2. *[Maximum mark: 6]*

Let $p = \sin 40^\circ$ and $q = \cos 110^\circ$. Give your answers to the following in terms of p and/or q.

(a) Write down an expression for

(i) $\sin 140^\circ$;

(ii) $\cos 70^\circ$. *[2 marks]*

(b) Find an expression for $\cos 140^\circ$. *[3 marks]*

(c) Find an expression for $\tan 140^\circ$. *[1 mark]*

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4. *[Maximum mark: 6]*

Consider $g(x) = 3\sin 2x$.

(a) Write down the period of g. *[1 mark]*

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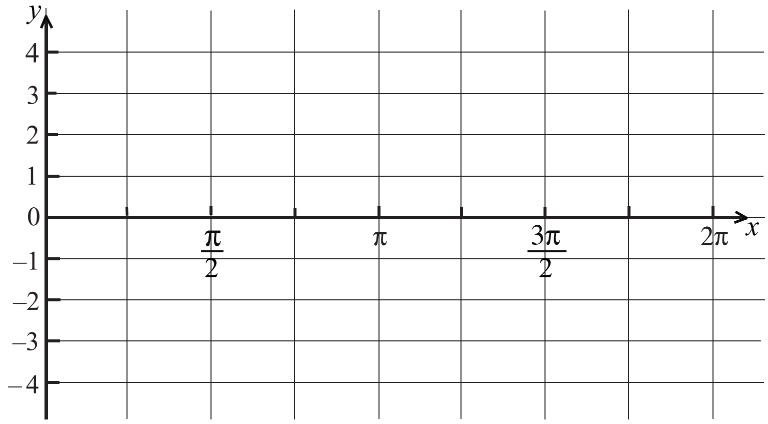
(b) On the diagram below, sketch the curve of g, for $0 \le x \le 2\pi$. *[3 marks]*



(c) Write down the number of solutions to the equation $g(x) = 2$, for $0 \le x \le 2\pi$. *[2 marks]*

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4. *[Maximum mark: 6]*

(a) Given that $\cos A = \frac{1}{3}$ and $0 \le A \le \frac{\pi}{2}$, find $\cos 2A$. *[3 marks]*

(b) Given that $\sin B = \frac{2}{3}$ and $\frac{\pi}{2} \le B \le \pi$, find $\cos B$. *[3 marks]*

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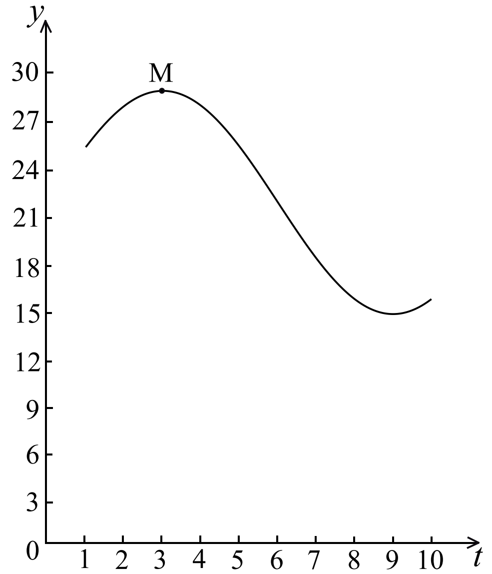
10. *[Maximum mark: 16]*

Let $f(t) = a\cos b(t-c)+d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.



When $t = 3$, there is a maximum value of 29, at M.
When $t = 9$, there is a minimum value of 15.

(a) (i) Find the value of a.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of d.

(iv) Write down a value for c. *[7 marks]*

The transformation P is given by a horizontal stretch of a scale factor of $\frac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$.

(b) Let M′ be the image of M under P. Find the coordinates of M′. *[2 marks]*

The graph of g is the image of the graph of f under P.

(c) Find $g(t)$ in the form $g(t) = 7\cos B(t-C)+D$. *[4 marks]*

(d) Give a full geometric description of the transformation that maps the graph of g to the graph of f. *[3 marks]*

6. *[Maximum mark: 7]*

Solve $\cos 2x - 3\cos x - 3 - \cos^2 x = \sin^2 x$, for $0 \le x \le 2\pi$.

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4. *[Maximum mark: 7]*

The straight line with equation $y = \frac{3}{4}x$ makes an acute angle θ with the x-axis.

