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Evolutionary Algorithms for Finding Nash Equilibria in Electricity Markets

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***Abstract*—Determining the Nash equilibria (NEs) in a competitive electricity market is a challenging economic game problem. Although finding one equilibrium has been well studied, detecting multiple ones is more practical and difficult, with a few attempts to solve such discrete game problems. However, most of the real-life game problems, such an energy market is a continuous one containing infinite sets of strategy that can be adopted by each player. Therefore, in this paper, a co-evolutionary approach is proposed for detecting multiple NEs in a single run involving continuous games among $N-$players. Five standard test functions and three *IEEE* energy market problems in three different scenarios are solved, and their results are compared with those obtained from state-of-the-art algorithms. The results clearly show the benefits of the proposed approach in terms of both the quality of solutions and efficiency.**

***Index Terms*—energy market, game theory, genetic algorithm, differential evolution**

I. Introduction

Over the last decade, applications of game theory have proliferated in economics and engineering. Game theory portrays socio-economic phenomena with interactions among decision-makers, called players, whose actions affect each other [1]. Accordingly, a game is played by a number of players, each of whom chooses one strategy from a set of known ones that have payoffs assigned to them by a profit function. Each player attempts to maximize their profit. The profit derived by a player depends on its own move and that of the actions of its rivals [1]. Once all players choose their actions to maximize their individual profits with respect to the actions of others, an outcome called the NE is reached. An NE is a stable state of a game in which a player cannot improve its profit unilaterally given that the actions of its rivals remain unchanged.

Determining an NE is usually formulated as an optimization problem, where its difficulty depends on a number of factors; for example, if the game has a global equilibrium with a single NE, it is comparatively easier than one with mixed equilibria that has multiple NEs. Most works have been dedicated to investigating the issues of the complexity related to the mixed equilibria of games with mixed strategies, where a player's choices are not deterministic but regulated by probability distributions. The problem becomes even more difficult when the game is solved with an aim to identity all NEs instead of just one [2]. The problem of finding one equilibrium is well studied in the literature, and there are several different methods for computing it numerically, such as use of linear programming based methods, bi-matrix and game trees [3] etc. In them, once a game is solved, multiple equilibria are found for the same expectation, and the resulting strategies are determined using a probabilistic approach. However, computing a single resulting NE is not sufficient for many practical applications as, even if it is perfect, the possibilities of having other equilibria cannot be ignored [4]. If one solution is not suitable for a player, an alternative one can be adopted immediately [5].

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While niching methods have been widely used to determine multiple optima of multimodal problems [5], they have not been used to identify multiple NEs. Recently, a problem of finding many NEs of a discrete game has been proposed [3], [4], [6]. In this discrete game, each player has a set of discrete strategies with the sizes of their payoff matrices defined using finite permutations of their strategies. Different algorithms, such as the polynomial matrix approach [4], linear programs [7], iterative line search algorithm [6] and tree search-based method [3] were used to find all their possible equilibria. A few computational intelligence (CI) based methods have been also used [1], [2] to determine multiple equilibria for a discrete game. Three of them, covariance matrix adaptation evolution strategies (CMA-ES), particle swarm optimization (PSO) and differential evolution (DE), which employ multi-start and deflection techniques, have been developed [1]. These algorithms obtain more than one local minima which are actually multiple NEs of the game. However, most of these methods require multiple runs to obtain multiple equilibria even though it is not guaranteed that an algorithm will not converge to a previously detected one. Furthermore, they are computationally expensive and only feasible for discrete games.

In a continuous game, a player has a mathematical function with its variables capable of taking any value within a given range. Many real-life problems have the form of continuous games (i.e., sets of infinite strategies); for example, the bidding strategies of generator companies (GENCOs) and strategic customers (e.g, large industries, distributor companies, residential loads, etc.) in a competitive energy market requires continuous formulations. In this example, all players (both GENCOs and customers) simultaneously submit their bids to an independent system operator (ISO) that determines the market clearing price (MCP) and power dispatch (PD) of each winning bidder

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by solving an optimal power flow (OPF) problem, with the aim of finding an optimal operating point of a power system by maximizing its community social welfare (CSW) metric subject to its network and physical constraints [8]. The CSW is defined as the difference between the profits obtained by trading electricity to consumers and the expenses incurred in purchasing it from GENCOs. Once a winning bidder is informed of the MCP and its allocated quantity of PD, its profit is calculated based on its actual cost and revenue [9]. As the profit of a bidder depends on both its own submitted bid and those of its rivals, each bidder plays a game by optimizing its bidding behavior with respect to those of its competitors and the constraints imposed by the power system. A high bid by a player may not be selected by the ISO, while a lower bid may not cover its own costs. Selecting an appropriate bidding strategy that maximizes the profits of all bidders is a challenging economic game problem [9]. Moreover, as the bidding parameters of this game are continuous and the payoff functions are non-smooth, non-convex and multi-modal, the game has multiple equilibria [10].

To solve a continuous strategic-game, many researchers have used the Nikaido-Isoda function [11] to transform an equilibrium problem into an optimization one. Then, a relaxation algorithm, which is a way of sequentially converging with a weighted average of the players' improvements, is often used to determine an NE. To solve a real-world energy market's game problems, both these approaches are widely used [12]–[14] but, although they are guaranteed to obtain a single NE quickly, they are only applicable in a case in which every player has convex strategic functions [14]. However, many of the real-world game problems have non-convex strategic functions; for example, a game model of an energy market is formulated as a non-convex, non-smooth, bi-level optimization problem [15]. Therefore, based on CI, such as genetic algorithms (GAs) [10], [16], [17], DE [18], and a bat-inspired algorithm [19] have been used to solve the energy market game. However, most detect a single NE from a number of equilibria which is not adequate for a player to make a decision [2] as if the energy market problem considers the capacity constraints of a transmission line (TL), the search space of the problem becomes discrete resulting in multiple NEs [10]. Furthermore, as such approaches solve the bidding problem iteratively (bidders bid one after the other), they may take too long to identify a solution in the presence of many bidders.

To address the abovementioned issues, in this paper, a new solution approach with a new ranking technique for solving continuous games in an energy market is developed. In it, a co-evolutionary (CE) technique that uses $N-$subpopulations for $N-$competitive players is used. Each subpopulation contains each player's actions which are updated by either a GA or self-adaptive DE operators during the optimization process. The payoff for the individuals of a player (subpopulation) is evaluated considering the best actions of the other players which are determined using a new ranking technique called Nash non-dominated sorting (NNDS) derived from well known fast non-dominated sorting (NDS) mechanism [20]. Two propositions are proven to validate that the best solutions from the NNDS are actually the NEs. The performance of the proposed CE approach for solving five standard test functions and three well-known real-world energy markets are compared with those of two conventional ones [21], [22] from the literature and the professional Gambit software [23]. Based on the comparison, it is found that the proposed CE approach determines multiple NEs in a single run faster than conventional methods. This is because the latter sequentially determine a solution for each player assuming the best actions of its rivals while the former simultaneously determines them for all players considering the multiple best solutions of their rivals (details are provided in section V).

The contributions of this paper can be summarized as follows: (i) a CE approach for detecting multiple NEs for a mixed strategy game; (ii) a new ranking technique referred as NNDS for ranking the individuals in a subpopulation; (iii) two propositions which prove that the best solutions obtained from the NNDS are actually NEs; and (iv) validation of the proposed method by using it to solve standard and real-life applications of energy market game problems.

The rest of this paper is organized as follows: section II presents the definition of the NE of a game, section III a literature survey of solving different games, section IV the proposed methodology, section V the experimental results, section VI conclusions and suggestions for future work, and in the supplementary material, mathematical formulations of the test problems and CE progress measures are presented.

