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Multihop teleportation of two-qubit state via the composite GHZ–Bell channel ☆

Zhen-Zhen Zou [a], Xu-Tao Yu [a,*], Yan-Xiao Gong [b], Zai-Chen Zhang [c]

[a] State Key Lab. of Millimeter Waves, Southeast University, Nanjing, 210096, China
[b] Department of Physics, Southeast University, Nanjing, 211189, China
[c] National Mobile Communications Research Lab, Southeast University, Nanjing, 210096, China

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ABSTRACT

A multihop teleportation protocol in quantum communication network is introduced to teleport an arbitrary two-qubit state, between two nodes without directly sharing entanglement pairs. Quantum channels are built among neighbor nodes based on a five-qubit entangled system composed of GHZ and Bell pairs. The von Neumann measurements in all intermediate nodes and the source node are implemented, and then the measurement outcomes are sent to the destination node independently. After collecting all the measurement outcomes at the destination node, an efficient method is proposed to calculate the unitary operations for transforming the receiver's states to the state teleported. Therefore, only adopting the proper unitary operations at the destination node, the desired quantum state can be recovered perfectly. The transmission flexibility and efficiency of quantum network with composite GHZ–Bell channel are improved by transmitting measurement outcomes of all nodes in parallelism and reducing hop-by-hop teleportation delay.

1. Introduction

Quantum teleportation is a useful technique for quantum communication to transmit unknown states by using shared entanglement and classical channels between the sender and the receiver. Teleportation of an arbitrary single qubit, based on an EPR pair shared between the source and destination nodes, was first proposed by Bennett et al. [1], which was experimentally demonstrated later [2]. After that, schemes of teleporting an arbitrary single-qubit state were proposed through three-particle GHZ state [3,4], four-particle GHZ state [5], n-particle GHZ state [6], W state [7], mixed state [8,9], etc. To teleport an arbitrary two-qubit state, many protocols were proposed using tensor products of two Bell states [10], genuine five-qubit entangled state [11], multi-qubit cluster state [12,13], partially entangled three-particle GHZ state and W state [14], genuine six-qubit entangled state [15], etc. Several theoretical studies were demonstrated experimentally in [16,17]. In addition, schemes to teleport an arbitrary N-qubit state were proposed through non-maximally entangled Bell state [18], the composite GHZ–Bell channel [19], genuine multipartite entanglement channel [20] and n pairs of EPR channel [21], respectively.

However, in practice, considering the inevitable losses on the quantum channel, teleportation is hard to be performed directly between two distant nodes. In a quantum network, intermediate nodes are introduced for multihop teleportation between two distant nodes. In the multihop route, there are entangled resources among adjacent nodes. Therefore, quantum channels can be built to transmit qubit states between the source and destination nodes. Yu et al. proposed a routing protocol for a wireless *ad hoc* quantum communication network and a distributed wireless network [22]. Shi et al. proposed quantum wireless multihop network in which an arbitrary single qubit can be teleported hop by hop based on shared W states [23]. In order to reduce the waiting time cost in hop-by-hop transmission, Cai et al. proposed a quantum bridging protocol with partially entangled states to teleport qubits [24]. And then a protocol for quantum communication through entanglement swapping based on arbitrary types of Bell pairs is demonstrated by Wang et al. [25]. These protocols improve transmission efficiency by sending the measurement outcomes and the types of Bell pairs to the destination node independently. In addition, Xiong

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* Corresponding author.
E-mail address: yuxutao@seu.edu.cn (X.-T. Yu).

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et al. proposed a multiple teleportation protocol of partially entangled GHZ state, with the help of auxiliary particles [26].

