Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Tuesday 12 May 2015 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2215 – 7305

12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

A bag contains eight marbles. Three marbles are red and five are blue. Two marbles are drawn from the bag without replacement.

(a) Write down the probability that the first marble drawn is red. [1]

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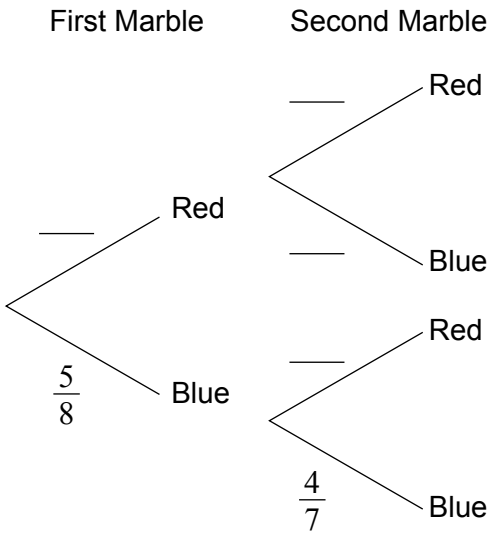
(b) Complete the following tree diagram. [3]



(c) Find the probability that both marbles are blue. [2]

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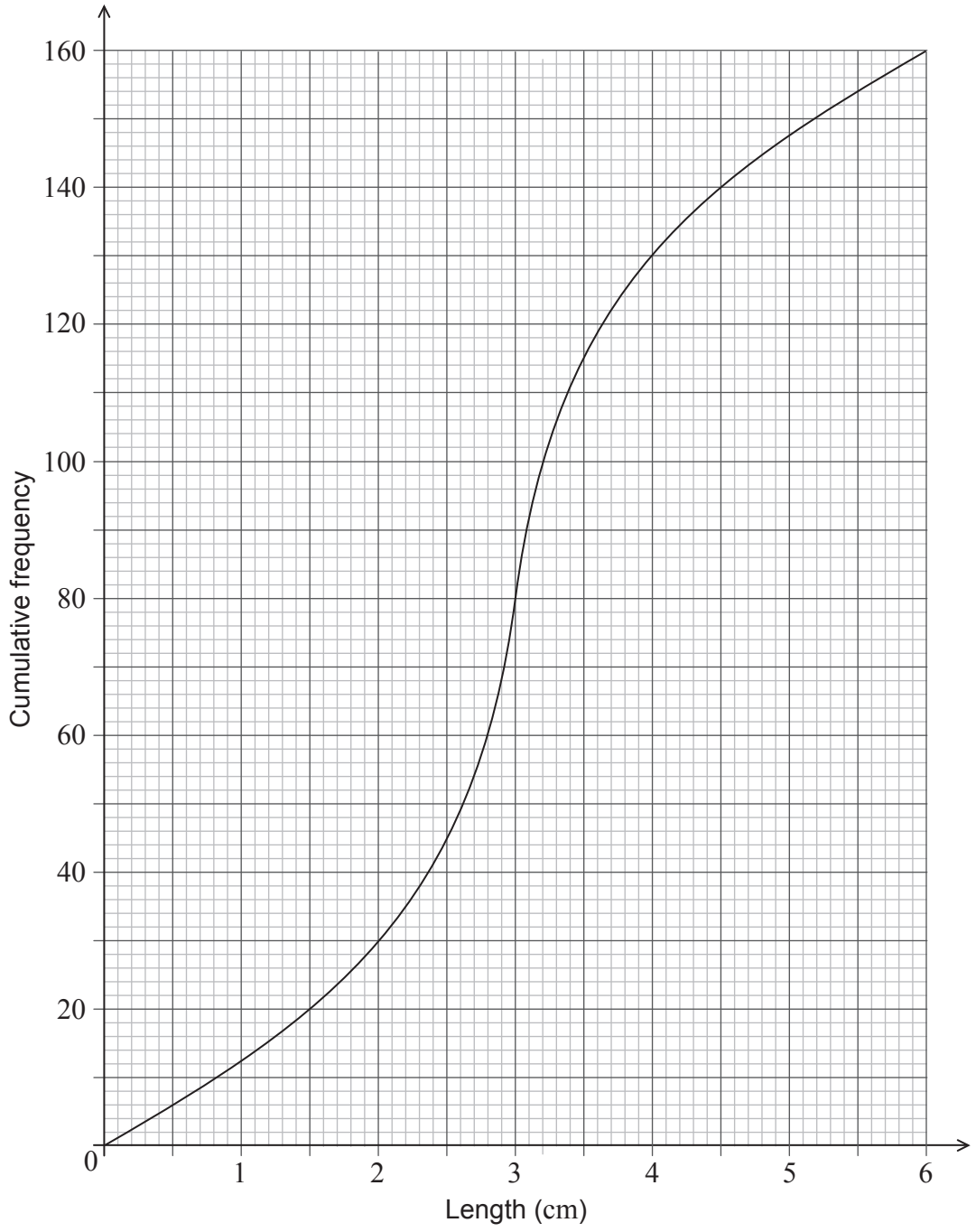
3. [Maximum mark: 6]

