Balancing Convergence and Diversity by Using Two Different Reproduction Operators in MOEA/D: Some Preliminary Work

Zhenkun Wang*, Qingfu Zhang†‡ and Hui Li§

*Key Laboratory of Intelligent Perception and Image Understanding of Ministry of Education of China, Xidian University

Email: wangzhenkun90@gmail.com

†Department of Computer Science, City University of Hong Kong, Kowloon, Hong Kong SAR

‡School of Computer Science and Electronic Engineering, University of Essex, Colchester CO4 3SQ, U.K.

§School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an, Shaanxi Province 710049, China

***Abstract*—This paper studies how to use two reproduction operators with different characteristics for balancing the convergence and the diversity in MOEA/D. We consider two operators. One is a differential evolution and polynomial mutation, and the other is a neighbor learning and inversion mutation. We show that these two operators have different search abilities. Then we propose a scheme to use these two operators in our recently proposed MOEA/D-GR framework. We test the proposed algorithm on some benchmark problems to demonstrate its effectiveness.**

***Index Terms*—Multiobjective optimization, MOEA/D, reproduction, convergence, diversity.**

I. Introduction

A multiobjective optimization problem (MOP) can be defined mathematically as follows:

$$\begin{array}{ll} \text{minimize} & F(x) = (f_1(x), f_2(x), \cdots, f_m(x))^T \\ \text{subject to} & x \in \Omega \end{array} \qquad (1)$$

where $x = (x_1, x_2, \cdots, x_n)^T \in \Omega$ is a decision vector, Ω is the feasible region of the search space, and $F : \Omega \to R^m$ consists of m objective functions $f_i(x) (i = 1, 2, \cdots, m)$.

To define the optimal solutions of an MOP, we need the following definitions:

- Let $x, y \in \Omega$, x is said to ***dominate*** y, denoted by $x \prec y$, if and only if $f_i(x) \leq f_i(y)$ for all $i \in \{1, 2, \cdots, m\}$, and $f_j(x) < f_j(y)$ for at least one $j \in \{1, 2, \cdots, m\}$.
- A point $x^* \in \Omega$ is called ***Pareto optimal*** if there is no other $x \in \Omega$ which *dominates* x^*.
- The set of all the Pareto optimal points in the decision space is called the ***Pareto set (PS)***. The ***Pareto front (PF)*** is defined as the set of objective vectors of all members of PS, i.e., $PF = \{F(x) | x \in PS\}$.

A good approximation to PF or PS is often required by decision makers. Multiobjective optimization evolutionary algorithms (MOEAs) are popular methods for approximating the whole PF in a single run. According to the selection strategies, MOEAs can be classified into three categories: Pareto domination based MOEAs (e.g., NSGA-II [1], SPEA-II [2] and PAES [3]), indicator based MOEAs (e.g., IBEA [4], SMS-EMOA [5] and HypE [6]) and decomposition based MOEAs (MOGLS [7], MOEA/D [8], C-MOGA [9] and NSGA-III [10]). Among these MOEAs, multiobjective evolutionary algorithm based on decomposition (MOEA/D) provides a generic multiobjective optimization framework. It decomposes an MOP into a number of simple subproblems, which could be single objective (e.g. [8]), or multiobjective ones [11]. All subproblems are optimized in a collaborative manner. To achieve this, neighborhood relationships among subproblems are defined and utilized.

The convergence and the population diversity are two important issues in the design of MOEAs. How to balance them is critical. In fact, replacement and reproduction are two main components in MOEAs for controlling the convergence and the diversity. In MOEA/D, the replacement mainly depends on the objective function of subproblems and replacement neighborhood size. Moreover, the distribution of weight vectors and the setting of reference points also have influence on the performance of MOEA/D in the convergence and the diversity. The following are some related research works on the convergence and the diversity on MOEA/D: The adaptive adjustment of reference points or weight vectors were used to achieve a good distribution of population in [12]. The simultaneous use of different decomposition approaches in MOEA/D s was studied in [13]. The matching strategies between subproblems and solutions have been investigated [14]. Reproduction mainly involves mating selection and reproduction operators. Under the framework of MOEA/D, various studies have been done on this issue. Some research work on the selection of mating parent were discussed in [15]. The use of multiple search operators was suggested in MOEA/D [16].