Recently, Saha et al. proposed a composite GHZ–Bell channel for quantum teleportation, information splitting and superdense coding, considering the GHZ state could be less prone to decoherence [19]. In this paper, we propose a quantum communication protocol to realize multihop quantum teleportation of arbitrary two-qubit states, with such channels between adjacent nodes in the link from the source to destination. In addition, in order to reduce large delay of hop-by-hop transmission [25], all nodes except the destination node can take von Neumann five-qubit measurement independently in this protocol. Note that these measurement outcomes do not depend on other nodes and can be carried out immediately after the nodes are selected on quantum paths. After measurement, the source and intermediate nodes send their measurement outcomes to the destination node independently. Finally, the unitary operations to perfectly recover the initial quantum state are performed at the destination node. Therefore, the transmission efficiency of quantum network would be improved effectively by transmitting measurement outcomes of all nodes in parallelism.

The efficiency and effectiveness of our protocol could be utilized in quantum satellite communications [27], which use satellite links to increase the quantum transmission distance and promote global quantum communications. In addition, the utility of our generalized scheme for multihop teleportation could be also applied in other hot research topics, like quantum cheques scheme (an alternate for current e-Payment Gateways) [28] and quantum secure direct communication [29], etc.

The rest of the paper is organized as follows. In Section 2, we revisit the scheme to teleport an arbitrary two-qubit, based on the GHZ–Bell entangled state shared between the source and destination node. In Section 3, one intermediate node is added to realize a two-hop teleportation protocol. Then the general multihop teleportation protocol is introduced in Section 4. The whole quantum communication protocol is described in Section 5. Finally, the conclusions are given in Section 6.

2. One-hop quantum teleportation

Suppose a sender Alice wishes to teleport an arbitrary two-qubit state to a receiver Bob. Alice begins with the arbitrary state

$$\left|\varphi^{src}\right\rangle_{1,2} = (a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle)_{1,2}, \tag{1}$$

where $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$, and subscripts 1 and 2 denote the different particles.

Alice and Bob share the five-qubit composite GHZ–Bell state given by

$$\begin{aligned} &|\zeta\rangle_{A_1,A_2,B_4,A_3,B_5} \\ &= |GHZ\rangle_{A_1,A_2,B_4} \otimes \left|\Phi^+\right\rangle_{A_3,B_5} \\ &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_1,A_2,B_4} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_3,B_5} \\ &= \frac{1}{2}(|00000\rangle + |00011\rangle + |11100\rangle + |11111\rangle)_{A_1,A_2,B_4,A_3,B_5}, \end{aligned} \tag{2}$$

where $|GHZ\rangle$ is the maximally entangled GHZ state and $\left|\Phi^+\right\rangle$ is one of the Bell states. Let us consider a situation where Alice has particles A_1, A_2 and A_3, while Bob has particles B_4 and B_5. The corresponding protocol for teleportation is shown in Fig. 1.

The combination of input and GHZ–Bell state can be written and decomposed into Alice and Bob systems

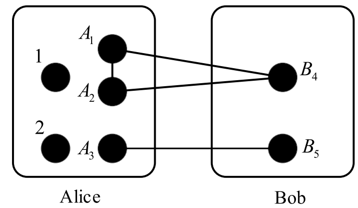

Fig. 1. Alice and Bob share a five-qubit entangled system composed of GHZ and Bell pairs as quantum channel.

