1. The number (n) of bacteria in a colony after h hours is given by the formula $n = 1200(3^{0.25h})$.
Initially, there are 1200 bacteria in the colony.

(a) Copy and complete the table below, which gives values of n and h.
Give your answers to the nearest hundred.

time in hours (h)	0	1	2	3	4
no. of bacteria (n)	1200		2100	2700	

(2)

(b) On graph paper, draw the graph of the above function. Use a scale of 3 cm to represent 1 hour on the horizontal axis and 4 cm to represent 1000 bacteria on the vertical axis. Label the graph clearly.

(5)

(c) Use your graph to answer each of the following, showing your method **clearly**.

(i) How many bacteria would there be after 2 hours and 40 minutes?
Give your answer to the nearest hundred bacteria.

(ii) After how long will there be approximately 3000 bacteria? Give your answer to the nearest 10 minutes.

(4)
(Total 11 marks)

2. The velocity, $v\text{ms}^{-1}$, of a kite, after t seconds, is given by

$$v = t^3 - 4t^2 + 4t, \qquad 0 \leq t \leq 4.$$

(a) What is the velocity of the kite after

(i) one second?

(ii) half a second?

(2)

(b) Calculate the values of a and b in the table below.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
v	0			a	0	0.625	b	7.88	16

(2)

(c) (i) Find $\frac{\mathrm{d}v}{\mathrm{d}t}$ in terms of t. Find the value of t at the local maximum and minimum values of the function.

(ii) Explain what is happening to the function at its local maximum point.
Write down the gradient of the tangent to its curve at this point.

(8)

(d) On graph paper, draw the graph of the function $v = t^3 - 4t^2 + 4t$, $0 \le t \le 4$.
Use a scale of 2 cm to represent 1 second on the horizontal axis and 2 cm to represent 2 ms^{-1} on the vertical axis.

(5)

(e) Describe the motion of the kite at different times during the first 4 seconds.
Write down the intervals corresponding to changes in motion.

(3)
(Total 20 marks)

3. Consider the function $f(x) = x^3 - 4x^2 - 3x + 18$

(a) (i) Find $f'(x)$.

(ii) Find the coordinates of the maximum and minimum points of the function.

(10)

(b) Find the values of $f(x)$ for a and b in the table below:

x	–3	–2	–1	0	1	2	3	4	5
$f(x)$	–36	a	16	b	12	4	0	6	28

(2)

(c) Using a scale of 1 cm for each unit on the x-axis and 1 cm for each 5 units on the y-axis, draw the graph of $f(x)$ for $-3 \le x \le 5$. Label clearly.

(5)

(d) The gradient of the curve at any particular point varies. Within the interval $-3 \le x \le 5$, state all the intervals where the gradient of the curve at any particular point is

(i) negative,

(ii) positive.

(3)
(Total 20 marks)

4. (a) Sketch the graph of the function $y = 2x^2 - 6x + 5$.

(b) Write down the coordinates of the local maximum or minimum of the function.

(c) Find the equation of the axis of symmetry of the function.

(Total 6 marks)

5. (a) Sketch a graph of $y = \dfrac{x}{2+x}$ for $-10 \le x \le 10$.

(b) Hence write down the equations of the horizontal and vertical asymptotes.

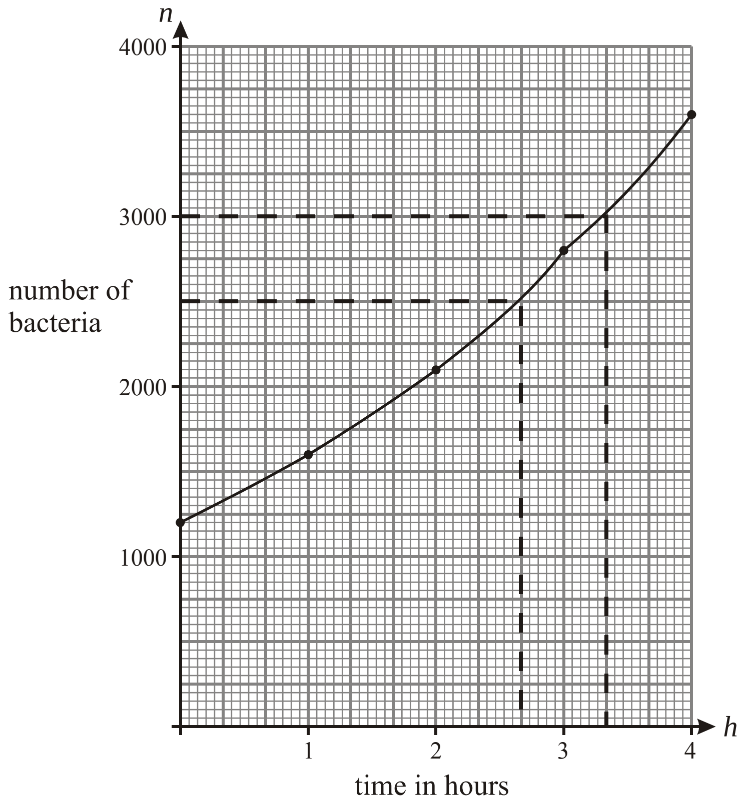
(Total 6 marks)

