MOEA/D-ACO: A Multiobjective Evolutionary Algorithm Using Decomposition and Ant Colony

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***Abstract*—Combining ant colony optimization (ACO) and the multiobjective evolutionary algorithm (EA) based on decomposition (MOEA/D), this paper proposes a multiobjective EA, i.e., MOEA/D-ACO. Following other MOEA/D-like algorithms, MOEA/D-ACO decomposes a multiobjective optimization problem into a number of single-objective optimization problems. Each ant (i.e., agent) is responsible for solving one subproblem. All the ants are divided into a few groups, and each ant has several neighboring ants. An ant group maintains a pheromone matrix, and an individual ant has a heuristic information matrix. During the search, each ant also records the best solution found so far for its subproblem. To construct a new solution, an ant combines information from its group's pheromone matrix, its own heuristic information matrix, and its current solution. An ant checks the new solutions constructed by itself and its neighbors, and updates its current solution if it has found a better one in terms of its own objective. Extensive experiments have been conducted in this paper to study and compare MOEA/D-ACO with other algorithms on two sets of test problems. On the multiobjective 0–1 knapsack problem, MOEA/D-ACO outperforms the MOEA/D with conventional genetic operators and local search on all the nine test instances. We also demonstrate that the heuristic information matrices in MOEA/D-ACO are crucial to the good performance of MOEA/D-ACO for the knapsack problem. On the biobjective traveling salesman problem, MOEA/D-ACO performs much better than the BicriterionAnt on all the 12 test instances. We also evaluate the effects of grouping, neighborhood, and the location information of current solutions on the performance of MOEA/D-ACO. The work in this paper shows that reactive search optimization scheme, i.e., the "learning while optimizing" principle, is effective in improving multiobjective optimization algorithms.**

***Index Terms*—Ant colony optimization (ACO), decomposition, multiobjective optimization, multiobjective traveling salesman problem (MTSP), multiobjective 0–1 knapsack problem (MOKP), reactive search optimization (RSO).**

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I. Introduction

In many real-life applications, a decision maker (DM) has several conflicting objectives to consider and wants to determine an optimal tradeoff among them. These applications can be modeled as *multiobjective optimization problems* (MOPs). A Pareto-optimal solution to an MOP is a candidate for the optimal tradeoff [1]. An MOP may have many, even an infinite number of Pareto-optimal solutions. The Pareto set (PS)/Pareto front (PF) is the set of all the Pareto-optimal solutions in the decision/objective space. Without the DM's preference information, all the Pareto-optimal solutions are equally good. The DM often wants to have a good approximation to the PS and/or the PF available to further investigate the problem and to make his final decision [1].

Many *evolutionary algorithms* (EAs) have been developed for MOPs [2]–[9]. The major advantage of these *multiobjective EAs* (MOEAs) over other methods is that they work with a population of candidate solutions and thus can produce a set of Pareto-optimal solutions to approximate the PF in a single run.

Reactive search optimization (RSO) [10], [11], i.e., the "learning while optimizing" principle, has been widely accepted as a basic design principle in metaheuristics. RSO advocates the integration of machine learning techniques into heuristics and self-tunes algorithm behaviors based on online information collected during the search. Several successful evolutionary computation paradigms, such as *estimation of distribution algorithms* (EDA) and *ant colony optimization* (ACO), adopt this principle. ACO was originally proposed for dealing with single-objective combinatorial optimization problems [12]–[17]. ACO represents its knowledge learned about the problem during the search as a pheromone matrix. The pheromone value of each solution component empirically measures how likely this component is present in a promising solution. To allow for the incorporation of problem specific knowledge, ACO also uses a heuristic information matrix, which is predetermined before the search. Typical ACO employs a set of ants (i.e., agents) to search. At each generation, each ant, in a probabilistic way, constructs a solution based on the pheromone matrix and heuristic information matrix; then, the pheromone values are updated by these newly constructed solutions. Encouraged by the successful applications of ACO in single-objective optimization, several *multiobjective ACOs* (mACOs) have been developed in recent years [18]–[24]. In the design of an mACO, the following two related ingredients must be carefully considered.

- **Pheromone and heuristic information matrices:** Because the goal of mACOs is to approximate the whole PF, pheromone and heuristic information matrices should contain information about the possible locations of *all* the Pareto-optimal solutions instead of a single solution. Only one pheromone matrix and one heuristic information matrix may not serve this purpose very well, particularly when the distribution of the Pareto-optimal solutions is complicated. For this reason, most mACOs use multiple pheromone and heuristic information matrices [19], [20]. Often, each individual objective has one pheromone matrix and one heuristic information matrix, which can reflect the merit of each solution component for this particular objective [19], [20]. However, one pheromone matrix and one heuristic information matrix per objective are not enough for dealing with some hard MOPs. To develop an efficient mACO for approximating the whole PF, it is necessary to develop an effective way to use a reasonable number of pheromone and heuristic information matrices for recording useful information.

- **Solution construction:** mACOs should be able to generate a set of diverse new solutions at each generation so that they can approximate the whole PF [24]. To this end, different ants should use different combinations of pheromone and heuristic information matrices in the solution construction phase, therefore focusing onto different parts of the PF. How to combine pheromone and heuristic information has not been fully explored yet.

MOEA based on decomposition (MOEA/D) [25] is a recent MOEA framework. Similar to multiobjective genetic local search (MOGLS) [26]–[28], multiple single-objective Pareto sampling [29], and adapting weighted aggregation [30], it is based on conventional aggregation approaches: an MOP is decomposed into a number of single-objective optimization subproblems. The objective of each subproblem is a (linearly or nonlinearly) weighted aggregation of the individual objectives. Neighborhood relations among these subproblems are defined based on the distances among their aggregation weight vectors. Each subproblem is optimized in MOEA/D by using information mainly from its neighboring subproblems. The MOEA/D framework has been studied and used for dealing with a number of multiobjective problems [31]–[46].

This paper proposes an mACO in the MOEA/D framework, which is called MOEA/D-ACO. In designing this algorithm, special attention has been paid to the two issues aforementioned. In MOEA/D-ACO, each ant is responsible for one single-objective subproblem, and it targets a particular point in the PF. An ant has its own heuristic information matrix for holding its *a priori* information about its subproblem. Based on the neighborhood structure of the subproblems, the ants are divided into groups. Each group is to approximate a particular part of the PF. The ants in the same group share one pheromone matrix, which contains learned information about the position of their Pareto subregion. During the search, each ant records a current solution to its subproblem. To generate a new solution, an ant first builds one matrix by combining information from its own heuristic information matrix, the pheromone matrix of its group, and its current solution. The matrix built in this way is then used to guide its solution construction. In this manner, the constructed solution is more likely to be a good solution to the ant's subproblem. The pheromone matrix of each ant group is updated at each generation by using information extracted from the new solutions constructed by the ants in its group. Based on its own objective, each individual ant updates its current solution if it finds a better one from its neighborhood. This enables collaboration among different ant groups in MOEA/D-ACO since two neighbors may not be in the same group.

The *BicriterionAnt with region update* (BicriterionAnt) [24] is probably the first mACO in which different ant groups target different subregions of the PF. The major differences between it and our proposal are as follows. First, each ant in MOEA/D-ACO records a solution to its subproblem, and this solution makes direct contribution to its solution construction, which in a sense, combines the location information of an individual solution and global statistical information, whereas BicriterionAnt has no such mechanism. Second, MOEA/D-ACO relies on neighborhood relations among individual ants for implementing collaboration among different ant groups. In contrast, different ant groups in BicriterionAnt collaborate via sharing a global solution pool. Finally, an ant in MOEA/D-ACO only updates its group's pheromone matrix, which is very easy to implement and does not involve heavy computational overhead, In BicriterionAnt, ant i updates the pheromone matrix of group k if ant i has found a nondominated solution in the range that group k is responsible for. Therefore, at each generation, BicriterionAnt has to decide which solutions are in the range of a particular ant group, which makes it hard to generalize to the case of more than two objectives [24].

The rest of this paper is organized as follows. Section II introduces multiobjective optimization, two test problems, and basic decomposition approaches used in this paper. Section III presents MOEA/D-ACO. Sections IV and V compare MOEA/D-ACO with other algorithms and study the behavior of MOEA/D-ACO. Section VI concludes the paper and outlines some avenues for future research.

II. Multiobjective Optimization

This section presents the general MOP definition and two special problems used in our experiments.

A. Problem Definition

A general *MOP* can be stated as follows:

$$\begin{aligned} &\text{minimize} \quad F(x) = (f_1(x), \ldots, f_m(x)) \\ &\text{subject to} \quad x \in \Omega \end{aligned} \tag{1}$$

where Ω is the *decision (variable) space*, $F : \Omega \to R^m$ consists of m real-valued objective functions, and R^m is called the *objective space*. The *attainable objective set* is defined as set $\{F(x)|x \in \Omega\}$.

Let $u, v \in R^m$, u is said to *dominate* v if and only if $u_i \leq v_i$ for every $i \in \{1, \ldots, m\}$ and $u_j < v_j$ for at least one index $j \in \{1, \ldots, m\}$.[1] Point $x^* \in \Omega$ is *Pareto optimal* if there is

[1]This definition of domination is for minimization. All the inequalities should be reversed if the goal is to maximize the objectives in (1).

no point $x \in \Omega$ so that $F(x)$ dominates $F(x^*)$. $F(x^*)$ is then called a *Pareto-optimal (objective) vector*. In other words, any improvement in one objective of a Pareto-optimal point must lead to deterioration to at least another objective. The set of all the Pareto-optimal points is called the *PS*, and the set of all the Pareto-optimal objective vectors is called the *PF* [1].

