GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LECTURE 5: PROBABILITY & LOGISTIC REGRESSION

LAST TIME:

- LINEAR REGRESSION
- REGULARIZATION

QUESTIONS?

I. REVIEW OF REGULARIZATION
II. PROBABILITY
III. LOGISTIC REGRESSION

These regularization problems can also be expressed as:

OLS: $min(\|y - x\beta\|^2)$

L1 regularization: $min(\|y - x\beta\|^2 + \lambda\|x\|)$

L2 regularization: $min(\|y - x\beta\|^2 + \lambda\|x\|^2)$

We are no longer just minimizing error but also an additional term.

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When do we use L1?

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Common case: Text Classification

X = [animal = 1, ..., carnival = 0, ..., xylophone =0, ...zebra = 0]
Y = Topic or Y = Important/Not Important or Y = Positive/Negative

I. INTRO TO PROBABILITY

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The probability of event A is denoted $P(A)$.

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The probability of the sample space $P(\Omega)$ is 1.

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A distribution can be *discrete* or *continuous*
Ex:
Discrete – Uniform distribution

$X \sim \{1, \dots, N\}$ – $P(X = x) = 1/N$

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A distribution can be *discrete* or *continuous*
Ex:
Continuous – Normal distribution – N(u, o)
$X \sim N(0, 1)$ - $P(X = x) = 0$

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For discrete distributions

$$E(X) = \Sigma\ x * p(x)$$

For continuous distributions

$$E(X) = \textit{integral}\ (\ x * p(x)\ )$$

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1) Linda is a bank teller.
2) Linda is a bank teller and active in the feminist movement.

Q: Consider two events A & B. How can we characterize the **intersection** of these events?

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A: With the **joint probability** of A and B, written $P(AB)$.

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Notice, with this we can also write $P(AB) = P(A|B) * P(B)$.

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Using the definition of the conditional probability, we can also write:

$$P(A|B) = P(AB) \,/\, P(B) = P(A) \;\rightarrow\; P(AB) = P(A) * P(B)$$

This result is called **Bayes' theorem**. Here it is again:

$$P(A|B) = P(B|A) * P(A) / P(B)$$

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Some facts:

- This is a simple algebraic relationship using elementary definitions.
- It's interesting because it's kind of a "wormhole" between two different "interpretations" of probability.
- It's a very powerful computational tool.

Each term in this relationship has a name, and each plays a distinct role in any probability calculation (including ours).

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **likelihood function**. It represents the joint probability of observing features $\{x_i\}$ given that that record belongs to class C.

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We can observe the value of the likelihood function from the training data.

This term is the **prior probability** of C. It represents the probability of a record belonging to class C before the data is taken into account.

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The value of the prior is also observed from the data.

This term is the **normalization constant.** It doesn't depend on C, and is generally ignored until the end of the computation.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Maximum likelihood estimator (MLE):

What parameters ***maximize*** the likelihood function?

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Maximum a posteriori estimate (MAP):

What parameters ***maximize*** the likelihood function **AND** prior?

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

II. LOGISTIC REGRESSION

	continuous	*categorical*
supervised	regression	classification
unsupervised	dimension reduction	clustering

Q: What is **logistic regression**?

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A: A generalization of the linear regression model to *classification* problems.

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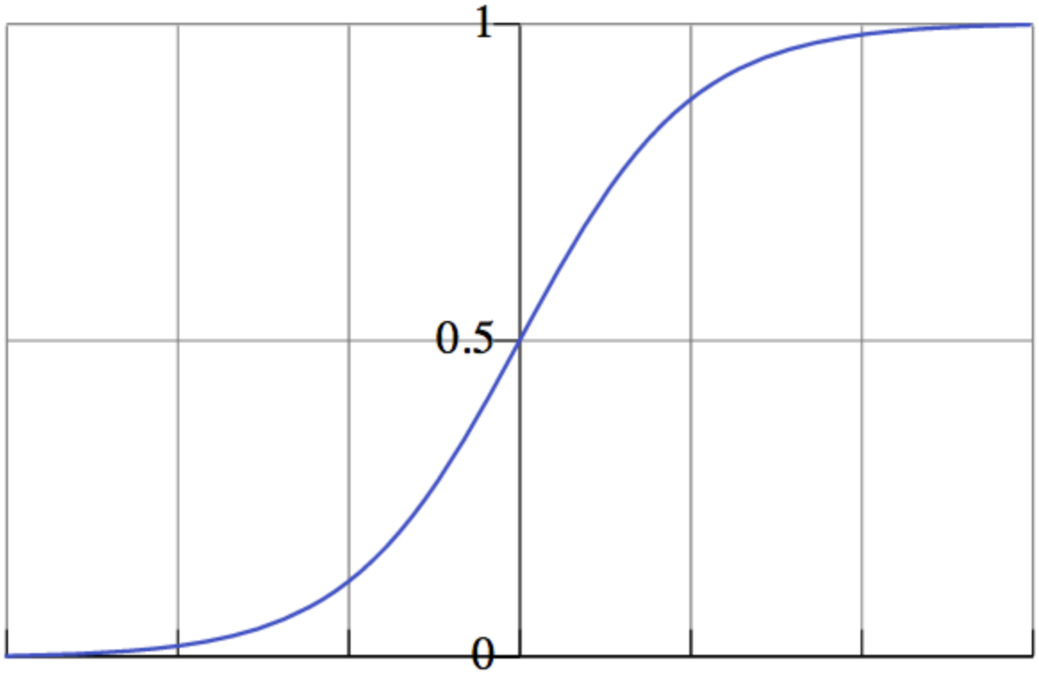
In logistic regression, we use a set of covariates to predict *probabilities* of (binary) class membership.

