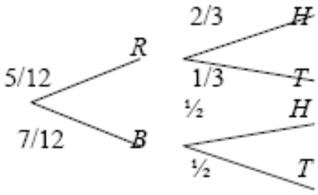
1. (a)



P(R) and P(B)	B1	
2nd set of probabilities	B1	2

Note

1st B1 for the probabilities on the first 2 branches. Accept $0.41\dot{6}$ and $0.58\dot{3}$

2nd B1 for probabilities on the second set of branches. Accept $0.\dot{6}$, $0.\dot{3}$, 0.5 and $\frac{1.5}{3}$

Allow exact decimal equivalents using clear recurring notation if required.

(b) P(H) = $\frac{5}{12}\times\frac{2}{3}+\frac{7}{12}\times\frac{1}{2}, = \frac{41}{72}$ or awrt 0.569 M1 A1 2

Note

M1 for an expression for P(H) that follows through their sum of two products of **probabilities** from their tree diagram

(c) $P(R|H) = \dfrac{\frac{5}{12}\times\frac{2}{3}}{"\frac{41}{72}"}, = \dfrac{20}{41}$ or awrt 0.488 M1 A1ft A1 3

Note

Formula seem

M1 for $\dfrac{P(R\cap H)}{P(H)}$ with denominator their (b) substituted e.g.

$\dfrac{P(R\cap H)}{P(H)} = \dfrac{\frac{5}{12}}{\text{(their (b))}}$ award M1.

Formula not seen

M1 for $\dfrac{\text{probability}\times\text{probability}}{\text{their } b}$ but M0 if fraction repeated e.g.

$\dfrac{\frac{5}{12}\times\frac{2}{3}}{\frac{2}{3}}$.

1st A1ft for a fully correct expression or correct follow through

2nd A1 for $\frac{20}{41}$ o.e.

(d) $\left(\dfrac{5}{12}\right)^2 + \left(\dfrac{7}{12}\right)^2$ M1 A1ft

$= \dfrac{25}{144} + \dfrac{49}{144} = \dfrac{74}{144}$ or $\dfrac{37}{72}$ or awrt 0.514 A1 3

Note

M1 for $\left(\dfrac{5}{12}\right)^2$ or $\left(\dfrac{7}{12}\right)^2$ can follow through their equivalent values from tree diagram

1st A1 for both values correct or follow through from their original tree and +

2nd A1 for a correct answer

Special Case $\dfrac{5}{12}\times\dfrac{4}{11}$ or $\dfrac{7}{12}\times\dfrac{6}{11}$ seen award M1A0A0s

[10]

2.	(a)	$\frac{2+3}{\text{their total}} = \frac{5}{\text{their total}} = \frac{1}{6}$ **(* * given answer * *)**	M1 A1cso	2

Note

M1 for $\frac{2+3}{\text{their total}}$ or $\frac{5}{30}$

(b)	$\frac{4+2+5+3}{\text{total}}$, $= \frac{14}{30}$ or $\frac{7}{15}$ or $0.4\dot{6}$	M1 A1	2

Note

M1 for adding at least 3 of "4, 2, 5, 3" and dividing by their total to give a probability
Can be written as separate fractions substituted into the completely correct Addition Rule

(c)	$\mathrm{P}(A \cap C) = 0$	B1	1

Note

B1 for 0 or 0/30

(d)	$\mathrm{P}(\mathrm{C} \mid \text{reads at least one magazine}) = \frac{6+3}{20} = \frac{9}{20}$	M1 A1	2

Note

M1 for a **denominator of 20** or $\frac{20}{30}$ leading to an answer with denominator of 20 $\frac{9}{20}$ only, 2/2

(e) $\text{P}(B) = \frac{10}{30} = \frac{1}{3}$, $\text{P}(C) = \frac{9}{30} = \frac{3}{10}$, $\text{P}(B \cap C) = \frac{3}{30} = \frac{1}{10}$

or $\text{P}(B|C) = \frac{3}{9}$ M1

$\text{P}(B) \times \text{P}(C) = \frac{1}{3} \times \frac{3}{10} = \frac{1}{10} = \text{P}(B \cap C)$ or $\text{P}(B|C) = \frac{3}{9} = \frac{1}{3} = \text{P}(B)$ M1

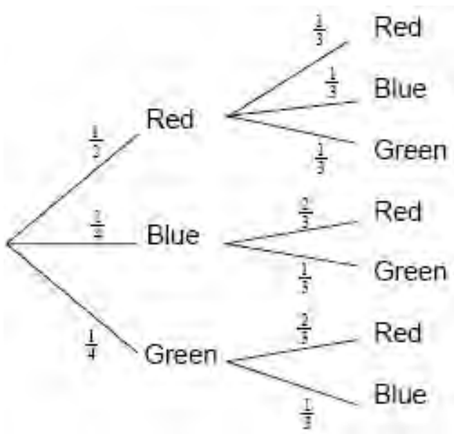
So yes they are statistically independent A1cso 3

Note

1st M1 for attempting all the required probabilities for a suitable test
2nd M1 for use of a correct test – must have attempted all the correct probabilities.
Equality can be implied in line 2.
A1 for fully correct test carried out with a comment

[10]

3. (a)



M1A1A1 3

Note

M1 for shape and labels: 3 branches followed by 3,2,2 with some *R*, *B* and *G* seen

Allow 3 branches followed by 3, 3, 3 if 0 probabilities are seen implying that 3, 2, 2 intended

Allow blank branches if the other probabilities imply probability on blanks is zero

Ignore further sets of branches

1st A1 for correct probabilities and correct labels on 1st set of branches.

2nd A1 for correct probabilities and correct labels on 2nd set of branches.

(accept 0.33, 0.67 etc or better here)

Special Case

With Replacement (This oversimplifies so do not apply Mis -Read: max mark 2/5)

B1 for 3 branches followed by 3, 3, 3 with correct labels and probabilities of $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ on each.

(b) P(Blue bead and a green bead) = $\left(\frac{1}{4}\times\frac{1}{3}\right)+\left(\frac{1}{4}\times\frac{1}{3}\right)=\frac{1}{6}$

(or any exact equivalent) M1 A1 2

Note

M1 for identifying the 2 cases *BG* and *GB* and adding 2 products of probabilities.

