3.   *[Maximum mark: 7]*

Let $f(x) = 3x - e^{x-2} - 4$, for $-1 \le x \le 5$.

(a)   Find the *x*-intercepts of the graph of f.   *[3 marks]*

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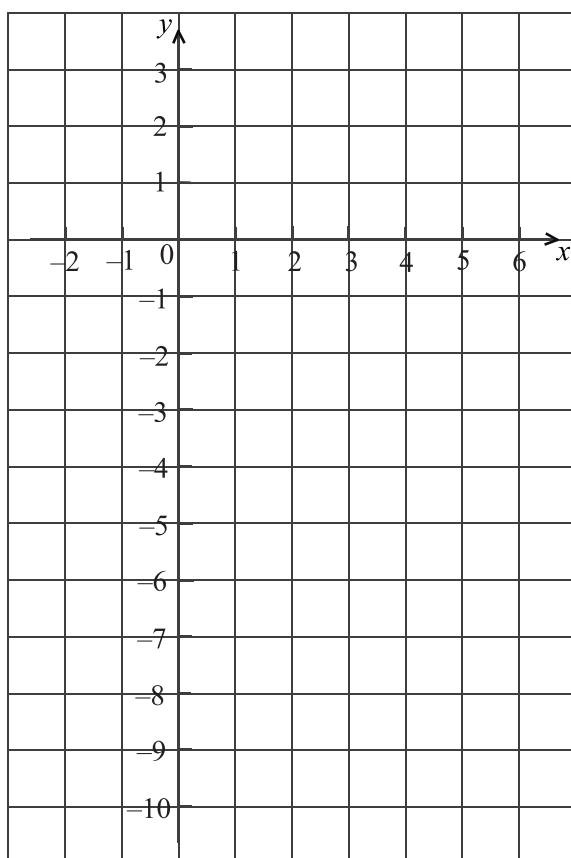
(b)   On the grid below, sketch the graph of f.   *[3 marks]*

(This question continues on the following page)

10.   *[Maximum mark: 17]*

A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After n years the number of taxis, T, in the city is given by

$$T = 280 \times 1.12^n.$$

(a)   (i)   Find the number of taxis in the city at the end of 2005.

   (ii)   Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.   *[6 marks]*

(b)   At the end of 2000 there were 25 600 people in the city who used taxis. After n years the number of people, P, in the city who used taxis is given by

$$P = \frac{2\,560\,000}{10 + 90e^{-0.1n}}.$$

   (i)   Find the value of P at the end of 2005, giving your answer to the nearest whole number.

   (ii)   After seven complete years, will the value of P be double its value at the end of 2000? Justify your answer.   *[6 marks]*

(c)   Let R be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$.

   (i)   Find the value of R at the end of 2000.

   (ii)   After how many complete years will the city first reduce the number of taxis?   *[5 marks]*

7. *[Maximum mark: 8]*

The number of bacteria, n, in a dish, after t minutes is given by $n = 800e^{0.13t}$.

(a) Find the value of n when $t = 0$. *[2 marks]*

(b) Find the rate at which n is increasing when $t = 15$. *[2 marks]*

(c) After k minutes, the rate of increase in n is greater than $10\,000$ bacteria per minute. Find the least value of k, where $k \in \mathbb{Z}$. *[4 marks]*

