Project 3 Report

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September 25, 2024

Results for Problem: 1

The output is obtained by implementing the **Local SQP method** (Algorithm 18.1 in the book) without any warm-start strategies. (the output of all iterations for this section and following sections are in the Jupyter notebook file) For the first problem, the solution in the book for starting point $x_0 = (-1.71, 1.59, 1.82, -0.763, -0.763)^T$ is given as:

$$x^* = (-1.8, 1.7, 1.9, -0.8, -0.8)^T \tag{1}$$

the output of my simulation run for the same starting point is not exactly the same but it is very close.

```
Starting point [0 0 0 0 0]
Iteration 1

f_k = 0.500000000000000

nabla f_k = [0. 0. 0. 0. 0.]

nabla_xx^2 L_k =
[[0. 0. 0. 0. 0.]
    [0. 0. 0. 0. 0.]
    [0. 0. 0. 0. 0.]
    [0. 0. 0. 0. 0.]
    [0. 0. 0. 0. 0.]]

c_k = [-10.   0.   1.]

A_k =
[[0 0 0 0 0]
    [0 0 0 0 0]
    [0 0 0 0 0]]
```

```
KKT_matrix =
[[1.e-10 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00]
    [0.e+00 1.e-10 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00]
    [0.e+00 0.e+00 1.e-10 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00]
    [0.e+00 0.e+00 0.e+00 1.e-10 0.e+00 0.e+00 0.e+00 0.e+00]
    [0.e+00 0.e+00 0.e+00 0.e+00 1.e-10 0.e+00 0.e+00 0.e+00]
    [0.e+00 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00]
    [0.e+00 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00]
    [0.e+00 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00 0.e+00]]
============================================================
Determinant of KKT matrix = 0!
Solution did not converge!
============================================================
Starting point [1 0 3 0 0]
Iteration 1

f_k = -1.00000000000000

nabla f_k = [-6.  0.  0.  0.  0.]

nabla_xx^2 L_k =
[[-21.   0.   0.   0.   0.]
    [  0.   0.   0.   0.   0.]
    [  0.   0.   0.   0.   0.]
    [  0.   0.   0.   0.   0.]
    [  0.   0.   0.   0.   0.]]

c_k = [18.  0.  2.]

A_k =
[[ 2  0 27  0  0]
    [ 0  3  0  0  0]
    [ 3  0  0  0  0]]

KKT_matrix =
[[-2.1e+01  0.0e+00  0.0e+00  0.0e+00  0.0e+00 -2.0e+00  0.0e+00 -3.0e+00]
    [ 0.0e+00  1.0e-10  0.0e+00  0.0e+00  0.0e+00  0.0e+00 -3.0e+00  0.0e+00]
    [ 0.0e+00  0.0e+00  1.0e-10  0.0e+00  0.0e+00 -2.7e+01  0.0e+00  0.0e+00]
    [ 0.0e+00  0.0e+00  0.0e+00  1.0e-10  0.0e+00  0.0e+00  0.0e+00  0.0e+00]
    [ 0.0e+00  0.0e+00  0.0e+00  0.0e+00  1.0e-10  0.0e+00  0.0e+00  0.0e+00]
    [ 2.0e+00  0.0e+00  2.7e+01  0.0e+00  0.0e+00  0.0e+00  0.0e+00  0.0e+00]
    [ 0.0e+00  3.0e+00  0.0e+00  0.0e+00  0.0e+00  0.0e+00  0.0e+00  0.0e+00]
    [ 3.0e+00  0.0e+00  0.0e+00  0.0e+00  0.0e+00  0.0e+00  0.0e+00  0.0e+00]]
============================================================================
```

```
f_k = 0.462277091906722

nabla f_k = [-0.34567901  0.          0.          0.          0.        ]

nabla_xx^2 L_k =
[[-7.51851852e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
    [ 0.00000000e+00  4.57247371e-12  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  3.26847195e-11  0.00000000e+00
    0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  4.57247371e-12
    0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    4.57247371e-12]]

c_k = [3.63859017 0.         1.03703704]

A_k =
[[ 0.66666667  0.         17.03200732  0.          0.        ]
    [ 0.          2.38271605  0.         -0.         -0.        ]
    [ 0.33333333  0.          0.          0.          0.        ]]

KKT_matrix =
[[-7.51851852e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00 -6.66666667e-01 -0.00000000e+00 -3.33333333e-01]
    [ 0.00000000e+00  1.04572474e-10  0.00000000e+00  0.00000000e+00
    0.00000000e+00 -0.00000000e+00 -2.38271605e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  1.32684719e-10  0.00000000e+00
    0.00000000e+00 -1.70320073e+01 -0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.04572474e-10
    0.00000000e+00 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    1.04572474e-10 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [ 6.66666667e-01  0.00000000e+00  1.70320073e+01  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 0.00000000e+00  2.38271605e+00  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 3.33333333e-01  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]
====================================================================

f_k = -207.763358491348

nabla f_k = [472.99700249   0.          -0.          -0.          -0.        ]
```

```
nabla_xx^2 L_k =
[[ 2.75868770e+02  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
    [ 0.00000000e+00  1.43119652e-12  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  9.83600676e-12 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00  1.43119652e-12
    0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00  0.00000000e+00
    1.43119652e-12]]

c_k = [  9.7385518    0.          -20.43347051]

A_k =
[[-5.55555556  0.          15.7440999   0.          0.        ]
    [ 0.          2.29085864  0.          -0.          -0.        ]
    [23.14814815  0.          0.          0.          0.        ]]

KKT_matrix =
[[ 2.75868770e+02  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  5.55555556e+00 -0.00000000e+00 -2.31481481e+01]
    [ 0.00000000e+00  1.01431197e-10  0.00000000e+00  0.00000000e+00
    0.00000000e+00 -0.00000000e+00 -2.29085864e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  1.09836007e-10  0.00000000e+00
