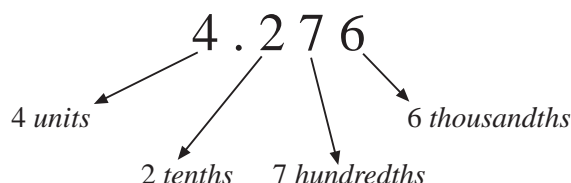# 17 Arithmetic: Decimals, Fractions and Percentages

17.1 Conversion: Decimals into Fractions

In this section we revise ideas of decimals and work on writing decimals as fractions.

Recall that the number

$$4 . 2\ 7\ 6$$

means

4 *units*   2 *tenths*   7 *hundredths*   6 *thousandths*

The table below shows how to write the fractions you need to know in order to write decimals as fractions:

Decimal	Words	Fraction
0.1	1 tenth	$\dfrac{1}{10}$
0.01	1 hundredth	$\dfrac{1}{100}$
0.001	1 thousandth	$\dfrac{1}{1000}$

Example 1

Write these numbers in order, with the smallest first:

$$0.7, \quad 0.17, \quad 0.77, \quad 0.71, \quad 0.701, \quad 0.107$$

Solution

Note: It is, perhaps, easier to see this if we first write *all* the numbers to 3 decimal places,

i.e. 0.700, 0.170, 0.770, 0.710, 0.701, 0.107

The required order is:

0.107, 0.17, 0.7, 0.701, 0.71, 0.77

Example 2

Write these numbers as fractions, where possible giving them in their simplest form:

(a)   0.7   (b)   0.09   (c)   0.004

(d)   0.47   (e)   0.132   (f)   1.75

Solution

(a) $0.7 = \dfrac{7}{10}$

(b) $0.09 = \dfrac{9}{100}$

(c) $0.004 = \dfrac{4}{1000} = \dfrac{1}{250}$

(d) $0.47 = \dfrac{47}{100}$

(e) $0.132 = \dfrac{132}{1000} = \dfrac{33}{250}$

(f) $1.75 = \dfrac{175}{100} = \dfrac{7}{4}$ (note that fractions larger than 1, such as this, are often referred to as *improper* or *vulgar fractions)*

Exercises

1.   What is the value of the 7 in each of these numbers:

(a)   0.714   (b)   0.070   (c)   7.042

(d)   0.007   (e)   0.471   (f)   0.157

2.   Write each list of numbers in order with the smallest first:

(a)   0.61, 0.16, 0.601, 0.106, 0.661, 0.616

(b)   0.47, 0.82, 0.4, 0.78, 0.28

(c)   0.32, 0.23, 0.2, 0.301, 0.3

(d)   0.17, 0.19, 0.9, 0.91, 0.79

3. Write each of these decimals as a fraction, giving them in their simplest form:

    (a) 0.1   (b) 0.9   (c) 0.3

    (d) 0.07   (e) 0.25   (f) 0.001

    (g) 0.05   (h) 0.003   (i) 0.017

    (j) 0.71   (k) 0.87   (l) 0.201

4. Write each of these decimals as a fraction and simplify where possible:

    (a) 0.4   (b) 0.08   (c) 0.54

    (d) 0.006   (e) 0.012   (f) 0.162

    (g) 0.048   (h) 0.84   (i) 0.328

    (j) 0.014   (k) 0.006   (l) 0.108

5. Write down the missing numbers:

    (a) $0.6 = \dfrac{?}{5}$ (b) $0.14 = \dfrac{?}{50}$ (c) $0.18 = \dfrac{?}{50}$

    (d) $0.008 = \dfrac{?}{125}$ (e) $0.012 = \dfrac{?}{250}$ (f) $0.016 = \dfrac{?}{125}$

6. Write these numbers as *improper fractions* in their simplest form:

    (a) 1.2   (b) 3.02   (c) 4.12

    (d) 3.62   (e) 4.008   (f) 5.015

7. Calculate, giving your answers as decimals *and* as fractions:

    (a) 0.7 + 0.6   (b) 0.8 − 0.3

    (c) 0.71 + 0.62   (d) 8.21 − 0.31

    (e) 0.06 + 0.3   (f) 1.7 + 0.21

    (g) 8.06 − 0.2   (h) 0.42 − 0.002

8. Write the missing numbers as decimals and convert them to fractions in their simplest form:

    (a) 0.20 + ? = 0.81   (b) 0.42 + ? = 0.53

    (c) 0.91 − ? = 0.47   (d) 0.92 − ? = 0.58

9. Convert these decimals to fractions:

    (a)   0.0001   (b)   0.0009

    (c)   0.00021   (d)   0.123491

10. Convert these decimals to fractions in their simplest form:

    (a)   0.00008   (b)   0.02222

    (c)   0.00102   (d)   0.000004

    (e)   0.000224   (f)   0.0000002

17.2 Conversion: Fractions into Decimals

In this section we consider how to write fractions as decimals.

