Construction of a Fuzzy Relation with Reduced Dimension for Multivariable Systems by using Genetic Algorithm

Kunihiko OURA*, Ryo KITAZAWA*, Kageo AKIZUKI,* and Michio MIYAZAKI**

* **Waseda University, 3-4-1, Ohkubo, Shinjuku-ku, Tokyo, 169-8555 , JAPAN**
E-mail: kuni@akizuki.elec.waseda.ac.jp

** **Kanto-Gakuin University, 4834, Mutsuura-cho, Kanazawa-ku, Yokohama, Kanagawa, 236-0032, JAPAN**
E-mail: miyazaki@kanto-gakuin.ac.jp

ABSTRACT

Recently, fuzzy control is widely used in the fields of industry. It is often applied by describing both of the input and output in fuzzy variables, and composing the fuzzy relation "R" represented by multi-dimensional matrix, which is based on the "if-then" fuzzy rules. The method of Mamdani and the methods using GA (genetic algorithm) were proposed to solve the fuzzy relation equation. However, for most of the multivariable system such as traffic system or process control system, there occurs a problem of insufficient memory for calculation. It is caused by enormous amount of on-line calculations due to many matrix elements, which makes it impossible to use these methods in real-time control.

To cope with this problem, Gegov proposed a method using two-dimensional fuzzy relation for multivariable system. Though it is effective in the sense of reducing the calculation, but it does not often satisfy all of the fuzzy rules. Especially in the case when outputs are not likely in shape in spite of their similar inputs, it is difficult to find the fuzzy relation representing the system correctly. Therefore, another method suitable for multivariable cases is expected.

In this paper, we propose a technique to construct the reduced (not only two) dimensional fuzzy relation for multivariable system. The optimal fuzzy relation can be chosen among the candidates that satisfy all of the rules, by evaluating the number of matrix elements. Here, GA is adopted for efficient search. In case that we have several solutions from the search, the set that gives the lowest dimension is selected. Then we decide the value of matrix elements by least square errors that are obtained by applying the same inputs to the resulted (reduced dimensional) fuzzy relation and original one. Finally we show some simulation examples, which will show usefulness of the proposed method, in the point that it enables us to construct a fuzzy relation for multivariable systems.

1. INTRODUCTION

Let us consider the case when input and output is fuzzy variable in fuzzy control system. In order to calculate an output, we first construct the fuzzy relation given by multi-dimensional matrix, which is based on the fuzzy rules, and synthesize it with input. But the memory shortage becomes a problem in case of the multivariable system, because the number of matrix elements increases exponentially. For this problem, some methods those using the two-dimensional fuzzy relation for multivariable system are proposed, but there often happens that resulted fuzzy relation does not satisfy all of the fuzzy rules. So that we propose another method which construct a fuzzy relation with reduced dimension, by applying GA. Numerical example and simulation example for traffic signal control will show usefulness of our method.

2. FUZZY RELATION FOR MULTIVARIABLE SYSTEMS

Consider the following "if-then" fuzzy model .

$$L^h: \text{if } x_1 \text{ is } A_{1(h)} \text{ and } \cdots \text{ and } x_n \text{ is } A_{n(h)}$$
$$\text{then } u_1 \text{ is } B_{1(h)} \text{ and } \cdots \text{ and } u_m \text{ is } B_{m(h)} \quad (h = 1, \cdots, l)$$

where L^h denotes h-th fuzzy rule, $A_{i(h)}, B_{j(h)}$ are fuzzy sets, x_i, u_j are fuzzy input - output variables.

Both of the input and output variables are defined in universal sets of equal power f, then the fuzzy sets are represented by membership value as follows:

$$A_n = \{\mu^1_{A_n}, \mu^2_{A_n}, \cdots, \mu^f_{A_n}\}$$
$$B_m = \{\mu^1_{B_m}, \mu^2_{B_m}, \cdots, \mu^f_{B_m}\}$$

With the above definition, the $(n+1)$ dimensional fuzzy relation R_i for each rule can be determined for n-input $\{x_1, x_2, \cdots, x_n\}$ and one output u_i.

