21 | Probability of One Event

21.1 | Introduction to Probability

A *probability* describes mathematically how likely it is that something will happen. We can talk about the probability that it will rain tomorrow or the probability that England will win their next football match.

Example 1

Which of the words,

certain, likely, unlikely or *impossible*

best describes how likely each of the events below is to take place?

(a)   It will rain tomorrow.

(b)   It will snow tomorrow.

(c)   Manchester United will win the next FA Cup.

(d)   Exeter City will win the next FA Cup.

(e)   It is your teacher's birthday tomorrow.

(f)   You will obtain a 7 when rolling a dice.

Solution

(a)   This can be *likely* or *unlikely* depending on the current weather pattern.

(b)   This is *unlikely.*

(c)   This is *likely.*

(d)   This is *unlikely*, some would say *impossible*!

(e)   This is *unlikely,* but it could be true.

(f)   *Impossible* (as only numbers 1 to 6 can be obtained).

Probability Line

```
|----------------------|----------------------|
0                     1/2                     1
```
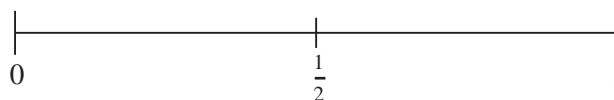
Probability is 0 for an *impossible* event

Probability is 1 for a *certain* event

Probability is $\dfrac{1}{2}$ for an *even chance*
(for example, getting a head when you toss an unbiased coin)

Example 2

For each event below, mark an estimate of the probability that it will happen on a probability scale.

(a)   It will snow on 19 August next year in London.

(b)   Your maths teacher will give you homework this week.

(c)   You will go to school tomorrow.

(d)   You will go to bed before midnight tonight.

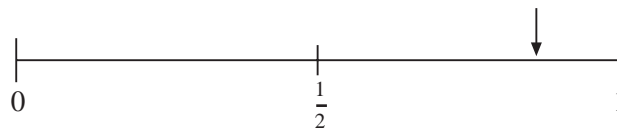(e)   You throw an unbiased dice and get an even number.

Solution

(a)   This is virtually impossible, so the probability will be very close to zero.
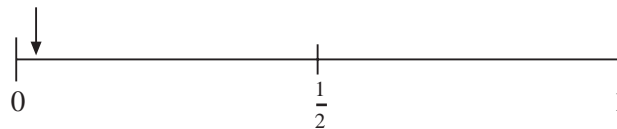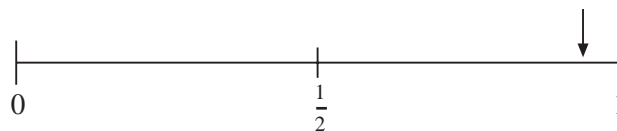

(b)   This is very likely (at least for most teachers) and so the probability will be quite high.
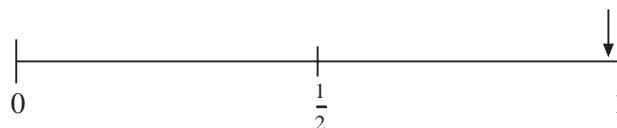

(c)   The answer to this depends on what day of the week it is. On a Friday or Saturday it is very unlikely that you will go to school the next day, so the probability is low.
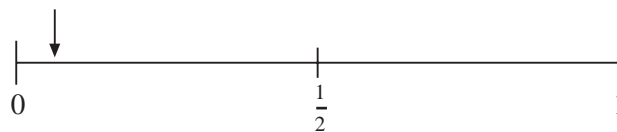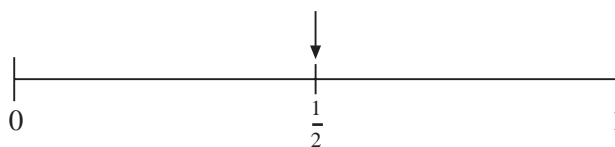

On any other day the probability is much higher as it is very likely that you will go to school the next day.

(d)   This is almost certain. The probability is very close to 1.

(e)   There are 3 even numbers and 3 odd numbers on a dice, so there is an even

chance that you will get an even number. The probability is $\frac{1}{2}$.

Exercises

1.   Use one of the words, *certain, likely, unlikely* or *impossible*, to describe
      each event below. Give a reason for each of your answers.

(a)   You are more than 8 years old.

(b)   You will miss the school bus tomorrow.

(c)   Your favourite football team will win their next match.

(d)   You will arrive at school on time tomorrow.

2.   The probability line shows the probability of 5 events, A, B, C, D and E.
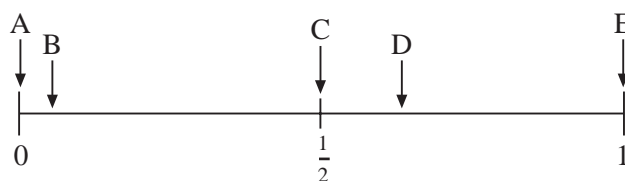
(a)   Which event is *certain*?

(b)   Which event is *impossible*?

(c)   Which event is *unlikely* but *possible*?

(d)   Which event is *most likely*, but *not certain* to occur?

3.   For each event below, draw a probability line and mark in the probability of
      the event:

(a)   England will win the most medals at the next Olympic Games.