1. (a)

Time in hours *(h)*	0	1	2	3	4
No. of bacteria *(n)*	1200	**1600**	2100	2700	**3600**

(A1)(A1) 2

(b)



(A2)(A3)

Note: *Award (A1) for the axes correctly labelled and (A1) for the correct scales.*

Award (A2) for 4 or 5 points correctly plotted, (A1) for 2 or 3 correct and (A1) for connecting points with a smooth curve.

(c) (i) 2500 (M1)(A1)

(ii) 3hrs 20min (M1)(A1)

Note: *Use follow through from graph. If no method is shown from graph give (C1) only for correct answer.*

[11]

2. (a) (i) $v(1) = 1^3 - 4(1)^2 + 4(1)$

$= 1 \text{ ms}^{-1}$ (A1)

(ii) $v(0.5) = (0.5)^3 - 4(0.5)^2 + 4(0.5)$

$= 1.125 \text{ ms}^{-1}$ accept 1.13 (3 s.f.) (A1) 2

(b) $a = v(1.5) = 1.5^3 - 4(1.5) + 4(1.5)$
$= 0.375$ (A1)

$b = v(3) = 3^3 - 4(3^2) + 4(3)$
$= 3$ (A1) 2

Table (not required)

t	0	0.5	1	1.5	2	2.5	3	3.5	4
v	0	1.125	1	0.375	0	0.625	3	7.875	16

(c) (i) $\frac{dv}{dt} = 3t^2 - 8t + 4$ (A1)

$3t^2 - 8t + 4 = 0$ (M1)

$(3t - 2)(t - 2) = 0$ (M1)

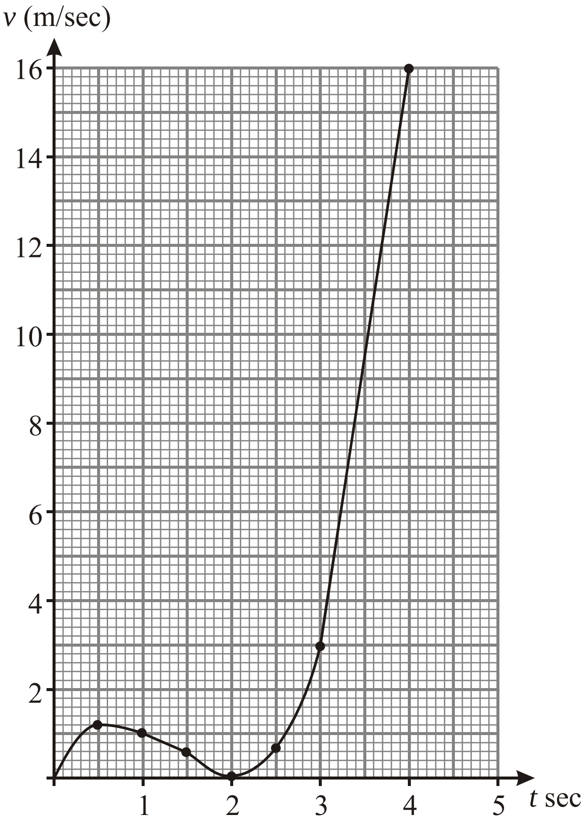
$t = \frac{2}{3}, t = 2$ (A1)(A1)

(ii) The function is changing from acceleration to deceleration
or velocity changes from increasing to decreasing
or kite is stationary *or* velocity is zero (R1)(R1)

Note: *Award (R1) for acceleration, (R1) for deceleration.*

Gradient = 0 (A1) 8

(d)



(A5) 5

Note: *Award (A1) for axes correctly labelled, (A1) if scales correct, (A1) for correct general shape of curve, (A1) for each turning point in approximately the correct place.*

(e)

time t	motion		
$t = 0$	stopped		
$0 < t < \frac{2}{3}$	accelerating (increasing in velocity)	(A1)	
$t = \frac{2}{3}$	stopped accelerating		
$\frac{2}{3} < t < 2$	decelerating (decreasing in velocity)	(A1)	
$t = 2$	stopped decelerating	(A1)	3
$2 < t \le 4$	accelerating		