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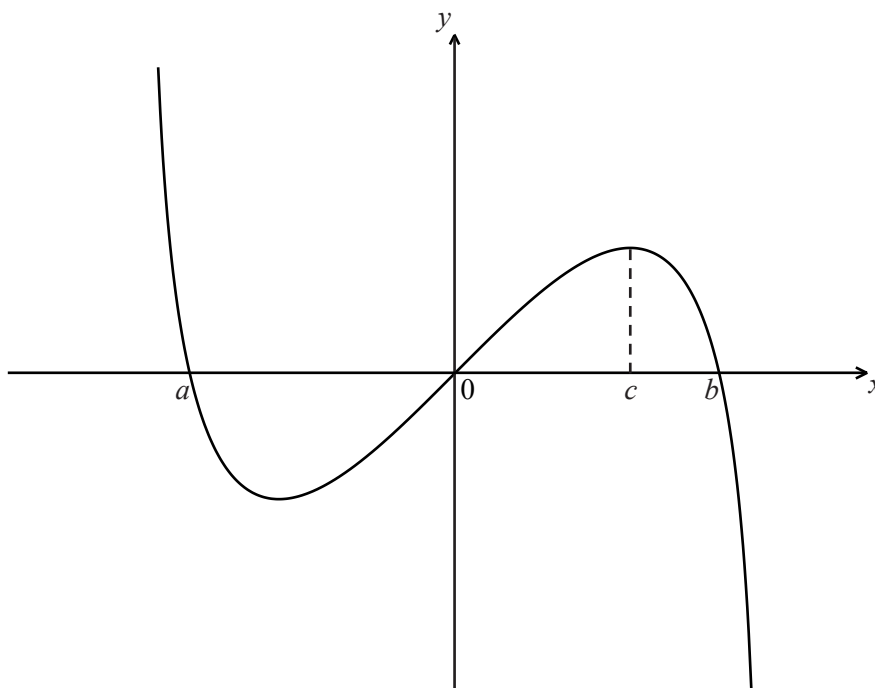
*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8.   *[Maximum mark: 12]*

Let $f(x) = x \ln(4 - x^2)$, for $-2 < x < 2$. The graph of f is shown below.

The graph of f crosses the x-axis at $x = a$, $x = 0$ and $x = b$.

(a)   Find the value of a and of b.   *[3 marks]*

$$x = c$$

$$x = 0 \qquad x = c \qquad\qquad 360°$$

$$x = c$$

$$x = a \qquad x = c$$

2.   *[Maximum mark: 6]*

Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down. The graph of g is the image of the graph of f after this translation.

(a)   Write down the coordinates of the vertex of the graph of g.   *[2 marks]*

(b)   Express g in the form $g(x) = 3(x - p)^2 + q$.   *[2 marks]*

The graph of h is the reflection of the graph of g in the x-axis.

(c)   Write down the coordinates of the vertex of the graph of h.   *[2 marks]*

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10. *[Maximum mark: 16]*

Let $f(x) = \log_3 \dfrac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

(a) Show that $f(x) = \log_3 2x$. *[2 marks]*

(b) Find the value of $f(0.5)$ and of $f(4.5)$. *[3 marks]*

The function f can also be written in the form $f(x) = \dfrac{\ln ax}{\ln b}$.

(c) (i) Write down the value of a and of b.

    (ii) Hence on graph paper, **sketch** the graph of f, for $-5 \le x \le 5$, $-5 \le y \le 5$, using a scale of 1 cm to 1 unit on each axis.

    (iii) Write down the equation of the asymptote. *[6 marks]*

(d) Write down the value of $f^{-1}(0)$. *[1 mark]*

The point A lies on the graph of f. At A, $x = 4.5$.

(e) On your diagram, sketch the graph of f^{-1}, noting clearly the image of point A. *[4 marks]*

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided.*

1.   *[Maximum mark: 5]*

Let $f(x) = 3x$, $g(x) = 2x - 5$ and $h(x) = (f \circ g)(x)$.

(a)   Find $h(x)$.   *[2 marks]*

(b)   Find $h^{-1}(x)$.   *[3 marks]*

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1.   *[Maximum mark: 7]*

Let $f(x) = 2x + 4$ and $g(x) = 7x^2$.

(a)   Find $f^{-1}(x)$.   *[3 marks]*

(b)   Find $(f \circ g)(x)$.   *[2 marks]*

(c)   Find $(f \circ g)(3.5)$.   *[2 marks]*

6. *[Maximum mark: 8]*

Jose takes medication. After t minutes, the concentration of medication left in his bloodstream is given by $A(t) = 10(0.5)^{0.014t}$, where A is in milligrams per litre.