By using Mamdani's method, $(a_1, a_2, \cdots a_n, b_i)$-th element of R_i is given by (1).

$$r_i^{a_1, a_2, \cdots, a_n, b_i} = \mu^{a_1}_{A_1} \cap \mu^{a_2}_{A_2} \cap \cdots \cap \mu^{a_n}_{A_n} \cap \mu^{b_i}_{B_i} \qquad (1)$$

where symbol "$\cap$","$\cup$" denote the max and min operator. If we assume that all rules are combined by "or"-rule, fuzzy relation R_i for h rules are calculated by (2),

$$R_i = R_{i1} \cup R_{i2} \cup \cdots \cup R_{ih} \qquad (2)$$

and its element is given by (3).

$$r_i^{a_1, a_2, \cdots, a_n, b_i} = \bigcup_{s=1}^{h} \left\{ \mu^{a_1}_{A_1} \cap \mu^{a_2}_{A_2} \cap \cdots \cap \mu^{a_n}_{A_n} \cap \mu^{b_i}_{B_i} \right\} \qquad (3)$$

When fuzzy value (4) are added to the model,

$$\begin{aligned} x_1 &= \{\mu^1_{x_1}, \mu^2_{x_1}, \cdots, \mu^f_{x_1}\} \\ &\vdots \\ x_n &= \{\mu^1_{x_n}, \mu^2_{x_n}, \cdots, \mu^f_{x_n}\} \end{aligned} \qquad (4)$$

output fuzzy value (5) can be calculated by (6).

$$u_i = \{\mu_{u_i}^1, \mu_{u_i}^2, \cdots, \mu_{u_i}^f\} \quad (5)$$

$$u_1 = (x_1 \cap x_2 \cap \cdots \cap x_n) \circ R_i \quad (6)$$

where "$\circ$" denotes the max-min composition. Similarly, the detailed presentation of the above equation is given by (7).

$$\mu_{u_i}^t = (\mu_{x_1}^1 \cap \cdots \cap \mu_{x_n}^1 \cap r_i^{1,\cdots,1,t}) \cup \cdots$$
$$\cup (\mu_{x_1}^f \cap \cdots \cap \mu_{x_n}^f \cap r_i^{f,\cdots,f,t}) \; t = 1, \cdots, f \quad (7)$$

The number of matrix elements R_i is f^{n+1}, when the number of mount cluster is f. So that it becomes enormous order for multivariable systems. It is clearly seen that in (1), fuzzy relation is defined for all inputs, that it considers all combination of fuzzy inputs and states. This is why fuzzy relation is effective to represent nonlinear systems, but it has disadvantage of increasing the number of matrix elements. Considering the application to the real plant, it is better to attack this problem.

3. USE OF TWO DIMENSIONAL FUZZY RELATION FOR MULTIVALIABLE SYSTEM

In section2, we saw that output of fuzzy model can be calculated by (6) for multi-variable system, but it needs enormous amount of online calculations. Therefore, we think of using (8) instead of (6) in this section, that two-dimensional fuzzy relation is introduced.

$$u_i = (x_1 \circ R_{1i}) \cap (x_2 \circ R_{2i}) \cap \cdots \cap (x_n \circ R_{ni}) \quad (8)$$

where R_{ji} denotes two-dimensional fuzzy relation of x_i corresponding to u_j. (8) means that the model output can be calculated by each input and fuzzy relation, and common value is selected.