(a) Write down the value of $\tan\theta$. *[1 mark]*

(b) Find the value of

(i) $\sin 2\theta$;

(ii) $\cos 2\theta$. *[6 marks]*

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4. *[Maximum mark: 7]*

Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

(a) Find $f\left(\frac{\pi}{2}\right)$. *[2 marks]*

(b) Find $(g \circ f)\left(\frac{\pi}{2}\right)$. *[2 marks]*

(c) Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of k, $k \in \mathbb{Z}$. *[3 marks]*

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6. *[Maximum mark: 7]*

Solve the equation $2\cos x = \sin 2x$, for $0 \le x \le 3\pi$.

6. *[Maximum mark: 8]*

Let $\sin\theta = \frac{2}{\sqrt{13}}$, where $\frac{\pi}{2} < \theta < \pi$.

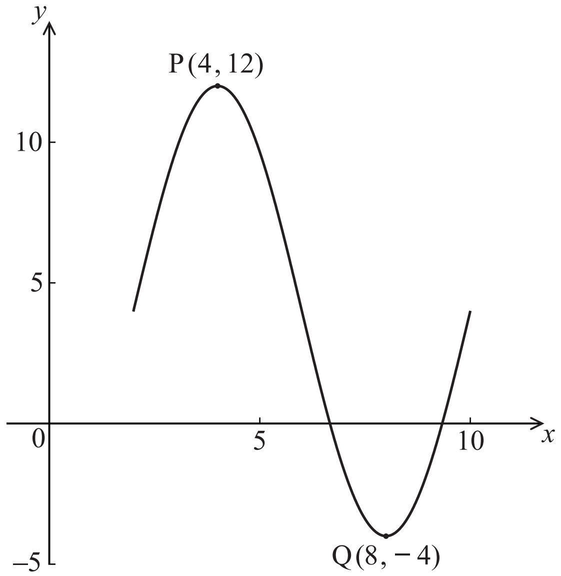
(a) Find $\cos\theta$. *[3 marks]*

(b) Find $\tan 2\theta$. *[5 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9. *[Maximum mark: 14]*

The following diagram shows the graph of $f(x) = a\sin\left(b(x-c)\right) + d$, for $2 \le x \le 10$.



There is a maximum point at $\mathrm{P}(4, 12)$ and a minimum point at $\mathrm{Q}(8, -4)$.

(a) Use the graph to write down the value of

(i) a;

(ii) c;

(iii) d. *[3 marks]*

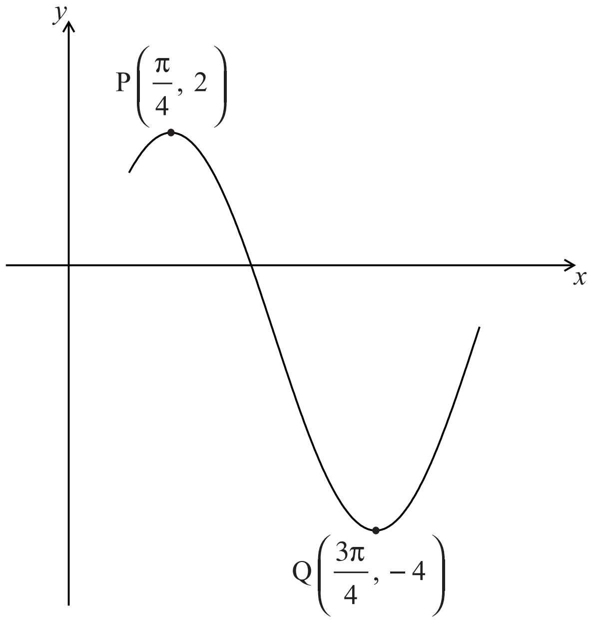
(b) Show that $b = \frac{\pi}{4}$. *[2 marks]*

(c) Find $f'(x)$. *[3 marks]*

(d) At a point R, the gradient is -2π. Find the x-coordinate of R. *[6 marks]*

5. *[Maximum mark: 7]*

The diagram below shows part of the graph of $f(x) = a\cos\left(b(x-c)\right) - 1$, where $a > 0$.



The point $P\left(\frac{\pi}{4}, 2\right)$ is a maximum point and the point $Q\left(\frac{3\pi}{4}, -4\right)$ is a minimum point.

(a) Find the value of a. *[2 marks]*

(b) (i) Show that the period of f is π.

