

## Summary of Lecture 5 – APPLICATIONS OF NEWTON’S LAWS – I

1. An obvious conclusion from  $F = ma$  is that if  $F = 0$  then  $a = 0$  ! How simple, yet how powerful ! This says that for any body that is not accelerating the *sum of all the forces acting upon it* must vanish.
2. Examples of systems in equilibrium: a stone resting on the ground; a pencil balanced on your finger; a ladder placed against the wall, an aircraft flying at a constant speed and constant height.
3. Examples of systems out of equilibrium: a stone thrown upwards that is at its highest point; a plane diving downwards; a car at rest whose driver has just stepped on the car's accelerator.
4. If you know the acceleration of a body, it is easy to find the force that causes it to accelerate. Example: An aircraft of mass  $m$  has position vector,

$$\vec{r} = (at + bt^3)\hat{i} + (ct^2 + dt^4)\hat{j}$$

What force is acting upon it?

SOLUTION:

$$\begin{aligned}\vec{F} &= m \frac{d^2x}{dt^2} \hat{i} + m \frac{d^2y}{dt^2} \hat{j} \\ &= 6bt\hat{i} + m(2c + 12dt^2)\hat{j}\end{aligned}$$

5. The other way around is not so simple: suppose that you know  $F$  and you want to find  $x$ . For this you must solve the equation,

$$\frac{d^2x}{dt^2} = \frac{F}{m}$$

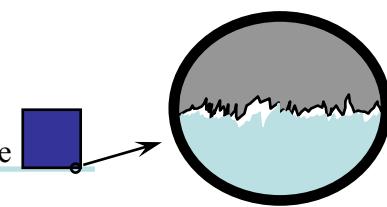
This may or may not be easy, depending upon  $F$  (which may depend upon both  $x$  as well as  $t$  if the force is not constant).

6. Ropes are useful because you can pull from a distance to change the direction of a force. The tension, often denoted by  $T$ , is the force you would feel if you cut the rope and grabbed the ends. For a massless rope (which may be a very good approximation in many situations) the tension is the same at every point along the rope. Why? Because if you take any small slice of the rope it weighs nothing (or very little). So if the force on one side of the slice was any different from the force on the other side, it would be accelerating hugely. All this was for the "ideal rope" which has no mass and never breaks. But this idealization is often good enough.

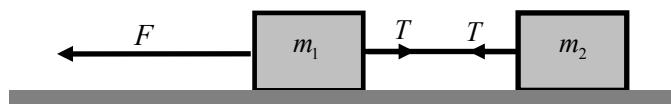
7. We are all familiar with frictional force. When two bodies rub against each other, the frictional force acts upon each body separately opposite to its direction of motion (i.e it acts to slow down the motion). The harder you press two bodies against each other, the greater the friction. Mathematically,  $\vec{F} = \mu \vec{N}$ , where  $\vec{N}$  is the force with which you press the two bodies against each other (normal force). The quantity  $\mu$  is called the coefficient of friction (obviously!). It is large for rough surfaces, and small for smooth ones. Remember that  $\vec{F} = \mu \vec{N}$  is an empirical relation and holds only approximately. This is obviously true: if you put a large enough mass on a table, the table will start to bend and will eventually break.

8. Friction is caused by roughness at a microscopic level

- if you look at any surface with a powerful microscope you will see unevenness and jaggedness. If these big bumps are levelled somehow, friction will still not disappear because there will still be little bumps due to atoms. More precisely, atoms from the two bodies will interact each other because of the electrostatic interaction between their charges. Even if an atom is neutral, it can still exchange electrons and there will be a force because of surrounding atoms.



9. Consider the two blocks below on a frictionless surface:



We want to find the tension and acceleration: The total force on the first mass is  $F - T$  and so  $F - T = m_1 a$ . The force on the second mass is simply  $T$  and so  $T = m_2 a$ . Solving the above, we get:  $T = \frac{m_2 F}{m_1 + m_2}$  and  $a = \frac{F}{m_1 + m_2}$ .

10. There is a general principle by which you solve equilibrium problems. For equilibrium, the sum of forces in every direction must vanish. So  $F_x = F_y = F_z = 0$ . You may always choose the  $x$ ,  $y$ ,  $z$  directions according to your convenience. So, for example, as in the lecture problem dealing with a body sliding down an inclined plane, you can choose the directions to be along and perpendicular to the surface of the plane.

