Answers

7 Mensuration

7.1 Units and Measuring

1. All ± mm (a) 125 mm (b) 24 mm (c) 70 mm (d) 107 mm (e) 7 mm

2. (a) km or miles (b) cm (c) mg or grams (d) kg (e) ml (f) m^3 or litres

3. (a) 12 300 g (b) 4 700 mm (c) 164 mm (d) 3 400 m (e) 370 cm (f) 6 000 ml

4.

Length in m	*Length in cm*	*Length in mm*
4	400	4 000
3.11	311	3 110
1.5	150	1 500
3.74	374	3 740
8.62	862	8 620

5. (a) 15 (b) 10.3 (c) 130 (d) 45 (e) 56 (f) 18.2 (g) 6.6 (h) 3.4 (i) 11.2 (j) 2.6 (k) 36 (l) 84 (m) 220

6. (a) 850 ml (b) 5

7. (a) cm (b) m (c) m (d) cm/mm (e) m (f) cm/mm (g) mm

8. 42 mm , 56 mm , 21 mm

9. (a) 4 cm (b) 7 cm (c) 11 cm (d) 0 cm (e) 1 cm (f) 1 cm

10. (a) m (b) 10 m

7.2 Estimating Areas

1. (a) 6 cm^2 (b) 10 cm^2 (c) 10 cm^2 (d) 14 cm^2 (e) 14 cm^2 (f) 7 cm^2

2. (a) 16 cm^2 (b) 14 cm^2 (c) 8 cm^2 (d) 12 cm^2 (e) 9 cm^2 (f) 12 cm^2

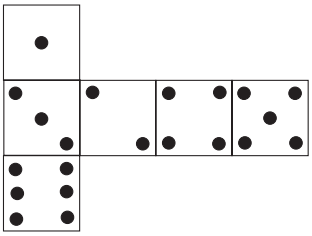
3. (a) 8 cm^2 (b) 9 cm^2 (c) between 10 cm^2 and 12 cm^2 (11 cm^2)
   (d) between 8 cm^2 and 10 cm^2 (9 cm^2)
   (e) between 6 cm^2 and 8 cm^2 (7 cm^2)
   (f) betweeen 16 cm^2 and 18 cm^2 (17 cm^2)
   (g) between 10 cm^2 and 12 cm^2 (11 cm^2)
   (h) between 9 cm^2 and 10 cm^2
   (i) between 9 cm^2 and 11 cm^2 (10 cm^2)

4. The area of each island is between $25\ \text{km}^2$ and $27\ \text{km}^2$
(Remark: The left island is slightly bigger than the right one)

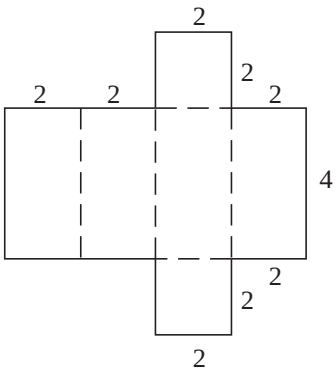
5. $47\ \text{cm}^2$

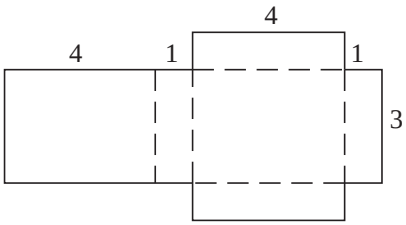
7.3 Making Solids Using Nets

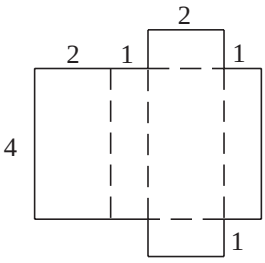
2. (a) square-based pyramid (b) cuboid (c) tetrahedron (d) hexahedron
(e) hexagonal prism (f) octahedron

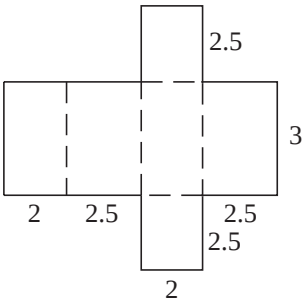
3. 

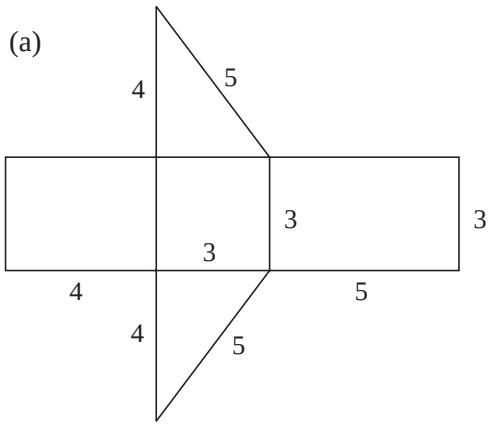
7.4 Constructing Nets

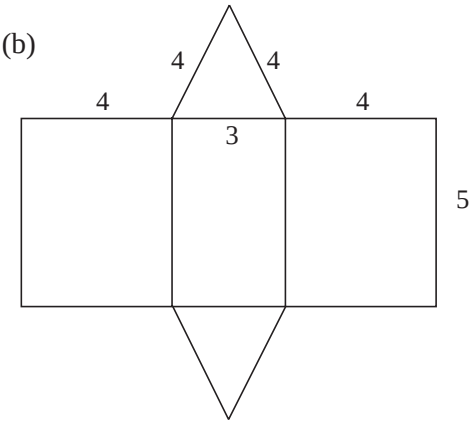
1. (a) 

(b) 

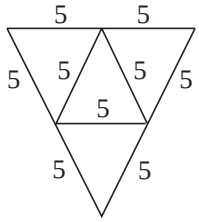
(c) 

(d) 

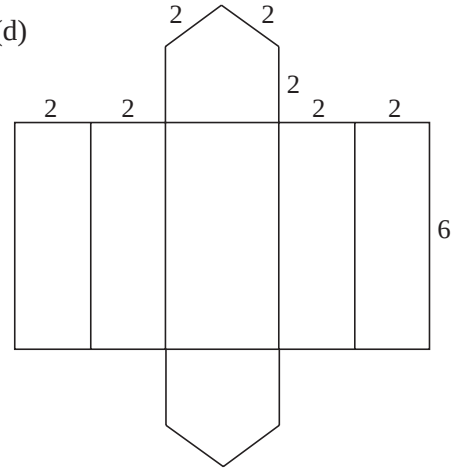
2. (a) 

(b) 

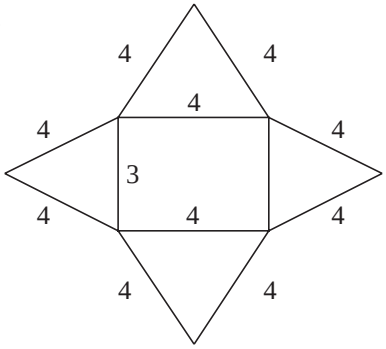
(c)



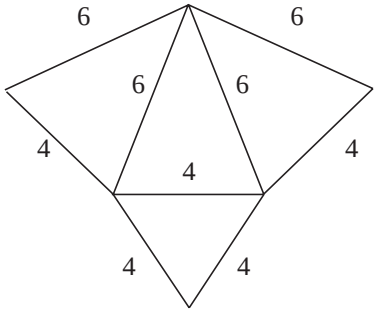
(d)

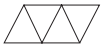


(e)



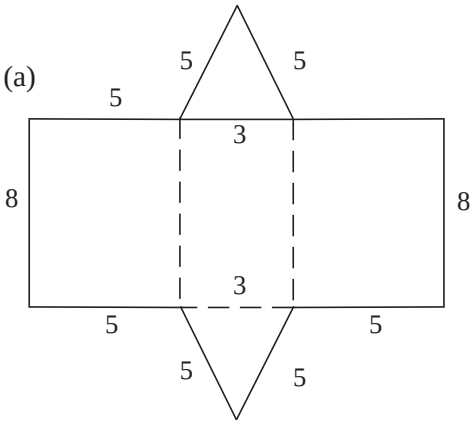
(f)



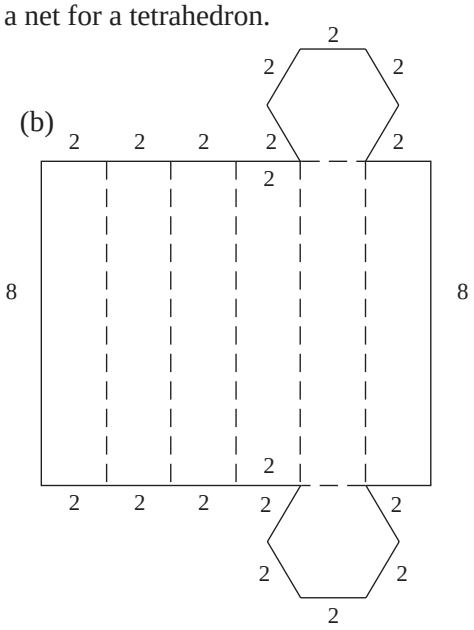
3. (b) Three ways; as in question and  and  .

Only the latter two configurations form a net for a tetrahedron.

4. (a)

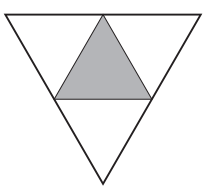


(b)



5. S

6. (a) Equilateral triangle (b) 60° (c)



7.5 Conversion of Units

1. (a) 7.5 cm (b) 252 lbs (c) 96 ounces (d) 75 inches (e) 33 lbs
   (f) 108 inches (g) 90 cm (h) 22.5 litres (i) 300 cm (j) 99 lbs
   (k) 15.75 pints (l) 202.5 litres (m) 14 pints (n) 48 pints

2. (a) 3.6 kg (b) 1.4 kg (c) 9.1 litres (d) 4.0 inches (e) 13.3 feet
   (f) 5.0 lbs (g) 13.0 stones (h) 11.1 gallons (i) 7.0 feet (j) 20.8 inches
   (k) 3.6 gallons (l) 1.7 litres (m) 2.7 kg (n) 7.1 feet

3.

30	Great Yarmouth		Norwich
43	18	Lowestoft	
29	32	14	Beccles

4. (a) 65.6 km (b) 6.56 litres (c) 1.46 gallons

5. 0.43 litres of orange juice 1.36 kg of flour
   0.23 kg of butter 0.91 kg of mixed fruit

6. Total length = 354 inches , 885 cm, 8.85 m

7. James is both taller and heavier than Michael (James is 185 cm tall and weighs over 70 kg).

8. Jane picked the greater weight; she picked about 17.6 lbs.

9. The area of the sheet of glass is 2250 cm^2. Its dimensions are 37.5 cm by 60 cm.

10. The first car consumes 0.1125 litres per km, and the second car consumes 0.12 litres per km. Hence, the first car is the more economical.

11. (a) 48 km (b) $K = \dfrac{M \times 8}{5}$ (c) = 62.5

12. (a) 48 000 g (b) 171 g (to the nearest gram) (c) 105.6 lbs

13. (a) cm (b) litres

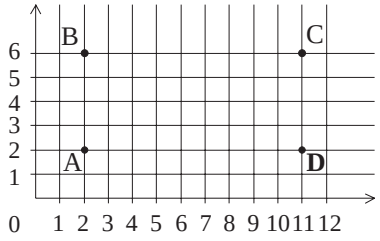
14. (a) 160 cm (to the nearest cm) (b) 63.5 kg

15. (a) 0.2 kg (b) kg/cm^3

16. Yes he will. He needs less than 1 lb of flour for this recipe, and he has more than 1 lb of flour (500 g = $\frac{1}{2}$ kg $\cong$ 1.1 lbs).

7.6 Squares, Rectangles and Triangles

1. (a) 28 cm^2 (b) 12.4 cm^2 (c) 12 cm^2 (d) 9.46 cm^2
   (e) 4.68 cm^2 (f) 14.4 cm^2
2. (a) 14.4 cm, 12.96 cm^2 (b) 22.8 cm, 31.49 cm^2
   (c) 38 cm, 60 cm^2 (d) 44 cm, 80 cm^2 (e) 32 cm, 36 cm^2
   (f) 28 cm, 28 cm^2
3. (a) 100 cm^2 (b) 60.5 cm^2 (c) 63 cm^2 (d) 24 cm^2
4. (a) 42 000 cm^2 (b) 41 blocks 5. $57\frac{1}{2}$ feet 6. 8 m^2
7. 3.99 m^2 + 1.5 m = 5.49 m^2 8. 40 cm^2 9. 1800 cm^2
10. (a) (i) 3.9 m^2 (ii) 5.2 m^2 (b) 4.32 m^2
11. (a) 38 cm (b) 40 cm^2 12. (a) 24 cm (b) 18 cm^2
13. (a) 18 cm^2 (b) 30 cm^2 14. 22 cm^2
15. AB = 7 cm. Perpendicular height from C to AB = 2.4 cm ; area $\approx$ 8.4 cm^2.
16. (a) (6.5, 4)

    (b) 

    D is (11, 2)

    (c) (i) 26 cm (ii) 36 cm^2 (d) Rotational symmetry

7.7 Area and Circumference of Circles

1. (a) C = 31.4 cm (to 1 d.p.)
   A = 78.5 cm^2 (to 1 d.p.)
   (b) C = 1.26 m (to 2 d.p.)
   A = 0.13 m^2 (to 2 d.p.)
   (c) C = 3.77 m (to 2 d.p.)
   A = 1.13 m^2 (to 2 d.p.)
   (d) C = 75.40 cm (to 1 d.p.)
   A = 452.39 cm^2
   (e) C = 8.80 m (to 2 d.p.)
   A = 6.16 m^2 (to 2 d.p.)
   (f) C = 62.83 m (to 2 d.p.)
   A = 314.16 m^2 (to 2 d.p.)
2. (a) r = 6.7 cm (to 1 d.p.) (b) r = 2.9 cm (to 1 d.p.)
   (c) r = 4.7 cm (to 1 d.p.) (d) r = 5.4 cm (to 1 d.p.)
3. (a) 357 m (to the nearest m)
   (b) $\left(\frac{50}{2}\right)^2 \pi + 50 \times 100 = 6963$ m^2 (to the nearest m^2)

4. $(1.8)^2\pi - (0.5)^2\pi \approx 9.39\ \text{cm}^2$

5. (a) 12.57 cm^2 (to 2 d.p.) (b) 78.54 cm^2 (to 2 d.p.)
   (c) 66 cm^2 (to the nearest cm^2)

6. (a) 28.27 cm^2 (b) 37.70 cm^2 7. $10 \times 7 - \left(\frac{5}{2}\right)^2\pi \approx 50.37\ \text{cm}$

8. $16 \times 8 - 2 \times 4^2 \times \pi \approx 27.47\ \text{cm}^2$

9. (a) 61.91 cm^2 (b) $r^2\pi - (4.2)^2 = 50\ \text{cm}^2 \Rightarrow r \approx 4.64\ \text{cm}$

10. (a) 5.78 m^2 (b) 1.6 m, 6.55 m^2

11. 201 cm^2 (to the nearest cm^2) 12. 707 cm^2 (to the nearest cm^2)

13. (a) C = 50π = 157 cm (to the nearest cm)
    (b) 31848 revs (C = 157 cm) or 31831 revs (C = 50π)

14. 16 times

15. (a) 3.82 m (b) 3.82 m is slightly less than 4 m and $\pi = 3.14$ is slightly more than 3 m, hence their product (which gives the length of the rope) is about 12 m.
    (c) 4.52 cm^2

16. (a) 400 m (to the nearest m) (b) 10148.5 m^2 (to 1 d.p.)

7.8 Volumes of Cubes, Cuboids, Cylinders and Prisms

1. (a) 125 cm^3 (b) 360 cm^3 (c) 8.96 m^3
   (d) 1005 mm^3 (to the nearest mm^3) (e) 2 m^3 (to the nearest m^3)
   (f) 1508 cm^3 (to the nearest cm^3) (g) 504 cm^3 (h) 144 cm^3
   (i) 2.4 m^3

2. (a) 0.25 m^3 (b) 0.20 m^3 (to the nearest m^3)

3. (a) 3.864 m^3 (b) 3.312 m^3

4. (a) 251327 cm^3 (b) 203.6 cm^3, 2036 cm^3
   (c) 249291 cm^3 (to the nearest cm^3)

5. (a) 120 cm^3 (b) 105 cm^3

6. (a) 168 cm^3 (b) 45 cm^3 (c) 176 cm^3 (d) 1600 cm^3

7. (a) 1200 cm^3 (b) 130 cm^3 8. 20 000 cm^3

9. 67.5 m^3 10. 15 m^3 11. 26 m^3 (to the nearest m^3)

12. (a) 15 m^2 (b) 3 m^3 (c) 200 mm

13. (a) 462 cm^3 (to the nearest cm^3) (b) 3 cm

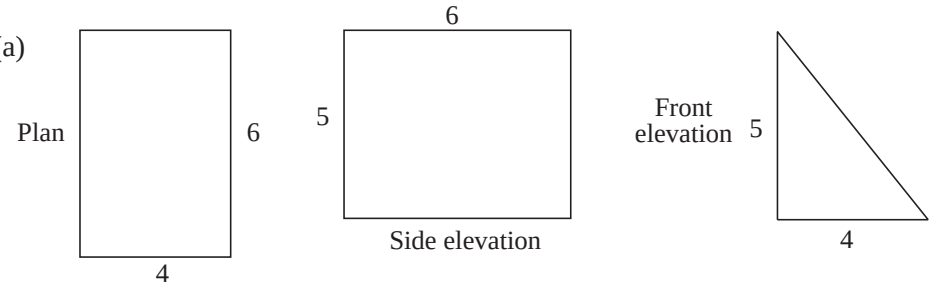
14. (a) 36 .9 m^2 (b) 295.2 m^3 (c) 0.32 m 15. 1575 cm^3

16. (a) 7680 cm^3
(b) (i) 50.3 cm (ii) 2011 cm^3 (to the nearest cm^3)

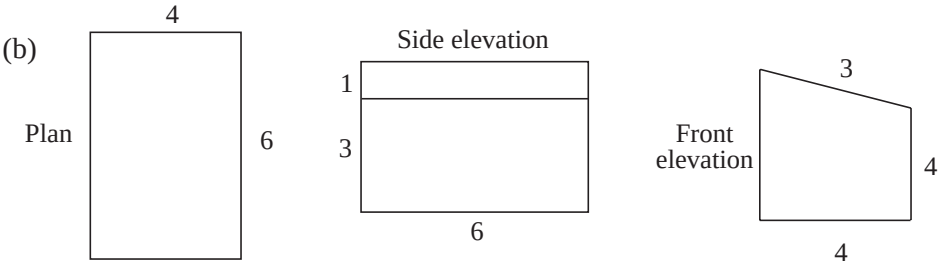
17. (a) 500 cm^3 (b) (i) 400 cm^3 (ii) 80%
(c) (i) 42 cm^2 (ii) 9.5 cm (to the nearest mm)

7.9 Plans and Elevations

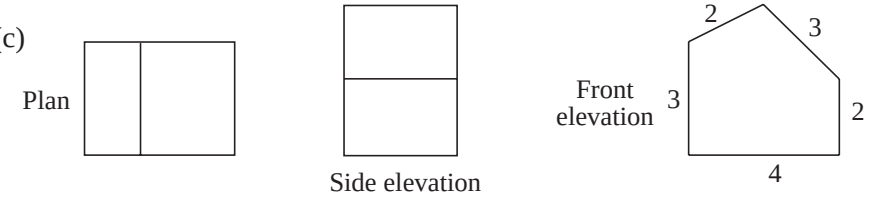
1. (a)



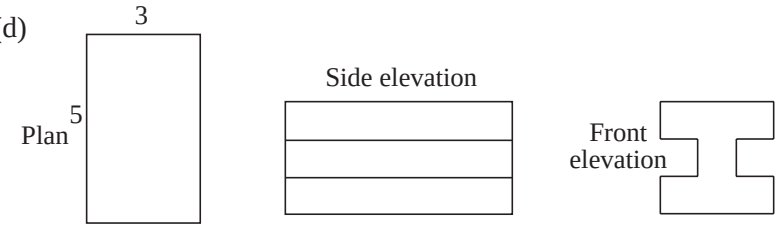
(b)



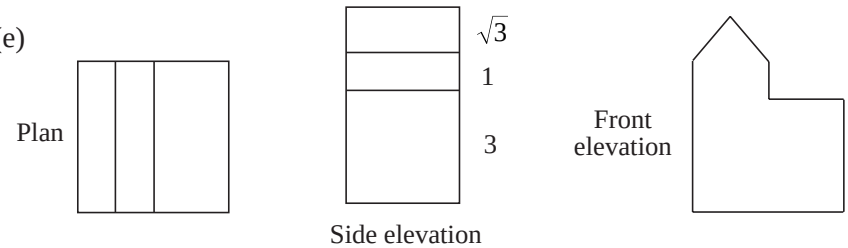
(c)



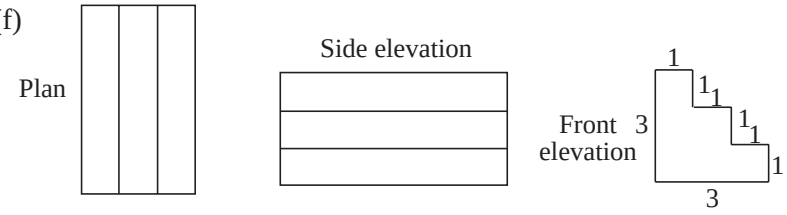
(d)



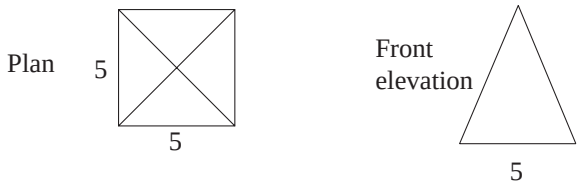
(e)



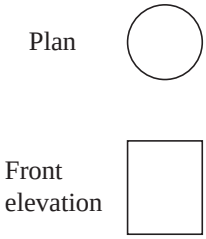
(f)



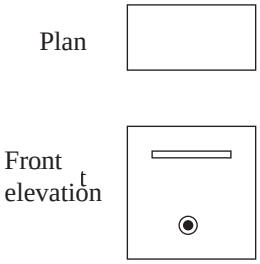
2.



