Cart-Pole Optimal Control under Earthquake Disturbances

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1 Introduction

The cart-pole system consists of an inverted pendulum mounted on a cart that moves along a horizontal track. The objective of this assignment is to design and analyze a Linear Quadratic Regulator (LQR) controller to stabilize the pole in the upright position while keeping the cart within the allowable range of ± 2.5 m. The system must remain stable even under realistic earthquake-like disturbances.

2 System Description

2.1 Physical Setup

The system consists of:

- An inverted pendulum mounted on a cart
- Cart traversal range: ± 2.5 m (total 5 m)
- Pole length: 1 m
- Cart mass: 1.0 kg
- Pole mass: 1.0 kg

The goal is to maintain the pole in the upright position while ensuring that the cart remains within its physical limits.

2.2 Disturbance Model

To simulate realistic operating conditions, an earthquake disturbance generator is used. The disturbance introduces external forces acting on the cart. These disturbances are generated using:

- Superposition of sine waves to create continuous earthquake-like motion

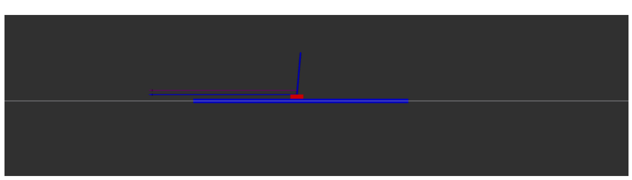

Figure 1: Inverted pendulum on cart system

- Base amplitude of 15.0 N
- Frequency range of 0.5–4.0 Hz
- Random variations in amplitude and phase
- Additional Gaussian noise

3 Controller Overview

The provided controller is a Linear Quadratic Regulator (LQR), which uses state feedback to stabilize the system. The controller minimizes a quadratic cost function that balances state error and control effort. The state vector includes:

$$[x, \dot{x}, \theta, \dot{\theta}] \tag{1}$$

The default cost matrices are:

$$Q = \text{diag}(1, 1, 10, 10), \quad R = 0.1 \tag{2}$$

The Q matrix penalizes deviations in cart position, velocity, pole angle, and angular velocity, while R penalizes control effort.

4 Objective of the Assignment

The main objectives of this work are:

- Stabilize the pendulum in the upright position
- Keep the cart within ± 2.5 m
- Achieve stability under earthquake disturbances
- Analyze the effect of LQR parameters on system behavior

4.1 Parameter Study with Fixed R

To better understand the influence of the Q matrix, the control cost was fixed at $R = 0.5$, and two focused studies were conducted.

- In the first study, the pole-related weights (q_θ, $q_{\dot{\theta}}$) were kept constant while the cart-related weights (q_x, $q_{\dot{x}}$) were varied. Increasing the cart weights reduced cart displacement and improved constraint satisfaction. However, overly large cart penalties sometimes resulted in slower pole stabilization and increased oscillations under disturbances.
- In the second study, the cart-related weights (q_x, $q_{\dot{x}}$) were kept constant while the pole-related weights (q_θ, $q_{\dot{\theta}}$) were varied. Increasing the pole weights significantly improved upright stability and reduced pole angle deviation. However, aggressive pole stabilization increased control effort and caused slightly larger cart movements.
- These experiments showed a clear trade-off between pole stability, cart motion, and control efficiency. The insights from this study were used to guide the Bayesian Optimization process and improve overall controller performance.

5 Methodology

5.1 Baseline Analysis

Initially, the default LQR parameters were tested in the simulation environment. The system exhibited instability under strong disturbances, particularly due to insufficient weighting of the pole angle. This indicated the need for improved tuning.

5.2 Bayesian Optimization for LQR Tuning

Manual tuning of the LQR parameters is time-consuming and may not lead to optimal results. Therefore, Bayesian Optimization was used to automatically search for suitable values of the Q and R matrices. This approach efficiently explores the parameter space and balances exploration and exploitation.

The optimization process was designed to minimize a cost function based on:

- Pole angle deviation
- Cart position error
- Control effort
- Stability duration

Higher importance was given to maintaining the pole upright.

5.3 Cost Function Formulation

A customized cost function was designed to guide the Bayesian Optimization process toward stabilizing the inverted pendulum while satisfying cart position constraints. The pole angle was given the highest priority since maintaining the upright position is the primary control objective.

The cost function is defined as:

$$J = w_1\theta_{\text{rms}}^2 + w_2\theta_{\text{max}}^2 + w_3x_{\text{rms}}^2 + w_4u_{\text{rms}}^2 - w_5T + J_{\text{penalty}} \tag{3}$$

where θ_{rms} and θ_{max} represent the RMS and maximum pole angle, x_{rms} is the RMS cart displacement, u_{rms} is the RMS control effort, and T is the duration of stable operation. Larger weights were assigned to the pole angle terms to ensure that the controller strongly prioritizes stability.

