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THEORETICAL ANALYSIS

- Step 1: Input Size

The input for each analysis is an array length of n.

- Step 2: Basic Operation for (1)

Basic operation is the comparison marked as (1)

Analyze B(n)

- Step 3: Count

(1) is executed exactly n times regardless of the input.

$$B(n) = \sum_{i=1}^{n} 1$$

- Step 4: Closed-Form

$$B(n) = \sum_{i=1}^{n} 1 = n$$

- Step 5: Asymptotic Notation

$$\lim_{n \to \infty} \frac{B(n)}{g(n)} = \lim_{n \to \infty} \frac{n}{n} = 1 \to \ B(n) \in \Theta(n)$$

Analyze W(n)

- Step 3: Count

(1) is executed exactly n times regardless of the input.

$$W(n) = \sum_{i=1}^{n} 1$$

- Step 4: Closed-Form

$$W(n) = \sum_{i=1}^{n} 1 = n$$

- Step 5: Asymptotic Notation

$$\lim_{n \to \infty} \frac{W(n)}{g(n)} = \lim_{n \to \infty} \frac{n}{n} = 1 \to \ W(n) \in \Theta(n)$$

Analyze A(n)

- Step 3: Count

(1) is executed exactly n times regardless of the input.

$$A(n) = \sum_{i=1}^{n} 1$$

- Step 4: Closed-Form

$$A(n) = \sum_{i=1}^{n} 1 = n$$

- Step 5: Asymptotic Notation

$$\lim_{n \to \infty} \frac{A(n)}{g(n)} = \lim_{n \to \infty} \frac{n}{n} = 1 \to \ A(n) \in \Theta(n)$$

- Step 2: Basic Operation for (2)

Basic operations are the three assignments marked as (2)

Analyze B(n)

- Step 3: Count

$$B(n) = \sum_{i=1}^{n} (i \cdot [\log_2(n+1) + 1]) = [\log_2(n+1) + 1] \sum_{i=1}^{n} i$$

- Step 4: Closed-Form

$$B(n) = [\log_2(n+1) + 1] \sum_{i=1}^{n} i = [\log_2(n+1) + 1] \frac{n(n+1)}{2} = [\log_2(n+1) + 1] \frac{n^2 + n}{2}$$
$$= \frac{1}{2} n^2 \log_2(n+1) + \frac{1}{2} n \log_2(n+1) + \frac{1}{2} n^2 + \frac{1}{2} n$$

- Step 5: Asymptotic Notation

$$B(n) = \frac{1}{2} n^2 \log_2(n+1) + \frac{1}{2} n \log_2(n+1) + \frac{1}{2} n^2 + \frac{1}{2} n \in \Theta(n^2 \log n)$$

Analyze W(n)

- Step 3: Count

$$W(n) = \sum_{i=1}^{n} \sum_{j=1}^{n} j^2$$

- Step 4: Closed-Form

$$W(n) = \sum_{i=1}^{n} \sum_{j=1}^{n} j^2 = \sum_{i=1}^{n} \frac{n(n+1)(2n+1)}{6} = \sum_{i=1}^{n} \frac{2n^3 + 3n^2 + n}{6} = \frac{2n^4 + 3n^3 + n^2}{6}$$
$$= \frac{n^4}{3} + \frac{n^3}{2} + \frac{n^2}{6}$$

- Step 5: Asymptotic Notation

$$W(n) = \frac{n^4}{3} + \frac{n^3}{2} + \frac{n^2}{6} \in \Theta(n^4)$$

Analyze A(n)

- Step 3: Count

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$$A(n) = \sum\nolimits_{I \in T_n} \tau(I) \cdot p(I)$$
$$= \sum_{i=1}^{n} \left[\left(\frac{1}{3} \cdot i \cdot [\log_2(n+1) + 1] \right) + \left(\frac{1}{3} \cdot n^2 [\log_2(n+1) + 1] \right) + \left(\frac{1}{3} \cdot \sum_{j=1}^{n} j^2 \right) \right] = \frac{1}{3} \sum_{j=1}^{n} \left[i[\log_2(n+1)+1] + n^2[\log_2(n+1)+1] + \sum_{j=1}^{n} j^2 \right]$$

- Step 4: Closed-Form

$$A(n) = \frac{1}{3} \sum_{j=1}^{n} \left[i[\log_2(n+1)+1] + n^2[\log_2(n+1)+1] + \sum_{j=1}^{n} j^2 \right]$$
$$= \frac{1}{3} \sum_{j=1}^{n} \left[i[\log_2(n+1)+1] + n^2[\log_2(n+1)+1] + \frac{n(n+1)(2n+1)}{6} \right]$$
$$= \frac{1}{3} \sum_{j=1}^{n} \left[i[\log_2(n+1)+1] + n^2[\log_2(n+1)+1] + \frac{2n^3+3n^2+n}{6} \right]$$
$$= \frac{1}{3} \left[\frac{n(n+1)}{2} [\log_2(n+1)+1] + n^3 \log_2(n+1) + n^3 + \frac{2n^4+3n^3+n^2}{6} \right]$$
$$= \frac{1}{3} \left[\frac{n^2+n}{2} [\log_2(n+1)+1] + n^3 \log_2(n+1) + n^3 + \frac{2n^4+3n^3+n^2}{6} \right]$$
$$= \frac{1}{6} n^2 \log_2(n+1) + \frac{1}{6} n \log_2(n+1) + \frac{1}{6} n^2 + \frac{1}{6} n + \frac{1}{3} n^3 \log_2(n+1) + \frac{1}{3} n^3 + \frac{2n^4+3n^3+n^2}{18}$$
$$= \frac{1}{6} n^2 \log_2(n+1) + \frac{1}{6} n \log_2(n+1) + \frac{1}{3} n^3 \log_2(n+1) + \frac{2n^4+9n^3+4n^2+3n}{18}$$
$$= \frac{1}{6} n^2 \log_2(n+1) + \frac{1}{6} n \log_2(n+1) + \frac{1}{3} n^3 \log_2(n+1) + \frac{n^4}{9} + \frac{n^3}{2} + \frac{2n^2}{9} + \frac{n}{6}$$

- Step 5: Asymptotic Notation

$$A(n) = \frac{1}{6} n^2 \log_2(n+1) + \frac{1}{6} n \log_2(n+1) + \frac{1}{3} n^3 \log_2(n+1) + \frac{n^4}{9} + \frac{n^3}{2} + \frac{2n^2}{9} + \frac{n}{6} \in \Theta(n^4)$$

- Step 2: Basic Operation for (3)

Basic operation is two assignments marked as (3)

Analyze B(n)

- Step 3: Count

When the input array does not contain any 0, (5) is never executed.

$$B(n) = \sum_{i=1}^{n} 0$$

- Step 4: Closed-Form

$$B(n) = \sum_{i=1}^{n} 0 = 0$$

- Step 5: Asymptotic Notation

$$Constant\ Order \rightarrow\ B(n) \in \Theta(1)$$

Analyze W(n)

