

14 Straight Line Graphs

14.1 Coordinates

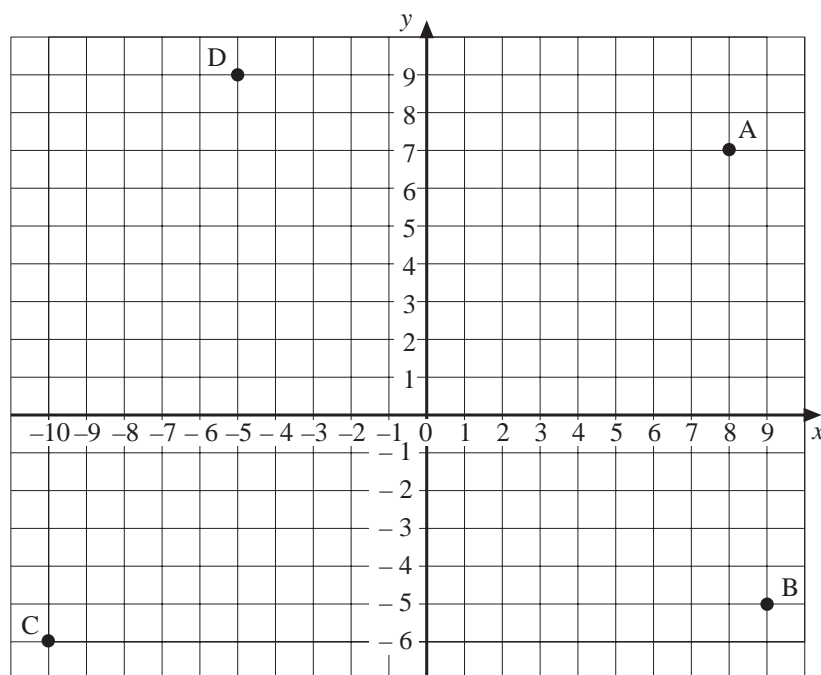
You will have used coordinates in Unit 3 of Book Y7A. In this section, we revisit *coordinates* before starting work on *lines* and *graphs*.

Remember that the first number is the x -coordinate and the second number is the y -coordinate.



Example 1

What are the coordinates of the points marked on the following grid:



Solution

The coordinates are:

- A (8, 7)
- B (9, -5)
- C (-10, -6)
- D (-5, 9)



Example 2

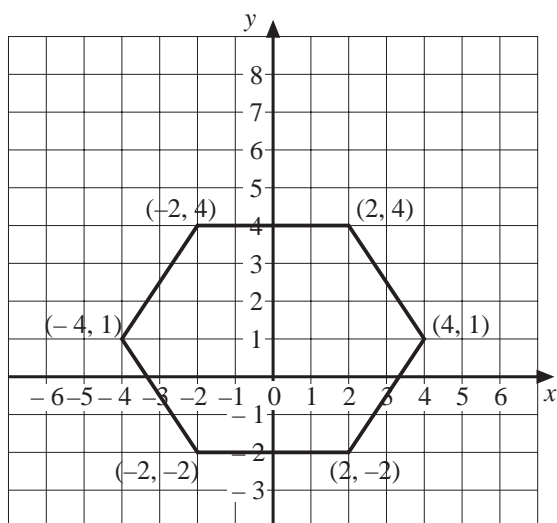
The coordinates of the corners of a shape are (2, 4), (4, 1), (2, -2), (-2, -2), (-4, 1) and (-2, 4).

- (a) Draw the shape.
- (b) What is the name of the shape?



Solution

(a)

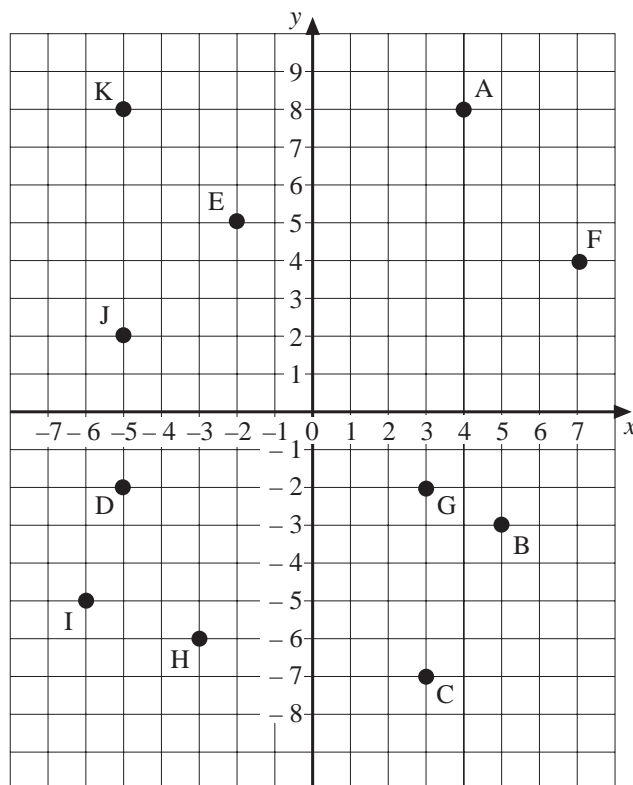


(b) The shape has six sides and is called a *hexagon*.



