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Teleportation of two-qubit entangled state via non-maximally entangled GHZ state

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Abstract

Quantum teleportation is of significant meaning in quantum information. We study the probabilistic teleportation of unknown two-qubit entangled state utilizing non-maximally entangled GHZ state as quantum channel. We formulate it as unambiguous state discrimination problem and derive exact optimal POVM operator for maximizing the success probability of unambiguous state discrimination. Only one three-qubit POVM for the sender, one two-qubit unitary operation for the receiver and two cbits for outcome notification are required in this scheme. The unitary operation is given in the form of concise formula and average fidelity is calculated. We show that our scheme is applicable in the situation where the information of quantum channel is only available for the sender.

Keywords: Quantum information theory; Probabilistic teleportation; Optimal POVM; Unambiguous state discrimination

1. Introduction

Quantum teleportation, since proposed by Bennett [1] in 1993, has been in the heart of quantum information theory [2]. It utilizes the entangled state as quantum channel to transmit quantum states between distinct nodes with the aid of classical information. Owing to the potential applications in the sphere of quantum information, a growing number of theoretical and experimental progresses [3, 4, 5] have been made in this domain.

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Probabilistic teleportation was introduced by Li et al. [6] for using non-maximally entangled state to transmit quantum state. Several other works [7, 8, 9] were conducted to explore this area. In most schemes, the sender carries out standard measurement (e.g., Bell-state measurement) in general, and the receiver introduces an auxiliary particle and makes corresponding unitary transformation to reconstruct the original quantum state with the aid of full information of the shared non-maximally entangled state. When only sender has the coefficients information of quantum channel, these probabilistic teleportation schemes are inapplicable.

Exploring the one-to-one correspondence between quantum state measurement result and the state collapsed into is the basic task in designing schemes. The correct discrimination of quantum state guides the receiver reconstruct the original state exactly. There are two main kinds of state discrimination strategy. One is minimum error discrimination (MED) [10] which aims to minimize the average probability of identifying an inaccurate state. Numerous results for MED have been proposed [11, 12, 13], some of which are quite interesting. Unambiguous state discrimination (USD) [14] is another important kind of state discrimination strategy, which expects no-error identification of the states at the cost of obtaining an inclusive result with non-zero probability. An unambiguous measurement is called optimal when it maximizes average correct probability and closed-form analytical expressions are given in some cases [15, 16, 17]. Chefles [18] showed that only linearly independent m states can be distinguished unambiguously. He further proposed equal-probability measurement (EPM) for unambiguous state discrimination which is proved to be optimal by Eldar in [16].

In this paper, probabilistic teleportation of two-qubit entangled state via non-maximally entangled GHZ state is studied. We show that distinguishing states unambiguously is crucial for successful teleportation, the connection between teleportation and quantum state discrimination could be utilized for transmitting quantum state conclusively. The average fidelity of our scheme is separated into two parts accordingly and worked out to show how much information transmitted. Besides, we prove our scheme is useful in the case that only sender has all information of quantum channel's coefficients.

We organize rest of paper as follows: In Section 2, we give the whole scheme of teleportation and necessary elaboration on implementing the scheme. The average fidelity is calculated in Section 3 to show transmission efficiency. In Section 4 we make discussions and conclude the whole paper.

2. Teleportation scheme

In this section, we introduce the detailed scheme for teleportation of unknown two-qubit entangled state. The transmission scenario considered is firstly presented and followed by problem formulation and optimal positive operator-valued measure (POVM) construction. The exact unitary operation is given for original state recovery at the end.

2.1. Transmission Scenario

In this paper, two nodes, conveniently called Alice and Bob, share one non-maximally entangled GHZ state. The entanglement is used as quantum channel, through which Alice wishes to transmit unknown two-qubit entangled state to Bob. The state and the quantum channel are described as follows,

$$|\mu\rangle = \alpha|00\rangle + \beta|11\rangle \tag{1}$$

$$|GHZ\rangle = a|000\rangle + b|111\rangle \tag{2}$$

where α and β are normalized complex numbers. Without any loss of generality, we assume a and b to be real with $|a| \geq |b|$ and satisfies $|a|^2 + |b|^2 = 1$. For the convenience of description, as shown in Fig. 1, we assume particle 1 and 2 are in possession of Alice, particle 3 from non-maximally entangled GHZ state is with Alice while Bob owns particle 4 and 5. Wired or wireless classical communication channel is also equipped between Alice and Bob.