The detailed representation of (8) is given as (9).

$$\mu_{u_i}^{b_i} = \{\bigcup_{a_1}[\mu_{x_1}^{a_1} \cap r_{1i}^{a_1,b_i}]\} \cap \{\bigcup_{a_2}[\mu_{x_2}^{a_2} \cap r_{2i}^{a_2,b_i}]\} \cap \quad (9)$$
$$\cdots \cap \{\bigcup_{a_n}[\mu_{x_n}^{a_n} \cap r_{ni}^{a_n,b_i}]\}$$

where $r_{j,i}^{a_j,b_i}$ denote the (a_j, b_i)-th element. By using (9), one output is calculated by corresponding input, then fuzzy relation and common value is selected, whereas for the multidimensional fuzzy relation (7), the output is calculated considering the states of all inputs $\{x_1, x_2, x_3 \cdots, x_n\}$ at the same time.

There are some methods proposed to solve two-dimensional fuzzy relation, e.g. Mamdani's method, and methods using GA. By using Mamdani's method, $r_{j,i}^{a_j,b_i}$ is given by (10).

$$r_{j,i}^{a_j,b_i} = \mu_{A_j}^{a_j} \cap \mu_{B_i}^{b_i} \quad (10)$$

In this case, the number of elements "r" in all matrices is nf^2 and the number of max and min operation is $(2nf-1)$. Therefore, the amount of calculation is much smaller than in the case when using the multidimensional fuzzy relation. Simple example for the case when $(n = 8, f = 5)$ is shown in table1.

	Multidimensional Fuzzy Relation	Two-dimensional Fuzzy Relation
"r"	1.95*10^6	200
"min","max,	3.51*10^6	79

Table1: The number of "r","$\cap$","$\cup$" for fuzzy model when $(n = 8, f = 5)$

From table1, it is proven that the number of matrix elements is drastically decreased for two-dimensional fuzzy relation. Then fuzzy relation for multivariable systems can be constructed for the case which was impossible when using the multi dimensional fuzzy relation.

However, the above method is not always effective for multivariable systems. For example, when there are some rules that has some common fuzzy values. Simple example will be useful for understanding them.

Assume that the following fuzzy rules are given for the case when $(n = 3, f = 3)$.

Rule1: if x_1 is S and x_2 is S and x_3 is B, then u_1 is S
Rule2: if x_1 is S and x_2 is M and x_3 is B, then u_1 is M
Rule3: if x_1 is S and x_2 is B and x_3 is S, then u_1 is M
Rule4: if x_1 is M and x_2 is B and x_3 is S, then u_1 is B
Rule5: if x_1 is S and x_2 is M and x_3 is S, then u_1 is B

Fuzzy output values for fuzzy input values are shown in table2.

	x_1	x_2	x_3
S	S,M,B	S	M,B
M	B	M,B	
B		M,B	S,M

Table2: Fuzzy rules

It is easy to show that Rule3 and Rule5 cannot be distinguished because fuzzy value "M" and "B" are common. Therefore, we cannot represent these rules in two-dimensional fuzzy relation.

4. CONSTRUCTION OF A FUZZY RELATION WITH REDUCED DIMENSION

In section3, we found that two-dimensional fuzzy relation can be applied to multivariable systems, but it sometimes lacks ability to represent all the rules. This problem appears more frequently as multivariable system becomes complicated. However, the number of fuzzy elements should be reduced considering the application to practical cases. In this section, we propose a method to construct a reduced (not only two) dimensional fuzzy relation. The procedure consists of two steps. The first step is to decide a fuzzy relation, and the second step is to decide matrix elements.

4.1 Method to decide a fuzzy relation

We propose to use (11), that the model output is calculated with several inputs, each of them are given by combination of fuzzy relation.

$$u_i = (x_1 \circ R_1) \cap \{(x_2 \cap x_5) \circ R_2\} \cap$$
$$\{(x_3 \cap x_4 \cap x_6) \circ R_3\} \cap \cdots \quad (11)$$

The detailed representation is (12).

$$\mu_{u_i}^{b_i} = \{\bigcup_{a_1}[\mu_{x_1}^{a_1} \cap r_{1i}^{a_1,b_i}]\} \cap \{\bigcup_{a_2,a_5}[\mu_{x_2}^{a_2} \cap \mu_{x_5}^{a_5} \cap r_{2i}^{a_2,a_5,b_i}]\}$$
$$\cap \{\bigcup_{a_3,a_4,a_6}[\mu_{x_3}^{a_3} \cap \mu_{x_4}^{a_4} \cap \mu_{x_6}^{a_6} \cap r_{3i}^{a_3,a_4,a_6,b_i}]\} \cap \cdots \quad (12)$$

The number of matrix elements f_{all} is (13)

$$f_{all} = f^2 + f^3 + f^4 + \cdots \quad (13)$$

Fuzzy relation is chosen among various sets, which satisfies the following conditions, C1) It agrees with the fuzzy output of original rules.

