

1. (a) $2a + \frac{2}{5} + \frac{1}{10} = 1$ (or equivalent) M1

$$\underline{a = \frac{1}{4} \text{ or } 0.25} \quad \text{A1} \quad 2$$

Note

M1 for a clear attempt to use $\sum P(X = x) = 1$

Correct answer only 2/2.

NB Division by 5 in parts (b), (c) and (d) seen scores 0.

Do not apply ISW.

(b) $E(X) = \underline{1}$ B1 1

Note

B1 for 1

(c) $E(X^2) = 1 \times \frac{1}{5} + 1 \times \frac{1}{10} + 4 \times \frac{1}{4} + 9 \times \frac{1}{5} \quad (= 3.1)$ M1

$$\text{Var}(X) = 3.1 - 1^2, \quad \underline{= 2.1 \text{ or } \frac{21}{10} \text{ oe}} \quad \text{M1 A1} \quad 3$$

Note

1st M1 for attempting $\sum x^2 P(X = x)$ at least two terms correct.

Can follow through.

2nd M1 for attempting $E(X^2) - [E(X)]^2$ or allow subtracting 1 from their attempt at $E(X^2)$ provided no incorrect formula seen.

Correct answer only 3/3.

(d) $\text{Var}(Y) = (-2)^2 \text{Var}(X), \quad \underline{= 8.4 \text{ or } \frac{42}{5} \text{ oe}} \quad \text{M1 A1} \quad 2$

Note

M1 for $(-2)^2 \text{Var}(X)$ or $4\text{Var}(X)$

Condone missing brackets provided final answer correct for their $\text{Var}(X)$.

Correct answer only 2/2.

(e) $X \geq Y$ when $X = 3$ or 2 , so probability = $\frac{1}{4} + \frac{1}{5}$ M1 A1ft

$$\underline{= \frac{9}{20} \text{ oe}} \quad \text{A1} \quad 3$$

Note

Allow M1 for distribution of $Y = 6 - 2X$ and correct attempt at $E(Y^2) - [E(Y)]^2$

M1 for identifying $X = 2, 3$

1st A1ft for attempting to find their $P(X = 2) + P(X = 3)$

2nd A1 for $\frac{9}{20}$ or 0.45

[11]

2. (a) $k + 4k + 9k = 1$ M1

$$14k = 1$$

$$k = \frac{1}{14} \quad ** \text{ given } ** \quad \text{cso} \quad \text{A1} \quad 2$$

Note

M1 for clear attempt to use $\sum p(x) = 1$, full expression needed and the “1” must be clearly seen. This may be seen in a table.

A1cso for no incorrect working seen. The sum and “= 1” must be explicitly seen somewhere.

A verification approach to (a) must show addition for M1 and have a suitable comment e.g. “therefore $k = \frac{1}{14}$ ” for A1 cso

(b) $P(X \geq 2) = 1 - P(X=1) \quad \text{or } P(X=2) + P(X=3) \quad \text{M1}$

$$= 1 - k = \frac{13}{14} \quad \text{or } 0.92857... \quad \text{awrt } 0.929 \quad \text{A1} \quad 2$$

Note

M1 for $1 - P(X \leq 1)$ or $P(X=2) + P(X=3)$

A1 for awrt 0.929. Answer only scores 2/2

(c) $E(X) = 1 \times k + 2 \times k \times 4 + 3 \times k \times 9 \quad \text{or } 36k \quad \text{M1}$

$$= \frac{36}{14} = \frac{18}{7} \text{ or } 2\frac{4}{7} \quad (\text{or exact equivalent}) \quad \text{A1} \quad 2$$

Note

M1 for a full expression for $E(X)$ with at least two terms correct.

