UNIT 10 *Sequences* Teaching Notes

Historical Background and Introduction

Please refer to the information given in Year 7A, Unit 7. Points which should be stressed are:

1. Sequences covered in this unit are always defined by a rule.
2. When given 3 or 4 terms of a sequence, it is not always possible to determine the actual rule (there could, for example, be more than one possible rule that fits with the given members of the sequence).
3. If the first differences are constant and non-zero, then the general term is of the form $u_n = an + b$; if the second differences are constant and non-zero, then it is a quadratic of the form $u_n = an^2 + bn + c$, etc.

Point 2 can be illustrated with a sequence such as

1, 2, 4, ... , ... , ...

which can be extended as

(1, 2, 4,) 7, 11, 16, ... , (i.e. $u_n = \frac{1}{2}\left(n^2 - n + 2\right)$)

or as

(1, 2, 4,) 8, 16, 32, ... , (i.e. $u_n = 2^{n-1}$)

It is important that students realise that there is not always a unique answer when given only a few terms of a sequence. In fact, given, say, the sequence

1, 2, 3, 4, ?, ... , ... ,

the next term can be any number. For example, suppose $u_5 = c$, then

$$u_n = n + (n-1)(n-2)(n-3)(n-4)\times\frac{(c-5)}{24} \qquad (n = 1,\ 2,\ 3,\ 4,\ ...)$$

will give the sequence

1, 2, 3, 4, *c*, ... , ...

Routes	**Standard**	**Academic**	**Express**
10.1 Sequences: Constant Differences	✓	✓	✓
10.2 Finding the Formula for a Linear Sequence	(✓)	✓	✓
10.3 Second Differences and Quadratic Sequences	×	(✓)	✓
10.4 Special Sequences	×	✓	✓

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Language	**Standard**	**Academic**	**Express**
Linear sequence	✓	✓	✓
Second difference	×	(✓)	✓
Quadratic sequence	×	(✓)	✓
Exponential sequence	×	✓	✓
Fibonacci sequence	×	✓	✓

Misconceptions

- pupils must realise that there can be more than one rule giving rise to the same starting values,
  e.g. 5, 10, 15, ... , ... can be generated by
  $u_n = 5n$ (in which the next numbers in the sequence are 20, 25, 30, etc.)
  and also by
  $u_n = n^3 - 6n^2 + 16n - 6$ (in which case the next numbers in the sequence are 26, 49, 90, etc.)

- it must be realised that many sequences are neither linear nor quadratic,
  e.g. Fibonacci (1, 1, 2, 3, 5, 8, ... ,)
  exponential (1, 2, 4, 8, 16, 32, ... ,)
  cubic (1, 8, 27, 64, 125, ... ,)

Challenging Questions

The following questions are more challenging than others in the same section:

	Section	*Question No.*	*Page*
Practice Book Y9B	10.1	12	32
" "	10.2	10, 11	39
" "	10.3	11	46
" "	10.4	12	54