Practical 4

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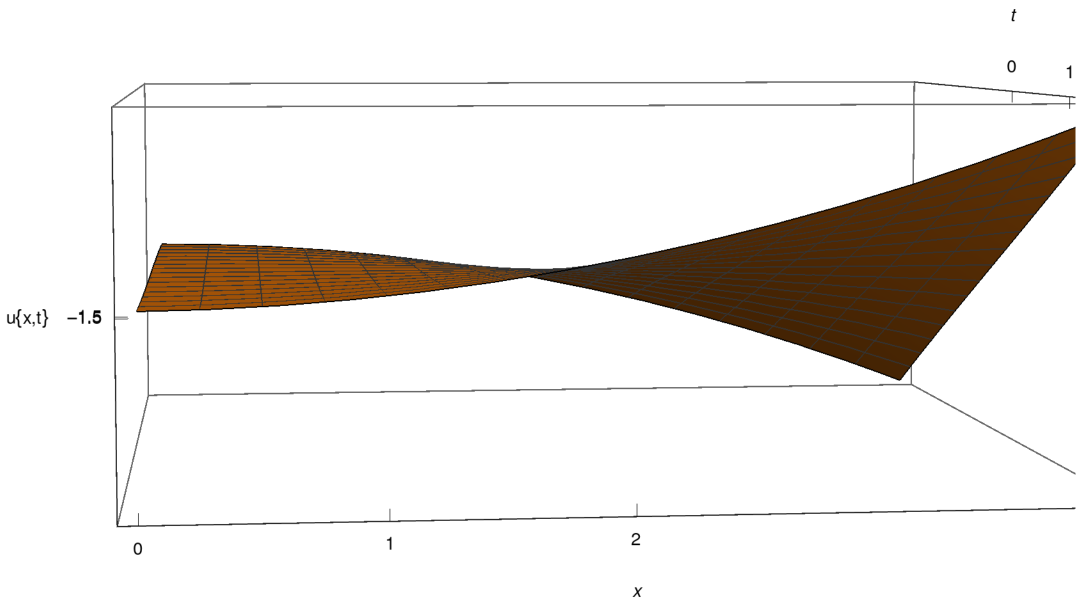
Ques .1 Solve wave equation u_tt – 4 u_xx = 0 with cauchy data u (x, 0) = x ^ 3, u_t(x, 0) = x; –inf < x < inf, t > 0 .

Method 1

In[19]:=
```
pde1 = {D[u[x, t], {t, 2}] - 4 D[u[x, t], {x, 2}] == 0, u[x, 0] == x^3,
    Derivative[0, 1][u][x, 0] == x};
Sol1 = DSolve[pde1, u[x, t], {x, t}]
Plot3D[u[x, t] /. Sol1, {x, -2, 2}, {t, 0, 4}, AxesLabel → {t, x, "u{x,t}"},
 Ticks → {{0, 1, 2, 3, 4, 5}, {-2, -1, 0, 1, 2}, {-1.5, 1.5}}, PlotStyle → {Dashed, Thick}]
```

Out[20]= $\left\{\left\{u[x,\ t] \to \frac{1}{4}\left(-\frac{1}{2}(-2\,t+x)^2+\frac{1}{2}(2\,t+x)^2\right)+\frac{1}{2}\left((-2\,t+x)^3+(2\,t+x)^3\right)\right\}\right\}$

Out[21]=



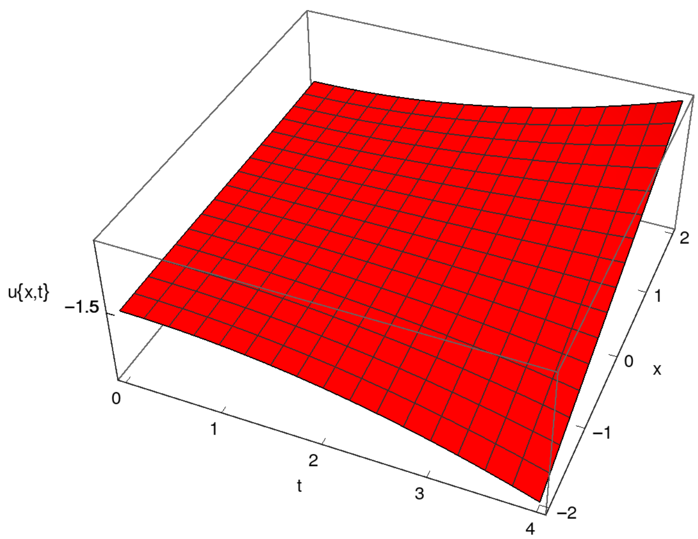
Method 2

In[25]:=

```
npde1 = {D[u[x, t], {t, 2}] - 4 D[u[x, t], {x, 2}] == 0, u[x, 0] == x^3,
    Derivative[0, 1][u][x, 0] == x};
nsol1 = u[x, t] /. NDSolve[npde1, u[x, t], {x, -2, 2}, {t, 0, 4}, PrecisionGoal → 4]
Plot3D[nsol1, {t, 0, 4}, {x, -2, 2}, AxesLabel → {"t", "x", "u{x,t}"},
 Ticks → {{0, 1, 2, 3, 4, 5}, {-2, -1, 0, 1, 2}, {-1.5, 1.5}}, PlotStyle → Red]
```

NDSolve: Warning: an insufficient number of boundary conditions have been specified for the direction of independent variable x. Artificial boundary effects may be present in the solution.