C2) It minimizes the number of matrix elements

The result depends on the combination in (11), and we are going to use GA to search it efficiently. For parameters of GA, we set the gene codes as figure1, when searching the combination of optimum fuzzy variable. In order to avoid the lethal gene, the number of gene code is $(n-1)$ for n inputs.

Gene Index	Gene(1)	Gene(2)	···	Gene(n-1)
Value	1 to n	1 to (n-1)	···	1 or 2

Figure1: Gene codes

For the cost function in searching with GA, the following rules are conformed.

R1) If the fuzzy value of an output, which is obtained from the fuzzy value of the input do not satisfy the rule, penalty is imposed on the cost function.

R2) If the first values of the variable groups are not arranged by its size, penalty is imposed on the cost function.

R3) The number of all matrix elements is calculated, and its logarithm is taken.

The solution can be chosen among the candidates which has the smallest cost function. In case that there are several solutions from the search, the set which has the smallest number of fuzzy variable is selected.

4.2 Method to decide parameters of fuzzy relation

After the combination of variables are decided, we have come to decide the parameters of matrix elements. We again, apply GA to search the optimum value.

As for encoding, the real-number expression is converted into the individual bit steam. In order to improve search efficiency, the grey code is used here. If the number of bits in bit steam is M for N matrix elements, it is encoded in the new bit steam of the size $(M \times N)$.

In searching with GA in this case, we use (14) as the cost function of the kth fuzzy relation for u_i.

$$\sum_{s,b_i}\{\mu^{b_i}_{B_{i,s}} - \mu^{b_i}_{k(s)}\}^2 \quad (14)$$

where $\mu^{b_i}_{B_{i,s}}$ denotes the b_i-th membership value for i-th output of the s-th rule, $\mu^{b_i}_{k(s)}$ denotes the b_i-th membership value for i-th output of the s-th rule in k-th variable group.

However, if we use (14), the values are not decided to the singularity, and sometimes they have upper and lower limits. For such cases, the value which satisfies (15) is adopted.

$$u \subseteq \overline{u} \quad (15)$$

The reason for (15) is, if we denote the fuzzy relation when adopting the upper limit as $\overline{R}$, that is $(R \subseteq \overline{R})$, (16) is established for unknown input x.

$$x \circ R \subseteq x \circ \overline{R} \quad (16)$$

From considerations above, we use the cost function (17).

$$J = \sum_{s,b_i}\{\mu^{b_i}_{B_{i,s}} - \mu^{b_i}_{k(s)}\}^2 \times W_1 + \sum(1-r_k)\times W_2 \quad (17)$$

In (17), W_1 should be larger in case than membership value is "1", because fuzzy value is decided by the mount cluster where membership value is "1". The second term of (17) is to deduce the upper value, and chosen as to maximize the sum of matrix elements $r^{a_j,b_i}_{j,i}$. Therefore, W_2 should be enough smaller than W_1 so that it does not affect the error of the first term. The elements of fuzzy relation are searched by GA so as to minimize (17).

5. SIMULATION EXAMPLES

Some examples will help you to understand the usefulness of the proposed method. We are going to show two examples. The first one is to confirm the construction of fuzzy relation with reduced dimension. The other is to consider the application of the method for traffic signal control.

5.1 Numerical example

Suppose that 7 input and one output fuzzy model whose rule is given by table3, and universal set have five elements like(18).

