



Improving interdependent networks robustness by adding connectivity links



Xingpei Ji^a, Bo Wang^{a,*}, Dichen Liu^a, Guo Chen^b, Fei Tang^a, Daqian Wei^a, Lian Tu^a

^a School of Electrical Engineering, Wuhan University, Wuhan 430072, China

^b School of Electrical and Information Engineering, The University of Sydney, NSW 2006, Australia

HIGHLIGHTS

- Considering interdependent relationships, two novel connectivity link addition strategies are proposed.
- Performance comparisons among six link addition strategies are conducted in three types of interdependent networks.
- The robustness of interdependent networks can be improved by adding connectivity links.
- Double-network link allocation strategy yields better performance to single-network link allocation strategy.
- Simulation results indicate that our proposed methods are superior to the current link addition strategies.

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ABSTRACT

Compared with a single and isolated network, interdependent networks have two types of links: connectivity link and dependency link. This paper aims to improve the robustness of interdependent networks by adding connectivity links. Firstly, interdependent networks failure model and four frequently used link addition strategies are briefly reviewed. Furthermore, by defining inter degree–degree difference, two novel link addition strategies are proposed. Finally, we verify the effectiveness of our proposed link addition strategies by comparing with the current link addition strategies in three different network models. The simulation results show that, given the number of added links, link allocation strategies have great effects on the robustness of interdependent networks, i.e., the double-network link allocation strategy is superior to single-network link allocation strategy. Link addition strategies proposed in this paper excel the current strategies, especially for BA interdependent networks. Moreover, our work can provide guidance on how to allocate limited resources to an existing interdependent networks system and optimize its topology to avoid the potential cascade failures.

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1. Introduction

In the past fifteen years, single-layer complex network theory has been studied extensively [1–5]. By modelling a complex system into a network, many useful outcomes have already been applied in the areas of vulnerability analysis [6–16], optimal bandwidth allocation [17], network evolution evaluation [18], isolated communities detection and predictive control strategies [19,20]. However, in modern society, critical infrastructures are not self-sufficient but highly interdependent [21].

* Corresponding author. Tel.: +86 15972976215.

E-mail address: whdwb@whu.edu.cn (B. Wang).

Local failures within one system may be deteriorated by spreading to its dependent networks and cause recursive damages, such as the 2003 North America blackout [22], the 2003 Roman blackout [23] and the 2004 Italia blackout [24]. Hence, it is of theoretical significance and practical value to improve the robustness of interdependent networks system.

Several enhancement methods have been conducted to protect interdependent networks. (1) The first approach is to protect the critical nodes. Ruj et al. [25] point out that interdependent networks disintegrate faster under high degree node attacks compared to random attacks. Liu et al. [26] show that attack strategy considering the connectivity of nodes in both networks will be more effective than only considering the degree of one single network. Further, Nguyen et al. [27] prove that critical node identification problem in interdependent networks is a NP-hard problem and propose a greedy framework to identify the critical nodes in a timely manner. Sen et al. [28] propose a new model to identify the K most vulnerable nodes of interdependent networks. (2) The second approach is to deploy autonomous nodes. Shao et al. [29] conclude that interdependent network's robustness can be improved by deploying autonomous nodes which do not need any support from other network. Schneider et al. [30] propose the strategy based on degree centrality and betweenness centrality to decrease the necessary number of autonomous nodes that guarantee the robustness of the system. (3) The third approach is to adjust dependency link allocation. Parshani et al. [31] find that as the networks become more inter-similar, the system becomes significantly more robust to random failure. Yagan et al. [32] show that the regular allocation strategy that allots exactly the same number of bi-directional interlink to all the nodes in the system yields better performance compared to random allocation and unidirectional interlinks. Reis et al. [33] demonstrate that if interconnections are provided by network hubs, and the connection between networks are moderately convergent, the system of networks is stable and robust to failures. (4) The fourth approach is to refigure the topology of single network by rewiring. Zhou et al. [34] indicate that decreasing assortativity within single network by rewiring could improve the robustness of entire system.

It is worth mentioning that for an existing interdependent networks system, e.g., power grid and its communication network, the current enhancement strategies may have some limitations. Protecting critical nodes is a passive approach since it only aims to decrease the possibility of damages rather than improve the robustness of network. Deploying autonomous nodes, which can effectively increase the robustness of interdependent networks, is expensive and needs to make major technical modifications. Furthermore, adjusting interdependent relationships or refiguring its topology by rewiring in a wide range is difficult to realize due to economic or other various constraints.

In order to improve the robustness of interdependent networks system, adding some connectivity links will be a proper strategy since it is both technically feasible and without impairing the current services. However, previous link addition strategies, e.g., random addition strategy [35–37], low degree addition strategy [36,37], low betweenness addition strategy [36] and algebraic connectivity based addition strategy [38,39], are all designed to optimize single-layer network. In interdependent networks, the vulnerability of isolated network is both affected by its own topology and coupled network. Thus, the effectiveness of the single-layer network link addition strategies should be validated and link addition strategies designed for interdependent networks should be investigated to address the problem.

The rest of this paper is organized as follows: in Section 2, interdependent networks failure model and four frequently used link addition strategies are briefly reviewed. In Section 3, by defining inter degree–degree difference and average inter degree–degree difference, two link addition strategies, i.e., low inter degree–degree difference addition strategy and random inter degree–degree difference addition strategy, are proposed. In Section 4, the impacts of different addition strategies on the robustness of interdependent networks are compared under both single-network link allocation scenario and double-network link allocation scenario. Some conclusions and summaries are shown in Section 5.

2. The model

In Section 2, we review the interdependent networks failure model and make slight modification to better illustrate the vulnerability of interdependent networks under random failures from the perspective of industrial demands. Then, four frequently used single network link addition strategies, namely random addition (RA) link addition strategy, low degree (LD) link addition strategy, low betweenness (LB) link addition strategy and algebraic connectivity based (ACB) link addition strategy, are introduced.