    0.00000000e+00 -1.57440999e+01 -0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.01431197e-10
    0.00000000e+00 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    1.01431197e-10 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [-5.55555556e+00  0.00000000e+00  1.57440999e+01  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 0.00000000e+00  2.29085864e+00  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 2.31481481e+01  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]
==========================================================================
Iteration 4

f_k = -15.8522111729763

nabla f_k = [62.54705902  0.         -0.         -0.         -0.        ]

nabla_xx^2 L_k =
[[ 1.69866837e+02  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
```

```
    [ 0.00000000e+00  4.28442276e-12  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  2.54981854e-11 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00  4.28442276e-12
    0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00  0.00000000e+00
    4.28442276e-12]]

c_k = [ 1.39827417  0.         -5.80555099]

A_k =
[[-3.7901037   0.         11.80626862  0.          0.        ]
    [ 0.          1.98379003  0.         -0.         -0.        ]
    [10.77366456  0.          0.          0.          0.        ]]

KKT_matrix =
[[ 1.69866837e+02  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  3.79010370e+00 -0.00000000e+00 -1.07736646e+01]
    [ 0.00000000e+00  1.04284423e-10  0.00000000e+00  0.00000000e+00
    0.00000000e+00 -0.00000000e+00 -1.98379003e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  1.25498185e-10  0.00000000e+00
    0.00000000e+00 -1.18062686e+01 -0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.04284423e-10
    0.00000000e+00 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    1.04284423e-10 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [-3.79010370e+00  0.00000000e+00  1.18062686e+01  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 0.00000000e+00  1.98379003e+00  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 1.07736646e+01  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]
======================================================================
Iteration 5

f_k = -0.116551047031228

nabla f_k = [ 8.2454543  0.        -0.        -0.        -0.       ]

nabla_xx^2 L_k =
[[ 7.37700588e+01  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
    [ 0.00000000e+00 -1.15979714e-12  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00 -7.09219698e-12 -0.00000000e+00
```

```
    -0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00 -1.15979714e-12
    0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00  0.00000000e+00
    -1.15979714e-12]]

c_k = [ 0.30825003  0.         -1.49435675]

A_k =
[[-2.71237366  0.         12.46454012  0.          0.        ]
    [ 0.          2.03834411  0.         -0.         -0.        ]
    [ 5.51772815  0.          0.          0.          0.        ]]

KKT_matrix =
[[ 7.37700588e+01  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  2.71237366e+00 -0.00000000e+00 -5.51772815e+00]
    [ 0.00000000e+00  9.88402029e-11  0.00000000e+00  0.00000000e+00
    0.00000000e+00 -0.00000000e+00 -2.03834411e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  9.29078030e-11  0.00000000e+00
    0.00000000e+00 -1.24645401e+01 -0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  9.88402029e-11
    0.00000000e+00 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    9.88402029e-11 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [-2.71237366e+00  0.00000000e+00  1.24645401e+01  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 0.00000000e+00  2.03834411e+00  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 5.51772815e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]
======================================================================
Iteration 6

f_k = 0.961203312914790

nabla f_k = [ 0.98441914  0.         -0.         -0.         -0.        ]

nabla_xx^2 L_k =
[[ 1.90079570e+01  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
    [ 0.00000000e+00 -5.09897784e-13  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00 -3.17036310e-12 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00 -5.09897784e-13
    0.00000000e+00]
```

```
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00  0.00000000e+00
    -5.09897784e-13]]

c_k = [ 0.08054204  0.         -0.27855587]

A_k =
[[-2.17071713  0.         12.88636729  0.          0.        ]
    [ 0.          2.07254813  0.         -0.         -0.        ]
    [ 3.53400963  0.          0.          0.          0.        ]]

KKT_matrix =
[[ 1.90079570e+01  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  2.17071713e+00 -0.00000000e+00 -3.53400963e+00]
    [ 0.00000000e+00  9.94901022e-11  0.00000000e+00  0.00000000e+00
    0.00000000e+00 -0.00000000e+00 -2.07254813e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  9.68296369e-11  0.00000000e+00
    0.00000000e+00 -1.28863673e+01 -0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  9.94901022e-11
    0.00000000e+00 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    9.94901022e-11 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [-2.17071713e+00  0.00000000e+00  1.28863673e+01  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 0.00000000e+00  2.07254813e+00  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 3.53400963e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]
