Practical 1:

Solution of cauchy problem for first order partial differential equations

In[ ]:=
```
pde1 = D[u[x, y], x] + x * D[u[x, y], y] == 0
c1 = u[0, y] == Sin[y]
DSolve[{pde1, c1}, u[x, y], {x, y}]
```

Out[ ]= $x\, u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$

Out[ ]= $u[0, y] == \mathrm{Sin}[y]$

Out[ ]= $\left\{\left\{u[x, y] \to -\mathrm{Sin}\left[\frac{x^2}{2} - y\right]\right\}\right\}$

In[ ]:=
```
pde2 = x * D[u[x, y], x] + y * D[u[x, y], y] == 2 * x * y
c2 = u[x, x^2] == 2
DSolve[{pde2, c2}, u[x, y], {x, y}]
```

Out[ ]= $y\, u^{(0,1)}[x, y] + x\, u^{(1,0)}[x, y] == 2\, x\, y$

Out[ ]= $u\left[x, x^2\right] == 2$

Out[ ]= $\left\{\left\{u[x, y] \to \frac{2\, x^3 + x^4\, y - y^3}{x^3}\right\}\right\}$

In[ ]:=
```
pde3 = 3 * x * D[u[x, t], x] + t * D[u[x, t], t] == 0
c3 = u[x, 1] == Exp[-x^2]
DSolve[{pde3, c3}, u[x, t], {x, t}]
```

Out[ ]= $t\, u^{(0,1)}[x, t] + 3\, x\, u^{(1,0)}[x, t] == 0$

Out[ ]= $u[x, 1] == e^{-x^2}$

Out[ ]= $\left\{\left\{u[x, t] \to e^{-\frac{x^2}{t^6}}\right\}\right\}$

In[ ]:=
```
pde4 = D[u[x, y], x] + x * D[u[x, y], y] == 0
c4 = u[0, y] == Sin[y]
DSolve[{pde4, c4}, u[x, y], {x, y}]
```

Out[ ]= $x\, u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$

Out[ ]= $u[0, y] == \mathrm{Sin}[y]$

Out[ ]= $\left\{\left\{u[x, y] \to -\mathrm{Sin}\left[\frac{x^2}{2} - y\right]\right\}\right\}$

In[ ]:=
```
pde5 = 3 * D[u[x, y], x] + 2 * D[u[x, y], y] == 0
c5 = u[x, 0] == Sin[x]
DSolve[{pde5, c5}, u[x, y], {x, y}]
```

Out[ ]= $2\, u^{(0,1)}[x, y] + 3\, u^{(1,0)}[x, y] == 0$

Out[ ]= $u[x, 0] == \mathrm{Sin}[x]$

Out[ ]= $\left\{\left\{u[x, y] \to -\mathrm{Sin}\left[\frac{3}{2}\left(-\frac{2\,x}{3} + y\right)\right]\right\}\right\}$

In[ ]:=
```
pde6 = x * D[u[x, y], x] + y * D[u[x, y], y] == u[x, y] + 1
c6 = u[x, x^2] == x^2
DSolve[{pde6, c6}, u[x, y], {x, y}]
```

Out[ ]= $y\, u^{(0,1)}[x, y] + x\, u^{(1,0)}[x, y] == 1 + u[x, y]$

Out[ ]= $u\left[x, x^2\right] == x^2$

Out[ ]= $\left\{\left\{u[x, y] \to \frac{x^2 - y + y^2}{y}\right\}\right\}$

Practical 2:

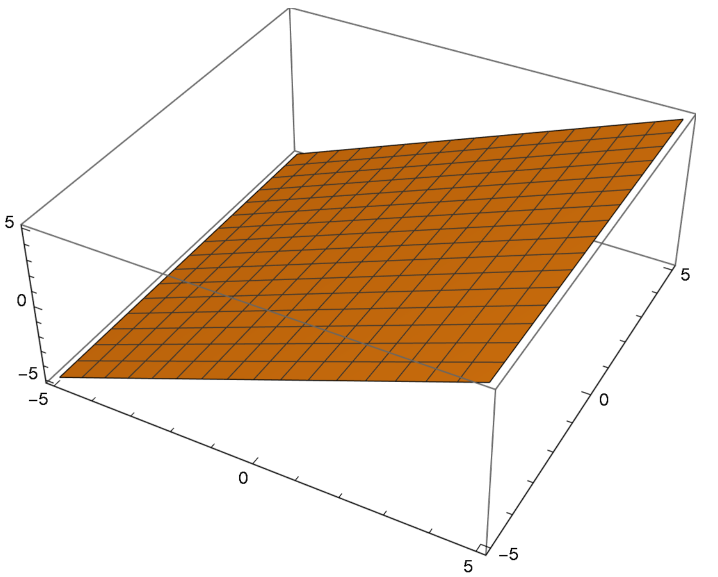
Plotting the Characteristics of first order PDE

In[ ]:=
```
pde1 = x * D[u[x, y], x] + y * D[u[x, y], y] == u[x, y]
DSolve[pde1, u[x, y], {x, y}]
```

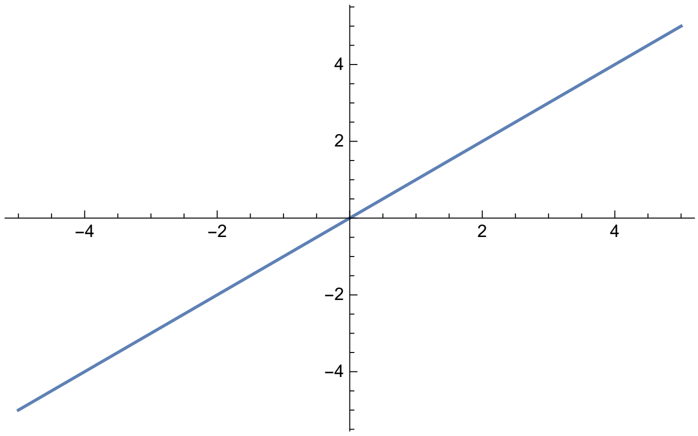
Out[ ]= $y\, u^{(0,1)}[x, y] + x\, u^{(1,0)}[x, y] == u[x, y]$

Out[ ]= $\left\{\left\{u[x, y] \to x\, \mathbb{c}_1\left[\frac{y}{x}\right]\right\}\right\}$

In[ ]:=
```
Plot3D[x, {x, -5, 5}, {y, -5, 5}]
```

Out[ ]=


In[∘]:= `Plot[x, {x, -5, 5}]`

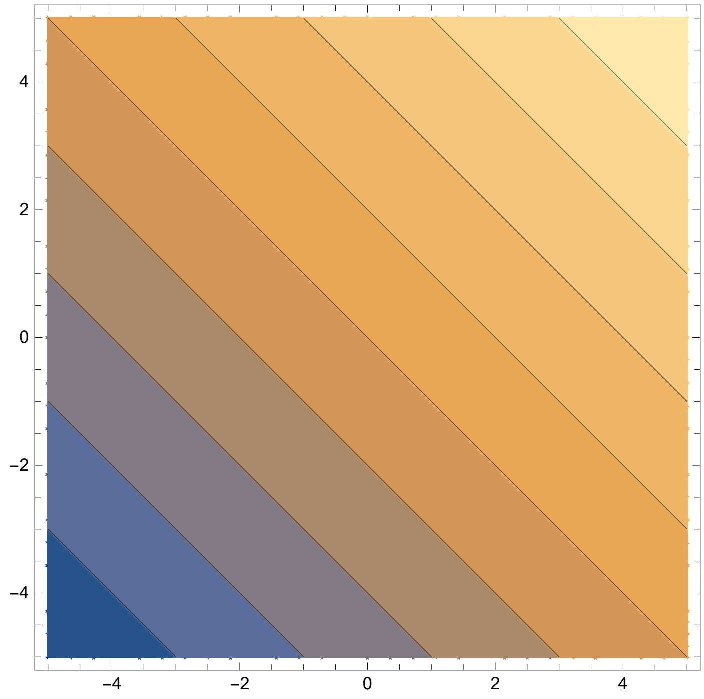
Out[∘]= 

In[∘]:=
```
pde2 = D[u[x, y], x] - D[u[x, y], y] == 1
DSolve[pde2, u[x, y], {x, y}]
```