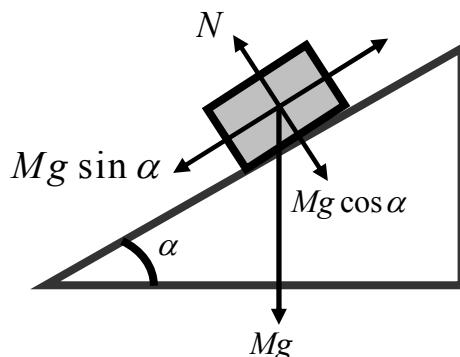
## QUESTIONS AND EXERCISES – 5

Q.1 The relation  $F = \mu N$  is an approximate one only. Discuss various circumstances in which a) it would be correct to use the relation, b) where it would not be correct.

Q.2 Force is a vector quantity but  $F = \mu N$  is written as a relation between the magnitudes of two forces. So what is the direction of the frictional force?

Q.3 In (9), a frictionless surface was considered. Now assume that the coefficient of friction is  $\mu$ . What will be the acceleration and the tension now. Assume that the body is pulled with sufficient force so that it moves.

Q.4 For a body resting on an inclined plane as below:

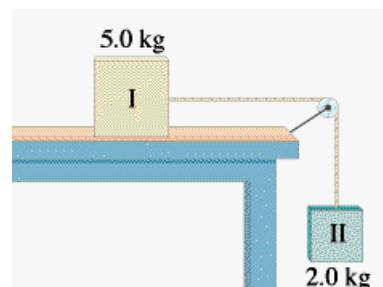


- Find the minimum angle at which the body starts to slide.
- If the angle exceeds the above, find the acceleration.
- Find the time taken to slide a length  $d$  down the slope if the body starts at rest.

Q5. Consider the system of two masses shown. Find

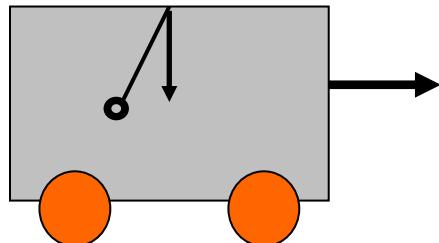
the acceleration of the two masses (are they the same?) for the following two situations:

- The table is frictionless.
- The coefficient of friction  $\mu = 2$ .



## Summary of Lecture 6 – APPLICATIONS OF NEWTON’S LAWS – II

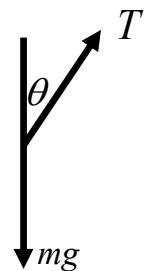
1. As a body moves through a body it displaces the fluid. it has to exert a force on the fluid to push it out of the way. By Newton's third law, the fluid pushes back on the body with an equal and opposite force. The direction of the fluid resistance force on a body is always opposite to the direction of the body's velocity relative to the fluid.
2. The magnitude of the fluid resistance force usually increases with the speed of the body through the fluid. Typically,  $f = kv$  (an empirical law!). Imagine that you drop a ball bearing into a deep container filled with oil. After a while the ball bearing will approach its maximum (terminal) speed when the forces of gravity and friction balance each other:  $mg = kv$  from which  $v_{\text{final}} = mg / k$ .
3. The above was a simple example of equilibrium under two forces. In general, while solving problems you should a)draw a diagram, b)define an origin for a system of coordinates, c)identify all forces (tension, normal, friction, weight, etc) and their  $x$  and  $y$  components, d)Apply Newton's law separately along the  $x$  and  $y$  axes. e) find the accelerations, then velocities, then displacements. This sounds very cook-book, and in fact it will occur to you naturally how to do this while solving actual problems.
4. Your weight in a lift: suppose you are in a lift that is at rest or moving at constant velocity. In either case  $a=0$  and the normal force  $N$  and the force due to gravity are exactly equal,  $N - Mg = 0 \Rightarrow N = Mg$ . But if the lift is accelerating downwards then  $Mg - N = Ma$  or  $N = M(g - a)$ . So now the normal force (i.e. the force with which the floor of the lift is pushing on you) is decreased. Note that if the lift is accelerating downwards with acceleration  $a$  (which it will if the cable breaks!) then  $N=0$  and you will experience weightlessness just like astronauts in space do. Finally, if the lift is accelerating upwards then  $a$  is negative and you will feel heavier.
5. Imagine that you are in a railway wagon and want to know how much you are accelerating. You are not able to look out of the windows. A mass is hung from the roof. Find the acceleration of the car from the angle made by the mass.



