

Lesson 2-9: Volume of Pyramids and Cones

Learning Goal: How do I find the volume of a Pyramid and a Cone?

Volume of Pyramids

The volume of a pyramid can be found using the following formula:

It can also be written as:

$$V = \frac{1}{3} (l \cdot w) H$$



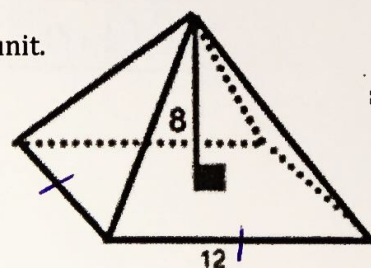
Watch Me!

Example 1: Below is a regular pyramid. Find the volume to the nearest cubic unit.

→ all sides of base are \cong .

$$V = \left(\frac{1}{3}\right) (12 \times 12) (8)$$

$$= \boxed{384 \text{ units}^2}$$



Volume of Cones

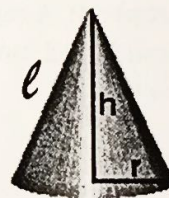
The volume of a cone can be found using the following formula:

$$V = \frac{1}{3} (\pi r^2) H$$

How are cones and pyramids similar? How are they different?

- $\frac{1}{3}$ in both formulas
- base shapes are different -

Cone



h = height (altitude)
r = radius
l = slant height

Example 2: The base of a cone has a radius of 7 inches. The height of the cone is 5 inches. Find the volume of the cone. Round your answer to the nearest 10^{th} of a cubic centimeter.

$$V = \frac{1}{3} (\pi r^2) (H)$$

$$= \left(\frac{1}{3}\right) \pi (7)^2 (5)$$

$$= 256.5634$$

$$= \boxed{256.6 \text{ in}^3}$$

→ (nearest 10^{th} !)



You Try!

Example 3: In the diagram below, a right circular cone has a diameter of 8 inches and a height of 12 inches.

What is the volume of the cone to the *nearest cubic inch*?

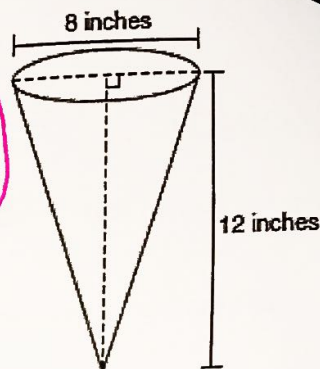
$$V = \frac{1}{3}(\pi r^2)(h)$$

$$= \frac{1}{3} \pi (4)^2 (12)$$

$$= 201.0619298$$

$$= \boxed{201 \text{ in}^3}$$

(nearest whole #.)

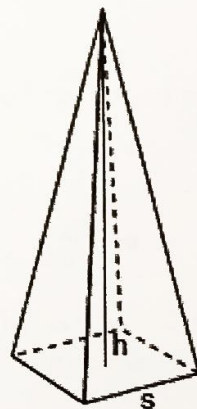


Example 4: A regular pyramid with a square base is shown in the diagram to the left. A side, s , of the base of the pyramid is 12 meters, and the height, h , is 42 meters. What is the volume of the pyramid, to the nearest cubic meter?

$$V = \frac{1}{3}(12 \times 12)(42)$$

$$= \frac{1}{3}(6048)$$

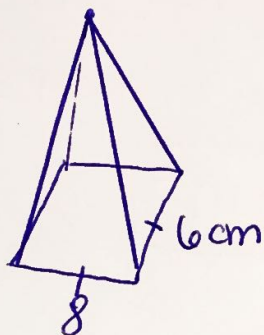
$$= \boxed{2016 \text{ m}^3}$$



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Date:

Lesson 2-9: Homework

1. The base of a pyramid is a rectangle with a width of 6 cm and a length of 8 cm. If the volume is 288cm³, find the height, in centimeters, of the pyramid.



$$V = \frac{1}{3}(l \times w)(h)$$

$$288 = \frac{1}{3}(8 \times 6)(h)$$

$$\frac{288}{16} = \frac{48h}{16}$$

$$h = 18 \text{ cm}$$

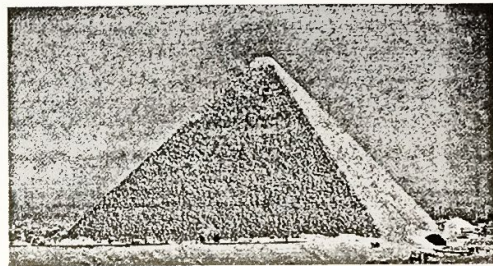
$$V = 288$$

*WORKING BACKWARDS!

2. The largest pyramid in the world was built around 2500 B.C. by Khufu, or Cheops, a king of ancient Egypt. The pyramid had a square base of 230 meters on each side, and a height of 147 meters. Find the volume of Cheops' pyramid.

$$V = \frac{1}{3}(230 \times 230)(147)$$

$$= 2,592,100 \text{ m}^3$$



3. If the volume of a cone is 4000 ft³ and the height is 25.4 ft., find the radius of the base.

$$V = \frac{1}{3}(\pi r^2)h$$

$$4000 = \frac{1}{3}(\pi(r)^2)(25.4)$$

$$\frac{4000}{\left(\frac{1}{3}(25.4)\right)} = \frac{\pi r^2}{\left(\frac{1}{3}(25.4)\right)}$$

$$\frac{472.4409449}{\pi} = \frac{\pi r^2}{\pi}$$

$$\sqrt{150.3826234} = \sqrt{r^2}$$

$$12.2630593 = r$$

$$r = 12.3 \text{ ft}$$

4. Error Analysis -- Consider the following student work:

Question: Larry is shipping a package in a cardboard box that measures 9.5 inches long, 13 inches high and 7 inches wide. He would like to wrap the box with paper. How much paper does Larry need?

Student Answer:

$$\begin{aligned} V &= 9.5 \times 13 \times 7 \\ &= 123.5 \times 7 \\ &= 864 \end{aligned}$$

Larry will need 864.5 square inches of paper to wrap the cardboard box.

a) What mistake(s) did the student make above?

The student found volume but the question asks for amount of wrapping paper. Wrapping paper goes on the outside, which is surface area!

b) Correct the error and perform the question the right way!

Your Correction:

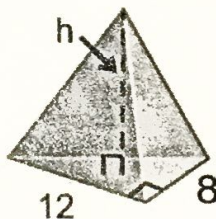
$$\begin{aligned} SA &= 2(lw) + 2(lh) + 2(wh) \\ &= 2(9.5 \times 13) + 2(9.5 \times 7) \\ &\quad + 2(13 \times 7) \\ &= 562 \text{ in}^2 \end{aligned}$$

5. Error Analysis -- Consider the following student work:

Question: The volume of the pyramid is 448 in³, the length of the base is 12 inches and the width of the base is 8 inches. What is the height?

Student Answer:

$$\begin{aligned} V &= \left(\frac{1}{3}\right) l \times w \times h \\ ? &= \left(\frac{1}{3}\right) (12)(8) h \\ . &= 32 h \end{aligned}$$



The height is 32 inches.

a) What mistake(s) did the student make below?

The student did not fill in 448 for "volume" on the left side of the equation.

b) Correct the error and perform the question the right way!

Your Correction:

$$\begin{aligned} V &= \frac{1}{3} (l \times w) \times h \\ 448 &= \frac{1}{3} (12 \times 8) (h) \\ 448 &= 32 h \\ \frac{448}{32} &= \frac{32 h}{32} \\ 14 &= h \\ h &= 14 \text{ in} \end{aligned}$$