$$\begin{aligned} &\left|\varphi^{src}\right\rangle_{1,2} \otimes |\zeta\rangle_{A_1,A_2,B_4,A_3,B_5} \\ &= \frac{1}{2}(a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle)_{1,2} \\ &\otimes (|00000\rangle + |00011\rangle + |11100\rangle + |11111\rangle)_{A_1,A_2,B_4,A_3,B_5} \\ &= \sum_{i=1}^{16} |\Upsilon_i\rangle_{1,2,A_1,A_2,A_3} \otimes |\varphi_i\rangle_{B_4,B_5} \\ &= \{|\Upsilon_1\rangle\, I \otimes I + |\Upsilon_2\rangle\, \sigma_z \otimes I + |\Upsilon_3\rangle\, I \otimes \sigma_z + |\Upsilon_4\rangle\, \sigma_z \otimes \sigma_z \\ &+ |\Upsilon_5\rangle\, \sigma_x \otimes \sigma_x + |\Upsilon_6\rangle \left(i\sigma_y\right) \otimes \sigma_x \\ &+ |\Upsilon_7\rangle\, \sigma_x \otimes \left(i\sigma_y\right) + \Upsilon \left(i\sigma_y\right) \otimes \left(i\sigma_y\right) \\ &+ |\Upsilon_9\rangle\, \sigma_x \otimes I + |\Upsilon_{10}\rangle \left(i\sigma_y\right) \otimes I \\ &+ |\Upsilon_{11}\rangle\, \sigma_x \otimes \sigma_z + |\Upsilon_{12}\rangle \left(i\sigma_y\right) \otimes \sigma_z \\ &+ |\Upsilon_{13}\rangle\, I \otimes \sigma_x + |\Upsilon_{14}\rangle\, \sigma_z \otimes \sigma_x \\ &+ |\Upsilon_{15}\rangle\, I \otimes \left(i\sigma_y\right) + |\Upsilon_{16}\rangle\, \sigma_z \otimes \left(i\sigma_y\right)\} \\ &\otimes (a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle) \\ &= \sum_{i=1}^{16} |\Upsilon_i\rangle_{1,2,A_1,A_2,A_3} \otimes (U_i\,|\varphi_1\rangle)_{B_4,B_5}, \end{aligned} \tag{3}$$

where $|\Upsilon_i\rangle_{1,2,A_1,A_2,A_3}$ $(i = 1, 2, ..., 16)$ are mutually orthonormal five particle states that belong to Alice, and $|\varphi_i\rangle_{B_4,B_5}$ $(i = 1, 2, ..., 16)$ are two-qubit states of Bob. In addition, $|\varphi_i\rangle_{B_4,B_5}$ and $|\varphi_1\rangle_{B_4,B_5}$ can be transformed into each other through the unitary operations U_i with Pauli matrices $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, and identity matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. $|\Upsilon_i\rangle$, $|\varphi_i\rangle$ and U_i are respectively given in Table 1 and Table 2 (see Supplementary Material).

Now Alice performs a five-qubit von Neumann measurement in a basis that includes the orthonormal states $\left\{|\Upsilon_i\rangle_{1,2,A_1,A_2,A_3} (i = 1, 2, ..., 16)\right\}$ and sends the measurement outcomes using four classical bits to Bob through classical channel. According to the measurement outcomes, Bob applies suitable unitary operations to recover the initial two-qubit state. For example, if Bob receives the measurement outcome $|\Upsilon_{11}\rangle$, Bob needs to apply σ_x and σ_z unitary operations to recover the initial input state based on

$$\left|\varphi^{src}\right\rangle_{1,2} \rightarrow |\varphi_{11}\rangle_{B_4,B_5} = \sigma_x \otimes \sigma_z \left|\varphi^{src}\right\rangle_{1,2}. \tag{4}$$

3. Two-hop quantum teleportation

In quantum network, there are usually no direct entangle pairs shared between any source and destination nodes. In order to communicate between two nodes which do not share entanglement, the source node can teleport quantum information to destination via intermediate nodes. Consider the simplest case, as shown in Fig. 2, Alice wants to teleport arbitrary two-qubit to Cindy who does not share direct entangle with Alice, while Bob shares the five-qubit composite GHZ–Bell state with Alice and Cindy respectively. Therefore Bob can be used as intermediate note to teleport

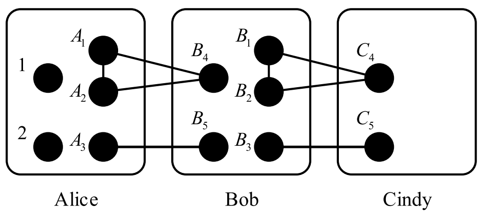

Fig. 2. Teleportation with the help of Bob who share the five-qubit composite GHZ–Bell state as quantum channel with Alice and Cindy respectively.