We first balance the forces vertically:  $T \cos \theta = mg$   
 And then horizontally:  $T \sin \theta = ma$

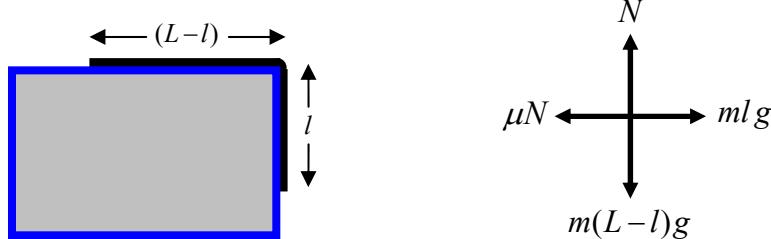
From these two equations we find that:  $\tan \theta = \frac{a}{g}$

Note that the mass  $m$  doesn't matter - it cancels out!



6. Friction is a funny kind of force. It does not make up its mind which way to act until some other force compels it to decide. Imagine a block lying on the floor. If you push it forward, friction will act backward. And if you push it to the left, friction will act to the right. In other words, the direction of the frictional force is always in the opposite direction to the applied force.

7. Let us solve the following problem: a rope of total length  $L$  and mass per unit length  $m$  is put on a table with a length  $l$  hanging from one edge. What should be  $l$  such that the rope just begins to slip?



To solve this, look at the balance of forces in the diagram below: in the vertical direction, the normal force balances the weight of that part of the rope that lies on the table:

$N = m(L - l)g$ . In the horizontal direction, the rope exerts a force  $mlg$  to the right, which is counteracted by the friction that acts to the left. Therefore  $\mu N = ml g$ . Substituting  $N$

from the first equation we find that  $l = \frac{\mu L}{\mu + 1}$ . Note that if  $\mu$  is very small then even a small

piece of string that hangs over the edge will cause the entire string to slip down.

9. In this problem, we would like to calculate the minimum force

$F$  such that the small block does not slip downwards. Clearly,  $F \rightarrow m M$  since the 2 bodies move together,  $F = (m + M)a$ . This gives

$a = \frac{F}{(m + M)}$ . We want the friction  $\mu N$  to be at least as large as the downwards force,  $mg$ .

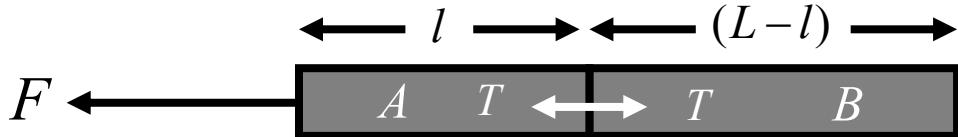
So, we put  $N = ma = m \left( \frac{F}{(m + M)} \right)$  from which the minimum horizontal force needed to prevent slippage is  $F = \frac{(m + M)g}{\mu}$ .

## QUESTIONS AND EXERCISES – 6

- Q.1 a) Why is it necessary to choose a reference frame?
- b) If you used spherical coordinates to solve a problem instead of rectangular cartesian coordinates, would it make a difference?
- c) Why do we pretend that the earth is an inertial frame when we know it is not?
- d) Without making measurements by looking outside, can you know whether or not your frame is inertial?
- e) Look up the meaning of "inertial guidance".
- f) Newton thought that one frame that was fixed to the distant stars was the best (or most preferred frame). Was this a sensible thought at that time? Discuss.
- Q.2 For a particular liquid and a certain shape of body the relation between the frictional resistance and velocity is of the form  $F = av + bv^2$ .
- a) What are the dimensions of  $a$  and  $b$ ?
- b) If a body of mass  $M$  is dropped into a deep vessel containing the liquid, what is the maximum speed that it will attain?
- c) What will be the initial acceleration just as it enters the liquid?

Q.3 A rope is pulled with force  $F$  on the floor.

- (a) Find its acceleration and the tension  $T$  at distance  $l$ .  
 (b) Repeat if the coefficient of friction is  $\mu$ .



Solution to part (a):

Take a small piece of the rope and look at the forces acting to the left and the right.