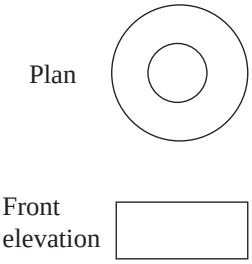
3. (a)



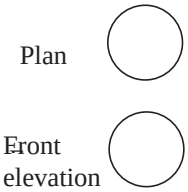
(b)



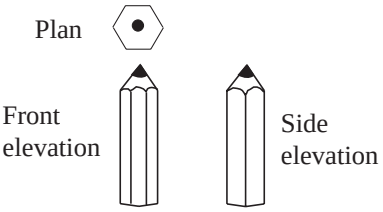
(c)



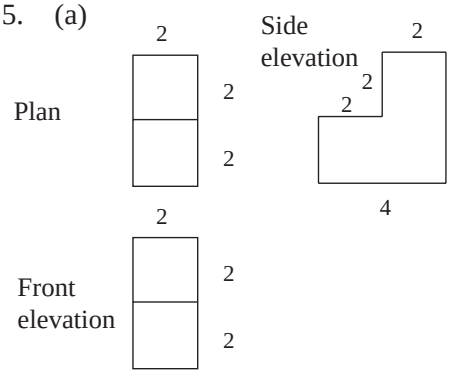
(b)



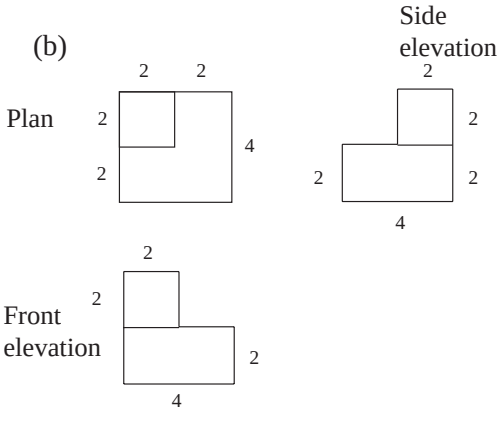
4.



5. (a)



(b)



7.10 Using Isometric Paper

7. (a) 34 cm (b) 44 cm^2 (d) 2 cm, 3 cm, 4 cm (e) 24 cm^3

7.11 Discrete and Continuous Measures

1. (a) continuous (b) discrete (c) discrete (d) continuous
   (e) continuous (f) discrete (g) continuous (h) discrete
   (i) continuous (j) continuous (k) discrete (l) continuous

3. (a) 4.625 miles $\leq$ 4.63 miles < 4.635 miles (b) exact
   (c) 124.5 g $\leq$ 125 g < 125.5 g (d) 161.5 cm $\leq$ 162 cm < 162 5 cm
   (e) exact (f) 52.15 $cm^3 \leq$ 52.2 cm^3 < 52.25 cm^3
   (g) 22.15 cm $\leq$ 22.2 cm < 22.25 cm (h) exact
   (i) 53.5 g $\leq$ 54 g < 54.5 g (j) exact

7.12 Areas of Parallelograms, Trapeziums, Kites and Rhombuses

1. (a) 12 m^2 (b) 5.5 m^2 (c) 80 cm^2 (d) 75 cm^2
   (e) 72 cm^2 (f) 110 m^2 (g) 37.5 cm^2 (h) 7.2 m^2
   (i) 10.2 cm^2

2. (a) 3.5 m^2 (b) 4.5 m^2 (c) 17.5 m^2

3. (a) 13.5 m^2 (b) 6 m^2 (c) 32 m^2 (d) 75 m^2

4. 18 m^2 5. 2500 cm^2

6. (a) 4816 cm^2 (b) 4816 cm^2
   (c) The area of the wasted plastic would be equal to the area of the kite (each of them would be 5600 cm^2, which is half the area of the rectangular sheet used to make the kite).

7. (a) 64 cm^2 (b) 58 cm^2 (c) 94.25 cm^2 (d) 44 cm^2
   (e) 32 cm^2 (f) 44 cm^2

8. 132 cm^2 , 144 cm^2 . Total area: 552 cm^2

9. (a) (i) 36 cm^2 (ii) 72 cm^2 (iii) 90 cm^2
   (b) No, since h is a side of a right-angled triangle whose hypotenuse is 6 cm long .
   (c) The maximum area is reached when h = 6 cm, and is equal to 108 cm^2 .
   (The maximum area is reached when the parallelogram is actually a rectangle.)

10. (a) 28 cm^2 (b) 14 cm^2 (half the area of the parallelogram)

11. The kite ABCD is made of the two congruent triangles ABD and CBD. Hence, its area covers twice the area of each of these triangles.

7.13 Surface Area

1. (a) 96 cm^2 (b) 136 cm^2 (c) 236 cm^2 (d) 62 cm^2
   (e) 250 cm^2 (f) 30.4 cm^2

2. (a) 108.38 cm^2 (to 2 d.p.) (b) 283 cm^2 (to the nearest cm^2)
   (c) 207 cm^2 (to the nearest cm^2) (d) 69 m^2 (to the nearest m^2)
   (e) 12.6 m^2 (to 1 d.p.) (f) 115 cm^2 (to the nearest cm^2)

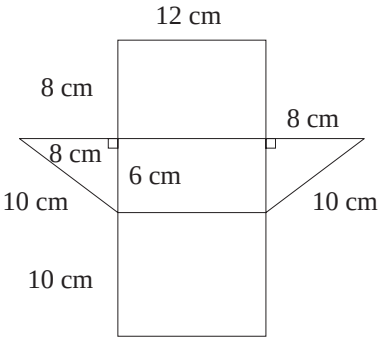
3. The surface area of each cylinder is 96π cm^2 (A = 301.44 when π = 3.14, and 301.60 using the π key on your calculator.)
   (a) V = 402 cm^3 (b) V = 276 cm^3 (c) V = 368 cm^3

4. The volume of each cuboid is 64 cm^3. The surface area of each cuboid is:
   (a) 96 cm^2, which is the smallest area, (b) 112 cm^2, (c) 168 cm^2

5. (a) 13 195 cm^2 (to the nearest cm^2) (b) 35 000 cm^2

6. $6 \times 7^2 = 294$ cm^2 7. $\left(\sqrt{\frac{150}{6}}\right)^3 = 5^3 = 125$ cm^3

8. (a) 54 cm^2 (b) 66 cm^2 (c) 120 cm^2

9. 

   A = 336 cm^2

10. 11 027 cm^2
    (Total surface area = $(2 \times 25\pi \times 12) + 2(40^2\pi - 25^2\pi) + (2 \times 40\pi \times 12)$)

11. (a) (i) R(–1, 9, 0) (ii) B(3, 9, 6) (b) (i) PQ = 5 cm (ii) QR = 4 cm
    (c) CR = 6 cm, hence the total surface area is $2(4 \times 6 + 4 \times 5 + 6 \times 5) = 148$ cm^2.

7.14 Mass, Volume and Density

1. (a) 96 cm^3 (b) 88 cm^3 (c) 88 g

2. (a) 8000 cm^3 (b) 0.75 g/cm^3 3. 0.794 g/cm^3

4. (a) 402 123.86 cm^3 (b) 402.124 kg

5. (a) 120 cm^3 (b) 2.5 g/cm^3 (c) 375 g (d) 1000 g (1 kg)

6. 80 g

7. (a) 60 m^3 (b) 60 000 kg (c) volume: 52 m^3; mass: 52 000 kg

8. (a) 6.28 cm^2 (to 2 d.p.) (b) 1414 g (c) 40 cm

9. (a) 4188.79 cm^3 (b) 0.048 g/cm^3 (c) 83 g (to the nearest g)

10. (a) 6 g/cm^3 (b) 89 cm

7.15 Volumes, Areas and Lengths

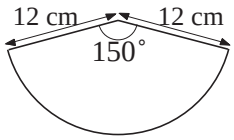
1. (a) 4.38 cm^2 (to 2 d.p.) (b) 5.27 cm^2 (c) 24.13 cm^2
   (d) 33.5 cm^2 (e) 64.14 cm^2 (f) 5.14 cm^2 (g) 4.47 cm^2
   (h) 12.57 cm^2 (i) 12.5 cm^2 (j) 94.6 cm^2

2. (a) 4.80 cm (b) 26.9 cm (c) $\theta = \dfrac{360^\circ}{2\pi} = 57.30^\circ$

   (d) $r = \dfrac{6\times 15}{2\pi} = 14.32$ cm (e) 9.42 cm (f) 4.2 cm

3. The volume needed to cover all the three edges and the top is 278 cm^3.

4. 40.84 cm^2 5. (a) 13.3 m (b) 11.53 cm^2

6. (a) 

The angle within the sector is equal to $150°$.

   (b) 188.5 cm^2 (c) 286 cm^3

7. (a) 283 cm^3 (to the nearest cm^3) (b) 314 cm^3 (to the nearest cm^3)
   (c) 257 cm^3 (to the nearest cm^3) (d) 1.6 m^3 (to 1 d.p.)
   (e) 1018 cm^3 (to the nearest cm^3) (f) 2399 cm^3 (to the nearest cm^3)
   (g) 905 cm^3 (h) 262 cm^3 (to the nearest cm^3) (i) 500 cm^3 (j) 144 cm^3

8. r = 6.08 cm (to 2 d.p.) 9. d = 3 cm (to the nearest cm)

10. (a) 1131 cm^3 (to the nearest cm^3) (b) 1283 cm^3 (to the nearest cm^3)
    (c) 115 cm^3 (to the nearest cm^3) (d) 816 cm^3 (to the nearest cm^3)

11. 0.1 cm (to 1 d.p.) 12. 81.9 cm^3 (to the nearest cm^3)

13. 20.6 cm^3 (to 1 d.p.)

14. (a) 150 cm^2 (to the nearest cm^2) (b) $x = 129^\circ$ (to the nearest degree)
    (c) r may take one of the two values: 20 cm , 30 cm.

15. (a) AB = $2.5\cos 55° = 1.43$ m (to 2 d.p.)
    (b) Area of the sector BDE is 3.82 m^2
    (c) 6.75 m^2 (d) 10 m (to the nearest m)

16. (a) p = 7.57 cm (to 2 d.p.) (b) $59°$

17. (a) (i) OX = 10.5 m (ii) AB = 20 m
    (b) The angle BOX is $44°$ (to the nearest degree) (c) 55 m^2

7.16 Dimensions

1. (a) $L^2 \Rightarrow$ Area (b) None (c) $L^3 \Rightarrow$ Volume
   (d) $L^2 \Rightarrow$ Area (e) $L^2 \Rightarrow$ Area (f) None
   (g) mixed dimensions (h) $L^3 \Rightarrow$ Volume (i) $L^3 \Rightarrow$ Volume

2. (a) Yes (b) No (c) No (d) Yes (e) Yes (f) Yes

3. A = $ab + ac + bc + cd$

4. $\sin\theta$ has dimension zero (it is the quotient of two lengths so it is a number)

5. $V = \frac{13}{17}\pi r^2 d$ (This is the only formula with dimension L^3.)

6. $V = \pi R^2 h - \pi r^2 h$ (This formula is the only one with dimension L^3.)

7. (a) The formulae that can give a volume are the ones with dimension L^3:
   $\frac{4}{3}\pi r^3$, $\frac{3}{4}\pi r^3$.
   (b) The only formula that can give an area is the one with dimension L^2:
   $\frac{4}{3}\pi r^2$.

8. $6x^2 + 4y^2$ has dimension L^2 and hence, can give a surface area.
   $x^3 + y^3$ has dimension L^3 and hence, can give a volume.

9. $V = \frac{\pi d^2 h}{4} + \frac{\pi d^2 a}{12}$, since it is the only formula with dimension L^3.

10. (a) L (b) $L^2 \Rightarrow$ area

11. (a) Dimension $L \Rightarrow$ perimeter (b) Dimension $L^3 \Rightarrow$ volume

12.

$2\pi r$	$4\pi r^2$	$\pi r^2 h$	πr^2	lbh	$\frac{1}{2}bh$
		✓		✓	

13. The formulae which represent areas are the formulae with dimension L^2.
    These are: πab, $\pi(a+b)l$.

14. (iii) $\pi h(a+b)$. It is the only formula with dimension L^2, the dimension of an area.

7.17 Areas of Triangles

1. (a) 12 cm^2 (to the nearest cm^2) (b) 11 cm^2 (to the nearest cm^2)
   (c) 10 cm^2 (to the nearest cm^2) (d) 13 cm^2 (to the nearest cm^2)

2. (i) $A = 28$ mm^2 (to the nearest mm^2), $\theta = 44°$ (to the nearest degree)
   (ii) $A = 56$ cm^2 (to the nearest cm^2), $\theta = 61°$ (to the nearest degree)
   (iii) $A = 19$ cm^2 (to the nearest cm^2), $\theta = 102°$ (to the nearest degree)
   (iv) $A = 15$ cm^2 (to the nearest cm^2), $\theta = 44°$ (to the nearest degree)

8 Data Handling

8.1 Tables and Timetables

1. (a) 0800 (b) 0923
   (c) He should catch the next train and get off at Exeter St. Davids and walk.
   (d) For each journey - catch the 0723 from Paignton and change at either Newton Abbot or Exeter St. Davids - arriving at Bristol at 0932 or London at 1110.

2. (a) 2106 (b) 1 hr, 39 mins. (c) Yes
   (d) The 2027 train from Reading.

3. (a) No (b) 1235 (c) 1141

4. (a) (i) 109 miles (ii) 34 miles (iii) 78 miles (b) (i) 149 miles
   (ii) The travelling distance would be reduced by 29 miles because Manchester is "en route" travelling from Birmingham to Leeds.
   (c) The route via Sheffield.

5. (a) St. Malo (b) 3984 km (c) 2781 km
   (d) Quimper is closest to St. Malo, and Ile de Re is closest to Calais.

6. (a) 9 (b) 11 (c) "D" grade.

7. (a) 7 (b) 6 (c) Years 7, 8, 9 and 11
   (d) (i) Year 10 (ii) 43 students

8. (a) £305 (b) £156 (c) £276 (d) (i) £260 (ii) £45

9. (a)

	Male	*Female*	*Total*
Standard	64	48	112
Senior	20	8	28
Total	84	56	140

   (b) Men are more likely to become senior conductors than women.

10. (a) 51 (b) 11
    (c) 52 outdoor and 50 indoor shows a roughly even split (only marginally in favour of outdoor sports)

11. (a) 4 (b) 19
    (c) People seem to achieve better English results than French.

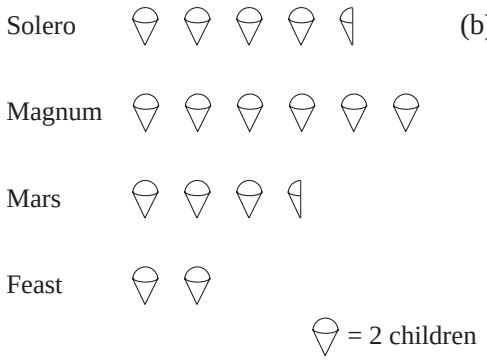
12. (a) 31 (b) 90 (c) 30%

8.2 Pictograms and Bar Charts

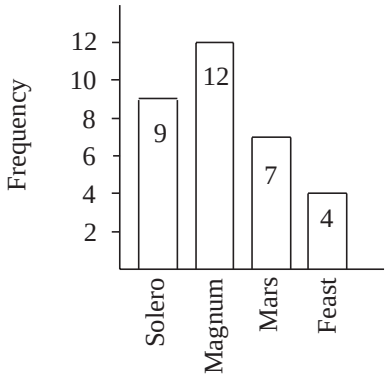
1. (a) 1996 (b) (i) 10 (ii) 7 (iii) 12 (c) 1995

2. (a) 400 (b) 250 (c) 700 (d) $5\frac{1}{2}$ (e) 3300

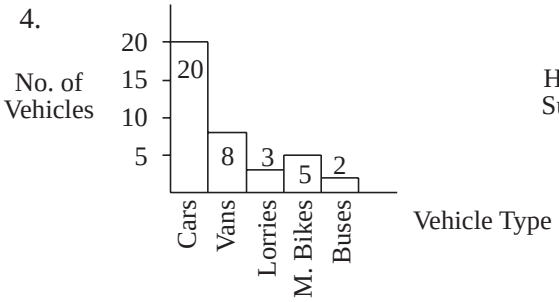
3. (a)



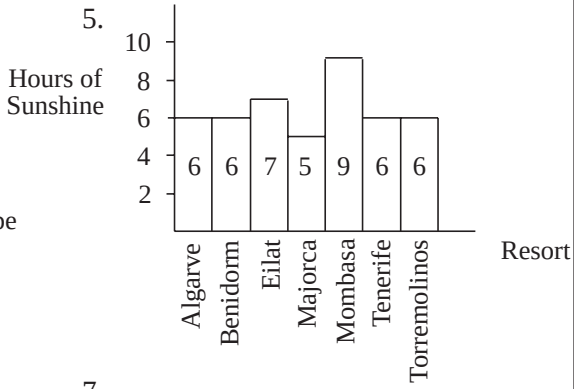
(b)



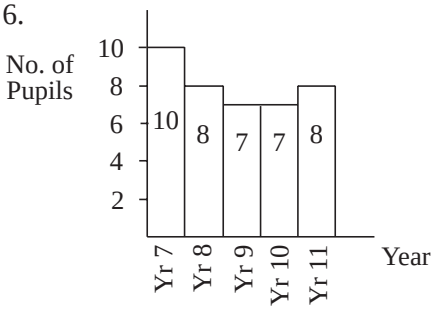
4.



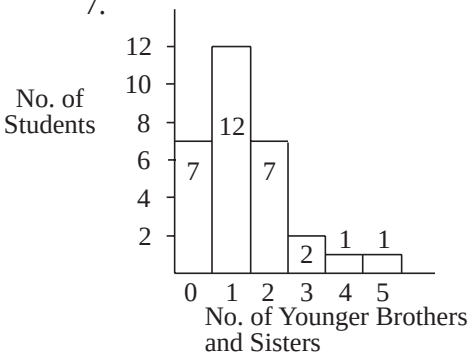
5.