NB If there is evidence of division (usually by 3) then score M0

A1 for any exact equivalent – answer only scores 2/2

(d) $\text{Var}(X) = 1 \times k + 4 \times k \times 4 + 9 \times k \times 9, - \left(\frac{18}{7}\right)^2$ M1 M1

$\text{Var}(1-X) = \text{Var}(X)$ M1

$\frac{19}{49}$ or 0.387755... awrt 0.388 A1 4

Note

1st M1 for clear attempt at $E(X^2)$, need at least 2 terms correct in $1 \times k + 4 \times 4k + 9 \times 9k$ or $E(X^2) = 7$

2nd M1 for their $E(X^2) - (\text{their } \mu)^2$

3rd M1 for clearly stating that $\text{Var}(1-X) = \text{Var}(X)$, wherever seen

A1 accept awrt 0.388. All 3 M marks are required.

Allow 4/4 for correct answer only but must be for $\text{Var}(1-X)$.

[10]

3. (a)

$$\begin{array}{cccc} 0 & 1 & 2 & 3 \\ 3a & 2a & a & b \end{array}$$

B1 1

Note

Condone a clearly stated in text but not put in table.

(b) $3a + 2a + a + b = 1$ or equivalent, using Sum of probabilities = 1 M1

$2a + 2a + 3b = 1.6$ or equivalent, using $E(X) = 1.6$ M1

$14a = 1.4$ Attempt to solve M1dep

$a = 0.1$ cao B1

$b = 0.4$ cao B1 5

Note

Must be attempting to solve 2 different equations so third M dependent upon first two Ms being awarded.

Correct answers seen with no working
B1B1 only, 2/5

Correctly verified values can be awarded
M1 for correctly verifying sum of probabilities

=1, M1 for using $E(X)=1.6$ M0 as no attempt to solve and B1B1 if answers correct.

(c)	$P(0.5 < x < 3) = P(1) + P(2)$	3a or their 2a+their a	M1	
	$= 0.2 + 0.1$			
	$= 0.3$	Require $0 < 3a < 1$ to award follow through	A1 ft	2

(d)	$E(3X - 2) = 3E(X) - 2$		M1	
	$= 3 \times 1.6 - 2$			
	$= 2.8$	cao	A1	2

Note

2.8 only award M1A1

(e)	$E(X^2) = 1 \times 0.2 + 4 \times 0.1 + 9 \times 0.4 (= 4.2)$		M1	
	$\text{Var}(X) = "4.2" - 1.6^2$		M1	
	$= 1.64$	* * given answer * *	cs0	A1 3

Note

Award first M for at least two non-zero terms correct. Allow first M for correct expression with a and b e.g. $E(X^2) = 6a + 9b$
Given answer so award final A1 for correct solution.

(f)	$\text{Var}(3X - 2) = 9 \text{Var}(X)$		M1	
	$= 14.76$	awrt 14.8	A1	2

Note

14.76 only award M1A1

[15]

4.	(a)	$E(X) = 0 \times 0.4 + 1 \times 0.3 + \dots + 3 \times 0.1, = 1$	M1, A1	2
----	-----	--	--------	---

Note

M1 for at least 3 terms seen. Correct answer only scores M1A1.
Dividing by $k(\neq 1)$ is M0.

(b)	$F(1.5) = [P(X \leq 1.5) =] P(X \leq 1), = 0.4 + 0.3 = 0.7$	M1, A1	2
-----	---	--------	---

Note

M1 for $F(1.5) = P(X \leq 1)$. [Beware: $2 \times 0.2 + 3 \times 0.1 = 0.7$ but scores M0A0]

- (c) $E(X^2) = 0^2 \times 0.4 + 1^2 \times 0.3 + \dots + 3^2 \times 0.1 = 2$ M1, A1
 $\text{Var}(X) = 2 - 1^2 = 1$ (*) M1, A1 cso 4

Note

1st M1 for at least 2 non-zero terms seen. $E(X^2) = 2$ alone is M0.
 Condone calling $E(X^2) = \text{Var}(X)$.

ALT

1st A1 is for an answer of 2 or a fully correct expression.

