

Numerical Optimization 2024 - Homework 2

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Problem 1.

(a)

$$x_k = 1 / k$$

$$x_{k+1} = 1 / (k + 1)$$

$$x^* = 0$$

$$l_k = \frac{\| x_{k+1} - x^* \|}{\| x_k - x^* \|}$$

$$l_k = \frac{\| 1 / (k + 1) - 0 \|}{\| 1 / k - 0 \|}$$

$$l_k = 1 - \frac{1}{k + 1}$$

$$\lim_{k \rightarrow \infty} l_k = 1, \text{ sublinear}$$

(b)

$$x_k = 1 / k!$$

$$\lim_{k \rightarrow \infty} x_k = 0$$

$$l_k = \frac{\| 1 / (k + 1)! - 0 \|}{\| 1 / k! - 0 \|}$$

$$l_k = \frac{k!}{(k + 1)!}$$

$$l_k = \frac{1}{k+1}$$

$$\lim_{k \rightarrow \infty} l_k = 0 \quad \text{superlinear}$$

$$\text{Check for quadratic fractions: } \frac{1}{(k+1)^2}$$

$$\lim_{k \rightarrow \infty} \frac{1}{(k+1)^2} = 0$$

(c)

$$x_k = a^k \text{ for some } a \in (0, 1)$$

$$x^* = 0$$

$$l_k = \frac{\|a^{k+1} - 0\|}{\|a^k - 0\|} = \frac{a^{k+1}}{a^k} = a, \text{ converges linearly}$$

(d)

$$x_k = 1 + (0.5)^{2^k}$$

$$x^* = 1$$

$$l_k = \frac{\|1 + (0.5)^{2^{k+1}} - 1\|}{\|1 + (0.5)^{2^k} - 1\|} = \frac{(0.5)^{2^{k+1}}}{(0.5)^{2^k}} = (0.5)^{2^k}, \text{ superlinear}$$

$$\text{Check for quadratic fractions: } \frac{\|1 + (0.5)^{2^{k+1}} - 1\|}{\|1 + (0.5)^{2^k} - 1\|^2} = \frac{(0.5)^{2^{k+1}}}{((0.5)^{2^k})^2} = 1, \text{ quadratic}$$

Problem 2.

$$x_{k+1} = x_k - \gamma(4x_k^3 - 20x_k) \text{ for } \gamma = 1/88 \text{ converges to } x^* = \sqrt{5}$$

Convergence of x_{k+1} is shown in Figure: 1, the l_k values for 100 iterations are in Figure: 2 and the corresponding plot is in Figure: 3. The values indicate linear convergence approaching to the constant 0.55 after 10 iterations.

