1.) (a) attempt to find d (M1)

e.g. $\frac{u_3 - u_1}{2}$, $8 = 2 + 2d$

$d = 3$ A1 N2 2

(b) correct substitution (A1)

e.g. $u_{20} = 2 + (20 - 1)3$, $u_{20} = 3 \times 20 - 1$

$u_{20} = 59$ A1 N22

(c) correct substitution (A1)

e.g. $S_{20} = \frac{20}{2}(2 + 59)$, $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$

$S_{20} = 610$ A1 N22

[6]

2.) (a) evidence of choosing the formula for 20th term (M1)

e.g. $u_{20} = u_1 + 19d$

correct equation A1

e.g. $64 = 7 + 19d$, $d = \frac{64 - 7}{19}$

$d = 3$ A1 N23

(b) correct substitution into formula for u_n A1

e.g. $3709 = 7 + 3(n - 1)$, $3709 = 3n + 4$

$n = 1235$ A1 N12

[5]

3.) (a) common difference is 6 A1 N1

(b) evidence of appropriate approach (M1)

e.g. $u_n = 1353$

correct working A1

e.g. $1353 = 3 + (n - 1)6$, $\frac{1353 + 3}{6}$

$n = 226$ A1N2

(c) evidence of correct substitution A1

e.g. $S_{226} = \frac{226(3 + 1353)}{2}$, $\frac{226}{2}(2 \times 3 + 225 \times 6)$

$S_{226} = 153\ 228$ (accept 153 000) A1N1

[6]

4.) (a) evidence of equation for u_{27} M1

e.g. $263 = u_1 + 26 \times 11$, $u_{27} = u_1 + (n - 1) \times 11$, $263 - (11 \times 26)$

$u_1 = -23$ A1 N1

(b) (i) correct equation A1

e.g. $516 = -23 + (n-1) \times 11$, $539 = (n-1) \times 11$

$n = 50$ A1 N1

(ii) correct substitution into sum formula A1

e.g. $S_{50} = \dfrac{50(-23+516)}{2}$, $S_{50} = \dfrac{50(2\times(-23)+49\times 11)}{2}$

$S_{50} = 12325$ (accept 12300) A1N1

[6]

5.) (a) $r = \dfrac{16}{32}\left(=\dfrac{1}{2}\right)$ A1 N1

(b) correct calculation or listing terms (A1)

e.g. $32 \times \left(\dfrac{1}{2}\right)^{6-1}, 8\times\left(\dfrac{1}{2}\right)^{3}$, 32, ... 4, 2, 1

$u_6 = 1$ A1N2

(c) evidence of correct substitution in S_∞ A1

e.g. $\dfrac{32}{1-\frac{1}{2}}, \dfrac{32}{\frac{1}{2}}$

$S_\infty = 64$ A1N1

[5]

6.) (a) $d = 2$ A1 N1

(b) (i) $5 + 2n = 115$ (A1)

$n = 55$ A1N2

(ii) $u_1 = 7$ (may be seen in above) (A1)

correct substitution into formula for sum of arithmetic series (A1)

e.g. $S_{55} = \dfrac{55}{2}(7+115), S_{55} = \dfrac{55}{2}(2(7)+54(2)), \sum_{k=1}^{55}(5+2k)$

$S_{55} = 3355$ (accept 3360) A1N3

[6]

7.) (a) attempt to substitute into sum formula for AP (accept term formula) (M1)

e.g. $S_{20} = \dfrac{20}{2}\{2(-7)+19d\}, \left(\text{or } \dfrac{20}{2}(-7+u_{20})\right)$

setting up correct equation using sum formula A1

e.g. $\dfrac{20}{2}\{2(-7) + 19d\} = 620$ A1 N2

(b) correct substitution $u_{78} = -7 + 77(4)$ (A1)

$= 301$ A1N2

[5]

