Investigating the Effect of Imbalance Between Convergence and Diversity in Evolutionary Multiobjective Algorithms

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***Abstract*—There are two main tasks involved in addressing a multiobjective optimization problem (MOP) by evolutionary multiobjective (EMO) algorithms: 1) make the population converge close to the Pareto-optimal front and 2) maintain adequate population diversity. However, most state-of-the-art EMO algorithms are designed based on the "convergence first and diversity second" principle. It has been observed that although these EMO algorithms have been successful in optimizing many real-world MOPs, they fail to solve certain problems that feature a severe *imbalance* between diversity preservation and achieving convergence. This paper characterizes an imbalanced MOP by clearly defining properties and indicating the reasons for the existing EMO algorithms' difficulties in solving them. We then present 14 imbalanced problems, with and without constraints. Computational results using four existing EMO algorithms—elitist non-dominated sorting genetic algorithm (NSGA-II), multiobjective evolutionary algorithm based on decomposition (MOEA/D), strength Pareto evolutionary algorithm 2 (SPEA2), and S metric selection EMO algorithm (SMS-EMOA) and a proposed generalized vector-evaluated genetic algorithm are then presented. It is seen that these EMO algorithms cannot solve these imbalanced problems, but they are able to solve the problems when augmented by multiobjective to multiobjective (M2M), an approach that decomposes the population into several interacting subpopulations. These results and the successful application of the EMO methods with the M2M approach even on standard so-called balanced problems indicate the usefulness of using the M2M approach.**

***Index Terms*—Evolutionary algorithm, imbalanced problems, multiobjective evolutionary algorithm based on decomposition (MOEA/D)-multiobjective to multiobjective (M2M) decomposition, multiobjective optimization.**

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I. Introduction

Multiobjective optimization problems (MOPs) involve more than one conflicting objective and give rise to a set of Pareto-optimal solutions. An MOP is described as follows:

$$\begin{aligned} &\text{minimize} \quad F(\mathbf{x}) = \{f_1(\mathbf{x}), \ldots, f_m(\mathbf{x})\} \\ &\text{subject to} \quad \mathbf{x} \in \mathbf{X} \end{aligned} \tag{1}$$

where $F \subset R^m$ is an objective vector in the m-dimensional objective space and $\mathbf{X} \in R^n$ is the n-dimensional feasible variable search space for which $\mathbf{X} = \prod_{i=1}^{n} [a_i, b_i] \subset R^n$ is a special case. The objective function $F : \prod_{i=1}^{n} [a_i, b_i] \to R^m$ performs a mapping from search space to objective space, which contains m real-valued objective functions $f_1, \ldots, f_m$. Let $\mathbf{u} = (u_1, u_2, \ldots, u_m)$ and $\mathbf{v} = (v_1, v_2, \ldots, v_m)$ be two solutions in the search space. Then, if and only if $u_i \le v_i$ for each $i = 1, 2, \ldots, m$ and there exists at least one index $j \in \{1, 2, \ldots, m\}$ such that $u_j < v_j$, we say that $\mathbf{u}$ dominates $\mathbf{v}$. If there is no $\mathbf{x} \in \mathbf{X}$ such that $F(\mathbf{x})$ dominates $F(\mathbf{x}^*)$, we call $\mathbf{x}^*$ a global Pareto-optimal point. The Pareto-optimal points of an MOP are generally more than one, and the set of these Pareto-optimal points is called a Pareto-optimal set (PS). The set of these Pareto-optimal objective vectors is called a Pareto-optimal front (PF). In general, both PS and PF of an MOP with m objectives are a $(m-1)$-dimensional manifold [1].

Recently, a number of evolutionary algorithms have emerged to solve MOPs in various fields [2]–[5]. Compared to single-objective optimization problems, which usually have only one optimum, MOPs have a set of Pareto-optima, so population-based evolutionary algorithms are particularly well suited to addressing them, since they can find multiple Pareto-optimal points in a single run. Every Pareto-optimal point represents a tradeoff among different objectives, and a good approximation to the PF is of great importance to various decision makers, so that they can evaluate multiple tradeoff solutions before choosing the most satisfactory or preferred solution for their MOP.

Generally, an evolutionary multiobjective (EMO) algorithm executes two main tasks when optimizing an MOP: 1) push its population toward the PF, thereby ensuring convergence and 2) maintain diversity of the population, thereby ensuring finding a wide variety of solutions [39]. Therefore, EMO algorithms must be designed in such a way that both of the above tasks are effectively accomplished. Despite the equal importance of both tasks, most state-of-the-art EMO algorithms follow a "convergence first and diversity second" procedure. That means solutions contributing more to convergence than to diversity are preferred when selecting the next generation population. For example, elitist non-dominated sorting genetic algorithm (NSGA-II) [6] and strength Pareto evolutionary algorithm 2 (SPEA2) [7] emphasize nondominated solutions for convergence first before applying a niching method for emphasizing less-crowded nondominated solutions for the purpose of maintaining a diverse population, while decomposition-based methods such as multiobjective evolutionary algorithm based on decomposition (MOEA/D) also preferentially select nondominated solutions rather than the most diverse solutions. It has been observed that although these EMO algorithms have achieved great success in optimizing many real-world MOPs, they encounter difficulties when optimizing certain problems requiring that more importance be placed on diversity preservation [31], [32]. We call these problems imbalanced problems due to the fact that these problems produce an imbalance between the emphasis on convergence and diversity and demand that equal or more importance be given to diversity preservation than to convergence.

Imbalanced problems refer to MOPs whose PFs offer different degrees of search difficulty in different regions of the front. Some parts of the PF can be more easily searched than other parts, and moreover the easy-to-find part of PF often dominates a relatively large area in the search space, thereby making it difficult to find "supporting" solutions for discovering the rest of the PF. Simulation results [31] have shown that this kind of imbalanced problem can be effectively optimized if a convergence and diversity balance enhancement mechanism, such as that introduced by the multiobjective to multiobjective (M2M) population decomposition strategy [31], can be reasonably applied to balance the population diversity and convergence during the evolutionary process. However, this former study only shows us by computational experiments that the M2M population decomposition strategy can make EMO algorithms achieve a better balance between population convergence and diversity and that the general "convergence preferred" EMO algorithms fail to maintain the population diversity when optimizing imbalanced problems. It does not focus on any theoretical analysis. We are still unable to know how the M2M population decomposition affects the population distribution in the process of optimization and why the properties of imbalanced problems make them hard to optimize with state-of-the-art EMO algorithms. With this in mind, we try to study why the M2M population decomposition strategy can make EMO algorithms achieve better performance and how the convergence preferred EMO algorithms fail to preserve the population diversity in the process of evolution. Theoretical analysis is an important aspect of EMO algorithm development research, but most of the existing studies focus on either the algorithm convergence or computational complexity.

In this paper, the population convergence and diversity of an EMO algorithm are analyzed simultaneously. First, we give a formal definition for the imbalanced problem and study its properties. After that, we summarize three types of imbalanced problems, providing reasons that cause the imbalance between convergence and diversity, and construct 14 imbalanced problems of the three types for computational experiments. Based on the definition, we use a simplified EMO algorithm model to analyze the population convergence and diversity in the process of optimizing imbalanced problems and show that the population convergence and diversity are equally important when optimizing imbalanced problems. We also study the influence on the EMO algorithm performance brought about by M2M population decomposition when optimizing balanced MOPs that are often used as standard EMO test problems. In order to achieve a balance between the population convergence and diversity, M2M decomposes the population into a number of subpopulations and makes each subpopulation search a different multiobjective subproblem. This decomposition strategy can reduce the computational complexity by limiting the selection operator to focus on each subpopulation; however, this strategy may damage the population integrity to some extent. To better understand the roles that the M2M population decomposition strategy plays in the evolutionary process, we study the performance of five EMO algorithms by trying to optimize a series of balanced multiobjective optimization test problems, including the ZDT [42] and DTLZ [43] series. By analyzing and comparing computational results, we find that the M2M population decomposition strategy can effectively maintain the performance of the five EMO algorithms. It is also found that all five EMO algorithms with the M2M framework have similar performance in solving the balanced problems.

Among the five EMO methods, we introduce and include a generalized vector evaluated GA (GVEGA) procedure that extends the original vector-evaluated genetic algorithm (VEGA) [27] to better solve MOPs. GVEGA uses a decomposition-based VEGA for solving each subpopulation and shows promise for its future use. This remains as another contribution of this paper.

The remainder of this paper is organized as follows. Section II gives a comprehensive description of five state-of-the-art EMO algorithms and the M2M population decomposition framework. Section III describes the concept of an imbalanced problem and discusses its properties in detail. In Section IV, we analyze a simplified multiobjective model to see a case in which the convergence-first strategy is not always suitable for imbalanced MOPs, while the M2M can achieve a better balance between population convergence and diversity. In Section V, a test suite of imbalanced MOPs is constructed and optimized by NSGA-II, MOEA/D, SPEA2, S metric selection EMO algorithm (SMS-EMOA), and GVEGA with and without M2M. The computational results are analyzed, and the effectiveness of M2M is confirmed by the experimental comparisons. Section VI gives the computational results on a series of commonly-used multiobjective optimization test problems by the above-mentioned EMO algorithms with and without M2M. Section VIII concludes the findings of this paper.

II. Related Work

In this section, we first discuss five convergence-first EMO algorithms and then describe the M2M strategy, as this strategy is then used with these five standard EMO algorithms in the rest of this paper.

A. Convergence-First EMO Algorithms

Most state-of-the-art EMO algorithms are designed based on the convergence first and diversity second principle. This means that convergence is emphasized in these EMO algorithms; about four categories of implementations can be found in the literature.

The first category of EMO algorithms is based on the Pareto dominance concept. These algorithms use the dominance relationship as the criterion to select the next generation population, and nondominated individuals are preferred. The next-level strategy considers a niche-preservation method applied to achieve population diversity. The NSGA series of algorithms are nondominated-sorting-based EMO algorithms. In NSGA-II, a mechanism called crowding distance sorting is used to preserve the population diversity. Only when the nondominance sorting cannot completely select the required solutions, crowding distance sorting is applied. This leads to the result that the convergence is superior to the diversity in NSGA-II. Zitzler and Thiele proposed another Pareto-dominance-based EMO algorithm, called strength Pareto evolutionary algorithm (SPEA) [7]. They improved SPEA and proposed SPEA2. Both SPEA and SPEA2 use an external archive to preserve nondominated solutions. Newly found nondominated solutions are compared with the solutions in the external archive based on their fitness, called the "Pareto strength," and the resulting solutions with high Pareto strength are preserved. When the size of the nondominated solution set exceeds the predefined capacity of the external set, an archive truncation procedure is conducted to maintain the diversity of the archive population. Convergence is also emphasized more than diversity preservation in SPEA2, because it is those solutions that dominate at least one of the archive solutions, rather than those that improve diversity, that will enter the archive set. Besides NSGA-II and SPEA2, there are also some other Pareto-dominance-based EMO algorithms. Horn *et al.* [8] proposed the niched Pareto genetic algorithm, which uses a dynamic niche updating strategy to emphasize convergence first and then maintain diversity of population. Yen and Lu [9] and Tan *et al.* [10] proposed Pareto-dominance-based algorithms with a cell-based and a dynamic population strategy, respectively, as secondary mechanisms to maintain population diversity. Zhu *et al.* [11] generalized the Pareto-dominance relationship by expanding the dominance area of solutions to enhance the convergence of existing Pareto-dominance-based algorithms.

The second category of EMO algorithms is based on the decomposition (i.e., aggregation) strategy. By optimizing the aggregated objectives of an MOP, we can get a Pareto-optimal point of the problem. Therefore, an MOP can be decomposed into a number of single-objective optimization subproblems and optimized at the same time. MOEA/D [12] and MOGLS [13] are two representative decomposition-based EMO algorithms. In MOEA/D, an MOP can be decomposed into a number of single-objective optimization problems by aggregation methods such as the weighted-sum approach, Tchebycheff approach, or the boundary intersection (BI) approach [14]. Each subproblem can exchange information with its neighborhood. The neighborhood of each single-objective optimization problem is defined by the distance between the weight vectors, and every single-objective subproblem can share information with its neighbors during evolution to improve the search efficiency. The decomposition can make sure that MOEA/D converges to the PF and the diversity of MOEA/D is guaranteed by the distribution of weights. In MOEA/D, nondominated solutions, instead of solutions with good distribution, are preferred first when optimizing each single-objective subproblem. MOGLS [13] also decomposes the MOP into single-objective optimization problems, but unlike MOEA/D, MOGLS uses randomly selected aggregation weights at each stage to facilitate the local search. Some other decomposition-based EMO algorithms are proposed to solve many-objective optimization problems [15]–[17].

