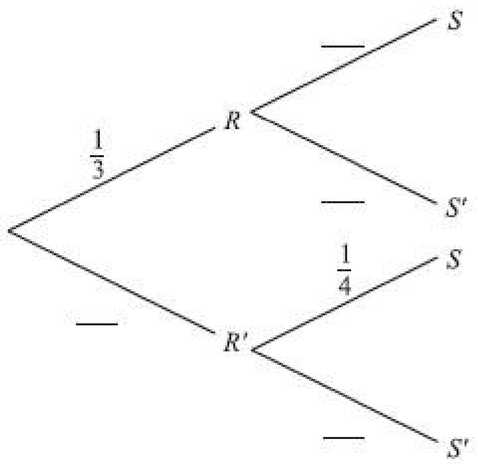
....

(Total 6 marks)

73.) The following probabilities were found for two events R and S.

$$P(R) = \frac{1}{3},\ P(S \mid R) = \frac{4}{5},\ P(S \mid R') = \frac{1}{4}.$$

(a) **Copy** and **complete** the tree diagram.



(3)

(b) Find the following probabilities.

(i) $P(R \cap S)$.

(ii) $P(S)$.

(iii) $P(R \mid S)$.

(7)
(Total 10 marks)

74.) The heights, H, of the people in a certain town are normally distributed with mean 170 cm and standard deviation 20 cm.

(a) A person is selected at random. Find the probability that his height is less than 185 cm.

(3)

(b) Given that $P(H > d) = 0.6808$, find the value of d.

(3)
(Total 6 marks)

75.) Let A and B be events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{7}{8}$.

(a) Calculate $P(A \cap B)$.

(b) Calculate $P(A \mid B)$.

(c) Are the events A and B independent? Give a reason for your answer.

Working:

Answers:

(a) …………………………………………..

(b) …………………………………………..

(c) …………………………………….........

(Total 6 marks)

76.) Dumisani is a student at IB World College.

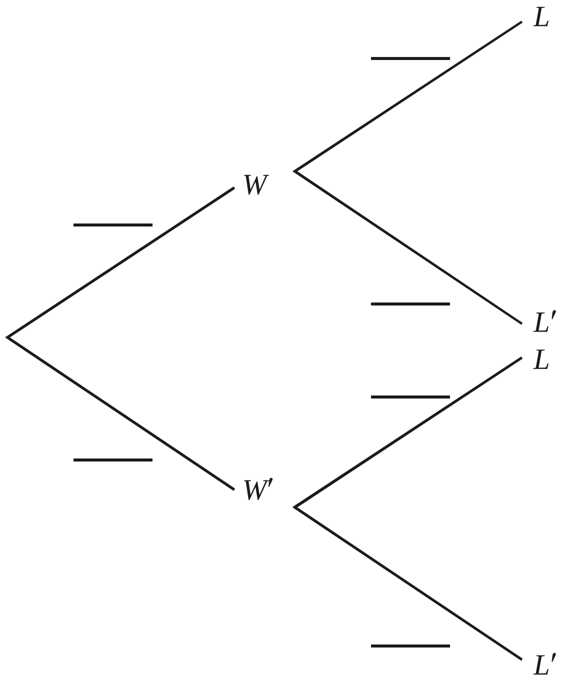
The probability that he will be woken by his alarm clock is $\frac{7}{8}$.

If he is woken by his alarm clock the probability he will be late for school is $\frac{1}{4}$.

If he is not woken by his alarm clock the probability he will be late for school is $\frac{3}{5}$.

Let W be the event "Dumisani is woken by his alarm clock".
Let L be the event "Dumisani is late for school".

(a) Copy and complete the tree diagram below.



