Physica A 443 (2016) 212–220

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Optimal network topology for structural robustness based on natural connectivity

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HIGHLIGHTS

- We optimize the network robustness based on natural connectivity.
- We propose a tabu search algorithm to find the optimal network topology.
- We discover that the optimal network exhibits a roughly eggplant-like topology.
- We propose an improved algorithm by employing the assortative rewiring strategy.

ARTICLE INFO

Article history:
Received 4 January 2015
Received in revised form 4 May 2015
Available online 26 September 2015

Keywords:
Robustness optimization
Natural connectivity
Scale-free networks
"Eggplant-like" topology

ABSTRACT

The structural robustness of the infrastructure of various real-life systems, which can be represented by networks, is of great importance. Thus we have proposed a tabu search algorithm to optimize the structural robustness of a given network by rewiring the links and fixing the node degrees. The objective of our algorithm is to maximize a new structural robustness measure, natural connectivity, which provides a sensitive and reliable measure of the structural robustness of complex networks and has lower computation complexity. We initially applied this method to several networks with different degree distributions for contrast analysis and investigated the basic properties of the optimal network. We discovered that the optimal network based on the power-law degree distribution exhibits a roughly "eggplant-like" topology, where there is a cluster of high-degree nodes at the head and other low-degree nodes scattered across the body of "eggplant". Additionally, the cost to rewire links in practical applications is considered; therefore, we optimized this method by employing the assortative rewiring strategy and validated its efficiency.

1. Introduction

Real networks, such as the Internet, power supply networks [1], optical communication networks [2], and social networks, play a necessary role in the practical life. Thus the networks need to be robust and capable of enduring random failures or intentional attacks. The structural robustness of a network is the ability of the network to maintain its connectivity after a random failure or an intentional attack, meaning that the nodes (or links) deletions [3]. Most real networks, also called scale-free networks, display a surprisingly high degree of tolerance against random failure; however, they are vulnerable to malicious attack [4]. Consequently, the development of methods to improve the structural robustness of networks, especially the scale-free network, has become an important issue.

Previously, many studies have been devoted to optimizing the structural robustness by designing the topology of the network. Several special optimal models were presented to generate robust networks that could withstand random failures

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http://dx.doi.org/10.1016/j.physa.2015.09.023

and attacks [5–7]. However, in reality, it is impractical to redesign a complex large-scale network. For instance, because the internet and the power supply network have existed for such a long time, it is impossible to redesign their structures. An alternative solution would be the addition of a certain number of links to the network or the rewiring of the links to obtain a special topology that will enhance robustness [8,9]. However, the addition of links is not a practical solution due to the high cost of addition. For example, the installation of power lines between each pair of power plants would cause skyrocket costs and transmission losses. We also assumed that changing the degree of a node can be more expensive than changing the edge of a network. These assumptions suggest that the best solution would involve maintaining an invariant number of links and degree of each node. Therefore, under these constraints, we designed the optimization algorithm for robustness by rewiring links.

In addition to the aforementioned constraints, another essential factor in optimization is the selection of an appropriate measure of structural robustness because different measures will lead to different optimal robust network. Most of the previous work are based on the percolation threshold f_c, which is used to characterize the critical fraction of the least nodes that need to be removed until the network is collapsed, during the malicious attacks or random failures [4]. However, this measure has proven to be inapplicable in many realistic cases [10].

In a recent work [11], the authors proposed a new measure, R, for network robustness under malicious attack on nodes, which corresponds to the failure of the stressed nodes in physical networks [12]. This new measure R, considers the size of the largest connected cluster during the malicious attack [10]. With this measure, they designed an algorithm, based on greedy algorithm, to enhance the structural robustness of a given network by swapping its links while keeping its degree distribution fixed. The R-optimized network has an "onion-like" topology consisting of a core of highly connected nodes that are hierarchically surrounded by rings of nodes with decreasing degree [11].