Exercises

- Write down the coordinates of each of the points marked on the following axes:



2.
 - (a) Plot the points with coordinates $(3, -2)$, $(-1, 6)$ and $(-5, -2)$.
 - (b) Join the points to form a triangle.
 - (c) What type of triangle have you drawn?
3.
 - (a) Plot the points with coordinates $(-1, 4)$, $(2, 5)$, $(5, 4)$ and $(2, -1)$.
 - (b) Join these points, in order, to form a shape.
 - (c) What is the name of the shape that you have drawn?
4. The coordinates of 3 corners of a square are $(3, 1)$, $(-1, 1)$ and $(3, -3)$. What are the coordinates of the other corner?
5. The coordinates of 3 corners of a rectangle are $(-1, 6)$, $(-4, 6)$ and $(-4, -5)$. What are the coordinates of the other corner?
6. A shape has corners at the points with coordinates $(3, -2)$, $(6, 2)$, $(-2, 2)$ and $(-5, -2)$.
 - (a) Draw the shape.
 - (b) What is the name of the shape?
7. A shape has corners at the points with coordinates $(3, 1)$, $(1, -3)$, $(3, -7)$ and $(5, -3)$.
 - (a) Draw the shape.
 - (b) What is the name of the shape?
8.
 - (a) Join the points with the coordinates below, in order, to form a polygon:
 $(-5, 0)$, $(-3, 2)$, $(-1, 2)$, $(1, 0)$, $(1, -2)$, $(-1, -4)$, $(-3, -4)$ and $(-5, -2)$.
 - (b) What is the name of the polygon?
9. Three of the corners of a parallelogram have coordinates $(1, 5)$, $(4, 4)$ and $(6, -3)$.
 - (a) Draw the parallelogram.
 - (b) What are the coordinates of the other corner?
10. Ben draws a pattern by joining, in order, the points with the following coordinates:
 $(-2, 1)$, $(-2, 2)$, $(0, 2)$, $(0, -1)$, $(-4, -1)$, $(-4, 4)$, $(2, 4)$ and $(2, -3)$.
What are the coordinates of the next three points he would use?

14.2 Plotting Points on Straight Lines

In this section we plot points that lie on a straight line, and look for relationships between the coordinates of these points.



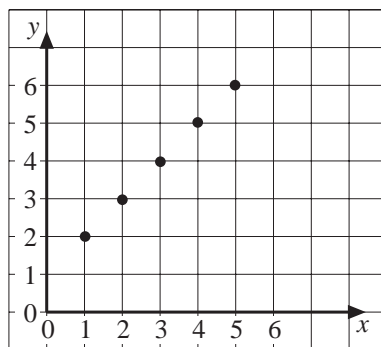
Example 1

- Plot the points with coordinates:
(1, 2), (2, 3), (3, 4), (4, 5) and (5, 6).
- Draw a straight line through these points.
- Describe how the x - and y -coordinates of these points are related.

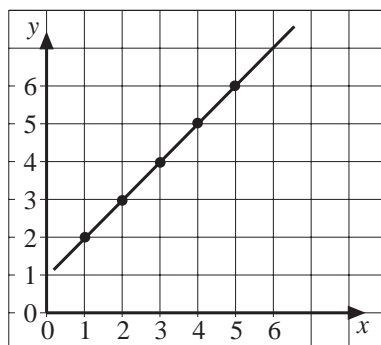


Solution

- The points are plotted below:



- A straight line can be drawn through these points:



- The y -coordinate is always one more than the x -coordinate, so we can write $y = x + 1$.



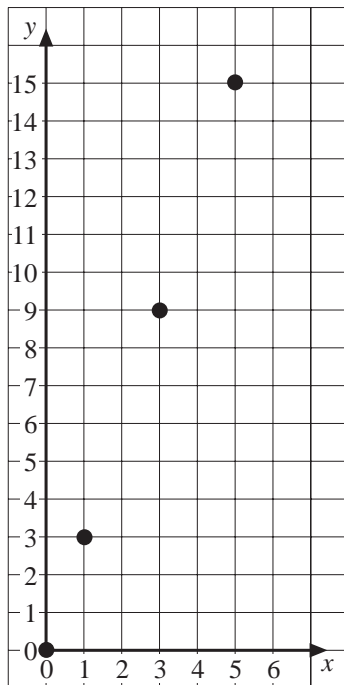
Example 2

- Plot the points with coordinates:
(0, 0), (1, 3), (3, 9) and (5, 15).
- Draw a straight line through these points
- Write down the coordinates of two other points on this line.
- Describe how the x - and y -coordinates are related.

