Preference-Based Solution Selection Algorithm for Evolutionary Multiobjective Optimization

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***Abstract*—Since multiobjective evolutionary algorithms (MOEAs) provide a set of nondominated solutions, decision making of selecting a preferred one out of them is required in real applications. However, there has been some research on MOEA in which the user's preferences are incorporated for this purpose. This paper proposes preference-based solution selection algorithm (PSSA) by which user can select a preferred one out of nondominated solutions obtained by any one of MOEAs. The PSSA, which is a kind of multiple criteria decision making (MCDM) algorithm, represents user's preference to multiple objectives or criteria as a degree of consideration by fuzzy measure and globally evaluates obtained solutions by fuzzy integral. The PSSA is also employed in each and every generation of evolutionary process to propose multiobjective quantum-inspired evolutionary algorithm with preference-based selection (MQEA-PS). To demonstrate the effectiveness of PSSA and MQEA-PS, computer simulations and real experiments on evolutionary multiobjective optimization for the fuzzy path planner of mobile robot are carried out. Computer simulation and experiment results show that the user's preference is properly reflected in the selected solution. Moreover, MQEA-PS shows improved performance for the DTLZ problems and fuzzy path planner optimization problem compared to MQEA with dominance-based selection and other MOEAs like NSGA-II and MOPBIL.**

***Index Terms*—Fuzzy integral, fuzzy path planning, multiobjective quantum-inspired evolutionary algorithm, multiple criteria decision making (MCDM), preference-based MOEA.**

I. Introduction

In most practical applications, there are multiple objectives to be optimized at the same time, which are known as multiobjective optimization problems (MOPs). For example, robotics research has many MOPs such as problems on path planning, path following, gait generation, and footstep planning. In order to solve the MOPs in robotics research, multiobjective evolutionary algorithms (MOEAs) were developed [1]–[3]. In addition to robotics, MOEAs have been devised to provide high quality solutions to the MOPs of other various engineering research fields, such as aircraft, bioinformatics, system design, and power system.

The strength Pareto evolutionary algorithm2 (SPEA2) was developed by employing a refined fitness assignment and an enhanced archive truncation technique [4]. Also, the nondominated sorting genetic algorithm-II (NSGA-II) was proposed, which is a strong elitist method with a mechanism to maintain diversity efficiently using nondominated sorting and crowding distance assignment [5]. Motivated by population-based incremental learning (PBIL) [6], multiobjective PBIL (MOPBIL) algorithm was proposed. It provides a wider search space by randomly selecting nondominated solutions in archive when each element in probability vector gets updated [2]. Multiobjective quantum-inspired evolutionary algorithm (MQEA) was proposed to improve proximity to the Pareto-optimal set, while preserving diversity [7], [8]. The quality of solutions was improved by probabilistic representation based on the concepts of quantum computing, since it allowed a good balance between global and local searches [9], [10]. Also, maintenance of the global archive provided the balance of evolution among subpopulations. Research on quantum-inspired evolutionary algorithm (QEA) for MOPs has included from NP-hard problems to real-world applications [11], [12].

Recently, many-objective optimization problem which deals with more than three objectives has been an issue. Since the number of nondominated solutions increases exponentially when the number of objectives increases, existing dominance-based MOEAs are less effective in many-objective problems [13]. There are several approaches in the evolutionary multiobjective optimization (EMO), which attempt to solve this issue by using the user's preference [14]–[22].

As one of the MOEAs using preference-based approach, Fonseca and Fleming [14] proposed a decision making framework based on goals and priorities to represent user's preferability. Tan *et al.* [15] proposed a goal-sequence domination scheme to allow advanced hard and soft priorities for better decision of user in multiobjective optimization. Molina *et al.* [16] proposed a variation of the concept of Pareto dominance, called g-dominance, which is based on the information included in a reference point and designed to be used with any multiobjective evolutionary method or any multiobjective metaheuristic. Zitzler *et al.* [17], [18] proposed a quality measure design based on the hypervolume measure

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to capture different preferences of decision maker and also proposed a set preference algorithm for multiobjective optimization. Branke and Deb [19] proposed a guided MOEA (G-MOEA), where the definition of dominance was modified based on the user's preference. Deb *et al.* [20]–[22] proposed reference point-based, reference direction-based, and light beam search-based EMOs. These methods have focused on finding a set of preferred solutions or a preferred region during evolutionary process.

In the previous research, a single preferred solution was selected by an aggregation method such as weighted sum method, epsilon, hypervolume, and Tchebycheff measure [23], [24]. Considering the multiobjectives in the problem, following multiple criteria decision making (MCDM) methods could be also used. ELECTRE method, proposed by Roy, models preferences by using binary outranking relations [25]. There are two distinct sets of parameters, the importance coefficients which are the same as weights and the veto thresholds which express the power attributed to a given criterion. It aggregates preference relations on pair of alternatives. Multiattribute utility theory (MAUT), proposed by Edwards, aggregates unidimensional utility functions into a single global utility function combining all the criteria using a suitable aggregation rule and weighting procedure [26]. Analytic hierarchy process (AHP), proposed by Saaty, is a theory of measurement through pairwise comparisons and relies on the judgments of experts to derive priority scales [27]. It assists the decision maker to describe the general decision operation by decomposing a complex problem into a multilevel hierarchy structure of criteria, subcriteria, and alternatives. PROMETHEE method, proposed by Brans, is an outranking method in multicriteria analysis [28]. It has two ways of treatment, either a partial preorder (PROMETHEE I) or a complete one (PROMETHEE II). A visual interactive method for solving the multicriteria problem, proposed by Korhonen, does not rely on explicit knowledge of the utility function [29]. It employs reference goals or aspiration levels for reflecting a decision maker's preference. Also, there are several aggregation tools for multicriteria. Ordered weighted averaging operators, introduced by Yager, provided an aggregation which lies in between the cases of requiring all the criteria to be satisfied and requiring at least one of the criteria to be satisfied [30]. The desirability function, introduced by Harrington, is a method for multicriteria optimization in industrial quality management [31]. It is based on the quality of alternatives and the alternative which is outside of some desired limits is completely unacceptable.

However, these aggregation methods do not effectively represent the interactions among criteria. Also, most of the MCDM methods, such as ELECTRE method, MAUT, AHP, and PROMETHEE method, have been studied much more on the modeling of what could be called the choice procedure, rather than on the aggregation step. On the contrary, since fuzzy measure and fuzzy integral [32] effectively represent the interactions among criteria or objectives compared to other methods [25]–[31], it is a suitable aggregation method for MCDM of MOEAs. It has been applied to the evaluation of sustainable-shing development strategies, the sensor network problem, and others [33], [34].

In this paper, preference-based solution selection algorithm (PSSA) is proposed to select a preferred one out of nondominated solutions obtained by any one of MOEAs. It is based on both of partial evaluation and global evaluation for each solution. The former is obtained by normalizing the objective function value of the solution to 1.0. For the latter, user's preference for the objectives or criteria is represented by a degree of consideration using fuzzy measure. Global evaluation of each solution is calculated by fuzzy integral of the partial evaluation with respect to the user's preference. This paper also proposes MQEA with preference-based selection (MQEA-PS) by employing PSSA in MQEA in each and every generation of evolutionary process. Considering that dominance-based sorting is not effective for many-objective problems, in this algorithm, global population is sorted by preference-based sorting, while subpopulation is sorted by fast nondominated sorting. Since this algorithm can lay emphasis on specific objectives by the degree of consideration, it can select preferred solutions considering the preference for the specific ones in many-objective optimization problems. To demonstrate the effectiveness of the proposed PSSA and MQEA-PS, computer simulations and real experiments on evolutionary multiobjective optimization for the fuzzy path planner for a shooting behavior of soccer robot [35] are carried out. Moreover, MQEA-PS shows improved performance for the DTLZ problems, which are test problems for MOEA using bottom-up approach and constraint surface approach [36], and fuzzy path planner optimization problem compared to MQEA with dominance-based selection and other MOEAs such as NSGA-II and MOPBIL.

This paper is organized as follows. Section II describes fuzzy measure and fuzzy integral and proposes PSSA. Section III briefly introduces QEA and proposes MQEA-PS. In Section IV, a problem definition of multiobjective approach for the fuzzy path planner for the shooting behavior of soccer robot is described and the overall structure of the proposed scheme is presented. Also, both simulation and experiment results are discussed. Finally, concluding remarks follow in Section V.

II. PSSA

In the process of selecting a preferred one out of nondominated solutions, it is required to have a global evaluation for each one considering both of partial evaluation over objectives and user's degree of consideration for objectives. The one with the highest value of global evaluation is selected as a preferred solution. In this paper, fuzzy measure is employed to represent user's degree of consideration and global evaluation is calculated by fuzzy integral. The fuzzy measure and fuzzy integral are briefly described in the following and then detailed description of the proposed PSSA follows.