(b)   You will forget to take your packed lunch to school.

(c)   You will get a 6 when you roll an unbiased dice once.

(d)   It will rain tomorrow.

(e)   It will not rain tomorrow.

4.   Describe an event that is:

(a)   impossible,   (b)   certain,

(c)   very unlikely,   (d)   very likely.

5. The events A, B, C and D have probabilities as shown on this probability
    line:
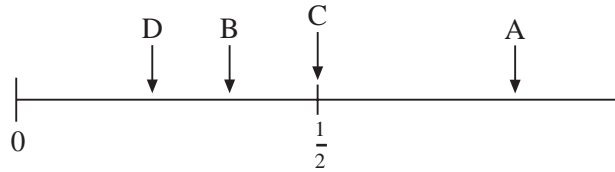
    (a)   Which event is *most likely* to take place?

    (b)   Which event is *most unlikely* to take place?

    (c)   Which events are *more likely* to take place than event B?

    (d)   Which events are *less likely* to take place than event A?

6. (a)   What is the probability that a pupil in Year 7 will be 12 years old on
          their next birthday?

    (b)   What is the probability that a pupil in Year 7 will be 13 years old on
          their next birthday?

7. Estimate, by marking a probability line, the probability that you will get all
    your next maths homework correct.

8. The events A, B, C, D and E are listed below:

          A :   You will live to be 70 years old.

          B :   You will live to be 80 years old.

          C :   You will live to be 90 years old.

          D :   You will live to be 100 years old.

          E :   You will live to be 110 years old.

    Mark an estimate of the probability of each event on a probability line.

9. When you toss an unbiased coin, the probability of getting a head is $\frac{1}{2}$,
    because you have an equal (or even) chance of getting a head or a tail.
    What other events have a probability of $\frac{1}{2}$?

10. Make a list of some events that have a probability of *more than* $\frac{1}{2}$, but that
    are *not certain.*

21.2 Calculating the Probability of a Single Event

In this section we calculate the probabilities of single events. We consider cases where all the possible outcomes are equally likely. For example, when you roll a fair dice you are equally likely to get *any* of the six numbers. (The words 'fair' or 'unbiased' mean that all outcomes are equally likely.)

$$\text{Probability of an event} \ = \ \frac{number \ of \ successful \ outcomes}{total \ number \ of \ outcomes}$$

Example 1

When you roll a fair dice, what is the probability of getting:

(a)   a five,

(b)   an even number,

(c)   a four or a five?

Solution

The possible outcomes when you roll a dice are the scores

$$1, \ 2, \ 3, \ 4, \ 5, \ 6$$

so there are 6 possible outcomes.

(a)   In this case there is only one successful outcome, that is, a 5.

$$\text{Probability of a five} = \frac{number \ of \ successful \ outcomes}{total \ number \ of \ outcomes}$$

$$= \frac{1}{6}$$

(b)   In this case there are 3 successful outcomes, 2, 4 or 6.

$$\text{Probability of an even number} \ = \ \frac{number \ of \ successful \ outcomes}{total \ number \ of \ outcomes}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

(c)   In this case there are 2 successful outcomes, 4 or 5.

$$\text{Probability of a 4 or a 5} \ = \ \frac{number \ of \ successful \ outcomes}{total \ number \ of \ outcomes}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

Example 2

A bag of sweets contains 6 mints and 4 eclairs. One sweet is taken at random from the bag. What is the probability that it is:

(a)   a mint,   (b)   an eclair?

Solution

The total number of possible outcomes is 10 as there are 10 sweets in the bag.

(a)   As there are 6 mints in the bag, there are 6 successful outcomes.

Probability of a mint $= \dfrac{\text{number of successful outcomes}}{\text{total number of outcomes}}$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

(b)   As there are 4 eclairs, there are 4 successful outcomes.

Probability of an eclair $= \dfrac{\text{number of successful outcomes}}{\text{total number of outcomes}}$

$$= \frac{4}{10}$$

$$= \frac{2}{5}$$

Exercises

1.   When you roll a fair dice, what is the probability that you obtain:

(a)   an odd number,

(b)   a 2,

(c)   a multiple of 3,

(d)   a number less than 5,

(e)   a number greater than 4,

(f)   a 3 or a number less than 3?

2.   A bag contains 6 red sweets and 14 blue sweets. A sweet is taken at random from the bag. What is the probability that it is:
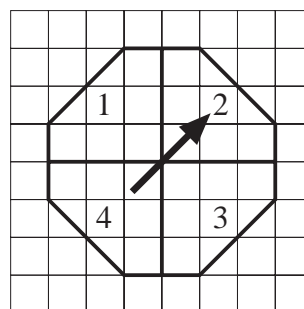
   (a)   a red sweet,   (b)   a blue sweet?

3.   You toss a fair coin. What is the probability that you obtain a tail?

4.   The diagram shows a spinner from a game. The black arrow spins and ends up pointing to one of the four numbers.
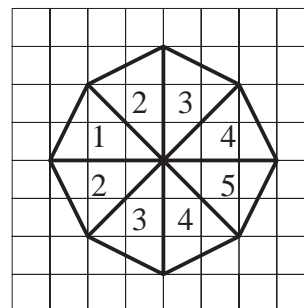
   What is the probability that the arrow points to:

   (a)   the number 1,

   (b)   an even number.