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	u_1
L1	B	M	B	M	S	B	S	M
L2	M	B	B	S	B	S	M	M
L3	B	M	S	M	S	B	S	S
L4	M	B	B	B	B	W	M	M
L5	M	S	S	S	M	M	M	B
L6	S	B	M	M	M	B	M	M
L7	M	M	S	B	B	B	B	M
L8	S	B	M	M	B	S	S	S
L9	M	M	B	S	B	B	M	B
L10	S	S	S	S	S	B	B	B

Table3: Fuzzy Rules

$$\begin{aligned} S &= \{1.0, 0.5, 0.0, 0.0, 0.0\} \\ M &= \{0.0, 0.5, 1.0, 0.5, 0.0\} \\ B &= \{0.0, 0.0, 0.0, 0.5, 1.0\} \end{aligned} \quad (18)$$

We set GA parameters as table4.

Size of indivisual group	50
Mutation probabiilty	0.3
Cross probability	0.6
Search frequency	10000

Table4: GA parameters

After the search, we have the results shown in table5. The third column is the number of the fuzzy output value required for the fuzzy input value for each combination.

1	$x_6, x_4, (x_3, x_5), (x_2, x_7), x_1$	35
2	$x_6, x_5, (x_3, x_4), (x_2, x_7), x_1$	38
3	$x_6, x_5, (x_3, x_7), (x_2, x_1), x_4$	39

Table5: Search results

Among these, we calculate the fuzzy relation for group1 and group3, because their numbers of fuzzy output value are maximum and minimum. Then model output is calculated for the same input as the original rule, where GA parameters are set as talble6.

Size of indivisual group	50
Gene length	5*elements
Mutation probabiilty	0.3
Cross probability	0.6
Search frequency	10000

Table6: GA parameters

After the search, we have fuzzy output shown in table7 and table8, where the outputs are calculated by (19) and (20), respectively.

	Fuzzy output of the rule	Fuzzy output estimates
L1	M	(0.07,0.40,0.93,0.27,0.07)
L2	M	(0.00,0.13,0.87,0.40,0.00)
L3	S	(0.80,0.33,0.13,0.27,0.07)
L4	M	(0.07,0.33,1.00,0.40,0.00)
L5	B	(0.00,0.13,0.13,0.47,0.93)
L6	M	(0.20,0.40,0.93,0.33,0.20)
L7	M	(0.07,0.33,0.93,0.47,0.07)
L8	S	(0.93,0.40,0.07,0.07,0.00)
L9	B	(0.00,0.13,0.13,0.47,0.93)
L10	B	(0.00,0.07,0.13,0.40,0.93)

Table7: Fuzzy output of case1

	Fuzzy output of the rule	Fuzzy output estimates
L1	M	(0.13,0.40,0.93,0.27,0.07)
L2	M	(0.00,0.40,1.00,0.33,0.00)
L3	S	(0.80,0.40,0.13,0.27,0.07)
L4	M	(0.00,0.40,1.00,0.33,0.00)
L5	B	(0.00,0.07,0.07,0.47,0.93)
L6	M	(0.07,0.40,0.93,0.47,0.50)
L7	M	(0.00,0.40,0.93,0.47,0.50)
L8	S	(0.87,0.40,0.13,0.33,0.00)
L9	B	(0.00,0.13,0.50,0.47,0.80)
L10	B	(0.20,0.07,0.07,0.40,0.93)

Table8: Fuzzy output of case2

$$u_1 = (x_6 \circ R_1) \cup (x_4 \circ R_2) \cup (x_1 \circ R_3) \cup \{(x_3 \cup x_5) \circ R_4\} \cup \{(x_2 \cup x_7) \circ R_5\} \tag{19}$$

$$u_1 = (x_6 \circ R_1) \cup (x_5 \circ R_2) \cup (x_1 \circ R_4) \cup \{(x_3 \cup x_7) \circ R_4\} \cup \{(x_2 \cup x_1) \circ R_5\} \tag{20}$$

Figure2 and figure3 shows the shape of membership function of each result, together with the original rules.