2.1. Interdependent networks failure model

To model interdependent networks, the “one-to-one correspondence” model proposed by Buldyrev et al. [24] is adopted in this paper. For simplicity, we assume networks A and B have the same number of nodes N , namely $N_A = N_B$, and share the same topology. Each node in network A is randomly interdependent with only one node in network B and vice versa. If node i in network A stops functioning as a result of failure or attack, its coupled node j in network B stops functioning too. Thus, the interdependent relationship between coupled nodes can be described as a bidirectional link $A_i \leftrightarrow B_j$. Furthermore, notice that only the nodes belonging to the largest connected component are valid. Since the interdependent networks model we use in this paper is one-to-one correspondence, the number of nodes remaining in network A at the final stage is equal to that remaining in network B , namely $N'_A = N'_B$.

To model the robustness of interdependent networks under random failures, we define p , instead of $(1 - p)$ adopted by Ref. [24], as the fraction of nodes is randomly removed from network A at the first stage since it is more obvious to observe

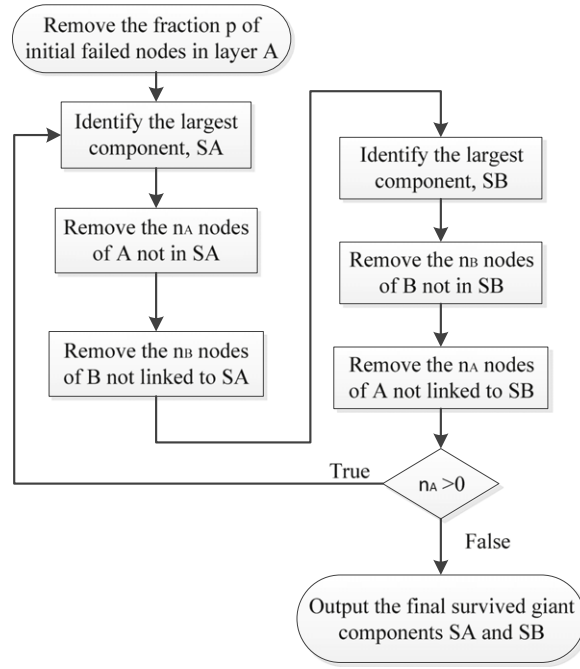


Fig. 1. Vulnerability analysis algorithm of interdependent networks.

the relative drop of performances of interdependent networks under failures. G represents the relative size of the largest connected component of interdependent networks.

$$G = (N'_A + N'_B) / (N_A + N_B) \quad (1)$$

where N_A and N_B are the number of nodes in network A and network B respectively. N'_A and N'_B are the number of survival nodes in network A and network B after attacks.

It needs to be noticed that as p increases, G will eventually decrease to zero. We define the value of critical p as p_c . The vulnerability analysis algorithm used in this paper is shown in Fig. 1.

2.2. Connectivity link addition strategies

• Strategy 1—Random addition (RA)

Random addition strategy is often selected as a reference to compare with other link addition strategies [35–37]. RA link addition algorithm is as follows: at each step, two nodes are chosen randomly and a new link is added. Self-loop and parallel edges are not allowed. This procedure is repeated until the demanded number of links is added.

• Strategy 2—Low degree (LD)

Degree centrality which is the simplest centrality metric reflects a node's importance in its locality [39]. For an undirected network, the degree of a node is equal to the number of links connected to it. LD link addition algorithm is as follows: at each step, degrees of the nodes are calculated and ranked in an increasing order. A connectivity link is added between a pair of nodes with the lowest degree. Self-loop and parallel edges are not allowed. This procedure is repeated until the demanded number of links is added.

• Strategy 3—Low betweenness (LB)

Betweenness centrality is one of the most important metrics to evaluate the routing strategy performance of the network [36]. Betweenness centrality of a node is defined in Ref. [40] as

$$B(v) = \sum_{i \neq j} \frac{\sigma_{ij}(v)}{\sigma_{ij}} \quad (2)$$

where σ_{ij} is the number of shortest paths going from node i to node j , and $\sigma_{ij}(v)$ is the number of shortest paths going from node i to node j passing through the node v .

LB link addition algorithm is as follows: at each step, betweenness of the nodes are calculated and ranked in an increasing order. A connectivity link is added between a pair of nodes with the lowest betweenness. Self-loop and parallel edges are not allowed. This procedure is repeated until the demanded number of links is added.