A. Fuzzy Measure and Fuzzy Integral

Fuzzy measure on the power set of X, denoted $P(X)$, in the finite space $X = \{x_1, \cdots, x_n\}$ is defined as follows.

Definition 1: A fuzzy measure g defined on $(X, P(X))$ is a set function $g : P(X) \rightarrow [0, 1]$ satisfying the following axioms.

1) Boundary condition

$$g(\emptyset) = 0, \quad g(X) = 1. \tag{1}$$

2) Monotonicity

$$\forall A, B \subseteq P(X), \;\; if \;\; A \subseteq B, \;\; then \;\; g(A) \leq g(B). \tag{2}$$

Fuzzy measures are classified as belief measure, plausibility measure, probability measure, and others. Belief measure, *Bel* is a set function, $Bel : P(X) \rightarrow [0, 1]$, satisfying the additional axiom as follows:

$$Bel(A_1 \cup A_2 \cup \cdots \cup A_n) \geq \sum_i Bel(A_i) - \sum_{i>j} Bel(A_i \cap A_j) + \cdots + (-1)^{n+1} Bel(A_1 \cap A_2 \cap \cdots \cap A_n). \tag{3}$$

Since $Bel(A \cup \overline{A}) = 1$ and $Bel(A \cap \overline{A}) = 0$, $Bel(A) + Bel(\overline{A}) \leq 1$. In other words, the sum of all the belief measures is less than or equal to 1. Plausibility measure, *Pl* is a set function, $Pl : P(X) \rightarrow [0, 1]$, satisfying the additional axiom as follows:

$$Pl(A_1 \cap A_2 \cap \cdots \cap A_n) \leq \sum_i Pl(A_i) - \sum_{i>j} Pl(A_i \cup A_j) + \cdots + (-1)^{n+1} Pl(A_1 \cup A_2 \cup \cdots \cup A_n). \tag{4}$$

Since $Pl(A \cup \overline{A}) = 1$ and $Pl(A \cap \overline{A}) = 0$, $Pl(A) + Pl(\overline{A}) \geq 1$. It means that the sum of all the plausibility measures is greater than or equal to 1. Probability measure can also be defined as a special case of either belief measure or plausibility measure, which satisfies an additional axiom on additivity property.

Note that belief and plausibility measures are mutually dual and can be derived from one another, such as $Pl(A) = 1 - Bel(\overline{A})$. Belief measure indicates one's confidence of making a decision with certainty. Plausibility measure, on the contrary, represents one's confidence considering all the plausible cases in making a decision. Therefore, $Bel(A)$ is always less than or equal to $Pl(A)$.

As a general representation of fuzzy measure, λ-fuzzy measure, $g : P(X) \rightarrow [0, 1]$, is defined, which additionally satisfies the axiom [32] as follows:

$$\forall A_{i,j} \in P(X), \; i, j = 1, \cdots, n, \; A_i \cap A_j = \emptyset \; and \; -1 < \lambda$$
$$g(A_i \cup A_j) = g(A_i) + g(A_j) + \lambda g(A_i) g(A_j) \tag{5}$$

where λ represents a degree of interaction between A_i and A_j. λ-fuzzy measure is considered as belief measure, plausibility measure, or probability measure depending on the value of λ. If $\lambda > 0$, $\lambda < 0$, and $\lambda = 0$, they are considered respectively as belief measure, plausibility measure, and probability measure.

Note that each kind of fuzzy measures indicates a different interaction between criteria [37]. Belief measure indicates a positive interaction, since $g(A_i \cup A_j) > g(A_i) + g(A_j)$. On the contrary, plausibility measure indicates a negative interaction, since $g(A_i \cup A_j) < g(A_i) + g(A_j)$. Probability measure does not represent any interactions among criteria, since it is the same as conventional weighted sum which satisfies the additivity.

For global evaluation of each solution over criteria with respect to a degree of consideration for each of criterion, either Sugeno fuzzy integral or Choquet fuzzy integral can be used, which are defined in the following.

Definition 2: Let $h : X \rightarrow [0, 1]$, where X can be any set. The Sugeno fuzzy integral of evaluated value, h over a subset of $X \in P(X)$ with respect to a fuzzy measure g is defined as follows:

$$\int_X h \circ g = \max_i \; min[h(x_i), g(E_i)] \tag{6}$$

where $h(x_1) \leq h(x_2) \leq \cdots \leq h(x_n)$ and $E_i = \{x_i, x_{i+1}, \cdots, x_n\}$ for $x_i \in X$ and $i = 1, \cdots, n$.

Definition 3: Let $h : X \rightarrow [0, 1]$, where X can be any set. The Choquet fuzzy integral of evaluated value, h over a subset of $X \in P(X)$ with respect to a fuzzy measure g is defined as follows:

$$\int_X h \circ g = \sum_{i=1}^{n} (h(x_i) - h(x_{i-1})) g(E_i) \tag{7}$$

where $h(x_1) \leq h(x_2) \leq \cdots \leq h(x_n)$, $E_i = \{x_i, x_{i+1}, \cdots, x_n\}$ and $h(x_0) = 0$, for $x_i \in X$ and $i = 1, \cdots, n$.

Note that x_i, $i = 1, \cdots, n$, denotes ith criterion which corresponds to ith objective in MOP and then $h(x_i)$ is the partial evaluation value over x_i. The fuzzy measure g represents the degree of consideration for each objective. Thus, fuzzy integral can be used for global evaluation of each solution.

B. Proposed Algorithm Using Fuzzy Integral

The preferred solution can be selected using classical MCDM methods [25]–[29] and aggregation methods [30], [31]. However, there are some problems for using these methods to MOEA. First, since the aggregation methods which are used in these MCDM methods are actually the same as conventional weighted method, they perform well only when all criteria are independent from each other. The problem is, however, they are dependent on each other in most cases of real-world applications. For example, when people want to buy a house, they select one based on multiple criteria such as price, size, location, and others. It is quite certain that price (c_1) and size (c_2) of house are dependent on each other. Even if c_1 and c_2 are independent, the degrees of consideration for them, represented by fuzzy measure, may not satisfy additivity property, i.e., $g(\{c_1, c_2\}) \neq g(\{c_1\}) + g(\{c_2\})$, where $g(\{c_i\})$, $i = 1, 2$, is the degree of consideration for criterion c_i. Second, the existing aggregation methods do not properly represent the interactions among criteria. As mentioned above, since criteria are dependent on each other in most real-world applications, there are some interactions among them. For more accurate decision-making in multicriteria problems, the aggregation method should represent the interactions among criteria. Last, in some of the existing methods, it is difficult to set the degree of consideration or preference to each criterion. For example, in Tchebycheff measure, it is difficult to set proper weights, since each weight has a boundary which is changed in every generation and the sum of weights has to be one.

Because of these reasons, the conventional methods are not appropriate to select a preferred one out of candidate solutions. On the contrary, fuzzy integral requires neither criteria (or objectives in MOP) to be independent nor fuzzy measure to be additive for any subset in power set of criteria because it can effectively represent the interactions, i.e., positive interactions

Procedure PSSA

$C = \{c_1, c_2, \cdots, c_n\}$: a set of criteria
n: the number of criteria
$P(C)$: a power set of C
$h_k(c_i)$: partial evaluation value of k-th solution, $k = 1, \cdots, m$, over c_i
m: the number of solutions
e_k: global evaluation value of k-th solution

i) Define a set of objectives in MOP as C.
ii) Calculate λ-fuzzy measures g's of $P(C)$.
  1) Make a pairwise comparison matrix, P.
  2) Calculate normalized weights of c_i, $\forall i$.
  3) Calculate λ-fuzzy measures of $P(C)$.
iii) Normalize the solutions to get the partial evaluation value $h_k(c_i)$, $\forall i, k$.
iv) Calculate the global evaluation value e_k, $\forall k$ using fuzzy integral for partial evaluation value and λ-fuzzy measures.
v) Sort and select the one with the highest global evaluation value as the preferred solution.

Fig. 1. Procedure of PSSA using fuzzy integral.

and negative interactions among criteria. Also, it is easy to set a preference for each criterion. It just needs to set the comparative preference between two criteria and to decide either plausibility measure or belief measure to be used. Thus, considering general MOPs, it is a more suitable method for selecting the preferred solution compared to other existing methods. Overall selection algorithm using λ-fuzzy measure and fuzzy integral is summarized in Fig. 1 and each step is described in the following.