Example 1

Write these fractions as decimals:

(a) $\dfrac{7}{10}$ (b) $\dfrac{81}{100}$ (c) $\dfrac{9}{1000}$ (d) $\dfrac{407}{1000}$

Solution

(a) $\dfrac{7}{10} = 0.7$

(b) $\dfrac{81}{100} = 0.81$

(c) $\dfrac{9}{1000} = 0.009$

(d) $\dfrac{407}{1000} = 0.407$

Example 2

Write these fractions as decimals:

(a) $\dfrac{2}{5}$ (b) $\dfrac{3}{50}$ (c) $\dfrac{6}{25}$ (d) $\dfrac{5}{4}$ (e) $\dfrac{7}{250}$

Solution

In each case, determine the equivalent fraction with the denominator as either 10, 100 or 1000. The fractions can then be written as decimals.

(a) $\dfrac{2}{5} = \dfrac{4}{10} = 0.4$

(b) $\dfrac{3}{50} = \dfrac{6}{100} = 0.06$

(c) $\dfrac{6}{25} = \dfrac{24}{100} = 0.24$

(d) $\dfrac{5}{4} = \dfrac{125}{100} = 1.25$

(e) $\dfrac{7}{250} = \dfrac{28}{1000} = 0.028$

Example 3

(a) Calculate $18 \div 5$, then write $\dfrac{18}{5}$ as a decimal.

(b) Calculate $5 \div 8$, then write $\dfrac{5}{8}$ as a decimal.

