Q1)

let $u \times v$ be equal to $\vec{w}$. $\vec{w}$ is orthagonal to both $\vec{u}$ and $\vec{v}$

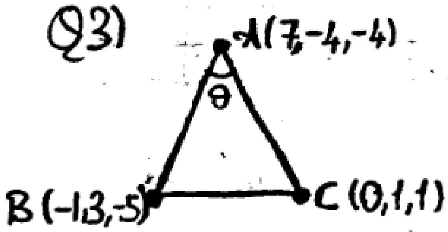
$\vec{u} . \vec{w} = |u|.|w|. \underbrace{\cos 90}_{0} = 0$

Q2)

$u = \langle 1,4,1 \rangle \quad v = \langle 0,-2,3 \rangle$

$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 4 & 1 \\ 0 & -2 & 3 \end{vmatrix} = \underbrace{\langle 14,-3,-2 \rangle}_{\text{This vector is orthagonal to both } \vec{u} \text{ and } \vec{v}}$

Q3)



$\vec{AB} = \langle -8, 7, -1 \rangle$
$\vec{AC} = \langle -7, 5, 5 \rangle$

$Area = |\vec{AB}|.|\vec{AC}|. \sin\theta . \frac{1}{2}$

$Area = |\vec{AB} \times \vec{AC}| . \frac{1}{2}$

$Area = \frac{1}{2} \left| \begin{vmatrix} i & j & k \\ -8 & 7 & -1 \\ -7 & 5 & 5 \end{vmatrix} \right| = |\langle 40, 47, 9 \rangle| . \frac{1}{2} = 31,2$

Q4) $\left.\begin{matrix} \vec{AB} = \langle -8, 7, -1 \rangle \\ \vec{AC} = \langle -7, 5, 5 \rangle \\ \vec{AO} = \langle -7, 4, 4 \rangle \end{matrix}\right\} V = |\vec{AO}.(\vec{AB} \times \vec{AC})| = |\langle -7,4,4 \rangle . \langle 40,47,9 \rangle|$

$= 56$

Since $V \neq 0$, these vectors are NOT coplanar

Q5)

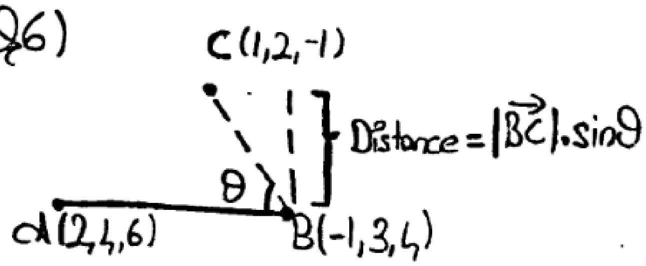
$\vec{N} = \begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \langle 4,2,-2 \rangle$ } Normal vector of the determined plane

let $\vec{p} = \langle a,b,c \rangle$, $\vec{p} \perp \vec{N} \Rightarrow \vec{p}.\vec{N} = 0$, $\vec{p} \perp \vec{w} \Rightarrow \vec{p} . \vec{w} = 0$

$\left.\begin{matrix} 4a + 2b - 2c = 0 \\ 5a + 7b - 4c = 0 \end{matrix}\right\} \begin{matrix} a = b \\ c = 3a \end{matrix} \Rightarrow \vec{p_1} = \langle a, a, 3a \rangle, \vec{p_2} = \langle -a, -a, -3a \rangle$

convert $\vec{p}$ into unit vector $\Rightarrow \vec{p_1}' = \langle \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}} \rangle$, $\vec{p_2}' = \langle \frac{-1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{-3}{\sqrt{11}} \rangle$

Q6)



$\overrightarrow{AB} = \langle -3,-1,-2 \rangle \Rightarrow |\overrightarrow{AB}| = \sqrt{14}$
$\overrightarrow{BC} = \langle 2,-1,-5 \rangle \Rightarrow |\overrightarrow{BC}| = \sqrt{30}$
$|\overrightarrow{AB} \times \overrightarrow{BC}| = |\overrightarrow{AB}| \cdot |\overrightarrow{BC}| \cdot \sin\theta$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} i & j & k \\ -3 & -1 & -2 \\ 2 & -1 & -5 \end{vmatrix} = \langle 3,-19,5 \rangle \text{ , } |\overrightarrow{AB} \times \overrightarrow{BC}| = \sqrt{395}$$

$$\sqrt{395} = \sqrt{14} \cdot \sqrt{30} \cdot \sin\theta \Rightarrow \sin\theta = \sqrt{0,94}$$

$$\text{Distance} = |\overrightarrow{BC}| \cdot \sin\theta = \sqrt{30} \cdot \sqrt{0,94} = 5,31$$

Q7)

a) $N_1 = \langle 1,2,-1 \rangle$ and $N_2 = \langle 2,-1,1 \rangle$

$$\left. \overrightarrow{N_1} \cdot \overrightarrow{N_2} = |N_1| \cdot |N_2| \cdot \cos\theta = \sqrt{6} \cdot \sqrt{6} \cdot \cos\theta = -1 \right\} \cos\theta = \frac{-1}{6}$$

$$\overrightarrow{N_1} \times \overrightarrow{N_2} = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = \langle 1,-3,-5 \rangle \text{ , } |N_1 \times N_2| = \sqrt{35} = N_1 N_2 . \sin\theta$$

$$\left. \sqrt{35} = \sqrt{6} \cdot \sqrt{6} \cdot \sin\theta \right\} \sin\theta = \frac{\sqrt{35}}{6}$$