These cases may be identified by their probabilities e.g. $\left(\frac{1}{4}\times\frac{1}{3}\right)+\left(\frac{1}{4}\times\frac{1}{3}\right)$

NB $\frac{1}{6}$ (or exact equivalent) with no working scores 2/2

Special Case

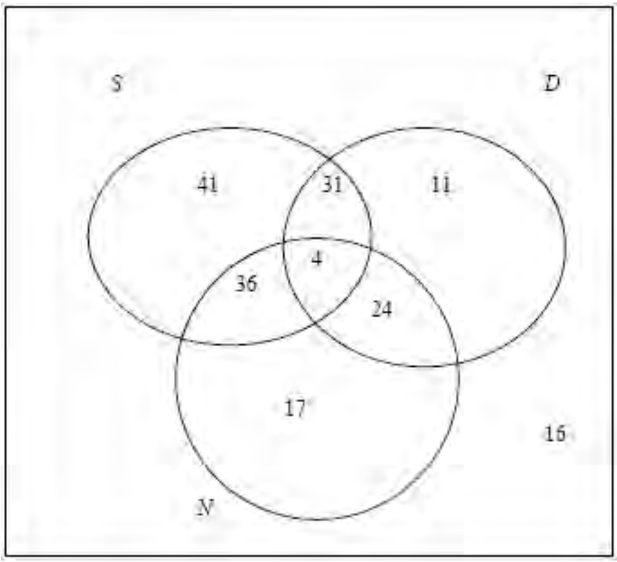
M1 for identifying 2, possibly correct cases and adding 2 products of probabilities but A0 for wrong answer

$\left[\left(\frac{1}{4}\times\frac{1}{4}\right)+\left(\frac{1}{4}\times\frac{1}{4}\right)\right]$ will be sufficient for M1A0 here

but $\frac{1}{4}\times\frac{1}{2}+\ldots$ would score M0

[5]

4. (a)



3 closed curves and 4 in centre	M1	
Evidence of subtraction	M1	
31,36,24	A1	
41,17,11	A1	
Labels on loops, 16 and box	B1	5

Note

2nd M1 There may be evidence of subtraction in "outer" portions, so with 4 in the centre then 35, 40 28 (instead of 31,36,24) along with 33, 9, 3 can score this mark but A0A0

N.B. This is a common error and their "16" becomes 28 but still scores B0 in part (a)

(b) P(None of the 3 options) $= \dfrac{16}{180} = \dfrac{4}{45}$ B1ft 1

Note

B1ft for $\frac{16}{180}$ or any exact equivalent. Can ft their "16" from their box. If there is no value for their "16" in the box only allow this mark if they have shown some working.

(c) P(Networking only) = $\dfrac{17}{180}$ B1ft 1

Note

B1ft ft their "17". Accept any exact equivalent

(d) P(All 3 options/technician) = $\dfrac{4}{40} = \dfrac{1}{10}$ M1 A1 2

Note

If a probability greater than 1 is found in part (d) score M0A0

M1 for clear sight of $\dfrac{P(S \cap D \cap N)}{P(S \cap N)}$ and an attempt at one of the probabilities, ft their values.

Allow P(all 3 | $S \cap N$) = $\dfrac{4}{36}$ or $\dfrac{1}{9}$ to score M1 A0.

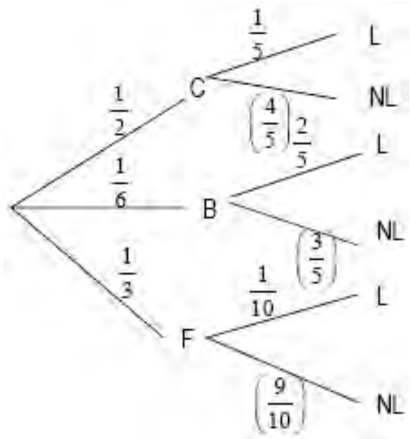
Allow a correct ft from their diagram to score M1A0 e.g. in 33,3,9 case in (a): $\frac{4}{44}$ or $\frac{1}{11}$ is M1A0 A ratio of probabilities with a product of probabilities on top is M0, even with a correct formula.

A1 for $\dfrac{4}{40}$ or $\dfrac{1}{10}$ or an exact equivalent

Allow $\dfrac{4}{40}$ or $\dfrac{1}{10}$ to score both marks if this follows from their diagram, otherwise some explanation (method) is required.

[9]

5. (a)



Correct tree	B1	
All labels	B1	
Probabilities on correct branches	B1	3

Note

Exact decimal equivalents required throughout if fractions not used e.g. 2(b)(i) 0.03& Correct path through their tree given in their probabilities award Ms

All branches required for first B1. Labels can be words rather than symbols for second B1. Probabilities from question enough for third B1 i.e. bracketed probabilities not required. Probabilities and labels swapped i.e. labels on branches and probabilities at end can be awarded the marks if correct.

(b) (i) $\frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$ or equivalent — M1 A1 — 2

Note

Correct answer only award both marks.

(ii) CNL + BNL + FNL $= \frac{1}{2} \times \frac{4}{5} + \frac{1}{6} \times \frac{3}{5} + \frac{1}{3} \times \frac{9}{10}$ — M1

$= \frac{4}{5}$ or equivalent — A1 — 2

Note

At least one correct path identified and attempt at adding all three multiplied pairs award M1

(c)	$\mathrm{P}(F'/L) = \dfrac{\mathrm{P}(F' \cap L)}{\mathrm{P}(L)}$	Attempt correct conditional probability **but see notes**	M1	
	$= \dfrac{\frac{1}{6}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{5}}{1-(ii)}$	$\dfrac{\text{numerator}}{\text{denominator}}$	$\dfrac{\text{A1}}{\text{A1ft}}$	
	$= \dfrac{\frac{5}{30}}{\frac{1}{5}} = \dfrac{5}{6}$ or equivalent	cao	A1	4

Note

Require probability on numerator and division by probability for M1.Require numerator correct for their tree for M1.

Correct formula seen and used, accept denominator as attempt and award M1

No formula, denominator must be correct for their tree or 1-(ii) for M1

1/30 on numerator only is M0, P(L/F') is M0.

[11]

6.	(a)	(i)	$\mathrm{P}(A \cup B) = a + b$	cao	B1	

<u>**Note**</u>

Accept $a + b - 0$ for B1

(ii)	$\mathrm{P}(A \cup B) = a + b - ab$	or equivalent	B1	2

<u>Special Case</u>

If answers to (i) and (ii) are

(i) P(A)+P(B) and (ii) P(A)+P(B) -P(A)P(B)

award B0B1

7(a)(i) and (ii) answers must be clearly labelled or in correct order for marks to be awarded.

(b)	$\mathrm{P}(R \cup Q) = 0.15 + 0.35$			
	$= 0.5$	0.5	B1	1

(c) $P(R \cap Q) = P(R|Q) \times P(Q)$

$= 0.1 \times 0.35$ M1

$= 0.035$ **0.035** A1 2

(d) $P(R \cup Q) = P(R) + P(Q) - P(R \cap Q)$ OR $P(R) = P(R \cap Q')$ $+ P(R \cap Q)$ M1

$= 0.15 +$ their (c)

$0.5 = P(R) + 0.35 - 0.035$ $= 0.15 + 0.035$

$P(R) = 0.185$ $= 0.185$ 0.185 A1 2

[7]

7. (a) E = take regular exercise B = always eat breakfast

$P(E \cap B) = P(E|B) \times P(B)$ M1

$= \frac{9}{25} \times \frac{2}{3} = 0.24$ or $\frac{6}{25}$ or $\frac{18}{75}$ A1 2

Note

M1 for $\frac{9}{25} \times \frac{2}{3}$ or $P(E \mid B) \times P(B)$ and at least one correct value seen.
A1 for 0.24 or exact equiv.
NB $\frac{2}{5} \times \frac{2}{3}$ alone or $\frac{2}{5} \times \frac{9}{25}$ alone scores M0A0. Correct answer scores full marks.