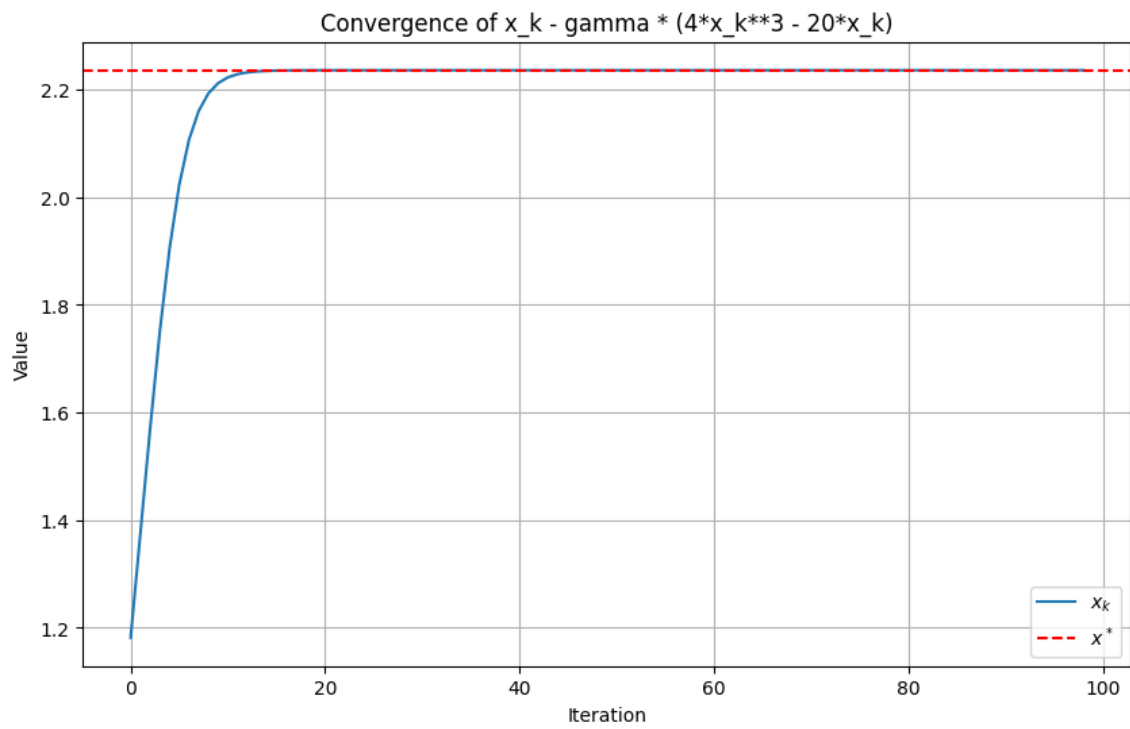


Figure 1: Convergence of $x_k - \gamma(4x_k^3 - 20x_k)$

0 value	for iteration 1:	0.8529060010227368278987225856571811512289469103149939850102
1 value	for iteration 2:	0.816394545160664829841867983535851396865368797497594279546
2 value	for iteration 3:	0.7742211709365829438881944446093661538840795990159122093036
3 value	for iteration 4:	0.7284561092754020080965924503513277023515733080079208527088
4 value	for iteration 5:	0.68275566028773918490426157080859977676595695977061142710
5 value	for iteration 6:	0.641517677708264845621065915514361566869570222991867666708
6 value	for iteration 7:	0.6082289264595967843073974564845327348193565887993461459083
7 value	for iteration 8:	0.5841253447506927112242947796625137879117493386610128214982
8 value	for iteration 9:	0.568227798080966285123093412640880390526534197071567532671
9 value	for iteration 10:	0.558458568014160622232664573297649629435056656785329402537
10 value	for iteration 11:	0.55273740248469983461089491156207372933472603736291703237
11 value	for iteration 12:	0.54948650954032250148851355154834925084374189744141342740
12 value	for iteration 13:	0.5476720309242912868719996833635881055994577744734448397
13 value	for iteration 14:	0.546695968702601951002771252069108953408445301107708069
14 value	for iteration 15:	0.54611895620568974794017945566360029105841187172851708807
15 value	for iteration 16:	0.5458174629018135410955145354885381521614179091093043129
16 value	for iteration 17:	0.54565263902871703257087316688845339183427771898564065799
17 value	for iteration 18:	0.54556264049313463366740758831123203734126391818183950587
18 value	for iteration 19:	0.54551351948562343401420276141648601837656939417402228732
19 value	for iteration 20:	0.54548671700738028182344265965622933015152494417557005102
20 value	for iteration 21:	0.5454720947347404278652132516374156480271580710197284470
21 value	for iteration 22:	0.54546411813450546210221395706668902804118591573255342338
22 value	for iteration 23:	0.54545976701908580153855221054564322273759853354509722723
23 value	for iteration 24:	0.545457930112277650220038474800615953993725121666436112
24 value	for iteration 25:	0.5454560990364925229792727164525668512377205154287656856
25 value	for iteration 26:	0.5454553928476717644766925725373808497715830801389245109
26 value	for iteration 27:	0.54545506766978309606353699845871895265116615737351233548
27 value	for iteration 28:	0.5454547975721873377752467929357849683410452669968067032
28 value	for iteration 29:	0.54545468297333047124942143066966509297401250515398866049
29 value	for iteration 30:	0.5454546204648130252677537169378503968102125363367844279
30 value	for iteration 31:	0.5454545863692431595861458395257782150146240482042336639
31 value	for iteration 32:	0.545454567771655164837216294570969608841280138077176521131
32 value	for iteration 33:	0.54545455762751494012074135051176261482338956702340302401
33 value	for iteration 34:	0.5454545520943715253989387154383561834377500116631992453
34 value	for iteration 35:	0.54545454907625551532954893289583067181076522605815065102
35 value	for iteration 36:	0.5454545474300236784906708857538829150870896574614346225
36 value	for iteration 37:	0.5454545465320790298837684861190584196478409457964013226
37 value	for iteration 38:	0.5454545460422910366608184842565861645694657993232700709
38 value	for iteration 39:	0.54545454577513394853380134261516436838919093344461436875
39 value	for iteration 40:	0.5454545456294119001924372682067899567508136359781226253
40 value	for iteration 41:	0.54545454554992714647074315804371123280938955732281743135
41 value	for iteration 42:	0.54545454550657182623482568924274174497933891888924784228
42 value	for iteration 43:	0.54545454548292346973534148798304407109630833893580161471
43 value	for iteration 44:	0.5454545454709243661880357876201792862456118932755056804
44 value	for iteration 45:	0.545454545462984915252356687878233741699724072364283632
45 value	for iteration 46:	0.5454545454591507417089738408797552783241271114682771910