$\therefore$ The planes are neither parallel nor perpendicular

b) $N_1 = \langle 3,1,-1 \rangle$ and $N_2 = \langle 1,2,5 \rangle$

$$\left. \overrightarrow{N_1} \cdot \overrightarrow{N_2} = |N_1| \cdot |N_2| \cdot \cos\theta = \sqrt{11} \cdot \sqrt{30} \cdot \cos\theta = 0 \right\} \begin{matrix} \cos\theta = 0 \\ \theta = 90^\circ \end{matrix}$$

$\therefore$ The planes are perpendicular

c) $N_1 = \langle 1,1,-3 \rangle$ and $N_2 = \langle 2,2,-6 \rangle$

$$\overrightarrow{N_1} \times \overrightarrow{N_2} = \begin{vmatrix} i & j & k \\ 1 & 1 & -3 \\ 2 & 2 & -6 \end{vmatrix} = \langle 0,0,0 \rangle \text{ , } |\overrightarrow{N_1} \times \overrightarrow{N_2}| = 0$$

$$\left. |\overrightarrow{N_1} \times \overrightarrow{N_2}| = |N_1| \cdot |N_2| \cdot \sin\theta = \sqrt{11} \cdot \sqrt{44} \cdot \sin\theta = 0 \right\} \begin{matrix} \sin\theta = 0 \\ \theta = 0^\circ \end{matrix}$$

$\therefore$ The planes are parallel

Q8)

$P = (1, 0, -1)$ and $M = (x, y, z)$

$\vec{PM} = \langle x-1, y, z+1 \rangle$ and $\vec{n} \perp \vec{PM}$

$\vec{PM} \cdot \vec{n} = |PM| \cdot |n| \cdot \cos 90 = 0$

$5x - 5 + y - 2z - 2 = 0 \Rightarrow \underbrace{5x + y - 2z = 7}_{}$ Plane's equation

Q9) Find the position vector by calculating $\vec{QP}$

$\vec{QP} = \langle 0, 2, 3 \rangle \rightarrow$ Position vector
The point $P = (0, 2, 1)$ is on the plane
Find the parametric equation

$$\left.\begin{aligned} x &= 0 + 0.t \\ y &= 2 + 2.t \\ z &= 1 + 3.t \end{aligned}\right\} \text{line eq.} : (0, 2+2t, 1+3t)$$

Q10)

let the plane equation be : $ax + by + cz - d = 0$

$\underbrace{ax + by + cz - d = 0}_{\vec{N_1} = \langle a, b, c \rangle} \; // \; \underbrace{x + 2y + 5z - 3 = 0}_{\vec{N_2} = \langle 1, 2, 5 \rangle}$

Since these lines are parallel,

$\frac{a}{1} = \frac{b}{2} = \frac{c}{5} = k \Rightarrow$ plane: $kx + 2ky + 5kz - d = 0$

P is on the plane $\Rightarrow 5k - 4k + 5k - d = 6k - d = 0$

$\forall$ With every $k \in R$ ve get a different plane

For $k = 1$, $d = 6$. and plane: $x + 2y + 5z - 6 = 0$

Q1) Let the plane equation be : $ax+by+cz+d=0$

Normal of the plane is $\vec{N}=\langle a,b,c\rangle$

line equation $\Rightarrow (1+3t,-1-2t,3-t)=P_0+\vec{V}.t$ $\left.\begin{cases} P_0=(1,-1,3) \\ \vec{V}=\langle 3,-2,-1\rangle \end{cases}\right.$

① $\vec{N}\perp\vec{V} \Rightarrow \vec{N}.\vec{V}=3a-2b-c=0$

② P_0 is on the plane $\Rightarrow a-b+3c+d=0$

The plane is orthagonal to the plane given by $x+y-z=0$ those normal vector is $\vec{N_1}=\langle 1,1,-1\rangle$

③ $\vec{N}\perp\vec{N_1}=\vec{N}.\vec{N_1}=a+b-c=0$

Using the equations ①, ② and ③:

$2a=3b$, $c=a+b$

$a-b+3(a+b)+d=0$

$4a+2b+d=0$

$8b+d=0$

* Since we do not have enough data, there is an infinite amount of planes that can be written

for $b=2$;

The plane equation is:

$3x+2y+5z-16=0$

Q12)

let the equation of the plane be $ax+by+cz+d=0$

Normal vector of the plane is $\vec{N}=\langle a,b,c\rangle$

$\vec{P_2P_1}=\langle 1,0,4\rangle$, $\vec{P_2P_1}\perp\vec{N}\Rightarrow\vec{P_2P_1}\cdot\vec{N}$

① $a+4c=0$

The plane is perpendicular to the plane $2x+2y+3z=1$ whose normal vector is $\vec{N_1}=\langle 2,2,3\rangle$

$\vec{N}\perp\vec{N_1}\Rightarrow\vec{N}\cdot\vec{N_1}=0$

② $2a+2b+3c=0$

The points P_1 and P_2 are on the plane

③ $a+2b+3c+d=0$

④ $2b-c+d=0$

Using ① and ② $\Rightarrow 2b=5c$ ... ⑤

Using ⑤ and ④ $\Rightarrow 4c+d=0$

Since we do not have enough data, there is an infinite amount of planes that can be written

For $c=-2\Rightarrow$ The plane eq: $8x-5y-2z+8=0$

Q13)

The line described $\Rightarrow(1-2t,1+t,5-2t)=P_0+\vec{V}\cdot t$ $\left\}\begin{array}{l}P_0=(1,1,5)\\ \vec{V}=\langle -2,1,-2\rangle\end{array}\right.$