Common Errors

$\frac{9}{25}$ is M0A0

(b) $P(E \cup B) = \frac{2}{3} + \frac{2}{5} - \frac{6}{25}$ or $P(E' \mid B')$ or $P(B' \cap E)$ or $P(B \cap E')$ M1

$= \frac{62}{75}$ $= \frac{13}{25}$ $= \frac{12}{75}$ $= \frac{32}{75}$ A1

$P(E' \cap B') = 1 - P(E \cup B) = \frac{13}{75}$ or $0.17\dot{3}$ M1 A1 4

<u>Note</u>

1st M1 for use of the addition rule. Must have 3 terms and some values, can ft their (a) <u>Or</u> a full method for P(E'|B') requires 1 – P(E|B') and equation for P(E|B'): (a) + $\frac{x}{3}=\frac{2}{5}$ <u>Or</u> a full method for P($B' \cap E$) <u>or</u> P($B \cap E'$) [or other valid method]

2nd M1 for a method leading to answer e.g. 1 – P($E \cup B$) <u>or</u> P(B') × P($E'|B'$)or P(B') – P($B' \cap E$) <u>or</u> P(E') – P($B \cap E'$)

<u>Venn Diagram</u> 1st M1 for diagram with attempt at $\frac{2}{5} - \text{P}(B' \cap E)$ or $\frac{2}{3} - \text{P}(B \cap E)$. Can ft their (a)

1st A1 for a correct first probability as listed or 32, 18 and 12 on Venn Diagram

2nd M1 for attempting 75 – their (18 +32 + 12)

(c) P(E | B) = 0.36 ≠ 40 = P(E) or P($E \cap B'$) = $\frac{6}{25} \neq \frac{2}{5} \times \frac{2}{3} = \text{P}(E) \times \text{P}(B)$ M1

A1 2

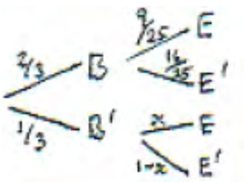
So E and B are <u>not</u> statistically independent

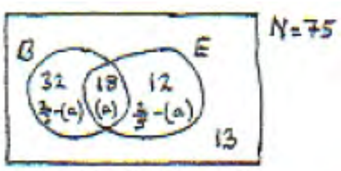
<u>Note</u>

M1 for identifying suitable values to test for independence e.g. P(E) = 0.40 and P(E|B) = 0.36 <u>Or</u> P(E) × P(B) = ... and P($E \cap B$) = their (a) [but their (a) $\neq \frac{2}{5} \times \frac{2}{3}$]. Values seen somewhere

A1 for correct values and a correct comment

<u>Diagrams</u> You may see these or find these useful for identifying probabilities.





<u>Common Errors</u>

(a) $\frac{9}{25}$ is M0A0

(b) P(EUB) = $\frac{53}{75}$ scores M1A0 1– P(E ∪ B) = $\frac{22}{75}$ scores M1A0

(b) P(B') × P(E') = $\frac{1}{3} \times \frac{3}{5}$ scores 0/4

[8]

8.	(a)	$E(X) = 0 \times 0.4 + 1 \times 0.3 + ... + 3 \times 0.1, = 1$	M1, A1	2

Note

M1 for at least 3 terms seen. Correct answer only scores M1A1. Dividing by $k(\neq 1)$ is M0.

(b)	$F(1.5) = [P(X \leq 1.5) =]\ P(X \leq 1), = 0.4 + 0.3 = 0.7$	M1, A1	2

Note

M1 for $F(1.5) = P(X \leq 1)$.[**Beware**: $2 \times 0.2 + 3 \times 0.1 = 0.7$ but scores M0A0]

(c)	$E(X^2) = 0^2 \times 0.4 + 1^2 \times 0.3 + ... + 3^2 \times 0.1, = 2$	M1, A1	
	$Var(X) = 2 - 1^2, = 1$ (*)	M1, A1cso	4

Note

1st M1 for at least 2 non-zero terms seen. $E(X^2) = 2$ alone is M0. Condone calling $E(X^2) = Var(X)$.

ALT

1st A1 is for an answer of 2 or a fully correct expression.

2nd M1 for $-\mu^2$, condone $2 - 1$, unless clearly $2 - \mu$ Allow $2 - \mu^2$ with $\mu = 1$ even if $E(X) \neq 1$

2nd A1 for a fully correct solution with no incorrect working seen, **both** Ms required.

$\underline{\Sigma\,(x - \mu)^2 \times P(X = x)}$

1st M1 for an attempt at a full list of $(x - \mu)^2$ values and probabilities. 1st A1 if all correct

2nd M1 for at least 2 non-zero terms of $(x - \mu)^2 \times P(X = x)$ seen. 2nd A1 for $0.4 + 0.2 + 0.4 = 1$

(d)	$Var(5 - 3X) = (-3)^2\ Var(X), = 9$	M1, A1	2

Note

M1 for use of the correct formula. $-3^2\ Var(X)$ is M0 unless the final answer is >0.

(e)

Total	Cases	Probability		
4	$(X=3)\cap(X=1)$	$0.1 \times 0.3 = 0.03$		
	$(X=1)\cap(X=3)$	$0.3 \times 0.1 = 0.03$		
	$(X=2)\cap(X=2)$	$0.2 \times 0.2 = 0.04$	B1B1B1	
5	$(X=3)\cap(X=2)$	$0.1 \times 0.2 = 0.02$		
	$(X=2)\cap(X=3)$	$0.2 \times 0.1 = 0.02$	M1	
6	$(X=3)\cap(X=3)$	$0.1 \times 0.1 = 0.01$	A1	
Total probability = 0.03 + 0.03 + 0.04 +0.02 + 0.02 + 0.01 = 0.15			A1	6

Note

Can follow through their Var(X) for M1

ALT

1st B1 for all cases listed for a total of 4 or 5 or 6 . e.g. (2,2) counted twice for a total of 4 is B0
2nd B1 for all cases listed for 2 totals }
3rd B1 for a complete list of all 6 cases } These may be highlighted in a table

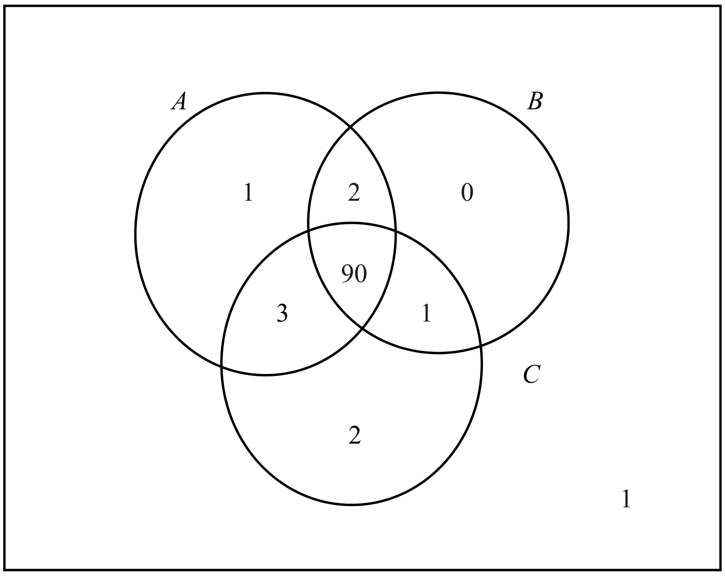
Using Cumulative probabilities

1st B1 for one or more cumulative probabilities used e.g.2 then 2 or more or 3 then 1 or more
2nd B1 for both cumulative probabilities used. 3rd B1 for a complete list 1, 3; 2, ≥2; 3, ≥1
M1 for one correct pair of correct probabilities multiplied
1st A1 for all 6 correct probabilities listed (0.03, 0.03, 0.04, 0.02, 0.02, 0.01) needn't be added.
2nd A1 for 0.15 or exact equivalent only as the final answer.