Following our previous work [17], a new version of global replacement (GR) is firstly proposed in this paper. This version focuses on the diversity of population in order to provide various options for mating selection in reproduction. The major contribution of this paper are as follows:

- Two reproduction operators, i.e., differential evolution operator and polynomial mutation (denoted as DE), and neighbor learning operator and inversion mutation (denot-

978-1-4799-8697-2/15 $31.00 © 2015 IEEE
DOI 10.1109/SMC.2015.496

ed as NL), are employed.

- We also study the search characteristics of the proposed two operators. A new variant of MOEA/D, called MOEA/D-NL&DE is proposed. It employs the new GR operator and a greedy strategy.
- Comprehensive experiments on some benchmark test problems have been carried out to study the effectiveness of MOEA/D-NL&DE.

The rest of the paper is organized as follows. In Section II, MOEA/D is briefly reviewed. Section III introduces the new GR and studies the search behaviors of NL and DE. Section IV gives the details of the proposed MOEA/D-NL&DE. Section V presents the experimental results. The paper is concluded in Section VI.

II. MOEA/D

A. Decomposition

The MOEA/D framework used in this paper decomposes an MOP into a number of single objective subproblems. There are a number of approaches for decomposition [8]. Among them, the weighted Tchebycheff approach is adopted in this paper. Let $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_m)$ be a weight vector with $\lambda_i \geq 0$ for all $i = 1, 2, \ldots, m$ and $\sum\limits_{i=1}^{m} \lambda_i = 1$, a single objective subproblem is defined as follows:

$$\begin{array}{ll} \text{minimize} & g(x|\lambda, z) = \max\limits_{1 \leq i \leq m} \{\lambda_i |f_i(x) - z_i|\} \\ \text{subject to} & x \in \Omega \end{array} \quad (2)$$

where z is a reference objective vector, i.e.,

$$z_i \leq \min\{f_i(x)|x \in \Omega\}, i = 1, \ldots, m.$$

The direction vector for this subproblem is

$$\left(\frac{\frac{1}{\lambda_1}}{\sum_{i=1}^{m} \frac{1}{\lambda_i}}, \frac{\frac{1}{\lambda_2}}{\sum_{i=1}^{m} \frac{1}{\lambda_i}}, \cdots, \frac{\frac{1}{\lambda_m}}{\sum_{i=1}^{m} \frac{1}{\lambda_i}} \right)$$

Apart from the above reference point, ideal point, utopian point, nadir point or points sampled from a reference plane [10] can also be used as reference points. Moreover, different types of distributions of direction vectors, such as gathered, scattered or adaptively adjusted ones [12] were studied. Following [12], this paper predefines the direction vectors with uniform distribution.

B. Neighborhood

A basic assumption in MOEA/D is that subproblems with neighboring direction vectors have similar optimal solutions. MOEA/D introduces a neighborhood concept among subproblems [8]. A subproblem's T-neighborhood consists of subproblems with the T closest direction vectors to its direction vector in terms of Euclidean distance. This subproblem itself is always belongs to its neighborhood.

The mating neighborhood and the replacement neighborhood are two commonly-used neighborhoods used in MOEA/D variants. The first one is used for selecting parent solutions while the second is used for determining which solutions should be updated. It has also been proved that the settings of different neighborhood sizes can affect the performance of MOEA/D [17].

C. Algorithmic Framework

At each generation (i.e., iteration), MOEA/D maintains N solutions $x^1, \ldots, x^N$, where x^i is the current solution to subproblem i. For convenience, the objective function of subproblem i is denoted by $g^i(x)$. The T-neighborhood of solution x^i (denoted as $T(i)$) consists of the solutions of the subproblems in T-neighborhood of subproblem i. A solution in the T-neighborhood of solution x^i is called a T-neighbor of x^i. The major steps in MOEA/D framework are as follows:

1) Initialize the reference points and direction vectors, and determine the mating neighborhood size T_m and the replacement neighborhood size T_r.
2) For each subproblem i, do:
   a) Mating: Conduct reproduction operators on mating solutions selected from the $T_m(i)$ to generate $\bar{x}^i$.
   b) Replacement: For solutions x^j in the $T_r(i)$, if $\bar{x}^i$ is better than x^j in terms of $g^j(x)$, then replace x^j by $\bar{x}^i$.
   c) Update: Update the reference points, direction vectors, T_m and T_r if necessary.

III. Balancing Convergence and Diversity in MOEA/D

This section will first review some related studies on the convergence and diversity and then present our work under the framework of MOEA/D.