(4)

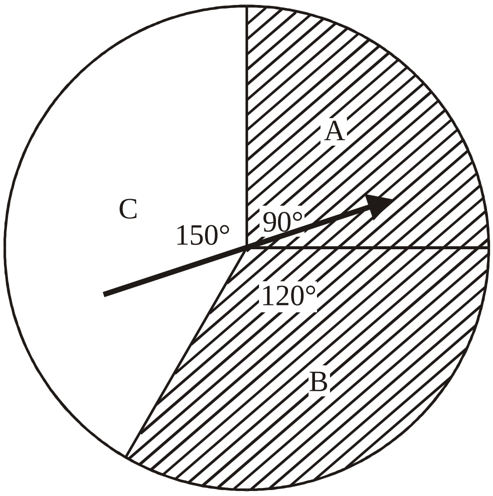
(b) Calculate the probability that Dumisani will be late for school.

(3)

(c) Given that Dumisani is late for school what is the probability that he was woken by his alarm clock?

(4)
(Total 11 marks)

77.) The following diagram shows a circle divided into three sectors A, B and C. The angles at the centre of the circle are 90°, 120° and 150°. Sectors A and B are shaded as shown.



The arrow is spun. It cannot land on the lines between the sectors. Let *A*, *B*, *C* and *S* be the events defined by

A: Arrow lands in sector A

B: Arrow lands in sector B

C: Arrow lands in sector C

S: Arrow lands in a shaded region.

Find

(a) P(*B*);

(b) P(*S*);

(c) $P(A \mid S)$.

Working:

Answers:

(a) ……………………………………………..

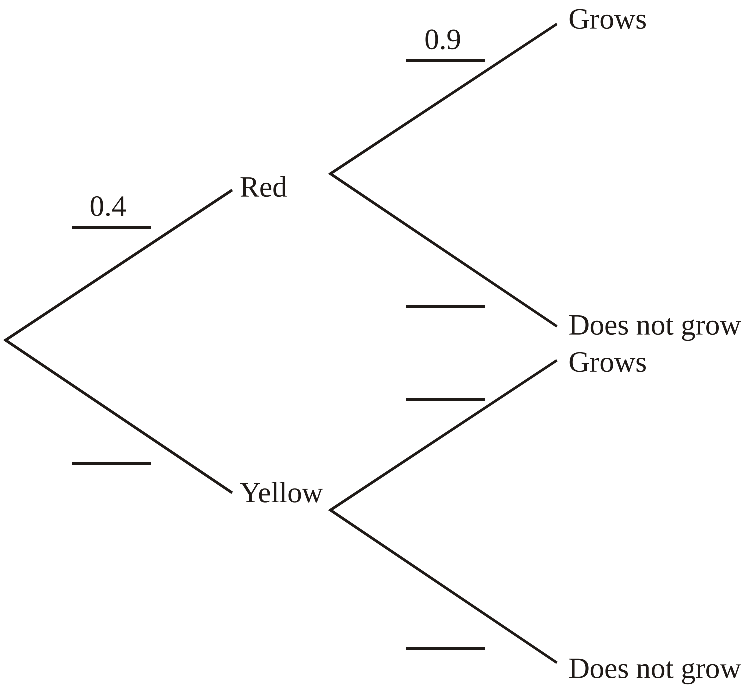
(b) …………………………………………..

(c) …………………………………………..

(Total 6 marks)

78.) A packet of seeds contains 40% red seeds and 60% yellow seeds. The probability that a red seed grows is 0.9, and that a yellow seed grows is 0.8. A seed is chosen at random from the packet.

(a) Complete the probability tree diagram below.



(3)

(b) (i) Calculate the probability that the chosen seed is red and grows.

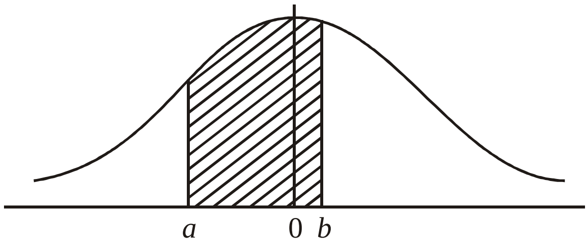
(ii) Calculate the probability that the chosen seed grows.

(iii) Given that the seed grows, calculate the probability that it is red.