   (c)   a multiple of 3?

5.   The diagram shows a spinner that is used in a board game. When the spinner is spun, what is the probability that it lands on:

   (a)   1,

   (b)   5,

   (c)   4,

   (d)   an even number,

   (e)   a number less than 4?

6.   A bag of sweets contains 8 mints, 6 toffees and 2 boiled sweets. A sweet is taken at random from the bag. What is the probability that it is:

   (a)   a mint,

   (b)   a toffee,

   (c)   a boiled sweet,

   (d)   not a mint,

   (e)   not a toffee?

7.   In a class there are 18 boys and 12 girls. One child is chosen at random to represent the class. What is the probability that this child is:

   (a)   a girl,

   (b)   a boy?

8.   The diagram shows a piece of card
     that is folded to form a dice.

     When the dice is rolled, what is the
     probability that it shows:

     (a)   a blue face,

     (b)   a red face,

     (c)   a yellow face,

     (d)   a face that is not red,

     (e)   a face that is not yellow?

	Blue		
Red	Yellow	Red	Red
	Blue		
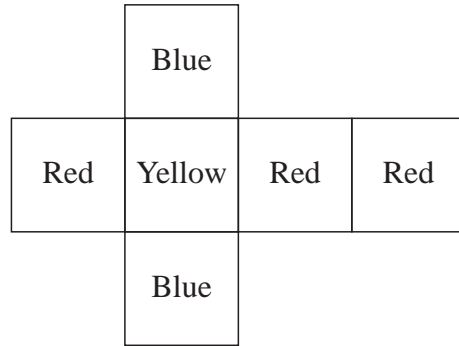
9.   The children in a class were asked to name their favourite colour. The
     results are given in the table:

Colour	*Number of Children*
Red	6
Black	2
Yellow	3
Green	4
Blue	10
Pink	7

     If a child is picked at random from the class, what is the probability that
     their favourite colour is:

     (a)   red,   (b)   yellow,

     (c)   pink   (d)   black,

     (e)   not pink,   (f)   not green?

10. A bag contains 6 red balls and some white balls. When a ball is taken from

     the bag at random, the probability that it is red is $\frac{3}{5}$. How many white balls

     are in the bag?

21.3 Relative Frequency

Some probabilities cannot be calculated as in the last section; for example, the probability that it will rain on 20 November cannot be found in this way. Probabilities can, however, be estimated using *relative frequencies* found from observations or from experiments.

$$Relative\ frequency\ =\ \frac{number\ of\ successful\ trials}{total\ number\ of\ trials}$$

Example 1

Matthew decides to try to estimate the probability that toast lands butter-side-down when dropped. He drops a piece of buttered toast 50 times and observes that it lands butter-side-down 30 times.

Estimate the probability that the toast lands butter-side-down.

Solution

An estimate of the probability is given by the relative frequency. In this case this is

$$\frac{30}{50}\ =\ \frac{3}{5}$$

Example 2

Sarah tosses a coin 200 times. She gets 108 heads and 92 tails. Using her results, estimate the probability of obtaining:

(a)   a head when the coin is tossed,

(b)   a tail when the coin is tossed.

Solution

The relative frequency gives an estimate of the probability.

(a)   Relative frequency $=\ \dfrac{108}{200}\ =\ \dfrac{27}{50}$

(b)   Relative Frequency $=\ \dfrac{92}{200}\ =\ \dfrac{23}{50}$

We would expect both these probabilities to be $\dfrac{1}{2}$, and here the estimates are close to that value, indicating that her coin may be a fair one.

Example 3

Rachel was testing a coin to see if it was fair. She tossed the coin 50 times and recorded 36 HEADS. She tossed it another 50 times and recorded 32 HEADS. She continued in this way, and recorded her results on the following table:

No. of Tosses	No. of Heads
50	36
50	32
50	30
50	38
50	30
50	36
50	34
50	30

(a) Calculate the total frequency (total number of HEADS) after 50, 100, 150, . . . , 400 throws and also calculate, at each stage, the relative frequency.

(b) Plot the points on a relative frequency graph, and hence estimate the probability of obtaining a head. What should be Rachel's conclusion?

Solution

(a)

No. of Tosses	50	100	150	200	250	300	350	400
Total Frequency	36	68	98	136	166	202	236	266
Relative Frequency	$\frac{36}{50} = 0.72$	$\frac{68}{100} = 0.68$	0.653	0.68	0.664	0.673	0.674	0.665