II. Nash Equilibrium

In this section, we define an NE for a non-cooperative game. In a cooperative game, the participants coordinate their strategies in order to maximize the profits of all players while, in a non-cooperative one, each player maximizes its own profit regardless of those of its rivals, with no commitment to coordinating their strategies [18]. The most popular method for solving a non-cooperative strategic game is the NE concept [24] whereby, if s, is a strategic profile of S, a collective strategy profile, s^* is an NE when no player has anything to gain by changing only its own action or strategy. Letting, π_i be a profit and s^*_{-i} a set of best actions of its rivals, then s^* is an NE if no deviation strategy by any single player is profitable, that is, if for all players, $i \in \{1, \ldots, N\}$ and all strategies $s_{i_j} \in S_i$ are the inequality ones as [24]:

$$\pi_i(s_{i_j}, s^*_{-i}) \leq \pi_i(s^*),\ i \in N,\ j = 1, 2, \ldots, NS_i \quad (1)$$

where NS_i is the number of strategies of player, $i \in N$. However, it is not obvious that a game has a single NE as there could be more than one for a mixed strategic game [25]. For example, consider the two-player prisoner's

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Single NE

A \ B	s_1	s_2
s_1	(1,1)	(-1,2)
s_2	(2,-1)	**(0,0)**

Multiple NEs

A \ B	s_1	s_2
s_1	**(8,7)**	(4,6)
s_2	(6,5)	**(7,8)**

A: First Player
B: Second Player
s_1 and s_2: Players' strategies
(: , :)- Payoffs of player -A and B, respectively.

Fig. 1: Strategies and payoffs of a two-player game

dilemma game [26] with two different payoff matrices illustrated in Fig. 1 in which each player has two different strategies, s_1 and s_2. The first value in each cell of the matrices corresponds to the payoff of player-A, and the second value corresponds to the payoff of player-B. It can be seen that the left matrix has only one equilibrium point $(0, 0)$ while the right has two different ones $(8, 7)$ and $(7, 8)$. The set of NEs is defined as the set of points $p \in s$ that satisfies the following inequalities [2]

$$p \in \Delta \text{ and } z(p) \leq 0 \tag{2}$$

where Δ is defined by a set of linear inequalities (as shown in Eqn. (1)) derived from the normal form game π, and z is a polynomial in p [2].

III. Literature Review

Computing an NE is generally difficult in either a discrete or continuous game and it has been proven that solving an $N-$player game is a $NP-$hard problem [27]. Most relevant methods presented in the literature were developed to find a single NE and, to solve a mixed strategic game, the algorithms were basically designed to find at least one equilibrium but not all. To find multiple equilibria of a discrete game, Fearnley et. al. [28] developed a bi-matrix approach, Datta et. al [4] a polynomial algebra and Berg et al. [3] a tree search-based method. In all these methods, it is assumed that each player has a finite set of strategies ($x \in \{x_1, x_2, \dots x_n\}$), with their payoffs determined using all combinations of these strategies. In other words, their algorithms require at least n^N fitness function evaluations (FFEs) for an $N-$player game with n strategies to each player. Therefore, it is clear that, when a game has many players, it is inherently challenging to solve using conventional optimization methods [18]. To overcome this issue, a CI-based approach with a multi-start and deflection technique was developed [1]. However, although it obtains multiple NEs of large games, it requires multiple runs.

To solve continuous games (those with an infinite number of strategies for each player), such as real-world energy market problems, numerous studies have been conducted to determine the optimal bidding strategy based on different game theory-based equilibrium models [15], [29]–[31]. Of them, the Cournot and Supply Function Equilibrium (SFE) are the most popular due to their realistic characteristics [32], [33]. In the former, the amount of power to be produced by each player is considered a strategic variable while a linear function is used in the latter [32]. To solve the Cournot equilibrium model, a number of researchers [12]–[14] employed Nikaido-Isoda functions with this model represented as an optimization one. For example, in [34], a three-step methodology for finding all possible equilibria for the electricity market considering multi-period bidding options is presented. In it, firstly, the model is reformulated as an optimization problem using a Nikaido-Isoda function and then solved iteratively using a relaxation algorithm considering an initial given solution. Although this method is very computationally efficient, selecting a proper initial solution is non trivial.

While the game problem of the modern energy market can be formulated as a bi-level optimization problem (as described in Appendix B in the supplementary document) [32], [33], it is difficult to solve it using conventional mathematical optimization based approaches as it requires to deal with a nested optimization task with constraints on the outer problem [18]. Heuristic based approaches are increasingly being used to deal with the non-convex characteristics of the objective functions [35]. For example, Azadeh et al. [10] applied a GA to determine the optimal bidding strategies of GENCOs in both cooperative and non-cooperative electricity markets. In [16], another GA was used to solve a scenario-based bi-level strategic game in which a player optimizes its bidding strategy by predicting the possible bidding scenarios of its rivals determined from historical data. However, as there are risk factors associated with assuming opponents' bids when a player optimizes its own, an information-gap decision theory (IGDT) has been used to formulate a risk-based optimal bidding strategy optimization problem solved using a modified PSO (MPSO) [36]. Considering the uncertainty inherent in power generation, Kharrati et al. [37] developed a decomposition algorithm and Dai et al. [38] formulated the same as a stochastic bi-level optimization problem with an aim to determine an NE. A mixed-integer linear programming (MILP) based approach with the duality theory and Karush–Kuhn–Tucker (KKT) condition was used to solve it, with the NEs of the models obtained by a diagonalization algorithm. However, since the problem is non-convex, satisfying the KKT condition does not guarantee the optimality of the solution [32]. Furthermore, in game-based bidding strategies in the abovementioned methods, the bids are represented as discrete quantities, such as bidding high, medium or low, with the payoff matrices determined by computing all possible combinations of the strategies and, subsequently, an equilibrium state of the bidding game corresponding to the optimal bidding strategies are obtained.

Considering a continuous definite bidding function for each player, various solution approaches based on an EA and Monte Carlo (MC) simulation technique for a bi-level SFE model have been proposed [39]. In them, rivals' bidding strategies are estimated using the MC technique while an EA determines each bidder's optimal bidding strategy. Wu et al. [39] converted a bi-level problem into a MILP one using a prime-dual formulation with linearized constraints and assuming a convex objective function. However, as previously mentioned, for an electricity market

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with TL constraints, the problem becomes non-convex [18]. Moreover, in many of the above methods, consumers were considered non-strategic, i.e., they had no option to participate in the market. Considering a consumer strategy, in [32], [33], the SFE model was solved using four different types of bidding parameters, such as intercept, slope, slope-and-intercept, and slope intercept parameterizations, with the one between the slope and intercept of the bidding curve as a strategic variable in order to achieve a definite equilibrium. In their model, IT solution approaches based on a GA [40] and bat-inspired algorithm [19], in which the bidding strategy of each player is iteratively updated, have been developed to obtain the NE. However, as the bidding strategy of each player is updated sequentially in each iteration, approaching the NE is very difficult, even for a small problem, and it requires very significant computational effort to deal with larger instances.

The CE algorithm is an alternative approach that simultaneously determines the optimal bidding strategies for all players and which reduces the computational effort when compared with iterative methods [41]. In the literature, several CE methods for solving different competitive energy markets have been suggested [18], [42], [43]; for example, a CE approach based on a GA was developed to determine the multi-period optimal bidding strategy for an oligopoly electricity market in [44]. In it, each agent (subpopulation) used a reinforcement learning algorithm to increase its own profit from one trading period to the next based on experience from past trading hours. Chen et al. [21] developed a CE approach based on a GA for solving the real-world electricity market in which two different SFE models, the affine and piece-wise affine cost functions, were solved and analyzed, with the solution rapidly converging to the former one. In order to obtain an NE, another CE approach was applied in two different competitive electricity markets, spot and settlement, with the simulation results indicating the effectiveness of the CE algorithm for finding optimal strategies in both markets [45].

However, although most of the abovementioned approaches were successfully used to find an NE of a continuous or discrete game, they are not applicable for solving games which may have multiple equilibria and that can be significantly different in terms of profit, stability, and commitment. Although, some researchers developed few techniques to determine multiple NEs for a discrete game, to the best of our knowledge, computing them for a real-world continuous game, such as an energy market one, has not yet been explored.