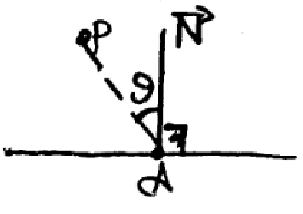
The plane given by the normal vector of $\vec{N}=\langle 1,2,0\rangle$

$\vec{V}\cdot\vec{N}=|\vec{V}|\cdot|\vec{N}|\cdot\cos\theta=3\cdot\sqrt{3}\cdot\cos\theta=0\Rightarrow\cos\theta=0\}\ \theta=90°$

The line and the plane are perpendicular

Q14) let $A=(0,0,-3)$ be a point on the plane $\Rightarrow\vec{AP}=\langle 1,3,3\rangle$

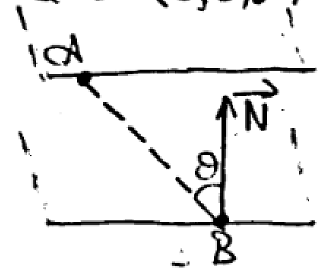
The normal vector of the plane is $\vec{N}=\langle 5,4,-1\rangle$



$\vec{AP}\cdot\vec{N}=|\vec{AP}|\cdot|\vec{N}|\cdot\cos\theta=14$

$\cos\theta=\frac{14}{\sqrt{19}\cdot\sqrt{42}}$, distance is $|\vec{AP}|\cdot\cos\theta=2{,}16$

Q15) let $A=(0,0,-3)$ and $B=(0,0,-5)$ be points on the planes respectively.



$\vec{BA}=\langle 0,0,2\rangle$ and the normal vectors for the planes $\vec{N}=\langle 5,4,-1\rangle$

$\vec{BA}\cdot\vec{N}=|\vec{BA}|\cdot|\vec{N}|\cdot\cos\theta=-2 \Big\}\cos\theta=\frac{-2}{2\cdot\sqrt{42}}$

$$\text{distance}=|\vec{AB}|\cdot\cos\theta$$
$$=\left|\cancel{2}\cdot\frac{-2}{\cancel{2}\cdot\sqrt{42}}\right|$$
$$=0,31$$

Q16) let $P=(a,b,c)$

$$\left.\begin{array}{l} ①\ a-4b-c=-3 \\ ②\ 2a-3b+2c=0 \end{array}\right\} \begin{array}{ll} 1)\ \text{for } c=-1 & 2)\ \text{for } c=-6 \\ P_1=(4,2,-1) & P_2=(15,6,-6) \end{array}$$

$\vec{P_1P_2}=\langle 11,4,-5\rangle$ } Location vector

The equations for the line are:
$$x=4+11t$$
$$y=2+4t$$
$$z=-1-5t$$

Q17) The location vector of the given line is $\vec{V}=\langle -2,1,-2\rangle$

The normal vector of the given plane is $\vec{N}=\langle 1,1,-1\rangle$

The angle between the normal vector and the line is equal to the angle between the plane and the line.

$$\vec{V}\cdot\vec{N}=\underbrace{|\vec{V}|}_{3}\cdot\underbrace{|\vec{N}|}_{\sqrt{3}}\cdot\cos\theta=1 \Big\}\cos\theta=\frac{1}{3\sqrt{3}} \Rightarrow \theta\approx 79^\circ$$

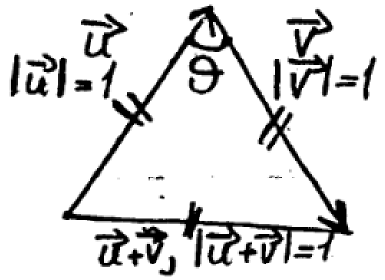
18) If the lines intersect, they must have a common point

$$\begin{array}{l} 1-2t=3s \\ 1+2t=1-s \\ 5-2t=1+2s \end{array} \Rightarrow \left.\begin{array}{l} 3s+2t=1 \\ s+2t=0 \\ 2s+2t=4 \end{array}\right\} \text{Solve for } s,t \quad \begin{bmatrix} 3 & 2 \\ 1 & 2 \\ 2 & 2 \end{bmatrix}\cdot\begin{bmatrix} s \\ t \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 3 & 2 & 1 \\ 1 & 2 & 0 \\ 2 & 2 & 4 \end{array}\right] \begin{array}{l} E_{2,1}(-3) \\ E_{2,3}(-2) \end{array} \rightarrow \left[\begin{array}{cc|c} 0 & -4 & 1 \\ 1 & 2 & 0 \\ 0 & -2 & 4 \end{array}\right] E_3(-1/2) \rightarrow \left[\begin{array}{cc|c} 0 & -4 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & -2 \end{array}\right]$$

$$E_{3,1}(4) \rightarrow \left[\begin{array}{cc|c} 0 & 0 & -7 \\ 1 & 0 & 0 \\ 0 & 1 & -2 \end{array}\right] \Big\}$$ We got an inconsistent system $\Rightarrow$ The lines don't intersect

Q19)



When we add the two vectors, we get a triangle those sides are equal.