6.



7.



8. (a) (i) 90% (ii) About 97% (b) (i) 0% (ii) About 22%

(c) The percentage of households with some sort of TV hasn't changed very much, but there has been an increase in the number of homes recieving satellite.

9. (a) 50 (b) 35

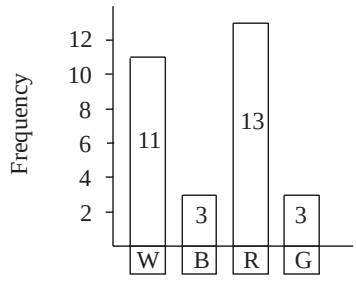
10. (a) 23 (b) We would expect there to be many more boys with shoe sizes around 8 and 9 than for 5 or 12, so the results are surprising.

11. (a) £14 (b) £375 (c) 16 coins

(d) Only whole coins are used, and so the number is rounded up/down.

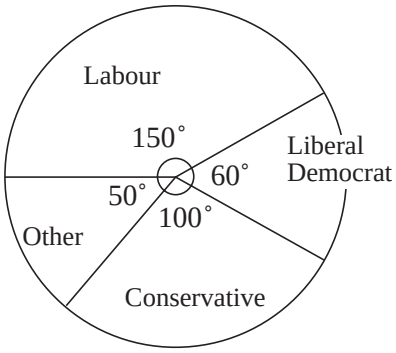
12.

Colour	*Tally*	*Frequency*
White	𝍸 𝍸 I	11
Blue	III	3
Red	𝍸 𝍸 III	13
Green	III	3

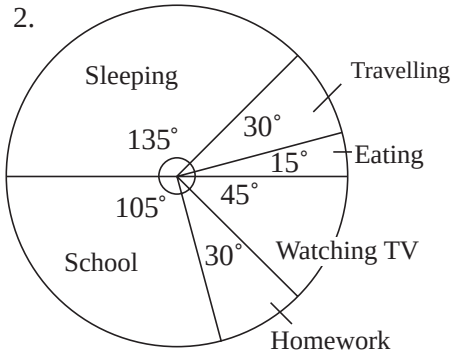


8.3 Pie Charts

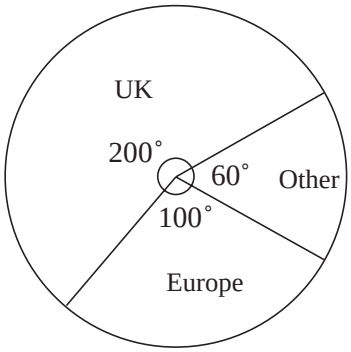
1.



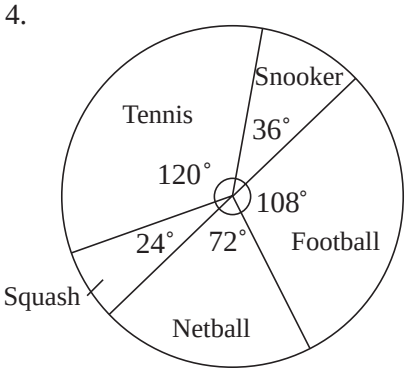
2.



3.



4.



5. (a) 2 hours (b) 5 hours

6. (a) £10 (b) £15 (c) £35

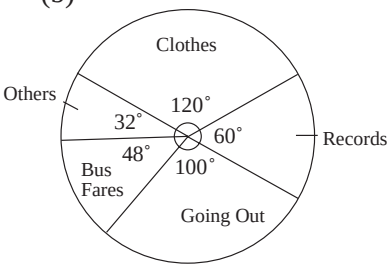
7. (a) 96° (b) 6 (c) 30 (d) 13

8. Airmail = 50, 1st class = 320, 2nd class = 350

9. (a)

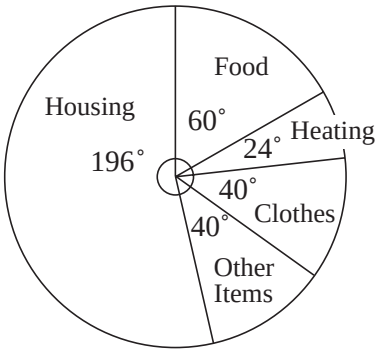
Items	Angle of Sector
Bus fares	48°
Going out	100°
Clothes	120°
Records	60°
Others	32°
Total of angles	360°

(b)



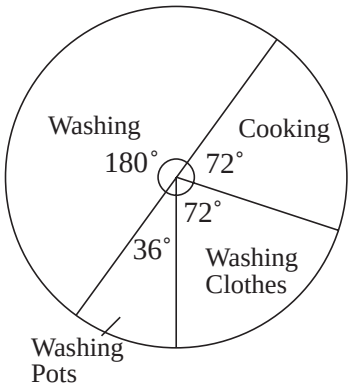
(c) $\frac{1}{3}$

10. (a)



(b) Arthur spends more money on housing.

11. (a) (i) 81 litres (ii) $\frac{15}{100} = \frac{3}{20}$ (b)



12. (a) (i) Five hundred thousand (ii) 200 000

(b) (i) 39% (ii) Over 21 year olds. (iii) 0.61 or $\frac{61}{100}$

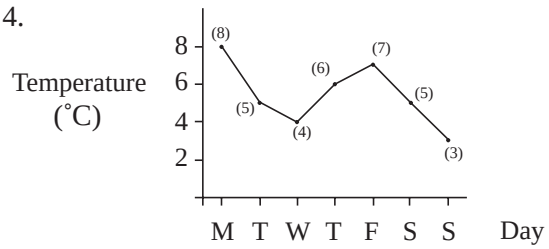
8.4 Line Graphs

1. (a) 4 cm (b) April (c) February & December (d) July & August

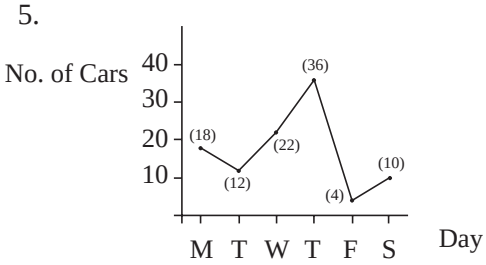
2. (a) 40°C (b) 80°C (c) 20 mins after filling the mug. (d) 25 mins

3. (a) 8 cm (b) 22 cm (c) 84 cm (d) 3 weeks

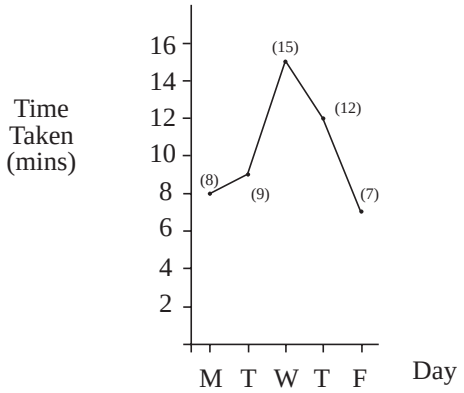
4.



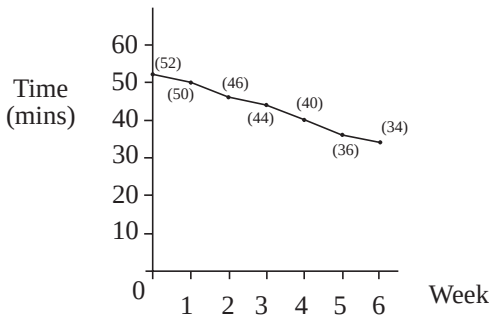
5.



6.



7.



8.5 Questionnaires and Surveys

7. Question (A) gives genuine answers – although they may be difficult to analyse. Question (B) will give precise answers to just three aspects of life on this housing estate.

8. (a) Cycling and Swimming

10. (a) The percentage late is given by:

Bus	Cycle	Car	Walk
27%	20%	22%	13%

so her conclusion is supported by the data.

(b) Choose pupils randomly, do not always use Tuesdays, increase sample size.

(c) Wrong – because you do not know whether there was an equal number of Y7 and Y8 pupils in the survey.

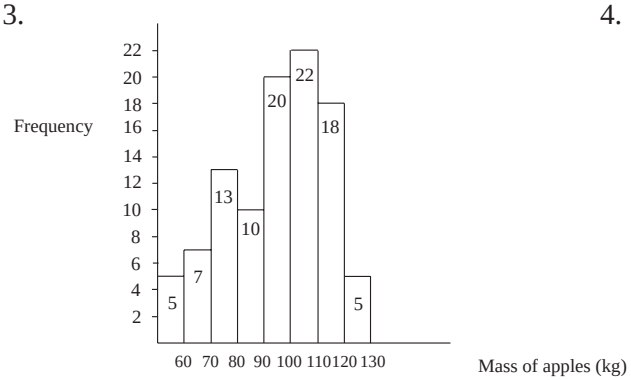
12. Sample is not random; sample value of 19% is not necessarily true for the whole population; they may only have travelled once on a bus in the last week – not each day.

8.6 Frequency Graphs

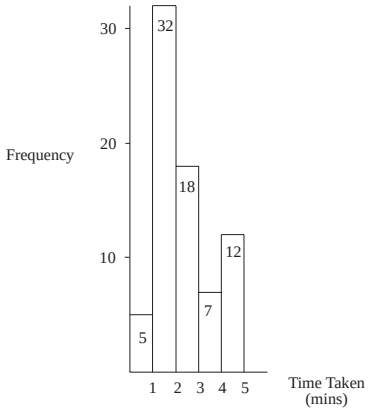
1. (a) 5 (b) 55 (c) 15 (d) 81 pupils in year group.

2. (a) 10 (b) 49 (c) 73 (d) largest = £549.99, smallest = £50.

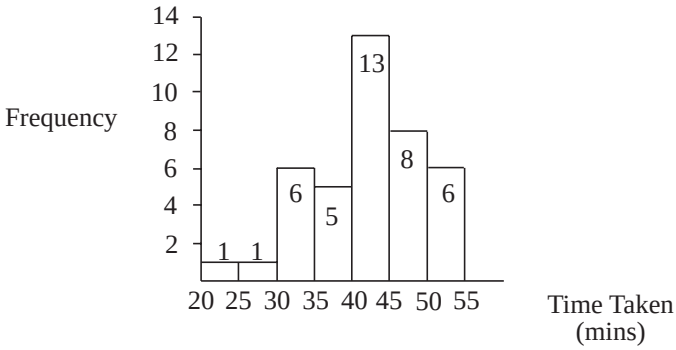
3.



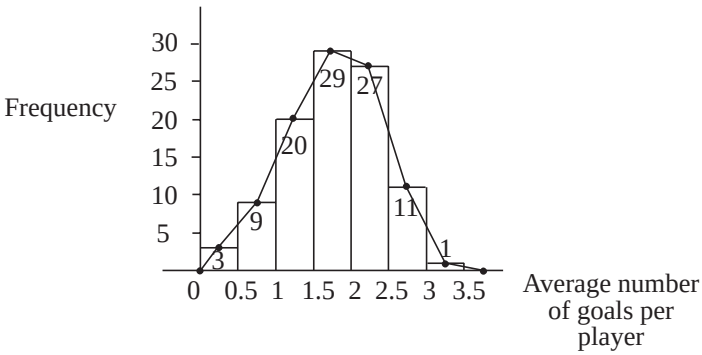
4.



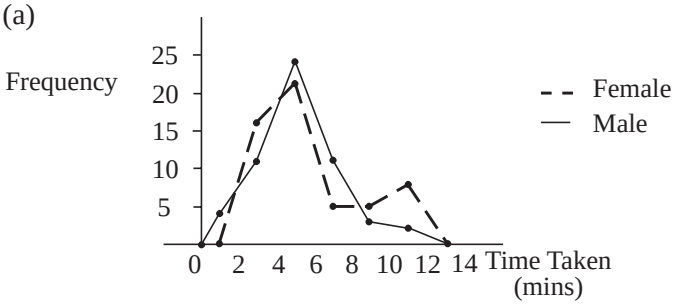
5.



6.

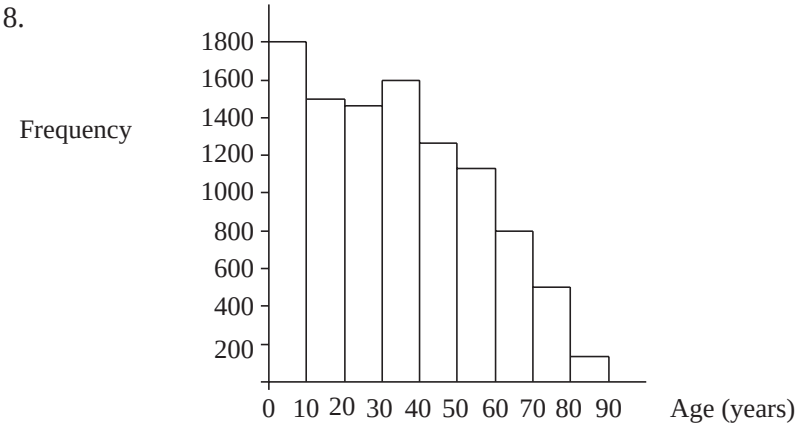


7. (a)

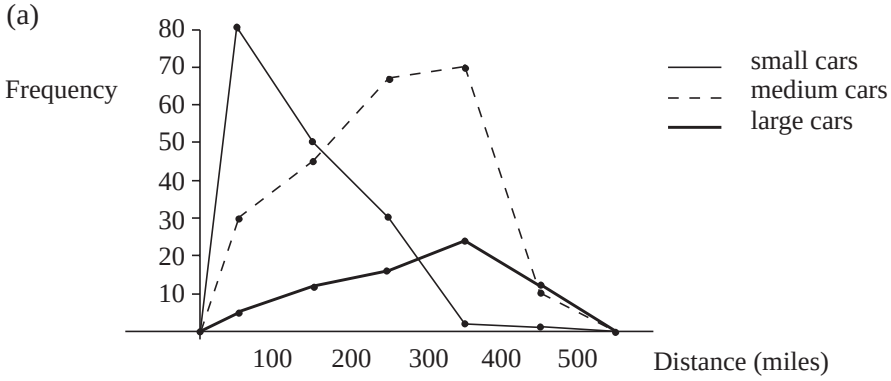


(b) Any reasonable answer.

8.



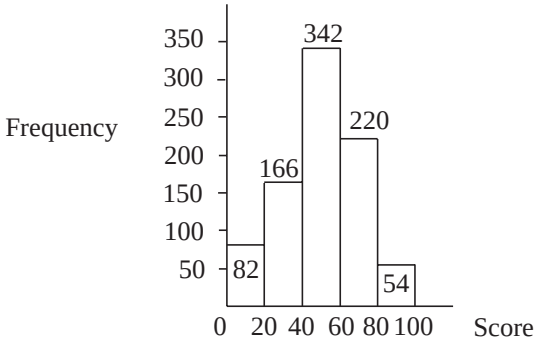
9. (a)



(b) There is a downward trend in the small cars, and an upward trend in the medium and large cars.

i.e. People use small cars for short journeys and the larger ones for long distances.

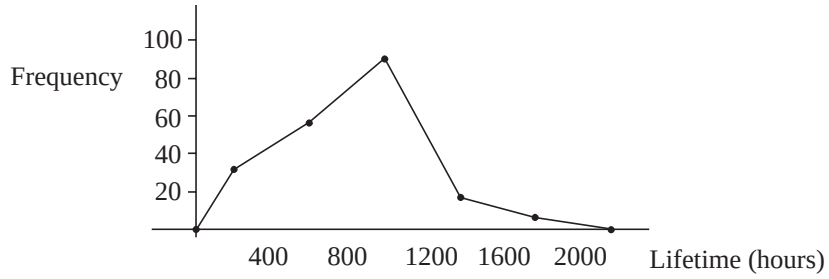
10.



11. 62 + 70 = 132 pupils.

12. (a) (i) prob. $= \frac{32}{200} = 0.16$ (ii) prob. $= \frac{106}{200} = 0.53$

(b)



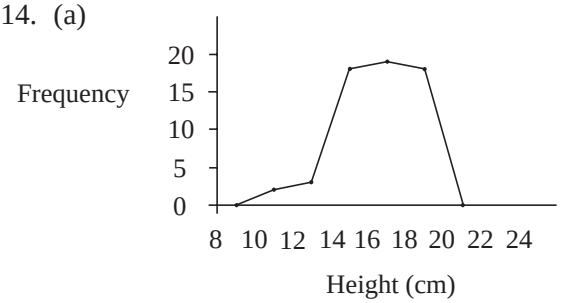
13. (a)

Height	*Frequency*
120 - 130	7
130 - 140	22
140 - 150	20
150 - 160	4

(b) The columns should be adjacent (no gaps).
The second column $(130 \le h < 140)$ should be 22 not 24.

(c) The last category.

14. (a)



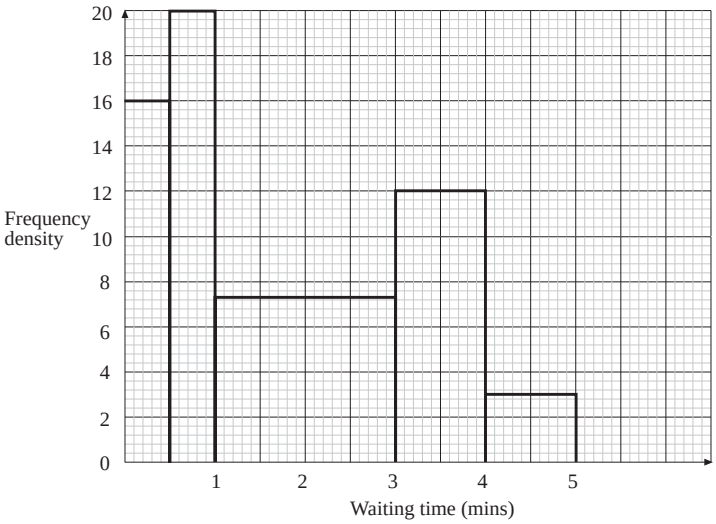
(b) Type B has an even spread of plants between 10 and 22, whereas Type A has more taller plants.
The maximum number interval is 14 for Type B plants and 19 for Type A plants.

8.7 Histograms with Unequal Class Intervals

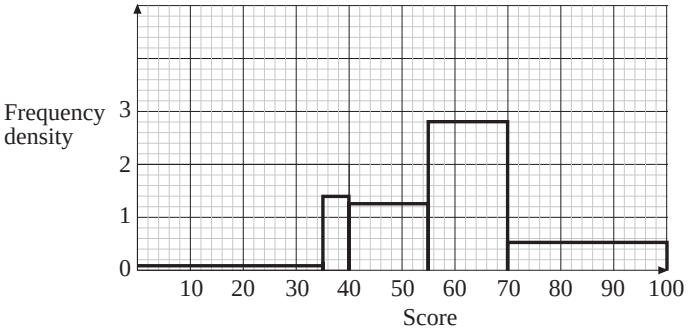
1. (a) 9 (b) 4 (c) 23

2. The area for the 20 - 25 interval is $0.9 \times 5 = 4.5$. This should be a whole number.

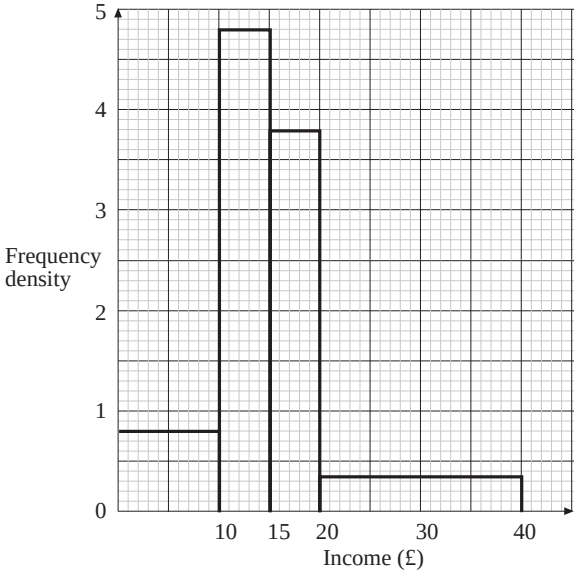
3.



4.