The following cumulative frequency diagram shows the lengths of 160 fish, in cm.



(This question continues on the following page)

(Question 3 continued)

(a) Find the median length. [2]

The following frequency table also gives the lengths of the 160 fish.

Length x cm	$0 \leq x \leq 2$	$2 < x \leq 3$	$3 < x \leq 4.5$	$4.5 < x \leq 6$
Frequency	p	50	q	20

(b) (i) Write down the value of p.

(ii) Find the value of q. [4]

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6. [Maximum mark: 8]

Let $f(x) = ax^3 + bx$. At $x = 0$, the gradient of the curve of f is 3. Given that $f^{-1}(7) = 1$, find the value of a and of b.

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7. [Maximum mark: 7]

A bag contains black and white chips. Rose pays $\$10$ to play a game where she draws a chip from the bag. The following table gives the probability of choosing each colour chip.

Outcome	black	white
Probability	0.4	0.6

Rose gets no money if she draws a white chip, and gets $\$k$ if she draws a black chip. The game is fair. Find the value of k.

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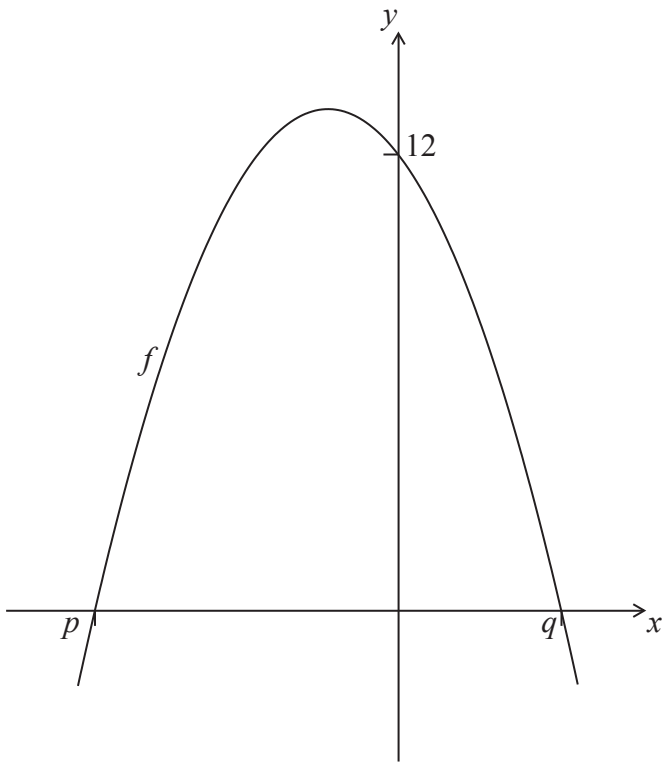
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Let $f(x) = a(x+3)(x-1)$. The following diagram shows part of the graph of f.