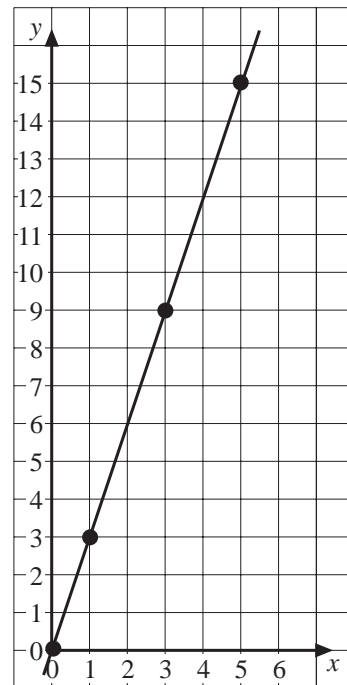


Solution

- The points are plotted below:



- A line can then be drawn through these points:



- The points (2, 6), and (4, 12) also lie on the line (and many others).
- The y -coordinate is 3 times the x -coordinate. So we can write $y = 3x$



Exercises

1. (a) Plot the points with coordinates
(0, 4), (1, 5), (3, 7) and (5, 9).
(b) Draw a straight line through the points.
(c) Write down the coordinates of 3 other points that lie on this line.

2. (a) Plot the points with coordinates
(0, 6), (2, 4), (3, 3) and (5, 1)
and draw a straight line through them.
(b) On the same graph as used for question 2 (a), plot the points with coordinates
(1, 8), (2, 7), (5, 4) and (7, 2)
and draw a straight line through them.
(c) Copy and complete the sentence:
"These two lines are p.....".

3. (a) Plot the points with coordinates
(2, 6), (3, 5), (4, 4) and (7, 1)
and draw a straight line through them.
(b) On the same set of axes, plot the points with coordinates
(0, 1), (1, 2), (3, 4) and (5, 6)
and draw a straight line through them.
(c) Copy and complete this sentence:
"These two lines are p.....".

4. (a) Plot the points with coordinates
(1, 1), (2, 2), (4, 4) and (5, 5)
and draw a straight line through them.
(b) Write down the coordinates of two other points on the line.
(c) Describe the relationship between the x - and y -coordinates.

5. The points (1, 3), (2, 4), (3, 5) and (5, 7) lie on a straight line.
(a) Plot these points and draw the line.
(b) Write down the coordinates of 3 other points on the line.
(c) Describe the relationship between the x - and y -coordinates.

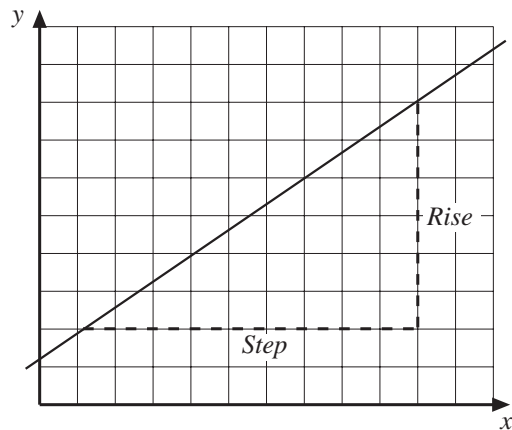
6. (a) Plot the points $(0, 5)$, $(2, 3)$, $(4, 1)$ and $(5, 0)$. Draw a straight line through them.
- (b) Write down the coordinates of two other points on the line.
- (c) The relationship between the x - and y -coordinates can be written as $x + y = \boxed{}$. What is the missing number?
7. (a) Plot the points with coordinates $(-3, -4)$, $(-1, -2)$, $(1, 0)$, $(4, 3)$
- (b) Draw a straight line graph through these points.
- (c) Describe the relationship between the x - and y -coordinates.
8. The points with coordinates $(-2, -4)$, $(2, 4)$, $(3, 6)$ and $(4, 8)$ lie on a straight line.
- (a) Draw the line.
- (b) Describe the relationship between the x - and y -coordinates of points on the line.
9. The points with coordinates $(-6, -3)$, $(-1, 2)$, $(2, 5)$ and $(4, 7)$ lie on a straight line.
- (a) Draw the line.
- (b) Complete the missing numbers in the coordinates of other points that lie on the line:
- $(-7, \boxed{})$, $(\boxed{}, -1)$, $(3, \boxed{})$, $(\boxed{}, 4)$, $(100, \boxed{})$
- (c) Describe the relationship between the x - and y -coordinates of the points on the line.
- (d) Will the point with coordinates $(25, 27)$ lie on the line? Give a reason for your answer.
10. Each set of points listed below lies on a straight line. Plot the points, draw the line, and complete the statement about the relationship between the x - and y -coordinates.
- (a) $(1, 6)$, $(3, 4)$, $(8, -1)$ $x + y = \boxed{}$
- (b) $(-4, 2)$, $(-1, 5)$, $(3, 9)$ $y = x + \boxed{}$
- (c) $(-2, -8)$, $(0, 0)$, $(3, 12)$ $y = \boxed{}x$
- (d) $(-4, -6)$, $(-1, -3)$, $(3, 1)$ $y = x - \boxed{}$

14.3 Plotting Graphs Given Their Equations

In this section we see how to plot a graph, given its equation. We also look at how steep it is and use the word *gradient* to describe this. There is a simple connection between the equation of a line and its gradient, which you will notice as you work through this section.