The net force on part A of the rope gives its acceleration:  $F - T = m_l a$

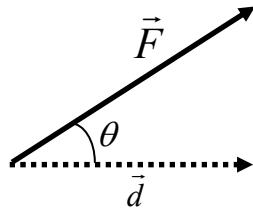
Similarly, the net force on part B of the rope gives its acceleration:  $T = m(L-l)a$

From these two equations it follows that  $T$  depends on  $l$ :  $T = F \left(1 - \frac{l}{L}\right)$

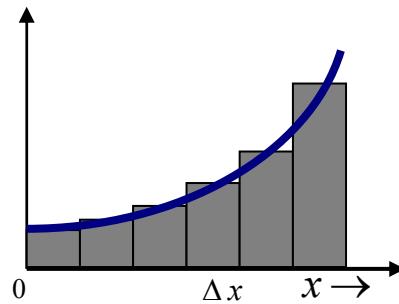
## Summary of Lecture 7 – WORK AND ENERGY

1. Definition of work: force applied in direction of displacement  $\times$  displacement. This means that if the force  $F$  acts at an angle  $\theta$  with respect to the direction of motion, then  

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$



2. a) Work is a scalar - it has magnitude but no direction.  
 b) Work has dimensions:  $M \times (L T^{-2}) \times L = M L^2 T^{-2}$   
 c) Work has units: 1 Newton  $\times$  1 Metre  $\equiv$  1 Joule (J)
3. Suppose you lift a mass of 20 kg through a distance of 2 metres. Then the work you do is  $20 \text{ kg} \times 9.8 \text{ Newtons} \times 2 \text{ metres} = 39.2 \text{ Joules}$ . On the other hand, the force of gravity is directed opposite to the force you exert and the work done by gravity is -39.2 Joules.
4. What if the force varies with distance (say, a spring pulls harder as it becomes longer). In that case, we should break up the distance over which the force acts into small pieces so that the force is approximately constant over each bit. As we make the pieces smaller and smaller, we will approach the exact result:



Now add up all the little pieces of work:

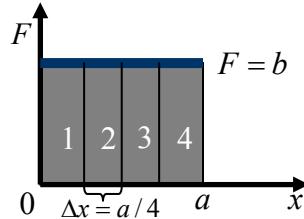
$$W = \Delta W_1 + \Delta W_2 + \dots + \Delta W_N = F_1 \Delta x + F_2 \Delta x + \dots + F_N \Delta x \equiv \sum_{n=1}^N F_n \Delta x$$

To get the exact result let  $\Delta x \rightarrow 0$  and the number of intervals  $N \rightarrow \infty$ :  $W = \lim_{\Delta x \rightarrow 0} \sum_{n=1}^{\infty} F_n \Delta x$

Definition:  $W = \lim_{\Delta x \rightarrow 0} \sum_{n=1}^{\infty} F_n \Delta x \equiv \int_{x_i}^{x_f} F(x) dx$  is called the integral of  $F$  with respect to  $x$  from

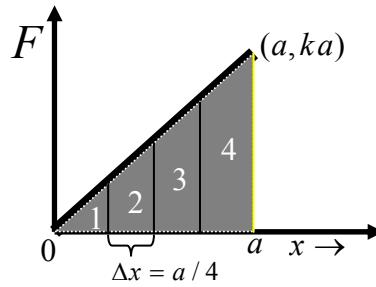
from  $x_i$  to  $x_f$ . This quantity is the work done by a force, constant or non-constant. So if the force is known as a function of position, we can always find the work done by calculating the definite integral.

5. Just to check what our result looks like for a constant force, let us calculate  $W$  if  $F = b$ ,



$$\frac{1}{4}a(b) + \frac{1}{4}a(b) + \frac{1}{4}a(b) + \frac{1}{4}a(b) = ab \quad \therefore \int_0^a F dx = ab$$

6. Now for a less trivial case: suppose that  $F = kx$ , i.e. the force increases linearly with  $x$ .



$$\text{Area of shaded region} = \frac{1}{2}(a)(ka) = k \frac{a^2}{2} \quad \therefore \int_0^a F dx = k \frac{a^2}{2}$$

7. Energy is the capacity of a physical system to do work:

- it comes in many forms – mechanical, electrical, chemical, nuclear, etc
- it can be stored
- it can be converted into different forms
- it can never be *created* or *destroyed*

8. Accepting the fact that energy is conserved, let us derive an expression for the kinetic energy of a body. Suppose a *constant force* accelerates a mass  $m$  from speed  $0$  to speed  $v$  over a *distance*  $d$ . What is the work done by the force? Obviously the answer is:

$$W = Fd. \text{ But } F = ma \text{ and } v^2 = 2ad. \text{ This gives } W = (ma)d = \frac{mv^2}{2d} d = \frac{1}{2}mv^2. \text{ So, we}$$