1) *Define objectives in MOP as criteria:* MOPs have predefined objectives to be optimized simultaneously. Partial evaluation over each objective is conducted for each candidate solution, which corresponds to calculating its normalized objective function value. The preferred solution is selected considering both the partial evaluation and user's degree of consideration for each objective. Thus, the objectives of MOPs can be defined as the criteria of fuzzy integral for global evaluation.
2) *Calculation of fuzzy measure:* In this paper, λ-fuzzy measure is used to represent the degree of consideration for each criterion. According to (1) and (5), λ-fuzzy measure has to satisfy as follows:

$$\begin{aligned} g(C) &= g(\{c_1, c_2, \cdots, c_n\}) \\ &= g(\{c_1, \cdots, c_{n-1}\}) + g_n + \lambda g(\{c_1, c_2, \cdots, c_{n-1}\})g_n \\ &\qquad \vdots \\ &= (g_1+g_2+\cdots+g_n) + \lambda(g_1g_2 + g_1g_3 + \cdots + g_{n-1}g_n) \\ &\quad +\lambda^2(g_1g_2g_3 + g_1g_2g_4 + \cdots + g_{n-2}g_{n-1}g_n)+\cdots+ \\ &\quad \lambda^{n-1}(g_1g_2\cdots g_n) \\ &= 1 \end{aligned} \tag{8}$$

where C is a set of criteria, $\{c_1, c_2, \cdots, c_n\}$ and for notational simplicity $g_i = g(\{c_i\})$. Since (8) is an (n-1)th order equation of λ, it is quite difficult to solve the equation for λ given g_is, if the number of criteria is more than three. Thus, the following procedure is employed to calculate the fuzzy measures [38].

a) *Make a pairwise comparison matrix:* The pairwise comparison matrix of criteria, P, which represents preference degrees between criteria, is defined [27] as follows:

$$\begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix} \tag{9}$$

where p_{ij} represents the preference degree between ith criterion, c_i and jth criterion, c_j, p_{ii} is 1 and $p_{ji} = 1/p_{ij}$. For example, if p_{12} is 5, it means c_1 is five times more preferred to c_2.

b) *Calculate normalized weight:* The normalized weight, w_i of ith criterion, c_i, $i, j = 1, \cdots, n$ is calculated as follows:

$$w_i = \frac{\sum_{j=1}^{n} p_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij}}. \tag{10}$$

There are some other methods to derive priority vectors, like normalized weight, from pairwise comparison matrix [39]. Any one of them can be used in this step.

c) *Calculate λ-fuzzy measures:* $\phi_{\lambda+1}$ transformation is employed to calculate λ-fuzzy measures [38]. The transformation, $\phi_{\lambda+1} : [0, 1] \times [0, 1] \to [0, 1]$ is defined as follows:

$$\phi_{\lambda+1}(\xi, w_i) = \begin{cases} 1, & \text{if } \xi = 1 \text{ and } w_i > 0 \\ 0, & \text{if } \xi = 1 \text{ and } w_i = 0 \\ 1, & \text{if } \xi = 0 \text{ and } w_i = 1 \\ 0, & \text{if } \xi = 0 \text{ and } w_i < 1 \\ w_i, & \text{if } \xi = 0.5 \\ \frac{(\lambda+1)^{w_i}-1}{\lambda}, & \text{other cases} \end{cases} \tag{11}$$

where ξ is another interaction degree of which value lies in [0, 1]. Then, λ is determined by ξ, where $\lambda = \frac{(1-\xi)^2}{\xi^2} - 1$. It means $\xi \in (0, 1)$ has one to one correspondence with $\lambda \in (-1, \infty)$. Using (11), λ-fuzzy measure of each element of $P(C)$, $g(A)$ is calculated as follows:

$$g(A) = \phi_{\lambda+1}\left(\xi, \sum_{c_i \in A} w_i\right), \quad \forall A \in P(C) \tag{12}$$

where A is an element of $P(C)$.

3) *Partial evaluation of solutions:* The function, h in (6) and (7) is a normalized objective function which represents partial evaluation of each solution over each criterion. The objective function values need to be normalized to 1, since h is defined from 0 to 1. This step calculates $h_k(c_i)$ of kth solution over c_i.

TABLE I
EXAMPLE OF VALUES OF FUZZY MEASURES AND CONVENTIONAL WEIGHTS

$P(C)$	Plausibility Measure ($\xi = 0.75$)	Belief Measure ($\xi = 0.25$)	Weight Method
$\{c_1\}$	0.917	0.553	0.769
$\{c_2\}$	0.323	0.050	0.154
$\{c_3\}$	0.175	0.023	0.077
$\{c_1, c_2\}$	0.977	0.825	0.923
$\{c_2, c_3\}$	0.447	0.083	0.231
$\{c_1, c_3\}$	0.950	0.677	0.846
$\{c_1, c_2, c_3\}$	1	1	1

4) *Global evaluation of solutions:* The global evaluation value of each candidate solution is calculated by fuzzy integral using (6) or (7). g and h_k are λ-fuzzy measure and partial evaluation value obtained at Steps 2) and 3), respectively. It means the global evaluation value is calculated by considering both user's degree of consideration for each criterion and partial evaluation of the candidate solution.
5) *Select the preferred solution:* The preferred one is selected out of candidate solutions based on their global evaluation value. The one with the highest global evaluation value is selected as the preferred solution. If user wants to select more than one solution, one may sort the solutions in descending order by their global evaluation value and then select preferred ones accordingly.

C. Comparison of Fuzzy Measure and Other MCDM Methods

As already mentioned above, the distinguishing characteristic of fuzzy measure and fuzzy integral compared to other methods is that it is able to represent interactions among criteria. Belief measure represents the positive interaction, i.e., "and" like evaluation and plausibility measure represents the negative interaction, i.e., "or" like evaluation. To show different behavior and benefit of the fuzzy measure and fuzzy integral, interaction index is employed [37]. The interaction index of power set A of criteria with respect to fuzzy measure g is defined as follows:

$$I(g, A) = \sum_{T \subseteq C \setminus A} \frac{(n - |T| - |A|)!|T|!}{(n - |A| + 1)!} (\triangle_A g)(T) \tag{13}$$

$$(\triangle_A g)(T) = \sum_{L \subseteq A} (-1)^{|A|-|L|} g(L \cup T) \tag{14}$$

where A is a power set of C with $|A| \geq 2$. Table I shows fuzzy measure values and weight values of the conventional weight method. Table II shows the interaction indices when the number of criteria is three and P is set as follows:

$$P = \begin{bmatrix} 1 & 5 & 10 \\ 0.2 & 1 & 2 \\ 0.1 & 0.5 & 1 \end{bmatrix}. \tag{15}$$

As shown in Table I, fuzzy measure does not need to satisfy additivity condition unlike the conventional weight method. As shown in Table II, the interaction indices of cases using plausibility and belief measures are negative and positive, respectively. On the contrary, the interaction indices of case using the conventional weight method are all zeros, since it does not consider any interactions among criteria as if they are independent from each other. Therefore, fuzzy measure and fuzzy integral is a more proper method to aggregate multicriteria than the conventional weight method. It has the same advantage over other aggregation methods, such as epsilon measure, hypervolume measure, Tchebycheff measure, and desirability function, since these methods can represent only either user's preference or desirability level, but not the interactions among criteria.

TABLE II
COMPARISON OF INTERACTION INDICES OF PLAUSIBILITY MEASURE, BELIEF MEASURE, AND CONVENTIONAL WEIGHT

Measure	c_1,c_2	c_2,c_3	c_1,c_3
Plausibility ($\xi = 0.75$)	−0.243	−0.030	−0.122
Belief ($\xi = 0.25$)	0.243	0.030	0.122
Weight method	0.000	0.000	0.000

III. MQEA-PS

This section briefly summarizes QEA and proposes MQEA-PS by employing PSSA into MQEA in each and every generation.

A. QEA

Building block of classical digital computer is represented by two binary states, "0" or "1," which is a finite set of discrete and stable state. In contrast, QEA utilizes a novel representation, called Q-bit, for the probabilistic representation based on the concept of qubits in quantum computing [40]. Q-bit is defined as the smallest unit of information in QEA, which is defined with a pair of number, (α, β), as follows:

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{16}$$

where $|\alpha|^2 + |\beta|^2 = 1$. $|\alpha|^2$ gives the probability that the Q-bit will be found in the "0" state and $|\beta|^2$ gives the probability that the Q-bit will be found in the "1" state. It can be illustrated as a unit vector in 2-D space which can be changed by rotation as shown in Fig. 2, where $|0\rangle$ and $|1\rangle$ mean the "0" state and the "1" state, respectively, and $\Delta\theta$ is the rotation angle from $q = \alpha|0\rangle + \beta|1\rangle$ to $q' = \alpha'|0\rangle + \beta'|1\rangle$. Q-bit individual is defined as a string of Q-bits as follows:

$$\mathbf{q}_j^t = \left[\begin{array}{c|c|c|c} \alpha_{j1}^t & \alpha_{j2}^t & \cdots & \alpha_{jm}^t \\ \beta_{j1}^t & \beta_{j2}^t & \cdots & \beta_{jm}^t \end{array} \right] \tag{17}$$

where m is the number of Q-bits, i.e., the string length of the Q-bit individual, and $j = 1, 2, \ldots, n$ for population size n. The population of Q-bit individuals at generation t is represented as $Q(t) = \{\mathbf{q}_1^t, \mathbf{q}_2^t, \ldots, \mathbf{q}_n^t\}$.