In reality, most networks are large-scale and complex, which indicates that the optimization has a request for the computational complexity of robustness measure. The measure R, however, does not pay enough attention to its computational complexity or to f_c. Unfortunately, each time of calculating R at least costs the time complexity of $O(m^2)$, where m is the number of links in a network [13]. As the size of network increases, the number of times for rewiring increases rapidly and the computation time would be unacceptable. Moreover, the structural robustness improvement based on measure R depends strongly on the specific attack strategy, instead of the inner properties of a complex network. Thus, a more appropriate measure which can be computed fast and measure structural robustness accurately is of great importance for the optimization.

Natural connectivity $\bar{\lambda}$, a new measure of robustness in complex networks based on graph spectra, has been proposed and received growing attention [14–20]. It is derived from the graph spectrum as an average eigenvalue, and can be interpreted as the Helmholtz free energy of a network [21]. The measure decreases the running time of computing the robustness obviously, and it does not rely on the attack strategy. Consequently, the optimization of robustness based on natural connectivity is more suitable and reliable for the practical applications.

Moreover, the method based on greedy algorithm [11] usually fails to find the global optima due to the complexity of this kind of combinatorial optimization problem. Given that meta-heuristic algorithms can efficiently solve the combination optimization problem in consideration of both global and local searches, we employ the tabu search algorithm to search the optimal solution.

Therefore, in this paper, we design a tabu search algorithm to determine the optimal network structure in terms of its structural robustness $\bar{\lambda}$, by repeatedly rewiring links. We employ the method to different types of initial networks and compare the results with the R-optimized network (the "onion-like" network). Finally, an improved rewiring strategy is proposed to reduce the times of rewiring.

2. Structural robustness measure: natural connectivity

Before attempting to optimize robustness, we first introduce the natural connectivity, which is employed to measure the structural robustness of networks in this paper.

Natural connectivity is presented to describe the redundancy of alternative paths, which can be associated with the robustness of a network [14]. This measure is defined as an "average eigenvalue" of the graph adjacency matrix [14]:

$$\bar{\lambda} = \ln\left(\frac{1}{N}\sum_{i=1}^{N} e^{\lambda_i}\right), \tag{1}$$

where N is the number of nodes in a network G and λ_i is the ith element of the set $\{\lambda_1, \lambda_2, \ldots, \lambda_N\}$, which is called the spectrum of G. The natural connectivity has a physical meaning and a simple mathematical formulation. It characterizes the redundancy of alternative paths by quantifying the weighted number of closed walks of all lengths and can also be interpreted as the Helmholtz free energy of a network [21]. Mathematically, the natural connectivity can be derived from the graph spectrum as an average eigenvalue. It is shown that the natural connectivity has a strong discrimination in measuring the structural robustness of complex networks and can sensitively exhibit the variation of robustness. Moreover, the amount of time needed to compute natural connectivity is much less than that required to compute the measure R.

The aim of this research is to determine an optimal network topology for structural robustness based on natural connectivity. The study will track how the properties of the networks change during the optimization process, and will

