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Enhancing the network robustness against cascades by rewiring edges

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Abstract **– The network robustness against cascades is crucial for many networked systems. In the paper, we enhance the network robustness against cascades by rewiring edges of networks. Three edge-rewiring strategies are compared, including random rewiring strategy (RRS), assortative rewiring strategy (ARS), and low-polarization rewiring strategy (LPRS). The results show that ARS is more effective than RRS to improve the network robustness against cascades while the network achieves the strongest robustness level under LPRS. We also investigate the effects of the total cost of edge-rewiring. It is found that the superiority of LPRS becomes more evident than ARS and RRS as the total cost grows. Our work might shed light on the design of networked systems.**

Keywords - cascading; network robustness; edge-rewiring strategy

I. INTRODUCTION

We live in a modern world supported by large, complex networks. Examples range from power grids to the internet, communication, and transportation systems. However, a large-scale collapse might break out in these networked systems when a disastrous incident is triggered. In those systems which the flow of physical quantities are supported, the fail of a single node or edge will cause the redistribution of physical flows over the remaining nodes or edges, and then some nodes or edges will breakdown if they are overloaded. The procedure will propagate until no overloaded nodes or edges exist in the networked systems. This phenomenon has been called “cascading failures” or “avalanche” [1-2].

The research of defense and control strategy against cascading failures has attracted a lot of interest in recent years [3-5]. To enhance the network robustness against cascades, one method is to design efficient load redistribution strategies. In ref. [6], the authors proposed a proactive algorithm to improve the robustness of heterogeneously loaded networks against avalanche. It is based on load-dependent weights. Compared to simple hop weights, respective shortest flow paths turn a previously heterogeneous load distribution into a more homogeneous one for the nodes and edges of the network. The use of these flow paths greatly increases the networks robustness. The authors in Ref. [7] research cascade defense strategy on complex networks via routing strategy. They propose a new routing strategy which can effectively defend avalanche break down without decreasing the efficiency of the network. In ref. [8], the authors study the behaviour of cascading reaction on weighted networks. The results show that, in the case of the value of weight parameter is 1, the weighted network will achieve its strongest robustness level. The evolution of cascading failures on heterogeneous weighted networks is studied by Wu et al. [9]. They adopt a local weighted flow redistribution rule, where the node weight corresponds to its link degree.

The other method for improving the network robustness against cascades is to make appropriate changes to underlying network structures. In ref. [5], the authors introduce a new strategy which based on a selective removal of nodes and edges. The selective removal of network nodes and edges is shown to significantly reduce the avalanche size. The authors in Ref. [10] investigate scale-free traffic networks with dynamic weights. They focus on the efficiency of three removal strategies (flow-based removal, betweenness-based removal and mix-based removal) against cascades. Simulation results demonstrate that the mix-based removal can delay the time of network breakdown and reduce the damage of cascade.

Many previous works have found that adding edges can significantly enhance the dynamic behaviors on complex networks, such as epidemic spreading [11-12], information traffic [13]. Actually, rewiring edges can also enhance the robustness of networks [14]. Thus it is valuable to find an optimal edge-rewiring strategy which can maximally enhance the network robustness against avalanche. In this paper, we compared the effects of three edge-rewiring strategies: random rewiring strategy (RRS), assortative rewiring strategy (ARS), and low-polarization rewiring strategy (LPRS). Simulation results show that ARS can enforce the network robustness against cascades more remarkably than RRS while the strongest network robustness level is achieved by LPRS.

The paper is organized as follows. In the next section we describe the cascading model and edge-rewiring strategies in detail. In Section 3, simulation results and correspondent theoretical analysis are provided, and the work is summarized in Section 4.

II. THE MODEL

A. Network model:

Many realistic networks have been found to be scale-free, such as WWW, the Internet, and airline routes. Following common practices [15-16], we adopt the well-known Barabási-Albert (BA) model [17] in the paper. The BA model is generated by two general rules (i.e., growth rule and preferential attachment rule). Starting from a small amount of m_0 fully connected nodes and the network increases by adding a new node at each time step. The new node is preferentially connected to m $(m \le m_0)$ existing ones in a way that the likelihood of connecting to an old node depends on the node's degree.

B. Cascading model:

We use the relative size of the giant component $G = N'/N$ to measure the network robustness against cascading failures, where N' is the size of the giant component after cascades and N is the initial network size. When $G \approx 1$ the network integrity is remained, while breakdown when $G \approx 0$. In the paper, the node's load is denoted by its betweenness [2], which is characterized as the total number of shortest paths passing through it [18]:

$$B_i = \sum_{j \ne l \ne i} \frac{n_{jl}(i)}{n_{jl}}, \qquad (1)$$

where n_{jl} is the number of shortest paths connecting j and l, while $n_{jl}(i)$ is the number of shortest paths going from j to l and passing through i. In our model, each node has a finite ability to process the load and the node capacity is the maximum load that the node can handle [2]:

$$C_j = (1+\beta)B_j, \qquad (2)$$

where $\beta \ge 0$ is a tolerance parameter, and B_j is the betweenness of node j in the initial network. Obviously, the value of β is related to the capability of nodes to process the load. Larger β value means higher node's security margin.

