Adaptive Nonsingular Terminal Sliding Mode Control Design for Near Space Hypersonic Vehicles

Ruimin Zhang Lu Dong Changyin Sun

***Abstract*—This paper presents an adaptive nonsingular terminal sliding mode approach for the attitude control of near space hypersonic vehicles (NSHV) in the presence of parameter uncertainties and external disturbances. Firstly, a novel nonsingular terminal sliding surface is developed and its finite-time convergence is analyzed. Then, an adaptive nonsingular terminal sliding mode control law is proposed, which is chattering free. In the proposed approach, all parameter uncertainties and external disturbances are lumped into one term, which is estimated by an adaptive uncertainty estimation for eliminating the boundary requirement needed in the conventional control design. Subsequently, stability of the closed-loop system is proven based on Lyapunov theory. Finally, the proposed approach is applied to the attitude control design for NSHV. Simulation results show that the proposed approach attains a satisfactory performance in the presence of parameter uncertainties and external disturbances.**

***Index Terms*—Near space hypersonic vehicle, sliding mode control, nonsingular terminal sliding mode control, tracking control.**

I. INTRODUCTION

NEAR space is the region of earth's atmosphere that lies between 20 km and 100 km above sea level. NSHV is a vehicle which operates in near space. Compared with ordinary airplanes, it is characterized by large flight envelope, high altitude, fast velocity, complicated flight environment, strong external disturbances, etc. Therefore, control design for NSHV faces significant difficulties such as strong nonlinearity, highly time-varying dynamics, and large parameter uncertainties[1, 2]. Various control approaches have been presented to tackle these problems, such as linear parameter varying control[3−5], dynamic inversion[6], backstepping control[7−9], predictive control[10−12], sliding mode control (SMC)[13−15] and fuzzy control[15, 16].

Among these control approaches, sliding mode control is a well-known and powerful control scheme which has many attractive characteristics such as good transient, fast response and insensitivity to parameter uncertainties and external disturbances. Therefore, SMC has been successfully applied to flight control systems. In [17], SMC was used in ascent and reentry attitude control design for X-33 in the presence of external disturbances. Durmaz et al.[18] proposed a sliding mode controller with an adaptive sliding surface for NSHV under external disturbances. In [14], an adaptive sliding mode controller combined with feedback linearization was designed for the longitudinal dynamics of a generic hypersonic vehicle. In these papers, the sliding mode controllers were designed based on linear sliding surfaces, and thus the system state only could reach the equilibrium point asymptotically. To overcome this drawback, terminal sliding mode control (TSMC) with nonlinear sliding surface was proposed based on the concept of terminal attractor[19]. Compared with conventional SMC with linear sliding surface, TSMC has some improved characteristics such as faster convergence in finite time and higher control precision. Nevertheless, the originally proposed TSMC method has two disadvantages. The first is singularity problem, which may cause infinitely large control values, and the second is chattering phenomenon, which is caused by high-frequency control switching. To address the first issue, some nonsingular TSMC (NTSMC) approaches have been proposed[20−22]. NTSMC technique was employed to design attitude controller for aerospace vehicle, and tracking performance was improved in [23]. For the second problem, various methods have been proposed to reduce or eliminate the chattering, such as the boundary layer method[24], high order sliding mode[25]. Yu et al.[26] proposed a continuous NTSMC approach for rigid robotic manipulators based on a novel continuous reaching law, so that the finite-time reachability to a boundary layer was guaranteed under external disturbances. In [27−29], NTSMC was combined with the second order sliding mode to design flight control systems for hypersonic vehicles, and thus, the designed control systems made the system track desired commands in less time and eliminate chattering phenomena. An alternative simple way to solve the chattering problem is to replace the discontinuous term with an estimate of the uncertainties in an adaptive manner. Based on the effective online estimation of uncertainties and using the dynamic behaviour of a sliding variable, a chattering-free SMC was constructed[30].

Motivated by the above discussion, in this study, an adaptive nonsingular terminal sliding mode control approach is developed for the attitude control of NSHV under parameter uncertainties and external disturbances. Meanwhile, the proposed approach can effectively solve the chattering phenomena.

Firstly, a novel nonsingular terminal sliding surface is proposed, and its finite-time convergence to zero is analytically proved. Then, an adaptive law for nonsingular terminal sliding mode control is derived based on an online uncertainty estimation. Thereinto, the adaption law of uncertainty estimation is derived based on the Lyapunov technique. Thus the priori

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knowledge of the upper bound of lumped uncertainty is not required, and the controller design is greatly simplified. Finally, the proposed approach is used to design attitude control system for NSHV, and simulation results are presented to verify efficiency and robustness of the proposed approach.

The major contributions of this paper are stated as follows: 1) Proposing a novel nonsingular terminal sliding surface, which not only can provide a finite-time convergence, but also makes the tracking error in the sliding mode converge to the origin within a maximum settling time; 2) Developing an adaptive estimation approach for the lumped uncertainty under the fast-varying condition, and eliminating the requirement for the upper bound of the lumped uncertainty; 3) Developing a continuous control law that eliminates the chattering phenomena.