The third category is indicator-based EMO algorithms. The indicator is originally used to measure the goodness of the approximation to the PF. Later, it is found that these indicators can be effectively used for selection, for they can measure the convergence to the PF and the distribution of solutions across the PF at the same time. Zitzler and Künzli [18] first proposed a general indicator-based EMO algorithm framework called the indicator-based evolutionary algorithm (IBEA). IBEA uses the indicator to compare the solutions and choose the next-generation population, and does not require any additional diversity-preserving mechanism. The hyper-volume (HV) indicator [19], [20] is one of the most commonly used indicators. An HV indicator selects solutions according to their contributions to the dominated HV. However, each time a solution is discarded, the whole population's contribution to the dominated HV must be recalculated. This leads to the high computational complexity of HV-indicator-based EMO algorithms [21]–[23]. Many EMO algorithms based on the HV indicator have been proposed, and among them, SMS-EMOA [24] is one of the most popular. SMS-EMOA is designed to lower the computational complexity by limiting the application of HV indicator selection to the resulting last front of fast-nondominated-sorting selection. In SMS-EMOA, nondominated sorting is first conducted, and then HV indicator selection is performed. This strategy can best make use of nondominated sorting to reduce the computational complexity of HV indicator selection, because not every solution's contribution to the dominated HV has to be calculated in SMS-EMOA. Many studies have reported that HV-selection-based EMO algorithms can achieve better results than MOEA/D and NSGA-II in some MOPs [25], [26]. However, as we have discussed above, the fast-nondominated-sorting procedure emphasizes the convergence of the population. Besides, according to the definition of the solution's contribution to the dominated HV, nondominated solutions are also preferred in the HV-based selection, thereby giving double importance to nondominated solutions.

The fourth category is objective-based EMO algorithms, such as the VEGA [27] and coevolutionary multiswarm particle swarm optimization (CMPSO) [28]. VEGA is a simple strategy and is easy to implement. VEGA divides the population into equal-sized subpopulations on the fly, and each subpopulation corresponds to an objective of the MOP. Each subpopulation is then evaluated according to a single objective function. Due to the lack of evaluating other objective functions, it is likely that a solution preferred by one objective is not preferred by another objective. Although it was assumed that the crossover operator may lead to the generation of tradeoff solutions, VEGA did not work very well in practice. The crossover could not maintain population diversity, and eventually VEGA converged near each individual objective's minimum solution. Here, we proposed a modified VEGA approach based on the concept of M2M and demonstrate that VEGA-M2M can be a viable algorithm for solving difficult problems. CMPSO was developed based on the coevolutionary technique named multiple populations for multiple objectives (MPMOs). In MPMO, multiple populations cooperate to optimize all objectives of the MOPs, and each population is corresponded with only one objective. An information-sharing strategy is applied to the MPMO as a remedy for the possible loss of population diversity, and it has been adapted by many other researchers [29], [30].

B. MOEA/D-M2M

We describe the MOEA/D-M2M approach first as it provides a generic framework which can integrate with most standard EMO methods, such as NSGA-II, SPEA2, MOEA/D, and SMS-EMOA. Unlike MOEA/D [12], M2M decomposes an MOP into a set of multiobjective optimization subproblems. Each multiobjective subproblem in M2M has its own subpopulation, and these subproblems can be solved in a collaborative manner. For this purpose, K direction vectors $\mathbf{v}^1, \ldots, \mathbf{v}^K$ in $\mathbf{R}_+^m$ are first generated in the first quadrant of the objective space. They divide $\mathbf{R}_+^m$ into K subregions $\Omega_1, \ldots, \Omega_K$

$$\Omega_k = \left\{ \mathbf{u} \in \mathbf{R}_+^m \middle| \left\langle \mathbf{u}, \mathbf{v}^k \right\rangle \leq \left\langle \mathbf{u}, \mathbf{v}^j \right\rangle \text{ for any } j = 1, \ldots, K \right\}$$

where $\langle \mathbf{u}, \mathbf{v}^j \rangle$ is the acute angle between solution $\mathbf{u}$ and the jth direction vector $\mathbf{v}^j$. Accordingly, the population is decomposed into K subpopulations, and each subpopulation corresponds to a multiobjective subproblem. For each multiobjective subproblem, there is a subpopulation $\mathbf{P}_k$, $(k = 1, \ldots, K)$ with S_k individuals to be used to optimize it.

Many selection strategies can be used to independently select the S_k solutions for each $\mathbf{P}_k$. If the solutions assigned to subpopulation $\mathbf{P}_k$ number fewer than S_k, then $S_K - |\mathbf{P}_k|$ solutions can be randomly selected from the entire population and added to $\mathbf{P}_k$; if the solutions assigned to subpopulation $\mathbf{P}_k$ number more than S_k, then the best S_k solutions can be selected according to the rule of selection strategies. The MOEA/D-M2M population decomposition strategy has a strong ability to maintain the population diversity because of the independent selection in each subregion.

C. Related Theoretical Studies

The study of EMO algorithms is mainly focused on algorithm design and practical application. The past two decades have seen great success achieved by EMO algorithm in optimizing multiobjective problems. However, the lack of theoretical analysis has inhibited the further development and application of EMO algorithms. Theoretical study can help us better understand how EMO algorithms work, and thus to design better algorithms. Therefore, theoretical study continues to draw wide attention and is becoming a hot issue among researchers today.

It had been proved that an EMO algorithm can converge to the Pareto-optima of a particular dispersed MOP with probability one [33]. Hanne [34] studied the convergence of real MOPs, and proposed a convergence with probability one theorem for multiobjective problems. He and Yao [35] proposed a Markov-chain-based framework for the computational complexity analysis of MOPs. The computational complexity of some particular MOPs can be analyzed and compared in this framework, which can give an effective guidance for efficient algorithm design. Li *et al.* [36] theoretically studied decomposition-based multiobjective evolutionary algorithms by analyzing their runtime complexities on two simple and two difficult instances. He and Lin [37] analyzed the limit of average convergence rate and then proposed the asymptotic average convergence rate to measure the convergence rate of evolutionary algorithms.

III. Imbalanced Problems in Multiobjective Optimization

In this section, we provide a definition of imbalanced MOPs and discuss their properties.

A. Definition of Imbalanced Problem

Definition 1 (Imbalanced Problem): An MOP is defined to be an imbalanced MOP, if a specific subset (here called the "favored" subset) of its PF satisfies the following conditions.

1) The complexity of the optimization subproblem corresponding to the favored subset is significantly lower than the complexity of the subproblem corresponding to the other part (the "unfavored" subset) of the PF.
2) The Pareto-set (PS) of the favored subset dominates a significantly larger part of the feasible variable space than the PS of the unfavored subset.

Although the above definition does not explicitly quantify the extent of favor of one PF subset over the other, it is clear that in such problems, if the favored PF is relatively easy to find and the corresponding favored PS dominates most of the feasible search space, both conditions make it difficult for an algorithm to find the entire PF. In such problems, diversity preservation must be given more importance than convergence so that despite the natural favoring of one PF subset, the diversity preservation forces the unfavored PF to be found and maintained. The exact extent of favoring of one subset over the other that will cause an algorithm difficulty may depend on the search power of the algorithm and hence is difficult to quantify. But in this paper, we suggest some imbalanced test problems and demonstrate that they cause significant difficulties to standard EMO algorithms. Those MOPs that do not satisfy the above two conditions are termed "balanced" problems.

The favoring of one subset in the PF can be introduced in the following manner.

1) *Imbalanced Mapping:* The favored part of the PF can be easily approached from a large part of the feasible

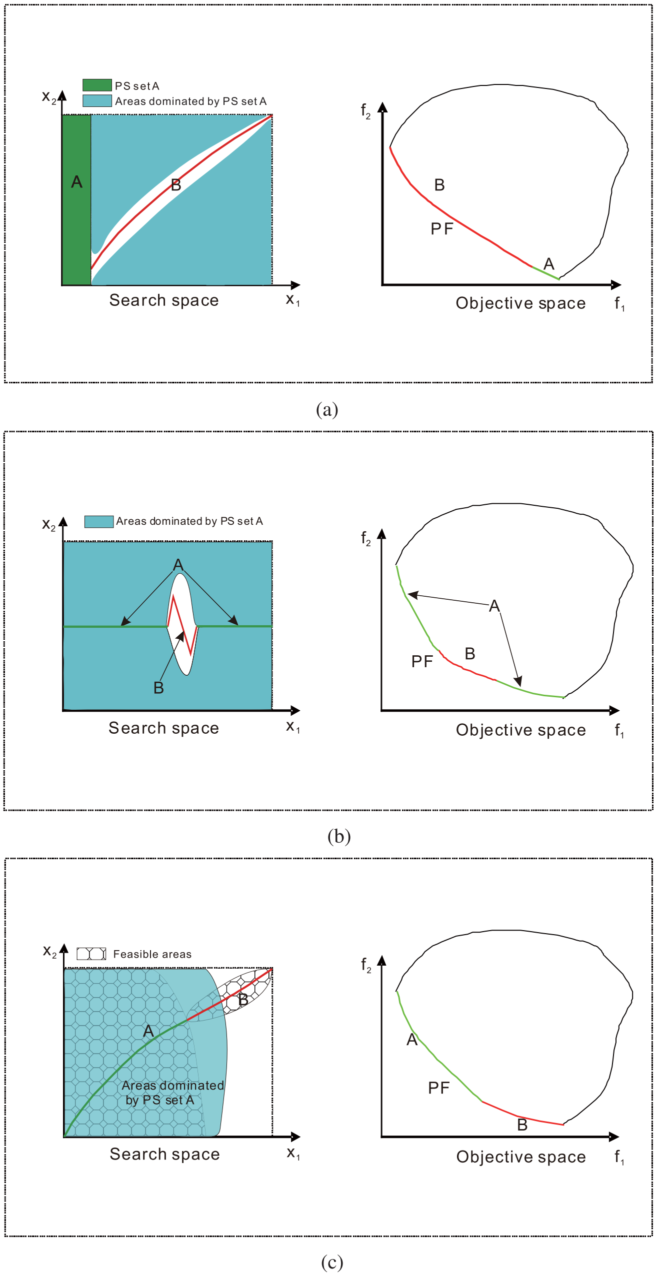


Fig. 1. Illustration of imbalanced problems. (a) Imbalanced mapping. (b) Variable linkages. (c) Constraint isolation.

search space, whereas the unfavored part of the PF can only be found from a relatively small search region. One way to achieve this would be to have a many-to-one mapping from the search space to the favored PF so that the favored PF can be arrived at easily.

2) *Variable Linkages:* Variable linkages can cause some part of the PF to be difficult to find. Certain imbalanced problems can be constructed so that the favored PS comes from relatively uncorrelated variable interactions, while the unfavored part of the PS is formed from extreme correlation among variables. These properties will make it easy for an algorithm to find the favored part of the PF.

3) *Constraint Isolation:* The unfavored PS subset can be close to constraint boundaries, while the favored PS can be well inside the constraint boundaries, thereby causing algorithms difficulty in finding and maintaining feasible solutions near the unfavored PS.

We illustrate the above three scenarios in Fig. 1. Fig. 1(a) illustrates a problem for which the complete region A maps to a subset on the PF (marked by A). Thus, any point in A in the search space automatically becomes a Pareto-optimal point and lies in the favored subset of the PF. The PS region A also dominates the light blue shaded region. Thus, there is a narrow search region (shown in white) that may lead to the PF subset B (unfavored part). Both aspects make it difficult for an algorithm to find the entire PF (A and B).

Fig. 1(b) illustrates a problem for which the favored part of the PF comes from two line segments in the search space, but they dominate a major part of the search space. To make the matter worse, the unfavored part of the PF (marked as B) requires variables to follow a specific nonlinear linkage among them.

Fig. 1(c) illustrates a problem in which the difficulty of arriving at the unfavored PF comes from constraints. The search space near subset B is highly constrained, thereby making it difficult for an algorithm to find and maintain PS points from B. In contrast, PS points at A are far from constraint boundaries and due to the fact that once these points are discovered they also dominate most of the search space, they pose an imbalance in the search process against maintaining a variety of PS points in both A and B.

The properties of these problems can be summarized as follows.

1) PS A is relatively easy to find.
2) PS A dominates most of the search space.
3) PS B is difficult to find.

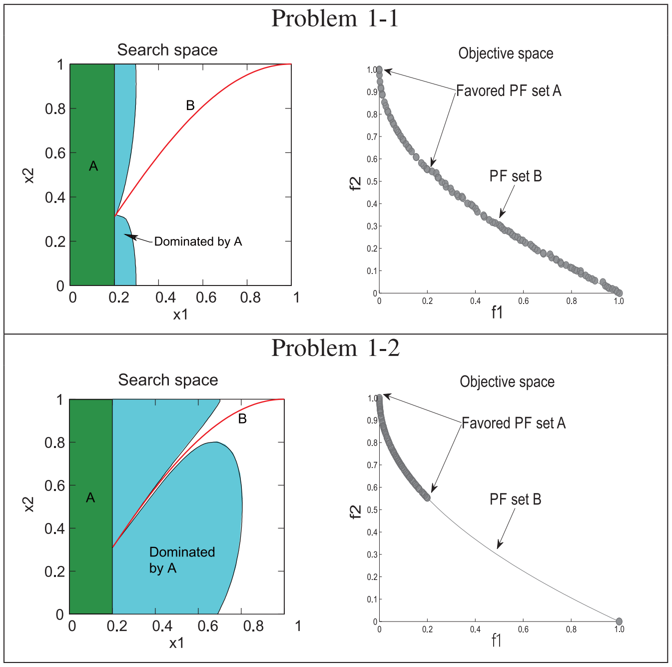
B. Illustrative Problems

Different problems may show different degrees of imbalance depending on the relative bias toward the favored and unfavored PF and the sizes of areas dominated by the favored PF. Study of the degree of imbalance serves to better clarify how the problem makes an imbalance between the diversity and convergence of a population. As a matter of fact, a change of the dominated areas by the favored PF in the search space can affect the degree of imbalance. We design four bi-objective problems to illustrate the above-mentioned properties of imbalanced problems. To demonstrate their solution difficulties, NSGA-II is applied to optimize these problems. A population size of 100 is used in each case and NSGA-II is run for 1000 generations. Replicate runs are not used here as these examples are meant only to provide illustrations of the types of problems encountered by the algorithms. The SBX operator [38] with $p_c = 0.9$ and $\eta_c = 20$, and polynomial mutation [39] with $p_m = 0.5$ and $\eta_m = 50$ are used. The four MOPs are as follows:

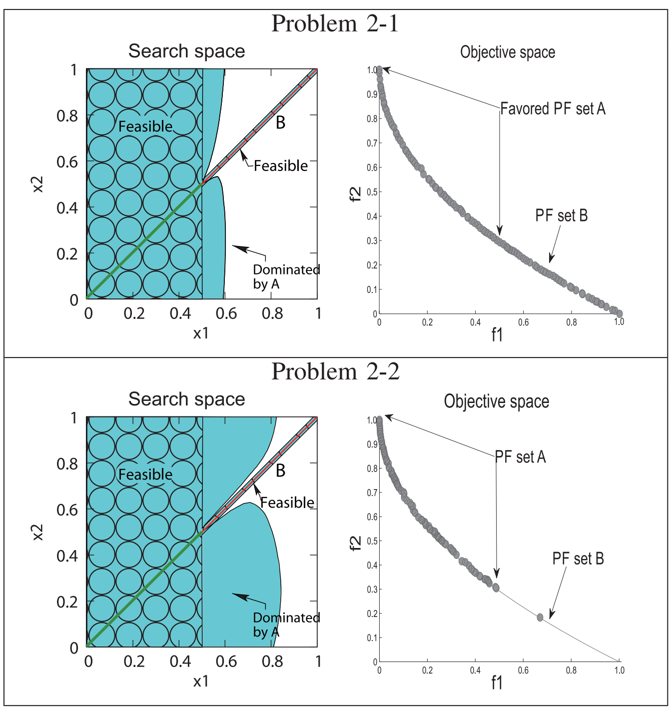
Problem 1-1

$$\text{Min: } \begin{cases} f_1(\mathbf{x}) = (1+g(\mathbf{x}))x_1 \\ f_2(\mathbf{x}) = (1+g(\mathbf{x}))\left(1-\sqrt{x_1}\right) \end{cases}$$

$$\text{where } \begin{cases} g(\mathbf{x}) = \begin{cases} 0 & \text{if } 0 \le x_1 \le 0.2 \\ 0.5\left(-0.9t^2+|t|^{0.6}\right) & \text{otherwise} \end{cases} \\ t = x_2 - \sin(0.5\pi x_1), x_i \in [0,1], i = 1,2. \end{cases} \quad (2)$$



(a)



(b)

Fig. 2. Search space and the nondominated front obtained by NSGA-II for four illustrative problems. (a) Problems 1-1 and 1-2. (b) Problems 2-1 and 2-2.

Its PF is $f_2 = 1 - \sqrt{f_1}$ in the range $0 \le f_1 \le 1$. Its PS comes from all points that meet $0 \le x_1 \le 0.2$ or $x_2 = \sin(0.5\pi x_1)$ in the range $x_1 > 0.2$. The top part of Fig. 2(a) shows the search space and PS for favored (A) and unfavored (B) PFs (left figure) and the NSGA-II solutions obtained. Although the favored PF comes from the entire region A in PS, these points dominate only a small part of the rest of the search space (shown in light blue color). This does not satisfy the second condition stated in the definition of imbalanced problems. As a result, NSGA-II with population size 100 has no difficulty in finding both PFs (A and B)

Problem 1-2

$$\text{Min:} \quad \begin{cases} f_1(\mathbf{x}) = (1 + g(\mathbf{x}))x_1 \\ f_2(\mathbf{x}) = (1 + g(\mathbf{x}))\big(1 - \sqrt{x_1}\big) \end{cases}$$

$$\text{where} \quad \begin{cases} g(\mathbf{x}) = \begin{cases} 0 \quad \text{if} \ \ 0 \le x_1 \le 0.2 \\ 10\big(-0.9t^2 + |t|^{0.2}\big) \quad \text{otherwise} \end{cases} \\ t = x_2 - \sin(0.5\pi x_1), x_i \in [0, 1], \ \ i = 1, 2. \end{cases}$$

This problem is different from the previous problem only in the $g(\mathbf{x})$ function, but its PS and PF are identical to that of the previous problem. As shown in the bottom left part of Fig. 2(a), although an identical PS for A is retained, this region dominates a much larger part of the search space. Most points of the PS for B are now surrounded by dominated points of A. Thus, even if some neighboring points to B are found, they can be dominated by points of A, making it difficult for an EMO to find and maintain subset B. The right-hand figure shows the NSGA-II results. Even after 1000 generations, 100 population members are unable to find any point from B

Problem 2-1

$$\text{Min:} \quad \begin{cases} f_1(\mathbf{x}) = (1 + g(\mathbf{x}))x_1 \\ f_2(\mathbf{x}) = (1 + g(\mathbf{x}))\big(1 - \sqrt{x_1}\big) \end{cases}$$

$$\text{s.t.} \quad G(\mathbf{x}) = 0$$

$$\text{where} \quad \begin{cases} g(\mathbf{x}) = 0.5\big(-0.9t^2 + |t|^{0.6}\big) \\ t = x_2 - x_1, x_i \in [0, 1], \ \ i = 1, 2 \\ G(\mathbf{x}) = \begin{cases} 1 + 10g(\mathbf{x}), & \text{if } x_1 > 0.5, g(\mathbf{x}) > 0.01 \\ 0, & \text{otherwise.} \end{cases} \end{cases}$$

The PF for problem 2-1 is $f_2 = 1 - \sqrt{f_1}$ in the range $0 \le f_1 \le 1$. The favored PS (A) is $x_2 = x_1$ within $0 \le x_1 \le 0.6$ [top part of Fig. 2(b)]. The search space around this PS is feasible, but this PS dominates a small part of the remaining search space. The unfavored PS (B) also comes from $x_2 = x_1$ but in the range $x_1 > 0.5$. However, the feasible region around this PS is very small, as shown by a narrow band of feasible region around B. Despite this bias against B, NSGA-II with 100 population members can discover both A and B after 1000 generations. The eventual success of NSGA-II can be attributed to the fact that, despite the difficulties in approaching B, once some feasible points near B are found, they can survive in the NSGA-II population, as favored PS points from A do not dominate them

Problem 2-2

$$\text{Min:} \quad \begin{cases} f_1(\mathbf{x}) = (1 + g(\mathbf{x}))x_1 \\ f_2(\mathbf{x}) = (1 + g(\mathbf{x}))\big(1 - \sqrt{x_1}\big) \end{cases}$$

$$\text{s.t.} \quad G(\mathbf{x}) = 0$$

$$\text{where} \quad \begin{cases} g(\mathbf{x}) = 8\big(-0.9t^2 + |t|^{0.6}\big) \\ t = x_2 - x_1, x_i \in [0, 1], \ \ i = 1, 2 \\ G(\mathbf{x}) = \begin{cases} 1 + 10g(\mathbf{x}), & \text{if } x_1 > 0.5, g(\mathbf{x}) > 0.01 \\ 0, & \text{otherwise.} \end{cases} \end{cases}$$

This problem differs from the previous problem only in its $g(\mathbf{x})$ function. Its PF and PS are identical to those of the previous problem. As shown in the lower part of Fig. 2(b), here the PS of A dominates a larger part of the rest of the search space. The NSGA-II solutions are now mostly found in the favored part of the PF rather than in the entire PF.

It can be seen from the above results that with an increasing dominated region for the favored PF, the degree of imbalance increases and the problem becomes harder to solve. The "convergence first" strategy of EMO methods such as NSGA-II focuses on feasibility and convergence to the PF, thereby providing a major imbalance between diversity preservation and convergence in solving such problems. To solve such problems, we require algorithms that provide a better balance between these two aspects.

IV. Critical Review of Convergence-First EMO Methods for Imbalanced Problems

Most state-of-the-art EMO algorithms are designed based on the principle of convergence first and diversity second. These algorithms are effective when optimizing balanced problems. However, it has been found recently [31] that these EMO algorithms face great difficulties in optimizing imbalanced MOPs, while EMO algorithms which strike a balance between convergence and diversity, such as MOEA/D-M2M [31], can be more effective when optimizing imbalanced problems. This section will investigate the theoretical reasons behind this phenomenon.

Some basic assumptions must be made when analyzing EMO algorithms in theory [40]. The three basic assumptions about the EMO algorithms are as follows.

1) *Assumption 1:* $\forall \mathbf{x}^* \in \mathrm{PS}$, there is a neighborhood of x^*, denoted as $\delta(\mathbf{x}^*) = \{\mathbf{x} \in \Omega || F(\mathbf{x}) - F(\mathbf{x}^*)| < \varepsilon\}$, satisfying $L(\delta(\mathbf{x}^*)) > 0$, where ε represents any small positive number and L represents the Lebesgue measure.
2) *Assumption 2:* Let $\mathbf{x}$ and $\mathbf{x}'$ be two random points in the search space, and $\mathrm{MC}(\mathbf{x})$ represent the operation of creating a point from $\mathbf{x}$ by crossover and mutation operators; then we suppose the probability of getting $\mathbf{x}'$ from $\mathbf{x}$ is greater than zero, namely, $P(\mathrm{MC}(\mathbf{x}) = \mathbf{x}') > 0$.
3) *Assumption 3:* In this paper, the convergence of an EMO algorithm means that its population converges to a very small neighborhood of PS.

To facilitate the theoretical analysis, the evolutionary process is simplified by a new EMO model in this paper. Suppose that the PS of a multiobjective problem only contains two points A and B, where A is the favored PF point and B is the unfavored PF point. A convergence first and diversity second selection strategy can be explained as whenever the newly generated individual dominates the parent individual, the parent individual will be replaced by the new individual in this model. The following theorems are based on the above assumptions.

Theorem 1: When optimizing an imbalanced problem, the population of a convergence first and diversity second-selection-strategy-based EMO algorithm is more likely to converge to the favored PF.

Proof: First, we consider an initial solution $\mathbf{x}$ in the search space, which is located in the shared dominating area of A and B [see Fig. 3(a)]. The evolutionary process of $\mathbf{x}$ can be considered as a map from one state to another state, which can be described as

$$\mathbf{x} \xrightarrow{\text{Crossover}} \mathbf{u}^1 \xrightarrow{\text{Mutation}} \mathbf{u}^2 \xrightarrow{\text{Selection}} \mathbf{x}_{\text{new}}.$$

Favored PF point A and the areas dominated by A are denoted as $\mathrm{Dom}(A)$. The entire search space is denoted as Ω. Let us say that the probability of a creating an offspring $\mathbf{u}^2$ that dominates $\mathbf{x}$ and lies in $\mathrm{Dom}(A)$ is λ. According to the first condition of the imbalanced problem's definition, the probability of finding favored PF point A is significantly larger than finding unfavored PF point B. That is, $\lambda \approx 1$ for imbalanced problems. In the following evolutionary process, the offspring individual ($\mathbf{u}^2$) of $\mathbf{x}$ may have two possible states.

1) *State 1:* Located in $\mathrm{Dom}(A)$, and its probability is denoted as $P_{S1}(\mathbf{u}^2)$.
2) *State 2:* Located in $\Omega - \mathrm{Dom}(A)$, and its probability is denoted as $P_{S2}(\mathbf{u}^2)$.

$P_{S1}(\mathbf{u}^2)$ and $P_{S2}(\mathbf{u}^2)$ can be calculated by the following equations:

$$P_{S1}\left(\mathbf{u}^2\right) = \frac{L(\mathrm{Dom}(A))}{L(\Omega)} \tag{3}$$

$$P_{S2}\left(\mathbf{u}^2\right) = \frac{L(\Omega - \mathrm{Dom}(A))}{L(\Omega)} \tag{4}$$

where $L(C)$ denotes the Lebesgue measure of the search region C. According to the second condition of the imbalanced problem's definition, $L(\mathrm{Dom}(A)) \gg L(\Omega - \mathrm{Dom}(A))$ and we get

$$P_{S1}\left(\mathbf{u}^2\right) \gg P_{S2}\left(\mathbf{u}^2\right). \tag{5}$$

When the convergence first and diversity second selection strategy is adopted, the probability of creating solutions closer to A is

$$\hat{P}(A) = \lambda P_{S1}\left(\mathbf{u}^2\right) \tag{6}$$

and the only way $\mathbf{u}^2$ can get closer to B is if it dominates $\mathbf{x}$ and falls in $\Omega - \mathrm{Dom}(A)$—that is, with probability

$$\tilde{P}(A) = (1 - \lambda) P_{S2}\left(\mathbf{u}^2\right). \tag{7}$$

Since $\lambda \approx 1$ and $P_{S1}(\mathbf{u}^2) \gg P_{S2}(\mathbf{u}^2)$, we can write

$$\begin{aligned}\hat{P}(A) = \lambda P_{S1}\left(\mathbf{u}^2\right) &\gg \lambda P_{S2}\left(\mathbf{u}^2\right) \\ &\gg (1 - \lambda) P_{S2}\left(\mathbf{u}^2\right) = \tilde{P}(A).\end{aligned} \tag{8}$$

This means the next generation individual of $\mathbf{x}$ is more likely to remain in A and get closer to A than $\mathbf{x}$. Therefore, the population of the EMO algorithm are more likely to converge to the favored PF point A. ■

Theorem 2: The M2M population decomposition strategy can effectively aid EMO algorithms to converge into both favored and unfavored PFs when optimizing imbalanced problems.