(ii) Hence, find the value of b. *[4 marks]*

(c) Given that $0 < c < \pi$, write down the value of c. *[1 mark]*

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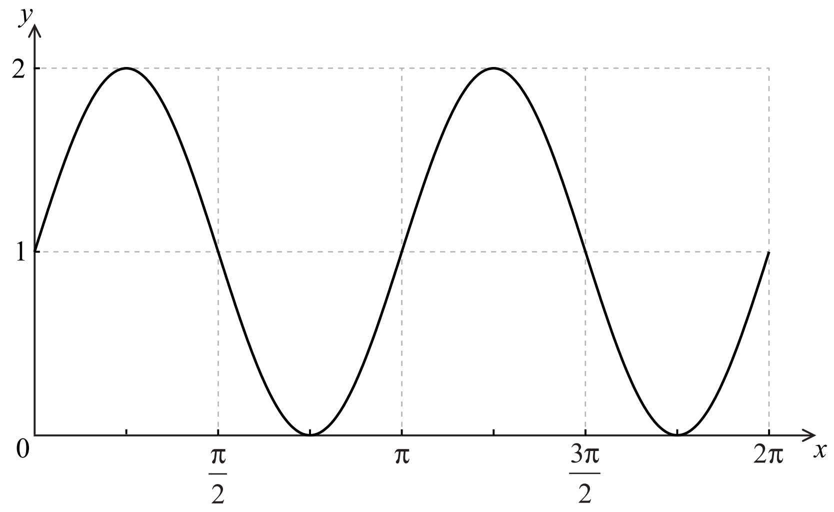
7. *[Maximum mark: 6]*

Let $f(x) = (\sin x + \cos x)^2$.

(a) Show that $f(x)$ can be expressed as $1 + \sin 2x$. *[2 marks]*

The graph of f is shown below for $0 \le x \le 2\pi$.



(b) Let $g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of g for $0 \le x \le 2\pi$. *[2 marks]*

The graph of g can be obtained from the graph of f under a horizontal stretch of scale factor p followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$.

(c) Write down the value of p and a possible value of k. *[2 marks]*

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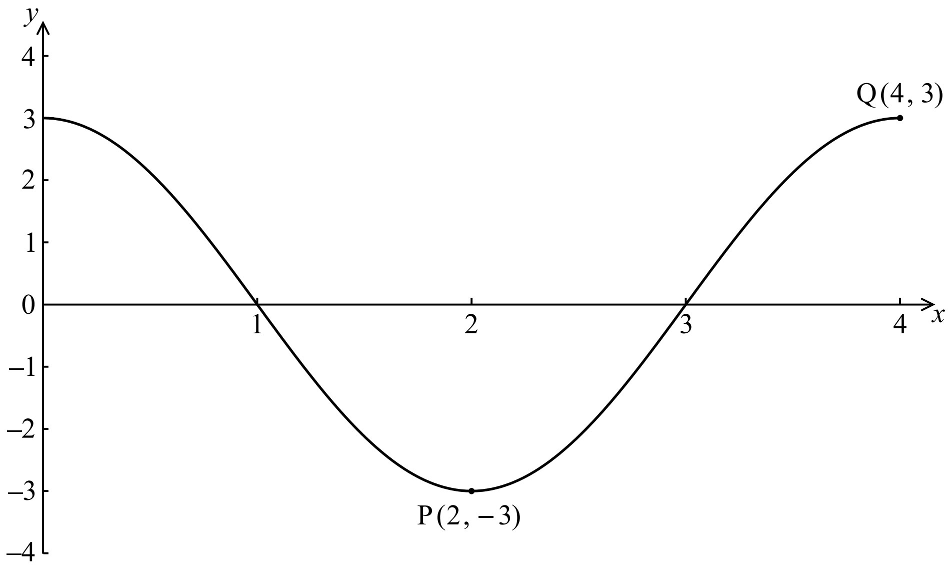
3. *[Maximum mark: 6]*

The following diagram shows the graph of $f(x) = a\cos(bx)$, for $0 \le x \le 4$.



There is a minimum point at $\mathrm{P}(2, -3)$ and a maximum point at $\mathrm{Q}(4, 3)$.

(a) (i) Write down the value of a.

