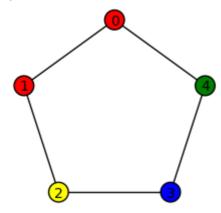
Graph dynamics examples

```
attach "/Users/raichev/graph_dynamics/graph_dynamics.py"
# attach "/Users/arai021/graph_dynamics/graph_dynamics.py"
# For a list of Sage's graph generators, see
http://wiki.sagemath.org/graph_generators.
```

```
# Example: Use color()

G = graphs.CycleGraph(5)
coloring = color(G, ['red', 'red', 'yellow', 'blue', 'green'])
print(coloring)
G.show(vertex_colors=invert_dict(coloring), figsize=3)
```

```
{0: 'red', 1: 'red', 2: 'yellow', 3: 'blue', 4: 'green'}
```



```
# Example: Use color_randomly() and color_count()

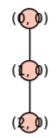
G = graphs.CycleGraph(30)
coloring = color_randomly(G, {'red': 0.6, 'yellow': 0.3,
   'purple': 1/10})
print(coloring)
print('Color count = {!s}'.format(color_count(coloring)))
G.show(vertex_colors=invert_dict(coloring), figsize=3)
```

```
{0: 'red', 1: 'red', 2: 'yellow', 3: 'red', 4: 'yellow', 5:
'yellow', 6: 'yellow', 7: 'red', 8: 'red', 9: 'purple', 10: '
11: 'red', 12: 'red', 13: 'red', 14: 'red', 15: 'red', 16: 'r
17: 'red', 18: 'yellow', 19: 'red', 20: 'red', 21: 'yellow',
'red', 23: 'yellow', 24: 'purple', 25: 'yellow', 26: 'yellow'
'red', 28: 'purple', 29: 'red'}
Color count = Counter({'red': 18, 'yellow': 9, 'purple': 3})
```



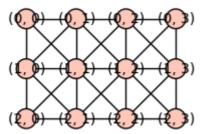
```
# Test the Moore lattice generator
G = moore lattice(1, 1)
G.show(fiqsize=2)
G = moore lattice(3, 1)
G.show(figsize=2)
G = moore lattice(1, 3)
G.show(figsize=2)
G = moore lattice(3, 4)
print('num edges = {!s}'.format(G.num_edges()))
print(G.edges())
G.show(figsize=2)
G = moore_lattice(3, 4, toroidal=True)
print('num edges = %s' % G.num edges())
print('degrees = %s' % G.degree())
print(G.edges())
G.show(figsize=2)
G = moore lattice(2, 4)
coloring = color(G, ['red', 'red', 'yellow', 'blue', 'green',
'green', 'blue', 'blue'])
print(coloring)
G.show(vertex colors=invert dict(coloring), figsize=3)
```







num edges = 29
[((0, 0), (0, 1), None), ((0, 0), (1, 0), None), ((0, 0), (1, None), ((0, 1), (0, 2), None), ((0, 1), (1, 0), None), ((0, 1), None), ((0, 1), (1, 2), None), ((0, 2), (0, 3), None), ((1, 1), None), ((0, 2), (1, 2), None), ((0, 2), (1, 3), None), ((1, 1), None), ((0, 3), (1, 3), None), ((1, 0), (1, 1), None), ((1, 0), (2, 0), None), ((1, 0), (2, 1), None), ((1, 1), (1, None), ((1, 1), (2, 0), None), ((1, 1), (2, 1), None), ((1, 1, 2), None), ((1, 2), (1, 3), None), ((1, 2), (2, 1), None), ((2, 2), None), ((1, 2), (2, 3), None), ((2, 2), None), ((2, 2), (2, 3), None)]



num edges = 48

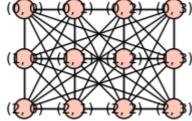
degrees = [8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8]