2nd M1 for $-\mu^2$, condone $2 - 1$, unless clearly $2 - \mu$ Allow $2 - \mu^2$
 with $\mu = 1$ even if $E(X) \neq 1$

2nd A1 for a fully correct solution with no incorrect working seen,
both Ms required.

$$\underline{\sum (x - \mu)^2 \times P(X = x)}$$

1st M1 for an attempt at a full list of $(x - \mu)^2$ values and probabilities.

1st A1 if all correct

2nd M1 for at least 2 non-zero terms of $(x - \mu)^2 \times P(X = x)$ seen. 2nd
 A1 for $0.4 + 0.2 + 0.4 = 1$

- (d) $\text{Var}(5 - 3X) = (-3)^2 \text{Var}(X) = 9$ M1, A1 2

Note

M1 for use of the correct formula. $-3^2 \text{Var}(X)$ is M0 unless the final answer is >0 .

(e)

Total	Cases	Probability	
4	$(X = 3) \cap (X = 1)$	$0.1 \times 0.3 = 0.03$	B1B1B1
	$(X = 1) \cap (X = 3)$	$0.3 \times 0.1 = 0.03$	
	$(X = 2) \cap (X = 2)$	$0.2 \times 0.2 = 0.04$	
5	$(X = 3) \cap (X = 2)$	$0.1 \times 0.2 = 0.02$	M1
	$(X = 2) \cap (X = 3)$	$0.2 \times 0.1 = 0.02$	
6	$(X = 3) \cap (X = 3)$	$0.1 \times 0.1 = 0.01$	A1
Total probability = $0.03 + 0.03 + 0.04 + 0.02 + 0.02 + 0.01 = 0.15$			A1 6

Note

Can follow through their $\text{Var}(X)$ for M1

ALT

1st B1 for all cases listed for a total of 4 or 5 or 6 . e.g. (2,2) counted twice for a total of 4 is B0

2nd B1 for all cases listed for 2 totals }

3rd B1 for a complete list of all 6 cases } These may be highlighted in a table

Using Cumulative probabilities

1st B1 for one or more cumulative probabilities used e.g.2 then 2

or more or 3 then 1 or more

2nd B1 for both cumulative probabilities used. 3rd B1 for a complete list 1, 3; 2, ≥ 2 ; 3, ≥ 1

M1 for one correct pair of correct probabilities multiplied

1st A1 for all 6 correct probabilities listed (0.03, 0.03, 0.04, 0.02, 0.02, 0.01) needn't be added.

2nd A1 for 0.15 or exact equivalent only as the final answer.

[16]

5. (a) $-1 \times p + 1 \times 0.2 + 2 \times 0.15 + 3 \times 0.15 = 0.55$ M1dM1
 $p = 0.4$ A1
 $p + q + 0.2 + 0.15 + 0.15 = 1$ M1
 $q = 0.1$ A1 5

M1 for at least 2 correct terms on LHS

Division by constant e.g. 5 then M0

dM1 dependent on first M1 for equate to 0.55 and attempt to solve.

Award M1M1A1 for $p=0.4$ with no working

M1 for adding probabilities and equating to 1.

All terms or equivalent required e.g. $p + q = 0.5$

Award M1A1 for $q = 0.1$ with no working

- (b) $\text{Var}(X) = (-1)^2 \times p + 1^2 \times 0.2 + 2^2 \times 0.15 + 3^2 \times 0.15, -0.55^2$ M1A1,M1
 $= 2.55 - 0.3025 = 2.2475$ awrt 2.25 A1 4

M1 attempting $E(X^2)$ with at least 2 correct terms

A1 for fully correct expression or 2.55

Division by constant at any point e.g. 5 then M0

M1 for subtracting their mean squared

A1 for awrt 2.25

Award awrt 2.25 only with no working then 4 marks

- (c) $E(2X - 4) = 2E(X) - 4$ M1
 $= -2.9$ A1 2

M1 for $2x(\text{their mean}) - 4$

Award 2 marks for -2.9 with no working

[11]

6. (a) $F(4) = 1$
 $(4 + k)^2 = 25$ M1
 $k = 1$ as $k > 0$ A1 2

M1 for use of $F(4) = 1$ only If $F(2) = 1$ and / or $F(3) = 1$ seen then M0.