The following two MOPs are used in this paper as benchmarks.

1) MOKP: Given a set of n items and a set of m knapsacks, the multiobjective 0–1 knapsack problem (MOKP) can be stated as

$$\begin{aligned} \text{maximize} \quad & f_i(x) = \sum_{j=1}^{n} p_{i,j} x_j,\ i = 1, \ldots, m \\ \text{subject to} \quad & \sum_{j=1}^{n} w_{i,j} x_j \le c_i,\ i = 1, \ldots, m \\ & x = (x_1, \ldots, x_n) \in \{0,1\}^n \end{aligned} \tag{2}$$

where $p_{i,j} \ge 0$ is the profit of item j in knapsack i, $w_{i,j} \ge 0$ is the weight of item j in knapsack i, and c_i is the capacity of knapsack i. $x_i = 1$ means that item i is selected and put into all knapsacks.

The MOKP is NP-hard and can model a variety of applications in resource allocation. A set of nine test instances of the given problem have been proposed in [3] and widely used in testing multiobjective heuristics.

2) MTSP: Given a set of n cities and m symmetric matrices $(c^i_{j,k})_{n\times n}$, $i = 1, \ldots, m$, where $c^i_{j,k} > 0$ can be interpreted as the ith cost of traveling between cities j and k. The problem is as follows:

$$\begin{aligned} \text{minimize} \quad & f_i(x) = \sum_{j=1}^{n-1} c^i_{x_j, x_{j+1}} + c^i_{x_1, x_n} \\ & i = 1, \ldots, m \end{aligned} \tag{3}$$

where x is a permutation of n cities, representing a tour that visits each city exactly once. The multiobjective traveling salesman problem (MTSP) is also NP-hard. In real-world applications, the different costs can be length, gasoline consumption, time, risk, etc.

B. MOP Decomposition

Several approaches have been proposed for decomposing an MOP into a number of single-objective optimization subproblems [1], [47]–[49]. In the following, we introduce two most commonly used approaches.

1) Weighted Sum Approach: Let $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_m)$ be a weight vector, i.e., $\sum_{i=1}^{m} \lambda_i = 1$ and $\lambda_i \ge 0$ for all $i = 1, \ldots, m$. Then, the optimal solutions to the following single-optimization problems:[2]

$$\begin{aligned} \text{minimize} \quad & g^{ws}(x|\boldsymbol{\lambda}) = \sum_{i=1}^{m} \lambda_i f_i(x) \\ \text{subject to} \quad & x \in \Omega \end{aligned} \tag{4}$$

are Pareto optimal to (1) if the PF of (1) is convex, where we use $g^{ws}(x|\lambda)$ to emphasize that λ is a weight vector in this objective function, whereas x is the variable to be optimized. However, when the PF is not convex, the weighted sum approach may not be able to find some Pareto-optimal solutions.

2) Tchebycheff Approach: The single-objective functions to be minimized are in the following form:

$$g^{te}(x|\lambda) = \max_{1\le i\le m} \{\lambda_i (f_i(x) - z^*_i)\} \tag{5}$$

where $z^* = (z^*_1, \ldots, z^*_m)$ is the reference point, i.e.,[3]

$$z^*_i = \min_{x\in\Omega} f_i(x) \tag{6}$$

for each $i = 1, \ldots, m$. Under some mild condition [1], for each Pareto-optimal point x^*, there exists a weight vector λ such that x^* is the optimal solution of (5), and each optimal solution of (5) is Pareto-optimal to (1).

To obtain a set of different Pareto-optimal solutions of (1), one can solve a set of single-objective optimization problems with different weight vectors defined by (4) and (5) or any other decomposition approaches.

III. New Proposal: MOEA/D-ACO

This section describes the MOEA/D-ACO and its implementations for the MOKP and MTSP.

A. Algorithmic Framework

MOEA/D-ACO first decomposes the MOP into N single-objective subproblems by choosing N weight vectors $\lambda^1, \ldots, \lambda^N$. Subproblem i is associated with weight vector λ^i, and its objective function is denoted as $g(x|\lambda^i)$. MOEA/D-ACO employs N ants for solving these single-objective subproblems. Ant i is responsible for subproblem i.

Because $g(x|\lambda)$ is a continuous function of λ, two subproblems are likely to have similar solutions if their weight vectors are close. Motivated by this observation, MOEA/D-ACO has the following concept.

- **Neighborhood**: The neighborhood $B(i)$ of ant i contains T ants whose subproblem's weight vectors are the T closest to λ^i among all the N weight vectors. We always assume that $i \in B(i)$; in other words, ant i is its own neighbor.

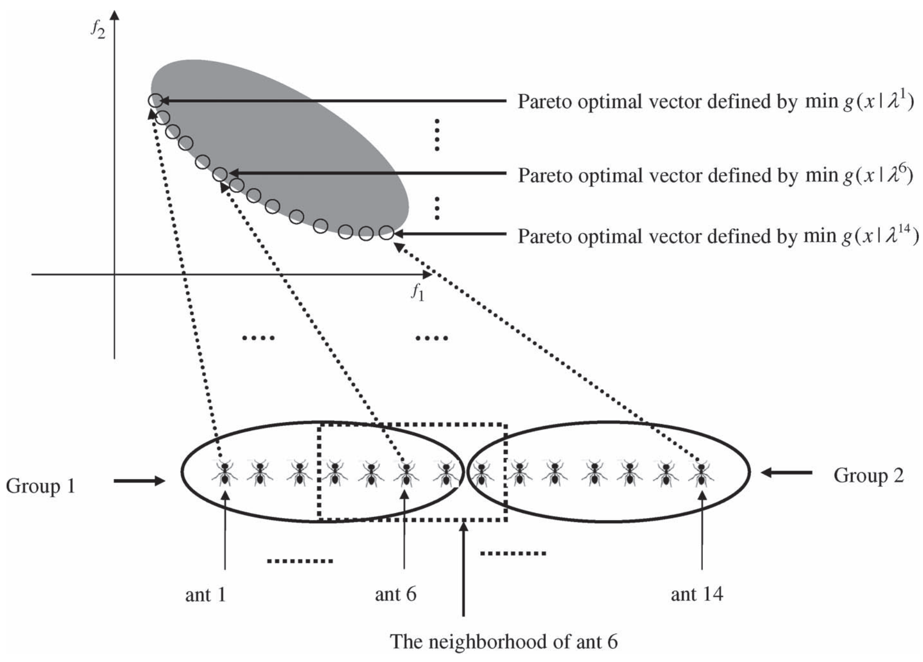
In addition, MOEA/D-ACO introduces the group concept.

- **Group:** The N ants are divided into K groups by clustering their corresponding weight vectors. Each group is intended to approximate a small range of the PF.

Fig. 1 illustrates the ideas of decomposition, neighborhoods, and groups in MOEA/D-ACO. To the best of our knowledge, the group concept was first used in BicriterionAnt [24] for biobjective optimization. BicriterionAnt is a Pareto-dominance based method, whereas MOEA/D-ACO adopts a decomposition approach.

[2]In the case of maximization, "minimize" should be replaced by "maximize."

[3]In the case of maximization, $z^*_i = \max_{x\in\Omega} f_i(x)$.



f_2
f_1
Pareto optimal vector defined by $\min g(x \mid \lambda^1)$
Pareto optimal vector defined by $\min g(x \mid \lambda^6)$
Pareto optimal vector defined by $\min g(x \mid \lambda^{14})$
Group 1
Group 2
ant 1
ant 6
ant 14
The neighborhood of ant 6

Fig. 1. In this illustrative example, the number of objectives (i.e., m) is 2, the number of ants (i.e., N) is 14, the number of groups (i.e., K) is 2, and the neighborhood size (i.e., T) is 5, The MOP is decomposed into 14 single-objective subproblems. Ant i is responsible for finding the optimal solution of subproblem i. Group 1 contains ant 1, 2, . . ., 7, and Group 2 contains ant 8, 9, . . ., 14. Each ant has five neighbors (including itself). As shown in this figure, the neighbors of ant 6 are ants 4, 5, 6, 7, and 8.

At each generation, MOEA/D-ACO with N ants maintains the following:

- N solutions $x^1, \ldots, x^N \in \Omega$, where x^i is the current solution to subproblem i;
- $F^1, \ldots, F^N$, where F^i is the F-value of x^i, i.e., $F^i = F(x^i)$;
- $\tau^1, \ldots, \tau^K$, where τ^j is the current pheromone matrix for group j, storing its learned knowledge about the subregion of the PF that it aims at approximating;
- $\eta^1, \ldots, \eta^N$, where η^i is the heuristic information matrix for subproblem i, which is predetermined before the search;
- EP, which is the external archive containing all the nondominated solutions found so far.

MOEA/D-ACO works as follows:

Step 0 Initialization: For $i = 1, \ldots, N$, generate initial solution x^i to subproblem i and set $F^i = F(x^i)$. For $i = 1, \ldots, N$, initialize η^i, the heuristic information matrix to subproblem i. For $j = 1, \ldots, K$, initialize pheromone information matrix τ^j for group j. Initialize EP as the set of all the nondominated vectors in $\{F^1, \ldots, F^N\}$.

Step 1 Solution Construction: For each $i = 1, \ldots, N$, ant i constructs a new solution y^i by using a probabilistic rule to choose solution components. The rule is a function of x^i, η^i, and τ^j, where ant i is in group j. Compute $F(y^i)$.

Step 2 Update of EP**:** For each $i = 1, \ldots, N$, if no vector in EP dominates $F(y^i)$, add $F(y^i)$ to EP and remove from EP all the vectors dominated by $F(y^i)$.