The rest of this paper is organized as follows. In Section II, the problem statement and preliminaries are formulated. In Section III, we propose an adaptive nonsingular terminal sliding mode control approach. The proposed approach is applied to designing the attitude control system for NSHV in Section IV. Section V presents simulation to validate the proposed approach. Finally, some conclusions are made in Section VI.

II. Problem formulation and preliminaries

A. Problem Formulation

A nonlinear system under model uncertainties and external disturbances is described as

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + \Delta f(\boldsymbol{x}) + (g(\boldsymbol{x}) + \Delta g(\boldsymbol{x}))\boldsymbol{u} + \boldsymbol{d}(t), \tag{1}$$

where $\boldsymbol{x} \in \mathbf{R}^n$ is state vector of the system, $\boldsymbol{u} \in \mathbf{R}^n$ is control input vector, $f(\boldsymbol{x}) \in \mathbf{R}^n$ and $g(\boldsymbol{x}) \in \mathbf{R}^{n\times n}$ are known nonlinear system functions of state variables and time, $\Delta f(\boldsymbol{x})$ and $\Delta g(\boldsymbol{x})$ represent model uncertainties, and $\boldsymbol{d}(t)$ denotes external disturbances.

In this study, the aim of control is to design an appropriate sliding mode controller for the nonlinear system (1), so that system output $\boldsymbol{x}$ can track its desired trajectory $\boldsymbol{x}_c \in \mathbf{R}^n$ under parameter uncertainties and external disturbances. To develop the desired controller, the following assumptions are made.

Assumption 1. Function $g(\boldsymbol{x})$ is invertible.

Assumption 2. The lumped uncertainty $\boldsymbol{\psi} = \Delta f + \Delta g\boldsymbol{u} + \boldsymbol{d}$ and its derivative are bounded as

$$\|\boldsymbol{\psi}\| \le D_1,\ \ \|\dot{\boldsymbol{\psi}}\| \le D_2, \tag{2}$$

where D_1 and D_2 are unknown positive constants.

B. Preliminaries

Some notations and lemmas which will be useful later are introduced in this subsection.

Notation. Define $sig^{\gamma}(x) = |x|^{\gamma}\,\mathrm{sign}(x)$, where $\gamma > 0$, $x \in \mathbf{R}$. For vector $\boldsymbol{x} = [x_1, \cdots, x_n]^{\mathrm{T}}$, the notation $sig^{\gamma}(\boldsymbol{x})$ represents the vector $sig^{\gamma}(\boldsymbol{x}) = [sig^{\gamma}(x_1), \cdots, sig^{\gamma}(x_n)]^{\mathrm{T}}$.

Lemma 1[31]**.** Consider a nonlinear system

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t)),\ f(0) = 0,\ \boldsymbol{x} \in \mathbf{R}^n, \tag{3}$$

where $f: D \to \mathbf{R}^n$ is continuous on an open neighbourhood D of the origin $\boldsymbol{x} = 0$.

Suppose there exists a continuous function $V(\boldsymbol{x}) : D \to \mathbf{R}$ such that the following conditions hold:

1) V is positive definite.

2) There exist real numbers $c > 0$, $a \in (0, 1)$ and an open neighbourhood $\mho \subseteq D$ of the origin such that

$$\dot{V}(\boldsymbol{x}) + cV^a \le 0, \quad \forall \boldsymbol{x} \in \mho \backslash \{0\}\,. \tag{4}$$

Then the origin is a finite-time stable equilibrium of system (3). The settling time, depending on the initial state $\boldsymbol{x}(0) = \boldsymbol{x}_0$, satisfies $T(\boldsymbol{x}_0) \le \frac{V^{1-a}(\boldsymbol{x}_0)}{c(1-a)}$. If $D = \mathbf{R}^n$ and V is radially unbounded, system (3) is globally finite-time stable.

Lemma 2[22]**.** Suppose $a_1, a_2, \cdots, a_n \in \mathbf{R}$ and $0 < q < 2$, then the following inequality holds:

$$|a_1|^q + |a_2|^q + \cdots + |a_n|^q \ge (a_1^2 + a_2^2 \cdots + a_n^2)^{q/2}. \tag{5}$$

III. Adaptive nonsingular terminal sliding mode control design

In this section, an adaptive scheme for nonsingular terminal sliding mode control is proposed with an adaptive uncertainty estimator for the uncertain system (1). The proposed control scheme can make the resulting closed-loop system stable.

Firstly, a novel nonsingular terminal sliding surface is designed as

$$\boldsymbol{s} = \boldsymbol{e} + \int_0^t (k_1 sig^{1+\frac{1}{\gamma}}(\boldsymbol{e}) + k_2 sig^{1-\frac{1}{\gamma}}(\boldsymbol{e}))\mathrm{d}\tau, \tag{6}$$

where $\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{x}_c$, $k_1 = \mathrm{diag}\{k_{11}, \cdots, k_{1n}\}$, $k_2 = \mathrm{diag}\{k_{21}, \cdots, k_{2n}\}$, $k_{ij} > 0$ $(i = 1, 2,\ j = 1, \cdots, n)$ and $\gamma > 1$ are constants.