$F(2) + F(3) + F(4) = 1$ M0

A1 for $k = 1$ and ignore $k = -9$

(b)

x	2	3	4
-----	---	---	---

$P(X=x)$	$\frac{9}{25}$	$\frac{7}{25}$	$\frac{9}{25}$
----------	----------------	----------------	----------------

B1ftB1B1 3

B1ft follow through their k for $P(X=2)$ either exact or 3sf between 0 and 1 inclusive.

B1 correct answer only or exact equivalent

B1 correct answer only or exact equivalent

[5]

7. (a) $p + q = 0.45$ B1
 $\sum xP(X=x) = 4.5$ M1
 $3p + 7q = 1.95$ A1 3

$0.55 + p + q = 1$ award B1. Not seen award B0.

$0.2 + 3p + 1 + 7q + 1.35 = 4.5$ or equivalent award M1A1

$3p + 7q + k = 4.5$ award M1.

- (b) Attempt to solve equations in (a) M1
 $q = 0.15$ A1
 $p = 0.30$ A1 3

Attempt to solve must involve 2 linear equations in 2 unknowns

Correct answers only for accuracy.

Correct answers with no working award 3/3

- (c) $P(4 < X < 7) = P(5) + P(7)$ M1
 $= 0.2 + q = 0.35$ A1ft 2

Follow through accuracy mark for their q , $0 < q < 0.8$

- (d) $\text{Var}(X) = E(X^2) - [E(X)]^2 = 27.4 - 4.5^2$ M1
 $= 7.15$ A1 2

Attempt to substitute given values only into correct formula for M1.

7.15 only for A1

7.15 seen award 2/2

- (e) $E(19 - 4X) = 19 - 4 \times 4.5 = 1$ B1 1

- (f) $\text{Var}(19 - 4X) = 16\text{Var}(X)$ M1
 $= 16 \times 7.15 = 114.4$ A1 2

Accept 'invisible brackets' i.e. $-4^2 \text{Var}(X)$ provided answer positive.

Anything that rounds to 114 for A1.

[13]

S1 Discrete random variables

8. (a) $E(X) = 3$; B1
- $$\text{Var}(X) = \frac{25-1}{12} = 2 \quad \text{AG}$$
- $$\text{Var}(X) = 1^2 \times \frac{1}{5} + 2^2 \times \frac{1}{5} + 3^2 \times \frac{1}{5} \dots - 3^3 = 11 - 9 = 2 \quad \text{AG} \quad \text{M1 A1} \quad 3$$
- Accept (55 / 5) – as minimum evidence.*
- (b) $E(3X - 2) = 3E(X) - 2 = 7$ M1 A1ft 2
- (c) $\text{Var}(4 - 3x) = 3^2 \text{Var}(X) = 18$ M1 A1 2
- [7]**
-
9. (a) $p + q = 0.4$ B1
- $$2p + 4q = 1.3 \quad \text{Consider with (b).} \quad \text{M1 A1} \quad 3$$
- (b) Attempt to solve $p = 0.15, q = 0.25$ M1
- If both seen, award 3. A1 A1 3
- (c) $E(X^2) = 1^2 \times 0.10 + 2^2 \times 0.15 + \dots + 5^2 \times 0.30 = 14$ M1A1ft
- $$\text{Var}(X) = 14 - 3.5^2 = 1.75 \quad \text{M1A1} \quad 4$$
- (d) $\text{Var}(3 - 2X) = 4\text{Var}(X) = 7.00$ M1A1ft 2
- [12]**
-
10. (a) $k + 2k + 3k + 5k + 6k = 1$ M1
- use of $\sum P(X = x) = 1$*
- $$17k = 1$$
- $$k = \frac{1}{17} = 0.0588 \quad \text{A1} \quad 2$$
- (b) $E(X) = 1 \times \frac{1}{17} + 2 \times \frac{2}{17} + \dots + 5 \times \frac{6}{17} = \frac{64}{17}$ M1
- use of $\sum xP(X = x)$ and at least 2 prob correct*
- $$= 3 \frac{13}{17} \quad \text{A1} \quad 2$$
- Do not ignore subsequent working*
- (c) $E(X) = 1^2 \times \frac{1}{17} + 2^2 \times \frac{2}{17} + \dots + 5^2 \times \frac{6}{17} = \left(\frac{266}{17} = 15.6 \right)$ M1 A1