Proof: We consider two initial individuals $\mathbf{x}_A$ and $\mathbf{x}_B$ located in the shared dominated areas of favored PF point A and unfavored PF point B [see Fig. 3(b)]. M2M assigns the two individuals into two different subregions R_A and R_B in which A and B are located, respectively. In the process of evolution,

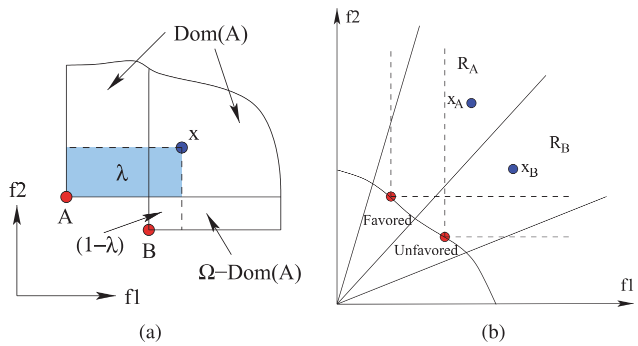


Fig. 3. Illustrative sketches for two theorems. (a) Sketch for Theorem 1. (b) Sketch for Theorem 2.

$\mathbf{x}_A$ will converge to favored PF point A. The offspring individual of $\mathbf{x}_B$ will have two possible states at the beginning of evolution.

1) *State 1:* Located in subregion R_A.
2) *State 2:* Located in subregion R_B.

Although $\mathbf{x}_B$ also has a very high probability to find favored PF point A, $\mathbf{x}_B$ will not be replaced by the newly generated individual if it is located in subregion R_A, according to the M2M selection strategy. In other words, the next generation individual of $\mathbf{x}_B$ will always stay in subregion R_B, which means that $\mathbf{x}_B$ will converge to unfavored PF point B instead of the favored PF point A. ■

V. Numerical Computations on Imbalanced Problems

First, we describe the five EMO-M2M algorithms used in this paper and then present the test suite of imbalanced problems.

A. EMO Algorithms in the M2M Framework

1) NSGA-II-M2M: Deb *et al.* [6] proposed an elitist nondominated sorting genetic algorithm: NSGA-II in 2002. In NSGA-II, the offspring population is generated from the parent population and put together with the parent population. Then, a nondominated sorting of the combined population is used to rank the population members. Solutions from the best ranks are chosen until all members of the last acceptable rank cannot be all chosen. The remaining population members having worse ranks are deleted. Solutions from the last acceptable rank are assigned a crowding distance value based on their sparsity in the objective space, and solutions with higher crowding distance values are chosen to complete the next-generation population. Clearly, NSGA-II emphasizes nondominated solutions first and when two solutions have an identical rank, their sparsity is checked to break the tie. Thus, the NSGA-II procedure comes under the category of convergence first, diversity second strategies. In our proposed NSGA-II-M2M algorithm, each subpopulation uses NSGA-II's selection strategy to select the next-generation subpopulation and all subpopulations are put together to form the next-generation population.

2) MOEA/D-M2M: MOEA/D decomposes an MOP into a series of single-objective optimization subproblems. Decomposition methods may use the Tchebycheff approach, the weighted-sum approach or the BI approach. MOEA/D is based on the assumption that each Pareto optimum can be achieved by optimizing the respective single-objective optimization subproblem defined by a weight vector. Therefore, a set of weight vectors needs to be generated in the beginning of the optimization. The MOP is decomposed into single-objective subproblems by those weight vectors, and each single-objective subproblem is associated with a weight vector. The subproblems represented by the closest weight vectors to this vector are defined as the neighbors of this subproblem. Every subproblem can share information with its neighbors by genetic operators during the evolutionary process. In MOEA/D-M2M, the three decomposition approaches can be used to select the next-generation subpopulations during the evolution.

3) SPEA2-M2M: SPEA2 is an elitist domination-based EMO algorithm proposed by Zitzler *et al.* [7]. It maintains a fixed evolving population and an external archive. At each generation, newly generated solutions are first used to update the parent population, and then newly found nondominated solutions are used to update the external archive. A new solution will enter the external population if it dominates some solution in the archive. The dominated solutions in the archive are discarded. If the archive size exceeds a predefined size, a truncation procedure is applied to discard some overcrowded solutions. In SPEA2-M2M, each subproblem uses an equal-sized external archive. In each generation, the newly generated subpopulation is used to update its respective external archive. Finally, the external archive is declared as the problem solution.

4) SMS-EMOA-M2M: SMS-EMOA combines the HV measure and the nondominated sorting approach in such a way as to maximize the HV during the evolutionary process. In SMS-EMOA, a nondominated sorting process is conducted first, and the solutions with better rank are selected to fill the next generation population until its size exceeds the predefined population size. The solutions in the last selected nondominated front are truncated by maximizing the HV measure, instead of NSGA-II's crowding distance measure. In SMS-EMOA-M2M, the SMS-EMOA selection strategy is used to update each subpopulation solving a different subproblem.

5) GVEGA-M2M: The original VEGA procedure [27] divides N population members into m equal subpopulations (of size N/m each) and uses an individual objective function f_i to assign fitness to all members of the ith subpopulation. Then a selection operation is used to construct a mating pool of every subpopulation, independently. Then all mating pools are combined to form an overall mating pool of size N and the usual recombination and mutation operators are applied to the mating pool without paying any further attention to which subpopulation each member comes from. VEGA was found to converge near individual objective-minimum solutions rather than to the PF, mainly due to selection of individuals based on individual objective functions. However, the simplicity of the VEGA approach was the intriguing aspect.

Here, we modify the VEGA procedure to construct a generalized VEGA procedure, which is described next.

The proposed GVEGA works with the concept of an achievement scalarizing function (ASF) [44]. First, a set of equally spaced H reference directions, as in MOEA/D [12] or NSGA-III [45], is defined. Each reference direction is represented using an m-dimensional weight vector: $\mathbf{w}^k = (w_1^k, w_2^k, \ldots, w_m^k)$ for $k = 1, 2, \ldots, H$. Every feasible population member, $\mathbf{x}$, can then be assigned an ASF value for each weight vector, $\mathbf{w}^k$, as follows:

$$\mathrm{ASF}(\mathbf{x}, k) = \max_{i=1}^{m}\left(\frac{f_i(\mathbf{x}) - z_i}{w_i^k}\right) \tag{9}$$

where $\mathbf{z}$ is any fixed utopian vector. The GVEGA procedure is then applied as follows. First, we divide the population P_t at generation t into H subpopulations based on the ASF values. For this purpose, we calculate $\mathrm{ASF}(\mathbf{x}, k)$ for every population member, $\mathbf{x}$, for each weight vector $\mathbf{w}^k$. Then, we sort the population members from the smallest ASF to largest ASF for each $\mathbf{w}^k$. Thereafter, we select the top N/H members from each sorted list and call it the kth subpopulation and perform a tournament selection using $\mathrm{ASF}(\mathbf{x}, k)$ as the fitnesses of the subpopulation members to create a mating pool of size N/H. Then, we combine all H mating pools and form the overall mating pool of size N. Next, we perform a recombination operation on the overall mating pool and then mutation as usual to form the next generation population P_{t+1} of size N. Note that in the above process, one population member can be selected multiple times for different weight vectors. This means that such a solution turns out to be good for multiple weight vectors and should have more than one copy in the overall mating pool. Also note that there is no explicit domination check in the above procedure, but the ASF fitness calculation establishes an indirect pressure for nondominated solutions to be selected.

In GVEGA-M2M, the above-described GVEGA is applied to each subproblem independently at each generation. Thereafter, all subpopulations are collected together to form the overall population for the next generation.

B. Proposed Imbalanced Multiobjective Test Suite

The proposed imbalanced problems have the following structure, similar to the DTLZ test problem construction procedure [43]:

$$\min \begin{cases} f_1(\mathbf{x}) = (1 + g(\mathbf{x}_{II}))\phi_1(\mathbf{x}_I) \\ f_2(\mathbf{x}) = (1 + g(\mathbf{x}_{II}))\phi_2(\mathbf{x}_I) \\ \cdots \\ f_m(\mathbf{x}) = (1 + g(\mathbf{x}_{II}))\phi_m(\mathbf{x}_I) \end{cases}$$

where m is the dimension of the objective space, $\mathbf{x}_I = (x_1, x_2, \ldots, x_{m-1})$, $\mathbf{x}_{II} = (x_m, x_{m+1}, \ldots, x_n)$, and $\mathbf{x} = (\mathbf{x}_I, \mathbf{x}_{II})$. The parameter n is the dimension of the search space. The function $g(\mathbf{x}_{II})$ is a positive function having a minimum value of zero. The functions $\phi_i(\mathbf{x}_{II})$ are also positive functions and are mutually conflicting. The construction ensures that the PS occurs for $\mathbf{x}_{II}^*$ that makes $g(\mathbf{x}_{II}^*) = 0$. The PS for $\mathbf{x}_I$ occurs from the tradeoff among ϕ_i conflicting functions.

A little thought will reveal that the convergence of points close to the PF is determined by the complexity in solving the problem $g(\mathbf{x}_I) = 0$, and the diversity in maintaining the PF is determined by the t_i functions. To introduce an imbalance between the ease of convergence and diversity maintenance, we divide the g function into an easy and a difficult part. This way the entire search space gets divided into an easy and a difficult part. We construct 14 imbalanced test problems, including all three types of imbalances discussed earlier. Problems 1–6 are caused by "imbalanced mapping," 7–10 are caused by "variable linkages" and 11–14 are caused by "constraint isolation." All these test functions are to be minimized by default, and we recommend $n = 10$ for all problems.

Imbalanced problems IMB1–IMB6 introduce the imbalance mapping difficulty described before

Problem IMB1

$$\text{Min:} \begin{cases} f_1(\mathbf{x}) = (1 + g(\mathbf{x}))x_1 \\ f_2(\mathbf{x}) = (1 + g(\mathbf{x}))\left(1 - \sqrt{x_1}\right) \end{cases}$$

$$\text{s.t. } x_i \in [0, 1] \quad i = 1, 2, \ldots, n$$

$$\text{where } \begin{cases} g(\mathbf{x}) = \begin{cases} 0, & \text{if } 0 \le x_1 \le 0.2 \\ \sum_{i=2}^{n} 0.5\left(-0.9t_i^2 + |t_i|^{0.6}\right), & \text{otherwise} \end{cases} \\ t_i = x_i - \sin(0.5\pi x_1), \ i = 2, 3, \ldots, n. \end{cases}$$

The PF is given by $f_2 = 1 - \sqrt{f_1}$ for $0 \le f_1 \le 1$. The part of the PF within $0 \le f_1 \le 0.2$ is the favored PF subset and is easy to find. Notice that the $g()$ function is constructed in such a way that makes any solution in $0 \le x_1 \le 0.2$ to be automatically on the favored PS irrespective of other variable values, thereby making 20% of the search space lie on the favored PS. This property makes the favored part of the PF easy to find. On the other hand, the unfavored part of the PS comes from nonlinear combinations of variables: $x_i = \sin(0.5\pi x_1)$ for $0.2 \le x_1 \le 1$ for $j = 2, 3, \ldots, n$, and is relatively difficult to find

Problem IMB2

$$\text{Min:} \begin{cases} f_1(\mathbf{x}) = (1 + g(\mathbf{x}))x_1 \\ f_2(\mathbf{x}) = (1 + g(\mathbf{x}))(1 - x_1) \end{cases}$$

$$\text{s.t. } x_i \in [0, 1] \quad i = 1, 2, \ldots, n$$

$$\text{where } \begin{cases} g(\mathbf{x}) = \begin{cases} 0, & \text{if } 0.4 \le x_1 \le 0.6 \\ \sum_{i=2}^{n} 0.5\left(-0.9t_i^2 + |t_i|^{0.6}\right), & \text{otherwise} \end{cases} \\ t_i = x_i - \sin(0.5\pi x_1), \ i = 2, 3, \ldots, n. \end{cases}$$

Its PF is given by $f_2 = 1 - f_1$ in the range $0 \le f_1 \le 1$. The favored PF lies in $0.4 \le f_1 \le 0.6$. The favored PS comes from all points that satisfy $0.4 \le x_1 \le 0.6$ and the unfavored PS comes from $x_j = \sin(0.5\pi x_1)$ for $0 \le x_1 \le 1$ and for all $j = 2, \ldots, n$

Problem IMB3

$$\text{Min:} \begin{cases} f_1(\mathbf{x}) = (1 + g(\mathbf{x}))\cos(\pi x_1/2) \\ f_2(\mathbf{x}) = (1 + g(\mathbf{x}))\sin(\pi x_1/2) \end{cases}$$

$$\text{s.t. } x_i \in [0, 1] \quad i = 1, 2, \ldots, n$$

$$\text{where } \begin{cases} g(\mathbf{x}) = \begin{cases} 0, & \text{if } 0.8 \le x_1 \le 1 \\ \sum_{i=2}^{n} 0.5\left(-0.9t_i^2 + |t_i|^{0.6}\right), & \text{otherwise} \end{cases} \\ t_i = x_i - \sin(0.5\pi x_1), \ i = 2, 3, \ldots, n. \end{cases}$$

Its PF satisfies $f_1^2 + f_2^2 = 1$. The favored PF comes from $0.8 \le f_1 \le 1$, which is easy to find. The favored PS comes from all the points that meet $0.8 \le x_1 \le 1$ and the unfavored PS comes from $x_j = \sin(0.5\pi x_1)$ for $0 \le x_1 \le 1$ and for $j = 2, \ldots, n$