Since the Q-bit individual represents the linear superposition of all possible states probabilistically, diverse individuals are generated during the evolutionary process. The procedure of QEA and the overall structure for SOPs are described in [9] and [10].

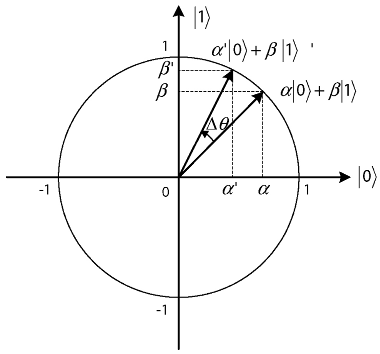


Fig. 2. Representation and rotation of Q-bit in 2-D space.

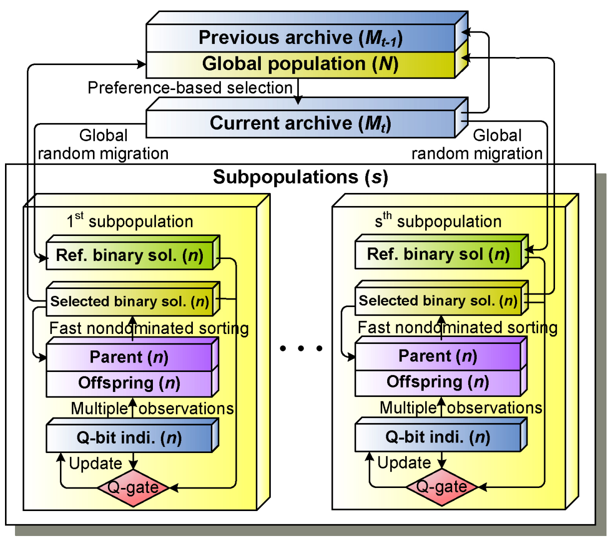


Fig. 3. Overall structure of MQEA-PS.

Procedure MQEA-PS

Begin

$t \leftarrow 0$

i) Initialize Q-bit individuals

ii) Make binary solutions of offspring by observing the states of Q-bit individuals and evaluate them

iii) Store all binary solutions into global population and each parent

iv) Store nondominated solutions in the global population to current archive

v) **While** (**not** termination condition) **do**

Begin

$t \leftarrow t+1$

vi) Make binary solutions of offspring by observing the states of Q-bit individuals and evaluate them

vii) Run the fast nondominated sorting and crowding distance assignment for parent and offspring

viii) Select first n binary solutions and sort their corresponding Q-bit individuals

ix) Store the n selected binary solutions of every subpopulation into global population

x) Sort the solutions in the previous archive and global population based on user's preference, and form the current archive by top M_t solutions among them

xii) Migrate randomly selected solutions in the current archive to every reference binary solution

xiii) Update each Q-bit individual using Q-gates referring to the corresponding reference binary solution

End

End

Fig. 4. Procedure of MQEA-PS.

B. Overall Procedure of the Proposed MQEA-PS

This section describes overall structure and procedure of the proposed MQEA-PS. MQEA-PS is designed by incorporating the proposed PSSA and QEA with the NSGA-II framework [5]. Fig. 3 illustrates overall structure of MQEA-PS. Global population and archive maintain the elitism and each subpopulation evolves concurrently, where global population size, N, is equal to $n \cdot s$, where n is the subpopulation size and s is the number of subpopulations, and current archive size, M_t is equal to or smaller than N. Fig. 4 shows the whole procedure of MQEA-PS, where t is the generation number. jth Q-bit individual, $\mathbf{q}_j^t$ is defined as a string of Q-bits as follows:

$$\mathbf{q}_j^t = \left[\begin{array}{c|c|c|c} \alpha_{j1}^t & \alpha_{j2}^t & \cdots & \alpha_{jl}^t \\ \beta_{j1}^t & \beta_{j2}^t & \cdots & \beta_{jl}^t \end{array} \right] \tag{18}$$

where l is the string length of the Q-bit individual.

The following describes each and every step of the algorithm.

1) This step initializes each Q-bit individual with the same value of $1/\sqrt{2}$. It represents the linear superposition of all possible states with the same probability.
2) One binary solution is formed to reproduce offspring by selecting either 0 or 1 for each bit using the initial probability of Q-bit. Then, each binary solution is evaluated for its objective function values.
3) The initial global population is filled with the binary solutions in every subpopulation. Also, the binary solutions in each subpopulation are stored into each parent. Then, the nondominated solutions in the global population are copied to current archive.
4) Until the termination condition is satisfied, it is running in the while loop. The termination criterion used is the maximum number of generations.
5) In the while loop, binary solutions are formed to reproduce offspring by multiple observation of the states of each Q-bit individual as in Step 2), and each binary solution is evaluated for the objective function value. After that, the newly observed solution replaces the old one, if it dominates the old. Therefore, one binary solution from one Q-bit individual survives as an offspring based on the dominance relationship at the last observation. The following two conditions are used to determine the superiority between two solutions, i.e., solution i is better than solution j:
   a) if solution i dominates solution j based on Pareto dominance or;
   b) if solution i is closer to the Pareto-optimal front than solution j when they are indifferent. Note that only first condition was employed in the fuzzy path

planner problem of three objectives because dominance relations can be easily obtained. In DTLZ problems of seven objective, however, it is difficult to get such dominance relations such that both conditions were used.

6) Both parent and offspring, i.e., previous and current binary solutions are sorted by fast nondominated sorting and crowding distance assignment to select n binary solutions [5].
7) Higher ranked n binary solutions are selected and their corresponding Q-bit individuals are sorted according to their order.
8) Global population is filled with the selected binary solutions in every subpopulation.
9) The solutions in the previous archive and global population are sorted by their global evaluation value obtained by PSSA in descending order and then top M_t solutions are selected. Note that the dominance-based sorting and selection are employed in conventional MQEA [8].
10) Global random migration is to replace all the reference binary solutions by randomly selected ones in the current archive. Note that each reference binary solution is used as a reference to update the corresponding Q-bit individual.
11) The bit values of reference binary solution and selected binary solution in each subpopulation are compared to calculate the rotation angle of rotation gate which is one of quantum gates [9]. If the bit value of the reference binary solution differs from that of the selected binary solution, the corresponding Q-bit moves toward the bit value of the reference binary solution by the rotation angle. If they are the same, Q-bit is not updated. Note that instead of crossover and mutation, rotation gate, $U(\Delta\theta)$ is employed as an update operator for the Q-bit individuals defined as follows:

$$\mathbf{q}_j^t = U(\Delta\theta) \cdot \mathbf{q}_j^{t-1} \tag{19}$$

$$U(\Delta\theta) = \begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta) \\ \sin(\Delta\theta) & \cos(\Delta\theta) \end{bmatrix} \tag{20}$$

where $\Delta\theta$ is a rotation angle of each Q-bit.

Note that computational complexity of the proposed algorithm is governed by the sorting algorithms, i.e., fast nondominated sorting and preference-based sorting. The computational complexity of fast nondominated sorting is $O(nlog(n))$. Since preference-based sorting is done by quick sorting with global evaluation value of solutions, its computational complexity is also $O(nlog(n))$. Therefore, the proposed algorithm has the computational complexity of $O(nlog(n))$.

IV. Computer Simulations and Real Experiments

To show the effectiveness of the proposed PSSA, fuzzy path planner for a shooting behavior of soccer robot was employed as an application problem. It inherently has multiple objectives such as reducing elapsed time to get to the target and heading angle and posture angle errors at the target position. In computer simulations and real experiments, MQEA was applied for the fuzzy path planner optimization and PSSA was employed to select one out of the obtained nonmodinated solutions. Also, the performance of MQEA-PS was compared with that of NSGA-II, MOPBIL, and MQEA for seven-objective DTLZ problems along with the fuzzy path planner optimization problem using three performance metrics.

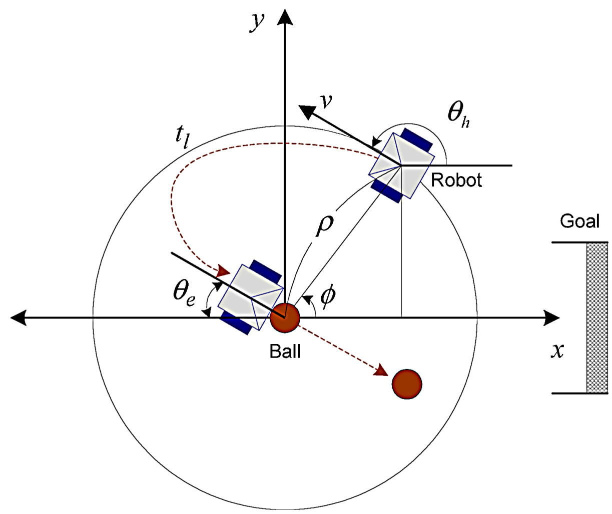


Fig. 5. Localization variables in robot soccer system. The origin is the location of the ball.