Gradient of a Line

$$\text{Gradient} = \frac{\text{Rise}}{\text{Step}}$$

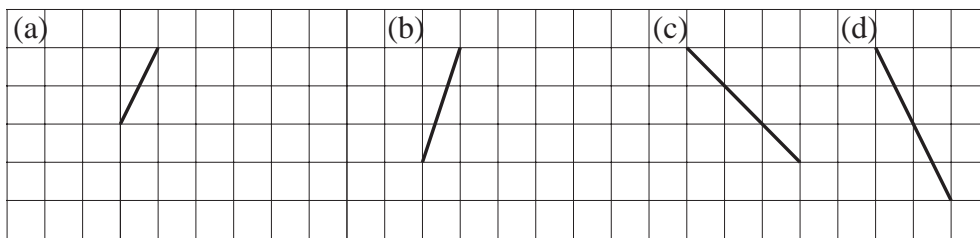


You can draw any triangle using the sides to determine the rise and step, but the triangle must have *one side horizontal* and *one side vertical*.

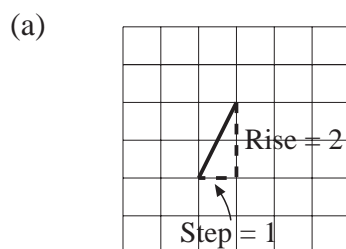


Example 1

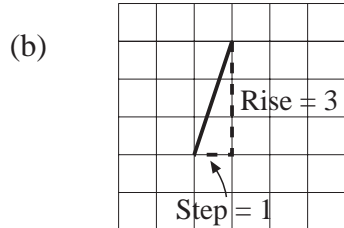
Determine the gradient of each of the following lines:



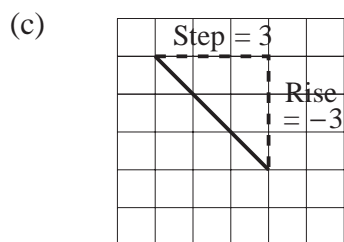
Solution



$$\begin{aligned} \text{Gradient} &= \frac{\text{Rise}}{\text{Step}} \\ &= \frac{2}{1} \\ &= 2 \end{aligned}$$

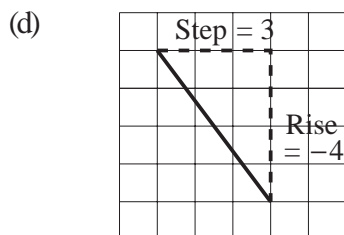


$$\begin{aligned}\text{Gradient} &= \frac{\text{Rise}}{\text{Step}} \\ &= \frac{3}{1} \\ &= 3\end{aligned}$$



$$\begin{aligned}\text{Gradient} &= \frac{\text{Rise}}{\text{Step}} \\ &= \frac{(-3)}{3} \\ &= -1\end{aligned}$$

Note that in (c) the rise is *negative* although the step is *positive*, so the gradient of the line is *negative*.



$$\begin{aligned}\text{Gradient} &= \frac{\text{Rise}}{\text{Step}} \\ &= \frac{(-4)}{3} \\ &= -\frac{4}{3}\end{aligned}$$

Note that in (d) once again the rise is *negative*, and the step is *positive*, so the gradient of the line is *negative*.

(In both (c) and (d) you will see that the lines slope in a different direction to the lines in (a) and (b), which have a positive gradient.)



Example 2

(a) Complete the table below for $y = 2x + 1$.

x	-2	-1	0	1	2
y					

(b) Use the information in the table to plot the graph with equation $y = 2x + 1$.

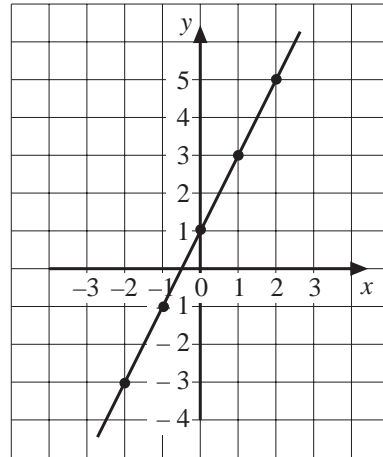


Solution

(a)

x	-2	-1	0	1	2
y	-3	-1	1	3	5

- (b) The points
 $(-2, -3)$, $(-1, -1)$, $(0, 1)$
 $(1, 3)$ and $(2, 5)$
 can then be plotted, and a
 straight line drawn through
 these points.



Example 3

- (a) Draw the graph of the line with equation $y = x + 1$.
 (b) What is the gradient of the line?