53 value	for iteration 54:	0.54545454545458152690583733058788036408038836943900647741
54 value	for iteration 55:	0.545454545454565124643767550451877783232026496431360536
55 value	for iteration 56:	0.545454545454556177953585811704923160115802650033092084
56 value	for iteration 57:	0.5454545454545512979435102785301695211680639346716807327
57 value	for iteration 58:	0.5454545454545486361188619388174530155688372990085493643
58 value	for iteration 59:	0.5454545454545471842145802440172207778811386365204770885
59 value	for iteration 60:	0.545454545454546392266789563857837748346682349713091350
60 value	for iteration 61:	0.54545454545454596029513570859848273187266640886264443675
61 value	for iteration 62:	0.545454545454545724674293969984519445085675580825795295
62 value	for iteration 63:	0.54545454545454559615383478885277194035547878302638124596
63 value	for iteration 64:	0.5454545454545455260517661259143059278199516725158190989
64 value	for iteration 65:	0.5454545454545454878142741846368109910330809944116064617
65 value	for iteration 66:	0.5454545454545454669574603704702671931484130830502851791
66 value	for iteration 67:	0.54545454545454545558101648420894912197521781923511018107
67 value	for iteration 68:	0.54545454545454544937568345096313891512083781094525610866
68 value	for iteration 69:	0.545454545454545454990956341924332792431205411543193448
69 value	for iteration 70:	0.54545454545454544414474155517589384052083341772408409975
70 value	for iteration 71:	0.5454545454545454431377153078585632478799154310712775964
71 value	for iteration 72:	0.54545454545454544258842826386729202436925964976398845776
72 value	for iteration 73:	0.54545454545454544228881714896296227187044031966698834140
73 value	for iteration 74:	0.54545454545454544212539290446969150121791821074241989893
74 value	for iteration 75:	0.5454545454545454420362524074733610919128821569025025329
75 value	for iteration 76:	0.54545454545454544189763031020263806611384663431671740167
76 value	for iteration 77:	0.5454545454545454411961189178600423069092304825685734897
77 value	for iteration 78:	0.5454545454545454419466431024537610309206029814925953649
78 value	for iteration 79:	0.54545454545454544193875251546467264669034128282025081492
79 value	for iteration 80:	0.545454545454545441934448558925160991754961832699598252231
80 value	for iteration 81:	0.54545454545454544193210094626725929820807460364009038089
81 value	for iteration 82:	0.54545454545454544193082043027203353812118753293992168639
82 value	for iteration 83:	0.5454545454545454419301219670019131780899454289316814945
83 value	for iteration 84:	0.5454545454545454419297409870363919998989325680639198854
84 value	for iteration 85:	0.54545454545454544192953317978247135725558067391917668854
85 value	for iteration 86:	0.54545454545454544192941983087124019150158502224481141489
86 value	for iteration 87:	0.5454545454545454419293580834196622048651805606503627064
87 value	for iteration 88:	0.5454545454545454419293242796278914775803895269743954573
88 value	for iteration 89:	0.54545454545454544192935884832380163268134916349214889
89 value	for iteration 90:	0.54545454545454544192929585130755501000751801419041376866
90 value	for iteration 91:	0.5454545454545454419292903784758334550171914302903209105
91 value	for iteration 92:	0.5454545454545454419292873932948939584862516846078093216
92 value	for iteration 93:	0.545454545454545441929287650143815164969752074874204617
93 value	for iteration 94:	0.54545454545454544192928487686137472995064122597506764981
94 value	for iteration 95:	0.54545454545454544192928439241428011910719753205519805967
95 value	for iteration 96:	0.54545454545454544192928412817041033137441017451074310535
96 value	for iteration 97:	0.545454545454545441929283860373904471565307677746455406
97 value	for iteration 98:	0.5454545454545454419292839054193706012195165461478388292
98 value	for iteration 99:	0.54545454545454544192928386253682823070804150802626712931
99 value	for iteration 100:	0.54545454545454544192928383914634564975086646099015059