(b) The relative frequency graph follows, from which we can see that the probability looks to be about 0.665 (or about $\frac{2}{3}$). Rachel should conclude that her coin is probably *not* fair, i.e. the coin probably *is* biased (since, for a fair coin, we would expect the probability to be 0.5; so although the evidence is fairly strong, Rachel cannot be certain that her coin is biased).
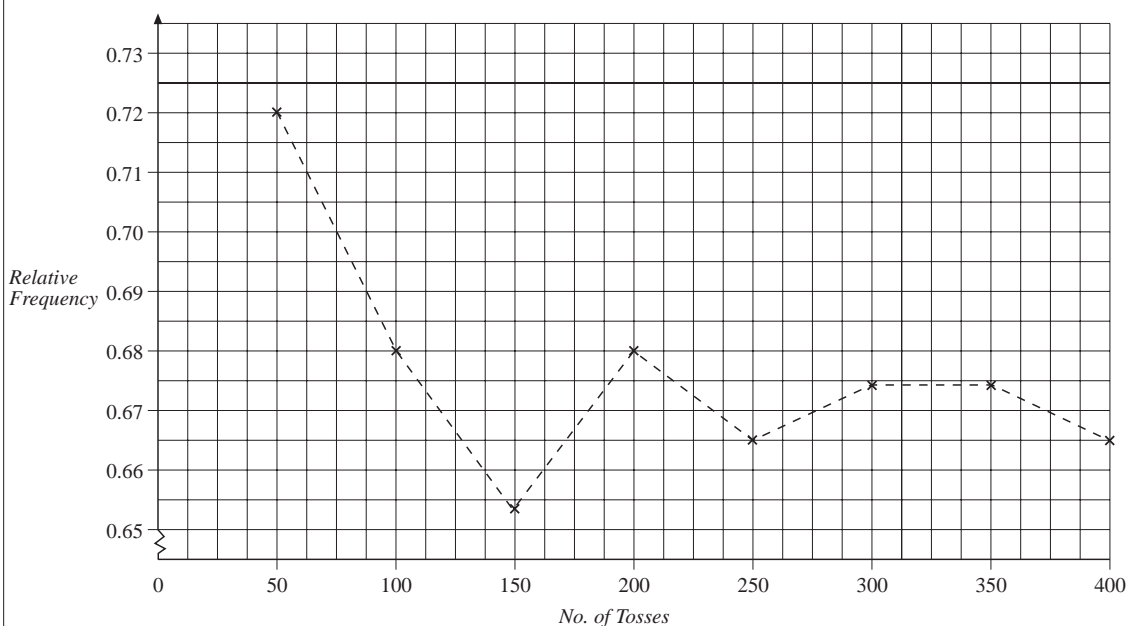
Exercises

1. (a) Toss a coin 100 times. Record your results after every 10 tosses. Plot a relative frequency graph and estimate the probability of obtaining a head when you toss the coin.

    (b) Is your answer to (a) close to $\frac{1}{2}$?

    (c) Put all the results for your class together and obtain a new estimate of the probability of obtaining a head.

    (d) Is your new estimate closer to $\frac{1}{2}$ than the estimate in (a)?

2. A drawing pin can land 'point up' or 'point down' when dropped. Carry out an experiment to find an estimate of the probability that a drawing pin lands 'point up', using a relative frequency graph.

3. (a) Roll a dice 100 times and record the results you obtain.

    (b) Estimate the probability of obtaining each of the numbers on the faces of the dice.

    (c) Do you think that the probabilities that you obtain are reasonable?

    (d) Obtain more results by rolling the dice another 100 times. How do your probability estimates change as you use more results?

4. By considering the people in your class, estimate the probability that a person chosen at random is left-handed.

5.   If it rained on 12 days in November last year, estimate the probability that it will rain on 20 November next year.

6.   You can make a biased dice out of a hollow cube by sticking a small lump of blu-tac inside the cube.

Make a biased dice and use it to estimate the probability of obtaining a 6 on your dice.
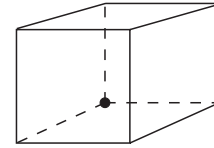
7.   A calculator can be used to generate random digits. Halim generates 100 random digits with his calculator. He lists the results in the following table:

0	ⵢⵢⵢ ‖	5	ⵢⵢⵢ ⵢⵢⵢ
1	ⵢⵢⵢ ‖‖	6	ⵢⵢⵢ ‖‖‖
2	ⵢⵢⵢ ‖	7	ⵢⵢⵢ ⵢⵢⵢ ‖
3	ⵢⵢⵢ ⵢⵢⵢ ‖	8	ⵢⵢⵢ ⵢⵢⵢ
4	ⵢⵢⵢ ⵢⵢⵢ ‖	9	ⵢⵢⵢ ⵢⵢⵢ ‖‖‖

Based on Halim's results, estimate the probability that the calculator produces:

(a)   9,

(b)   2,

(c)   a digit that is an odd number,

(d)   a digit that is a prime number.

8.   Tony estimates that the probability that there will be an empty space in the car park when he arrives at work is $\frac{4}{5}$. His estimate is based on 50 observations. On how many of these 50 days was he *unable* to find an empty space in the car park?

9.   Paul draws the bar chart opposite to show the results for his football team so far this season.

(a)   Use the bar chart to estimate the probability that his team will win their next match.