Therefore all of the interior angles of the triangle is θ, and since $3\theta = 180°$, θ is $60°$

$|\vec{v}\times\vec{u}| = |\vec{v}|.|\vec{u}|.\sin\theta = 1.1.\sin 60 = \frac{\sqrt{3}}{2}$

Q20) Gauss-Schwarz Inequality : $|\vec{u}.\vec{v}| \le \|\vec{u}\|.\|\vec{v}\|$

Verify the inequality using $\vec{u}=\langle 2,0,-1,4\rangle$ and $\vec{v}=\langle 0,2,1,1\rangle$

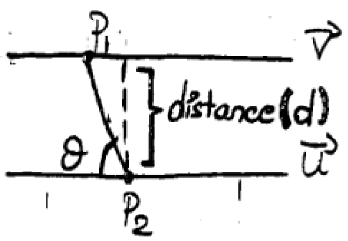
$|\vec{u}.\vec{v}| = 2.0 + 0.2 + (-1).(1) + 4.1 = 3 = \sqrt{9}$

$|\vec{u}| = \sqrt{21}$, $|\vec{v}| = \sqrt{6}$ $\Rightarrow |\vec{u}|.|\vec{v}| = \sqrt{126}$

As shown above, $|\vec{u}.\vec{v}| = \sqrt{9} < \sqrt{126} = |\vec{u}|.|\vec{v}|$.

Q21) location vector for line 1 $= \vec{v} = \langle -2,3,-2\rangle$ and $P_1 = (1,1,5)$

location vector for line 2 $= \vec{u} = \langle 3,-1,2\rangle$ and $P_2 = (0,1,1)$



$\overrightarrow{P_2P_1} = \langle 1,0,4\rangle$, $\vec{u}.\overrightarrow{P_2P_1} = |\vec{u}|.|\overrightarrow{P_2P_1}|.\cos\theta$

$\sqrt{14}.\sqrt{17}.\cos\theta = 11 \Rightarrow \cos\theta = \frac{11}{\sqrt{14}.\sqrt{17}} \Rightarrow \sin\theta = 0{,}7$

$d = |\overrightarrow{P_2P_1}|.\sin\theta = 2{,}89$

Q22) * I have assumed you meant to write :

$\underline{c_1}(1,0,2,0) + \underline{c_2}(1,-2,-2,0) + \underline{c_3}(0,1,1,-2) = (2,-1,3,1)$

$$\left.\begin{array}{l} c_1 + c_2 = 2 \\ -2c_2 + c_3 = -1 \\ 2c_1 - 2c_2 + c_3 = 3 \\ -2c_3 = 1 \end{array}\right\} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -2 & 1 & -1 \\ 2 & -2 & 1 & 3 \\ 0 & 0 & -2 & 1 \end{array}\right] \xrightarrow[E_{2,3}(-1)]{E_4(-\frac{1}{2})} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -2 & 1 & -1 \\ 2 & 0 & 0 & 4 \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right]$$

$$\xrightarrow[E_{4,2}(-1)]{E_3(\frac{1}{2})} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -2 & 0 & -\frac{1}{2} \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right] \xrightarrow[E_2(-\frac{1}{2})]{E_{3,1}(-1)} \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right] \xrightarrow{E_{1,2}(-1)} \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right]$$

The system is inconsistent. therefore there are no c_1, c_2, c_3 that that meet the requirements.

Q23)

The x variables differ between $x_1 \dots x_3$ } Domain is R^3
The w results differ betwen $w_1 \dots w_3$ } Codomain is R^3] $f: R^3 \to R^3$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 3 & -1 \\ 1 & -3 & -1 \\ 5 & 3 & -3 \end{bmatrix}}_{\text{Standard Matrix}} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

∇ To find the range, find the constraint on w_i when they are in reduced echolon form.

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & w_1 \\ 1 & -3 & -1 & w_2 \\ 5 & 3 & -3 & w_3 \end{array}\right] \xrightarrow[E_{2,3}(-5)]{E_{2,1}(-2)} \left[\begin{array}{ccc|c} 0 & 9 & 1 & w_1-2w_2 \\ 1 & -3 & -1 & w_2 \\ 0 & 18 & 2 & w_3-5w_2 \end{array}\right] \xrightarrow{E_{1,3}(-2)} \left[\begin{array}{ccc|c} 0 & 9 & 1 & w_1-2w_2 \\ 1 & -3 & -1 & w_2 \\ 0 & 0 & 0 & w_3-w_2-2w_1 \end{array}\right]$$

$$E(1/9) \longrightarrow \left[\begin{array}{ccc|c} 0 & 1 & 1/9 & (w_1-2w_2)/9 \\ 1 & -3 & -1 & w_2 \\ 0 & 0 & 0 & w_3-w_2-2w_1 \end{array}\right] \xrightarrow{E_{1,2}(3)} \left[\begin{array}{ccc|c} 0 & 1 & 1/9 & (w_1-2w_2)/9 \\ 1 & 0 & -2/3 & (w_1+w_2)/3 \\ 0 & 0 & 0 & w_3-w_2-2w_1 \end{array}\right]$$

$$E_{1,2} \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2/3 & (w_1+w_2)/9 \\ 0 & 1 & 1/9 & (w_1-2w_2)/3 \\ 0 & 0 & 0 & w_3-w_2-2w_1 \end{array}\right] \Bigg\} \; w_3-w_2-2w_1=0$$

∇ Constraint

Q24)

a) Standart matrix for the equation is: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$ } Result is $(2,-1,3)$

b) Standard matrix for the equation is: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$$ } Result is $(1,0,5)$

c) Standard matrix for the operation is: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$ } Result is $(1,0,0)$

d) The standard matrix for the operation is: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix}$

$$\left.\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -\sqrt{3}-3/2 \\ -1+3\sqrt{3}/2 \end{bmatrix}\right\} \text{Result: } \left(1, -\sqrt{3}-\frac{3}{2}, -1+\frac{3\sqrt{3}}{2}\right)$$

f)

$$= \underbrace{\begin{bmatrix} \cos 90 & 0 & \sin 90 \\ 0 & 1 & 0 \\ -\sin 90 & 0 & \cos 90 \end{bmatrix}}_{\text{Rotation around y axis by } 90^\circ} \cdot \underbrace{\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}}_{\text{Dilation with factor 2}} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\text{Orthogonal pr. onto x-axis}} \cdot \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ -1/2 \end{bmatrix}$$

e)