use of $\sum x^2 P(X=x)$ and at least 2 prob correct

$$\text{Var}(X) = \frac{266}{17} - \left(\frac{64}{17}\right)^2 \quad \text{M1}$$

use of $\sum x^2 P(X=x)$ –

$$(E(X))^2 = 1.4740\dots \quad \text{A1} \quad 4$$

awrt 1.47

(d) $\text{Var}(4-3X) = 9 \text{Var}(X) = 9 \times 1.47 = 13.23 \Rightarrow 13.2$ M1 A1 2
 or $9 \times 1.4740\dots = 13.266 \Rightarrow 13.3$
 cao 9 Var X

[10]

11. (a) $k + 2k + 3k + 4k + 5k = 1$ M1
 $15k = 1$

verification / use of $\sum P(X=x) = 1$

$$** k = \frac{1}{15} ** \quad \text{A1} \quad 2$$

CSO

(b) $P(X < 4) = P(1) + P(2) + P(3) = \frac{1}{15} + \frac{2}{15} + \frac{3}{15}$ M1

sum of 3 probabilities

$$= \frac{2}{5} \quad \text{A1 seen (2)} \quad 2$$

$$0.4 \text{ or } \frac{6}{15} \text{ or } \frac{2}{5}$$

(c) $E(X) = 1 \times \frac{1}{15} + 2 \times \frac{2}{15} + 3 \times \frac{3}{15} + 4 \times \frac{4}{15} + 5 \times \frac{5}{15}$ M1

use of $\sum xP(X=x)$

$$= \frac{11}{3} \quad \text{A1} \quad 2$$

$$\frac{55}{15} \text{ or } \frac{11}{3} \text{ or } 3\frac{2}{3} \text{ or } 3.\dot{6} \text{ or } 3.67$$

$$(d) \quad E(3X - 4) = 3E(X) - 4 = 11 - 4$$

3 × theirs - 4

M1

$$= 7$$

A1 seen (2)

(OR)

$$E(3X - 4) = -1 \times \frac{1}{15} + 2 \times \frac{2}{15} + 5 \times \frac{3}{15} + 8 \times \frac{4}{15} + 11 \times \frac{5}{15}$$

M1

$$\Sigma(3x - 4)kx$$

$$= 7$$

A1

2

*cao***[8]**

12. (a) $0.5 + b + a = 1$ M1 A1

use of $\Sigma P(X = x) = 1$

$$0.3 + 2b + 3a = 1.7$$

M1 A1

$$\text{use of } E(x) = \Sigma xP(X = x)$$

$$\therefore \underline{a = 0.4 \text{ \& } b = 0.1}$$

B1

5

(b) $P(0 < X < 1.5) = P(X = 1) = \underline{0.3}$

B1

1

(c) $E(2X - 3) = 2E(X) - 3$

M1

$$\text{Use of } E(aX + b)$$

$$= 2 \times 1.7 - 3 = \underline{0.4}$$

A1

2

(d) $\text{Var}(X) = (1^2 \times 0.3) + (2^2 \times 0.1) + (3^2 \times 0.4) - 1.7^2$

