1 Expressions

1.1 Expressions

$$
\begin{aligned}
e \Coloneqq\; & x \quad (\textbf{variable}) \\
\parallel\; & \lambda \quad (\textbf{literal}) \\
\parallel\; & [e_1, ..., e_n] \\
\parallel\; & \{k_1 : e_1 \;;\; ... \;;\; k_n : e_n\} \\
\parallel\; & (x_1, ..., x_n) \to e \\
\parallel\; & e_1[e_2] \\
\parallel\; & f(e_1, ..., e_n) \\
\parallel\; & \text{if } e \text{ elif } f_1 ... \text{ elif } f_n \text{ else } g \text{ end} \\
\parallel\; & \text{match } e \;;\; p_1 \Rightarrow e_1 \;;\; ... \;;\; p_n \Rightarrow e_n \text{ end} \\
\parallel\; & (e : \tau) \\
\parallel\; & e \text{ as } \tau
\end{aligned}
$$

2 Typing

2.1 Types

$$
\begin{aligned}
\tau \Coloneqq\; & T \quad (\textbf{type variable}) \\
\parallel\; & \lambda \quad (\textbf{literal}) \\
\parallel\; & \mathit{int} \parallel \mathit{number} \parallel \mathit{string} \parallel \mathit{bool} \parallel \mathit{null} \parallel \mathit{void} \\
\parallel\; & [\tau_1, ..., \tau_n] \parallel \tau[] \\
\parallel\; & \{k_1 : \tau_1 \;;\; ... \;;\; k_n : \tau_n\} \\
\parallel\; & \tau_1 \mid \tau_2 \\
\parallel\; & (\tau_1, ..., \tau_n) \to \sigma \\
\parallel\; & \mathit{any} \parallel \mathit{some}
\end{aligned}
$$

2.2 Type Equivalence

$$(\tau_0 \mid \tau_1) \mid \tau_2 \overset{c}{\sim} \tau_0 \mid (\tau_1 \mid \tau_2)$$

$$\tau_0 \mid \tau_1 \overset{c}{\sim} \tau_1 \mid \tau_0$$

$$\tau \mid \tau \overset{c}{\sim} \tau$$

$$\mathit{any} \mid \tau \overset{c}{\sim} \tau$$

$$\mathit{some} \mid \tau \overset{c}{\sim} \mathit{some}$$

2.3 Subtype Relation

$$\frac{}{\tau \preceq \tau}$$

$$\frac{}{any \preceq \tau} \quad \frac{}{\tau \preceq some}$$

$$\frac{\text{typeof}(\lambda) \preceq \tau}{\lambda \preceq \tau}$$

$$\frac{}{int \preceq number}$$

$$\frac{\tau_1 \preceq \sigma_1 \quad ... \quad \tau_n \preceq \sigma_n}{[\tau_1, ..., \tau_n] \preceq [\sigma_1, ..., \sigma_n]} \quad \frac{\tau_1 \preceq \sigma \quad ... \quad \tau_n \preceq \sigma}{[\tau_1, ..., \tau_n] \preceq \sigma[]} \quad \frac{\tau \preceq \sigma}{\tau[] \preceq \sigma[]}$$

$$\frac{\tau_1 \preceq \sigma \quad \tau_2 \preceq \sigma}{\tau_1 \mid \tau_2 \preceq \sigma} \quad \frac{\tau \preceq \sigma_1}{\tau \preceq \sigma_1 \mid \sigma_2}$$

$$\frac{\tau_1 \preceq \sigma_1 \quad ... \quad \tau_n \preceq \sigma_n}{\{k_1 : \tau_1 \ ; \ ... \ ; \ k_n : \tau_n \ ; \ k'_1 : \tau'_1 \ ; \ ... \ ; \ k'_m : \tau'_m\} \preceq \{k_1 : \sigma_1 \ ; \ ... \ ; \ k_n : \sigma_n\}}$$

$$\frac{\tau_2^1 \preceq \tau_1^1 \quad ... \quad \tau_2^n \preceq \tau_1^n \quad \sigma_1 \preceq \sigma_2}{(\tau_1^1, ..., \tau_1^n) \to \sigma_1 \preceq (\tau_2^1, ..., \tau_2^n) \to \sigma_2}$$

2.4 Expression Typing

$$\frac{}{\Gamma, x : T \vdash x : T}$$

$$\frac{}{\Gamma \vdash \lambda : \text{typeof}(\lambda)}$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad ... \quad \Gamma \vdash e_n : T_n}{\Gamma \vdash [e_1, ..., e_n] : [T_1, ..., T_n]}$$

$$\frac{\Gamma \vdash e : T}{\Gamma \vdash \{k : e\} : \{k : T\}}$$

$$\frac{\Gamma, x_1 : \tau_1, ..., x_n : \tau_n \vdash e : \sigma}{\Gamma \vdash (x_1, ..., x_n) \to e : (\tau_1, ..., \tau_n) \to \sigma}$$

$$\frac{\Gamma \vdash e_1 : [\tau_1, ..., \tau_n] \quad \Gamma \vdash e_2 : i}{\Gamma \vdash e_1[e_2] : \tau_i}$$

$$\frac{\Gamma \vdash e_1 : [\tau_1, ..., \tau_n] \quad \Gamma \vdash e_2 : null}{\Gamma \vdash e_1[e_2] : null}$$

$$\frac{\Gamma \vdash e_1 : [\tau_1, ..., \tau_n] \quad \Gamma \vdash e_2 : \sigma \quad \sigma \preceq int}{\Gamma \vdash e_1[e_2] : \tau_1 \mid ... \mid \tau_n \mid null}$$

$$\frac{\Gamma \vdash f : (\sigma_1, ..., \sigma_n) \to \rho \quad \Gamma \vdash e_1 : \tau_1 \quad ... \quad \Gamma \vdash e_n : \tau_n \quad \tau_1 \preceq \sigma_1 \quad ... \quad \tau_n \preceq \sigma_n}{\Gamma \vdash f(e_1, ..., e_n) : \rho}$$

$$\frac{\Gamma \vdash e : bool \quad \Gamma \vdash f_1 : \tau_1 \quad ... \quad \Gamma \vdash f_n : \tau_n \quad \Gamma \vdash g : \tau'}{\Gamma \vdash \text{if } e \text{ elif } f_1 ... \text{ elif } f_n \text{ else } g \text{ end} : \tau_1 \mid ... \mid \tau_n \mid \tau'}$$

$$\frac{\Gamma \vdash e : bool \quad \Gamma \vdash f_1 : \tau_1 \quad ... \quad \Gamma \vdash f_n : \tau_n}{\Gamma \vdash \text{match } e \ ; \ p_1 \Rightarrow f_1 \ ; \ ... \ ; \ p_n \Rightarrow f_n \text{ end} : \tau_1 \mid ... \mid \tau_n}$$

$$\frac{\Gamma \vdash e : \tau \quad \Gamma \vdash \tau \preceq \sigma}{\Gamma \vdash (e : \sigma) : \sigma}$$

$$\frac{}{\Gamma \vdash e \text{ as } \sigma : \sigma}$$