(7)
(Total 10 marks)

79.) Reaction times of human beings are normally distributed with a mean of 0.76 seconds and a standard deviation of 0.06 seconds.

(a) The graph below is that of the **standard** normal curve. The shaded area represents the probability that the reaction time of a person chosen at random is between 0.70 and 0.79 seconds.



(i) Write down the value of a and of b.

(ii) Calculate the probability that the reaction time of a person chosen at random is

(a) greater than 0.70 seconds;

(b) between 0.70 and 0.79 seconds.

(6)

Three percent (3%) of the population have a reaction time less than c seconds.

(b) (i) Represent this information on a diagram similar to the one above. Indicate clearly the area representing 3%.

(ii) Find c.

(4)
(Total 10 marks)

80.) Consider events A, B such that P (A) $\neq 0$, P (A) $\neq 1$, P (B) $\neq 0$, and P (B) $\neq 1$.

In each of the situations (a), (b), (c) below state whether A and B are

mutually exclusive (M);
independent (I);
neither (N).

(a) $P(A|B) = P(A)$

(b) $P(A \cap B) = 0$

(c) $P(A \cap B) = P(A)$

Working:

Answers:

(a) ....

(b) ....

(c) ....

(Total 6 marks)

81.) A family of functions is given by

$$f(x) = x^2 + 3x + k, \text{ where } k \in \{1, 2, 3, 4, 5, 6, 7\}.$$

One of these functions is chosen at random. Calculate the probability that the curve of this function crosses the x-axis.

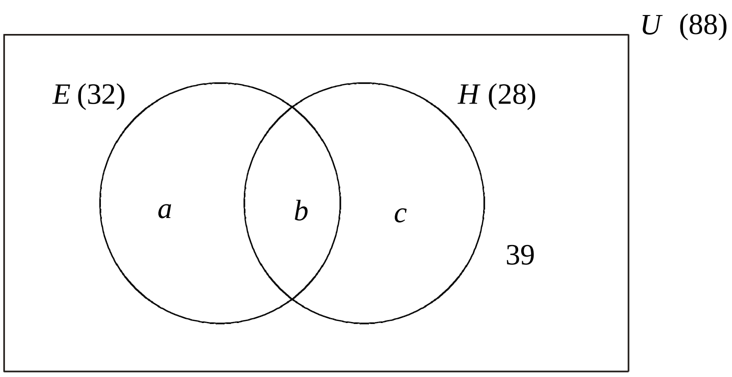
Working:

Answer:

....

(Total 6 marks)

82.) In a school of 88 boys, 32 study economics (E), 28 study history (H) and 39 do not study either subject. This information is represented in the following Venn diagram.



(a) Calculate the values a, b, c.

(4)

(b) A student is selected at random.

(i) Calculate the probability that he studies **both** economics and history.

(ii) Given that he studies economics, calculate the probability that he does **not** study history.

(3)

(c) A group of three students is selected at random from the school.

(i) Calculate the probability that none of these students studies economics.

(ii) Calculate the probability that at least one of these students studies economics.

(5)
(Total 12 marks)

83.) A company manufactures television sets. They claim that the lifetime of a set is normally distributed with a mean of 80 months and standard deviation of 8 months.

(a) What proportion of television sets break down in less than 72 months?

(2)

(b) (i) Calculate the proportion of sets which have a lifetime between 72 months and 90 months.

(ii) Illustrate this proportion by appropriate shading in a sketch of a normal distribution curve.

(5)

(c) If a set breaks down in less than *x* months, the company replace it free of charge. They replace 4% of the sets. Find the value of *x*.

(3)
(Total 10 marks)

84.) A painter has 12 tins of paint. Seven tins are red and five tins are yellow. Two tins are chosen at random. Calculate the probability that both tins are the same colour.

Working:

Answer:

....

(Total 6 marks)

85.) It is claimed that the masses of a population of lions are normally distributed with a mean mass of 310 kg and a standard deviation of 30 kg.

(a) Calculate the probability that a lion selected at random will have a mass of 350 kg or more.

