3.3 Open Loop Plant Analysis

1. BIBO Stability

The transfer function of the open-loop plant is given by:

$$G(s) = \frac{1}{ms^2 + bs} = \frac{1}{s(ms+b)}$$

where $m = 0.958$ kg and $b = 5.56 \times 10^{-5}$ N·s/m

The poles of $G(s)$ are 0 and $-\frac{b}{m}$ (5.8×10^{-5}). Since there is a pole at 0, the system is not BIBO stable.

2. Simulation Behavior

The step response shown in Figure 1 does not settle to a steady-state value. Instead, the output continuously increases over time, which is consistent with the presence of a pole at the origin in the transfer function (type-1 plant), causing the step input to produce a ramp-like output. The second pole at $s = -\frac{b}{m}$ is very close to the origin, resulting in a slow transient before the output becomes nearly linear. Overall, the simulated behavior aligns with theoretical expectations based on the system's poles.

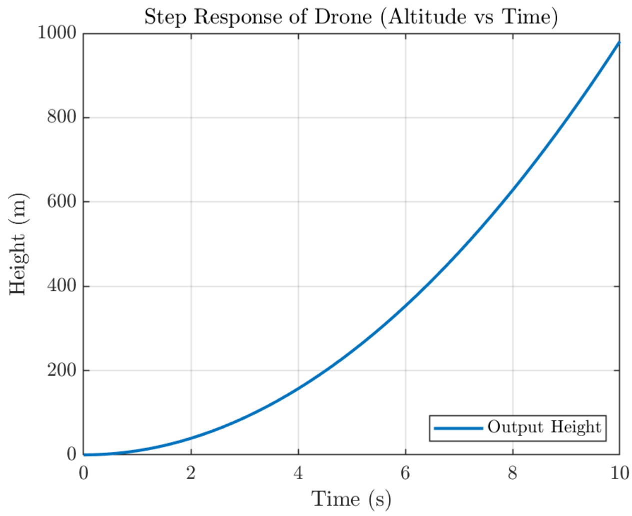

Figure 1: Step response of the open-loop plant $G(s)$.

3. Disturbance and Modeling

Yes, the plant will likely face external disturbances. For a drone's vertical motion, these include air turbulence, rotor thrust changes, or sudden payload shifts. Such disturbances alter the net force on the drone, affecting its acceleration and height. The configuration in Figure 2 models this accurately: disturbances enter the plant input, subtract from the control signal, and pass through G(s), effectively representing how external forces influence the dynamics of the real-world system.

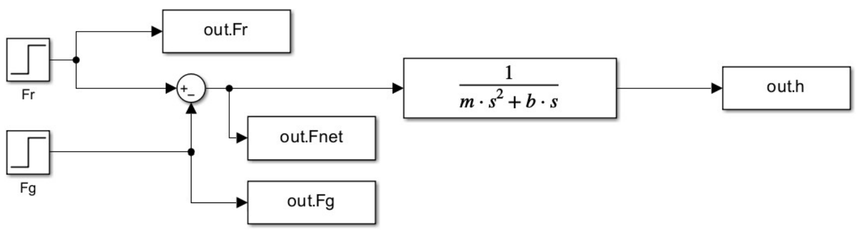

Figure 2: Simulink Block Diagram of Plant Function $G(s)$.

4. Additional Questions

- How sensitive is the system to uncertainty in mass (m) or the drag coefficient (b)?
- What is the steady-state error for standard inputs (step, ramp)?
- Which controller type will be most optimal for the system given the existing poles?

References:

- Kathryn Johnson, *EENG307: Semester Project*, Colorado School of Mines, Fall 2025.

3.2.1: Transfer Function Derivation

We model the vertical motion $h(t)$ with three forces: rotor thrust $f_r(t)$ (upward), linear drag $f_{\text{drag}}(t) = b\dot{h}(t)$ (opposing motion), and gravity $f_g(t) = mg$ (downward). Applying Newton's Second Law gives

$$m\ddot{h}(t) = f_r(t) - b\dot{h}(t) - f_g(t),$$

Taking the Laplace transform (zero initial conditions) yields

$$ms^2H(s) + bsH(s) = F_r(s) - F_g(s),$$

$$H(s) = \frac{F_r(s) - F_g(s)}{ms^2 + bs} = \frac{F_r(s)}{ms^2 + bs} - \frac{F_g(s)}{ms^2 + bs}.$$