These probabilities are then mapped to *class labels*, thus solving the classification problem.



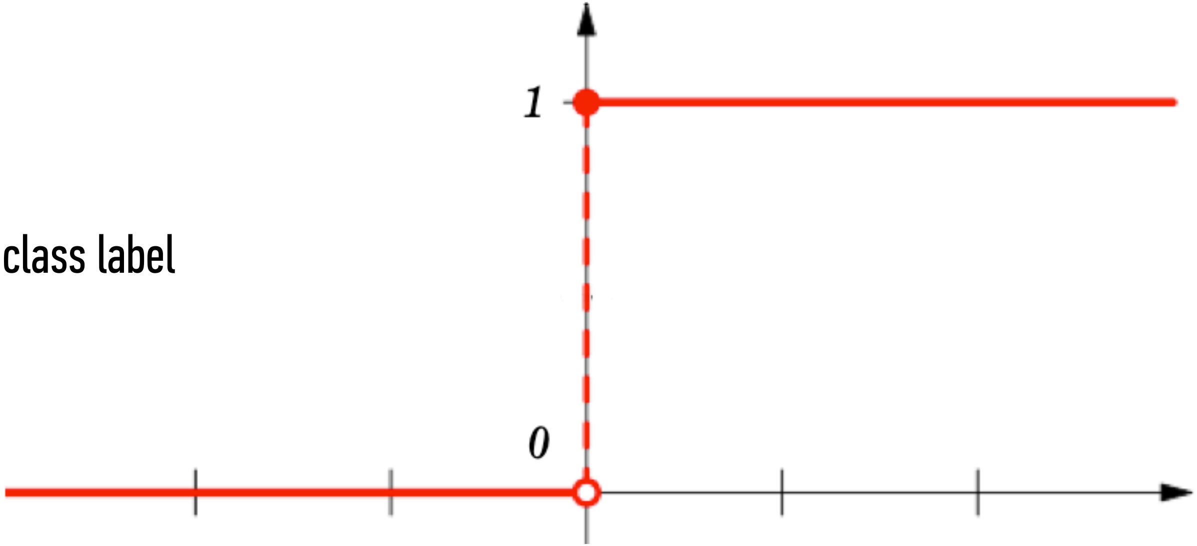
probability of belonging to class

value of independent variable

NOTE

Probability predictions look like this.

class label



value of independent variable

NOTE

Probabilities are "snapped" to class labels (eg by threshholding at 50%).

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The main difference is in the outcome variable.

The key variable in any regression problem is the **response type** of the outcome variable y given the value of the covariate x:

$$E(y|x)$$

The key variable in any regression problem is the **conditional mean** of the outcome variable y given the value of the covariate x:

$$E(y|x)$$

In linear regression, we assume that this conditional mean is a linear function taking values in $(-\infty, +\infty)$:

$$E(y|x) = \alpha + \beta x$$

In logistic regression, we’ve seen that the conditional mean of the outcome variable takes values only in the unit interval $[0, 1]$.

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The first step in extending the linear regression model to logistic regression is to map the outcome variable $E(y|x)$ into the unit interval.

Q: How do we do this?

A: By using a transformation called the **logistic function**:

$$E(y|x) = \pi(x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$$

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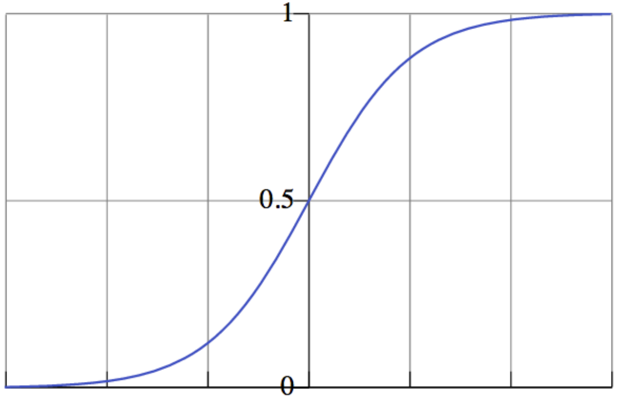

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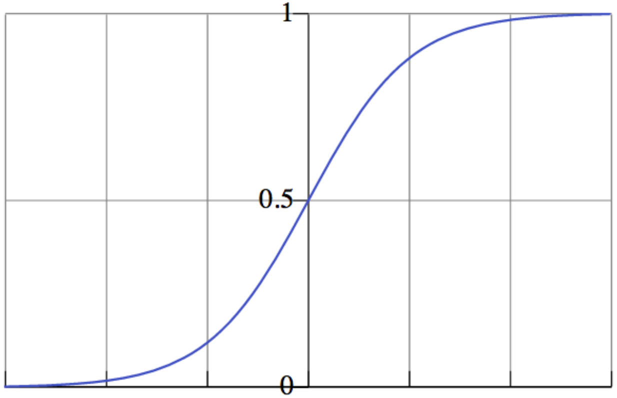

NOTE

For any value of x, y is in the interval [0, 1]

This is a nonlinear transformation!

The **logit function** is an important transformation of the logistic function. Notice that it returns the linear model!

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NOTE

This name hints at its usefulness in interpreting our results.

We will see why shortly.

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