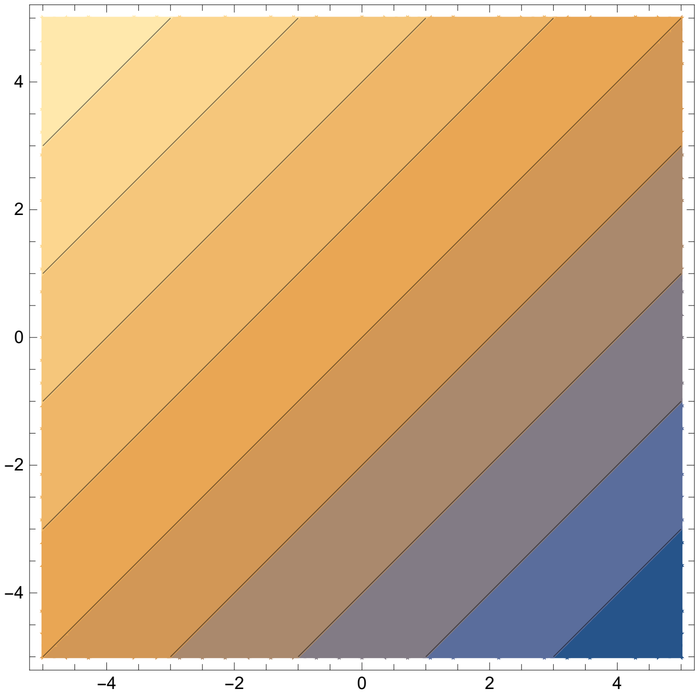
Out[∘]= $-u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$

Out[∘]= $\{\{u[x, y] \to x + c_1[x + y]\}\}$

In[∘]:= `ContourPlot[x + y, {x, -5, 5}, {y, -5, 5}]`

Out[∘]= 

In[ ]:= ContourPlot[u - x, {x, -5, 5}, {u, -5, 5}]

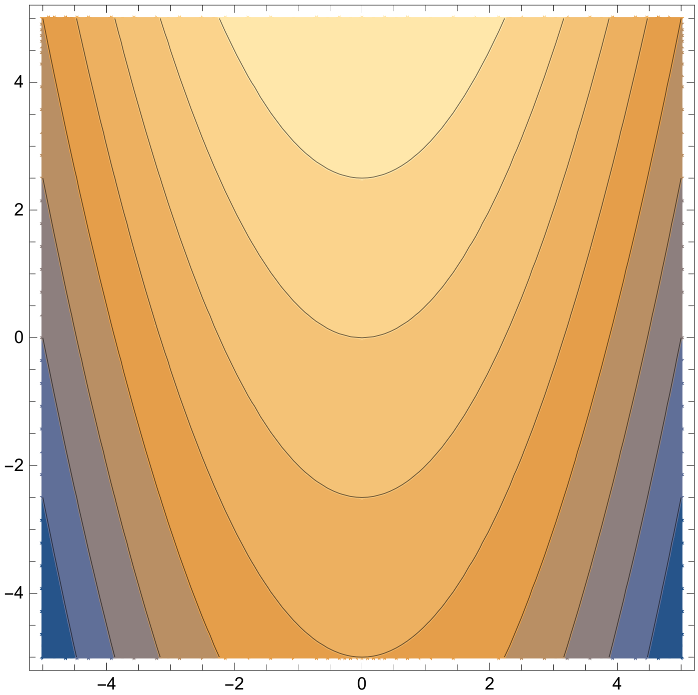
Out[ ]=


In[ ]:= pde3 = D[u[x, y], x] + x * D[u[x, y], y] == 0
DSolve[pde3, u[x, y], {x, y}]

Out[ ]= $x\, u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$

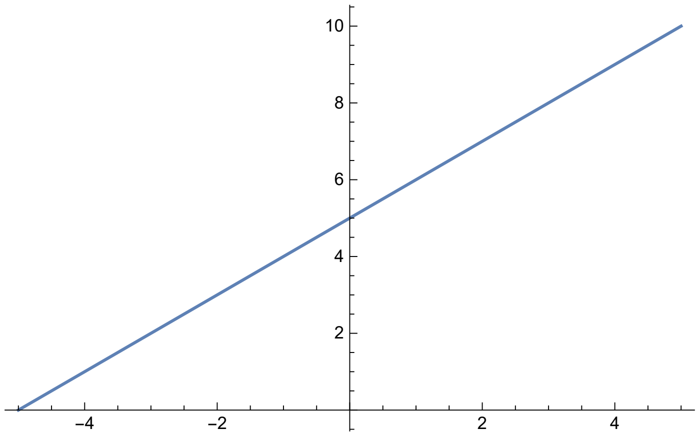
Out[ ]= $\left\{\left\{u[x, y] \to c_1\left[-\frac{x^2}{2} + y\right]\right\}\right\}$

In[ ]:= ContourPlot[(-(x^2) / 2) + y, {x, -5, 5}, {y, -5, 5}]

Out[ ]=


In[∘]:= `Plot[u + 5, {u, -5, 5}]`

Out[∘]=



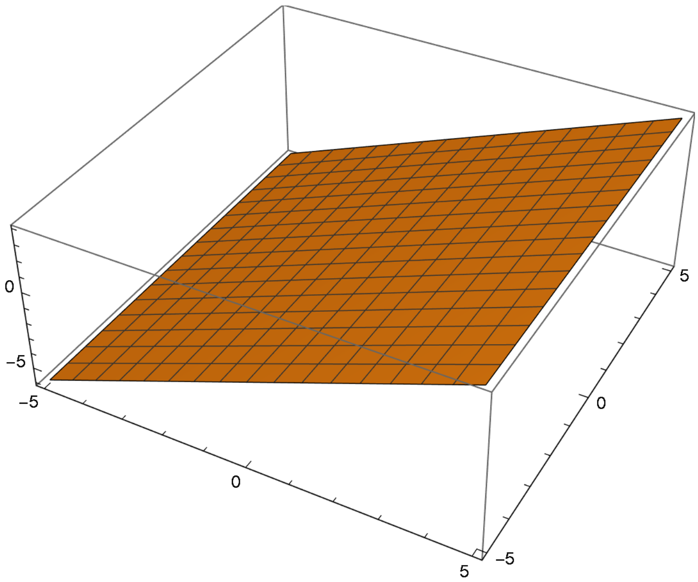
In[∘]:= `pde4 = x * D[u[x, y], x] + y * D[u[x, y], y] == u[x, y] + 1`
`DSolve[pde4, u[x, y], {x, y}]`

Out[∘]= $y\,u^{(0,1)}[x, y] + x\,u^{(1,0)}[x, y] == 1 + u[x, y]$

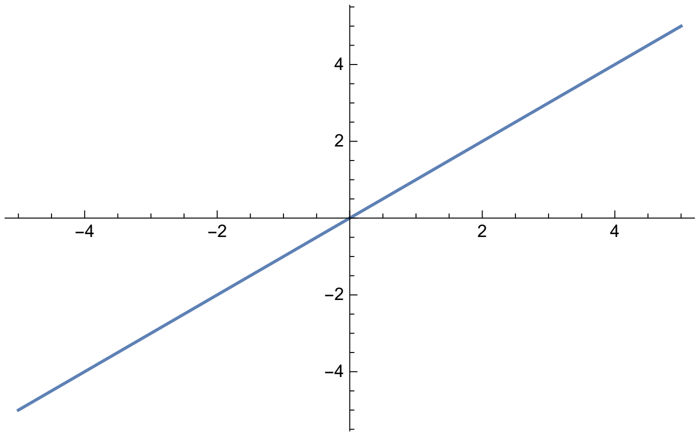
Out[∘]= $\left\{\left\{u[x, y] \to -1 + x\,\mathbb{c}_1\left[\frac{y}{x}\right]\right\}\right\}$

In[∘]:= `Plot3D[-1 + x, {x, -5, 5}, {y, -5, 5}]`

Out[∘]=



In[ ]:=
```
Plot[x, {x, -5, 5}]
```

Out[ ]=


Practical 3:

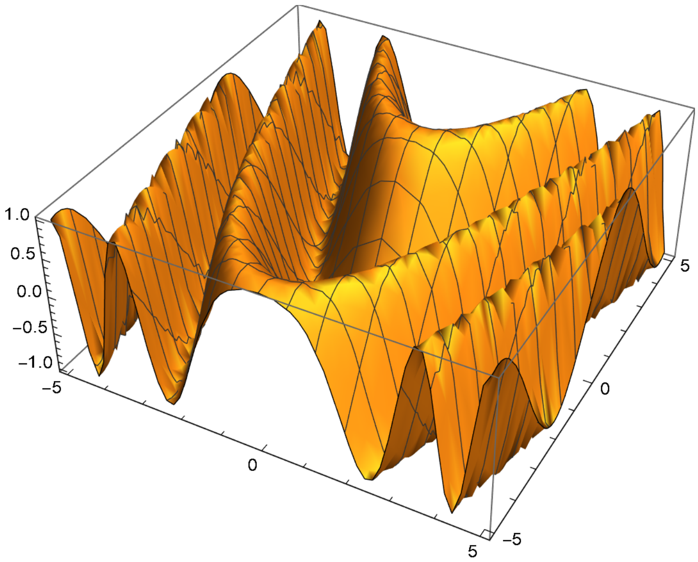
Plotting the integral surface of first order PDE with initial data

In[ ]:=
```
pde1 = D[u[x, y], x] + x * D[u[x, y], y] == 0
c1 = u[0, y] == Sin[y]
s1 = DSolve[{pde1, c1}, u[x, y], {x, y}]
Plot3D[u[x, y] /. s1, {x, -5, 5}, {y, -5, 5}]
```

Out[ ]= $x\, u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$

Out[ ]= $u[0, y] == \text{Sin}[y]$

Out[ ]= $\left\{\left\{u[x, y] \to -\text{Sin}\left[\frac{x^2}{2} - y\right]\right\}\right\}$

Out[ ]=


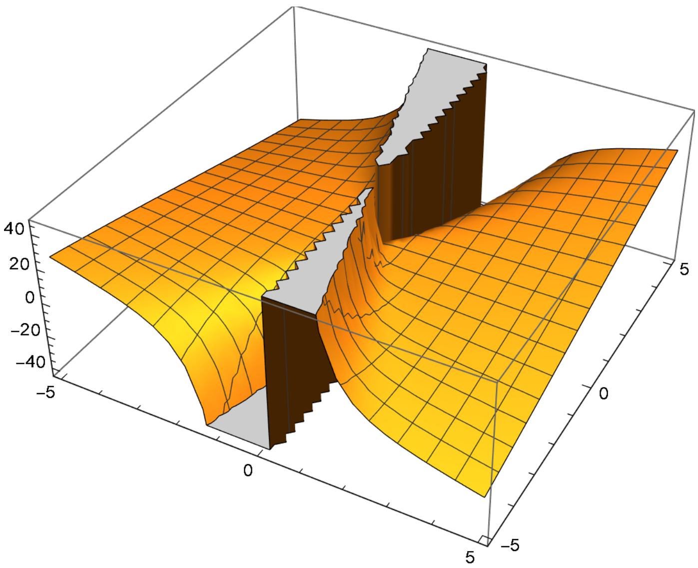
In[ ]:=
```
pde2 = x * D[u[x, y], x] + y * D[u[x, y], y] == 2 * x * y
c2 = u[x, x^2] == 2
s2 = DSolve[{pde2, c2}, u[x, y], {x, y}]
Plot3D[u[x, y] /. s2, {x, -5, 5}, {y, -5, 5}]
```

Out[ ]= $y\, u^{(0,1)}[x, y] + x\, u^{(1,0)}[x, y] == 2\, x\, y$

Out[ ]= $u[x, x^2] == 2$

Out[ ]= $\left\{\left\{u[x, y] \to \frac{2 x^3 + x^4 y - y^3}{x^3}\right\}\right\}$

Out[ ]=



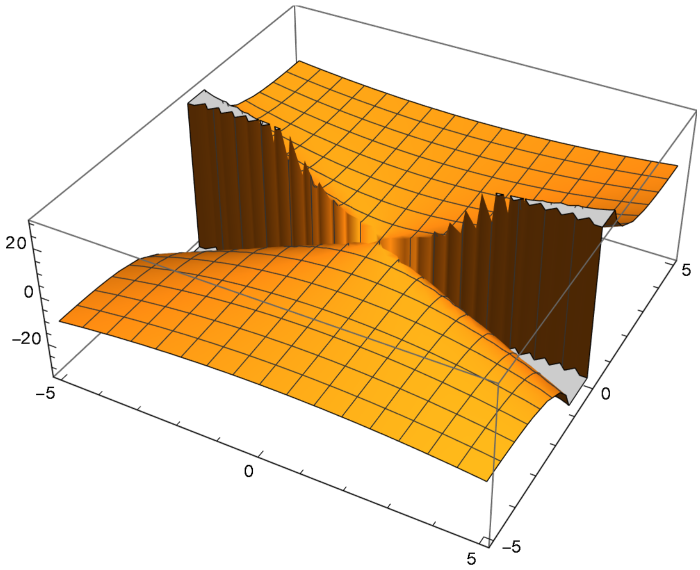
In[ ]:=
```
pde3 = x * D[u[x, y], x] + y * D[u[x, y], y] == u[x, y] + 1
c3 = u[x, x^2] == x^2
s3 = DSolve[{pde3, c3}, u[x, y], {x, y}]
Plot3D[u[x, y] /. s3, {x, -5, 5}, {y, -5, 5}]
```

Out[ ]= $y\, u^{(0,1)}[x, y] + x\, u^{(1,0)}[x, y] == 1 + u[x, y]$

Out[ ]= $u[x, x^2] == x^2$

Out[ ]= $\left\{\left\{u[x, y] \to \frac{x^2 - y + y^2}{y}\right\}\right\}$

Out[ ]=

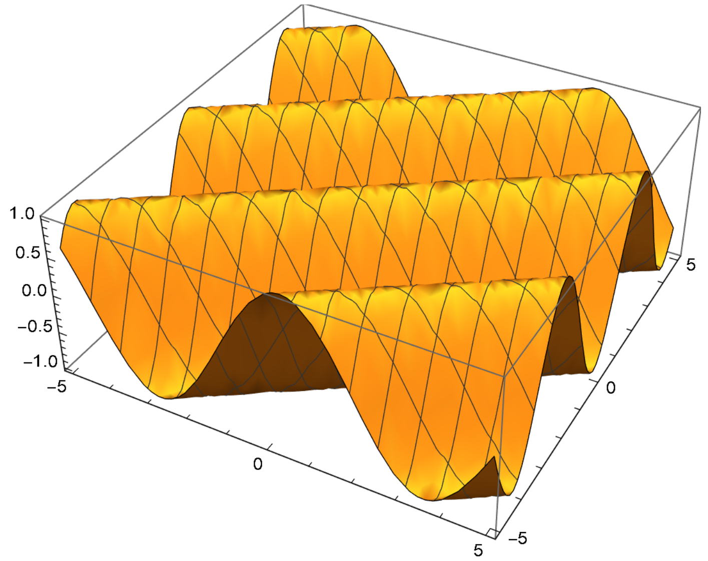


In[•]:=
```
pde4 = 3 * D[u[x, y], x] + 2 * D[u[x, y], y] == 0
c4 = u[x, 0] == Sin[x]
s4 = DSolve[{pde4, c4}, u[x, y], {x, y}]
Plot3D[u[x, y] /. s4, {x, -5, 5}, {y, -5, 5}]
```

Out[•]= $2\,u^{(0,1)}[x, y] + 3\,u^{(1,0)}[x, y] == 0$

Out[•]= $u[x, 0] == \mathrm{Sin}[x]$

Out[•]= $\left\{\left\{u[x, y] \to -\mathrm{Sin}\left[\frac{3}{2}\left(-\frac{2x}{3} + y\right)\right]\right\}\right\}$