8.) (a) evidence of substituting into formula for nth term of GP (M1)

e.g. $u_4 = \frac{1}{81}r^3$

setting up correct equation $\frac{1}{81}r^3 = \frac{1}{3}$ A1

$r = 3$ A1 N2

(b) **METHOD 1**

setting up an inequality (accept an equation) M1

e.g. $\frac{\frac{1}{81}(3^n - 1)}{2} > 40; \frac{\frac{1}{81}(1 - 3^n)}{-2} > 40; 3^n > 6481$

evidence of solving M1

e.g. graph, taking logs

$n > 7.9888...$ (A1)

$n = 8$ A1N2

METHOD 2

if $n = 7$, sum = 13.49...; if $n = 8$, sum = 40.49... A2

$n = 8$ (is the smallest value) A2N2

[7]

9.) (a) $\sum_{r=4}^{7} 2^r = 2^4 + 2^5 + 2^6 + 2^7$ (accept 16 + 32 + 64 + 128) A1 N1

(b) (i) **METHOD 1**

recognizing a GP (M1)

$u_1 = 2^4$, $r = 2$, $n = 27$ (A1)

correct substitution into formula for sum (A1)

e.g. $S_{27} = \frac{2^4(2^{27} - 1)}{2 - 1}$

$S_{27} = 2147483632$ A1N4

METHOD 2

recognizing $\sum_{r=4}^{30} = \sum_{r=1}^{30} - \sum_{r=1}^{3}$ (M1)

recognizing GP with $u_1 = 2$, $r = 2$, $n = 30$ (A1)

correct substitution into formula for sum

$S_{30} = \frac{2(2^{30} - 1)}{2 - 1}$ (A1)

$= 2147483646$

$\sum_{r=4}^{30} 2^r = 2147483646 - (2 + 4 + 8)$

$= 2147483632$ A1N4

(ii) valid reason (*e.g.* **infinite** GP, diverging series), **and** $r \geq 1$ (accept $r > 1$) R1R1 N2

[7]

10.) **METHOD 1**

substituting into formula for S_{40} (M1)

correct substitution A1

e.g. $1900 = \frac{40(u_1 + 106)}{2}$

$u_1 = -11$ A1 N2

substituting into formula for u_{40} or S_{40} (M1)

correct substitution A1

e.g. $106 = -11 + 39d$, $1900 = 20(-22 + 39d)$

$d = 3$ A1 N2

METHOD 2

substituting into formula for S_{40} (M1)

correct substitution A1

e.g. $20(2u_1 + 39d) = 1900$

substituting into formula for u_{40} (M1)

correct substitution A1

e.g. $106 = u_1 + 39d$

$u_1 = -11$, $d = 3$ A1A1 N2N2

[6]

11.) (a) $d = 3$ (A1)

evidence of substitution into $u_n = a + (n - 1) d$ (M1)

e.g. $u_{101} = 2 + 100 \times 3$

$u_{101} = 302$ A1 N3

(b) correct approach (M1)

e.g. $152 = 2 + (n - 1) \times 3$

correct simplification (A1)

e.g. $150 = (n - 1) \times 3$, $50 = n - 1$, $152 = -1 + 3n$

$n = 51$ A1 N2

[6]

12.) (a) evidence of dividing two terms (M1)

e.g. $-\frac{1800}{3000}, -\frac{1800}{1080}$

$r = -0.6$ A1 N2

(b) evidence of substituting into the formula for the 10th term (M1)

e.g. $u_{10} = 3000(-0.6)^9$

$u_{10} = -30.2$ (accept the exact value -30.233088) A1 N2

(c) evidence of substituting into the formula for the infinite sum (M1)

$e.g.\ S = \dfrac{3000}{1.6}$

$S = 1875$ A1 N2

[6]

13.) (a) $u_{10} = 3(0.9)^9$ A1 N1

(b) recognizing $r = 0.9$ (A1)
correct substitution A1

$e.g.\ S = \dfrac{3}{1 - 0.9}$

$S = \dfrac{3}{0.1}$ (A1)