(b)   Give reasons why this estimate of the probability that they will win their next match may not be very reliable.
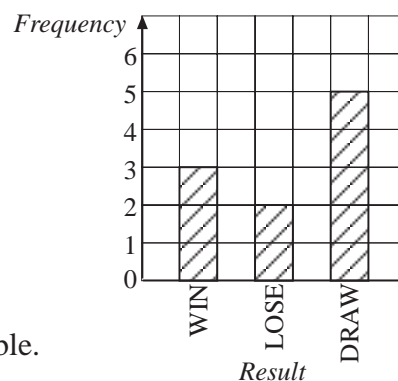
10. Sasha carries out the drawing pin experiment described in question 2. She shows her results in this pie chart:
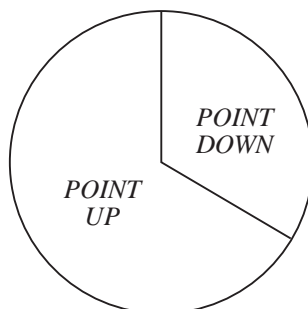
Use her results to estimate the probability that the pin lands 'point up'.

21.4 Complementary Events

Two events are described as *complementary* if they are the *only two* possible outcomes. For example, the events A and B below are complementary:

    A : It rains.

    B : It does not rain.

If A is an event and A' is the complementary event,

$$p(A) + p(A') = 1$$

or

$$p(A') = 1 - p(A)$$

Example 1

If the probability that it will rain tomorrow is $\frac{1}{5}$, what is the probability that it will not rain tomorrow?

Solution

As these are two complementary events,

probability that it will not rain tomorrow $= 1 - \frac{1}{5}$

$$= \frac{4}{5}$$

Example 2

A dice has been renumbered so that the probability of obtaining an even number is now $\frac{2}{3}$. What is the probability of obtaining an odd number?
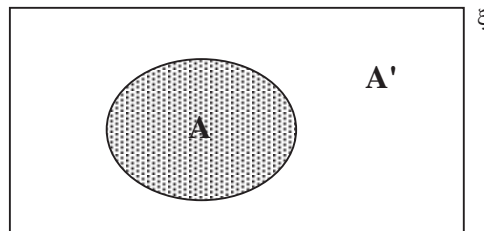
Solution

As these are two complementary events,

$$\text{probability of obtaining an odd number} \quad = 1 - \frac{2}{3}$$

$$= \frac{1}{3}$$

Complementary events can be illustrated in a Venn diagram:

Exercises

1. The probability that Scot will win his next tennis match is $\frac{3}{5}$. What is the probability that he will *not* win?

2. The probability that it will snow on Christmas Day is $\frac{1}{8}$. What is the probability that it will *not* snow on Christmas Day?

3. The probability that a child is left-handed is $\frac{1}{20}$. What is the probability that a child is right-handed?

4. The probability that Natasha is late for school is 0.1. What is the probability that she is *not* late?

5. The probability that Sergio gets all his spellings correct in his next test is 0.75. What is the probability that he does *not* get them all correct?

6.   If you take a card at random from a pack of playing cards, the probability
     that you get a king is $\dfrac{1}{13}$. What is the probability that you do *not* get a king?

7.   One of the numbers in the Venn diagram below is chosen at random.
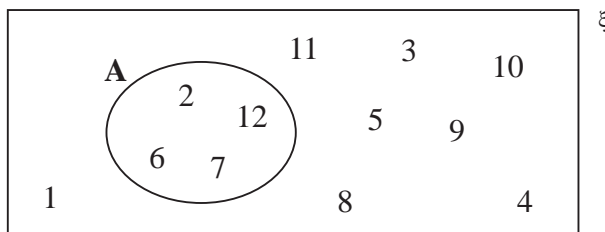
     (a)   What is the probability that the number is in the set A?
     (b)   What is the probability that the number is *not* in the set A?

8.   (a)   What is the probability that you will get a prime number when you
           roll a fair dice?
     (b)   What is the probability that you will *not* get a prime number when
           you roll a fair dice?

9.   In one year it rains on 12 days in the month of September.
     (a)   Use this information to estimate the probability that it will rain on
           19 September next year.
     (b)   Use your answer to (a) to make an estimate of the probability that it
           will not rain on 19 September next year.

10. A bag contains 100 balls, each marked with a number from 1 to 100. A ball
     is taken from the bag at random.
     (a)   What is the probability that the number on the ball is a multiple
           of 3?
     (b)   What is the probability that the number on the ball is *not* a multiple
           of 3?

21.5 Estimating the Number of Outcomes

If we know the probability of an event we can estimate the number of times we expect that event to take place.

> *Expected number of successful outcomes*
> *= probability of success × total number of outcomes*

Example 1

You toss an unbiased coin 500 times. How many heads should you expect to obtain?

Solution

Probability of a head $= \dfrac{1}{2}$

Expected number of heads $= \dfrac{1}{2} \times 500$

$= 250$

Example 2

You roll a fair dice 120 times. How many times would you expect to obtain:

(a) a 6,

(b) a multiple of 3?

Solution

(a) Probability of a 6 $= \dfrac{1}{6}$

Expected number of sixes $= \dfrac{1}{6} \times 120$

$= 20$

(b) Probability of a multiple of 3 $= \dfrac{1}{3}$

Expected number of multiples of 3 $= \dfrac{1}{3} \times 120$

$= 40$

Exercises

1. If you roll an unbiased dice 600 times, how many times would you expect to obtain:

   (a) a one,

   (b) an *even* number,

   (c) an *odd* number,

   (d) a number less than 3?