Figure 2: 100 iterations for l_k

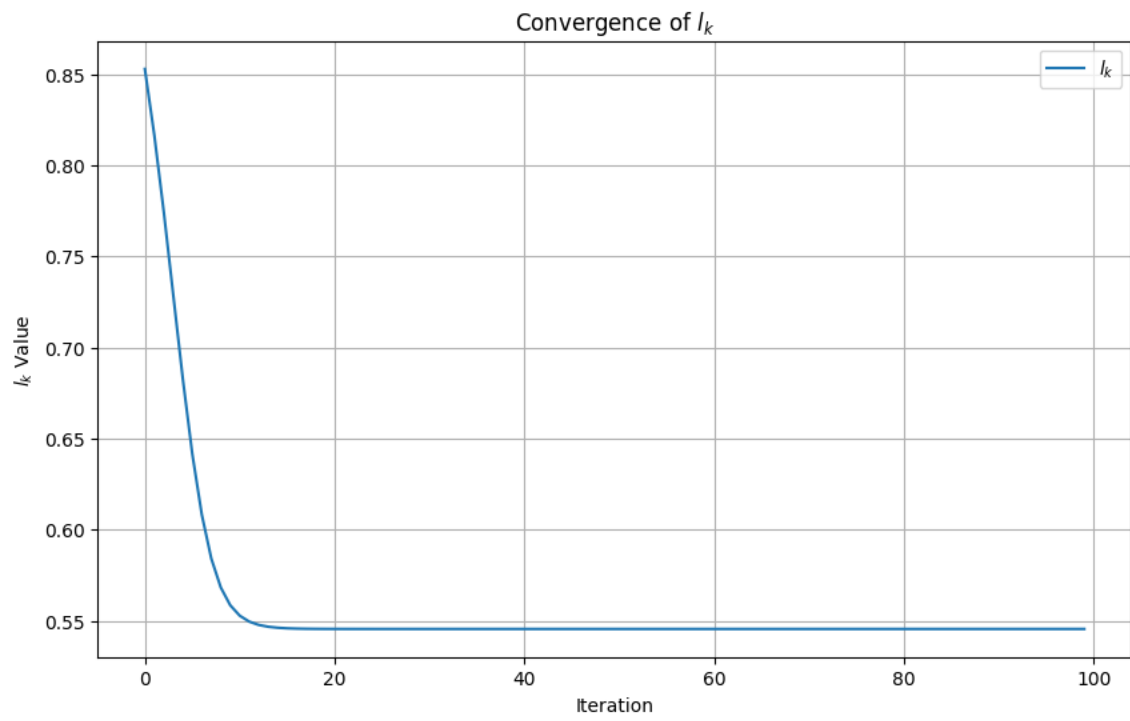


Figure 3: Convergence of l_k

Problem 3.

$x_{k+1} = x_k + \gamma \cos(x_k)$ for any $\gamma > 0$ either converges to $x^* = \pi/2$ or does not converge

Figure: 4 shows 100 iterations for $\gamma = 0.5$

I plotted the graph (Figure: 5) for the first 10 iterations where it is easier to see that l_k linearly converges to 0.5.

Figure: 6 shows 100 iterations for $\gamma = 3$ and Figure: 7 shows the convergence of l_k for $\gamma = 3$ reaching to 1.

Figure: 8 shows 100 iterations for $\gamma = 2$ and Figure: 9 shows the convergence of l_k for $\gamma = 2$ reaching to 1 slower than $\gamma = 3$.

Figure: 10 shows 100 iterations for $\gamma = 1$ and Figure: 11 shows the convergence of l_k for $\gamma = 1$ reaching to 1 faster than $\gamma = 3$ and $\gamma = 2$.

For gamma values 3, 2 and 1 l_k converges sublinearly.