Once the tracking error reaches the sliding surface, $\boldsymbol{s}$ satisfies the equation $\dot{\boldsymbol{s}} = 0$. Then the sliding mode dynamics is derived as follows:

$$\dot{\boldsymbol{s}} = \dot{\boldsymbol{e}} + k_1 sig^{1+\frac{1}{\gamma}}(\boldsymbol{e}) + k_2 sig^{1-\frac{1}{\gamma}}(\boldsymbol{e}) = 0, \tag{7}$$

$$\dot{\boldsymbol{e}} = -k_1 sig^{1+\frac{1}{\gamma}}(\boldsymbol{e}) - k_2 sig^{1-\frac{1}{\gamma}}(\boldsymbol{e}). \tag{8}$$

Theorem 1. Consider the terminal sliding mode dynamics (8). The system is finite-time stable and its equilibrium point $\boldsymbol{e} = 0$ is reached in a finite time T_1, given by

$$T_1 \le \frac{\gamma \|\boldsymbol{e}(0)\|^{\frac{1}{\gamma}} 2^{\frac{1}{\gamma}}}{\underline{k}_2}, \tag{9}$$

where $\underline{k}_2 = \min\limits_{i=1,\cdots,n}\{k_{2i}\} > 0$.

Proof. The Lyapunov function is defined as follows:

$$V = \frac{1}{2}\sum_{i=1}^{n} e_i^2. \tag{10}$$

Differentiating V with respect to time and using (8), one can obtain

$$\begin{aligned}\dot V = &\sum_{i=1}^{n} e_i\dot e_i = \sum_{i=1}^{n} e_i(-k_{1i}sig^{1+\frac{1}{\gamma}}(e_i) - k_{2i}sig^{1-\frac{1}{\gamma}}(e_i)) = \\ &\sum_{i=1}^{n}(-k_{1i}\left|e_i\right|^{2+\frac{1}{\gamma}} - k_{2i}\left|e_i\right|^{2-\frac{1}{\gamma}}) = \\ &-\sum_{i=1}^{n}k_{1i}\left|e_i\right|^{2+\frac{1}{\gamma}} - \sum_{i=1}^{n}k_{2i}\left|e_i\right|^{2-\frac{1}{\gamma}} \le \\ &-\underline{k}_1\sum_{i=1}^{n}\left|e_i\right|^{2+\frac{1}{\gamma}} - \underline{k}_2\sum_{i=1}^{n}\left|e_i\right|^{2-\frac{1}{\gamma}} \le \\ &-\underline{k}_2\sum_{i=1}^{n}\left|e_i\right|^{2-\frac{1}{\gamma}},\end{aligned} \tag{11}$$

where $\underline{k}_1 = \min\limits_{i=1,\cdots,n}\{k_{1i}\} > 0$ and $\underline{k}_2 = \min\limits_{i=1,\cdots,n}\{k_{2i}\} > 0$.

According to Lemma 2, we can obtain

$$\begin{aligned}\dot V \le &-\underline{k}_2\sum_{i=1}^{n}\left|e_i\right|^{2-\frac{1}{\gamma}} \le \\ &-\underline{k}_2(\sum_{i=1}^{n}e_i^2)^{1-\frac{1}{2\gamma}} = \\ &-\underline{k}_2(2V)^{1-\frac{1}{2\gamma}} = \\ &-2^{1-\frac{1}{2\gamma}}\underline{k}_2V^{1-\frac{1}{2\gamma}}.\end{aligned} \tag{12}$$

So, from Lemma 1, the tracking error $\boldsymbol{e}$ will converge to zero in a finite time T_1 ($T_1 \le \dfrac{\gamma\|\boldsymbol{e}(0)\|^{\frac{1}{\gamma}}2^{\frac{1}{\gamma}}}{\underline{k}_2}$). □

Remark 1. The novel nonsingular terminal sliding surface (6) is different from some existing terminal sliding surfaces that are expressed as

$$s = \dot e + ke^{p/q}, s = e + k_1e + k_2e^{p/q}, \tag{13}$$

where $e \in \mathbf{R}$, $k, k_1, k_2 > 0$, and $q > p > 0$ are odd integers. It is noted that for an error e, if $e < 0$, the fractional power p/q may lead to the term $e^{p/q} \notin \mathbf{R}$, which leads to $\dot e \notin \mathbf{R}$. In contrast, the new nonsingular terminal sliding surface (6) overcomes this drawback.

Remark 2. A nonsingular terminal sliding surface has been proposed in [22] as

$$\sigma = e + c\int_0^t sig^a(e)\mathrm{d}\tau, \tag{14}$$

where $c > 0$ and $0 < a < 1$ are constants. The proposed nonsingular terminal sliding surface (6) can guarantee a high convergence rate even if the tracking error is far away from the equilibrium point. It can be regarded as an extension of the sliding surface (14).