IV. Proposed Co-evolutionary Approach

In this research, the strategic games are solved with the aim of determining more than one NE by maximizing an individual's profit and anticipating the actions of each player with respect to those of the others. To achieve this, we propose a competitive CE approach in which individual fitness of a player is evaluated through a competition with its rivals' individuals in their populations. It is developed based on an evolutionary algorithm (EA) (either GA or DE) considering $N-$subpopulations for $N-$players in which, in each subpopulation, the actions of a player ($x \in [q, P, D]$) are optimized with respect to those of its rivals. In the initial generation, N_P actions (say, $\vec{x}$) of each subpopulation are randomly assigned based on Eqn. (3) and then evaluated. As, to evaluate an individual of a player ($n \in N$), it is necessary to know the set of best individuals (let x_n^*, $n = 1, 2, \ldots, N$) of its rivals, we firstly assume that the initial individuals are the best individuals of the other players, that is, $x_n^* \in x_n$, $n = 1, 2, .., N$. Once all the individuals in a subpopulation are evaluated, the NNDS algorithm is used to determine their ranks, as shown in Algorithm 4. Subsequently, the best individuals of a player are updated based on the first non-dominated rank ($nd\_rank = 1$) and their fitness (or, objective function) values (FVs), as described in subsection IV-E. In subsequent generations, the offspring are generated by evolving the parents using either a self-adaptive DE (variant-1) [46] or real coded GA (variant-2) [46], [47] with this process continuing until a stopping criterion is met. The pseudo-code of the proposed CE solution approach is shown in Algorithm 1 and details of its components provided in subsequent subsections.

Algorithm 1 CE solution approach

Require: N_P, $N_G > 1$ and N
1: Set $g = 0$
2: Randomly generate initial individuals in each subpopulation using Eqn. (3)
3: Set random best individuals, as $x_n^* \subseteq x_n \forall n$
4: **for** $n = 1 : N$ **do**
5: Evaluate FVs of all N_P individuals using Algorithm-2
6: Determine the rank of the individuals and updating best ones (x_n^*) by performing NNDS, in Algorithm-4.
7: **end for**
8: **for** $g = 1 : N_G$ **do** ▷ g and N_G are the current and maximum generation number, respectively.
9: **for** $n = 1 : N$ **do**
10: **for** $p = 1 : N_P$ **do**
11: Replace the redundant individuals as in section IV-F
12: Generate a child $y_{n,p}$ by evolving x_n using either GA or DE operators
13: Evaluate FVs of both $x_{n,p}$ and $y_{n,p}$ using Algorithm-2.
14: Accept $x_{n,p}$ or $y_{n,p}$ based on Algorithm-3.
15: **end for**
16: Repeat step 6
17: **end for**
18: Terminate, if a stopping criterion is met, described in section IV-E
19: **end for**

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	Player-1	Player-2	....	Player-N
Individual	Sub-pop-1	Sub-pop-2	....	Sub-pop-N
1	$\overrightarrow{x_{1,1}}$	$\overrightarrow{x_{1,2}}$	....	$\overrightarrow{x_{1,N}}$
2	$\overrightarrow{x_{2,1}}$	$\overrightarrow{x_{2,2}}$	....	$\overrightarrow{x_{2,N}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_P	$\overrightarrow{x_{N_P,1}}$	$\overrightarrow{x_{N_P,2}}$	....	$\overrightarrow{x_{N_P,N}}$

Fig. 2: Encoding scheme

A. Initial generation and encoding

A CE algorithm starts with a number of subpopulations, each of which has N_P random individuals, as shown in Fig. 2. In this research, a real-value encoding is used. Considering that N players participate in a game, each of them would have their own subpopulation. Therefore, the CE-algorithm has $N-$number of subpopulations and the individuals in the n^{th} subpopulation is initialized as:

$$\vec{x}_{p,n} = \vec{x}^n_{\min} + (\vec{x}^n_{\max} - \vec{x}^n_{\min}) \text{ LHS}(N_P) \qquad (3)$$
$$p = 1, 2, \ldots, N_P,\ n = 1, 2, \ldots, N$$

where $\vec{x}_{p,n}$ is a vector to represent the decision variables of the p^{th} individual in n^{th} subpopulation. Since this research considers different types of test problems, the decision variables ($\vec{x}_{p,n}$) might be different depending on the problem. For example, in the standard benchmark problems (discussed in Appendix A in the supplementary document), the $\vec{x}_{p,n}$ would be q; for the Cournot model of the energy market problems (shown in Appendix B.1 in the supplementary document), $\vec{x}_{p,n}$ are the output of the generators, P; and for the SFE of the energy market (discussed in Appendix B.2 in the supplementary document), $\vec{x}_{p,n}$ are the bidding coefficients (discussed later) k_g and k_d. The minimum ($\vec{x}_{min}$) and maximum ($\vec{x}_{max}$) limits are found from their respective limits in Eqns. (S.1) to (S.14) in the supplementary document. The number of players (N) depends on the problem, that is, $N = 2$, n_g, $n_g + n_d$, for the test functions, Cournot, and SFE models, respectively. $LHS(N_P)$ represents the N_P random individuals generated using Latin hypercube sampling (LHS) [47].

B. Evaluation

As previously mentioned, the objective function of a player is to maximize its own profit by modifying its bidding action with respect to those of its rivals. From the profit functions in Eqns. (S.1) to (S.14) in the supplementary document, it can be seen that, when a player evaluates its own objective function, the values of the others must be known. Therefore, the FFE of an individual ($p \in N_P$) in a subpopulation ($n \in N$) depends on both its own and its opponents' bidding actions. Since selecting rivals' bidding actions is difficult, we use the sets of best bidding actions found in the previous generation. This process is illustrated by an example that assumes the market has two strategic players: the individuals of player 1 in generation $g+1$ are evaluated by taking the set of best bidding actions of player 2 from its previous generation (g); if there is more than one but less than N_P best individuals, to evaluate all of those of player-1 in generation $g+1$, its rivals' strategies are randomly chosen ensuring that each is selected at least once. For an $N-$player game, the FFEs of N_P individuals of a player in subpopulation$-n$ are presented in Algorithm 2.

Algorithm 2 The process of FFEs of single player

Require: $x_{i,n}\quad i = 1, 2, ..., N_x$ and $n \in N$
Require: A set of best bidding actions of n^{th} player's rivals, as $\{x^*_k\},\quad k = 1, 2, ..., N, \quad k \neq n$

```
Determine the size of each {x*_k}, as N_{best_k} = size({x*_k}),  k = 1,2,...,N,  k ≠ n
for i = 1 : N_x do
    Set an empty vector, xo = ∅          ▷ xo represents operating x and ∅ an empty set
    for k = 1 : N do
        if k ≠ n then
            Update, xo_k = x*_{k,r}, where ℤr ∈ [1, N_{best_k}]
        else                             ▷ take the current individual
            Update, xo_k = x_{i,k}
        end if
        Evaluate FV by supplying a xo to the respective player's profit function
    end for
    Update, x_{i,k} = xo_k    k = 1,2,..N
end for
Return, FVs and x
```

C. Update bidding actions

To update the bidding actions of each player, we use the GA and DE search operators, as briefly described in the following subsections.

1) GA search operators: Of the various GA crossovers the widely used simulated binary crossover (SBX) is employed here to generate child bidding actions from those of its parents by randomly choosing two through a binary tournament as:

$$\vec{y}^1_p = 0.5\left[(1+\beta_{qp})\vec{x}^1_p + (1-\beta_{qp})\vec{x}^2_p\right] \qquad (4)$$

$$\vec{y}^2_p = 0.5\left[(1-\beta_{qp})\vec{x}^1_p + (1+\beta_{qp})\vec{x}^2_p\right] \qquad (5)$$

where

$$\beta_{qp} = \begin{cases} (2u_p)^{1/\eta_c+1} & u_p \leq 0.5, \\ \left(\frac{1}{2(1-u_p)}\right)^{1/\eta_c+1} & u_p > 0.5 \end{cases} \qquad (6)$$

where u_p is a uniform random number $u_p \in [0, 1]$ and η_c a user-defined parameter distribution factor.

To maintain diversity in a subpopulation, a non-uniform mutation (NUM) is used as it is very popular for constrained optimization problems [47]. In it, the new value ($\acute{\vec{y}}_p$) of parameter $\vec{y}_p$ after mutation in generation g is given as:

$$\acute{y}^j_{p,g+1} = y^j_{p,g} + \delta_{p,j} \qquad (7)$$

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$$\delta_{p,j} = \begin{cases} \left(x_p^{max} - x_{p,g}^j\right)\left(1 - [u]^{1-(g/N_G)^b}\right) & u \leq 0.5, \\ \left(x_p^{mpn} - x_{p,g}^j\right)\left(1 - [u]^{1-(g/N_G)^b}\right) & u > 0.5 \end{cases} \tag{8}$$

$$\forall p \quad \text{and} \quad j \in N_P$$

As in [46], the speed of the step length is $b = 5$.