Q25)

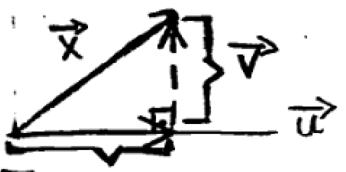
$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$T_1 \cdot T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $T_2 \cdot T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

∇ $T_1.T_2$ is firstly projects to x-z plane, then projects to x-y plane
$T_2.T_1$ is firstly projects to x-y plane, then projects to x-z plane

Since those two operations give the same results, $T_1.T_2 = T_2.T_1$

Q26)



$T(x) = \text{proj}_u x$

$\vec{v} \perp \vec{u}$ and $T(x) // \vec{u} \Rightarrow \vec{v} \perp T(x)$ ①

$T(x) + \vec{v} = x$, $\text{proj}_u x + \vec{v} = x$

$v = x - \text{proj}_u x$, and according to ①

$(x - \text{proj}_u x) \perp \text{proj}_u x$

Q27)

a) $\underbrace{\begin{bmatrix} 1 & 3 & -1 \\ 2 & -3 & -1 \\ 5 & 6 & -4 \end{bmatrix}}_{A} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$ } For the $T(x): R^3 \to R^3$ to be one-to-one, A must be invertible. Therefore $|A| \neq 0$

$|A| = \begin{vmatrix} 1 & 3 & -1 \\ 2 & -3 & -1 \\ 5 & 6 & -4 \end{vmatrix} \xrightarrow[E_{1,3}(-5)]{E_{1,2}(-2)} \begin{vmatrix} 1 & 3 & -1 \\ 0 & -9 & 1 \\ 0 & -9 & 1 \end{vmatrix} \xrightarrow{E_{2,3}(-1)} \begin{vmatrix} 1 & 3 & -1 \\ 0 & -9 & 1 \\ 0 & 0 & 0 \end{vmatrix} = 0$

A is <u>not</u> invertible. Therefore, the transformation is <u>not</u> one-to-one

b) $\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 2 & 0 & -3 \end{bmatrix}}_{A} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$ } For the $T(x): R^3 \to R^3$ to be one-to-one A must be invertible. Therefore $|A| \neq 0$

$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 2 & 0 & 3 \end{vmatrix} \xrightarrow[E_{1,3}(-2)]{E_{1,2}(-1)} \begin{vmatrix} 1 & 1 & 1 \\ 0 & -2 & -2 \\ 0 & -2 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = -6 \neq 0$

A is Invertible, therefore the transformation is one-to-one

Range of the transformation is R^3

a₂) Find the range

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & w_1 \\ 2 & -3 & -1 & w_2 \\ 5 & 6 & -4 & w_3 \end{array}\right] \xrightarrow[E_{1,3}(-5)]{E_{1,2}(-2)} \left[\begin{array}{ccc|c} 1 & 3 & -1 & w_1 \\ 0 & -9 & 1 & w_2-2w_1 \\ 0 & -9 & 1 & w_3-5w_1 \end{array}\right] \xrightarrow{E_{2,3}(-1)} \left[\begin{array}{ccc|c} 1 & 3 & -1 & w_1 \\ 0 & -9 & 1 & w_2-2w_1 \\ 0 & 0 & 0 & w_3-3w_1-w_2 \end{array}\right]$$

For the system to be consistent, $w_3 - w_2 - 3w_1$ must be 0

The constraint for the range is $w_3 - w_2 - 3w_1 = 0$

There is not an inverse transformation for this transformation

b₂) To find the inverse transformation, first find A^{-1}

$$A^{-1} = \frac{adj(A)}{|A|} = -\frac{1}{6} \cdot \begin{bmatrix} -3 & -3 & 0 \\ -5 & 1 & 2 \\ 2 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 5/6 & -1/6 & -1/3 \\ -1/3 & -1/3 & 1/3 \end{bmatrix}$$

Therefore the inverse transformation $T_{A^{-1}}\ R^3 \to R^3$ is :

$w_1 = \frac{x_1}{2} + \frac{x_2}{2}$ $\quad w_2 = \frac{5x_1}{6} - \frac{x_2}{6} - \frac{x_3}{3}$ $\quad w_3 = \frac{-x_1}{3} - \frac{x_2}{3} + \frac{x_3}{3}$

Q28) We can notate the given transformation as:

$w_1 = 2x_1 + 3x_2$

$\underbrace{w_2 = 1}$

w_2 doesn't get effected as x_1, x_2 changes

} We can not define a matrix A where $A \cdot x = w$, so the transformation is not linear

Q29) let the base vectors be: $e_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$, $e_2 = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $e_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$

① Rotation around z axis by 45°

$e_1' = \begin{bmatrix}\cos 45\\ \sin 45\\ 0\end{bmatrix} = \begin{bmatrix}\sqrt{2}/2\\ \sqrt{2}/2\\ 0\end{bmatrix}$ $\quad e_2' = \begin{bmatrix}-\sin 45\\ \cos 45\\ 0\end{bmatrix} = \begin{bmatrix}-\sqrt{2}/2\\ \sqrt{2}/2\\ 0\end{bmatrix}$ $\quad e_3' = \begin{bmatrix}0\\0\\1\end{bmatrix}$