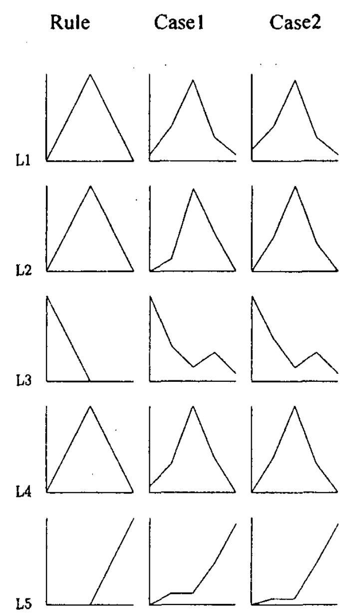

Figure2: Results for rule1 to 5

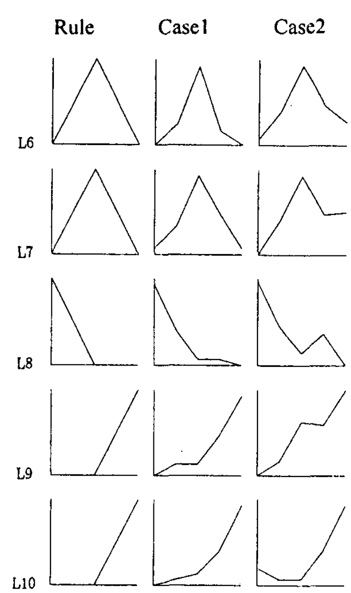

Figure3: Results for rule 6 to 10

From these figures, it is proven that both results express the fuzzy value appropriately, which shows usefulness of our method.

5.2 Application to traffic signal control

We apply the method to traffic signal control. For the traffic signal control, three parameters exist. They are cycle time, sprit time and offset time. In this example, the cycle time and offset time are fixed to simplify the problem. Then only the sprit time should be decided. We consider the network shown in figure4, which includes three junctions. The purpose is to decrease the queue length at junctions.

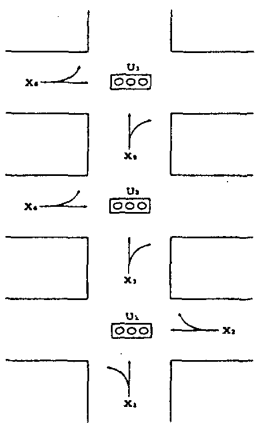

Figure4: Model for traffic signal control

In figure3, $x_1 \cdots x_6$ denote queue length at junctions, and u_1, u_2, u_3 denote sprit time associated with vertical queues. All of these variables are described by fuzzy value, e.g. "x_1 is approximately ten". Other parameters for this model are: the car passes junction in 1.5s, cycle time is 57s, offset time is 57s, passage time between two signals are 57s, and the time of clearance loss is 6s.

Under these conditions, fuzzy rules shown in table9 are constructed through the expert's knowledge and some simulations.

If we use multidimensional fuzzy relation for the model, the number of matrix elements are approximately 1.4×10^9, then it is hard to calculate them. So that we use the proposed method, as given in section4. After using GA whose parameters are the same as example in 5.1, we have the fuzzy relation (21) -(23).

$$u_1 = \{(x_1 \cup x_2) \circ R_{11}\} \cup (x_3 \circ R_{12}) \cup (x_4 \circ R_{13}) \cup (x_5 \circ R_{14}) \cup (x_6 \circ R_{15}) \quad (21)$$

$$u_2 = \{(x_3 \cup x_4) \circ R_{21}\} \cup (x_1 \circ R_{22}) \cup (x_2 \circ R_{23}) \cup (x_5 \circ R_{24}) \cup (x_6 \circ R_{25}) \quad (22)$$

$$u_1 = \{(x_5 \cup x_6) \circ R_{31}\} \cup (x_1 \circ R_{32}) \cup (x_2 \circ R_{33}) \cup (x_3 \circ R_{34}) \cup (x_4 \circ R_{35}) \quad (23)$$

The number of matrix elements is now decreased to 3159, we can calculate the fuzzy relation efficiently. Parameters of matrix elements are also decided by GA. Controlled input to the signals are obtained by defuzzification of the estimated fuzzy output.