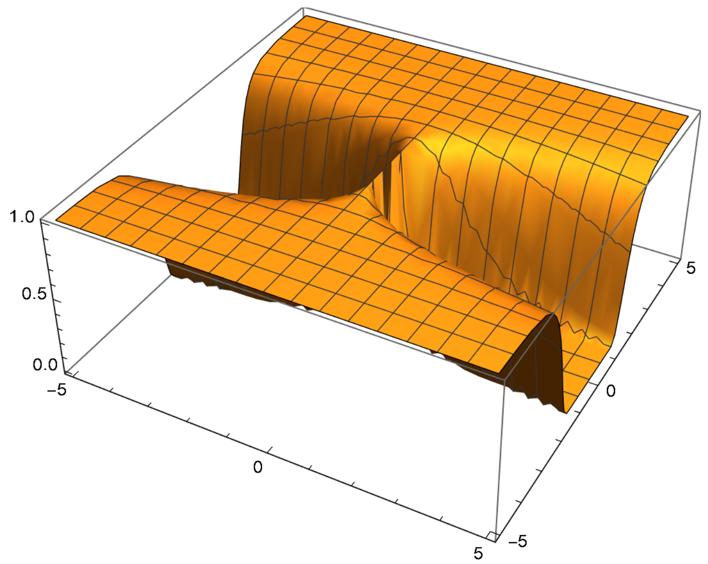
Out[•]=


In[•]:=
```
pde5 = 3 * x * D[u[x, y], x] + y * D[u[x, y], y] == 0
c5 = u[x, 1] == Exp[-x^2]
s5 = DSolve[{pde5, c5}, u[x, y], {x, y}]
Plot3D[u[x, y] /. s5, {x, -5, 5}, {y, -5, 5}]
```

Out[•]= $y\,u^{(0,1)}[x, y] + 3\,x\,u^{(1,0)}[x, y] == 0$

Out[•]= $u[x, 1] == e^{-x^2}$

Out[•]= $\left\{\left\{u[x, y] \to e^{-\frac{x^2}{y^6}}\right\}\right\}$

Out[•]=


Practical 4:

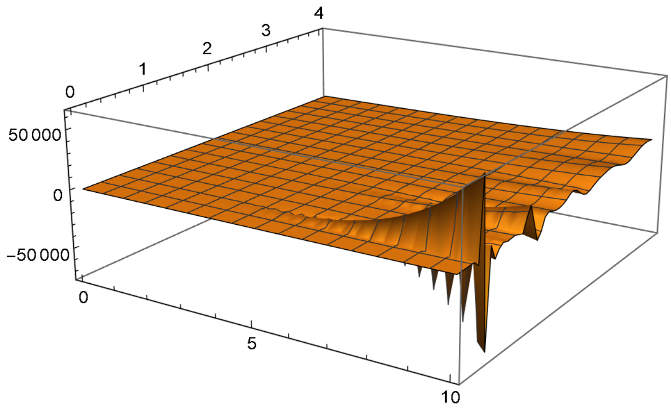
Solutions of Wave Equations

In[ ]:=
```
pde = D[u[x, t], {t, 2}] - 4 * D[u[x, t], {x, 2}] == 0
NDSolve[{pde, u[x, 0] == Sin[x], Derivative[0, 1][u][x, 0] == x^2, u[0, t] == 0},
 u, {x, 0, 1}, {t, 0, 4}]
Plot3D[Evaluate[u[x, t] /. First[%]], {x, 0, 10}, {t, 0, 4}, PlotRange → All]
```

Out[ ]= $u^{(0,2)}[x, t] - 4\,u^{(2,0)}[x, t] == 0$

Out[ ]= {{u → InterpolatingFunction[Domain: {{0., 1.}, {0., 4.}} Output: scalar]}}

Out[ ]=

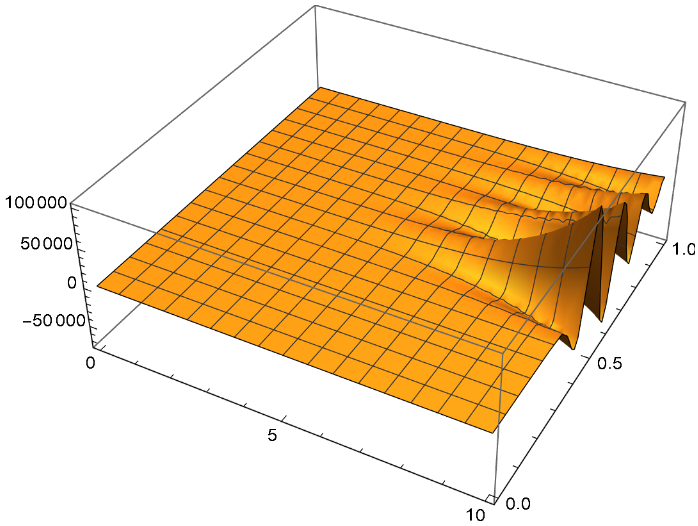


In[∘]:=
```
pde = D[u[x, t], {t, 2}] - 4 * D[u[x, t], {x, 2}] == 0
NDSolve[{pde, u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x (1 - x), u[0, t] == 0},
 u, {x, 0, 1}, {t, 0, 1}]
Plot3D[Evaluate[u[x, t] /. First[%]], {x, 0, 10}, {t, 0, 1}, PlotRange → All]
```

Out[∘]= $u^{(0,2)}[x, t] - 4 u^{(2,0)}[x, t] == 0$

Out[∘]= {{u → InterpolatingFunction[Domain: {{0., 1.}, {0., 1.}} Output: scalar]}}

Out[∘]=

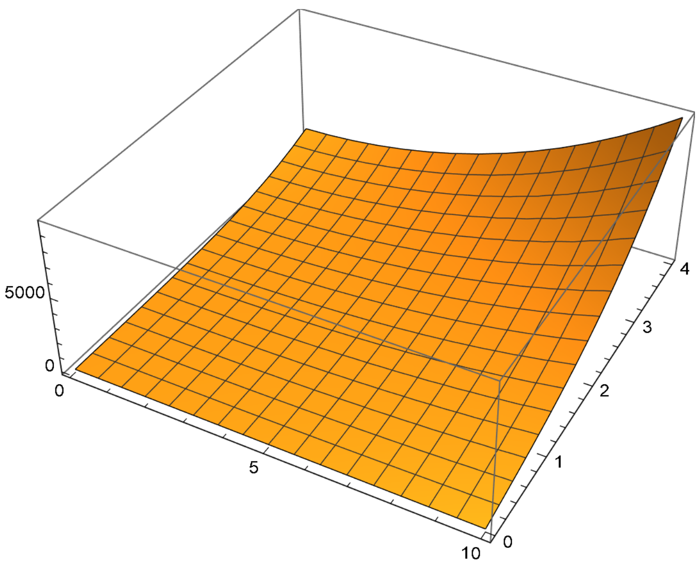


In[∘]:=
```
pde = D[u[x, t], {t, 2}] - 9 * D[u[x, t], {x, 2}] == 0
NDSolve[{pde, u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x^3,
  Derivative[1, 0][u][0, t] == 0}, u, {x, 0, 10}, {t, 0, 9}]
Plot3D[Evaluate[u[x, t] /. First[%]], {x, 0, 10}, {t, 0, 4}, PlotRange → All]
```

Out[∘]= $u^{(0,2)}[x, t] - 9 u^{(2,0)}[x, t] == 0$

Out[∘]= {{u → InterpolatingFunction[Domain: {{0., 10.}, {0., 9.}} Output: scalar]}}

Out[∘]=

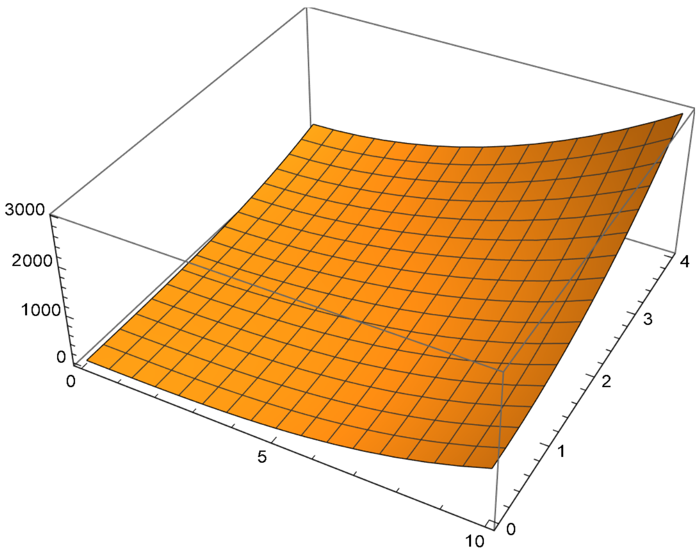


In[ ]:=
```
pde = D[u[x, t], {t, 2}] - 4 * D[u[x, t], {x, 2}] == 0
NDSolve[{pde, u[x, 0] == x^3, Derivative[0, 1][u][x, 0] == x,
   Derivative[1, 0][u][0, t] == 0}, u, {x, 0, 10}, {t, 0, 9}]
Plot3D[Evaluate[u[x, t] /. First[%]], {x, 0, 10}, {t, 0, 4}, PlotRange → All]
```

