Practical - 1

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Ques.1) Solve yu_y - xu_x=1

In[ ]:= `DSolve[y * D[u[x, y], y] - x * D[u[x, y], x] == 1, u[x, y], {x, y}]`

Out[ ]= $\{\{u[x, y] \to -\mathrm{Log}[x] + c_1[x\, y]\}\}$

Ques.2) Solve u_x - yu_y=0

In[ ]:= `DSolve[D[u[x, y], x] - y * D[u[x, y], y] == 0, u[x, y], {x, y}]`

Out[ ]= $\{\{u[x, y] \to c_1[e^x\, y]\}\}$

Ques.3) Solve (1+x^2) u_x - u_y=0

In[ ]:= `DSolve[(1 + x^2) * D[u[x, y], y] - D[u[x, y], y] == 0, u[x, y], {x, y}]`

Out[ ]= $\{\{u[x, y] \to c_1[x]\}\}$

Ques.4) Solve y^2u_x - xyu_y=x(u-2y)

In[ ]:= `DSolve[y^2 * D[u[x, y], x] - x * y * D[u[x, y], y] == x (u[x, y] - 2 y), u[x, y], {x, y}]`

Out[ ]= $$\left\{\left\{u[x, y] \to \frac{-x^2\sqrt{-y^2} + \sqrt{y^2}\, c_1\left[\frac{1}{2}(x^2+y^2)\right]}{\sqrt{-y^2}\,\sqrt{y^2}}\right\}, \left\{u[x, y] \to \frac{x^2\sqrt{-y^2} + \sqrt{y^2}\, c_1\left[\frac{1}{2}(x^2+y^2)\right]}{\sqrt{-y^2}\,\sqrt{y^2}}\right\}\right\}$$

In[ ]:= `Clear All`

Out[ ]= All Clear

Ques.5) Solve xu_x - yu_y=u

In[ ]:= `DSolve[x * D[u[x, y], x] + y * D[u[x, y], y] == u[x, y], u[x, y], {x, y}]`

Out[ ]= $$\left\{\left\{u[x, y] \to x\, c_1\left[\frac{y}{x}\right]\right\}\right\}$$

Ques.6) Solve xu_x - yu_y=u

In[ ]:= `Clear All`

Out[ ]= All Clear

Practical-2

Ques.1) Solve z_x + 2xz_y=1+z with Cauchy data z(x,y)=x^2 on y=3x+1

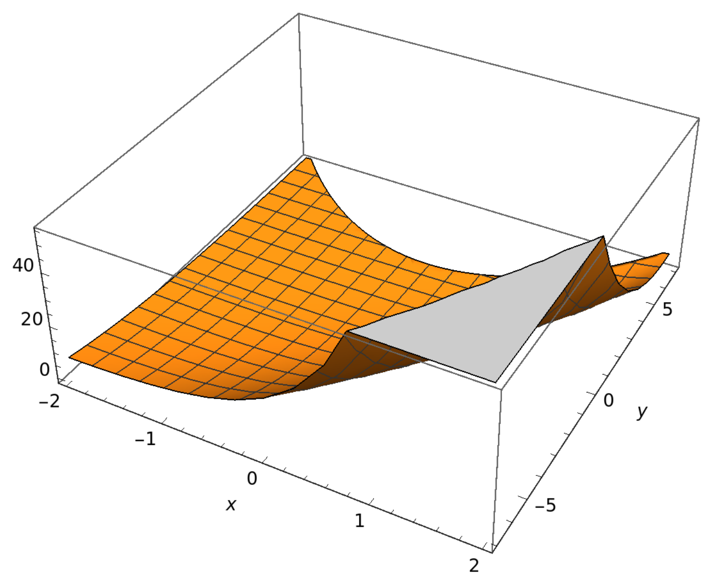
In[ ]:= `A = D[z[x, y], x] + 2 * x * D[z[x, y], y] == 1 + z[x, y]`

Out[ ]= $2\,x\,z^{(0,1)}[x,y] + z^{(1,0)}[x,y] == 1 + z[x,y]$

In[ ]:= `solb = DSolve[{A, z[x, 3 * x + 1] == x^2}, z[x, y], {x, y}]`

Out[ ]= $$\left\{\left\{z[x,y] \to -\frac{1}{2\,e^{3/2}}\left(2\,e^{3/2} - 13\,e^{x+\frac{1}{2}\sqrt{13+4x^2-4y}} - 2\,e^{x+\frac{1}{2}\sqrt{13+4x^2-4y}}\,x^2 + 3\,e^{x+\frac{1}{2}\sqrt{13+4x^2-4y}}\sqrt{13+4x^2-4y} + 2\,e^{x+\frac{1}{2}\sqrt{13+4x^2-4y}}\,y\right)\right\}\right\}$$