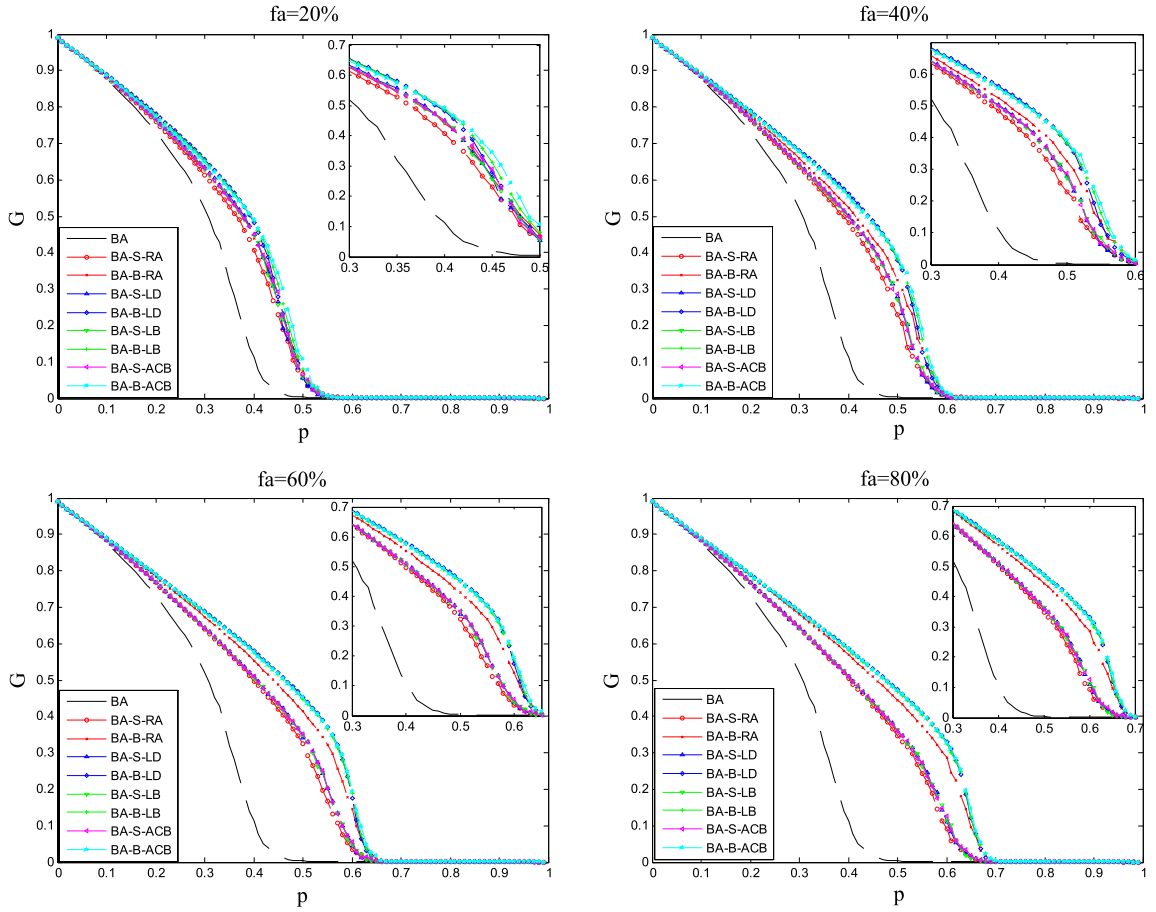


Fig. 2. Performance comparisons among RA, LD, LB and ACB link addition strategy on BA interdependent networks under both-networks scenario and double-network scenario.

• Strategy 4—Algebraic connectivity based (ACB)

Algebraic connectivity, the second smallest eigenvalue of the Laplacian matrix of a network, is a widely used indicator of network robustness. More links need to be removed to disintegrate the network with larger algebraic connectivity. Link addition strategy proposed by Wang et al. [41] is adopted in this paper in order to maximize the algebraic connectivity in a timely manner.

ACB link addition algorithm is as follows: at each step, a pair of nodes, with the maximal absolute difference between the elements of Fiedler vector, is chosen and a connectivity link is added between them. Self-loop and parallel edges are not allowed. This procedure is repeated until the demanded number of links is added.

3. Inter degree–degree difference based addition strategies

Ref. [31] shows that inter-similarity can effectively guarantee the robustness of interdependent networks. To quantitatively evaluate the inter-similarity of interdependent networks from the perspective of degree centrality, inter degree–degree difference and average inter degree–degree difference are defined in Section 3.1. Two corresponding link addition strategies, i.e., low inter degree–degree difference addition strategy and random inter degree–degree difference addition strategy are proposed in Section 3.2.

3.1. Inter degree–degree difference

Inter degree–degree difference (IDD) is defined to evaluate the degree difference between two coupled nodes.

$$IDD_{AB}(u, v) = k_u^A - k_v^B \quad (3)$$

where $IDD_{AB}(u, v)$ is the degree difference between node u in network A and its dependent node v in network B . k_u^A and k_v^B are the degree of node u in network A and degree of node v in network B respectively. Hence, $d_{AB}(u, v) = -d_{BA}(v, u)$.