Solution

(a) $18 \div 5 = 3.6$, since

$$5 \overline{\smash{\big)}\, 18.{}^{3}0} \quad \overset{3.6}{}$$

So $\dfrac{18}{5} = 18 \div 5$

$= 3.6$

(b) $5 \div 8 = 0.625$, since

$$8 \overline{\smash{\big)}\, 5.0{}^{2}0{}^{4}0} \quad \overset{0.625}{}$$

So $\dfrac{5}{8} = 5 \div 8$

$= 0.625$

Exercises

1. Write these fractions as decimals:

   (a) $\dfrac{3}{10}$ (b) $\dfrac{7}{100}$ (c) $\dfrac{9}{1000}$

   (d) $\dfrac{13}{100}$ (e) $\dfrac{131}{1000}$ (f) $\dfrac{47}{1000}$

   (g) $\dfrac{21}{100}$ (h) $\dfrac{183}{1000}$ (i) $\dfrac{19}{100}$

   (j) $\dfrac{19}{1000}$ (k) $\dfrac{11}{100}$ (l) $\dfrac{81}{1000}$

2. Calculate the missing numbers:

   (a) $\dfrac{?}{2} = \dfrac{5}{10}$ (b) $\dfrac{?}{20} = \dfrac{35}{100}$

   (c) $\dfrac{?}{25} = \dfrac{8}{100}$ (d) $\dfrac{?}{4} = \dfrac{25}{100}$

   (e) $\dfrac{2}{?} = \dfrac{4}{100}$ (f) $\dfrac{6}{?} = \dfrac{12}{1000}$

   (g) $\dfrac{8}{?} = \dfrac{32}{100}$ (h) $\dfrac{7}{?} = \dfrac{28}{100}$

3. Write these fractions as decimals:

   (a) $\dfrac{1}{2}$ (b) $\dfrac{4}{5}$ (c) $\dfrac{9}{50}$

   (d) $\dfrac{3}{25}$ (e) $\dfrac{3}{20}$ (f) $\dfrac{3}{500}$

   (g) $\dfrac{1}{250}$ (h) $\dfrac{7}{20}$ (i) $\dfrac{61}{200}$

   (j) $\dfrac{18}{25}$ (k) $\dfrac{9}{125}$ (l) $\dfrac{1}{4}$

4. Write these improper fractions as decimals:

   (a) $\dfrac{12}{10}$ (b) $\dfrac{212}{100}$ (c) $\dfrac{5218}{1000}$

(d) $\dfrac{2008}{100}$ (e) $\dfrac{2008}{1000}$ (f) $\dfrac{418}{10}$

5. Write these improper fractions as decimals:

(a) $\dfrac{7}{2}$ (b) $\dfrac{21}{20}$ (c) $\dfrac{33}{20}$

(d) $\dfrac{31}{25}$ (e) $\dfrac{16}{5}$ (f) $\dfrac{1001}{500}$

6. Write as a fraction and as a decimal:

(a) $3 \div 5$ (b) $3 \div 8$ (c) $25 \div 4$

(d) $16 \div 5$ (e) $26 \div 4$ (f) $30 \div 8$

7. (a) Calculate $7 \div 8$.

(b) Write $\dfrac{7}{8}$ as a decimal.

8. (a) Calculate $41 \div 5$.

(b) Write $\dfrac{41}{5}$ as a decimal.

9. Write $\dfrac{1}{8}$ as a decimal by using division.

10. Write $\dfrac{13}{16}$ as a decimal.

17.3 Introduction to Percentages

The word 'percentage' means 'per hundred'. In this section we look at how percentages can be used as an alternative to fractions or decimals.

$$100\% \;=\; \frac{100}{100} \;=\; 1$$

$$50\% \;=\; \frac{50}{100} \;=\; \frac{1}{2}$$

$$1\% \;=\; \frac{1}{100}$$

Example 1

Draw diagrams to show:

(a)   71%

(b)   20%

(c)   5%

Solution

These percentages can be shown by shading a suitable fraction of a 10 by 10 square shape.

(a) $71\% = \dfrac{71}{100}$, so $\dfrac{71}{100}$ of a shape needs to be shaded:
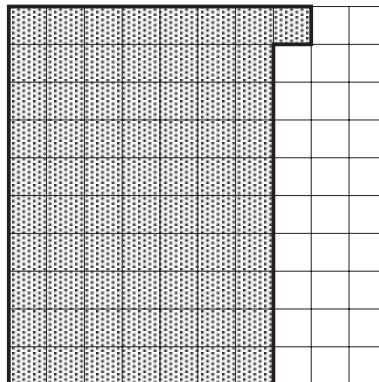
(b) $20\% = \dfrac{20}{100} = \dfrac{1}{5}$, so $\dfrac{1}{5}$ of a shape needs to be shaded:
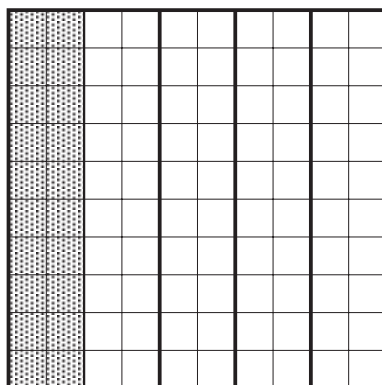
(c) $5\% = \dfrac{5}{100} = \dfrac{1}{20}$, so $\dfrac{1}{20}$ of a shape needs to be shaded:
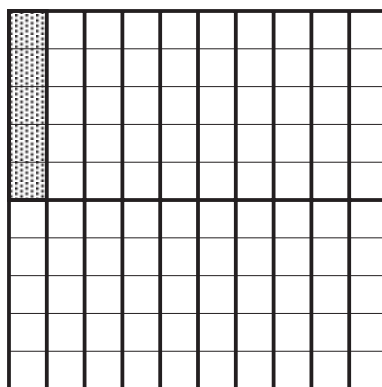
Example 2

(a)   What percentage of this shape is shaded?

(b)   What percentage of this shape is *not* shaded?

Solution

(a) $\dfrac{3}{5}$ of the shape is shaded, and

$$\dfrac{3}{5} = \dfrac{6}{10}$$

$$= \dfrac{60}{100},$$

so 60% is shaded.

(b) $100 - 60 = 40$,   so 40% is *not* shaded.

Example 3

Find:

(a)   5% of 100 kg,

(b)   20% of 40 m,

(c)   25% of £80.