(2)

(b) The probability that the mass of a lion lies between *a* and *b* is 0.95, where *a* and *b* are symmetric about the mean. Find the value of *a* and of *b*.

(3)

(Total 5 marks)

86.) A box contains 22 red apples and 3 green apples. Three apples are selected at random, one after the other, without replacement.

(a) The first two apples are green. What is the probability that the third apple is red?

(b) What is the probability that exactly two of the three apples are red?

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

87.) Two fair dice are thrown and the number showing on each is noted. The sum of these two numbers is S. Find the probability that

(a) S is less than 8;

(2)

(b) at least one die shows a 3;

(2)

(c) at least one die shows a 3, given that S is less than 8.

(3)

(Total 7 marks)

88.) The mass of packets of a breakfast cereal is normally distributed with a mean of 750 g and standard deviation of 25 g.

(a) Find the probability that a packet chosen at random has mass

(i) less than 740 g;

(ii) at least 780 g;

(iii) between 740 g and 780 g.

(5)

(b) Two packets are chosen at random. What is the probability that both packets have a mass which is less than 740 g?

(2)

(c) The mass of 70% of the packets is more than x grams. Find the value of x.

(2)
(Total 9 marks)

89.) For events A and B, the probabilities are $P(A) = \frac{3}{11}$, $P(B) = \frac{4}{11}$.

Calculate the value of $P(A \cap B)$ if

(a) $P(A \cup B) = \frac{6}{11}$;

(b) events A and B are independent.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

90.) In a country called *Tallopia*, the height of adults is normally distributed with a mean of 187.5 cm and a standard deviation of 9.5 cm.

(a) What percentage of adults in *Tallopia* have a height greater than 197 cm?

(3)

(b) A standard doorway in *Tallopia* is designed so that 99% of adults have a space of at least 17 cm over their heads when going through a doorway. Find the height of a standard doorway in *Tallopia*. Give your answer to the nearest cm.

(4)
(Total 7 marks)

91.) A bag contains 10 red balls, 10 green balls and 6 white balls. Two balls are drawn at random from the bag without replacement. What is the probability that they are of different colours?

Working:

Answer:

....

(Total 4 marks)

92.) The table below represents the weights, W, in grams, of 80 packets of roasted peanuts.

Weight (W)	$80 < W \le 85$	$85 < W \le 90$	$90 < W \le 95$	$95 < W \le 100$	$100 < W \le 105$	$105 < W \le 110$	$110 < W \le 115$
Number of packets	5	10	15	26	13	7	4

(a) Use the midpoint of each interval to find an estimate for the standard deviation of the weights.