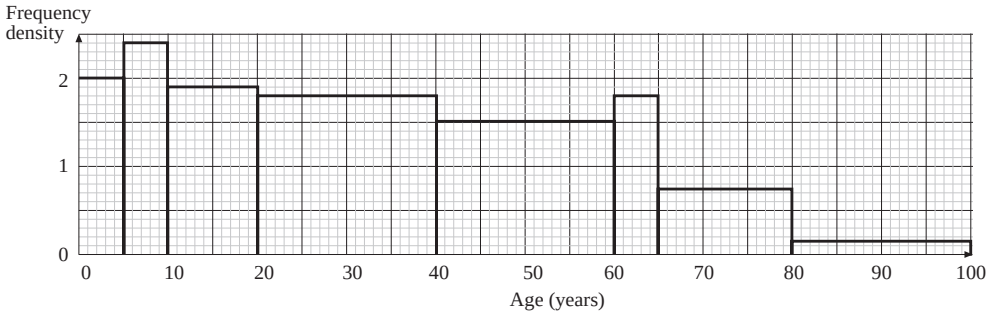
5.



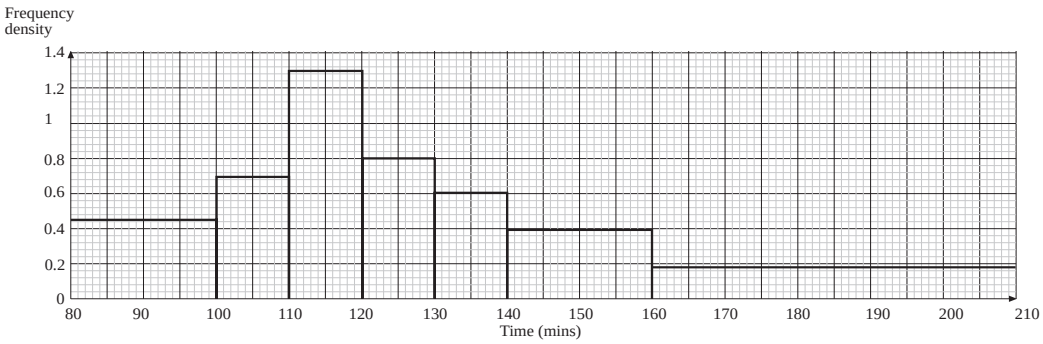
6. (a) The width is 5 because age 0–4 is actually age 0 up to age 5, i.e. 5 years.

(b) The widths are (from the 2nd interval) - 5, 10, 20, 20, 5, 15, 20.

(c)

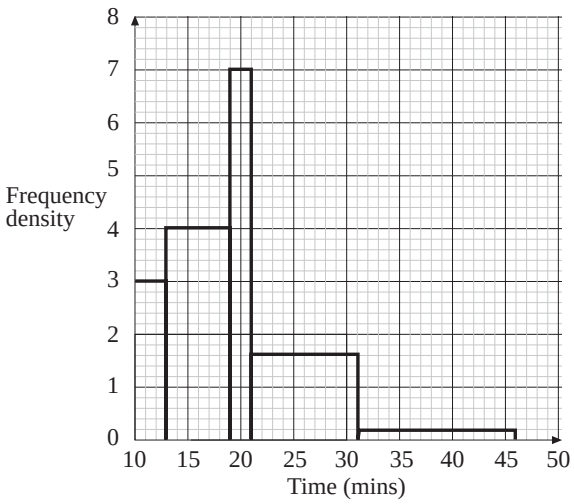


7.



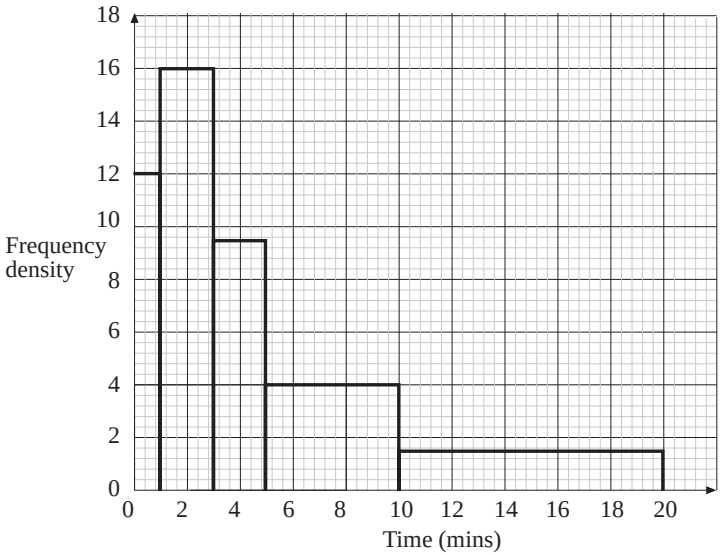
8. (a) Width of class intervals: 3, 6, 2, 10, 15.

(b)



9. (a) 4 people (b) 23 people

10.



11. (a) Frequencies- 4, 6, 7, 4, 6, 1. (b) 28 matches.

8.8 Sampling

1. (a) Possible answers: (i) C, F, I, L, O, R, U, X. (ii) D, H, L, P, T, X.
(iii) F, L, R, X.

(c) Number 24 balls and place in a bag, then draw out the relevant number of balls to correspond with the number of machines.

2. 13, 13, 14, 14, 13, 7, 6 (taking the integer part of each answer plus the two greatest remainders to give a sample of size 80)

3. (a) 1, 2, 1, 13, 3. (b) 12

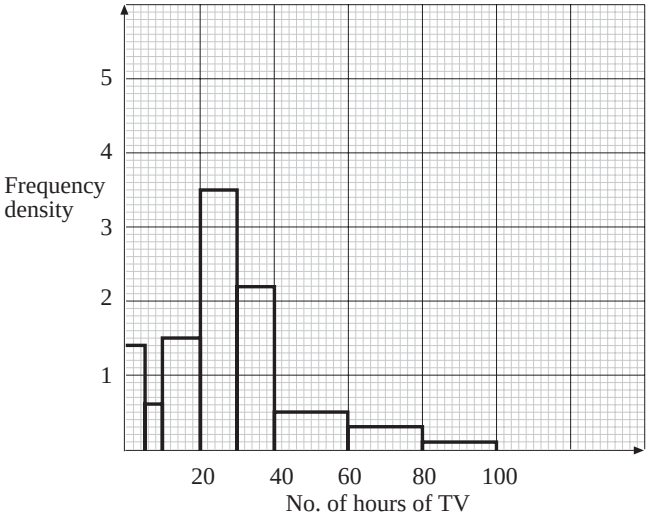
4. (a) No (b) Change 5 to 6 and change 4 to 5.

5. (a) Working either across the rows or down the columns, choose the 10th square in each case.

9. (a) (i) 49, 36, 15.

(ii) This ensures that each age cohort has a fair representation.

(b)



9 Data Analysis

9.1 Mean, Median, Mode and Range

1. (a) mean = 5 median = 4.5 mode = 3 range = 8
   (b) mean = 9.9 median = 9.5 mode = 7 range = 11
   (c) mean = 16.25 median = 16 5 mode = 17 range = 10
   (d) mean = 107.6 median = 108 mode = 108 range = 13
   (e) mean = 63 median = 64 mode = 61 range = 10
   (f) mean = 21.75 median = 21.5 mode = 16 range = 14

2. (a) 6.8 (b) 6.75 (c) 6 (d) $5\frac{1}{2}$

3. (a) (i) 4.875 (ii) 3.5 (iii) £3 (b) (i) mean (ii) mode
   (c) range = £13

4. (a) Fred: mean = 20.8, range = 2. Harry: mean = 24.2, range = 24.
   (b) Fred (c) Harry

5. (a) A: mean = 15.9, median = 20, mode = 20
   B: mean = 16.9, median = 17, mode = 17
   (b) Mode suggests A (c) Mean suggests B (d) Range: A = 15, B = 3.

6. (a) mean = 68, median = 68.5 (b) His mean increases to 68.4.
   (c) The median. It increases from 68.5 to 70, whereas the mean only increases by 0.4.

7. (a) He objects because the mode = 0 = median (b) mean = 2.30, range = 18
   (c) 15 fish

8. (a) 268.4 cars (b) The mean decreases.

9. (a) mean = 2.035, median = 2, mode = 2.
   (b) Either median or mode (whole numbers).

10. The mean will increase. 11. (a) 19°C (b) Archangel (c) 27°C

12. 225 grams 13. (a) mean = 2 (b) range = 4

14. (a) 77 kg (b) Hereward House, because they have a much heavier team.

15. (a) modal class = 24 pupils (b) mean class = 26 pupils
   (c) There is more of an even spread of pupils in Year 9.

16. (a) Pat: mean = 25.3, range = 18
   (b) They both have approximately the same mean, but Kim's scores are more consistent as shown by the smaller range; thus Kim should be selected.

17. (a) (i) 5.6 (ii) 0.5 (b) Leaving out the two extreme marks probably gives a less biased measure of performance.
   (c) Mean + Range = 5.2 + 0.6 = 5.8, therefore no mark could possibly exceed 5.8.

9.2 Finding the Mean using Tables and Tally Charts

1. mean = 1.25 2. mean = 1.93 3. mean = 4.08 4. mean = 3.56

5. (a) mean = 1.95 (b) 22 times 6. mean number = 1.15 trains

7. mean = 2.30 8.(a) 6 (b) 2.04

9. Missing frequencies are 1, 5, 0. Missing frequencies of tickets are 0, 20, 21, 10, 0. mean = 2.8 (1 d.p.)

10. (a)

Weight Range (w)	*Tally*	*Frequency*
$30 \le w < 40$	𝍸	4
$40 \le w < 50$	𝍸 II	7
$50 \le w < 60$	𝍸 II	7
$60 \le w < 70$	𝍸 III	8
$70 \le w < 80$	I	1
$80 \le w < 90$	III	3

(b) class $60 \le w < 70$

11. (a) 3 (b) frequency = 21, total = 48, mean = 2.29.
(c) the number of children per family has decreased on average (2.29 < 2.7), and there is less variation from family to family (today's range is 3, whilst in 1960 it was 7).

9.3 Calculations with the Mean

1. mean = 161 55 2. mean = 2 3. mean = 4 4. mean = 60.15 kg

5. 7 6. 84% 7. £6000 8. 9.5 9. mean $\approx$ 4.47 10. 320

9.4 Mean, Median and Mode for Grouped Data

1. (a) 33.09 (b) 33.42 (c) 30 - 39
2. (a) 40.90 (b) 41 (c) $40 \le w < 45$
3. (a) Yes (b) 20.54 (c) 20.95 (d) Median is greater than the mean.
4. (a) No (b) median = 0 72, mean = 0.78. (c) The mean is the largest.
5. 11.45 years 6. (a) 11.95 (b) 10.92
7. (a) 9.65 (b) 9.92 (c) 11 - 15
8. (a) (i) 26.78 (ii) 27.17 (iii) 21 - 30
(b) (i) 21.5 (ii) 22.25 (iii) 21 - 30
(c) The second class have a lower mean but similar range.
9. (a) 0 - £1.00 (b) £1.44
10. (a) 21, 7, 2 (b) People would spend more time watching television than in summer. (c) 23.83 (24 hours)

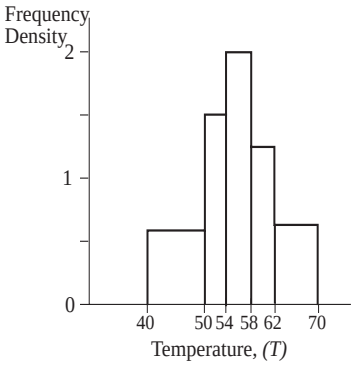
11. (a) 24 cm (b) 24.32 cm

12 (a)

Temperature , T	*Mid-point*	*Frequency*
$40 < T \leq 50$	45	6
$50 < T \leq 54$	52	6
$54 < T \leq 58$	56	8
$58 < T \leq 62$	60	5
$62 < T \leq 70$	66	5

(b) $\frac{1660}{30} \approx 55.3$ (c)

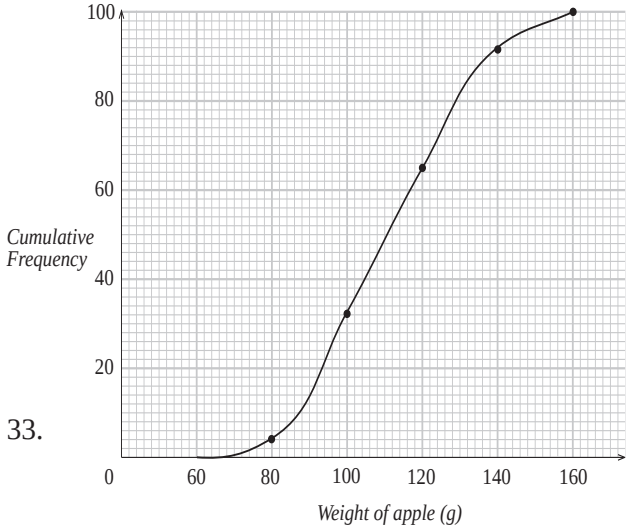
Interval	*Freq. Density*
40-50	$\frac{6}{10} = 0.6$
50-54	$\frac{6}{4} = 1.5$
54-58	$\frac{8}{4} = 2$
58-62	$\frac{5}{4} = 1.25$
62-70	$\frac{5}{8} = 0.625$



9.5 Cumulative Frequency

1. (a)

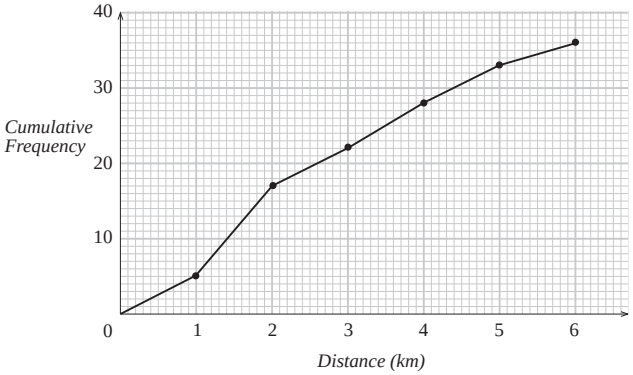
Weight in grams (w)	*Cumulative Frequency*
$60 < w \leq 80$	4
$80 < w \leq 100$	32
$100 < w \leq 120$	65
$120 < w \leq 140$	92
$140 < w \leq 160$	100



Median = 111, Inter-quartile range = 33.

(b)

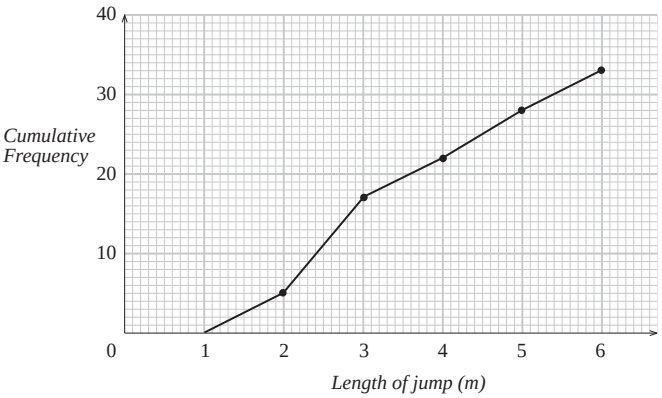
Distance in km (d)	*Cumulative Frequency*
$0 < d \leq 1$	5
$1 < d \leq 2$	17
$2 < d \leq 3$	22
$3 < d \leq 4$	28
$4 < d \leq 5$	33
$5 < d \leq 6$	36



Median = 2.2, Inter-quartile range – any answer between 2 and 3 is acceptable.

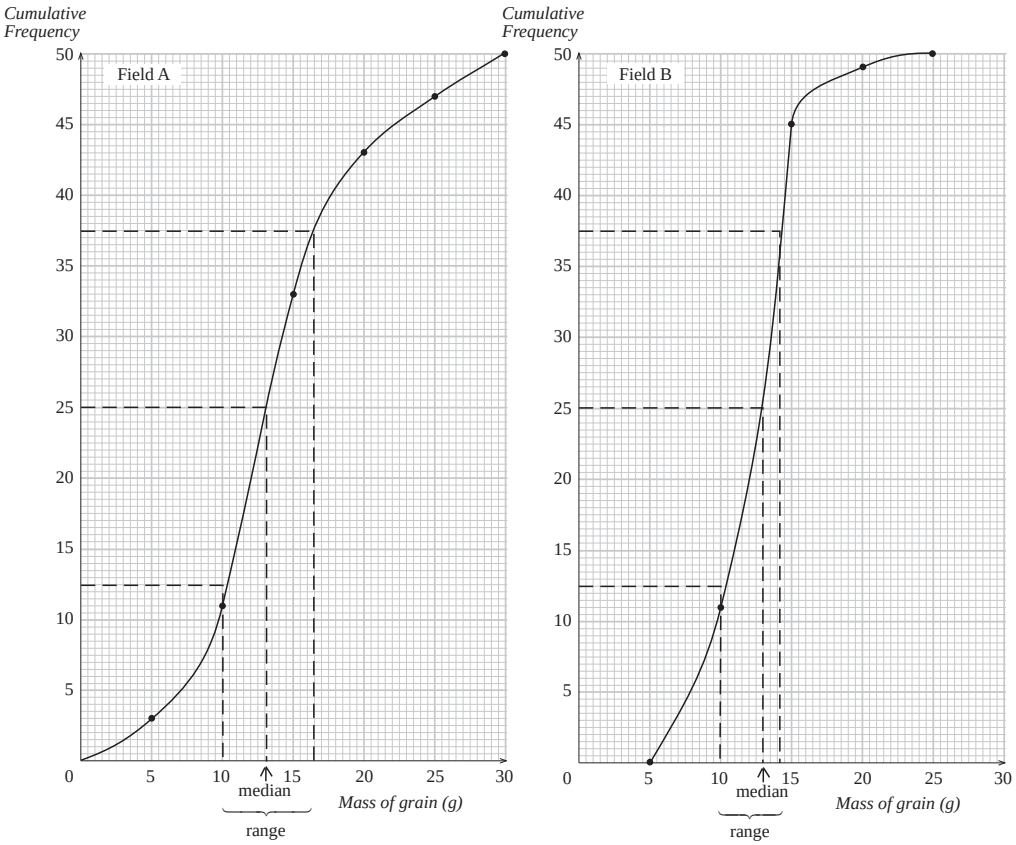
(c)

Length in metres (d)	*Cumulative Frequency*
$1 < d \leq 2$	5
$2 < d \leq 3$	17
$3 < d \leq 4$	22
$4 < d \leq 5$	28
$5 < d \leq 6$	33



Median = 3, Inter-quartile range – any answer between 2 and 2.5 is acceptable.

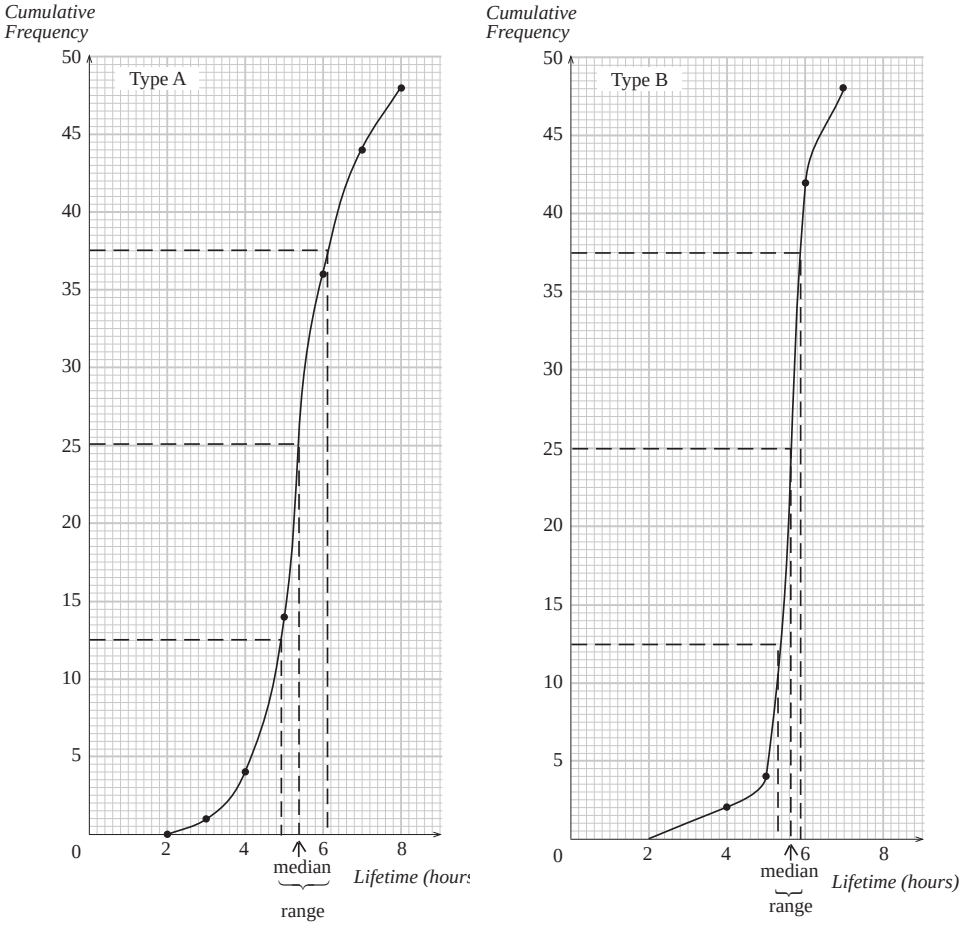
2. (a)



(b) The median of field A is 13g, the median of field B is 12g.
The inter-quartile range for field A is around 7, for B it is around 4.