2) DE search operators: To compare results, we also use another EA variant, DE which is similar to a GA as it shares information among the population members but differs from it in that it applies mutation first to generate a trial vector which is then used within the crossover operator to produce one offspring. In this paper, we use two mutation operators, *i.e.*, 'DE/rand/1' and 'DE/rand-to-best/1', and one binomial crossover to maintain a balance between diversity and convergence. Such a combination have demonstrated good search abilities [47]. For each parent ($\vec{x}_p$), the trial vector ($\vec{y}_{p,g}$) is generated as:

$$\vec{y}_{p,g+1} = \begin{cases} \vec{x}_{r_3,g} + F_p\left(\vec{x}_{r_1,g} - \vec{x}_{r_2,g}\right), \\ \qquad \text{if} \quad rand_1 \leq Cr_p \text{ and } rand_2 \leq prob_1, \\ \vec{x}_{p,g} + F_p\left((\vec{x}_{r_1,g} - \vec{x}_{r_2,g}) + (\vec{x}_{best,t} - \vec{x}_{p,g})\right), \\ \qquad \text{if} \quad rand_1 \leq Cr_p \text{ and } rand_2 > prob_1, \\ \vec{x}_{p,g}, \quad \text{otherwise} \end{cases} \tag{9}$$

In this mutation, three parents are selected randomly from the entire population such that $p \neq r_1 \neq r_2 \neq r_3$, where F_p is the amplification factor for the mutation operator, Cr_p the crossover rate and $prob_1$ a predefined probability of selecting the mutation operator (it is set to a value of 0.5).

As the performance of DE depends on the values of F_p and Cr_p, an adaptive mechanism for setting them is employed [47]. In it, F_p and Cr_p are initially set using the normal distribution with a mean of 0 and standard deviation of 0.1 as, $\{\dot{F}, \dot{Cr}\} \in N(0.5, 0.1)$. Then, in subsequent generations are respectively updated as:

$$F_p = \begin{cases} \dot{F}_{r_1} + rand_1(\dot{F}_{r_2} - \dot{F}_{r_3}), \text{ if } (rand_2 < \tau_1), \\ rand_3, \text{ otherwise} \end{cases} \tag{10}$$

$$Cr_p = \begin{cases} \dot{Cr}_{r_1} + rand_4(\dot{Cr}_{r_2} - \dot{Cr}_{r_3}), \text{ if } (rand_5 < \tau_1), \\ rand_6, \text{ otherwise} \end{cases} \tag{11}$$

where $rand_k \in [0,1], k = 1, 2, .., 6$ and $\tau_1 = 0.75$ [46]. The values of both F_p and Cr_p are in the range from 0.1 to 1 and, , if less than 0.1 or greater than 1, are fixed to 0.1 and 1, respectively.

Based on the selection criteria (discussed in the next subsection), if an offspring (y) is better than its parent (x), the parent's $\dot{F}_p$ and $\dot{Cr}_p$ are replaced by its offspring's F_p and Cr_p and vice versa. This process is repeated until all the individuals are selected which means that, at the end of the current generation, the better-performing F_p and Cr_p survive to the next generation.

D. Selection

In the literature, many types of selection methods for the CE approaches are available; for example to select an individual, a greedy scheme is used in which the fittest (according to the FVs) of two candidates, a parent and its child, is chosen [18]. However, as the problem considered in this paper involves maximizing the profits of a number of players ($N > 1$), a direct greedy method is not appropriate. Therefore, the new selection criteria described below are proposed.

Consider an $N-$player game with a player's two strategies are x_1 and x_2, and a set of best actions of its rivals, y^*. The payoffs for x_1 and x_2 are $FV(x_1, y^*)$ and $FV(x_2, y^*)$, respectively, assuming an operator, M where $M(x_1, x_2) = FV(x_1, y^*) > FV(x_2, y^*)$ represents the number of players that benefit if a player uses a x_1 strategy compared with when that player uses of x_2 with the same best bidding actions of its rivals, *i.e.*, y^*. Then, the potential relationships between x_1 and x_2 are:

1) $M(x_1, x_2) = N$: x_1 is strictly non-dominated by x_2, *i.e.*, all player are clearly benefited (not even same) if they use x_1 individual instead of using x_2
2) $M(x_2, x_1) = N$: x_2 is strictly non-dominated by x_1
3) $M(x_1, x_2) = M(x_2, x_1) \neq N$: neither x_1 strictly non-dominated by x_2 nor x_2 strictly non-dominated by x_1

Proposition 1. *A strictly non-dominated solution is a global NE.*

Proof. Let $x^* \in x$ be a solution strictly non-dominated by another solution, $x_1 \in x$. Suppose that, x^* is not a global NE, but, x_1 is an equilibrium *i.e.*, there must be at least one player, $n \in N$ that benefits when using x_1 but not x^*, as:

$$M(x_1, x^*) = FV(x_1, y^*) > FV(x^*, y^*) \geq 1 \tag{12}$$

However, $M(x_1, x^*) = 0$ when x^* is strictly nondominated by x_1 which means that x^* is a global NE.

As, for criterion 3, if since none of the solutions are in an equilibria, we use the following, called the self-benefit criteria (shown in Algorithm 3), to select the individuals to drive the solutions towards an NE. For a clearer understanding, an example of a $2-$player game is given in Table I. Let us assume that an offspring of player-1 is generated after applying the search operators to its parent. This parent is x_1 at the stage that the best individual of player-2 is y^* and their FVs are $FV_1(x_1, y^*) = [10, 8]$, where 10 and 8 are the profits of players 1 and 2, respectively. Once the offspring (x_{2i}, $i = 1, 2, \ldots, 5$) is evaluated, the FVs must be one of five types, considering the same best individual of player-2 as y^*. Based on the values of M, one of two individuals is selected for the next generation. Details of these criteria for selecting an individual from a parent (x_1) and its child (x_2) are provided in Algorithm 3.

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Table I: Example of selection operator

Parent x_1 and its $FVs = [10, 8]$					
offspring	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}
FVs	[10,8]	[11,9]	[9,7]	[11,7]	[9,12]
M	0	2	-2	1	1
Select	y_{11}	y_{12}	x_1	y_{14}	x_1

Algorithm 3 Criteria for selecting parent or child

Require: A parent, x_1, and a child, x_2, with their FVs, as $FV(x_1, y^*)$ and $FV(x_2, y^*)$, respectively
Require: $n \leftarrow$ Index of player is currently operating
Determine, $M(x_1, x_2)$ and $M(x_2, x_1)$
if $M(x_1, x_2) = N$ **then** ▷ parent is good
    $x = x_1$ ▷ select parent
else if $M(x_2, x_1) = N$ **then** ▷ child is good
    $x = x_2$ ▷ select child
else if $FV_n(x_1, y^*) > FV_n(x_2, y^*)$ **then** ▷ is FV of n^{th} player better for parent?
    $x = x_1$ ▷ select parent
else ▷ the FV of n^{th} player better for child
    $x = x_2$ ▷ select child
end if
Return, x

E. Ranking individuals

Once all the better-performing individuals in a subpopulation are selected, they are further ranked to determine the best ones for evaluation in the next generations. This is achieved using Algorithm 4 which is developed based on the concept of the NDS approach [20] and called the NNDS.

Algorithm 4 Ranking individuals using NNDS

Require: The individuals, x of a subpopulation, n
Determine $nd\_rank(x)$ using the NDS approach [20]
Take the n^{th} player's individuals, $x_n \subset x$
Set, $X = \varnothing$ and $xBest = \varnothing$ ▷ $\varnothing$ is a symbol for the empty set
for $k = 1 : \max(nd\_rank)$ **do**
    Get, $x_{n,k} \subset x_n$ based on the k^{th} nd_rank
    Update, $x_{n,k} = \text{sort}(x_{n,k})$ based on n^{th} player's FVs
    Update, $X = [X, x_{n,k}]$
    if $k = 1$ **then** ▷ Update the best individuals
        $xBest_n = xn_k$
    end if
end for
Return, $x \leftarrow X$ and $x^* \leftarrow xBest$

In the NNDS, firstly, the individuals are ranked based on the conventional NDS approach which means that those better for all players receive nd_rank=1 and those worse for all players a maximum nd_rank, and so on, as explained in following example.