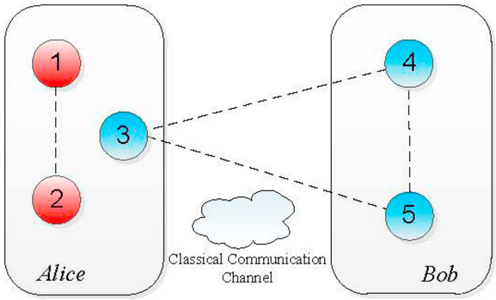

Fig. 1. Quantum state transmission scenario.

Therefore, the combined three-qubit system state could be written down in the following way,

$$\left|\Psi_{sys}\right\rangle = \left|\mu\right\rangle_{12} \otimes \left|GHZ\right\rangle_{345} \tag{3}$$

To expand the above equation, we could rewritten it as

$$\begin{aligned}\left|\Psi_{sys}\right\rangle &= \left(\alpha\left|00\right\rangle + \beta\left|11\right\rangle\right)_{12} \otimes \left(a\left|000\right\rangle + b\left|111\right\rangle\right)_{345} \\ &= \frac{1}{2}\left(a\left|000\right\rangle + b\left|111\right\rangle\right)_{123} \otimes \left(\alpha\left|00\right\rangle + \beta\left|11\right\rangle\right)_{45} + \frac{1}{2}\left(a\left|000\right\rangle - b\left|111\right\rangle\right)_{123} \otimes \left(\alpha\left|00\right\rangle - \beta\left|11\right\rangle\right)_{45} \\ &\quad + \frac{1}{2}\left(b\left|001\right\rangle + a\left|110\right\rangle\right)_{123} \otimes \left(\alpha\left|11\right\rangle + \beta\left|00\right\rangle\right)_{45} + \frac{1}{2}\left(b\left|001\right\rangle - a\left|110\right\rangle\right)_{123} \otimes \left(\alpha\left|11\right\rangle - \beta\left|00\right\rangle\right)_{45}\end{aligned} \tag{4}$$

For realizing teleportation, A joint measurement is carried out by Alice on particles (1, 2, 3) so that the state on particles (4, 5) would collapse into one of the four possible states. Observing Eq. 4, we find that the correct discrimination of the quantum states on particles (1, 2, 3) will indicate state on particles (4, 5) precisely. This correspondence relationship is utilized to give instruction on proper unitary operations Bob should perform to recover original state. It has been known that standard von Neumann measurement can only determine the states from orthogonal set infallibly. No strategy will correctly identify the state with unit probability if discriminating states from non-orthogonal set.

We extract these four states needs to be distinguished from Eq. 4 as follows, and find out that they are non-orthogonal and linearly independent pure states.

$$\begin{aligned}&\left|\varphi_1\right\rangle = [a,0,0,0,0,0,0,b]^T, \qquad \left|\varphi_2\right\rangle = [a,0,0,0,0,0,0,-b]^T, \\ &\left|\varphi_3\right\rangle = [0,b,0,0,0,0,a,0]^T, \qquad \left|\varphi_4\right\rangle = [0,b,0,0,0,0,-a,0]^T.\end{aligned} \tag{5}$$

Our task is to specify the exact POVM that Alice should make to discriminate state. Since the state discrimination here is crucial for the whole teleportation process, the measurement must distinguish the states unambiguously without error. We accept the failure of the teleportation with inclusive result other than recovering the state with erroneous information. Therefore, we formulate it as a typical unambiguous state discrimination problem, and then describe the procedure of getting the exact optimal measurements.

2.2. Formulation

Suppose that the possible state of particles (1, 2, 3) would be taken from a set of $|\varphi_i\rangle$ with priori probabilities η_i in Hilbert space $\mathcal{H}$ where $1 \leq i \leq 4$, and the states span a subspace $\mathcal{U}$ of $\mathcal{H}$. The m states are linearly independent and the occurrence probabilities η_i are non-zero satisfies $\sum_{i=1}^{4} \eta_i = 1$. A measurement is described as positive operator-valued measure (POVM) which should be constructed comprising m+1 measurement operators $\{E_i,\ 0 \le i \le 4\}$ that $\sum_{i=0}^{4} E_i = I$. Based on these measurement operators, either the state is discriminated correctly or an inconclusive result is returned. In this way, each E_i corresponds to the detection of the state $|\varphi_i\rangle$, $1 \le i \le 4$, and E_0 corresponds to the inclusive result. Suppose the actual quantum state is $|\varphi_i\rangle$, the outcome k would be obtained with probability $\langle\varphi_i|E_k|\varphi_i\rangle$. Thus, to guarantee of obtaining either error-free or inclusive measurement result, we must have

$$\langle\varphi_i \mid E_k \mid \varphi_i\rangle = p_i\delta_{ik}, \qquad 1 \le i,k \le 4 \tag{6}$$