Problem IMB4

$$\text{Min:} \begin{cases} f_1(\mathbf{x}) = (1+g(\mathbf{x}))x_1x_2 \\ f_2(\mathbf{x}) = (1+g(\mathbf{x}))x_1(1-x_2) \\ f_3(\mathbf{x}) = (1+g(\mathbf{x}))(1-x_1) \end{cases}$$

s.t. $x_i \in [0, 1] \quad i = 1, 2, \ldots, n$

where

$$g(\mathbf{x}) = \begin{cases} 0, & \text{if } 2/3 \le x_1 \le 1 \\ 2\cos\left(\frac{\pi x_1}{2}\right)\sum_{i=3}^{n}\left(-0.9t_i^2 + |t_i|^{0.6}\right), & \text{otherwise} \end{cases}$$

$$t_i = x_i - (x_1 + x_2)/2, \; i = 3, 4, \ldots, n.$$

Its PF is a linear hyperplane, given as follows: $f_1 + f_2 + f_3 = 1$ for $0 \le (f_1, f_2, f_3) \le 1$. The favored PF comes from $0 \le f_3 \le 1/3$ and the corresponding PS satisfies $2/3 \le x_1 \le 1$. However, the unfavored part of the PS comes from linear relationships among variables: $x_j = (x_1 + x_2)/2$ in the range $0 \le (x_1, x_2) \le 1$ and for $j = 3, 4, \ldots, n$

Problem IMB5

$$\text{Min:} \begin{cases} f_1(\mathbf{x}) = (1+g(\mathbf{x}))\cos(\pi x_1/2)\cos(\pi x_2/2) \\ f_2(\mathbf{x}) = (1+g(\mathbf{x}))\cos(\pi x_1/2)\sin(\pi x_2/2) \\ f_3(\mathbf{x}) = (1+g(\mathbf{x}))\sin(\pi x_1/2) \end{cases}$$

s.t. $x_i \in [0, 1] \quad i = 1, 2, \ldots, n$

where

$$g(\mathbf{x}) = \begin{cases} 0, & \text{if } 0 \le x_1 \le 0.5 \\ 2\cos\left(\frac{\pi x_1}{2}\right)\sum_{i=3}^{n}\left(-0.9t_i^2 + |t_i|^{0.6}\right), & \text{otherwise} \end{cases}$$

$$t_i = x_i - (x_1 + x_2)/2, \; i = 3, 4, \ldots, n.$$

Its PF is a part of the sphere given by $f_1^2 + f_2^2 + f_3^2 = 1$ in the range $0 \le (f_1, f_2, f_3) \le 1$. The favored part of the PF satisfies $0 \le f_3 \le \sqrt{2}/2$ and $(0, 0, 1)^T$ point in the objective space. The favored part of the PS lies in $0 \le x_1 \le 0.5$ and $x_1 = 1$ irrespective of other variable values. The unfavored part of the PS comes from specific relationships among variables: $x_j = (x_1 + x_2)/2$ in the range $0 \le (x_1, x_2) \le 1$ and for $j = 3, 4, \ldots, n$

Problem IMB6

$$\text{Min:} \begin{cases} f_1(\mathbf{x}) = (1+g(\mathbf{x}))x_1x_2 \\ f_2(\mathbf{x}) = (1+g(\mathbf{x}))x_1(1-x_2) \\ f_3(\mathbf{x}) = (1+g(\mathbf{x}))(1-x_1) \end{cases}$$

s.t. $x_i \in [0, 1] \quad i = 1, 2, \ldots, n$

where

$$g(\mathbf{x}) = \begin{cases} 0, & \text{if } 0 \le x_1 \le 0.75 \\ 2\cos\left(\frac{\pi x_1}{2}\right)\sum_{i=3}^{n}\left(-0.9t_i^2 + |t_i|^{0.6}\right), & \text{otherwise} \end{cases}$$

$$t_i = x_i - (x_1 + x_2)/2, \; i = 3, 4, \ldots, n.$$

Its PF is a linear hyperplane: $f_1 + f_2 + f_3 = 1$ in the range $0 \le (f_1, f_2, f_3) \le 1$. The favored PF satisfies $0 \le f_3 \le 0.75$ and $(0, 0, 1)^T$ point in the objective space. The favored part of the PS comes from $0 \le x_1 \le 0.75$ and $x_1 = 1$, irrespective of other variable values, but the unfavored part of the PS comes from multiple linear relationships among variables, as follows: $x_j = (x_1 + x_2)/2$ for $0 \le (x_1, x_2) \le 1$ and for $j = 3, 4, \ldots, n$.

Imbalanced problems IMB7–IMB10 are constructed to introduce the variable linkage difficulty in imbalance problems

Problem IMB7

$$\text{Min:} \begin{cases} f_1(\mathbf{x}) = (1+g(\mathbf{x}))x_1 \\ f_2(\mathbf{x}) = (1+g(\mathbf{x}))\left(1-\sqrt{x_1}\right) \end{cases}$$

s.t. $x_i \in [0, 1] \quad i = 1, 2, \ldots, n$

where

$$g(\mathbf{x}) = \begin{cases} \sum_{i=2}^{n}\left(-0.9s_i^2 + |s_i|^{0.6}\right), & \text{if } 0.5 \le x_1 \le 0.8 \\ \sum_{i=2}^{n}|t_i|^{0.6}, & \text{otherwise} \end{cases}$$

$$s_i = x_i - \sin(0.5\pi x_1), \; i = 2, 3, \ldots, n$$

$$t_i = x_i - 0.5, \; i = 2, 3, \ldots, n.$$

The PF has a parabolic relationship between two objectives: $f_2 = 1 - \sqrt{f_1}$ in the range $0 \le f_1 \le 1$. The favored part of the PF comes from $0 \le f_1 \le 0.5$ and $0.8 \le f_1 \le 1$ and the unfavored part of the PF is the parabola within $0.5 < f_1 < 0.8$. The unfavored part of the PS must satisfy $x_j = \sin(0.5\pi x_1)$ for all $j = 2, 3, \ldots, n$ in the range $0.5 < x_1 < 0.8$; however, the favored PS comes from all points for which $x_j = 0.5$ for all $j = 2, 3, \ldots, n$ and for the rest of x_1 values

Problem IMB8

$$\text{Min:} \begin{cases} f_1(\mathbf{x}) = (1+g(\mathbf{x}))x_1 \\ f_2(\mathbf{x}) = (1+g(\mathbf{x}))(1-x_1) \end{cases}$$

s.t. $x_i \in [0, 1] \quad i = 1, 2, \ldots, n$

where

$$g(\mathbf{x}) = \begin{cases} \sum_{i=2}^{n}\left(-0.9s_i^2 + |s_i|^{0.6}\right), & \text{if } 0.5 \le x_1 \le 0.8 \\ \sum_{i=2}^{n}|t_i|^{0.6}, & \text{otherwise} \end{cases}$$

$$s_i = x_i - \sin(0.5\pi x_1), \; i = 2, 3, \ldots, n$$

$$t_i = x_i - 0.5, \; i = 2, 3, \ldots, n.$$

The PF has a linear relationship between two objectives: $f_2 = 1 - f_1$ in the range $0 \le f_1 \le 1$. The favored and unfavored parts of the PF and PS are identical to those of problem IMB7

Problem IMB9

$$\text{Min:} \begin{cases} f_1(\mathbf{x}) = (1+g(\mathbf{x}))\cos(\pi x_1/2) \\ f_2(\mathbf{x}) = (1+g(\mathbf{x}))\sin(\pi x_1/2) \end{cases}$$

s.t. $x_i \in [0, 1] \quad i = 1, 2, \ldots, n$

where

$$g(\mathbf{x}) = \begin{cases} \sum_{i=2}^{n}\left(-0.9s_i^2 + |s_i|^{0.6}\right), & \text{if } 0.5 \le x_1 \le 0.8 \\ \sum_{i=2}^{n}|t_i|^{0.6}, & \text{otherwise} \end{cases}$$

$$s_i = x_i - \sin(0.5\pi x_1), \; i = 2, 3, \ldots, n$$

$$t_i = x_i - 0.5, \; i = 2, 3, \ldots, n.$$

The PF has a parabolic relationship between two objectives: $f_2 = \sqrt{1 - f_1^2}$ in the range $0 \le f_1 \le 0.309$ and $0.707 \le f_1 \le 1$. The favored part of the PF comes from these ranges and the unfavored part of the PF is the parabola within $0.309 < f_1 < 0.707$. The unfavored part of the PS must satisfy $x_j = \sin(0.5\pi x_1)$ for all $j = 2, 3 \ldots, n$ in the range $0.5 < x_1 < 0.8$; however, the favored PS comes from all points for which $x_j = 0.5$ for all $j = 2, 3, \ldots, n$ and for the rest of the x_1 values

Problem IMB10

$$\text{Min:} \begin{cases} f_1(\mathbf{x}) = (1+g(\mathbf{x}))x_1x_2 \\ f_2(\mathbf{x}) = (1+g(\mathbf{x}))x_1(1-x_2) \\ f_3(\mathbf{x}) = (1+g(\mathbf{x}))(1-x_1) \end{cases}$$

s.t. $x_i \in [0, 1] \quad i = 1, 2, \ldots, n$

where

$$g(\mathbf{x}) = \begin{cases} \sum_{i=3}^{n} 2\left(-0.9s_i^2 + |s_i|^{0.6}\right), & \text{if } 0.2 \le (x_1, x_2) \le 0.8 \\ \sum_{i=3}^{n} |t_i|^{0.6}, & \text{otherwise} \end{cases}$$

$$s_i = x_i - (x_1 + x_2)/2, \ i = 3, 4, \ldots, n$$

$$t_i = x_i - x_1 x_2, \ i = 3, 4, \ldots, n.$$

The PF is a linear hyperplane: $f_1 + f_2 + f_3 = 1$ in the range $0 \le (f_1, f_2, f_3) \le 1$. The favored PF lies in $0.04 \le f_1 \le 0.64$ and $0.2 \le f_3 \le 0.8$. The unfavored PS comes from relationships: $x_j = \sin(0.5\pi x_1)$ in the range $0.2 \le (x_1, x_2) \le 0.8$, and the favored PS comes from $x_j = 0.5$ and x_1 values in the remainder of the x_1-space.

Imbalanced problems IMB11–IMB14 are constrained problems in which the unfavored PS lies close to constraint boundaries

Problem IMB11

$$\text{Min: } \begin{cases} f_1(\mathbf{x}) = (1 + g(\mathbf{x}))x_1 \\ f_2(\mathbf{x}) = (1 + g(\mathbf{x}))\left(1 - \sqrt{x_1}\right) \end{cases}$$

$$\text{s.t. } \begin{cases} G(\mathbf{x}) = 0 \\ x_i \in [0, 1] \quad i = 1, 2, \ldots, n \end{cases}$$

where

$$G(\mathbf{x}) = \begin{cases} 10g(\mathbf{x}), & x_1 > 0.6 \text{ and } g(\mathbf{x}) > 0.001 \\ 0, & \text{otherwise} \end{cases}$$

$$g(\mathbf{x}) = \sum_{i=2}^{n} 0.5\left(-0.9t_i^2 + |t_i|^{0.6}\right)$$

$$t_i = x_i - x_1, \ i = 2, 3, \ldots, n.$$

Its PF is $f_2 = 1 - \sqrt{f_1}$ in the range $0 \le f_1 \le 1$. The favored part of the PF comes from $0 \le f_1 \le 0.6$ and the corresponding PS is $x_j = x_1$ for $0 \le x_1 \le 0.6$ for all $j = 2, 3, \ldots, n$. However, the unfavored part of the PF is in the range $f_1 > 0.6$ and its corresponding PS comes from the same relationship $x_j = x_1$, but in the range $x_1 > 0.6$. The unfavored PS region is close to the infeasible search space

Problem IMB12

$$\text{Min: } \begin{cases} f_1(\mathbf{x}) = (1 + g(\mathbf{x}))x_1 \\ f_2(\mathbf{x}) = (1 + g(\mathbf{x}))(1 - x_1) \end{cases}$$

$$\text{s.t. } \begin{cases} G(\mathbf{x}) = 0 \\ x_i \in [0, 1] \quad i = 1, 2, \ldots, n \end{cases}$$

where

$$G(\mathbf{x}) = \begin{cases} 10g(\mathbf{x}), & x_1 < 0.2 \text{ or } x_1 > 0.8 \text{ and } g(\mathbf{x}) > 0.002 \\ 0, & \text{otherwise} \end{cases}$$

$$g(\mathbf{x}) = \sum_{i=2}^{n} 0.5\left(-0.9t_i^2 + |t_i|^{0.6}\right)$$

$$t_i = x_i - x_1, \ i = 2, 3, \ldots, n.$$

Its PF is $f_2 = 1 - f_1$ in the range $0 \le f_1 \le 1$. Its favored PF is at $0.2 \le f_1 \le 0.8$ and comes from $x_j = x_1$ for $j = 2, 3, \ldots, n$. The unfavored PS follows the same relationship but comes from two disconnected regions: $x_1 < 0.2$ and $x_1 > 0.8$