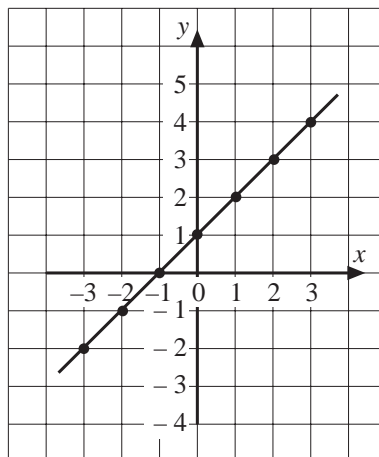


Solution

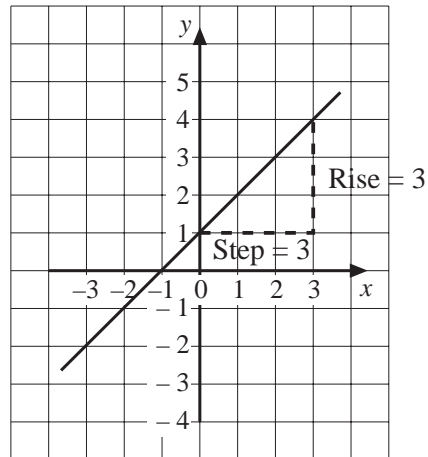
- (a) The table shows how to calculate the coordinates of some points on the line.

x	-3	-2	-1	0	1	2	3
y	-2	-1	0	1	2	3	4

The points with coordinates $(-3, -2)$, $(-2, -1)$, $(-1, 0)$, $(0, 1)$, $(1, 2)$, $(2, 3)$ and $(3, 4)$ can then be plotted and a line drawn as shown:



- (b) To calculate the gradient of the line, draw a triangle under the line as shown in the diagram on the next page. The triangle can be of any size, but must have one horizontal side and one vertical side.

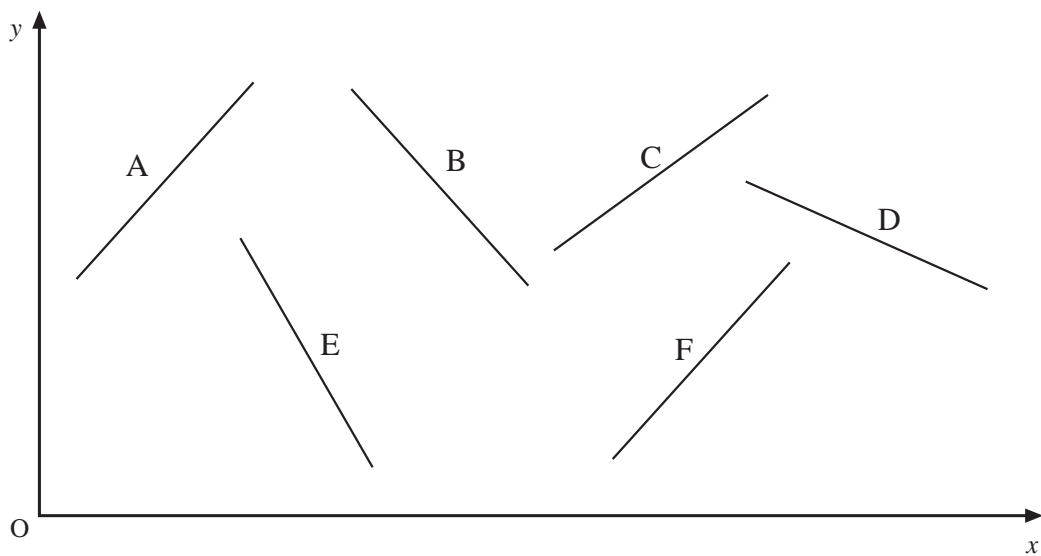


$$\begin{aligned}\text{Gradient} &= \frac{\text{Rise}}{\text{Step}} \\ &= \frac{3}{3} \\ &= 1\end{aligned}$$

