22147304

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 2

Wednesday 14 May 2014 (morning)

1 hour 30 minutes

Candidate session number

Examination code

| 2 | 2 | 1 | 4 | – | 7 | 3 | 0 | 4 |

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer all questions in the boxes provided. Working may be continued below the lines if necessary.

1. *[Maximum mark: 6]*

   The following diagram shows triangle ABC.
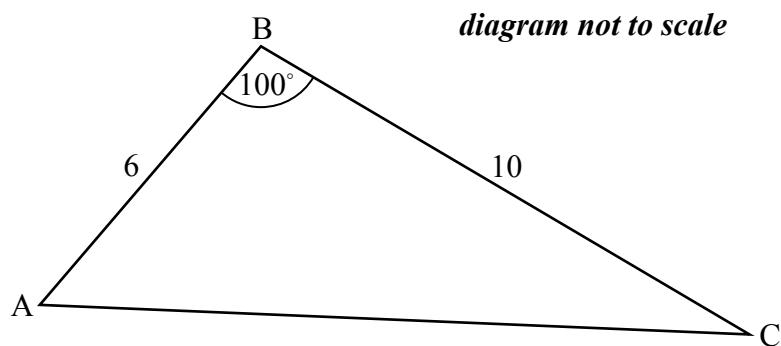
   $AB = 6\,cm$, $BC = 10\,cm$, and $A\hat{B}C = 100°$.

   (a) Find AC. *[3]*

   (b) Find $B\hat{C}A$. *[3]*

Turn over

2. *[Maximum mark: 5]*

Consider the expansion of $(x+3)^{10}$.

(a) Write down the number of terms in this expansion. *[1]*

(b) Find the term containing x^3. *[4]*

. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .

3. *[Maximum mark: 7]*

The following table shows the average weights (y kg) for given heights (x cm) in a population of men.

Heights (x cm)	165	170	175	180	185
Weights (y kg)	67.8	70.0	72.7	75.5	77.2

(a) The relationship between the variables is modelled by the regression equation $y = ax + b$.

   (i) Write down the value of a and of b.

   (ii) Hence, estimate the weight of a man whose height is 172 cm. *[4]*

(b) (i) Write down the correlation coefficient.

   (ii) State which **two** of the following describe the correlation between the variables. *[3]*

   strong   zero   positive

   negative   no correlation   weak
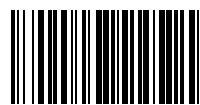
Turn over

4.   *[Maximum mark: 6]*

The following diagram shows two perpendicular vectors **u** and **v**.
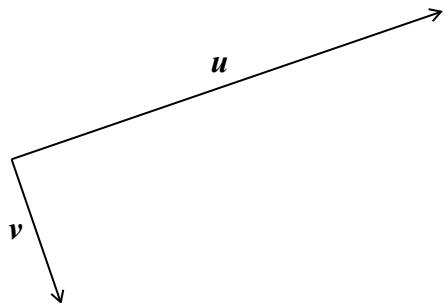
(a)   Let $w = u - v$. Represent w on the diagram above.   *[2]*

(b)   Given that $u = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $v = \begin{pmatrix} 5 \\ n \\ 3 \end{pmatrix}$, where $n \in \mathbb{Z}$, find n .   *[4]*

5. *[Maximum mark: 6]*

The population of deer in an enclosed game reserve is modelled by the function $P(t) = 210\sin(0.5t - 2.6) + 990$, where t is in months, and $t = 1$ corresponds to 1 January 2014.

(a) Find the number of deer in the reserve on 1 May 2014. *[3]*

(b) (i) Find the rate of change of the deer population on 1 May 2014.

(ii) Interpret the answer to part (i) with reference to the deer population size on 1 May 2014. *[3]*

Turn over

6. *[Maximum mark: 8]*

Ramiro and Lautaro are travelling from Buenos Aires to El Moro.

Ramiro travels in a vehicle whose velocity in $\mathrm{m\,s^{-1}}$ is given by $V_R = 40 - t^2$, where t is in seconds.

Lautaro travels in a vehicle whose displacement from Buenos Aires in metres is given by $S_L = 2t^2 + 60$.

When $t = 0$, both vehicles are at the same point.

Find Ramiro's displacement from Buenos Aires when $t = 10$.

7.   *[Maximum mark: 7]*

Let $f(x) = \dfrac{g(x)}{h(x)}$, where $g(2) = 18$, $h(2) = 6$, $g'(2) = 5$, and $h'(2) = 2$. Find the equation of

the normal to the graph of f at $x = 2$.