A. EMO of Fuzzy Path Planning for a Shooting Behavior

Fuzzy navigation system is composed of fuzzy path planner and fuzzy path follower [41]. The fuzzy path planner generates a desired path from the current posture to the ball position. It is assumed in this paper that the fuzzy path follower has been well designed such that it adequately tracks the planned path. Thus, the main focus is to design the fuzzy path planner using the evolutionary approach. The fuzzy path planner uses fuzzified information describing the relative position of robot with respect to the ball. Note that $(\rho,\ \phi)$ represents the posture of robot as shown in Fig. 5, where ρ is the distance between robot and ball and ϕ is the angel from x-axis to the location of robot. In the figure, v is the velocity of robot, t_l is the elapsed time, θ_h is the heading angle, and θ_e is the heading angle error at the moment of kicking the ball or at the last moment of time limit.

The fuzzified information partitioning the map of the pitch was the input of fuzzy rule set and real values were used as consequent part, as shown in Table III. The consequent part in the table represents each of appropriate heading angles ($0° \sim 360°$) corresponding to an input fuzzy set. The table shows an example of consequent part which is to be obtained by an evolutionary approach. The inputs were divided into seven membership functions of isosceles triangle, which were at intervals of 20 cm for ρ and 30° for ϕ, respectively. Inputs were constrained to $0\,\text{cm} \leq \rho \leq 120\,\text{cm}$ and $0° \leq \phi \leq 180°$ (due to geometrical symmetry).

The key objectives of path planning are that robot should approach the ball as quickly as possible and kick the ball accurately toward the goal. For the former, elapsed time to get to the ball should be minimized, whereas drift errors, i.e., heading angle and posture angle errors, at the moment

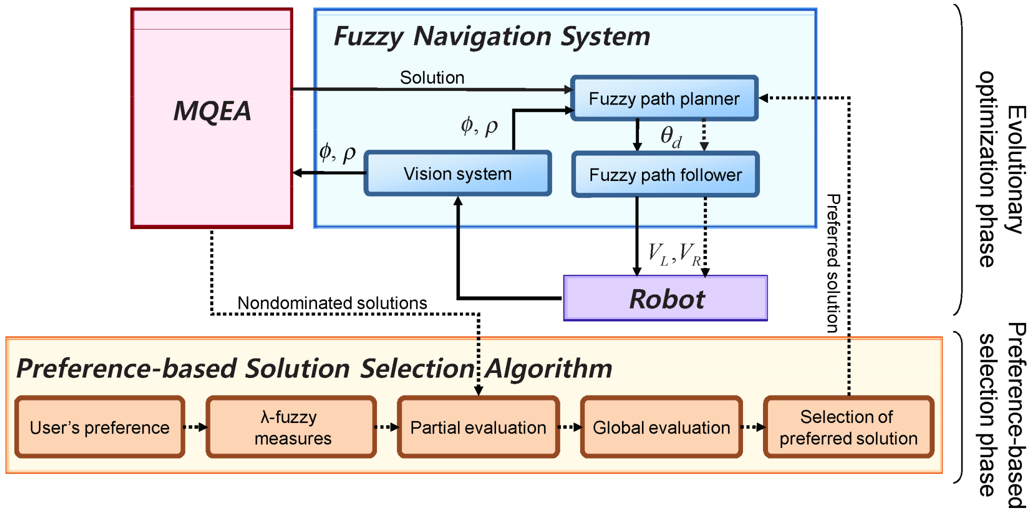


Fig. 6. Overall data flow of MQEA, fuzzy navigation system, and PSSA. Solid line represents data flow of evolutionary optimization phase and dashed line represents data flow of preference-based selection phase.

TABLE III
EXAMPLE OF FUZZY INFERENCE RULES FOR HEADING ANGLE

ϕ ρ	VN	AN	SN	MD	SF	AF	VF
VS	284	45	343	263	220	147	279
LS	10	157	268	180	257	34	152
SS	165	28	210	290	230	283	218
MD	93	245	187	321	209	288	257
SL	216	256	348	321	308	76	48
AL	296	3	265	259	81	355	44
VL	159	25	355	345	291	193	175

VN: very near; AN: average near; SN: somewhat near; MD: medium; SF: somewhat far; AF: average far; VF: very far; VS: very small; AS: average small; SS: somewhat small; SL: somewhat large; AL: average large; VL: very large.

of kicking the ball or at the last moment of time limit should be minimized for the latter one.

In conventional single-objective evolutionary approaches, the objective function values are summed up for the evaluation. Since there exists no solution, which satisfies both the fastest movement and the highest accuracy simultaneously, multiobjective evolutionary approach is needed for the path planning problem. In this paper, objective functions for a MOP of path planning were defined as follows:

$$f_1 = t_l \tag{21}$$

$$f_2 = |\theta_e| \tag{22}$$

$$f_3 = |\pi - \phi| \tag{23}$$

where f_1, f_2, and f_3 correspond to the objective function of elapsed time, heading angle error, and posture angle error, respectively. The consequent part of fuzzy inference rules in Table III was encoded as a chromosome or individual of MOEA which was to obtain the nondominated solutions as close to Pareto-optimal front as possible.

B. Structure of Overall Algorithm

Fig. 6 shows the overall data flow of MQEA, fuzzy navigation system, and PSSA. The overall algorithm consists of two phases: evolutionary optimization phase is to obtain nondominated solutions and preference-based selection phase is to select a preferred one among them. They are represented as solid and dashed lines in the figure, respectively. In the first phase, each solution, i.e., a fuzzy rule set is implanted to fuzzy path planner, vision system provides the relative posture information of robot to the ball and fuzzy path planner calculates the desired heading angle, θ_d, of robot. Then, fuzzy path follower calculates left and right wheel velocities, V_L and V_R. MQEA evaluates the solution for the objective function values at the moment of kicking the ball or at the last moment of time limit. In the second phase, PSSA is employed. Finally, the preferred solution is applied to the fuzzy path planner and then robot is to move toward the ball as per the user's preference.

C. Performance Metrics

Three performance metrics such as the size of the dominated space, the coverage of two sets, and the diversity metric, were employed to evaluate the results of NSGA-II, MOPBIL, MQEA, and MQEA-PS. Brief explanations of the metrics are in the following [42]. The size of the dominated space, $\vec{\mathcal{S}}$ is defined by the hypervolume of nondominated solutions. The quality of obtained solution set is high if this space is large. The coverage of two sets, $\vec{\mathcal{C}}$ is defined for two sets of obtained solutions, A, $B \subseteq X$, as follows:

$$\vec{\mathcal{C}}(A, B) = \frac{|\{b \in B | \exists a \in A : a \succeq b\}|}{|B|} \tag{24}$$

where $\vec{\mathcal{C}}(A, B) = 1$ means that all solutions in B are weakly dominated by A. In contrast, $\vec{\mathcal{C}}(A, B) = 0$ represents that none of the solutions in B are weakly dominated by A.

The diversity metric, $\vec{\mathcal{D}}$ is to evaluate the spread of nondominated solutions, which is defined [43] as follows:

$$\vec{\mathcal{D}} = \frac{\sum_{k=1}^{n}(f_k^{(max)} - f_k^{(min)})}{1 + \sqrt{\frac{1}{|N_0|}\sum_{i=1}^{|N_0|}(d_i - \bar{d})^2}} \tag{25}$$

TABLE IV
PARAMETER SETTING OF NSGA-II, MOPBIL, MQEA, AND MQEA-PS FOR FUZZY PATH PLANNING PROBLEM

Algorithms	Parameters	Values
NSGA-II	Population size (N)	20
	No. of generations	100
	Mutation probability (p_m)	0.1
MOPBIL	Population size (N)	20
	No. of generations	100
	No. of probability vectors	49
	Max. archive size	20
	Learning rate	0.1
	Amount of shift mutation (ms)	0.2
	Mutation probability (p_m)	0.06
MQEA/ MQEA-PS	Global population size ($N = n \cdot s$)	20
	No. of generations	100
	Subpoppulation size (n)	5
	No. of subpopulations (s)	4
	No. of multiple observations	10
	The rotation angle ($\Delta\theta$)	0.23π

where N_0 is the set of nondominated solutions, d_i is the minimal distance between ith solution and the nearest neighbor, and $\bar{d}$ is the mean value of all d_i. $f_k^{(max)}(f_k^{(min)})$ represents the maximum (minimum) objective function value of kth objective. A larger value means a better diversity of the nondominated solutions.

D. Simulation Environment

Simulation program for robot soccer was used for algorithm verification. It was assumed that the simulated robot did not slip and had a limit in acceleration speed. The max–min fuzzy reasoning and weighted average method for defuzzification were employed [44]. Parameters used in simulations are given in Table IV. The results of fuzzy path planning problem were averaged over ten runs. Forty-eight initial training points were selected evenly for angles radially from 0° to 180° in the upper part of the pitch considering the geometrical symmetry. The initial rule set at the training points were randomly generated. The averaged objective function value of each solution at all training points was used for evaluation. Also, parameters used in DTLZ problems were the same as those in Table IV except that population size and the number of generation were 100 (subpopulation size was 20 and the number of subpopulation was 5 in MQEA and MQEA-PS) and 300, respectively. The results of DTLZ problems were averaged over 30 runs.