② Orthogonal projection onto the x axis

$e_1'' = \begin{bmatrix}\sqrt{2}/2\\0\\0\end{bmatrix}$ $\quad e_2'' = \begin{bmatrix}-\sqrt{2}/2\\0\\0\end{bmatrix}$ $\quad e_3'' = \begin{bmatrix}0\\0\\0\end{bmatrix}$

To find the standard matrix, write the e_i'' together

$$\text{St. Matrix} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Q30) Let the standard matrices for the transformations be A, B respectively

$T_1 \to$ One-to-one $\Rightarrow |A| \neq 0$
$T_2 \to$ Not one-to-one $\Rightarrow |B| = 0$ $\Big\}$ Let the st. matrix for $T_1 . T_2$ be C. Let the st. matrix for $T_2 . T_1$ be D

$|C| = |A.B| = |A|.\underbrace{|B|}_{0} = 0 \Rightarrow T_1 T_2$ is not one-to-one

$|D| = |B.A| = \underbrace{|B|}_{0}.|A| = 0 \Rightarrow T_2 . T_1$ is not one-to-one

Q31)

a) $V: R^2 \to (2x, x)$ $\}$ Test the axioms one by one

✓ 1- let $u = (2a, a)$ and $v = (2b, b) \Rightarrow u+v = (2(a+b), (a+b)) \in V$

✓ 2- let $u = (2a, a)$, $v = (2b, b)$ and $w = (2c, c)$

$$\left.\begin{aligned} u+(v+w) &= (2a,a) + (2(b+c),(b+c)) = (2(a+b+c),(a+b+c)) \\ (u+v)+w &= (2(a+b),(a+b)) + (2c,c) = (2(a+b+c),(a+b+c)) \end{aligned}\right\} \text{equal}$$

✓ 3- $u+v = (2(a+b), a+b)$, $v+u = (2(b+a), b+a)$ $\}$ equal

✓ 4- $u = (2a, a)$ and $0 = (0,0) \in V \Rightarrow u+0 = 0+u = (2a, a) = u$

✓ 5- $u = (2a, a)$ and $-u = (-2a, -a) \in V \Rightarrow u + (-u) = (0,0) = 0$

✓ 6- $u = (2a, a)$ and $ku = (2ak, ak) \in V$

✓ 7-
$$\left.\begin{aligned} &u+v = (2(a+b), a+b),\ k(u+v) = (2k(a+b), k(a+b)) \\ &k.u = (2ka, ka),\ kv = (2kb, kb),\ ku + kv = (2k(a+b), k(a+b)) \end{aligned}\right] \text{equal}$$

✓ 8- $(k+m)u = (2(k+m)a, (k+m)a)$, $ku = (2ka, ka)$, $mu = (2ma, ma)$

$ku + mu = (2(k+m)a, (k+m)a) = (k+m)u$

✓ 9-
$$\left.\begin{aligned} k(mu) &= k.(2ma, ma) = (2kma, kma) \\ m(ku) &= m.(2ka, ka) = (2mka, mka) \end{aligned}\right] \text{equal}$$

✓ 10- $1.u = (1.2a, 1.a) = (2a, a) = u$

! axioms are true for every $u \in V \Rightarrow V$ is a vector space

b) $V: R^2 \to (2x+1, x)$ $\}$ Test axioms one by one

X 1- $u = (2a+1, a)$, $v = (2b+1, b) \Rightarrow u+v = (2(a+b+1), (a+b)) \notin V$

! $2(a+b+1) = 2a+2b+2 \neq 2(a+b)+1$. Since the axiom is not true for V, it is not a vector space

c) $V: \mathbb{R}^2 \rightarrow (x,y),\ x \geq 0$ } Test axioms one by one

✓1- $u=(a,b)$, $v=(d,e)$ and $a \geq 0, d \geq 0$

$u+v = (a+d,\ b+e) \in V$

✓2- $u+v = (a+d,\ b+e)$ and $v+u = (d+a,\ e+b)$ } equal

✓3- $u=(a,b)$, $v=(d,e)$, $w=(f,g)$ and $a \geq 0,\ d \geq 0,\ f \geq 0$

$$\left.\begin{aligned} u+(v+w) &= (a,b)+(d+f,\ e+g) = (a+d+f,\ b+e+g) \\ (u+v)+w &= (a+d,\ b+e)+(f,g) = (a+d+f,\ b+e+g) \end{aligned}\right\} \text{equal}$$

✓4- $u=(a,b)$ and $0=(0,0) \in V$

$u+0 = 0+u = (a,b) = u$

X5- $u=(a,b)$, $-u=(-a,-b) \notin V$ (if $a \neq 0$)

∇ For any $a > 0$ the axiom is not met.