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0 value for iteration 1: 0.5267118230227453918832061568492826396412220345581978701997118
1 value for iteration 2: 0.5074983252821068191517120328679822251230738547475547825849458
2 value for iteration 3: 0.5019371195793932674526403180428638778445196148015042330214146
3 value for iteration 4: 0.500486172106920613928222968919795809768279708232346868747247
4 value for iteration 5: 0.5001224200711785494672395314178600617166764708457197574062822
5 value for iteration 6: 0.5000386216930269140209388755068292074331155375211555955561626
6 value for iteration 7: 0.5000076564664719774783478356919859386601121399294940520161626
7 value for iteration 8: 0.5000019141818294261497168428810366894982475291259216455673737
8 value for iteration 9: 0.5000004785495224897475322837494275438473481359582374257232091
9 value for iteration 10: 0.500000119637613037767628634475961615798495767370451824673843
10 value for iteration 11: 0.500000029969734466780783203654214217597260140521004753016
11 value for iteration 12: 0.5000000074772551796798926224865715852463541194398126684544
12 value for iteration 13: 0.50000000186913499258565685015458329718744075059977224815775
13 value for iteration 14: 0.50000000046692602298923847936759002665584126279510820944847
14 value for iteration 15: 0.500000000011601604790090075309978108990672412050210383184000
15 value for iteration 16: 0.500000000002757309581169831191055987817011400398762022642603
16 value for iteration 17: 0.500000000000403144159646227913650284814895029442253454117499
17 value for iteration 18: 0.4999999999528419568432389876420425499766814740611227610227
18 value for iteration 19: 0.49999999990727371949122821329680403938013417449165182959706
19 value for iteration 20: 0.49999999973848778101245540747357923448938891921746360249038
20 value for iteration 21: 0.4999999994769787502984446563586888792183842447449602508226
21 value for iteration 22: 0.499999999053458330065365419465075007261583659840196067183
22 value for iteration 23: 0.49999999979067918737047140841766624835845784434050057143975
23 value for iteration 24: 0.49999999951352547689242197436806650134296427754344237079
24 value for iteration 25: 0.499999991627097288876046483541339298094704261526889810670
25 value for iteration 26: 0.4999999832541925998369144816497775273881182473600805315918
26 value for iteration 27: 0.49999996605081846080167561330512683653296462362064861873738
27 value for iteration 28: 0.4999999330167680295311260394947148938662587561893507369622
28 value for iteration 29: 0.4999998668335354338941481005483887455379687351588308947389
29 value for iteration 30: 0.4999997326670636813659608156112703135556945472516374511269
30 value for iteration 31: 0.4999996461349896456028641219576890582810576227815876764266
31 value for iteration 32: 0.499999892868882429811681181762274630763687262240197113476
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33 value for iteration 34: 0.499999571306957305487774335684040096766093016789174883493
34 value for iteration 35: 0.4999991426131749944531793640979983630309240958430103714773
35 value for iteration 36: 0.499998203222410545208365240683236783164077507505289004609
36 value for iteration 37: 0.4999965704350752151814844692260861721741447177406227932359
37 value for iteration 38: 0.4999914082318244536146183875491278892881955303400152340948
38 value for iteration 39: 0.499986281458009678138391874032345159399726772903420335656027
39 value for iteration 40: 0.4999725621632039238227190342059595232876243612125537015312
40 value for iteration 41: 0.49994512131490383709798578774285808947218867101091636268513
41 value for iteration 42: 0.4998802305818834052204628908517820044430524857214248987534
42 value for iteration 43: 0.4997804129557262270573350875371263390615685516919314842053
43 value for iteration 44: 0.499660329520227604734908547476914798511419046702216579409
44 value for iteration 45: 0.49912649305290486865006029228511800184783529688037636454640
45 value for iteration 46: 0.49823788652372186061390580932026232146559237509394897890
46 value for iteration 47: 0.49646380945506610937588897890342234934860893449285002134
47 value for iteration 48: 0.49287622873479742386852316073973578610669973670510879392330
53 value for iteration 54: -3.9521474513679549140209187119009536945716170666227229489806
54 value for iteration 55: 1.62651349833289626837286307314686174179959074107752074717080
55 value for iteration 56: 1.1925940052065498504415030657627744348531440244170790761801
56 value for iteration 57: 1.08074583821725361516459624492206952646641444374965985929669
57 value for iteration 58: 1.0373565344236939503520252509211927267002823101255066552831
58 value for iteration 59: 1.01800563894092953665794156594757482201180960790215832925
59 value for iteration 60: 1.00884358507073767102791128090154647427913579394938753643449
60 value for iteration 61: 1.0043830829361709348672776375131279728313956407520016508873
61 value for iteration 62: 1.00210195105244341220603419324664385820120411069296194272651
62 value for iteration 63: 1.0010886006520124764240302474286168894647400119969397570064
63 value for iteration 64: 1.0005437084446388996164509884710534745901120016444854878986
64 value for iteration 65: 1.0002717064932046312284323937711729457533594149760432446680
65 value for iteration 66: 1.00013581634441965347778407313088715385523676163651648709851
66 value for iteration 67: 1.000067898950422590411376671201529727102552394895796451817
67 value for iteration 68: 1.00003394717023406649557042190141685824010692128986217483876
68 value for iteration 69: 1.00001697300893140966889012163350538219381676343825874330868
69 value for iteration 70: 1.0000040836402663351032791150802608609769101340171575769
70 value for iteration 71: 1.000004243144204465697793374236911861986324285668144565687
71 value for iteration 72: 1.0000021156310013467617249389532927860594241658197201234976
72 value for iteration 73: 1.0000010607792959571187353783798723941364525352417617079076
73 value for iteration 74: 1.00000053038908715279500500770114036709292518387489494337345
74 value for iteration 75: 1.0000002651944029201802196389498782186053316071207051384318
75 value for iteration 76: 1.00000013259716629666376502700021309114990448858636828913544
76 value for iteration 77: 1.0000000629857435762879330260439415597747415965272470288758
77 value for iteration 78: 1.000000031492049807640614717782552440744227775725010452866
78 value for iteration 79: 1.00000001657464194094450158138547916086832719550615766301463
79 value for iteration 80: 1.000000002873208331128753321219477142474892217283424684261
80 value for iteration 81: 1.0000000041436603821659455712929211149422145300476117362
81 value for iteration 82: 1.0000000020718301825233668158393826368748801939346837825421
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83 value for iteration 84: 1.00000000051795754402154066942954347587556250336343484973
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97 value for iteration 98: 1.0000000000000316136195974503647282365841096714544182981423
98 value for iteration 99: 1.00000000000001580608979872468265364620587540882651334551485
99 value for iteration 100: 1.000000000000007903404899362216399205126410353847940913283