The graph has x-intercepts at $(p, 0)$ and $(q, 0)$, and a y-intercept at $(0, 12)$.

(a) (i) Write down the value of p and of q.

(ii) Find the value of a. [6]

(b) Find the equation of the axis of symmetry of the graph of f. [3]

(c) Find the largest value of f. [3]

The function f can also be written as $f(x) = a(x-h)^2 + k$.

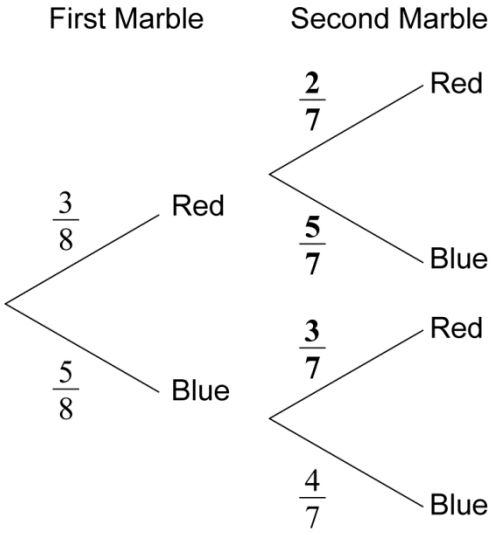
(d) Find the value of h and of k. [3]

Section A

1. (a) $\frac{3}{8}$ ***A1*** ***N1***

[1 mark]

(b)



A1A1A1 ***N3***

Note: Award ***A1*** for each correct **bold** value.

[3 marks]

(c) multiplying along the blue branches ***(M1)***

eg $\frac{5}{8}\times\frac{4}{7}$

$\frac{20}{56}\left(=\frac{5}{14}\right)$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

2. (a) (i) valid approach ***(M1)***

eg two cycles is 2π, $2\times\left(\pi-\frac{\pi}{2}\right)$

period is π ***A1*** ***N2***

(ii) amplitude is 3 ***A1*** ***N1***

[3 marks]

(b) (i) $a=3$ ***A1*** ***N1***

(ii) valid approach to find b ***(M1)***

eg correctly substituting the coordinates of a point, $b=\frac{2\pi}{\text{period}}$, $\text{period}=\frac{2\pi}{|b|}$

$b=2$ ***A1*** ***N2***

Note: If no working shown, award ***N3*** for $3\sin 2x$. ***[3 marks]***

[3 marks]

Total [6 marks]

3. (a) evidence of approach (may be seen on graph) ***(M1)***

eg 80, $(3,80)$

Note: Award ***M0*** for an incorrect approach such as $\frac{0+6}{2}$, which leads to the correct answer, even if $(3,80)$ is indicated on graph.

median $=3$ ***A1*** ***N2***

[2 marks]

(b) (i) $p=30$ ***A1*** ***N1***

(ii) attempt to set up an expression to find q ***(M1)***

eg cumulative frequency for 4.5 indicated on graph

correct expression to find q ***(A1)***

eg $160-20-50-30$, $140-50-p$, $140-80$

$q=60$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

5. (a) $f'(x) = -2e^{-2x}$, $f''(x) = 4e^{-2x}$, $f^{(3)}(x) = -8e^{-2x}$ ***A1A1A1*** ***N3***

[3 marks]

(b) $f^{(n)}(x) = (-2)^n e^{-2x}$ $\left(\text{accept } (-1)^n 2^n e^{-2x}, (-2)^n f(x)\right)$ ***A2A1*** ***N3***

[3 marks]

Total [6 marks]