$S = 30$ A1 N3

[5]

14.) (a) (i) attempt to set up equations (M1)
$-37 = u_1 + 20d$ and $-3 = u_1 + 3d$ A1
$-34 = 17d$
$d = -2$ A1 N2

(ii) $-3 = u_1 - 6 \Rightarrow u_1 = 3$ A1N1

(b) $u_{10} = 3 + 9 \times -2 = -15$ (A1)

$S_{10} = \dfrac{10}{2}(3 + (-15))$ M1

$= -60$ A1N2

[7]

15.) (a) $u_1 = 1$, $u_2 = -1$, $u_3 = -3$ A1A1A1 N3

(b) Evidence of using appropriate formula M1

correct values $S_{20} = \dfrac{20}{2}(2 \times 1 + 19 \times -2)$ $(= 10(2 - 38))$ A1

$S_{20} = -360$ A1N1

[6]

16.) (a) Recognizing an AP (M1)
$u_1 = 15$ $d = 2$ $n = 20$ (A1)
substituting into $u_{20} = 15 + (20 - 1) \times 2$ M1
$= 53$ (that is, 53 seats in the 20th row) A1 N2

(b) Substituting into $S_{20} = \dfrac{20}{2}(2(15) + (20 - 1)2)$ (or into $\dfrac{20}{2}(15 + 53)$) M1

= 680 (that is, 680 seats in total) A1N2

[6]

17.) (a) $5000(1.063)^n$ A1 N1

(b) Value = \$ $5000(1.063)^5$ (= \$ 6786.3511...)
= \$ 6790 to 3 s.f. (accept \$ 6786, or \$ 6786.35) A1N1

(c) (i) $5000(1.063)^n > 10\,000$ or $(1.063)^n > 2$ A1 N1

(ii) Attempting to solve the inequality $n\log(1.063) > \log 2$ (M1)
$n > 11.345$ (A1)
12 years A1N3

Note: *Candidates are likely to use TABLE or LIST on a GDC to find n.*
A good way of communicating this is suggested below.

Let $y = 1.063^x$ (M1)
When $x = 11$, $y = 1.9582$, when $x = 12$, $y = 2.0816$ (A1)
$x = 12$ *i.e.* 12 years A1N3

[6]

18.) (a) $\frac{1}{5}$ (0.2)A1 N1

(b) (i) $u_{10} = 25\left(\frac{1}{5}\right)^9$ (M1)

$= 0.0000128 \left(\left(\frac{1}{5}\right)^7, 1.28\times10^{-5}, \frac{1}{78125}\right)$ A1 N2

(ii) $u_n = 25\left(\frac{1}{5}\right)^{n-1}$ A1 N1

(c) For attempting to use infinite sum formula for a GP $\left(\frac{25}{1-\left(\frac{1}{5}\right)}\right)$ (M1)

$S = \frac{125}{4} = 31.25 \ (= 31.3 \text{ to } 3\,s\,f)$ A1 N2

[6]

19.) (a) For taking three ratios of consecutive terms (M1)

$\frac{54}{18} = \frac{162}{54} = \frac{486}{162} \ (=3)$ A1

hence geometric AG N0

(b) (i) $r = 3$ (A1)

$u_n = 18 \times 3^{n-1}$ A1 N2

(ii) For a valid attempt to solve $18 \times 3^{n-1} = 1062882$ (M1)

eg trial and error, logs

$n = 11$ A1 N2

[6]