[16]

9. Diagram may be drawn with $B \subset (A \cup C)$ or with the 0 for $B \cap (A \cup C)'$ simply left blank

(a)



3cc	M1	
90, 3, 2, 1	A1	
1, (0), 2	M1A1	
1 outside	A1	
Box	B1	6

Accept decimals or probs. in Venn diagram

1st M1 for 3 closed, labelled curves that overlap.
A1 for the 90, 3, 2 and 1

2nd M1 for one of 1, 0 or 2 correct <u>or</u> a correct sum of 4 values for A, B or C

2nd A1 for all 7 values correct. Accept a blank instead of 0.

NB final mark is a B1 for the box not an A mark as on EPEN

(b) P(none) = 0.01 B1ft 1

B1ft Follow through their '1'from outside divided by 100

(c) P(A but not B) = 0.04 M1 A1ft 2

M1 for correct expression eg $P(A \cup B) - P(B)$ or calculation e.g. 3 + 1 or 4 on top

A1 for a correct probability, follow through with their '3 + 1' from diagram

(d)	P(any wine but C) = 0.03	M1A1ft	2

M1 for correct expression or calculation e.g. 1 + 2 + 0 or 99 – 96 or 3 on top

A1 for a correct probability, follow through their '2 + 1 + 0' from diagram

(e)	P(exactly two) = 0.06	M1A1ft	2

M1 for a correct expression or calculation e.g. 3 + 2 + 1 or 6 on top

(f)	$P(C \mid A) = \dfrac{P(C \cap A)}{P(A)} =, \dfrac{93}{96}$ or $\dfrac{31}{32}$ or AWRT 0.969	M1A1ft,A1	3

M1 for a correct expression upto "," and <u>some</u> correct substitution, ft their values.
One of these probabilities must be correct or correct ft.
If P(C) on bottom M0

1st A1ft follow through their $A \cap C$ and their A but the ratio must be in (0, 1)

2nd A1 for correct answer only.
Answer only scores 3/3, but check working
P($A \cap C$)/P(C) is M0

In parts (b) to (f) full marks can be scored for correct answers or correct ft

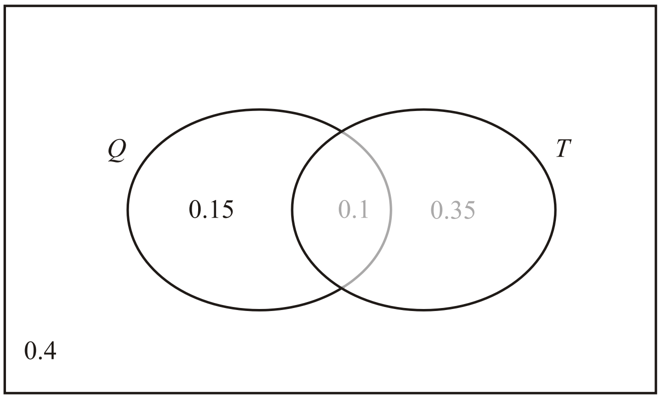
For M marks in (c) to (e) they must have a fraction

[16]

10. (a)	$P(Q \cup T) = 0.6$	B1	
	$P(Q) + P(T) - P(Q \cap T) = 0.6$	M1	
	$P(Q \cap T) = 0.1$	A1	3

B1 for 0.6
M1 for use of $P(Q) + P(T) - P(Q \cap T) = P(Q \cup T)$
0.1 Correct answer only for A1
Alternative:
(25 + 45 + 40 =) 110% B1
110 – 100 = 10% M1A1
0.1 stated clearly as the final answer with no working gets 3/3

(b)



Venn	M1	
0.15, 0.35	A1ft	
0.4 and box	B1	3

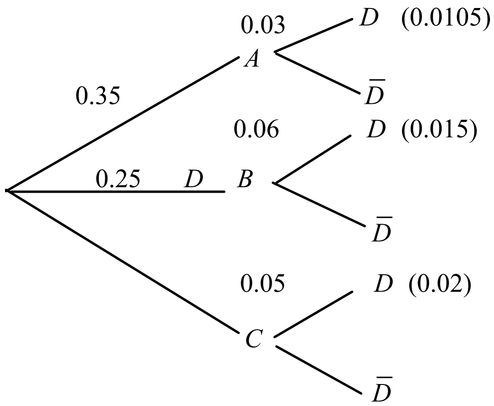
Two intersecting closed curves for M1, no box required.
At least one label (Q or T) required for first A1.
Follow through (0.25 – 'their 0.1') and (0.45 – 'their 0.1') for A1.
0.4 and box required, correct answer only for B1
Using %, whole numbers in Venn diagram without % sign,
whole numbers in correct ratio all OK

(c) $P(Q \cap T' \mid Q \cup T) = \dfrac{0.15}{0.60} = \dfrac{1}{4}$ or 0.25 or 25% M1A1ftA1 3

require fraction with denominator 0.6 or their equivalent from
Venn diagram for M1
Follow through their values in fraction for A1
Final A1 is correct answer only.
<u>No working no marks.</u>

[9]

11. (a)



Correct tree shape	M1	
A, B and *C* and 0.35 and 0.25	A1	
D (x3) and 0.03, 0.06, 0.05 (May be implied by seeing P($A \cap D$) etc at the ends)	A1	3

M1 for tree diagram, 3 branches and then two from each.
At least one probability attempted.

(b) (i) $P(A \cap D) = 0.35 \times 0.03$, = **0.0105** or $\frac{21}{2000}$ M1, A1

P(C) = 0.4 (anywhere) B1

(ii) P(D) = (i) + 0.25 × 0.06 + (0.4 × 0.05) M1

= **0.0455** or $\frac{91}{2000}$ A1 5

1st M1 for 0.35 × 0.03. Allow for equivalent from <u>their</u> tree diagram,
B1 for P(C) = 0.4, can be in correct place on tree diagram or implied by 0.4 × 0.05 in P(D).
2nd M1 for all 3 cases attempted and <u>some</u> correct probabilities seen, including +. Can ft their tree.

Condone poor use of notation if correct calculations seen.
E.g. P(C | D) for P($C \cap D$).

(c) $P(C|D) = \frac{P(C \cap D)}{P(D)}, = \frac{0.4 \times 0.05}{\text{(ii)}}$ M1, A1ft

$= 0.43956...$ or $\frac{40}{91}$ **<u>0.44</u>** or awrt **<u>0.440</u>** A1 3

[Correct answers only score full marks in each part]

[11]

M1 for attempting correct ratio of probabilities.
There must be an attempt to substitute some values in a correct formula.
If no correct formula and ration not correct ft score M0.
Writing P(*D*|*C*) and attempting to find this is M0.
Writing P(*D*|*C*) but calculating correct ratio – ignore notation and mark ratios.

