GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LECTURE 12: DIMENSIONALITY REDUCTION

I. DIMENSIONALITY REDUCTION
II. PRINCIPAL COMPONENTS ANALYSIS
III. SINGULAR VALUE DECOMPOSITION
IV. OTHER METHODS

EXERCISE:
IV. DIMENSIONALITY REDUCTION IN SCIKIT-LEARN

I. DIMENSIONALITY REDUCTION

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Dimensionality reduction is frequently performed as a pre-processing step before another learning algorithm is applied.

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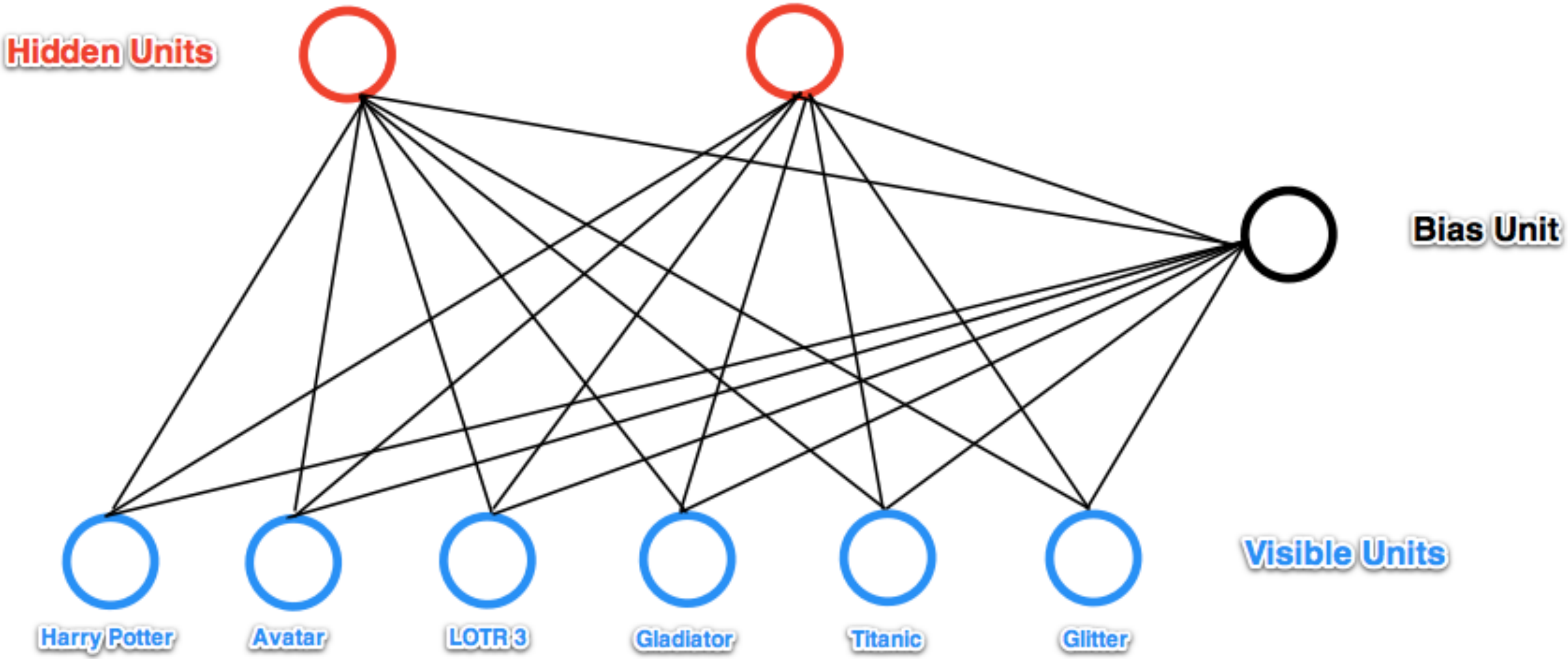
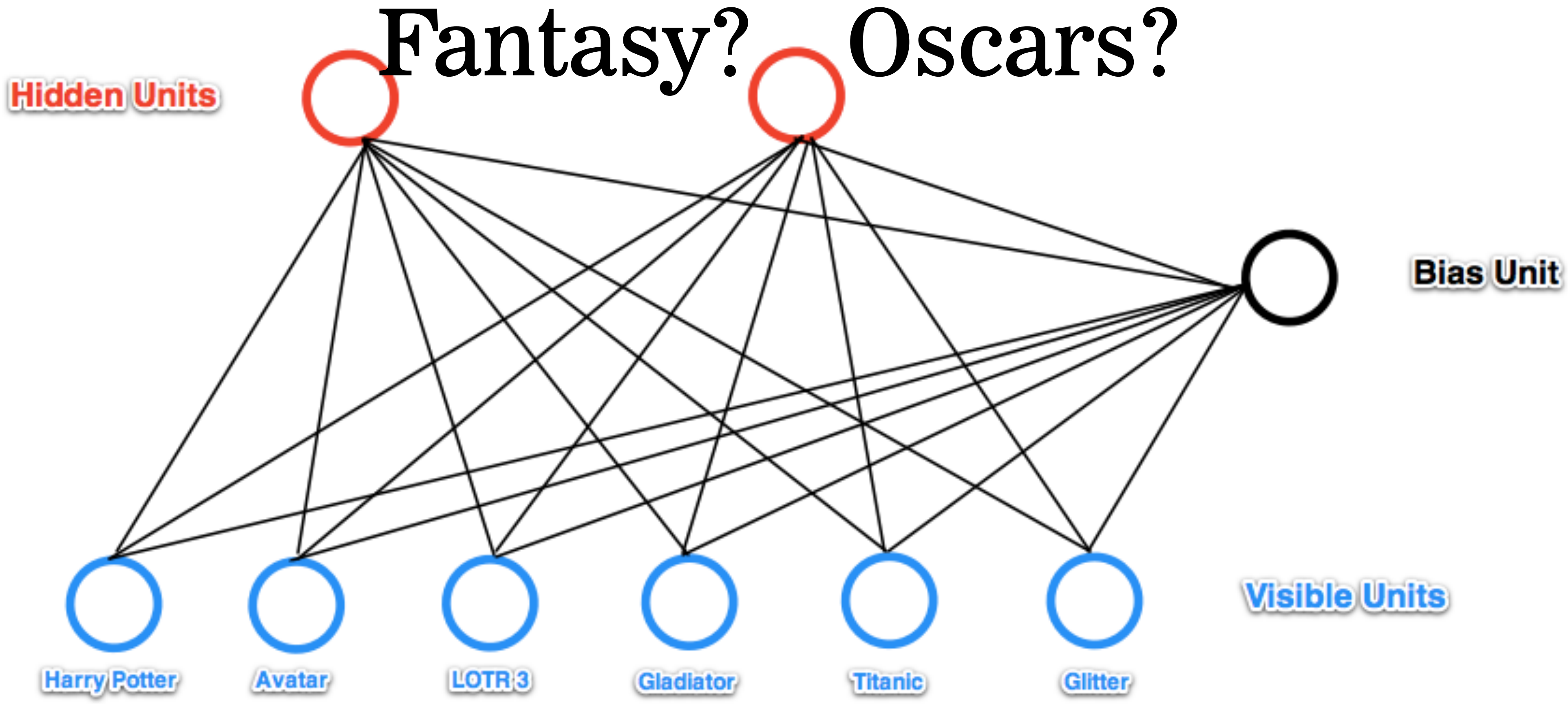
The number of features in our dataset can be difficult to manage, or even misleading (eg, if the relationships are actually simpler than they appear).