20.) (a) 3, 6, 9 A1 N1

(b) (i) Evidence of using the sum of an AP M1

eg $\frac{20}{2}\ 2\times 3+(20-1)\times 3$

$\sum_{n=1}^{20} 3n = 630$ A1 N1

(ii) **METHOD 1**

Correct calculation for $\sum_{n=1}^{100} 3n$ (A1)

eg $\frac{100}{2}(2\times 3+99\times 3), 15150$

Evidence of subtraction (M1)

eg $15150 - 630$

$\sum_{n=21}^{100} 3n = 14520$ A1 N2

METHOD 2

Recognising that first term is 63, the number of terms is 80 (A1)(A1)

eg $\frac{80}{2}(63+300),\ \frac{80}{2}(126+79\times 3)$

$\sum_{n=21}^{100} 3n = 14520$ A1 N2

[6]

21.) (a) For taking an appropriate ratio of consecutive terms (M1)

$r = \frac{2}{3}$ A1 N2

(b) For attempting to use the formula for the n^{th} term of a GP (M1)

$u_{15} = 1.39$ A1 N2

(c) For attempting to use infinite sum formula for a GP (M1)

$S = 1215$ A1 N2

[6]

22.) (a) (i) $r = -2$ A1 N1

(ii) $u_{15} = -3\,(-2)^{14}$ (A1)

$= -49152$ (accept -49200) A1 N2

(b) (i) 2, 6, 18 A1 N1

(ii) $r = 3$ A1 N1

(c) Setting up equation (or a sketch) M1

$\frac{x+1}{x-3} = \frac{2x+8}{x+1}$ (or correct sketch with relevant information) A1

$x^2 + 2x + 1 = 2x^2 + 2x - 24$ (A1)

$x^2 = 25$

$x = 5$ or $x = -5$

$x = -5$ A1 N2

***Notes**: If "trial and error" is used, work must be documented with several trials shown.*
Award full marks for a correct answer with this approach.
*If the work is **not** documented, award N2 for a correct answer.*

(d) (i) $r = \frac{1}{2}$ A1 N1

(ii) For attempting to use infinite sum formula for a GP (M1)

$S = \frac{-8}{1-\frac{1}{2}}$

$S = -16$ A1 N2

***Note:** Award M0A0 if candidates use a value of r where r > 1, or r < - 1.*

[12]

23.) (a) (i) $S_4 = 20$ A1 N1

(ii) $u_1 = 2$, $d = 2$ (A1)

Attempting to use formula for S_n M1

$S_{100} = 10100$ A1 N2

(b) (i) $\boldsymbol{M}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$ A2 N2

(ii) For writing $\boldsymbol{M}^3$ as $\boldsymbol{M}^2 \times \boldsymbol{M}$ or $\boldsymbol{M} \times \boldsymbol{M}^2$ $\left(\text{or} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}\right)$ M1

$\boldsymbol{M}^3 = \begin{pmatrix} 1+0 & 4+2 \\ 0+0 & 0+1 \end{pmatrix}$ A2

$\boldsymbol{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$ AG N0

(c) (i) $M^4 = \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix}$ A1 N1

(ii) $T^4 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix}$ (M1)

$= \begin{pmatrix} 4 & 20 \\ 0 & 4 \end{pmatrix}$ A1A1 N3

(d) $T_{100} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} + \ldots + \begin{pmatrix} 1 & 200 \\ 0 & 1 \end{pmatrix}$ (M1)

$= \begin{pmatrix} 100 & 10100 \\ 0 & 100 \end{pmatrix}$ A1A1 N3

[16]

24.) ***Note:*** *Throughout this question, the first and last terms are interchangeable.*

(a) For recognizing the arithmetic sequence (M1)

$u_1 = 1$, $n = 20$, $u_{20} = 20$ ($u_1 = 1$, $n = 20$, $d = 1$) (A1)

Evidence of using sum of an AP M1

$S_{20} = \frac{(1+20)20}{2}$ (or $S = \frac{20}{2}(2\times 1 + 19\times 1)$) A1

$S_{20} = 210$ AG N0

(b) Let there be n cans in bottom row

Evidence of using $S_n = 3240$ (M1)

eg $\frac{(1+n)n}{2} = 3240, \frac{n}{2}(2+(n-1)) = 3240, \frac{n}{2}(2n+(n-1)(-1)) = 3240$