Remark 3. For the terminal sliding mode dynamics (8), the tracking error e_i can converge to the origin within a maximum settling time $\overline{T}_i$ independent of initial state $e_i(0)$, given by

$$\overline{T}_i < \frac{\pi\gamma}{2\sqrt{k_{1i}k_{2i}}}. \tag{15}$$

Consider the following differential equation

$$\dot e_i = -k_{1i}sig^{1+\frac{1}{\gamma}}(e_i) - k_{2i}sig^{1-\frac{1}{\gamma}}(e_i). \tag{16}$$

The settling time of (16) is determined by

$$\overline{T}_i = \int_0^{|e_i(0)|}\frac{\mathrm{d}e_i}{k_{1i}{e_i}^{1+\frac{1}{\gamma}} + k_{2i}{e_i}^{1-\frac{1}{\gamma}}}. \tag{17}$$

Based on the study of Parsegov and Polyakov[32], we choose an alternate variable $z_i = {e_i}^{\frac{1}{\gamma}}$. Then, we obtain

$$\begin{aligned}\overline{T}_i = &\int_0^{|e_i(0)|^{1/\gamma}}\frac{\gamma{z_i}^{\gamma-1}\mathrm{d}z_i}{k_{1i}{z_i}^{\gamma+1} + k_{2i}{z_i}^{\gamma-1}} = \\ &\gamma\int_0^{|e_i(0)|^{1/\gamma}}\frac{\mathrm{d}z_i}{k_{1i}{z_i}^2 + k_{2i}} = \\ &\frac{\gamma}{\sqrt{k_{1i}}k_{2i}}\arctan\left(\sqrt{\frac{k_{1i}}{k_{2i}}}|e_i(0)|^{1/\gamma}\right) \le \\ &\frac{\pi\gamma}{2\sqrt{k_{1i}k_{2i}}}.\end{aligned} \tag{18}$$

Inequality (15) implies that the tracking error can reach the origin in a desired time by choosing the appropriate parameters γ, k_1 and k_2.

After the appropriate sliding surface has been chosen, the next step is to design a sliding mode control law such that the closed-loop system is stable.

Differentiating $\boldsymbol{s}$ with respect to time and utilizing (1), one can obtain

$$\dot{\boldsymbol{s}} = f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{\psi} - \dot{\boldsymbol{x}}_c + k_1sig^{1+\frac{1}{\gamma}}(\boldsymbol{e}) + k_2sig^{1-\frac{1}{\gamma}}(\boldsymbol{e}). \tag{19}$$

To implement NTSMC controller, a fast terminal-sliding-mode type reaching law is selected as[26]

$$\dot{\boldsymbol{s}} = -l_1\boldsymbol{s} - l_2sig^{\eta}(\boldsymbol{s}), \tag{20}$$

where design matrices $l_1, l_2 \in \mathbf{R}^{n\times n}$ are constant, diagonal and positive definite, and $0 < \eta < 1$ is a constant.

By combining (19) with (20), the control law can be designed as

$$\begin{aligned}\boldsymbol{u} = &-g^{-1}[f - \dot{\boldsymbol{x}}_c + k_1sig^{1+\frac{1}{\gamma}}(\boldsymbol{e}) + k_2sig^{1-\frac{1}{\gamma}}(\boldsymbol{e}) + \boldsymbol{\psi} + \\ &l_1\boldsymbol{s} + l_2sig^{\eta}(\boldsymbol{s})].\end{aligned} \tag{21}$$

Unfortunately, since the lumped uncertainty $\boldsymbol{\psi}$ is unknown, the control law (21) cannot be applied directly. In conventional sliding mode control, a discontinuous term $D_1\mathrm{sign}(\boldsymbol{s})$ is usually employed to suppress the effect of uncertainty $\boldsymbol{\psi}$. However, the discontinuous term would cause chattering phenomena. Moreover, the upper bound D_1 is usually unknown and difficult to be estimated. In this study, a continuous adaptive term $\hat{\boldsymbol{\psi}}$, which can be updated in an online manner to estimate the lumped uncertainty, is utilized to replace the term $D_1\mathrm{sign}(\boldsymbol{s})$.

Thus, an adaptive nonsingular terminal sliding mode control law (ANTSMC) is proposed as

$$\begin{aligned}\boldsymbol{u} = &-g^{-1}[f - \dot{\boldsymbol{x}}_c + k_1sig^{1+\frac{1}{\gamma}}(\boldsymbol{e}) + k_2sig^{1-\frac{1}{\gamma}}(\boldsymbol{e}) + \hat{\boldsymbol{\psi}} + \\ &l_1\boldsymbol{s} + l_2sig^{\eta}(\boldsymbol{s})],\end{aligned} \tag{22}$$

where $\hat{\boldsymbol{\psi}}$ is an adaptive term to estimate the lumped uncertainty based on the dynamics of the sliding variable $\boldsymbol{s}$. The updating law is chosen as

$$\dot{\hat{\boldsymbol{\psi}}} = \Lambda\boldsymbol{s}, \tag{23}$$

where $\Lambda = \mathrm{diag}\{\lambda_1, \cdots, \lambda_n\}$ is a positive definite diagonal constant matrix that determines the learning rate.

Next, the closed-loop stability with the proposed control law is analyzed based on the Lyapunov method.

Theorem 2. Consider the nonlinear system (1), if the nonsingular terminal sliding surface and the control law are chosen as (6) and (22), respectively, and the adaptive law is designed as (23), then the stability of the closed-loop system in the presence of parameter uncertainties and external disturbances is guaranteed.