Turn over

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8.   *[Maximum mark: 16]*

The weights in grams of 80 rats are shown in the following cumulative frequency diagram.
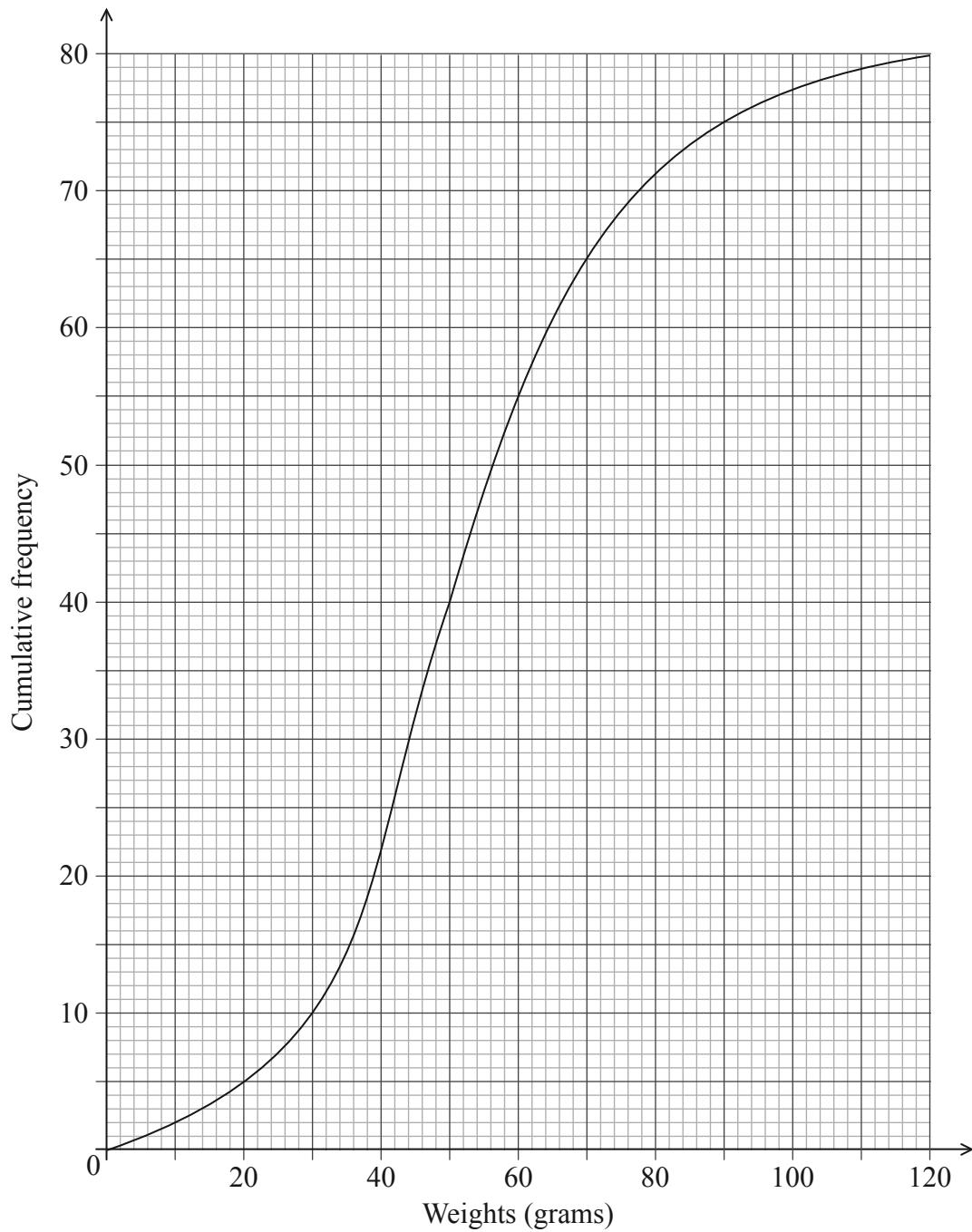
(This question continues on the following page)
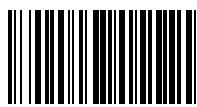
*Do **NOT** write solutions on this page.*

(Question 8 continued)

(a) (i) Write down the median weight of the rats.

    (ii) Find the percentage of rats that weigh 70 grams or less.   *[4]*

The same data is presented in the following table.

Weights w grams	$0 \leq w \leq 30$	$30 < w \leq 60$	$60 < w \leq 90$	$90 < w \leq 120$
Frequency	p	45	q	5

(b) (i) Write down the value of p.

    (ii) Find the value of q.   *[4]*

(c) Use the values from the table to estimate the mean and standard deviation of the weights.   *[3]*

Assume that the weights of these rats are normally distributed with the mean and standard deviation estimated in part (c).

(d) Find the percentage of rats that weigh 70 grams or less.   *[2]*

(e) A sample of five rats is chosen at random. Find the probability that at most three rats weigh 70 grams or less.   *[3]*

Turn over

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

Let $f(x) = \cos\left(\dfrac{\pi}{4}x\right) + \sin\left(\dfrac{\pi}{4}x\right)$, for $-4 \le x \le 4$.

(a) Sketch the graph of f. *[3]*

(b) Find the values of x where the function is decreasing. *[5]*

(c) The function f can also be written in the form $f(x) = a\sin\left(\dfrac{\pi}{4}(x+c)\right)$, where $a \in \mathbb{R}$, and $0 \le c \le 2$. Find the value of

(i) a;

(ii) c. *[7]*

10. *[Maximum mark: 14]*

Let $f(x) = \dfrac{3x}{x-q}$, where $x \ne q$.

(a) Write down the equations of the vertical and horizontal asymptotes of the graph of f. *[2]*

The vertical and horizontal asymptotes to the graph of f intersect at the point $Q(1, 3)$.

(b) Find the value of q. *[2]*

(c) The point $P(x, y)$ lies on the graph of f. Show that $PQ = \sqrt{(x-1)^2 + \left(\dfrac{3}{x-1}\right)^2}$. *[4]*

(d) Hence find the coordinates of the points on the graph of f that are closest to $(1, 3)$. *[6]*