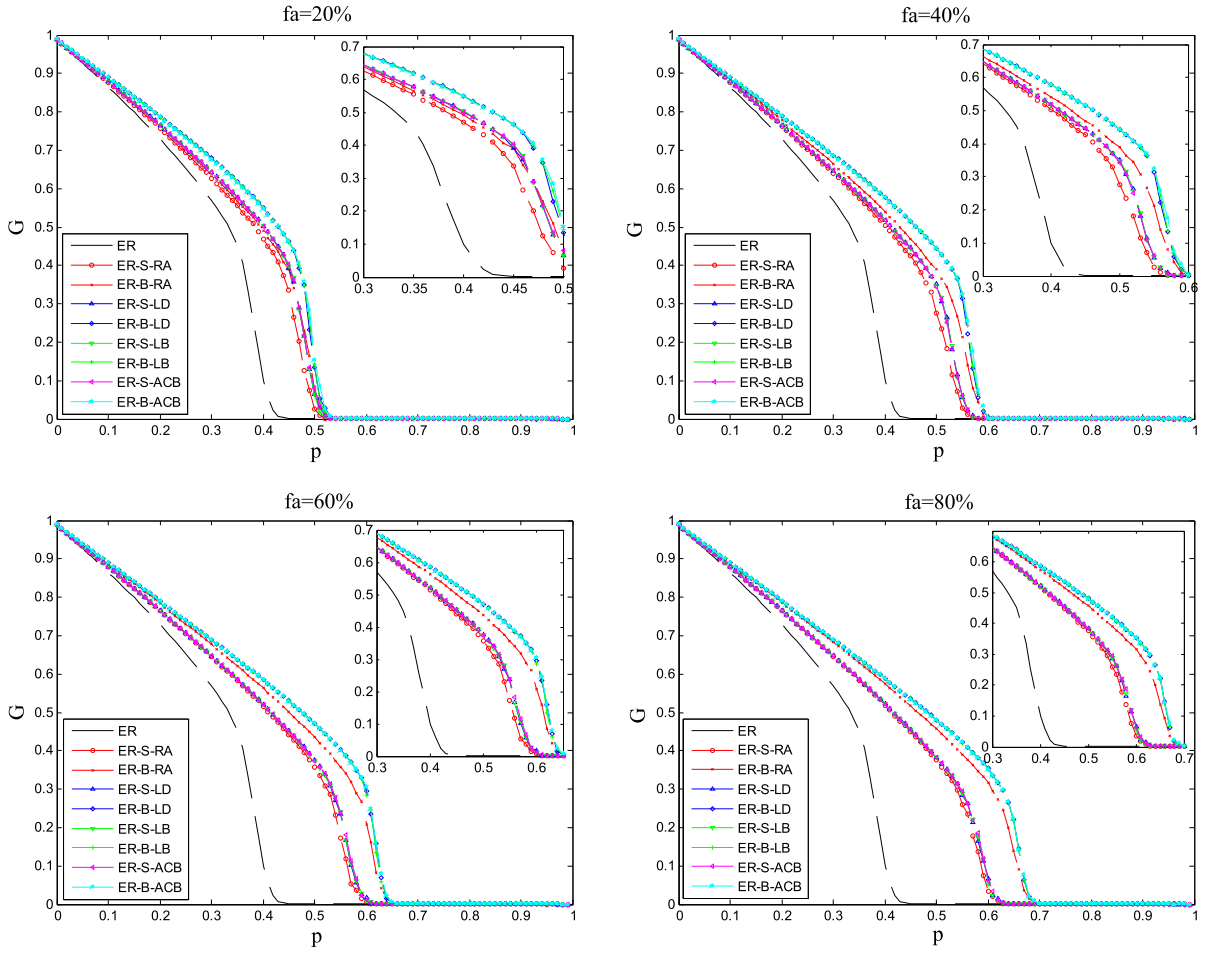


Fig. 3. Performance comparisons among RA, LD, LB and ACB link addition strategy on ER interdependent networks under both single-networks scenario and double-network scenario.

$IDD_{AB}(u, v) < 0$ indicates that a low degree node u in network A is coupled with a higher degree node v in network B . High degree nodes, which are regarded as critical nodes, play an important role in maintaining the connectivity of isolated network. In interdependent networks, the vulnerability of a node is both affected by its own degree and the coupled node. From this point, it implies that a vulnerable node in network A is interdependent with an important node in network B . In order to compensate the unbalance between the coupled nodes, connectivity links between two unconnected nodes with negative inter degree–degree difference value should be added to network A . Similarly, when $IDD_{AB}(u, v) > 0$, connectivity links between two unconnected nodes with negative inter degree–degree difference in network B should be added. If $IDD_{AB}(u, v) = 0$, node u in network A and its corresponding node v in network B share the same degree.

Average inter degree–degree difference (AIDD) is used to evaluate the overall degree difference of the entire interdependent networks system. AIDD is defined as the average absolute inter degree–degree difference per node in an interdependent network.

$$AIDD = \sum_{i=1}^N |IDD_i| / N \quad (4)$$

where N denotes the number of interdependent links.

Since heterogeneous network, e.g., scale-free network, has a broader degree distribution than homogeneous network, e.g., Erdős–Rényi network, the value of AIDD for randomly coupled heterogeneous interdependent networks is larger than that of randomly coupled homogeneous interdependent networks.

3.2. Inter degree–degree difference based addition strategies

Taking into account the inter degree–degree difference between isolated networks, we propose two novel link addition strategies.