Simulation results by the model is shown in figure4 and figure5, where we did two different simulations for different initial values.

In both cases, we set the probability of the car's turning right or left is 0.2, and that of the car's flowing from other road is 0.2. We processed 4 stage simulations, each stage consists of 1 cycle.

	x_1	x_2	x_3	x_4	x_5	x_6	u_1	u_2	u_3
L1	B	B	B	B	B	B	M	M	M
L2	M	M	B	M	B	M	M	B	B
L3	S	S	M	M	B	M	M	M	B
L4	M	S	M	M	B	M	M	M	B
L5	M	M	M	M	M	M	M	M	M
L6	S	B	B	B	S	B	S	M	S
L7	M	B	B	B	M	B	S	M	S
L8	S	S	S	S	S	S	M	M	M
L9	S	S	S	S	M	S	M	M	M
L10	S	S	M	S	M	S	M	M	M

Table9: GA parameters

Simulation results for the first set of initial values (24) are shown in table10 (sprit) and figure5 (car length).

$$(x_1, x_2, x_3, x_4, x_5, x_6) = (25, 25, 25, 25, 25, 25) \quad (24)$$

stage	u_1	u_2	u_3
1	50	50	50
2	50	70	70
3	50	50	70
4	50	50	50

Table10: Sprits for case1

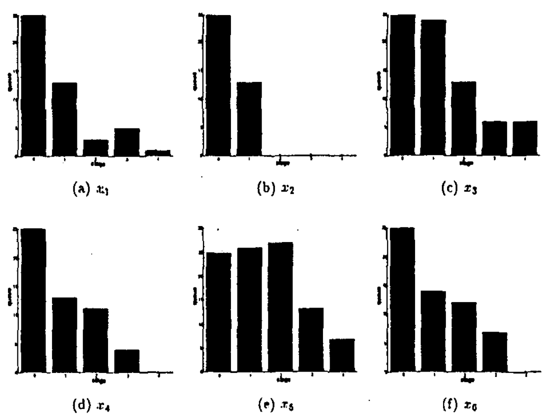

Figure5: Car length for case1

Simulation results for the second set of initial values (25) are shown in table11 (sprit) and figure6 (car length).

$$(x_1, x_2, x_3, x_4, x_5, x_6) = (3, 25, 25, 25, 3, 25) \quad (25)$$

stage	u_1	u_2	u_3
1	30	55	30
2	50	55	60
3	45	45	50
4	50	45	50

Table11: Sprits for case2

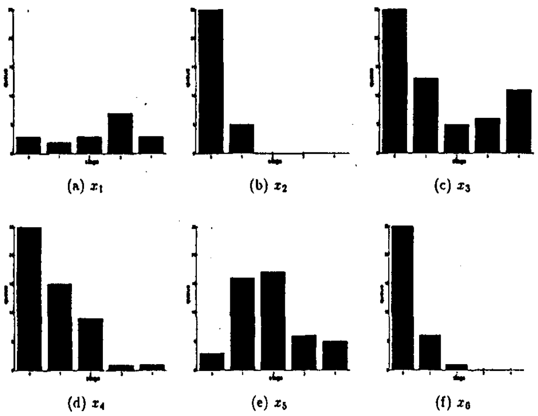

Figure6: Car length for case2

Both of the simulation results show the usefulness of the proposed method.

6. CONCLUSION

In this paper, we have proposed the method to decide fuzzy relation with reduced dimension by using GA. It can reduce the enormous amount of elements in multivariable fuzzy relation ,without losing any fuzzy rules, which helps the problem of insufficient memory for calculation. Simulation examples showed usefulness of the proposed method.

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