Problem IMB13

$$\text{Min: } \begin{cases} f_1(\mathbf{x}) = (1 + g(\mathbf{x}))x_1 \\ f_2(\mathbf{x}) = (1 + g(\mathbf{x}))\left(1 - x_1^2\right) \end{cases}$$

$$\text{s.t. } \begin{cases} G(\mathbf{x}) = 0 \\ x_i \in [0, 1] \quad i = 1, 2, \ldots, n \end{cases}$$

where

$$G(\mathbf{x}) = \begin{cases} 10g(\mathbf{x}), & x_1 < 0.2 \text{ or } x_1 > 0.8 \text{ and } g(\mathbf{x}) > 0.001 \\ 0, & \text{otherwise} \end{cases}$$

$$g(\mathbf{x}) = \sum_{i=2}^{n} 0.5\left(-0.9t_i^2 + |t_i|^{0.6}\right)$$

$$t_i = x_i - x_1, \ i = 2, 3, \ldots, n.$$

This constrained imbalance problem has a PF relationship: $f_2 = 1 - f_1^2$ for $0 \le f_1 \le 1$. The favored and unfavored subsets are identical to those of IMB12. Here, the constraint boundaries are closer to the unfavored PS than in IMB12

Problem IMB14

$$\text{Min: } \begin{cases} f_1(\mathbf{x}) = (1 + g(\mathbf{x}))x_1 x_2 \\ f_2(\mathbf{x}) = (1 + g(\mathbf{x}))x_1(1 - x_2) \\ f_3(\mathbf{x}) = (1 + g(\mathbf{x}))(1 - x_1) \end{cases}$$

$$\text{s.t. } \begin{cases} G(\mathbf{x}) = 0 \\ x_i \in [0, 1] \quad i = 1, 2, \ldots, n \end{cases}$$

where

$$G(\mathbf{x}) = \begin{cases} 10g(\mathbf{x}), & x_1 > 0.75 \text{ and } g(\mathbf{x}) > 0.001 \\ 0, & \text{otherwise} \end{cases}$$

$$g(\mathbf{x}) = \sum_{i=3}^{n} 0.5\left(-0.9t_i^2 + |t_i|^{0.6}\right)$$

$$t_i = x_i - (x_1 + x_2)/2, \ i = 3, 4, \ldots, n.$$

Finally, this problem has a PF satisfying $f_1 + f_2 + f_3 = 1$ in the range $0 \le (f_1, f_2, f_3) \le 1$. The favored PF comes from $0.25 \le f_3 \le 1$ and the respective PS comes from $x_j = (x_1 + x_2)/2$, for $0 \le x_1 \le 0.75$ and for $j = 3, 4, \ldots, n$. The unfavored PS has the same relationship, but comes from $x_1 > 0.75$.

The above test problems have two or three objectives, however, the same principle can be used to construct an imbalanced problem having any arbitrary number of objectives.

VI. Results of EMO and EMO-M2M Methods

In this section, the effectiveness of five EMO algorithms—NSGA-II, MOEA/D, SPEA2, SMS-EMOA, and GVEGA—for optimizing all 14 imbalanced problems described above is studied. The five EMO algorithms are very popular, and they are typically representative of the four categories of convergence first EMO algorithms. The optimized solutions from each EMO algorithm and the respective EMO-M2M approach are compared in this section. The constraint handling strategy for handling constrained problems is based on the feasibility rules proposed in [41].

The parameter settings are as follows.

1) In order to be fair, the same crossover and mutation operators that are used in [31] are also used in all the M2M version of the EMO methods.
2) For all algorithms, population sizes of 100 and 300 are used for bi-objective and tri-objective problems, respectively.
3) All EMO algorithms are terminated after 2000 generations to give algorithms enough time to attempt to find the entire PF.
4) For M2M versions, the number of subpopulations is 10 for bi-objective unconstrained test problems

TABLE I
BEST/MEAN/WORST HV-METRIC VALUES OF NSGA-II-M2M AND NSGA-II FROM 30 INDEPENDENT RUNS FOR EACH IMBALANCED TEST PROBLEM

HV-metric	NSGA-II-M2M			NSGA-II		
Instance	best	mean	worst	best	mean	worst
IMB1	**0.6375**	**0.6360**	**0.6353**	0.4168	0.4167	0.4167
IMB2	**0.4605**	**0.4577**	**0.4537**	0.3397	0.3397	0.3396
IMB3	**0.1828**	**0.1815**	**0.1801**	0.0387	0.0387	0.0387
IMB4	**0.7445**	**0.7424**	**0.7398**	0.6514	0.6390	0.6242
IMB5	**0.3874**	**0.3842**	**0.3802**	0.3406	0.3324	0.3175
IMB6	**0.7700**	**0.7686**	**0.7675**	0.7478	0.7418	0.7316
IMB7	**0.6499**	**0.6482**	**0.6464**	0.6289	0.6284	0.6280
IMB8	**0.4798**	**0.4774**	**0.4756**	0.4468	0.4463	0.4459
IMB9	**0.1925**	**0.1912**	**0.1896**	0.1701	0.1695	0.1691
IMB10	**0.7636**	**0.7617**	**0.7598**	0.7200	0.7153	0.7049
IMB11	**0.6469**	**0.6460**	**0.6436**	0.6082	0.6071	0.6060
IMB12	**0.4832**	**0.4803**	**0.4777**	0.4478	0.4473	0.4463
IMB13	**0.3141**	**0.3118**	**0.3104**	0.2825	0.2819	0.2814
IMB14	**0.7498**	**0.7441**	**0.7403**	0.5922	0.5825	0.5749

(IMB1–IMB10) and is 30 for tri-objective unconstrained test problems. However, for constrained problems (IMB11–IMB14), 30 subpopulations are used for bi-objective problems and 60 for tri-objective problems. To make a fair comparison between EMO and EMO-M2M methods, each subproblem of an EMO-M2M approach is assigned a subpopulation size of N/K, where N is the EMO population size and K is the number of subproblems. This ensures that both EMO and EMO-M2M use an identical number of function evaluations. It is worth noting that only the contrasting results of MOEA/D and MOEA/D-M2M are displayed in Fig. 4 for the sake of readability.

First, we apply NSGA-II and NSGA-II-M2M to all 14 imbalanced problems. Table I presents the HV-metric values obtained using both methods. It is clear that NSGA-II-M2M performs better than the original NSGA-II in solving the imbalanced problems. The M2M approach introduces a balance between convergence and diversity preservation that is essential in solving these problems.

Next, we apply MOEA/D and MOEA/D-M2M to all 14 imbalanced problems. Fig. 4 shows the performance of MOEA/D and MOEA/D-M2M methods side by side for all 14 problems for a median performing run. The left figure is the performance of MOEA/D and the right figure is obtained using the corresponding M2M approach. It can be seen that in all 14 problems, the MOEA/D-M2M approach is able to find a better spread of solutions on the entire PF. It is clear that MOEA/D-M2M performs better than the original MOEA/D in solving the imbalanced problems. The M2M approach introduces a balance between convergence and diversity preservation that is essential in solving these problems. Although the superior performance is visible from this figure, Table II compares the two approaches with the HV-metric. It is clear that the best, median, and worst HV-metric values for the 30 MOEA/D-M2M runs are better on all 14 imbalanced test problems.

TABLE II
BEST/MEAN/WORST HV-METRIC VALUES OF MOEA/D-M2M AND MOEA/D FROM 30 INDEPENDENT RUNS FOR EACH IMBALANCED TEST PROBLEM

HV-metric	MOEA/D-M2M			MOEA/D		
Instance	best	mean	worst	best	mean	worst
IMB1	**0.6387**	**0.6375**	**0.6354**	0.4684	0.4207	0.4154
IMB2	**0.4627**	**0.4608**	**0.4583**	0.4436	0.3840	0.3390
IMB3	**0.1851**	**0.1836**	**0.1824**	0.0385	0.0385	0.0385
IMB4	**0.7803**	**0.7795**	**0.7785**	0.7361	0.7122	0.7030
IMB5	**0.4266**	**0.4229**	**0.4202**	0.3916	0.3913	0.3910
IMB6	**0.7916**	**0.7909**	**0.7904**	0.7783	0.7758	0.7751
IMB7	**0.6545**	**0.6540**	**0.6534**	0.6327	0.6325	0.6322
IMB8	**0.4840**	**0.4830**	**0.4820**	0.4504	0.4500	0.4496
IMB9	**0.1975**	**0.1960**	**0.1946**	0.1716	0.1713	0.1712
IMB10	**0.7849**	**0.7842**	**0.7835**	0.7815	0.7812	0.7807
IMB11	**0.6516**	**0.6503**	**0.6493**	0.6098	0.6083	0.6069
IMB12	**0.4836**	**0.4823**	**0.4808**	0.4471	0.4461	0.4453
IMB13	**0.3148**	**0.3136**	**0.3119**	0.2810	0.2802	0.2793
IMB14	**0.7707**	**0.7656**	**0.7583**	0.6471	0.6464	0.6453

TABLE III
BEST/MEAN/WORST HV-METRIC VALUES OF SMS-EMOA-M2M AND SMS-EMOA FROM 30 INDEPENDENT RUNS FOR EACH IMBALANCED TEST PROBLEM

HV-metric	SMS-EMOA-M2M			SMS-EMOA		
Instance	best	mean	worst	best	mean	worst
IMB1	**0.6402**	**0.6386**	**0.6363**	0.4098	0.4004	0.3833
IMB2	**0.4639**	**0.4592**	**0.4411**	0.3356	0.3268	0.3162
IMB3	**0.1845**	**0.1834**	**0.1819**	0.0378	0.0354	0.0308
IMB4	**0.7792**	**0.7786**	**0.7783**	0.6259	0.6128	0.5985
IMB5	**0.4169**	**0.4140**	**0.4119**	0.4064	0.3713	0.2983
IMB6	**0.7859**	**0.7856**	**0.7853**	0.7816	0.7812	0.7807
IMB7	**0.6559**	**0.6550**	**0.6542**	0.6568	0.6564	0.6559
IMB8	**0.4852**	**0.4835**	**0.4820**	0.4865	0.4863	0.4860
IMB9	**0.1974**	**0.1961**	**0.1947**	0.1998	0.1994	0.1988
IMB10	**0.7834**	**0.7829**	**0.7824**	0.7852	0.7851	0.7847
IMB11	**0.6550**	**0.6541**	**0.6531**	0.6119	0.6113	0.6108
IMB12	**0.4868**	**0.4856**	**0.4848**	0.4508	0.4504	0.4500
IMB13	**0.3168**	**0.3160**	**0.3150**	0.2852	0.2847	0.2841
IMB14	**0.7717**	**0.7649**	**0.7603**	0.6540	0.6536	0.6530

SMS-EMOA is applied next. Table III presents statistics of HV-metric values for 30 runs, and the table makes it clear that the original SMS-EMOA is unable to find unfavorable PF parts, whereas SMS-EMOA-M2M is able to perform better in maintaining a spread of solutions.

Table IV compares the HV-metric values of 30 runs of SPEA2 and SPEA2-M2M approaches. It is amply clear that SPEA2-M2M is able to better distribute its points on the entire PF. The hierarchical emphasis of nondominated solutions first and then diverse solutions is not adequate in solving these imbalanced problems.

Finally, we apply the proposed GVEGA and its M2M version on all 14 imbalanced problems. Table V clearly shows the advantage of the M2M version in solving these problems.

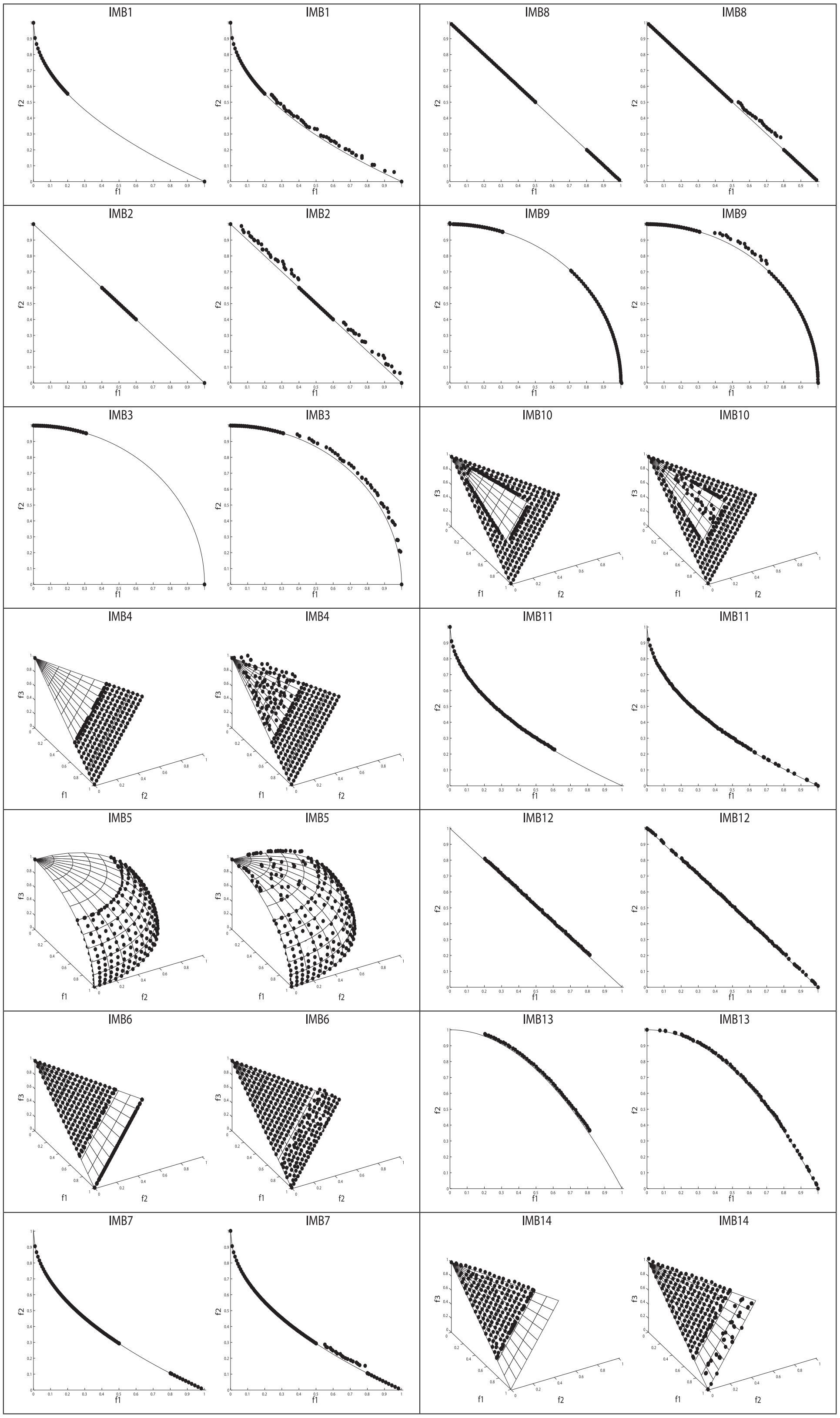


Fig. 4. Obtained nondominated solutions for the median HV-metric value run using MOEA/D (the left panel) and MOEA/D-M2M (the right panel) for all 14 imbalanced test problems.