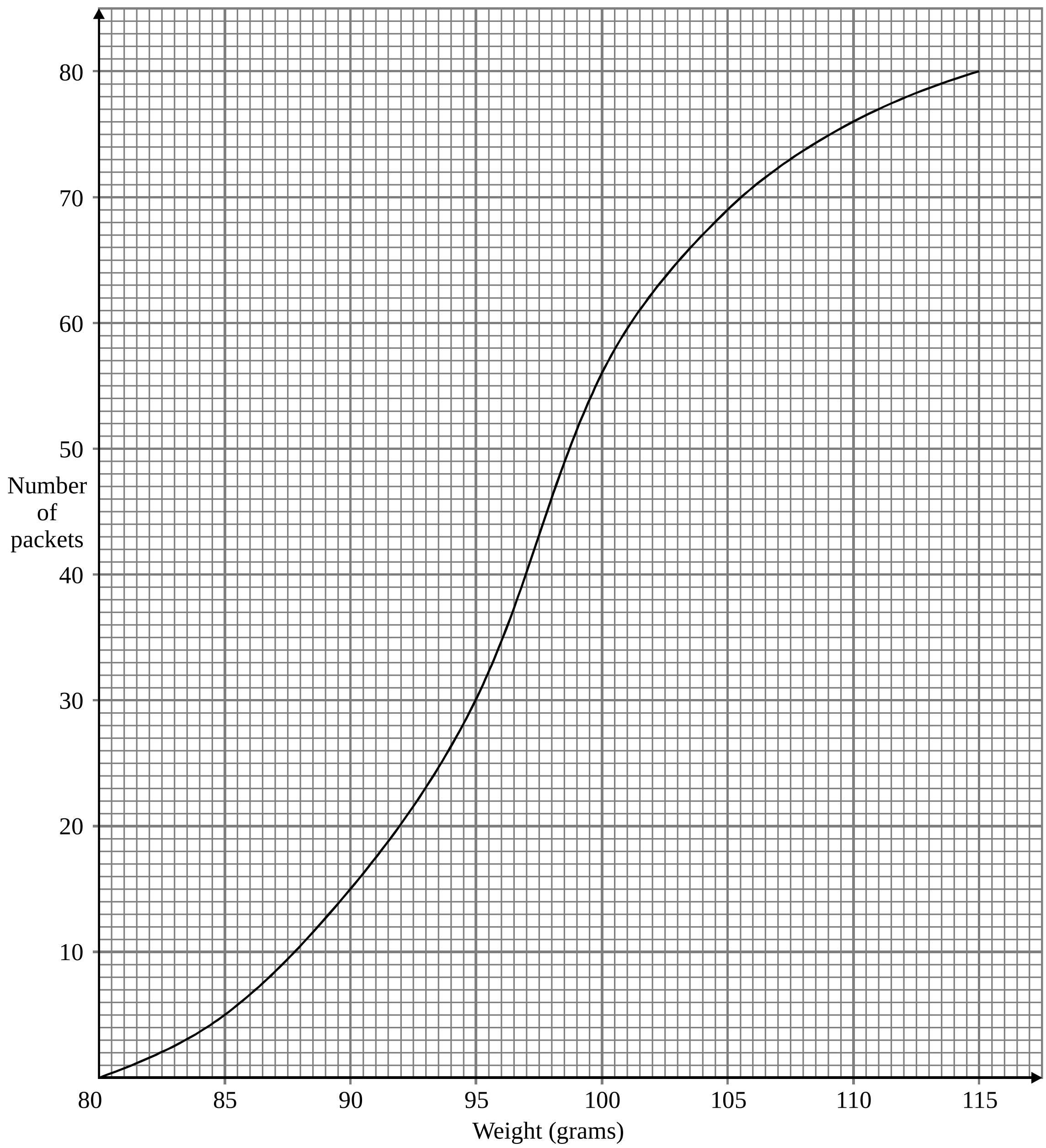
(3)

(b) Copy and complete the following cumulative frequency table for the above data.

Weight (W)	$W \le 85$	$W \le 90$	$W \le 95$	$W \le 100$	$W \le 105$	$W \le 110$	$W \le 115$
Number of packets	5	15					80

(1)

(c) A cumulative frequency graph of the distribution is shown below, with a scale 2 cm for 10 packets on the vertical axis and 2 cm for 5 grams on the horizontal axis.



Use the graph to estimate

(i) the median;

(ii) the upper quartile (that is, the third quartile).

Give your answers to the nearest gram.

(4)

(d) Let $W_1, W_2, ..., W_{80}$ be the individual weights of the packets, and let $\overline{W}$ be their mean. What is the value of the sum

$$(W_1 - \overline{W}) + (W_2 - \overline{W}) + (W_3 - \overline{W}) + \ldots + (W_{79} - \overline{W}) + (W_{80} - \overline{W})?$$

(2)

(e) One of the 80 packets is selected at random. Given that its weight satisfies $85 < W \leq 110$, find the probability that its weight is greater than 100 grams.

(4)
(Total 14 marks)

93.) Intelligence Quotient (IQ) in a certain population is normally distributed with a mean of 100 and a standard deviation of 15.

(a) What percentage of the population has an IQ between 90 and 125?