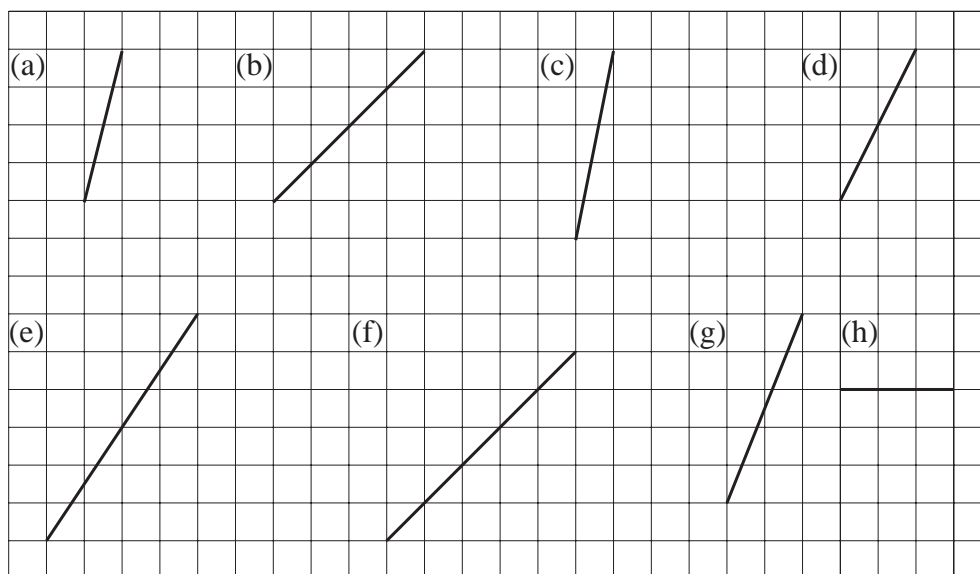


Exercises

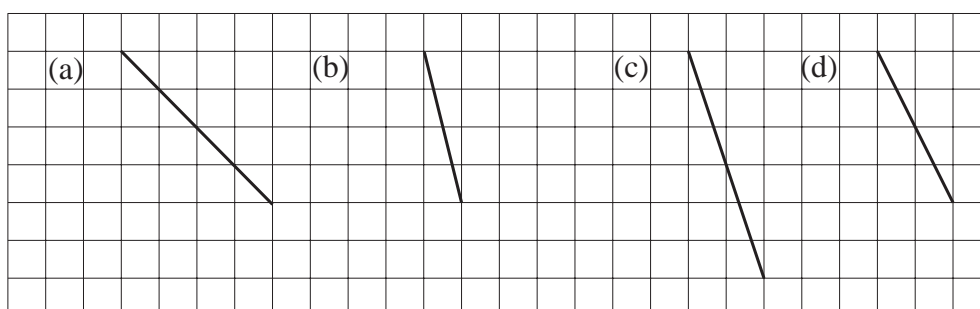
- Which of the following lines have a *positive* gradient and which have a *negative* gradient:



2. Determine the gradient of each of the following lines:



3. Determine the gradient of each of the following lines:



4. (a) Copy and complete the following table for $y = x - 2$.

x	-3	-2	-1	0	1	2	3
y				-2			

- (b) Draw the line with equation $y = x - 2$.

5. (a) Copy and complete the following table for $y = 2x + 3$.

x	-3	-2	-1	0	1	2	3
y							9

- (b) Draw the line with equation $y = 2x + 3$.

6.
 - (a) Draw the line with equation $y = 2x - 1$.
 - (b) Determine the gradient of this line.

7.
 - (a) Draw the line with equation $y = \frac{1}{2}x + 2$.
 - (b) Determine the gradient of this line.

8.
 - (a) Draw the lines $y = 3x + 1$ and $y = 4x - 5$.
 - (b) Determine the gradient of each of these lines.

9. Without drawing the lines, state the gradients of the lines with the following equations:
 - (a) $y = 2x + 4$
 - (b) $y = 3x - 9$
 - (c) $y = 10x + 1$
 - (d) $y = 5x + 3$

10.
 - (a) Draw the lines and equations $y = 2x + 1$ and $y = 3x - 2$.
 - (b) Write down the coordinates of the point where these two lines cross.

11. Determine the coordinates of the point where the lines $y = x + 3$ and $y = 7 - x$ cross.

12.
 - (a) Draw the line with equation $y = 6 - 2x$.
 - (b) Explain why the gradient of this line is -2 .

13.
 - (a) Explain why the lines with equations $y = 2 - 2x$ and $y = 5 - 2x$ are parallel.
 - (b) Write down the equation of another line that would be parallel to these lines.
 - (c) Draw all three lines.

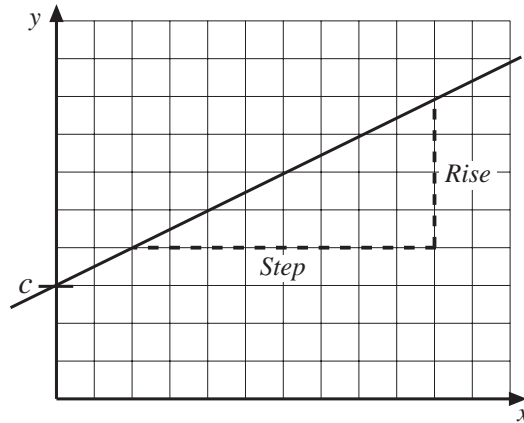
14.4 The Equation of a Straight Line

In this section we examine how the equation of a straight line contains information about the gradient of the line and the point where it crosses the y-axis.

The intercept is c , that is the point where the line crosses the y-axis.