Step 3 Termination: If a problem-specific stopping condition is met, stop and output EP.

Step 4 Update of Pheromone Matrices: For each $j = 1, \ldots, K$, update pheromone matrix τ^j by using information extracted from those new solutions that were constructed by ants in group j in Step 1 and have just been added to EP in Step 2.

Step 5 Update of x^i**:** For $i = 1, \ldots, N$, ant i finds y, i.e., the solution with the smallest $g(\cdot|\lambda^i)$ value among all the new solutions that were constructed by its neighbors and have not been used to replace any other old solutions; replace x^i by y if $g(y|\lambda^i) < g(x^i|\lambda^i)$. Go to Step 1.

At each generation, each ant constructs a solution in Step 1. These newly constructed solutions update the external archive in Step 2. An ant updates the pheromone matrix of its group in Step 4 if it has found a new nondominated solution. In Step 5, each ant updates its current solution if there is a solution that: 1) is better than its current solution; 2) has not been used for updating other old solutions; and 3) was generated by its neighbors. Step 3 is termination. MOEA/D-ACO is an anytime algorithm; it is always able to return complete and feasible solutions.

In the above top-level description, the details of some steps are problem specific. In the following, we take the MOKP and MTSP as examples to show how these steps can be implemented. We would like to point out that our implementation is not unique. There are many possible ways to instantiate the above framework.

B. MOEA/D-ACO for the MOKP

1) Basic Setting:

- **Setting of** N and $\lambda^1, \ldots, \lambda^N$: This is controlled by parameter H. More precisely, $\lambda^1, \ldots, \lambda^N$ are all the weight vectors in which each individual weight component takes a value from

$$\left\{\frac{0}{H}, \frac{1}{H}, \dots, \frac{H}{H}\right\}.$$

Therefore, the number of the weight vectors is

$$N = C_{H+m-1}^{m-1}. \tag{7}$$

- **Setting of Group:** K representative weight vectors $\xi^1, \dots, \xi^K$ are generated by using the same method as above with a smaller value of H. Then, we cluster $\lambda^1, \dots, \lambda^N$ into K groups so that ξ^i is the closest representative vector, in terms of Euclidean distance, to all the weight vectors in group i.
- **Setting of Neighborhood:** The Euclidean distance is used to compute the distance between any two weight vectors.
- **Decomposition Approach:** Both the weighted sum and Tchebycheff approaches are used in this paper. In the Tchebycheff approach, the reference point z^* is substituted by $z = (z_1, \dots, z_m)$, where z_i is the best value of function f_i found so far.
- **The Data Structure of Pheromone and Heuristic Information Matrices:** Because each candidate solution to the MOKP is an n-Dimensional 0–1 vector, we represent pheromone and heuristic information matrices by using n-dimensional real-valued vectors. More precisely, the pheromone matrix for group j is

$$\tau^j = \left(\tau_1^j, \dots, \tau_n^j\right)$$

where τ_k^j can be interpreted as a figure of merit, which is learned from the previous search, to measure the benefit of $x_k = 1$ in a solution for the task of group j.

The heuristic information matrix for ant i is

$$\eta^i = \left(\eta_1^i, \dots, \eta_n^i\right)$$

where η_k^i is to measure the desirability, learned from the domain knowledge before the search, that $x_k = 1$ in a solution for subproblem i.

2) Initialization:

- The value of the kth element in η^i, the heuristic information matrix for ant i, is set as

$$\eta_k^i = \frac{\sum_{l=1}^m \lambda_l^i p_{l,k}}{\sum_{l=1}^m \gamma_l^i w_{l,k}} \tag{8}$$

where γ_l^i is the shadow price of constraint l in the linear programming relaxation of (4) with weight vector λ^i. η_k^i is the pseudoutility ratio [50], which has been used as a heuristic information value in [51].
- All the pheromone values are initialized to be the same large value. In our experiments, we have

$$\tau_k^j = 1 \tag{9}$$

for all $j = 1, \dots, K$ and $k = 1, \dots, n$. The motivation is to encourage the search to focus on exploration at its early stage.

