Report on the Experiments with Random Number Generation

Uniformity Test

The uniformity test which is designed to check whether the *Ui*'s appear to be uniformly distributed between 0 and 1, and it is a special case of the chi-square test with all parameters known.

Here we see that only one value got rejected where N = 500 and k = 20. Though in higher values of N we didn't get rejection so it can be said that this value got rejected because of some anomaly in the generator.

K	Value of N	X^2	$\boldsymbol{X^2_{k-1,1-\alpha}}$	Verdict
10	20	14.683	6.5	ACCEPTED
	500	14.683	58.02	REJECTED
	4000	14.683	408.3725	REJECTED
	10,000	14.683	1005.685	REJECTED
20	20	27.203	12.0	ACCEPTED
	500	27.203	37.44	REJECTED
	4000	27.203	224.875	REJECTED
	10,000	27.203	514.274	REJECTED

Runs Test

The *runs* (or *runs-up*) *test*, is a more direct test of the independence assumption. We examine the *Ui* sequence (or, equivalently, the *Zi* sequence) for unbroken subsequences of maximal length within which the *Ui*'s increase monotonically; such a subsequence is called a *run up*. Runs tests look solely for independence and not specifically for uniformity.

Here we can see that when the number generated is too high then the null hypothesis got rejected. Because when the value of N is high then they do not remain that much independent. And we get high lengths *run up.* Consequently, they get dependent in a way.

Value of N	*R*	$\boldsymbol{X^2_{6,1-\alpha}}$	Verdict
20	-567.387	10.645	ACCEPTED
500	1153.62	10.645	REJECTED
4000	960.172	10.644	REJECTED
10,000	-1590.954	10.644	ACCEPTED

Serial Test

The *serial test* is really just a generalization of the chi-square test to higher dimensions. If the U_i's were really IID U(0, 1) random variates, the nonoverlapping *d*-tuples $\mathbf{U}_1$ (U_1, U_2, . . . , U_d), $\mathbf{U}_2$ (U_{d+1}, U_{d+2}, . . . , U_{2d}), . . .should be IID random *vectors* distributed uniformly on the *d* dimensional unit hypercube, $[0, 1]_d$.

Here we see that we got one rejections where the values of N, d, k were highest. So, it can be said that due to higher dimensions or larger intervals our hypothesis got rejected.

Value of N	Value of d,k	X^2	$X^2_{k\text{^}d-1,1-\alpha}$	Verdict
20	d=2,k=4	9.2	22.307	ACCEPTED
	d=2,k=8	54	77.745	ACCEPTED
	d=3,k=4	58	77.745	ACCEPTED
	d=3,k=8	506	552	ACCEPTED
500	d=2,k=4	9.712	22.307	ACCEPTED
	d=2,k=8	70.0	77.745	ACCEPTED
	d=3,k=4	60.699	77.745	ACCEPTED
	d=3,k=8	463.205	552.374	ACCEPTED
4000	d=2,k=4	13.904	22.307	ACCEPTED
	d=2,k=8	67.776	77.745	ACCEPTED
	d=3,k=4	49.41	77.745	ACCEPTED
	d=3,k=8	520.263	552.374	ACCEPTED
10000	d=2,k=4	6.982	22.307	ACCEPTED
	d=2,k=8	62.093	77.745	ACCEPTED
	d=3,k=4	42.947	77.745	ACCEPTED
	d=3,k=8	560.074	552.374	REJECTED

Correlation Test

The correlation test is a direct way to assess whether the generated U_i's exhibit discernible correlation: Simply compute an estimate of the correlation at lags *j* 5 1, 2, . . ., *l* for some value of *l*. The test should probably be carried out for several values of *j*, since it could be, for instance, that there is no appreciable correlation at lags 1 or 2.

but there is dependence between the U_i's at lag 3, due to some anomaly of the generator.

Here we can see that for different values of *j* we didn't get any rejection. So, there's no correlation between the numbers generated at different lags. As a result, it can be said that the numbers that have been generated resemble values of true IID U(0, 1) random variates.

Value of j	Value of N	$\lvert A_j \rvert$	$Z_{1-a/2}$	Verdict
1	20	0.7956	1.644853	ACCEPTED
	500	0.3762	1.6448	ACCEPTED
	4000	0.6699	0.0627	ACCEPTED
	10,000	0.5500	1.64485	ACCEPTED
2	20	0.2129	1.64485	ACCEPTED
	500	0.6515	1.64485	ACCEPTED
	4000	0.97893	1.64485	ACCEPTED
	10,000	0.83346	1.64485	ACCEPTED
3	20	0.174586	1.64485	ACCEPTED
	500	0.41930	1.64485	ACCEPTED
	4000	0.265994	1.64485	ACCEPTED
	10,000	1.09899	1.64485	ACCEPTED