1.) **METHOD 1**

using double-angle identity (seen anywhere) A1

e.g. $\sin 2x = 2\sin x \cos x$, $2\cos x = 2\sin x \cos x$

evidence of valid attempt to solve equation (M1)

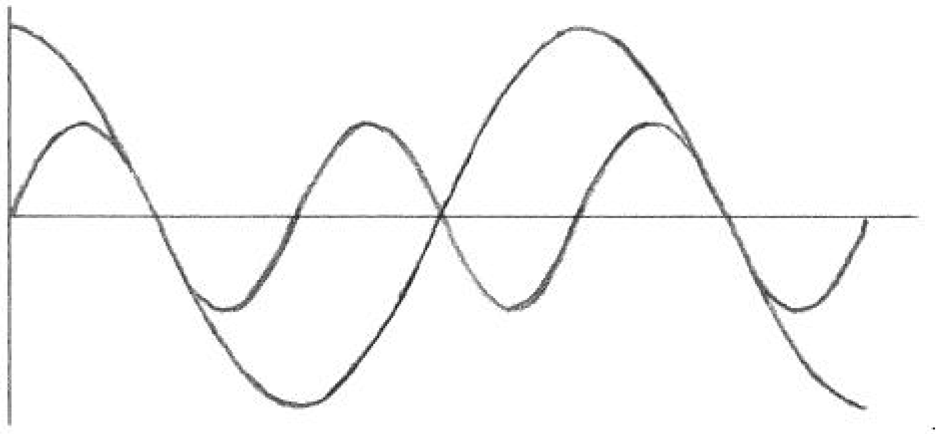
e.g. $0 = 2\sin x \cos x - 2\cos x$, $2\cos x\,(1 - \sin x) = 0$

$\cos x = 0$, $\sin x = 1$ A1A1

$x = \frac{\text{®}}{2}, x = \frac{3\text{®}}{2}, x = \frac{5\text{®}}{2}$ A1A1A1 N4

[7]

METHOD 2



A1A1M1A1

Notes: *Award A1 for sketch of sin 2x, A1 for a sketch of 2 cos x, M1 for at least one intersection point seen, and A1 for 3 approximately correct intersection points. Accept sketches drawn outside [0, 3π], even those with more than 3 intersections.*

$x = \frac{\pi}{2}, x = \frac{3\pi}{2}, x\frac{5\pi}{2}$ A1A1A1 N4

[7]

2.) (a) $\tan\theta = \frac{3}{4}\left(\text{do not accept } \frac{3}{4}x\right)$ A1 N1

(b) (i) $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ (A1)(A1)

correct substitution A1

e.g. $\sin 2\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$\sin 2\theta = \frac{24}{25}$ A1 N3

(ii) correct substitution A1

e.g. $\cos 2\theta = 1 - 2\left(\frac{3}{5}\right)^2, \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$\cos 2\theta = \frac{7}{25}$ A1 N1

[7]

3.) (a) attempt to substitute $1 - 2\sin^2\theta$ for $\cos 2\theta$ (M1)

correct substitution A1

e.g. $4 - (1 - 2\sin^2\theta) + 5\sin\theta$

$4 - \cos 2\theta + 5\sin\theta = 2\sin^2\theta + 5\sin\theta + 3$ AG N0

(b) evidence of appropriate approach to solve (M1)

e.g. factorizing, quadratic formula

correct working A1

e.g. $(2\sin\theta + 3)(\sin\theta + 1)$, $(2x + 3)(x + 1) = 0$, $\sin x = \frac{-5 \pm \sqrt{1}}{4}$

correct solution $\sin\theta = -1$ $\left(\text{do not penalise for including } \sin^{-} = -\frac{3}{2}\right)$ (A1)

$\theta = \frac{3\pi}{2}$ A2N3

[7]

4.) evidence of substituting for $\cos 2x$ (M1)

evidence of substituting into $\sin^2 x + \cos^2 x = 1$ (M1)

correct equation in terms of $\cos x$ (seen anywhere) A1

e.g. $2\cos^2 x - 1 - 3\cos x - 3 = 1$, $2\cos^2 x - 3\cos x - 5 = 0$

evidence of appropriate approach to solve (M1)

e.g. factorizing, quadratic formula

appropriate working A1

e.g. $(2\cos x - 5)(\cos x + 1) = 0$, $(2x - 5)(x + 1)$, $\cos x = \frac{3 \pm \sqrt{49}}{4}$

correct solutions to the equation

e.g. $\cos x = \frac{5}{2}$, $\cos x = -1$, $x = \frac{5}{2}$, $x = -1$ (A1)

$x = \pi$ A1 N4

[7]

