Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 13 May 2015 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2215 – 7306

3. [Maximum mark: 6]

The following table shows the sales, y millions of dollars, of a company, x years after it opened.

Time after opening (x years)	2	4	6	8	10
Sales (y millions of dollars)	12	20	30	36	52

The relationship between the variables is modelled by the regression line with equation $y = ax + b$.

(a) (i) Find the value of a and of b.

(ii) Write down the value of r. [4]

(b) Hence estimate the sales in millions of dollars after seven years. [2]

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4. [Maximum mark: 5]

The third term in the expansion of $(x+k)^8$ is $63x^6$. Find the possible values of k.

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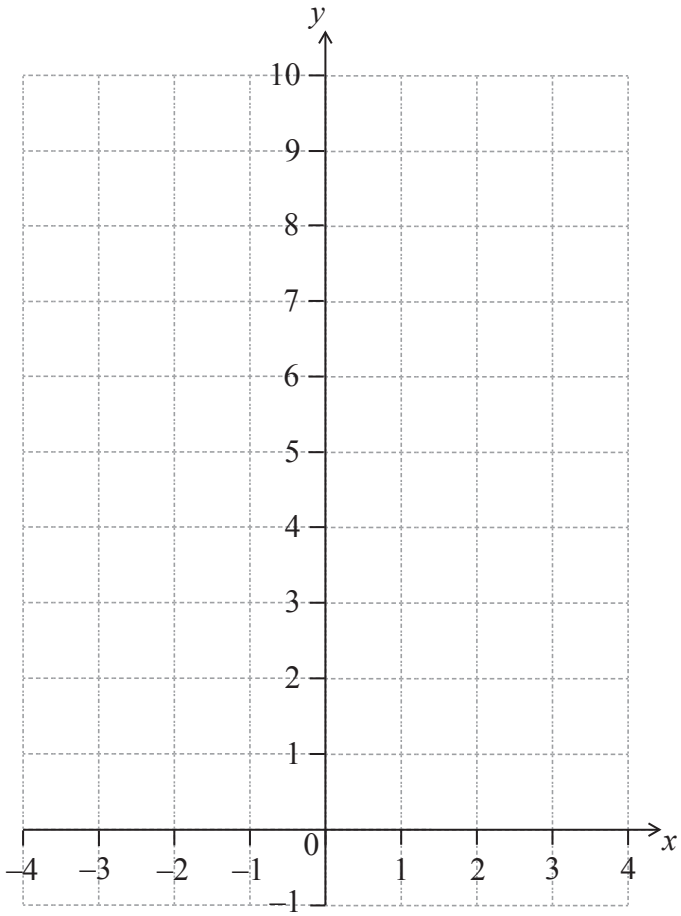
5. [Maximum mark: 6]

Let $f(x) = e^{x+1} + 2$, for $-4 \leq x \leq 1$.

(a) On the following grid, sketch the graph of f. [3]



(b) The graph of f is translated by the vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to obtain the graph of a function g.

Find an expression for $g(x)$. [3]

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6. [Maximum mark: 7]

Ramiro walks to work each morning. During the first minute he walks 80 metres. In each subsequent minute he walks $90\,\%$ of the distance walked during the previous minute. The distance between his house and work is 660 metres. Ramiro leaves his house at $08{:}00$ and has to be at work by $08{:}15$.

Explain why he will not be at work on time.

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7. [Maximum mark: 8]

Let $f(x) = kx^2 + kx$ and $g(x) = x - 0.8$. The graphs of f and g intersect at two distinct points. Find the possible values of k.

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Do **not** write solutions on this page.

9. [Maximum mark: 16]

A machine manufactures a large number of nails. The length, L mm, of a nail is normally distributed, where $L \sim \mathrm{N}\left(50, \sigma^2\right)$.

(a) Find $\mathrm{P}(50-\sigma < L < 50+2\sigma)$. [3]

(b) The probability that the length of a nail is less than $53.92\,\text{mm}$ is 0.975.
Show that $\sigma = 2.00$ (correct to three significant figures). [2]

All nails with length at least t mm are classified as large nails.