$n^2 + n - 6480 = 0$ A1

$n = 80$ or $n = -81$ (A1)

$n = 80$ A1 N2

(c) (i) Evidence of using $S = \frac{(1+n)n}{2}$ (M1)

$2S = n^2 + n$ A1

$n^2 + n - 2S = 0$ AG N0

(ii) **METHOD 1**

Substituting $S = 2100$

eg $n^2 + n - 4200 = 0$, $2100 = \frac{(1+n)n}{2}$ A1

EITHER

$n = 64.3$, $n = -65.3$ A1

Any valid reason which includes reference to integer being needed, R1 and pointing out that integer not possible here. R1 N1

eg n must be a (positive) integer, this equation does not have integer solutions.

OR

Discriminant = 16 801 A1

Valid reason which includes reference to integer being needed, R1

and pointing out that integer not possible here. R1 N1

eg this discriminant is not a perfect square, therefore no integer solution as needed.

METHOD 2

Trial and error

$S_{64} = 2080$, $S_{65} = 2145$ A1A1

Any valid reason which includes reference to integer being needed, R1

and pointing out that integer not possible here. R1 N1

[14]

25.) (a) Recognizing an AP (M1)
$u_1 = 15$ $d = 2$ $n = 20$ (A1) 4
substituting into $u_{20} = 15 + (20 - 1) \times 2$ M1
= 53 (that is, 53 seats in the 20th row) A1

(b) Substituting into $S_{20} = \frac{20}{2}(2(15) + (20-1)2)$ (or into $\frac{20}{2}(15 + 53)$) M1
= 680 (that is, 680 seats in total) A1 2

[6]

26.) (a) $5000(1.063)^n$ A1 1

(b) Value = \$5000$(1.063)^5$ (= \$6786.3511...)
= \$6790 to 3 sf (Accept \$6786, or \$6786.35) A1 1

(c) (i) $5000(1.063)^n > 10000$ or $(1.063)^n > 2$ A1 1

(ii) Attempting to solve the inequality «log (1.063) > log 2 (M1)
$n > 11.345...$ (A1)
12 years A1 3

***Note**: Candidates are likely to use TABLE or LIST on a GDC to find n. A good way of communicating this is suggested below.*

Let $y = 1.063^x$ (M1)
When $x = 11$, $y = 1.9582$, when $x = 12$, $y = 2.0816$ (A1)
$x = 12$ *ie* 12 years A1 3

[6]

27.) (a) $u_1 = S_1 = 7$ (A1) (C1)

(b) $u_2 = S_2 - u_1 = 18 - 7$

$= 11$ (A1)

$d = 11 - 7$ (M1)

$= 4$ (A1) (C3)

(c) $u_4 = u_1 + (n-1)d = 7 + 3(4)$ (M1)

$u_4 = 19$ (A1) (C2)

[6]

28.) For using $u_3 = u_1 r^2 = 8$ (M1)

$8 = 18r^2$ (A1)

$r^2 = \frac{8}{18}\left(= \frac{4}{9}\right)$

$r = \pm\frac{2}{3}$ (A1)(A1)

$S_\infty = \frac{u_1}{1-r},$

$S_\infty = 54, \frac{54}{5} (=10.8)$ (A1)(A1)(C3)(C3)

[6]

29.) (a) (i) Neither

(ii) Geometric series

(iii) Arithmetic series

(iv) Neither (C3)

Note: *Award (A1) for geometric correct, (A1) for arithmetic correct and (A1) for* ***both*** *"neither". These may be implied by blanks* ***only*** *if GP* ***and*** *AP correct.*

(b) (Series (ii) is a GP with a sum to infinity)

Common ratio $\frac{3}{4}$ (A1)

$S_\infty = \frac{a}{1-r}\left(= \frac{1}{1-\frac{3}{4}}\right)$ (M1)