E. Preferred Solution Selection Using PSSA

The PSSA was used to select a preferred one out of the nondominated solutions obtained by MQEA. Choquet fuzzy integral was used for global evaluation of the nondominated solutions. To show the selected preferred solutions according to the degrees of consideration for objectives, three cases were considered by employing the pairwise comparison matrices, P_1, P_2, and P_3, as follows:

$$P_1 = \begin{bmatrix} 1 & 5 & 10 \\ 0.2 & 1 & 2 \\ 0.1 & 0.5 & 1 \end{bmatrix} \tag{26}$$

TABLE V
NORMALIZED WEIGHTS OF EACH CASE

	w_1	w_2	w_3
Case 1	**0.7692**	0.1538	0.0769
Case 2	0.0870	**0.8696**	0.0435
Case 3	0.0769	0.1538	**0.7692**

TABLE VI
OBJECTIVE FUNCTION VALUES OF EACH SELECTED SOLUTION

	Measure	f_1 (s)	f_2 (deg)	f_3 (deg)
Case 1	Plausibility	**66.10**	54.48	83.31
	Belief	**83.00**	15.67	28.40
Case 2	Plausibility	99.27	**1.73**	52.28
	Belief	93.90	**2.56**	9.31
Case 3	Plausibility	109.42	7.25	**8.11**
	Belief	93.90	2.56	**9.31**

$$P_2 = \begin{bmatrix} 1 & 0.1 & 2 \\ 10 & 1 & 20 \\ 0.5 & 0.05 & 1 \end{bmatrix} \tag{27}$$

$$P_3 = \begin{bmatrix} 1 & 0.5 & 0.1 \\ 2 & 1 & 0.2 \\ 10 & 5 & 1 \end{bmatrix} \tag{28}$$

where P_1, P_2 and P_3 represent the degrees of consideration for objectives, i.e. $f_1 : f_2 : f_3 = 10 : 2 : 1$, $f_1 : f_2 : f_3 = 2 : 20 : 1$, and $f_1 : f_2 : f_3 = 1 : 2 : 10$, respectively. Table V shows the normalized weights of each case by (10) according to each pairwise comparison matrix. To obtain the partial evaluation of nondominated solutions, their objective function values were normalized to 1, where minimum and maximum values were mapped, respectively, to 1 and 0 because all criteria should be minimized in this problem.

Fig. 7 describes the selected solutions out of the nondominated solutions obtained by MQEA in each case. Table VI shows objective function values of each selected solution according to the three cases and employed fuzzy measures, where ξ in (12) was set as 0.75 for plausibility measure and 0.25 for belief measure. In Case 1, the selected solution with plausibility measure had the minimum value of f_1 and larger values of f_2 and f_3, since f_1 had the highest degree of consideration and f_2 and f_3 have the lower degree of consideration than f_1. Note that plausibility measure provided a lower value of f_1 than belief measure. In Case 2, the selected solution with plausibility measure had the minimum value of f_2 and larger values of f_1 and f_3, since f_2 had the highest degree of consideration and f_1 and f_3 had the lower degree of consideration than f_2. In Case 3, the selected solution with plausibility measure had the minimum value of f_3 and larger value of f_1, since f_3 had the highest degree of consideration.

As shown in Table VI, a different solution was selected as the preferred one according to plausibility measure or belief measure in the same case. When the plausibility measure was used, the most optimized solution to the objective with the highest degree of consideration was selected even though it was less optimized to other objectives. When the belief measure was used, on the other hand, the preferred solution might not be the most optimized one to the objective with the

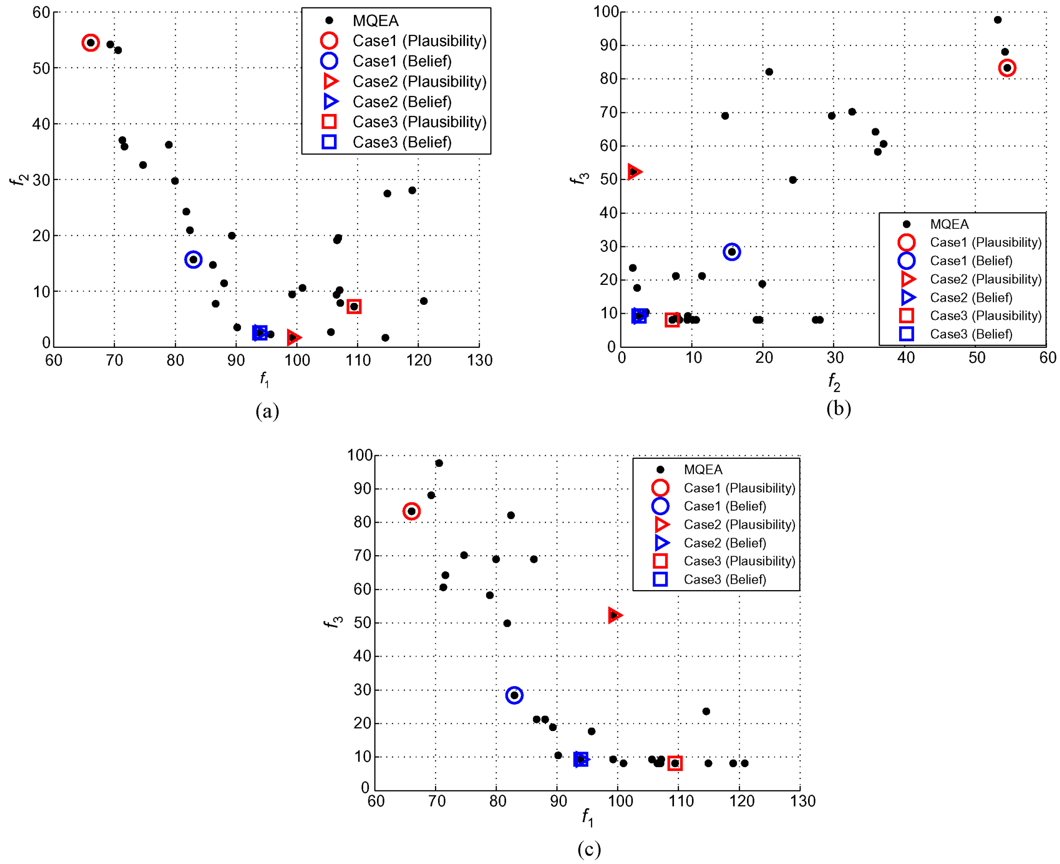


Fig. 7. Selected nondominated solutions of MQEA using PSSA. (a) f_1–f_2 objective space. (b) f_2–f_3 objective space. (c) f_1–f_3 objective space.

highest degree of consideration, because it was selected by considering other objectives with a given amount of certainty. The reason for this result comes from the negative and positive interactions of plausibility and belief measures, respectively, among criteria, which influence the global evaluation, as mentioned in Sections II-B and II-C. Thus, belief measure less encourages the selection of solution which is the best to the objective with the highest degree of consideration than plausibility measure. Therefore, if user wants to have a solution satisfying the preference with a given amount of certainty, belief measure should be used. However, if the most optimized one to the objective with the highest degree of consideration is needed, plausibility measure should be employed.

Note that the above property is a distinctive advantage of PSSA. When the conventional utility function method like weighted sum method is used in selection process, the weights need to be set very carefully if a preferred solution is not only optimized more for some criteria than the others but also is optimized to a certain level for the other criteria. On the contrary, the proposed PSSA can solve this problem by employing two kinds of fuzzy measure, i.e., plausibility measure and belief measure.

In computer simulations and experiments, plausibility measure was used. Fig. 8 shows the trajectories of each case starting from each of four initial postures. Fig. 8(a) shows the elapsed time of Case 1 was shorter than that of Cases 2 and 3, but heading and posture angle errors were larger than those of other cases. Since f_1 had the highest degree of consideration in Case 1, the robot approached the ball quickly with larger heading and posture angle errors. Fig. 8(b) shows the heading angle error of Case 2 was smaller than that of Cases 1 and 3, but the elapse time was longer than that of Case 1 and the posture angle error was larger than that of Case 3. Since f_2 had the highest degree of consideration in Case 2, the heading angle error to the ball was smaller than that of other cases. In Fig. 8(c), the posture angle error of Case 3 was smaller than that of Cases 1 and 2, but the elapse time was longer than that of Cases 1 and 2, and the heading angle error was larger than that of Case 2. Since f_3 had the highest degree of consideration in Case 3, the robot approached the ball with the smallest posture angle error. In summary, each selected solution provides the robot with preferred trajectories satisfying the degrees of consideration for the three objectives.