(ii) Find the value of b. *[3 marks]*

(b) Write down the gradient of the curve at P. *[1 mark]*

(c) Write down the equation of the normal to the curve at P. *[2 marks]*

5. *[Maximum mark: 6]*

Let $\sin 100^\circ = m$. Find expressions for each of the following in terms of m

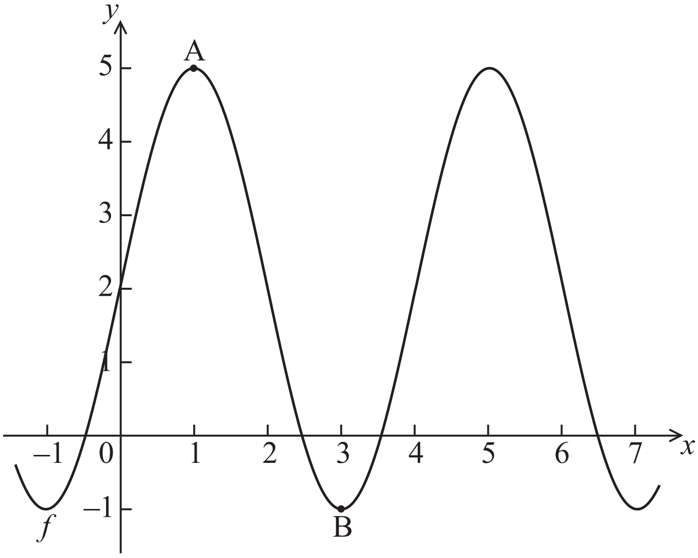
(a) $\cos 100^\circ$; *[3 marks]*

(b) $\tan 100^\circ$; *[1 mark]*

(c) $\sin 200^\circ$. *[2 marks]*

5. *[Maximum mark: 6]*

The diagram below shows part of the graph of a function f.



The graph has a maximum at $A(1, 5)$ and a minimum at $B(3, -1)$.

The function f can be written in the form $f(x) = p\sin(qx) + r$. Find the value of

(a) p; *[2 marks]*

(b) q; *[2 marks]*

(c) r. *[2 marks]*

5. *[Maximum mark: 7]*

Let $f(x) = \sin\left(x + \frac{\pi}{4}\right) + k$. The graph of f passes through the point $\left(\frac{\pi}{4}, 6\right)$.

(a) Find the value of k. *[3]*

(b) Find the minimum value of $f(x)$. *[2]*

Let $g(x) = \sin x$. The graph of g is translated to the graph of f by the vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

(c) Write down the value of p and of q. *[2]*

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

The following diagram shows a right-angled triangle, ABC, where $\sin A = \frac{5}{13}$.

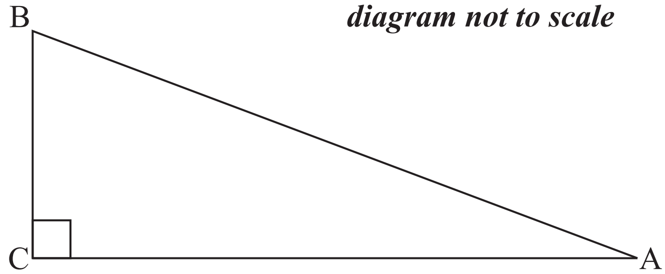


diagram not to scale

(a) Show that $\cos A = \frac{12}{13}$. *[2]*

(b) Find $\cos 2A$. *[3]*

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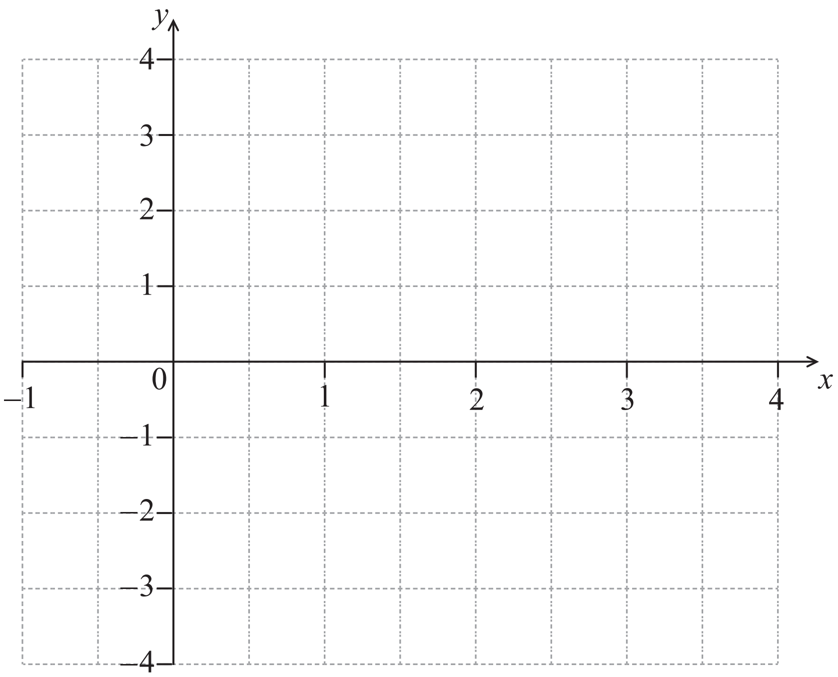
4. [Maximum mark: 7]