3) Solution Construction: Assume that ant i is in group j and that its current solution is $x^i = (x_1^i, \dots, x_n^i)$. In Step 1 of MOEA/D-ACO, ant i constructs a new solution $y^i = (y_1^i, \dots, y_n^i)$ as follows.

```
1. For $k = 1, \dots, n$, set
      $$\phi_k = \left(\tau_k^j + \Delta \times x_k^i\right)^\alpha \left(\eta_k^i\right)^\beta \quad (10)$$
   where $\Delta$, $\alpha$, and $\beta > 0$ are three control parameters. $\phi_k$ represents the desirability that $y_k^i = 1$ in $y^i$ (i.e., item $k$ is selected) for subproblem $i$.
2. Initialize $y^i = (0, \dots, 0)$ (i.e., the knapsack is empty). Set $I = \{1, \dots, n\}$.
Do while $I \neq \emptyset$:
    2.1 For each $s \in I$, remove $s$ from $I$ if setting $y_s^i = 1$ (i.e., adding $s$ to the knapsack) will make $y^i$ infeasible.
    2.2 If $I = \emptyset$, return $y^i$.
    2.3 Generate a uniformly random number from (0, 1). If it is smaller than a control parameter $r$, set $s$ to be the index in $I$ with the largest $\phi$ value; otherwise, select one index $s$ randomly from $I$ according to the following probability by the roulette wheel selection:
          $$\frac{\phi_s}{\sum_{k \in I} \phi_k}.$$
    2.4 Remove $s$ from $I$ and set $y_s^i = 1$.
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In (10), τ_k^j is the pheromone trail of item k shared by all the ants in group j. $x_k^i = 1$ means that item k is selected in the current solution of subproblem i. x_k^i is private to ant i. Thus, $\tau_k^j + \Delta \times x_k^i$ is a combination of the group and private knowledge. This combination can be regarded as a private pheromone trail of item k for ant i. As in other ACO algorithms, α and β are for balancing the contributions from the pheromone trails and heuristic information values.

In Step 2.3, the selection of s follows the so-called pseudo-random proportional rule [13]. The control parameter r is used to balance exploration and exploitation. If the generated random number is smaller than r, the ant does exploitation and selects the item with the largest desirability; otherwise, it conducts random exploration biased toward items with large desirability.

4) Update of Pheromone Matrices: Let Π be the set of all the new solutions:

- that were constructed by the ants in group j in Step 1 of the current iteration;
- that were just added to EP in Step 2;
- in which $x_k = 1$.

τ_k^j, i.e., the pheromone trail value of item k for group j, is updated as follows:

$$\tau_k^j := \rho \tau_k^j + \sum_{x \in \Pi} \frac{1}{\sum_{i=1}^m \sum_{j=1}^n p_{ij} - g(x|\lambda^j)} \tag{11}$$

where ρ is the persistence rate of the old pheromone trails. With this update scheme, pheromone matrix τ^j stores some statistical information of good solutions found so far for the task of group j.

To avoid abnormally large or small pheromone trails, the upper and lower bounds of pheromone values $\tau_{\max}$ and

$\tau_{\min}$ are used to limit the range of pheromone trails in our implementation.

Following the max–min ant system [14], MOEA/D-ACO, at each generation, updates $\tau_{\max}$ as follows:

$$\tau_{\max} = \frac{B+1}{(1-\rho)\left(\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij} - g_{\max}\right)} \tag{12}$$

where B is the number of nondominated solutions found at the current iteration, and $g_{\max}$ is the largest value obtained for the objectives functions of all the N subproblems. $\tau_{\min}$ is always set as

$$\tau_{\min} = \varepsilon\tau_{\max} \tag{13}$$

where $0 < \varepsilon < 1$ is a control parameter. Then, for τ_k^j obtained in (11), reset $\tau_k^j = \tau_{\min}$ if $\tau_k^j < \tau_{\min}$, and reset $\tau_k^j = \tau_{\max}$ if $\tau_k^j > \tau_{\max}$.

C. MOEA/D-ACO for the MTSP

1) Basic Setting: It is the same as that for the MOKP, except the data structures of pheromone and heuristic information matrices, and solutions are as follows.

- Each group j has pheromone trail $\tau_{k,l}^j$ for a link between two different cities k and l.
- Each ant i has a heuristic information value $\eta_{k,l}^i$ for a link between cities k and l.
- Each individual solution x is a tour represented by permutation.

2) Initialization: Heuristic information values are initialized as

$$\eta_{k,l}^i = \frac{1}{\sum_{j=1}^{m} \lambda_j^i c_{k,l}^j}. \tag{14}$$

Similar to the implementation for the MOKP, $\tau_{i,j}^k$ is initialized to 1.

3) Solution Construction: Assume that ant i is in group j, and its full-scale current solution $x^i = (x_1^i, \ldots, x_n^i)$. In Step 1 of MOEA/D-ACO, ant i constructs its new solution $y^i = (y_1^i, \ldots, y_n^i)$ as follows.

1. For $k, l = 1, \ldots, n$, set

$$\phi_{k,l} = \left[\tau_{k,l}^i + \Delta \times In\left(x^i, (k,l)\right)\right]^\alpha \left(\eta_{k,l}^i\right)^\beta \tag{15}$$

where Δ, α, and $\beta > 0$ are three control parameters. $\phi_{k,l}$ represents the attractiveness of the link between cities k and l to ant i. The indicator function $\mathrm{In}(x^i, (k,l))$ is equal to 1 if link (k,l) is in tour x^i or 0 if otherwise.

2. Ant i first randomly selects a city as its starting point and then builds its tour city by city. Suppose that its current position is l and it has not completed its tour. It chooses city k to visit next from C, i.e., the set of the cities not visited so far, as follows:

Do while there are unvisited cities:

2.1 Generate a uniformly random number from (0, 1). If it is smaller than a control parameter r, choose k to be the city in C with the largest ϕ value; otherwise, select k randomly from C according to the following probability by the roulette wheel selection:

$$\frac{\phi_{k,l}}{\sum_{s\in C}\phi_{s,l}}.$$

2.2 If the ant has visited all the cities, return its tour.

4) Update of Pheromone Matrices: Let Π be the set of all the new solutions that:

- were constructed by the ants in group j in Step 1 of the current iteration;
- were just added to EP in Step 2;
- contain the link between cities k and l.

$\tau_{k,l}^j$, i.e., the pheromone trail value of link (k,l) for group j, is updated as follows:

$$\tau_{k,l}^j := \rho\tau_{k,l}^j + \sum_{x\in\Pi}\frac{1}{g(x|\lambda^j)} \tag{16}$$

where ρ is the persistence rate of the old pheromone trails. As in the implementation for the MOKP, $\tau_{\max}$ and $\tau_{\min}$ are used to limit the range of pheromone trails in our implementation. At each generation, $\tau_{\max}$ is set:

$$\tau_{\max} = \frac{B+1}{(1-\rho)g_{\min}} \tag{17}$$

where B is the number of nondominated solutions found at the current iteration, and $g_{\min}$ is the smallest value obtained for the objectives functions of all the N subproblems. $\tau_{\min}$ is

$$\tau_{\min} = \varepsilon\tau_{\max} \tag{18}$$

where $0 < \varepsilon < 1$ is a control parameter. Then, for $\tau_{k,l}^j$ obtained in (16), reset $\tau_{k,l}^j = \tau_{\min}$ if $\tau_{k,l}^j < \tau_{\min}$, and $\tau_{k,l}^j = \tau_{\max}$ if $\tau_{k,l}^j > \tau_{\max}$.

IV. Comparison With MOEA/D-GA on the MOKP

In [25], an implementation of MOEA/D with conventional genetic operators and local search (denoted by MOEA/D-GA in the following) is proposed for the MOKP. The crossover operator used is the one-point crossover and the mutation operator mutates each position of a solution with a very low probability. The local search used in MOEA/D-GA was developed by Jaszkiewicz [28].

The major reasons for this comparison are twofold: First, MOEA/D-GA outperforms some other popular and efficient EAs, such as MOGLS on the MOKP [28], and second, both MOEA/D-GA and MOEA/D-ACO use the MOEA/D framework. This comparison is necessary for studying the effect of using ACO in MOEA/D.

A. Experimental Setup

The nine instances from [3] are used in our experiments.

The parameter setting in MOEA/D-GA is the same as in [25], which demonstrated that the weighted sum approach performs

TABLE I
THE SETTINGS OF THE NUMBER OF ANTS, $N(H)$, AND THE NUMBER OF GROUPS, $K(H)$, IN MOEA/D-ACO FOR THE MOKP

Instance		$N(H)$	$K(H)$
n	m		
250	2	150 (149)	9 (8)
500	2	200 (199)	9 (8)
750	2	250 (249)	9 (8)
250	3	351 (25)	10 (3)
500	3	351 (25)	10 (3)
750	3	351 (25)	10 (3)
250	4	455 (12)	20 (3)
500	4	455 (12)	20 (3)
750	4	455 (12)	20 (3)

where n is the number of items and m is the number of objectives.

much better than the Tchebycheff approach in MOEA/D-GA for the MOKP. In our comparison, MOEA/D-GA uses the weighted sum approach as its decomposition approach.

To make a fair comparison, the parameters with respect to the MOEA/D framework in MOEA/D-ACO are the same as those in MOEA/D-GA. Both the weighted sum and Tchebycheff approaches are tested in MOEA/D-ACO. The settings of the number of ants and the number of groups for all instances are given in Table I. The other parameters are set as follows.

- $T = 10$.
- $\alpha = 1$.
- $\beta = 10$.
- $\rho = 0.95$.
- $r = 0.9$.
- $\varepsilon = (1/2n)$.
- $\Delta = 0.05 \times \tau_{\max}$.

Both algorithms stop after 300 generations. All the experiments have been carried out on the same computer (a Pentium IV 3-GHz CPU and 2 GB RAM). The programming language is C++. All the statistics are based on 30 independent runs.

B. Performance Metric

The inverted generational distance (IGD) [25] is used in assessing the performance of the algorithms in our experimental studies.

Let P^* be a set of uniformly distributed points in the objective space along the PF. Let P be an approximation to the PF, the IGD from P^* to P is defined as

$$D(P^*, P) = \frac{\sum_{v \in P^*} d(v, P)}{|P^*|}$$

where $d(v, P)$ is the minimum Euclidean distance between v and the points in P. If P^* is large enough to represent the PF very well, $D(P^*, P)$ measures both the diversity and convergence of P. To have a low value of $D(P^*, P)$, P must be very close to the PF and cannot miss any part of the whole PF. We chose the upper approximations obtained by Jaszkiewicz [28] as P^* in our experiments on the MOKP.

C. Comparison Results

Table II presents the mean and standard deviation of the IGD values of the final approximations obtained by each algorithm

TABLE II
THE IGD STATISTICS OF THE FINAL APPROXIMATIONS OBTAINED BY MOEA/D-ACO^W, MOEA/D-ACO^T, AND MOEA/D-GA

Instance		MOEA/D-ACO^W			MOEA/D-ACO^T			MOEA/D-GA	
n	m	mean	std dev	*W*-test	mean	std dev	*W*-test	mean	std dev
250	2	13.8	0.3	+	13.4	0.4	+	59.1	6.3
500	2	12.7	0.5	+	12.7	0.6	+	157.1	16.5
750	2	14.0	0.5	+	14.4	0.5	+	401.7	36.4
250	3	46.6	1.4	+	47.3	1.5	+	119.7	8.2
500	3	73.9	2.6	+	77.2	2.3	+	357.1	24.3
750	3	101.2	3.1	+	106.1	3.9	+	696.1	30.7
250	4	90.3	1.4	+	88.7	1.2	+	195.2	6.3
500	4	175.4	1.5	+	170.3	1.9	+	534.0	15.2
750	4	280.7	1.9	+	270.8	2.1	+	1064.5	28.1

W-test (Wilcoxon signed-rank test) has been made out at the 5% significance level to compare the IGD values of MOEA/D-ACO and MOEA/D-GA. "+" denotes that the mean IGD value of the corresponding algorithm is significantly smaller than that of MOEA/D-GA.

TABLE III
AVERAGE CPU TIME (IN SECONDS) USED BY MOEA/D-ACO^W, MOEA/D-ACO^T, AND MOEA/D-GA

Instance		MOEA/D-ACO^W		MOEA/D-ACO^T		MOEA/D-GA	
n	m	mean	std dev	mean	std dev	mean	std dev
250	2	6.4	0.0	6.2	0.0	0.9	0.0
500	2	10.9	0.0	10.7	0.0	2.8	0.0
750	2	15.7	0.0	15.5	0.1	5.8	0.1
250	3	15.9	0.9	16.2	0.0	2.3	0.1
500	3	24.6	0.1	24.3	0.0	7.2	0.2
750	3	33.7	0.1	34.0	0.1	13.1	0.3
250	4	26.1	0.0	25.3	0.5	6.4	0.1
500	4	43.1	0.0	42.3	0.0	16.2	0.2
750	4	54.6	0.1	55.3	0.1	29.4	0.4

among 30 runs for each test instance. Table III shows the average CPU time used by each algorithm. Fig. 2 plots the distribution of the final approximation with the lowest IGD value among 30 runs of each algorithm for each biobjective test instance. MOEA/D-ACO^W and MOEA/D-ACO^T represent MOEA/D-ACO with the weighted sum approach and the Tchebycheff approach, respectively. Figs. 3–5 show the evolution of the mean of the IGD value with the number of generations.

It is clear from Table II that the final approximations obtained by both variants of MOEA/D-ACO are much better than those obtained by MOEA/D-GA in terms of the IGD metric value on all the test instances. For example, the average IGD values in both MOEA/D-ACO variants are about 23%, 22%, and 25% of those obtained by MOEA/D-GA on instances 250-2, 500-3, 750-4, respectively. From these figures, it is also clear that the differences in the final approximations between two MOEA/D-ACO variants are very small and therefore difficult to distinguish visually.