(c) Field B is more reliable than field A (its inter-quartile range is narrower), although it is less productive in 50% of the cases (its median is lower than the median of field B).

3. (a)

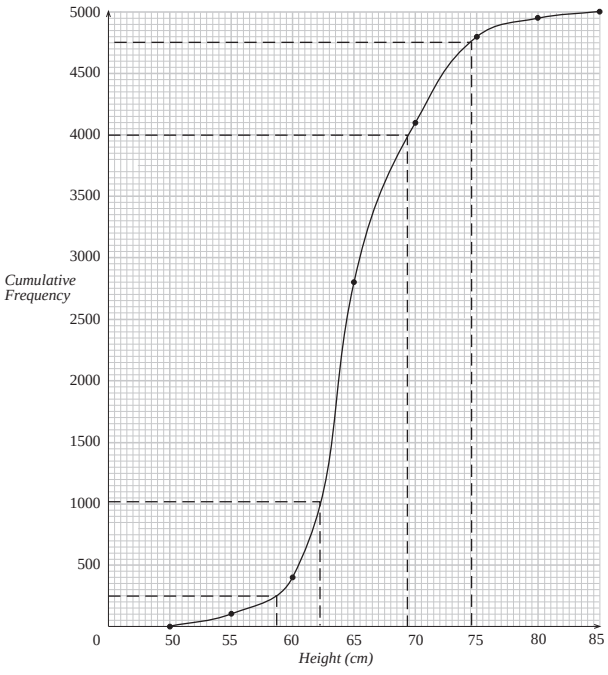


Median for type A = 5.6, Inter-quartile range for type A = 1.2.
Median for type B = 5.8, Inter-quartile range for type B = 0.8.

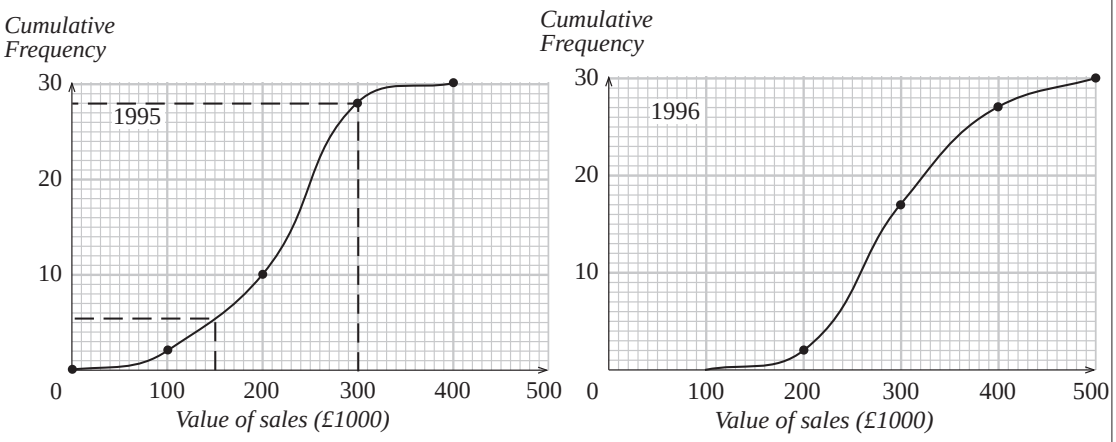
(b) Type B - although both types have quite similar medians, type B is more predictable (its inter-quartile range is narrower) and most of the time its lifetime is above 5.

4. The heights of children in each category (to the nearest cm) are:

Very tall	-	$75 < h \leq 85$
Tall	-	$70 < h \leq 75$
Normal	-	$62 < h \leq 70$
Short	-	$58 < h \leq 62$
Very short	-	$50 < h \leq 58$



5.



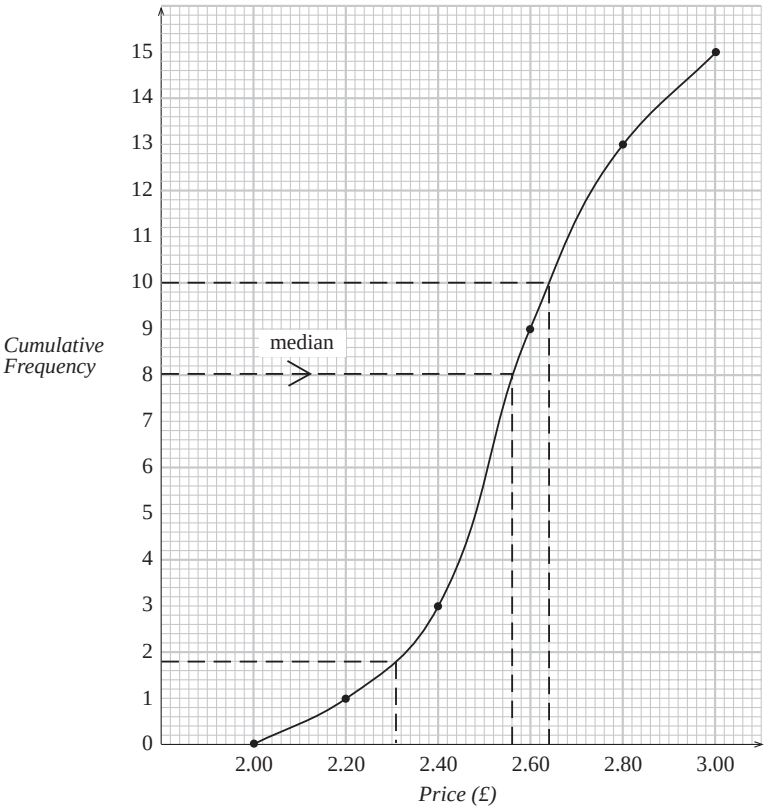
For 1995:

Bonus	*Value of sales*
£50	$300 < V \leq 400$
£250	$140 < V \leq 300$
£500	$0 < V \leq 140$

For 1996:

Bonus	*Value of sales*
£50	$100 < V \leq 230$
£250	$230 < V \leq 400$
£500	$400 < V \leq 500$

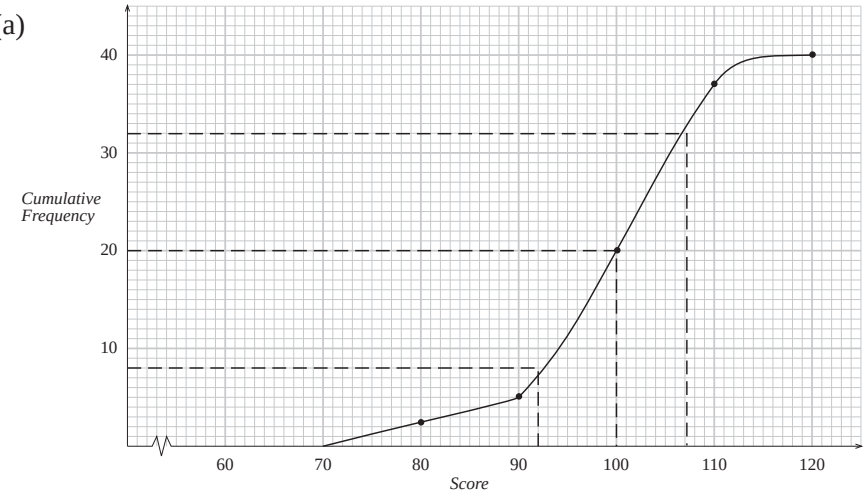
6. (a)



(i) 5 or 6 shops (ii) About £2.50 (iii) 2 shops
(iv) 8 shops (v) 5 or 6 shops

(b) The only exact answer is (iv). The other answers are estimates since they relate to prices for which we do not have exact information.

7. (a)



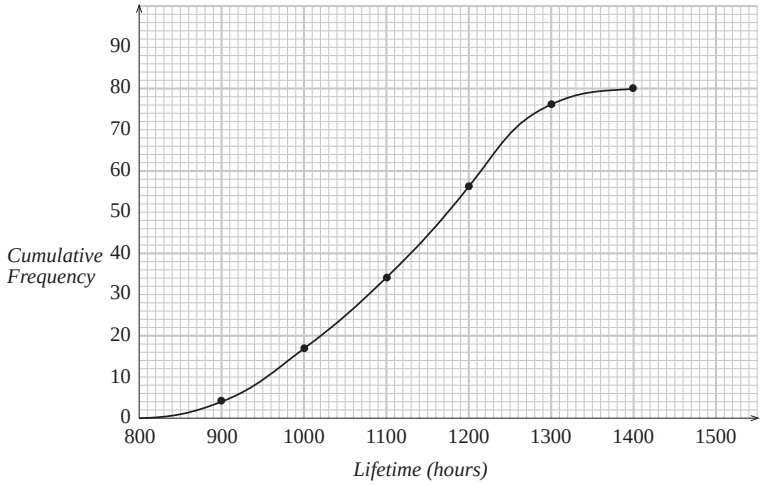
(b) (i) Laura's median = 100 (ii) Inter-quartile range - between 14 and 16.

(c) (i) Joy was the more consistent player because her inter-quartile range is lower.

(ii) Laura won most of the games. She scored less than 100 in 20 matches and more than 103 in only 16 matches, whereas Joy scored more than 103 in 20 matches and her inter-quartile scores were quite consistent.

8. (a) Cumulative frequencies - 34, 56, 76, 80, 80.

(b)



(c) 58 bulbs (d) Inter-quartile range – between 250 and 260

(e) The bulbs from the second sample are more reliable than those from the first.

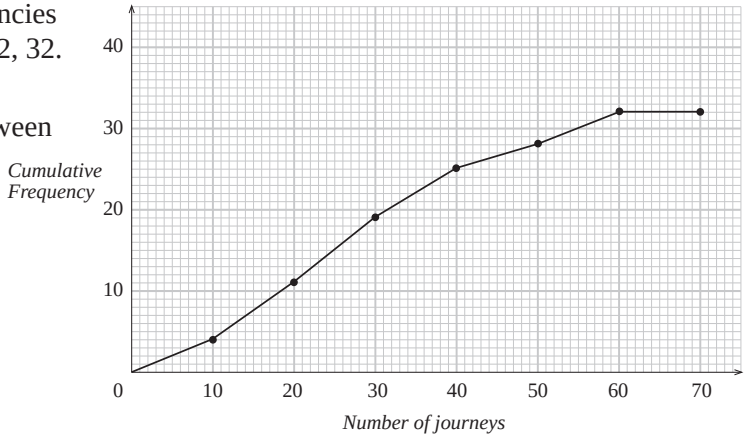
9. (a) Cumulative frequencies 4, 11, 19, 25, 28, 32, 32.

(b) (i) as graph.

(ii) Median – between 25 and 28

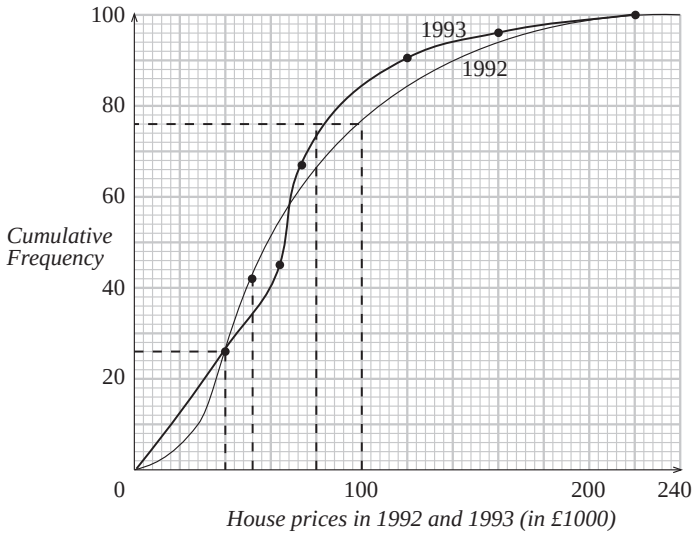
(iii) 6 people

(c) The second group travelled more .



10. (a) Cumulative frequencies – 26%, 45%, 67%, 82%, 91%, 96%, 100%.

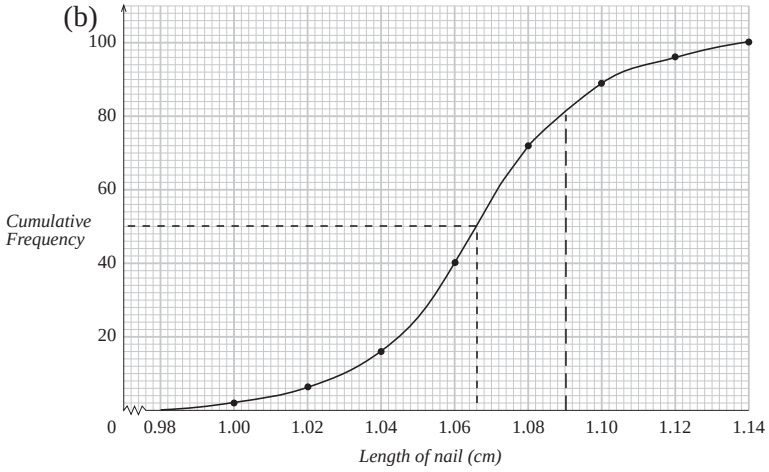
(b)



(c) In 1992, 76% of houses cost up to £100 000, whilst in 1993 76% of houses cost up to £80 000. Hence, in 1993 the price of the house should be around £80 000.

11. (a) Cumulative frequencies: 2, 6, 16, 40, 72, 89, 96, 100.

(b)



(i) Median: between 1.06 cm and 1.07 cm

(ii) Inter-quartile range : between 0.04 and 0.05

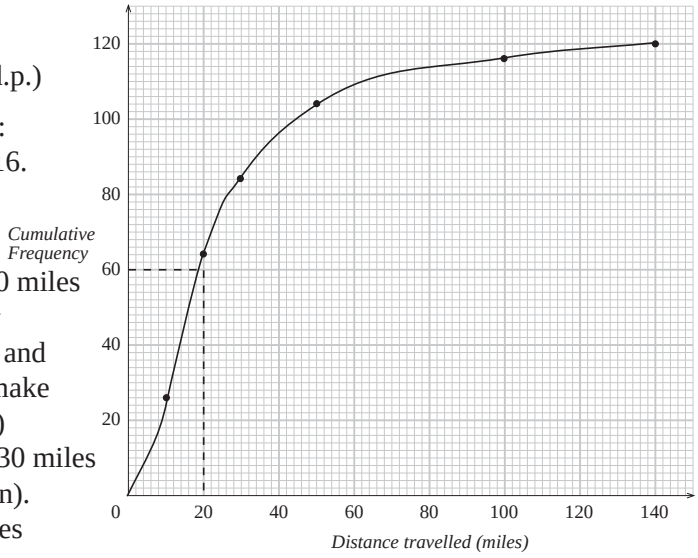
12. (a) Mean distance = 28.17 miles (to 2 d.p.)

(b) (i) Number of guests: 26, 64, 84, 104, 116.

(ii) as graph

(c) (i) Median: between 18 and 20 miles

(ii) The range is very large(140 miles), and 36 people (who make up almost a third) travel more than 30 miles (above the median). These upper values influenced the mean which is far above the median.



9.6 Standard Deviation

1. (a)

	Mean	*Standard Deviation*
A	53.67	4.31
B	73.67	4.31
C	107.33	8.62

(b) Adding same number to values: the mean increases but the s.d does not;
× values by a scale factor: the mean increases and so does the s.d.

2. (a)

	Mean	*Standard Deviation*
A	2.24 kg	0.04
B	2.64 kg	0.07

(b) On average, the boxes filled by A weigh less than the boxes filled by B, but A is more accurate (0.04 < 0.07).

3.

	Mean	*Standard Deviation*
A	10.02	0.43
B	9.6	0.14

The experiments done by B are more accurate and more reliable (0.14 < 0.49).

4. 45.61; 1.49 5. 6.7; 1.85

6. The estimated mean is 1.22, the estimated standard deviation is 0.96.

7. The estimated mean is 61.30, the estimated standard deviation is 51.61.

8.

	Estimated Mean	*Estimated Standard Deviation*
A	1.57	1.15
B	3.2	1.99

On average, the sizes of the families in A are smaller and less spread out than in B.

9. (a) mean = 5, standard deviation = $\sqrt{2}$ $(\cong 1.41)$
(b) Any five consecutive integers have the following pattern:
$n-2$, $n-1$, n, $n+1$, $n+2$, where n is the middle integer.
There are two integers which are 2 units away from n, ($n-2$ and $n+2$), and two integers which are 1 unit away from n, ($n-1$and n + 1). Since n is the mean, the standard deviation will be:

$$\sqrt{\frac{2^2+1^2+0^2+1^2+2^2}{5}} = \sqrt{\frac{10}{5}} = \sqrt{2}$$

10. (a) mean = 30, standard deviation = 16.11
    (b) The means in both Mathematics and English are identical, but the marks in English are less spread out (3.6 < 16.11) than in Mathematics.

11. (a) (i) 32 (ii) 11.32
    (b) On average, the girls performed less than boys (30 < 32), but their scores are less spread out (6.5 < 11.32) around the mean.

12. (a) 8.98
    (b) Class A has, on average, higher and less spread out I.Q. scores than Class B.
    (c) 0

13. (a) (i) 6 (ii) 2.16
    (b) The second test has 6 as mean and 1.48 as standard deviation; e.g.the means are identical on both tests but the second group performed more homogeneously than the first group. On average, the scores of the second group were closer to 6 than the scores of the first group.

14. (a) mean = 4.5, s.d. = 1.86
    (b) Yes it does. The values within one s.d. of the mean lie between 2.64 and 6.36. In our data these values are 3, 4, 5, 6. The total frequency of these values in our data is 7 + 8 + 9 + 10 = 34. The percentage of values within one s.d. of the mean is therefore:

$$\frac{34}{50} \times 100 = 68\%.$$

10 Equations

10.1 Negative Numbers

1. $-4°C$; $-2°C$; $-8°C$; $-14°C$; $-8°C$; $-5°C$; $-7°C$.

2. (a) –10, –6, –5, –4, 2, 3 (b) –4, –1, 0, 3, 7
   (c) –6, –4, –2, 0, 3, 7 (d) –7, –4, –2, 0, 5, 7
   (e) –5, –4, –3, –1, 0, 6

3. (a) –7, –8, –9 (b) –3, –2, –1, 0, 1 (c) –2, –1 (d) –2
   (e) –8, –7, –6, –5, -4, –3, –2, –1

4. (a) –7, –8 or –9 (b) –7, –8, . . . (c) –1, –2, –3, or –4
   (d) –9, –8, . . . (e) –7 or –8 (f) –5, –6, . . .