M1

$$\text{Use of } E(x^2) - \{E(x)\}^2$$

$$= 4.3 - 2.89$$

A1ft

$$= \underline{1.41} (*)$$

A1

3

cs0

(e) $\text{Var}(2X - 3) = 2^2 \text{Var}(X)$

M1

$$\text{Use of } \text{Var}$$

$$= 4 \times 1.41 = \underline{5.64}$$

A1

2

[13]

13. (a) $P(1 < X \leq 3) = P(X = 2) + P(X = 3)$

$$= \frac{1}{12} + \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$$

$$\frac{2}{12}; \frac{1}{6}; 0.167; 0.1\dot{6}\dot{6}; 0.1\dot{6}$$

M1

A1 2

(b) $F(2.6) = P(X \leq 2) = 1 - P(X = 3) = 1 - \frac{1}{12} = \frac{11}{12}$

$$\frac{11}{12}; 0.917; 0.91\dot{6}$$

B1 1

(or: $P(X \leq 2) = \frac{1}{3} + \frac{1}{2} + \frac{1}{12} = \frac{11}{12}$)

(c) $E(X) = \left(0 \times \frac{1}{3}\right) + \dots + \left(3 \times \frac{1}{12}\right) = \frac{11}{12}$

x) M1

Use of $\Sigma xP(X =$

$$\frac{11}{12}; AWR T$$

$$0.917 \text{ A1 } 2$$

(d) $E(2X - 3) = 2E(X) - 3$

Use of $E(ax + b)$

M1

$$= 2 \times \frac{11}{12} - 3 = -\frac{14}{12} = -\frac{7}{6}$$

A1 2

$$-\frac{7}{6}; -1\frac{1}{6};$$

AWRT -1.17

(e) $\text{Var}(X) = 1^2 \times \frac{1}{2} + \dots + 3^2 \times \frac{1}{12} - \left(\frac{11}{12}\right)^2$

$$E(X^2) - \{E(X)\}^2 \quad \text{M1}$$

Use of

substitution A1ft

Correct

$$= \frac{107}{144}$$

$$\frac{107}{144};$$

AWRT 0.743

$$\text{A1 } 3$$

[10]

14. (a) $P(\text{scores 30 points}) = P(\text{hit, hit, hit}) = 0.6^3 = 0.216 = \frac{27}{125} \quad 0.6^3 \quad \text{M1}$

$\frac{27}{125}; 0.216 \quad \text{A1} \quad 2$

(b)

x	0	10	20	30
	0.4	0.6×0.4	$0.6^2 \times 0.4$	
$P(X=x)$	0.4	0.24	0.144	(0.216)
	$\frac{4}{10}$	$\frac{6}{25}$	$\frac{18}{225}$	

$x = 0, 10, 20, 30$

B1

One correct

$P(X=x)$

M1

$0.4; 0.24; 0.144$

A1; A1; A1 5

(c) $E(X) = (0 \times 0.4) + \dots + (30 \times 0.216) = \underline{11.76}$

$\sum xP(X=x)$

Their distribution M1

AWRT 11.8 A1

$E(X^2) = (0^2 \times 0.4) + \dots + (30^2 \times 0.216) = 276$

B1

Std Dev = $\sqrt{276 - 11.76^2} = 11.7346\dots \quad \sqrt{E(X^2) - (E(X))^2}$

M1

3 s.f. 11.7

A1 5

(d) $P(\text{Linda scores more in round 2 than in round 1})$

$= P(X_1 = 0 \ \& \ X_2 = 10, 20, 30) \ X_2 > X_1$

M1

$+ P(X_1 = 10 \ \& \ X_2 = 20, 30)$

Can be implied

All possible

A1

$+ P(X_1 = 20 \ \& \ X_2 = 30)$

A1

$= 0.4 \times (0.24 + 0.144 + 0.216) = 0.24$

A1 ft

$+ 0.24 \times (0.144 + 0.216) = 0.0864$

A1

$+ (0.144 \times 0.216) = 0.031104$

A1 ft

$= 0.357504$

A1 6

AWRT 0.358

[18]