A. Replacement

Different decomposition approaches were adaptively adopted for selection in [13]. A stable matching technique was proposed to associate subproblems with solutions in [18]. In our previous work, a global replacement (GR) strategy was proposed [14]. Within this strategy, the trade-off between convergence and diversity can be easily controlled by T_r. We further studied the strategies for balancing this trade-off by adapting T_r in [17]. However, the former GR may cause duplicate solutions during the search such that MOEA/D becomes less stable. In addition, diverse parent solutions for mating selection could cater to a wider range of reproduction operators. Inspired by this, a new version of GR is designed in the following. For each new solution $\bar{x}^i$:

1) Find the most suitable subproblem j for $\bar{x}^i$.
2) Compare $\bar{x}^i$ with x^j. If $\bar{x}^i$ is better than x^j in terms of $g^j(x)$, set $x^{rep} = x^j$ and replace x^j by $\bar{x}^i$. Otherwise, set $x^{rep} = \bar{x}^i$.
3) Compare x^{rep} with other solutions in the $T_r(j)$. If x^{rep} is better than some of them in terms of their corresponding $g(x)$, randomly replace one by x^{rep}.

The following are some remarks on this new GR scheme.

- Several approaches have been proposed to define the most appropriate subproblem for each solution [14]. The objective functions of subproblems are used in this paper to find the most appropriate subproblem for one solution.

- Like previous GR, any new solution $\bar{x}^i$ has the priority to replace its most suitable subproblems rather than its own neighboring ones. The mismatch between subproblems and solutions can be reduced. Unlike other GR schemes, $\bar{x}^i$ first competes with x^j, and then the loser could replace at most one worse solution from $T_r(j)$. That is, $\bar{x}^i$ prefers to subproblem j while the worst solution of $T_r(j)$ could be replaced. Due to this machanism, this GR can improve the diversity of population by avoiding the duplicates.
- T_r in the former GR versions is a key parameter for controlling the balance between convergence and diversity. However, the influence of T_r on population diversity and convergence becomes less important in this version. Therefore, a fixed T_r with small value is used to facilitate diversity and provide various mating selection for reproduction operators.

B. Reproduction

In general, two main issues in reproduction are the selection of mating parents and reproduction operator. Many studies have been done for MOEA/D on these two issues. For example, it was suggested in [19] that part of parent solutions should be selected from the whole population with probability $(1-\delta)$. In [16], MOEA/D adaptively selects operators with bandits from a pool of operators for reproduction. In this paper, we consider two operators with different search characteristics. The first one is the differential evolution operator and polynomial mutation (denoted as DE). The second one is the neighbor learning operator and inversion mutation (denoted as NL). In the following, we introduce the details of these two operators and analyze their search behaviors.

1) DE: The differential evolution operator with polynomial mutation in [19] has shown its effectiveness on some MOPs with complicated PS shapes. For each subproblem i, choose its current solution x^i as the first mating parent x^{r_1}, and randomly select two other different mating parents x^{r_2} and x^{r_3} from the mating pool. In the DE operator, an intermediate solution $y^i = (y_1^i, \ldots, y_n^n)$ is first produced by:

$$y_k^i = \begin{cases} x_k^{r_1} + F \times (x_k^{r_2} - x_k^{r_3}) & \text{if } rand \le CR \\ x_k^{r_1} & \text{otherwise} \end{cases} \tag{3}$$

Then, a new solution $\bar{x}^i = (\bar{x}_1^i, \bar{x}_2^i, \ldots, \bar{x}_n^i)$ is generated by polynomial mutation as follows:

$$\bar{x}_k^i = \begin{cases} y_k^i + \sigma_k \times (b_k - a_k) & \text{if } rand \le p_m \\ y_k^i & \text{otherwise} \end{cases} \tag{4}$$

with

$$\sigma_k = \begin{cases} (2 \times rand)^{\frac{1}{\eta+1}} - 1 & \text{if } rand < 0.5 \\ 1 - (2 - 2 \times rand)^{\frac{1}{\eta+1}} & \text{otherwise} \end{cases} \tag{5}$$

where CR are F two parameters in the differential evolution operator, the distribution index η and the mutation rate p_m are control parameters in polynomial mutation. $rand$ is a uniform random number from $[0, 1]$. a_k and b_k are the lower and upper bounds of the k-th decision variable.