It is evident in Fig. 2 that almost all the final solutions with the lowest IGD values obtained by MOEA/D-GA are dominated by those obtained by MOEA/D-ACO variants on instances 250-2, 500-2, and 750-2. Clearly, the larger the number of decision variables is, the larger the difference between MOEA/D-ACO and MOEA/D-GA is. Table III suggests that both MOEA/D-ACO variants consume more CPU time than MOEA/D-GA. This should be because the ACO component involves more computational overhead than crossover and mutation operators. When concentrating the attention on function evaluations, i.e., the number of candidate solutions visited, Figs. 3–5 indicate that both MOEA/D-ACO variants are always more efficient in terms of the number of function evaluations than MOEA/D-GA in minimizing the IGD values on all the

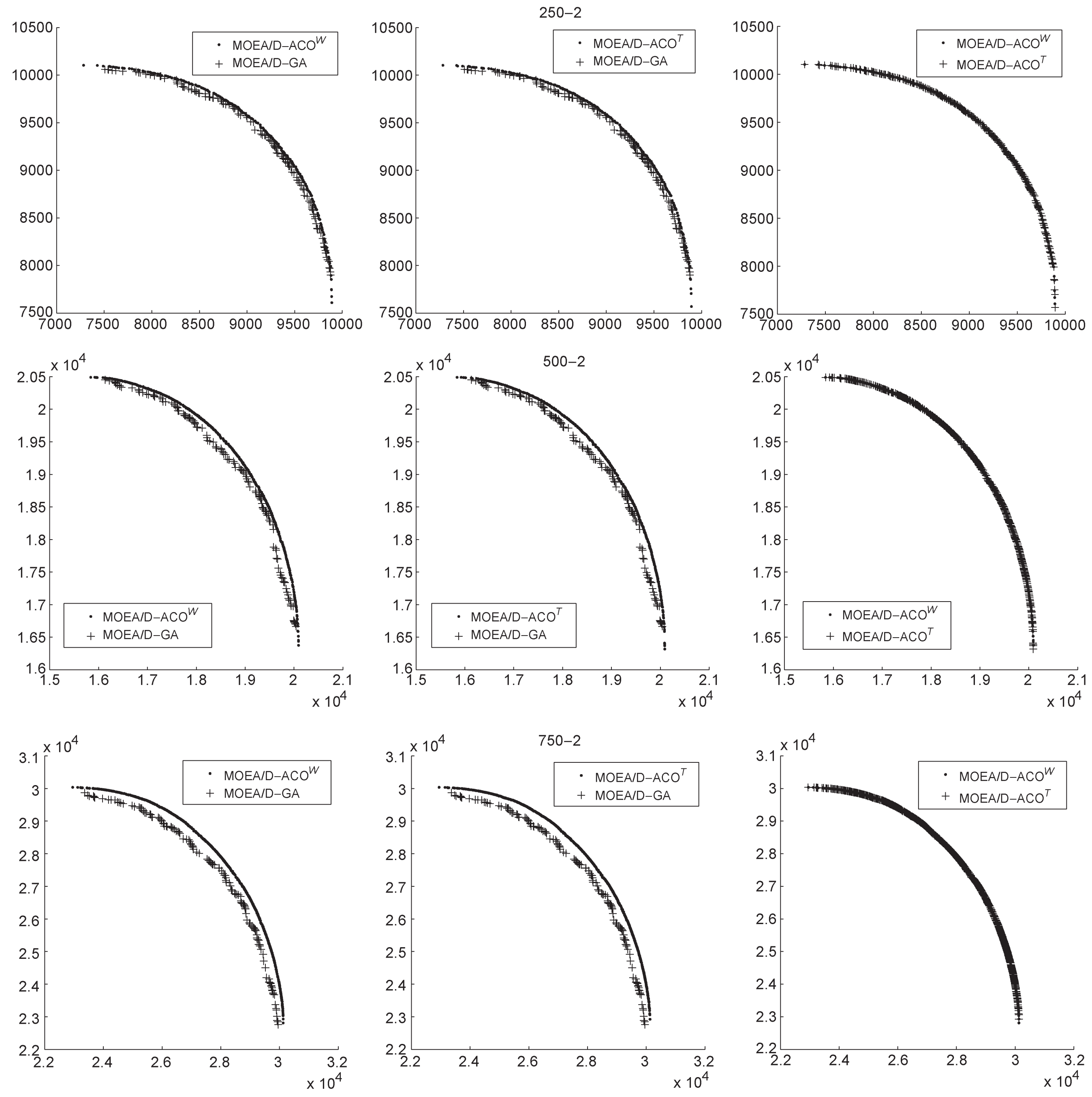

Fig. 2. Final approximations with the lowest IGD value among 30 runs obtained by MOEA/D-ACO^W, MOEA/D-ACO^T, and MOEA/D-GA on instance 250-2, 500-2, and 750-2. In each subplot, the horizontal and longitudinal axes correspond to the first and second objective functions, respectively.

three biobjective test instances. The final IGD values obtained by MOEA/D-GA are larger than the final IGD values obtained by MOEA/D-ACO variants.

D. More Discussions

Both MOEA/D-ACO and MOEA/D-GA adopt the same MOEA/D framework. Therefore, the success of MOEA/D-ACO should therefore be credited to its ACO part. A distinct feature of ACO is that it makes good use of the heuristic information matrices and uses them to guide its solution construction. From (10), $\beta = 0$ in MOEA/D-ACO means that the heuristic information will make no contribution to solution construction. To investigate the effect of the heuristic information matrices on the performance of MOEA/D-ACO, we have set $\beta = 0$ and kept all the other parameters the same as in Section IV-A, and tested MOEA/D-ACO on all the MOKP instances. The IGD statistics of 30 independent runs obtained by MOEA-ACO variants without heuristic information are presented and compared in Table IV. Clearly, MOEA/D-ACO variants without heuristic information performs much poorer than those with heuristic information on all the instances. For example, on instance 250-2, the average IGD-value obtained by MOEA/D-ACO^W without heuristic information is about

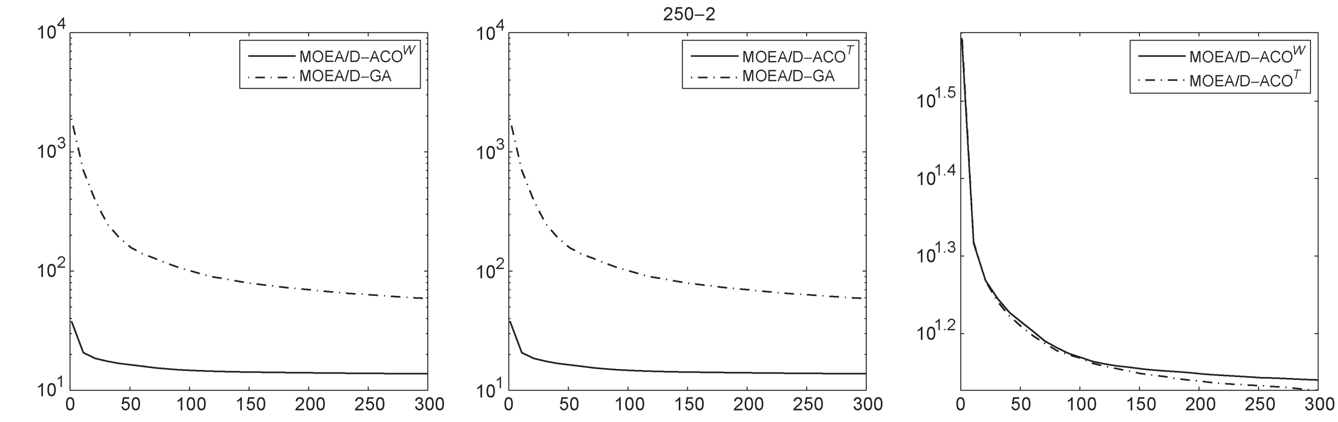

Fig. 3. Evolution of average IGD values with the number of generations in instance 250-2. In each subplot, the horizontal and longitudinal axes correspond to the number of generations and the average IGD value, respectively.

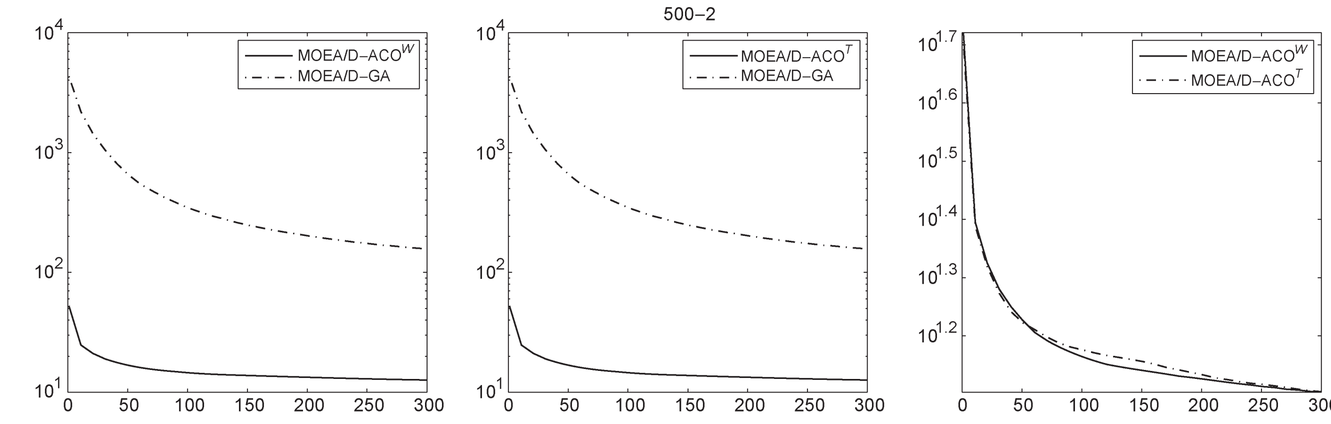

Fig. 4. Evolution of average IGD values with the number of generations in instance 500-2. In each subplot, the horizontal and longitudinal axes correspond to the number of generations and the average IGD value, respectively.

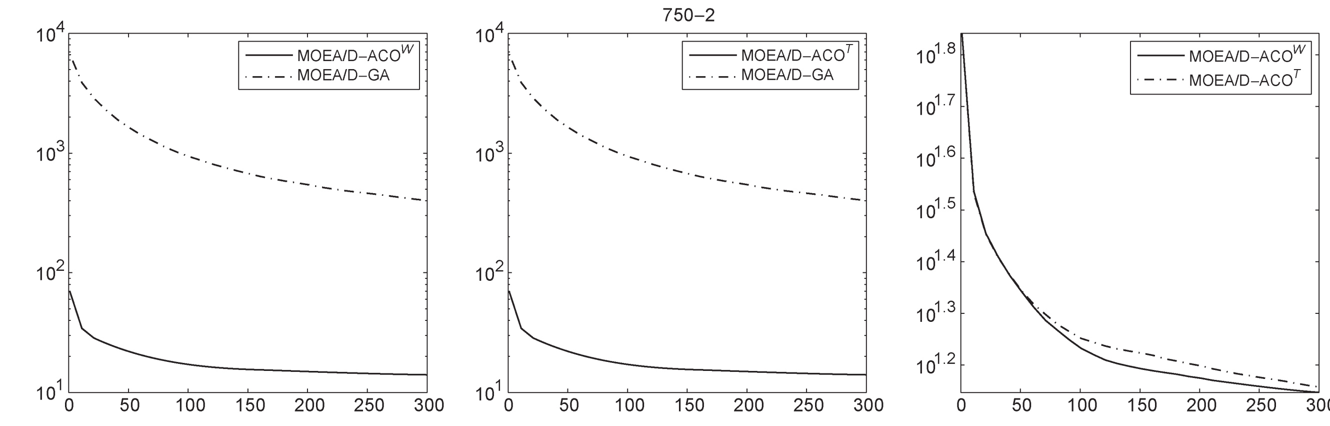

Fig. 5. Evolution of average IGD values with the number of generations in instance 750-2. In each subplot, the horizontal and longitudinal axes correspond to the number of generations and the average IGD value, respectively.

79 times as large as that obtained with heuristic information. Comparing the results in Tables II and IV, one can find that MOEA/D-ACO variants without heuristic information are also poorer than MOEA/D-GA on all the instances. Thus, one can claim that the use of heuristic information significantly improves the performance of MOEA/D-ACO. As it is the case for most heuristic approaches, ACO is not a black-box method, it requires intuition and intelligence of designers to be embodied into the heuristic information matrix for achieving good performance.

V. Comparison With BicriterionAnt on the MTSP

BicriterionAnt [24] is one of the best existing mACOs on the biobjective TSP [19], [22]. As pointed out in the introduction, BicriterionAnt also uses the group concept. It relies on Pareto dominance for guiding its search, whereas MOEA/D-ACO employs decomposition for dealing with MOPs. Both of them use ACO approaches for constructing new solutions. Therefore, this comparison should be useful for understanding the benefit of the decomposition approach.

TABLE IV
IGD STATISTICS OBTAINED BY MOEA/D-ACO FOR STUDYING THE INFLUENCE OF HEURISTIC INFORMATION

Instance		MOEA/D-ACO^W					MOEA/D-ACO^T				
		default setting		$\beta = 0$			default setting		$\beta = 0$		
n	m	mean	std dev	mean	std dev	*W*-test	mean	std dev	mean	std dev	*W*-test
250	2	13.8	0.3	1101.9	137.3	+	13.4	0.4	1030.5	142.0	+
500	2	12.7	0.5	3728.9	349.8	+	12.7	0.6	3690.5	428.0	+
750	2	14.0	0.5	5154.7	658.7	+	14.4	0.5	5171.4	688.6	+
250	3	46.6	1.4	776.0	28.4	+	47.3	1.5	803.5	92.1	+
500	3	73.9	2.6	2658.4	287.3	+	77.2	2.3	2847.5	295.4	+
750	3	101.2	3.1	5830.8	865.6	+	106.1	3.9	4803.9	580.9	+
250	4	90.3	1.4	873.7	38.0	+	88.7	1.2	857.7	82.1	+
500	4	175.4	1.5	2658.9	89.7	+	170.3	1.9	2619.4	140.1	+
750	4	280.7	1.9	3947.8	381.8	+	270.8	2.1	4104.4	418.3	+

W-test has been made out at the 5% significance level to compare the IGD values of MOEA/D-ACO using heuristic information or not. "+" denotes that the mean IGD value of the corresponding algorithm is significantly larger than that of MOEA/D-ACO using heuristic information.

TABLE V
THE IGD STATISTICS OF THE FINAL APPROXIMATIONS OBTAINED BY MOEA/D-ACO^W, MOEA/D-ACO^T, AND BICRITERIONANT

Instance		MOEA/D-ACO^W			MOEA/D-ACO^T			BicriterionAnt	
$name$	n	mean	std dev	*W*-test	mean	std dev	*W*-test	mean	std dev
kroab100	100	2008.2	330.3	+	1865.7	318.6	+	12318.5	382.9
kroac100	100	1733.0	254.9	+	1697.8	288.4	+	13209.0	373.1
kroad100	100	1494.6	217.3	+	1440.2	200.8	+	11888.0	403.1
kroae100	100	1887.3	303.5	+	1669.6	290.3	+	12526.1	368.9
krobc100	100	1802.5	345.3	+	1452.9	172.8	+	13881.1	387.9
krobd100	100	1777.3	298.3	+	1655.1	215.7	+	13625.8	271.0
krobe100	100	2285.9	292.8	+	1864.9	301.8	+	15042.7	458.3
krocd100	100	1897.3	295.4	+	1650.2	299.4	+	11688.2	274.0
kroce100	100	1852.2	311.2	+	1710.3	275.9	+	13050.4	324.7
krode100	100	2017.1	340.5	+	1904.6	375.7	+	12451.9	328.1
kroab200	200	4476.7	598.6	+	3910.1	393.8	+	32691.2	908.1
kroab300	300	8735.9	1121.2	+	6441.4	595.2	+	51776.2	1171.5

W-test has been made out at the 5% significance level to compare the IGD values of MOEA/D-ACO and BicriterionAnt. "+" denotes that the mean IGD value of the corresponding algorithm is significantly smaller than that of Bicriterion

A. Experimental Setup

The 12 instances from [52] are used in the comparison.

The parameter setting of BicriterionAnt is the same as in [22]. Specifically, we have the following.

- number of groups = 3.
- number of ant in each group = 8.