Proof. Substituting the control law (22) into (19) will yield

$$\dot{\boldsymbol{s}} = -l_1\boldsymbol{s} - l_2 sig^{\eta}(\boldsymbol{s}) - \tilde{\boldsymbol{\psi}}, \tag{24}$$

where $\tilde{\boldsymbol{\psi}} = \hat{\boldsymbol{\psi}} - \boldsymbol{\psi}$ is the estimation error.

Consider the following Lyapunov function candidate

$$V = \frac{1}{2}(\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s} + \tilde{\boldsymbol{\psi}}^{\mathrm{T}}\Lambda^{-1}\tilde{\boldsymbol{\psi}}). \tag{25}$$

Taking the time derivative of (25) and using (24), one can get

$$\begin{aligned}\dot{V} = &\boldsymbol{s}^{\mathrm{T}}\dot{\boldsymbol{s}} + \tilde{\boldsymbol{\psi}}^{\mathrm{T}}\Lambda^{-1}\dot{\tilde{\boldsymbol{\psi}}} = \\ &\boldsymbol{s}^{\mathrm{T}}[-l_1\boldsymbol{s} - l_2 sig^{\eta}(\boldsymbol{s}) - \tilde{\boldsymbol{\psi}}] + \tilde{\boldsymbol{\psi}}^{\mathrm{T}}\Lambda^{-1}(\dot{\hat{\boldsymbol{\psi}}} - \dot{\boldsymbol{\psi}}).\end{aligned} \tag{26}$$

Here, it is assumed that the uncertainty $\boldsymbol{\psi}$ is fast-varying, i.e., $\dot{\boldsymbol{\psi}} \neq 0$. Then, if the adaptive law for $\hat{\boldsymbol{\psi}}$ is chosen as (23), the following equation can be obtained:

$$\dot{V} = -\boldsymbol{s}^{\mathrm{T}}l_1\boldsymbol{s} - \boldsymbol{s}^{\mathrm{T}}l_2 sig^{\eta}(\boldsymbol{s}) - \tilde{\boldsymbol{\psi}}^{\mathrm{T}}\Lambda^{-1}\dot{\boldsymbol{\psi}}. \tag{27}$$

It is known that the complete knowledge of the term $\tilde{\boldsymbol{\psi}}^{\mathrm{T}}\Lambda^{-1}\dot{\boldsymbol{\psi}}$ is unknown. So, if $\tilde{\boldsymbol{\psi}}^{\mathrm{T}}\Lambda^{-1}\dot{\boldsymbol{\psi}} \geq 0$,

$$\dot{V} = -\boldsymbol{s}^{\mathrm{T}}l_1\boldsymbol{s} - \boldsymbol{s}^{\mathrm{T}}l_2 sig^{\eta}(\boldsymbol{s}) - \tilde{\boldsymbol{\psi}}^{\mathrm{T}}\Lambda^{-1}\dot{\boldsymbol{\psi}} \leq 0. \tag{28}$$

Thus, the stability with fast convergence of the closed-loop system is obtained.

If $\tilde{\boldsymbol{\psi}}^{\mathrm{T}}\Lambda^{-1}\dot{\boldsymbol{\psi}} < 0$, which is the worse case, (27) can be rewritten as

$$\begin{aligned}\dot{V} = &-\boldsymbol{s}^{\mathrm{T}}l_1\boldsymbol{s} - \boldsymbol{s}^{\mathrm{T}}l_2 sig^{\eta}(\boldsymbol{s}) + \zeta \leq \\ &- \underline{l}_1\|\boldsymbol{s}\|^2 - \underline{l}_2\|\boldsymbol{s}\|^{\eta+1} + \zeta,\end{aligned} \tag{29}$$

where $\underline{l}_1 = \min\limits_{i=1,\cdots,n}\{l_{1i}\}$, $\underline{l}_2 = \min\limits_{i=1,\cdots,n}\{l_{2i}\}$, and $\zeta = -\tilde{\boldsymbol{\psi}}^{\mathrm{T}}\Lambda^{-1}\dot{\boldsymbol{\psi}}$ is a positive scalar value. From (29), $\dot{V} < 0$, if

$$\|\boldsymbol{s}\| > \left(\frac{\zeta}{\underline{l}_1}\right)^{\frac{1}{2}} \tag{30}$$

or

$$\|\boldsymbol{s}\| > \left(\frac{\zeta}{\underline{l}_2}\right)^{\frac{1}{\eta+1}}. \tag{31}$$

The inequalities given in (30) and (31) imply that the system trajectories will converge to the vicinity of the sliding surface $\boldsymbol{s} = 0$, i.e.,

$$\|\boldsymbol{s}\| \leq \min\left\{\left(\frac{\zeta}{\underline{l}_1}\right)^{\frac{1}{2}}, \left(\frac{\zeta}{\underline{l}_2}\right)^{\frac{1}{\eta+1}}\right\}. \tag{32}$$

Thus, it can be concluded that the system can approach to any arbitrary neighbourhood of the sliding surface by selecting the appropriate controller parameters l_1, l_2. □

Remark 4. The control law (22) is continuous, and therefore eliminates the chattering phenomena. It does not include any negative fractional power, hence is singularity-free.