V is not a vector space

d) $V: \mathbb{R}^2 \rightarrow (1,y)$ } Test axioms one by one

X1- $u=(1,b)$ $v=(1,e)$

$u+v = (2,\ b+e) \notin V$

∇ Since $2 \neq 1$ the axiom is not met

V is not a vector space

Q32) $V: M_{2\times 2} \rightarrow \begin{bmatrix} x & 2x \\ 3x & 4x \end{bmatrix}$, $x \in \mathbb{R}$ } Test the axioms

✓1- $u = \begin{bmatrix} a & 2a \\ 3a & 4a \end{bmatrix}$, $v = \begin{bmatrix} b & 2b \\ 3b & 4b \end{bmatrix}$ and $u+v = \begin{bmatrix} (a+b) & 2(a+b) \\ 3(a+b) & 4(a+b) \end{bmatrix} \in V$

✓2- $u+v = \begin{bmatrix} (a+b) & 2(a+b) \\ 3(a+b) & 4(a+b) \end{bmatrix}$, $v+u = \begin{bmatrix} (b+a) & 2(b+a) \\ 3(b+a) & 4(b+a) \end{bmatrix}$ } equal

✓3- $u = \begin{bmatrix} a & 2a \\ 3a & 4a \end{bmatrix}$, $v = \begin{bmatrix} b & 2b \\ 3b & 4b \end{bmatrix}$, $w = \begin{bmatrix} c & 2c \\ 3c & 4c \end{bmatrix}$

$$\left.\begin{aligned} u+(v+w) &= \begin{bmatrix} a & 2a \\ 3a & 4a \end{bmatrix} + \begin{bmatrix} b+c & 2(b+c) \\ 3(b+c) & 4(b+c) \end{bmatrix} = \begin{bmatrix} a+b+c & 2(a+b+c) \\ 3(a+b+c) & 4(a+b+c) \end{bmatrix} \\ (u+v)+w &= \begin{bmatrix} a+b & 2(a+b) \\ 3(a+b) & 4(a+b) \end{bmatrix} + \begin{bmatrix} c & 2c \\ 3c & 4c \end{bmatrix} = \begin{bmatrix} a+b+c & 2(a+b+c) \\ 3(a+b+c) & 4(a+b+c) \end{bmatrix} \end{aligned}\right\} \text{equal}$$

✓ 4- $0=\begin{bmatrix}0&0\\0&0\end{bmatrix}\in$ and $u+0=0+u=\begin{bmatrix}a&2a\\3a&4a\end{bmatrix}+\begin{bmatrix}0&0\\0&0\end{bmatrix}=\begin{bmatrix}a&2a\\3a&4a\end{bmatrix}=u$

✓ 5- $u=\begin{bmatrix}a&2a\\3a&4a\end{bmatrix}$ and $-u=\begin{bmatrix}-a&-2a\\-3a&-4a\end{bmatrix}\in V$ $\quad u+(-u)=\begin{bmatrix}0&0\\0&0\end{bmatrix}=0$

✓ 6- $k.u=\begin{bmatrix}ka&2ka\\3ka&4ka\end{bmatrix}\in V$

✓ 7- $\left.\begin{array}{l} k.(u+v)=k.\begin{bmatrix}a+b&2(a+b)\\3(a+b)&4(a+b)\end{bmatrix}=\begin{bmatrix}k(a+b)&2k(a+b)\\3k(a+b)&4k(a+b)\end{bmatrix} \\ ku+kv=\begin{bmatrix}ka&2ka\\3ka&4ka\end{bmatrix}+\begin{bmatrix}kb&2kb\\3kb&4kb\end{bmatrix}=\begin{bmatrix}k(a+b)&2k(a+b)\\3k(a+b)&4k(a+b)\end{bmatrix}\end{array}\right\}$ equal

✓ 8- $\left.\begin{array}{l} (k+m).u=(k+m).\begin{bmatrix}a&2a\\3a&4a\end{bmatrix}=\begin{bmatrix}(k+m)a&2(k+m)a\\3(k+m)a&4(k+m)a\end{bmatrix} \\ ku+mu=\begin{bmatrix}ka&2ka\\3ka&4ka\end{bmatrix}+\begin{bmatrix}ma&2ma\\3ma&4ma\end{bmatrix}=\begin{bmatrix}(k+m)a&2(k+m)a\\3(k+m)a&4(k+m)a\end{bmatrix}\end{array}\right\}$ equal

✓ 9- $\left.\begin{array}{l} k(mu)=k.\begin{bmatrix}ma&2ma\\3ma&4ma\end{bmatrix}=\begin{bmatrix}kma&2kma\\3kma&4kma\end{bmatrix} \\ (km).(u)=\begin{bmatrix}kma&2kma\\3kma&4kma\end{bmatrix}\end{array}\right\}$ equal

✓ 10- $1.u=1.\begin{bmatrix}a&2a\\3a&4a\end{bmatrix}=\begin{bmatrix}a&2a\\3a&4a\end{bmatrix}=u$

$\therefore$ All axioms are met for every $u\in V$

V is a vector space

Q33) Test the axioms:

✓ 1- $u=a$, $v=b$ and $a,b\in R \Rightarrow u+v=a+b\in V$

✓ 2- $u+v=a+b$, $v+u=b+a \Rightarrow u+v=v+u$

✓ 3- $u=a, v=b, w=c \Rightarrow \left.\begin{array}{l} u+(v+w)=a+(b+c)=a+b+c \\ (u+v)+w=(a+b)+c=a+b+c\end{array}\right\}$ equal

✓ 4- $\vec{0}=0\in V$ and $u+0=a+0=a=u$

✓ 5- $u=a$ $\quad -u=-a\in V \Rightarrow u+(-u)=a-a=0\in V$

X 6- $k.u=\sqrt{k}.a \Rightarrow$ if $k<0$ the expression becomes undefined in R

Ex: $-1.u=\sqrt{-1}.a=?$

$\therefore$ Since all axioms are not met the set is not a vector space

Q34) a) $W = M_{n\times n}$, $|M| = 0$

let $u = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & \cdots & 0 \end{bmatrix}_{n\times n}$ and $v = \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ \vdots & 1 & & \vdots \\ & & \ddots & \\ 0 & \cdots & \cdots & 1 \end{bmatrix}_{n\times n}$

$|u| = 1.0 \ldots 0 = 0$ and $|v| = 0.1 \ldots 1 = 0$

$u+v = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & \cdots & 1 \end{bmatrix}_{n\times n}$ and $|u+v| = 1 \ldots 1 = 1 \neq 0$.