1) x_1 is non-dominated by x_2 if $M(x_1, x_2) > M(x_2, x_1)$, *i.e.*, $nd\_rank(x_1) = 1$ and $nd\_rank(x_2) = 2$
2) x_2 is non-dominated by x_1 if $M(x_2, x_1) > M(x_1, x_2)$, *i.e.*, $nd\_rank(x_2) = 1$ and $nd\_rank(x_1) = 2$
3) x_1 and x_2 are indifferent if $M(x_1, x_2) = M(x_2, x_1)$, *i.e.*, $nd\_rank(x_1) = nd\_rank(x_2) = 1$

To determine the best set of actions of each player, we select the individuals with $nd\_rank = 1$. These solutions are actually the NEs found so far, as proven in proposition-2.

Proposition 2. *The solutions with $nd\_rank = 1$ are NEs.*

***Proof*.** Suppose that $x^* \in x$ is a solution with $nd\_rank = 1$ but not an NE. Let $x_1 \in x$ be another solution with $nd\_rank > 1$ but an NE, *i.e.*, there must be at least one player that obtain a benefit using x_1 rather than x^*, such as:

$$M(x_1, x^*) = FV(x_1, y^*) > FV(x^*, y^*) \geq 1 \tag{13}$$

However, it is necessary that $M(x_1, x^*) = 0$ satisfies the conditions of the NNDS approach. Therefore, $M(x^*, x_1) = FV(x^*, y^*) > FV(x_1, y^*) \geq 1$ which indicates that x^* is an NE because $\nexists x_1 \in x$, $x_1 \neq x^*$ such that $M(x_1, x^*) > M(x^*, x_1)$.

F. Maintaining diversity

As, if there are redundant individuals in a subpopulation, which is possible, the performance of the optimization algorithm can be affected. Any redundant individual is replaced by a random solution generated using Eqn. (3). To allow the algorithm to concentrate on high-quality solutions during the later stages of the evolution, the process continues until the algorithm reaches half the N_G.

G. Stopping criteria

In this study, we consider two stopping criteria:

1) the maximum number of generations (N_G) is reached; and
2) both the number of generations reach $N_G/2$ and the best bidding actions are no longer improved for a predefined number of generations, θ.

Criteria 1 and 2 affect the run-time and solution quality, respectively. When only criterion 1 is used, N_G is difficult to define because different systems have different convergence characteristics. Therefore, when both criteria 1 and 2 are used simultaneously, the best solution may possibly be found [47].

V. Experimental results

For the experimental study, the wide range of test problems, including standard benchmark and real-life electricity game ones, considered are:

1) five standard test problems;
2) a Cournot model of the *IEEE* 2-bus system;
3) a SFE model of the *IEEE* 3- and 30-bus systems; and
4) a real-world energy market.

Each test problem is solved using the proposed CE approaches based on the (i) GA (called the CE-GA) and (ii) DE (called the CE-DE) algorithms, and the results from their median runs reported. Furthermore, to validate these methods, we adopt three conventional ones based on a Nikaido–Isoda function [12] with a relaxation algorithm [13] and two iterative (IT) line search algorithms [18], [26], one based on GA (called IT-GA) [18] and the other on DE (called IT-DE) [18].

The Nikaido–Isoda function is very attractive for solving the continuous game [12]. In it, the game problem is transformed into an optimization one and, subsequently, a sequential improvement in the Nikaido–Isoda function is achieved using a relaxation algorithm. Let $\pi_i(\mathbf{x})$ be the payoff function of player$-i$ and $\mathbf{x}$ the set of strategies of all the players in a game. Then, the Nikaido–Isoda function $\psi(\mathbf{x}, \mathbf{y})$ is defined as:

$$\psi(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} [\pi_i(y_i|x) - \pi_i(x)] \tag{14}$$

where n is the number of players in the game and y their alternative strategies. At an NE, $\psi(\mathbf{x}, \mathbf{y}) = 0$ and, in other generations, its value is sequentially improved using a relaxation algorithm as:

$$\mathbf{x}^{k+1} = (1 - \alpha)\mathbf{x}^k + \alpha Z(\mathbf{x}^k) \tag{15}$$

where k is the iteration number,$0 < \alpha \leq 1$ (typically set to 0.5 [13]) and Z the optimum response function that can be obtained after making the first partial derivatives of Eqn.(14). w.r.t. $\mathbf{y}$. The algorithm is stopped when $\left|\mathbf{x}^{k+1} - \mathbf{x}^k\right| < 0.0001$ is satisfied [13].

Iterative line search algorithms are very popular for non-convex problems [18], [22], [26]. In them, in each iteration, a player optimizes its own bidding strategy given that its rivals have fixed actions. To start a game, each player randomly chooses its action assuming that random strategies are used by the other players and then optimizes its own action with respect to those of the other players. In the following generations, either GA or DE search operators are used to generate a better-quality strategy from its parents. Once a player obtains its optimal bidding strategy, the first iteration is complete. Then, the next player begins to optimize its own strategy using the same process while considering the best bidding strategies of its rivals found so far. This iterative process is terminated when either no player is able to change its action in θ generations or the maximum number of iterations ($MaxIt$) is reached.

A. Experimental setup

The GA parameters, the probabilities of crossover and mutation, are set to 0.9 and 0.1, respectively. Based on the empirical analysis discussed in subsection V-D, the values of θ, N_P, N_G, and $MaxIt$ are set to 5, 40, 100 and 100, respectively. Thirty independent runs are performed for each test case and the solutions recorded, with the median one based on the nNE reported in this paper. Moreover, the NEs obtained by the CE algorithms are verified by performing a Gambit simulation [23] in which the final results consider the pure strategies of the players, with their payoff matrices further evaluated to determine all the equilibrium points through configuring the Gambit software as "Compute all Nash Equilibria" [23][1].

The CE, IT and Nikaido–Isoda-based algorithms are implemented on a computer with a 3.4 GHZ Intel Core i7 processor with 16 GB of RAM in the MATLAB (R2017a) environment.

B. Test problems

In this subsection, we firstly solve four standard infinite games in which each has two competitive players with their convex strategic functions. The strategy of a player (decision variables) can be set within a range to maximize their own profits with respect to those of its rivals [24]. They are described in Appendix A in the supplementary document, and their analytical results *i.e.*, known NEs are $(5, 5)$, $(1.25, 1.25)$, $(0.5, 0 \sim 1)$, and $(48.98, 48.98)$ for test problems 1, 2, 3 and 4, respectively [24]. Then, we solve a simple electricity market (without TL constraints) that uses the IEEE 30-bus system with three players in which player-1 has a single generator (#1), player-2 two (#2 and #3) and player-3 three (#4, #5 and #6). Each player has a non-convex strategic function with decision variables that are the power outputs from each generator that can vary within a range. The data and payoff function of each player can be found in [12].

All these problems are solved using the CE-based (CE-GA and CE-DE), IT-based (IT-GA and IT-DE) and Nikaido–Isoda function-based relaxation algorithms. When test problems 1, 2 and 4 are solved, all the algorithms, including the proposed ones, obtain the same NEs, that are, (5,5), (1.25,1.25) and (50,50) for problems 1, 2 and 4, respectively. However, for test problem 3, which has multiple NEs, conventional methods find a single NE and CE-based ones identify several, as shown in Fig. 3. Moreover, when the non-convex problem (test problem 5) is solved using these algorithms, conventional methods detect different local NEs in different runs (Fig. 4) while the proposed algorithms always delivered the global NE (as proven in [12]). Note that Nikaido–Isoda-1 to Nikaido–Isoda-5 represent the results obtained from 5 random runs with different initial values.