qubits between Alice and Cindy. From Fig. 2, the total state can be written as

$$\begin{aligned}
&\left|\varphi^{src}\right\rangle_{1,2} \otimes |\zeta\rangle_{A_1,A_2,B_4,A_3,B_5} \otimes |\zeta\rangle_{B_1,B_2,C_4,B_3,C_5} \\
&= \frac{1}{2}(a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle)_{1,2} \\
&\otimes (|00000\rangle + |00011\rangle + |11100\rangle + |11111\rangle)_{A_1,A_2,B_4,A_3,B_5} \\
&\otimes |\zeta\rangle_{B_1,B_2,C_4,B_3,C_5} \\
&= \sum_{i=1}^{16} |\Upsilon_i\rangle_{1,2,A_1,A_2,A_3} |\varphi_i\rangle_{B_4,B_5} \otimes |\zeta\rangle_{B_1,B_2,C_4,B_3,C_5}.
\end{aligned} \tag{5}$$

According to Eq. (3), the possible states of particles B_4 and B_5 are $|\varphi_i\rangle_{B_4,B_5}$ $(i = 1, 2, ..., 16)$, which have sixteen different situations. So the combined state of particles B_4, B_5 and GHZ–Bell state between Bob and Cindy will get sixteen different decomposition.

Assuming the state of particles B_4 and B_5 is $|\varphi_k\rangle_{B_4,B_5}$, corresponding state of five particles that belong to Alice is $|\Upsilon_k\rangle_{1,2,A_1,A_2,A_3}$. Note that the sender Alice do not employ measurement now. The combined state of particles B_1, B_2, B_3, B_4, B_5, C_4 and C_5 can be decomposed into Bob and Cindy system

$$\begin{aligned}
&|\varphi_k\rangle_{B_4,B_5} \otimes |\zeta\rangle_{B_1,B_2,C_4,B_3,C_5} \\
&= \frac{1}{2}(d\,|00\rangle + c\,|01\rangle + b\,|10\rangle + a\,|11\rangle)_{B_4,B_5} \\
&\otimes (|00000\rangle + |00011\rangle + |11100\rangle + |11111\rangle)_{B_1,B_2,C_4,B_3,C_5} \\
&= \sum_{i=1}^{16} |\Upsilon_i\rangle_{B_4,B_5,B_1,B_2,B_3} \otimes \left|\varphi_i^k\right\rangle_{C_4,C_5} \\
&= \sum_{i=1}^{16} |\Upsilon_i\rangle_{B_4,B_5,B_1,B_2,B_3} \otimes \left(U_i \left|\varphi_1^k\right\rangle\right)_{C_4,C_5},
\end{aligned} \tag{6}$$

where $|\varphi_i^k\rangle_{C_4,C_5}$ $(i = 1, 2, ..., 16)$ are two particle states that belong to Cindy, which vary with $|\varphi_k\rangle_{B_4,B_5}$. According to Eq. (6), no matter what state of particles B_4 and B_5, $|\varphi_1^k\rangle_{C_4,C_5}$ (or $|\varphi_k\rangle_{B_4,B_5}$) and $|\varphi_i^k\rangle_{C_4,C_5}$ can always be transformed into each other through the unitary operations U_i which are same as above in Table 2. And $|\Upsilon_i\rangle_{B_4,B_5,B_1,B_2,B_3}$ are also same as in Table 1.

The quantum communication scheme proposed in this paper employs von Neumann measurement in all nodes except destination node, and then these nodes send measurement outcomes to destination node independently. Therefore, both Alice and Bob's measurements are directly sent to Cindy where the local operations to recover initial quantum state are required. For general case, the recovering process is

$$\begin{aligned}
\left|\varphi^{src}\right\rangle_{1,2} &\to \left|\varphi_{k_1}\right\rangle_{B_4,B_5} = U_{k_1}\left|\varphi^{src}\right\rangle_{1,2} \\
&\to \left|\varphi_{k_2}^{k_1}\right\rangle_{C_4,C_5} = U_{k_2}\left|\varphi_{k_1}\right\rangle_{B_4,B_5}, \\
\left|\varphi_{k_2}^{k_1}\right\rangle_{C_4,C_5} &= U_{k_2}\left|\varphi_{k_1}\right\rangle_{B_4,B_5} = U_{k_2}U_{k_1}\left|\varphi^{src}\right\rangle_{1,2} \\
&(k_1, k_2 = 1, 2, ..., 16),
\end{aligned} \tag{7}$$