Hidden Units
Bias Unit
Visible Units
Harry Potter
Avatar
LOTR 3
Gladiator
Titanic
Glitter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Q: What is the goal of dimensionality reduction?

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- reduce computational expense
- reduce susceptibility to overfitting
- reduce noise in the dataset
- enhance our intuition

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feature selection – selecting a subset of features using an external criterion (*filter*) or the learning algo accuracy itself (*wrapper*)

feature extraction – mapping the features to a lower dimensional space

Feature selection is important, but typically when people say dimensionality reduction, they are referring to *feature extraction*.

The goal of feature extraction is to create a new set of coordinates that *simplify the representation* of the data.

Q: What are some applications of dimensionality reduction?

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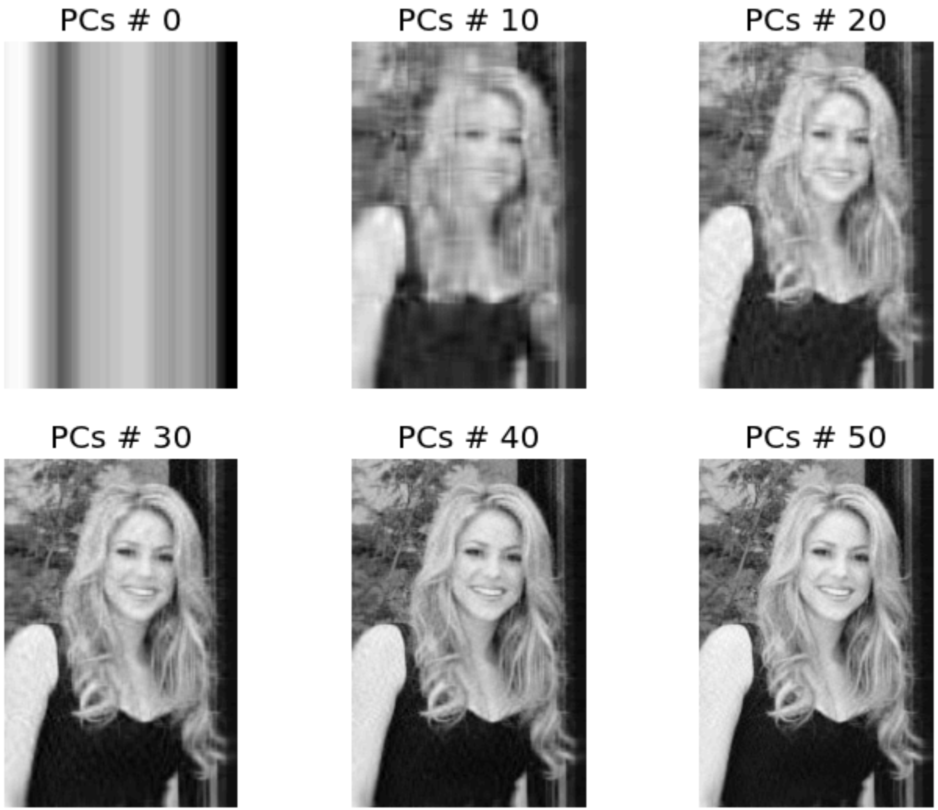
- topic models (document clustering)
- image recognition/computer vision
- recommender systems

DIMENSIONALITY REDUCTION



source: http://glowingpython.blogspot.it/2011/07/pca-and-image-compression-with-numpy.html

II. PRINCIPAL COMPONENT ANALYSIS

Principal component analysis is a dimension reduction technique that can be used on a matrix of any dimensions.

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The PCA of a matrix A boils down to the **eigenvalue decomposition** of the **covariance matrix** of A.