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Figure 4: 100 iterations for $\gamma = 1/2$

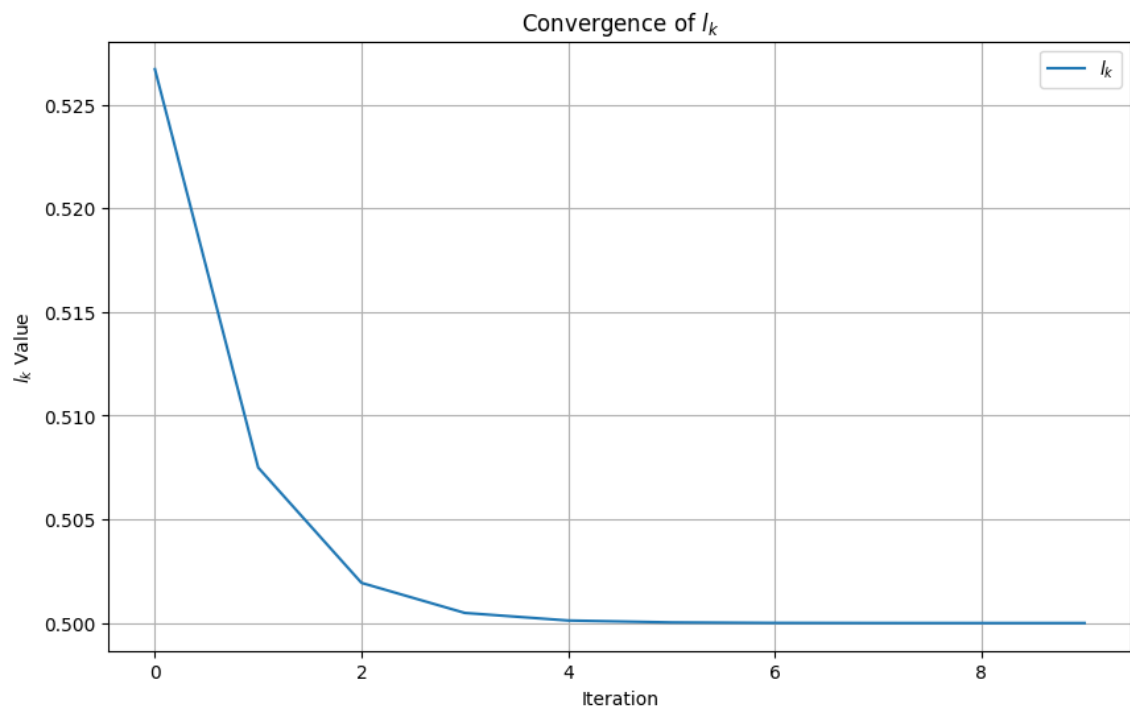


Figure 5: Convergence of l_k for $\gamma = 1/2$

0	value for iteration 1:	1.83972966186352764870076305890430416215266779265081277	53	value for iteration 54:	1.000000158401667459298018806421136653026641567951242
1	value for iteration 2:	1.4782482568975477733362371464282097043430782892241012	54	value for iteration 55:	0.999999877212324469680159848801898714194088080277015
2	value for iteration 3:	0.93225634079314193675157072351152870360518686169982415	55	value for iteration 56:	1.000000095180936492236236561372392216300333969555033
3	value for iteration 4:	1.0571971844176288723978606902781659463894667941154020	56	value for iteration 57:	0.999999926218914598955482258663704405269099015385651
4	value for iteration 5:	0.95922757730905848426594552057340333809530725642963403	57	value for iteration 58:	1.000000057192648225420036569275940178789655438764716
5	value for iteration 6:	1.0333275771872406162772096884648179046771858951262488	58	value for iteration 59:	0.99999955666165550103505401820964533864224748515556
6	value for iteration 7:	0.97536154921727013755964873927987012788728378166270558	59	value for iteration 60:	1.000000034366115406028719939355487957992718885152196
7	value for iteration 8:	1.01973323783935996988439907109012476284161623659070278	60	value for iteration 61:	0.999999973360532071126610337922664115318652662668582
8	value for iteration 9:	0.98511985580732167927770269308225732538405988182342193	61	value for iteration 62:	1.00000002605002989504293075407138350720635211502503
9	value for iteration 10:	1.01176715787209539204177484137679673092147346273	62	value for iteration 63:	0.999999983992784384531577052007229427701621770143716
10	value for iteration 11:	0.991025956035454958291018761094481146802140392278295	63	value for iteration 64:	1.00000001240826118912769512969007980849949614920161
11	value for iteration 12:	1.007041200469846062824013926937608423938661151521225	64	value for iteration 65:	0.99999990381528968207393711940290738619579881357075
12	value for iteration 13:	0.994594545490470605459666085336283221982036614772	65	value for iteration 66:	1.0000000074559187548685644151063362485684596470676
13	value for iteration 14:	1.004220971032826355798190494723473882037404761842298	66	value for iteration 67:	0.99999994220419914266695970015107539450163241245362
14	value for iteration 15:	0.996746944565895923058707051475368740485372408667084	67	value for iteration 68:	1.000000004480138195183075845218734352866228936045798
15	value for iteration 16:	1.002532843256742737915086845554075270753651266474255	68	value for iteration 69:	0.9999999652714597858496978858912410261188677800552
16	value for iteration 17:	0.99804342395051297325951344253795824327685939532846	69	value for iteration 70:	1.00000000269204887932881751024420003514719308755456
17	value for iteration 18:	1.001520720823721614447300534445791264405899238502884	70	value for iteration 71:	0.9999999791321950882090286218824216474591786589907
18	value for iteration 19:	0.998823637717991293624110862388509434590916850461339	71	value for iteration 72:	1.000000001617602789422483885447781430274857276300585
19	value for iteration 20:	1.000913340841541273702090937876250416552271892915957	72	value for iteration 73:	0.999999998746088161387828438048281139908319936719608
20	value for iteration 21:	0.999292892012390259293846630794967886104668499692765	73	value for iteration 74:	1.000000000971998719083554378339220920217764690234303
21	value for iteration 22:	1.000548655661474352809631208367538550826213539986537	74	value for iteration 75:	0.999999999246545155065330510976714386865753508424635
22	value for iteration 23:	0.99957501912213596957054231853812822667332168797835	75	value for iteration 76:	1.000000000584053108249435299311805000089938235676136
23	value for iteration 24:	1.000329622175378911098079096458559125985470885154164	76	value for iteration 77:	0.999999998472614824793576304873299552193528105002
24	value for iteration 25:	0.99974460288797836989783317800636258936680126473555	77	value for iteration 78:	1.0000000035094781660203142709540501520269324735055
25	value for iteration 26:	1.000198044380310420528828687098007152715859466034614	78	value for iteration 79:	0.999999997279569482108451702345633452335567906645
26	value for iteration 27:	0.999846524107771312732567616404920210297479330450847	79	value for iteration 80:	1.00000000218078703716720260723377490058921043426084
27	value for iteration 28:	1.000118994336792046327156555902758451262358553436401	80	value for iteration 81:	0.9999999983665386391033576255911208011357066949195
28	value for iteration 29:	0.999907774338896814385423090447701725499059019855355	81	value for iteration 82:	1.0000000012671349130102688584728959819989206001156
29	value for iteration 30:	1.000071499111837776892715226491662089184086626358735	82	value for iteration 83:	0.9999999990177593611200115916138045697453919365226
30	value for iteration 31:	0.9999445816790804293344539701436751746612711707218494	83	value for iteration 84:	1.00000000076140007714629894099406583226423904908226
31	value for iteration 32:	1.000042961689322578114339474871778028547536792980780	84	value for iteration 85:	0.999999999409788178435194804520000808833384431277345
32	value for iteration 33:	0.9996660948170005834062212693913411777473447209961	85	value for iteration 86:	1.00000000045751238189081003150039268991344955291073
33	value for iteration 34:	1.0000250146275963221080320660706090822216484565896	86	value for iteration 87:	0.9999999996453518084806967025240259173225611967736
34	value for iteration 35:	0.99979900665033540298490689942054605265773799516	87	value for iteration 88:	1.0000000002749112937675774975276616634325642549331
35	value for iteration 36:	1.00001551149626734904110382267808075966696761907735	88	value for iteration 89:	0.9999999997868980962928986046484612679694538927472
36	value for iteration 37:	0.999987976207302165680990288478986896406616590578123	89	value for iteration 90:	1.00000000016518932113073177856893013321335288872876
37	value for iteration 38:	1.000009320535048744964006169019824140689742477985746	90	value for iteration 91:	0.99999999987195091869264762105851615383842150357738
38	value for iteration 39:	0.999927751233110749851114463050008069416537804263977	91	value for iteration 92:	1.00000000009925920316976514918284298180774121843075
39	value for iteration 40:	1.000005600539269907592861683240803097886663811807885	92	value for iteration 93:	0.99999999992305775243835914584555113258462550879984
40	value for iteration 41:	0.99999568681507263576091689712966474352370168215875	93	value for iteration 94:	1.000000000005964288612896153218785954417955962212156
41	value for iteration 42:	1.0000033652655965959011297534520500362190816622266292	94	value for iteration 95:	0.9999999999537670832522263619261260909257270715008
42	value for iteration 43:	0.999997391370069881156286153449647424361110756234682	95	value for iteration 96:	1.0000000000358309668793953857853570849948712225031
43	value for iteration 44:	1.000002022130328045830107640540978241104449655296205	96	value for iteration 97:	0.9999999999722196839688311150458873325498617216596
44	value for iteration 45:	0.999998432516180534142703214301463810266149169376802	97	value for iteration 98:	1.00000000002153419257135667172005186107279533757303
45	value for iteration 46:	1.00000121506390270655528398091489527930651340737280	98	value for iteration 99:	0.9999999999830759450407814603652057785332997311117
46	value for iteration 47:	0.9999990581244543237620226111826521788004084506122	99	value for iteration 100:	1.0000000000012039213254727529751630315214114130698
47	value for iteration 48:	1.000000730111522983373311453186580505932281510676246			