- Step 3: Count

When the input array is full of 2's:

$$W(n) = \sum_{i=1}^{n}\sum_{j=1}^{n} j^2$$

- Step 4: Closed-Form

$$W(n) = \sum_{i=1}^{n}\sum_{j=1}^{n} j^2 = \sum_{i=1}^{n} \frac{n(n+1)(2n+1)}{6} = n \cdot \frac{2n^3 + 3n^2 + n}{6} = \frac{2n^4 + 3n^3 + n^2}{6}$$
$$= \frac{n^4}{3} + \frac{n^3}{2} + \frac{n^2}{6}$$

- Step 5: Asymptotic Notation

$$W(n) = \frac{n^4}{3} + \frac{n^3}{2} + \frac{n^2}{6} \in \Theta(n^4)$$

Analyze A(n)

- Step 3: Count

$$A(n) = \sum\nolimits_{I \in T_n} \tau(I) \cdot p(I) = \sum_{i=1}^{n}\left[\left(\frac{1}{3} \cdot 0\right) + \left(\frac{1}{3} \cdot n^2[\log_2(n+1) + 1]\right) + \left(\frac{1}{3} \cdot \sum_{j=1}^{n} j^2\right)\right]$$
$$= \frac{1}{3}\sum_{i=1}^{n}\left[n^2(\log_2(n+1) + 1) + \sum_{j=1}^{n} j^2\right]$$

- Step 4: Closed-Form

$$A(n) = \frac{1}{3}\sum_{j=1}^{n}\left[n^2(\log_2(n+1)+1) + \sum_{i=1}^{n} i^2\right]$$
$$= \frac{1}{3}\sum_{j=1}^{n}\left[n^2(\log_2(n+1)+1) + \frac{n(n+1)(2n+1)}{6}\right]$$
$$= \frac{1}{3}\sum_{j=1}^{n}\left[n^2(\log_2(n+1)+1) + \frac{2n^3+3n^2+n}{6}\right]$$
$$= \frac{1}{3}\cdot n\cdot\left[n^2\log_2(n+1) + n^2 + \frac{2n^3+3n^2+n}{6}\right]$$
$$= \frac{1}{3}n^3\log_2(n+1) + \frac{n^4}{9} + \frac{n^3}{2} + \frac{n^2}{18}$$

- Step 5: Asymptotic Notation

$$A(n) = \frac{1}{3}n^3\log_2(n+1) + \frac{n^4}{9} + \frac{n^3}{2} + \frac{n^2}{18} \in \Theta(n^4)$$

- Step 2: Basic Operation for (4)

Basic operations are the two loop incrementations marked as (4)

Analyze B(n)

- Step 3: Count

When the input array is full of 0's, assignments marked as (4) are never executed:

$$B(n) = \sum_{i=1}^{n} 0$$

- Step 4: Closed-Form

$$B(n) = \sum_{i=1}^{n} 0 = 0$$

- Step 5: Asymptotic Notation

$$Constant\ Order \rightarrow\ B(n) \in \Theta(1)$$

Analyze W(n)

When the input array is full of 2's:

- Step 3: Count

$$W(n) = \sum_{i=1}^{n}\sum_{j=1}^{n} j^2$$

- Step 4: Closed-Form

$$W(n)=\sum_{i=1}^{n}\sum_{j=1}^{n}j^2=\sum_{i=1}^{n}\frac{n(n+1)(2n+1)}{6}=\sum_{i=1}^{n}\frac{2n^3+3n^2+n}{6}=\frac{2n^4+3n^3+n^2}{6}$$
$$=\frac{n^4}{3}+\frac{n^3}{2}+\frac{n^2}{6}$$

- Step 5: Asymptotic Notation

$$W(n)=\frac{n^4}{3}+\frac{n^3}{2}+\frac{n^2}{6}\in\Theta(n^4)$$

Analyze A(n)

- Step 3: Count

$$A(n)=\sum_{I\in T_n}\tau(I)\cdot p(I)==\sum_{i=1}^{n}\left[\left(\frac{1}{3}\cdot 0\right)+\left(\frac{1}{3}\cdot n^2[\log_2(n+1)+1]\right)+\left(\frac{1}{3}\cdot\sum_{j=1}^{n}j^2\right)\right]$$
$$=\frac{1}{3}\sum_{i=1}^{n}\left[n^2[\log_2(n+1)+1]+\left(\sum_{j=1}^{n}j^2\right)\right]$$

- Step 4: Closed-Form

$$A(n)=\frac{1}{3}\sum_{i=1}^{n}\left[n^2[\log_2(n+1)+1]+\left(\sum_{j=1}^{n}j^2\right)\right]$$
$$=\frac{1}{3}\sum_{i=1}^{n}\left[n^2[\log_2(n+1)+1]+\frac{n(n+1)(2n+1)}{6}\right]=$$
$$=\frac{1}{3}\sum_{i=1}^{n}\left[n^2[\log_2(n+1)+1]+\frac{2n^3+3n^2+n}{6}\right]$$
$$=\frac{1}{3}\sum_{i=1}^{n}\left[n^2[\log_2(n+1)+1]+\frac{2n^3+3n^2+n}{6}\right]$$
$$=\frac{1}{3}\left[n^3[\log_2(n+1)+1]+\frac{2n^4+3n^3+n^2}{6}\right]$$
$$=\frac{1}{3}n^3\log_2(n+1)+\frac{1}{3}n^3+\frac{2n^4+3n^3+n^2}{18}=$$
$$=\frac{1}{3}n^3\log_2(n+1)+\frac{2n^4+9n^3+n^2}{18}=\frac{1}{3}n^3\log_2(n+1)+\frac{n^4}{9}+\frac{n^3}{2}+\frac{n^2}{18}$$

- Step 5: Asymptotic Notation

$$A(n)=\frac{1}{3}n^3\log_2(n+1)+\frac{n^4}{9}+\frac{n^3}{2}+\frac{n^2}{18}\in\Theta(n^4)$$

- Step 2: Basic Operation for (5)

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Basic operation is the assignment marked as (5)

Analyze B(n)

- Step 3: Count

When the input array does not contain any 0, (5) is never executed.

$$B(n) = \sum_{i=1}^{n} 0$$

- Step 4: Closed-Form

$$B(n) = \sum_{i=1}^{n} 0 = 0$$

- Step 5: Asymptotic Notation

$$Constant\ Order \rightarrow\ B(n) \in \Theta(1)$$

Analyze W(n)

- Step 3: Count

When the input array is full of 0's:

$$W(n) = \sum_{i=1}^{n} i$$

- Step 4: Closed-Form

$$W(n) = \sum_{i=1}^{n} i = \frac{n \cdot (n+1)}{2} = \frac{n^2 + n}{2}$$

- Step 5: Asymptotic Notation

$$\lim_{n \to \infty} \frac{W(n)}{g(n)} = \lim_{n \to \infty} \frac{\frac{n^2+n}{2}}{n^2} = \frac{1}{2} \rightarrow W(n) \in \Theta(n^2)$$

Analyze A(n)