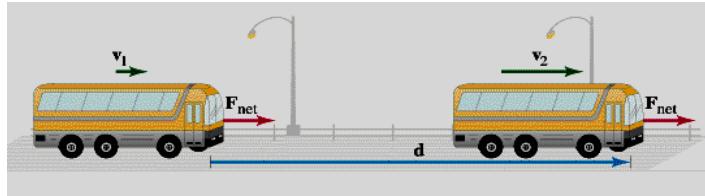
conclude that the work done by the force has gone into creating kinetic energy. and that the amount of kinetic energy possessed by a body moving with speed  $v$  is  $\frac{1}{2}mv^2$ .

9. The work done by a force is just the force multiplied by the distance – it does not depend upon time. But suppose that the same amount of work is done in half the time. We then say that the *power* is twice as much. We define:

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

If the force does not depend on time:  $\frac{\text{Work}}{\text{Time}} = \frac{F \Delta x}{\Delta t} = F v$ . Therefore, Power =  $F v$ .

10. Let's work out an example. A constant force accelerates a bus (mass  $m$ ) from speed  $v_1$  to speed  $v_2$  over a distance  $d$ . What work is done by the engine?



Recall that for constant acceleration,  $v_2^2 - v_1^2 = 2a(x_2 - x_1)$  where:  $v_2$  = final velocity,  $x_2$  = final position,  $v_1$  = initial velocity,  $x_1$  = initial position. Hence,  $a = \frac{v_2^2 - v_1^2}{2d}$ . Now calculate the work done:  $W = Fd = mad = m \frac{v_2^2 - v_1^2}{2d} d = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ . So the work done has resulted in an increase in the quantity  $\frac{1}{2}mv^2$ , which is kinetic energy.

### QUESTIONS AND EXERCISES – 7

Q.1 A stone tied to a string is whirled around. Suppose the string has tension  $T$ . How much work will be done when the stone goes around one complete revolution?

Q.2 In (6) above, calculate the areas 1,2,3,4 separately and then add them up. Is your answer equal to  $k \frac{a^2}{2}$ ?

Q.3 A 1000 kg trolley is pulled up a  $45^\circ$  inclined plane at 1.5 m/sec. How much power is needed?

Q. 4 In the previous question, if the coefficient of friction is  $\mu = 0.5$  what will be the power needed now?

Q.5 Suppose the air friction acting on a car increases as  $kv^2$ . What is the engine power needed to keep the car moving at speed  $v$ ?

Q.6 In the example solved in (10) above, calculate the power of the bus engine.

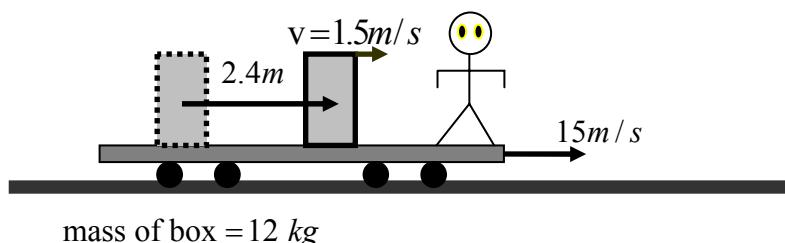
## Summary of Lecture 8 – CONSERVATION OF ENERGY

- Potential energy is, as the word suggests, the energy “locked up” up somewhere and which can do work. Potential energy *kam karnay ki salahiat hai!* Potential energy can be converted into kinetic energy,  $\frac{1}{2}mv^2$ . As I showed you earlier, this follows directly from Newton’s Laws.
- If you lift a stone of mass  $m$  from the ground up a distance  $x$ , you have to do work against gravity. The (constant) force is  $mg$ , and so  $W = mgx$ . By conservation of energy, the work done by you was transformed into gravitational potential energy whose values is exactly equal to  $mgx$ . Where is the energy stored? Answer: it is stored neither in the mass or in the earth - it is stored in the gravitational field of the combined system of stone+earth.
- Suppose you pull on a spring and stretch it by an amount  $x$  away from its normal (equilibrium) position. How much energy is stored in the spring? Obviously, the spring gets harder and harder to pull as it becomes longer. When it is extended by length  $x$  and you pull it a further distance  $dx$ , the small amount of work done is  $dW = Fdx = kxdx$ . Adding up all the small bits of work gives the total work:

$$W = \int_0^x Fdx = \int_0^x kxdx = \frac{1}{2}kx^2$$

This is the work you did. Maybe you got tired working so hard. What was the result of your working so hard? Answer: this work was transformed into energy stored in the spring. The spring contains energy exactly equal to  $\frac{1}{2}kx^2$ .

