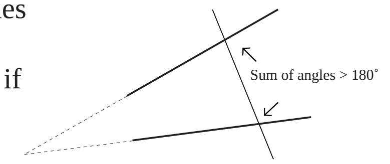
UNIT 7 *Transformations* Teaching Notes

Historical Background and Introduction

The 'traditional' approach to geometry, which predominated all school geometry up to the 1960s, was based essentially on *Euclid's 'Elements'*, written about 300 BC. Euclid's theorems were based on five assumptions or postulates:

1. For every point P, and for every point Q not equal to P, there exists a unique line, *l*, which passes through P and Q.
2. For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.
3. For every point O and every point A not equal to O, there exists a circle with Centre O and radius OA.
4. All right angles are equal to each other.
5. If a straight line falls on two other straight lines to make the interior angles on the same side less than two right angles, then the two lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.



(See: *Euclidean and Non-Euclidean Geometries* by M. J. Greenberg (Freeman) ISBN: 0 7167 1103 6)

The basic tools were congruent triangles and parallel lines and the subject matter mainly concerned

triangles, parallelograms and circles

and their properties.

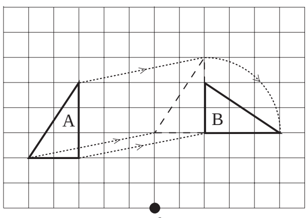
However, the German mathematician, *Felix Klein* (1849–1925), in his inaugural lecture as Professor of Mathematics at the University of Enlanger, gave a description of geometry as:

those properties of figures in space which remain unchanged under some fixed group of transformations.

This influential address led directly to the *Erlanger Programme*, which has changed radically the style of geometry taught in schools today. It led to a shift in emphasis away from congruence as the fundamental idea.

(See: *The Mathematics Curriculum: Geometry* by W.W. Willson (Blackie) ISBN: 0 216 90337 8)

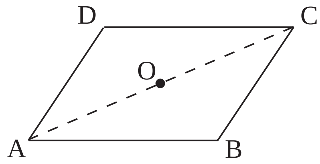
To say that two plane figures are *congruent* means that one can be moved to fit exactly onto the other. Klein's approach would be to view this as a translation and (possible) rotation. For example, A and B as shown opposite are congruent but one shape can be obtained from the other by a translation, followed by a rotation or, indeed, by a single rotation about the centre of rotation as shown.



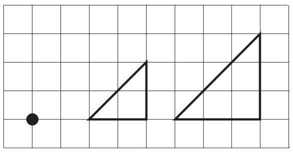
Centre of Rotation

UNIT 7 *Transformations* Teaching Notes

As another example, consider a parallelogram ABCD, as shown opposite. The triangles ABC and CDA are congruent. You can prove this either by *SAS* or *SSS* in the traditional way. Klein geometry, though, would consider rotating ADC through 180˚ about the midpoint, O, of the line AD and, in so doing, show that the triangle ADC fits exactly onto ABC. This is essentially the same mathematics but by a very different approach.



Also the traditional work on similarity can, in Klein geometry, be thought of as an enlargement with different scale factors (2 or $\frac{1}{2}$ shown opposite).



Some people may feel that geometry managed very well for more than 2000 years without transformations and that the introduction of transformation geometry is just a fad – but there are strong reasons for the use of transformations in school geometry.

One reason is that rotation, reflection, etc. can be introduced in a practical way and so should be more accessible to some pupils than the more theoretical traditional geometry. Another reason is that this geometry is, in fact, fundamental to future work, when the use of vectors becomes an integral part. It should also be noted that this approach does still provide logical and powerful analysis, although it is rather different in nature to that of traditional geometry.

This unit first revises shapes and then deals with the following transformations:

- *translations*
- *enlargements*
- *reflections*
- *rotation*

finally dealing with combinations of these transfromations.

Routes		**Standard**	**Academic**	**Express**
7.1	Shapes	✓	(✓)	✕
7.2	Translations	✓	✓	(✓)
7.3	Enlargements	✓	✓	(✓)
7.4	Reflections	✓	✓	✓
7.5	Rotations	(✓)	✓	✓
7.6	Combining Transformations	✕	(✓)	✓

UNIT 7 *Transformations* Teaching Notes

Language	**Standard**	**Academic**	**Express**
Translation	✓	✓	✓
Vector	✓	✓	✓
Enlargement	✓	✓	✓
Scale factor	✓	✓	✓
Centre of enlargement	✓	✓	✓
Reflection	✓	✓	✓
Mirror line	✓	✓	✓
Rotation	✓	✓	✓
Centre of rotation	(✓)	✓	✓

Misconceptions

- pupils need to realise that the mirror line for a reflection does not need to be vertical or horizontal
- rotations are not always about the origin - they can be about *any* point
- the direction, 'clockwise' or 'anticlockwise', for a rotation, *must* be stated (but note that $180°$ *clockwise* is, in fact, the same as $180°$ *anticlockwise*)