Additional penalty terms were introduced to discourage large pole deviations and cart motion near the physical limits. Soft and hard angular thresholds were used to penalize unsafe pole angles, and a boundary penalty was applied when the cart approached its allowable range. A fixed penalty was also added when stability constraints were violated.

5.4 Performance Metrics

The controller performance was evaluated using:

- Maximum and RMS pole angle deviation
- Maximum and RMS cart displacement
- Control effort (RMS force)
- Duration of stable operation

5.5 Performance Metrics

The following metrics were used:

- Maximum pole angle deviation
- RMS pole angle
- Maximum cart displacement
- RMS cart position
- Control effort (RMS force)
- Duration of stable operation

These metrics help evaluate stability, constraint satisfaction, and control efficiency.

6 Challenges Faced

Several challenges were encountered during this work:

- Initially, the optimization cost function used pole angle in degrees, which resulted in very large and unstable cost values. This prevented the optimizer from converging effectively.
- The issue was resolved by converting the pole angle from degrees to radians, which normalized the scale and significantly improved optimization performance.
- The earthquake disturbances caused strong nonlinear behavior, making it difficult for the controller to stabilize the system consistently.
- Proper balancing between control aggressiveness and stability was required to avoid excessive cart motion.
- ROS2 and Gazebo synchronization also required careful debugging during data collection.

7 Results and Discussion

The optimized LQR controller showed significant improvement in stability and disturbance rejection compared to the baseline controller. The main observations are summarized below:

- The cart position remained close to the equilibrium and stayed well within the constraint of ± 2.5 m throughout the simulation.
- The pole angle was maintained near the upright position, with small oscillations typically within $\pm 1^\circ$ even under strong earthquake disturbances.
- The system remained stable for the entire simulation duration, indicating robust disturbance rejection.
- The earthquake disturbance had large amplitude and varying frequency, yet the controller successfully maintained stability.

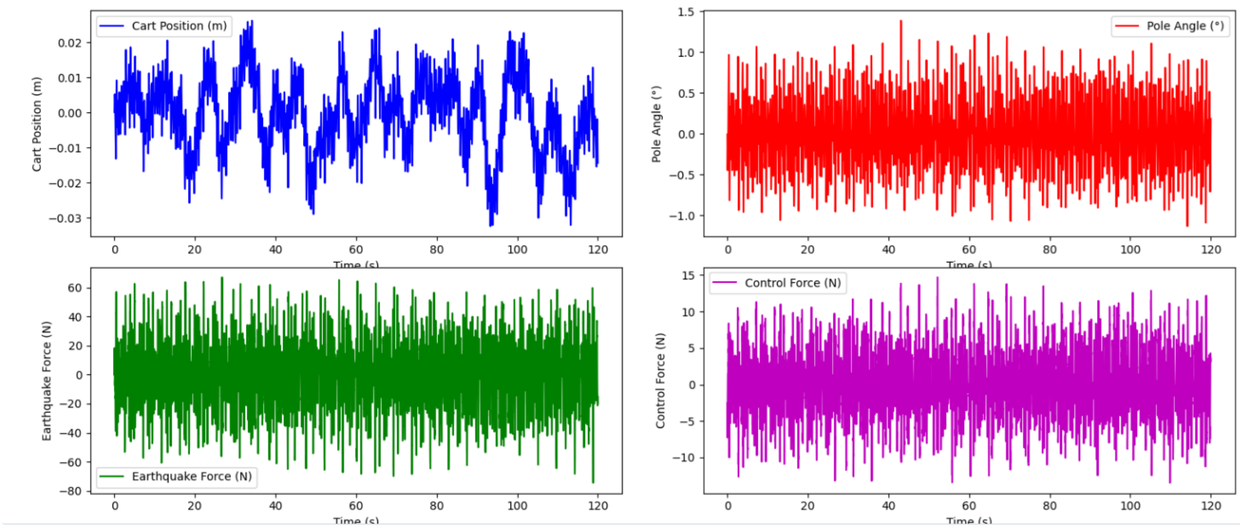

Figure 2: Plot

- The control effort was moderate and smooth, without excessive spikes, showing a good balance between stability and efficiency.
- The results demonstrate that Bayesian Optimization effectively tuned the LQR parameters and improved overall system performance.

8 Conclusion

This assignment provided practical experience in optimal control and disturbance rejection. The LQR controller was successfully analyzed and tuned using Bayesian Optimization. The final controller achieved improved stability, better disturbance handling, and satisfied the physical constraints of the system.