Solution

(a)   5% of 100 kg $= \dfrac{5}{100} \times 100$

$= \dfrac{1}{20} \times 100$

$= 5 \text{ kg}$

(b)   20% of 40 m $= \dfrac{20}{100} \times 40$

$= \dfrac{1}{5} \times 40$

$= 8 \text{ m}$

(c)   25% of £80 $= \dfrac{25}{100} \times 80$

$= \dfrac{1}{4} \times 80$

$= £20$

Exercises

1.   For each diagram, state the percentage that is shaded:

(a)
(b)

(c)

(d)

(e)

(f)
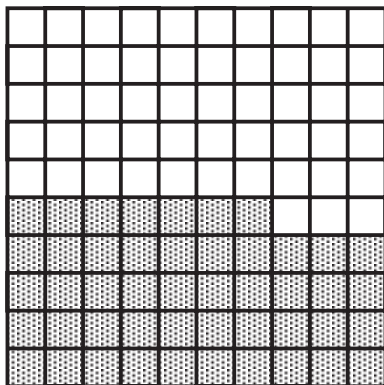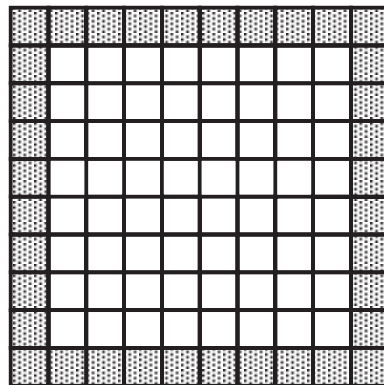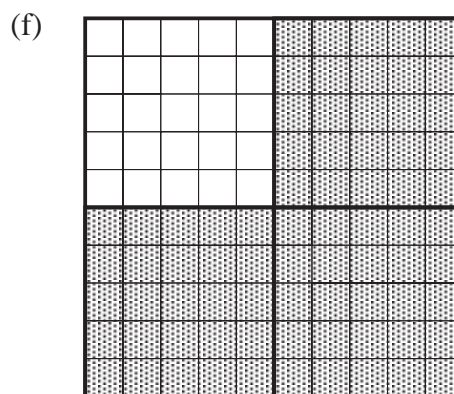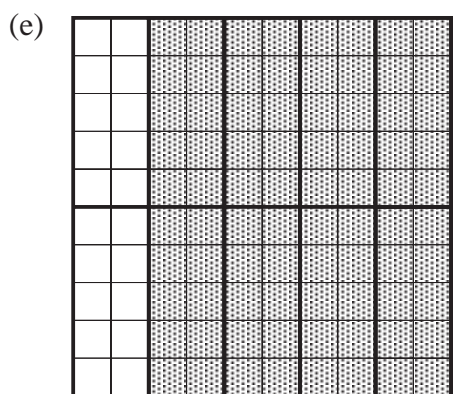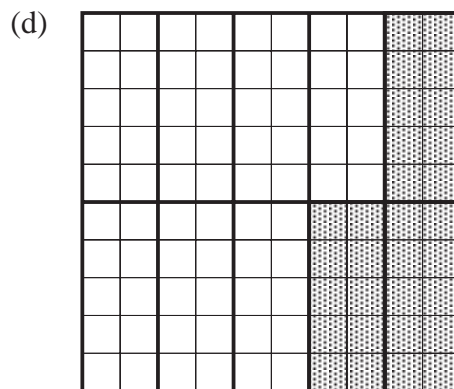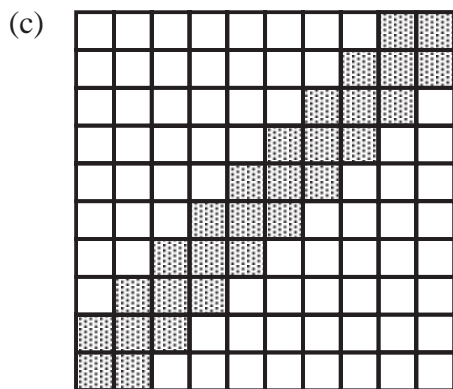
2.   For each diagram in question 1, state the percentage that is *not* shaded.