2. A spinner is marked with the numbers 1 to 5, each of which is equally likely to occur when the spinner is spun. If it is spun 200 times, how many times would you expect to obtain:

   (a) a five,

   (b) an even number,

   (c) a number less than 3,

   (d) a prime number?

3. If the probability that it rains on a day in September is $\frac{1}{5}$, on how many days in September would you expect it to rain?

4. When you open a packet of sweets and take one out at random, the probability that it is blue is $\frac{1}{8}$. If you open 40 packets of sweets, how many times would you expect to take out a blue sweet *first*?

5. Some crisp packets contain prizes. The probability that you find a prize in a crisp packet is $\frac{1}{25}$. How many prizes would you expect to find if you opened:

   (a) 50 packets,

   (b) 200 packets,

   (c) 1000 packets?

6. The probability that Joshua misses the school bus is $\frac{3}{10}$. In a school year there are 40 weeks, each of 5 days.

   How many times can you expect Joshua to miss the bus in:

   (a) a 12-week term,

   (b) a school year?

7. The probability that a person, selected at random, has been trained in First Aid is $\frac{1}{50}$. How many people trained in First Aid would you expect to find in:

   (a) a crowd of 50 000 spectators at a football match,

   (b) an audience of 300 at a theatre,

   (c) a group of 50 onlookers at the scene of an accident?

8. The probability that a certain type of seed germinates is 0.7. How many seeds would you expect to germinate if you planted:

   (a) 20 seeds,

   (b) 70 seeds,

   (c) 1000 seeds?

9. The probability that Emma wins a game of 'Freecell' on her computer is $\frac{2}{5}$. She wants to be able to say that she has won 5 games. How many games should she expect to play before she wins 5 games?

10. Prakesh says that the probability that he misses the school bus is $\frac{1}{10}$.

    (a) How many times would you expect him to miss the bus in 4 weeks?

    (b) In 4 weeks he actually misses the bus 3 times, which is not the same as your answer to (a). Explain why your answer to (a) is still correct.

21.6 Addition Law for Mutually Exclusive Events

Two events are *mutually exclusive* if only *one* can take place at any given time. For example, if a bag contains red balls and yellow balls, when a ball is taken out it is either red or yellow, but it cannot be both. The events 'red ball' and 'yellow ball' are therefore *mutually exclusive*.

> If two events, A and B, are mutually exclusive,
>
> $$p(\text{A or B}) = p(\text{A}) + p(\text{B})$$
>
> where $\quad p(\text{A}) = $ probability of A
>
> and $\quad p(\text{B}) = $ probability of B

Example 1

A bag contains 6 red balls, 8 yellow balls and 4 green balls. One ball is taken at random from the bag.

What is the probability that the ball is:

(a)   yellow,

(b)   green,

(c)   yellow or green?

Solution

(a) $p(\text{yellow}) = \dfrac{8}{18}$

$= \dfrac{4}{9}$

(b) $p(\text{green}) = \dfrac{4}{18}$

$= \dfrac{2}{9}$

(c) $p(\text{yellow or green}) = p(\text{yellow}) + p(\text{green})$

$= \dfrac{4}{9} + \dfrac{2}{9}$

$= \dfrac{6}{9}$

$= \dfrac{2}{3}$

Example 2

Ten children were asked to state their favourite sport. Their responses are listed in this table:

Favourite Sport	Number of Children
Swimming	4
Hockey	1
Tennis	1
Volleyball	1
Football	3

What is the probability that, if one of the children is chosen at random, their favourite sport will be:

(a)   volleyball,   (b)   swimming,   (c)   volleyball or swimming?

Solution

(a) $p(\text{volleyball}) = \dfrac{1}{10}$

(b) $p(\text{swimming}) = \dfrac{4}{10}$

$\phantom{p(\text{swimming})} = \dfrac{2}{5}$

(c) $p(\text{volleyball or swimming}) = p(\text{volleyball}) + p(\text{swimming})$

$$= \dfrac{1}{10} + \dfrac{4}{10}$$

$$= \dfrac{5}{10}$$

$$= \dfrac{1}{2}$$

Exercises

1. A bag contains 6 red balls, 5 blue balls and 9 yellow balls. A ball is taken at random from the bag. What is the probability that it is:

    (a) red,

    (b) blue,

    (c) yellow,

    (d) red or blue,

    (e) red or yellow,

    (f) blue or yellow?

2. In a packet of sweets there are 20 mints, 10 fudges and 20 toffees. A sweet is taken at random from the packet. What is the probability that it is:

    (a) a fudge,

    (b) a toffee,

    (c) a mint,

    (d) a mint or a toffee,

    (e) a mint or a fudge,

    (f) a toffee or a fudge?

3. A group of children were asked their ages. These are recorded in the table opposite:

Age (in years)	Number of Children
10	4
11	14
12	16
13	6

What is the probability that a child selected at random from the group, is:

(a)   age 10,

(b)   age 10 or 11,

(c)   age 12 or 13,

(d)   age 11 or 12?

4.   In the school canteen, children can choose one of baked potato, chips or rice for lunch. The probability that a child chooses a baked potato is $\frac{1}{6}$, the probability that they choose chips is $\frac{2}{3}$ and the probability that they choose rice is $\frac{1}{6}$.