(c) A nail is chosen at random. The probability that it is a large nail is 0.75.
Find the value of t. [3]

(d) (i) A nail is chosen at random from the large nails. Find the probability that the length of this nail is less than $50.1\,\text{mm}$.

(ii) Ten nails are chosen at random from the large nails. Find the probability that at least two nails have a length that is less than $50.1\,\text{mm}$. [8]

3. (a) (i) evidence of set up ***(M1)***

eg correct value for a, b or r

$a = 4.8$, $b = 1.2$ ***A1A1*** ***N3***

(ii) $r = 0.988064$

$r = 0.988$ ***A1*** ***N1***

[4 marks]

(b) correct substitution into **their** regression equation ***(A1)***

eg $4.8 \times 7 + 1.2$

34.8 (millions of dollars) (accept 35 and 34 800 000) ***A1*** ***N2***

[2 marks]

Total [6 marks]

4. valid approach to find the required term ***(M1)***

eg $\binom{8}{r} x^{8-r} k^r$, Pascal's triangle to 8th row, $x^8 + 8x^7k + 28x^6k^2 + \ldots$

identifying correct term (may be indicated in expansion) ***(A1)***

eg $\binom{8}{2} x^6k^2$, $\binom{8}{6} x^6k^2$, $r = 2$

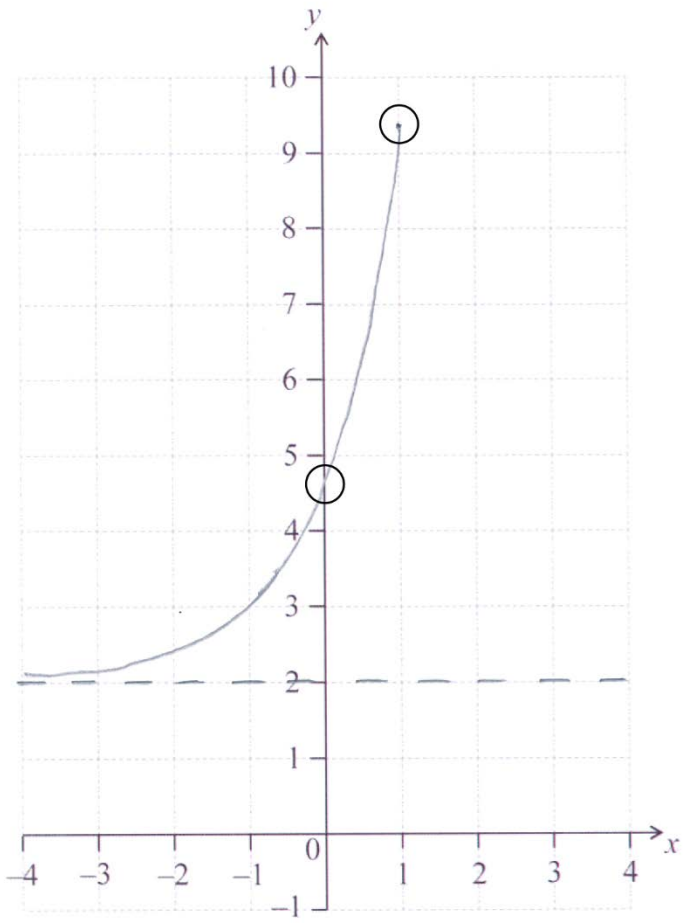
setting up equation in k with **their** coefficient/term ***(M1)***

eg $28k^2x^6 = 63x^6$, $\binom{8}{6} k^2 = 63$

$k = \pm 1.5$ (exact) ***A1A1*** ***N3***

[5 marks]

5. (a)



A1A1A1 ***N3***

Note: Curve must be approximately correct exponential shape (increasing and concave up). Only if the shape is approximately correct, award the following:
A1 for right end point in circle,
A1 for y-intercept in circle,
A1 for asymptotic to $y = 2$, (must be above $y = 2$).

