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Level by Level

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Standard form

5.36×10^4 ↔ Means move the decimal point 4 places to the right

5 . 3 6 0 0 . = 53 600

8.31×10^{-3} ↔ Means move the decimal point 3 places to the left

0 . 0 0 8 . 3 1 = 0.00831

Questions

1 Write 8.4×10^3 as an ordinary number.

2 Write 3.24×10^{-2} as an ordinary number.

3 Write 3820 in standard form.

4 Write 0.00236 in standard form.

Answers

1 8 . 4 0 0 . = 8400

2 0 . 0 3 . 2 4 = 0.0324

3 **Note:** In standard form the decimal point is always placed after the first whole number.

3 . 8 2 0 The decimal point has moved 3 places to the left.

We write the number in standard form as 3.82×10^3.

4 0 . 0 0 2 . 3 6 The decimal point has moved 3 places to the right.

We write the number in standard form as 2.36×10^{-3}.

Standard form

Exercises

1 Write the following in standard form:

a	387	b	4360	c	251.8
d	683000	e	29000	f	0.036
g	0.78	h	0.000423	i	0.0507
j	862000	k	5370	l	0.00082
m	0.003	n	8000	o	0.306

2 Write the following as ordinary numbers:

a	5.27×10^3	b	6.25×10^4	c	1.36×10^6
d	4.2×10^4	e	3×10^5	f	4.2×10^{-2}
g	6.8×10^{-1}	h	8.32×10^{-4}	i	6.82×10^{-3}
j	7×10^{-3}	k	6.2×10^6	l	4.8×10^{-5}
m	3.82×10^4	n	6.27×10^{-3}	o	5.82×10^4

3 Convert the following into the form $z \times 10^n$. Give the value of:

(i)	z	(ii)	n		
a	5800	b	47300	c	624
d	30700	e	480000	f	36820
g	527.3	h	124.62	i	0.03
j	0.0000072	k	0.00301	l	0.0042
m	0.0036	n	0.0721	o	0.0000038

Using a calculator for powers, roots and standard form

Power keys [x^y] or [y^x]

Root keys [$x^{\frac{1}{y}}$] or [$\sqrt[x]{}$]

Standard form key [EXP]

What is the value of 5^4?

Calculator keys [5] [x^y] [4] [=] Answer: 625

What is the value of 2^{-3}?

Calculator keys [2] [x^y] [3] [+/−] [=] Answer: 0.125

What is the value of $\sqrt[5]{32}$?

Calculator keys [3] [2] [$x^{\frac{1}{y}}$] [5] [=] Answer: 2

Calculate $3.82 \times 10^4 \times 4.26 \times 10^6$

Calculator keys

[3] [.] [8] [2] [EXP] [4] [x] [4] [.] [2] [6] [EXP] [6] [=]

The calculator display shows 1.62732^{11}

This means 1.62732×10^{11}

Questions

Calculate the value of:

1 8^3 2 $\sqrt[4]{16}$ 3 $7.3 \times 10^8 \div 6.4 \times 10^{-7}$

Answers

1 [8] [x^y] [3] [=] 512

2 [1] [6] [$x^{\frac{1}{y}}$] [4] [=] 2

3 [7] [.] [3] [EXP] [8] [÷] [6] [.] [4] [EXP] [7] [+/−] [=] 1.140625×10^{15}

Using a calculator for powers, roots and standard form

Exercises

Calculate the value of the following:

1	6^3	2	5^5	3	2^9
4	4^5	5	3^6	6	10^4
7	16^{-2}	8	8^{-1}	9	2^{-5}
10	$(1/8)^{-4}$	11	$(1/5)^{-3}$	12	$(2/7)^{-2}$
13	$\sqrt[3]{27}$	14	$\sqrt[4]{16}$	15	$\sqrt[4]{256}$
16	$\sqrt[8]{256}$	17	$\sqrt[10]{1024}$	18	$\sqrt[5]{243}$
19	15^3	20	12^4	21	$(1/2)^3$
22	$(0.2)^4$	23	$(1/8)^{-1/3}$	24	$(1/256)^{-1/4}$

Calculate the value of the following. (Write your answer in standard form to three significant figures.)

25 $6.3 \times 10^4 + 8.4 \times 10^3$

26 $5.8 \times 10^3 \div 4.6 \times 10^5$

27 $6.8 \times 10^{-4} \times 4.3 \times 10^{-3}$

28 $5.7 \times 10^5 \times 3.6 \times 10^{-8}$

29 $$\frac{4.8 \times 10^4 \times 3.6 \times 10^5}{3.1 \times 10^3 \times 5.2 \times 10^2}$$

30 $$\frac{2.93 \times 10^4 \times 6.8 \times 10^3}{9.82 \times 10^4 + 3.6 \times 10^2}$$

Proportional change

To calculate a 6% increase, multiply by 1.06 [ie 1 + 0.06]

To calculate a 12% increase, multiply by 1.12 [ie 1 + 0.12]

To calculate a 6% decrease, multiply by 0.94 [ie 1 - 0.06]

To calculate a 12% decrease, multiply by 0.88 [ie 1 - 0.12]

Examples

1 A man earns £12 000 per annum. He receives a 4% increase each year. How much does he earn after five years?

Method: $12\,000 \times 1.04^5 = £14\,599.83$

2 A television costs £200 + 17.5% VAT. What is the total cost?

$200 \times 1.175 = £235$

Now try this:

3 A television costs £235 including 17.5% VAT. Calculate the cost before VAT was added.

£235 is 117.5%. We need to find 100%.

It is example 2 reversed.

$235 \div 1.175 = £200$

Question

A car is bought for £15 000. It depreciates by 10% each year.

How much is it worth after eight years? (Give your answer to the nearest £.)

Answer

$15\,000 \times 0.90^8 = £6457$

Proportional change

Exercises

1 A man earns £20 000 per annum. He receives a 6% increase each year.
How much does he earn after eight years?
How many years does it take for his salary to double?

2 A house is bought for £80 000. Its value increases by 3% per annum.
   a How much is the house worth in four years?
   b How many years will it take for the value to reach £100 000?

3 There are 2000 elephants in a national park. The numbers are decreasing by 5% per annum.
   a How many elephants will be in the park in six years?
   b When the number of elephants falls below 800 the elephant will be declared an endangered species. How many years will this take?

4 A meal, including 17.5% VAT cost £35.25. How much was the VAT?

5 A woman received an 8% pay rise. Her new pay was £345.60.
How much was her original wage?

6 The value of a car fell by 18% after one year. Its value at the end of the first year was £9840.
   a What was the original value of the car?
   b The value of the car continues to fall by 18% each year. What is its value after six years?
   c When the value of the car falls below £400 it is sold for scrap. How many years is the car used before it is sold for scrap?

7 A man invests £4000 at 6% per annum compound interest. How many years does it take for the sum of money to double in value?

8 The value of a car decreases by ⅛ each year. Its original value is £8000.
How many years does it take for its value to fall below £3000?

Using algebraic formulae 1

$5a^2$ means 5 x a x a

$(5a)^2$ means 5 x a x 5 x a

$\sqrt{\left(\frac{8.4}{1.6}\right)}$ First find the value of 8.4 ÷ 1.6 = 5.25
Then find √5.25 = 2.29 (approx)

Brackets

Always work out the brackets first.

Example

a = 4 b = 5 c = 7

$3a^2 (b^2 + c)$

$(b^2 + c) \longrightarrow (5^2 + 7) \longrightarrow 32$

$3a^2 (b^2+c) = 3 \times 4^2 \times 32 = 1536$

Question

Given a = 8.4 b = 3.2

Find the value of $\sqrt{\left(\frac{a^2+b^2+2(a+b)}{3\,(a - b)}\right)}$

Answer

$a^2 + b^2 + 2\,(a + b)$ means $8.4^2 + 3.2^2 + 2 \times (8.4 + 3.2)$

$70.56 + 10.24 + 23.2 = 104$

3(a - b) means 3 x (8.4 - 3.2) = 15.6

104 ÷ 15.6 = 6.6667

√6.6667 = 2.58 (approx)

Using algebraic formulae 1

Exercises

Given $a = \frac{3}{8}$, $b = \frac{1}{5}$, $c = 3.27$, $d = -1.74$, $e = -0.3$

Evaluate the following:

1 $3(4a - 2b)$

2 $-6a(a^2 + ab)$

3 $\sqrt{\left(\frac{a}{b}\right)}$

4 $3\pi\sqrt{(2a)}$

5 $\frac{2a - 3b}{ab^2}$

6 $\frac{5a + 2b}{3(ab)^3}$

7 $3c^2d^3$

8 $\frac{2cd^2}{2c - d}$

9 $\frac{cde}{3}$

10 $\sqrt{(2de)}$

11 $\frac{c^2d}{\sqrt{c}}$

12 $\frac{4cd}{3\sqrt{(d/e)}}$

13 $\frac{c^2 + d^3}{3cd - d^2}$

14 $\frac{(c - d)(c + d)}{3c(d - e)}$

15 $\sqrt{\frac{a + b}{3(c - d)}}$

16 $\frac{3a\sqrt{(c^3 - d)}}{5cde}$

17 $\frac{(a - b)^2}{(c + d)^2 - 2d}$

18 $\frac{a - b}{c + d} - 3\pi\sqrt{(a + 8b)}$

19 $\sqrt{\left(\frac{a^2 + b^2 + c^2}{d^2 - e^2}\right)}$

20 $\left(\frac{3c - 2d}{3de}\right)^2$

Using algebraic formulae 2

Questions

1 Calculate the value of a, given b = 3.2, c = 5.4:

$a = \sqrt{((b + c)^2 + 2bc)}$

2 Calculate the value of w, given y = 3, z = 8:

$1/w = 1/y - 1/z$

3 Calculate the value of r, given v = 90, h = 6:

$v = 1/3\pi r^2 h$

Answers

1 $(b + c)^2 \rightarrow (3.2 + 5.4)^2 \rightarrow (8.6)^2 \rightarrow 73.96$

$2bc \rightarrow 2 \times 3.2 \times 5.4 \rightarrow 34.56$

$(b + c)^2 + 2bc \rightarrow 73.96 + 34.56 \rightarrow 108.52$

$\sqrt{(b + c)^2 + 2bc} \rightarrow \sqrt{108.52} \rightarrow 10.4$ (approx)

2 $1/w = 1/3 - 1/8$

$1/w = 5/24$

$w = 24/5 = 4.8$

3 $v = 1/3 \times \pi \times r^2 \times h$

$90 = 1/3 \times \pi \times r^2 \times 6$

$90 = 6.283185 \times r^2$

$$r^2 = \frac{90}{6.283185}$$

$r^2 = 14.3239$

$r = \sqrt{14.3239}$

$r = 3.78$ (approx)

Using algebraic formulae 2

Exercises

The volume of a cylinder is given by the formula $V = \pi r^2 h$.

Calculate V given:

1	r = 5 m	h = 8 m	2	r = 7 m	h = 12 m
3	r = 8 m	h = 6 m	4	r = 3¼ m	h = 2⅜ m
5	r = 3.5 m	h = 2.8 m	6	r = 7.4 m	h = 8.2 m

Calculate r given:

7	V = 120 m^3	h = 3 m	8	V = 400 m^3	h = 6m
9	V = 500 m^3	h = 8 m	10	V = 324 m^3	h = 5.4 m
11	V = 764 m^3	h = 7.6 m	12	V = 478 m^3	h = 3.8 m

The surface area of a cylinder is given by the formula $A = 2\pi r(r + h)$.

Calculate A given:

13	r = 3 m	h = 7 m	14	r = 7 m	h = 5 m
15	r = 5 m	h = 6 m	16	r = 3.8 m	h = 7 m
17	r = 2.6 m	h = 3.8 m	18	r = 8.4 m	h = 6.4 m

Calculate h given:

19	A = 160 m^2	r = 3 m	20	A = 300 m^2	r = 4 m
21	A = 200 m^2	r = 5 m	22	A = 1258 m^2	r = 7.6 m
23	A = 430 m^2	r = 5.4 m	24	A = 872 m^2	r = 9.7 m

The total surface area of a cone is given by the formula $A = \pi r(l + r)$.
l is calculated using the formula $l = \sqrt{(h^2 + r^2)}$.