5. (a) $3 > 2$ (b) $-5 > -6$ (c) $0 > -1$ (d) $-7 > -10$ (e) $-2 > -4$
   (f) $-1 > -6$ (g) $-5 < 0$ (h) $-9 < -6$ (i) $-8 < -2$

10.2 Arithmetic with Negative Numbers

1. (a) 12 (b) –14 (c) –12 (d) -20 (e) –7 (f) –6
   (g) –1 (h) 28 (i) 1 (j) 32 (k) -2 (l) 7
   (m) –10 (n) 2 (o) –13 (p) –12 (q) 56 (r) 1
   (s) –6 (t) –15 (u) –8 (v) –8 (w) 5 (x) –2

10.3 Simplifying Expressions

1. (a) $6a$ (b) $3a + 12$ (c) $11a + 6b$ (d) $4x + 11y$
   (e) $13x - y$ (f) $2a + 10b$ (g) $3a + b + 6$ (h) $6p - 7q$
   (i) $-7x + 3y$ (j) $2x - p$ (k) $15x - 9z$ (l) $-14x + 5z$
   (m) $8a + 4q - 6x$ (n) $-p - q + x + z$ (o) $3x - 2y + 10$ (p) $21x - 32q$
   (q) $2y$ (r) $2x$ (s) $-19x + 11y$ (t) $23x - 18y - 4$
   (u) $-7x - 10y$ (v) $-4p + 4q$

2. (a) $6x^2 + 8x$ (b) $x^2 + 13x + 10$ (c) $2x^2 + 10x$
   (d) $5x^2 + 2x + 10$ (e) $-x^2 + 7x$ (f) $3x^2 + 4y^2$
   (g) $2x^2 + y^2 - x - y$ (h) $5x^2 - 3x - 10$ (i) $3x^2 - 3y^2 - x - y$
   (j) $5x^2 + 2y - 4$ (k) $5ab + cd$ (l) $2xy + 5xz$
   (m) $11ab - 3ad$ (n) $9pq - 3qr$

3. (a) $3x + 15$ (b) $24 + 4x$ (c) $7x + 14$ (d) $2x + 12$
   (e) $5x + 10$ (f) $8x + 12$ (g) $15x + 10$ (h) $40x + 24$
   (i) $7x - 42$ (j) $40 - 8x$ (k) $8x - 28$ (l) $35x - 21$
   (m) $18x - 30$ (n) $4x - 8y$ (o) $5x + 10y + 15z$ (p) $5x + x^2$

(q) $2a - a^2$ (r) $4b - 12$ (s) $2x^2 - 12x$ (t) $8x^2 + 12x$
(u) $21x - 6x^2$ (v) $8x^2 - 40x$

4. (a) $7a + 26$ (b) $7x + 23$ (c) $8x + 22$ (d) $2x^2 + 7x$
(e) $x^2 + 9x + 4$ (f) $12a + 17b$

10.4 Simple Equations

1. (a) $x = 4$ (b) $x = 9$ (c) $x = 3$ (d) $x = 12$ (e) $x = 14$
(f) $x = -2$ (g) $x = 9$ (h) $x = 4$ (i) $x = 18$ (j) $x = 8$
(k) $x = 34$ (l) $x = 7$ (m) $x = 10$ (n) $x = 24$ (o) $x = 50$
(p) $x = 12$ (q) $x = 21$ (r) $x = 66$ (s) $x = 8$ (t) $x = 6$
(u) $x = 26$ (v) $x = 36$ (w) $x = 15$ (x) $x = 4$

2. (a) $x = -4$ (b) $x = -5$ (c) $x = 3$ (d) $x = -6$ (e) $x = -4$
(f) $x = -14$ (g) $x = -6$ (h) $x = -3$ (i) $x = -12$ (j) $x = -16$
(k) $x = -3$ (l) $x = -20$ (m) $x = 8$ (n) $x = 8$ (o) $x = 6$
(p) $x = -5$ (q) $x = -10$ (r) $x = -26$

3. (a) $x + 110° = 180°$; $x = 70°$ (b) $x + 92° = 180°$; $x = 88°$
(c) $x + 110° = 180°$; $x = 149°$ (d) $42° + x + 40° = 180°$; $x = 98°$

4. $x - 2 = 16$; $x = 18$ 5. $12 \times x = 54$; $x =$ £4.50 6. $x + 3 = 41$; $x =$ £38

7. 52 is double Majid's number, so the number is half of 52. It is 26.
Alternatively, $x \times 2 = 52$, so $x = 26$.

8. 24 is double Jim's number, so his number is half of 24. It is 12.
Alternatively, $x \times 2 = 24$, so $x = 12$.

9. Phillip is 11 years old. Sue is $11 - 2 = 9$ years old.
Ali is twice as old as Sue, so Ali is $9 \times 2 = 18$ years old.

10.5 Solving Equations

1. (a) $x = 14$ (b) $x = 9$ (c) $x = 14$ (d) $x = 8$ (e) $x = 4$
(f) $x = 1\frac{1}{3}$ (g) $x = -4$ (h) $x = -1$ (i) $x = \frac{-5}{3}$ (j) $x = 7$
(k) $x = 6$ (l) $x = -2$ (m) $x = 10$ (n) $x = 1$ (o) $x = \frac{3}{5}$
(p) $x = -2\frac{1}{2}$ (q) $x = \frac{1}{5}$ (r) $x = \frac{-3}{4}$ (s) $x = -3$ (t) $x = 0$
(u) $x = \frac{-2}{5}$ (v) $x = 36$ (w) $x = 15$ (x) $x = 5$

2. (a) $x = -3$ (b) $x = -6$ (c) $x = \frac{13}{4}$ (d) $x = 14$

(e) $x = 4$ (f) $x = 10$ (g) $x = 1$ (h) $x = -3$ (i) $x = 13$

(j) $x = 3$ (k) $x = 4$ (l) $x = \frac{2}{3}$ (m) $x = 3$ (n) $x = 2$

(o) $x = \frac{5}{3}$ (p) $x = 4$ (q) $x = 2$ (r) $x = \frac{1}{2}$ (s) $x = 8$

(t) $x = 12$ (u) $x = 32$ (v) $x = 14$ (w) $x = 39$ (x) $x = 12$

3. (a) $(3x + 40) + 80 + x = 360$; $x = 60°$

(b) $(x + 10) + (x + 30) + (2x + 30) + x = 360$; $x = 58°$

(c) $(x + 10) + (x + 30) + (x - 10) = 360$; $x = 110°$

(d) $(140 - x) + (120 - x) + (180 - x) = 360°$; $x = 26\frac{2}{3}°$

4. $x + x + (x + 1) + (x + 1) = 10$; $x = 2$ m 5. $2x + 10 = 42$; $x = 16$

6. $6x - 8 = 34$; $x = 7$ 7. $4m + 2 = 6$; $m = 1$; Mary drives for 1 hour.

8. (a) $80 + (60 \times x) = 290$; $x = 3\frac{1}{2}$ (3 hours and 30 mins).

(b) $80 + (60 \times x) = 220$; $x = 2\frac{1}{3}$ (2 hours and 20 mins).

9. (a) $(x + 6) \times 2 = 18$; $x = 3$ (b) $(x \div 2) + 10 = 16$; $x = 12$

(c) $[(x \div 2) + 2] \times 2 = 9$; $x = 5$ (d) $[(x - 7) \div 2] \times 10 = 115$; $x = 30$

10. $x + (x + 1) + (x + 2) + (x + 3) = 114$; $x = 27$ 11. 6

12. (a) $y = 2x + 1$ (b) $x = 4$ 13. (a) 7 (b) 4 14. $x = 2$

15. (a) $x = 4\frac{1}{2}$ (b) $y = 4$

16. (a) $(3x + 1)$ cm (b) (i) $3x + 1 = 22$ (ii) 7 cm, 5 cm and 10 cm.

17. (a) $2x$ pence (b) $(x + 10)$ pence (c) 30 pence

10.6 Trial and Improvement Method

1. (a) $x = 5$ (b) $x = 27$ (c) $x = 5$ (d) $x = 12$ (e) $x = 3$
(f) $x = 64$ (g) $x = 100$ (h) $x = 11$

2. (a) $x = 2.2$ (to 1 decimal place) (b) $x = 4.8$ (to 1 d.p.)
(c) $x = 1.8$ (to 1 d.p.) (d) $x = 5.6$ (to 1 d.p.)
(e) $x = 2.6$ (to 1 d.p.) (f) $x = 2.4$ (to 1 d.p.)

3. (a) $x = 1.82$ (to 2 d.p.) (b) 10.1 is a good starting number; $x = 10.07$ (2 d.p.)
(c) 7.05 (to 2 d.p.)

4. (a) $x = 7$ or -8 (b) $x = 9$ or -8 (c) $x = 1$ or -1
   (d) $x = 10$ or -8 (e) $x = 7$ or -1 (f) $x = 3$ or -8

5. (a) $x = 1.14$ or 6.14 (b) $x = 5.24$ or 0.76 (c) $x = 2.32$ or -4.32
   (d) $x = 1.12$ or -7.12

6. (a) x cannot be negative because, so far, we have no meaning for the square root of a negative number.
   (b) $x = 5.628$ to 3 d,p,

7. (a) $x = 7.87$ or 0.13 (b) $x = 4.24$ or -0.24 (c) $x = 3.41$ or 0.59
   (d) $x = 23.31$ (e) $x = 6.46$ or 0.06 (f) $x = 6.19$ or 0.81

8. $x = 1.8$ to 1 d.p. 9. $x = 2.9$ gives the result nearest 11.

10. (a) $x = 5$
    (b) (i) $4^3 = 64$; $5^3 = 125$ (ii) $x = 4.6$ (to 1 d.p.) is the nearest value.

11. (a) 17.3 m (b) 1730 cm (to the nearest 10 centimetres)

12. (a) 2 and 3 (b) 2.7

10.7 Expanding Brackets

1. (a) $x^2 + x$ (b) $2x - 3x^2$ (c) $6a^2 - 27a$ (d) $x^2 - x^3$
   (e) $4x^2 - 28x$ (f) $3x + 11$ (g) $x^2 - 2x$ (h) $x + x^3$
   (i) $-4x^2 - 3$ (j) $-5x$ (k) $5x^2 - 20x$ (l) $5x^2 - 8$
   (m) $5x^2 - x^3$ (n) $3x + 16$ (o) $7x - 3x^2$

2. (a) $x^2 + 7x + 6$ (b) $x^2 + 11x + 28$ (c) $x^2 + 6x - 16$
   (d) $x^2 + 3x - 4$ (e) $x^2 - 8x + 7$ (f) $6x^2 + x - 1$
   (g) $8x^2 + 34x + 21$ (h) $10x^2 + 11x - 6$ (i) $24x^2 + 14x - 3$
   (j) $12x^2 - 13x + 3$ (k) $15x^2 - 58x + 48$ (l) $15a^2 - 29a - 14$
   (m) $12a^2 - 35a + 25$ (n) $24n^2 - 8n - 2$ (o) $20x^2 - 17x - 24$

3. (a) $x^2 + 2x + 1$ (b) $x^2 - 2x + 1$ (c) $x^2 + 16x + 64$
   (d) $4x^2 + 4x + 1$ (e) $9x^2 + 24x + 16$ (f) $36x^2 - 12x + 1$
   (g) $4x^2 - 20x + 25$ (h) $36x^2 - 84x + 49$ (i) $25x^2 + 90x + 81$

4. (a) $ac + ad + bc + bd$ (b) $2a^2 + 3ac + c^2$ (c) $3a^2 - 14ad - 5d^2$
   (d) $12x^2 + xy - y^2$ (e) $4a^2 + 13ad + 3d^2$ (f) $2a^2 + ab + 8ac + 4bc$
   (g) $9xy - y^2 - 18x^2$ (h) $q^2 - 2p^2 - pq$ (i) $45x^2 - xy - 2y^2$
   (j) $4x^2 + 4xy - 2x - 2y$ (k) $4a^2 - 2ac + 2ab - bc$
   (l) $4x^2 - 29xy + 30y^2$ (m) $p^2 - 4q^2$ (n) $5a^2 - 13ab - 6b^2$
   (o) $10x^2 - 27xy + 18y^2$

5. (a) $x^2 - 1$; $x^2 - 16$; $x^2 - 36$ (b) $x^2 - 25$ (c) $a^2 - b^2$

   (d) $(5x)^2 - 2^2 = 25x^2 - 4$

6. (a) $x^2 + x - 2$ (b) $2x^2 - 6x + 4$ (c) $2x^2 + 4x + 2$

7. (a) $x^3 - 3x^2 - 10x$ (b) $x^3 + 4x^2 - 4x - 16$

   (c) $x^3 + 2x^2 - 5x - 6$ (d) $6x^3 + 37x^2 - 39x - 28$

   (e) $x^3 + 3x^2 + 3x + 1$ (f) $x^3 - 12x^2 + 48x - 64$

10.8 Simultaneous Linear Equations

1. (a) $x = 1, y = 2$ (b) $x = 5, y = 2$ (c) $x = 10, y = 3$

   (d) $x = 4, y = 2$ (e) $x = \frac{46}{43}, y = \frac{-13}{43}$ (f) $x = 5, y = -2$

   (g) $x = -2, y = 12$ (h) $x = 10, y = 12$ (i) $x = 5, y = 6$

   (j) $x = \frac{5}{7}, y = \frac{9}{28}$ (k) $x = \frac{7}{45}, y = \frac{13}{36}$ (l) $x = 7, y = \frac{1}{5}$

   (m) $x = \frac{1}{2}, y = 3$ (n) $x = -3, y = -1$ (o) $x = \frac{43}{33}, y = \frac{48}{11}$

   (p) $x = 0.2, y = 0.4$ (q) $x = 10, y = 20$ (r) $x = -20, y = 30$

2. (a) (3, 5) (b) (3, 7) (c) $\left(\frac{20}{9}, \frac{16}{9}\right)$

3. It is not possible to eliminate either x or y because of their coefficients. If the equations had been used to produce graphs it would be seen that the two straight lines produced are parallel. Therefore, there can be no point of intersection and it is impossible to solve the equations simultaneously.

4. (b) There is no way of eliminating x or y in order to find the value of the other variable.

   (c) $x + 2y = 12 \quad (1)$ gives $y = 6 - \frac{x}{2}$

   $3x + 6y = 36 \quad (2)$ gives $y = 6 - \frac{x}{2}$

   So equations (1) and (2) are different versions of the same equation.

5. $v + c = 3.7$; $v - c = 1.3$; $v = 2.5$, $c = 1.2$.

6. 72 return tickets and 28 single tickets were sold.

7. (a) 79 ten-pound notes; 121 five-pound notes
   (b) 78 ten-pound notes; 122 five-pound notes

8. 16

9. The length of the flight is 9 hours; the time difference is 5 hours.

10. $x = -11, y = 15$ 11. $a = \frac{7}{2}, c = \frac{3}{2}$

12. (a) 2 metres

(b) (i) $t = 1$ when $h = 37$, so $37 = (a \times 1) + (b \times 1) + 2$ and $35 = a + b$

$t = 2$ when $h = 62$, so $62 = (a \times 4) + (b \times 2) + 2$ and $60 = 4a + 2b$

(ii) $a = -5$; $b = 40$

(c) 20.75 metres

10.9 Factorisation 1

1. (a) $5x + 10 = 5(x + 2)$ (b) $6x - 8 = 2(3x - 4)$

(c) $15x + 25 = 5(3x + 5)$ (d) $12x + 8 = 4(3x + 2)$

(e) $18 - 6x = 6(3 - x)$ (f) $6x - 21 = 3(2x - 7)$

(g) $16a + 24 = 8(2a + 3)$ (h) $33x - 9 = 3(11x - 3)$

2. (a) $6(x + 4)$ (b) $5(x - 4)$ (c) $8(2 - x)$ (d) $4(2n + 3)$

(e) $2(6x - 7)$ (f) $3(a - 8)$ (g) $11(x - 6)$ (h) $5(2 + 5x)$

(i) $20(5x - 2)$ (j) $10(5 - 4x)$ (k) $6(x - 5)$ (l) $5(y - 9)$

(m) $12(1 + 3x)$ (n) $16(x + 2)$ (o) $3(9x - 11)$

3. (a) $x(x + 1)$ (b) $x(x + 2)$ (c) $a(2a - 5)$ (d) $x(4x + 1)$

(e) $x(x + 4)$ (f) $x(a + b)$ (g) $3x(2x + 1)$ (h) $2x(2x - a)$

4. (a) $x(5x + 1)$ (b) $a(a + 3)$ (c) $n(5n + 2)$ (d) $3n(2n + 1)$

(e) $5n(n - 2)$ (f) $3x(x + 2)$ (g) $15x(x + 2)$ (h) $7x(2x + 3)$

(i) $8x(2x + 3)$ (j) $6x(5x - 3)$ (k) $5(1 + n^2)$ (l) $5(2n^2 - 3)$

(m) $3n(n^2 + 3)$ (n) $9x(x + 3)$ (o) $5x^2(2x - 1)$

5. (a) $ax(1 + x)$ (b) $x(b + cx)$ (c) $2q(p - 2r)$ (d) $5y(3x - y)$

(e) $8p(2q + 3p)$ (f) $6x(x + 3y)$ (g) $3p(p - 3x)$

(h) $8x(3p + 7x)$ (i) $2xy(8x - 9y)$

6. (a) Yes; $2x(3x + 1)$ (b) Yes; $8x^2(2x + 1)$ (c) No

(d) Yes; $3xy(x - 6y)$

10.10 Factorisation 2

1. (a) $(x+2)^2$ (b) $(x+4)(x+3)$ (c) $(x+4)(x+2)$
   (d) $(x+6)(x+1)$ (e) $(x+8)(x+2)$ (f) $(x+3)(x+1)$
   (g) $(x+5)(x+3)$ (h) $(x+2)(x+1)$ (i) $(x+4)(x+1)$
   (j) $(x+3)(x+8)$ (k) $(x+11)(x+1)$ (l) $(x+8)(x+7)$
   (m) $(x+3)(x+3)$ (n) $(x+5)(x+2)$ (o) $(x+7)(x+2)$
   (p) $(x+6)(x+5)$ (q) $(x+8)(x+1)$ (r) $(x+8)(x+4)$

2. (a) $(x+2)(x-1)$ (b) $(x-4)(x+3)$ (c) $(x-5)(x+2)$
   (d) $(x+5)(x-1)$ (e) $(x-7)(x+2)$ (f) $(x-4)(x+2)$
   (g) $(x+5)(x-3)$ (h) $(x-2)(x-1)$ (i) $(x-5)(x-4)$
   (j) $(x-7)(x-3)$ (k) $(x-7)(x-2)$ (l) $(x-5)(x-2)$
   (m) $(x-8)(x+2)$ (n) $(x-9)(x-8)$ (o) $(x-8)(x+3)$

3. (a) $(x+1)(x-1)$ (b) $(x+4)(x-4)$ (c) $(x+9)(x-9)$
   (d) $(3x+2)(3x-2)$ (e) $(4x+6)(4x-6)$ (f) $(2x+10)(2x-10)$
   (g) $\left(x^2+10\right)\left(x^2-10\right)=\left(x^2+10\right)\left(x+\sqrt{10}\right)\left(x-\sqrt{10}\right)$
   (h) $\left(x^2+2\right)\left(x^2-2\right)=\left(x^2+2\right)\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)$
   (i) $\left(2x^2+3\right)\left(2x^2-3\right)=\left(2x^2+3\right)\left(\sqrt{2}x+\sqrt{2}\right)\left(\sqrt{2}x-\sqrt{2}\right)$

4. (a) $(2x+1)(x+1)$ (b) $(3x+1)(x+2)$ (c) $(2x+3)(x+1)$
   (d) $(3x+2)(x+4)$ (e) $(2x-1)(x+5)$ (f) $(4x+3)(x-2)$
   (g) $(3x-5)(x+2)$ (h) $(3x-2)(x-7)$ (i) $(3x+1)(2x+5)$
   (j) $(4x-1)(2x-1)$

5. (a) $(3x+1)(x-1)$ (b) $(3x+1)(x+1)$ (c) $(2x+1)(x+2)$
   (d) $(3x+2)(x+2)$ (e) $(3x-1)(x+3)$ (f) $(4x+1)(x-3)$
   (g) $(5x-2)(x+1)$ (h) $(3x-2)(x-2)$ (i) $(5x-2)(x+3)$
   (j) $(3x+1)(2x+1)$ (k) $(2x-1)(3x-2)$ (l) $(5x+1)(2x-1)$
   (m) $(4x-1)(2x+3)$ (n) $(2x+7)(3x-1)$ (o) $(3x-4)(2x-3)$