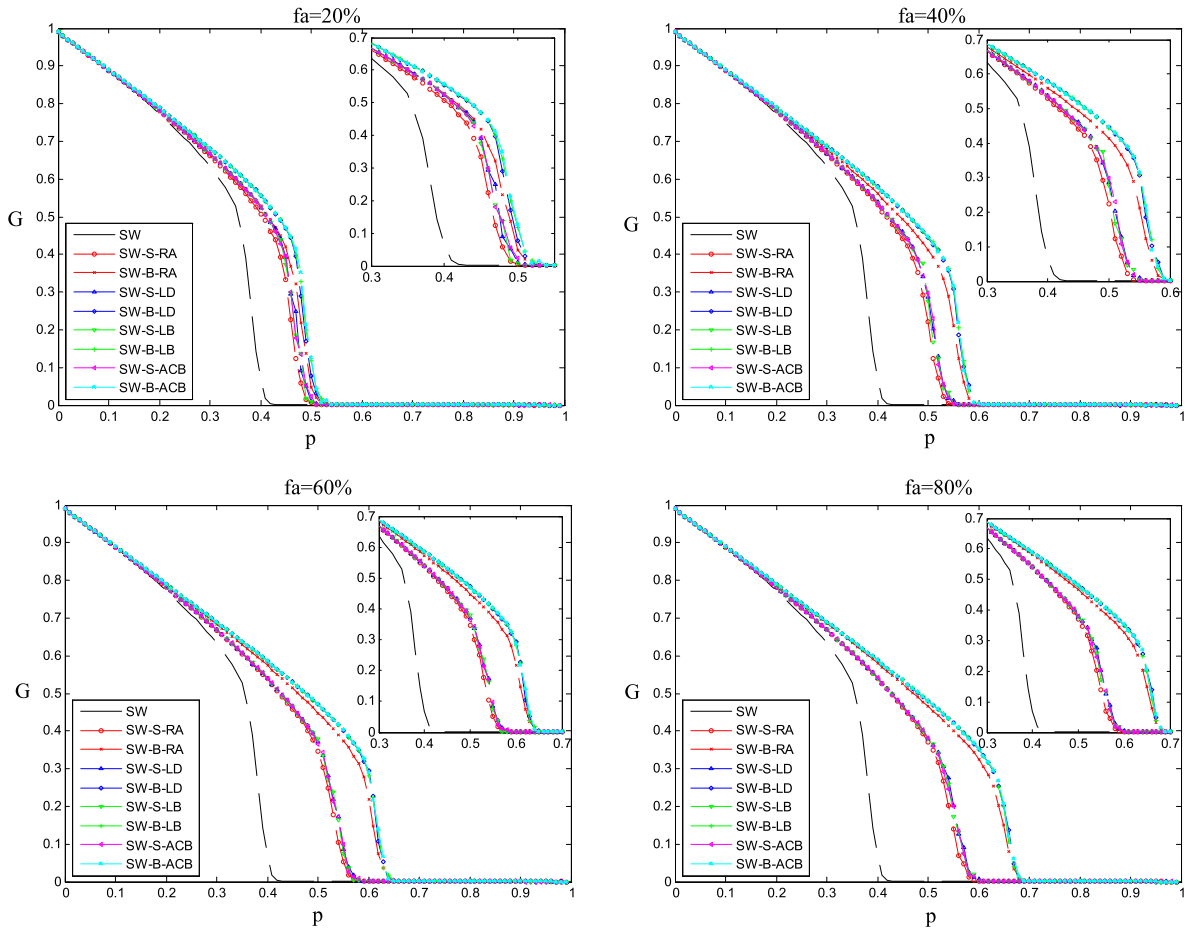


Fig. 4. Performance comparisons among RA, LD, LB and ACB link addition strategy on WS interdependent networks under both single-networks scenario and double-network scenario.

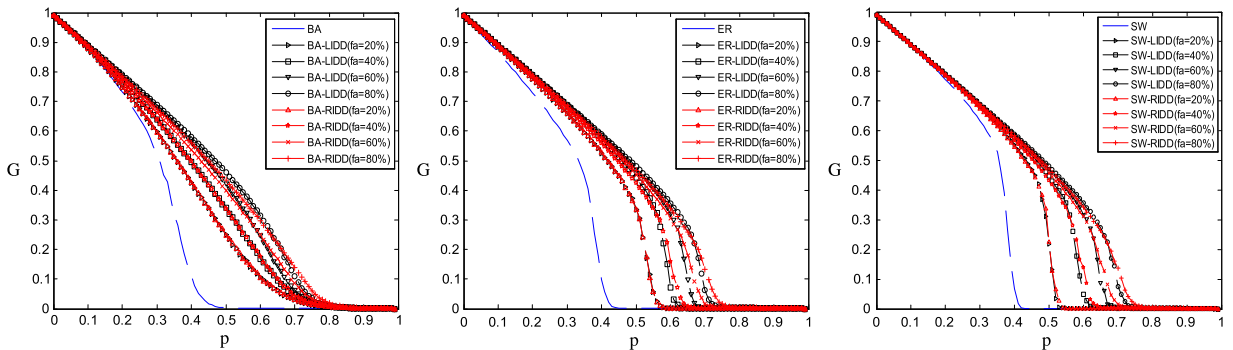


Fig. 5. Performance comparison between LIDD link addition strategy and RIDD link addition strategy.

- Strategy 5—Low inter degree–degree difference (LIDD)

At each step, inter degree–degree differences of all nodes are calculated and ranked in an increasing order. For each isolated network, network *A* and network *B*, a link is added between a pair of unconnected nodes with the lowest negative inter degree–degree difference; otherwise, a link is added between a pair of unconnected nodes with the lowest degree. Self-loop and parallel edges are not allowed. This procedure is repeated until the demanded number of links is added.

- Strategy 6—Random inter degree–degree difference (RIDD)

At each step, inter degree–degree differences of all nodes are calculated. For each isolated network, network *A* and network *B*, a link is randomly added between a pair of unconnected nodes with negative inter degree–degree difference;