======================================================================
Iteration 7

f_k = 0.999805171587724

nabla f_k = [ 0.05999595  0.         -0.         -0.         -0.        ]

nabla_xx^2 L_k =
[[-5.11428977e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
    [ 0.00000000e+00 -1.05608728e-13  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00 -6.58863964e-13 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00 -1.05608728e-13
    0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00  0.00000000e+00
    -1.05608728e-13]]
```

```
c_k = [ 0.00652022  0.         -0.01973973]

A_k =
[[-2.01307416  0.         12.97390266  0.          0.        ]
    [ 0.          2.07957549  0.         -0.         -0.        ]
    [ 3.0393507   0.          0.          0.          0.        ]]

KKT_matrix =
[[-5.11428977e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  2.01307416e+00 -0.00000000e+00 -3.03935070e+00]
    [ 0.00000000e+00  9.98943913e-11  0.00000000e+00  0.00000000e+00
    0.00000000e+00 -0.00000000e+00 -2.07957549e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  9.93411360e-11  0.00000000e+00
    0.00000000e+00 -1.29739027e+01 -0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  9.98943913e-11
    0.00000000e+00 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    9.98943913e-11 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [-2.01307416e+00  0.00000000e+00  1.29739027e+01  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 0.00000000e+00  2.07957549e+00  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 3.03935070e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]
==========================================================================

f_k = 0.999999991923128

nabla f_k = [ 0.00038132  0.         -0.         -0.         -0.        ]

nabla_xx^2 L_k =
[[-8.94941886e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
    [ 0.00000000e+00 -7.73628866e-15  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00 -4.82763134e-14 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00 -7.73628866e-15
    0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00  0.00000000e+00
    -7.73628866e-15]]

c_k = [ 4.37736480e-05  0.00000000e+00 -1.27097377e-04]

A_k =
[[-2.00008473  0.         12.98020675  0.          0.        ]
```

```
    [ 0.          2.08008067  0.         -0.         -0.        ]
    [ 3.00025419  0.          0.          0.          0.        ]]

KKT_matrix =
[[-8.94941886e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  2.00008473e+00 -0.00000000e+00 -3.00025419e+00]
    [ 0.00000000e+00  9.99922637e-11  0.00000000e+00  0.00000000e+00
    0.00000000e+00 -0.00000000e+00 -2.08008067e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  9.99517237e-11  0.00000000e+00
    0.00000000e+00 -1.29802068e+01 -0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  9.99922637e-11
    0.00000000e+00 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    9.99922637e-11 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [-2.00008473e+00  0.00000000e+00  1.29802068e+01  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 0.00000000e+00  2.08008067e+00  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 3.00025419e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]
======================================================================
Iteration 9

f_k = 1.00000000000000

nabla f_k = [ 1.61514622e-08  0.00000000e+00 -0.00000000e+00 -0.00000000e+00
    -0.00000000e+00]

nabla_xx^2 L_k =
[[-8.99999568e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
    [ 0.00000000e+00 -4.85911379e-17  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00 -3.03220920e-16 -0.00000000e+00
    -0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00 -4.85911379e-17
    0.00000000e+00]
    [ 0.00000000e+00 -0.00000000e+00 -0.00000000e+00  0.00000000e+00
    -4.85911379e-17]]

c_k = [ 1.85668192e-09  0.00000000e+00 -5.38382072e-09]

A_k =
[[-2.          0.         12.98024613  0.          0.        ]
    [ 0.          2.08008382  0.         -0.         -0.        ]
    [ 3.00000001  0.          0.          0.          0.        ]]
```

```
KKT_matrix =
[[-8.99999568e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  2.00000000e+00 -0.00000000e+00 -3.00000001e+00]
    [ 0.00000000e+00  9.99999514e-11  0.00000000e+00  0.00000000e+00
    0.00000000e+00 -0.00000000e+00 -2.08008382e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  9.99996968e-11  0.00000000e+00
    0.00000000e+00 -1.29802461e+01 -0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  9.99999514e-11
    0.00000000e+00 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    9.99999514e-11 -0.00000000e+00  0.00000000e+00 -0.00000000e+00]
    [-2.00000000e+00  0.00000000e+00  1.29802461e+01  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 0.00000000e+00  2.08008382e+00  0.00000000e+00 -0.00000000e+00
    -0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
    [ 3.00000001e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]
====================================================================
Converged @ 9 iterations
x* = [-1.          0.          2.08008382  0.          0.        ]
====================================================================
```