$= 4$ (A1) (C3)

Note: *Do* ***not*** *allow* ***ft*** *from an incorrect series.*

[6]

30.) (a) (i) $11400, $11800(A1) 1

(ii) Total salary $=\frac{10}{2}(2\times 11000 + 9\times 400)$ (A1)

= $128000 (A1) (N2) 2

(b) (i) $10700, $11449(A1)(A1)

(ii) 10^{th} year salary $=10\,000(1.07)^9$ (A1)

= $18384.59 or $18400 or $18385 (A1) (N2) 4

(c) **EITHER**

Scheme A $S_A=\frac{n}{2}\left(2\times 11\,000+(n-1)400\right)$ (A1)

Scheme B $S_B=\frac{10\,000(1.07^n-1)}{1.07-1}$ (A1)

Solving $S_B > S_A$ (accept $S_B = S_A$, giving $n=6.33$) (may be implied) (M1)

Minimum value of n is 7 years. (A1) (N2)

OR

Using trial and error (M1)

	Arturo	Bill
6 years	$72 000	$71532.91
7 years	$85 400	$86 540.21

(A1)(A1)

Note: *Award (A1) for* ***both*** *values for 6 years, and (A1) for* ***both*** *values for 7 years.*

Therefore, minimum number of years is 7. (A1) (N2)4

[11]

31.) Arithmetic sequence d = 3 (may be implied) (M1)(A1)

n = 1250 (A2)

$S = \frac{1250}{2}(3 + 3750)$ $\left(\text{or } S=\frac{1250}{2}(6+1249\times 3)\right)$ (M1)

= 2 345 625 (A1) (C6)

[6]

32.)

x	f	Σf
4	2	2
5	5	7
6	4	11
7	3	14
8	4	18

10	2	20
12	1	21

(a) m = 6 (A2) (C2)

(b) $Q_1 = 5$ (A2) (C2)

(c) $Q_3 = 8$ (A1)
IQR = 8 – 5 (M1)
= 3 (accept 5 – 8 or [5, 8]) (C2)

[6]

33.) Arithmetic sequence (M1)
$a = 200$ $d = 30$ (A1)

(a) Distance in final week = 200 + 51 × 30 (M1)
= 1730 m (A1) (C3)

(b) Total distance = $\frac{52}{2}$ [2.200 + 51.30] (M1)
= 50180 m (A1) (C3)

Note: *Penalize once for absence of units ie award A0 the first time units are omitted, A1 the next time.*

[6]

34.) (a) (i) Area B = $\frac{1}{16}$, area C = $\frac{1}{64}$ (A1)(A1)

(ii) $\dfrac{\frac{1}{16}}{\frac{1}{4}} = \dfrac{1}{4}$ $\dfrac{\frac{1}{64}}{\frac{1}{16}} = \dfrac{1}{4}$ (Ratio is the same.) (M1)(R1)

(iii) Common ratio = $\frac{1}{4}$ (A1) 5

(b) (i)Total area (S_2) = $\frac{1}{4} + \frac{1}{16} = \frac{5}{16}$ = (= 0.3125) (0.313, 3 sf) (A1)

(ii) Required area = S_8 = $\dfrac{\frac{1}{4}\left(1-\left(\frac{1}{4}\right)^8\right)}{1-\frac{1}{4}}$ (M1)

= 0.333328 2(471...) (A1)
= 0.333328 (6 sf) (A1) 4

Note: *Accept result of adding together eight areas correctly.*

(c) Sum to infinity = $\dfrac{\frac{1}{4}}{1-\frac{1}{4}}$ (A1)

= $\frac{1}{3}$ (A1) 2

[11]

35.) (a) $u_4 = u_1 + 3d$ or $16 = -2 + 3d$ (M1)

$d = \frac{16-(-2)}{3}$ (M1)

$= 6$ (A1) (C3)

(b) $u_n = u_1 + (n-1)6$ or $11998 = -2 + (n-1)6$ (M1)