[((0, 0), (0, 1), None), ((0, 0), (0, 3), None), ((0, 0), (1, None), ((0, 0), (1, 1), None), ((0, 0), (1, 3), None), ((0, 0), None), ((0, 0), (2, 1), None), ((0, 0), (2, 3), None), ((0, 2), None), ((0, 1), (1, 0), None), ((0, 1), (1, 1), None)

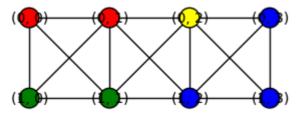
1), (1, 2), None), ((0, 1), (2, 0), None), ((0, 1), (2, 1), None), ((0, 1), (2, 2), None), ((0, 2), (0, 3), None), ((0, 2), (1, None), ((0, 2), (1, 2), None), ((0, 2), (1, 3), None), ((0, 2), (1, 3), None), ((0, 2), (2, 2), None), ((0, 2), (2, 3), None), ((1, 0), None), ((0, 3), (1, 2), None), ((0, 3), (1, 3), None)

3), (2, 0), None), ((0, 3), (2, 2), None), ((0, 3), (2, 3), None), ((1, 0), (1, 1), None), ((1, 0), (2, 3), None), ((1, 1), (2, 1), None), ((1, 2), None), ((1, 1), (2, 1), None), ((2, 2), None), ((1, 2), (1, 3), None), ((1, 2), (2, 1), None), ((2, 2), None), ((1, 2), (1, 3), None), ((1, 2), (2, 1), None), ((2, 2), None), ((1, 2), (1, 3), None), ((1, 2), (2, 1), None), ((2, 2), None), ((1, 2), (1, 3), None), ((1, 3), (2, 0), None), ((1, 2), (2, 1), None), ((2, 2), None), ((1, 2), (2, 3), None), ((1, 3), (2, 0), None), ((1,

```
((1, 3), (2, 2), None), ((1, 3), (2, 3), None), ((2, 0), (2, None), ((2, 0), (2, 3), None), ((2, 1), (2, 2), None), ((2, 2, 3), None)]
```



```
{(0, 1): 'red', (1, 2): 'blue', (0, 0): 'red', (0, 2): 'yellc'; 'blue', (1, 0): 'green', (0, 3): 'blue', (1, 1): 'green'}
```

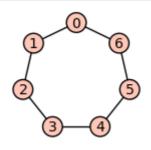


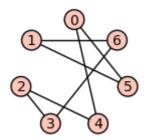
```
# Test the Maslov-Sneppen rewiring method

G = graphs.CycleGraph(7)
G.show(figsize=2)

H = maslov_sneppen(G)
H.show(figsize=2)

G = graphs.RandomBarabasiAlbert(50, 5)
H = maslov_sneppen(G)
print(G.num_verts() == H.num_verts())
print(G.num_edges() == H.num_edges())
print(G.degree() == H.degree())
```





True

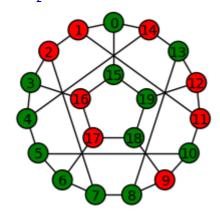
True True

```
# Example: Run the majority rule

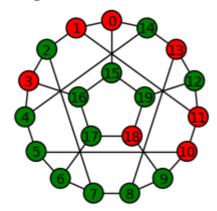
G = graphs.FlowerSnark()
color_bias = {'green': 0.7, 'red': 0.3}
ur = majority_rule
ur_kwargs = {}
initial_coloring = color_randomly(G, color_bias)
s, stabilized = run_rule(ur, ur_kwargs, G, initial_coloring)
print('Stabilized?\n %s' % stabilized)
print(s)
show_colorings(G, s, vertex_labels=True)
```