F. Experiment Results

Experiments were carried out in a similar environment on a pitch as in simulations, where the ball was located in front of goal and starting posture of robot was almost the same as the fourth one in Fig. 8. To show the heading angle clearly, the robot was programmed to move straight forward after kicking the ball. Fig. 9 shows the experiment results for each of three cases as in simulations. The robot in Fig. 9(a) approached the ball very quickly, but it was not

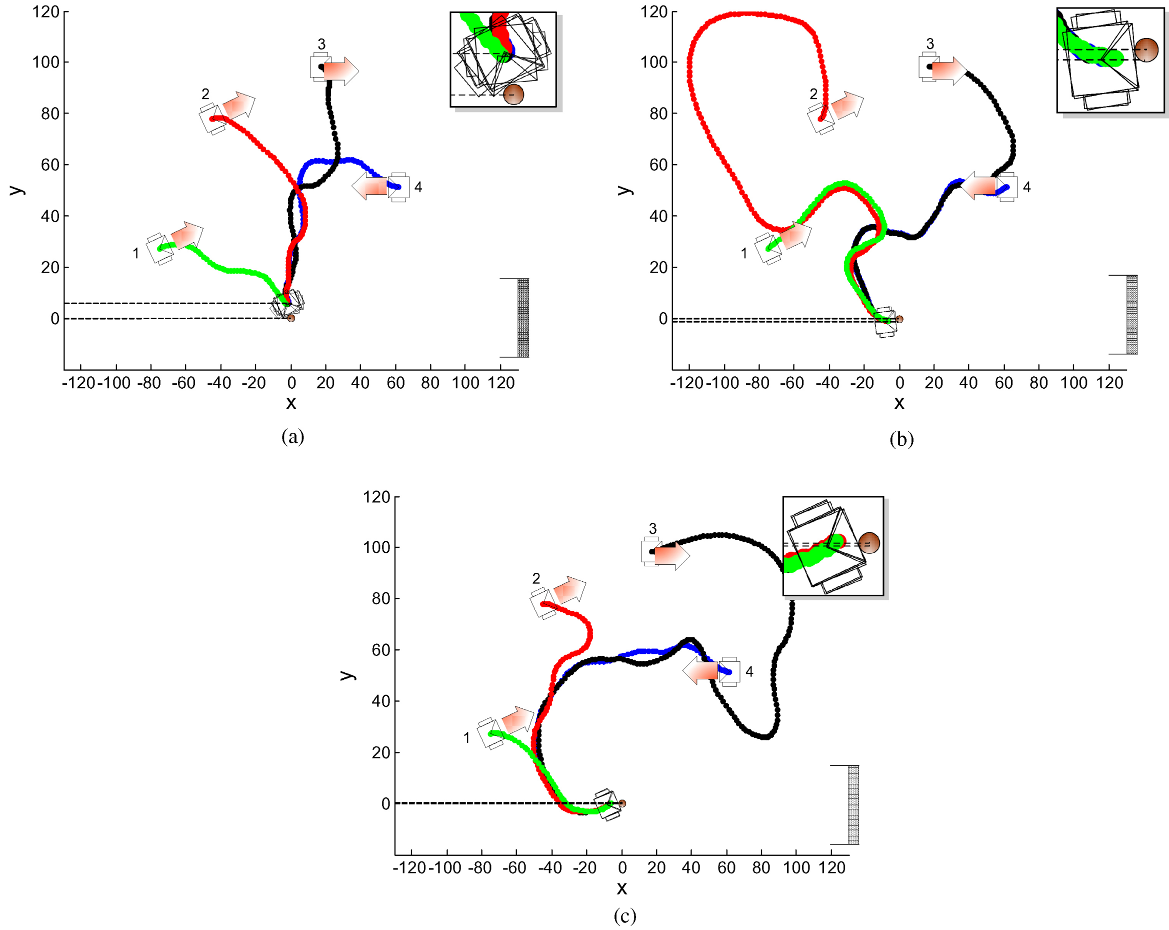


Fig. 8. Simulation results for three cases with plausibility measure. (a) Case 1. (b) Case 2. (c) Case 3. The width between two dashed lines is proportional to the posture angle error. The heading angle error is shown in detail at right up side with enlarged figure.

able to kick the ball toward the goal because there were large heading and posture angle errors. In Fig. 9(b), on the contrary, the robot spent more time to approach the ball, but it could kick the ball precisely toward the goal with a small heading angle error. The robot in Fig. 9(c) spent more time than that in Fig. 9(b), but it was not able to kick the ball precisely toward the goal, since the heading angle error was large even though the posture angle error was small.

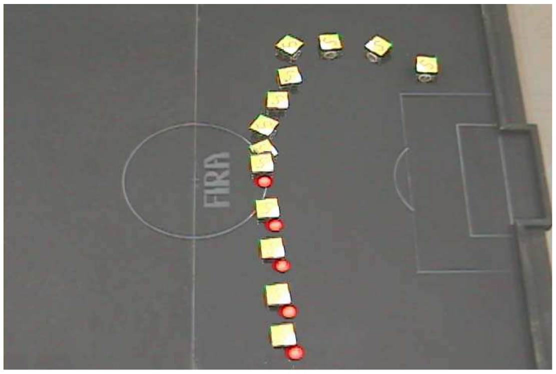
It should be noted that user has to assign the proper degree of consideration for each objective to robot considering the game situation, such as defense or offense, in real robot soccer. For example, the defense robot should generate a fast path for active defending by assigning the high degree of consideration for f_1. On the contrary, the offense robot should generate a path for precise shooting by assigning the high degrees of consideration for f_2 and f_3.

G. Comparison Results of NSGA-II, MOPBIL, MQEA, and MQEA-PS

Above sections are about conventional MQEA with dominance-based selection (MQEA) to provide nondominated solutions and then PSSA to select a preferred one out of them according to user's preference. In this section, the performance of MQEA with preference-based selection (MQEA-PS) was compared with that of NSGA-II, MOPBIL, and MQEA for seven-objective DTLZ problems along with the three-objective robot shooting problem. Belief measure ($\xi = 0.25$) was employed in MQEA-PS because PSSA using belief measure selects solutions satisfying the preference with a given amount of certainty. If plausibility measure is used in PSSA, it selects more optimized solutions to the specific objectives which are emphasized by user's high degree of consideration. If such solutions are selected in every generation, entire population converges toward local search area. Thus, it can give negative influence to the performance, i.e., proximity to Pareto-optimal solutions and solution diversity. Parameters used in simulations were the same those in Table VII. The number of variables was $\{(\text{no. of objectives})+K-1\}$, where K is the number of variables in constraint function of each DTLZ problem [36].

Only two out of the seven objectives in DTLZ problems were chosen as preferred ones. The degree of consideration for them was set as $f_1 : f_2 : f_3 : f_4 : f_5 : f_6 : f_7 = 10 : 10 : 1 : 1 : 1 : 1 : 1$. The normalized weights according to pairwise comparison matrix were obtained as (0.4, 0.4, 0.04, 0.04, 0.04, 0.04, 0.04).

The hypervolume and diversity results of NSGA-II, MOPBIL, MQEA, and MQEA-PS are shown in Table VIII. Since coverage metric is a dominance-based performance metric and the number of nondominated solutions increases



(a)

(b)

(c)

Fig. 9. Experiment results for three cases with plausibility measure. Robot started from the fourth initial posture in Fig. 8. (a) Case 1. (b) Case 2. (c) Case 3.

TABLE VII
PARAMETER SETTINGS OF MQEA-PS AND MQEA FOR DTLZ PROBLEMS

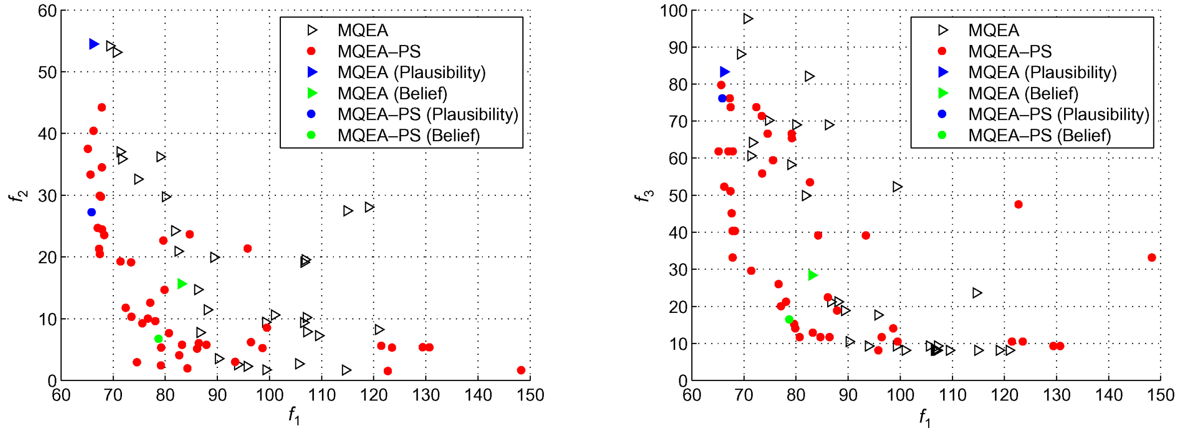
Problem	$\Delta\theta$	K	Sorting Criteria for $P(t) \cup A(t-1)$
DTLZ1	0.20π	5	Dominance and proximity
DTLZ2	0.20π	10	Dominance and proximity
DTLZ3	0.20π	10	Dominance and proximity
DTLZ4	0.23π	10	Dominance and proximity
DTLZ5	0.20π	10	Dominance and proximity
DTLZ6	0.20π	10	Dominance only
DTLZ7	0.23π	20	Dominance only