where $\left|\varphi^{src}\right\rangle_{1,2}$, $\left|\varphi_{k_1}\right\rangle_{B_4,B_5}$ and $\left|\varphi_{k_2}^{k_1}\right\rangle_{C_4,C_5}$ are the states belonging to Alice, Bob and Cindy respectively. After collecting the information from Alice and Bob, Cindy would perform the proper unitary operations $U_{k_2}U_{k_1}$ to recover the original state $\left|\varphi^{src}\right\rangle$.

For instance, if the measurement outcomes of Alice and Bob are $|\Upsilon_5\rangle$ and $|\Upsilon_9\rangle$ respectively, the change process of two particles output state is $\left|\varphi^{src}\right\rangle_{1,2} \to |\varphi_5\rangle_{B_4,B_5} \to \left|\varphi_9^5\right\rangle_{C_4,C_5}$. The unitary operation with Pauli matrices between $\left|\varphi^{src}\right\rangle$ and $\left|\varphi_9^5\right\rangle$ is given by

$$\left|\varphi_9^5\right\rangle = U_9|\varphi_5\rangle_{B_4,B_5} = U_9U_5\left|\varphi^{src}\right\rangle = (\sigma_x \otimes I)(\sigma_x \otimes \sigma_x)\left|\varphi^{src}\right\rangle. \tag{8}$$

Therefore, Cindy has to apply unitary operation $(\sigma_x \otimes I) \times (\sigma_x \otimes \sigma_x)$ to get the desired state $\left|\varphi^{src}\right\rangle$.

4. Multihop quantum teleportation

The above scheme can be generalized to multihop quantum teleportation based on the composite GHZ–Bell channel between intermediate nodes. The multihop quantum teleportation protocol is shown in Fig. 3, where N − 1 pairs entangled GHZ–Bell particles are shared by N nodes, and the source node A wishes to teleport an arbitrary two-qubit state to the destination node N through N − 2 intermediate nodes. The entire system state is written as

$$\begin{aligned}
&\left|\varphi^{src}\right\rangle_{1,2} \otimes |\zeta\rangle^{N-1} \\
&= \left|\varphi^{src}\right\rangle_{1,2} \otimes |\zeta\rangle_{A_1,A_2,B_4,A_3,B_5} \otimes |\zeta\rangle^{N-2} \\
&= \sum_{i=1}^{16} |\Upsilon_i\rangle_{1,2,A_1,A_2,A_3} \left|\varphi_i^B\right\rangle_{B_4,B_5} \otimes |\zeta\rangle^{N-2} \\
&= \sum_{i=1}^{16} |\Upsilon_i\rangle_{1,2,A_1,A_2,A_3} \left(U_i \left|\varphi_1^B\right\rangle\right)_{B_4,B_5} \otimes |\zeta\rangle^{N-2}.
\end{aligned} \tag{9}$$

We can regard $|\varphi_i^B\rangle_{B_4,B_5}$ $(i = 1, 2, ..., 16)$ as the input state of the following subsystem with N − 1 nodes. As well proven above, there are sixteen different decompositions between adjoining nodes, all decomposition processes are similar to Eq. (6). No matter what states of particles B_4 and B_5 are, unitary operations U_i and $|\Upsilon_i\rangle_{B_4,B_5,B_1,B_2,B_3}$ are same as above in Table 1 and Table 2.