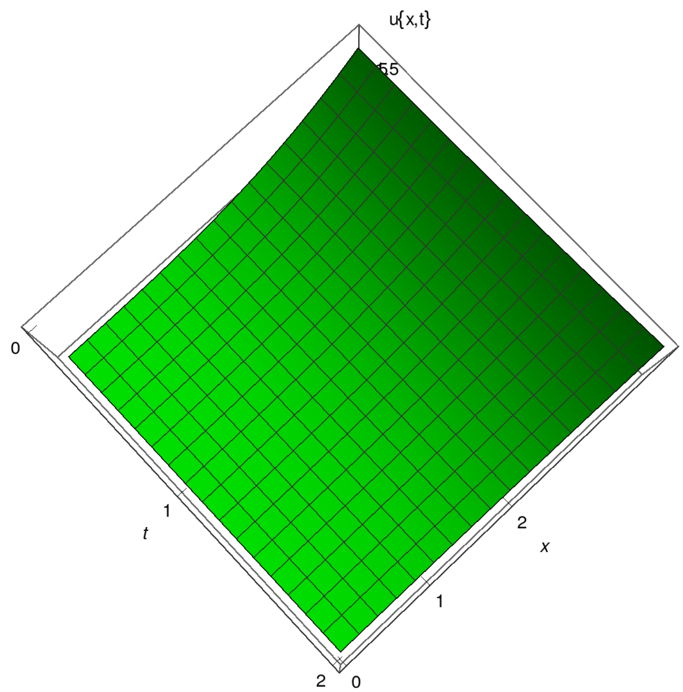
Out[26]= {InterpolatingFunction[Domain: {{-2., 2.}, {0., 4.}} Output: scalar][x, t]}



Ques .2 Solve wave equation u_tt - 4 u_xx = 0 with cauchy data u (x, 0) = x ^ 3, u_t(x, 0) = x; 0 < x < inf, t > 0 .

```
In[22]:= pde2 = {D[u[x, t], {t, 2}] - 4 D[u[x, t], {x, 2}] == 0, u[x, 0] == x^3,
            Derivative[0, 1][u][x, 0] == x, Derivative[1, 0][u][0, t] == 0};
         sol2 = DSolve[pde2, u[x, t], {x, t}]
         Plot3D[u[x, t] /. sol2, {x, 0, 2}, {t, 0, 4}, AxesLabel → {t, x, "u{x,t}"},
          Ticks → {{0, 1, 2, 3, 4, 5}, {0, 1, 2}, {-1.5, 1.5}}, PlotStyle → Green]
```

Out[23]= $\left\{\left\{u[x, t] \to t\, x + 12\, t^2\, x + x^3 + 2\, \mathbb{c}_1\, \text{DiracDelta}[2\, t - x] + 2\, \mathbb{c}_1\, \text{DiracDelta}[2\, t + x] + (2\, t - x)^3\, \text{HeavisideTheta}\left[t - \frac{x}{2}\right] + \left(t - \frac{x}{2}\right)^2 \text{HeavisideTheta}\left[t - \frac{x}{2}\right]\right\}\right\}$



Ques. 3 $\partial_{tt} u - 9\, \partial_{xx} u = 0$, $u[x, 0] = \mathrm{Sin}[x]$, $\frac{\partial u}{\partial t}[x, 0] = x^3$, $\frac{\partial u}{\partial x}[0, t] = 0$, $0 < x < 1$, $t > 0$.

In[28]:=
```
aA = {D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] == 0, u[x, 0] == Sin[x],
    Derivative[0, 1][u][x, 0] == x^3, Derivative[1, 0][u][0, t] == 0};
sol = u[x, t] /. NDSolve[aA, u[x, t], {x, 0, 1}, {t, 0, 4}, PrecisionGoal → 3]
Plot3D[sol, {x, 0, 1}, {t, 0, 4}, AxesLabel → {t, x, "u{x,t}"},
 Ticks → {{0, 1, 2, 3, 4, 5}, {0, 1}, {-3, 0}}, PlotStyle → Gray]
```

NDSolve: Warning: boundary and initial conditions are inconsistent.

Out[29]= {InterpolatingFunction[Domain: {{0., 1.}, {0., 4.}} Output: scalar][x, t]}

Out[30]=