Figure 6: 100 iterations for $\gamma = 3$

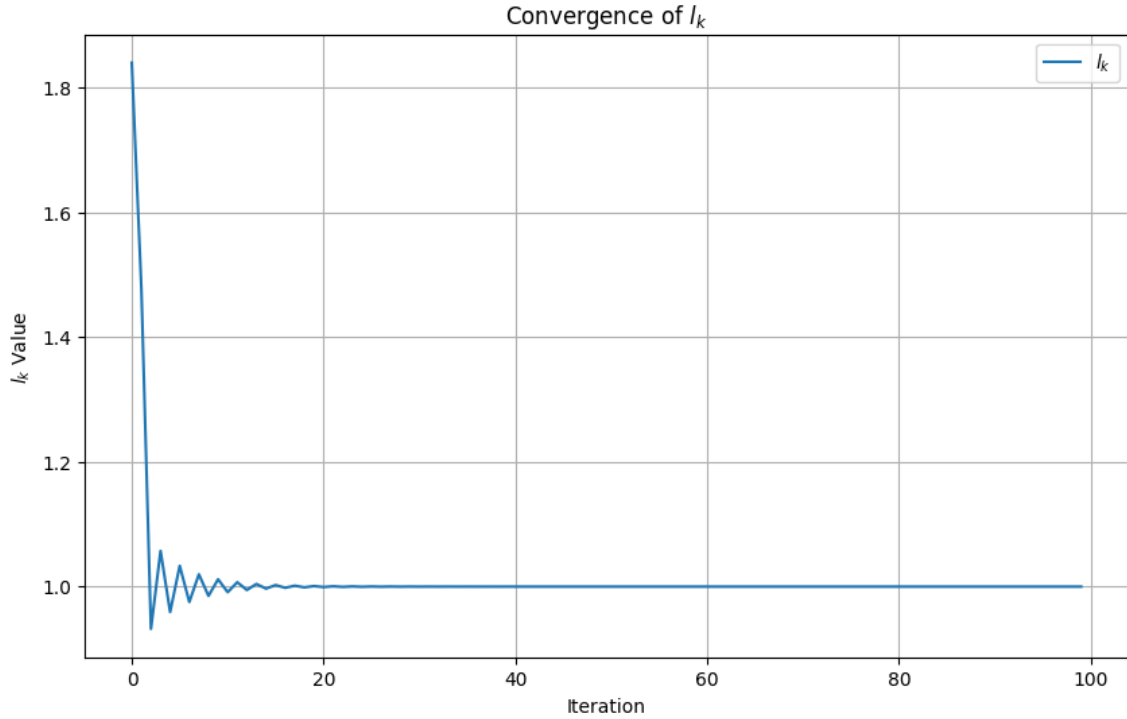


Figure 7: Convergence of l_k for $\gamma = 3$

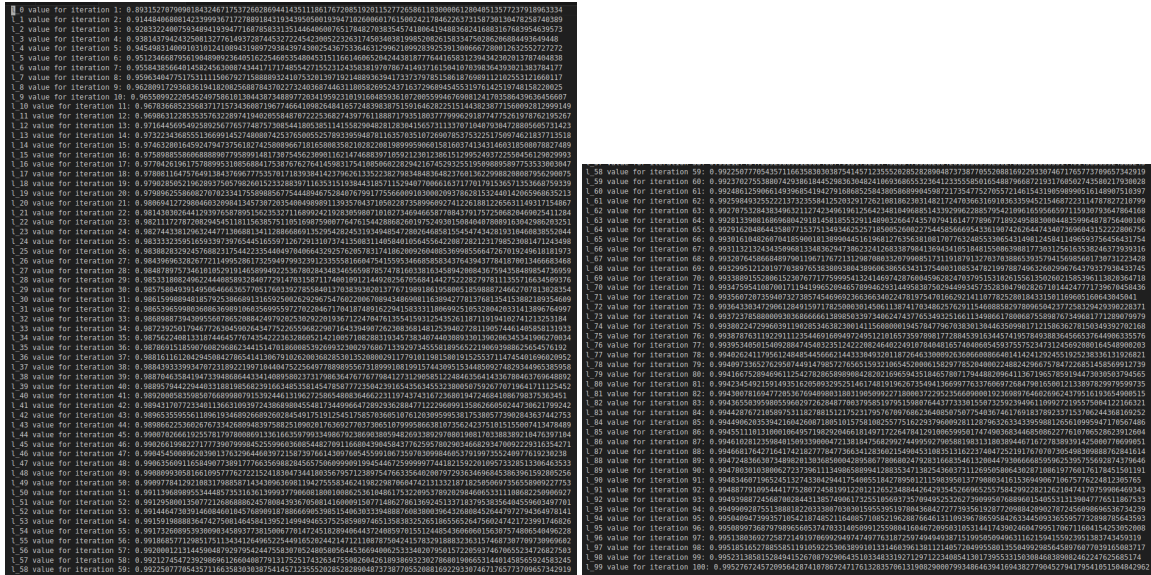


Figure 8: 100 iterations for $\gamma = 2$

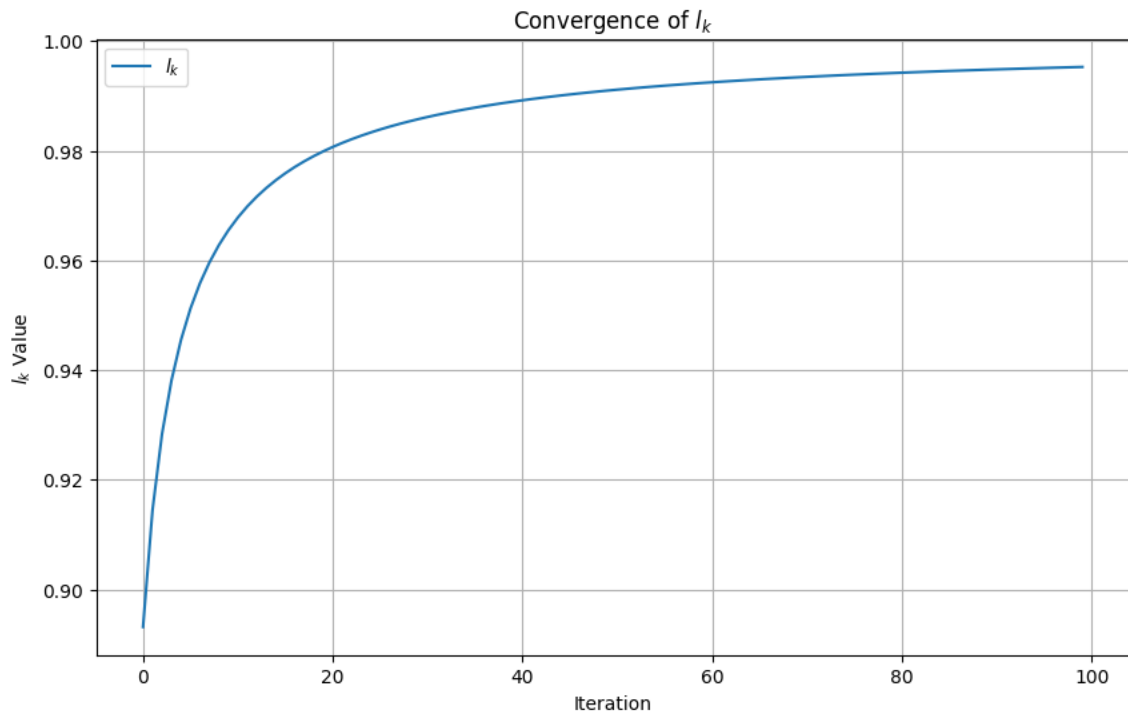


Figure 9: Convergence of l_k for $\gamma = 2$

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_l60 value for iteration 61: 1.0
_l61 value for iteration 62: 1.0
_l62 value for iteration 63: 1.0
_l63 value for iteration 64: 1.0
_l64 value for iteration 65: 1.0
_l65 value for iteration 66: 1.0
_l66 value for iteration 67: 1.0
_l67 value for iteration 68: 1.0
_l68 value for iteration 69: 1.0
_l69 value for iteration 70: 1.0
_l70 value for iteration 71: 1.0
_l71 value for iteration 72: 1.0
_l72 value for iteration 73: 1.0
_l73 value for iteration 74: 1.0
_l74 value for iteration 75: 1.0
_l75 value for iteration 76: 1.0
_l76 value for iteration 77: 1.0