Assuming the state of particles B_4 and B_5 is $\left|\varphi_{k_1}^B\right\rangle_{B_4,B_5}$, the state of the remaining nodes can be described as

$$\begin{aligned}
&\left|\varphi_{k_1}^B\right\rangle_{B_4,B_5} \otimes |\zeta\rangle^{N-2} \\
&= \left|\varphi_{k_1}^B\right\rangle_{B_4,B_5} \otimes |\zeta\rangle_{B_1,B_2,C_4,B_3,C_5} \otimes |\zeta\rangle^{N-3} \\
&= \sum_{i=1}^{16} |\Upsilon_i\rangle_{B_4,B_5,B_1,B_2,B_3} \left|\varphi_i^C\right\rangle_{C_4,C_5} \otimes |\zeta\rangle^{N-3} \\
&= \sum_{i=1}^{16} |\Upsilon_i\rangle_{B_4,B_5,B_1,B_2,B_3} \left(U_i \left|\varphi_1^C\right\rangle\right)_{C_4,C_5} \otimes |\zeta\rangle^{N-3},
\end{aligned} \tag{10}$$

where $|\varphi_i^C\rangle_{C_4,C_5}$ $(i = 1, 2, ..., 16)$ is the possible state of particles C_4 and C_5 that belong to intermediate node C and can also be transformed into $|\varphi_1^C\rangle_{C_4,C_5}$ by unitary operations,

$$\begin{aligned}
&\left|\varphi_{k_{n-2}}^M\right\rangle_{M_4,M_5} \otimes |\zeta\rangle_{M_1,M_2,N_4,M_3,N_5} \\
&= \sum_{i=1}^{16} |\Upsilon_i\rangle_{M_4,M_5,M_1,M_2,M_3} \left|\varphi_i^N\right\rangle_{N_4,N_5}.
\end{aligned} \tag{11}$$

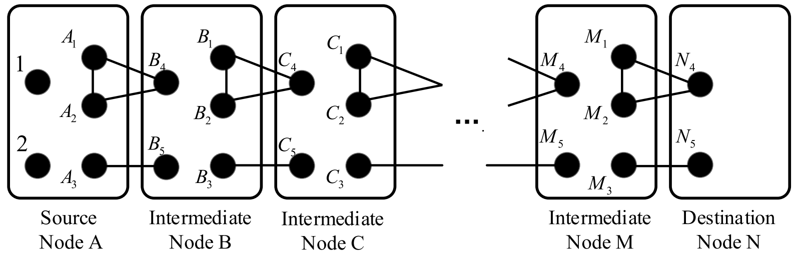

Fig. 3. N − 1 pairs of five-qubit composite GHZ–Bell particles shared between N nodes. Assume that N is the number of nodes.

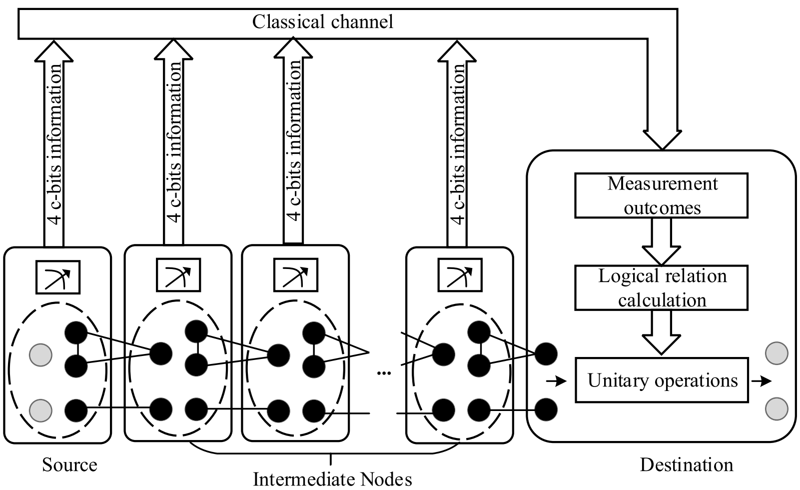

Fig. 4. The quantum communication protocol realizes multihop teleportation of arbitrary two-qubit through GHZ–Bell quantum channels and classical channel.