- Step 3: Count

$$A(n) = \sum\nolimits_{I \in T_n} \tau(I) \cdot p(I) = \sum_{i=1}^{n} \left[\left(\frac{1}{3} \cdot i\right) + \left(\frac{1}{3} \cdot 0\right) + \left(\frac{1}{3} \cdot 0\right) \right] = \sum_{i=1}^{n} \left(\frac{1}{3} i\right) = \frac{1}{3} \sum_{i=1}^{n} i$$

- Step 4: Closed-Form

$$A(n) = \sum_{I \in T_n} \tau(I) \cdot p(I) = \sum_{i=1}^{n} \left[\left(\frac{1}{3} \cdot i\right) + \left(\frac{1}{3} \cdot 0\right) + \left(\frac{1}{3} \cdot 0\right) \right] = \sum_{i=1}^{n} \left(\frac{1}{3} i\right) = \frac{1}{3} \sum_{i=1}^{n} i$$
$$= \frac{1}{3} \cdot \frac{n \cdot (n+1)}{2} = \frac{n^2 + n}{6}$$

- Step 5: Asymptotic Notation

$$\lim_{n \to \infty} \frac{A(n)}{g(n)} = \lim_{n \to \infty} \frac{\frac{n^2 + n}{6}}{n^2} = \frac{1}{6} \rightarrow A(n) \in \Theta(n^2)$$

IDENTIFICATION OF BASIC OPERATION(S)

Here, state clearly which operation(s) in the algorithm must be the basic operation(s). Also, you should provide a simple explanation about why you have decided on the basic operation you choose. (1-3 sentences)

Three assignments in (2) are basic operations. Because at least one of these operations always contributes to an algorithm's total running time. Also, second assignment in (2) is typically the most time-consuming operation in the algorithm's innermost loop by being inside of 3 for loops and 1 while loop.

REAL EXECUTION

Best Case

N Size	Time Elapsed
1	0.00000476837158203125
5	0.000010967254638671875
10	0.00003814697265625
25	0.0002429485321044922
50	0.0011188983917236328
75	0.002679109573364258
100	0.004364013671875
150	0.011707067489624023
200	0.02629995346069336
250	0.036992788314819336

Worst Case

N Size	Time Elapsed
1	0.0000011920928955078125
5	0.000041961669921875
10	0.00040078163146972656
25	0.013779878616333008
50	0.18341803550720215
75	0.9667437076568604
100	2.958270788192749
150	15.30362319946289
200	48.0098979473114

250	121.85824203491211

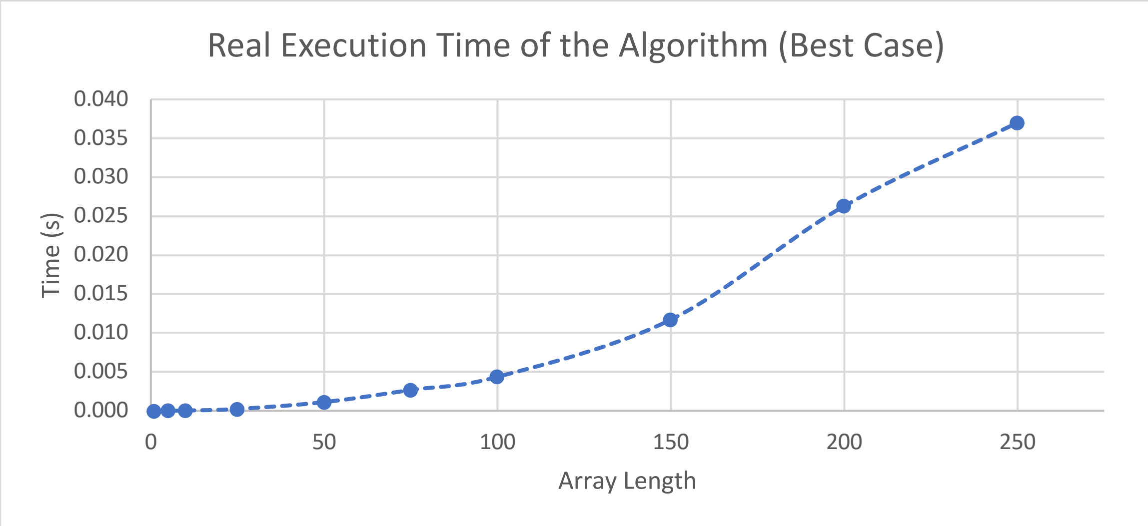
Average Case

N Size	**Time Elapsed**
1	0.0000023841857910156250
5	0.000033775965372721350
10	0.000313361485799153630
25	0.008852005004882812
50	0.09887814521789551
75	0.4282924334208171
100	1.199751853942871
150	6.5252476533253985
200	19.19698127110799
250	41.276151180267334