What is the probability that a child chooses:

(a)   rice or baked potato,

(b)   rice or chips,

(c)   chips or baked potato?

5.   Zahra buys a mobile phone with 5 different coloured covers. The probability of her using each cover is given in the table opposite.

What is the probability that she uses:

(a)   a red or a pink cover,

(b)   a green or a red cover,

(c)   a yellow or a blue cover,

(d)   a pink or a yellow cover?

Colour	Probability
Red	$\frac{1}{8}$
Blue	$\frac{1}{4}$
Green	$\frac{1}{16}$
Yellow	$\frac{1}{16}$
Pink	$\frac{1}{2}$

6.   In his pocket, Andy has the coins below:
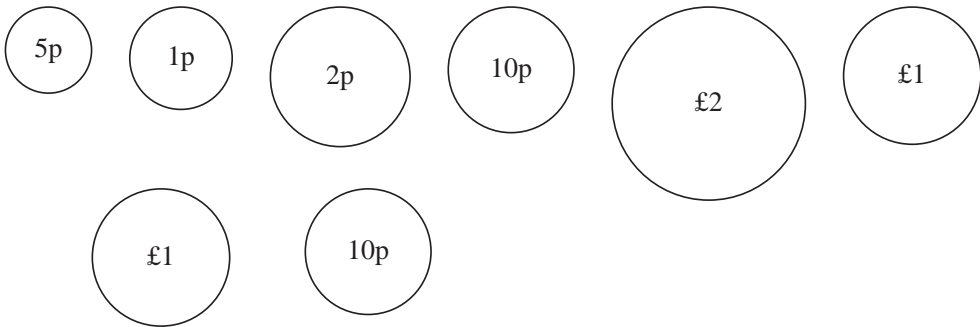
He takes a coin at random from his pocket.

What is the probability that it is:

(a)   a £1 or a £2 coin,

(b)   a 1p or a 2p coin,

(c)   a 5p or a 10p coin,

(d)   a 1p or a £1 coin?

7.   Alex can walk to school, cycle to school or go by bus. The probability that he walks or cycles to school is $\frac{3}{4}$.

(a)   If the probability that he cycles is $\frac{1}{4}$, what is the probability that he walks?

(b)   What is the probability that he goes by bus?

(c)   What is the probability that he cycles or goes by bus?

8.   A bus can arrive early, on time or late. The probability that it is late is $\frac{1}{4}$.

The probability that it is on time or late is $\frac{2}{3}$.

(a)   What is the probability that the bus is on time?

(b)   What is the probability that it is early or on time?

9.   A bag contains 30 coloured balls. Of these, 10 are red, 6 are blue and the rest are green or yellow.

A ball is taken at random from the bag. The probability that this ball is yellow is $\frac{1}{6}$.

What is the probability that a ball taken at random from the bag is:

(a)   green,

(b)   green or yellow,

(c)   red or blue,

(d)   red, blue or yellow?

10.   A bag contains red, yellow and green balls. One ball is taken at random from the bag. The probability that it is red or green is $\frac{1}{2}$. The probability that it is yellow or green is $\frac{3}{4}$.

What is the probability that a ball taken at random from the bag is:
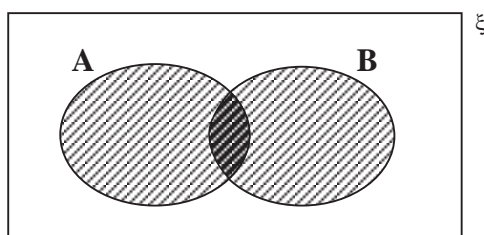
(a)   green,

(b)   red or yellow?

21.7 General Addition Law

Events may not always be mutually exclusive. For example, if you roll a dice, the events 'getting a six' and 'getting an even number' are *not* mutually exclusive. In this section we consider examples of this type.

$$p(\text{A or B}) = p(\text{A}) + p(\text{B}) - p(\text{A and B})$$

This can be shown using a *Venn diagram*.

The *union* (A or B) is shaded, whilst the *intersection* (A and B) is double shaded.

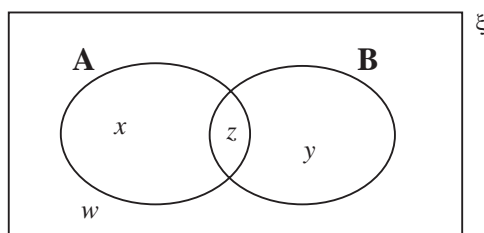So, if there are n possible events, with

number of events in only A $= x$

number of events in only B $= y$

number of events in both A and B $= z$

number of events in neither A nor B $= w,$

this can be illustrated in the following Venn diagram:

Thus

$$p(A) \quad = \frac{x + z}{n}$$

$$p(B) \quad = \frac{y + z}{n}$$

$$p(A \text{ and } B) \ = \frac{z}{n}$$

and $\quad p(A \text{ or } B) \quad = \frac{x + z + y}{n}$

Now $\quad p(A) + p(B) - p(A \text{ and } B) \ = \ \left(\frac{x + z}{n}\right) + \left(\frac{y + z}{n}\right) - \left(\frac{z}{n}\right)$

$$= \ \frac{x + z + y + z - z}{n}$$

$$= \ \frac{x + z + y}{n}$$

$$= \ p(A \text{ or } B)$$

and so the result is true.