Calculate A given:

25	r = 5 m	h = 7 m	26	r = 4 m	h = 3 m
27	r = 8 m	h = 5 m	28	r = 4.8 m	h = 3.6 m
29	r = 7.2 m	h = 12 m	30	r = 3.6 m	h = 5.2 m

31 Calculate the value of t in the formula $\frac{1}{t} = \frac{1}{r} - \frac{1}{s}$, given r = 7 and s = 12.

Transformation of formulae

The following questions show several useful techniques.
In each question make A the subject.

Questions

1 $\sqrt{A} = B$

2 $A^2 = B$

3 $3C\sqrt{A} = B$

4 $C = B + A$

5 $C = B - A$

6 $C = \frac{A}{B}$

7 $C = \frac{B}{A}$

8 $AB + C = D$

9 $3B = \frac{Y}{2A} - 7$

10 $3B = \frac{Y - 7}{2A}$

11 $4A - B = 3AC + D$

12 $B = \frac{C}{D + A}$

Answers

1 $\sqrt{A} = B$

$A = B^2$

2 $A^2 = B$

$A = \sqrt{B}$

3 $3C\sqrt{A} = B$

$\sqrt{A} = \frac{B}{3C}$

$A = \left(\frac{B}{3C}\right)^2$

4 $C = B + A$

$B + A = C$

$A = C - B$

5 $C = B - A$

$C + A = B$

$A = B - C$

6 $C = \frac{A}{B}$

$\frac{A}{B} = C$

$A = BC$

7 $C = \frac{B}{A}$

$AC = B$

$A = \frac{B}{C}$

8 $AB + C = D$

$AB = D - C$

$A = \frac{D - C}{B}$

9 $3B = \frac{Y}{2A} - 7$

$3B + 7 = \frac{Y}{2A}$

$A(3B + 7) = \frac{Y}{2}$

$A = \frac{Y}{2(3B + 7)}$

10 $3B = \frac{Y - 7}{2A}$

$3AB = \frac{Y - 7}{2}$

$A = \frac{Y - 7}{2(3B)}$

$A = \frac{Y - 7}{6B}$

11 $4A - B = 3AC + D$

$4A - 3AC = B + D$

$A(4 - 3C) = B + D$

$A = \frac{B + D}{4 - 3C}$

12 $B = \frac{C}{D + A}$

$B(D + A) = C$

$D + A = \frac{C}{B}$

$A = \frac{C}{B} - D$

Transformation of formulae

Exercises

Make Y the subject.

1 $A = B + Y$

2 $3A = 2B - Y$

3 $4A = CB - 2Y$

4 $3C = \frac{Y}{D}$

5 $5A = \frac{C}{Y}$

6 $DE = \frac{CY}{B}$

7 $7C = \frac{D + Y}{AB}$

8 $B + C = A + \frac{Y}{2}$

9 $C = \frac{D + B}{Y}$

10 $A + 5C = 3Y - 7$

11 $3Y - 4 = C$

12 $4 - Y = \frac{A}{B}$

13 $ABC = \frac{3Y}{D}$

14 $B^2C = \frac{A + D}{Y}$

15 $b = \frac{c}{\sqrt{y}}$

16 $a = \sqrt{(y + c)}$

17 $a + b = y^2 - 3c$

18 $y^2 = a^2 + b^2$

19 $a - b = \sqrt{y} + c$

20 $d - 4 = \frac{c^2}{a^2+y^2}$

Equations

The following questions show several useful techniques. In each question find the value of y correct to three significant figures.

Questions

1 $3(y + 4) = 5(3y - 7)$

2 $5(y + 4) - 3(2y - 4) = 3(4y - 2)$

3 $\frac{3y}{7} = 5y + 6$

4 $\frac{7y}{4} + \frac{3y}{2} = 9$

5 $\frac{8}{y} = 7$

6 $\frac{5}{y+3} = 3$

7 $y^2 = 8$

8 $\sqrt{y} = 5.32$

Answers

1 $3(y + 4) = 5(3y - 7)$
$3y + 12 = 15y - 35$
$3y - 15y = -35 -12$
$-12y = -47$
$y = \frac{-47}{-12}$
$y = 3.92$

2 $5(y + 4) - 3(2y - 4) = 3(4y - 2)$
$5y + 20 - 6y + 12 = 12y - 6$
$5y - 6y - 12y = -6 -20 -12$
$-13y = -38$
$y = \frac{-38}{-13}$
$y = 2.92$

Note: -3 x -4 = 12

3 $\frac{3y}{7} = 5y + 6$
$3y = 7(5y + 6)$
$3y = 35y + 42$
$3y - 35y = 42$
$-32y = 42$
$y = \frac{42}{-32}$
$y = -1.31$

4 $\frac{7y}{4} + \frac{3y}{2} = 9$
What do we have on this bottom line? 4 x 2 = 8.
Therefore multiply everything by 8.
$\frac{^{2}8(7y)}{\cancel{4}} + \frac{^{4}8(3y)}{\cancel{2}} = 8(9)$
$2(7y) + 4(3y) = 72$
$14y + 12y = 72$
$26y = 72$
$y = \frac{72}{26}$
$y = 2.77$

5 $\frac{8}{y} = 7$
$8 = 7y$
$\frac{8}{7} = y$
$1.14 = y$

6 $\frac{5}{y+3} = 3$
$5 = 3(y + 3)$
$5 = 3y + 9$
$5 - 9 = 3y$
$-4 = 3y$
$\frac{-4}{3} = y$
$-1.33 = y$

7 $y^2 = 8$
$y = \sqrt{8}$
$y = 2.83$

8 $\sqrt{y} = 5.32$
$y = 5.32^2$
$y = 28.3$

Equations

Exercises

Find the value of a in each equation:

1 $\frac{4a}{5} = 2$

2 $\frac{a}{6} = -2$

3 $5 - 3a = 7a + 12$

4 $\sqrt{a} = 9$

5 $3 + \sqrt{a} = 9$

6 $a^3 = 64$

7 $6a - 4 = 2a + 12$

8 $\frac{6}{a} = 12$

Find the value of a correct to three significant figures:

9 $3a - 5 = 4(5a - 3)$

10 $2(a - 4) = 3(5a - 7)$

11 $\frac{2a+3}{5} = 7a$

12 $\frac{6a+3}{2} = 7$

13 $\frac{a}{5} + \frac{2a}{7} = 6$

14 $\frac{5a}{4} - \frac{2a}{3} = 4$

15 $a^2 - 8 = 5$

16 $\sqrt{a} = 7.24$

17 $5(a + 2) - 3(a - 2) = 6$

18 $4(a + 2) - 3(a + 5) = 7$

19 $4(3a + 2) + 5(2a - 3) = 0$

20 $3(a - 4) - 4(3a + 6) = 10$

21 $\frac{6}{a+2} = 5$

22 $\frac{5}{a} = \frac{3}{2a} + 4$

Expansion of brackets

Questions

Expand the following:

1 $5(2a - 3)$

2 $3a(5 - 6a)$

3 $4y^6(2y^3 + 4y^2)$

4 $-3(a^3 + 2y^2)$

5 $4a^3b^2cd^2(3ab^4 - 6ac^3d)$

6 $(3a + 2)(5a - 3)$

7 $(6a - 7)(4a - 3)$

8 $(4y - 3)(7y + 6)$

9 $(6a - 4)^2$

Answers

1 $5(2a - 3)$
$10a - 15$

2 $3a(5 - 6a)$
$15a - 18a^2$

3 $4y^6(2y^3 + 4y^2)$
$8y^9 + 16y^8$

Note: Indices are added – $y^6 + y^3 = y^9$.

4 $-3(a^3 + 2y^2)$
$-3a^3 - 6y^2$

5 $4a^3b^2cd^2(3ab^4 - 6ac^3d)$
$12a^4b^6cd^2 - 24a^4b^2c^4d^3$

6 $(3a + 2)(5a - 3)$
$3a(5a - 3) + 2(5a - 3)$
$15a^2 - 9a + 10a - 6$
$15a^2 + a - 6$

7 $(6a - 7)(4a - 3)$
$6a(4a - 3) - 7(4a - 3)$
$24a^2 - 18a - 28a + 21$
$24a^2 - 46a + 21$

Note: $-7 \times -3 = 21$

8 $(4y - 3)(7y + 6)$
$4y(7y + 6) - 3(7y + 6)$
$28y^2 + 24y - 21y - 18$
$28y^2 + 3y - 18$

Note: $-3 \times 6 = -18$

9 $(6a - 4)^2$
This means
$(6a - 4)(6a - 4)$
$6a(6a - 4) - 4(6a - 4)$
$36a^2 - 24a - 24a + 16$
$36a^2 - 48a + 16$

Expansion of brackets

Exercises

Expand and simplify the following expressions:

1 $4(3a + 5)$

2 $7(6x - 3y)$

3 $a(7a - 3)$

4 $5a(4a - 2)$

5 $3y^2 (2y + 6)$

6 $5x^3 (3x^2 + 2x + 4)$

7 $2a^2b(3a - 2b)$

8 $5y^6 (3x^2 - 2y^4)$

9 $4(3x + 6) + 2(5x - 3)$

10 $7(4a - 2) - (3a - 4)$

11 $7(2a - 7) - 3(5a + 6)$

12 $6a(3a - 2) + 4a (3a^2 - a)$

13 $5y^2z (3y - 2yz)$

14 $(a + 3)(a + 2)$

15 $(a - 6)(a + 2)$

16 $(a - 5)(a - 3)$

17 $(4a + 6y)(3a - 5y)$

18 $(3y - 2a)(4y + 3a)$

19 $(7a - 3z)(4a - 6z)$

20 $5abc (3c^2 - 2abc)$

21 $7(3c - 4d) + 5d$

22 $6(7a + 3d) - (a-d)$

23 $a^2 - b^2 (3a)$

24 $x^2 - 9x (3x + 2)$

25 $(a + 7)^2$

26 $(x - 3)^2$

27 $(3a + 4)^2$

28 $(5a - 2)^2$

29 $(5a - 3y)^2$

30 $(4c + 2a)^2$

Factorisation

You must do expansion of brackets before you do factorisation. Factorisation is the reverse operation to expansion of brackets.