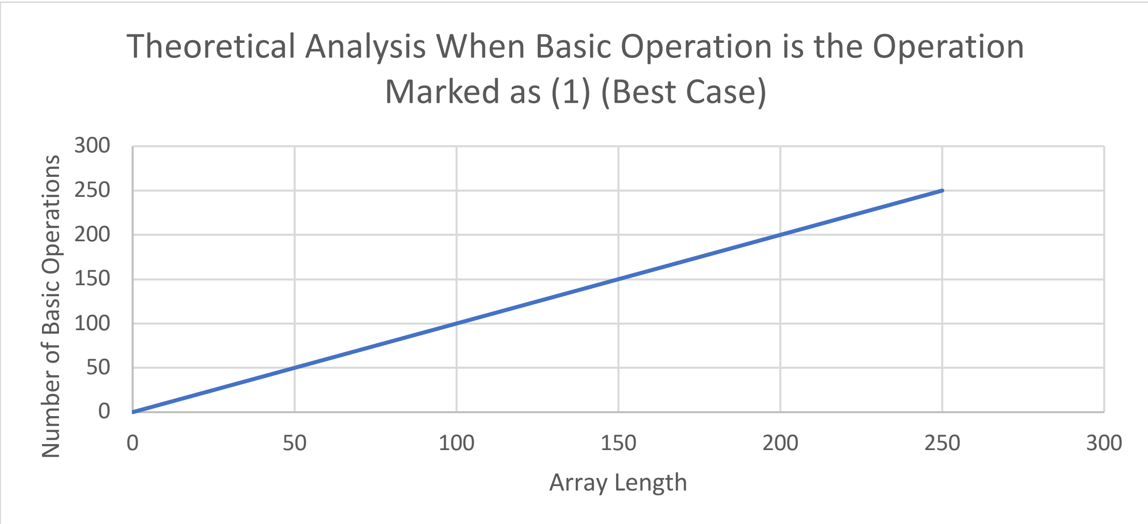
COMPARISON

Best Case

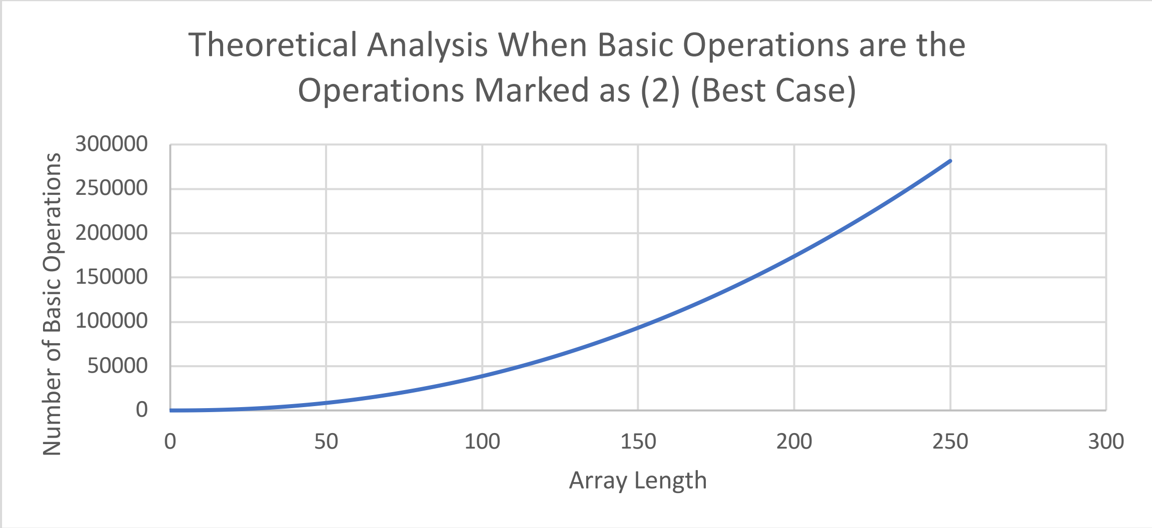
Graph of the real execution time of the algorithm



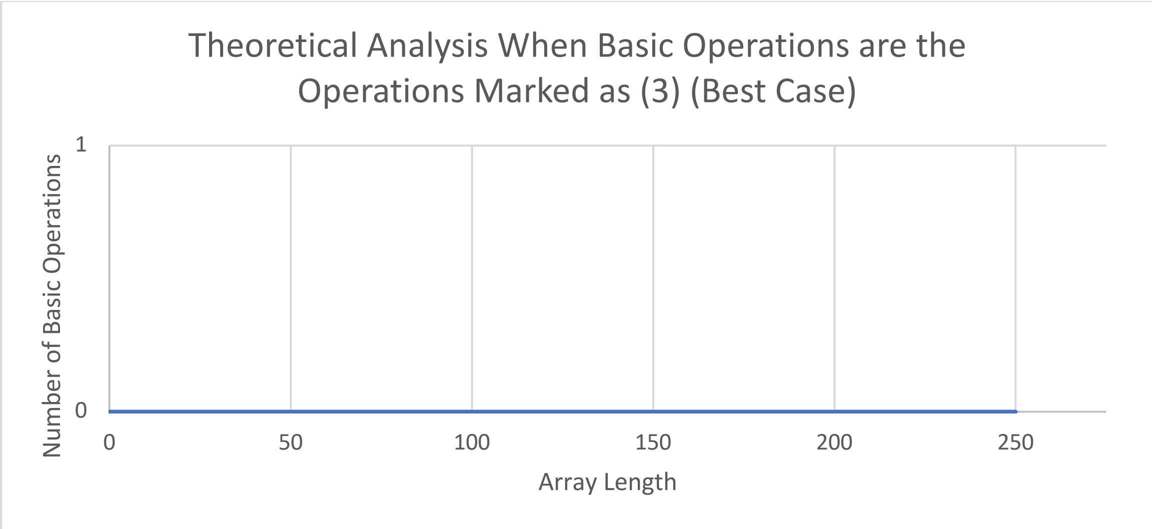
Graph of the theoretical analysis when basic operation is the operation marked as (1)



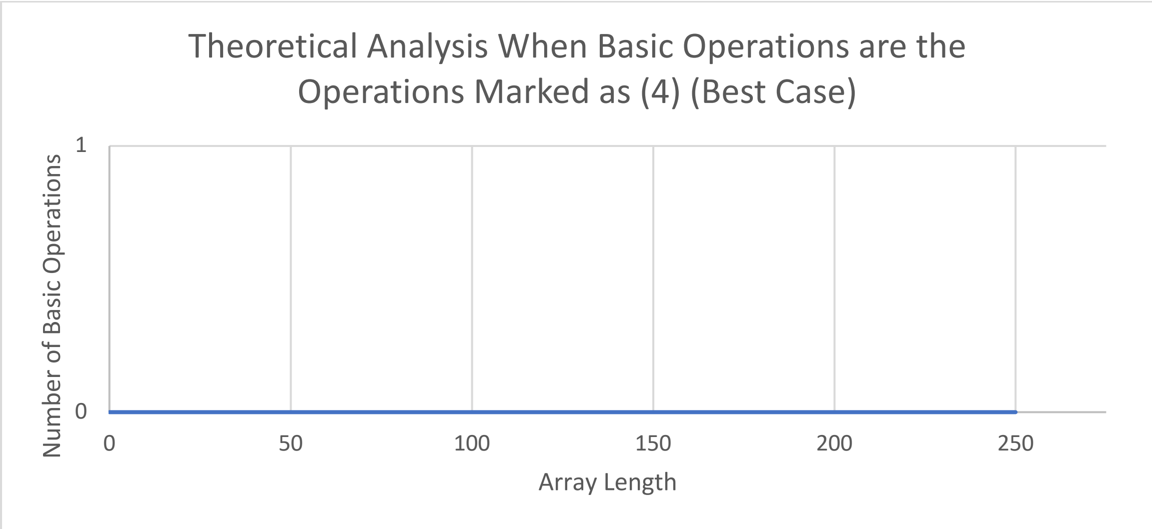
Graph of the theoretical analysis when basic operation is the operation marked as (2)



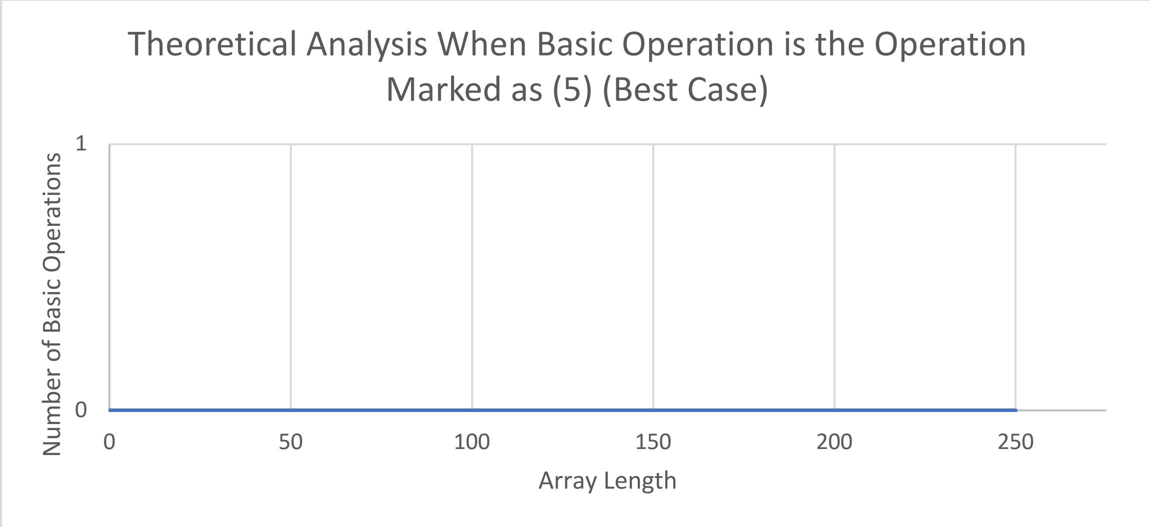
Graph of the theoretical analysis when basic operation is the operation marked as (3)