$n = \frac{11998+2}{6} + 1$ (A1)

$= 2001$ (A1) (C3)

[6]

36.) (a) Ashley

AP $12 + 14 + 16 + \ldots$ to 15 terms (M1)

$S_{15} = \frac{15}{2}[2(12) + 14(2)]$(M1)

$= 15 \times 26$

$= 390$ hours (A1) 3

(b) Billie

GP $12, 12(1.1), 12(1.1)^2\ldots$ (M1)

(i) In week 3, $12(1.1)^2$ (A1)

$= 14.52$ hours (AG)

(ii) $S_{15} = \frac{12[(1.1)^{15} - 1]}{1.1 - 1}$ (M1)

$= 381$ hours (3 sf) (A1) 4

(c) $12(1.1)^{n-1} > 50$ (M1)

$(1.1)^{n-1} > \frac{50}{12}$ (A1)

$(n-1)\ln 1.1 > \ln\frac{50}{12}$

$n - 1 > \frac{\ln\frac{50}{12}}{\ln 1.1}$ (A1)

$n - 1 > 14.97$

$n > 15.97$

$\Rightarrow$ Week 16 (A1)

OR

$12(1.1)^{n-1} > 50$ (M1)

By trial and error

$12(1.1)^{14} = 45.6$, $12(1.1)^{15} = 50.1$ (A1)

$\Rightarrow n - 1 = 15$ (A1)

$\Rightarrow n = 16$ (Week 16) (A1) 4

[11]

37.) (a) (i) $PQ = \sqrt{AP^2 + AQ^2}$ (M1)

$= \sqrt{2^2 + 2^2} = \sqrt{4(2)} = 2\sqrt{2}$ cm (A1)(AG)

(ii) Area of PQRS $= (2\sqrt{2})(2\sqrt{2}) = 8\ \text{cm}^2$ (A1) 3

(b) (i) Side of third square = $\sqrt{\left(\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2}$ = √4 = 2 cm

Area of third square = 4 cm^2 (A1)

(ii) $\frac{1^{st}}{2^{nd}}=\frac{16}{8}$ $\frac{2^{nd}}{3^{rd}}=\frac{8}{4}$ (M1)

$\Rightarrow$ Geometric progression, $r=\frac{8}{16}=\frac{4}{8}=\frac{1}{2}$ (A1) 3

(c) (i) $u_{11}=u_1r^{10}=16\left(\frac{1}{2}\right)^{10}=\frac{16}{1024}$ (M1)

$=\frac{1}{64}$ (= 0.015625 = 0.0156, 3 sf) (A1)

(ii) $S_\infty=\frac{u_1}{1-r}=\frac{16}{1-\frac{1}{2}}$ (M1)

= 32 (A1) 4

[10]

38.) (a) u_1 = 7, d = 2.5 (M1)
$u_{41}=u_1+(n-1)d=7+(41-1)2.5$
= 107 (A1) (C2)

(b) $S_{101}=\frac{n}{2}[2u_1+(n-1)d]$

$=\frac{101}{2}[2(7)+(101-1)2.5]$ (M1)

$=\frac{101(264)}{2}$

= 13332 (A1) (C2)

[4]

39.) (a) $r=\frac{360}{240}=\frac{240}{160}=\frac{3}{2}=1.5$ (A1) 1

(b) 2002 is the 13th year. (M1)
$u_{13}=160(1.5)^{13-1}$ (M1)
= 20759 (Accept 20760 or 20800.) (A1) 3

(c) $5000=160(1.5)^{n-1}$

$\frac{5000}{160}=(1.5)^{n-1}$ (M1)

$\log\left(\frac{5000}{160}\right)=(n-1)\log1.5$ (M1)

$n-1=\frac{\log\left(\frac{5000}{160}\right)}{\log1.5}=8.49$ (A1)