The loads are exchanged between each pair of nodes and transmitted along the shortest path connecting them. In our model, we focus on the situation where cascading failures are caused by attack on a single node with the maximal betweenness. The cascading failures can lead to immediate breakdown of the network [2]. The node removal will change the shortest paths distribution of the network. The load at a special node is changed. If the load grows larger than its capacity, the node will breakdown. This breakdown will result in a new redistribution of loads and lead to subsequent failures. The process can stop after a few steps but it can also propagate and breakdown a considerable fraction of the entire network. When there are no more breakdowns, the relative size of the giant component G is calculated.

C. edge-rewiring strategies:

✧ Strategy 1 (random rewiring strategy, RRS):

At each time step, two edges ij and $i'j'$ are randomly chosen, then we swap these [(i, j) and (i', j') to (i, j') and (i', j)]. Neither self-connections nor double-connections are permitted during the edge-rewiring process. The procedure keeps the degree of every node strictly and the whole degree distribution of the network.

✧ Strategy 2 (assortative rewiring strategy, ARS):

Newman found that the network robustness is to be influenced by assortative mixing [19]. The assortative mixing level can be evaluated by the associativity coefficient [19]:

$$r = \frac{M^{-1}\sum_i j_i k_i - [M^{-1}\sum_i \frac{1}{2}(j_i + k_i)]^2}{M^{-1}\sum_i \frac{1}{2}(j_i^2 + k_i^2) - [M^{-1}\sum_i \frac{1}{2}(j_i + k_i)]^2}, \qquad (3)$$

where j_i, k_i are the degrees of the nodes at the ends of the ith edge, with $i = 1,\dots,M$ and M is the number of edges in the network. Obviously, $-1 \le r \le 1$. $r > 0$ corresponds to assortative networks, $r < 0$ corresponds to disassortative networks, and $r = 0$ corresponds to neutral networks. The value of r can be impacted by rewiring edges. For assortative rewiring strategy, we rewire two pairs of randomly chosen edges at each time step if and only if such a rewire would increase the value of r remarkably ($r(t+1) \ge r(t) + \lambda_r$). In our model, the threshold λ_r is not larger than 0.01. Self-connections and double-connections are not permitted during the whole edge-rewiring process.

✧ Strategy 3 (low-polarization rewiring strategy, LPRS):

It is found that the heterogeneity of the networks makes them particularly vulnerable to cascading failures [2]. In this paper, we proposed an edge-rewiring strategy to enhance the network robustness against cascading failures by making the network structure more homogeneous. The homogeneity of the network can be characterized by the polarization of the network [20]:

$$\pi = \frac{B_{\max} - \langle B \rangle}{\langle B \rangle}, \qquad (4)$$

where $\langle B \rangle$ is the average network betweenness, and $B_{\max}$ is the maximum betweenness of the network. Obviously, smaller π value corresponds to more homogeneous network.

For LPRS, we swap two pairs of randomly selected edges at each time step if and only if such a swap would decrease the value of π drastically ($\pi(t) \ge \pi(t+1) + \lambda_\pi$). In our model, the threshold λ_π is not larger than 0.05. During the entire edge-rewiring process, neither self-connections nor double-connections are permitted.

III. Simulation Results

We compare the efficiency of RRS, ARS and LPRS on BA network with the network size $N = 1000$ and $m_0 = m = 2$. Fig. 1 shows the network robustness against cascading failures under three edge-rewiring strategies and original network. We rewire 398 edges for each strategy. One can see that the value of G increases with the increment of tolerance parameter β, reflecting the network robustness against cascades has a more secure margin as β grows. One can see that ARS and LPRS can obviously improve the network robustness against cascades, and the value of G under LPRS is the largest.

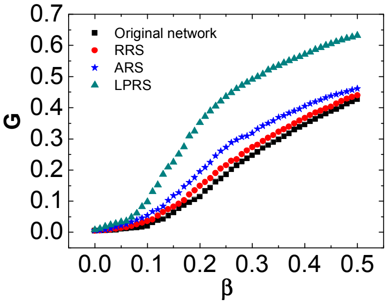

Fig. 1 (Color online) The relative size of the giant component G as a function of tolerance parameter β under three edge-rewiring strategies and original network. Here we set $N = 1000$ and $m_0 = m = 2$. Each data is averaged over 30 independent realizations.