Graph of the theoretical analysis when basic operation is the operation marked as (4)



Graph of the theoretical analysis when basic operation is the operation marked as (5)



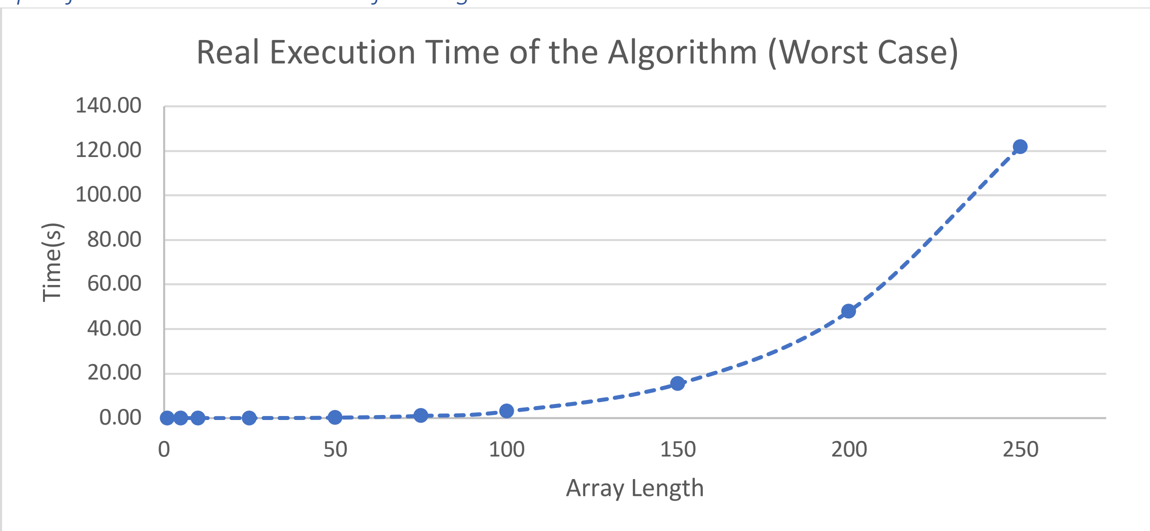
Comments

For the best case, we have 1 real measurement and 5 theoretical analyzes. To reach the real values, the algorithm is implemented in Python, and the execution time is measured for 10 different input sizes. Likewise, in the theoretical part, the number of basic operations executed are calculated with the closed-form formulas.

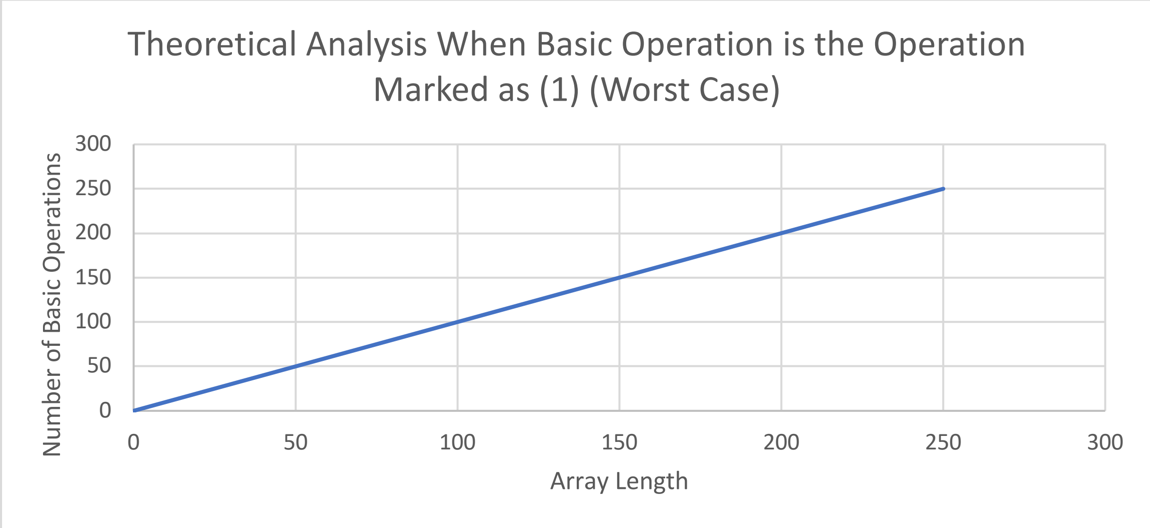
The number of real basic operations in this algorithm is proportional to the real execution time of the algorithm. Therefore, the time graph and the number of basic operations graphs can be interpreted on the same plane. The constant 0 value in the 3rd, 4th and 5th graphs, and the 1st graph is quite far from the real result. So as a result, basic operations marked as (2) seems to be the choice of basic operations that gives the closest result to reality due to the high correlation between the real execution time and the number of basic operations with respect to different input sizes.

Worst Case

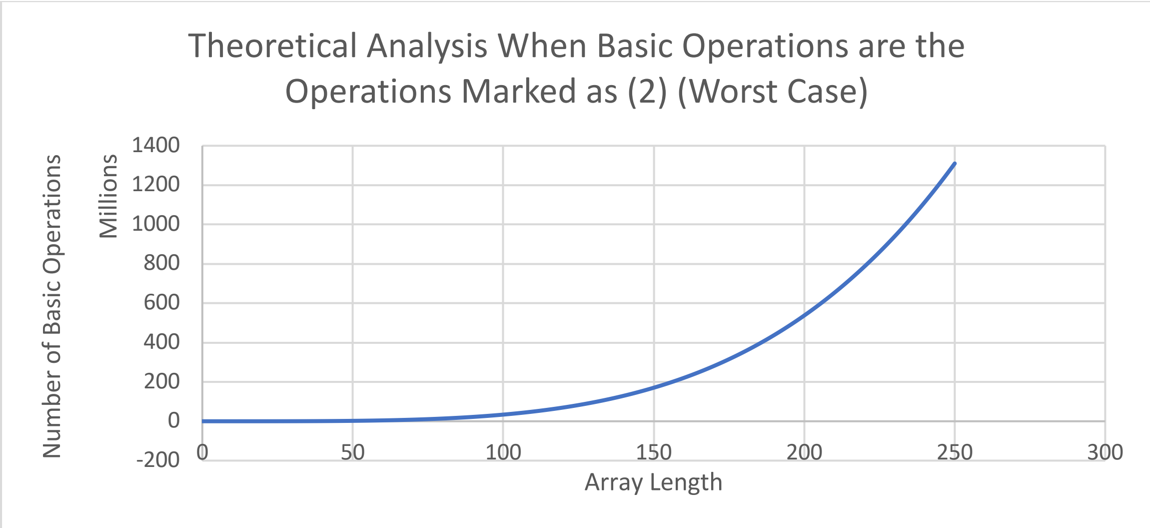
Graph of the real execution time of the algorithm



