1 - euclidean and non euclidean geometry

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1 euclidean and hyperbolic geometry

- flatness is defined as a space where the pythagorean theorem holds true.
- euclidean geometry, parallel axiom: Through any point p not on the line L, there exists exactly one line P that is parallel to L
- 1830, Lobachevsky announced the existence of hyperbolic geometry by denying the parallel axiom.
- hyperbolic geometry, hyperbolic axiom: There exists at least 2 parallel lines through p that do not meet L

2 spherical geometry

- parallel axiom states exactly one parallel exists. denying this means you're either stating there's less than 1 (aka 0) or more than 1. the first option defines spherical geometry and the second defines hyperbolic
- in spherical geometry, lines are "great circles", and these "lines" can be used to construct "triangles"

3 the angular excess of a spherical triangle

- denying the parallel axiom is logically equal to denying that the interior angles of a triangle sum to pi radians
- angular excess, $\mathcal{E} \equiv$ (angle sum of triagle) $- \pi$
- $\mathcal{E} = \frac{1}{R^2}\mathcal{A}$, for any triangle on a sphere with radius $\mathcal{R}$ and area $\mathcal{A}$

4 intrinsic and extrinsic geometry of curved surfaces

- these geodesics can be thought of as length minimizing, but only if the distances between the two points are sufficiently small
- this length minimizing property can then be used to define distance on a curved space.
- now that we have defined distance we can define a geodesic "circle" on the curved surface by specifying a point and radius
- intrinsic geometry: geometry that is knowable to a tiny ant living on the surface. thus nothing will seem different to the ant, provided that distances all remain the same
- extrinsic geometry: how the surface sits in space. here you can alter things so distances are the same, but the global differences are still noted.

5 constructing geodesics by their straightness

- to construct a geodesic on a surface, emanating from a point in direction v, stick out one end of a length of narrow sticky tape down at p and unroll it onto the surface, starting at direction v

6 the nature of space

- hyperbolic triangle is one where $\mathcal{E} < 0$
- $\mathcal{E}(\Delta) = \mathcal{K}\mathcal{A}(\Delta)$, where $\mathcal{K}$ a constant that's positive for spherical geometry and negative for hyperbolic geometry