(2)

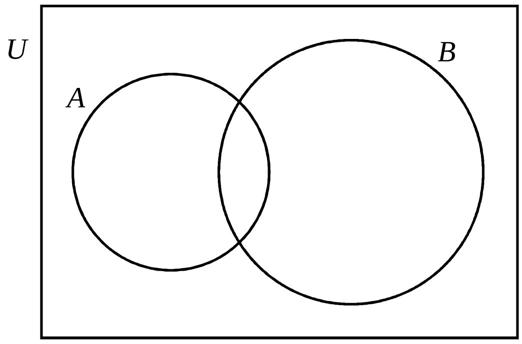
(b) If two persons are chosen at random from the population, what is the probability that both have an IQ greater than 125?

(3)

(c) The mean IQ of a random group of 25 persons suffering from a certain brain disorder was found to be 95.2. Is this sufficient evidence, at the 0.05 level of significance, that people suffering from the disorder have, on average, a lower IQ than the entire population? State your null hypothesis and your alternative hypothesis, and explain your reasoning.

(4)
(Total 9 marks)

94.) The following Venn diagram shows the universal set U and the sets A and B.



(a) Shade the area in the diagram which represents the set $B \cap A'$.

$n(U) = 100$, $n(A) = 30$, $n(B) = 50$, $n(A \cup B) = 65$.

(b) Find $n(B \cap A')$.

(c) An element is selected at random from U. What is the probability that this element is in $B \cap A'$?

Working:

Answers:

(b) ....

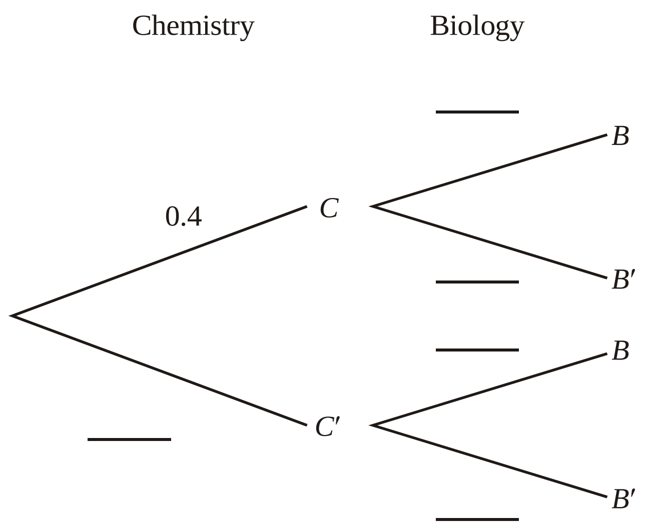
(c) ....

(Total 4 marks)

95.) The events B and C are dependent, where C is the event “a student takes Chemistry”, and B is the event “a student takes Biology”. It is known that

$$P(C) = 0.4,\ P(B \mid C) = 0.6,\ P(B \mid C') = 0.5.$$

(a) Complete the following tree diagram.



(b) Calculate the probability that a student takes Biology.

(c) Given that a student takes Biology, what is the probability that the student takes Chemistry?

Working:

Answers:

(b) ....

(c) ....

(Total 4 marks)

96.) Bags of cement are labelled 25 kg. The bags are filled by machine and the actual weights are normally distributed with mean 25.7 kg and standard deviation 0.50 kg.

(a) What is the probability a bag selected at random will weigh less than 25.0 kg?

(2)

In order to reduce the number of underweight bags (bags weighing less than 25 kg) to 2.5% of the total, the mean is increased without changing the standard deviation.

(b) Show that the increased mean is 26.0 kg.

(3)

It is decided to purchase a more accurate machine for filling the bags. The requirements for this machine are that only 2.5% of bags be under 25 kg and that only 2.5% of bags be over 26 kg.

(c) Calculate the mean and standard deviation that satisfy these requirements.

(3)

The cost of the new machine is $5000. Cement sells for $0.80 per kg.

(d) Compared to the cost of operating with a 26 kg mean, how many bags must be filled in order to recover the cost of the new equipment?

(3)
(Total 11 marks)

97.) In a survey of 200 people, 90 of whom were female, it was found that 60 people were unemployed, including 20 males.