A1ft must have their 0.4 × 0.05 divided by their (ii).

If ratio is incorrect ft (0/3) unless correct formula seen and part of ratio is correct then M1.

12. (a) N.B. Part (a) doesn't have to be in a table, could be a list B1, B1, B1 3
$P(X = 1) = \ldots$etc

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

0.0278, 0.0833, 0.139, 0.194, 0.25, 0.306 (Accept awrt 3 s.f)

1st B1 for $x = 1, \ldots 6$ and at least one correct probability
N.B. $\frac{3}{36} = \frac{1}{12}$ and $\frac{9}{36} = \frac{1}{4}$

2nd B1 for at least 3 correct probabilities

3rd B1 for a fully correct probability distribution.

(b) $P(3) + P(4) + P(5) =$, <u>$\frac{21}{36}$ or $\frac{7}{12}$ or awrt 0.583</u> M1, A1 2

M1 for attempt to add the correct three probabilities, ft their probability distribution

(c)	E(X) = $\frac{1}{36}+2\times\frac{3}{36}+....,=\underline{\frac{161}{36}}$ or $4.47\dot{2}$ or $4\frac{17}{36}$	M1, A1	2

M1 for a correct attempt at $E(X)$. Minimum is as printed.
Exact answer only scores M1A1.

[Division by 6 at any point scores M0, no ISW. Non-exact answers with no working score M0.]

(d)	E(X^2) = $\frac{1}{36}+2^2\times\frac{3}{36}+...,=\frac{791}{36}$ or full expression or $21\frac{35}{36}$ or awrt 21.97	M1, A1	
	Var(X) = $\frac{791}{36}-\left(\frac{161}{36}\right)^2$, = $\underline{\mathbf{1.9714...\ *}}$	M1, A1cso	4

1st M1 for a correct attempt at $E(X^2)$.
Minimum as printed.
$\frac{791}{36}$ or awrt 21.97 scores M1A1.

2nd M1 for their $E(X^2)$ – (their E(X))^2.

2nd A1 cso needs awrt 1.97 <u>and</u> $\frac{791}{35}-\left(\frac{161}{36}\right)^2$ or $\frac{2555}{1296}$ or any fully correct expression seen.

Can accept <u>at least 4 sf</u> for both. i.e. 21.97 for $\frac{791}{36}$,
4.472 for $\frac{161}{36}$, 20.00 for $\left(\frac{161}{36}\right)^2$.

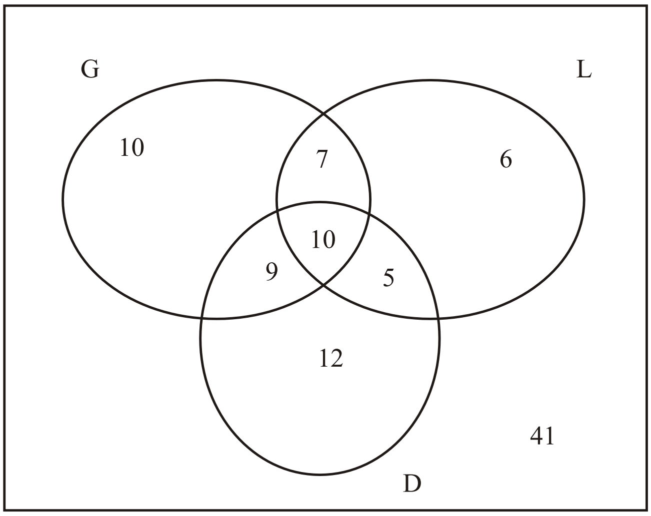
(e)	Var(2 – 3X) = 9 × 1.97 or $(-3)^2$ × 1.97, = 17.73 awrt $\underline{\mathbf{17.7}}$ or $\frac{2555}{144}$	M1, A1	2

M1 for correct use of Var($aX + b$) formula or a <u>full</u> method.

NB -3^2 × 1.97 followed by awrt 17.7 scores M1A1
<u>BUT</u> -3^2 × 1.97 alone, or followed by – 17.7, scores M0A0.

[13]

13. (a)



3 closed curves that intersect	M1	
Subtract at either stage	M1	
9, 7, 5	A1	
10, 6, 12	A1	
41 & box	A1	6

(b) $\mathrm{P}(\mathrm{G}, \overline{\mathrm{LH}}, \overline{\mathrm{D}}) = \frac{10}{100} = \frac{1}{10}$ B1ft 1

(c) $\mathrm{P}(\mathrm{G}, \overline{\mathrm{LH}}, \overline{\mathrm{D}}) = \frac{41}{100}$ B1ft 1

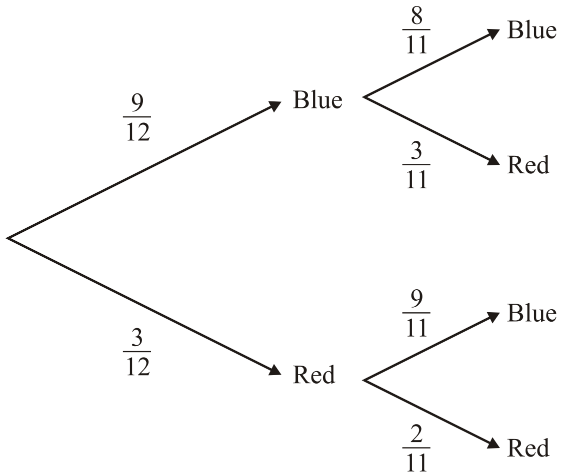
(d) P(only two attributes) $-\frac{9+7+5}{100} = \frac{21}{100}$ M1 A1 2

(e) $\mathrm{P}(\mathrm{G} \mid \mathrm{LH\&DH}) = \frac{\mathrm{P}(\mathrm{G}\ \&\ \mathrm{LH}\ \&\ \mathrm{DH})}{\mathrm{P}(\mathrm{LH}\ \&\ \mathrm{DH})} = \frac{\frac{10}{100}}{\frac{15}{100}} = \frac{10}{15} = \frac{2}{3}$ awrt 0.667 M1 A1ft A1 3

N.B. Assumption of independence M0

[13]

14. (a)



Tree	M1	
$\frac{9}{12}, \frac{3}{12}$	A1	
Complete & labels	A1	3

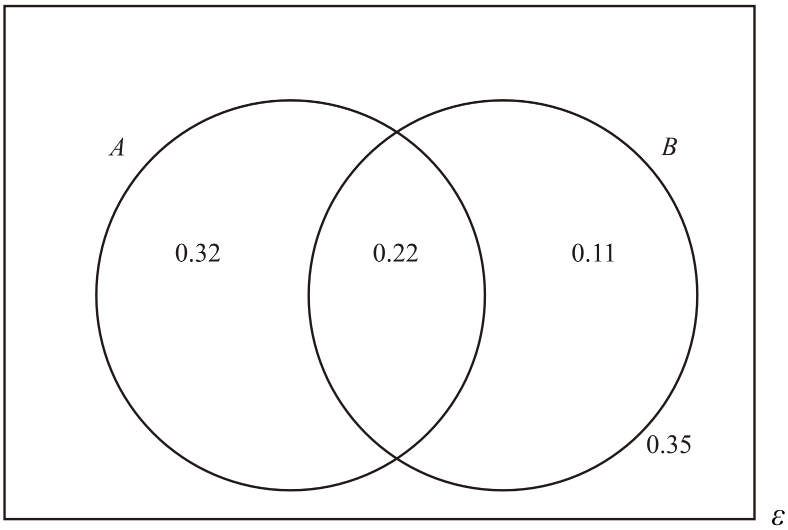
(b) P(Second ball is red) = $\frac{9}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{2}{11} = \frac{1}{4}$ M1 A1 2

