Solution to Homework Assignment I

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I. Introduction

In large coupled system, decentralized Linear feedback is used to reduce control design and numerical complexities. For the given system the eigen values are (4,-9 and -1). Two of them are stable as they lie of the left half plane (-9 and -1) while one is unstable (4). However the overall system is controllable and thus stabilizable since the rank of the controllability matrix is full. Now the task is to design a decentralized linear state feedback controller by placing all three eigen values equal to (-11) and dividing the system into N=2.

A. Design of Decentralized Linear State Feedback Controller

The state space equation for a continuous tine system is given by

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t),\\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t).\end{aligned} \tag{1}$$

The system is divided into two subsystems such that one subsystem contains 2 stable eigen modes that goes with the first column of input matrix and first row of output matrix. The other system of unstable eigen mode goes with the second column of input matrix B and second row of output matrix C. The Feedback matrix F is designed to be a sparse matrix. We also decompose the model's inputs and outputs into 2 parts

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \sum_{i=1}^{2}\boldsymbol{Biui}(t),\\ \boldsymbol{yi}(t) &= \boldsymbol{Cix}(t).\end{aligned} \tag{2}$$

The decomposition can be done as following;

- The input U = [u1 u2]'),
- Output Y = [y1 y2]',
- input Matrix B = [B1 B2] i.e B1 = [1 1 0]' and B2 = [0 0 1]',
- The Feedback matrix is decompose as F1 = [f1 f2 0] and F2 = [0 0 f3].

Now after adding the feedback matrix and modifying our state space equation, we will have

- Atilda = A*[x1;x2;x3]-B1*F1*[x1;x2;0]-B2*F2*[0;0;x3]
- Atilda = A*[x1;x2;x3]-[1;1;0]*[f1 f2 0]*[x1;x2;0]-[0;0;1]*[0 0 f3]*[0;0;x3]

By solving for the characteristic polynomial of Abar (coefficient Matrix of Atilda) we determine 3 equations and by comparing it with the characteristic polynomial equation of desired eigen values wihich are -11. we can find our approximate feedback Matrix F.

- f1= 225/13, f2 = -4/13, f3 = 10
- F1= [225/13 -4/13 0] and F2 = [0 0 10]

B. Design feedforward Gain Matrix V

Now considering the reference the tracking capability of the system, we will take a look at designing a feed forward gain matrix V and its impact on reference tracking.

In reference tracking (y =r). In this case

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}(t) &= \boldsymbol{Abarx}(t) + \boldsymbol{BVr}(t).\\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t).\\ \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}(t) &= \boldsymbol{Abarx}(t) + \boldsymbol{BVCx}(t).\end{aligned} \tag{3}$$

So at steady state, we will take derivative of state as zero and equate the whole equation to zero. We can calculate the V matrix. The numerical solution is as under. Since the intermediate matrix Fyr is invertible (p=m), we will make use of it to calculate matrix V. In contrast, if rank(Fyr) is less than p and/or m is less than p we cannot find V and if m is greater than p then equation becomes an over determined system of equations, i.e., we have to use the Moore-Penrose inverse.

Numerically it is as follows:

- at steady state

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}(t) = 0 \tag{4}$$

- Abar x(t) + B V r(t) = 0
- x(t) = - inv (Abar) B V r(t)
- since y (t) = C x(t)
- y(t) = - C inv (Abar) B V r(t)
- Since y (t) = r (t)
- r(t) = - C inv (Abar) B V r(t)
- I = - C inv (Abar) B V

Now we first calculate the whole static matrix as Fyr = -C inv (Abar) B and then calculate V Matrix using it. Using matlab, Fyr is calculated as Fyr = [-0.9109 0.0909;0.1012 -0.2727]. Now we take inverse to calculate V matrix and our V matrix is a 2 by 2 matrix. V = [-1.1400 -0.3800; -0.4231 -3.8077]

C. Simulation Task

In order to simulate, we make 2 seperate models with A, B1, C1 and F1 and stable states and with A,B2, F2 and C2 with one instable state. We take a reference as step input with zero initial conditions of the state and check the open loop response. then we add the V Pre filter. Before adding the V pre filter we divide it into V1 and V2 using the following formulas

- V1 = inv(-C1*inv(Abarr)*B1)
- V2 = inv(-C2*inv(Abarr)*B2)

In each simulation, we make use of the system Matrix along with their parts of B, C, F and V to simulate the system response against the step input. The behaviour can be observed with reference to the step input. The model and state evolution can be seen in the figures as follows:

We can see in the close loop that response the reference tracking behaviour is evident. In first case there is an overshoot and then the system became stable while in the second case there is no overshoot. Nevertheless system follows reference with seconds. Without the use of a pre filter or feed forward gain matrix, reference tracking is not achieved. The output do not follow the reference.

D. Figures

Figures are separately uploaded.

- Figure 1 Open Loop System
- Figure 2 Close Loop System
- Figure 3 First Response
- Figure 3 Second Response

E. Attached files

Please see this report in conjunction of these two files.

- 1. Home assignment Matlab file
- 2. SimHA

F. Additional resources on mathematical representations

- List of most common math commands and symbols: https://en.wikibooks.org/wiki/LaTeX/Mathematics
- Lecture notes and Exercise solutions