Example

Expand 4(2a + 1)
8a + 4

Factorise 8a + 4
4(2a + 1)

> Factorising means finding common factors
> 6a + 15
>
> 3 is a factor of 6 and 15
> 3(2a + 5)

Questions

Factorise:

1 $30c^2 - 12c$

2 $15c^2d + 20c^5d^4$

3 $6a^2bc^3 + 4a^4b^2d$

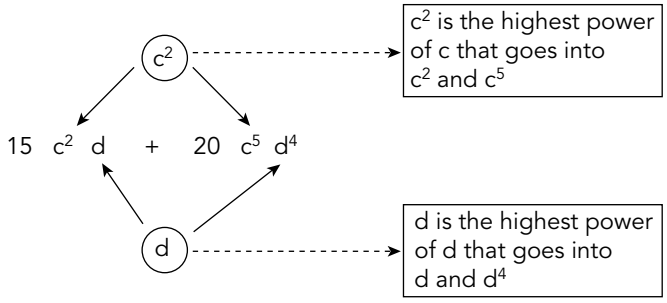
Answers

1 6 is the highest number that goes into 30 and 12 (ie highest factor)

c is the highest power of c that goes into c^2 and c

6c (5c - 2)

2



$5c^2d(3 + 4c^3d^3)$

3 2 is the highest number that goes into 6 and 4

a^2 is the highest power of a that goes into a^2 and a^4

b is the highest power of b that goes into b and b^2

$2a^2b(3c^3 + 2a^2bd)$

Factorisation

Exercises

Factorise:

1 $5a + 10$

2 $3y + 6$

3 $12y - 3$

4 $6x - 4$

5 $7a + 21$

6 $4a - 8$

7 $15a + 25$

8 $16a^2 + 8a + 4y$

9 $3y^2 + 6y + 12x$

10 $4y^3 + 6a + 8c$

11 $3a + 6b + 9c$

12 $12a + 15b - 18c$

13 $20x - 15y + 10$

14 $6a - 12b - 6c$

15 $7a^2 + 5a$

16 $12a^3 - 7a^2$

17 $5y - 7y^2$

18 $10a^2 - 12a$

19 $6ab - 3abc$

20 $7ab - 14bc$

21 $12ab + 8bc + 4bd$

22 $12a^3c^2 - 8a^2b$

23 $15a^4 + 10a^3 - 5a^2$

24 $15a^3b^2 + 10a^5b^3c^2$

25 $12a^3c - 8a^2c + 18ac$

26 $18a^3b^6c^3 - 12a^5bd^3$

27 $20a^6b^4c + 15a^4bcd$

28 $15abc + 10abcd$

29 $18x^3yz + 12xy^3$

30 $12xy^3z^2 - 6xy^4z^5$

Inequalities

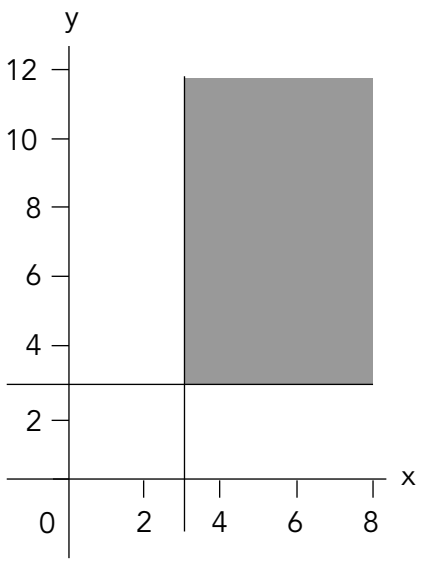
> means greater than
< means less than
≥ means greater than or equal to
≤ means less than or equal to

Note: The symbol always points to the smaller number.

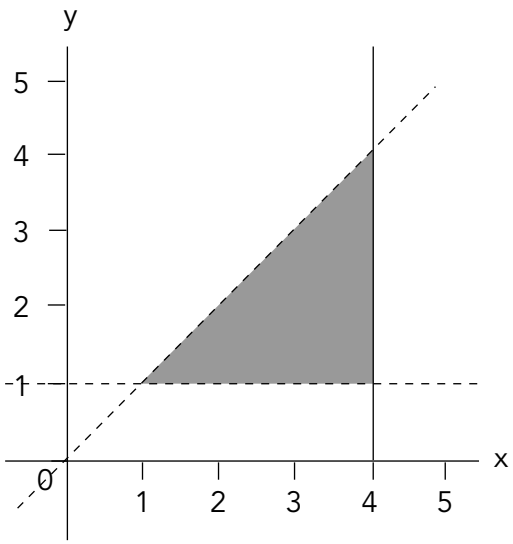
Questions

Describe each of the shaded regions:

1



2



Solve the inequalities:

3 $3 \leq 2x + 1 < 13$

4 $x^2 \geq 16$

Answers

1 $x \geq 3$ (if the line is solid it includes 'equal to')
$y \geq 3$

2 $x \leq 4$
$y > 1$ (if the line is dotted it does **not** include 'equal to')
$x > y$

3 Solve as an equation.

Subtract 1 from everything: $3 - 1 \leq 2x + 1 - 1 < 13 - 1$
$2 \leq 2x < 12$
Divide everything by 2: $1 \leq x < 6$

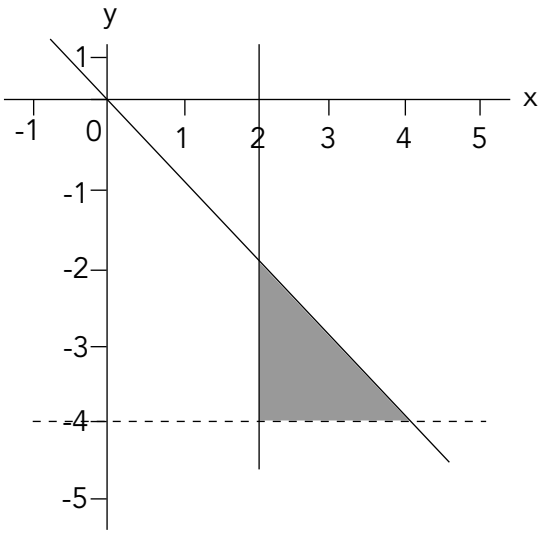
4 Remember, if $x^2 = 16$ then x can equal 4 or -4
$4 \times 4 = 16$, $-4 \times -4 = 16$
$x \geq 4$, $x \leq -4$

Inequalities

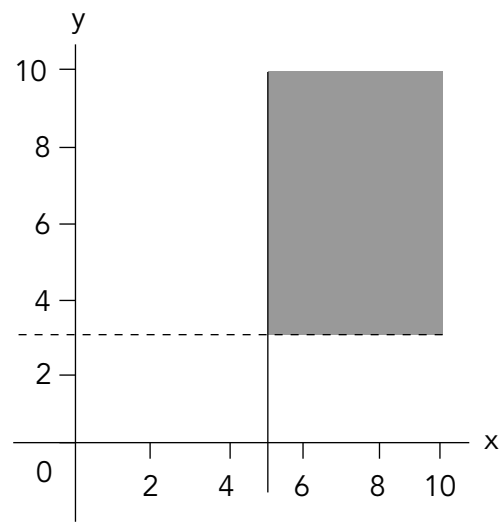
Exercises

Describe each of the shaded regions:

1



2



Solve the following inequalities:

3 $x^2 \geq 49$

4 $x^2 < 100$

5 $3x + 5 > 11$

6 $5x - 2 \leq 18$

7 $8 < 3x + 2 \leq 17$

8 $17 \leq 5x - 3 \leq 37$

9 $-11 \leq 2x - 3 \leq 7$

10 $-19 < 3x - 1 < -7$

11 a and b are both integers. Find all of the possible values of b.

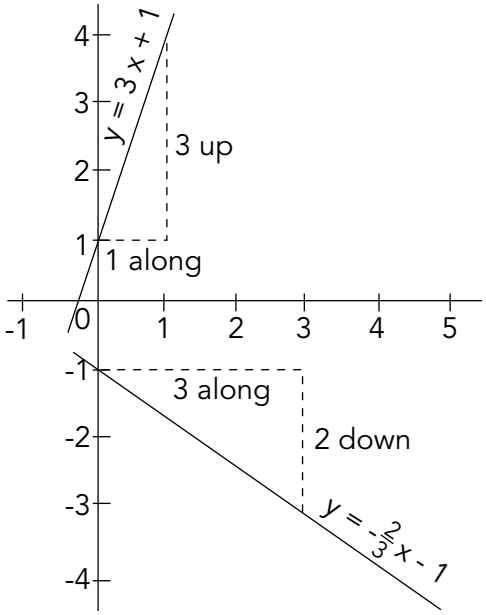
$a \geq 3$

$b < 11$

$b \geq 2a - 1$

Using the straight line equation y = mx + c

Examples



1 $y = \underset{m}{3}x + \underset{c}{1}$

$c = +1$ means that the line crosses the y axis at +1

$m = 3 = \frac{3}{1} \rightarrow \frac{\text{3 up the y axis}}{\text{1 along the x axis}}$

2 $y = \underset{m}{-\frac{2}{3}}x \underset{c}{-1}$

$c = -1$ means that the line crosses the y axis at -1

$m = -\frac{2}{3} \rightarrow \frac{\text{2 down the y axis}}{\text{3 along the x axis}}$

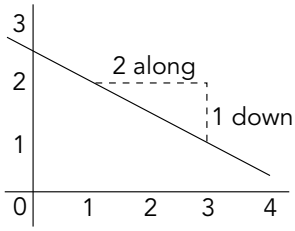
Questions

What is the equation of the line which passes through the points (1,2) and (3,1)?

Answer

Mark the points (1,2) and (3,1).

Draw a straight line through the points.



The equation of the line is $y = mx + c$ $\quad m = -\frac{1}{2}$ (down means -ve)

$c = 2.5$ $\quad y = -\frac{1}{2}x + 2.5$

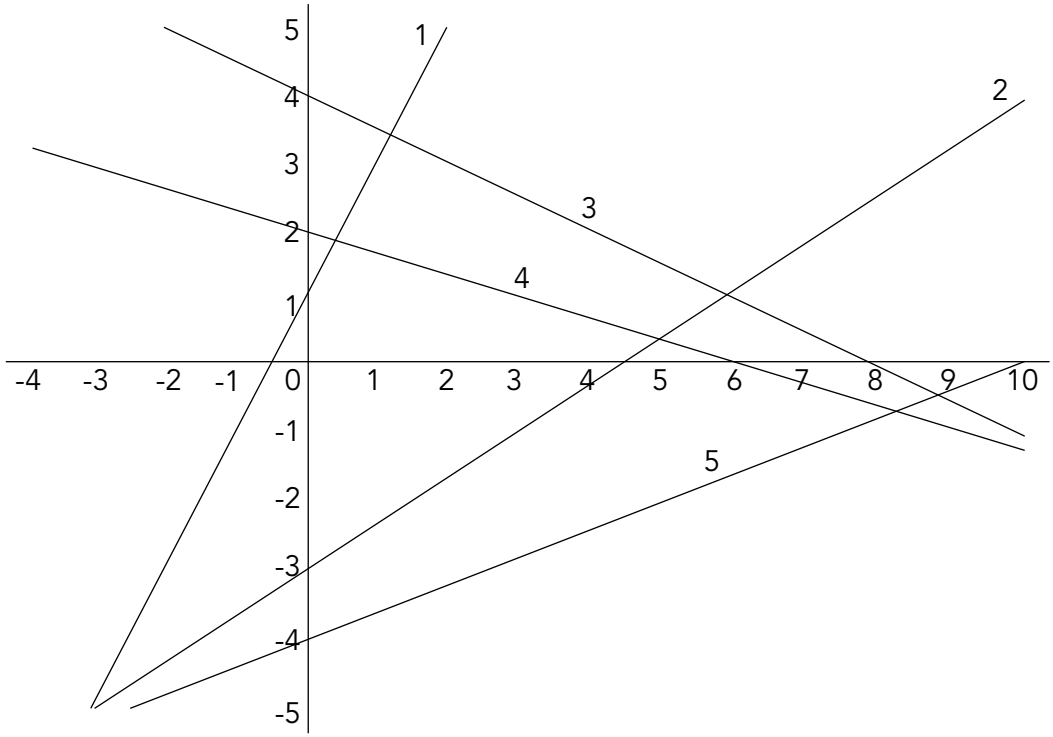
Note: If the line slopes up the gradient (m) is positive

If the line slopes down the gradient (m) is negative

Using the straight line equation y = mx + c

Exercises

Write down the equations of the lines shown:



6 a What is the equation of the line which passes through the points (-2,-2) and (4,1)?
b What is the gradient of the line?

7 a What is the equation of the line which passes through the points (-1,-3) and (1,3)?
b What is the gradient of the line?

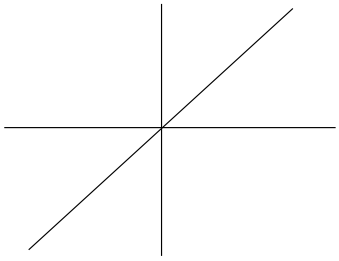
8 a What is the equation of the line which passes through the points (-3,5) and (3,1)?
b What is the gradient of the line?