(c) P(Both are red | Second ball is red) = $\dfrac{\frac{3}{12} \times \frac{2}{11}}{\frac{1}{4}} = \frac{2}{11}$

exact or awrt 0.182 M1 A1 2

[7]

15. (a)



Venn Diagram	M1	
0.32, 0.11 & A, B	A1	
0.22, 0.35 & box	A1	3

(b) $P(A) = 0.32 + 0.22 = 0.54$; $P(B) = 0.33$ — M1A1ft, A1ft — 3

(c) $P(A|B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{32}{67}$ — awrt 0.478 — M1 A1 — 2

(d) For independence $P(A \cap B') = P(A)P(B)$ — M1 A1ft

For these data $0.22 \neq 0.54 \times 0.33 = 0.1782$

(OR $P(A|B') \neq P(A)$ for M1A1ft

OR $\dfrac{2}{3} = P(A|B) \neq P(A) = 0.54$ for M1A1ft — A1ft — 3

$\therefore$ NOT independent

[11]

16.

	Glasses	No Glasses	Totals		
Science	18	**12**	30		
Arts	**27**	**23**	**50**	50 may be seen in (a)	
Humanities	44	**24**	68	23 may be seen in (b)	B1
					B1
Totals	89	**59**	148		

(a) P(Arts) = $\frac{50}{148} = \frac{25}{74}$ = 0.338 M1 A1 4

a number/148

(b) P(No glasses / Arts) = $\frac{23/148}{50/148} = \frac{23}{50} = 0.46$ M1 A1 2

$\frac{\textit{prob}}{\textit{their}(a)\,\textit{prob}}$ *or* $\frac{\textit{number}}{\textit{their } 50}$

(c) P(Right Handed) = $\left(\frac{30}{148}\times 0.8\right)+\left(\frac{50}{148}\times 0.7\right)+\left(\frac{68}{148}\times 0.75\right)$ M1 A1ft

attempt add three prob
A1 ft on their (a)

= $\frac{55}{74}$ = 0.743 A1 3

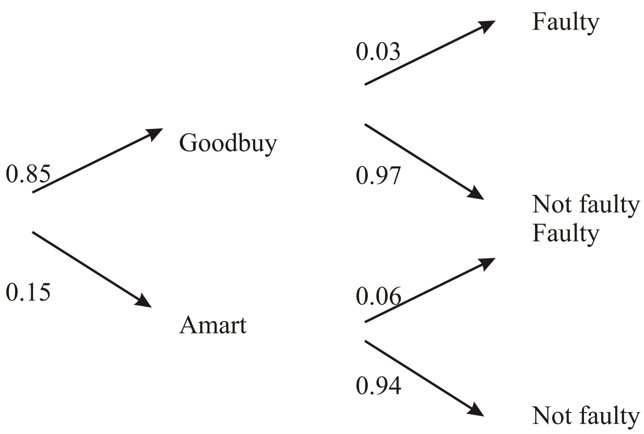
awrt 0.743

(d) P(Science /Right handed0 = $\frac{\frac{30}{148}\times 0.8}{(c)} = \frac{12}{55}$ = 0.218 M1 A1ft A1 3

ft on their (c)

[12]

17.

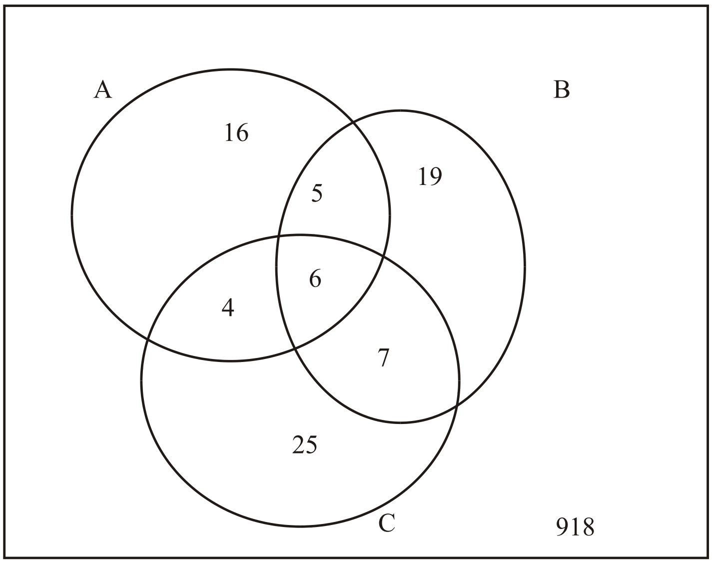


Tree (both sections) M1
labels & 0.85, 0.15 or equiv. A1
0.03,0.97,0.06,0.94 A1 3

(b)	P(Not faulty) = (0.85 × 0.97) + (0.15 × 0.94)	M1,A1∫	
	valid path & their values, correct		
	0.9655	A1	3
	% or 1931/2000 or equiv. or awrt 0.966		

[6]

18. (a)



6	B1	
subtract	M1	
4,5,7	A1	
subtract	M1	
16,19,25	A1	
box & 918	B1	6

(b) P(No defects) = $\frac{918}{1000}$ = 0.918 B1ft 1

(c) P(No more than 1) = $\frac{918+16+19+25}{1000}$ **OR** $1 - \frac{5+6+4+7}{1000}$ M1

=0.978 A1ft 2

0.978

(d) P(B | Only 1 defect) = $\dfrac{\text{P(B and 1 defect)}}{\text{P(1 defect)}} = \dfrac{\frac{19}{1000}}{\frac{16+19+25}{1000}}$ M1

conditional prob

$= \dfrac{19}{60}$ A1ft 2

$\dfrac{19}{60}$ *or* $0.31\dot{6}$ *or* 0.317

(e) P(Both had type B) = $\dfrac{37}{1000} \times \dfrac{36}{999}$ M1

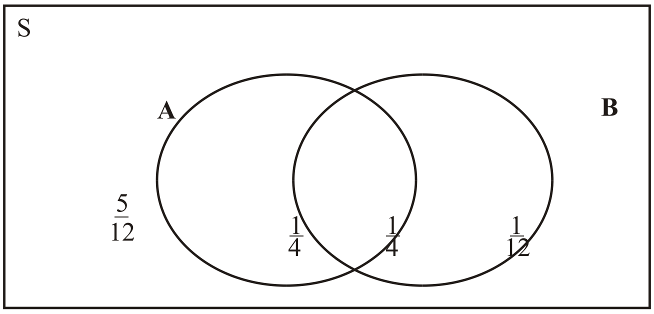
theirs from B ×

$= \dfrac{37}{27750}$ or $0.001\dot{3}$ or 0.00133 or equivalent A1 2

cao

[13]

19. (a)



2 intersecting closed curves in a box M1

both $\dfrac{1}{4}, \dfrac{1}{12}$ B1,B1

$\dfrac{5}{12}$ B1ft 4

(b) $\text{P}(A \cup B) = \dfrac{7}{12}$ B1ft 1

0.583 or $0.58\dot{3}$ *or* $\dfrac{7}{12}$

(c) $\mathrm{P}(A|B) = \dfrac{\mathrm{P}(A \cup B')}{\mathrm{P}(B')} = \dfrac{\frac{1}{4}}{\frac{2}{3}} = \dfrac{3}{8}$ or 0.375 M1,A1 2

their fractions divided, cao

[7]

20.