6. recognizing derivative ***(M1)***
eg $f'(x)$, $f'(0) = 3$

correct derivative $3ax^2 + b$ ***A1A1***

$b = 3$ ***A1*** ***N2***

recognizing inverse relationship (seen anywhere) ***(M1)***
eg $(1, 7)$, $f(1) = 7$, swapping x and y **and** substituting $(7, 1)$

correct equation
eg $a + b = 7$, $a + 3 = 7$ ***A1***

substituting **their** b ***(M1)***
eg $ax^3 + 3x$, $a + 3 = 7$

$a = 4$ ***A1*** ***N2***

Notes: If working shown, award relevant marks for $4x^3 + 3x$.
If no working shown, award ***N4*** for $4x^3 + 3x$.

[8 marks]

7. recognizing fair game (seen anywhere) ***(M1)***
eg $\mathrm{E}(X) = 10$, $\mathrm{E}(X) = 0$, money spent = money gained

correct substitution ***(A2)***
eg $0(0.6) + k(0.4)$, $0.4(k - 10) + 0.6(-10)$

correct equation ***(A2)***
eg $0(0.6) + k(0.4) = 10$, $0.4(k - 10) + 0.6(-10) = 0$, $k(0.4) = 10$

correct work towards solving equation ***(A1)***
eg $k = \frac{10}{0.4}, \frac{100}{4}$

$k = 25$ ***A1*** ***N3***

[7 marks]

Section B

8. **Note:** The values of p and q found in (a)(i) are used throughout the question. Please check ***FT*** carefully on **their** values.

(a) (i) recognizing intercepts occur when $f(x) = 0$ ***(M1)***

eg $p = 1,\ q = -3$

$p = -3,\ q = 1$ ***A1A1*** ***N3***

(ii) attempt to substitute $(0, 12)$ into **their** f to find a ***(M1)***

eg $f(0) = 12$

correct working ***(A1)***

eg $12 = a(3)(-1)$

$a = -4$ ***A1*** ***N2***

[6 marks]

(b) attempt to find x-value ***(M1)***

eg $\frac{p+q}{2},\ -\frac{b}{2a},\ f'(x) = 0$

correct working ***(A1)***

eg $\frac{-3+1}{2},\ \frac{8}{2(-4)},\ -1,\ -8x - 8 = 0$

$x = -1$ (must be equation) ***A1*** ***N3***

[3 marks]

continued

Question 8 continued

(c) **METHOD 1**

substituting **their** x to find y-value ***(M1)***

eg $f(-1),\ -4(-1+3)(-1-1)$

correct calculation ***(A1)***

eg $-4(2)(-2)$

largest value is 16 ***A1*** ***N2***

METHOD 2

valid attempt to complete the square ***(M1)***

eg $-4\left(x^2+2x+1\right)+12+4,\ -4\left(x^2+2x+1\right)+12-1$

correct vertex form ***(A1)***

eg $-4(x+1)^2+16$

largest value is 16 ***A1*** ***N2***

METHOD 3

valid approach (may be seen in (b)) ***(M1)***

eg $f'(x)=0,\ -8x-8=0$

substituting $x=-1$ into $f(x)$ ***(A1)***

eg $-4(-1)^2-8(-1)+12$

largest value is 16 ***A1*** ***N2***

[3 marks]

(d) **METHOD 1**

recognizing coordinates of vertex ***(M1)***

eg $(-1, 16)$

$h=-1,\ k=16$ $\left(\text{accept } -4(x+1)^2+16\right)$ ***A1A1*** ***N3***

METHOD 2

valid attempt to complete the square (may be seen in (c)) ***(M1)***

eg $-4\left(x^2+2x+1\right)+12+4,\ -4\left(x^2+2x+1\right)+12-1$

$h=-1,\ k=16$ $\left(\text{accept } -4(x+1)^2+16\right)$ ***A1A1*** ***N3***

[3 marks]

Total [15 marks]