_l77 value for iteration 78: 1.0
_l78 value for iteration 79: 1.0
_l79 value for iteration 80: 1.0
_l80 value for iteration 81: 1.0
_l81 value for iteration 82: 1.0
_l82 value for iteration 83: 1.0
_l83 value for iteration 84: 1.0
_l84 value for iteration 85: 1.0
_l85 value for iteration 86: 1.0
_l86 value for iteration 87: 1.0
_l87 value for iteration 88: 1.0
_l88 value for iteration 89: 1.0
_l89 value for iteration 90: 1.0
_l90 value for iteration 91: 1.0
_l91 value for iteration 92: 1.0
_l92 value for iteration 93: 1.0
_l93 value for iteration 94: 1.0
_l94 value for iteration 95: 1.0
_l95 value for iteration 96: 1.0
_l96 value for iteration 97: 1.0
_l97 value for iteration 98: 1.0
_l98 value for iteration 99: 1.0
_l99 value for iteration 100: 1.0

```

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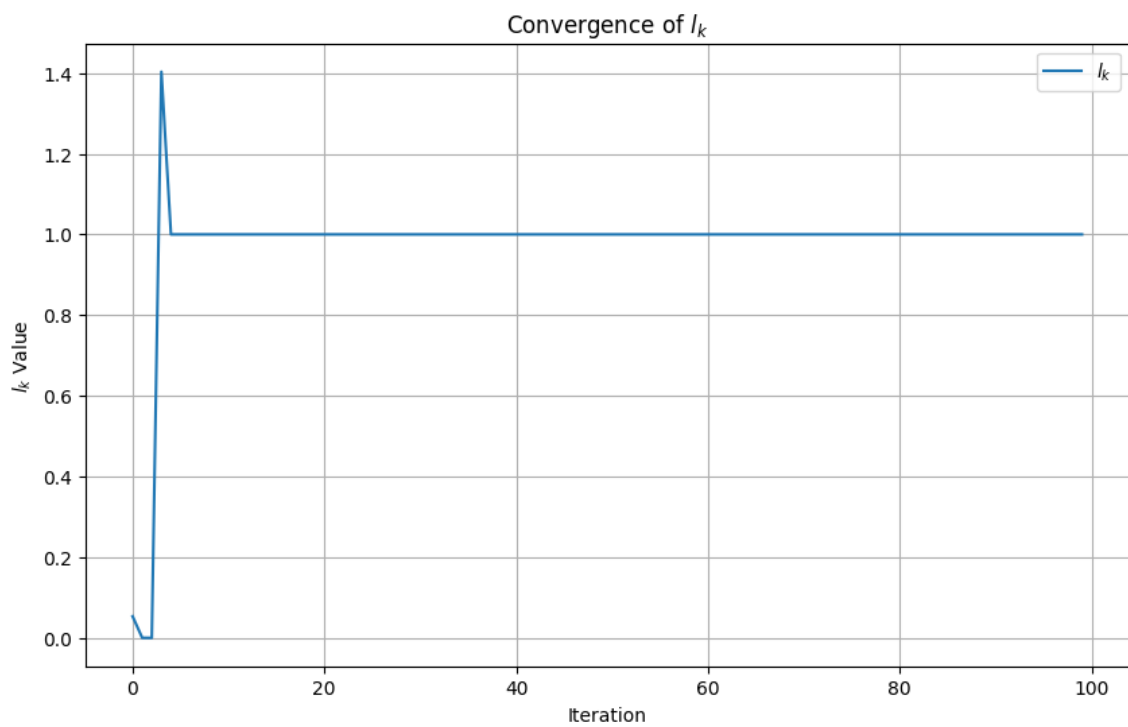


Figure 11: Convergence of l_k for $\gamma = 1$