Séries génératrices ordinaires élémentaires (SGO)

$1, 1, 1, 1, \ldots, 1, \ldots \qquad \frac{1}{1-z} = \sum_{N\geq 0} z^N$

$0, 1, 2, 3, 4, \ldots, N, \ldots \qquad \frac{z}{(1-z)^2} = \sum_{N\geq 1} N z^N$

$0, 0, 1, 3, 6, 10, \ldots, \binom{N}{2}, \ldots \qquad \frac{z^2}{(1-z)^3} = \sum_{N\geq 2} \binom{N}{2} z^N$

$0, \ldots, 0, 1, M+1, \ldots, \binom{N}{M}, \ldots \qquad \frac{z^M}{(1-z)^{M+1}} = \sum_{N\geq M} \binom{N}{M} z^N$

$1, M, \binom{M}{2} \ldots, \binom{M}{N}, \ldots, M, 1 \qquad (1+z)^M = \sum_{N\geq 0} \binom{M}{N} z^N$

$1, M+1, \binom{M+2}{2}, \binom{M+3}{3}, \ldots \qquad \frac{1}{(1-z)^{M+1}} = \sum_{N\geq 0} \binom{N+M}{N} z^N$

$1, 0, 1, 0, \ldots, 1, 0, \ldots \qquad \frac{1}{1-z^2} = \sum_{N\geq 0} z^{2N}$

$1, c, c^2, c^3, \ldots, c^N, \ldots \qquad \frac{1}{1-cz} = \sum_{N\geq 0} c^N z^N$

$1, 1, \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}, \ldots, \frac{1}{N!}, \ldots \qquad e^z = \sum_{N\geq 0} \frac{z^N}{N!}$

$0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{N}, \ldots \qquad \ln\frac{1}{1-z} = \sum_{N\geq 1} \frac{z^N}{N}$

$0, 1, 1+\frac{1}{2}, 1+\frac{1}{2}+\frac{1}{3}, \ldots, H_N, \ldots \qquad \frac{1}{1-z}\ln\frac{1}{1-z} = \sum_{N\geq 1} H_N z^N$

$0, 0, 1, 3(\frac{1}{2}+\frac{1}{3}), 4(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}), \ldots \qquad \frac{z}{(1-z)^2}\ln\frac{1}{1-z} = \sum_{N\geq 0} N(H_N - 1) z^N$

Opérations sur les SGO

$A(z) = \sum_{n\geq 0} a_n z^n \qquad a_0, a_1, a_2, \ldots, a_n, \ldots,$

$B(z) = \sum_{n\geq 0} b_n z^n \qquad b_0, b_1, b_2, \ldots, b_n, \ldots,$

décalage vers la droite

$zA(z) = \sum_{n\geq 1} a_{n-1} z^n \qquad 0, a_0, a_1, a_2, \ldots, a_{n-1}, \ldots,$

décalage vers la gauche

$\frac{A(z)-a_0}{z} = \sum_{n\geq 0} a_{n+1} z^n \qquad a_1, a_2, a_3, \ldots, a_{n+1}, \ldots,$

multiplication d'indice (différentiation)

$A'(z) = \sum_{n\geq 0} (n+1) a_{n+1} z^n \qquad a_1, 2a_2, \ldots, (n+1)a_{n+1}, \ldots,$

division d'indice (intégration)

$\int_0^z A(t)dt = \sum_{n\geq 1} \frac{a_{n-1}}{n} z^n \qquad 0, a_0, \frac{a_1}{2}, \frac{a_2}{3} \ldots, \frac{a_{n-1}}{n}, \ldots,$

mise à l'échelle

$A(\lambda z) = \sum_{n\geq 0} \lambda^n a_n z^n \qquad a_0, \lambda a_1, \lambda^2 a_2, \ldots, \lambda^n a_n, \ldots,$

addition

$A(z) + B(z) = \sum_{n\geq 0} (a_n + b_n) z^n \qquad a_0 + b_0, \ldots, a_n + b_n, \ldots,$

différence

$(1-z)A(z) = a_0 + \sum_{n\geq 1} (a_n - a_{n-1}) z^n \qquad a_0, a_1 - a_0, \ldots, a_n - a_{n-1}, \ldots,$

convolution

$A(z)B(z) = \sum_{n\geq 0} \Big(\sum_{0\leq k\leq n} a_k b_{n-k}\Big) z^n \qquad a_0 b_0, a_1 b_0 + a_0 b_1, \ldots, \sum_{0\leq k\leq n} a_k b_{n-k}, \ldots,$

somme partielle

$\frac{A(z)}{1-z} = \sum_{n\geq 0} \Big(\sum_{0\leq k\leq n} a_k\Big) z^n \qquad a_1, a_1 + a_2, \ldots, \sum_{0\leq k\leq n} a_k, \ldots,$