6. (a) $2x^2+9x\ 4$ (b) $2x(2x-3)$

7. (a) $3pq(4p-5q)$ (b) $2x^2+7x-15$ (c) $n=(\mathrm{C}-120)\div 40$

10.11 Solving Quadratic Equations by Factorisation

1. (a) $x = -4$ or 3 (b) $x = 5$ or -3 (c) $x = -6$ or 2 (d) $x = 6$ or 0

   (e) $x = 0$ or $\frac{4}{3}$ (f) $x = 0$ or $\frac{9}{4}$ (g) $x = 3$ or -3 (h) $x = 7$ or -7

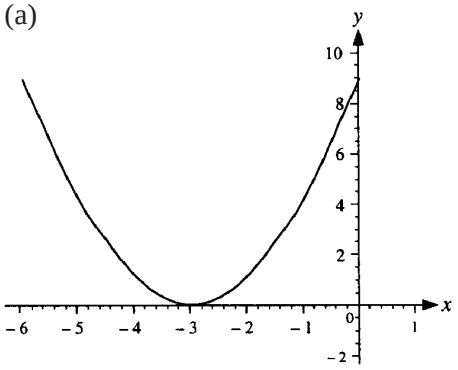
   (i) $x = \frac{8}{3}$ or $-\frac{8}{3}$ (j) $x = 4$ (both answers) (k) $x = -5$ (both answers)

   (l) $x = 6$ or -3 (m) $x = 4$ or 7 (n) $x = 6$ or -5 (o) $x = 10$ or 4

   (p) $x = -3$ or $-\frac{1}{2}$ (q) $x = \frac{3}{2}$ or -4 (r) $x = 1$ or $\frac{4}{3}$

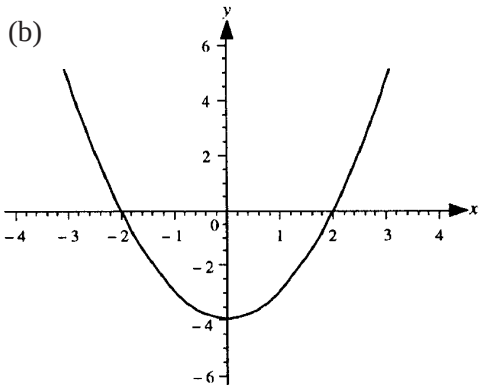
   (s) $x = \frac{3}{4}$ or -1 (t) $x = \frac{1}{2}$ or -3 (u) $x = 7$ or $\frac{5}{2}$

2. (a)



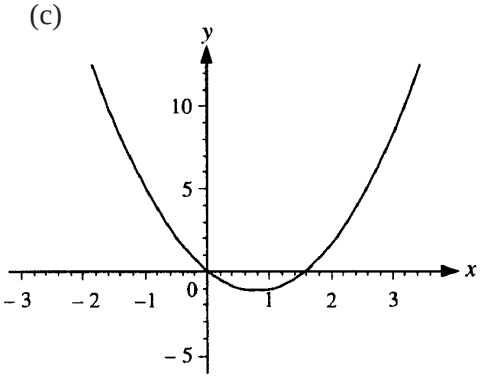
The curve touches the x-axis at the point where $x = -3$.

(b)



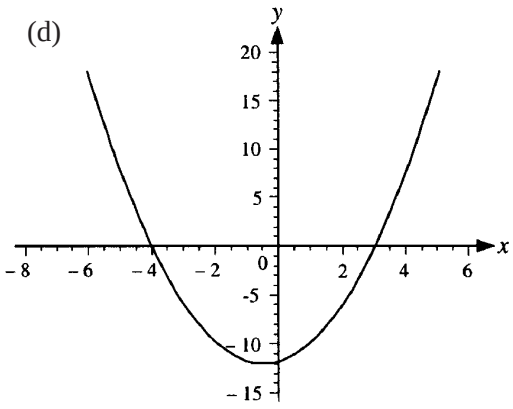
The curve cuts the x-axis at the points where $x = 2$ and $x = -2$.

(c)



The curve cuts the x-axis at the points where $x = 0$ and $x = \frac{3}{2}$.

(d)



The curve cuts the x-axis at the points where $x = 3$ and $x = -4$.

3. (a) $(x^2 + 4)(x + 2)(x - 2) = 0$, so $x = -2$ or 2

   (b) $(x^2 + 25)(x + 5)(x - 5) = 0$, so $x = -5$ or 5

   (c) $(x^3 + 1)(x^3 - 1) = 0$, $(x^3 + 1)(x - 1)(x^2 + x\,1) = 0$,
   $(x + 1)(x^2 - x + 1)(x - 1)(x^2 + x - 1) = 0$, so $x = -1$ or 1

4. (a) $x = 5$ (b) $x = 4$ (c) $x = 4$ (d) $x = 8$

5. $t = 0.8$ 6. (a) $L = \dfrac{2V}{W(E + R)}$ (c) $L = 5$ or $L = 10$

10.12 Solving Quadratic Equations using the Formula

1. (a) 1 or 0.75 (b) 2 or -2.5 (c) 1.49 or -0.82 (2 d.p.)
   (d) 2.57 or -0.91 (2d.p.) (e) 2.37 or -3.37 (2 d.p.)
   (f) 2.43 or 0.93 (g) 0.5 or -9 (h) 14 or 0
   (i) 2.32 or -4.32 (2 d.p.) (j) 0.12 or 2.79
   (k) $\sqrt{-6}$ is involved so we can give no answers. (l) 3.58 or 0.42
   (m) 1.69 or –0.44 (2 d.p.)
   (n) $\sqrt{-224}$ is involved so we can give no answers. (o) 6.74 or -0.74

2. (a) Area = $x(x + 2)$
   (b) The ticket is 2.32 cm wide and 4.32 cm long, both measures correct to 2 d.p.

3. The height is 1.69 m and the width is 1.19 m.

4. (a) The stone hits the ground after 2.04 seconds (correct to 2 d.p.)
   (b) 1.46 seconds, correct to 2 d.p.
   (c) The stone never reaches a height of 12 m above the ground level.
   (d) $m = 20t - 9.8t^2$. The maximum height is 10.20 m, correct to 2 d.p.

5. (a) The maximum stretch is 16.18 metres.
   (b) The rope would stretch by 18.11 metres, an increase of 1.93 m.

6. $x = 6.14$ or -1.14, both answers correct to 3 s.f.

10.13 Algebraic Fractions

1. (a) x^2 (b) $\dfrac{x + 2}{x(x - 2)}$ (c) $\dfrac{x - 3}{x - 2}$ (d) $\dfrac{2x - 1}{x(x + 1)}$
   (e) $\dfrac{x^2(x - 6)}{x + 6}$ (f) $\dfrac{1}{x}$ (g) $\dfrac{5x - 3}{x}$ (h) $\dfrac{2x + 1}{x}$ (i) $\dfrac{x - 3}{x + 2}$

2. (a) $\dfrac{1}{x}$ (b) x (c) $2x$ (d) $\dfrac{x^2 + 5x}{x + 5}$ (e) $x(x - 1)$
   (f) $x^2(x^2 + x + 1)$ (g) $\dfrac{x}{x - 1}$ (h) $\dfrac{x(x + 2)(x - 2)}{(x - 4)(x + 1)}$ (i) $\dfrac{x - 3}{x + 1}$
   (j) $\dfrac{x(x + 4)}{x - 2}$ (k) $\dfrac{x + 5}{x - 3}$ (l) $\dfrac{x(x - 2)}{x + 1}$ (m) $\dfrac{(x - 1)}{x(x - 3)}$

(n) $\frac{x(x-1)}{x+8}$ (o) $\frac{x-5}{x(x+6)}$ (p) $\frac{5x-6}{x+1}$ (q) $\frac{5x-2}{x-7}$

(r) $\frac{3x+1}{5x+1}$ (s) $\frac{2x-7}{3x+2}$ (t) $\frac{4x-3}{5x-1}$ (u) $\frac{x(3x+2)}{4x+3}$

3. (a) $\frac{x}{x+4}$ (b) $\frac{x-1}{x(x+1)}$ (c) $\frac{(x+2)^2}{x}$ (d) $\frac{3}{x^2(x+2)}$

(e) $\frac{(x+3)^2}{x^3}$ (f) $x(x+3)$ (g) $\frac{x^2(x-1)}{x+1}$ (h) $\frac{x^3(x+1)}{(x+4)^2}$

(i) $\left(\frac{x+3}{x-4}\right)^2$

4. (a) $\frac{3x-1}{(x+2)(x-5)}$ (b) $\frac{-x-14}{(x+2)(x-1)}$ (c) $\frac{x}{(x+1)(x-1)}$

(d) $\frac{8}{x+2}$ (e) $\frac{x+4}{(x+3)(x-3)}$ (f) $\frac{3(3x-13)}{(x-3)(x-6)}$

(g) $\frac{2x(x-1)}{(x-4)(x+2)}$ (h) x (i) $\frac{2x^2+3x-24}{(x-6)(2x-1)}$ (j) $\frac{x(5x-14)}{(x+2)(x-6)}$

(k) $\frac{x(2x-1)}{(x-1)(x+1)}$ (l) $\frac{x^2+8x+1}{x^2}$ (m) $\frac{x(5x+33)}{(x+6)(x+7)}$

(n) $\frac{2x^2+11x+11}{(x+2)(x+3)}$ (o) $\frac{2x^2-7x-17}{(x+1)(x+2)}$

10.14 Completing the Square

1. (a) $(x+2)^2-9$ (b) $(x+3)^2-10$ (c) $(x+5)^2-27$

(d) $(x-4)^2-14$ (e) $(x+6)^2-33$ (f) $(x-10)^2-90$

(g) $\left(x+\frac{3}{2}\right)^2-\frac{13}{4}$ (h) $\left(x-\frac{5}{2}\right)^2-\frac{17}{4}$ (i) $\left(x-\frac{1}{2}\right)^2+\frac{15}{4}$

2. (a) -1 and -3 (b) $3+\sqrt{13}$ and $3-\sqrt{13}$ (c) $-5+\sqrt{33}$ and $-5-\sqrt{33}$

(d) $\frac{-5}{2}+\frac{\sqrt{21}}{2}$ and $\frac{-5}{2}-\frac{\sqrt{21}}{2}$ (e) $-\frac{1}{2}+\frac{\sqrt{5}}{2}$ and $-\frac{1}{2}-\frac{\sqrt{5}}{2}$

(f) $-1+\sqrt{5}$ and $-1-\sqrt{5}$ (g) $-2+\sqrt{12}$ and $-2-\sqrt{12}$

(h) $-\frac{5}{2}+\frac{\sqrt{33}}{2}$ and $-\frac{5}{2}-\frac{\sqrt{33}}{2}$ (i) $-\frac{7}{2}+\frac{\sqrt{45}}{2}$ and $-\frac{7}{2}-\frac{\sqrt{45}}{2}$

3. (a) $2(x+2)^2-3^2$ (b) $2\left(x+\frac{5}{2}\right)^2-\frac{31}{2}$ (c) $2\left(x+\frac{1}{2}\right)^2+\frac{1}{2}$

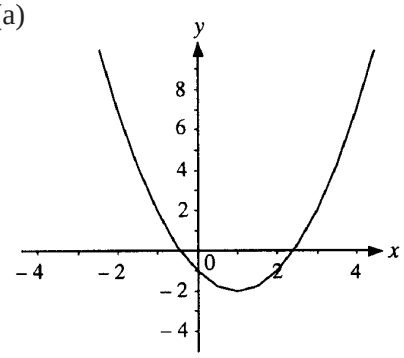
(d) $3(x+1)^2 - 5$ (e) $5\left(x+\frac{3}{2}\right)^2 - \frac{61}{4}$ (f) $7(x-1)^2 - 5$

(g) $3(x+2)^2 - 16$ (h) $4\left(x+\frac{5}{2}\right)^2 - 28$ (i) $2(x-3)^2 - 15$

4. (a) $x = -1 \pm \sqrt{\frac{7}{2}}$ (b) $x = -4 \pm \sqrt{\frac{35}{2}}$ (c) $x = -2 \pm \sqrt{\frac{20}{3}}$

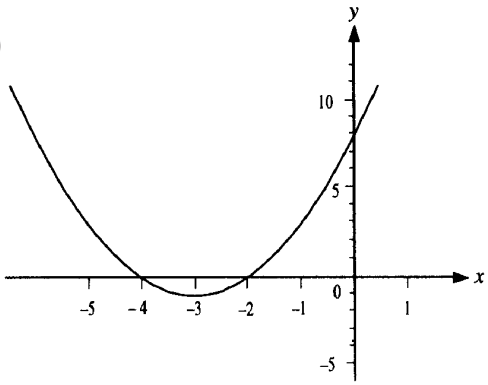
(d) $x = -\frac{1}{2} \pm \sqrt{\frac{1}{2}}$ (e) $x = \frac{13}{4}$ and $\frac{-15}{4}$ (f) $x = 2 \pm \sqrt{\frac{19}{5}}$

5. (a)



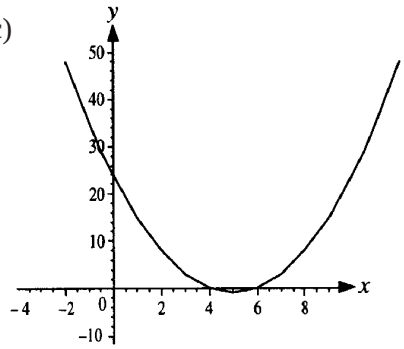
$y = 0$ when $x = -0.41$ and n 2.41;
the minimum value of y is -2.

(b)



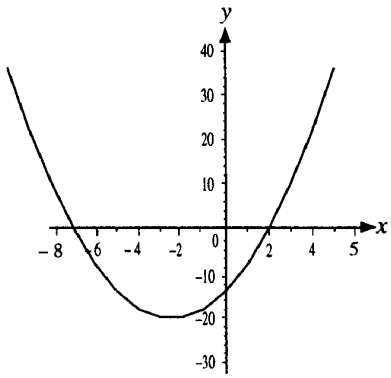
$y = 0$ when $x = -2$ and -4;
the minimum value of y is -1.

(c)



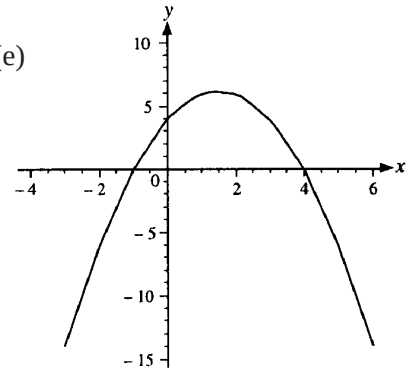
$y = 0$ when $x = 4$ and 6;
the minimum value of y is –1.

(d)



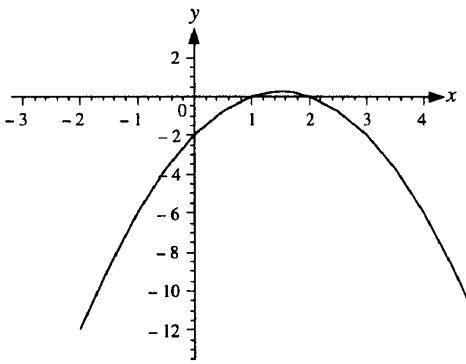
$y = 0$ when $x = 2$ and 7;
the minimum value of y is -20.25

(e)



$y = 0$ when $x = 4$ and -1;
the minimum value of y is 6.25.

(f)

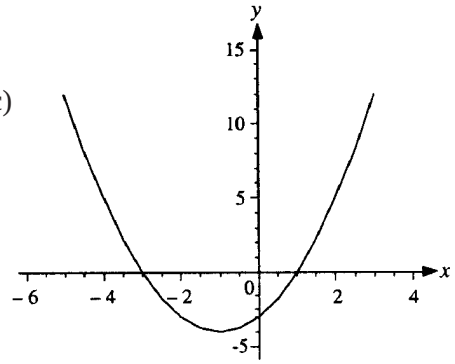


$y = 0$ when $x = 1$ and 2;
the minimum value of y is 0.25.

6. The maximum height of the ball is 11.

7. (a) $a = -2, \quad b = -2$; the minimum value is -2.
   (b) $x = 3.41$ and $x = 0.59$

8. (a) $(x+3)(x-1)$ (b) $(x+1)^2 - 4$ (c)



10.15 Algebraic Fractions and Quadratic Equations

1. Both answers are 4
2. 4 and –1
3. 5 and 15
4. 2 and $\frac{-2}{3}$
5. 3 and –5
6. 7 and $\frac{1}{4}$
7. 2 and $\frac{-23}{9}$
8. 10 and $\frac{-5}{2}$
9. 3 and $\frac{1}{3}$
10. 2 and $\frac{-48}{7}$
11. 8 and $\frac{5}{2}$
12. 1 and $\frac{9}{4}$
13. $\frac{-9 \pm \sqrt{113}}{8}$
14. 3 and $\frac{44}{3}$
15. $\frac{35 \pm \sqrt{845}}{10}$

11 Fractions and Percentages

11.1 Fractions, Decimals and Percentages

1. (a) $\frac{1}{10}$ (b) $\frac{4}{5}$ (c) $\frac{9}{10}$ (d) $\frac{1}{20}$ (e) $\frac{1}{4}$ (f) $\frac{3}{4}$
   (g) $\frac{7}{20}$ (h) $\frac{19}{50}$ (i) $\frac{1}{25}$ (j) $\frac{3}{25}$ (k) $\frac{41}{50}$ (l) $\frac{37}{50}$

2. (a) 0.32 (b) 0.5 (c) 0.34 (d) 0.2 (e) 0.15 (f) 0.81
   (g) 0.04 (h) 0.03 (i) 0.07 (j) 0.18 (k) 0.75 (l) 0.73

3. (a) 50% (b) 74% (c) 35% (d) 8% (e) 10% (f) 52%
   (g) 80% (h) 7% (i) 4% (j) 18% (k) 40% (l) 30%

4. (a) 50% (b) 70% (c) 20% (d) 75% (e) 10% (f) 90%
   (g) 80% (h) 8% (i) 32% (j) 35% (k) 28% (l) $66\frac{2}{3}\%$

5. (a) $\frac{2}{3} = \frac{10}{15} = \frac{16}{24}$ (b) 35% 6. (a) correct drawing (b) 75%

7. (a) 180 m^2 (to the nearest square metre) (b) (i) 120 m^2 (ii) 20 %

11.2 Simple Fractions and Percentages of Quantities

1. (a) 20 (b) £2.50 (c) £1.60 (d) £25 (e) £200
   (f) 180 (g) £4 (h) 600 (i) 750 (j) 16 kg
   (k) 3.5 kg (l) 15 kg (m) 5 m (n) 10 m (o) £7.50

2. (a) 32 (b) 90 (c) 18 (d) 90 (e) 120 (f) 150

3. (a) 400 g (or 0.4 kg) (b) 240 g (or 0.24 kg) 4. £3 000

5. £25 6. £150 7. £60 8. 360 m 9. £1.50

10. 9 kg 11. 80 p 12. 2.4 kg (or 240 g)

13. (a) £5.60 (b) £5 (c) 2.25 metres

14. (a) £90 (b) $\frac{1}{6}, \frac{2}{9}, \frac{1}{3}, \frac{5}{6}$ 15. 170 grams

16. 1400 headphones

17. No.
    $\frac{2}{5}$ of the price is equivalent to 40% of the price, therefore 30% of the price is not enough to pay the deposit.

11.3 Quantities as Percentages

1. (a) 16% (b) 12% (c) 40% (d) 30% (e) 12%
   (f) 15% (g) 15% (h) 30% (i) 90% (j) 40%
   (k) 42% (l) 65% (m) 40% (n) 30% (o) 85%

2. 40% of the class are girls and 60% are boys. 3. 20% 4. 8%

5. 56% *Manchester United* supporters and 44% *Tottenham* supporters.