C. Real-world energy market problems

As previously mentioned, the energy market is formulated as a bi-level optimization problem as shown in Appendix B in the supplementary document, with the upper-level profit maximization of each bidder solved using the proposed CE-GA and CE-DE, and conventional IT-GA and IT-DE algorithms. Since the electricity market has a number of non-differentiable strategic functions, the Nikaido–Isoda-based method is not used for simplicity. To solve the lower-level DC-OPF problem, a simple quadratic

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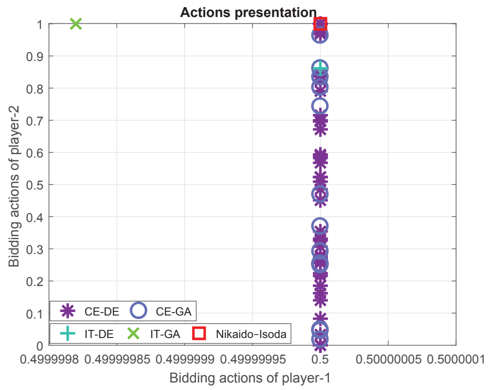

Fig. 3: Actions obtained by different approaches for test problem 3

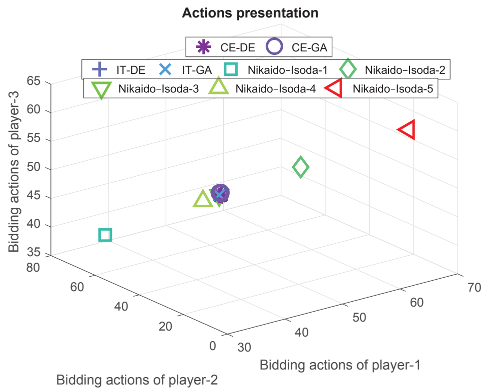

Fig. 4: Actions obtained by different approaches for test problem 5

programming (QP) approach that has shown superior performance in [22] is used with $MaxIt$ set to a value of 1000.

To demonstrate the performance of the proposed CE-DE and CE-GA algorithms in solving the real-world energy market problems, three energy test problems are considered with each has the following three cases.

- Case I: consumers are non-strategic, and TLs ignored
- Case II: consumers are non-strategic, and TLs considered
- Case III: consumers are strategic, and TLs considered

A strategic customer is one that can participate in the bidding process which increases the number of players in that game while, in a non-strategic customer game, the number of players is reduced to the number of GENCOs with customers only able to buy a predefined amount of electricity from the market. Furthermore, when capacity constraints of a TL are considered, the problems may have multiple NEs [10]. Details of each test problem are provided below.

1) IEEE 2-bus system: At first, we solve a modified IEEE 2-bus Cournot game [31] that is qualitatively similar to the California model [26]. Its formulation is discussed in Appendix B.1 in the supplementary document and, for cases I and II, the maximum TL flow is set to 80 and 55

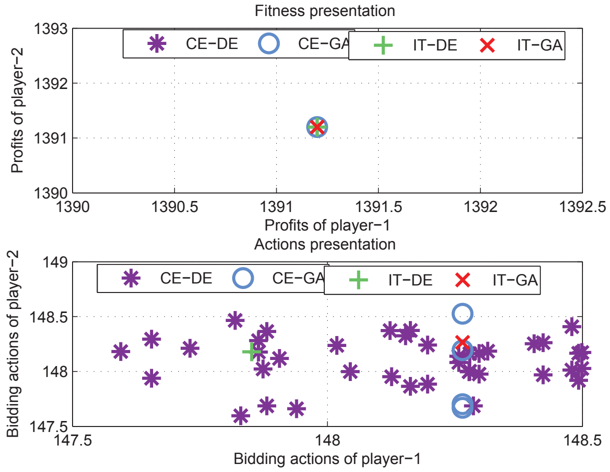

Fig. 5: Profits and actions obtained by CE and IT approaches for IEEE 2-bus system (case I)

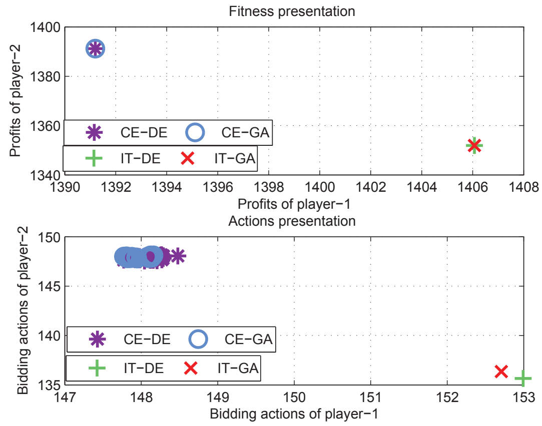

Fig. 6: Profits and actions obtained by CE and IT approaches for IEEE 2-bus system (case II)

MW, respectively [26]. For case I, a strategy of $(148, 148)$ with the payoff of $(1.39E+3, 1.39E+3)$ is found in [26] as well as obtained from the conventional IT-DE and IT-GA algorithms. The CE-DE and CE-GA algorithms determine several NEs with a fixed payoff, as shown in Fig. 5. For case II, although the local search algorithms obtain different NEs of $(148, 148)$, $(92, 136)$ and $(153, 136)$ from different runs, it is proven that, except for $(148, 148)$, the solutions are local NEs [26]. On the other hand, the proposed CE algorithms obtain a number of NEs, as shown in Fig. 6, all of which are verified through Gambit simulation.

By comparing the FFEs and simulation times (in minutes) of the IT- and CE-based algorithms for cases I and II, as shown in Table II, it is proven that the proposed CE algorithms are very efficient, even after obtaining multiple NEs, with CE-DE the best in terms of its nNEs, FFEs and computational time.

2) IEEE 3-bus system: The *IEEE* 3-bus test system is formulated as a SFE model assuming that the bidding

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Table II: Summary of results for IEEE 2 bus system

Algorithm	Case I			Case II		
	FFEs	Time	nNE	FFEs	Time	nNE
IT-GA [22]	41040	9.10	1	45600	10.84	1
IT-DE [22]	45560	9.05	1	48560	11.46	1
CE-GA	16080	3.63	4	16080	3.81	15
CE-DE	16080	2.61	40	16080	3.30	40

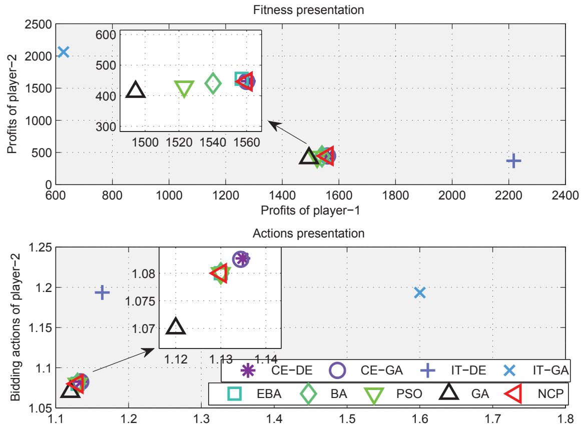

Fig. 7: Profits and actions obtained by CE and IT approaches for IEEE 3-bus system (case I)

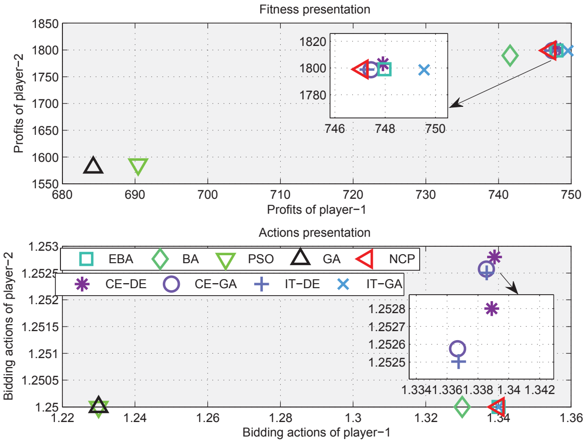

Fig. 8: Profits and actions obtained by CE and IT approaches for IEEE 3-bus system (case II)

Table III: Bidding actions and payoffs obtained by CE and IT approaches for IEEE 3-bus system (case III)

Alg.	Actions				Payoff			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
IT-GA [22]	1.36	1.18	0.92	0.82	762.36	1093.81	1988.80	1634.91
IT-DE [22]	1.31	1.16	0.90	0.78	766.58	861.24	2117.16	1657.94
CE-GA	1.31	1.16	0.90	0.78	766.98	859.96	2120.15	1656.04
CE-DE	1.31	1.16	0.90	0.78	766.66	861.39	2116.60	1658.40

action involves the cost coefficients of the respective generator and consumer as $k_g \in [1.0, 2.5]$, and $k_d \in [0.1, 1.0]$, respectively [19]. Its mathematical formulation is provided in Appendix B.2 in the supplementary document and its parameters could be found in [18]. It consists of two generators at nodes 1 and 3, two consumers at nodes 1 and 2, and three TLs considered lossless with equal reactance values of $x = 0.002$. The TL capacity of is set to 500 MW for case I and 25 MW for cases II and III while the capacity limits of the other TLs are ignored.