(a) Using this information, complete the table below.

	Males	Females	Totals
Unemployed			
Employed			
Totals			200

(b) If a person is selected at random from this group of 200, find the probability that this person is

(i) an unemployed female;

(ii) a male, given that the person is employed.

Working:

Answers:

(b) (i) ....

(ii) ....

(Total 4 marks)

98.) In a survey, 100 students were asked "do you prefer to watch television or play sport?" Of the 46 boys in the survey, 33 said they would choose sport, while 29 girls made this choice.

	Boys	Girls	Total
Television			
Sport	33	29	
Total	46		100

By completing this table or otherwise, find the probability that

(a) a student selected at random prefers to watch television;

(b) a student prefers to watch television, given that the student is a boy.

Working:

Answers:

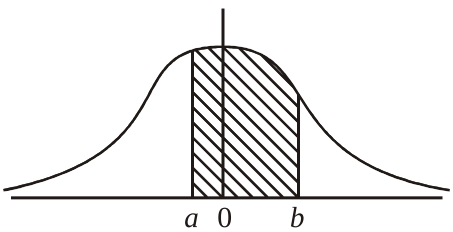
(a) ....

(b) ....

(Total 4 marks)

99.) The lifespan of a particular species of insect is normally distributed with a mean of 57 hours and a standard deviation of 4.4 hours.

(a) The probability that the lifespan of an insect of this species lies between 55 and 60 hours is represented by the shaded area in the following diagram. This diagram represents the standard normal curve.



(i) Write down the values of *a* and *b*.

(2)

(ii) Find the probability that the lifespan of an insect of this species is

(a) more than 55 hours;

(1)

(b) between 55 and 60 hours.

(2)

(b) 90% of the insects die after *t* hours.

(i) Represent this information on a standard normal curve diagram, similar to the one given in part (a), indicating clearly the area representing 90%.

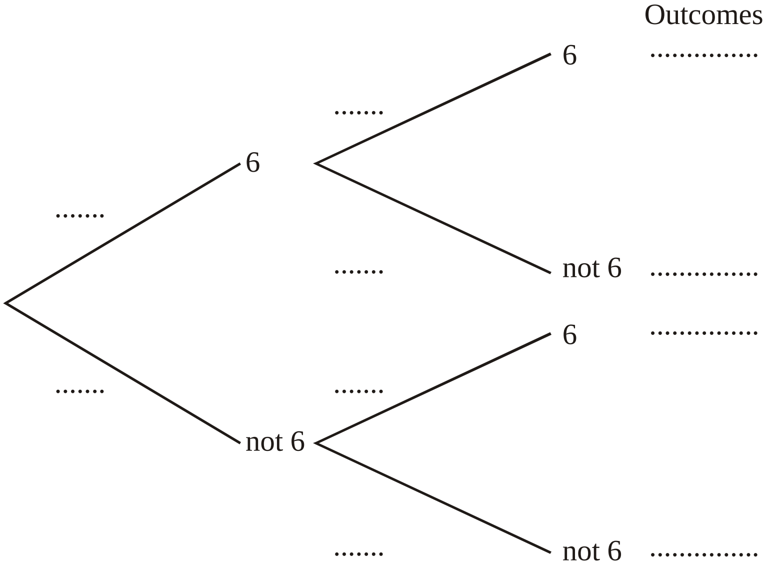
(2)

(ii) Find the value of *t*.

(3)
(Total 10 marks)

100.) Two ordinary, 6-sided dice are rolled and the total score is noted.

(a) Complete the tree diagram by entering probabilities and listing outcomes.



(b) Find the probability of getting one or more sixes.

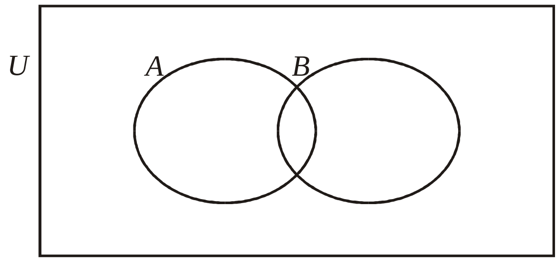
Working:

Answer:

(b) ....

