Chapter 6

- 01. The heavy pill

Because we can only use the scale once, we need to distinguish the 20 bottles by the exact weight showed by scale.

First, if we skip two or more bottles, we can not tell the difference between them.

Second, if we put the same amount of pills of each bottle, we can not tell which bottle the exceeded weight comes from.

Third, we know the exact weight of each pill. Therefore, we come out the solution to put different numbers of every bottle to the scale, such as one from first bottle, two from second and so on.

In this case, we put totally (1 + 20) * 20 / 2 = 210 pills to scale, and the heavy bottle is the bottle we put the following number to scale: (weight - 210 * 1.0) / 0.1

- 02 Basketball

Game one---one shot: p

Game two---two of three shot: p*p*(1-p)*3 + p^3 = 3*p*p - 2*p^3

when p = 3*p*p - 2*p^3 , which means p = 0.5, the probability of these two games are same.

- 03 Dominos

We can try to make some color flag to the board. For example, every black square is surrounded by only white squares, and vice versa. In that case, we can found the two missing corners are the same colors, which means that we left 30 squares of one color and 32 squares of the other. But as we know, one domino can cover two squares of different colors. Therefore, it is impossible to find a solution.

- 04 Ants on a triangle

only if all the ants go clockwise or anti-clockwise, they won't meet.

the probability is : 1 - 1/2*1/2*1/2 * 2 = 3/4 when it comes to n-vertex polygon, there is still only two situations they won't meet:

the probability is : 1 - (1/2)^n * 2 = 1 - (1/2)^(n-1) , n is the number of the ants.

- 05 Jugs of water

1. Fill the 5-quart jug and use it to fill the 3-quart jug, then 2 quart water left in 5-quart jug.

2. Pull away the water in the 3-quart jug, put the 2 quart water in 5-quart jug to it.

3. Fill the 5-quart jug, use it to fill 3-quart jug, then there will be 4 quart water in the 5-quart jug.

- 06 Blue-eyed island

1. If there are only one blue-eye people on the island, on the first day he saw no one else is with blue eyes, he will leave at the first night.

2. If there are two, they both saw one blue-eye people first day, and they saw each other again in the second day, they will know there are not only one blue-eye people, and both of them will leave at the second night.

3. If there are three, they all thought there are 2 blue-eye people, and they will leave at the second night. But during the third day, after they saw each other again, they will know there are not only 2 blue-eye people, and each of them will realize the left one is himself or herself, so all of them will leave at the third night.

4. If there are four, in the similar pattern, after they saw each other during the fourth day, they will know there are not only 3 blue-eye people, and each of them will realize the left one is himself or herself, so all of them will leave at the fourth night.

...

n. For n blue-eye people on the island, it will take n days for them to leave. And all of them will leave together at the nth night.

- 07 The apocalypse

By the java simulation, one can know the girl ratio of next generation is still 0.5, because the basic probability of the sex (male or female) of a baby is always 0.5.

- 08 The egg drop problem

The core of this problem is to minimize the number of drops for the worst case. We can try to find an algorithm to minimize the difference between the best case and the worst case.

Think about a simple case:

Drop the first egg every 10 floors: 10, 20, ..., 90, 100

Then we know the best case is that first egg broke in the first try, the number of drop should be 1 + 9 = 10.

The worst case is that first egg broke in the last try, the numbers of drop should be 10 + 9 = 19.

the number of drop of each case should be: drops of first egg + drops of second egg. And the drops of second egg equals the distance between the contiguous two drops of the first egg: the egg is not broken in the previous time while it is broken in the other time.

Assume one can make the cost of best case equals that of the worst case, which equals to N, we got all pairs of drops of two eggs: (0, N), (1, N-1), (2, N-2), ... (N-1, 1), (N, 0).

So the distance between two continuous of first egg should be: N, N-1, N-2, ... 1, 0.

As we all know, the sum of distance is 100, so that we come out an equation:

(N + N - 1 + N - 2 + ... + 1 + 0) = 100,   n = 13.65

If we pick n = 13, the floors of drops of first egg are: 1, 14, 26, 37, 47, 56, 64, 71, 77, 82, 86, 89, 91, 92. The sequence do not cover all the 100 floors and the cost of the worst case is 14 + 8 = 22.

If we pick n = 14, the floors of drops of first egg are: 1, 15, 28, 40, 51, 61, 70, 78, 85, 91, 96, 100. The cost of the worst case is 12 + 3 = 15, the cost of other general cases are 2 + 13 = 15 (first egg broke at 28 floor), 3 + 12 = 15 (first egg broke at 40 floor), ... which is equal to the worst case and verify the theory one mentioned before.

- 09 100 lockers

From the problem, one can know during the nth pass, all the numbers who is one multiply of n will be toggled. So the lockers left to be open is ones have odd numbers of factors. And only those numbers who are the square of a prime number have odd numbers of factors. Therefore, the numbers of open lockers are the number of such numbers within 100: the square of numbers from 1 to 10, 10 lockers totally.

- 10 Poison

Because there are 1000 bottles and 10 strips and one can know the result after exact 7 days. So one can put drops of different set of bottles to a single strip and put drops on different days to reduce the time cost.

The best solution is bit manipulation to use each strip as a bit of every bottle number. And 10 strips are enough to express 1024 - 1 = 1023 > 1000 bottles. If a number is 1 on the ith binary digit, one need to put a drop in this bottle to the ith strips. In the end, one can know the index of poison bottle by the result after 7 days.