In quantum paths, the transmission process of two-qubit state and the corresponding logical relation of destination node state and source node can be given. After the source and intermediate nodes make von Neumann five-qubit measurements and send measurement outcomes to the destination node independently, the destination node would perform the proper unitary operations $U_{k_{n-1}} \cdots U_{k_2} U_{k_1}$ to recover the initial state $\left|\varphi^{src}\right\rangle$.

$$\begin{aligned} &\left|\varphi^{src}\right\rangle_{1,2} \to \left|\varphi^{B}_{k_1}\right\rangle_{B_4,B_5} \to \left|\varphi^{C}_{k_2}\right\rangle_{C_4,C_5} \to \dots \to \left|\varphi^{M}_{k_{n-2}}\right\rangle_{M_4,M_5} \\ &\qquad \to \left|\varphi^{N}_{k_{n-1}}\right\rangle_{N_4,N_5}, \\ &\left|\varphi^{N}_{k_{n-2}}\right\rangle_{N_4,N_5} = U_{k_{n-1}} \cdots U_{k_2} U_{k_1} \left|\varphi^{src}\right\rangle_{1,2} \\ &\quad (k_1, k_2, \dots, k_{n-1} = 1, 2, \dots, 16)\,. \end{aligned} \tag{12}$$

For example, there are seven nodes in one selected quantum path, including a source node, five intermediate nodes and a destination node. Six pairs entangled GHZ–Bell particles are shared between adjacent nodes respectively. Assuming that the measurement outcomes on source node and intermediate nodes are $|\Upsilon_8\rangle$, $|\Upsilon_2\rangle$, $|\Upsilon_{14}\rangle$, $|\Upsilon_6\rangle$, $|\Upsilon_1\rangle$ and $|\Upsilon_5\rangle$, they send measurement outcomes to the destination node through classical channels independently. So that for the two particles at the destination node, the relation of the final state $\left|\varphi^{dest}\right\rangle$ and the state $\left|\varphi^{src}\right\rangle$ to be teleported is calculated by

$$\begin{aligned} &\left|\varphi^{dest}\right\rangle \\ &= U_5 U_1 U_6 U_{14} U_2 U_8 \left|\varphi^{src}_1\right\rangle \\ &= \left\{ (\sigma_x \otimes \sigma_x)(I \otimes I)\left[\left(i\sigma_y\right) \otimes \sigma_x\right](\sigma_z \otimes \sigma_x) \right. \\ &\quad \left. (\sigma_z \otimes I)\left[\left(i\sigma_y\right) \otimes \left(i\sigma_y\right)\right] \right\} \left|\varphi^{src}\right\rangle. \end{aligned} \tag{13}$$

Therefore, the destination node can apply unitary operations $(\sigma_x \otimes \sigma_x)(I \otimes I)\left[\left(i\sigma_y\right) \otimes \sigma_x\right](\sigma_z \otimes \sigma_x)(\sigma_z \otimes I)\left[\left(i\sigma_y\right) \otimes \left(i\sigma_y\right)\right]$ to get the desired state $\left|\varphi^{src}\right\rangle$.

5. Description of the whole scheme

The whole scheme for realizing multihop teleportation of an arbitrary two-qubit state through entangled channels composed of GHZ and Bell pairs, is shown in Fig. 4. The composite GHZ–Bell channels are built between adjacent nodes. In addition, the source node and all intermediate nodes have classical channels connecting the destination node. First, every node on quantum paths except the destination node performs the five-qubit von Neumann measurement in a basis that includes the orthonormal states $\left\{|\Upsilon_i\rangle_{1,2,A_1,A_2,A_3}\ (i = 1, 2, \dots, 16)\right\}$ independently. The measurements do not depend on any information from other nodes and can be carried out immediately after the nodes are selected on quantum paths. After measurements, each of the source and intermediate nodes sends measurement outcomes to the destination node through paths. Finally, when the destination node gathers all classical information, it will determine the unitary operations through the method introduced above, and then performs the unitary operations to perfectly recover the initial quantum state.