The covariance matrix C of a matrix A is always square:

$$\mathrm{C} = \begin{bmatrix} \mathrm{E}[(X_1-\mu_1)(X_1-\mu_1)] & \mathrm{E}[(X_1-\mu_1)(X_2-\mu_2)] & \cdots & \mathrm{E}[(X_1-\mu_1)(X_n-\mu_n)] \\ \mathrm{E}[(X_2-\mu_2)(X_1-\mu_1)] & \mathrm{E}[(X_2-\mu_2)(X_2-\mu_2)] & \cdots & \mathrm{E}[(X_2-\mu_2)(X_n-\mu_n)] \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{E}[(X_n-\mu_n)(X_1-\mu_1)] & \mathrm{E}[(X_n-\mu_n)(X_2-\mu_2)] & \cdots & \mathrm{E}[(X_n-\mu_n)(X_n-\mu_n)] \end{bmatrix}.$$

off-diagonal elements C_{ij} give the *covariance* between X_i, X_j $(i \neq j)$
diagonal elements C_{ii} give the *variance* of X_i

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NOTE

This relationship *defines* what it means to be an eigenvector of A.

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The eigenvectors form a basis of the vector space on which A acts (eg, they are orthogonal).

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Furthermore the basis elements are ordered by their eigenvalues (from largest to smallest), and these eigenvalues represent the amount of variance explained by each basis element.

III. SINGULAR VALUE DECOMPOSITION

Consider a matrix A with n rows and d features.

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→ $UU^T = I_n, \; VV^T = I_d$ → $\Sigma_{ij} = 0 \;\; (i \neq j)$

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NOTE

The number of singular values is equal to the *rank* of A.

The rank of a matrix measures its *non-degeneracy*.

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Q: How do you interpret the SVD?

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A: Recall that given a set of n points in d-dimensional space (eg, a matrix A), we want to find the best $k < d$ dimensional subspace to represent the data.

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For $k = 1$, this subspace is a line passing through the origin.

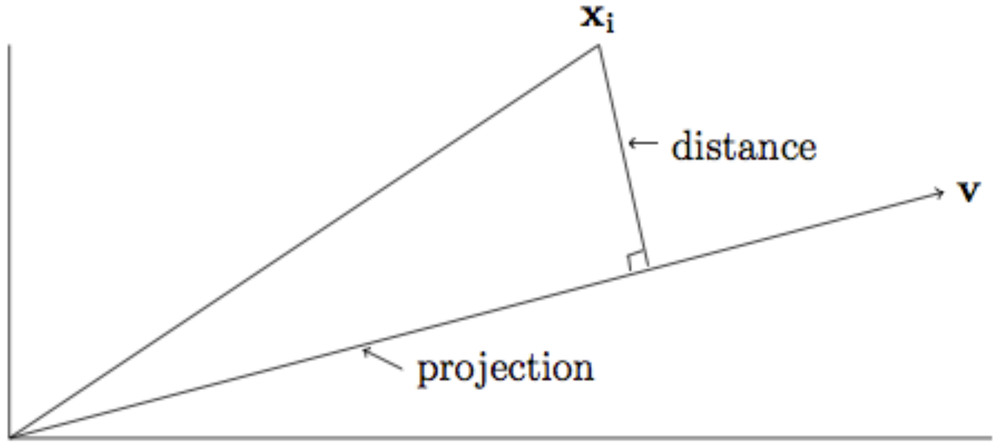

Figure 4.1: The projection of the point $\mathbf{x_i}$ onto the line through the origin in the direction of $\mathbf{v}$

source: http://www.cs.princeton.edu/courses/archive/spring12/cos598C/svdchapter.pdf doc-to-concept similarity matrix

concepts strengths

term-to-concept similarity matrix

$$\begin{array}{cc} & \begin{array}{ccccc}\text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung}\end{array} \\ \begin{array}{c}\\ \\ \text{CS}\\ \\ \\ \\ \text{MD}\\ \\ \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}\end{array} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

In any case, the key difficulties with dimensionality reduction are time/space complexity, randomness (eg different results for different runs), and selecting the number of dimensions in the lower-dim subspace.