Stabilized? False [{0: 'green', 1: 'red', 2: 'red', 3: 'green', 4: 'green', 5: green', 6: 'green', 7: 'green', 8: 'green', 9: 'red', 10: 'c 11: 'red', 12: 'red', 13: 'green', 14: 'red', 15: 'green', 16 'red', 17: 'red', 18: 'green', 19: 'green'}, {0: 'red', 1: 'r 'green', 3: 'red', 4: 'green', 5: 'green', 6: 'green', 7: 'gr 8: 'green', 9: 'green', 10: 'red', 11: 'red', 12: 'green', 13 'red', 14: 'green', 15: 'green', 16: 'green', 17: 'green', 18 'red', 19: 'green'}, {0: 'green', 1: 'red', 2: 'red', 3: 'gre 'green', 5: 'green', 6: 'green', 7: 'green', 8: 'green', 9: ' 10: 'green', 11: 'red', 12: 'red', 13: 'green', 14: 'red', 15 'green', 16: 'green', 17: 'green', 18: 'green', 19: 'green'}, 'red', 1: 'red', 2: 'green', 3: 'green', 4: 'green', 5: 'gree' 'green', 7: 'green', 8: 'green', 9: 'green', 10: 'red', 11: ' 12: 'green', 13: 'red', 14: 'green', 15: 'green', 16: 'green' 'green', 18: 'green', 19: 'green'}, {0: 'green', 1: 'red', 2: 'green', 3: 'green', 4: 'green', 5: 'green', 6: 'green', 7: ' 8: 'green', 9: 'green', 10: 'green', 11: 'red', 12: 'red', 13 'green', 14: 'red', 15: 'green', 16: 'green', 17: 'green', 18' 'green', 19: 'green'}, {0: 'red', 1: 'green', 2: 'green', 3: 'green', 4: 'green', 5: 'green', 6: 'green', 7: 'green', 8: '9: 'green', 10: 'green', 11: 'red', 12: 'green', 13: 'red', 1 'green', 15: 'green', 16: 'green', 17: 'green', 18: 'green', 'green', {0: 'green', 1: 'red', 2: 'green', 3: 'green', 4: ' 5: 'green', 6: 'green', 7: 'green', 8: 'green', 9: 'green', 1 'green', 11: 'green', 12: 'red', 13: 'green', 14: 'red', 15: 'green', 16: 'green', 17: 'green', 18: 'green', 19: 'green'}, 'red', 1: 'green', 2: 'green', 3: 'green', 4: 'green', 5: 'gr 6: 'green', 7: 'green', 8: 'green', 9: 'green', 10: 'green', 'red', 12: 'green', 13: 'red', 14: 'green', 15: 'green', 16: 'green', 17: 'green', 18: 'green', 19: 'green'}, {0: 'green', 'red', 2: 'green', 3: 'green', 4: 'green', 5: 'green', 6: 'gr 7: 'green', 8: 'green', 9: 'green', 10: 'green', 11: 'green', 'red', 13: 'green', 14: 'red', 15: 'green', 16: 'green', 17: 'green', 18: 'green', 19: 'green'}, {0: 'red', 1: 'green', 2: 'green', 3: 'green', 4: 'green', 5: 'green', 6: 'green', 7: ' 8: 'green', 9: 'green', 10: 'green', 11: 'red', 12: 'green', Loading [MathJax]/extensions/jsMath2jax.js | green', 16: 'green', 17: 'green', 18

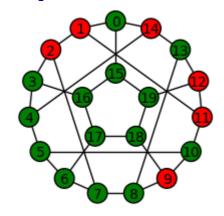
```
'green', 19: 'green'}, {0: 'green', 1: 'red', 2: 'green', 3: 'green', 4: 'green', 5: 'green', 6: 'green', 7: 'green', 8: '9: 'green', 10: 'green', 11: 'green', 12: 'red', 13: 'green', 'red', 15: 'green', 16: 'green', 17: 'green', 18: 'green', 19' 'green'}]
Step 0
```