	1	2	2	3	3	3
1	2	3	3	4	4	4
2	3	4	4	5	5	5
2	3	4	4	5	5	5
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6

$2 \times (1, 2, \ldots.., 3)$ M1

Adding M1

All ≥ 5 correctly indicated A1

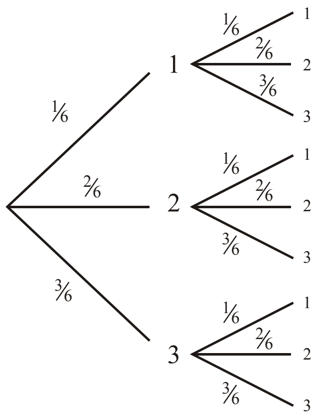
$\therefore$ P (sum at least 5) = $\dfrac{21}{36} = \dfrac{7}{12}$

Attempt to count ≥ 5 M1

$\dfrac{21}{36}$; $\dfrac{7}{12}$; $0.58\dot{3}$; *0.583* A1 5

[5]

Alt 1



Tree with relevant branches	M1	
All correct – $\frac{2}{6}$, $\frac{3}{6}$ on those branches	A1	
P(sum $\geq$ 5) = $\left(\frac{2}{6}\times\frac{3}{6}\right)+\left(\frac{3}{6}\times\frac{2}{6}\right)$ (At least 2 pairs & adding)	M1	
+ $\left(\frac{3}{6}\times\frac{3}{6}\right)$ all correct	A1	
= $\frac{21}{36}$; $\frac{7}{12}$; $0.58\dot{3}$; 0.583	A1	5

Alt 2

Outcomes (2, 3), (3, 3), (3, 2)		
Recognising 2 pairs		
Can be implied	M1	
All correct	A1	
P(sum $\geq$ 5) = $\left(\frac{2}{6}\times\frac{3}{6}\right)+\left(\frac{3}{6}\times\frac{3}{6}\right)+\left(\frac{3}{6}\times\frac{2}{6}\right)$		
Multiplying 2 pairs of 2 probs. & adding	M1	
All correct	A1	
= $\underline{\frac{21}{36}}$	A1	5

Alt 3

P (sum ≥ 5) = 12 $\left(\frac{1}{6}\times\frac{1}{6}\right)$ + 9 $\left(\frac{1}{6}\times\frac{1}{6}\right)$

$a(p_1 \times p_2)$ *or* $b(p_1 \times p_2)$	M1	
$p_1 = p_2 = \frac{1}{6}$	A1	
$a(\,) + b(\,)$	M1	

$= \underline{\frac{21}{36}}$

21 or 12 + 9	A1	
$\frac{21}{36}$; $\frac{7}{12}$ *etc*	A1	5

Alt 4

x	2	3	4	5	6
P($X = x$)	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

2, 3, 4, 5, 6	M1
Adding probabilities	M1
All correct	A1

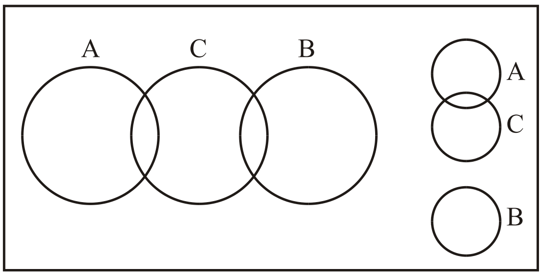
$\therefore P(X \geq 5) = \frac{12}{36} + \frac{9}{36}$

Adding P(5) & P(6)	M1

$= \underline{\frac{21}{36}}$

$\frac{21}{36}$; $\frac{7}{12}$ *etc*	A1	5

21. (a)



A, B, C inside S	B1	
A, B no overlap	B1	
A, C overlap	B1	3

(b) $\text{P}(A|C) = \dfrac{\text{P}(A \cap C)}{\text{P}(C)} = \dfrac{\text{P}(A)\text{P}(C)}{\text{P}(C)} = \text{P}(a)$ M1

Use of independence

$= \underline{0.2}$ A1 2

(c) $\text{P}(A \cup B) = \text{P(A)} + \text{P(B)} - \text{P}(A \cap B)$ M1

use of P(A ∪ B) & P(A ∩ B) = 0 can be implied

$= 0.2 + 0.4 - 0$

$= \underline{0.6}$ A1 2

SR: No working B1 only

(d) $\text{P}(A \cup C) = \text{P(A)} + \text{P(C)} - \text{P}(A \cap C)$ M1

Use of P(A ∪ C) & independence

$\therefore 0.7 = 0.2 + \text{P(c)} - 0.2\ \text{P(C)}$ A1

$\therefore 0.5 = \text{P(C)}\ \{1 - 0.2\}$ M1

Solving for P(C) from an equation with 2P(C) terms

$\therefore \underline{\text{P(c)} =}\ \frac{5}{8}$; 0.625 A1 4

[11]

<u>NB</u> P(B ∪ C) = P(B) + P(C) − P(B ∩ C)

= 0.4 + 0.625 − P(B ∩ C) ⇒ P(B ∩ C) > 0

22. (a) (i) $P(A \cap B') = P(A/B') = \frac{4}{5} \times \frac{1}{2} = \frac{4}{10} = \underline{\frac{2}{5}}$ M1

Use of P(A/B′)P(B′) A1

(ii) $P(A \cap B) = P(A) - P(A \cap B')$ M1

$= \frac{2}{5} - \frac{2}{5}$

$= \underline{0}$ A1

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ M1

$= \frac{2}{5} + \frac{1}{2} - 0$

$= \underline{\frac{9}{10}}$ A1ft

(iv) $P(A/B) = P\frac{(A \cap B)}{P(B)} = 0$ B1 7

(b) (i) since $P(A \cap B) = 0$ seen B1

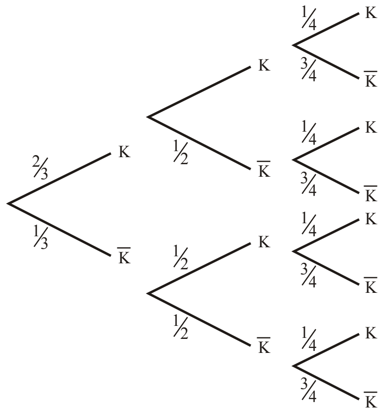
A and B are mutually exclusive B1 2

(ii) Since P (A/B) ≠ P (a) or equivalent B1

A and B are NOT independent B1 2

[11]