In quantum communication network, the delay is introduced mostly by von Neumann measurement, unitary operation and the transmission of measurements through classical channel. In order to reduce delay because of the limited decoherence time in quantum memory and QoS (quality of service) need, the quantum communication protocol described above adopts an efficient scheme by measuring and transmitting measurement outcomes in parallelism, compared with hop-by-hop transmission. Fig. 5 shows the overall delay of two different protocols in detail. Assume nodal processing delay of the ith node takes $d_{i,proc}$ seconds, which include measurement delay (d_{meas}) and unitary operation delay (d_{uoper}), and

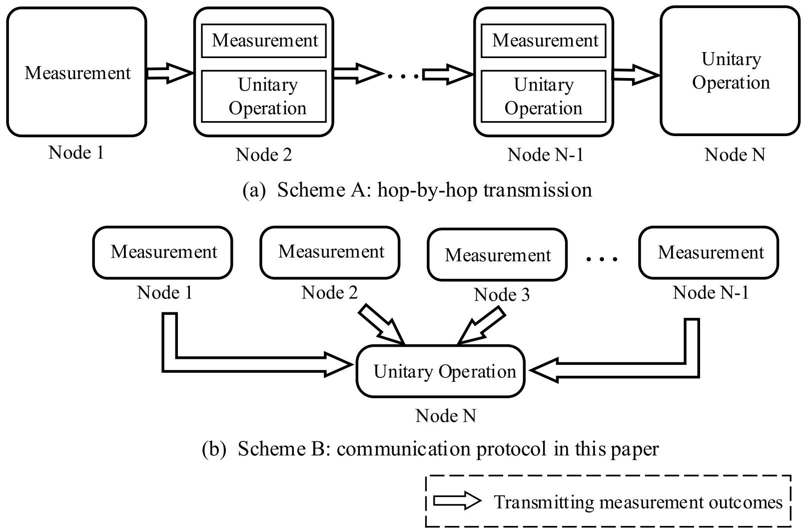

Fig. 5. The comparisons of quantum communication process between hop-by-hop transmission and communication protocol described in this paper.

each transmission of measurements through classical channel takes d_{trans} seconds.

For hop-by-hop transmission which is denoted by scheme A, shown in Fig. 5(a), the measurements and transmissions of measurement outcomes are performed in sequence, so the total quantum communication delay is written as

$$\begin{aligned} & d_{A,total} \\ & = (N-1)\cdot d_{trans} + \sum_{i=1}^{i=N} d_{i,proc} \\ & = (N-1)\cdot d_{trans} + d_{meas} + (N-2)\cdot\left(d_{meas} + d_{uoper}\right) + d_{uoper} \\ & = (N-1)\cdot\left(d_{trans} + d_{meas} + d_{uoper}\right). \end{aligned} \tag{14}$$

In addition, the quantum communication protocol proposed in this paper is described as scheme B, shown in Fig. 5(b). And the total delay of the system is given by

$$\begin{aligned} & d_{B,total} \\ & = (N-1)\cdot d_{trans} + \sum_{i=1}^{i=N} d_{i,proc} \\ & = (N-1)\cdot(d_{trans} + d_{meas}) + d_{uoper}. \end{aligned} \tag{15}$$

Therefore, when the differences of transmission delay through classical channel are ignored, scheme B requires less time for quantum communication, and it has shorter delay in general.

6. Conclusions

This paper proposes a protocol for multihop teleportation between two nodes which do not share entanglement directly in quantum network. Through the composite of GHZ and Bell pairs shared among intermediate nodes, a sender could teleport an arbitrary two-qubit state to a specified receiver. Quantum teleportation processes for one-hop, two-hop, and the general multihop cases are introduced in detail. Intermediate nodes and source node perform the von Neumann five-qubit measurements, and send measurement outcomes to the destination node independently. Then we show the desired quantum state can be recovered perfectly using proper unitary operations at the destination node after collecting all measurements outcomes. Compared with the hop-by-hop transmission schemes, in this scheme, the measurements are performed independently and measurement outcomes are transmitted independently in parallelism. Based on the analysis in section 5, communication delay can be reduced by $(N-1)\cdot d_{uoper}$ seconds in a multihop route which contains N nodes. And this multihop teleportation protocol provides more options for quantum communication networks. Therefore, the transmission flexibility and efficiency of quantum network can be improved significantly.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.physleta.2016.10.048.

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