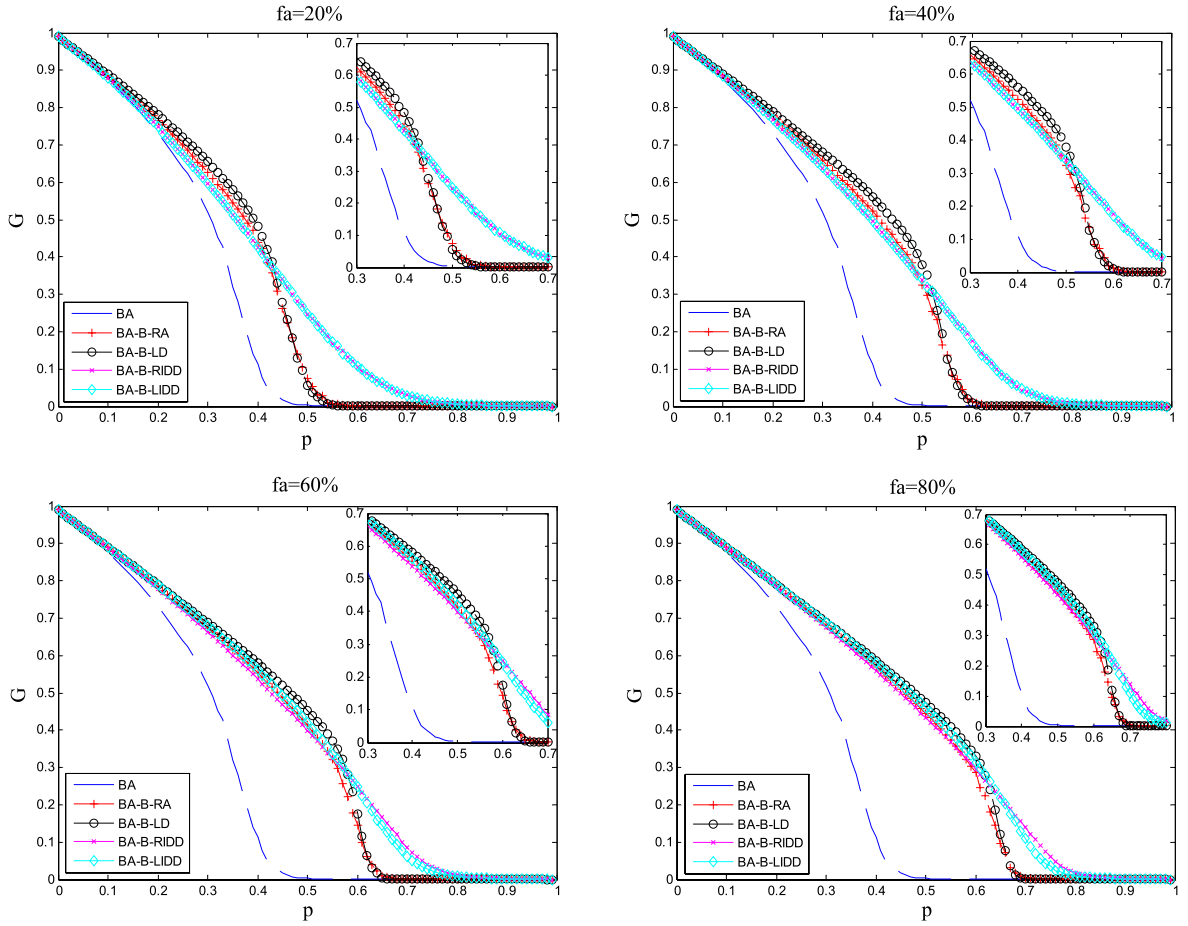


Fig. 6. Performance comparisons among RA, LD, RIDD and LIDD link addition strategy on BA interdependent networks under double-network scenario.

otherwise, a link is randomly added between a pair of unconnected nodes. Self-loop and parallel edges are not allowed. This procedure is repeated until the demanded number of links is added.

4. Numerical simulations

To simulate interdependent networks, we adopt three types of classic artificial networks, i.e., the scale-free network, the Erdős–Rényi (ER) random network and the small-world network, to generate the single network. For simplicity and without loss of generality, we build scale-free network with network size $N = 1000$, $m = 2$, $m_0 = 4$, average degree $\langle k \rangle \approx 4$ according to the classic Barabási–Albert (BA) network model [2], ER network model with $N = 1000$, connecting probability $\rho_{ER} = 0.04$ according to binomial model [3] and small-world network with network size $N = 1000$, $k = 4$, rewiring probability $\rho_{WS} = 0.5$ according to Watts–Strogatz (WS) network model [1]. We duplicate each single network to create two topologically identical monolayers (A and B) and randomly build the one-to-one correspondence interdependencies. The above three network models distinguish each other in the aspect of degree distribution. BA network has scale-free degree distribution, ER network has Poisson distribution and WS network's degree distribution interpolates between regular network and ER network.

Under the constraint of economic cost, it is not practical to add connectivity links infinitely. We define the fraction of connectivity links added as f_a .

$$f_a = M' / (M_A + M_B) \quad (5)$$

where M' is the number of connectivity links added, M_A and M_B are the number of connectivity links of network A and network B respectively. In this paper, $f_a = M' / 4000$.

Given the number of added links, the researchers should consider the allocation strategies. For simplicity, we assume that the links are allocated to two extreme scenarios, i.e., single-network allocation scenario which allocates all connectivity links to one single network and double-network allocation scenario which allocates exactly the same number of connectivity links to both isolated networks.