In[ ]:= `Plot3D[z[x, y] /. solb, {x, -2, 2}, {y, -7, 7}, AxesLabel → Automatic]`

Out[ ]= 

In[ ]:= `Clear All`

Out[ ]= `All Clear`

Ques.2) Solve xu_x + yu_y=2xy with Cauchy data u(x,y)=2 on y=x^2

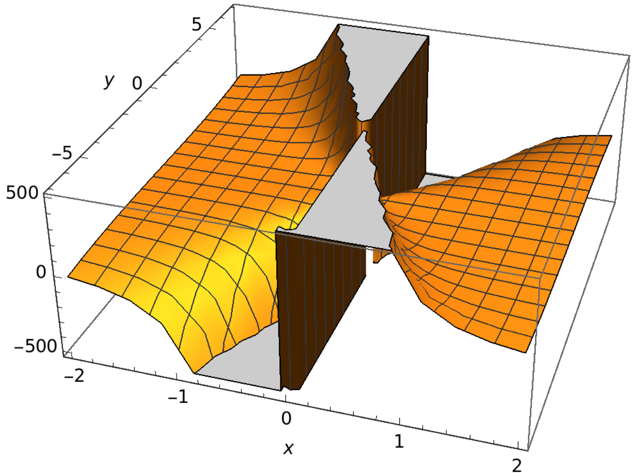
In[ ]:= `A = x * D[u[x, y], x] + y * D[u[x, y], y] == 2 * x * y`

Out[ ]= $y\,u^{(0,1)}[x,y] + x\,u^{(1,0)}[x,y] == 2\,x\,y$

In[ ]:= `solb = DSolve[{A, u[x, x^2] == 2}, u[x, y], {x, y}]`

Out[ ]= $$\left\{\left\{u[x,y] \to \frac{2\,x^3 + x^4\,y - y^3}{x^3}\right\}\right\}$$

In[ ]:= `Plot3D[u[x, y] /. solb, {x, -2, 2}, {y, -7, 7}, AxesLabel → Automatic]`

Out[ ]=


In[ ]:= `Clear All`

Out[ ]= `All Clear`

Ques.3) Solve xu_x + yu_y=u+1 with Cauchy data u(x,y)=x^2 on y=x^2

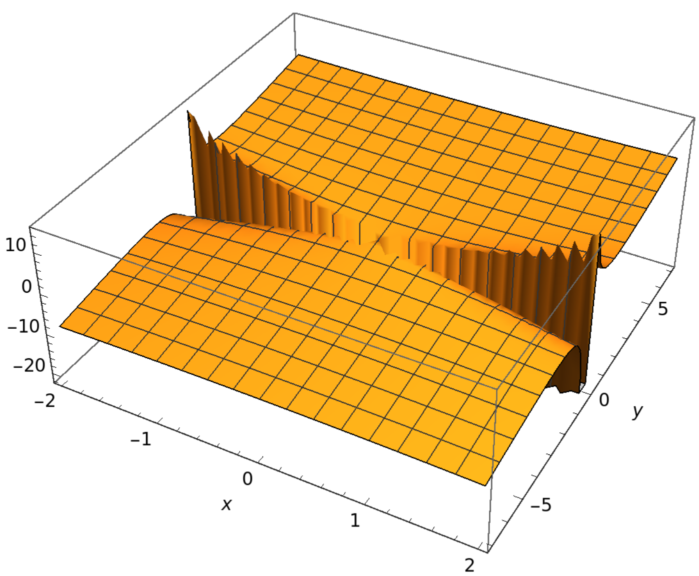
In[ ]:= `A = x * D[u[x, y], x] + y * D[u[x, y], y] == u[x, y] + 1`

Out[ ]= $y\, u^{(0,1)}[x, y] + x\, u^{(1,0)}[x, y] == 1 + u[x, y]$

In[ ]:= `solb = DSolve[{A, u[x, x^2] == x^2}, u[x, y], {x, y}]`

Out[ ]= $\left\{\left\{u[x, y] \to \frac{x^2 - y + y^2}{y}\right\}\right\}$

In[ ]:= `Plot3D[u[x, y] /. solb, {x, -2, 2}, {y, -7, 7}, AxesLabel → Automatic]`

Out[ ]=


In[ ]:=
```
Clear All
```
Out[ ]=
```
All Clear
```

Ques.4) Solve xu_x + (x+y)u_y=u+1 with Cauchy data u(x,y)=x^2 on y=0

In[ ]:=
```
A = x * D[u[x, y], x] + (x + y) * D[u[x, y], y] == u[x, y] + 1
```
Out[ ]= $(x+y)\,u^{(0,1)}[x, y] + x\,u^{(1,0)}[x, y] == 1 + u[x, y]$