Results for Problem: 2

For the starting point x_0 : $(0.8, 0.6)$ convergence is achieved after 9 iterations.

```
Starting point [0.8 0.6]
Iteration 1

f_k = 0.200000000000001

nabla f_k = [12.4 -8. ]

nabla_xx^2 L_k =
[[ 530. -320.]
    [-320.  200.]]

c_k = [ 1.11022302e-16 -6.00000000e-01]

A_k =
[[ 1.6  1.2]
    [ 0.  -1. ]]

KKT_matrix =
[[ 530.  -320.    -1.6   -0. ]
```

```
    [-320.   200.    -1.2    1. ]
    [   1.6    1.2    0.     0. ]
    [   0.    -1.     0.     0. ]]
================================================================================
Iteration 2

f_k = 244.203125000009

nabla f_k = [ 781.75 -312.5 ]

nabla_xx^2 L_k =
[[2430.625 -500.   ]
    [-500.     753.625]]

c_k = [5.62500000e-01 1.67643677e-14]

A_k =
[[ 2.50000000e+00 -3.35287353e-14]
    [ 0.00000000e+00 -1.00000000e+00]]

KKT_matrix =
[[ 2.43062500e+03 -5.00000000e+02 -2.50000000e+00 -0.00000000e+00]
    [-5.00000000e+02  7.53625000e+02  3.35287353e-14  1.00000000e+00]
    [ 2.50000000e+00 -3.35287353e-14  0.00000000e+00  0.00000000e+00]
    [ 0.00000000e+00 -1.00000000e+00  0.00000000e+00  0.00000000e+00]]
================================================================================
Iteration 3

f_k = 110.381914062500

nabla f_k = [ 430.80625 -210.125  ]

nabla_xx^2 L_k =
[[1450.6375 -410.    ]
    [-410.      387.8875]]

c_k = [5.06250000e-02 4.50219581e-18]

A_k =
[[ 2.05000000e+00 -9.00439162e-18]
    [ 0.00000000e+00 -1.00000000e+00]]

KKT_matrix =
[[ 1.45063750e+03 -4.10000000e+02 -2.05000000e+00 -0.00000000e+00]
    [-4.10000000e+02  3.87887500e+02  9.00439162e-18  1.00000000e+00]
    [ 2.05000000e+00 -9.00439162e-18  0.00000000e+00  0.00000000e+00]
```

```
    [ 0.00000000e+00 -1.00000000e+00  0.00000000e+00  0.00000000e+00]]
================================================================================
Iteration 4

f_k = 100.122007094174

nabla f_k = [ 400.36657497 -200.12196981]

nabla_xx^2 L_k =
[[1588.08067742 -400.12195122]
    [-400.12195122  585.34885857]]

c_k = [6.09849048e-04 4.50219581e-18]

A_k =
[[ 2.00060976e+00 -9.00439162e-18]
    [ 0.00000000e+00 -1.00000000e+00]]

KKT_matrix =
[[ 1.58808068e+03 -4.00121951e+02 -2.00060976e+00 -0.00000000e+00]
    [-4.00121951e+02  5.85348859e+02  9.00439162e-18  1.00000000e+00]
    [ 2.00060976e+00 -9.00439162e-18  0.00000000e+00  0.00000000e+00]
    [ 0.00000000e+00 -1.00000000e+00  0.00000000e+00  0.00000000e+00]]
================================================================================
Iteration 5

f_k = 100.000018584460

nabla f_k = [ 400.00005585 -200.00001858]

nabla_xx^2 L_k =
[[1601.76071109 -400.00001858]
    [-400.00001858  599.76059958]]

c_k = [ 9.29222970e-08 -8.34840175e-17]

A_k =
[[ 2.00000009e+00  1.66968035e-16]
    [ 0.00000000e+00 -1.00000000e+00]]

KKT_matrix =
[[ 1.60176071e+03 -4.00000019e+02 -2.00000009e+00 -0.00000000e+00]
    [-4.00000019e+02  5.99760600e+02 -1.66968035e-16  1.00000000e+00]
    [ 2.00000009e+00  1.66968035e-16  0.00000000e+00  0.00000000e+00]
    [ 0.00000000e+00 -1.00000000e+00  0.00000000e+00  0.00000000e+00]]
================================================================================
```

```
Converged @ 5 iterations
x* = [1.00000005e+00 8.34840175e-17]
```