Example 1

One of the numbers 1 to 10 is selected at random. What is the probability that it is:

(a)   an even number,

(b)   greater than 5,

(c)   an even number *and* greater than 5,
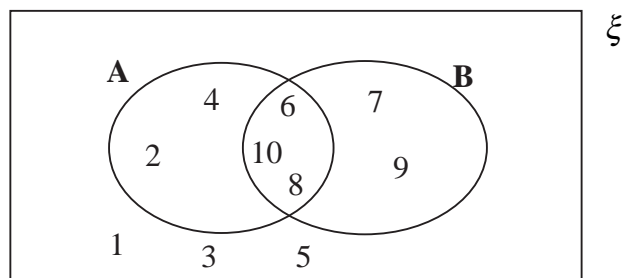
(d)   an even number *or* greater than 5.

Solution

A Venn diagram is helpful for problems like this.

Let:

A be the set of even numbers

and   B be the set of numbers greater than 5.

(a) $p(\text{an even number}) = p(A)$

$$= \frac{5}{10}$$

$$= \frac{1}{2}$$

(b) $p(\text{greater than 5}) = \frac{5}{10}$

$$= \frac{1}{2}$$

(c) $p(\text{even } and \text{ greater than 5}) = p(A \text{ and } B)$

$$= \frac{3}{10}$$

(d) $p(\text{even } or \text{ greater than 5}) = p(A) + p(B) - p(A \text{ and } B)$

$$= \frac{1}{2} + \frac{1}{2} - \frac{3}{10}$$

$$= \frac{7}{10}$$

Example 2

A dice is rolled. What is the probability of getting a prime number or an even number?

Solution

$p(\text{even number}) = \frac{3}{6}$ (since there are 3 even numbers, 2, 4 and 6)

$$= \frac{1}{2}$$

$$p(\text{prime number}) \;=\; \frac{3}{6} \qquad\qquad \text{(since there are 3 prime numbers, 2, 3 and 5)}$$

$$=\; \frac{1}{2}$$

$$p(\text{even } and \text{ prime number}) \;=\; \frac{1}{6} \quad \text{(since there is one even, prime number, namely 2)}$$

$$p(\text{even } or \text{ prime}) \;=\; p(\text{even}) + p(\text{prime}) - p(\text{even and prime})$$
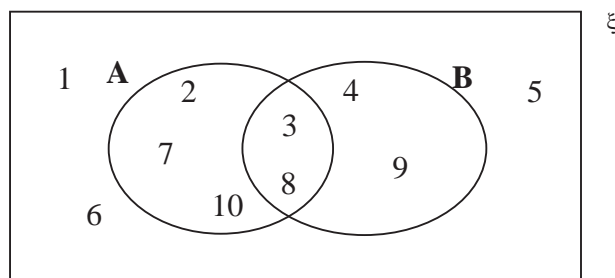
$$=\; \frac{1}{2} + \frac{1}{2} - \frac{1}{6}$$

$$=\; \frac{5}{6}$$

Exercises

1. One of the numbers 10 to 20 is selected at random. What is the probability that it is:

   (a) an even number,

   (b) a multiple of 5,

   (c) an even number *and* a multiple of 5,

   (d) an even number *or* a multiple of 5?

2. The numbers 1 to 10 are sorted into sets as shown in the Venn diagram:

   One of these numbers is selected at random. What is the probability that it is a member of:

   (a) A,

   (b) B,

   (c) A *and* B,

   (d) A *or* B?

3. A fair dice is rolled. What is the probability of getting a prime number *or* an odd number?

4. One of the numbers 1 to 20 is selected at random. What is the probability that the number is a multiple of 3 *or* a multiple of 4?

5. A bingo set contains 100 balls each marked with one of the numbers 1 to 100. One of these balls is selected at random. What it the probability that the number on this ball is a multiple of 7 or of 10?

6. If $p(A) = \dfrac{3}{4}$, $p(B) = \dfrac{3}{8}$ and $p(A \text{ and } B) = \dfrac{1}{3}$, find $p(A \text{ or } B)$.

7. If $p(A \text{ or } B) = \dfrac{3}{5}$, $p(A \text{ and } B) = \dfrac{2}{5}$ and $p(A) = \dfrac{1}{2}$, find $p(B)$.

8. If $p(A \text{ or } B) = \dfrac{1}{2}$, $p(A) = \dfrac{2}{5}$ and $p(B) = \dfrac{1}{3}$, find $p(A \text{ and } B)$.

9. The probability that Jai is late for school is 0.1. The probability that he forgets his lunch is 0.3. The probability that he forgets his lunch *and* is late is 0.05.

   What is the probability that he forgets his lunch or is late?

10. The faces of a dice are marked with the numbers 1, 2, 3 or 4. This is done so that the probability of rolling a 3 is $\dfrac{1}{6}$, the probability of rolling a 3 or a 4 is $\dfrac{1}{3}$ and the probability of rolling a prime or an even number is $\dfrac{2}{3}$. How often does each number appear on the dice?