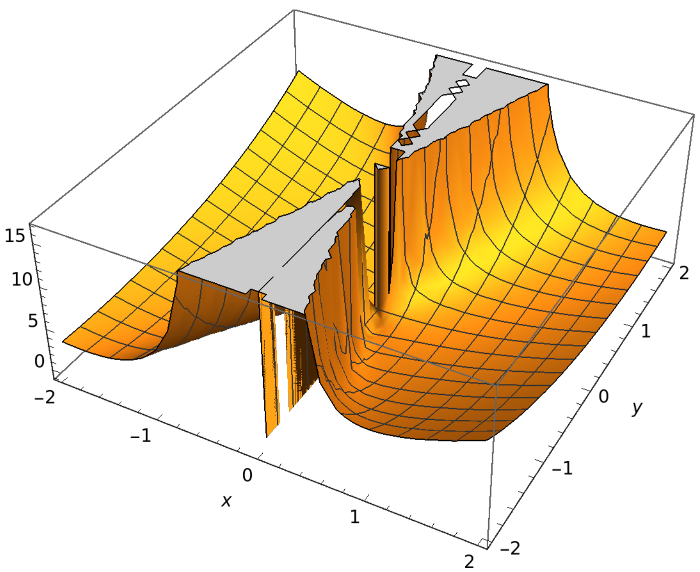
In[ ]:=
```
solb = DSolve[{A, u[x, 0] == x^2}, u[x, y], {x, y}]
```
Out[ ]= $\left\{\left\{u[x, y] \to e^{-\frac{y}{x}}\left(-e^{\frac{y}{x}} + e^{\frac{2y}{x}} + x^2\right)\right\}\right\}$

In[ ]:=
```
Plot3D[u[x, y] /. solb, {x, -2, 2}, {y, -2, 2}, AxesLabel → Automatic]
```
General: Exp[−768.] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[−1536.] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[−896.] is too small to represent as a normalized machine number; precision may be lost.

General: Further output of General::munfl will be suppressed during this calculation.

Out[ ]=


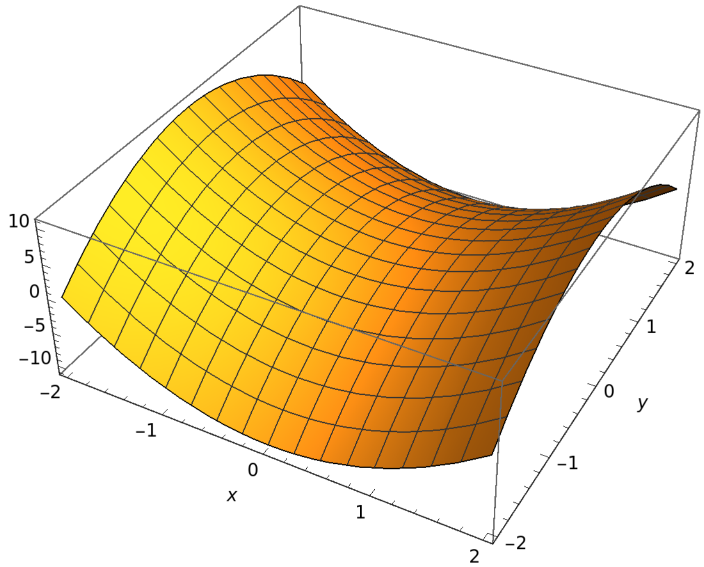
In[ ]:=
```
Clear All
```
Out[ ]=
```
All Clear
```

Ques.5) Solve yu_x + xu_y=0 with Cauchy data u(0,y)=exp(-y^2) on x=0

In[ ]:=
```
A = y * D[u[x, y], x] + x * D[u[x, y], y] == 0
```
Out[ ]= $x\,u^{(0,1)}[x, y] + y\,u^{(1,0)}[x, y] == 0$

In[ ]:=
```
solb = DSolve[{A, u[0, y] == e (-y^2)}, u[x, y], {x, y}]
```
Out[ ]= $\{\{u[x, y] \to e\,(x^2 - y^2)\}\}$

In[ ]:=
```
Plot3D[u[x, y] /. solb, {x, -2, 2}, {y, -2, 2}, AxesLabel → Automatic]
```

Out[ ]=


In[ ]:=
```
Clear All
```

Out[ ]= All Clear

Practical-3

Ques.1)Solve P.D.E. $u_x - u_y = 1$, with Cauchy data u(x,0)= x^2

In[ ]:=
```
sol1 = DSolve[
  {x'[t] == 1, y'[t] == -1, u'[t] == 1, x[0] == s, y[0] == 0, u[0] == s^2}, {x[t], y[t], u[t]}, t]
```

Out[ ]= $\{\{x[t] \to s + t,\ y[t] \to -t,\ u[t] \to s^2 + t\}\}$

```
Print["x[t]=", sol1[[1, 1, 2]]]
Print["y[t]=", sol1[[1, 2, 2]]]
Print["u[t]=", sol1[[1, 3, 2]]]
```