(a)   Write down $A(0)$.   *[1 mark]*

(b)   Find the concentration of medication left in his bloodstream after 50 minutes.   *[2 marks]*

(c)   At 13:00, when there is no medication in Jose's bloodstream, he takes his first dose of medication. He can take his medication again when the concentration of medication reaches 0.395 milligrams per litre. What time will Jose be able to take his medication again?   *[5 marks]*

7. *[Maximum mark: 7]*

Let $f(t) = 2t^2 + 7$, where $t > 0$. The function v is obtained when the graph of f is transformed by

> a stretch by a scale factor of $\dfrac{1}{3}$ parallel to the y-axis,
>
> followed by a translation by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

(a) Find $v(t)$, giving your answer in the form $a(t-b)^2 + c$. *[4 marks]*

(b) A particle moves along a straight line so that its velocity in ms^{-1}, at time t seconds, is given by v. Find the distance the particle travels between $t = 5.0$ and $t = 6.8$. *[3 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

Consider the function $f(x) = x^2 - 4x + 1$.

(a) Sketch the graph of f, for $-1 \le x \le 5$. *[4 marks]*

This function can also be written as $f(x) = (x - p)^2 - 3$.

(b) Write down the value of p. *[1 mark]*

The graph of g is obtained by reflecting the graph of f in the x-axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

(c) Show that $g(x) = -x^2 + 4x + 5$. *[4 marks]*

The graphs of f and g intersect at two points.

(d) Write down the x-coordinates of these two points. *[3 marks]*

Let R be the region enclosed by the graphs of f and g.

(e) Find the area of R. *[3 marks]*

6.   *[Maximum mark: 6]*

Let f and g be functions such that $g(x) = 2f(x+1) + 5$.

(a)   The graph of f is mapped to the graph of g under the following transformations:

vertical stretch by a factor of k, followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k;

(ii) p;

(iii) q.   *[3 marks]*

(b)   Let $h(x) = -g(3x)$. The point $A(6, 5)$ on the graph of g is mapped to the point A' on the graph of h. Find A'.   *[3 marks]*

10. *[Maximum mark: 14]*

Let $f(x) = \dfrac{3x}{x-q}$, where $x \neq q$.

(a) Write down the equations of the vertical and horizontal asymptotes of the graph of f. *[2]*

The vertical and horizontal asymptotes to the graph of f intersect at the point $Q(1, 3)$.

(b) Find the value of q. *[2]*

(c) The point $P(x, y)$ lies on the graph of f. Show that $PQ = \sqrt{(x-1)^2 + \left(\dfrac{3}{x-1}\right)^2}$. *[4]*

(d) Hence find the coordinates of the points on the graph of f that are closest to $(1, 3)$. *[6]*

*Do **NOT** write solutions on this page.*

9.   *[Maximum mark: 15]*

Let $f(x) = \cos\left(\dfrac{\pi}{4}x\right) + \sin\left(\dfrac{\pi}{4}x\right)$, for $-4 \leq x \leq 4$.

(a)   Sketch the graph of f.   *[3]*

(b)   Find the values of x where the function is decreasing.   *[5]*

(c)   The function f can also be written in the form $f(x) = a\sin\left(\dfrac{\pi}{4}(x+c)\right)$, where $a \in \mathbb{R}$, and $0 \leq c \leq 2$. Find the value of

(i) a;

(ii) c.   *[7]*

10.   *[Maximum mark: 14]*

Let $f(x) = \dfrac{3x}{x-q}$, where $x \neq q$.

(a)   Write down the equations of the vertical and horizontal asymptotes of the graph of f.   *[2]*

The vertical and horizontal asymptotes to the graph of f intersect at the point $Q(1, 3)$.

(b)   Find the value of q.   *[2]*

(c)   The point $P(x, y)$ lies on the graph of f. Show that $PQ = \sqrt{(x-1)^2 + \left(\dfrac{3}{x-1}\right)^2}$.   *[4]*

(d)   Hence find the coordinates of the points on the graph of f that are closest to $(1, 3)$.   *[6]*