exponentially when the number of objectives increases, the coverage metric is not appropriate to evaluate the performance of MOEAs in this kind of many-objective problem. Therefore, the coverage metric was not used for DTLZ problems. As the table shows, MQEA-PS outperformed other MOEAs for DTLZ1, DTLZ3, DTLZ4, and DTLZ5 and obtained competitive results compared with them for DTLZ2, DTLZ6, and DTLZ7 in hypervolume metric. This means that MQEA-PS had a larger dominating space than others such that it could obtain the solution set of better quality. In diversity metric, on the contrary, MQEA-PS did not show good results even though MQEA showed competitive results compared with others, because MQEA-PS selected more optimized solutions to the preferred objectives and the final solution set was crowded and biased toward the preferred ones.

NSGA-II, MOPBIL, MQEA, and MQEA-PS were also compared for the fuzzy path planning optimization problem of the robot shooting behavior with three objectives. Belief measure ($\xi = 0.25$) was employed for the same reason as in DTLZ problems. Parameters used here were the same as those in Table IV. The degree of consideration for the three objectives was set as $f_1 : f_2 : f_3 = 10 : 2 : 1$.

Table IX shows the coverage comparison between MQEA-PS and each of MOEAs and Table X shows the comparisons of hypervolume, diversity, and mean value of f_1 among them. As the tables show, MQEA-PS outperformed other MOEAs in coverage and hypervolume metric and also obtained competitive results in diversity. Moreover, MQEA-PS found solutions of more minimized value of f_1 which was set as a preferred objective.

Fig. 10(a) and (b) shows all the final solutions obtained by MQEA and MQEA-PS. As the figures show, the solutions of MQEA-PS in the left-hand side are distributed more toward smaller value of f_1 compared to those of MQEA. On the contrary, the solutions in the down side of the figures show that both algorithms found almost Pareto-optimal solutions to the objectives, f_2 and f_3. Since f_1 was more considered in every generation of evolutionary process, MQEA-PS could find more optimized solutions to f_1. Note that MQEA-PS also needs PSSA to select a preferred one out of obtained nondominated solutions for real application. As the figures show, the selected one of MQEA-PS using either plausibility measure or belief measure is better than that of MQEA in all the three objectives.



(a) (b)

Fig. 10. Comparison results of nondominated solutions of MQEA and MQEA-PS in the objective function space for robot shooting behavior. (a) f_1–f_2 space. (b) f_1–f_3 space.

TABLE VIII

COMPARISON RESULTS OF NSGA-II, MOPBIL, MQEA, AND MQEA-PS WITH THE HYPERVOLUME METRIC AND DIVERSITY METRIC FOR DTLZ PROBLEMS

Problem	$\vec{S}$ (NSGA-II)	$\vec{S}$ (MOPBIL)	$\vec{S}$ (MQEA)	$\vec{S}$ (MQEA-PS)
DTLZ1	$3.037495965 \cdot 10^{15}$	$3.031250548 \cdot 10^{15}$	$3.037485858 \cdot 10^{15}$	$\mathbf{3.037497436 \cdot 10^{15}}$
DTLZ2	$\mathbf{1.232777252 \cdot 10^{2}}$	$1.166474196 \cdot 10^{2}$	$1.151015520 \cdot 10^{2}$	$1.144135204 \cdot 10^{2}$
DTLZ3	$1.727774165 \cdot 10^{21}$	$1.706904794 \cdot 10^{21}$	$1.727667918 \cdot 10^{21}$	$\mathbf{1.727999998 \cdot 10^{21}}$
DTLZ4	$2.181412014 \cdot 10^{3}$	$1.867627497 \cdot 10^{3}$	$2.167845459 \cdot 10^{3}$	$\mathbf{2.183767143 \cdot 10^{3}}$
DTLZ5	$3.718400617 \cdot 10^{3}$	$2.901547851 \cdot 10^{3}$	$3.477029523 \cdot 10^{3}$	$\mathbf{3.722515165 \cdot 10^{3}}$
DTLZ6	$\mathbf{1.588829679 \cdot 10^{8}}$	$1.389913971 \cdot 10^{8}$	$1.549663344 \cdot 10^{8}$	$1.439051588 \cdot 10^{8}$
DTLZ7	$2.797104934 \cdot 10$	$\mathbf{3.240855952 \cdot 10}$	5.0904245	$2.815868438 \cdot 10^{1}$
Problem	$\vec{D}$ (NSGA-II)	$\vec{D}$ (MOPBIL)	$\vec{D}$ (MQEA)	$\vec{D}$ (MQEA-PS)
DTLZ1	**198.799958**	131.793983	46.366511	55.749381
DTLZ2	117.157433	82.968781	**122.471515**	100.489063
DTLZ3	**131.847799**	81.880466	69.370769	69.935475
DTLZ4	123.417024	**337.921809**	132.683831	128.651885
DTLZ5	142.760458	**181.710519**	120.522227	112.097071
DTLZ6	107.93717	93.412581	**129.980468**	124.155049
DTLZ7	168.657051	231.755959	**399.19066**	209.771355

TABLE IX

COMPARISON RESULTS OF NSGA-II, MOPBIL, MQEA, AND MQEA-PS WITH THE COVERAGE METRIC FOR ROBOT SHOOTING BEHAVIOR

$\vec{C}$(NSGA-II, MQEA-PS)	0.06383
$\vec{C}$(MQEA-PS, NSGA-II)	**0.981481**
$\vec{C}$(MOPBIL, MQEA-PS)	0.148936
$\vec{C}$(MQEA-PS, MOPBIL)	**0.71875**
$\vec{C}$(MQEA, MQEA-PS)	0.276596
$\vec{C}$(MQEA-PS, MQEA)	**0.724638**

TABLE X

COMPARISON RESULTS OF NSGA-II, MOPBIL, MQEA, AND MQEA-PS WITH THE HYPERVOLUME METRIC, DIVERSITY, AND MEAN VALUE OF f_1 FOR ROBOT SHOOTING BEHAVIOR

Algorithm	Hypervolume ($\vec{S}$)	Diversity ($\vec{D}$)	f_1 (mean)
NSGA-II	$1.632774803 \cdot 10^{6}$	**90.651375**	103.7103
MOPBIL	$1.644853722 \cdot 10^{6}$	49.578824	92.7194
MQEA	$1.710398279 \cdot 10^{6}$	36.902182	99.90037
MQEA-PS	$\mathbf{1.793636385 \cdot 10^{6}}$	64.694338	**91.97562**

V. CONCLUSION

In this paper, PSSA using fuzzy measure and fuzzy integral was proposed for EMO. It could select a preferred one out of nondominated solutions obtained by any of MOEAs as per user's degree of consideration for the objectives. As one of MOEAs, conventional MQEA with dominance-based selection was employed to provide nondominated solutions and then PSSA was applied to select a preferred one out of them according to user's preference. The effectiveness of this scheme was demonstrated through computer simulations and real experiments carried out for EMO of fuzzy path planner for a shooting behavior of soccer robot. The selected fuzzy path planner by PSSA generated a preferred behavior following user's preference. Also, MQEA-PS was proposed by employing PSSA in each generation of evolutionary process. The proposed MQEA-PS was compared with NSGA-II, MOPBIL, and conventional MQEA to show the improved performance for the DTLZ problems and the fuzzy path planner optimization problem. It could find more optimized solutions to specific preferred objectives. Since PSSA is a general scheme of multiple criteria decision making for MOPs, it can be used for any one of MOEAs to select a preferred solution or to sort the obtained solutions based on user's preference during evolutionary process.

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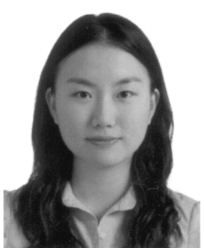

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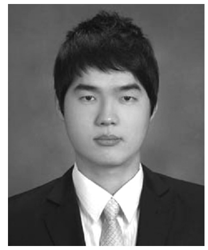

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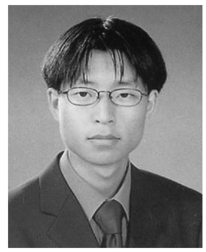

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