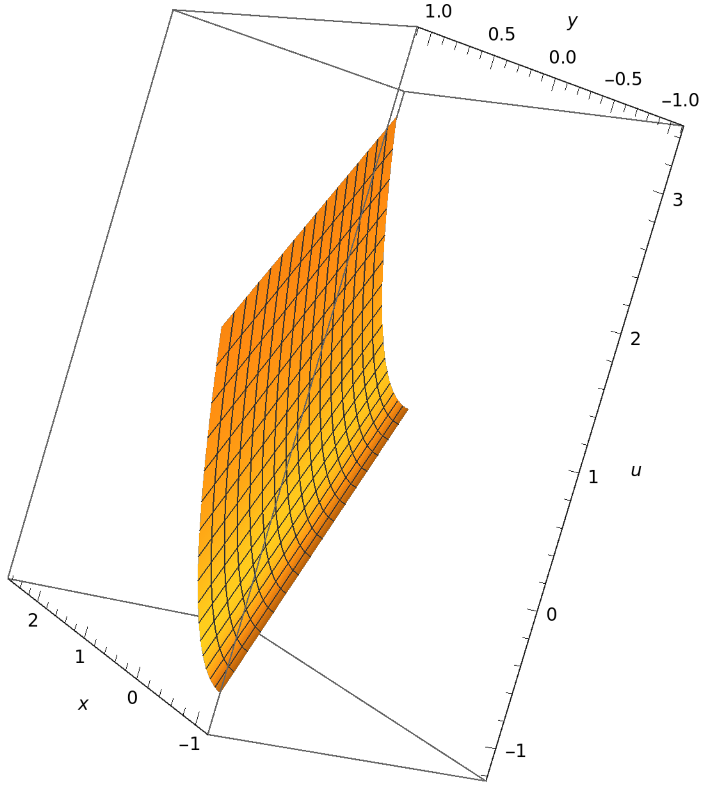
x[t]=s + t

y[t]=-t

u[t]=s^2 + t

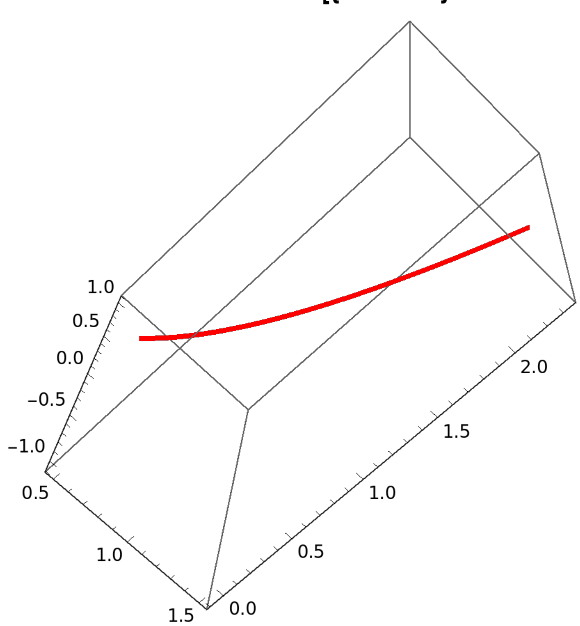
```
In[ ]:= nfig1 = ParametricPlot3D[{sol1[[1, 1, 2]], sol1[[1, 2, 2]], sol1[[1, 3, 2]]},
          {t, -1, 1}, {s, 0, 1.5}, AxesLabel → {x, y, u}]
```

Out[ ]=



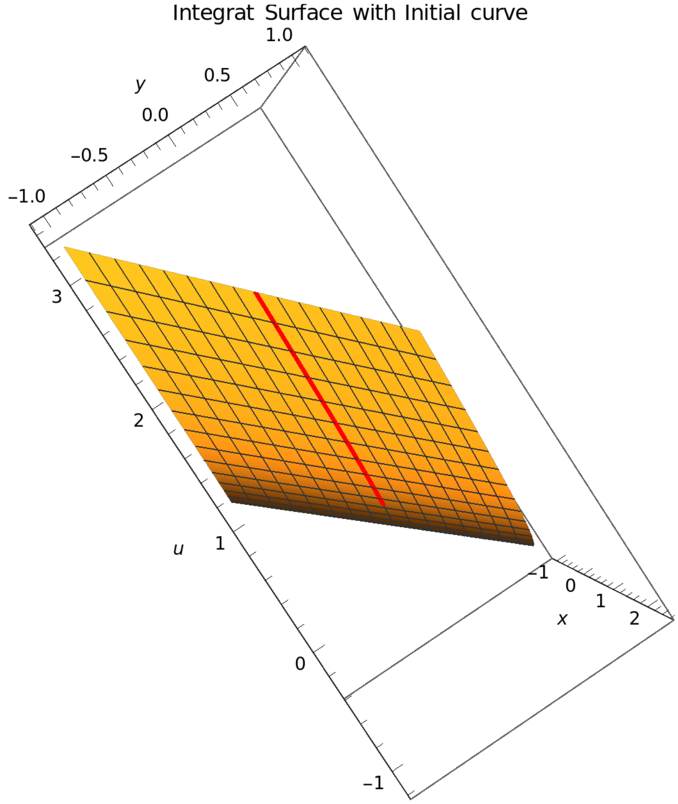

```
In[ ]:= nfig2 = ParametricPlot3D[{s, 0, s^2}, {s, 0.5, 1.5}, PlotStyle → {Thick, Red}]
```

Out[ ]=



In[∘]:= `Show[nfig1, nfig2, PlotLabel → "Integrat Surface with Initial curve"]`

Out[∘]=



In[∘]:= `Clear All`

Out[∘]= All Clear

Ques.2)Solve P.D.E. u_x +xu_y =0, with Cauchy data u(0,y)= Sinx

In[∘]:=
```
sol1 = DSolve[
  {x'[t] == 1, y'[t] == x[t], u'[t] == 0, x[0] == 0, y[0] == s, u[0] == Sin[s]}, {x[t], y[t], u[t]}, t]
```