Note: *Stops may be left out*

[20]

3. (a) (i) $f'(x) = 3x^2 - 8x - 3$ (A1)

(ii) $3x^2 - 8x - 3 = 0$ (M1)

$(3x + 1)(x - 3) = 0$ (A1)

$x = -\frac{1}{3}$ (A1)

$x = 3$ (A1)

Note: *Alternatively, award (G1) for 1 correct answer, (G3) for both.*

$f\left(-\frac{1}{3}\right) = \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 18$ (M1)

$= 18.5$ (A1)

$f(3) = (3)^3 - 4(3)^2 - 3(3) + 18$ (M1)

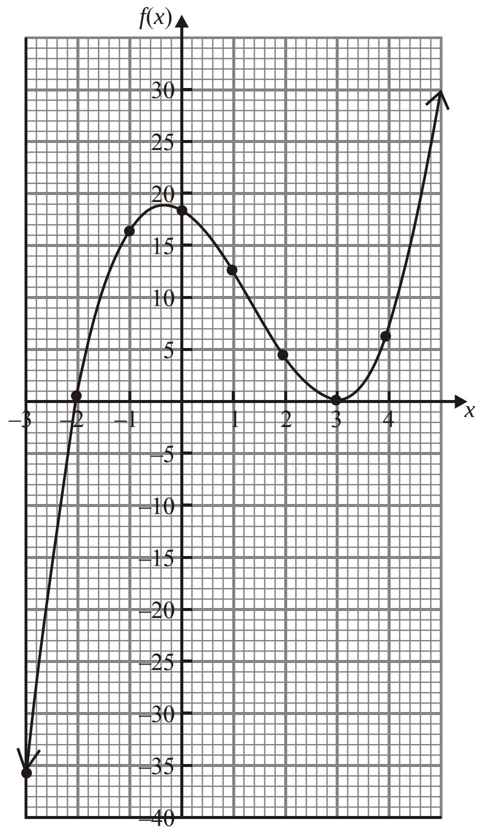
$= 0$ (A1)

Points are $\left(-\frac{1}{3}, 18.5\right)$ and (3, 0) (A1) 10

(b) $a = 0$ (A1)

$b = 18$ (A1) 2

(c)



(A5) 5

Note: *Award (A1) for scales and axes labelled correctly, (A1)(A1) for maximum and minimum placed correctly, (A1) for smooth curve, (A1) for all points plotted correctly.*

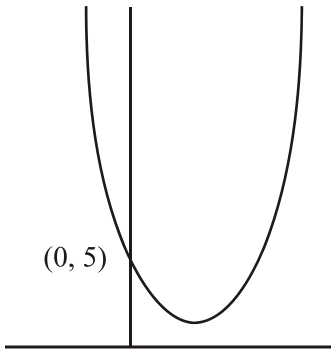
(d) (i) $\left(-\frac{1}{3}, 3\right)$ or $-\frac{1}{3} < x < 3$ all parts correct (A1)

(ii) $\left(-3, -\frac{1}{3}\right)$ or $-3 < x < -\frac{1}{3}$ $\left(\text{allow } x < -\frac{1}{3}\right)$ (A1)

and (3, 5) (allow ($x > 3$)) **ft** from error in (i). (A1) 3

[20]

4. (a)



(A3) (C3)

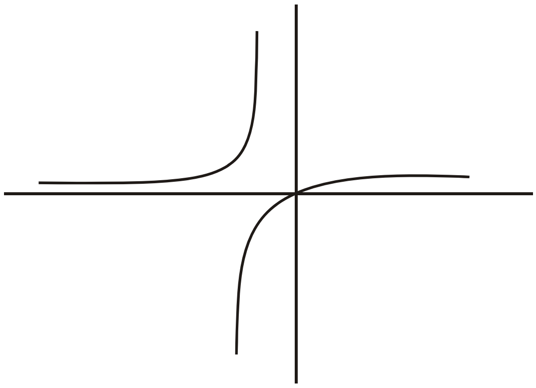
Notes: *Award (A1) for point (0,5) indicated.*
Award (A2) for correct shape.

(b) (1.5, 0.5) (A1)(A1)

(c) $x = 1.5$ (A1) (C1)

[6]

5. (a)



(A4) (C4)

Notes: *Award (A1) for correct scales.*
Award (A1)(A1) for two correct parts to the graph.
Award (A1) if asymptotes are shown.

(b) Horizontal asymptote $y = 1$. (A1) (C1)

[6]