- Kinetic energy obviously depends on the frame you choose to measure it in. If you are running with a ball, it has zero kinetic energy with respect to you. But someone who is standing will see that it has kinetic energy! Now consider the following situation: a box of mass 12kg is pushed with a constant force so that so that its speed goes from zero to 1.5m/sec (as measured by the person at rest on the cart) and it covers a distance of 2.4m. Assume there is no friction.



Let's first calculate the change in kinetic energy:

$$\Delta K = K_f - K_i = \frac{1}{2}(12\text{kg})(1.5\text{m/s})^2 - 0 = 13.5\text{J}$$

And then the (constant) acceleration:

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(1.5\text{m/s})^2 - 0}{2(2 \cdot 4\text{m})} = 0.469\text{m/s}^2$$

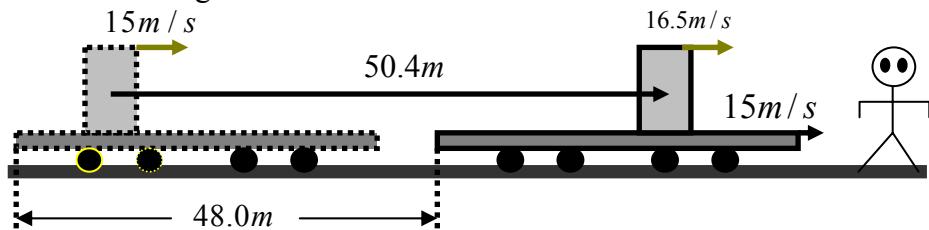
This acceleration results from a constant net force given by:

$$F = ma = (12\text{kg})(0.469\text{m/s}^2) = 5.63\text{N}$$

From this, the work done on the crate is:

$$W = F\Delta x = (5.63\text{N})(2.4\text{m}) = 13.5\text{J} \text{ (same as } \Delta K = 13.5\text{J !)}$$

5. Now suppose there is somebody standing on the ground, and that the trolley moves at 15 m/sec relative to the ground:



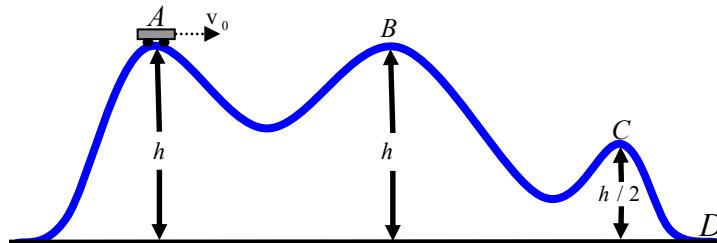
Let us repeat the same calculation:

$$\begin{aligned}\Delta K' &= K'_f - K'_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}(12\text{kg})(16.5\text{m/s})^2 - \frac{1}{2}(12\text{kg})(15.0\text{m/s})^2 = 284\text{J}\end{aligned}$$

This example clearly shows that work and energy have different values in different frames.

6. The total mechanical energy is:  $E_{\text{mech}} = KE + PE$ . If there is no friction then  $E_{\text{mech}}$  is conserved. This means that the sum does not change with time. For example: a ball is thrown upwards at speed  $v_0$ . How high will it go before it stops? The loss of potential energy is equal to the gain of potential energy. Hence,  $\frac{1}{2}mv_0^2 = mgh \Rightarrow h = \frac{v_0^2}{2g}$ .

Now look at the smooth, frictionless motion of a car over the hills below:



Even though the motion is complicated, we can use the fact that the total energy is a constant to get the speeds at the points B,C,D:

$$\text{At point A: } \frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2 + mgh \Rightarrow v_B = v_C$$

$$\text{At point C: } \frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_C^2 + mg\frac{h}{2} \Rightarrow v_C = \sqrt{v_A^2 + gh}$$

$$\text{At point D: } \frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_D^2 \Rightarrow v_D = \sqrt{v_A^2 + 2gh}$$

7. Remember that potential energy has meaning only for a force that is conservative. A conservative force is that for which the work done in going from point A to point B is independent of the path chosen. Friction is an example of a non-conservative force and a potential energy cannot be defined. For a conservative force,  $F = -\frac{dV}{dx}$ . So, for a spring,  $V = \frac{1}{2}kx^2$  and so  $F = -kx$ .