The gradient is m , where

$$m = \frac{\text{Rise}}{\text{Step}}$$

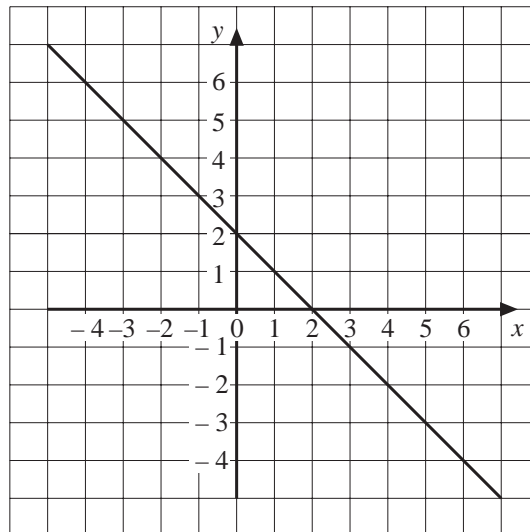


The equation of a straight line is $y = mx + c$.



Example 1

(a) Determine the equation of the line shown below:

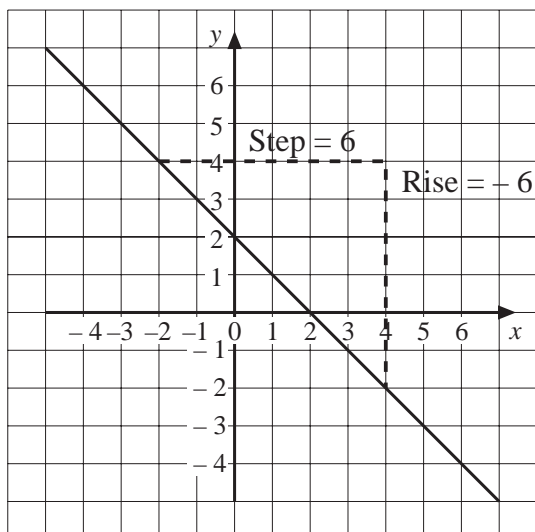


Solution

First note that the intercept is 2, so we write $c = 2$.

Next calculate the gradient of the line.

Note that the rise is -6 , as the line is going down as you move from left to right.



$$\text{Gradient} = \frac{-6}{6}$$

$$= -1$$

$$\text{So } m = -1$$

The equation of a straight line is $y = mx + c$, so here, with $m = -1$ and $c = 2$, we have

$$y = -x + 2$$

or

$$y = 2 - x.$$

Reminder

Recall that $-1 \times x = -1x$ is written as $-x$ for speed and convenience.

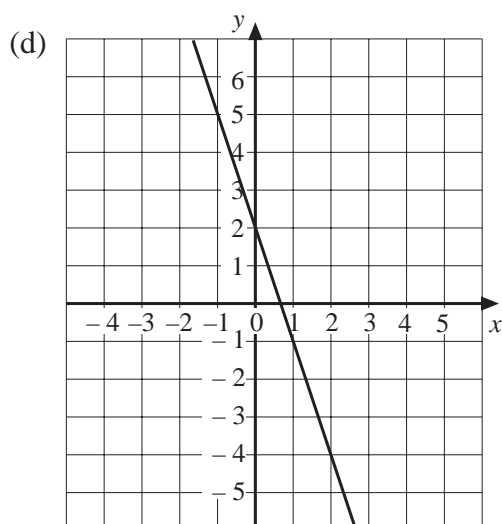
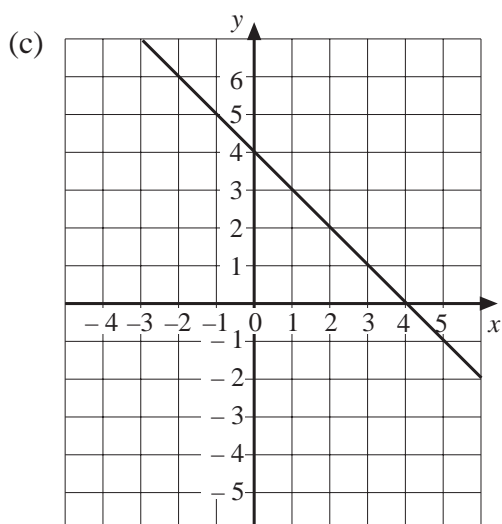
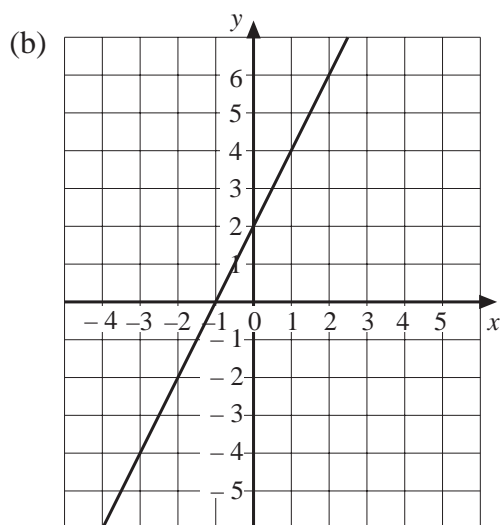
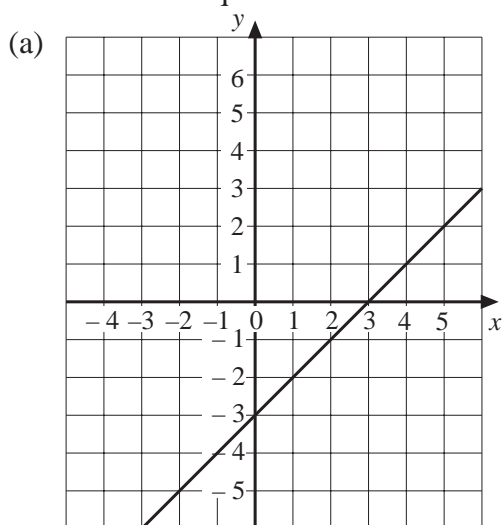