6. 20% 7. 60% 8. 81% 9. 92% 10. $33\frac{1}{3}\%$

11. 4% 12. 80% 13. 45%

14. (a) $\frac{3}{18} = \frac{1}{6}$ (b) 12 pieces (c) $33\frac{1}{3}\%$

11.4 More Complex Percentages

1. (a) £16 (b) £12.45 (c) £5.04 (d) £1.41 (e) £3.59
   (f) 69 p (g) £32 (h) £194.75 (i) 28 p

2. (a) £487.63 (b) £1184.74 (c) £15 240 (d) £205.37
   (e) £390.60 (f) £204 (g) £9359.22

3. (a) £132 (b) £153.75 4. 24 400 ice-creams 5. £1375

6. £147 7. 202.5 grams 8. £227.29 9. £94.50 10. £76

11. £61.25 12. (a) £41.40 (b) (i) £69 (ii) 945 (iii) £295.80

$$\begin{array}{r} 24 \times \\ \hline 18900 \\ 3780 \\ \hline 22680 \\ \hline \end{array}$$

13. (a) (i) £52.15 (ii) £350.15
    (b) (i) £7.50 (£360 – £352 50) (ii) *Berries' Store*

11.5 Percentage Increase and Decrease

1. 21.4% 2. 25% 3. 13.1% 4. 18.9%

5. Karen has a 23.5% saving and John has a 12.7% saving. 6. 40%

7. There is a 10.3% increase in the number of pupils and a 2.7% increase in the number of teachers. Therefore, class sizes will increase because the number of teachers has not increased at the same rate as the number of pupils.

8. 25% 9. 22.9%

10. 4.3% increase, therefore estimated value after another year is £50 086.96. (£50 100)

11. 4.2%

12. Percentage decrease in the value of the car is greatest during the first year. (16.7%)

13. (a) £7 (b) (i) £6.49 (ii) It would be £1.99 cheaper. (c) 21.5%

14. (a) £80 (b) 15% 15. 12.5%

11.6 Addition and Subtraction of Fractions

1. (a) $\frac{2}{5}$ (b) $\frac{1}{2}$ (c) $\frac{6}{7}$ (d) $\frac{3}{7}$ (e) $\frac{3}{13}$ (f) $\frac{1}{3}$
   (g) $\frac{5}{3}$ (h) $\frac{7}{5}$ (i) $\frac{11}{7}$ (j) $\frac{2}{5}$ (k) $\frac{1}{3}$ (l) $\frac{1}{5}$

2. (a) 14, 29 (b) 6, 5, 11 (c) 2, 3 (d) 3, 4, 7 (e) 12, 14, 26
   (f) 36, 35, 71

3. (a) $\frac{13}{24}$ (b) $\frac{39}{35}$ (c) $\frac{7}{32}$ (d) $\frac{13}{30}$ (e) $\frac{59}{56}$ (f) $\frac{7}{6}$
   (g) $\frac{17}{70}$ (h) $\frac{47}{24}$ (i) $\frac{32}{21}$ (j) $\frac{1}{14}$ (k) $\frac{13}{44}$ (l) $\frac{1}{2}$
   (m) $\frac{1}{12}$ (n) $\frac{5}{24}$ (o) $\frac{13}{24}$

4. $\frac{11}{15}$ hectare 5. £$1\frac{5}{12}$million 6. $\frac{1}{10}$ cm 7. $\frac{13}{30}$ 8. $\frac{3}{20}$

9. $\frac{1}{6}$ 10. $2\frac{7}{12}$ kg

11.7 Multiplication and Division of Fractions

1. (a) $\frac{15}{28}$ (b) $\frac{7}{40}$ (c) $\frac{1}{15}$ (d) $\frac{27}{70}$ (e) $\frac{5}{14}$ (f) $\frac{9}{14}$
   (g) $\frac{3}{28}$ (h) $\frac{2}{21}$ (i) $\frac{2}{3}$ (j) 2 (k) $10\frac{5}{12}$ (l) $3\frac{3}{4}$
   (m) $14\frac{2}{5}$ (n) $9\frac{5}{8}$ (o) $30\frac{5}{14}$ (p) $11\frac{11}{28}$ (q) $9\frac{41}{48}$ (r) $1\frac{43}{56}$

2. (a) $1\frac{1}{2}$ (b) $1\frac{1}{7}$ (c) $1\frac{2}{5}$ (d) $\frac{15}{32}$ (e) $\frac{10}{21}$ (f) $4\frac{3}{8}$
   (g) $1\frac{2}{3}$ (h) 22 (i) $\frac{64}{133}$ (j) $3\frac{3}{4}$ (k) $1\frac{1}{13}$ (l) $1\frac{71}{105}$

3. (a) $1\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $6\frac{33}{40}$ (d) $3\frac{1}{5}$

4. $\frac{1}{8}$ 5. $\frac{1}{8}$ 6. Volume = $3\frac{3}{8}$ cm^3, Surface area = $13\frac{1}{2}$ cm^2.

7. $4\frac{1}{8}$ litres 8. $37\frac{1}{2}$ miles 9. $2\frac{1}{2}$ m 10. 7 cakes

11. $4\frac{1}{2}$ miles per hour 12. 10 items

11.8 Compound Interest and Depreciation

1. (a) £1272 (b) £1348.32 (c) £1605.87

2. (a) £8144.47 (b) £7990.66 (c) £10 021.16

3. C (£51.94 interest) 4. (a) £900 (b) £284.77 (c) £90.10

5. £4608 6. (a) 6 years (b) (i) 5 years (ii) 8 years

7. £1.78 8. (a) 45.23 million (b) 8 years

9. (a) (i) £3132.04 (ii) 52.2% (b) (i) £6264.08 (ii) 52.2%

10. (a) £180 (b) £571.31

11.9 Reverse Percentage Problems

1. (a) £603.32 (b) £105.58 2. £192 3. £3.22

4. £81 200 5. 550 ml

6. Television - £320, Video recorder - £424, Computer - £1480, Calculator - £14.51

7. £9800

8. £500

9. (a) £10.29 (b) £6.58 (c) £3.37

10. £3501.78

12 Number Patterns

12.1 Simple Number Patterns

1. (a) 11, 13, 15, 17 (b) 24, 28, 32, 36 (c) 30, 35, 40, 45
   (d) 42, 49, 56, 63 (e) 54, 63, 72, 81 (f) 36, 42, 48, 54
   (g) 60, 70, 80, 90 (h) 66, 77, 88, 99 (i) 48, 56, 64, 72
   (j) 120, 140, 160, 180 (k) 90, 105, 120, 135 (l) 300, 350, 400, 450

2. (a) +3; 20, 23 (b) +8; 42, 50 (c) +5; 32, 37 (d) +11; 61, 72
   (e) +7; 43, 50 (f) $+\frac{1}{2}$; 5, $5\frac{1}{2}$ (g) +9; 49, 58 (h) –3; 11, 8
   (i) -4; 0, -4 (j) -4; –2, –6 (k) –3; -4, –7 (l) –3; –20, –23

3. (a) (i) 22 (ii) 242 (iii) 2442 (iv) 24442 (v) 244442
   (b) (i) 1089 (ii) 10989 (iii) 109989 (iv) 1099989 (v) 10999989
   (c) (i) 968 (ii) 9768 (iii) 97768 (iv) 977768 (v) 97777768
   (d) (i) 63 (ii) 693 (iii) 6993 (iv) 69993 (v) 6999993

4. (a) (i) $11 \times 11 \times 11 = 1331$, $11 \times 11 \times 11 \times 11 = 14641$
   (ii) Each one is a symmetric number (these are found in Pascal's triangle).
   (b) $11 \times 11 \times 11 \times 11 \times 11 = 11^5 = 161051$. This is *not* a symmetric number.

5. (a) (i) 5, 4, 3, 2, 1 (ii) 4, 5, 6, 7, 8
   (iii) In (i), the digits *increase* by 1 each time; in (ii), the numbers *decrease* by 1 each time.
   (b) The numbers sum to 9.

12.2 Recognising Number Patterns

1. (a) 35, 41; +6 (b) 36, 45; +8, +9 (c) 7, 4; –3

   (d) 0, –3;

30	22	15	9	4	0	–3
	–8	–7	–6	–5	–4	–3
		1	1	1	1	1

   (e) 90, 101;

50	56	63	71	80	90	101
	6	7	8	9	10	11
		1	1	1	1	1

(f) 32, 42;

2, 2, 4, 8, 14, 22, 32, 44
0, 2, 4, 6, 8, 10, 12

(g) 31, 36; +5

(h) 35, 48;

0, 3, 8, 15, 24, 35, 48
3, 5, 7, 9, 11, 13
2, 2, 2, 2, 2

(i) 96, 192;

3, 6, 12, 24, 48, 96, 192
× 2 × 2 × 2 × 2 × 2 × 2

(j) 94, 190;

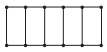
1, 4, 10, 22, 46, 94, 190
3, 6, 12, 24, 48, 96

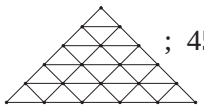
(k) –71, –101; –25, –30

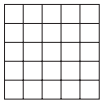
2. 216, 343;

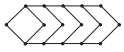
1, 8, 27, 64, 125, 216, 343
7, 19, 37, 61, 91, 127
12, 18, 24, 30, 36
6, 6, 6, 6

3. (a) (i) 3, 5, 7, 9 (ii) +2 (iii)  ; 11

(b) 4, 7, 10, 13 (ii) +3 (iii)  ; 16

(c) 3, 9, 18, 30
6, 9, 12
3, 3
(ii) +15 (iii)  ; 45

(d) 4, 12, 24, 40
8, 12, 16
4, 4
(ii) +20 (iii)  ; 60

(e) 6, 10, 14, 18 (ii) +4 (iii)  ; 22

4. (a) 1, 3, 6, 10, 15
2, 3, 4, 5
+1 to difference to get next number; 21.

(b) 1 4 8 12 16
3 4 4 4
If 3 is ignored, the sequence is constant, +4 for next number; 20.

(c) 5, 8, 11, 14, 17 +3 for next number; 20.

(d) 4, 6, 8, 10 +2 for next number; 12.

(e) 1 5 12 22
4 7 10
3 3
+3 for next difference; 35.

(f) 1 6 15 28 45
5 9 13 17
4 4 4
+4 for next difference; 66.

5. (a) 1^2, 2^2, 3^2, 4^2, . . .

(b) (i)$_n = n^2 + 2$ (ii)$_n = n^2 - 2$ (iii)$_n = 2n^2$ (iv)$_n = 4n^2$

6. (a) (ii)$_n$ = (i)$_n + 3$ (iii)$_n$ = (i)$_n - 2$

(b) (i) $13 + 21 = 34$ (iii) $16 + 24 - 3 = 37$ (iii) $11 + 19 + 2 = 32$

7. Circle 8. 1 2 4 7 11 16 22
1 2 3 4 5 6

8. (a) 35, 42 (b) The next term is obtained by adding 7 to the last term.

9. (a) 41, 122 (b) 125, 216

10. (a) $p = 15$ (b) $q = 52 + 15 = 67$

11. (a) It is increasing by 4. (b) (i) Sequence of square numbers. (ii) 49
(iii) 47; minus 2

12. (a) (i) 5, 10, 15, 20, 25, 30, 35, 40 (ii) 5, 0, 5, 0, ..., alternate
(b) Terms are the positive multiples of 3.
(c) $9 \rightarrow 9 + 1 = 10$, $10 \times 2 = 20 \rightarrow 20$
$12 \rightarrow 12 + 1 = 13$, $13 \times 2 = 26 \rightarrow 26$
$15 \rightarrow 15 + 1 = 16$, $16 \times 2 = 32 \rightarrow 32$
$18 \rightarrow 18 + 1 = 19$, $19 \times 2 = 38 \rightarrow 38$
(d) 85, 109 1 4 10 19 31 46 64 85 109
3 6 9 12 15 18 21 24

12.3 Extending Number Patterns

1. (a) 40; 80 (b) 50; 100 (c) 101; 201 (d) 25; 45
   (e) 41; 81 (f) 11; 1 (g) -4; -64 (h) 49; 79
   (i) -1; -11 (j) 32; 72 (k) 52; 102 (l) -42; -92

2. (a) (i) $10^2 + 2 = 102$ (ii) $4 + 10 \times 1 = 14$ or $5 + 9 \times 1 = 14$
   (b) (i) 116 (ii) 1428 (iii) 88

3. (a) (i) 99 (ii) 41 (b) 4059

4. (a) 90 (b) 990

5. (a) 28 dots (b) 40 dots (c) 60 dots

6. (a) 17 sticks (b) Pattern 58
   (c) (i) 401 sticks (ii) nth pattern needs $(4n + 1)$ sticks

12.4 Formulae and Number Patterns

1. (a) 5, 9, 13, 17, 21, 25 (b) –2, 3, 8, 13, 18, 23
   (c) 12, 22, 32, 42, 52, 62 (d) 0, 3, 8, 15, 24, 35
   (e) 3, 9, 19, 33, 51, 73 (f) 2, 4, 8, 16, 32, 64

2. (a) $u_n = n^3 - 1$ (b) $u_n = 3n + 5$ (c) $u_n = 8n - 2$ (d) $u_n = n^2 + 1$

3. (a) 80 (b) 10 (c) 87 (d) 0 (e) 396 (f) $\frac{1}{2}$

4. (a) Because the difference sequence is 7, 7, 7, ..., (b) $u_n = 7n + 1$

5. (a) $u_n = 5n - 1$ (b) $u_n = 3n + 8$ (c) $u_n = 6n - 8$
   (d) $u_n = -2n + 102$ (e) $u_n = \frac{1}{2}n + \frac{1}{2}$ (f) $u_n = -7n + 12$
   (g) $u_n = -\frac{1}{2}n + 10\frac{1}{2}$

6. (e) $u_n = 0.1n + 0.9$ (g) $u_n = 4n - 1$

7. 1, 4, 9, 16, 25, 36 (a) $u_n = n^2 + 2$ (b) $u_n = n^2 - 5$ (c) $u_n = 2n^2$
   (d) $u_n = 2n^2 - 2$ (e) $u_n = -n^2 + 100$

8. (a) $u_n = n^3 - 1$ (b) $u_n = n^3 + 9$ (c) $u_n = -n^3 + 200$

9. $u_n = 4n + 10$; 14, 18, 22, 26, 30

10. $u_n = 6n + 8$; 68

11. $u_n = 5n + 2$; 7

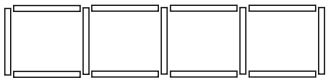
12. (a) $u_8 = 29$, add 4 (b) $n = 100$; 100th term is 397

13. $u_n = 4n + 3$

14. (b)

Number of enclosures	4	5	6	7	8
Number of posts	15	18	21	24	27

(c) 63 posts (d) $u_n = 3n + 3$

15. (a) (i)  (ii) 13 sticks (b) 3 more sticks

(c) number $= 3s + 1$

16. (a) (i) add 5 (ii) $u_n = 5n + 2$

(b) area $= n(n+1) = n^2 + n$

12.5 General Laws

1. (a) $u_n = \dfrac{n^2}{n+9}$ (b) $u_n = \dfrac{2n+3}{n+1}$ (c) $u_n = 2 \times 3^{n-1}$ (d) $u_n = 0.9^{n-1}$

(e) $u_n = 1.2^{n-1}$ (f) $u_n = \dfrac{2}{3^{n-1}}$ (g) $u_n = 2^n$ (h) $u_n = 2^n + 1$

(i) $u_n = \dfrac{3n}{n+3}$

(b), (d), (f), (i) converge ; (a), (c), (e), (g), (h) diverge;

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

2 0 0 3

2. (a) $u_1 = 8$; $u_{n+1} = u_n - 3$ (b) $u_1 = 5$; $u_{n+1} = 4u_n$

(c) $u_1 = 2$; $u_{n+1} = 2u_n - 1$ (d) $u_1 = 4000$; $u_{n+1} = \dfrac{u_n}{2}$

(e) $u_1 = 3$, $u_2 = 3$; $u_{n+2} = u_n + u_{n+1}$

(f) $u_1 = 1$, $u_2 = 1$, $u_3 = 1$

$u_{n+3} = u_n + u_{n+1} + u_{n+2}$

3. (a) 1, 3.5, 2.6071, 2.4543, 2.4495 (b) $\sqrt{6} \approx 2.449489743$

4. Yes; converges to 3. If u_1 is a different value it still converges to 3.

5. 0.1, 0.13, 0.1417, 0.14284777, 0.142857142; $\dfrac{1}{7}$

6. $\frac{1}{3}$; 0.5, 0.25, 0.3125, 0.33203125

7. (a) (i) $40 = 5 \times 8 = 5 \times (5 + 3)$ (ii) $n(n + 3)$

   (b) (i) $40 = 6 \times 7 - 2$ (ii) $(n + 1)(n + 2) - 2$

   (c) $(n + 1)(n + 2) - 2 = n^2 + 3n + 2 - 2 = n^2 + 3n = n(n + 3)$

8. (a) 64 (b) $u_n = 2^{n-1}$ (c) $u_{11} = 1024$, $u_{10} = \frac{1024}{2} = 512$

9. (a) Row 4: 13 15 17 19 Sum $= 64 = 4^3$

   (b) Row 10 $\rightarrow 10^3 = 1000$ (c) Row 20 $\rightarrow 20^3 = 8000$ (d) $x + 2$

12.6 Quadratic Formulae

1. (a) 50, 65 (b) 130, 180 (c) 142, 194 (d) 20, $26\frac{1}{2}$ (e) 14, 23

2. 1st term: 4, 2nd term: 4, 6th term: 24 3. 336 4. 22

5. (a) $u_n = n^2 - 4n + 6$ (c) $u_n = n^2 - n + 1$ (e) $u_n = n^2 + 2n - 8$
   (g) $u_n = 3n^2 - 9n + 7$

6. $u_1 = -1$, $u_5 = 15$ 7. $u_n = 2.5n^2 + 2.5n + 4$

8. (a)

	1	15	53	127	249	431 ...
1st difference	14	38	74	122	182	...
2nd difference	24	36	48	60	...	
3rd difference	12	12	12	...		

   The 1st difference sequence is a quadratic sequence; the 2nd difference sequence is an arithmetic sequence; the 3rd difference sequence is a non-zero constant sequence.

   (c) $6a$ (d) $u_n = n^3 + n^2 + n + 1$

9. (a) triangle $u_n = \frac{1}{2}n^2 + \frac{1}{2}n$, square: $u_n = n^2$, pentagon: $u_n = \frac{3}{2}n^2 - \frac{1}{2}n$

   (c) heptagonal : $u_n = \frac{5}{2}n^2 - \frac{3}{2}n$, octagonal: $u_n = 3n^2 - 2n$

   (d) decagonal: $u_n = 4n^2 - 3n$ and $u_8 = 232$

10. (a) (i) $r_4 = \sqrt{10^2 + 24^2} = 26$ (ii) $r_5 = \sqrt{12^2 + 35^2} = 37$

(b) (i) $p_{10} = 22$ (ii) $q_6 = 48$ (c) (i) $p_n = 2n + 2$

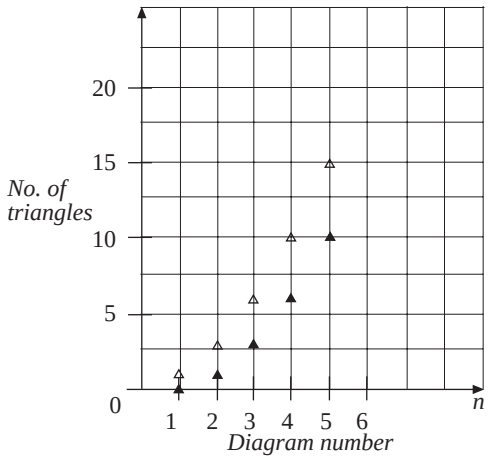
(ii) $k = 2$ so $q_n = n^2 + 2n$

11. (a)

Diagram no.	4	5
No. white triangles	10	15
No. black triangles	6	10
Total no. triangles	16	25

(b) $T_{10} = 10^2 = 100$

(c) (i) and (ii)



(iii) The two sets of points follow the same pattern, with the white triangles increasing more quickly than the black.

(d) No, this rule is not correct. Number $= \dfrac{d(d+1)}{2}$