UNIT 8 *Algebra: Brackets* Activities

Activities

8.1 Expansions and Area
8.2 Using Algebra to Solve Magic Squares
8.3 Expansion Crazy!
Notes and Solutions (2 pages)

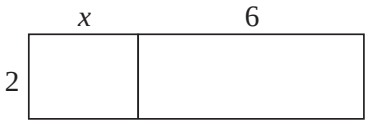
ACTIVITY 8.1 *Expansions and Area*

You can check expansions of brackets by relating them to areas.

1. (a) Expand $2(x+6)$.

   (b) Calculate the area of each part of the rectangle 2 by $(x+6)$, and hence find the total area.

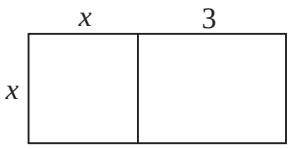
   (c) Compare your answers to (a) and (b).



2. (a) Expand $x(x+3)$.

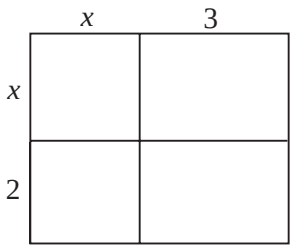
   (b) Calculate the area of each part of the rectangle x by $(x+3)$, and hence find the total area.

   (c) Compare your answers to (a) and (b).



3. (a) Expand $(x+2)(x+3)$.

   (b) Calculate the area of each part of the rectangle $(x+2)$ by $(x+3)$, and hence find the total area.



4. Draw similar diagrams to illustrate the link between area and expansion for each of the following, and hence find the expansions:

   (a) $3(x+5)$ (b) $x(x+7)$

   (c) $(x+5)(x+1)$ (d) $(x+3)^2$

   (e) $(x+a)(x+b)$ (f) $(x+a)^2$

ACTIVITY 8.2 *Using Algebra to Solve Magic Squares*

Magic squares have a property that, in each row, column and diagonal, the sum of the numbers is always equal to the *magic number* for that square.

1. Here is an example which happens to use 9 consecutive numbers.

   Check that the sum of the numbers in each row, column and diagonal is equal to the magic number, 12.

3	2	7
8	4	0
1	6	5

Solving magic squares

	11	7
9		
	5	10

This magic square is more challenging! The answer may be found by trial and error but really a more systematic method is required.

Let x be the unknown number in *column 1, row 1*;

y be the unknown number in *column 1, row 3*;

n be the *magic* number.

	11	7
9		
	5	10

Then for *row 1*, $n = x + 11 + 7 = x + 18$,

and for *column 1*, $n = x + 9 + y$.

So, $x + 18 = x + 9 + y$ (Subtracting x from each side.)

$18 = 9 + y$ (Subtracting 9 from each side.)

$y = 9$.

From *row 3*, $n = y + 5 + 10$ so $n = 24$. From *row a*, $x + 18 = 24$, so $x = 6$.

The other two missing numbers can then be found to be 8 (*column 2*) and 7 (*column 3*).

2. Use an algebraic approach to solve the following magic squares:

(a)

9	2	
12	8	

(b)

10	3	
5		9
	11	4

(c)

14		12
10		8

ACTIVITY 8.3 *Expansion Crazy!*

1. Show that $(x+1)^2 = x^2 + 2x + 1$ by completing the following table:

×	x	1
x		
1		

2. The expansion of $(x+1)^3$ can be found using $(x+1) \times (x+1)^2$ and the following table:

×	x^2	$2x$	1
x			
1			

Complete the table and determine $(x+1)^3$.

3. Calculate the following:

(a) $(x+1)^4$ (b) $(x+1)^5$ (c) $(x+1)^7$

Extension

Look carefully at the numbers that multiply the powers of x, etc. in your expansions. Can you see how they could be obtained without actually expanding the expressions. Find the expression for $(x+1)^8$ without expanding. Check your answer.

ACTIVITIES 8.1 - 8.2 *Notes for Solutions*

Notes and solutions given only where appropriate.

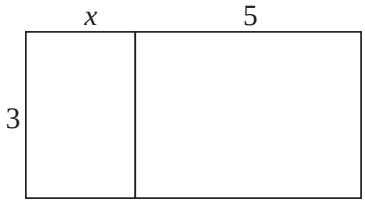
8.1 1. (a) $2x + 12$ (b) $2x$, 12, $2x + 12$ (c) Expansion gives total area.

2. (a) $x^2 + 3x$ (b) x^2, $3x$, $x^2 + 3x$ (c) Expansion gives total area.

3. (a) $x^2 + 5x + 6$ (b) x^2, $3x$, $2x$, 6, $x^2 + 5x + 6$

(c) Expansion gives total area.

4. (a)

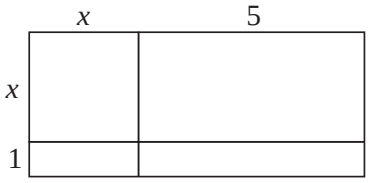


$3x + 15$

(b)

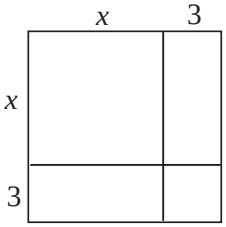
$x^2 + 7x$

(c)



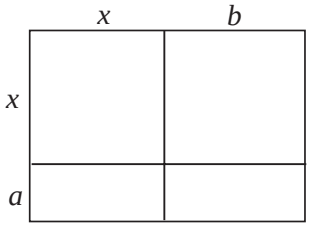
$x^2 + 6x + 5$

(d)



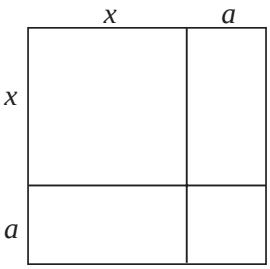
$x^2 + 6x + 9$

(e)



$x^2 + ax + bx + ab$

(f)



$x^2 + 2ax + a^2$

8.2 2. (a)

9	2	13
12	8	4
3	14	7

(b)

10	3	8
5	7	9
6	11	4

(c)

14	7	12
9	11	13
10	15	8

ACTIVITY 8.3 *Notes for Solutions*

8.3 2. $(x+1)^3 = x^3 + 3x^2 + 3x + 1$

3. (a) $(x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$

$(x+1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

$(x+1)^7 = x^7 + 7x^6 + 21x^5 + 35x^4 + 35x^3 + 21x^2 + 7x + 1$

Extension

Form Pascal's triangle:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

Next row is:

1 8 28 56 70 56 28 8 1

so the expansion is

$(1+x)^8 = 1 + 8x + 28x^2 + 56x^3 + 70x^4 + 56x^5 + 28x^6 + 8x^7 + x^8$