Graphs

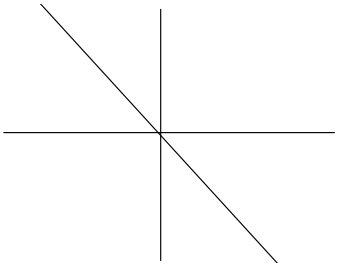
You should recognise these graphs.

Linear graphs such as $y = 3x + 6$, $y = -\frac{1}{2}x + 2$, etc

$y = x$

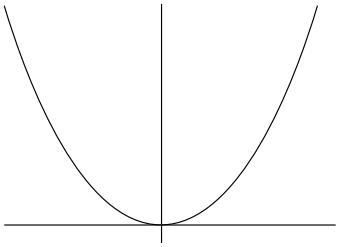


$y = -x$

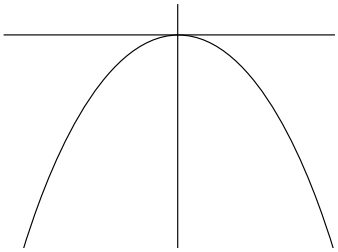


Quadratic graphs such as $y = 2x^2 + 3x - 6$, $y = -3x^2 + x + 4$, etc

$y = x^2$

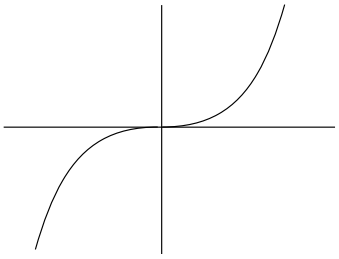


$y = -x^2$

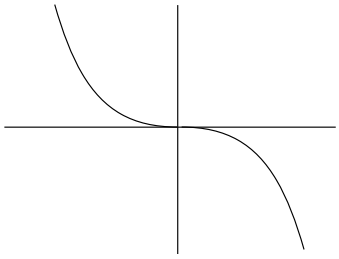


Cubic graphs such as $y = x^3 + 2x^2 + x - 1$, $y = -3x^3 + x^2 - 4$, etc

$y = x^3$

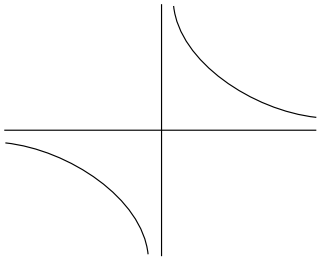


$y = -x^3$

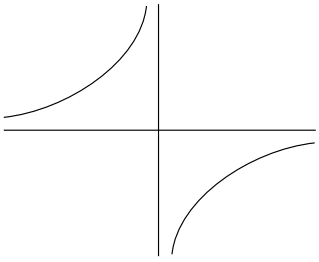


Reciprocal graphs such as $y = \frac{3}{2x}$, $y = -\frac{4}{x}$, etc

$y = \frac{1}{x}$



$y = -\frac{1}{x}$



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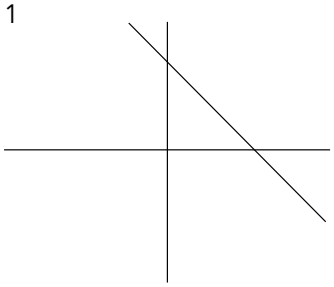
Graphs

Exercises

Label the following graphs. Choose from:

$y = x + 1$	$y = -x + 3$	$y = x^2 + 2$	$y = -x^2 + 2$
$y = x^3$	$y = -x^3$	$y = {}^1/_x$	$y = -{}^1/_x$

1



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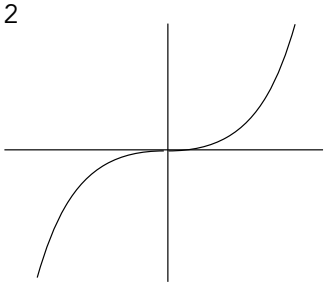
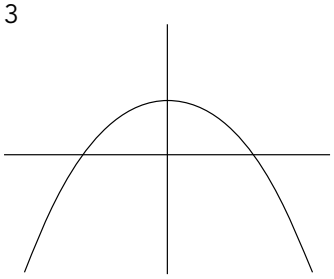
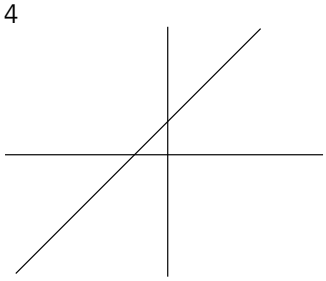


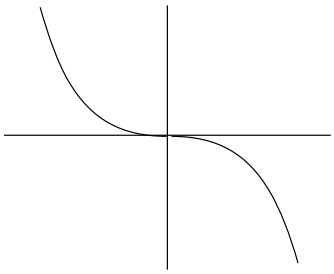
3



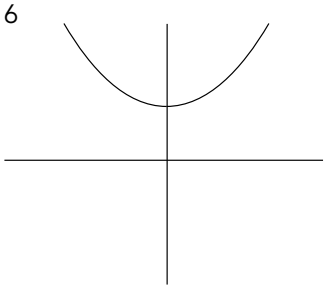
4



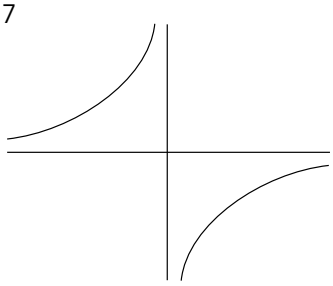
5



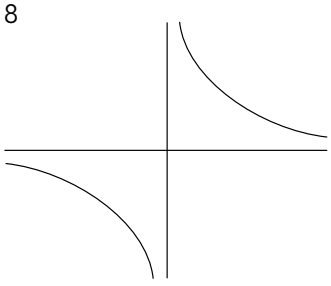
6



7



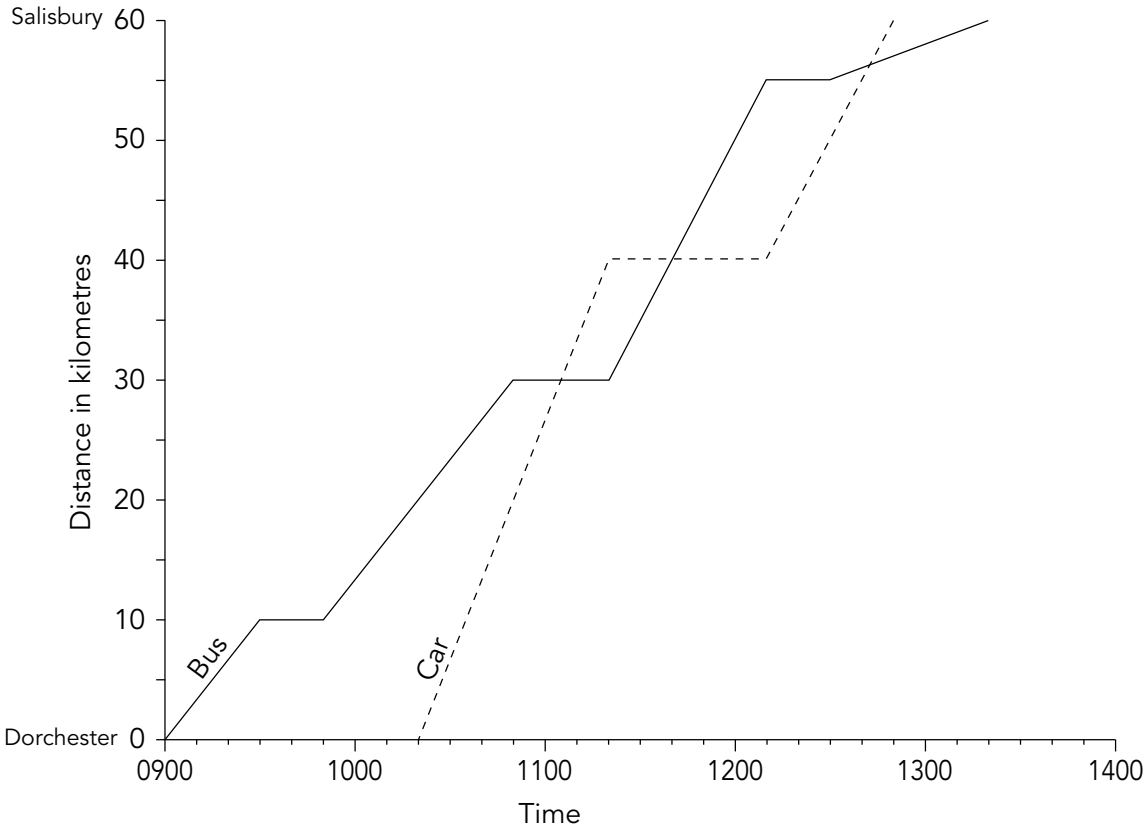
8



Graphs

Exercises

This graph shows the journeys made by a car and a bus. Both vehicles travelled from Dorchester to Salisbury.



1 What time did the bus leave Dorchester?

2 How many times did the bus stop?

3 How long was the first stop?

4 a Between which times did the bus travel fastest?

b How did you decide?

5 Describe what happened at 11.40.

6 How many times did the car pass the bus?

7 How long did the car stop for?

8 What was the speed of the car on the first part of its journey?

9 What was the speed of the bus at 10.20?

10 What was the time of arrival of the bus?

Graphs

Exercises

Complete the tables and draw the graphs of the following functions for $-3 \leq x \leq 3$:

1 $y = 3x^2 - 2x - 5$

x	-3	-2	-1	0	1	2	3
$3x^2$							
$-2x$							
-5							
y							

2 $y = \frac{1}{3}x^3 + x^2 - 4x - 1$

x	-3	-2	-1	0	1	2	3
$\frac{1}{3}x^3$							
x^2							
$-4x$							
-1							
y							

3 $y = \frac{12}{x}$

x	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2
y								

What happens when $x = 0$?

4 $y = x^3 - x^2$

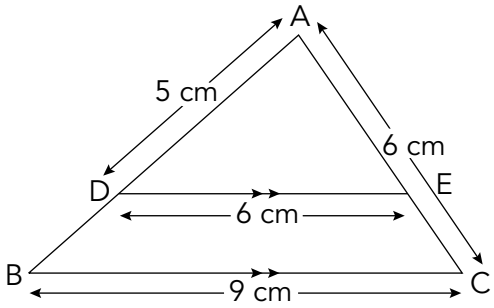
x	-3	-2	-1	0	1	2	3
x^3							
$-x^2$							
y							

Similarity

Two shapes are similar if the angles of one shape are equal to the angles of the other shape.

Question

Find the length of AB and AE.

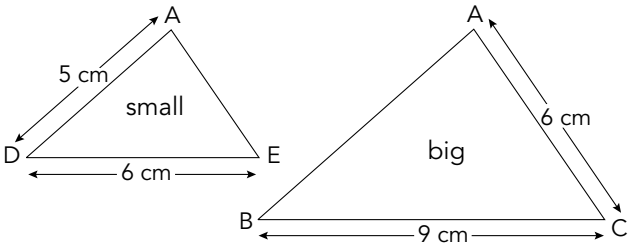


Answer

DE is parallel to BC. Therefore ADE is similar to ABC.

1 Draw the two triangles separately.

2 Identify the big triangle and the small triangle.



3 Find two sides which are in the same position on each triangle. In this example DE and BC.

DE = 6 cm, BC = 9 cm

4 The scale factor (SF) from small to big is $\frac{9}{6}$ $\left(\frac{\text{big number}}{\text{small number}}\right)$.

To convert any length on the small triangle to a length on the large triangle, multiply by SF $\frac{9}{6}$.

eg AD (small triangle) x SF $\frac{9}{6}$ = AB

5 cm x $\frac{9}{6}$ = 7.5 cm

5 The scale factor from big to small is $\frac{6}{9}$ $\left(\frac{\text{small number}}{\text{big number}}\right)$.

To convert any length on the large triangle to a length on the small triangle, multiply by SF $\frac{6}{9}$.