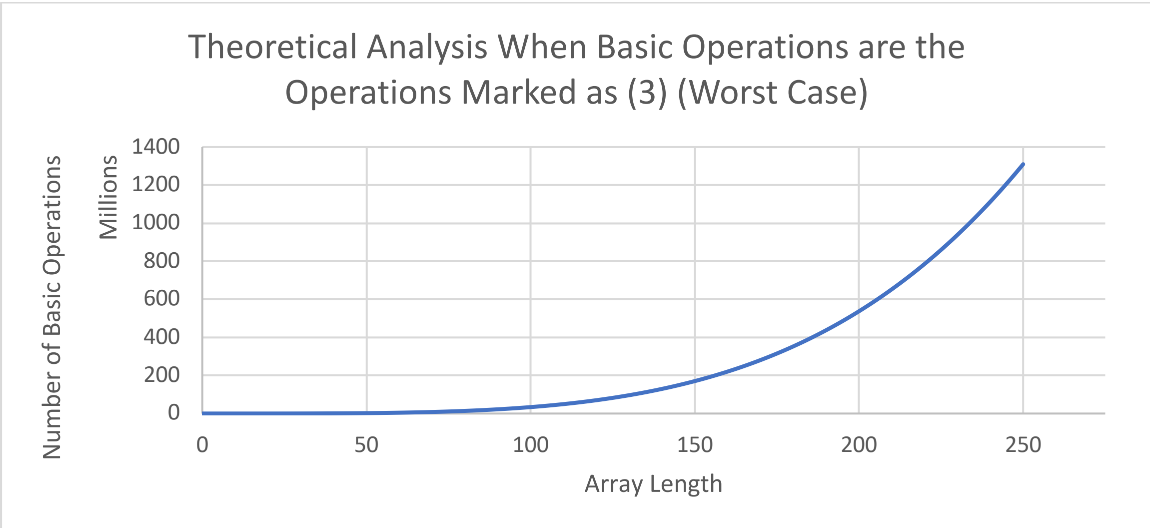
Graph of the theoretical analysis when basic operation is the operation marked as (1)



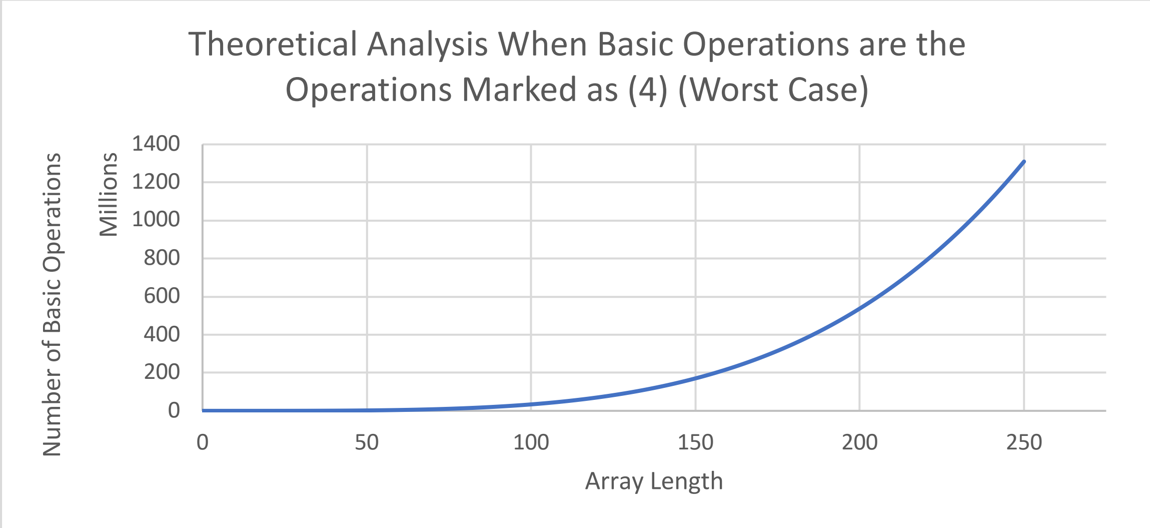
Graph of the theoretical analysis when basic operation is the operation marked as (2)



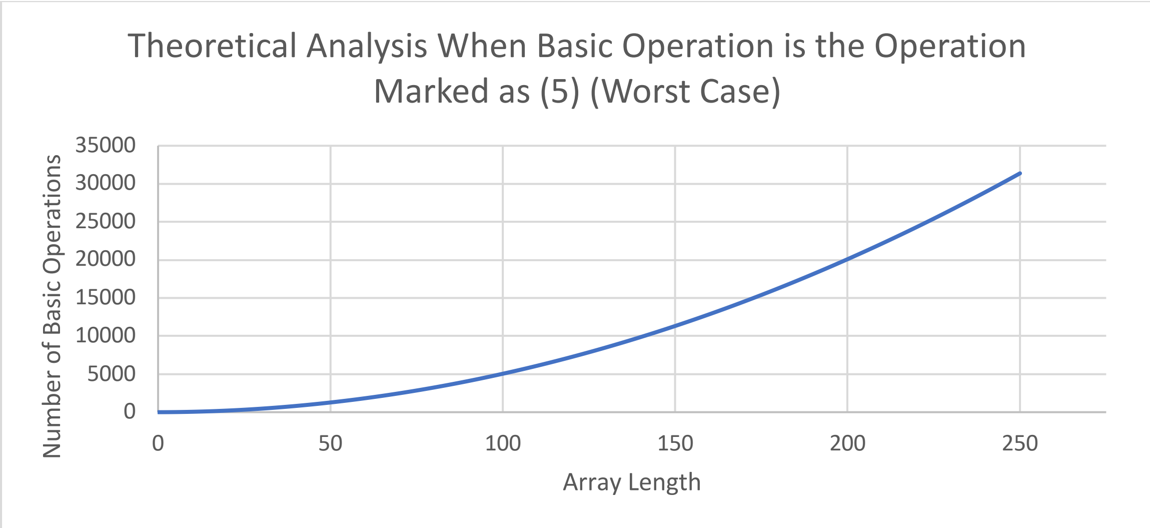
Graph of the theoretical analysis when basic operation is the operation marked as (3)



Graph of the theoretical analysis when basic operation is the operation marked as (4)



Graph of the theoretical analysis when basic operation is the operation marked as (5)