Hence, the open-loop transfer function from rotor force to altitude is

$$G(s) = \frac{H(s)}{F_r(s)} = \frac{1}{ms^2 + bs}, \qquad G_d(s) = \frac{H(s)}{F_g(s)} = -\frac{1}{ms^2 + bs}.$$

Here m (kg) is the vehicle mass, b (N·s/m) is the linear drag coefficient, $\dot{h}(t)$ (m/s) is vertical velocity, and $\ddot{h}(t)$ (m/s^2) is vertical acceleration. The denominator $ms^2 + bs$ represents inertial (ms^2) and damping (bs) effects of a second-order system (no stiffness), and gravity acts as a constant disturbance instead of the plant dynamics.

3.2.2: Paramater Estimates

The two transfer function parameters are the mass of the drone and the damping constant from air resistance. For consistency, we used the metrics of one of the most common commercial UAVs, the DJI Mavic 3 [2]. From [1], we are able to derive the following function for the dampening coefficient:

$$b = \beta D = 1.6 \times 10^{-4}\ \text{N·s/m}^2 \cdot 347.5 \cdot 10^{-3} m = 5.56 \times 10^{-5}\ \text{N·s/m}$$

Therefore, $m = 0.958$ kg and $b = 5.56 \times 10^{-5}$ N·s/m

3.2.3: Simulink Analysis

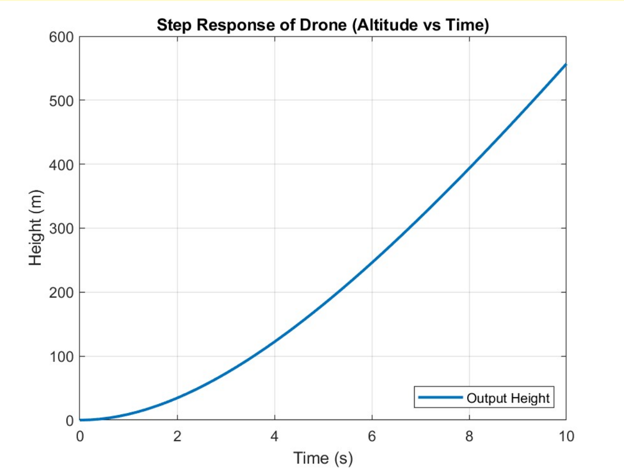

Figure 1: The rotor force F_r was modeled as a step input with a magnitude of $3mg$, while $F_g = mg$ acted as a constant disturbance. The resulting step response shows a continuously increasing height, matching the reference response from Section 8. Minor differences in slope are due to the specific parameter values of F_r, m, and b, which influence the rate of ascent.

[1] Classical Mechanics. Annai Academy Online School, Mar. 2025. [Online]. Available: https://annaiacademyonlineschool.com/wp-content/uploads/2025/03/classical-mechanics.pdf [Accessed: Oct. 8, 2025].
[2] "Mavic 3 Pro — Specifications," DJI, 2025. [Online]. Available: https://www.dji.com/mavic-3-pro/specs [Accessed: Oct. 8, 2025].

Reflection

Arael:
Our mistake was not taking advantage of the formatting to fit more into our report. Originally We had to cut down on our derivation and shrink our images to meet the space requirements. However we were able to reformat the way the text was places to include a longer more detailed derivation, and increase the plot size.

Kyle:
Our mistake was not taking into account the visibility of our step response plot. As well as missing some steps of our derivation process on the transfer function. To address this problem we were able to simplify some of our text as well as get rid of some of the white space in our doc.

Daniel:
After reviewing our document, we realized that the graph for the Simulink Analysis was hard to read and difficult to read due to the small font sizes. We adjusted our graph accordingly to the graphs. Additionally, we have found some errors in our transfer function derivations and fixed our derivation process by writing all the steps in an efficient manner.

Owen:
In retrospect, we definitely could have been more creative with our formatting. We initially wrote 3-4 pages that I think we didn't trim down very tactfully when we noticed the page limit. We pretty much entirely cut the derivation for our transfer function to fit the page limit. Going forward, we will begin better formatting our figures to make them more legible as well as to leave space to better explain our math.