Out[∘]= $\left\{\left\{x[t] \to t,\ y[t] \to \frac{1}{2}\left(2\,s + t^2\right),\ u[t] \to \mathrm{Sin}[s]\right\}\right\}$

In[∘]:=
```
Print["x[t]=", sol1[[1, 1, 2]]]
Print["y[t]=", sol1[[1, 2, 2]]]
Print["u[t]=", sol1[[1, 3, 2]]]
```

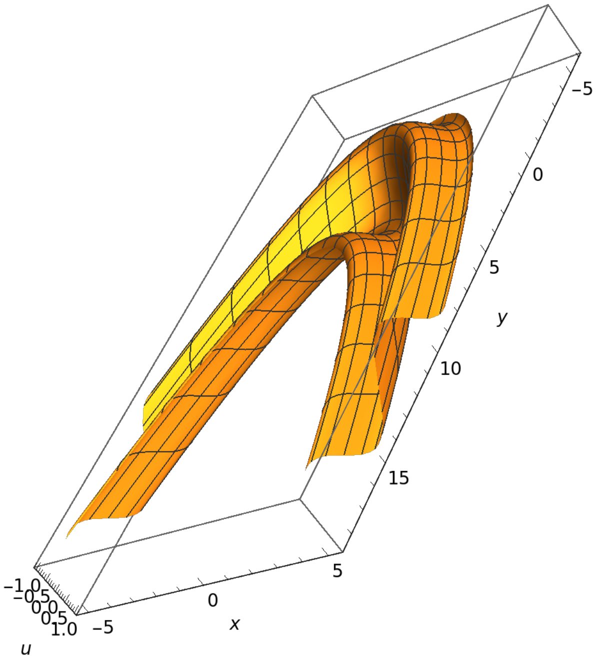
x[t]=t

y[t]=$\frac{1}{2}\left(2\,s + t^2\right)$

u[t]=Sin[s]

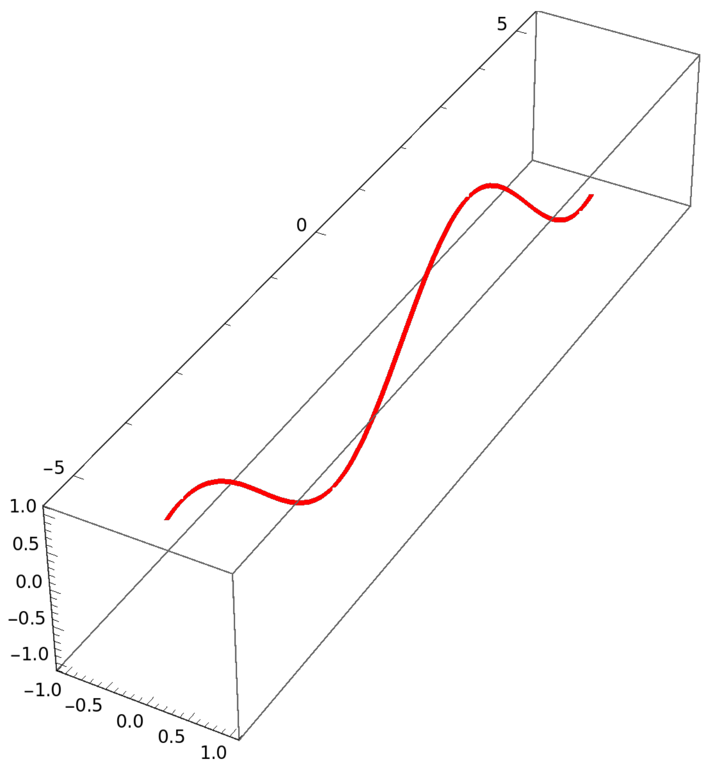
In[ ]:=
```
nfig1 = ParametricPlot3D[{sol1[[1, 1, 2]], sol1[[1, 2, 2]], sol1[[1, 3, 2]]},
  {t, -5, 5}, {s, -5, 5}, AxesLabel → {x, y, u}]
```

Out[ ]=

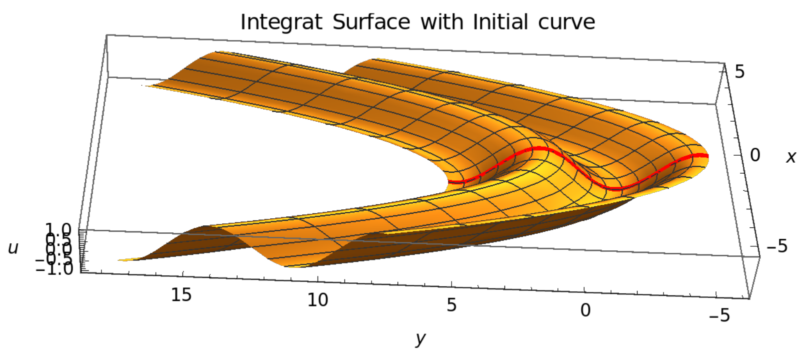


In[ ]:=
```
nfig2 = ParametricPlot3D[{0, s, Sin[s]}, {s, -5, 5}, PlotStyle → {Thick, Red}]
```

