Physics 203 Lecture: Electric force and potential

1. An earthbound sibling observers her twin to depart from earth at a speed 60% the speed of light. After 10.0 years, the travelling twin returns having travelled 3.0 light-years and back. (a) Calculate the proper time the travelling twin was aboard the spaceship. (b) From the traveler's vantage point, the earth-bound twin's clocks tick slow. How much time does she observe to pass on earth? (c) During the moment of acceleration 3.0 light-years out, the clocks aboard ship go out of synch with those on earth. Calculate how far ahead the desynchronization put the clocks on the ship.

(a) 8.0 yr
(b) 6.4 yr
(c) 3.6 yr

Solution

(a) The gamma factor for the travelling twin is

$$\gamma = \frac{1}{\sqrt{1-(0.60)^2}} = 1.25$$

The proper time of the travelling twin is

$$(10) = (1.25)\Delta\tau \implies \tau = 8.0$$

As the earth-bound twin watches, only 8.0 years pass on the ship.

(b) From the *traveler's* point of view it is the earth that is moving away with a speed of $0.60c$. The gamma factor is also 1.25. Since the travelling twin is measuring events on earth, her clocks are not measuring proper time on earth. Using the time dilation formula, the proper time is

$$(8) = (1.25)\Delta\tau \implies \tau = 6.4$$

In other words, as the travelling twin watches, only 6.4 years pass on earth.

(c) When the traveller returns either 6.4 years or 10.0 years have passed. Which is it? The truth is that the trip has two legs with a brief period of acceleration in the middle. As the question points out, this acceleration must cover the gap by introducing 3.6 years of percieved time for the traveller. This is the desynchronization effect. We can calculate the amount that the clocks on earth appear to shift as:

$$\Delta t = \frac{Lv}{c^2} = \frac{(3.0c)(0.60c)}{c^2} = 1.8$$

Notice the shortcut that $L = 3.0c$. This works because 3.0 light-years is the distance light travels in 3.0 years. Since $x = vt$, the distance is $L = (c)(3.0)$.

This is the amount of desynchronization introduced by going from rest to a speed of $0.60c$. But we are slowing down to rest then accelerating in the other direction up to the same speed so the time shift is double: 3.6 yr.

2. Halley's comet has a highly elongated orbit. The eccentricity of its orbit is 0.967 with a distance of closest approach equal to 0.586 AU. (a) What is the maximum distance Halley's comet reaches before it turns around toward the sun? Express your answer in units of AU. (For reference, Neptune's average distance from the sun is 30 AU.) (b) When distances are measured in astronomical units (AU) and time is measured in years, Kepler's Third Law simplifies to $T^2 = a^3$ for orbits around the sun. Determine the period of Halley's comet in years. (Hint: Halley's original estimate was 76 years).

(a) 34.9 AU
(b) 74.8 yr

Solution

(a) In terms of orbital parameters we are given $e = 0.967$ and $r_0 = 0.586$. The equation for the orbit (in polar coordinates) in general is

$$r = r_0 \frac{1+e}{1+e\cos\theta}$$

Clearly when θ is zero, $r = r_0$ which is the point of closest approach. When $\theta = 180°$, we have the point of farthest distance. Thus,

$$r_1 = r_0 \frac{1+e}{1-e} = (0.586)\frac{1.967}{0.033} = 34.93$$

To three significant digits we have $r_1 = 34.9$ AU.

(b) The semi-major axis is the average between closest and farthest distances. Thus,

$$a = (r_0 + r_1)/2 = 17.76$$

Using the modified form of Kepler's law we get:

$$T^2 = (17.76)^3 \implies T = 74.8$$

3. The pion used to be thought of as the particle that mediates the attractive interaction between nucleons. For example, a proton can convert into a neutron by absorbing a negative pion. Knowing that the quark content of the proton is uud and the neutron is udd, what must the quark content of the negative pion be? Justify your answer.

$\pi^- = \bar{u}d$

Solution

In symbols, the reaction we are interested in is:

$$p^+ + \pi^- \to n$$

The pion is a meson which means that it is a combination of a quark and an anti-quark. The anti-quark will annihilate one of the quarks in the proton. The left over quark will combine with the others to form the neutron. When we write the reaction with quarks, it looks like:

$$uud + \bar{u}d \to udd$$

The pion must destroy a u quark and bring a d.

4. Figure 1 illustrates a possible elastic collision of two identical particles. They both have an initial speed of 100 m/s, but the final speeds are different. This shows that even if we could make the molecular speeds in an ideal gas the same, over time the internal elastic collisions will spread the speed distribution.

(a) $p_x = 0$ kg-m/s

(b) $p_y = 0.0684$ kg-m/s

(c) $KE = 10$ joules

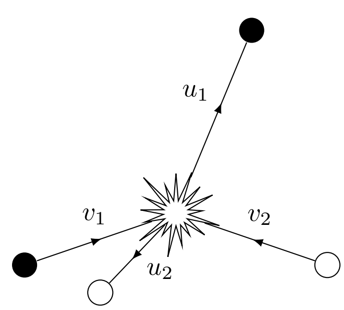

Figure 1: Elastic collision

In this example the initial velocity vectors are

$$v_1 = 100.0 \text{ at } 20° \qquad v_2 = 100.0 \text{ at } 160°$$

and the final velocity vectors are

$$u_1 = 66.6 \text{ at } 225° \qquad u_2 = 124.8 \text{ at } 68°$$

Confirm that this collision is elastic by showing that both momentum and kinetic energy are conserved. For your answer, assume each a mass is one gram.

Solution

We will need to calculate some components. Let's just do them all at once right now using $v_x = v\cos\theta$ and $v_y = v\sin\theta$:

Vector	**Mag.**	**Ang.**	x	y
v_1	100.0	20°	93.97	34.20
v_2	100.0	160°	-93.97	34.20
u_1	66.6	225°	-46.98	-47.18
u_2	124.8	68°	46.98	115.58

It is now pretty straight-forward to calculate the momentum and kinetic energy for each particle:

Vector	p_x	p_y	KE
v_1	0.09397	0.03420	5.0000
v_2	-0.09397	0.03420	5.0000
u_1	-0.04698	-0.04718	2.2166
u_2	0.04698	0.11558	7.7834

Adding up the first pair of rows give the initial quantities:

$$p_x = 0 \qquad \text{and} \qquad p_y = 0.06840$$

and

$$KE = 10.0000$$

Adding up the second pair of rows gives the same numbers which shows that both momentum and kinetic energy are conserved in this collision.

5. An electron is travelling east at a speed of 100 m/s. It encounters a magnetic field pointing north. What magnitude of magnetic field is required to overcome the acceleration due to gravity effectively levitating the electron? 5.57×10^{-13} tesla

Solution

The force experienced due to gravity is simply the weight:

$$W = mg = (9.109 \times 10^{-31})(9.8) = 8.927 \times 10^{-30}$$

The magnetic force needs to counter-balance this force. The magnitude of the magnetic force is given by $F = qvB$. In this case:

$$(8.927 \times 10^{-30}) = (1.602 \times 10^{-19})(100)(B) \implies B = 5.57 \times 10^{-13}$$

This is a miniscule amount of field. This shows how much more powerful the electromagnetic force is than gravity