2 - Gaussian Curvature

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1 introduction

- $\mathcal{E} = +\frac{1}{\mathcal{R}^2}$ is called *Gaussian Curvature* of the sphere
- $\mathcal{E} = -\frac{1}{\mathcal{R}^2}$ is also called *Gaussian Curvature*, in hyperbolic geometry
- Gauss wanted to measure the curvature at a particular point, so we can rearrange $\mathcal{E}(\Delta) = \mathcal{K}\mathcal{A}(\Delta)$ as $\mathcal{K} = \frac{\mathcal{E}(\Delta)}{\mathcal{A}(\Delta)}$, which makes it clear that $\mathcal{E}$ is *angular excess per unit area.* to define the curvature at a point, we define a limit as the area of a triange around the point shrinks to 0.
- there is nothing special about the triangle here, any n-gon will work, using the formula:
  $\mathcal{E}(\text{n-gon}) \equiv [\text{angle sum}] - (n-2)\pi$

2 the circumference and area of a circle

- we have defined curvature with respect to tiny triangles, but it is an abstract thing that has an "iron grip" over the geometry of a space. two examples:
- tiny circles determine curvature as well**
- area of circles determine curvature as well**

3 the local gauss-bonnet theorem

- angular excess is additive. $\mathcal{E}(\Delta) = \mathcal{E}(\Delta_1) + \mathcal{E}(\Delta_2)$. this allows for finer and finer subdivisions, and thus a limit to the curvature
- $\mathcal{E}(\Delta) = \iint_\Delta \mathcal{K}\, d\mathcal{A}$