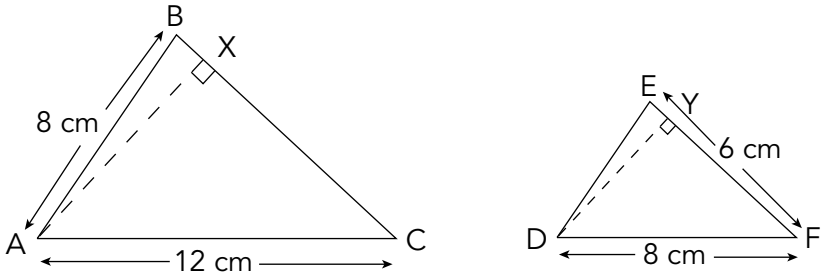
eg AC (large triangle) x SF $\frac{6}{9}$ = AE

6 cm x $\frac{6}{9}$ = 4 cm

Similarity

Exercises

1 The triangles ABC and DEF are similar.



a What is the length of BC?

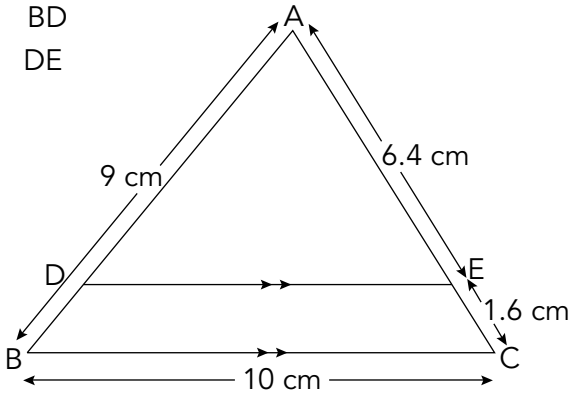
b What is the length of DE?

c What is the ratio of the length AX to the length DY?

2 Find a AD

b BD

c DE



AB = 9 cm

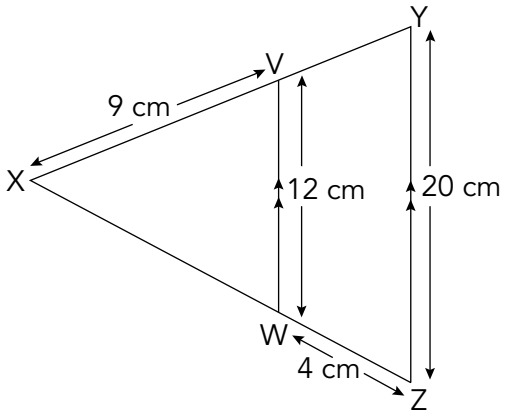
AE = 6.4 cm

BC = 10 cm

CE = 1.6 cm

d What is the ratio of length BC to length DE?

3



YZ = 20 cm

VW = 12 cm

WZ = 4 cm

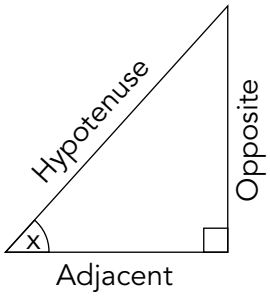
XV = 9 cm

Find a XZ

b XY

Trigonometry: Finding an angle

Information similar to this will be given on your examination paper.

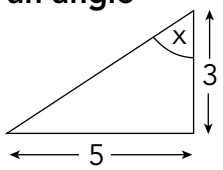


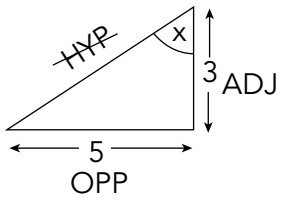
$$\text{SIN} = \frac{\text{OPP}}{\text{HYP}}$$

$$\text{COS} = \frac{\text{ADJ}}{\text{HYP}}$$

$$\text{TAN} = \frac{\text{OPP}}{\text{ADJ}}$$

To find an angle





Find x

Method

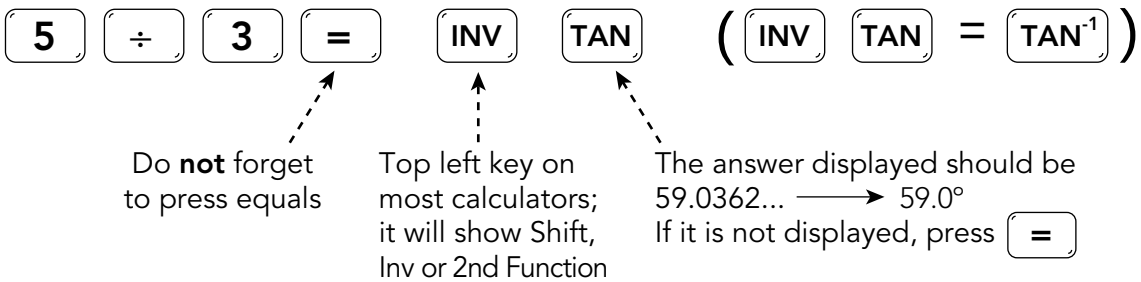
1 Label the triangle
Hypotenuse = the longest side, opposite the right angle
Opposite = opposite the angle being used
Adjacent = next to the angle being used

2 Cross out the side not being used.
In this question HYP.

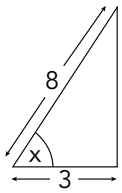
3 Look at the formulae in the box at the top.
Which uses OPP and ADJ?

4 $\text{TAN} = \frac{\text{OPP}}{\text{ADJ}} = \frac{5}{3}$

5 Calculator keys

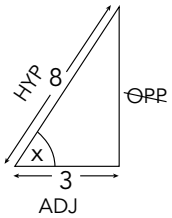


Question



Find x

Answer



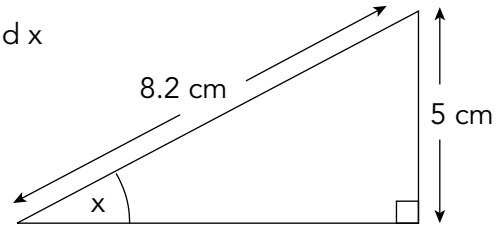
$$\text{COS} = \frac{\text{ADJ}}{\text{HYP}} = \frac{3}{8}$$

$$= 67.975687... \longrightarrow 68.0°$$

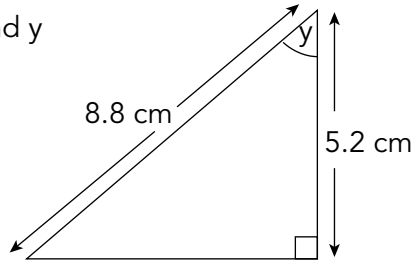
Trigonometry: Finding an angle

Exercises

1 Find x



2 Find y



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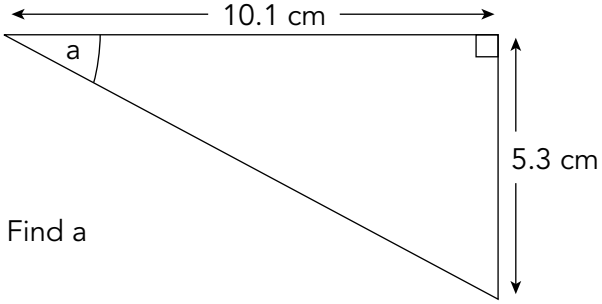
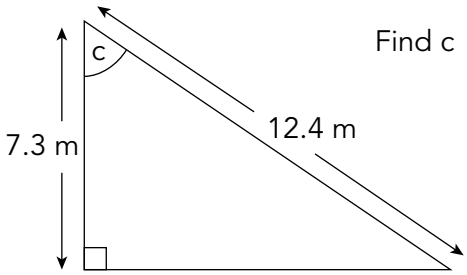


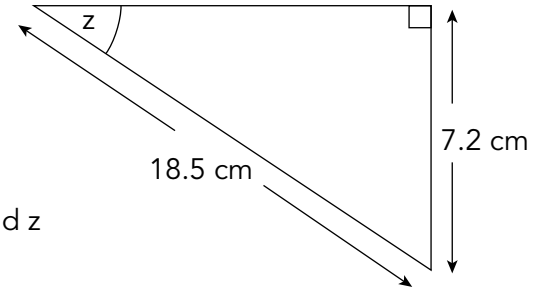
Find a

4



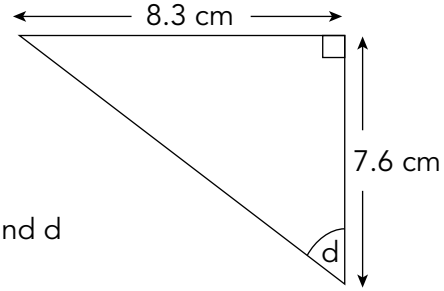
Find c

5



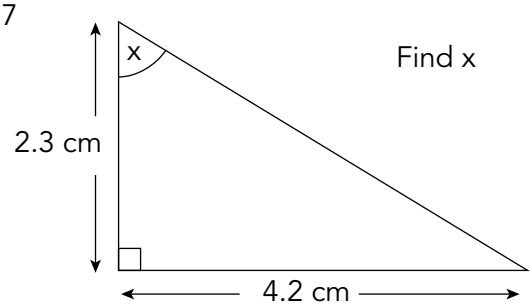
Find z

6



Find d

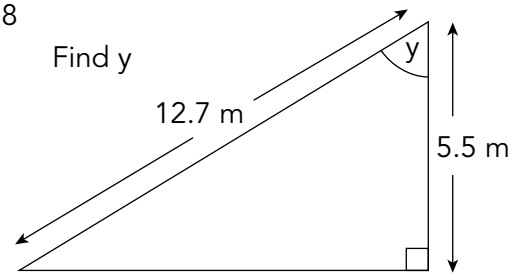
7



Find x

8

Find y



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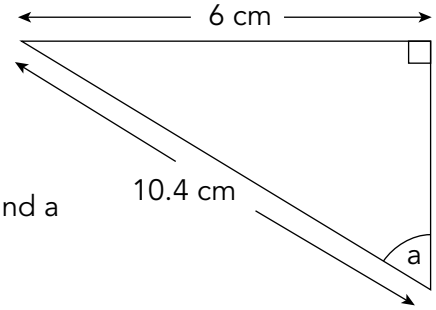
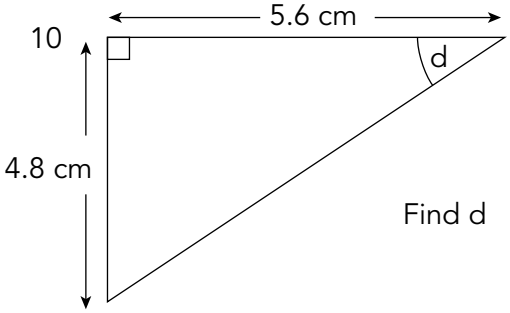


Find a

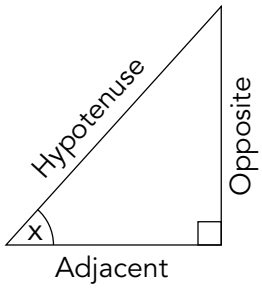
10



Find d

Trigonometry: Finding a side

Information similar to this will be given on your examination paper.



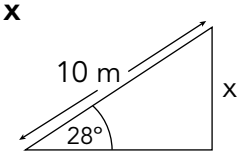
$$\text{SIN} = \frac{\text{OPP}}{\text{HYP}}$$

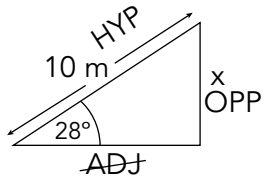
$$\text{COS} = \frac{\text{ADJ}}{\text{HYP}}$$

$$\text{TAN} = \frac{\text{OPP}}{\text{ADJ}}$$

To find a side

Find x





Method

1 Label the triangle
Hypotenuse = the longest side, opposite the right angle
Opposite = opposite the angle being used
Adjacent = next to the angle being used

2 You need the side you are finding (x).
You need the side you know (10 m).
Cross out the side not being used.
In this question ADJ.