- $\rho = 0.95$.
- $\alpha = 1$.
- $\beta = 2$.
- $r = 0.9$.

The parameter settings of MOEA/D-ACO are as follows.

- Total number of ants $N = 24$.
- Number of groups $K = 3$.
- Neighborhood size $T = 10$.
- $\rho = 0.95$.
- $\alpha = 1$.
- $\beta = 2$.
- $r = 0.9$.
- $\epsilon = (1/2n)$.
- $\Delta = 0.05 \times \tau_{\max}$.

In computing the IGD values, P^* for each instance is the set of all the nondominated solutions found by all the runs of all the algorithms. Both algorithms stop after 3000 generations. Therefore, they construct the same number of candidate solutions for each instance. All statistics are based on 30 independent runs.

B. Experimental Results

Table V shows that the IGD values of the final approximations obtained by both MOEA/D-ACO variants are nearly one order of magnitude lower than those by BicriterionAnt on all test instances. However, there is no substantial difference between two MOEA/D-ACO variants in terms of the IGD value. In Fig. 6, the quality differences between each MOEA/D-ACO variant and BicriterionAnt can be visually detected. Clearly, the final solutions obtained by MOEA/D-ACO variants dominate almost all the final solutions found by BicriterionAnt. Figs. 7–9 suggest that both MOEA/D-ACO are more efficient in reducing the IGD values than BicriterionAnt. These experimental results indicate that the MOEA/D framework does indeed help to improve the algorithm performance. The major reason could be that the decomposition strategy makes cooperation among different ants more efficient than Pareto dominance. It is also clear in Figs. 7–9 that MOEA/D-ACO^T performs better than MOEA/D-ACO^W.

Table VI shows that both MOEA/D-ACO variants require much less CPU time than BicriterionAnt. This is because BicriterionAnt needs to compare all the solutions to each other at each generation, which is very time-consuming.

C. More Discussions

Both MOEA/D-ACO and BicriterionAnt use ACO for solution construction. Unlike BicriterionAnt, each ant in MOEA/D-ACO records its current solution and uses it to compute ϕ_k in

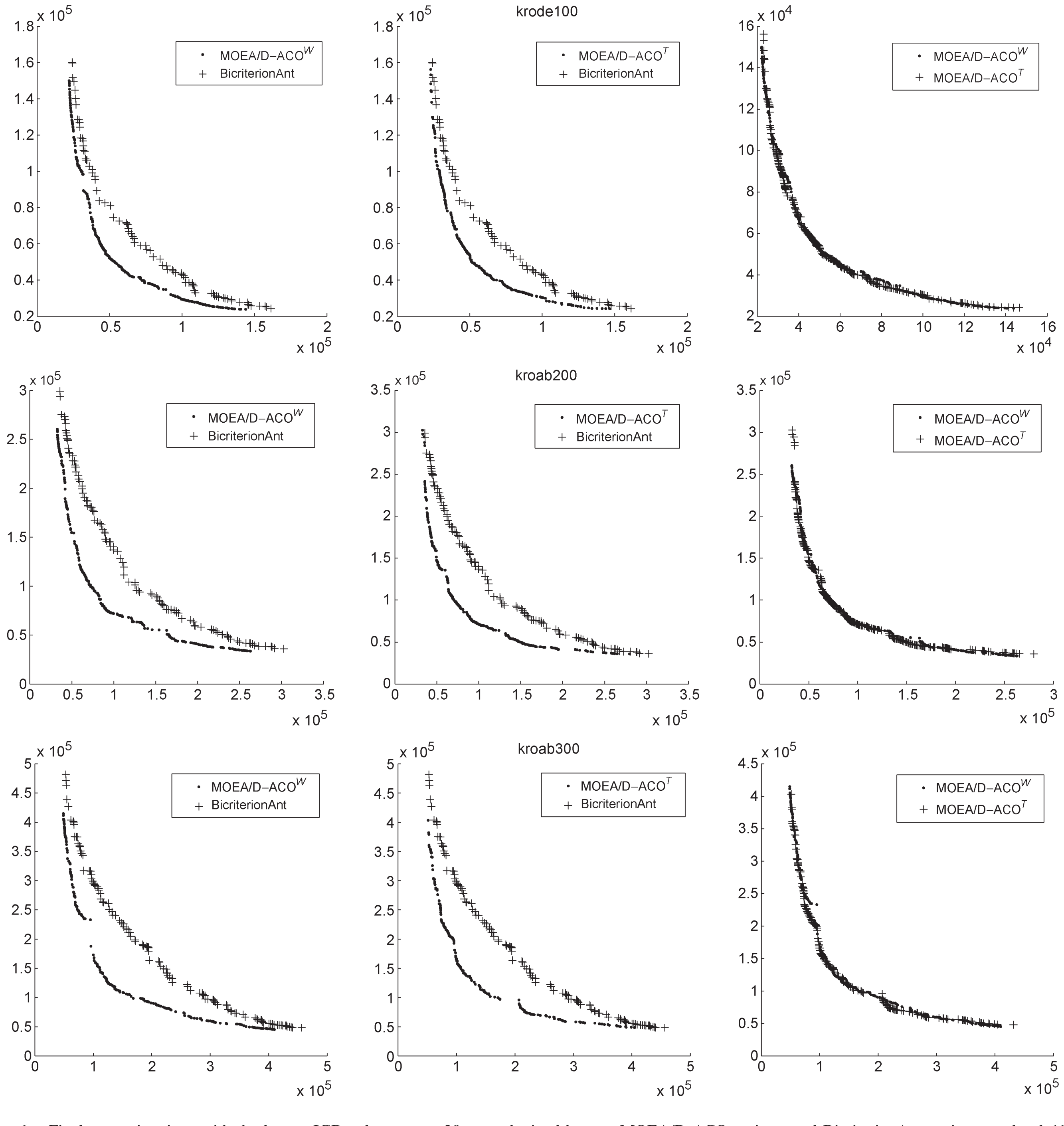

Fig. 6. Final approximations with the lowest IGD value among 30 runs obtained by two MOEA/D-ACO variants and BicriterionAnt on instance krode100, kroab200, and kroab300. In each subplot, the horizontal and longitudinal axes correspond to the first and second objective functions, respectively.

(15) for solution construction. MOEA/D-ACO also uses groups and neighborhoods to allow ants to exchange information. Thus, three issues arise naturally.

- What if grouping is not used in MOEA/D-ACO?
- What if the current solution makes no contribution to ϕ_k in (15) in MOEA/D-ACO?
- What if individual ants do not change information with their neighbors as described in Step 5 in MOEA/D-ACO?

Noting that $K = 1$ implies that no grouping is used, $\Delta = 0$ means that the current solution makes no contribution, and $T = 1$ means that an ant has no neighbor except itself, we tested MOEA/D-ACO variants in which $K = 1$, $\Delta = 0$, or $T = 1$, whereas the other parameters are the same as in Section V-A. The experimental results are listed in Tables VII and VIII, from which we can observe the following.

- Both MOEA/D-ACO variants with grouping (i.e., the default parameter setting) significantly outperform their counterparts without grouping. For example, in kroab200, the IGD value of MOEA/D-ACO^W with grouping is about 80% of that obtained without grouping.

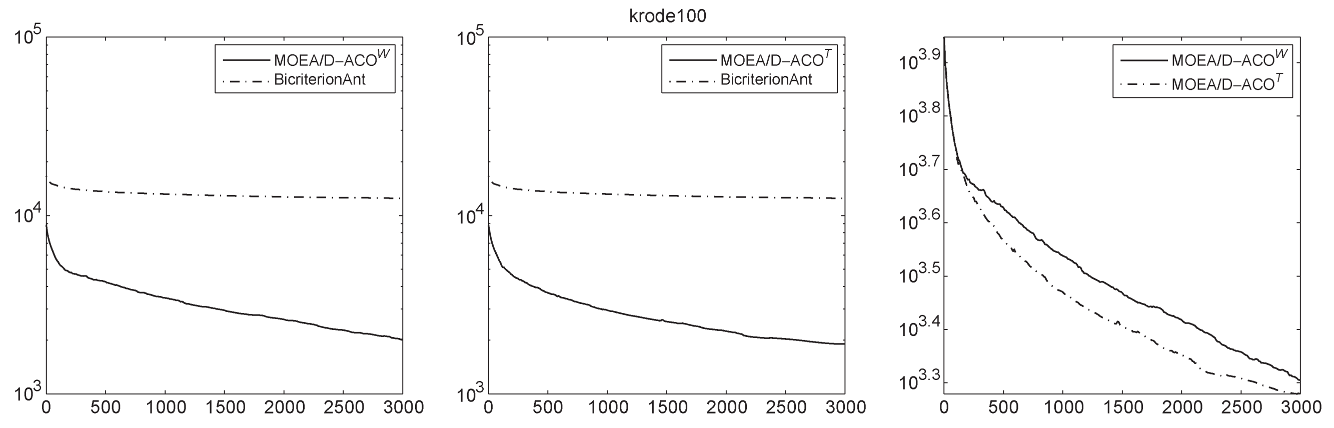

Fig. 7. Evolution of average IGD values with the number of generations in instance krode100. In each subplot, the horizontal and longitudinal axes correspond to the number of generations and the average IGD value, respectively.

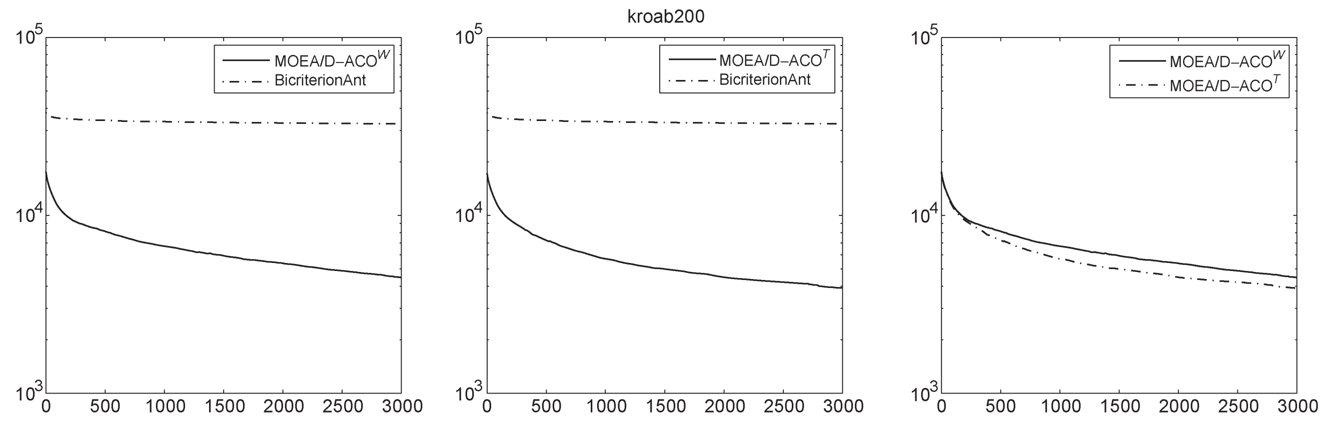

Fig. 8. Evolution of average IGD values with the number of generations in kroab200. In each subplot, the horizontal and longitudinal axes correspond to the number of generations and the average IGD value, respectively.

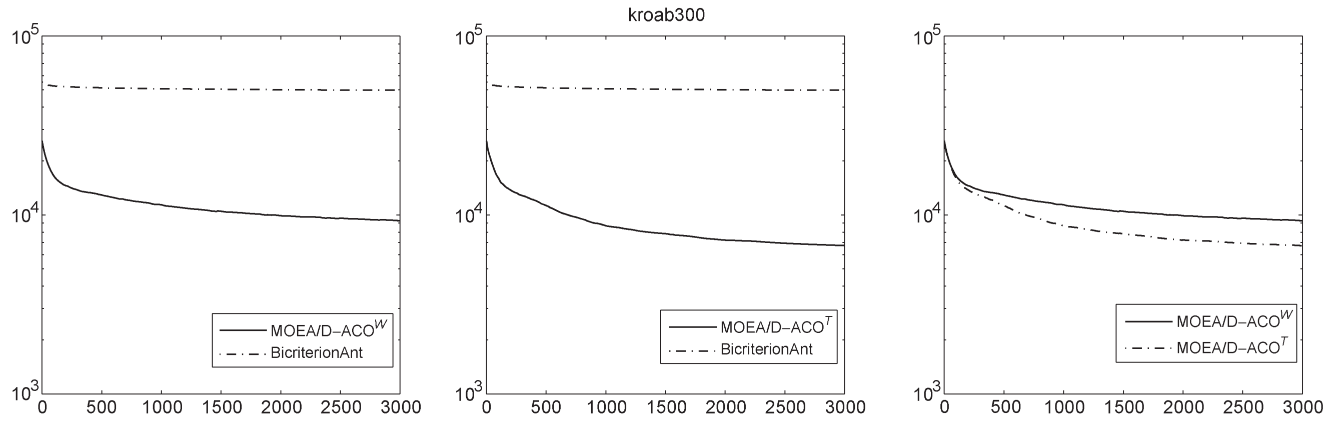

Fig. 9. Evolution of average IGD values with the number of generations in kroab300. In each subplot, the horizontal and longitudinal axes correspond to the number of generations and the average IGD value, respectively.

- Both MOEA/D-ACO variants with the default parameter setting are significantly better than those with $\Delta = 0$ on all the 12 test instances. Taking kroab100 as an example, the IGD value of MOEA/D-ACO^T with the default parameter setting is about 78% of that obtained without using location information of the current solutions.
- Both MOEA/D-ACO variants with the default parameter setting significantly outperform their counterparts with $T = 1$ on all test instances.

Therefore, one can claim that grouping, the location information of the current solutions, and the neighborhood significantly improve the algorithm performance. In our experiments, there is no instance in which these techniques cause any deterioration.

VI. Conclusion and Future Research Issues

Reactive search optimization principle, i.e., "learning while optimizing," has been widely used for designing and improving metaheuristics [11]. ACO is a successful heuristic using this

TABLE VI
AVERAGE RUNNING TIME (IN SECONDS) CONSUMED BY MOEA/D-ACO AND BICRITERIONANT

Instance		MOEA/D-ACO^W		MOEA/D-ACO^T		BicriterionAnt	
name	*n*	mean	std dev	mean	std dev	mean	std dev
kroab100	100	4.9	0.1	4.9	0.1	68.4	0.1
kroac100	100	4.9	0.1	4.9	0.1	68.3	0.0
kroad100	100	4.9	0.1	4.9	0.1	68.3	0.0
kroae100	100	4.9	0.1	4.9	0.0	68.3	0.1
krobc100	100	5.0	0.0	4.9	0.1	68.6	0.2
krobd100	100	4.9	0.1	4.9	0.0	68.8	0.1
krobe100	100	4.9	0.1	4.9	0.0	68.8	0.1
krocd100	100	4.9	0.0	4.9	0.1	68.7	0.0
kroce100	100	4.9	0.0	4.9	0.0	68.6	0.1
krode100	100	4.9	0.0	4.9	0.0	68.5	0.1
kroab200	200	20.4	0.2	20.3	0.1	276.3	0.3
kroab300	300	45.1	0.5	45.1	0.4	629.1	1.1

TABLE VII
THE IGD STATISTICS OF THE FINAL APPROXIMATIONS OBTAINED BY MOEA/D-ACO^W WHEN $K = 1$, $\Delta = 0$, AND $T = 1$

Instance		default setting		$K = 1$			$\Delta = 0$			$T = 1$		
name	*n*	mean	std dev	mean	std dev	*W*-test	mean	std dev	*W*-test	mean	std dev	*W*-test
kroab100	100	2008.2	330.3	2998.3	540.6	+	2417.2	294.1	+	2488.5	404.8	+
kroac100	100	1733.0	254.9	2916.1	477.7	+	2045.9	248.2	+	2045.1	276.6	+
kroad100	100	1494.6	217.3	2356.8	355.8	+	1957.4	182.8	+	1855.2	252.7	+