for specific $0 \le p_i \le 1$, where δ_{ik} is Kroncker Delta function that equal to one only when $i = k$. We get $E_0 = I - \sum_{i=1}^{4} E_i$ from Eq. 6. Assume the state is $|\varphi_i\rangle$, it is correctly discriminated with probability p_i so that the probability of getting an inconclusive result is $1 - p_i$. Therefore, the total probability of correctly detecting the state and obtain conclusive result is

$$P_{con} = \sum_{i=1}^{4} \eta_i \langle\varphi_i \mid E_i \mid \varphi_i\rangle = \sum_{i=1}^{4} \eta_i p_i \tag{7}$$

We need to decide the exact measurement operators E_i and corresponding probabilities $p_i \ge 0$ to maximize P_{con} considering the constraint in Eq. 6. It can be formulated as a problem of convex optimization, and we choose

$$E_i = p_i \mid \widetilde{\varphi_i}\rangle\langle\widetilde{\varphi_i} \mid \tag{8}$$

as the measurement operator, where $|\widetilde{\varphi}_i\rangle \in \mathcal{U}$ are the reciprocal state associated with $|\varphi_i\rangle$. We use the method described in [16] to determine the exact form of $|\widetilde{\varphi}_i\rangle$ and corresponding measurement operators E_i. Constructing the matrices Φ and $\widetilde{\Phi}$ with $|\varphi_i\rangle$ and $|\widetilde{\varphi}_i\rangle$ as columns, respectively, so that we have

$$\widetilde{\Phi} = \Phi(\Phi^*\Phi)^{-1} \tag{9}$$

The exact $|\widetilde{\varphi}_i\rangle$ is obtained by

$$|\widetilde{\varphi_i}\rangle = |\varphi_i\rangle(\Phi^*\Phi)^{-1} \tag{10}$$

Till now, we have prepared the elements for constructing the measurement operators with what Alice can measure the particles (1, 2, 3) conclusively and lay good foundation for the subsequent.

2.3. Optimal POVM

The goal of formulation is to construct POVM and obtain the optimal solution. For discriminating the four states as shown in Eq. 5 with equal priori probability $\eta_i = 1/4$ $(1 \le i \le 4)$, we construct matrix Φ and calculate corresponding reciprocal states. According to the Eq. 9, the $|\widetilde{\varphi}_i\rangle$ are given directly as follows for space saving,

$$\begin{aligned}
&|\widetilde{\varphi_1}\rangle = \frac{1}{2a}|000\rangle + \frac{1}{2b}|111\rangle, \qquad |\widetilde{\varphi_2}\rangle = \frac{1}{2a}|000\rangle - \frac{1}{2b}|111\rangle,\\
&|\widetilde{\varphi_3}\rangle = \frac{1}{2a}|001\rangle + \frac{1}{2b}|110\rangle, \qquad |\widetilde{\varphi_4}\rangle = \frac{1}{2a}|001\rangle - \frac{1}{2b}|110\rangle.
\end{aligned} \tag{11}$$

In this case, the states $|\varphi_i\rangle$ and the prior probabilities η_i satisfy the sufficient conditions for equal-probability measurement is optimal that maximize the total probability of correct discrimination P_{con} [16]. The optimal equal measurement probability $p_i = p = 2|b|^2$ for all $1 \le i \le 4$. Thus, we can specify the optimal POVM for detecting the four states unambiguously as

$$\begin{cases} E_i = 2|b|^2 |\widetilde{\varphi_i}\rangle\langle\widetilde{\varphi_i}| & 1 \le i \le 4 \\ E_0 = I - \sum_{i=1}^{4} E_i & i = 0 \end{cases} \tag{12}$$

The total probability of correctly distinguishing states is $P_{con} = \sum_{i=1}^{4} \eta_i p_i = 4 \times 1/4 \times 2|b|^2 = 2|b|^2$. Ignoring the possible error may arise accompany with unitary operation, it is the probability of successful teleportation with fidelity one. When $|b| = 1/\sqrt{2}$, this teleportation is always successful with certainty since the entangled state constituted quantum channel is maximal then.