Results for Problem: 3

For this problem, in some of the iterations, the KKT matrix becomes singular as the matrices are increasingly ill-conditioned. I did not understand exactly how to deal with this issue. Including the output in the report nonetheless.

```
Starting from point [0.1, 0.74]:
KKT matrix is singular at iteration 2
Solution: [0.0698554  0.58325633]
Lagrange multipliers: [-3.41131627e+16 -1.90764484e+16 -7.33276394e+15]
================================================
Starting from point [0, 0]:
KKT matrix is singular at iteration 0
Solution: [0 0]
Lagrange multipliers: [0 0 0]
================================================
Starting from point [0.5, 0.5]:
KKT matrix is singular at iteration 0
Solution: [0.5 0.5]
Lagrange multipliers: [0 0 0]
================================================
Starting from point [0, 0.76]:
KKT matrix is singular at iteration 5
Solution: [ 0.66319319 -1.00015295]
Lagrange multipliers: [3.31944071e+16 3.32079329e+16 2.16698969e+16]
================================================
Starting from point [-0.2, 0.8]:
KKT matrix is singular at iteration 1
Solution: [0.46579974 0.8       ]
Lagrange multipliers: [ 2.95833335e+17  1.77500001e+17 -8.13541672e+16]
================================================
/tmp/ipykernel_3421524/3342502933.py:60: LinAlgWarning: Ill-conditioned matrix
```

Results for Problem: 4

For local SQP the results for the first 10 variable LP problem with 2 feasible and non-feasible starting points are:

```
Problem 6 with SQP from starting_point 1:
Initial guess x0:
[0 0 0 0 0 0 0 0 0 0]
Converged after 20 iters.
```

```
Problem 6, solution for starting_point 1:
[  3.01351351  12.06756757  -3.77027027  14.21621622   8.48648649
  -9.18918919  -3.60810811  -6.35135135 -10.40540541  -7.93243243]
==================================================
Problem 6 with SQP from starting_point 2:
Initial guess x0:
[1 1 1 1 1 1 1 1 1 1]
Converged after 23 iters.
Problem 6, solution for starting_point 2:
[  3.01351351  12.06756757  -3.77027027  14.21621622   8.48648649
  -9.18918919  -3.60810811  -6.35135135 -10.40540541  -7.93243243]
==================================================
```

For the solution with the active set method from Project 2, the results for the same problem were:

```
Problem 1 (Feas. starting point):
+----------------------+-------------------------------------------------------+
|        Metric        |                         Value                         |
+----------------------+-------------------------------------------------------+
| Number of iterations |                           5                           |
|    Final iterate     | [0.   0.   0.5  0.   0.   0.5  0.25 0.   0.75 0.  ] |
|  Stopping criteria   |                Optimal solution found                 |
|     Running time     |                   0.000828 seconds                    |
+----------------------+-------------------------------------------------------+


Problem 1 (Non-feas. starting point):
+----------------------+-------------------------------------------------------+
|        Metric        |                         Value                         |
+----------------------+-------------------------------------------------------+
| Number of iterations |                           5                           |
|    Final iterate     | [0.   0.   0.5  0.   0.   0.5  0.25 0.   0.75 0.  ] |
|  Stopping criteria   |                Optimal solution found                 |
|     Running time     |                   0.000249 seconds                    |
+----------------------+-------------------------------------------------------+
```