8. Derivation of  $F = -\frac{dV}{dx}$ : If the particle moves distance  $\Delta x$  in a potential  $V$ , then

change in PE is  $\Delta V$  where,  $\Delta V = -F \Delta x$ . From this,  $F = -\frac{\Delta V}{\Delta x}$ . Now let  $\Delta x \rightarrow 0$ .

$$\text{Hence, } F = -\lim_{\Delta x \rightarrow 0} \frac{\Delta V}{\Delta x} = -\frac{dV}{dx}.$$

## QUESTIONS AND EXERCISES – 8

1. If energy is conserved, then why do you get tired simply standing in one place?
2. Suppose that the motion of the earth around the sun suddenly stopped. Using the conservation of energy, discuss what will happen to the earth after that.
3. In point 5 above, find the work done by the force (as seen by the ground observer) and show that it is equal to 284 J. So, even though work and energy are different in different frames, the law of conservation of energy holds in every frame.
4. The potential between 2 atoms as a function of distance is:

$$V(x) = -\frac{4}{x^6} + \frac{7}{x^{12}}$$

- a) Find the force as a function of distance. Where is the force zero?
- b) Calculate  $\frac{d^2V}{dx^2}$  at the value of  $x$  where the force is zero. Is it positive or negative?
- c) Note that the above expression for  $V(x)$  is not dimensionally correct if  $x$  is actually a length. How can it be made correct?

## Summary of Lecture 9 – MOMENTUM

1. Momentum is the "quantity of motion" possessed by a body. More precisely, it is defined as:

$$\text{Mass of the body} \times \text{Velocity of the body}.$$

The dimensions of momentum are  $MLT^{-1}$  and the units of momentum are kg-m/s.

2. Momentum is a vector quantity and has both magnitude and direction,  $p = m\vec{v}$ . We can easily see that Newton's Second Law can be reexpressed in terms of momentum. When

I wrote it down originally, it was in the form  $m\vec{a} = \vec{F}$ . But since  $\frac{d\vec{v}}{dt} = \vec{a}$ , this can also be

written as  $\frac{d\vec{p}}{dt} = \vec{F}$  (new form). In words, the rate of change of momentum of a body equals the total force acting upon it. Of course, the old and new are exactly the same,

$$\frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m \frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}.$$

3. When there are many particles, then the total momentum  $\vec{P}$  is,

$$\begin{aligned}\vec{P} &= \vec{p}_1 + \vec{p}_2 + \cdots + \vec{p}_N \\ \frac{d}{dt} \vec{P} &= \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} + \cdots + \frac{d\vec{p}_N}{dt} \\ &= \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N = \vec{F}\end{aligned}$$

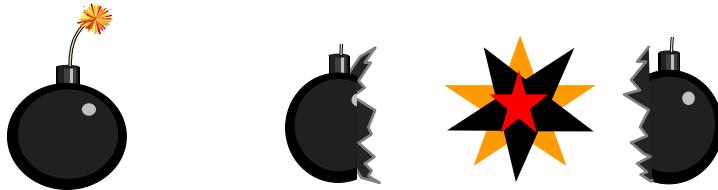
This shows that when there are several particles, the rate at which the total momentum changes is equal to the total force. It makes sense!

4. A very important conclusion of the above is that if the sum of the total external forces vanishes, then the total momentum is conserved,  $\sum \vec{F}_{ext} = 0 \Rightarrow \frac{d}{dt} \vec{P} = 0$ . This is quite independent of what sort of forces act between the bodies - electric, gravitational, etc. - or how complicated these are. We shall see why this is so important from the following examples.

5. Two balls, which can only move along a straight line, collide with each other. The initial momentum is  $P_i = m_1 u_1 + m_2 u_2$  and the final momentum is  $P_f = m_1 v_1 + m_2 v_2$ . Obviously one ball exerts a force on the other when they collide, so its momentum changes. But, from the fact that there is no external force acting on the balls,  $P_i = P_f$ , or  $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$ .