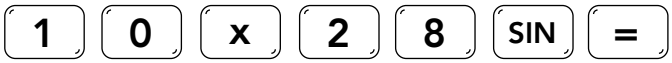
3 Look at the formulae in the box at the top. Which uses OPP and HYP?

4 $$\text{SIN} = \frac{\text{OPP}}{\text{HYP}}$$

$$\text{SIN}28° = \frac{x}{10}$$

$$10 \times \text{SIN}28° = x$$

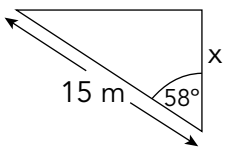
5 Calculator keys



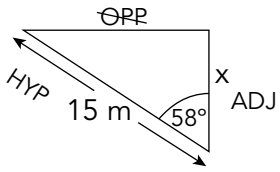
This should give you an answer 4.6947.. ⟶ 4.69 m

Note: If this does not work ask your teacher to show you how to work your calculator.

Question



Answer



$$\text{COS} = \frac{\text{ADJ}}{\text{HYP}}$$

$$\text{COS}58° = \frac{x}{15}$$

$$15 \times \text{COS}58° = x$$

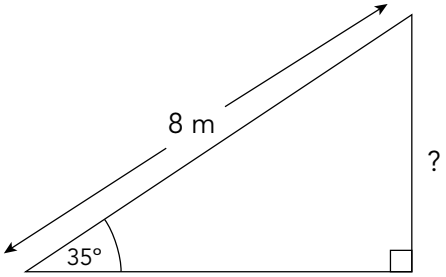
$$x = 7.95 \text{ m}$$

Trigonometry: Finding a side

Exercises

Find the indicated side:

1



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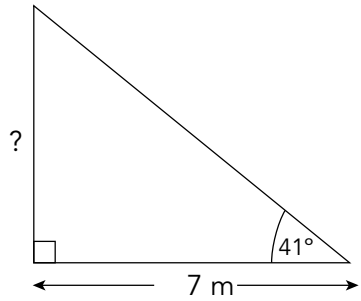

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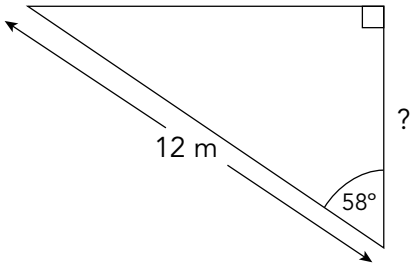

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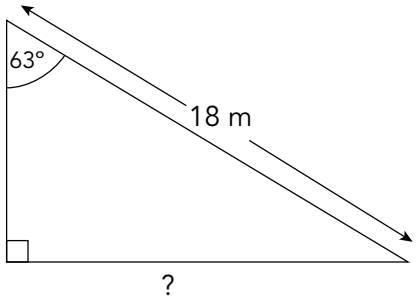

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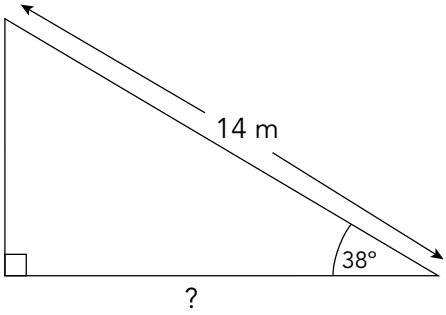

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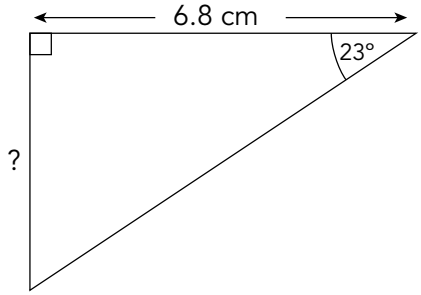

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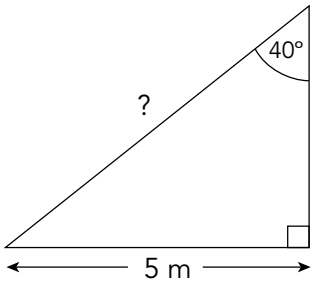

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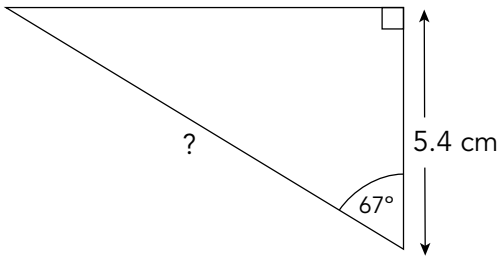

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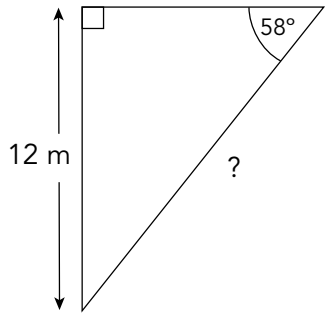

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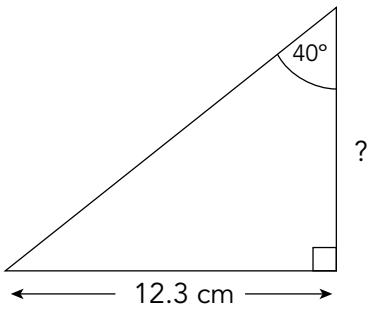
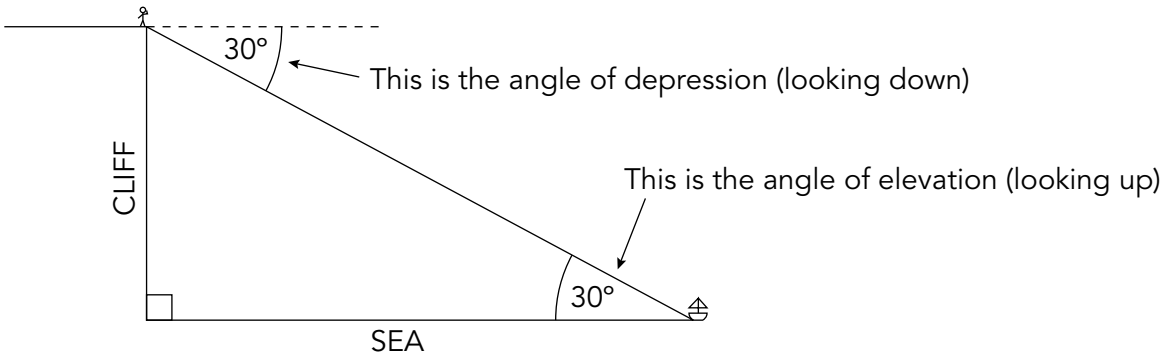


Trigonometry: Solving problems

Solving problems

This diagram shows a man at the top of a cliff looking down at a boat.



Note: The angle of depression from the top of the cliff is equal to the angle of elevation from the boat.

Angles of depression and angles of elevation are measured from the horizontal.

Answering questions

1 Read the question carefully.

2 It may help to visualise what is required. You can use objects such as pencils, rubbers, rulers to make a model of what is required.

3 Draw a diagram. Remember you need a right-angled triangle.

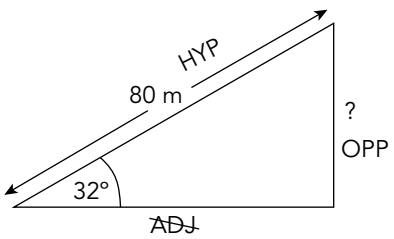
4 Read the question again. Check that your diagram is correct.

Question

Sarah is flying a kite. The string is 80 m long and the angle of elevation is 32°.
How high is the kite?

Answer

Draw a diagram.



$$\text{SIN} = \frac{\text{OPP}}{\text{HYP}}$$

$$\text{SIN}32° = \frac{?}{80}$$

$$80 \times \text{SIN}32° = ?$$

$$= 42.4 \text{ m}$$

Trigonometry: Solving problems

Exercises

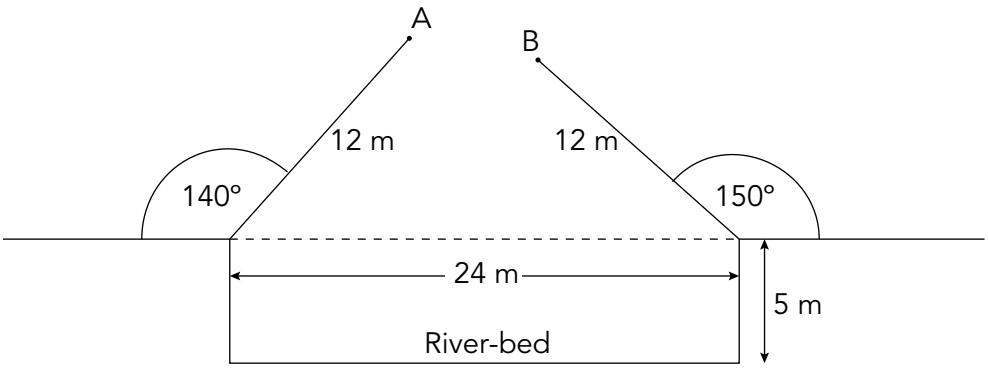
1 Andrea is standing 60 m from a building. She looks up to the top. The angle of elevation of the top is 53°. What is the height of the building?

2 A helicopter is hovering directly over a police car at a height of 300 m. A criminal is spotted on the ground. The angle of depression of the criminal from the helicopter is 21°. How far is the criminal from:
   a the police car?
   b the helicopter?

3 A ship is 5000 m from a vertical cliff, height 2000 m.
   a What is the angle of elevation of the top of the cliff from the ship?
   b What is the angle of depression of the ship from the top of the cliff?

4 A plane is flying at a height of 8000 m. The pilot looks down at the start of the runway. The angle of depression is 22°. What is the distance from the pilot to the start of the runway?

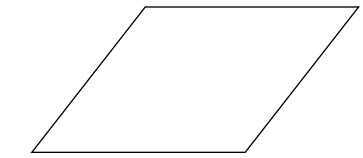
5 A bridge has two lifting sections. The dotted line shows the position when closed. Both sections are the same length, 12 m.



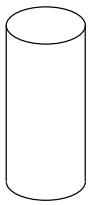
   a How far is point A above the river-bed?
   b How far is point B above the river-bed?
   c What is the angle of elevation of A from B?
   d What is the distance from A to B?

Distinguishing between formulae for length, area and volume

Length has 1 dimension
Area has 2 dimensions
Volume has 3 dimensions



Length x length = area
Length x length x length = volume
Length x area = volume



Length + length = length
Area + area = area
Volume + volume = volume

Different dimensions cannot be added. For example:

Length cannot be added to area
Volume cannot be added to area
Length cannot be added to volume

Numbers, eg 3, 7, π have no effect on the dimensions. For example:

r = radius
r is a length, $2\pi r$ is a length
r^2 is an area, πr^2 is an area

Questions

a, b, c and d are lengths.

State whether each formula gives a length, area, volume or none of these.