23. (a)



Tree with correct number of branches	M1	
$\frac{2}{3}, \frac{1}{3}$	A1	
$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	A1	
$\frac{1}{4}, \frac{3}{4} \ldots \frac{3}{4}$	A1	4

(b) P (All 3 Keys) = $\frac{2}{3} \times \frac{1}{2} \times \frac{1}{4} = \frac{2}{24} = \frac{1}{12}$ M1 A1 2

$\frac{1}{12}$; $0.08\dot{3}$; 0.0833

(c) P(exactly 1 key) = $\left(\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4}\right) + \left(\frac{1}{3} \times \frac{1}{2} \times \frac{3}{4}\right) + \left(\frac{1}{3} \times \frac{1}{2} \times \frac{1}{4}\right)$ 3 triples added M1

= $\underline{\frac{10}{24} = \frac{5}{12}}$

Each correct

$\frac{10}{24}$; $\frac{5}{12}$; $0.41\dot{6}$; 0.417 A1 A1 A1 A1 5

(d) P (Keys not collected on at least 2 successive stages)

$$= \left(\frac{2}{3}\times\frac{1}{2}\times\frac{3}{4}\right)+\left(\frac{1}{3}\times\frac{1}{2}\times\frac{1}{4}\right)+\left(\frac{1}{3}\times\frac{1}{2}\times\frac{3}{4}\right)$$

3 triples added M1
Each correct A1 A1 A1

$$= \frac{10}{24} = \frac{5}{12}$$ A1 5

$\frac{10}{24}$; $\frac{5}{12}$; $0.41\dot{6}$; 0.417

Alternative:
1 – P (Keys collected on at least 2 successive stages) M1

$$= 1 - \left\{\left(\frac{2}{3}\times\frac{1}{2}\times\frac{1}{4}\right)+\left(\frac{2}{3}\times\frac{1}{2}\times\frac{3}{4}\right)+\left(\frac{1}{3}\times\frac{1}{2}\times\frac{1}{4}\right)\right\}$$ A1 A1 A1

$$= \frac{5}{8}$$ A1 5

[16]

24. (a) P(scores 30 points) = P(hit, hit, hit,) = $0.6^3 = 0.216 = \frac{27}{125}$ 0.6^3 M1

$\frac{27}{125}$; 0.216 A1 2

(b)

x	0	10	20	30
	0.4	0.6×0.4	$0.6^2 \times 0.4$	
P ($X = x$)	0.4	0.24	0.144	(0.216)
	$\frac{4}{10}$	$\frac{6}{25}$	$\frac{18}{225}$	

x = 0, 10, 20, 30 B1
One correct
P($X = x$) M1
0.4; 0.24; 0.144 A1; A1; A1 5

(c) $E(X) = (0 \times 0.4) + \ldots + (30 \times 0.216) = \underline{11.76}$

$\Sigma xP(X = x)$
Their distribution M1
AWRT 11.8 A1

$E(X^2) = (10^2 \times 0.24) + \ldots + (30^2 \times 0.216) = 276$ B1

Std Dev = $\sqrt{276 - 11.76^2} = 11.7346\ldots$ $\sqrt{E(X^2) - (E(X))^2}$ M1

3 s.f. 11.7 A1 5

(d) P (Linda scores more in round 2 than in round 1)
$= P(X_1 = 0\ \&\ X_2 = 10, 20, 30)$ $X_2 > X_1$ M1
$+ P(X_1 = 10\ \&\ X_2 = 20, 30)$

Can be implied

All possible A1

$+P(X_1 = 20\ \&\ X_2 = 30)$ A1
$= 0.4 \times (0.24 + 0.144 + 0.216) = 0.24$ A1 ft
$+ 0.24 \times (0.144 + 0.216) = 0.0864$ A1
$+ (0.144 \times 0.216) = 0.031104$ A1 ft
$\underline{= 0.357504}$ A1 6

AWRT 0.358

[18]

25. (a) A list of <u>all possible outcomes</u> of an experiment B1 1

Accept examples

(b) A sub-set of outcomes of an experiment B1 1

(c) $P(A \cap B) = P(A)P(B) = \frac{1}{3} \times \frac{1}{4} = \underline{\frac{1}{12}}$ B1 1

(d) $P(A|B) = P(A) = \frac{1}{3}$ M1

Application of indep.
1/3 A1 cao 2

(e) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{3} + \frac{1}{4} - \frac{1}{12}$ M1

Application of P(A ∪ B)

$= \underline{\frac{1}{2}}$ A1 cao 2

[7]

26. (a) $P(A \cap B) = \frac{10}{100} = \frac{1}{10} = 0.1$ M1 A1 2

(b) $P(A') = \frac{75}{100} = 0.75$ M1 A1 2

(c) $P(B' | A) = \frac{P(B' \cap A)}{P(A)} = \frac{\frac{15}{100}}{\frac{25}{100}} = \frac{15}{25} = \frac{3}{5} = 0.6$ M1 A1 2

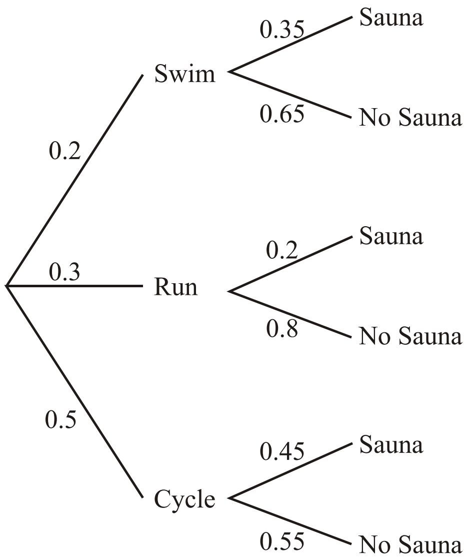
(d) $P(A' \cap B) = 0.4$; $P(A')P(B) = 0.75 \times 0.5 = 0.375$ M1

Since $P(A' \cap B) \neq P(A')P(B) \Rightarrow$ not independent A1

One of models is less reliable A1 3

[9]

27. (a)



Tree with correct number of branches	M1	
0.2, 0.3, 0.5	A1	
All correct	A1	3

(b) P(used sauna) = $(0.2 \times 0.35) + (0.3 \times 0.2) + (0.5 \times 0.45)$ M1 A1

$= 0.355$ A1 3

(c) P(swim | sauna used) = $\dfrac{\text{P(swim \& sauna)}}{\text{P(sauna)}}$

$= \dfrac{0.2 \times 0.35}{0.355}$ M1 A1

$= 0.19718$ (accept awrt 0.197) A1 3

(d) P(swim | sauna not used) = $\dfrac{\text{P(sauna not used} \mid \text{swim) P(swim)}}{\text{P(sauna not used)}}$ M1

P(sauna not used | swim) = $1 - 0.35 = 0.65$ B1

P(sauna not used) $= 1 - 0.355 = 0.645$ M1 A1 ft

$\therefore$ P(swim | sauna not used) $= \dfrac{0.65 \times 0.2}{0.645}$ M1

$= 0.20155$

(accept awrt 0.202) A16

[15]