1 - euclidean and non euclidean geometry

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Proof. example proof for later

Definition 1. example definition for later

1 euclidean and hyperbolic geometry

• flatness is defined as a space where the pythagorean theorem holds true.

- ullet euclidean geometry, parallel axiom: Through any point p not on the line L, there exists exactly one line P that is parallel to L
- 1830, Lobachevsky announced the existence of hyperbolic geometry by denying the parallel axiom.
- hyperbolic geometry, hyperbolic axiom: There exists at least 2 parallel lines through p that do not meet L

2 spherical geometry

- parallel axiom states exactly one parallel exists. denying this means you're either stating there's less than 1 (aka 0) or more than 1. the first option defines spherical geometry and the second defines hyperbolic
- in spherical geometry, lines are "great circles", and these "lines" can be used to construct "triangles"

3 the angular excess of a spherical triangle

- denying the parallel axiom is logically equal to denying that the interior angles of a triangle sum to pi radians
- angular excess, $\mathcal{E} \equiv \text{(angle sum of triagle)} \pi$
- $\mathcal{E} = \frac{1}{R^2} \mathcal{A}$, for any triangle on a sphere with radius \mathcal{R} and area \mathcal{A}

- 4 intrinsic and extrinsic geometry of curved surfaces
 - test
- 5 constructing geodesics by their straightness
 - \bullet test
- 6 the nature of space
 - test