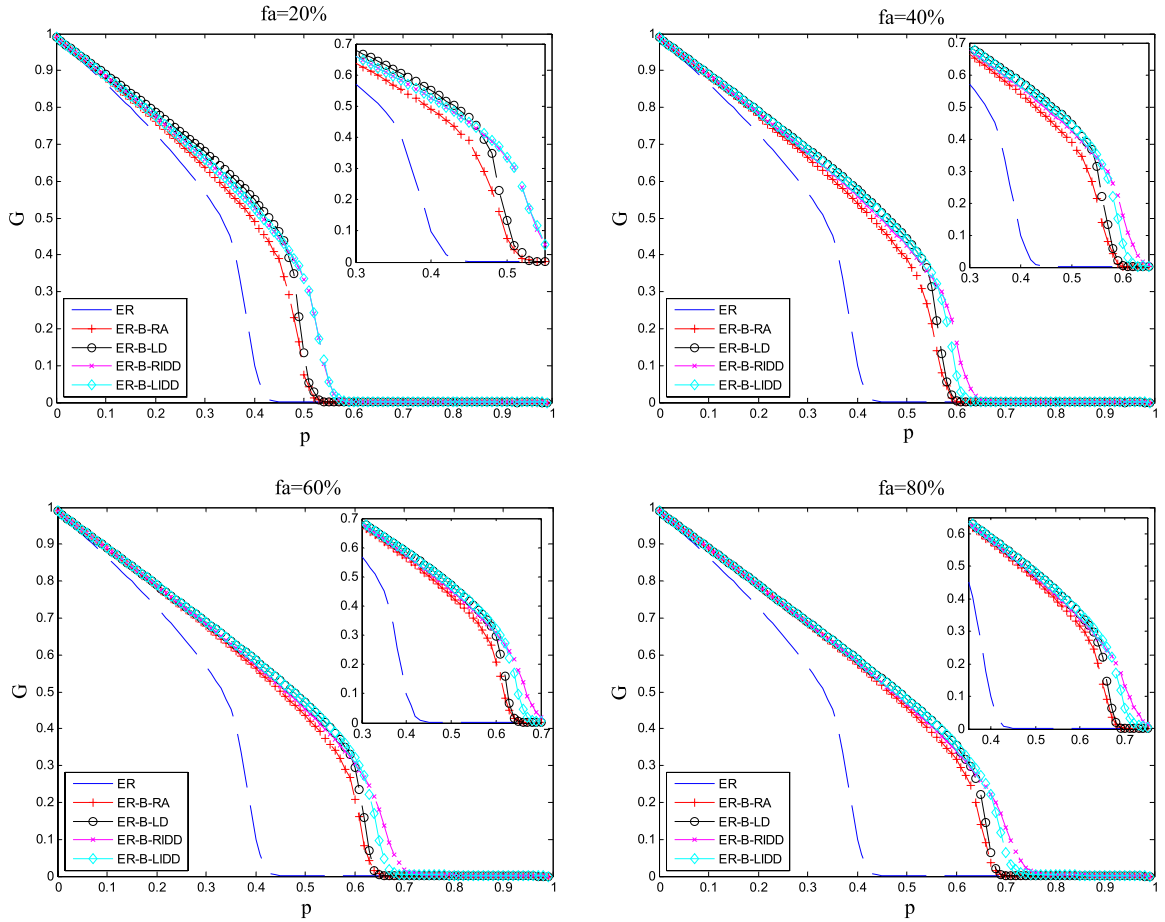


Fig. 7. Performance comparisons among RA, LD, RIDD and LIDD link addition strategy on ER interdependent networks under double-network scenario.

To compare the effects of different link addition strategies, we can evaluate the robustness of interdependent networks from two perspectives: G reflects the ability of interdependent networks to continue to provide services after initial failures and p_c represents the maximum tolerant ability against random failures. Each simulation result is obtained by averaging over simulations of 20 interdependent networks. 100 random interdependencies are independently built for each two-layer interdependent networks system and 20 independent simulations are performed for each link addition strategy. Thus, each curve is the average value of 40 000 simulations.

We use the form A–B–C to illustrate the interdependent networks after adding links. A, B and C denotes interdependent network type, link allocation scenario and link addition strategy respectively. For example, BA–S–RA represents the modified interdependent networks after applying RA addition strategy to one single network of the given BA interdependent networks.

Firstly, we conduct the performance comparisons among the existing four link addition strategies discussed in Section 2 when $f_a = 0.2$, $f_a = 0.4$, $f_a = 0.6$, and $f_a = 0.8$ respectively. In Figs. 2–4, we can clearly observe the following situations:

- (1) Adding connectivity links to an existing interdependent networks system does enhance its tolerance against cascading failures and higher robustness can be achieved as f_a increases.
- (2) Under the same number of added links, double-network allocation scenario yields better performance to single-network allocation scenario in improving the robustness of interdependent networks. It is understandable that, in interdependent networks, the robustness of single network is affected by both its own topology structure and the coupled network. Merely enhancing the robustness of one single network may not be as effective as we previously think. This finding has an important implication that it can provide guidance on how to allocate limited resources to an interdependent networks system to avoid the potential cascade failures.
- (3) Three link addition strategies, i.e., low degree addition strategy (LD), low betweenness addition strategy (LB) and algebraic connectivity based addition strategy (ACB), which mainly build links between low degree nodes, can be regarded as approximately equivalent in enhancing the robustness of interdependent networks. To make a reasonable explanation, we use Pearson correlation coefficient ρ_{D-B} to illustrate the correlation between degree centrality and

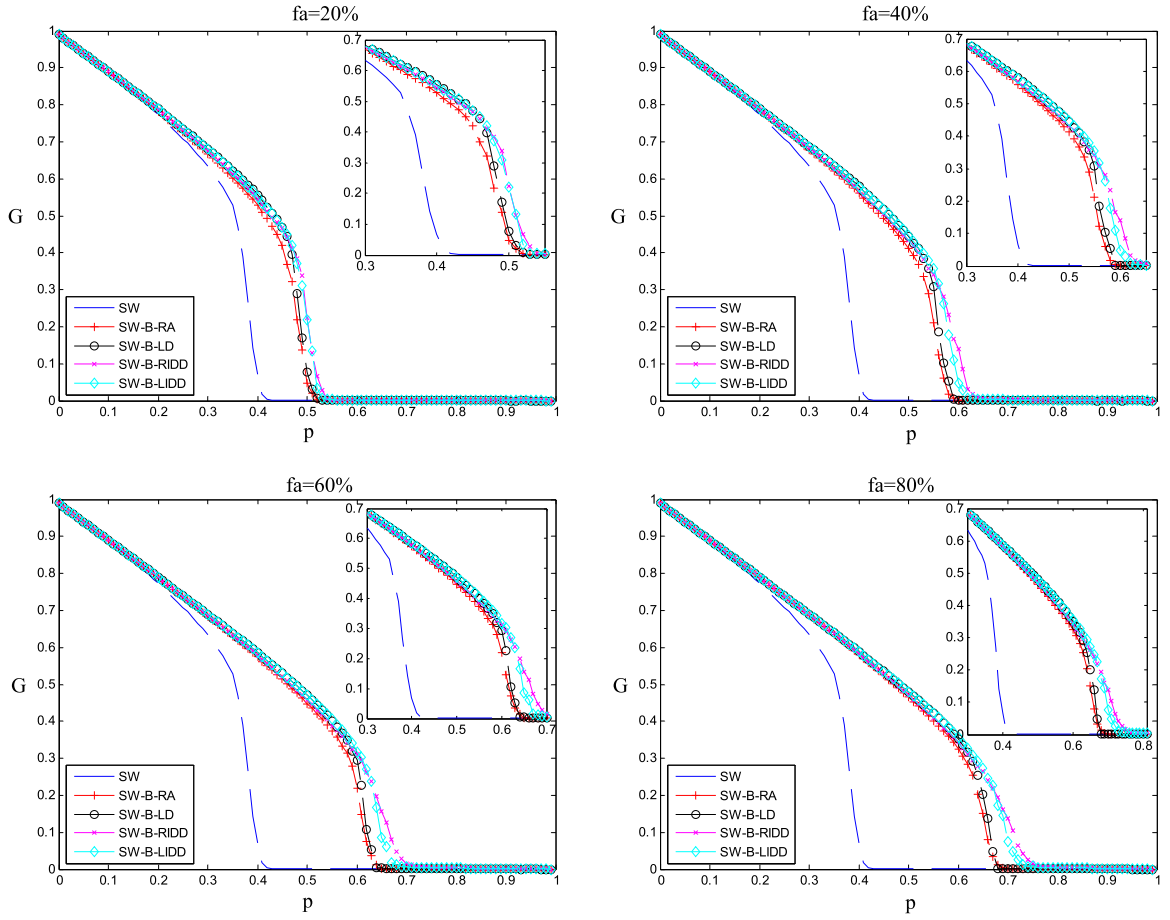


Fig. 8. Performance comparisons among RA, LD, RIDD and LIDD link addition strategy on WS interdependent networks under double-network scenario.