Since $u+v \notin W$, W is not a subspace

b) $W = M_{nn}$, $|M| \neq 0$

let $u = \begin{bmatrix} u_{11} & \cdots & u_{1n} \\ \vdots & \ddots & \vdots \\ u_{n1} & \cdots & u_{nn} \end{bmatrix}$ and $|u| \neq 0$

let $k = 0$

$k.u = 0. \begin{bmatrix} u_{11} & \cdots & u_{1n} \\ \vdots & \ddots & \vdots \\ u_{n1} & \cdots & u_{nn} \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} = \vec{0}$

$|k.u| = 0.\ 0 \ldots 0.0 = 0$

Since $|k.u| \notin W$, W is not a subspace

Q35)

a) $W = M_{n\times m}$, $a_{11} = a_{12} = \ldots = a_{1m} = 0$

let $u = \begin{bmatrix} 0 & \cdots & 0 \\ u_{21} & \cdots & u_{2n} \\ \vdots & \ddots & \vdots \\ u_{n1} & \cdots & u_{nm} \end{bmatrix}$ and $v = \begin{bmatrix} 0 & \cdots & 0 \\ v_{21} & \cdots & v_{2n} \\ \vdots & \ddots & \vdots \\ v_{n1} & \cdots & v_{nm} \end{bmatrix}$

Since W is closed under addition and mult, W is a subspace

$u+v = \begin{bmatrix} 0 & \cdots & 0 \\ u_{21}+v_{21} & \cdots & u_{2n}+v_{2n} \\ \vdots & \ddots & \vdots \\ u_{n1}+v_{n1} & \cdots & u_{nm}+v_{nm} \end{bmatrix} \in W$

$k.u = k. \begin{bmatrix} 0 & \cdots & 0 \\ u_{21} & \cdots & u_{2n} \\ \vdots & & \vdots \\ u_{n1} & \cdots & u_{nm} \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 \\ k.u_{21} & \cdots & k.u_{2n} \\ \vdots & \ddots & \vdots \\ k.u_{n1} & \cdots & k.u_{nm} \end{bmatrix} \in W$

Q36)

2x3 3x1

a) $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 4 \\ 1 & -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 2 & 1 & 4 & 0 \\ 1 & -1 & 3 & 0 \end{array}\right] \xrightarrow[E_{1,3}(-1)]{E_{1,2}(-2)} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & -1 & 5 & 0 \end{array}\right] \xrightarrow{E_{2,3}(1)} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 13 & 0 \end{array}\right]$$

$$E_3\left(\tfrac{1}{13}\right) \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \rightarrow \begin{matrix} E_{3,2}(-8) \\ E_{3,1}(2) \end{matrix} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \left.\begin{matrix} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{matrix}\right\}$$

Solution space for the given part is $W_{(x_1,x_2,x_3)}: x_1 = x_2 = x_3 = 0$

$W = \langle (0,0,0) \rangle$

b) $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 2 & 1 & 4 & 0 \end{array}\right] \xrightarrow{E_{1,2}(-2)} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 8 & 0 \end{array}\right]$$

$$\left.\begin{matrix} x_1 - 2x_3 = 0 \Rightarrow x_1 = 2x_3 \\ x_2 + 8x_3 = 0 \Rightarrow x_2 = -8x_3 \end{matrix}\right\} \text{let } x_3 = t \Rightarrow \begin{matrix} x_1 = 2t \\ x_2 = -8t \\ x_3 = t \end{matrix} \quad \text{Infinite solutions for } t$$

Solution space for the given part is $W(x_1,x_2,x_3): x_1 = 2t, x_2 = -8t, x_3 = t, t \in \mathbb{R}$

Ex: for $t=1$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \\ 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 4 \\ 0 & 1 & 8 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 2 & 1 & 4 & 0 \\ 0 & 1 & 8 & 0 \end{array}\right] \xrightarrow{E_{1,2}(-2)} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 1 & 8 & 0 \end{array}\right] \xrightarrow{E_{2,3}(-1)} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left.\begin{matrix} x_1 - 2x_3 = 0 \Rightarrow x_1 = 2x_3 \\ x_2 + 8x_3 = 0 \Rightarrow x_2 = -8x_3 \end{matrix}\right\} \text{let } x_3 = t \Rightarrow \begin{matrix} x_1 = 2t \\ x_2 = -8t \\ x_3 = t \end{matrix} \quad \text{Infinite solution for } t$$

Solution space for the given part is $W(x_1,x_2,x_3): x_1 = 2t, x_2 = -8t, x_3 = t, t \in \mathbb{R}$

Ex: for $t=2$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -16 \\ 2 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 0 & -4 \\ -1 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 2 & 0 & -4 & 0 \\ -1 & 0 & 2 & 0 \end{array}\right] \xrightarrow[E_{1,3}(1)]{E_{1,2}(-2)} \left.\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]\right\} : \text{Infinite solutions}$$

$x_1 - 2x_3 = 0 \Rightarrow x_1 = 2x_3$

The solution space for the given part is $W_{(x_1,x_2,x_3)} : x_1 = 2x_3$

Ex: for $x_1 = 1$ and $x_3 = 2$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$