3.   If 76% of a rectangle is shaded, what percentage is *not* shaded?
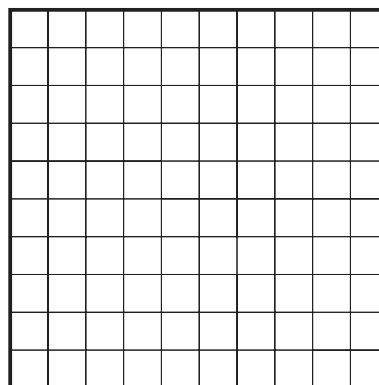
4.   Make 4 copies of this diagram
      and shade the percentage stated:
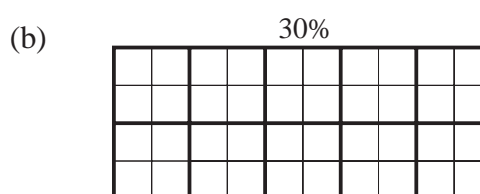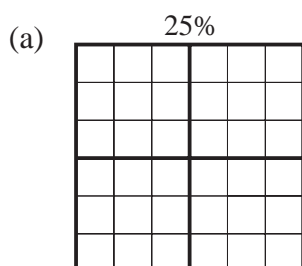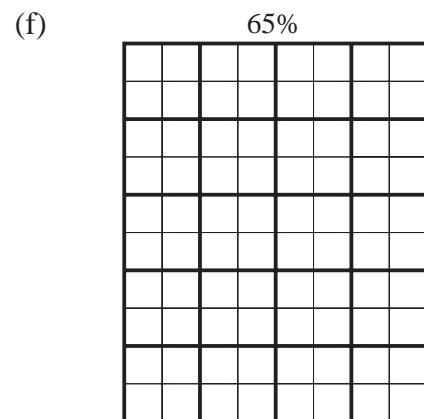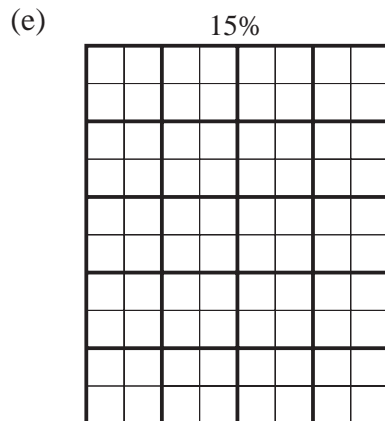
      (a)   23%

      (b)   50%

      (c)   79%

      (d)   87%

5.   Copy each diagram and shade the percentage stated:

      (a)   25%

      (b)   30%

(c)   90%

(d)   5%

(e)   15%

(f)   65%
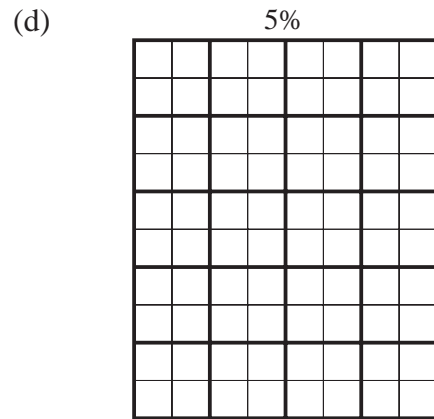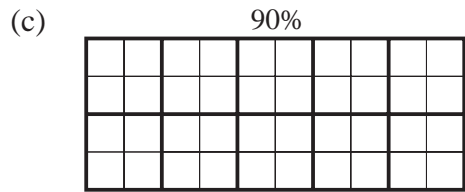
6.   State the shaded percentage of each shape:

(a)   (b)   (c)

(d)   (e)   (f)   (g)
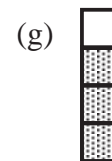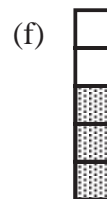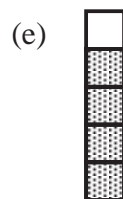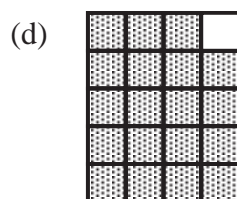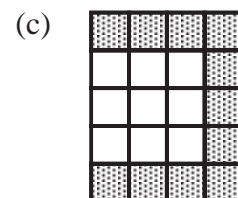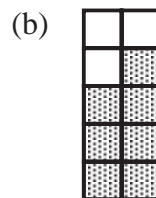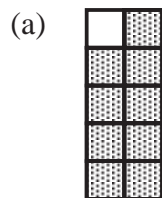
7.   If 35% of a class are girls, what percentage are boys?

8.   If 88% of a class pass a maths test, what percentage fail the test?

9.   Calculate:

(a)   50% of £200,   (b)   30% of 500 kg,

(c)   60% of 50p,   (d)   5% of £2,

(e)   15% of 10 kg,   (f)   25% of 120 m,

(g)   2% of £400,   (h)   26% of £2,

(i)   20% of £300,   (j)   75% of 200 kg.

10.   Ben and Adam spend their Saturdays cleaning cars. They agree that Adam
       will have 60% of the money they earn and that Ben will have the rest.

       (a)   What percentage of the money will Ben have?