Once cases I and II of this system are solved using the CE- and IT- based algorithms, their final results as well as those of the state-of-the-art algorithms, such as GA, PSO, Bat-inspired algorithm (BA), enhanced BA (EBA) [19] and mixed nonlinear complementarity problem's algorithm (NCP) [48] are illustrated in Fig. 7 and 8, respectively, with the proposed CE approaches identifying multiple equilibria. Nevertheless, the obtained solutions from the state-of-the-art algorithms are rechecked using the Gambit simulation and many of them found local. In other words, the algorithms are stopped too early and delivered a local solution. On the other hand, the solutions obtained from proposed algorithms are verified through the Gambit software and found all the solutions are NEs.

For case III, as the consumers participate in the bidding market, there are four players. Since it is difficult to present the results visually, we tabulate the mean values of the bidding actions and payoffs obtained from each algorithm in Table III. One can observe that the results are very consistent. The numbers of FFEs, computational times (in minutes) and nNEs presented in Table IV for all three cases indicate that CE-DE is the algorithm that performs the best.

3) IEEE 30-bus system: To demonstrate the effectiveness of the proposed algorithms for a large system, the *IEEE* 30-bus test system, which has up to 26 competitive players with 6 generators, 20 loads, 41 lines and 30 buses, is considered with all the TLs lossless and their reactance values set to 0.001. The system data and schematic diagram are described in [49]. The TLs' capacity limits are ignored for case I while, for cases II and III, are set to 10, 8 and 10 for three TLs, respectively, and the bidding coefficients of the generators and consumers $k_g \in [1.0, 2.5]$, and $k_d \in [0.1, 1.0]$, respectively. All the cases are solved using all the algorithms, with a summary of their results and computational time (in minutes) presented in Table V which demonstrates that there is a global NE in case I and multiple ones in cases II and III. However, to obtain these equilibria, conventional methods take longer and require more FFEs than the proposed CE-based approaches. Comparing CE-GA and CE-DE, again, CE-DE is best in terms of nNEs, FFEs and computational times for all cases.

4) Real-world energy market: In this subsection, we solve a real-world energy market's game problem of an Australian National Energy Market (NEM) system [50].

Table IV: Summary of results for IEEE 3-bus system

Alg.	Case I			Case II			Case III		
	FFEs	Time	nNE	FFEs	Time	nNE	FFEs	Time	nNE
IT -GA[22]	45600	5.70	1	27361	2.02	1	54720	4.27	1
IT -DE[22]	45600	4.49	1	27360	1.95	1	54720	4.04	1
CE-GA	**16080**	1.19	3	16080	1.19	**3**	32160	2.33	4
CE-DE	**16080**	**1.17**	**4**	**13040**	**0.95**	**3**	**18080**	**1.31**	**16**

Table V: Summary of results for IEEE 30-bus system

Alg.	Case-I			Case-II			Case-III		
	FFEs	Time	nNE	FFEs	Time	nNE	FFEs	Time	nNE
IT -GA[22]	95760	12.37	1	95760	13.13	1	592800	82.06	1
IT -DE[22]	82080	10.44	1	82080	10.71	1	592840	81.70	1
CE-GA	48240	6.88	1	48240	6.75	13	209040	28.25	16
CE-DE	48240	6.05	1	29520	3.89	21	209040	27.93	40

Table VI: Summary of the results for Australian NEM problem

	IT -GA[22]	IT -DE[22]	CE-GA	CE-DE
FFEs	95767	153367	56280	56280
Time (min)	27.66	44.28	17.20	15.20
nNE	1	1	36	40

Its mathematical formulation is shown in Appendix B.3 in the supplementary document. The problem is solved by both IT- and CE-based algorithms, and the results are shown in Table VI. It is found that the conventional IT-based approaches obtain a single NE, while the proposed CE-based ones determine multiple local NEs. In terms of computational effort, the CE-based approaches use fewer FFEs and computational time which is approximately 60% less compared with those of conventional IT-based approaches.

D. Parametric analysis

In this subsection, the robustness of the proposed CE approaches is evaluated by analyzing the parameters, (i) convergence plots (ii) N_P, (iii) stopping criteria (iv) selection operator with NNDS mechanism, (v) diversity operator, and (vi) statistical comparisons . To do this, the best-performing algorithm, CE-DE, is used to solve the test problems for case II by following a *ceteris paribus* strategy in which only one parameter is varied while all the others remain fixed to their best values [46].

1) Convergence: In this subsection, we illustrate the performance of the proposed CE-DE approach by comparing its convergence characteristics for the first 10 generations with those of the IT-DE and Nikaido–Isoda function-based relaxation algorithms when solving test problem 5, as shown in Fig. 9. It can be seen that the solutions obtained from the CE-based algorithm are very close to the global NEs (46.66, 32.16, 15.0, 22.13, 12.33, 12.33) even after 10 generations while those from other approaches are far from them. This is because conventional approaches determine the best solution for each player sequentially while the CE does so for all players simultaneously. Therefore, the proposed approach obtains the expected solution more quickly than conventional approaches.

2) Effect of N_P: Three different values of N_P are used to solve the test problems considered, with their results presented in Table VII demonstrating that all their mean fitness values (MFVs) are almost the same. However, in terms of nNEs, CE-DE with $N_P = 20$ is inferior to CE-DE with $N_P = 40$ and $N_P = 60$ although the computational times (in minutes) and FFVs are much higher for $N_P = 60$. Therefore, it can be stated that N_P does not significantly affect the algorithm's capability to obtain the best MFV, with its value of 40 saving computational time and producing good results.

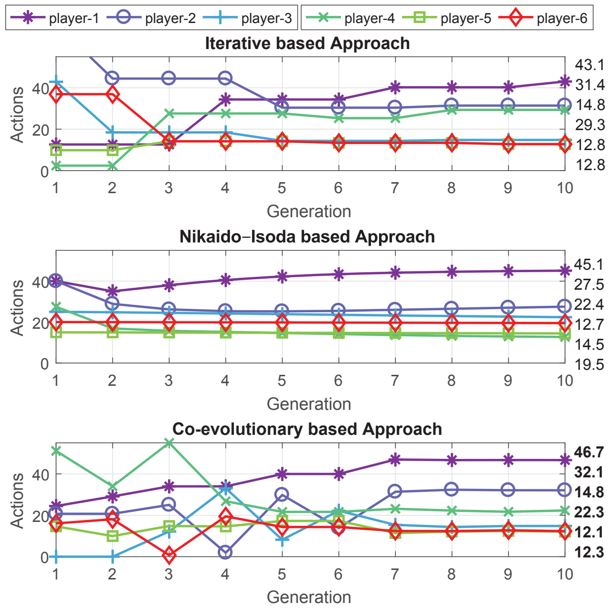

Fig. 9: Convergences plots of different algorithms

Table VIII: Effect of θ on CE-DE

Problem	$\theta = 2$				$\theta = 10$			
	$MFVs$	FFEs	Time	nNEs	FFEs	$MFVs$	Time	nNEs
1	25.00	9840	0.04	1	25.00	12400	0.05	1
2	4.07	10320	0.04	1	4.06	14480	0.06	1
3	0.25	16080	0.07	40	0.25	16080	0.07	40
4	3.16E+3	9040	0.04	1	3.16E+3	10800	0.05	1
5	4.42e+3	24120	0.05	1	4.42e+3	24120	0.05	1
IEEE 2	1.39E+3	16080	3.32	40	1.39E+3	16080	3.54	40
IEEE 3	1.28E+3	9200	0.68	3	1.28E+3	16080	1.23	3
IEEE 30	842.83	28560	3.03	12	842.76	35280	3.92	21

3) Effect of stopping criteria: As discussed in subsection IV-G, this paper uses two different stopping criteria, θ and N_G, the effects of which we analyze. Firstly, we test three different θ values (2, 5 and 10) and present their results in Table VIII and those of the default ones (i.e., $\theta = 5$) in Table VII which indicate that, although these values do not have a significant impact on the quality of solutions (MFVs), higher values increase the computational time. Furthermore, as it is seen that the nNEs are almost the same for θ values of 5 and 10, it is wise to choose one of 5.