betweenness centrality in ER network, BA network and WS network.

$$\rho_{D-B} = \frac{\sum_{i=1}^n [k(x_i) - \langle k(x_i) \rangle][B(x_i) - \langle B(x_i) \rangle]}{\sqrt{\sum_{i=1}^n [k(x_i) - \langle k(x_i) \rangle]^2} \sqrt{\sum_{i=1}^n [B(x_i) - \langle B(x_i) \rangle]^2}} \quad (6)$$

where $k(x_i)$ and $B(x_i)$ are the degree centrality and betweenness centrality of node i respectively, $\langle k(x_i) \rangle$ and $\langle B(x_i) \rangle$ stand for the average value of the degree centrality and betweenness centrality respectively.

We obtain that $\rho_{D-B}^{ER} \approx 0.938$, $\rho_{D-B}^{BA} \approx 0.942$ and $\rho_{D-B}^{WS} \approx 0.89$. The results indicate that, in ER network and WS network, degree centrality and betweenness centrality are highly positive correlated which means low degree nodes are commonly low betweenness nodes too. For the convenience of calculation, it is reasonable to choose LD addition strategy instead of the other two strategies.

- (4) Under single-network addition scenario, LD addition strategy only shows negligent advantages over RA addition strategy. While under double-network allocation scenario, it is obvious that LD addition strategy excels RA addition strategy.

Above all, given the number of added links, LD addition strategy under double-network allocation scenario should be the first choice. However, none of the four link addition strategies can avoid sharply dropping of survival nodes G near p_c . This makes the system difficult to be protected or controlled.

Secondly, in order to verify the effectiveness of our proposed link addition strategies, we compare the performances of RA, LD, LIDD and RIDD under double-network allocation scenario.

In Figs. 5(a) and 6, it is evident that both RIDD addition strategy and LIDD addition strategy can enhance the robustness of BA interdependent networks against cascading failures. Compared with the four link addition strategies previously discussed, RIDD addition strategy and LIDD addition strategy can not only avoid the sharply dropping of survival nodes G but also dramatically improve the largest failure tolerance ability p_c , even when f_a is low, i.e. $f_a = 20\%$. RIDD addition strategy

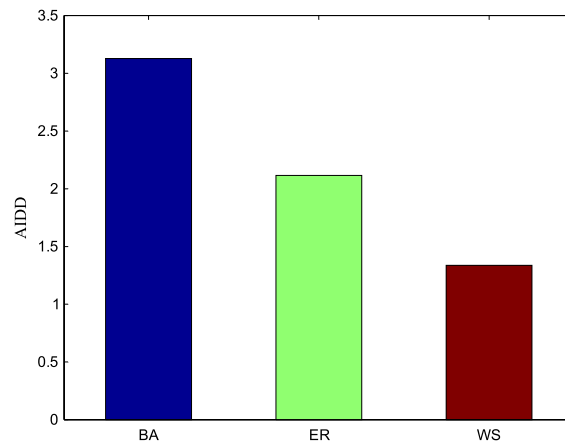


Fig. 9. Comparison of AIDD between BA interdependent networks, ER interdependent networks and WS interdependent networks.

and LIDD addition strategy can be regarded as equivalent when small number of links is added which means that there is not enough number of links to compensate all inter degree–degree difference. It implies that, regardless of the link addition strategies, the inter degree–degree difference plays an important role in improving the robustness of BA interdependent networks. Following the rising of f_a , RIDD addition strategy exhibits minor better performances associated with increasing p_c .

Analogously, in Figs. 5(b)–(c) and 7–8, RIDD addition strategy and LIDD addition strategy are more effective than the existing four link addition strategies in the aspect of increasing the largest failure tolerance ability p_c . However, unlike BA interdependent networks, RIDD addition strategy and LIDD addition strategy only show a minor advantage in improving the value of p_c due to the different network structure. In Fig. 9, we can clearly observe that the AIDD for BA interdependent networks is higher than that of ER interdependent networks and WS interdependent networks.

5. Conclusion

How to improve the robustness of interdependent networks is a young field up to now. Several efforts have been conducted to address the problem. However, none of them investigate the impacts of adding connectivity links on the robustness of interdependent networks. In this paper, we try to fill this gap. Given the number of connectivity links, we compare the performances of four frequently used link addition strategies in interdependent networks under two scenarios: single-network link allocation scenario and double-network link allocation scenario. We find that double-network link allocation strategy has considerable advantages over single-network link allocation strategy. Taking into account the interdependent relationships, inter degree–degree difference (IDD) is defined and two novel link addition strategies, low inter degree–degree difference (LIDD) addition strategy and random inter degree–degree difference (RIDD) addition strategy, are proposed. The simulation results show that our proposed link addition strategies are superior to the existing four link addition strategies in improving the robustness of interdependent networks, especially for interdependent networks with high average inter degree–degree difference. Our results may be useful in providing guidance on how to allocate limited resources to an existing interdependent networks system and optimize its topology to avoid the potential cascade failures.

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