2) NL: The NL operator is a modified version of neighborhood competition operator in the multi-agent genetic algorithm [20], where an agent makes use of the information of a better one in the neighborhood. This operator can be easily extended to MOEA/D. Set x^b to be the better one from $T_n(i)$ in terms of $g^j(x)$, A new solution $\bar{x}^i = (\bar{x}_1^i, \bar{x}_2^i, \ldots, \bar{x}_n^i)$ is generated as follows:

$$y_k^i = x_k^b + (1 - 2 \times rand) \times (x_k^b - x_k^i) \tag{6}$$

Afterwards, the inversion mutation is applied on y^i to generate $\bar{x}^i$ with the probability p_m.

$$y_k' = (y_k^i - a_k)/(b_k - a_k) \tag{7}$$

$$\begin{aligned}(x_1', x_2', \ldots, x_n') =& (y_1', y_2', \ldots, y_{i_1-1}', y_{i_2}', y_{i_2-1}', \ldots, \\ & y_{i_1+1}', y_{i_1}', y_{i_2+1}', y_{i_2+2}', \ldots, y_n')\end{aligned} \tag{8}$$

$$\bar{x}_k^i = a_k + x_k' \times (b_k - a_k) \tag{9}$$

where p_m is the mutation rate, and i_1, i_2 are two randomly generated indexes in $[1, n]$ with $i_1 < i_2$.

3) Search Behaviors of DE and NL: To analyze the search behaviors of both operators, an empirical study is conducted on the 2-D objective space. As shown in Fig. 1(a), Parents 1-3 are three parent solutions. Assume that:

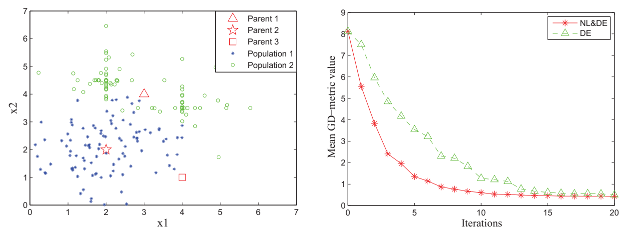
- Parent 1 is the current solution;
- Parent 2 has a better performance than parent 1;
- Parent 3 is a neighboring solution of Parent 1.

Then, we adopt NL and DE to generate 100 offspring solutions respectively. Population 1 is generated by the NL operator, whereas Population 2 is generated by the DE operator. It can be observed that all solutions of Population 1 surrounds parent 2. This could explain that NL has good search ability towards the solution with higher quality. Therefore, the NL operator could perform better than DE in convergence. Moreover, the distribution of solutions in Population 2 has no bias towards any parent solution. This is beneficial for the promotion of the population diversity. Since two operators have different search characteristics, it is reasonable to utilize both operators for balancing the trade-off between diversity and convergence for improving the performance of MOEA/D.

IV. MOEA/D-NL&DE

Since the above two operators have different search characteristics, the adaption of both operators in MOEA/D makes its ability of convergence and diversity adjustable. By adopting both NL and DE operators, MOEA/D-NL&DE is proposed. This is a preliminary attempt on balancing convergence and diversity by different operators. The pseudo-code of MOEA/D-NL&DE is given in Algorithm 1.

In [21], a probability mechanism is used to manage the use of two reproduction operator. Each of them is selected for producing new solutions with a certain probability, which is iteratively updated according to their success rate. In our work, the selection of reproduction operators is done by checking if there is better solution in the neighborhood of the current solution. That is, one solution is always learned



(a) (b)

Fig. 1. (a) Solutions generated by NL and DE. (b) Evolution of the mean GD-metric values of NL&DE and DE with iterations.

from the better one in its T_n neighboring solutions if possible in NL&DE. It has been demonstrated in Section III that NL can promote convergence while DE facilitates search along the population. Therefore, the population is expected to have a quick convergence to the PS, and extend the search along the PS afterwards. That is, a greedy strategy is used to manage the employment of these two operators. Actually, other strategies for operator management also can also be extended to MOEA/D-NL&DE. In addition, some parameters (e.g., T_n and T_r) can be adjustable to control the trade-off between convergence and diversity. To illustrate the search ability of NL&DE, a simple experiment is conducted by comparing NL&DE with DE in the following.