Out[ ]= $u^{(0,2)}[x, t] - 4\, u^{(2,0)}[x, t] == 0$

Out[ ]= {{u → InterpolatingFunction[Domain: {{0., 10.}, {0., 9.}} Output: scalar]}}

Out[ ]=



Practical 5:

Solving the Heat Equation

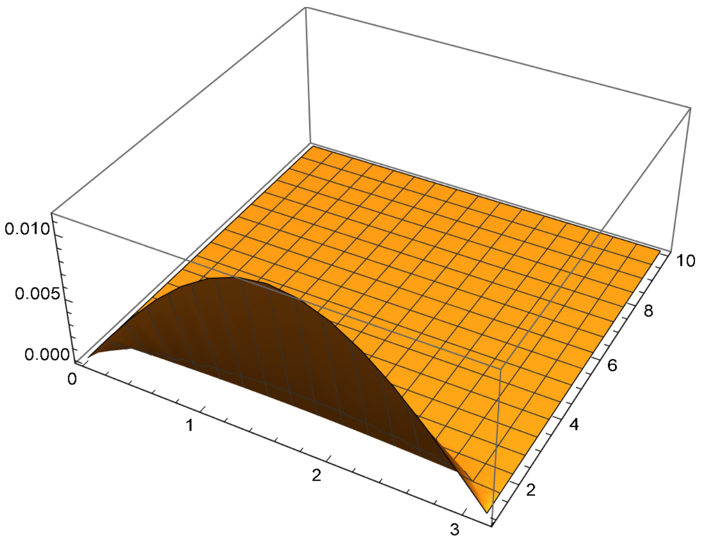
Find the solution of Heat equation $\frac{du}{dt} = 5\frac{d^2 u}{dx^2}$

u(x,0)=1-Cos2x,
u(0,t)=0
u(π,t)=0

In[ ]:=
```
pde1 = D[u[x, t], t] - 5 * D[u[x, t], x, x] == 0
NDSolve[{pde1, u[x, 0] == 1 - Cos[2 * x], u[0, t] == 0, u[Pi, t] == 0},
 u, {x, 0, Pi}, {t, 1, 10}]
Plot3D[Evaluate[u[x, t] /. %], {x, 0, Pi}, {t, 1, 10}, PlotRange → All]
```

Out[ ]= $u^{(0,1)}[x, t] - 5\,u^{(2,0)}[x, t] == 0$

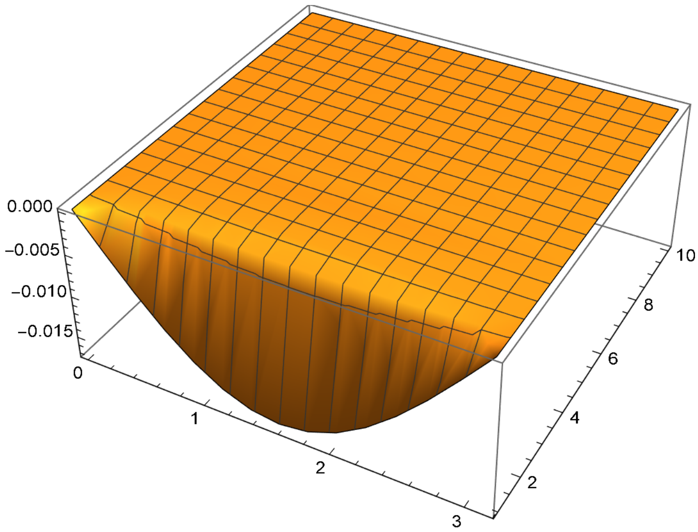
Out[ ]= {{u → InterpolatingFunction[Domain: {{0., 3.14}, {1., 10.}} Output: scalar]}}

Out[ ]=


In[•]:=
```
pde2 = D[u[x, t], t] - 5 * D[u[x, t], x, x] == 0
NDSolve[{pde2, u[x, 0] == x * (x - Pi), u[0, t] == 0, u[Pi, t] == 0},
 u, {x, 0, Pi}, {t, 1, 10}]
Plot3D[Evaluate[u[x, t] /. %], {x, 0, Pi}, {t, 1, 10}, PlotRange → All]
```

Out[•]= $u^{(0,1)}[x, t] - 5\, u^{(2,0)}[x, t] == 0$

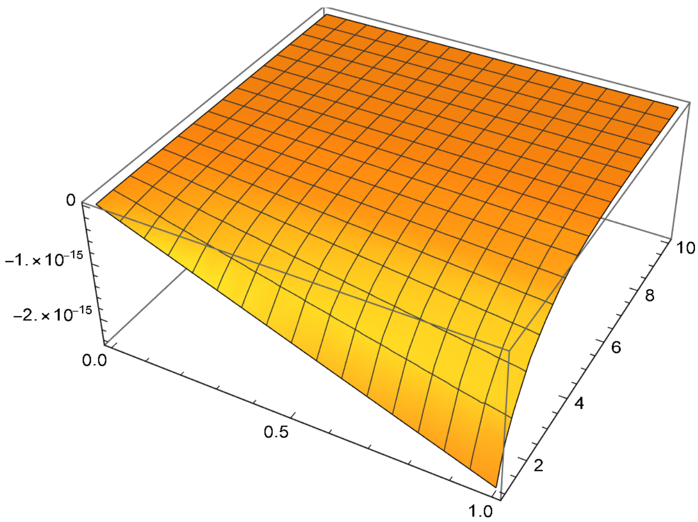
Out[•]= {{u → InterpolatingFunction[Domain: {{0., 3.14}, {1., 10.}} Output: scalar]}}

Out[•]=


In[•]:=
```
pde3 = D[u[x, t], t] - 50 * D[u[x, t], x, x] == 0
NDSolve[{pde3, u[x, 0] == 2 * Sin[2 * Pi * x] + 6 * Sin[8 * Pi * x], u[0, t] == 0, u[1, t] == 0},
 u, {x, 0, 1}, {t, 1, 10}]
Plot3D[Evaluate[u[x, t] /. %], {x, 0, 1}, {t, 1, 10}, PlotRange → All]
```

Out[•]= $u^{(0,1)}[x, t] - 50\, u^{(2,0)}[x, t] == 0$

Out[•]= {{u → InterpolatingFunction[Domain: {{0., 1.}, {1., 10.}} Output: scalar]}}

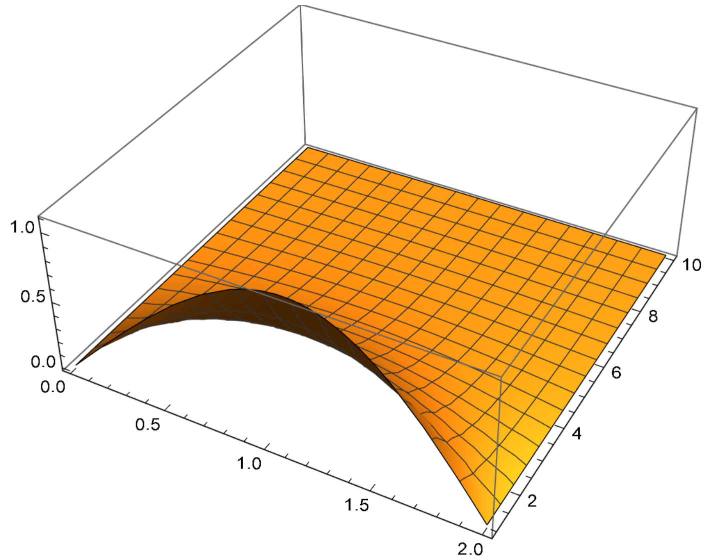
Out[•]=


In[∘]:=
```
pde4 = 4 * D[u[x, t], t] - D[u[x, t], x, x] == 0
NDSolve[{pde4, u[x, 0] == 2 * Sin[Pi * x / 2] - Sin[Pi * x] + 4 * Sin[2 * Pi * x],
  u[0, t] == 0, u[2, t] == 0}, u, {x, 0, 2}, {t, 1, 10}]
Plot3D[Evaluate[u[x, t] /. %], {x, 0, 2}, {t, 1, 10}, PlotRange → All]
```