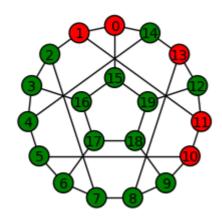
Step 1



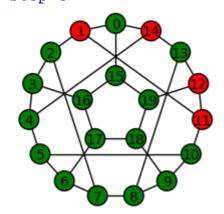
Step 2



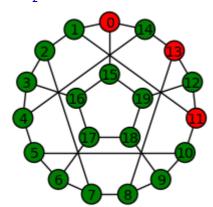
Step 3



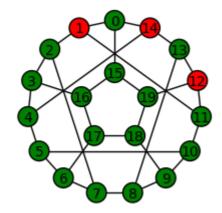
Step 4



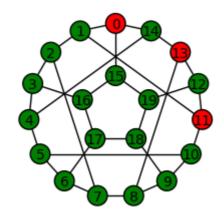
Step 5



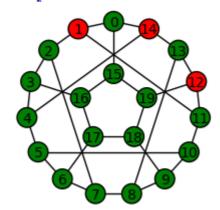
Step 6



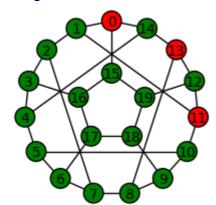
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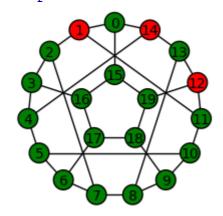
Step 8



Step 9



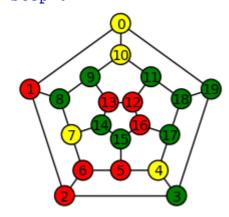
Step 10



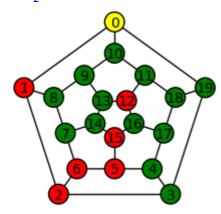
```
G = graphs.DodecahedralGraph()
ur = plurality_rule
ur_kwargs = {}
color_bias = {'green': 0.6, 'red': 0.3, 'yellow': 0.1}
initial_coloring = color_randomly(G, color_bias)

s, stabilized = run_rule(ur, ur_kwargs, G, initial_coloring)
print('Stabilized?\n %s' % stabilized)
show_colorings(G, s, vertex_labels=True)
```

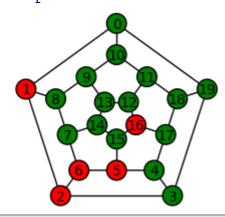
Stabilized?
True
Step 0



Step 1

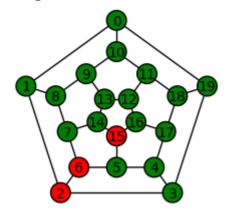


Step 2

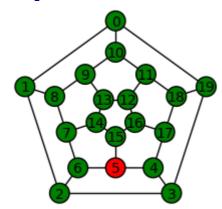


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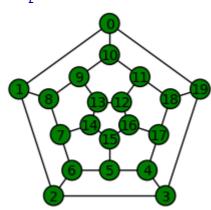
Step 3



Step 4



Step 5



```
# Example: Run the GSL2 rule.

G = graphs.Grid2dGraph(3, 10)
color_bias = {'green': 0.6, 'yellow': 0.4}
ur = gsl2_rule
ur_kwargs = {'palette': color_bias.keys(), 'T': 0.7}
initial_coloring = color_randomly(G, color_bias)

s, stabilized = run_rule(ur, ur_kwargs, G, initial_coloring)
print('Stabilized?\n %s' % stabilized)
show_colorings(G, s)
```

```
True
Step 0

Step 1
```

```
# Example: Run the GSL3 rule.

G = graphs.Grid2dGraph(3, 10)
color_bias = {'green': 1/3, 'red': 1/3, 'yellow': 1/3}
ur = gsl3_rule
ur_kwargs = {'palette': color_bias.keys(), 'T': 0.6}
initial_coloring = color_randomly(G, color_bias)

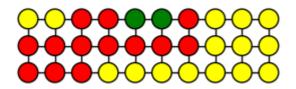
s, stabilized = run_rule(ur, ur_kwargs, G, initial_coloring)
print('Stabilized?\n %s' % stabilized)
show_colorings(G, s)
```

```
Stabilized?
True
Step 0

Step 1

Step 2

Step 3
```