kroae100	100	1887.3	303.5	2740.2	317.4	+	2051.8	285.2	+	2042.2	329.7	+
krobc100	100	1802.5	345.3	2617.2	486.3	+	2011.1	230.2	+	2141.9	307.8	+
krobd100	100	1777.3	298.3	2601.9	407.8	+	2175.3	216.2	+	2247.6	238.5	+
krobe100	100	2285.9	292.8	3015.1	641.5	+	2371.3	250.4	+	2723.2	352.7	+
krocd100	100	1897.3	295.4	2538.3	401.9	+	2081.9	299.8	+	2216.2	344.0	+
kroce100	100	1852.2	311.2	2800.4	577.2	+	2134.8	218.2	+	2283.2	291.3	+
krode100	100	2017.1	340.5	2835.9	554.3	+	2241.6	221.9	+	2308.8	274.0	+
kroab200	200	4476.7	598.6	5486.5	762.2	+	5612.1	556.3	+	5293.9	392.3	+
kroab300	300	8735.9	1121.2	8960.6	1534.7	+	9579.5	742.8	+	9580.3	901.8	+

W-test has been made out at the 5% significance level to compare the IGD values of MOEA/D-ACO^W under different settings. "+" denotes that the mean IGD value of the corresponding algorithm is significantly larger than that of MOEA/D-ACO with the default setting.

TABLE VIII
THE IGD STATISTICS OF THE FINAL APPROXIMATIONS OBTAINED BY MOEA/D-ACO^T WHEN $K = 1$, $\Delta = 0$, AND $T = 1$

Instance		default setting		$K = 1$			$\Delta = 0$			$T = 1$		
name	*n*	mean	std dev	mean	std dev	*W*-test	mean	std dev	*W*-test	mean	std dev	*W*-test
kroab100	100	1865.7	318.6	3217.1	592.1	+	2379.6	298.5	+	1963.6	408.1	+
kroac100	100	1697.8	288.4	3034.1	547.3	+	2125.7	281.2	+	1789.1	320.5	+
kroad100	100	1440.2	200.8	2356.6	317.9	+	1996.7	216.6	+	1592.4	187.8	+
kroae100	100	1669.6	290.3	2826.3	834.3	+	2142.0	330.8	+	1740.6	261.1	+
krobc100	100	1452.9	172.8	2544.2	336.4	+	2080.6	198.0	+	1756.4	289.3	+
krobd100	100	1655.1	215.7	2770.1	312.1	+	2133.9	222.8	+	1790.7	216.9	+
krobe100	100	1864.9	301.8	2963.9	542.6	+	2431.6	209.0	+	1976.8	266.0	+
krocd100	100	1650.2	299.4	2568.1	464.5	+	2074.6	289.9	+	1691.0	243.9	+
kroce100	100	1710.3	275.9	2590.6	389.2	+	2166.5	219.6	+	1925.3	307.4	+
krode100	100	1904.6	375.7	2714.3	487.0	+	2255.5	308.2	+	1947.8	324.6	+
kroab200	200	3910.1	393.8	5465.2	784.2	+	5487.8	545.2	+	4091.1	373.3	+
kroab300	300	6441.4	595.2	7768.9	808.2	+	8249.5	679.0	+	6477.2	604.6	+

W-test has been made out at the 5% significance level to compare the IGD values of MOEA/D-ACO^T under different settings. "+" denotes that the mean IGD value of the corresponding algorithm is significantly larger than that of MOEA/D-ACO^T with the default setting.

principle. ACO for single-objective optimization represents its problem specific knowledge as a heuristic information matrix and its knowledge learned from the previous search as a pheromone matrix. Because the goal of multiobjective optimization is to approximate the PF, instead of a single optimal solution, generalizing ACO to multiobjective optimization is not a trivial task.

MOEA/D decomposes an MOP into a number of single-objective subproblems and optimizes them simultaneously. Neighborhood relations among these subproblems are defined and used to make the search efficient and effective. With the MOEA/D framework, this paper designed an mACO, i.e., MOEA/D-ACO. In this proposed algorithm, each ant is responsible for one subproblem. The ants are divided into groups, and each ant has several neighboring ants. An ant group maintains a pheromone matrix, and an individual ant has a heuristic information matrix. During the search, each ant also records the best solution found so far for its subproblem. To construct a new solution, an ant combines information from its group's pheromone matrix, its own heuristic information matrix and its current solution.

We conducted extensive experimental results on two paradigmatic MOPs. The experimental results on the MOKP show that MOEA/D-ACO outperforms MOEA/D-GA on all the nine test instances. Furthermore, the heuristic information matrices in MOEA/D-ACO make a very significant contribution to its performance. In other words, MOEA/D-ACO successfully inherits the learning and adaptation ability of single-objective ACO. The comparison of MOEA/D-ACO with BicriterionAnt, which is a very efficient mACO proposed in the previous literature, on the biobjective TSP has shown that MOEA/D-ACO performs much better on all the 12 test instances. We also studied the effects of grouping, the neighborhood, and the use of location information of current solutions on the performance of MOEA/D-ACO.

We believe that our proposed hybrid approach can be extended in several ways. The future research avenues include the following.

- *The studies of MOEA/D-ACO on other hard or real-world multiobjective problems*. Specific attention should be paid for how to decompose these MOPs and how to define their heuristic information and pheromone matrices.
- *The investigation of online parameter-tuning and configuration techniques in MOEA/D-ACO*. Although the manual tuning in this paper provided excellent results, it is worthwhile studying automatic tuning techniques based on machine learning for extending MOEA/D-ACO to different problems [11], [53].
- *The combinations of MOEA/D-ACO with single objective search techniques*. Although MOEA/D framework provides a very natural framework for using single-objective search techniques, it is worthwhile studying when and how single-objective search techniques can be used in MOEA/D-ACO.
- *The use of MOEA/D-ACO in two-phase Pareto search*. A two-phase strategy first generates a set of high-quality solutions by a search method and then applies Pareto local search on them to generate approximate Pareto-optimal solutions [54]. It is very interesting to study how to use MOEA/D-ACO in the first phase.
- *The use of the ideas in MOEA/D-ACO for generating EDAs to multiobjective optimization*. Hybrids of EDAs with other techniques have been widely studied and used for single-objective optimization (e.g., in [55] and [56]). These experiences should be very useful for combining MOEA/D with EDA.
- *The studies of combination of MOEA/D-ACO with the DM's preference information*. The brain–computer evolutionary multiobjective optimization scheme approximates a utility function in an interactive manner [57] for reducing the cognitive burden on the DM. It is very interesting to study how the DM's preference can be integrated into MOEA/D-ACO in such a way.

The C++ source code of MOEA/D-ACO can be downloaded from Qingfu Zhang's homepage: http://dces.essex.ac.uk/staff/qzhang/.

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