Out[∘]= $4\,u^{(0,1)}[x, t] - u^{(2,0)}[x, t] == 0$

Out[∘]= {{u → InterpolatingFunction[Domain: {{0., 2.}, {1., 10.}} Output: scalar]}}

Out[∘]=



Practical 6:

Solution of System of ordinary differential equation

In[∘]:=
```
e1 = y'[x] + y[x] == z[x] + Exp[x];
e2 = z'[x] + z[x] == y[x] + Exp[x];
DSolve[{e1, e2}, {y[x], z[x]}, x]
```

Out[∘]= $$\left\{\left\{y[x] \to \frac{1}{2} e^{-x}\left(-1+e^{2x}\right) + \frac{1}{2} e^{-x}\left(1+e^{2x}\right) + \frac{1}{2} e^{-2x}\left(1+e^{2x}\right) c_1 + \frac{1}{2} e^{-2x}\left(-1+e^{2x}\right) c_2,\right.\right.$$
$$\left.\left. z[x] \to \frac{1}{2} e^{-x}\left(-1+e^{2x}\right) + \frac{1}{2} e^{-x}\left(1+e^{2x}\right) + \frac{1}{2} e^{-2x}\left(-1+e^{2x}\right) c_1 + \frac{1}{2} e^{-2x}\left(1+e^{2x}\right) c_2\right\}\right\}$$

In[∘]:=
```
e3 = x'[t] - 7 * x[t] + y[t] == 0;
e4 = y'[t] - 2 * x[t] - 5 * y[t] == 0;
DSolve[{e3, e4}, {x[t], y[t]}, t]
```

Out[∘]= $$\left\{\left\{x[t] \to -e^{6t} c_2 \operatorname{Sin}[t] + e^{6t} c_1 (\operatorname{Cos}[t] + \operatorname{Sin}[t]),\right.\right.$$
$$\left.\left. y[t] \to e^{6t} c_2 (\operatorname{Cos}[t] - \operatorname{Sin}[t]) + 2 e^{6t} c_1 \operatorname{Sin}[t]\right\}\right\}$$

In[∘]:=
```
e5 = x'[t] == 5 * x[t] + 3 * y[t];
e6 = y'[t] == 4 * x[t] + y[t];
c1 = x[0] == 0;
c2 = y[0] == 8;
DSolve[{e5, e6, c1, c2}, {x[t], y[t]}, t]
```

Out[∘]= $\left\{\left\{x[t] \to 3\, e^{-t}\left(-1+e^{8t}\right),\ y[t] \to 2\, e^{-t}\left(3+e^{8t}\right)\right\}\right\}$

In[∘]:=
```
e7 = x'[t] - 3 * x[t] == 0;
e8 = y'[t] - 4 * Exp[t] == 0;
c7 = x[0] == 0;
c8 = y[0] == Exp[5];
DSolve[{e7, e8, c7, c8}, {x[t], y[t]}, t]
```

Out[∘]= $\left\{\left\{x[t] \to 0,\ y[t] \to -4+e^{5}+4\,e^{t}\right\}\right\}$

In[∘]:=
```
e9 = 4 * x'[t] + x[t] + y[t] == 0;
e10 = y'[t] - 2 * y[t] + 5 * x[t] == 0;
DSolve[{e9, e10}, {x[t], y[t]}, t]
```

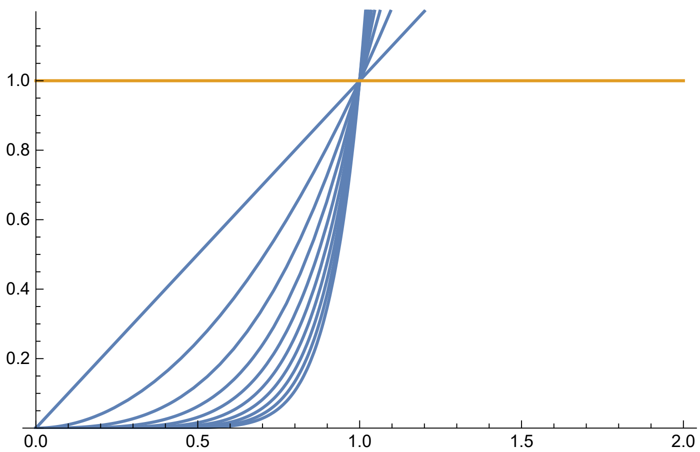
Out[∘]= $$\left\{\left\{x[t] \to -\frac{1}{322}\, e^{\frac{7t}{8}-\frac{\sqrt{161}\,t}{8}}\left(-161-9\sqrt{161}-161\, e^{\frac{\sqrt{161}\,t}{4}}+9\sqrt{161}\, e^{\frac{\sqrt{161}\,t}{4}}\right) C_1 - \frac{e^{\frac{7t}{8}-\frac{\sqrt{161}\,t}{8}}\left(-1+e^{\frac{\sqrt{161}\,t}{4}}\right) C_2}{\sqrt{161}},\ y[t] \to -\frac{20\, e^{\frac{7t}{8}-\frac{\sqrt{161}\,t}{8}}\left(-1+e^{\frac{\sqrt{161}\,t}{4}}\right) C_1}{\sqrt{161}} + \frac{1}{322}\, e^{\frac{7t}{8}-\frac{\sqrt{161}\,t}{8}}\left(161-9\sqrt{161}+161\, e^{\frac{\sqrt{161}\,t}{4}}+9\sqrt{161}\, e^{\frac{\sqrt{161}\,t}{4}}\right) C_2\right\}\right\}$$

Practical 7:

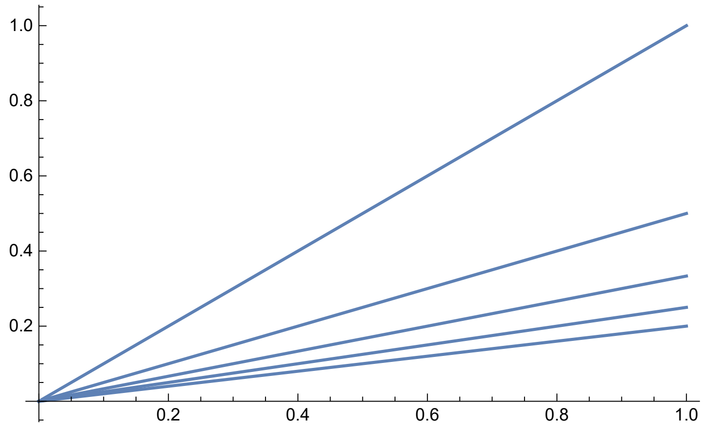
Draw the following sequence of functions on the given interval and discuss the point wise convergence

In[∘]:=
```
Plot[{Table[x^i, {i, 1, 10}], 1}, {x, 0, 2}, PlotRange → {0, 1.2}]
```

Out[∘]=



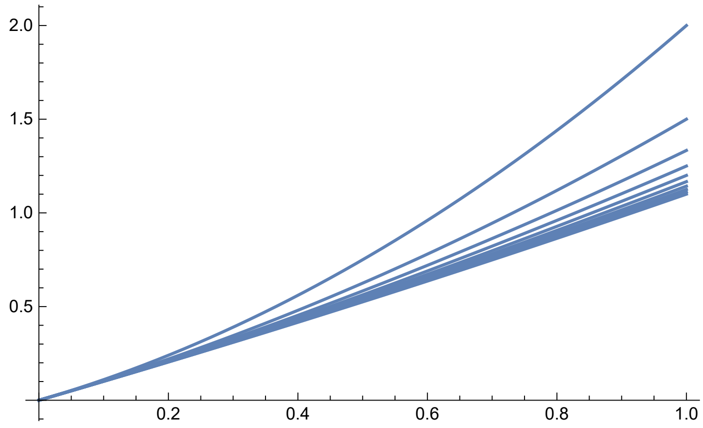
In[∘]:=
```
Plot[Table[x / i, {i, 1, 5}], {x, 0, 1}, PlotRange → All]
```