1 $3ab$ 2 $\frac{bcd}{3a}$ 3 $ab^2 + 3cd^2$ 4 $ab + d$

Answers

1 area 2 area 3 volume 4 none of these

Distinguishing between formulae for length, area and volume

Exercises

a, b, c and d are lengths

r = radius

In each question state whether the formulae gives a length, area, volume or none of these:

1 ab

2 $\frac{abc}{d}$

3 $\frac{a^2b}{cd}$

4 $\frac{4}{3}\pi r^2$

5 $\frac{a^2b^2}{c}$

6 $2\pi r + 3a$

7 $\pi r^2 + \frac{abc}{d}$

8 $\pi r^2 + 2\pi r$

9 $\frac{abc}{d} + cd$

10 $a^2b + abd$

11 $\frac{bc}{d} + 3r$

12 $ab + cd + r^2$

13 $\frac{4}{3}\pi r^3 + 2\pi r^2$

14 $\pi d + r + \frac{ab}{d}$

15 $\frac{b^2c}{d^2} + \frac{abc}{6ab}$

Cumulative frequency

Question

This table shows the ages of members of a cricket club:

Age	Frequency
6-15	3
16-25	10
26-35	14
36-45	28
46-55	20
56-65	5

1 What is the range of the ages?

2 Draw a cumulative frequency diagram.

3 What is the median age?

4 What is the upper quartile?

5 What is the lower quartile?

6 What is the interquartile range?

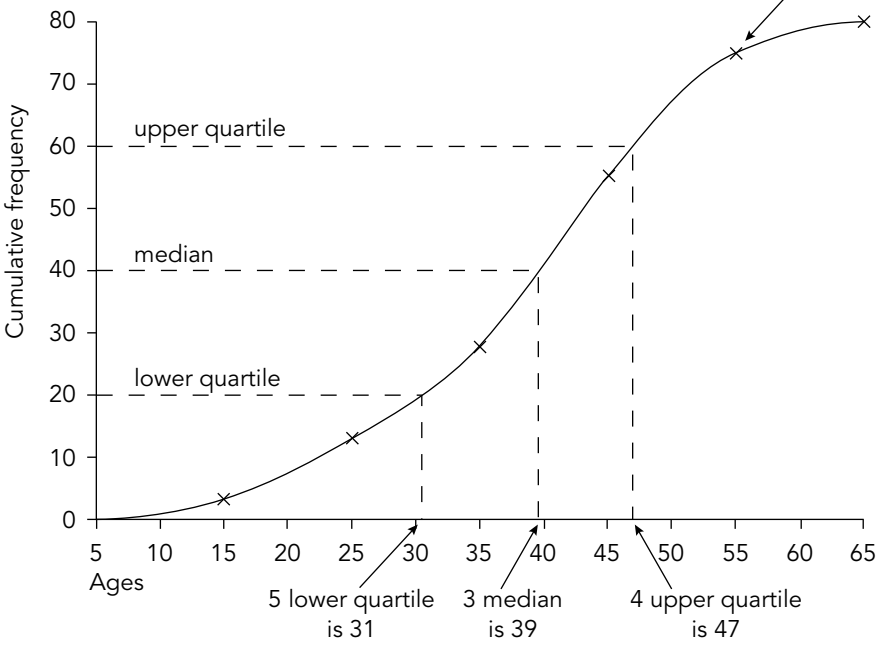
Answer

1 The range is 65 - 6 = 59 years.

2 First complete a cumulative frequency column.

Age	Frequency	Cumulative Frequency
6-15	3	3
16-25	10	3+10 = 13
26-35	14	3+10+14 = 27
36-45	28	3+10+14+28 = 55
46-55	20	3+10+14+28+20 = 75
56-65	5	3+10+14+28+20+5 = 80

Note: Points are plotted at the maximum value of the class interval, eg the 46-55 interval is plotted at (55,75) not (50,75).



6 interquartile range
47 - 31 = 16

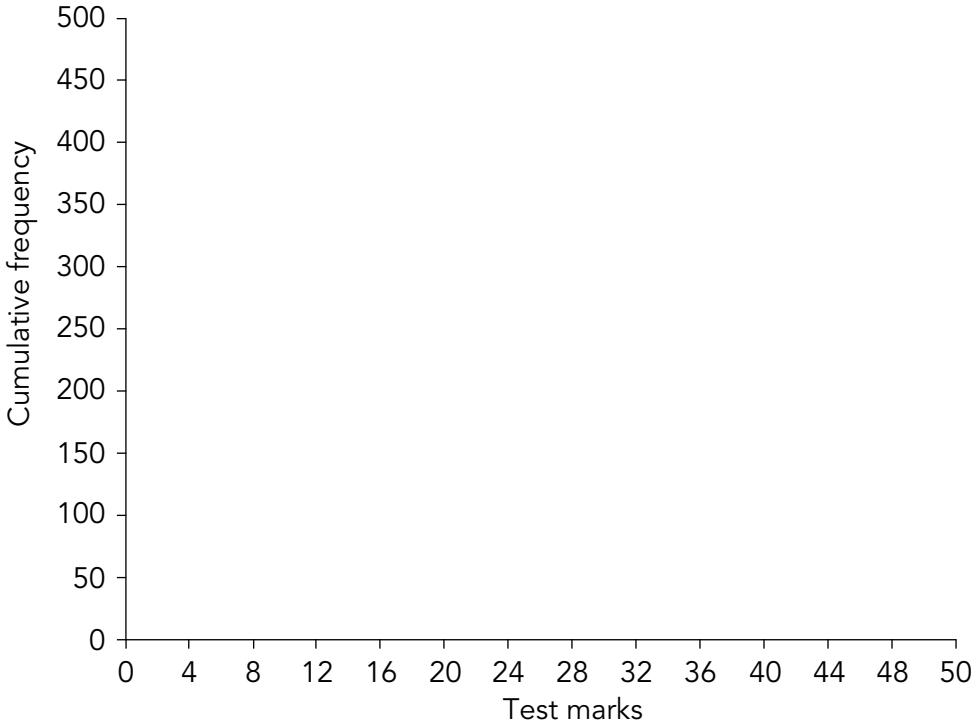
Cumulative frequency

Exercises

This table shows the test results (out of 50 marks) in Science.

Mark	Frequency	Cumulative Frequency
0-10	70	
11-20	90	
21-30	130	
31-40	160	
41-50	50	

1 Complete the cumulative frequency column.



2 Complete the cumulative frequency diagram.

3 What is the median mark?

4 What is the upper quartile?

5 What is the lower quartile?

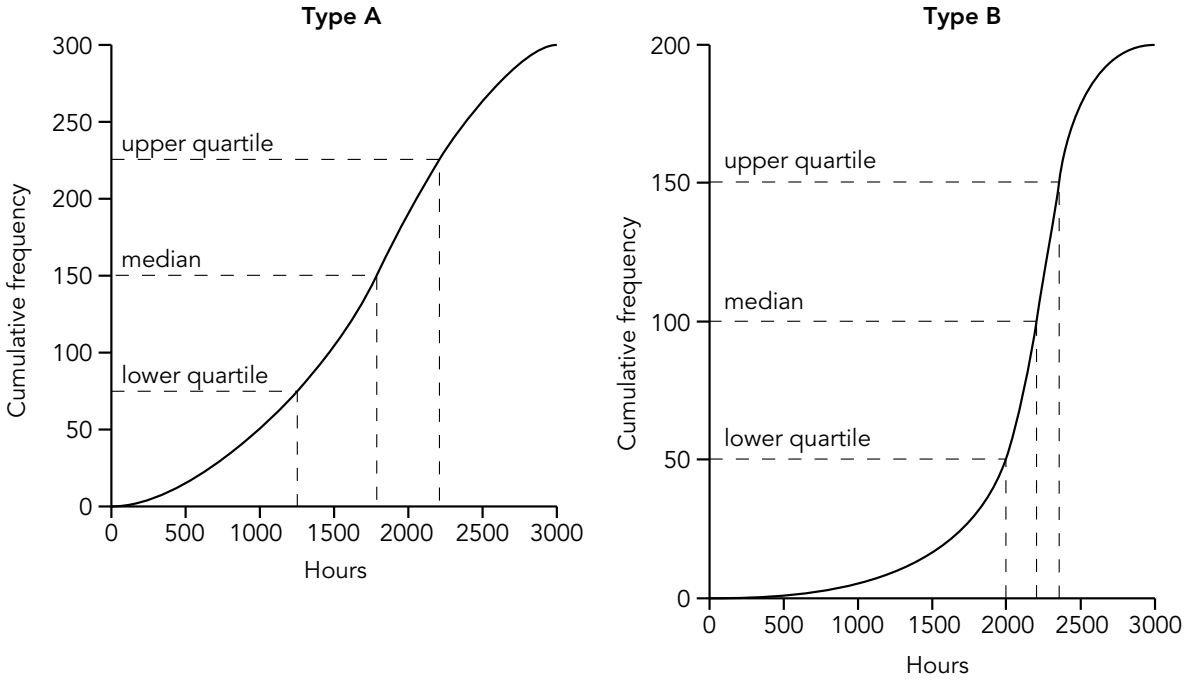
6 What is the interquartile range?

7 The top 5% of pupils receive an A grade. What mark is needed for an A grade?

Using cumulative frequency diagrams to compare distributions

Question

Two different makes of light bulbs were compared. The cumulative frequency diagrams show the number of hours the bulbs lasted.



Use the median and interquartile range to compare the two distributions.

Answer

Different numbers of bulbs were used in the tests but the median and interquartile range allow comparison between the two types of bulb. The interquartile range measures the range of the middle half of the distribution.

The median of bulb A is about 1800 hours.

The median of bulb B is about 2200 hours.

This implies that bulb B is better because the median bulb lasts 400 hours longer.

The interquartile range of bulb A is about (2200 - 1250) 950 hours.

The interquartile range of bulb B is about (2400 - 2000) 400 hours.

The middle half of bulb B is bunched together.

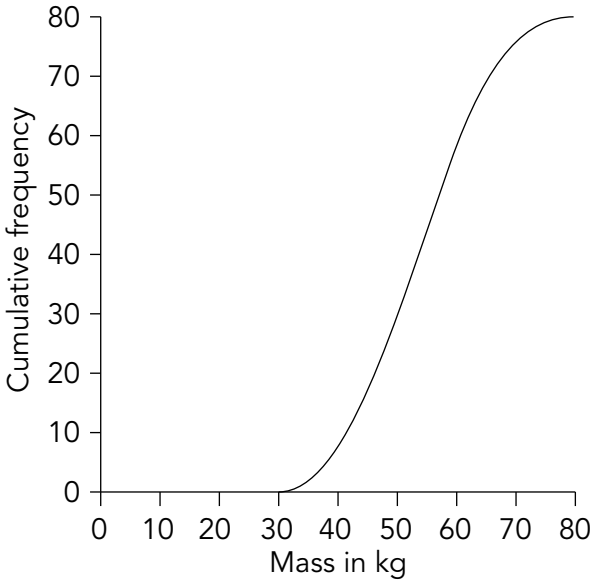
The middle half of bulb A is more spread out.

The information suggests that bulbs of type B are more consistent and have a longer lifetime.

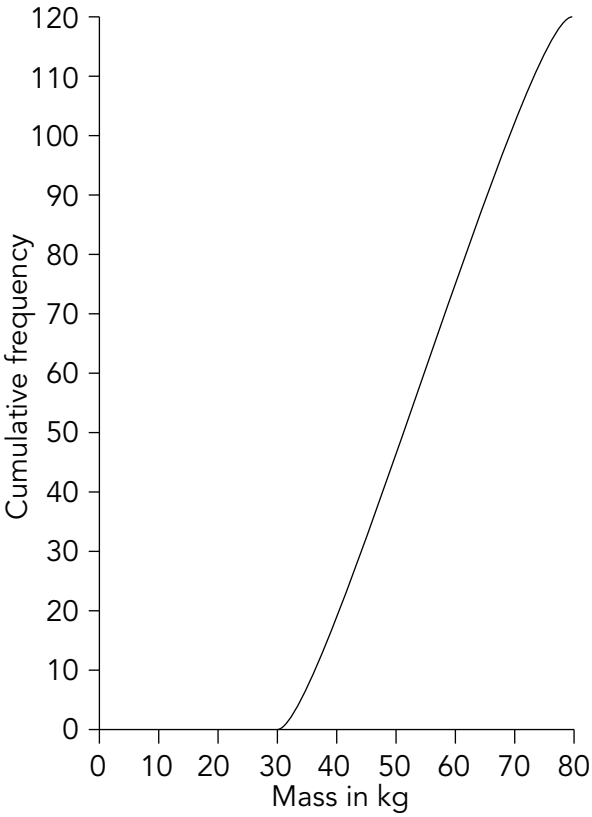
Using cumulative frequency diagrams to compare distributions

Exercises

Two farmers each had a flock of sheep. They decided to compare the success of the rearing methods they each used by weighing the sheep after one year.