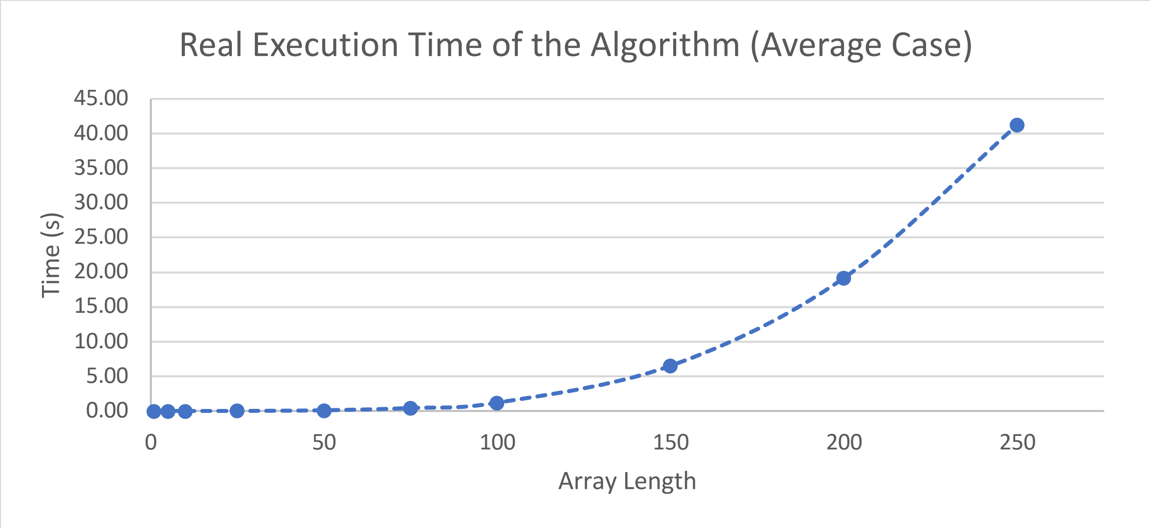
Comments

For the worst case, we have 1 real measurement and 5 theoretical analyzes. To reach the real values, the algorithm is implemented in Python, and the execution time is measured for 10 different input sizes. Likewise, in the theoretical part, the number of basic operations executed are calculated with the closed-form formulas.

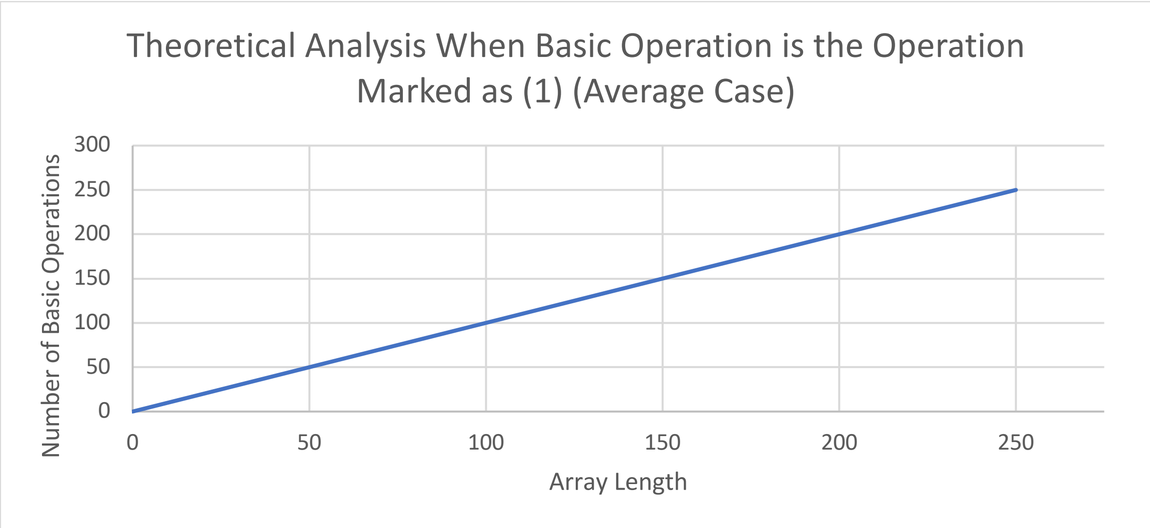
The number of real basic operations in this algorithm is proportional to the real execution time of the algorithm. Therefore, the time graph and the number of basic operations graphs can be interpreted on the same plane. It is clear that the 1st graph is quite far from the real graph, and the 2nd, 3rd, 4th and 5th are closer to the real execution time. The reason behind this is that the basic operations in the last 4 graphs take place in the inner loops compared to the 1st graph. Moreover, the basic operations marked as (2) seems to be the choice of basic operations that gives the closest result to reality among the last 4 graphs, due to the high correlation between the real execution time and the number of basic operations with respect to different input sizes.

Average Case

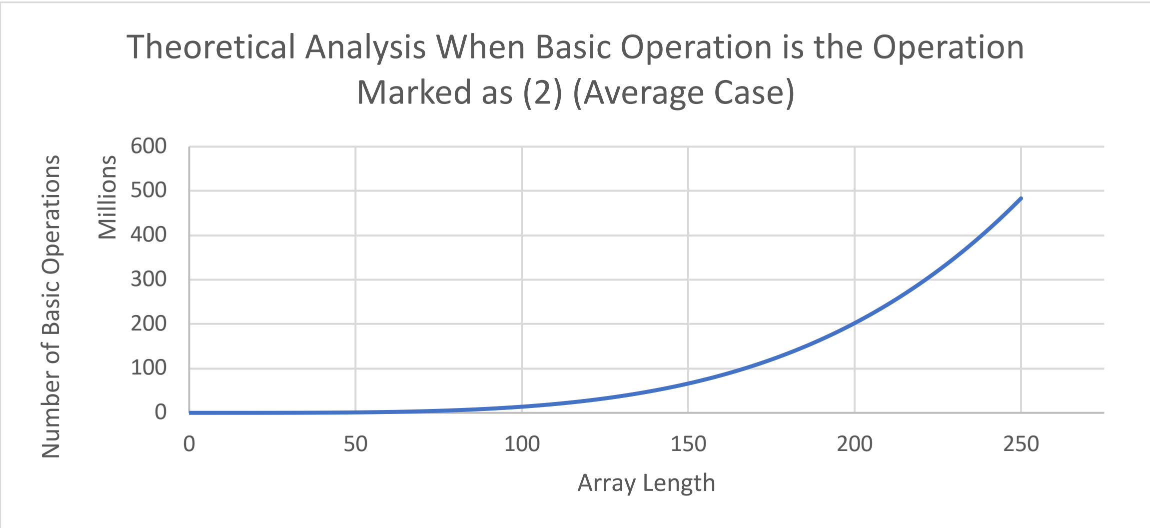
Graph of the real execution time of the algorithm