A. NL&DE vs DE

In order to illustrate the dynamic change on the distribution of solutions during the search, we take the following five 2-D Sphere functions as an example ($i = 1, 2, \cdots, 5$):

$$\begin{array}{ll} \text{minimize} & f_i(x) = (x_1 - r_1^i)^2 + (x_2 - r_2^i)^2 \\ \text{subject to} & x \in [-20, 20] \times [-20, 20] \end{array} \quad (10)$$

where $r^i = 0.25 \times (i - 1, 5 - 1)$ is the i-th ideal point.

Since these five functions have similar optimal solutions in the variable space, all of them are neighboring to each other. every function maintains one solution. GR in Section III-A is employed as the replacement strategy with either of two operators. Two GR variants are terminated within 20 iterations and run for 20 times. At each run, both operators start with the same initial population. Other parameters in both operators are the same.

In order to quantitatively assess the ability of two operators in convergence, the GD-metric is used here. Let $A = \{x^1, x^2, \cdots, x^5\}$ and $R^* = \{r^1, r^2, \cdots, r^5\}$. The GD is defined as:

$$GD(A, R^*) = \frac{\sum_{x \in A} d(x, R^*)}{|A|} \quad (11)$$

where $d(x, R^*)$ is the minimum Euclidean distance between x and the points in R^*.

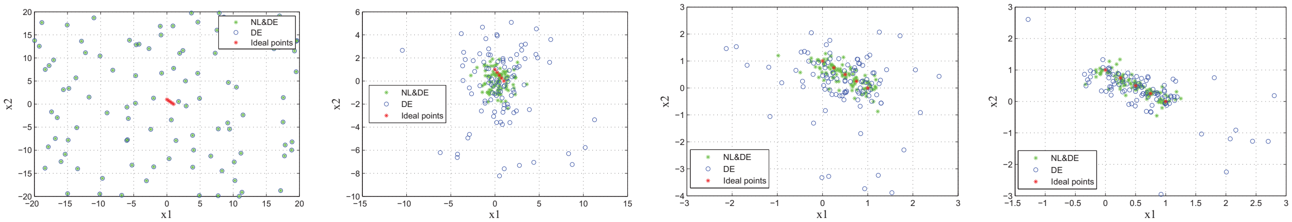
The evolution of the mean GD values of the population obtained by two operators over the number of iterations is plotted in Fig. 1(b). It is clear that NL&DE is more effective and efficient in reducing the GD values. The distributions of all the solutions found by NL&DE and DE with different iterations in 20 runs are shown in Fig. 2. We can observe that NL&DE has faster convergence speed toward the ideal points than DE. Therefore, we can conclude that the overall performance of NL&DE is better than that of DE in convergence.

Algorithm 1: MOEA/D-NL&DE

Input: MOP, a stopping criterion, N, T_m, T_r, T_n, δ;
Output: P;
Initialize a set of weight vectors $\lambda \leftarrow (\lambda^1, \lambda^2, \cdots, \lambda^N)$, the population $P \leftarrow \{x^1, x^2, \cdots, x^N\}$, ideal point z^* and work out the neighborhood relations between subproblems. Let $T(i)$ be the T neighboring solutions of x^i;
while *the stopping criterion is not satisfied* **do**
  for $i \leftarrow 1$ **to** N **do**
    if $\exists x^b \in T_n(i)$ with $g^i(x^b) < g^i(x^i)$ **then**
      Conduct the NL operator on x^b and x^i to generate a new solution $\bar{x}^i$;
    else
      if $rand < \delta$ **then**
        $E \leftarrow T_m(i)$;
      else
        $E \leftarrow P(i)$;
      end
      Set $x^{r_1} \leftarrow x^i$ and randomly select x^{r_2} and x^{r_3} from E, then generate a new solution $\bar{x}^i$ by applying the DE operator on them;
    end
    $j = \arg \min_{1 \le k \le N} \{g^k(x)\}$
    if $g^j(\bar{x}^i) < g^j(x^j)$ **then**
      $x^{rep} \leftarrow x^j$
      $x^j \leftarrow \bar{x}^i$;
    else
      $x^{rep} \leftarrow \bar{x}^i$;
    end
    if $\exists x^l \in T_r(j)$ with $g^j(x^{rep}) < g^j(x^l)$ **then**
      $x^l \leftarrow x^{rep}$;
    end
  end
end

V. Experimental Studies

To show the effectiveness of the new GR and NL&DE, MOEA/D-NL&DE is compared with MOEA/D-DE and MOEA/D-GR. Note that the only difference between MOEA/D-DE and MOEA/D-GR is the replacement scheme. MOEA/D-DE uses the original replacement in [19] while MOEA/D-GR adopts the new GR described in Section III-A. Moreover, MOEA/D-GR and MOEA/D-NL&DE only differ in reproduction. That is, DE is used in MOEA/D-GR while MOEA/D-NL&DE adopts NL&DE for reproduction.