From (19) and (22), the following closed-loop dynamics can be obtained

$$\dot{\boldsymbol{s}} + l_1\boldsymbol{s} + l_2 sig^{\eta}(\boldsymbol{s}) + \Lambda\int \boldsymbol{s}\mathrm{d}t = \boldsymbol{\psi}, \tag{33}$$

which shows the relation between uncertainty and sliding mode variable.

The approach described above will be used to design an attitude control system for NSHV in the next section.

IV. CONTROL DESIGN OF NSHV

The considered attitude control model of NSHV is derived from the twelve-state kinematic equations with six degrees of freedom[33, 34], which can be simplified as the affine nonlinear equation as follows:

$$\begin{cases}\dot{\boldsymbol{\Omega}} = f_s + \Delta f_s + (g_s + \Delta g_s)\boldsymbol{\omega},\\ \dot{\boldsymbol{\omega}} = f_f + \Delta f_f + (g_f + \Delta g_f)\boldsymbol{M}_c + \boldsymbol{d}(t).\end{cases} \tag{34}$$

Here, $\boldsymbol{\Omega} = [\alpha, \beta, \mu]^{\mathrm{T}}$ is state vector of the slow loop, i.e., attitude angle vector of NSHV, including angle of attack, angle of sideslip and angle of bank. $\boldsymbol{\omega} = [p, q, r]^{\mathrm{T}}$ is fast-loop state vector which is angular rate vector. $\boldsymbol{M}_c = g_{f\delta}\boldsymbol{\delta} = [l_{ctrl}, m_{ctrl}, n_{ctrl}]^{\mathrm{T}}$ is control torque vector. $\boldsymbol{\delta} = [\delta_e, \delta_a, \delta_r]^{\mathrm{T}}$ is aerodynamic control surface deflection where δ_e, δ_a and δ_r denote left elevator, right elevator and rudder, respectively. $g_{f\delta} \in \mathbf{R}^{3\times3}$ is control allocation matrix. $g_s, g_f \in \mathbf{R}^{3\times3}$ are invertible matrices, and $f_s, f_f \in \mathbf{R}^3$ are nonlinear vector functions. The concrete expressions of the above matrixes are specified in [33, 34]. $\Delta f_s, \Delta f_f, \Delta g_s$ and Δg_f are model uncertainties induced by system parameter uncertainties. $\boldsymbol{d}(t)$ is external disturbances.

System (34) satisfies the following assumptions: the unknown lumped uncertainties $\boldsymbol{\psi}_s = \Delta f_s + \Delta g_s\boldsymbol{\omega}$, $\boldsymbol{\psi}_f = \Delta f_f + \Delta g_f\boldsymbol{M}_c + \boldsymbol{d}(t)$ and their derivatives are bounded as $\|\boldsymbol{\psi}_i\| \leq D_{i1},\ \|\dot{\boldsymbol{\psi}}_i\| \leq D_{i2},\ i = s, f$.

The control aim is to design the slow-loop and fast-loop controllers such that attitude angle state $\boldsymbol{\Omega}$ can track the command $\boldsymbol{\Omega_c}$ stably and robustly.

Firstly, the slow-loop control design of NSHV is given. To design the slow-loop controller, the nonsingular terminal sliding surface is chosen as (6), that is,

$$\boldsymbol{s}_s = \boldsymbol{e}_s + \int_0^t (k_1^s sig^{1+\frac{1}{\gamma_s}}(\boldsymbol{e}_s) + k_2^s sig^{1-\frac{1}{\gamma_s}}(\boldsymbol{e}_s))\mathrm{d}\tau, \tag{35}$$

where $\boldsymbol{e}_s = \boldsymbol{\Omega} - \boldsymbol{\Omega}_c$, $k_1^s = \mathrm{diag}\{k_{11}^s, k_{12}^s, k_{13}^s\}$, $k_2^s = \mathrm{diag}\{k_{21}^s, k_{22}^s, k_{23}^s\}$, $k_{1i}^s, k_{2i}^s > 0\ (i = 1, 2, 3)$ and $\gamma_s > 1$.

Using the proposed control approach, the slow-loop controller is designed as

$$\begin{aligned}\boldsymbol{\omega}_c = &- g_s^{-1}[f_s - \dot{\boldsymbol{\Omega}}_c + k_1^s sig^{1+\frac{1}{\gamma_s}}(\boldsymbol{e}_s) + k_2^s sig^{1-\frac{1}{\gamma_s}}(\boldsymbol{e}_s) + \\ &l_1^s\boldsymbol{s}_s + l_2^s sig^{\eta_s}(\boldsymbol{s}_s) + \hat{\boldsymbol{\psi}}_s].\end{aligned} \tag{36}$$

Under the controller (36), a continuous control law $\boldsymbol{\omega}_c$ is produced, and serves as the command signal in the fast loop.