[3 marks]

(b) valid attempt to find g ***(M1)***

eg $f(x-3)-1$, $g(x) = e^{x+1-3}+2-1$, e^{x+1-3}, $2-1$, sketch

$g(x) = e^{x-2}+1$ ***A2*** ***N3***

[3 marks]

Total [6 marks]

6. METHOD 1

recognize that the distance walked each minute is a geometric sequence **(M1)**
eg $r = 0.9$, valid use of 0.9

recognize that total distance walked is the sum of a geometric sequence **(M1)**
eg S_n, $a\left(\frac{1-r^n}{1-r}\right)$

correct substitution into the sum of a geometric sequence **(A1)**
eg $80\left(\frac{1-0.9^n}{1-0.9}\right)$

any correct equation with sum of a geometric sequence **(A1)**
eg $80\left(\frac{0.9^n-1}{0.9-1}\right) = 660$, $1-0.9^n = \frac{66}{80}$

attempt to solve **their** equation involving the sum of a GP **(M1)**
eg graph, algebraic approach

$n = 16.54290788$ ***A1***

since $n > 15$ ***R1***
he will be late ***AG*** ***N0***

Note: Do not award the ***R*** mark without the preceding ***A*** mark.

continued...

Question 6 continued

METHOD 2

recognize that the distance walked each minute is a geometric sequence ***(M1)***
eg $r = 0.9$, valid use of 0.9

recognize that total distance walked is the sum of a geometric sequence ***(M1)***
eg S_n, $a\left(\frac{1-r^n}{1-r}\right)$

correct substitution into the sum of a geometric sequence ***(A1)***
eg $80\left(\frac{1-0.9^n}{1-0.9}\right)$

attempt to substitute $n = 15$ into sum of a geometric sequence ***(M1)***
eg S_{15}

correct substitution ***(A1)***
eg $80\left(\frac{0.9^{15}-1}{0.9-1}\right)$

$S_{15} = 635.287$ ***A1***

since $S < 660$ ***R1***
he will not be there on time ***AG*** ***N0***

Note: Do not award the ***R*** mark without the preceding ***A*** mark.

METHOD 3

recognize that the distance walked each minute is a geometric sequence ***(M1)***
eg $r = 0.9$, valid use of 0.9

recognize that total distance walked is the sum of a geometric sequence ***(M1)***
eg S_n, $a\left(\frac{1-r^n}{1-r}\right)$

listing at least 5 correct terms of the GP ***(M1)***

15 correct terms ***A1***
80, 72, 64.8, 58.32, 52.488, 47.2392, 42.5152, 38.2637, 34.4373, 30.9936, 27.8942, 25.1048, 22.59436, 20.3349, 18.3014

attempt to find the sum of the terms ***(M1)***

eg S_{15}, $80 + 72 + 64.8 + 58.32 + 52.488 + ... + 18.301433$

$S_{15} = 635.287$ ***A1***

since $S < 660$ ***R1***
he will not be there on time ***AG*** ***N0***

Note: Do not award the ***R*** mark without the preceding ***A*** mark.

[7 marks]

7. attempt to set up equation ***(M1)***

eg $f = g$, $kx^2 + kx = x - 0.8$

rearranging **their** equation to equal zero ***M1***

eg $kx^2 + kx - x + 0.8 = 0$, $kx^2 + x(k-1) + 0.8 = 0$

evidence of discriminant (if seen explicitly, not just in quadratic formula) ***(M1)***

eg $b^2 - 4ac$, $\Delta = (k-1)^2 - 4k \times 0.8$, $D = 0$

correct discriminant ***(A1)***

eg $(k-1)^2 - 4k \times 0.8$, $k^2 - 5.2k + 1$

evidence of correct discriminant greater than zero ***R1***

eg $k^2 - 5.2k + 1 > 0$, $(k-1)^2 - 4k \times 0.8 > 0$, correct answer

both correct values ***(A1)***

eg 0.2, 5

correct answer ***A2*** ***N3***

eg $k < 0.2$, $k \neq 0$, $k > 5$

[8 marks]

Section B

8. **Note:** The values of p and q found in (a) are used throughout the question. Please check ***FT*** carefully on **their** values.