(a) Iterations = 0 (b) Iterations = 7 (c) Iterations = 14 (d) Iterations = 20

Fig. 2. Plots 20 populations found by NL&DE and DE with different iterations.

A. Test Problems

In this work, we compare the performance of three algorithms on two sets of bi-objective problems i.e., MOP1-MOP5 proposed in [11] and F1-F5 in [19].

B. Performance Metric

The inverted generational distance metric (IGD) is employed to assess the algorithm performance. Assume that P^* is a set of uniformly distributed Pareto optimal points along the PF, and P is an approximattion of the PF obtained by an algorithm. The IGD from P^* to P is defined as

$$IGD(P^*, P) = \frac{\sum_{v \in P^*} d(v, P)}{|P^*|} \tag{12}$$

where $d(v, P)$ is the minimum Euclidean distance between v and the points in P. We select 500 evenly distributed points in the PF to be P^*.

C. Parameter Settings

1) *Parameters in reproduction operators*: Similar to the parameter setting used in [19], we also set $CR = 1.0$, $F = 0.5$ and $\eta = 20$, and $p_m = 1/n$.
2) *The population size* N: 300 for all the test problems.
3) *Number of runs and stopping condition*: For each test problem, all algorithms are run 30 times independently. The maximal number of function evaluations is set to be $150,000$ for F1-F5 and $300,000$ for MOP1-MOP5.
4) *The neighborhood sizes*: $T_m = 20$, $T_r = 5$ and $T_n = 5$.
5) *Other control parameters*: δ is set to 0.9 in all three algorithms. n_r is set to 2 in MOEA/D-DE.

D. Performance Comparisons

Experimental results on the comparisons of MOEA/D-NL&DE with MOEA/D-DE and MOEA/D-GR in terms of IGD metric values are given in Table I. It is evident from this table that MOEA/D-NL&DE significantly outperforms other two algorithms. It yields the best median IGD values on all the test problems. It also performs best in finding the minimal IGD-metric values on all test problems except F9. The IGD results in Table I also show that MOEA/D-GR performs reasonably well on all test problems. It can achieve best results on F1, F2, and F3, as well as competitive results on the rest. From the results in Table I, it is clear that MOEA/D-DE performs worst in terms of both the mean IGD-metric values and the min IGD-metric values on all test problems.

TABLE I
THE MEDIAN AND MINIMAL IGD-METRIC VALUES FOUND BY MOEA/D-DE, MOEA/D-GR AND MOEA/D-NL&DE

IGD	MOEA/D-DE		MOEA/D-GR		MOEA/D-NL&DE	
Problem	median	min	median	min	median	min
MOP1	0.4366	0.1451	0.0172	0.0154	**0.0071**	**0.0059**
MOP2	0.2796	0.1307	0.0061	0.0026	**0.0053**	**0.0024**
MOP3	0.5093	0.4240	0.0044	0.0032	**0.0036**	**0.0028**
MOP4	0.2510	0.2219	0.0106	0.0083	**0.0059**	**0.0017**
MOP5	0.3181	0.1618	0.0138	0.0119	**0.0080**	**0.0063**
F1	0.0014	0.0014	**0.0013**	**0.0013**	**0.0013**	**0.0013**
F2	0.0052	0.0043	**0.0030**	0.0027	**0.0030**	**0.0025**
F3	0.0063	0.0039	**0.0032**	0.0025	**0.0032**	**0.0023**
F4	0.0090	0.0065	0.0045	0.0028	**0.0031**	**0.0022**
F5	0.0143	0.0075	0.0113	**0.0066**	**0.0091**	0.0068

Comparing the results obtained by MOEA/D-DE and MOEA/D-GR, it is easy to conclude that MOEA/D-GR performs significantly better than MOEA/D-DE. Especially on MOP1-MOP5, the prformance of MOEA/D-DE is quite very poor while MOEA/D-GR is still acceptable. All these results show that the use of the new GR scheme can improve the performance of MOEA/D-DE. MOEA/D-NL&DE can achieve remarkably better results than MOEA/D-GR on MOP1, MOP4, MOP5, F4 and F5. On F1-F3, two algorithms have similar performance in terms of the mean IGD-metric values while the best results obtained by MOEA/D-NL&DE in 30 runs are better than those by MOEA/D-GR. Both algorithms differ in the reproduction operators. Therefore, NL&DE has better search ability in convergence than DE in MOEA/D.