Out[ ]=



In[ ]:= **Show[nfig1, nfig2, PlotLabel → "Integrat Surface with Initial curve"]**

Out[ ]=


In[ ]:= **Clear All**

Out[ ]= All Clear

Ques.3)Solve P.D.E. $3u_x + 2u_y$ =0, with Cauchy data u(x,0)= sin x

In[ ]:= **sol1 = DSolve[**
{x '[t] == 3, y '[t] == 2, u '[t] == 0, x[0] == s, y[0] == 0, u[0] == Sin[s]}, {x[t], y[t], u[t]}, t]

Out[ ]= {{x[t] → s + 3 t, y[t] → 2 t, u[t] → Sin[s]}}

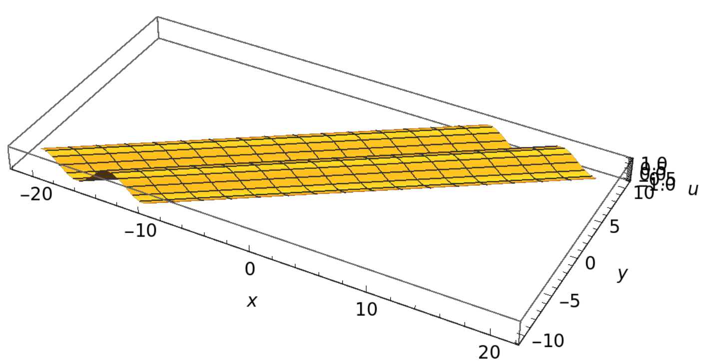
In[ ]:= **Print["x[t]=", sol1[[1, 1, 2]]]**
Print["y[t]=", sol1[[1, 2, 2]]]
Print["u[t]=", sol1[[1, 3, 2]]]

x[t]=s + 3 t

y[t]=2 t

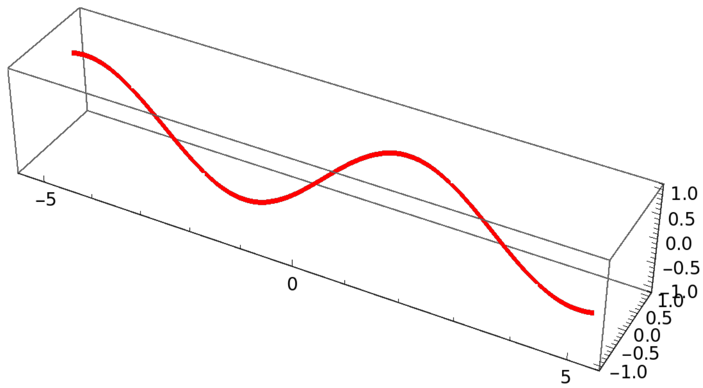
u[t]=Sin[s]

In[ ]:= **nfig1 = ParametricPlot3D[{sol1[[1, 1, 2]], sol1[[1, 2, 2]], sol1[[1, 3, 2]]},**
{t, −5, 5}, {s, −5, 5}, AxesLabel → {x, y, u}]

Out[ ]=


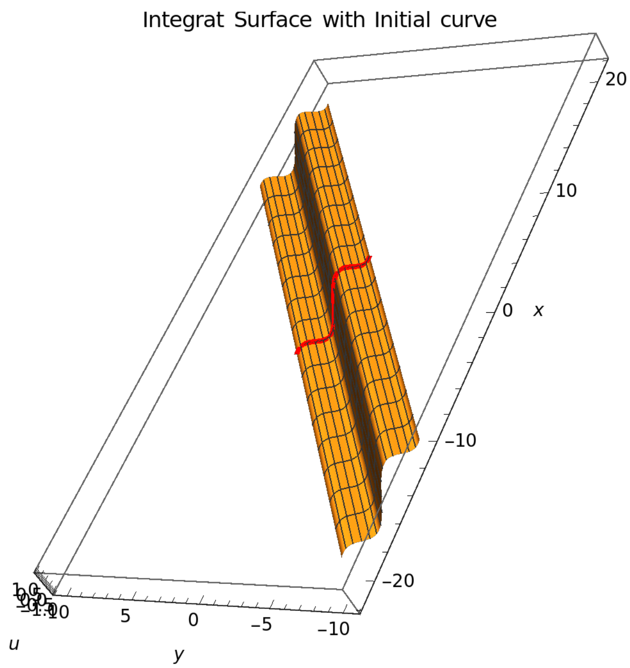
In[∘]:=
```
nfig2 = ParametricPlot3D[{s, 0, Sin[s]}, {s, -5, 5}, PlotStyle → {Thick, Red}]
```

Out[∘]=



In[∘]:=
```
Show[nfig1, nfig2, PlotLabel → "Integrat Surface with Initial curve"]
```