15. (a) $E(X) = \sum x \times P(X=x) = \frac{1}{n} + \frac{2}{n} + \dots + \frac{n}{n}$ Use of $E(X)$ M1

Accept Verify

i.e: - $= \frac{1}{n} \{1 + 2 \dots + n\}$

$$\frac{1}{9} + \frac{2}{9} + \dots + \frac{9}{9} = \frac{45}{9} = 5$$

or $\frac{n+1}{2} = 5 \quad = \frac{1}{n} \cdot \frac{1}{2} n(n+1) = \frac{n+1}{2}$ M1

Use of $\frac{1}{2}n(n+1)$

$\therefore \frac{n+1}{2} = 5 \Rightarrow \underline{n=9}$ A1 3

Must state $n = 9$ for final A1
c.s.o.

(b) $P(X < T) = \frac{1}{9} \times 6 = \frac{2}{3}$ M1 A1 2

$P(X \leq 6)$

Use of $E(X^2)$

(c) $\text{Var}(X) = E(X^2) - \{E(X)\}^2$ A1

$$\frac{95}{3} - \frac{285}{9} = 31\frac{2}{3}$$

$= \frac{1^2}{9} + \frac{2^2}{9} + \dots + \frac{9^2}{9} - 5^2$ M1

Use of $\text{Var}(X)$

$= \frac{1}{9} \times \frac{1}{6} \times 9 \times 10 \times 19 - 5^2$ M1

$= \frac{20}{3}$ A1 4

$6\frac{2}{3}; 6.6\dot{6}; 6.67; \frac{20}{3}$

OR

$\text{Var}(X) = \frac{n^2 - 1}{12} = \frac{80}{12} = \frac{20}{3}$ M2 A1 A1

[9]

16. (a) $k(16 - 9) + k(25 - 9) + k(36 - 9) = 1$
 $\therefore 7k + 16k + 27k = 1 \Rightarrow k = \frac{1}{50}$

M1 A1

A1 3

x	4	5	6
$P(X=x)$	$\frac{7}{50}$	$\frac{16}{50}$	$\frac{27}{50}$

(b) $E(X) = (4 \times \frac{7}{50}) + (5 \times \frac{16}{50}) + (6 \times \frac{27}{50}) = \frac{270}{50} = 5.4$
 $E(X^2) = (4^2 \times \frac{7}{50}) + (5^2 \times \frac{16}{50}) + (6^2 \times \frac{27}{50}) = \frac{1484}{50} = 29.68$
 $\therefore \text{Var}(X) = 29.68 - 5.4^2 = \frac{13}{25} = 0.52$

$$\sum xP(X=x)$$

M1

$$\frac{27}{5} \text{ or } 5.4$$

A1

$$\sum x^2P(X=x)$$

M1

$$29.68$$

A1

$$\text{Use of } E(X^2) - \{E(x)\}^2$$

M1

$$0.52$$

A1

6

(c) $\text{Var}(2X - 3) = 2^2 \text{Var}(X)$
 $= 4 \times 0.52 = \underline{2.08}$

$$\text{Use of } \text{Var}(x) = a^2 \text{Var}(x)$$

M1

+ve variance

A1 ft

2

[11]

17. (a) Discrete uniform

B1

1

(b) $P(X=x) = \frac{1}{6}, x = 1, 2, \dots, 6$

$$\therefore E(X) = \sum x P(X=x) = \frac{1}{6} + \frac{2}{6} + \dots + \frac{6}{6} = \frac{21}{6} = 3.5$$