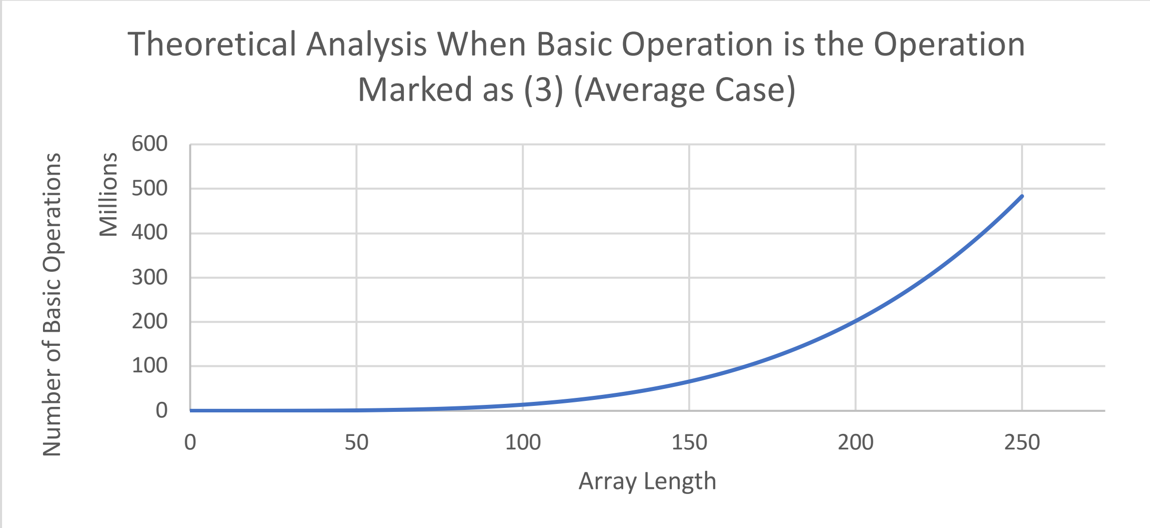
Graph of the theoretical analysis when basic operation is the operation marked as (1)



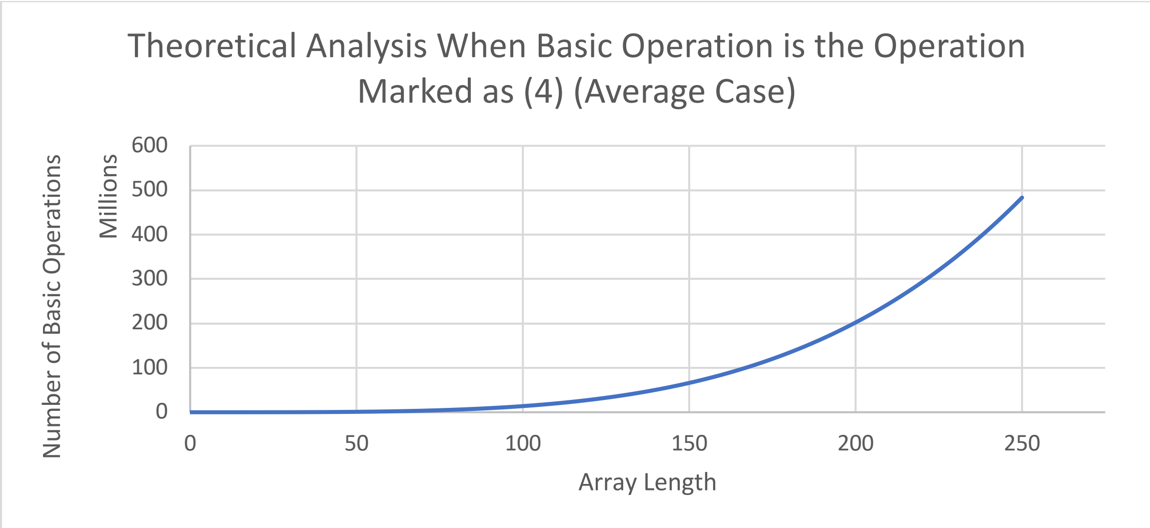
Graph of the theoretical analysis when basic operation is the operation marked as (2)



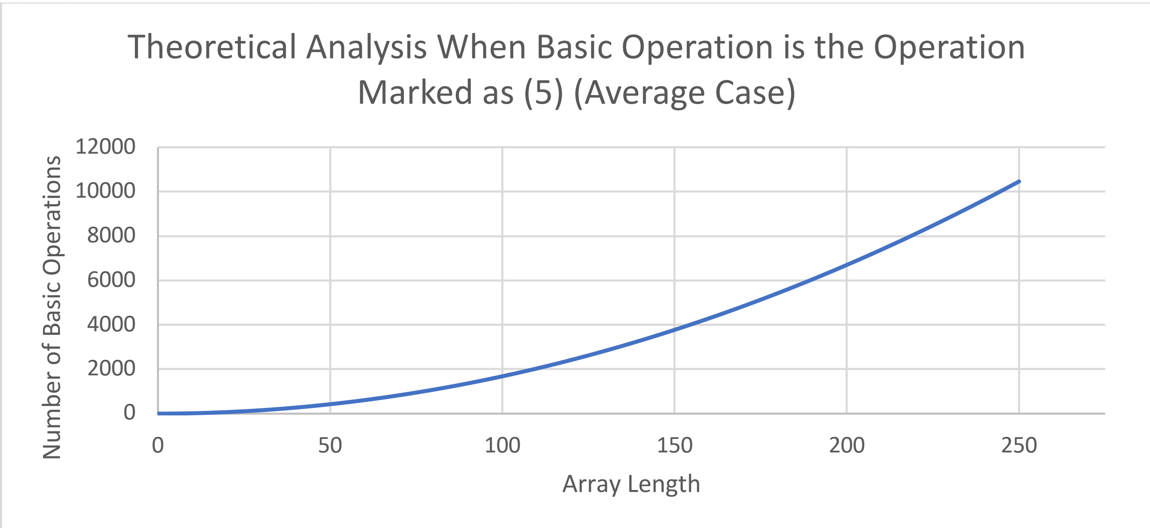
Graph of the theoretical analysis when basic operation is the operation marked as (3)



Graph of the theoretical analysis when basic operation is the operation marked as (4)



Graph of the theoretical analysis when basic operation is the operation marked as (5)



Comments

For the average case, we have 1 real measurement and 5 theoretical analyzes. To reach the real values, the algorithm is implemented in Python, and the execution time is measured for 10 different input sizes. Likewise, in the theoretical part, the number of basic operations executed are calculated with the closed-form formulas.

The number of real basic operations in this algorithm is proportional to the real execution time of the algorithm. Therefore, the time graph and the number of basic operations graphs can be interpreted on the same plane. The 1st graph is quite far from the real graph, and the 2nd, 3rd, 4th and 5th are closer to the real execution time. The reason behind this is that the basic operations in the last 4 graphs take place in the inner loops compared to the 1st graph. Moreover, the basic operations marked as (2) seems to be the choice of basic operations that gives the closest result to reality among the last 4 graphs, due to the high correlation between the real execution time and the number of basic operations with respect to different input sizes.