Out[∘]=



In[∘]:=
```
Clear All
```

Out[∘]= All Clear

Ques.4) Solve P.D.E. $y u_x + x u_y = 0$, with Cauchy data u(0,y)= y^2

In[1]:=
```
sol1 = DSolve[
   {x'[t] == y[t], y'[t] == x[t], u'[t] == 0, x[0] == 0, y[0] == s, u[0] == s^2}, {x[t], y[t], u[t]}, t]
```

Out[1]= $\left\{\left\{x[t] \to \frac{1}{2} e^{-t}\left(-1+e^{2t}\right) s,\ y[t] \to \frac{1}{2} e^{-t}\left(1+e^{2t}\right) s,\ u[t] \to s^2\right\}\right\}$

In[2]:=
```
Print["x[t]=", sol1[[1, 1, 2]]]
Print["y[t]=", sol1[[1, 2, 2]]]
Print["u[t]=", sol1[[1, 3, 2]]]
```

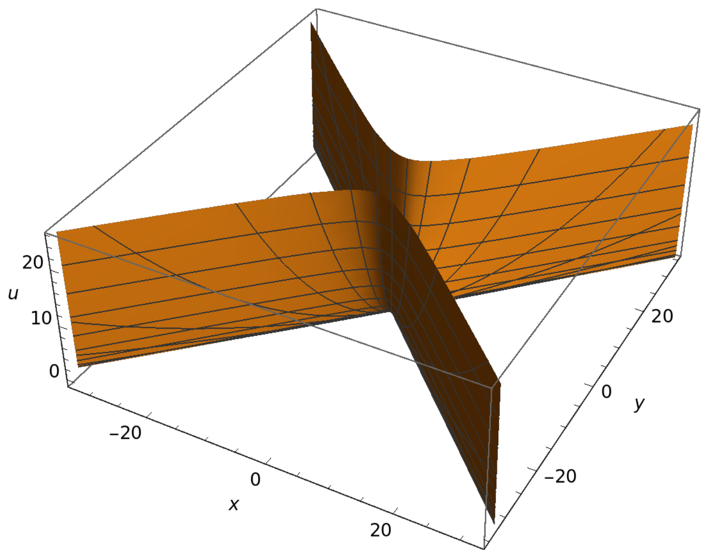
x[t]= $\frac{1}{2} e^{-t} \left(-1 + e^{2t}\right) s$

y[t]= $\frac{1}{2} e^{-t} \left(1 + e^{2t}\right) s$

u[t]= s^2

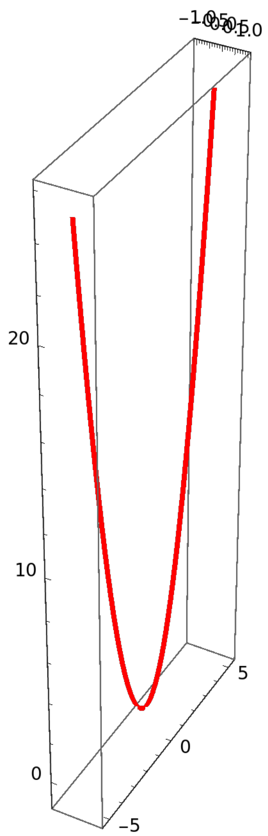
In[5]:=
```
nfig1 = ParametricPlot3D[{sol1[[1, 1, 2]], sol1[[1, 2, 2]], sol1[[1, 3, 2]]},
  {t, -5, 5}, {s, -5, 5}, AxesLabel → {x, y, u}]
```

Out[5]=



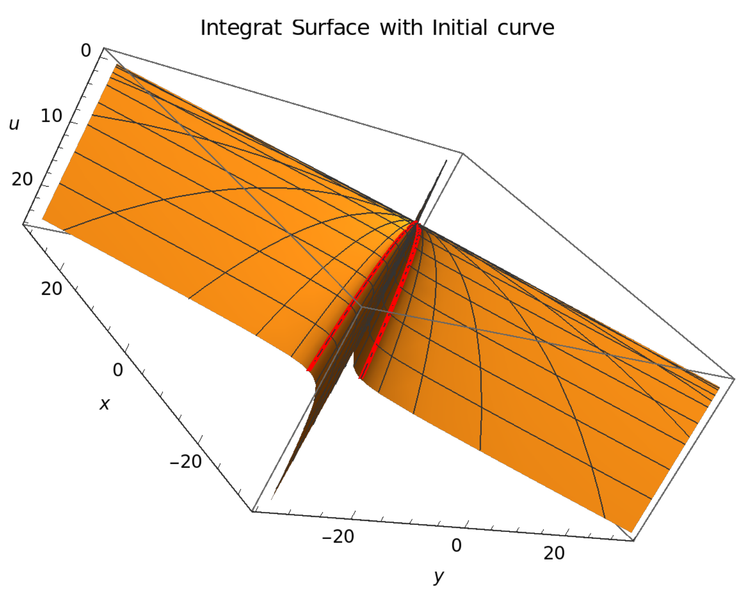
In[8]:=
```
nfig2 = ParametricPlot3D[{0, s, s^2}, {s, -5, 5}, PlotStyle → {Thick, Red}]
```