Out[∘]=


In[∘]:=
```
Limit[x / n, n → Infinity]
```

Out[∘]= 0

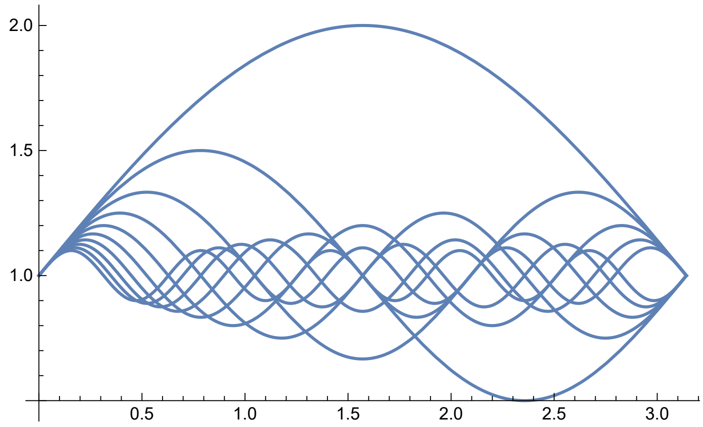
In[∘]:=
```
Plot[Table[{x^2 + i * x} / i, {i, 1, 10}], {x, 0, 1}]
```

Out[∘]=


In[∘]:=
```
Limit[{x^2 + n * x} / n, n → Infinity]
```

Out[∘]= {x}

In[∘]:=
```
Plot[Table[{Sin[i * x] + i} / i, {i, 1, 10}], {x, 0, Pi}]
```

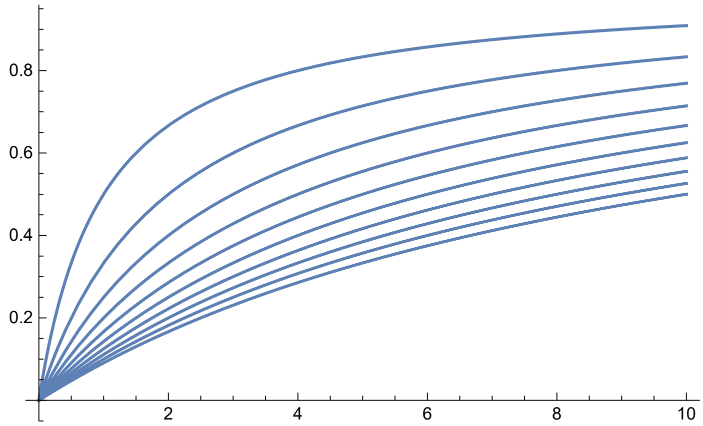
Out[∘]=


In[∘]:=
```
Limit[{Sin[n * x] + n} / n, n → Infinity]
```

Out[∘]= $\left\{ \boxed{1 \text{ if } x \in \mathbb{R}} \right\}$

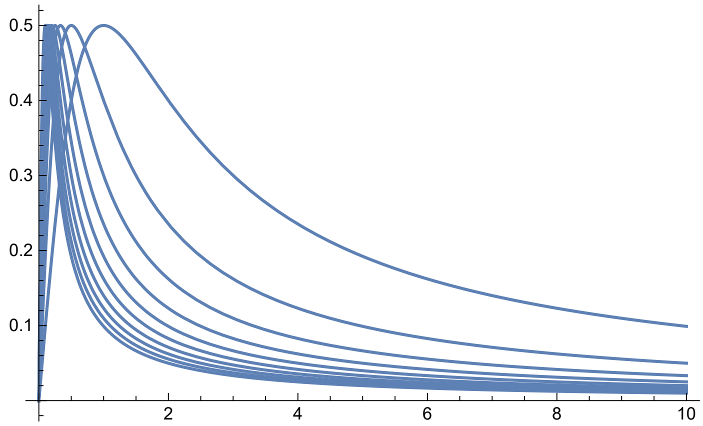
In[ ]:= `Plot[Table[x / {x + i}, {i, 1, 10}], {x, 0, 10}]`

Out[ ]=



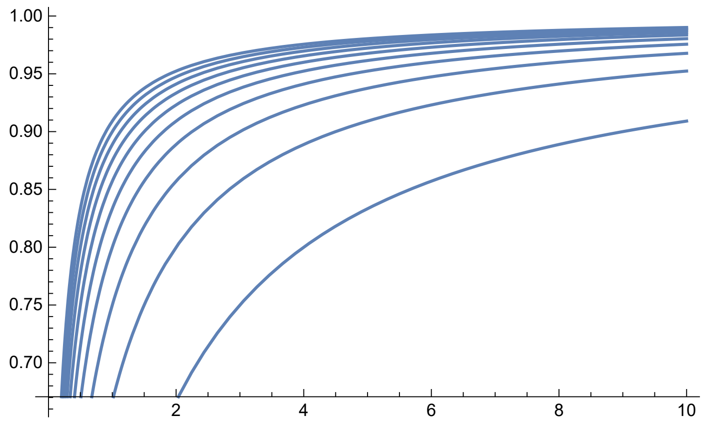
In[ ]:= `Plot[Table[{i * x} / {1 + i^2 * x^2}, {i, 1, 10}], {x, 0, 10}]`

Out[ ]=



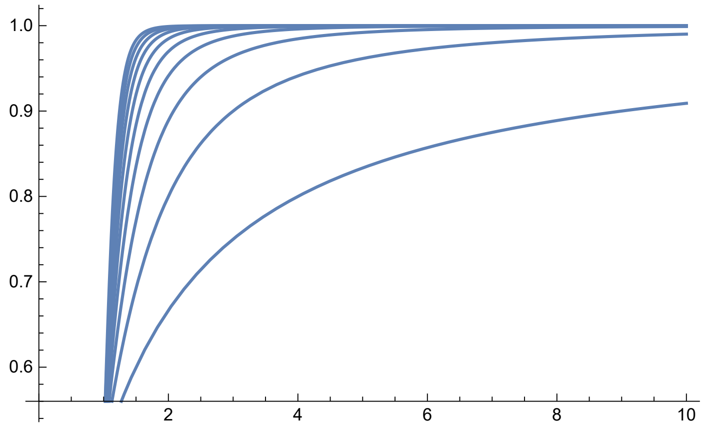
In[ ]:= `Plot[Table[{i * x} / {1 + i * x}, {i, 1, 10}], {x, 0, 10}]`

Out[ ]=



In[ ]:=
```
Plot[Table[{x^i} / {1 + x^i}, {i, 1, 10}], {x, 0, 10}]
```

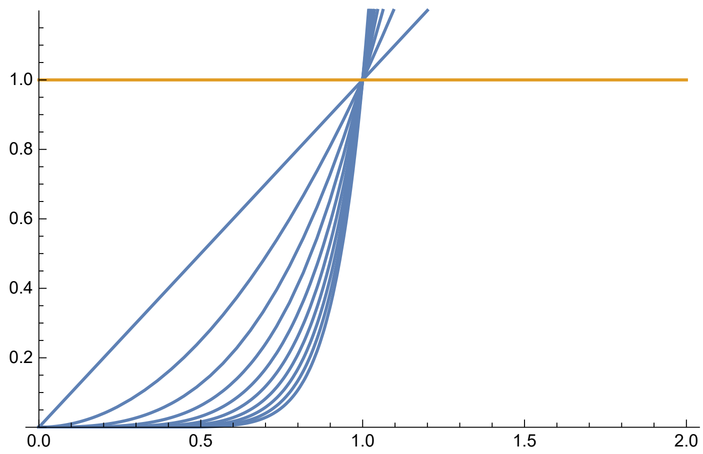
Out[ ]=



Practical 8
Discuss the uniform convergence of sequence of function:

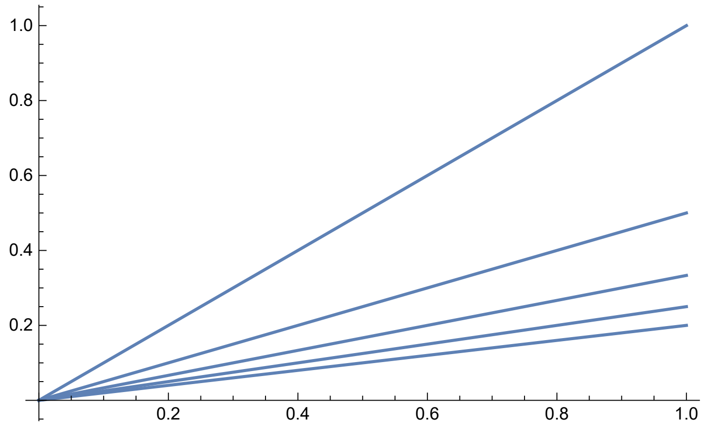
In[ ]:=
```
Plot[{Table[x^i, {i, 1, 10}], 1}, {x, 0, 2}, PlotRange → {0, 1.2}]
```

Out[ ]=



x^i do not converges uniformly.