(Total 4 marks)

101.) The following Venn diagram shows a sample space U and events A and B.



$n(U) = 36$, $n(A) = 11$, $n(B) = 6$ and $n(A \cup B)' = 21$.

(a) On the diagram, shade the region $(A \cup B)'$.

(b) Find

(i) $n(A \cap B)$;

(ii) $P(A \cap B)$.

(c) Explain why events A and B are not mutually exclusive.

Working:

Answers:

(b) (i) ....

(ii) ....

(c) ....

(Total 4 marks)

102.) An urban highway has a speed limit of 50 km h^{-1}. It is known that the speeds of vehicles travelling on the highway are normally distributed, with a standard deviation of l0 km h^{-1}, and that 30% of the vehicles using the highway exceed the speed limit.

(a) Show that the mean speed of the vehicles is approximately 44.8 km h^{-1}.

(3)

The police conduct a "Safer Driving" campaign intended to encourage slower driving, and want to know whether the campaign has been effective. It is found that a sample of 25 vehicles has a mean speed of 41.3 km h^{-1}.

(b) Given that the null hypothesis is

H_0: the mean speed has been unaffected by the campaign

State H_1, the alternative hypothesis.

(1)

(c) State whether a one-tailed or two-tailed test is appropriate for these hypotheses, and explain why.

(2)

(d) Has the campaign had significant effect at the 5% level?

(4)
(Total 10 marks)

103.) A box contains 35 red discs and 5 black discs. A disc is selected at random and its colour noted. The disc is then replaced in the box.

(a) In eight such selections, what is the probability that a black disc is selected

(i) exactly once?

(3)

(ii) at least once?

(3)

(b) The process of selecting and replacing is carried out 400 times.

What is the expected number of black discs that would be drawn?

(2)
(Total 8 marks)

104.) For the events A and B, $p(A) = 0.6$, $p(B) = 0.8$ and $p(A \cup B) = 1$.

Find

(a) $p(A \cap B)$;

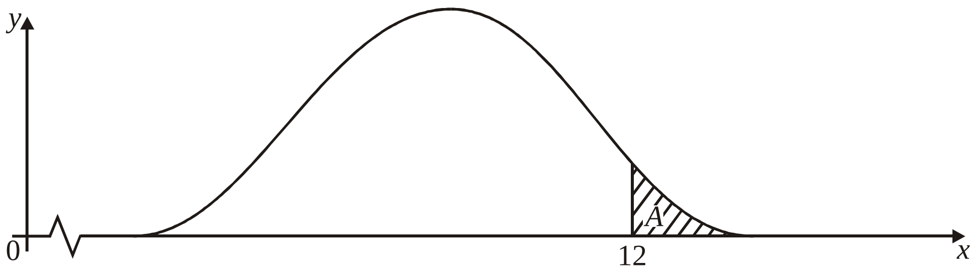
(b) $p(\complement A \cup \complement B)$.

Working:

	Answers: (a) .... (b) ....

(Total 4 marks)

105.) The graph shows a normal curve for the random variable X, with mean μ and standard deviation s.



It is known that $p\ (X \geq 12) = 0.1$.

(a) The shaded region A is the region under the curve where $x \geq 12$. Write down the area of the shaded region A.

(1)

It is also known that $p\ (X \leq 8) = 0.1$.

(b) Find the value of μ, explaining your method in full.

(5)

(c) Show that s = 1.56 to an accuracy of three significant figures.

(5)

(d) Find $p\ (X \leq 11)$.

(5)
(Total 16 marks)

106.) A fair coin is tossed eight times. Calculate

(a) the probability of obtaining exactly 4 heads;

(2)

(b) the probability of obtaining exactly 3 heads;

(1)

(c) the probability of obtaining 3, 4 or 5 heads.

(3)
(Total 6 marks)