Using the best value of $\theta = 5$, we run CE-DE with three different N_G, *i.e.*, 50, 100 and 200, to solve the same problems. The results presented in Tables IX and VII (for $N_G = 100$) indicate that the performance of this algorithm slightly improves when N_G is increased.

4) Effects of NNDS mechanism: In this subsection, we demonstrate the superiority of the proposed selection operator and NNDS algorithm over a traditional technique. The experiments are conducted using the third test problem (as it has multiple NEs) both with and without the selection operator and NNDS mechanism shown in the Algorithms 3 and 4, respectively. When the problem is solved without

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Table VII: Effect of N_P on CE-DE

Problem	$N_P = 20$				$N_P = 40$				$N_P = 60$			
	$MFVs$	FFEs	Time	nNEs	$MFVs$	FFEs	Time	nNEs	FFEs	$MFVs$	Time	nNEs
1	25.00	6521	0.05	1	25.00	10160	0.05	1	25.00	15480	0.05	1
2	4.06	4680	0.03	1	4.06	13520	0.05	1	4.06	16440	0.05	1
3	0.25	8040	0.06	20	0.25	16080	0.06	40	0.25	24120	0.08	60
4	3.16E+3	5160	0.04	1	3.16E+3	9900	0.05	1	3.16E+3	14280	0.05	1
5	4.41E+3	12060	0.03	1	4.42E+3	24120	0.05	1	36180	4.43E+3	0.07	1
IEEE 2	1.39E+3	8040	1.67	20	1.39E+3	16080	3.3	40	1.39E+3	24120	4.90	60
IEEE 3	1.27E+3	7160	0.58	2	1.27E+3	13040	0.95	3	1.27E+3	18120	1.28	5
IEEE 30	842.81	13800	1.31	14	842.83	29520	3.89	21	842.76	40680	3.84	20

Table IX: Effect of N_G on CE-DE

Problem	$N_G = 50$				$N_G = 200$			
	$MFVs$	FFEs	Time	nNEs	$MFVs$	FFEs	Time	nNEs
1	25.00	8080	0.04	1	25.00	17040	0.07	1
2	4.07	8080	0.04	1	4.06	17040	0.07	1
3	0.25	8080	0.04	40	0.25	32080	0.14	40
4	3.16E+3	8080	0.04	1	3.16E+3	17040	0.07	1
5	4.41e+3	12120	0.03	1	4.43e+3	48120	0.10	1
IEEE 2	1.39E+3	8080	1.74	40	1.39E+3	32080	6.69	40
IEEE 3	1.28E+3	8080	0.62	2	1.27E+3	18000	1.32	3
IEEE 30	842.96	24240	2.46	12	842.74	51120	5.16	21

Table X: Effect of diversity mechanism on CE-DE

Alg.	nNEs				FFEs				Time
	min.	mean	max	SD	min.	mean	max	SD	(min.)
CE-DE_1	3	**9.97**	**17**	**3.45**	2200	**2321.33**	**3720**	**316.44**	3.89
CE-DE_2	3	8.03	15	4.35	2200	2286.67	2960	577.05	**3.88**

them, the individuals are ranked using a conventional greedy approach, in which the fittest (according to their FVs) ones are selected for the next generation [18]. The individuals obtained from both approaches, in different generations, are illustrated in Fig. S.1 (in the supplementary document) It can be seen that the greedy method selects a single best individual and the proposed one selects several non-dominated ones (according to their FVs). Consequently, at the end of the evolutionary process, the proposed algorithm finds multiple NEs and the traditional approach identifies only one. In terms of time complexity, an algorithm with the proposed selection operator and NNDS technique requires around 0.04 minutes and that with a greedy scheme requires 0.03 minutes.

5) Effect of maintaining diversity: To demonstrate the performance of the diversity mechanism (subsection IV-F) presented in this paper, the IEEE 30-bus system (case II) is solved using the proposed CE-DE algorithm with (CE-DE_1) and without (CE-DE_2) considering this mechanism. Both approaches are run 30 times, with the minimum, mean, maximum and standard deviation (SD) values of the nNEs and FFS are reported in Table X. It can be seen that the algorithm finds more NEs by evaluating more solutions with the diversity mechanism in place. However, without diversity, the algorithm is often stuck and exits due to the stopping criteria. Furthermore, from the SD values, one can conclude that the algorithm with diversity provides more consistent results in all runs. Statistical performance is analyzed in the next subsection.

6) Statistical comparison: In this subsection, the proposed CE-based algorithms are statistically compared with the state-of-the-art ones. Three cases in CE-based algorithms are applied: (i) considering all the proposed parameters (e.g., initial generation, diversity operator, selection and ranking techniques), (ii) without considering the diversity operator, and (iii) without using the LHS technique in the proposed initial generation scheme but considers a conventional initialization technique [1]. Based on the results obtained for all the test problems and considering all relevant cases (a total of 13 instances), a Friedman test is carried out to rank all the algorithms based on their best (criterion 1) and average (criterion 2) nNEs. The Friedman test is a popular nonparametric test that is used to analysis the variances to identify the existence of different median values in a set of k algorithms (where $k > 2$). Consider, two hypotheses for the Friedman test: (i) the null hypothesis H_0 and (ii) the alternative hypothesis H_1. The former is defined as there is no significant difference between the k algorithms (here, $k = 8$) while the later is defined as the presence of a significant difference between algorithms. Another important factor in the Friedman test is a $p-$value, which represents the probability of obtaining a result at least as extreme as the one that was found with assuming H_0 is true. The value of p also provides an idea that the statistical hypothesis test is significant or not. Based on the value of p, the significance of the obtained results can be identified; smaller the $p-$value increases the evidences to against the null hypothesis, H_0.

Table S.1 (in the supplementary document) shows the Friedman test results of the mean ranks of the considered algorithms and $p-$values. Since in both cases, the $p-$values are less than 0.05, the null hypothesis (H_0) is rejected [46]. Therefore, the obtained results from the algorithms do not have identical effect (H_0 rejected), and based on the mean ranks it can be inferred that the proposed CE–DE algorithm (*i.e.*, case-1) is the best.

7) Analysis of CE progress measures: In this subsection, the dynamics of the proposed CE algorithm are evaluated by examining the well-known CIAO (*current individual against ancestral opponent*) plots [51] which is discussed in Appendix C in the supplementary document. The CIAO plot is used to visually convey the progress of two populations during co-evolution. The analysis shows that the CE process is resulted in cyclic trajectories with continuous improvement in both subpopulations.

VI. Conclusion and future work

Determining an NE in a continuous game is a challenging optimization problem which becomes more complex when the algorithm requires identification of more than one equilibria in a single run. In this paper, the effectiveness of two CE methods, CE-DE and CE-GA, for finding multiple NEs in continuous strategic games were investigated. Multi-populations were considered for multiple players in which each subpopulation represented a player that co-evolved with the each other and evaluated its own best performance considering the best individuals in the other subpopulations. Furthermore, a new sorting technique for ranking the individuals in a subpopulation in order to drive the solutions towards NEs was developed, with two propositions proven to justify that the best solutions obtained from the proposed methods were the actual NEs.

To validate the results, three conventional methods were also implemented and their results compared by solving a number of continuous strategic game problems, including five standard test functions and three real-world energy market problems. Comparisons of the simulation results revealed that the CE-based approaches had merit in terms of their nNEs detected, FFEs and computational times, with CE-DE the best of all the algorithms.

Possible future work could consider use of other EAs and means to configure two or more of them. Although the diversity of individuals was maintained by injecting some random individuals, another method for balancing convergence and diversity could be developed further. Moreover, in the application domain, a few more complex real-world problems, such as multi-period auctions over 24-hour time horizon with 5-minute intervals, could be considered.

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