$\Rightarrow n=9.49\Rightarrow 10^{th}$ year
$\Rightarrow$ 1999 (A1)

OR

Using a gdc with $u_1 = 160$, $u_{k+1} = \frac{3}{2}u_k$, $u_9 = 4100$, $u_{10} = 6150$ (M2)

1999 (G2) 4

(d) $S_{13} = 160\left[\frac{1.5^{13}-1}{1.5-1}\right]$ (M1)

$= 61958$ (Accept 61960 or 62000.) (A1) 2

(e) Nearly everyone would have bought a portable telephone so there would be fewer people left wanting to buy one. (R1)

OR

Sales would saturate. (R1) 1

[11]

40.) (a) $a_1 = 1000$, $a_n = 1000 + (n - 1)250 = 10000$ (M1)

$n = \frac{10\,000 - 1000}{250} + 1 = 37.$

She runs 10 km on the 37th day. (A1)

(b) $S_{37} = \frac{37}{2}(1000 + 10000)$ (M1)

She has run a total of 203.5 km (A1)

[4]

41.) $a = 5$

$a + 3d = 40$ (may be implied) (M1)

$d = \frac{35}{3}$ (A1)

$T_2 = 5 + \frac{35}{3}$ (A1)

$= 16\frac{2}{3}$ or $\frac{50}{3}$ or 16.7 (3 sf) (A1) (C4)

[4]

42.) $S = \frac{u_1}{1-r} = \frac{\frac{2}{3}}{1-\left(-\frac{2}{3}\right)}$ (M1)(A1)

$= \frac{2}{3} \times \frac{3}{5}$ (A1)

$= \frac{2}{5}$ (A1) (C4)

[4]

43.) (a) Plan A: 1000, 1080, 1160... Plan B: 1000, 1000(1.06), $1000(1.06)^2$…

2nd month: \$1060, 3rd month: \$1123.60 (A1)(A1) 2

(b) For Plan A, $T_{12} = a + 11d$

$= 1000 + 11(80)$ (M1)

$= \$1880$ (A1)

For Plan B, $T_{12} = 1000(1.06)^{11}$ (M1)

$= \$1898$ (to the nearest dollar) (A1) 4

(c) (i) For Plan A, $S_{12} = \frac{12}{2}[2000 + 11(80)]$ (M1)

$= 6(2880)$

$= \$17280$ (to the nearest dollar) (A1)

(ii) For Plan B, $S_{12} = \frac{1000(1.06^{12} - 1)}{1.06 - 1}$ (M1)

$= \$16870$ (to the nearest dollar) (A1) 4

[10]

44.) (a) $\$1000 \times 1.075^{10} = \$2061$ (nearest dollar) (A1) (C1)

(b) $1000(1.075^{10} + 1.075^{9} + ... + 1.075)$ (M1)

$= \frac{1000(1.075)(1.075^{10} - 1)}{1.075 - 1}$ (M1)

$= \$15208$ (nearest dollar) (A1) (C3)

[4]

45.) $17 + 27 + 37 + ... + 417$

$17 + (n - 1)10 = 417$ (M1)

$10(n - 1) = 400$

$n = 41$ (A1)

$S_{41} = \frac{41}{2}(2(17) + 40(10))$ (M1)

$= 41(17 + 200)$

$= 8897$ (A1)

OR

$S_{41} = \frac{41}{2}(17 + 417)$ (M1)

$= \frac{41}{2}(434)$

$= 8897$ (A1) (C4)

[4]

46.) $S_5 = \frac{5}{2}\{2 + 32\}$(M1)(A1)(A1)

$S_5 = 85$ (A1)

OR

$a = 2$, $a + 4d = 32$ (M1)

$\Rightarrow 4d = 30$

$d = 7.5$ (A1)

$S_5 = \frac{5}{2}(4 + 4(7.5))$ (M1)

$= \dfrac{5}{2}(4 + 30)$

$S_5 = 85$ (A1) (C4)

[4]