From all of these figures and tables, we conclude that the M2M population decomposition strategy can be effective in enhancing the balance between the population convergence and diversity, which makes the EMO algorithms using M2M population decomposition outperform the algorithms not using M2M. To illustrate further, we plot the HV-metric for all

TABLE IV
BEST/MEAN/WORST HV-METRIC VALUES OF SPEA2-M2M AND SPEA2 FROM 30 INDEPENDENT RUNS FOR EACH IMBALANCED TEST PROBLEM

HV-metric	SPEA2-M2M			SPEA2		
Instance	best	mean	worst	best	mean	worst
IMB1	**0.6384**	**0.6372**	**0.6351**	0.4170	0.4169	0.4169
IMB2	**0.4605**	**0.4564**	**0.4487**	0.3398	0.3398	0.3398
IMB3	**0.1824**	**0.1802**	**0.1783**	0.0387	0.0387	0.0387
IMB4	**0.7476**	**0.7421**	**0.7364**	0.7027	0.7024	0.7021
IMB5	**0.3973**	**0.3906**	**0.3832**	0.3912	0.3909	0.3904
IMB6	**0.7814**	**0.7807**	**0.7801**	0.7772	0.7771	0.7770
IMB7	**0.6515**	**0.6505**	**0.6494**	0.6310	0.6307	0.6303
IMB8	**0.4811**	**0.4795**	**0.4768**	0.4488	0.4487	0.4486
IMB9	**0.1930**	**0.1919**	**0.1911**	0.1710	0.1707	0.1704
IMB10	**0.7595**	**0.7558**	**0.7460**	0.7566	0.7530	0.7472
IMB11	**0.6496**	**0.6477**	**0.6460**	0.6088	0.6083	0.6072
IMB12	**0.4828**	**0.4811**	**0.4790**	0.4482	0.4476	0.4468
IMB13	**0.3147**	**0.3130**	**0.3112**	0.2837	0.2832	0.2825
IMB14	**0.7393**	**0.7316**	**0.7213**	0.6103	0.6082	0.6057

TABLE V
BEST/MEAN/WORST HV-METRIC VALUES OF GVEGA-M2M AND GVEGA FROM 30 INDEPENDENT RUNS FOR EACH IMBALANCED TEST PROBLEM

HV-metric	GVEGA-M2M			GVEGA		
Instance	best	mean	worst	best	mean	worst
IMB1	**0.6475**	**0.6408**	**0.5969**	0.4066	0.4066	0.4066
IMB2	**0.4750**	**0.4509**	**0.4224**	0.3354	0.3354	0.3354
IMB3	**0.1964**	**0.1950**	**0.1914**	0.0374	0.0374	0.0374
IMB4	**0.7798**	**0.7790**	**0.7784**	0.7117	0.6767	0.6724
IMB5	**0.4215**	**0.4209**	**0.4205**	0.3532	0.3524	0.3510
IMB6	**0.7837**	**0.7833**	**0.7828**	0.7505	0.7491	0.7476
IMB7	**0.6540**	**0.6537**	**0.6531**	0.6162	0.6162	0.6161
IMB8	**0.4863**	**0.4857**	**0.4848**	0.4370	0.4369	0.4368
IMB9	**0.2011**	**0.2005**	**0.1998**	0.1624	0.1623	0.1621
IMB10	**0.7803**	**0.7797**	**0.7785**	0.7524	0.7501	0.7491
IMB11	**0.6516**	**0.6503**	**0.6493**	0.6038	0.5998	0.5983
IMB12	**0.4836**	**0.4823**	**0.4808**	0.4427	0.4425	0.4422
IMB13	**0.3148**	**0.3136**	**0.3119**	0.3010	0.2844	0.2767
IMB14	**0.7707**	**0.7656**	**0.7583**	0.6689	0.6463	0.6342

five EMO methods and NSGA-II-M2M for problem IMB1 in Fig. 5.

It is clear that NSGA-II-M2M finds solutions with a better HV-metric measure than all five other EMO methods including the original NSGA-II approach.

Similarly, Fig. 6 shows the variation of the HV-metric measure for all five EMO methods without M2M and for NSGA-II-M2M for constrained problem IMB11. Since the unfavored parts of the PF add little more HV to the HV of the favored part of the PF, the differences in HV-metric value between EMO methods and one of their M2M variants is small. Nevertheless, the NSGA-II-M2M produces a larger HV-metric value than the original NSGA-II or other EMO methods not using M2M.

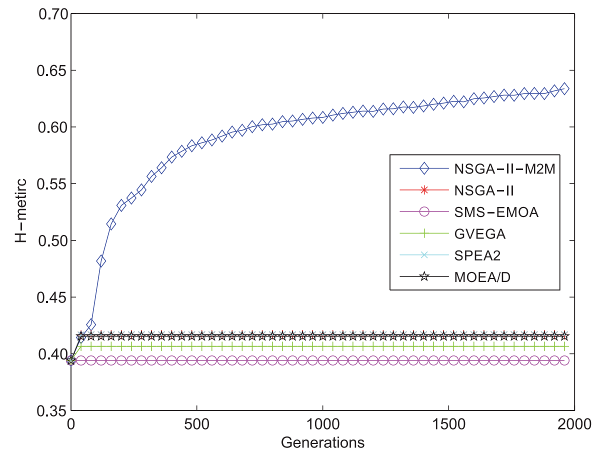


Fig. 5. Variation of HV-metric for five EMO methods and NSGA-II-M2M approach over generations for solving IMB1.

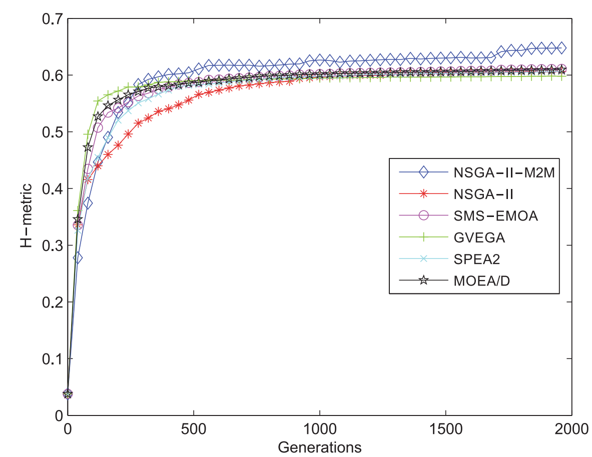


Fig. 6. HV-metric value change of NSGA-II-M2M and five EMO algorithms over generations for solving IMB11.

VII. EMO-M2M FOR BALANCED PROBLEMS

The above results of EMO-M2M methods on imbalanced problems are impressive and clearly portray the need for using a more balanced nonhierarchical algorithm between convergence achievement and diversity preservation that is so essential in solving imbalanced problems. However, the success of the original EMO algorithms on many test and real-world MOPs and the results presented in this paper so far may give the impression that EMO-M2M algorithms are esoteric algorithms that are specifically geared toward solving imbalanced problems. To evaluate their efficacy in solving other problems (which we call here balanced problems), we apply EMO-M2M methods to well-known test problems.

The effectiveness of five EMO algorithms with and without their M2M versions—NSGA-II, MOEA/D, SPEA2, SMS-EMOA, and GVEGA—are explored by numerical computation. The test problems include seven popular multiobjective optimization test problems belonging to the ZDT and DTLZ series. The HV-metric [46] and IGD-metric [47] are used to measure the performance of these algorithms. The experimental settings are as follows.

1) The crossover and mutation operators of the five EMO algorithms are all the same as in their originals, while the

TABLE VI
BEST, MEAN, AND WORST IGD-METRIC/HV-METRIC VALUES OF NSGA-II FOR SEVEN BALANCED MOPS WITH (Y) AND WITHOUT (N) THE M2M FRAMEWORK, IN 30 INDEPENDENT RUNS

MOP			ZDT1	ZDT2	ZDT3	ZDT4	ZDT6	DTLZ1	DTLZ2
IGD-metric	Best	Y	0.0069	0.0071	0.0115	0.0070	0.0453	**0.0542**	0.0611
		N	**0.0052**	**0.0060**	**0.0055**	**0.0050**	**0.0431**	13.5642	**0.0536**
	Mean	Y	**0.0084**	**0.0080**	0.0179	**0.0084**	**0.0485**	**0.6771**	0.0641
		N	0.0231	0.0568	**0.0109**	0.0932	0.0658	28.6024	**0.0610**
	Worst	Y	**0.0101**	**0.0092**	**0.0282**	**0.0105**	**0.0649**	**3.3076**	0.0689
		N	0.1760	0.4149	0.0672	0.9961	0.4147	49.8240	**0.0677**
HV-metric	Best	Y	0.6551	0.3228	0.7705	0.6552	0.3211	**0.7228**	**0.3918**
		N	**0.6588**	**0.3248**	**0.7776**	**0.6594**	**0.3212**	0.0000	0.3931
	Mean	Y	**0.6525**	**0.3214**	0.7625	**0.6523**	**0.3200**	**0.3985**	0.3802
		N	0.6319	0.2719	**0.7720**	0.5744	0.3038	0.0000	**0.3821**
	Worst	Y	**0.6486**	**0.3201**	0.7532	**0.6483**	**0.3161**	**0.0000**	0.3687
		N	0.5971	0.2145	**0.7628**	0.0000	0.0567	**0.0000**	**0.3698**

TABLE VII
BEST, MEAN, AND WORST IGD-METRIC/HV-METRIC VALUES OF MOEA/D FOR SEVEN BALANCED MOPS WITH (Y) AND WITHOUT (N) THE M2M FRAMEWORK, IN 30 INDEPENDENT RUNS

MOP			ZDT1	ZDT2	ZDT3	ZDT4	ZDT6	DTLZ1	DTLZ2
IGD-metric	Best	Y	**0.0042**	0.0039	0.0086	0.0042	**0.0438**	**0.0223**	**0.0280**
		N	**0.0042**	**0.0038**	**0.0085**	**0.0039**	0.0446	0.0297	0.0376
	Mean	Y	0.0053	0.0051	**0.0090**	0.0045	0.0456	0.1169	**0.0280**
		N	**0.0045**	**0.0040**	0.0600	**0.0039**	**0.0451**	**0.0297**	0.0378
	Worst	Y	0.0073	0.0095	0.0093	0.0052	**0.0476**	1.1207	**0.0281**
		N	**0.0049**	**0.0041**	0.1965	**0.0041**	**0.0476**	**0.0298**	0.0380
HV-metric	Best	Y	**0.6609**	0.3279	0.7748	0.6565	0.3225	**0.7934**	**0.4418**
		N	0.6601	**0.3280**	**0.7762**	**0.6612**	**0.3226**	0.7778	0.4188
	Mean	Y	0.6579	0.3244	**0.7708**	0.6521	0.3220	0.6340	**0.4416**
		N	**0.6592**	**0.3274**	0.7616	**0.6611**	**0.3225**	**0.7773**	0.4173
	Worst	Y	0.6541	0.3161	**0.7597**	0.6467	0.3214	0.0337	**0.4415**
		N	**0.6583**	**0.3267**	0.6993	**0.6609**	**0.3218**	**0.7767**	0.4164

crossover and mutation operations of the five EMO-M2M algorithms are kept same as in the original M2M algorithm [31].

2) The population size of all bi-objective problems is 100, and the population size of all tri-objective problems is 300.
3) All EMO algorithms are terminated after 300 generations for ZDT problems and 1000 generations for DTLZ problems.
4) The numbers of subpopulations for the M2M versions are used as they were chosen for solving the imbalanced problems: ten subproblems for bi-objective test problems and 20 for tri-objective test problems.

First, we apply NSGA-II and NSGA-II-M2M algorithms on all seven balanced test problems. Table VI shows the IGD and HV metrics of both algorithms in 30 runs. It is clear that in some problems the original NSGA-II performs statistically better and in some other problems NSGA-II-M2M produces better results. But, interestingly, the performances are not that different from each other. This indicates that the inclusion of additional subproblems in NSGA-II through the M2M framework does not change the performance of the algorithm on balanced problems. In contrast, as shown in Table VI, NSGA-II-M2M performs exceedingly better in solving the imbalanced problems compared to the original NSGA-II.

