Solutions for Linear Algebra Midterm

İbrahim Arol Özkaya
150200728
Özkaya

Q1) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 0 \\ 1 & 0 & -1 & -1 \end{array}\right] \xrightarrow[E_{3,1}(-1)]{E_{2,1}(-1)} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & -1 & -2 & -2 \end{array}\right] \xrightarrow{E_{3,2}(1)} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & -3 \end{array}\right]$

System is inconsistent

Q2) $A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 1 \\ 3 & -4 & 10 \end{bmatrix}$

i) $\left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 3 & -4 & 10 & 0 & 0 & 1 \end{array}\right] \xrightarrow{E_{3,1}(-3)} \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & -4 & -2 & -3 & 0 & 1 \end{array}\right]$

$\xrightarrow{E_{3,2}(2)} \left.\left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 2 & 1 \end{array}\right]\right\} A^{-1}$ does not exist

ii) We can normally the determinant of a matrix by $A^{-1} = \frac{adj(A)}{det(A)}$
Since A^{-1} does not exist, we can say that $det(A) = 0$

Q3)

① The standart matrix for rotation around y axis by θ degrees:

$\left.\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} = A \right\}$ Put $\theta = \frac{\pi}{6} \Rightarrow A = \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & \sqrt{3}/2 \end{bmatrix}$

② Find the standart matrix for projection onto $x - 2y = 0$

$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad \vec{N} = \langle 1, -2, 0 \rangle$

a) $e_1 - proj_N e_1 = \langle 1,0,0 \rangle - \left(\frac{\langle 1,0,0 \rangle \cdot \langle 1,-2,0 \rangle}{5} \cdot \langle 1,-2,0 \rangle \right) = \langle \frac{4}{5}, \frac{2}{5}, 0 \rangle$

İbrahim Arel Özkaya
150200728

b) $e_2 - proj_N e_2 = \langle 0,1,0\rangle - \left(\frac{\langle 0,1,0\rangle \cdot \langle 1,-2,0\rangle}{5} \cdot \langle 1,-2,0\rangle\right) = \langle \frac{2}{5}, \frac{1}{5}, 0\rangle$

c) $e_3 - proj_N e_3 = \langle 0,0,1\rangle - \left(\frac{\langle 0,0,1\rangle \cdot \langle 1,-2,0\rangle}{5} \cdot \langle 1,-2,0\rangle\right) = \langle 0,0,1\rangle$

To find the standard matrix, write the results from a, b, c together (vertically)

$$\text{St. Mat.} = A = \begin{bmatrix} 4/5 & 2/5 & 0 \\ 2/5 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

I) To find the standart matrix for the whole operation, multiply $A_2 \cdot A_1$

$$A_2 \cdot A_1 = \begin{bmatrix} 4/5 & 2/5 & 0 \\ 2/5 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 2\sqrt{3}/5 & 2/5 & 2/5 \\ \sqrt{3}/5 & 1/5 & 1/5 \\ -1/2 & 0 & \sqrt{3}/2 \end{bmatrix}$$

ii) let's assume we have the vector $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$A \cdot \vec{u} = \begin{bmatrix} 2\sqrt{3}/5 \\ \sqrt{3}/5 \\ -1/2 \end{bmatrix} = u' \Rightarrow |u'| \neq |u| = 1$ } Norm of the unit vector is not preserved

Q4)

$$\begin{vmatrix} 1 & -1 & 4 & 2 \\ 0 & 2 & 1 & 1 \\ 0 & -2 & 2 & 0 \\ 0 & -3 & 0 & 0 \end{vmatrix} = 1 \cdot \begin{vmatrix} 2 & 1 & 1 \\ -2 & 2 & 0 \\ -3 & 0 & 0 \end{vmatrix} = (-1)^4 \cdot 1 \cdot \begin{vmatrix} -2 & 2 \\ -3 & 0 \end{vmatrix} = 6$$

$$\det(A^5) = [\det(A)]^5 = 6^5 = 7776$$

Q5) line 1 $\Rightarrow x = 1 + t,\ y = 0 + 2t,\ z = -1 + 0.t$ } Position vector $v_1 = \langle 1,2,0\rangle$

line 2 $\Rightarrow x = 1 - t,\ y = 0,\ z = -1 + 2t$ } Position vector $v_2 = \langle -1,0,2\rangle$

Let the plane formed by these lines be : $ax + by + cz + d = 0$

$\vec{N} = \langle a, b, c\rangle \Rightarrow$ Find the normal vector by $v_1 \times v_2$

$$v_1 \times v_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ -1 & 0 & 2 \end{vmatrix} = \langle \overset{a}{4}, \overset{b}{-2}, \overset{c}{2}\rangle = \vec{N},\ 4x - 2y + 2z + d = 0$$

A point on the line 1, $P_0 = (1,0,-1)$ is on the plane

$4 - 2 + d = 0 \Rightarrow d = -2$ } The plane eq is : $4x - 2y + 2z - 2 = 0$