5.) (a) changing $\tan x$ into $\frac{\sin x}{\cos x}$ A1 *e.g.* $\sin^3 x + \cos^3 x \dfrac{\sin x}{\cos x}$

simplifying A1

e.g. $\sin x\,(\sin^2 x + \cos^2 x)$, $\sin^3 x + \sin x - \sin^3 x$

$f(x) = \sin x$ AG N0

(b) recognizing $f(2x) = \sin 2x$, seen anywhere (A1)

evidence of using double angle identity $\sin(2x) = 2\sin x\cos x$, seen anywhere (M1)

evidence of using Pythagoras with $\sin x = \dfrac{2}{3}$ M1

e.g. sketch of right triangle, $\sin^2 x + \cos^2 x = 1$

$\cos x = -\dfrac{\sqrt{5}}{3}\left(\text{accept } \dfrac{\sqrt{5}}{3}\right)$ (A1)

$f(2x) = 2\left(\dfrac{2}{3}\right)\left(-\dfrac{\sqrt{5}}{3}\right)$ A1

$f(2x) = -\dfrac{4\sqrt{5}}{9}$ AG N0

[7]

6.) ***Note**: Throughout this question, do **not** accept methods which involve finding* θ.

(a) Evidence of correct approach A1

eg $\sin\theta = \dfrac{BC}{AB}$, $BC = \sqrt{3^2 - 2^2} = \sqrt{5}$

$\sin\theta = \dfrac{\sqrt{5}}{3}$ AG N0

(b) Evidence of using $\sin 2\theta = 2\sin\theta\cos\theta$ (M1)

$= 2\left(\dfrac{\sqrt{5}}{3}\right)\left(\dfrac{2}{3}\right)$ A1

$= \dfrac{4\sqrt{5}}{9}$ AG N0

(c) Evidence of using an appropriate formula for $\cos 2\theta$ M1

eg $\dfrac{4}{9} - \dfrac{5}{9}$, $2\times\dfrac{4}{9} - 1$, $1 - 2\times\dfrac{5}{9}$, $\sqrt{\left(1 - \dfrac{80}{81}\right)}$

$\cos 2\theta = -\dfrac{1}{9}$ A2 N2

[6]

7.) (a) **METHOD 1**

Using the discriminant $\Delta = 0$ (M1)

$k^2 = 4 \times 4 \times 1$

$k = 4$, $k = -4$ A1A1 N3

METHOD 2

Factorizing (M1)

$(2x \pm 1)^2$

$k = 4$, $k = -4$ A1A1 N3

(b) Evidence of using $\cos 2\theta = 2\cos^2\theta - 1$ M1

eg $2(2\cos^2\theta - 1) + 4\cos\theta + 3$

$f(\theta) = 4\cos^2\theta + 4\cos\theta + 1$ AG N0

(c) (i) 1 A1 N1

(ii) **METHOD 1**

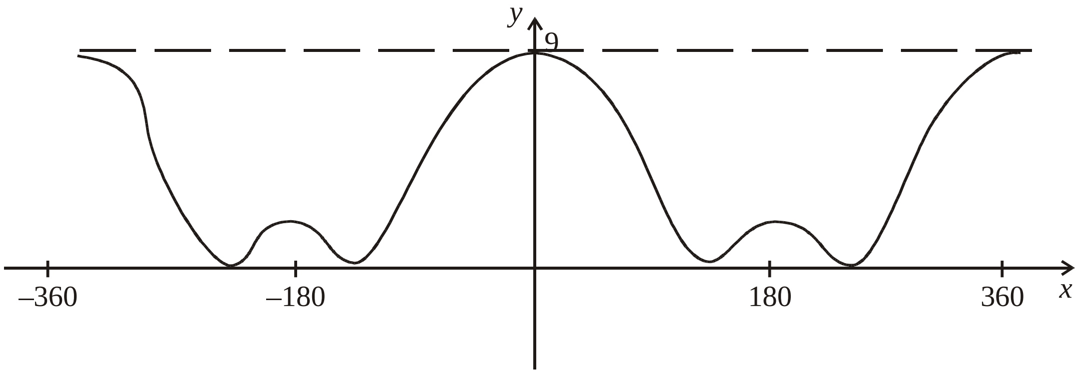
Attempting to solve for $\cos\theta$ M1

$\cos\theta = -\frac{1}{2}$ (A1)

$\theta = 240, 120, -240, -120$ (correct four values only) A2 N3

METHOD 2

Sketch of $y = 4\cos^2\theta + 4\cos\theta + 1$ M1



Indicating 4 zeros (A1)

$\theta = 240, 120, -240, -120$ (correct four values only) A2 N3

(d) Using sketch (M1)

$c = 9$ A1 N2

[11]

8.) (a) Vertex is (4, 8) A1A1 N2

(b) Substituting $-10 = a(7-4)^2 + 8$ M1

$a = -2$ A1 N1

(c) For y-intercept, $x = 0$ (A1)

$y = -24$ A1 N2

[6]

9.) (a) Evidence of choosing the double angle formula (M1)

$f(x) = 15 \sin (6x)$ A1 N2

(b) Evidence of substituting for $f(x)$ (M1)

eg $15 \sin 6x = 0$, $\sin 3x = 0$ **and** $\cos 3x = 0$

$6x = 0, \pi, 2\pi$

$x = 0, \frac{\pi}{6}, \frac{\pi}{3}$ A1A1A1 N4

[6]