Exercises

- Draw the line with equation $y = 2x + 3$.
 - Determine the gradient of this line.
 - What is the intercept of this line?
- Draw the lines with equations $y = x$, $y = -x$, $y = 2x$ and $y = -3x$.
 - Determine the gradient of each of these lines.
 - What is the intercept of each of these lines?
- The points with coordinates $(-2, 3)$, $(0, 5)$ and $(3, 8)$ lie on a straight line.

 - Plot the points and draw the line.
 - Determine the gradient of the line.
 - What is the intercept of the line?
 - Write down the equation of the line.

4. Determine the equation of each of the lines shown below:



5. Copy and complete the following table:

<i>Equation</i>	<i>Gradient</i>	<i>Intercept</i>
$y = 2x + 7$		
	8	-2
$y = 8 - 3x$		
	7	-5
	-3	2
	-5	-2

6. (a) Draw the lines with equations $y = x + 1$, $y = 1 - x$, $y = 2x + 1$ and $y = 3x + 1$ on the same set of axes.
 (b) Explain why these lines all pass through the same point on the y -axis.
7. The points with coordinates $(-2, -6)$, $(0, 0)$ and $(3, 9)$ all lie on a straight line.
 (a) What is the gradient of the line?
 (b) What is the intercept of the line?
 (c) What is the equation of the line?
8. Draw lines which have:
 (a) gradient 2 and intercept 3,
 (b) gradient $\frac{1}{2}$ and intercept 1,
 (c) gradient -4 and intercept 7.

14.5 The Equation of a Line Given Two Points

If you know the coordinates of two points on a line, it is possible to determine its equation *without drawing the line*.

If a line passes through the points with coordinates (x_1, y_1) and (x_2, y_2) , the gradient, m , of the line is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Example 1

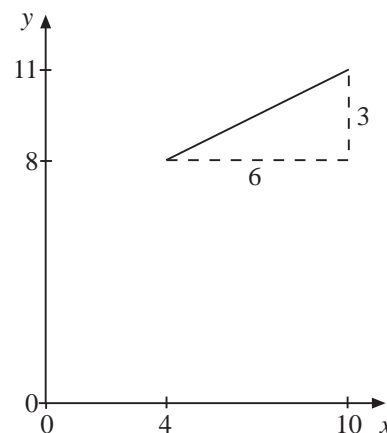
Determine the equation of the line that joins the points with coordinates $(4, 8)$ and $(10, 11)$.



Solution

First determine the gradient of the line:

$$\begin{aligned} m &= \frac{11 - 8}{10 - 4} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$



Now the equation must be $y = \frac{1}{2}x + c$.

To determine c , use the values of x and y from one of the points. Here $x = 4$ and $y = 8$, and substitute in the equation, giving:

$$8 = \frac{1}{2} \times 4 + c$$

$$8 = 2 + c$$

$$c = 6$$

So the equation of the line is given by $y = \frac{1}{2}x + 6$.



Exercises

- A straight line joins the points with coordinates $(1, 1)$ and $(4, 7)$.
 - Determine the gradient of the line.
 - Determine the equation of the line.
- Determine the equation of the line that passes through the points $(0, 0)$ and $(3, 21)$.
- Explain why a line that passes through the point $(0, 0)$ and any other point has equation $y = mx$.
- Determine the equation of a straight line that passes through the following pairs of points:

(a) $(0, 1)$ and $(5, 16)$	(b) $(3, 20)$ and $(7, 32)$
(c) $(0, 100)$ and $(50, 0)$	(d) $(-1, 9)$ and $(3, -3)$
(e) $(-6, -4)$ and $(10, 28)$	(f) $(-6, -2)$ and $(-2, -9)$
- A line has gradient -4 and passes through the point with coordinates $(5, 7)$. What is the equation of the line?
- A triangle has corners at the points with coordinates $(1, 2)$, $(-2, 3)$ and $(0, -1)$. Determine the equations of the lines that form the sides of the triangle.
- A parallelogram has corners at the points with coordinates $(-1, 1)$, $(0, 3)$, $(2, -1)$ and $(1, -3)$. Determine the equations of the lines that form the sides of the parallelogram.