Project Part 3.6: Robustness Analysis

We are analyzing the robustness of a closed-loop system consisting of the plant drone system and controller in the standard negative unity feedback configuration.

Our plant is modeled as the transfer function

$$G(s) = \frac{1}{ms^2 + bs},$$

where $R(s)$ is the desired height, $Y(s)$ is the output height, $E(s) = R(s) - Y(s)$ is the error, $C(s)$ generates the control force $F_r(s)$, $D(s) = -\frac{mg}{s}$ represents gravity, and $G(s) = \frac{1}{ms^2+bs}$ maps the net force to the mass position.

Our controller C(s) is

$$C(s) = K_p + \frac{K_i}{s} + K_d s$$

where K_p is the proportional gain, K_i is the integral gain, and K_d is the derivative gain. The final values for all 3 of these constants were 6, 2, and 4 respectively. This controller was developed by using MATLAB's Control System Designer and examining the open-loop transfer function, rise-time, and percent-overshoot targets on the root-locus plot, then adjusting pole and zero locations until the closed-loop step response met the required performance.

The Bode plot of G(s)C(s) is shown in Figure 1. From the Bode plot, it can be seen that the gain and phase margins are -21.96 dB and 70.34° respectively. Based on these stability margins, we calculate that the allowable time delay that our system can tolerate is 0.28 seconds. This value was calculated at a frequency of 4.31 rad/s. We have tested our calculated time delay using the closed-loop configuration shown in Figure 2.

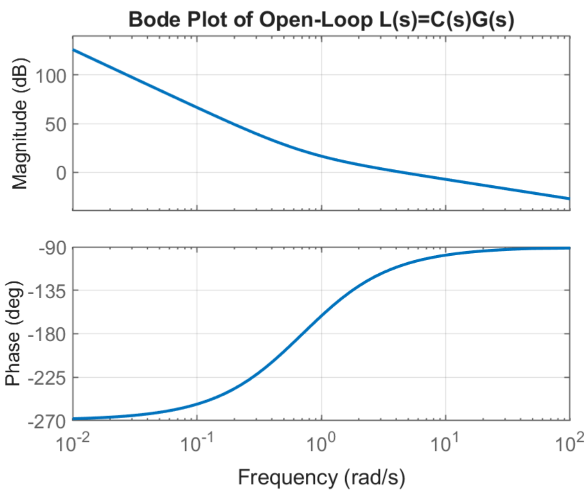

Figure 1: Bode plot of $L(s) = C(s)G(s)$

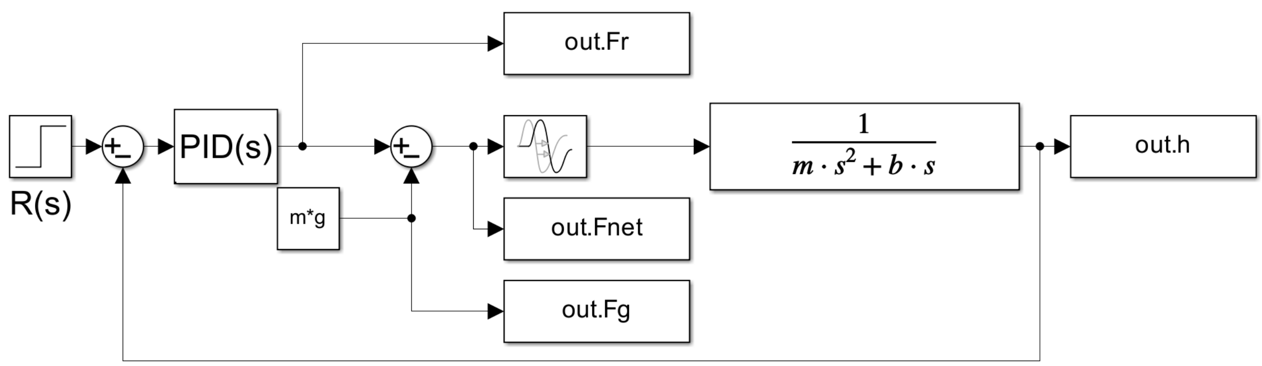

Figure 2: Closed-loop Simulink block diagram including inserted time delay after the PID and disturbance summing block.

Figures 3 and 4 show time series results using time delay values of 0.25 seconds and 0.33 seconds respectively.

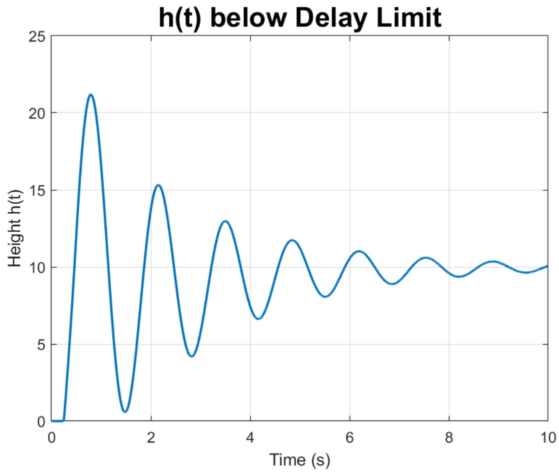

Figure 3: Closed-loop response with time delay $T = 0.25$ s (below stability margin)

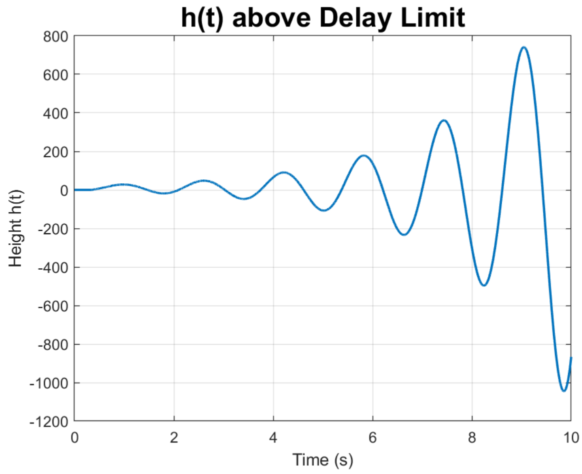

Figure 4: Closed-loop response with time delay $T = 0.33$ s (above stability margin)

The results confirm the theoretical prediction. When the delay is just below the calculated margin, the system remains stable with only a mild increase in settling time and overshoot. However, when the delay exceeds the margin, the response becomes oscillatory and diverges, demonstrating loss of closed-loop stability. This behavior verifies that the computed maximum delay of .28 seconds accurately represents the stability boundary for the given controller and plant.

References

[1] MathWorks, "Fixed-step size (fundamental sample time)," *Simulink Documentation*. Available: https://www.mathworks.com/help/simulink/gui/fixedstepsizefundamentalsampletime.html

[2] MathWorks, "margin — Gain and phase margins and crossover frequencies," *Control System Toolbox Documentation*. Available: https://www.mathworks.com/help/control/ref/dynamicsystem.margin.html

[3] MathWorks, "Variable Transport Delay," *Simulink Documentation*. Available: https://www.mathworks.com/help/simulink/slref/variabletransportdelay.html

[4] K. Johnson, *"EENG307: Semester Project"*, Colorado School of Mines, 2025.