B1

$$\text{or } E(X) = \frac{k+1}{2} = \frac{7}{2} = 3.5$$

$$\text{Var}(X) = \sum x^2 P(X=x) - \{E(X)\}^2$$

M1

$$= \frac{1}{6} + \frac{4}{6} + \dots + \frac{36}{6} - \left(\frac{21}{6}\right)^2$$

$$= \frac{105}{6} = \frac{35}{2} = 2\frac{11}{2} = 2.91\dot{6}$$

A1

3

$$\frac{35}{12}, 2\frac{11}{12}, 2.92$$

$$\text{Or } \text{Var}(X) = \frac{k^2 - 1}{12} = \frac{36 - 1}{12} = \frac{35}{12} \text{ etc}$$

(c) $P(\text{three 6s}) = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$

M1 A1

2

(d) $16 \Rightarrow (6, 5, 5); (5, 6, 5); (5, 5, 6)$
 $(6, 6, 4); (6, 4, 6); (4, 6, 6)$

B1 B1

B1 B1

4

$$(e) \quad P(16) = \frac{6}{216} = \frac{1}{36}$$

M1 A1 2

[12]

$$18. \quad (a) \quad 2k + k + 0 + k = 1$$

$$\therefore 4k = 1 \Rightarrow k = 0.25 (*)$$

M1
A1 2

(b)

x	0	1	2	3
$P(X=x)$	0.5	0.25	0	0.25
$xP(X=x)$	0	0.25	0	0.75
$x^2P(X=x)$	0	0.25	0	2.75

$$E(X) = \sum xP(X=x) = 0 + 0.25 + 0 + 0.75 = 1$$

M1 A1

$$E(X^2) = 0 + 0.25 + 0 + 2.25 = 2.5 (*)$$

M1 A1 4

$$(c) \quad \text{Var}(3X-2) = 3^2 \text{Var}(X)$$

$$= 9(2.5 - 1^2) = 13.5$$

M1
M1 A1 3

$$(d) \quad P(X_1 + X_2) = P(X_1 = 3 \cap X_2 = 2) + P(X_1 = 2 \cap X_2 = 3) = 0 + 0 = 0$$

B1 1

$$(e) \quad \text{Let } Y = X_1 + X_2$$

y	0	1	2	3	4	5	6
$P(Y=y)$	0.25	0.25	0.0625	0.25	0.125	0.0625	0.0625

B1
B2 3

$$(f) \quad P(1.3 \leq X_1 + X_2 \leq 3.2) = P(X_1 + X_2 = 2) + P(X_1 + X_2 = 3)$$

$$= 0.0625 + 0.25 = 0.3125$$

M1
A1 ft, A1 ft 3**[16]**

$$19. \quad (a) \quad P(\text{correct at third attempt}) = 0.4 \times 0.4 \times 0.6$$

$$= 0.096$$

M1
A1 2

(b)

a	1	2	3	4
$P(A=a)$	0.6	0.24	0.096	0.064

$$a = 1, 2, 3, 4$$

B1

$$\text{All } P(A=a) \text{ correct}$$

B1 2

$$(c) \quad P(\text{correct number}) = 1 - (0.4)^4$$

$$= 0.9744$$

M1

$$(\text{accept awrt } 0.974)$$

A12

(d) $E(A) = \sum a P(A = a) = (1 \times 0.6) + \dots + (4 \times 0.064)$	M1	
$= 1.624$		
<i>(accept awrt 1.62)</i>	A1	
$E(A^2) = \sum a^2 P(A = a) = (1^2 \times 0.6) + \dots + (4^2 \times 0.064)$	M1	
$= 3.448$	A1	
$\therefore \text{Var}(A) = 3.448 - (1.624)^2$	M1	
$= 0.810624$		
<i>(accept awrt 0.811)</i>	A16	
$F(1 + E(A)) = P(A \leq 1 + E(A))$		
$= P(A \leq 2.624)$	M1	
$= 0.84$	A1	2

[14]