Out[8]=



In[9]:=
```
Show[nfig1, nfig2, PlotLabel → "Integrat Surface with Initial curve"]
```

Out[9]=



In[10]:=
```
Clear All
```

Out[10]= All Clear

Ques.4) Solve P.D.E. $u u_x + u_y = 1/2$, with Cauchy data u(x,y)= 2x on y=x

In[11]:=
```
sol1 = DSolve[
  {x'[t] == u[t], y'[t] == 1, u'[t] == 1/2, x[0] == s, y[0] == s, u[0] == 2 s}, {x[t], y[t], u[t]}, t]
```

Out[11]= $\left\{\left\{u[t] \to \frac{1}{2}(4 s + t),\ x[t] \to \frac{1}{4}\left(4 s + 8 s t + t^2\right),\ y[t] \to s + t\right\}\right\}$

In[12]:=
```
Print["x[t]=", sol1[[1, 1, 2]]]
Print["y[t]=", sol1[[1, 2, 2]]]
Print["u[t]=", sol1[[1, 3, 2]]]
```

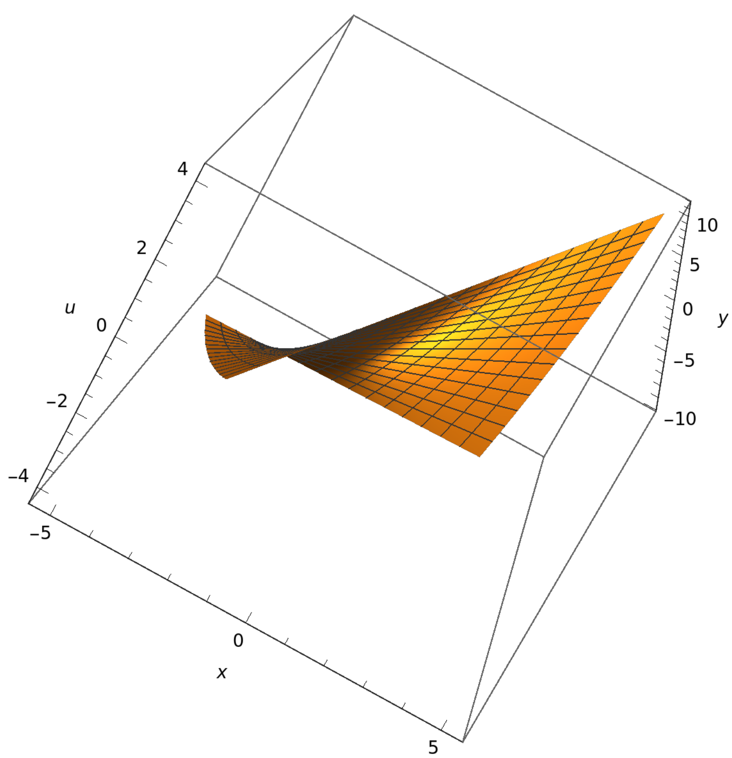
x[t]= $\frac{1}{2}(4 s + t)$

y[t]= $\frac{1}{4}\left(4 s + 8 s t + t^2\right)$

u[t]= $s + t$

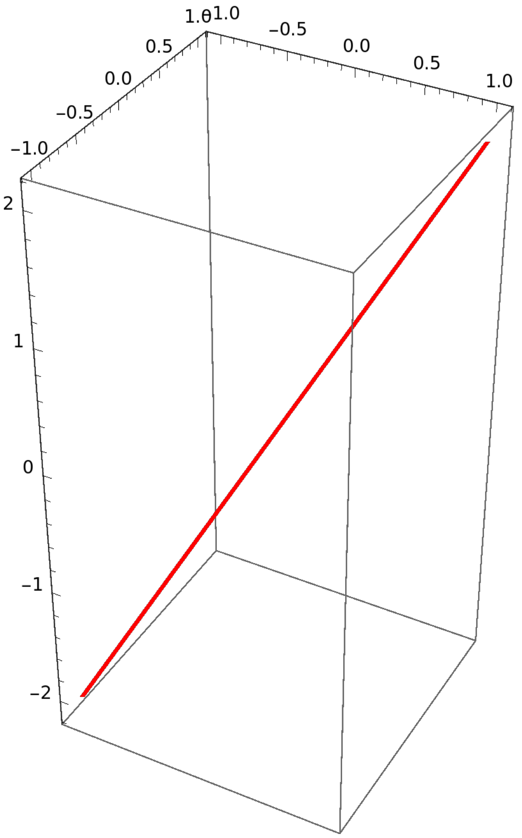
In[18]:=
```
nfig1 = ParametricPlot3D[{sol1[[1, 1, 2]], sol1[[1, 2, 2]], sol1[[1, 3, 2]]},
  {t, -2, 2}, {s, -2, 2}, AxesLabel → {x, y, u}]
```

Out[18]=



In[21]:= nfig2 = ParametricPlot3D[{s, s, 2 s}, {s, -1, 1}, PlotStyle → {Thick, Red}]

Out[21]=



In[22]:= `Show[nfig1, nfig2, PlotLabel → "Integrat Surface with Initial curve"]`

Out[22]=