       (b)   How much do they each have if they earn £25?

       (c)   How much do they each have if they earn £30?

17.4 Decimals, Fractions and Percentages

In this section we concentrate in converting between decimals, fractions and
percentages.

Example 1

Write these percentages as decimals:

(a)   72%   (b)   3%

Solution

(a)   72% $= \dfrac{72}{100}$

            $= 0.72$

(b)   3% $= \dfrac{3}{100}$

            $= 0.03$

Example 2

Write these decimals as percentages:

(a)   0.71   (b)   0.4   (c)   0.06

Solution

(a)   0.71 $= \dfrac{71}{100}$

             $= 71\%$

(b) 0.4 $= \dfrac{4}{10}$

$= \dfrac{40}{100}$

$= 40\%$

(c) 0.06 $= \dfrac{6}{100}$

$= 6\%$

Example 3

Write these percentages as fractions in their simplest possible form:

(a)   90%   (b)   20%   (c)   5%

Solution

(a) 90% $= \dfrac{90}{100}$

$= \dfrac{9}{10}$

(b) 20% $= \dfrac{20}{100}$

$= \dfrac{1}{5}$

(c) 5% $= \dfrac{5}{100}$

$= \dfrac{1}{20}$

Example 4

Write these fractions as percentages:

(a) $\dfrac{1}{2}$ (b) $\dfrac{2}{5}$ (c) $\dfrac{7}{20}$

Solution

(a) $\dfrac{1}{2}$ $= \dfrac{50}{100}$

$= 50\%$

(b) $\dfrac{2}{5} = \dfrac{40}{100}$

$\quad = 40\%$

(c) $\dfrac{7}{20} = \dfrac{35}{100}$

$\quad = 35\%$

Exercises

1. Write these percentages as decimals:

    (a)   42%   (b)   37%   (c)   20%

    (d)   5%   (e)   8%   (f)   10%

    (g)   22%   (h)   3%   (i)   15%

2. Write these decimals as percentages:

    (a)   0.14   (b)   0.72   (c)   0.55

    (d)   0.4   (e)   0.03   (f)   0.9

    (g)   0.18   (h)   0.04   (i)   0.7

3. Write these percentages as fractions in their simplest forms:

    (a)   50%   (b)   30%   (c)   80%

    (d)   70%   (e)   15%   (f)   25%

    (g)   64%   (h)   98%   (i)   56%

4. Write these fractions as percentages:

    (a) $\dfrac{7}{100}$ (b) $\dfrac{18}{100}$ (c) $\dfrac{3}{50}$

    (d) $\dfrac{17}{50}$ (e) $\dfrac{3}{20}$ (f) $\dfrac{7}{25}$

    (g) $\dfrac{3}{5}$ (h) $\dfrac{7}{10}$ (i) $\dfrac{3}{4}$

    (j) $\dfrac{1}{20}$ (k) $\dfrac{1}{2}$ (l) $\dfrac{3}{25}$

5.   Copy and complete this table:

Fraction	Decimal	Percentage
	0.04	
		10%
$\frac{1}{2}$		
		45%
$\frac{7}{50}$		
	0.84	

6.   There are 200 children in a school hall, eating lunch. Of these children, 124 have chosen chips as part of their lunch.

      (a)   What *fraction* of the children have chosen chips?

      (b)   What *percentage* of the children have chosen chips?

      (c)   What percentage of the children have *not* chosen chips?

7.   In a survey, $\frac{9}{10}$ of the children in a school said that maths was their favourite subject. What percentage of the children *did not* say that maths was their favourite subject?

8.   In a Year 7 class, $\frac{3}{4}$ of the children can swim more than 400 m and only $\frac{1}{10}$ of the children can not swim more than 200 m.

      What percentage of the class can swim:

      (a)   more than 400 m,

      (b)   less than 200 m,

      (c)   a distance between 200 m and 400 m?

9.   In the school canteen, children can choose chips, baked potato or rice. One day 50% choose chips and 26% choose baked potatoes.

      (a)   What percentage choose rice?

      (b)   What fraction of the children choose rice?

10.   In a car park, 40% of the cars are red and $\frac{7}{20}$ of the cars are blue.

      (a)   What *percentage* are blue?

      (b)   What *percentage* are *neither* red *nor* blue?

      (c)   What *percentage* are red or blue?

      (d)   What *fraction* are red?

      (e)   What *fraction* are *neither* red *nor* blue?

      (f)   What *fraction* are red or blue?