Farmer Giles' flock



Old McDonald's flock

1 Find the median of each distribution.

2 Find the upper quartile of each distribution.

3 Find the lower quartile of each distribution.

4 Find the interquartile range of each distribution.

5 Use the median, upper quartile and lower quartile of each distribution to compare the effectiveness of each farmer's methods.

Probability 1

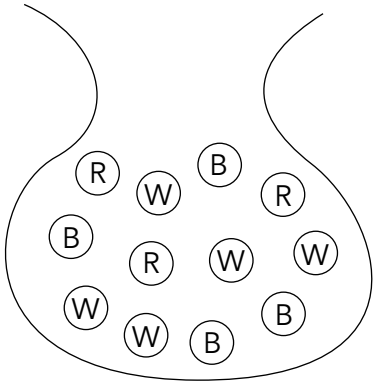
AND means **MULTIPLY**

OR means **ADD**

Example

A bag contains three red sweets, four blue sweets and five white sweets.

A boy is blindfolded. What is the probability he chooses a blue sweet, eats it, then chooses a red sweet?



Method

Try to rephrase the question using the key words:

AND **OR** **BLUE SWEET** **RED SWEET**

The boy needs **BLUE SWEET** **AND** **RED SWEET**

There are four blue sweets in the bag → $\frac{4}{12}$ ← There are 12 sweets in the bag

x

$\frac{3}{11}$ ← There are three red sweets in the bag

Remember, a blue sweet has been removed so there are only 11 sweets left in the bag

$$\frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$$

Questions

1 What is the probability of choosing two red sweets?

2 What is the probability of choosing a red sweet and a white sweet in any order?

1 Key words AND OR RED SWEET

Rephrase the question using the key words.

RED SWEET AND RED SWEET

$\frac{3}{12} \times \frac{2}{11} = \frac{1}{22}$

2 Key words AND OR RED SWEET WHITE SWEET

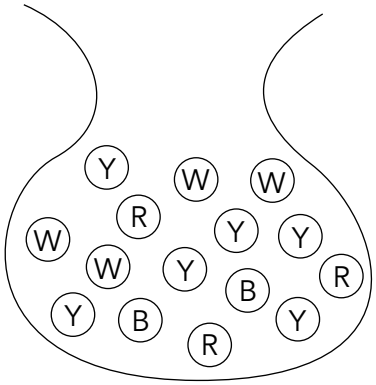
Rephrase the question using the key words.

RED SWEET AND WHITE SWEET OR WHITE SWEET AND RED SWEET

$\frac{3}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{3}{11} = \frac{5}{22}$

Answers

Probability 1



Exercises

1 A bag contains two blue counters, three red counters, four white counters and six yellow counters. Each time a counter is selected it is replaced.

Find the probability of:

a A white then a yellow.

b A white and a yellow in any order.

c Three reds.

d A red then a white.

e A blue, then a red, then a white, then a yellow.

f At least one blue in three attempts.

2 Repeat question 1 without replacing the counters.

3 A coin is tossed and a die is thrown.

Find the probability of:

a A head and a 6.

b A tail and an odd number.

c A head and a number over 4.

4 The word P R O B A B I L I T Y is written on cards. Cards are selected without replacement. What is the probability of choosing:

a The letters P then R then O then B?

b Two B s?

c A P and a Y in any order?

d Two identical letters, eg B and B?

e Three vowels?

Probability 2

Questions

Three coins are tossed.

What is the probability of:

1 Exactly one head?

2 At least one head?

Answers

1 We need:

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{8}$

2 Remember, the total probability for all of the possible ways three coins can land is 1.

We could say:

This will work but it takes a long time!

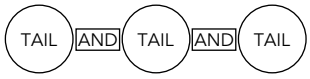
Think carefully

Sometimes it is quicker to work out the probability of what we do not want.

What don't we want?

We don't want three tails. Any other outcome will contain at least one head.

The probability of three tails is:



$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Total probability - Probability of three tails = Probability of at least one head

$1 - \frac{1}{8} = \frac{7}{8}$

Probability 2

Exercises

1 The probability of a dry day is 0.7. The probability of a rain day is 0.3. Sarah is on holiday for three days. What is the probability that:

a It rains on exactly one day?

b It rains every day?

c It rains on the second day?

d It rains on at least one day?

2 The chance that a light bulb is faulty is $\frac{1}{20}$.

a John selects a light bulb. What is the chance that it is not faulty?

b Jayne needs two light bulbs. If she buys three light bulbs what is the chance that at least two will work?

c Paul needs at least a 99.99% chance that one light bulb will work. How many light bulbs should he buy to ensure this probability? Show your working.

3 The probability of Carolyn's school bus not being late each day is 0.4. If she is late five times in any week she receives a detention.

What is the probability that she receives a detention in her first week of term for being late?

4 A biased coin has a 0.6 chance of landing on a head and a 0.4 chance of landing on a tail. The coin is tossed four times.

a What is the probability of at least one head?

b What is the probability of at least one tail?

Tree diagrams

Questions

A car driver passes through two sets of traffic lights on his way to work. The lights can either be red or green. The probability of red at the first lights is 0.6. The probability of red at the second lights is 0.3.

Draw a tree diagram to show this and hence calculate the probability that:

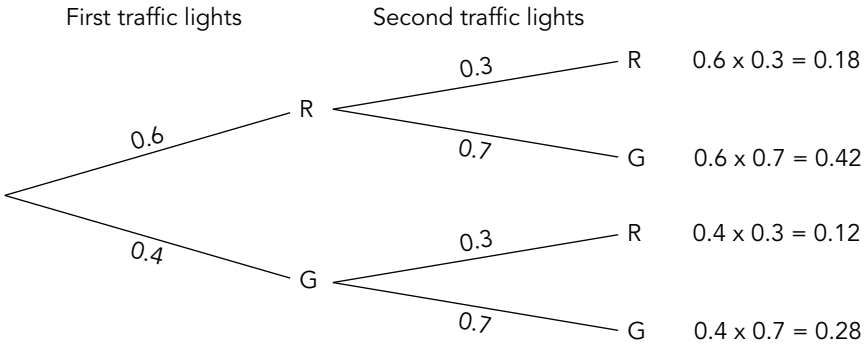
1 Both lights are red.

2 Both lights are green.

3 One set of lights is red and one is green.

4 At least one set of lights is red.

Answers



1 0.18

2 0.28

3 Red and green or green and red

0.42 + 0.12 = 0.54

4 Red and red or red and green or green and red

0.18 + 0.42 + 0.12 = 0.72

Alternative method

1 - green and green

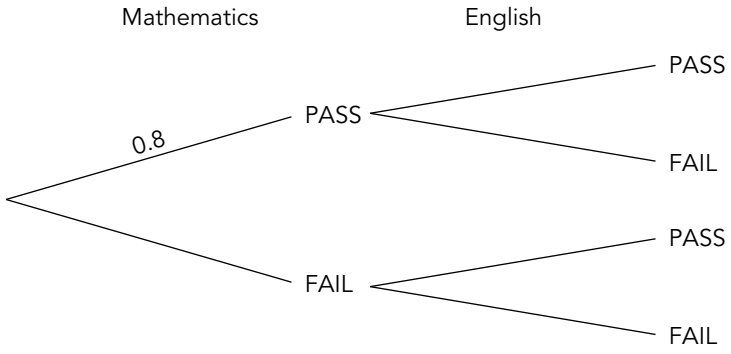
1 - 0.28 = 0.72

Tree diagrams

Exercises

1 The probability of passing a Mathematics examination is 0.8.
The probability of passing an English examination is 0.7.

a Complete this tree diagram:



b What is the probability that a pupil passes both examinations?
c What is the probability that a pupil fails both examinations?
d What is the probability that a pupil passes exactly one examination?

Three hundred pupils take the examinations.

e How many pupils would you expect to pass English and fail Mathematics?
f How many pupils would you expect to fail both examinations?

2 A fair coin is tossed three times.

a Draw a tree diagram to show all of the possible outcomes.

Use your tree diagram to find the probability of:

b Exactly two heads.
c At least two heads.
d Three tails.
e Exactly one head.

3 Two dice are tossed.

a Draw a tree diagram to show all of the possible outcomes.

Use your tree diagram to find the probability of:

b A total of 12.
c A total of 10.
d A total of less than 5.

Differences

Top left	Top right
Bottom left	Bottom right

Multiply top right by bottom left = ?
Multiply top left by bottom right = ?

Record the difference

Example

Going up in ones

1	2
4	3

2 x 4 = 8
1 x 3 = 3
Difference = 5

2	3
5	4

3 x 5 = 15
2 x 4 = 8
Difference = 7

3	4
6	5

4 x 6 = 24
3 x 5 = 15
Difference = 9

Going up in twos

1	3
7	5

3 x 7 = 21
1 x 5 = 5
Difference = 16

2	4
8	6

4 x 8 = 32
2 x 6 = 12
Difference = 20

3	5
9	7

5 x 9 = 45
3 x 7 = 21
Difference = 24

Investigate

1 Going up in threes, fours, fives.

2 Find formulae.

3 Try to find a general formula for:

a	a+t
a+3t	a+2t

Where a is the top left number and the pattern goes up in 't's.

4 Try square numbers.

5 Try cubed numbers.

6 Continue the investigation.

Puzzles

1 A mermaid was washed up on a beach.

Her head is 27 cm long

Her tail is as long as her head and half of her body.

Her head and her tail are the same length as her body.

Form equations to help you work out:

a The length of her body.

b The length of her tail.

c Her total length.

d Express the length of head to body to tail as a ratio.

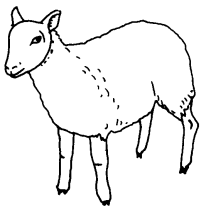
2 A farmer has 200 animals.

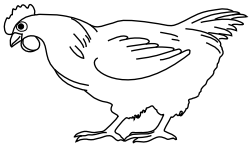
The animals have 632 legs between them.

A chicken is worth £4.

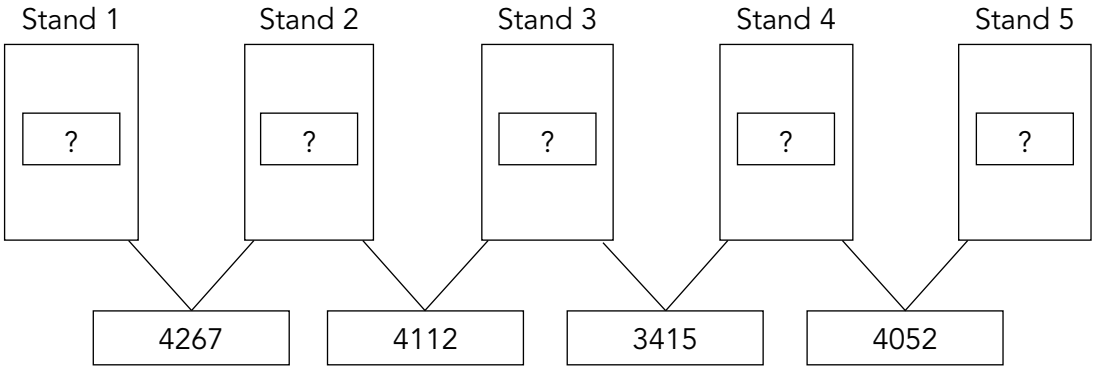
A sheep is worth £30.

What is the total value of his animals?





3 A football stadium has five stands.



10 000 people were in the stadium on Saturday.

Before the match could begin a safety check had to be made.

The safety regulations state:

1 Any two adjacent stands must not hold more than 4400 people.

2 Individual stands must not hold more than 2300 people.

The total number of people in stand 1 + stand 2 = 4267

The total number of people in stand 2 + stand 3 = 4112

The total number of people in stand 3 + stand 4 = 3415

The total number of people in stand 4 + stand 5 = 4052

a Were any safety regulations being broken?

b Which stands were overcrowded and by how many people?