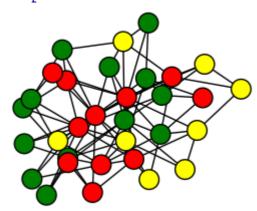


```
# Example: Run the GSL3 rule on a random graph

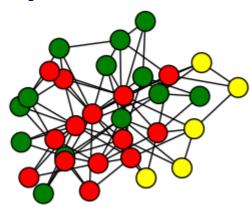
gg = graphs.RandomBarabasiAlbert
print(gg)
G = gg(32, 3)
color_bias = {'green': 1/3, 'red': 1/3, 'yellow': 1/3}
ur = gsl3_rule
ur_kwargs = {'palette': color_bias.keys(), 'T': 0.6}
initial_coloring = color_randomly(G, color_bias)

s, stabilized = run_rule(ur, ur_kwargs, G, initial_coloring)
print('Stabilized?\n %s' % stabilized)
show_colorings(G, s)
```

<function RandomBarabasiAlbert at 0x119a0baa0>
Stabilized?
True
Step 0



Step 1



```
# Example: Use run_rule_many_times() on one graph
Loading [MathJax]/extensions/jsMath2jax.js red': 1/3, 'yellow': 1/3}
```

```
ur = qs13 rule
 urk = {'palette': color bias.keys()}
 gg = graphs.Grid2dGraph
 qqk = {'n1': 3, 'n2': 10}
 cf = color randomly
 cfk = {'bias': color bias}
 num runs = 1000
 num stabilized, mean steps, mean initial, mean final =
 run_rule_many_times(ur, urk, qq, qqk, cf, cfk,
 num runs=num runs)
     <function gsl3 rule at 0x119955500>
     <function Grid2dGraph at 0x11996cd70>
     <function color randomly at 0x119955320>
     Number of runs: 1000
     Number of runs that stabilized: 815
     Mean number of steps required to stabilize: 4.41
     Mean initial color counts:
         green: 9.79
         red: 10.3
         yellow: 9.88
     Mean finial color counts:
         green: 11.5
         red: 6.85
         yellow: 11.6
 # Exploring the random Barabasi Albert graph
 G = graphs.RandomBarabasiAlbert(128, 4)
 n = G.num verts()
 degrees = G.degree()
 ave degree = sum(degrees)/n
 print('nvertices = {!s}'.format(n))
 print('degrees = {!s}'.format(degrees))
 print('ave degree = {:.3f}'.format(ave degree))
     nvertices = 128
     degrees = [7, 7, 18, 36, 34, 34, 37, 25, 19, 18, 9, 12, 20, 2
     15, 6, 12, 7, 9, 16, 12, 8, 6, 5, 6, 9, 10, 8, 10, 6, 9, 12,
     7, 4, 5, 8, 6, 9, 8, 7, 5, 5, 5, 12, 7, 6, 7, 6, 11, 4, 7, 6,
     7, 6, 6, 9, 4, 8, 5, 4, 6, 4, 5, 6, 4, 6, 5, 6, 8, 7, 7, 8, 5
     6, 4, 4, 6, 6, 4, 4, 6, 4, 5, 4, 4, 4, 4, 4, 4, 4, 4, 4, 6
     4, 4, 5, 4, 5, 4, 4, 4, 4, 4, 4, 5, 4, 4, 5, 4, 4, 4, 4
     4, 41
     ave degree = 7.000
 # Example: Use run rule many times() on many instances of a
 random graph
 color bias = {'green': 1/3, 'red': 1/3, 'yellow': 1/3}
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```

```
urk = {'palette': color_bias.keys(), 'T': 0.5, 't': 0.25, 's':
0.25}
gg = graphs.RandomBarabasiAlbert
ggk = {'n': 128, 'm': 4}
cf = color_randomly
cfk = {'bias': color_bias}

num_runs = 1000
num_stabilized, mean_steps, mean_initial, mean_final =
run_rule_many_times(ur, urk, gg, ggk, cf, cfk,
num_runs=num_runs)
<function gsl3_rule at 0x119955500>
```