Secondly, the controller design of the fast loop is similar to that of the slow loop, which is given as

$$\begin{aligned}\boldsymbol{M}_c = &- g_f^{-1}[f_f - \dot{\boldsymbol{\omega}}_c + k_1^f sig^{1+\frac{1}{\gamma_f}}(\boldsymbol{e}_f) + k_2^f sig^{1-\frac{1}{\gamma_f}}(\boldsymbol{e}_f) + \\ &l_1^f\boldsymbol{s}_f + l_2^f sig^{\eta_f}(\boldsymbol{s}_f) + \hat{\boldsymbol{\psi}}_f].\end{aligned} \tag{37}$$

V. SIMULATION RESULTS

In this section, numerical simulations are presented to evaluate performance of the proposed approach. The simulations are carried out with initial velocity $V_0 = 2.6\,\mathrm{km/s}$ and flight altitude $H_0 = 30\,\mathrm{km}$. The initial attitude and attitude angular velocity conditions are chosen as $\alpha_0 = 0.5°$, $\beta_0 = 0°$, $\mu_0 = 0.6°$ and $p_0 = q_0 = r_0 = 0°/\mathrm{s}$. The command signals are chosen to be $\alpha_c = 1.5°$, $\beta_c = 0°$, $\mu_c = 2°$. Assume that there exists $-30\,\% \sim +30\,\%$ random uncertainties in the aerodynamic coefficients and aerodynamic moment coefficients. Besides, the external disturbance moment is defined as $\boldsymbol{d}(t) = 10^5[\sin(2\pi t),\ \sin(\pi t),\ \sin(\pi t)]^{\mathrm{T}}\,\mathrm{N\cdot m}$.

The parameters of controllers (36) and (37) are set as

$$\gamma_s = 3,\ k_1^s = 0.5I,\ k_2^s = I,\ \eta_s = 0.8,\ l_1^s = 3I,\ l_2^s = 5I,$$
$$\gamma_f = 3,\ k_1^f = 0.5I,\ k_2^f = 0.8I,\ \eta_f = 0.8,\ l_1^f = 3I,\ l_2^f = 5I.$$

Some other methods are used to design the same control system for NSHV in order to compare with our method. Conventional terminal sliding mode control (CTSMC) with the terminal sliding surface, the same as (14), is employed as the controller:

$$\begin{aligned}\boldsymbol{\omega}_c &= -g_s^{-1}[f_s - \dot{\boldsymbol{\Omega}}_c + c_s sig^{a_s}(\boldsymbol{e}_s) + k_s\tanh(\boldsymbol{\sigma}_s)],\\ \boldsymbol{M}_c &= -g_f^{-1}[f_f - \dot{\boldsymbol{\omega}}_c + c_f sig^{a_f}(\boldsymbol{e}_f) + k_f\tanh(\boldsymbol{\sigma}_f)],\end{aligned} \tag{38}$$

where $\tanh(\cdot)$ is a hyperbolic tangent function, used to replace the sign function to avoid chattering. The parameters in CTSMC (38) are selected to be $c_s = c_f = 2I$, $a_s = a_f = 0.8$, and $k_s = k_f = 25I$.

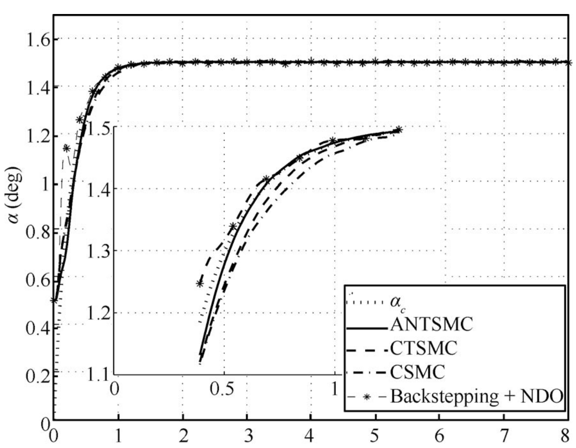
Conventional sliding mode control (CSMC) with a linear sliding surface $\Sigma = e(t) + \lambda\int_0^t e(\tau)\mathrm{d}\tau$ is adopted as the controller:

$$\begin{aligned}\boldsymbol{\omega}_c &= -g_s^{-1}[f_s - \dot{\boldsymbol{\Omega}}_c + \lambda_s\boldsymbol{e}_s + l_s\tanh(\boldsymbol{\Sigma}_s)],\\ \boldsymbol{M}_c &= -g_f^{-1}[f_f - \dot{\boldsymbol{\omega}}_c + \lambda_f\boldsymbol{e}_f + l_f\tanh(\boldsymbol{\Sigma}_f)].\end{aligned} \tag{39}$$

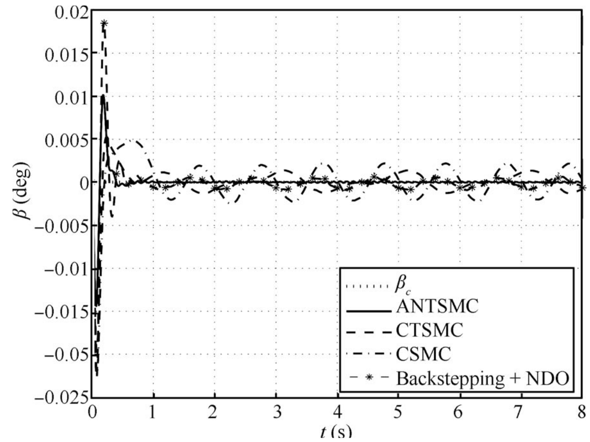
The parameters in CSMC (39) are chosen to be $\lambda_s = \lambda_f = 2I$, $l_s = l_f = 25I$.