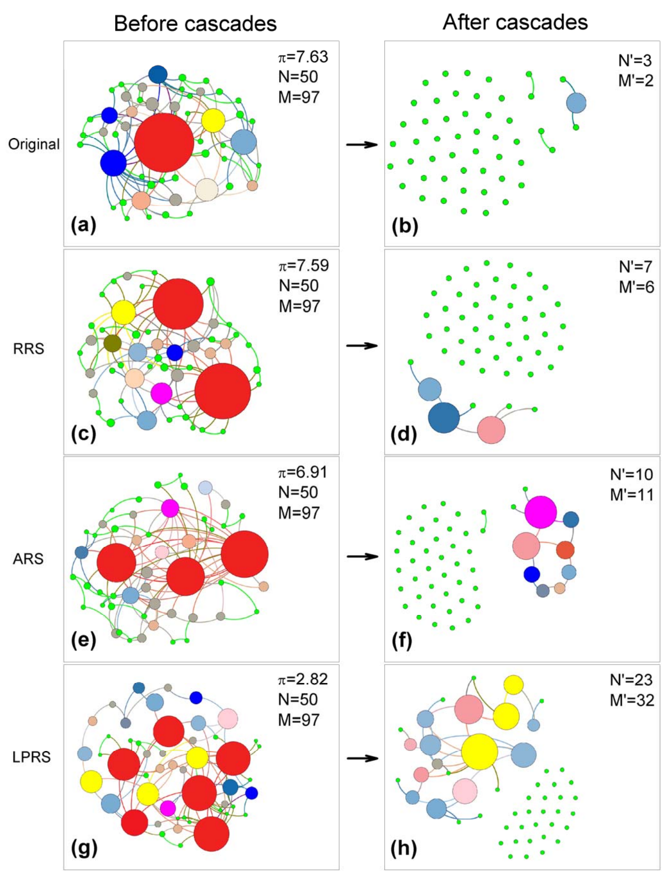

Fig. 2 (Color online) (a) The topological structure of the original BA network, where the number of total edges $M = 97$. Structures of the network after rewiring edges: (c) RRS, (e) ARS and (g) LPRS. Structures of the network after cascades with tolerance parameter $\beta = 0.3$: (b) the original BA network, (d) RRS, (f) ARS and (h) LPRS. Here we set $N = 50$ and $m_0 = m = 2$.

To better understand above phenomenon, we show the structure of the network before and after cascades on a BA scale-free network with $N = 50$ and $m_0 = m = 2$. As shown in Figure 2, M is the total number of edges of the network, M' is the total number of links of the giant component after cascades, π is the polarization of the network, N is the size of the network, and N' is the size of the giant component after cascades. The original network has 97 edges and $\pi = 7.63$ (Fig. 2(a)). There is a node with extremely high betweenness while the betweenness of others is relatively low, indicating that the load distribution of the original network is extremely heterogeneous. After cascading failures, there are only 2 edges and 3 nodes in the giant component. This means that the entire network is disintegrated (Fig. 2(b)). We rewired 38 edges under each strategy. For RRS, the value of $\pi(\pi = 7.59)$ is smaller than that of the original network. As shown in Fig. 2(c), there are 2 nodes with very high betweenness while the betweenness of others is relatively low, reflecting that the load distribution of RRS is still very heterogeneous. After cascades, we can see that only 7 nodes and 6 edges left in the giant component, which indicates that the whole network is almost collapsed (Fig. 2(d)). Under ARS, there are 3 nodes with high betweenness in the network and $\pi = 6.91$ (Fig. 2(e)), reflecting that the load distribution of ARS is more homogeneous than that of the original network and RRS. After cascades, there are 10 nodes and 11 edges remained in the giant component (Fig. 2(f)), which means that the whole network is seriously damaged. For LPRS, the value of π is only 2.82 (Fig. 2(g)). There are many nodes with relatively high betweenness in the network, indicating the loads are distributed evenly among the whole network. As a consequence, 32 edges and 23 nodes survive in the giant component after cascades (Fig. 2(h)).

It is known that the number of rewired edges is limited by cost in many real-world networks. We denote the total cost of edge-rewiring $\eta = M^{rewire} / M$, where M is the number of edges in the original network and M^{rewire} is the number of rewired edges. To understand how the total cost affects the efficiency of the three strategies, we display the relative size of giant component G as a function of the total cost η under three edge-rewiring strategies with tolerance parameter $\beta = 0.3$ (Fig. 3). We can find that the value of G under LPRS is increased more evidently than that of ARS and RRS as the increment of η.

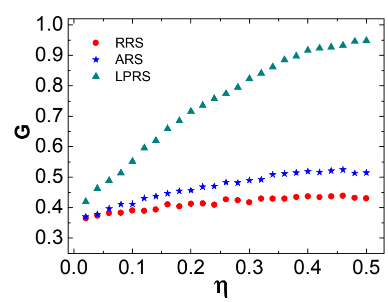

Fig. 3 (Color online) The relative size of the giant component G as a function of η under three edge-rewiring strategies with tolerance parameter $\beta = 0.3$. Here we set $N = 1000$ and $m_0 = m = 2$. Each point is averaged over 500 independent realizations.

IV. CONCLUSION

To summarize, we have compared the efficiency of three edge-rewiring strategies (RRS, ARS and LPRS) in the cascading model. The cascading failure is triggered by attacking on a single node with the largest betweenness in BA scale-free networks. It is found that LPRS is more effective than ARS and RRS for improving the network robustness against cascading failures. We also investigate the effects of the cost of edge-rewiring. Simulation results show that the network robustness against cascades under LPRS is obviously superior to ARS and RRS as the increment of the cost.

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