In Fig. 3, the evolution of the mean IGD values obtained by the three algorithms versus the number of function evaluations (FEs) is plotted. It is clear that MOEA/D-NL&DE is more effective and efficient in reducing the IGD values. On MOP1-MOP5, the convergence speed of MOEA/D-NL&DE is significantly faster than other two algorithms. MOEA/D-GR also performs competitively while MOEA/D-DE performs worst on MOP1-MOP5. The evolution of the mean IGD values of the three algorithms with FEs on F1-F5 also show that MOEA/D-DE performs the worst among three algorithms in reducing the IGD values. MOEA/D-NL&DE and MOEA/D-GR have similar performance on these five problems. In details, we can find that MOEA/D-NL&DE has a faster convergence speed at the early search stage, whereas MOEA/D-GR has good ability to search along the PS at the late search stage. However, fast convergence speed at the early search stage in

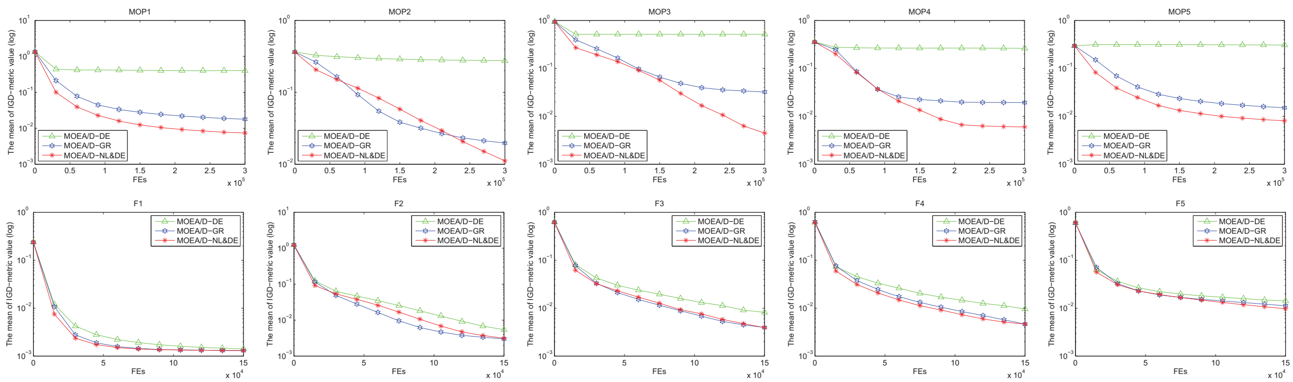


Fig. 3. Evolution of the mean IGD-metric values of MOEA/D-DE, MOEA/D-GR and MOEA/D-NL&DE during the evolutionary process.

MOEA/D-NL&DE may get trapped in the area away from some promising search regions. To overcome this weakness, the use of different operators adaptively can be considered in MOEA/D-NL&DE during the search.

VI. Conclusions

In this paper, we studied the use of different reproduction operators in MOEA/D in order to keep the balance between convergence and diversity. Following our previous work, a new GR was firstly proposed to ensure diverse solutions for mating selection. Then, we presented DE and NL and analyzed their search behaviors. A greedy strategy was adopted in the cooperation of two operators. Based on this strategy and the new GR, we have proposed MOEA/D-NL&DE. Our experimental results showed the advantages of both the new GR and NL&DE against reproduction operator based on pure DE.

This work is still very preliminary. In fact, a greedy strategy in MOEA/D-NL&DE may not work satisfactorily on some problems. In our future work, we will study more reproduction operators with different search abilities and design more advanced cooperation strategy.

References

[1] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002.

[2] E. Zitzler, M. Laumanns, and L. Thiele, "SPEA2: Improving the Strength Pareto Evolutionary Algorithm for Multiobjective Optimization," in *Evolutionary Methods for Design, Optimization and Control with Applications to Industrial Problems (EUROGEN 2001)*. K. Giannakoglou, et al., Eds. International Center for Numerical Methods in Engineering (CIMNE), 2001, pp. 95–100.