(a) attempt to find intersection ***(M1)***
eg $f = g$

$p = 1,\ q = 3$ ***A1A1*** ***N3***
[3 marks]

(b) $f'(p) = -1$ ***A2*** ***N2***
[2 marks]

(c) (i) correct approach to find the gradient of the normal ***(A1)***
eg $m_1 m_2 = -1,\ -\frac{1}{f'(p)}$, correct value of 1

EITHER

attempt to substitute coordinates (in any order) and correct normal gradient to find c ***(M1)***
eg $3 = -\frac{1}{f'(p)} \times 1 + c,\ 1 = 1 \times 3 + c$

$c = 2$ ***(A1)***

$y = x + 2$ ***A1*** ***N2***

OR

attempt to substitute coordinates (in any order) and correct normal gradient into equation of a straight line ***(M1)***
eg $y - 3 = -\frac{1}{f'(p)}(x - 1),\ y - 1 = 1 \times (x - 3)$

correct working
eg $y = (x - 1) + 3$ ***(A1)***

$y = x + 2$ ***A1*** ***N2***

(ii) $(0,\ 2)$ ***A1*** ***N1***
[5 marks]

(d) appropriate approach involving subtraction ***(M1)***
eg $\int_a^b (L - g)\,\mathrm{d}x,\ \int \left(3x^2 - (x + 2)\right)$

substitution of **their** limits or function ***(A1)***
eg $\int_0^p (L - g)\,\mathrm{d}x,\ \int \left((x + 2) - 3x^2\right)$

$\text{area} = 1.5$ ***A1*** ***N2***
[3 marks]
Total [13 marks]

9. **Note:** There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. In particular there are a number of ways of doing (d). Accept answers that are consistent with their working.

(a) valid approach ***(M1)***

eg $\frac{L-\mu}{\sigma}$, using a value for σ, using 68% and 95%

correct working

$\mathrm{P}(-1<Z<2)$, correct probabilities $(0.6826\ldots+0.1359\ldots)$ ***(A1)***

$\mathrm{P}(50-\sigma<L<50+2\sigma)=0.818594$

$\mathrm{P}(50-\sigma<L<50+2\sigma)=0.819$ ***A1*** ***N2***

[3 marks]

(b) $z=1.95996$ ***(A1)***

correct equation ***A1***

eg $\frac{53.92-50}{\sigma}=1.95996$, $\sigma=2.00004$

$\sigma=2.00$ ***AG*** ***N0***

[2 marks]

(c) valid set up ***M1***

eg $\mathrm{P}(L>t)=0.75$, right tail, 

, 0.25

$t=48.6510$

$t=48.7$ (do not accept 48.5 from using $z=-0.75$) ***A2*** ***N2***

[3 marks]

continued

Question 9 continued

(d) (i) correct approach **(A1)**
eg from t to $50.1,\ \mathrm{P}(48.7 < X < 50.1),\ 0.269942$

recognize conditional probability (seen anywhere, including in correct working) **R1**
eg $\mathrm{P}(\mathrm{A}|\mathrm{B})$

correct substitution **(A1)**
eg $\frac{\mathrm{P}(48.7 < X < 50.1)}{\mathrm{P}(X > 48.7)},\ \frac{0.269942}{0.75}$

0.359923
0.360 **A1** **N3**

(ii) $\mathrm{P}(X \geq 2)$ **(A1)**

attempt to find $\mathrm{P}(X \geq 2)$ **(M1)**
eg $1 - \mathrm{P}(X = 0) - \mathrm{P}(X = 1),\ \mathrm{P}(X = 2) + \mathrm{P}(X = 3) + \ldots$

recognize binomial distribution **(M1)**
eg $X \sim \mathrm{B}(n,\ p)$

0.923741
0.924 **A1** **N2**

[8 marks]

Total [16 marks]