Next, we consider MOEA/D with and without the M2M framework in solving several balanced problems. A similar conclusion to that above is made for this algorithm (see Table VII), as well.

Similar conclusions are drawn for SPEA2, SMS-EMOA and the newly proposed GVEGA procedures—inclusion of the M2M approach does not alter the performance of the original EMO algorithm in solving seven balanced test problems. The simulation results are shown in Tables VIII–X, respectively.

Next, we compare the performance of all five EMO-M2M methods on the same seven balanced test problems. Tables XI and XII show the best, mean, and

TABLE VIII
BEST, MEAN, AND WORST IGD-METRIC/HV-METRIC VALUE OF SPEA2 FOR SEVEN BALANCED MOPS WITH (Y) AND WITHOUT (N) M2M FRAMEWORK, IN 30 INDEPENDENT RUNS

MOP			ZDT1	ZDT2	ZDT3	ZDT4	ZDT6	DTLZ1	DTLZ2
IGD-metric	Best	Y	0.0060	0.0057	0.0063	0.0069	**0.0428**	0.0471	0.0570
		N	**0.0038**	**0.0038**	**0.0047**	**0.0039**	0.0445	**0.0221**	**0.0303**
	Mean	Y	0.0082	0.0072	0.0110	0.0086	0.0584	0.8470	0.0600
		N	**0.0038**	**0.0039**	**0.0048**	**0.0040**	**0.0446**	**0.0222**	**0.0309**
	Worst	Y	0.0091	0.0081	0.0249	0.0092	0.0647	3.6038	0.0648
		N	**0.0039**	**0.0039**	**0.0049**	**0.0043**	**0.0447**	**0.0225**	**0.0320**
HV-metric	Best	Y	0.6555	0.3251	**0.7781**	0.6541	0.3214	0.7320	0.3856
		N	**0.6619**	**0.3287**	0.7794	**0.6618**	**0.3217**	**0.7988**	**0.4369**
	Mean	Y	0.6522	0.3241	**0.7722**	0.6517	0.3175	0.3510	0.3738
		N	**0.6619**	**0.3286**	0.7793	**0.6612**	**0.3215**	**0.7985**	**0.4355**
	Worst	Y	0.6509	0.3228	0.7609	0.6503	0.3157	0.0000	0.3585
		N	**0.6618**	**0.3285**	**0.7793**	**0.6599**	**0.3214**	**0.7982**	**0.4327**

TABLE IX
BEST, MEAN, AND WORST IGD-METRIC/HV-METRIC VALUE OF SMS-EMOA FOR SEVEN BALANCED MOPS WITH (Y) AND WITHOUT (N) THE M2M FRAMEWORK, IN 30 INDEPENDENT RUNS

MOP			ZDT1	ZDT2	ZDT3	ZDT4	ZDT6	DTLZ1	DTLZ2
IGD-metric	Best	Y	0.0041	0.0045	0.0058	0.0041	0.0450	0.0294	0.0437
		N	**0.0036**	**0.0042**	**0.0044**	**0.1253**	**0.0445**	**0.0215**	**0.0412**
	Mean	Y	0.0047	0.0048	0.0069	0.0042	0.0450	0.2815	0.0448
		N	**0.0036**	**0.0043**	**0.0044**	**0.3259**	**0.0445**	**0.0213**	**0.0409**
	Worst	Y	0.0066	0.0067	0.0096	0.0043	0.0450	2.0310	0.0462
		N	**0.0036**	**0.0045**	**0.0045**	**0.8102**	**0.0446**	**0.0211**	**0.0407**
HV-metric	Best	Y	0.6612	0.3274	0.7772	0.6611	0.3219	0.7908	0.4323
		N	**0.6621**	**0.3289**	**0.7798**	**0.4952**	**0.3227**	**0.8002**	**0.4431**
	Mean	Y	0.6595	0.3261	0.7719	0.6607	0.3219	0.6034	0.4317
		N	**0.6621**	**0.3289**	**0.7798**	**0.2974**	**0.3227**	**0.7991**	**0.4421**
	Worst	Y	0.6552	0.3213	0.7620	0.6604	0.3218	0.0000	0.4310
		N	**0.6621**	**0.3289**	**0.7798**	**0.0036**	**0.3227**	**0.7990**	**0.4401**

TABLE X
BEST, MEAN, AND WORST IGD-METRIC/HV-METRIC VALUE OF GVEGA FOR SEVEN BALANCED MOPS WITH (Y) AND WITHOUT (N) THE M2M FRAMEWORK, IN 30 INDEPENDENT RUNS

MOP			ZDT1	ZDT2	ZDT3	ZDT4	ZDT6	DTLZ1	DTLZ2
IGD-metric	Best	Y	**0.0041**	**0.0038**	**0.0085**	**0.0039**	**0.0425**	**0.0210**	**0.0279**
		N	0.0210	0.0198	0.0413	0.0201	0.0528	0.0499	0.0658
	Mean	Y	**0.0063**	**0.0050**	**0.0094**	**0.0042**	**0.0455**	**0.0213**	**0.0284**
		N	0.0245	0.0221	0.0428	0.0208	0.0571	0.0499	0.0659
	Worst	Y	**0.0378**	**0.0102**	**0.0189**	**0.0082**	**0.0475**	**0.0302**	**0.0402**
		N	0.0715	0.0346	0.0487	0.0211	0.0708	0.0499	0.0660
HV-metric	Best	Y	**0.6609**	**0.3276**	**0.7761**	**0.6609**	**0.3225**	**0.7949**	**0.4417**
		N	0.6392	0.3081	0.7500	0.6391	0.3086	0.7573	0.3950
	Mean	Y	**0.6564**	**0.3247**	**0.7759**	**0.6605**	**0.3218**	**0.7942**	**0.4411**
		N	0.6279	0.3025	0.7499	0.6389	0.3069	0.7572	0.3944
	Worst	Y	**0.6063**	**0.3147**	**0.7702**	**0.6553**	**0.3190**	**0.7827**	**0.4270**
		N	0.6033	0.2726	0.7499	0.6387	0.3038	0.7572	0.3936

TABLE XI
BEST/MEAN/WORST HV-METRIC VALUE OBTAINED BY FIVE EMO-M2M ALGORITHMS FOR SOLVING SEVEN BALANCED MOPS, IN 30 INDEPENDENT RUNS

HV-metric		NSGA-II -M2M	MOEA/D -M2M	SPEA2 -M2M	SMS-EMOA -M2M	GVEGA -M2M
ZDT1	Best	0.6551	0.6609	0.6555	**0.6612**	0.6609
	Mean	0.6525	0.6579	0.6522	**0.6595**	0.6564
	Worst	0.6486	0.6541	0.6509	**0.6552**	0.6063
ZDT2	Best	0.3228	**0.3279**	0.3251	0.3274	0.3276
	Mean	0.3214	0.3244	0.3241	**0.3261**	0.3247
	Worst	0.3201	0.3161	**0.3228**	0.3213	0.3147
ZDT3	Best	0.7705	0.7748	**0.7781**	0.7772	0.7761
	Mean	0.7625	0.7708	0.7722	0.7719	**0.7759**
	Worst	0.7532	0.7597	0.7609	**0.7620**	0.7702
ZDT4	Best	0.6552	0.6565	0.6541	**0.6611**	0.6609
	Mean	0.6523	0.6521	0.6517	**0.6607**	0.6605
	Worst	0.6483	0.6467	0.6503	**0.6604**	0.6553
ZDT6	Best	0.3211	**0.3225**	0.3214	0.3219	**0.3225**
	Mean	0.3200	**0.3220**	0.3175	0.3219	0.3218
	Worst	0.3161	0.3214	0.3157	**0.3218**	0.3190
DTLZ1	Best	0.7228	0.7934	0.7320	0.7908	**0.7949**
	Mean	0.3985	0.6340	0.3510	0.6034	**0.7942**
	Worst	0.0000	0.0337	0.0000	0.0000	**0.7827**
DTLZ2	Best	0.3918	**0.4418**	0.3856	0.4323	0.4417
	Mean	0.3802	**0.4416**	0.3738	0.4317	0.4411
	Worst	0.3687	**0.4415**	0.3585	0.4310	0.4270

worst HV-metric and IGD-metric, respectively, for the five EMO-M2M algorithms in 30 independent runs for solving each balanced test problem. By comparing the HV and IGD metrics of the five EMO algorithms, we observe that all EMO algorithms in the M2M framework have similar performance. It is interesting that although they are reported to produce different performance on different problems, with the M2M framework they perform almost equally well.

VIII. CONCLUSION

In this paper, we have addressed a key issue for developing evolutionary algorithms for solving MOPs. Most existing EMO methods use a convergence first and diversity second approach in which an unbounded importance to selecting nondominated solutions is given over diversity preservation. Most algorithms select nondominated solutions and only when two

TABLE XII
BEST/MEAN/WORST IGD-METRIC VALUES OBTAINED BY FIVE EMO-M2M ALGORITHMS FOR SOLVING SEVEN BALANCED MOPS, IN 30 INDEPENDENT RUNS

IGD-metric		NSGA-II -M2M	MOEA/D -M2M	SPEA2 -M2M	SMS-EMOA -M2M	GVEGA -M2M
ZDT1	Best	0.0069	0.0042	0.0060	**0.0041**	**0.0041**
	Mean	0.0084	0.0053	0.0082	**0.0047**	0.0063
	Worst	0.0101	0.0073	0.0091	**0.0066**	0.0378
ZDT2	Best	0.0071	0.0039	0.0057	0.0045	**0.0038**
	Mean	0.0080	0.0051	0.0072	**0.0048**	0.0050
	Worst	0.0092	0.0095	0.0081	**0.0067**	0.0102
ZDT3	Best	0.0115	0.0087	0.0063	**0.0058**	0.0085
	Mean	0.0179	0.0096	0.0110	**0.0069**	0.0094
	Worst	0.0282	0.0125	0.0249	**0.0096**	0.0189
ZDT4	Best	0.0070	0.0060	0.0069	0.0041	**0.0039**
	Mean	0.0084	0.0086	0.0086	**0.0042**	**0.0042**
	Worst	0.0105	0.0120	0.0092	**0.0043**	0.0082
ZDT6	Best	0.0453	0.0438	0.0428	0.0450	**0.0425**
	Mean	0.0485	0.0456	0.0584	**0.0450**	0.0455
	Worst	0.0649	0.0476	0.0647	0.0450	**0.0475**
DTLZ1	Best	0.0542	0.0225	0.0471	0.0294	**0.0210**
	Mean	0.6771	0.1491	0.8470	0.2815	**0.0213**
	Worst	3.3076	0.7260	3.6038	2.0310	**0.0302**
DTLZ2	Best	0.0611	0.0280	0.0570	0.0437	**0.0279**
	Mean	0.0641	**0.0281**	0.0600	0.0448	0.0284
	Worst	0.0689	**0.0282**	0.0648	0.0462	0.0402

solutions under comparison come from the same nondominated rank, the tie is broken using the crowding of other population members in their neighborhood. However, both convergence and diversity preservation are equally important in solving MOPs. Although existing EMO methods were criticized for this imbalance in their actions, in this paper, we have formally defined the concept of "imbalanced problems" and demonstrated that the existing EMO algorithms can face difficulties from three different sources—imbalanced mapping, variable linkages, and constraint isolation. To facilitate further research and understanding of imbalanced problems, we have suggested a test suite with 14 imbalanced problems arising from all three difficulties. The problems are scalable to any number of variables and in practice extensible to any number of objectives.

Thereafter, extensive computational studies are performed using five well-known existing EMO algorithms and their decomposition-based versions using the M2M strategy. Results have clearly demonstrated the following.

1) Existing convergence first, diversity second EMO methods are unable to solve imbalanced problems to find the unfavored part of the PF in all 14 imbalanced problems.
2) Interestingly, their M2M versions have been able to find a wide spread of solutions near the entire PF in two and three-objective imbalanced test problems. These facts have been demonstrated with two performance measures—HV and IGD metrics.
3) EMO-M2M methods have also been applied to existing ZDT and DTLZ test problems and have been found to produce similar performance compared to their original versions on all of these balanced test problems.
4) Moreover, the performance of all five existing EMO-M2M methods are more or less similar on balanced problems.

These results indicate that EMO-M2M methods solve imbalanced problems better than their original versions and solve balanced problems equally as well as their original versions. Thus, the additional decomposition task of M2M versions does not cost more in solving balanced problems and helps in solving imbalanced problems better than their original counterparts.

As a by-product of defining and suggesting algorithms for imbalanced problems, we have also introduced an efficient version (GVEGA) of the first-ever proposed EMO algorithm—VEGA—using the achievement scalarization function concept. Based on the performance of GVEGA here, we recommend its use in solving balanced and other MOPs.

The imbalanced test problems of this paper have clearly shown that they induce difficulties for the most well-known EMO algorithms, and that special decomposition-based (not aggregation-based) methods were required to focus on the unfavored Pareto-optimal regions in order to solve these problems. It will be interesting to launch a study determining how often imbalanced properties are encountered in practical problems. However, the clear definition of imbalanced problems and the test suite proposed in this paper should encourage EMO researchers to devise modified algorithms balancing convergence and diversity-preservation aspects. As demonstrated here, the new algorithms need not be specialized to solve only imbalanced problems, but may also be equally efficient in solving balanced problems.

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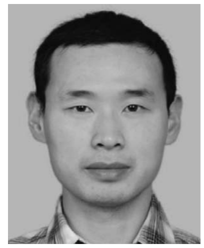

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