In[•]:= `Plot[Table[x / i, {i, 1, 5}], {x, 0, 1}, PlotRange → All]`

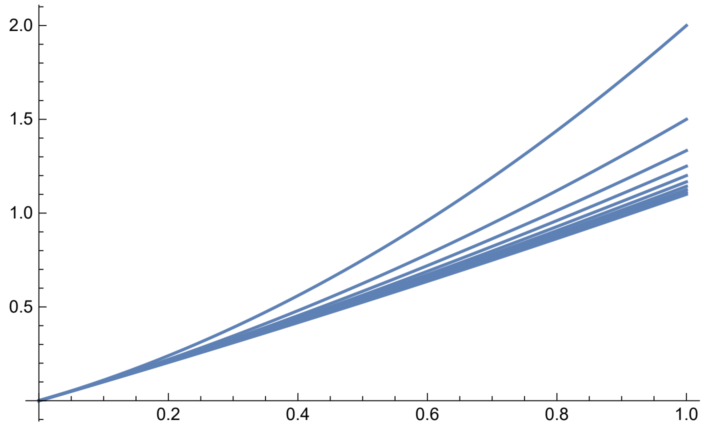
Out[•]=


In[•]:= `Limit[x / n, n → Infinity]`

Out[•]= 0

$\frac{x}{i}$ do not converge uniformly.

In[•]:= `Plot[Table[{x^2 + i * x} / i, {i, 1, 10}], {x, 0, 1}]`

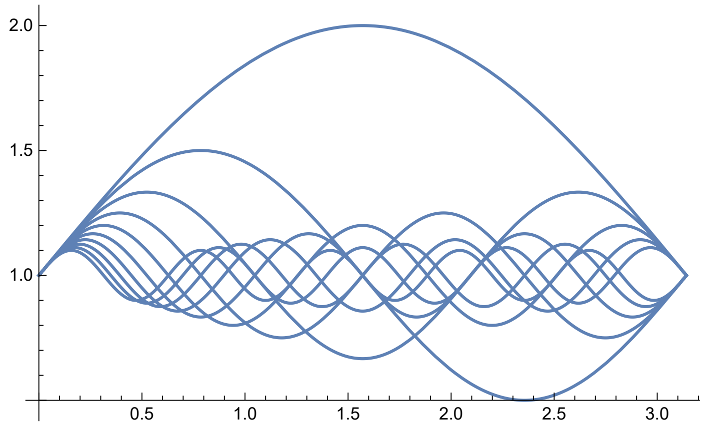
Out[•]=


In[•]:= `Limit[{x^2 + n * x} / n, n → Infinity]`

Out[•]= {x}

$\frac{x^2+ix}{i}$ do not converge uniformly.

In[•]:= `Plot[Table[{Sin[i * x] + i} / i, {i, 1, 10}], {x, 0, Pi}]`

Out[•]=


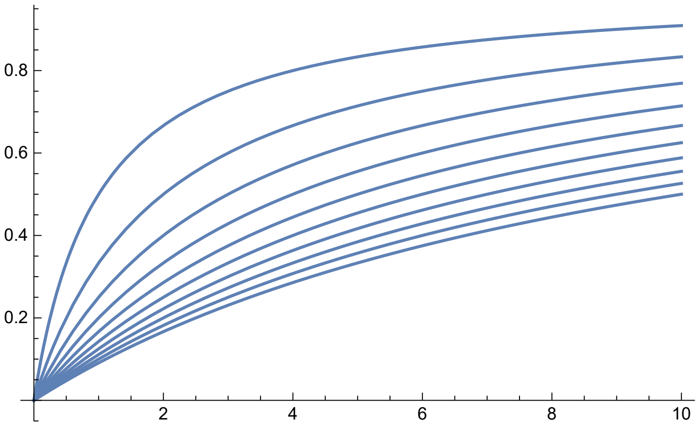
In[∘]:=
```
Limit[{Sin[n * x] + n} / n, n → Infinity]
```

Out[∘]= $\left\{\boxed{1 \text{ if } x \in \mathbb{R}}\right\}$

$\frac{\text{Sin}[ix]+i}{i}$ do not converges uniformly.

In[∘]:=
```
Plot[Table[x / {x + i}, {i, 1, 10}], {x, 0, 10}]
```

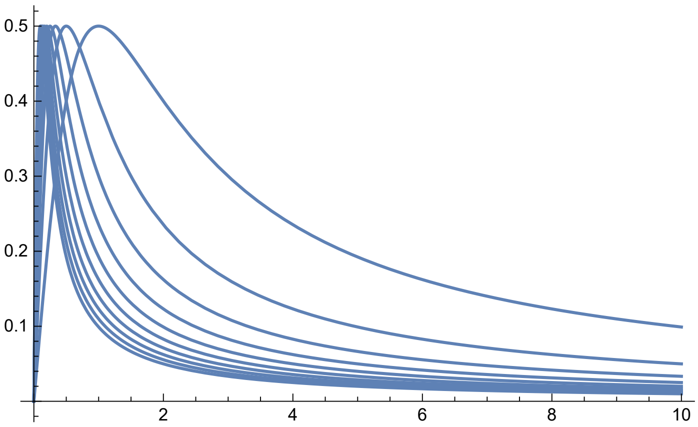
Out[∘]=



$\frac{x^i}{x+i}$ do not converge uniformly.

In[∘]:=
```
Plot[Table[{i * x} / {1 + i^2 * x^2}, {i, 1, 10}], {x, 0, 10}]
```

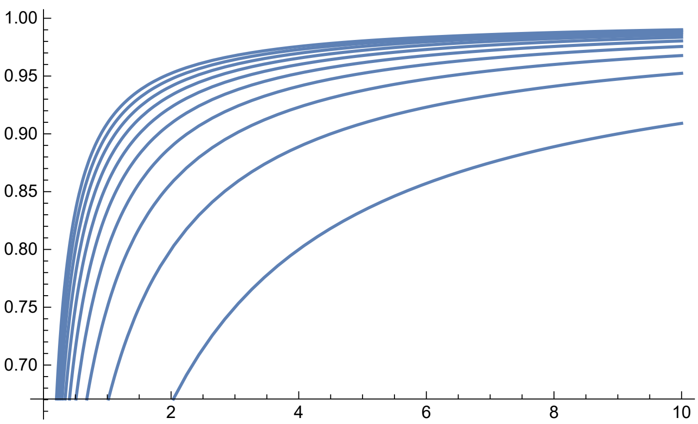
Out[∘]=



$\frac{\text{ix}}{1+i^2\,x^2}$ converges uniformly to 0.

In[∘]:=
```
Plot[Table[{i * x} / {1 + i * x}, {i, 1, 10}], {x, 0, 10}]
```

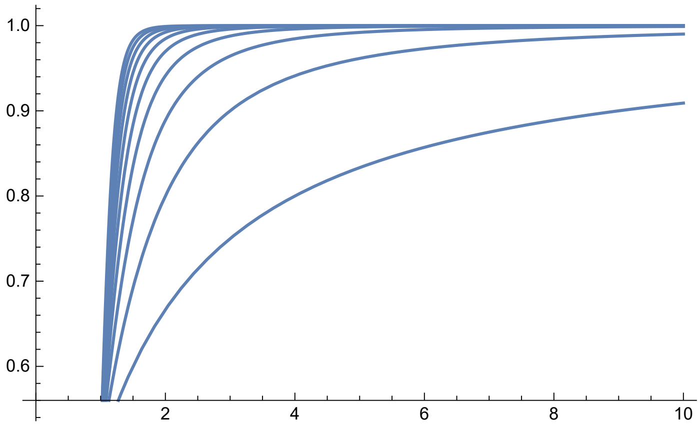
Out[∘]=



$\frac{\text{ix}}{i+\text{ix}}$ converge uniformly to 1.

In[ ]:=
```
Plot[Table[{x^i} / {1 + x^i}, {i, 1, 10}], {x, 0, 10}]
```

Out[ ]=



$\frac{x^i}{1+x^i}$ converges uniformly.