Let $f(x) = 3\sin(\pi x)$.

(a) Write down the amplitude of f. [1]

(b) Find the period of f. [2]

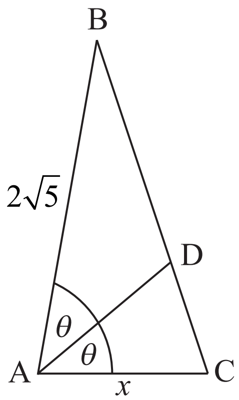
(c) On the following grid, sketch the graph of $y = f(x)$, for $0 \le x \le 3$. [4]



6. [Maximum mark: 7]

The following diagram shows triangle ABC. The point D lies on [BC] so that [AD] bisects $\hat{\text{BAC}}$.

diagram not to scale



$$\text{AB} = 2\sqrt{5}\ \text{cm},\ \text{AC} = x\,\text{cm},\ \text{and}\ \text{D}\hat{\text{A}}\text{C} = \theta,\ \text{where}\ \sin\theta = \frac{2}{3}$$

The area of triangle ABC is $5\,\text{cm}^2$. Find the value of x.

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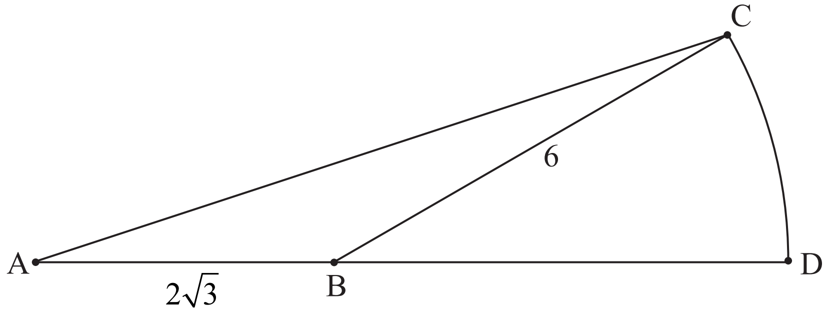
5. [Maximum mark: 8]

The following diagram shows a triangle ABC and a sector BDC of a circle with centre B and radius $6\,\mathrm{cm}$. The points A, B and D are on the same line.

diagram not to scale



$\mathrm{AB} = 2\sqrt{3}\,\mathrm{cm}$, $\mathrm{BC} = 6\,\mathrm{cm}$, area of triangle $\mathrm{ABC} = 3\sqrt{3}\,\mathrm{cm}^2$, $\mathrm{A\hat{B}C}$ is obtuse.

(a) Find $\mathrm{A\hat{B}C}$. [5]

(b) Find the exact area of the sector BDC. [3]

2. [Maximum mark: 5]

Let $\sin\theta = \frac{\sqrt{5}}{3}$, where θ is acute.

(a) Find $\cos\theta$. [3]

(b) Find $\cos 2\theta$. [2]

7. [Maximum mark: 7]

Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$.

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Do **not** write solutions on this page.

10. [Maximum mark: 15]

The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

a	1	2
$\mathrm{P}(A=a)$	$\cos\theta$	$2\cos 2\theta$

(a) Show that $\cos\theta = \frac{3}{4}$. [6]

(b) Given that $\tan\theta > 0$, find $\tan\theta$. [3]

(c) Let $y = \frac{1}{\cos x}$, for $0 < x < \frac{\pi}{2}$. The graph of y between $x = \theta$ and $x = \frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed. [6]