10.) **METHOD 1**

$2 \cos^2 x = 2 \sin x \cos x$ (M1)

$2 \cos^2 x - 2 \sin x \cos x = 0$

$2 \cos x(\cos x - \sin x) = 0$ (M1)

$\cos x = 0$, $(\cos x - \sin x) = 0$ (A1)(A1)

$x = \frac{\pi}{2}, x = \frac{\pi}{4}$ (A1)(A1) (C6)

METHOD 2

Graphical solutions

EITHER

for both graphs $y = 2 \cos^2 x$, $y = \sin 2x$, (M2)

OR

for the graph of $y = 2 \cos^2 x - \sin 2x$. (M2)

THEN

Points representing the solutions clearly indicated (A1)

1.57, 0.785 (A1)

$x = \frac{\pi}{2}, x = \frac{\pi}{4}$ (A1)(A1) (C6)

Notes: *If no working shown, award (C4) for one correct answer.*
Award (C2)(C2) for each correct decimal answer 1.57, 0.785.
Award (C2)(C2) for each correct degree answer 90°, 45°.
Penalize a total of [1 mark] for any additional answers.

[6]

11.) (a) x is an acute angle => $\cos x$ is positive. (M1)

$\cos^2 x + \sin^2 x = 1$ => $\cos x = \sqrt{1 - \sin^2 x}$ (M1)

=> $\cos x = \sqrt{1-\left(\frac{1}{3}\right)^2}$ (A1)

$= \sqrt{\frac{8}{9}}$ $\left(= \frac{2\sqrt{2}}{3}\right)$ (A1) (C4)

(b) $\cos 2x = 1 - 2\sin^2 x = 1 - 2\left(\frac{1}{3}\right)^2$ (M1)

$= \frac{7}{9}$ (A1) (C2)

Notes: *(a) Award (M1)(M0)(A1)(A0) for*

$\cos\left(\sin^{-1}\left(\frac{1}{3}\right)\right) = 0.943$.

(b) Award (M1)(A0) for $\cos\left(2\sin^{-1}\left(\frac{1}{3}\right)\right) = 0.778$.

[6]

12.) (a) $2\sin^2 x = 2(1 - \cos^2 x) = 2 - 2\cos^2 x = 1 + \cos x$ (M1)

=> $2\cos^2 x + \cos x - 1 = 0$ (A1) (C2)

Note: *Award the first (M1) for replacing $\sin^2 x$ by $1 - \cos^2 x$.*

(b) $2\cos^2 x + \cos x - 1 = (2\cos x - 1)(\cos x + 1)$ (A1) (C1)

(c) $\cos x = \frac{1}{2}$ or $\cos x = -1$

=> $x = 60°, 180°$ or $300°$ (A1)(A1)(A1) (C3)

Note: *Award (A1)(A1)(A0) if the correct answers are given in radians (ie $\frac{\text{®}}{3}$, p , $\frac{5\text{®}}{3}$, or 1.05, 3.14, 5.24)*

[6]

13.) (a) $3\sin^2 x + 4\cos x = 3(1 - \cos^2 x) + 4\cos x$

$= 3 - 3\cos^2 + 4\cos x$ (A1) (C1)

(b) $3\sin^2 x + 4\cos x - 4 = 0 \Rightarrow 3 - 3\cos^2 x + 4\cos x - 4 = 0$

$\Rightarrow 3\cos^2 x - 4\cos x + 1 = 0$ (A1)

$(3\cos x - 1)(\cos x - 1) = 0$

$\cos x = \frac{1}{3}$ or $\cos x = 1$

$x = 70.5°$ or $x = 0°$ (A1)(A1) (C3)

Note: *Award (C1) for each correct radian answer, ie $x = 1.23$ or $x = 0$.*

[4]

14.) (a) $2\cos^2 x + \sin x = 2(1 - \sin^2 x) + \sin x$

$= 2 - 2\sin^2 x + \sin x$ (A1)

(b) $2\cos^2 x + \sin x = 2$

$\Rightarrow 2 - 2\sin^2 x + \sin x = 2$

$\sin x - 2\sin^2 x = 0$

$\sin x(1 - 2\sin x) = 0$

$\sin x = 0$ or $\sin x = \frac{1}{2}$ (M1)

$\sin x = 0 \Rightarrow x = 0$ or π (0° or 180°) (A1)

Note: *Award (A1) for both answers.*

$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ (30° or 150°) (A1)

Note: *Award (A1) for both answers.*

[4]

15.) $\sin A = \frac{5}{13} \Rightarrow \cos A = \pm\frac{12}{13}$ (A1)

But A is obtuse $\Rightarrow \cos A = -\frac{12}{13}$ (A1)

$\sin 2A = 2\sin A\cos A$ (M1)

$= 2 \times \frac{5}{13} \times \left(-\frac{12}{13}\right)$

$= -\frac{120}{169}$ (A1) (C4)

[4]