Moreover, a backstepping control with nonlinear disturbance observer[8] (Backstepping+NDO) is employed for attitude control, in which the parameters are set as $K_1 = K_2 = 25I$, $K_d = 10I$.

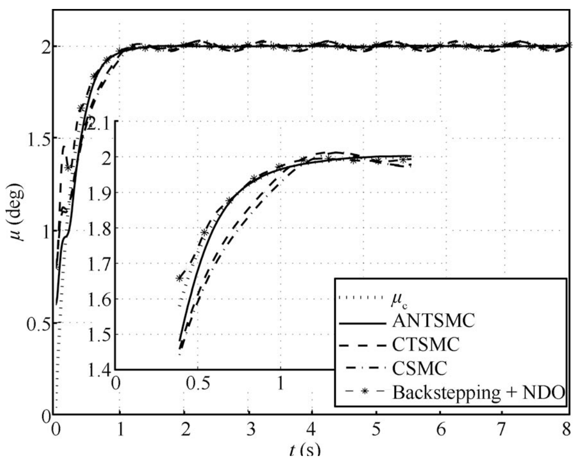
The experimental results of four control systems mentioned above are shown in Figs. 1 and 2.



(a) Angle of attack



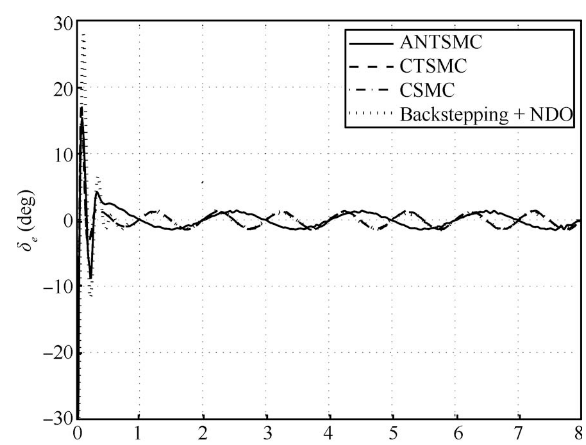
(b) Angle of sideslip



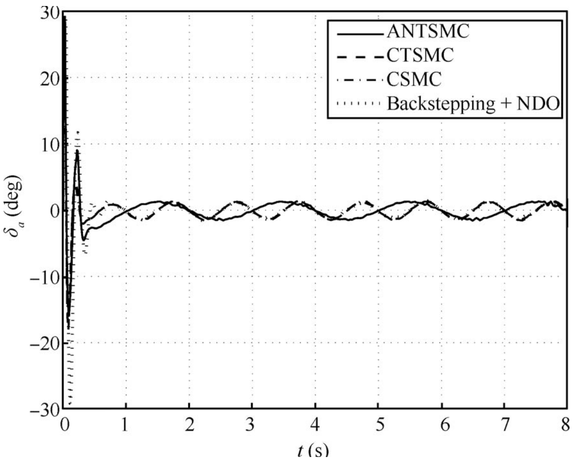
(c) Angle of bank

Fig. 1. Comparison of attitude control performance.

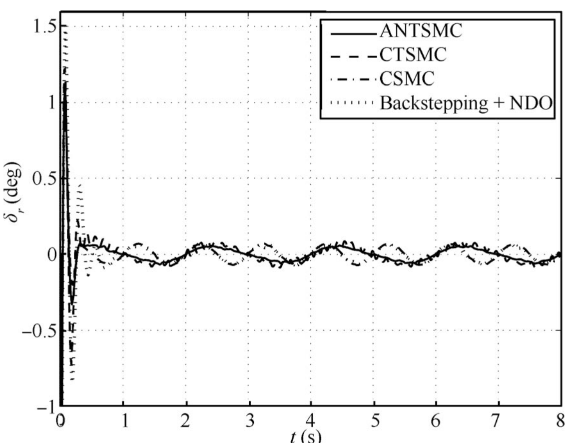
From Fig. 1, it can be seen that the tracking performance using ANTSMC is much improved, compared with the other control systems in the presence of parameter uncertainties and external disturbances. ANTSMC attains a higher tracking precision and better dynamic performance than the others. From Fig. 2, it can be inferred that the proposed approach overcomes chattering phenomena.



(a) Left elevator



(b) Right elevator



(c) Rudder

Fig. 2. Comparison of control surface deflections.

VI. CONCLUSION

In this paper, we have described the design of an adaptive nonsingular terminal sliding mode controller for NSHV. Firstly, a novel nonsingular terminal sliding surface is introduced, and its finite-time convergence is proved. Then, an adaptive nonsingular terminal sliding mode controller is presented based on an uncertainty estimator. The proposed approach relaxes the requirement for the bound of the lumped uncertainty in control design and eliminates chattering phenomenon. The stability of the closed-loop system is analyzed. Finally, simulation results show better performance of the presented approach for NSHV attitude control. Moreover, the robustness to parameter uncertainties and external disturbances rejection is successfully accomplished.

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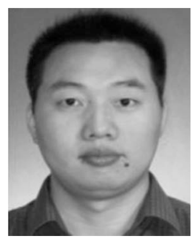

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