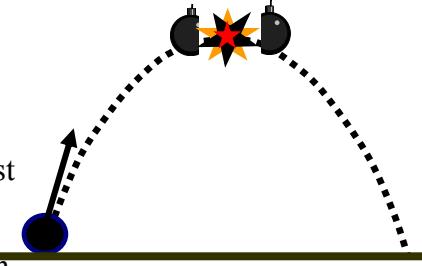
6. A bomb at rest explodes into two fragments. Before the explosion the total momentum is

zero. So obviously it is zero after the explosion as well,  $\mathbf{P}_f = \mathbf{0}$ . During the time that the explosion happens, the forces acting upon the pieces are very complicated and changing rapidly with time. But when all is said and done, there are two pieces flying away with a total zero final momentum  $\mathbf{P}_f = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$ . Hence  $m_1\mathbf{v}_1 = -m_2\mathbf{v}_2$ . In other words, the fragments fly apart with equal momentum but in opposite directions. The centre-of-mass stays at rest. So, knowing the velocity of one fragment permits knowing the velocity of the other fragment.



7. If air resistance can be ignored, then we can do some interesting calculations with what we have learned.

So, suppose a shell is fired from a cannon with a speed 10 m/s at an angle  $60^\circ$  with the horizontal. At the highest point in its path it explodes into two pieces of equal masses. One of the pieces retraces its path to the cannon. Let us find the velocity of the other piece immediately after the explosion.



Solution: After the explosion:  $P_{1x} = -5 \frac{M}{2}$  (why?). But  $P_{1x} + P_{2x} = P_x = M \times 10 \cos 60^\circ$

$$\Rightarrow P_{2x} = 5M + 5 \frac{M}{2}. \text{ Now use: } P_{2x} = \frac{M}{2}v_{2x} \Rightarrow v_{2x} = 15 \text{ m/s.}$$

8. When you hit your thumb with a hammer it hurts, doesn't it? Why? Because a large amount of momentum has been destroyed in a short amount of time. If you wrap your thumb with foam, it will hurt less. To understand this better, remember that force is the rate of change of momentum:  $F = \frac{dp}{dt} \Rightarrow dp = Fdt$ . Now define the **impulse  $I$**  as:

**force × time over which the force acts.**

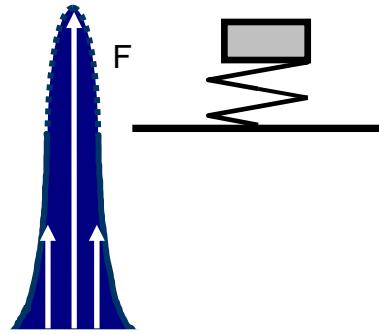
If the force changes with time between the limits , then one should define  $I$  as,

$$I = \int_{t_1}^{t_2} F dt. \text{ Since } \int_{t_1}^{t_2} F dt = \int_{p_i}^{p_f} dp, \text{ therefore } I = p_f - p_i. \text{ In words, the change of momentum}$$

equals the impulse, which is equal to the area under the curve of force versus time. Even if you wrap your thumb in foam, the impulse is the same. But the force is definitely not!

9. Sometimes we only know the force numerically (i.e. there is no expression like  $F=$ something).

But we still know what the integral means: it is the area under the curve of force versus time. The curve here is that of a hammer striking a table. Before the hammer strikes, the force is zero, reaches a peak, and goes back to zero.



### QUESTIONS AND EXERCISES – 9

Q.1 A stream of bullets, each of mass  $m$ , is fired horizontally with a speed  $v$  into a large wooden block of mass  $M$  that is at rest on a horizontal table. If the coefficient of friction is  $\mu$ , how many bullets must be fired per second so that the block just begins to move? [Hint, calculate the momentum destroyed in one second.]

Q. 2 In point 5, we have one equation but two unknowns  $v_1, v_2$ . Obviously more information needs to be supplied. So consider two extreme cases. In both cases, find the final velocities:

- The collision is perfectly elastic, meaning that the sum of initial kinetic energies is exactly equal to the sum of final kinetic energies.
- The collision results in the two bodies sticking together and moving off as one body.

Q.3 a) Would you rather land with your legs bending or stiff?

- Why do cricket fielders move their hands backwards when catching a fast ball?
- Why do railway carriages have dampers at the front and back?

Q.4 A rocket of mass  $M_0$  is at rest in space. Then at  $t = 0$  it starts to eject hot gas at speed  $v$  from the nozzle and the mass of the rocket is  $m(t) = M_0 - \alpha t$ . Find the acceleration of the rocket at time  $t = 0$ .

Q.5 Sand drops onto an open railway carriage at  $\rho$  kg/sec. If the engine pulling it is working at power  $P_0$  watts at time  $t = 0$ , find the additional power of the engine pulling it such that the speed of the train remains constant at  $v_0$ .