[3] J. D. Knowles and D. W. Corne, "Approximating the nondominated front using the pareto archived evolution strategy," *Evolutionary computation*, vol. 8, no. 2, pp. 149–172, 2000.

[4] E. Zitzler and S. Künzli, "Indicator-based selection in multiobjective search," in *Parallel Problem Solving from Nature-PPSN VIII*. Springer, 2004, pp. 832–842.

[5] N. Beume, B. Naujoks, and M. Emmerich, "SMS-EMOA: Multiobjective selection based on dominated hypervolume," *European Journal of Operational Research*, vol. 181, no. 3, pp. 1653–1669, 2007.

[6] J. Bader and E. Zitzler, "HypE: An algorithm for fast hypervolume-based many-objective optimization," *Evolutionary Computation*, vol. 19, no. 1, pp. 45–76, 2011.

[7] H. Ishibuchi and T. Murata, "A multi-objective genetic local search algorithm and its application to flowshop scheduling," *IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews*, vol. 28, no. 3, pp. 392–403, 1998.

[8] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *IEEE Transactions on Evolutionary Computation*, vol. 11, no. 6, pp. 712–731, 2007.

[9] T. Murata, H. Ishibuchi, and M. Gen, "Specification of genetic search directions in cellular multi-objective genetic algorithms," in *Evolutionary Multi-Criterion Optimization (EMO)*, vol. LNCS 1993. Springer, 2001, pp. 82–95.

[10] K. Deb and H. Jain, "An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part i: Solving problems with box constraints," *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 4, pp. 577–601, 2014.

[11] H.-L. Liu, F. Gu, and Q. Zhang, "Decomposition of a multiobjective optimization problem into a number of simple multiobjective subproblems," *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 3, pp. 450–455, 2014.

[12] Y. Qi, X. Ma, F. Liu, L. Jiao, J. Sun, and J. Wu, "MOEA/D with adaptive weight adjustment," *Evolutionary computation*, vol. 22, no. 2, pp. 231–264, 2014.

[13] H. Ishibuchi, Y. Sakane, N. Tsukamoto, and Y. Nojima, "Adaptation of scalarizing functions in moea/d: An adaptive scalarizing function-based multiobjective evolutionary algorithm," in *Evolutionary Multi-Criterion Optimization (EMO), 2009*. Springer, 2009, pp. 438–452.

[14] Z. Wang, Q. Zhang, M. Gong, and A. Zhou, "A replacement strategy for balancing convergence and diversity in moea/d," in *IEEE Congress on Evolutionary Computation (CEC), 2014*. IEEE, 2014, pp. 2132–2139.

[15] T.-C. Chiang and Y.-P. Lai, "MOEA/D-AMS: Improving MOEA/Dby an adaptive mating selection mechanism," in *IEEE Congress on Evolutionary Computation (CEC)*. IEEE, 2011, pp. 1473–1480.

[16] K. Li, A. Fialho, S. Kwong, and Q. Zhang, "Adaptive operator selection with bandits for multiobjective evolutionary algorithm based decomposition," *IEEE Transactions on Evolutionary Computation*, vol. PP, no. 99, p. 1, 2013.

[17] Z. Wang, Q. Zhang, A. Zhou, M. Gong, and L. Jiao, "Adaptive replacement strategies for MOEA/D," *IEEE Transactions on Cybernetics*, 2015, in press.

[18] K. Li, Q. Zhang, S. Kwong, M. Li, and R. Wang, "Stable matching based selection in evolutionary multiobjective optimization," *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 6, pp. 909–923, 2014.

[19] H. Li and Q. Zhang, "Comparison between NSGA-II and MOEA/D on a set of multiobjective optimization problems with complicated Pareto sets," *IEEE Transactions on Evolutionary Computation*, vol. 13, no. 2, pp. 284–302, 2009.

[20] W. Zhong, J. Liu, M. Xue, and L. Jiao, "A multiagent genetic algorithm for global numerical optimization," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 34, no. 2, pp. 1128–1141, 2004.

[21] W. Khan and Q. Zhang, "Moea/d-dra with two crossover operators," in *Computational Intelligence (UKCI), 2010 UK Workshop on*. IEEE, 2010, pp. 1–6.