2.4. Original State Recovery

For convenience, we use classical bit strings m_1m_2 to denote the measurement outcomes of E_i , and the correspondence relationship

$$E_1 \to 00, \quad E_2 \to 01, \quad E_3 \to 10, \quad E_4 \to 11 \tag{13}$$

After optimal POVM on particles (1, 2, 3), Alice sends the obtained 2cbits classical information of measurement outcome m_1m_2 to Bob through classical communication channel. If the inclusive result is obtained, Alice sends nothing and starts another teleportation. For Bob, if he received the classical information, he performs the unitary operation on particles (4, 5) to reconstruct the original state according to the classical information received as

$$T = \left(X^{m_1}\right)_4 \otimes \left(Z^{m_2} X^{m_1}\right)_5 \tag{14}$$

where X and Z are *Pauli matrices*. Through this operation, Bob yields original state successfully. Otherwise, if Bob don't get any information of this teleportation after a fixed time period, he would know Alice obtained inclusive result that the teleportation is failed. With the help of unambiguous state discrimination and optimal POVM, the total success probability of teleportation is $2|b|^2$ and reach up to 1 when $|b| = 1/\sqrt{2}$.

Thus, the whole teleportation is completed and we simply summarize it as follows. When the sender Alice wants to send two-qubit entangled state to the receiver Bob through shared non-maximally entangled GHZ state, she should make an optimal POVM on particles (1, 2, 3) and sends 2cbit classical information of measurement outcome obtained to Bob. Upon receiving the information, Bob makes proper unitary operation on particles (4, 5), in the light of Eq. 14, to reconstruct the unknown original state exactly. For constructing the optimal POVM, we may formulate it as unambiguous state discrimination problem and obtain the optimized solution.

3. Average fidelity

We also need to study how much information Alice transferred to Bob. The measurement outcome k determines corresponding density operator ρ_k of teleported pure state, and the average fidelity should be

$$F \equiv \frac{1}{V} \int d\vec{\Omega} \sum_k p_k(\vec{\Omega}) f_k(\vec{\Omega}) \tag{15}$$

The pure state ρ_k is parameterized by a real vector $\vec{\Omega}$ so that we have $f_k(\vec{\Omega}) = \langle \Psi(\vec{\Omega}) | \rho_k | \Psi(\vec{\Omega}) \rangle$. For clear expression, we separate the average fidelity into two parts, conclusive part and inclusive part, and rewrite it as

$$F = F_{con} + F_{inc} = \frac{1}{V}\int d\vec{\Omega}\left[\sum_{k=1}^{4} p_k^{con}(\vec{\Omega}) f_k^{con}(\vec{\Omega}) + p_0^{inc}(\vec{\Omega}) f_0^{inc}(\vec{\Omega})\right] \tag{16}$$

where F_{con} indicates the average fidelity conclusive events with $f_k^{con} = 1$ while F_{inc} denotes the average fidelity of inconclusive parts with $f_k^{con} = 2/3$. We have the average fidelity

$$F = \sum_{k=1}^{4}\left(\frac{1}{4}\times 2|b|^2 \times 1\right) + (1-2|b|^2)\times\frac{2}{3} = \frac{2}{3} + \frac{2|b|^2}{3} \tag{17}$$

If $|b| = 1/\sqrt{2}$, the average fidelity is 1, the teleportation changes to be faithful. For $|b| = 0$, the success probability will equal to 0 implying that it is impossible for Bob to reconstruct the original state in that situation. However, the average fidelity is still 2/3 then, *i.e.*, no matter which result we obtained from POVM, at least 2/3 information in the original state were transmitted.

4. Discussions and conclusions

In traditional schemes for teleporting state probabilistically, the sender usually makes standard measurement in general, such as Bell measurement, receiver needs to introduce auxiliary particle and makes corresponding unitary operation to recover the initial quantum state with aid of full information about the shared non-maximal quantum channel. When only sender has the coefficients information of quantum channel, these schemes are inapplicable. However, in our scheme, only Alice needs to perform POVM that contains the coefficients of the quantum channel while Bob only needs to make unitary operation consists of specific *Pauli matrices*. Therefore, our scheme is quit suitable for the situation that only Alice has the full knowledge of the coefficients of the quantum channel. In theory, we can implement any POVM by adding an ancilla in the known state and performing a standard measurement in enlarged Hilbert space [18]. It has already been experimentally demonstrated in [19].

In this paper, we proposed one novel scheme for teleporting unknown two-qubit entangled state via non-maximally entangled GHZ state probabilistically. We relate the quantum teleportation with quantum state discrimination, and thereby, derive optimal POVM operators to maximize the probability of correct detection. In our scheme, to realize this teleportation process, Alice makes the derived optimal POVM on her three particles, and Bob performs unitary operation according to the 2cbit classical information sent from Alice to yield original state. No auxiliary particle is required and the unitary operation for recovering is provided in the form of concise formula. Furthermore, the average fidelity of this scheme is calculated to show the quantum information transmission efficiency. The application of our scheme is reachable and convenient.

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