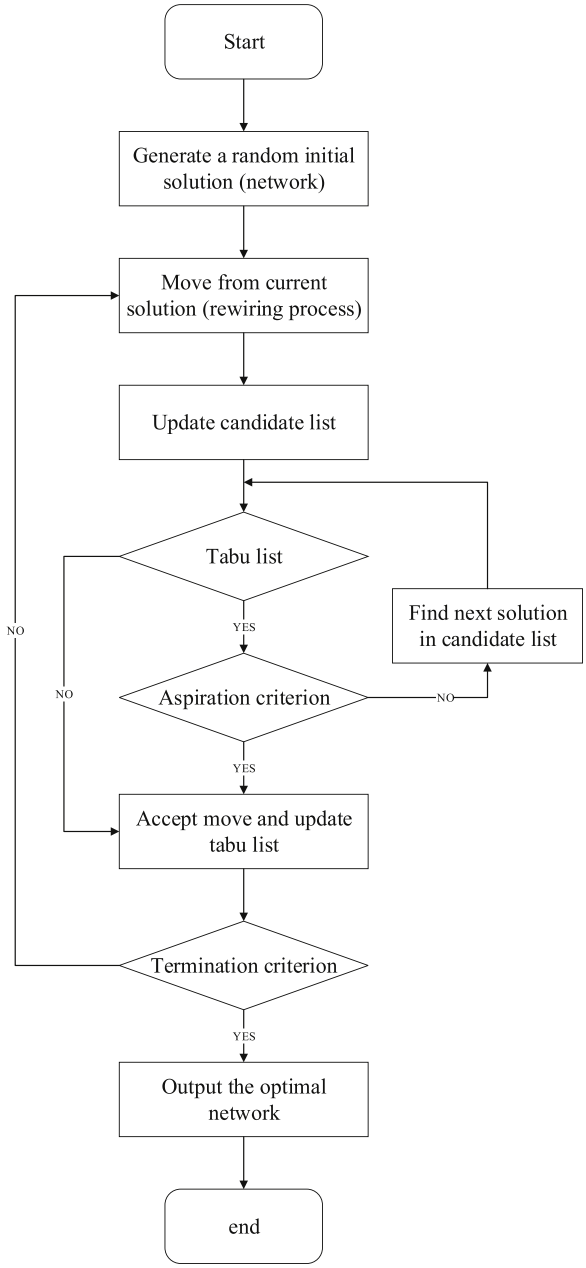

Fig. 1. The flow diagram of optimization algorithm.

determine the primary features of an optimal network topology. Additionally, the optimal network topology will be compared with the "onion-like" network [11].

3. Tabu search algorithm for improving the structural robustness

Tabu search (TS) is a meta-heuristic approach proposed by Glover (1986). It is currently applied to solve many combinatorial optimizations [22]. In particular, TS has been applied to electrical systems to solve network reconfiguration problems [23], optimally distributed generation in distribution networks [24], optimal power flow [25], among others.

In this section, we will give the operational procedures of tabu search algorithm. The objective is to maximize the structural robustness measure defined in Eq. (1). We describe the five primary parts of our algorithm: variable encoding, move mechanism, tabu list, aspiration criterion and termination criterion. And the whole framework of the algorithm is summarized in Fig. 1.

Variable encoding: Each variable means a solution, i.e., an adjacent matrix of a graph. A graph G with N nodes has $N(N-1)/2$ edges at most. Let x_i be the flag of the ith edge. Thus, x_i satisfy the conditions $x_i = 0, 1$ and $\sum_i x_i = W(W \le N(N-1)/2)$, where W is the number of edges in G.

Move mechanism: The "move" mechanism represents the process of pointing the current solution state toward another one. It decides the form of producing solution in neighborhood and the relation between solutions. Since we need to maintain an invariant number of links and the degree of each node, we consider the swap operation proposed in Ref. [11] as a move operation in tabu search algorithm. The swap operation is that swapping the connections of two randomly chosen edges which have no common nodes. The rewiring process is considered acceptable only if the network is still connected. Thus, at each iteration, the neighborhood of a current solution is obtained by applying the swap operation to the current solution. However, the number of possible solutions in neighborhood of a solution is very large. In order to simplify the issue, at each iteration, we only produce $n_{candidate}$ candidates randomly in neighborhood of the current solution.

Tabu list: Tabus are one of the distinctive elements of tabu search. It records previously encountered moves (solutions) to prevent cycling back to the previously visited solutions. The key realization here is that when the current solution is moved, we need to prevent the search from tracing back its steps to where it came from. In our algorithm, each time we swap the edges, the tabu list will record the change of the corresponding edges so that the tabu edges would not be allowed for rewiring in a certain number of iterations. This certain number of iterations is called the length of Tabu list, which is often determined by experience.

Aspiration criterion: The setting of an aspiration criterion is to avoid the loss of good solution and stimulate the local search for good solution, and conduce to global optimization. We follow the rule that tabus can be disregarded if the better solution is found and cycling cannot occur. Meanwhile, if both candidates are tabu, choose the best candidate to be the next solution.

Termination criterion: The proposed algorithm has two termination criteria: the first criterion is to terminate after a given number of iterations; and the second criterion is to terminate after a given number of iterations without improvement of the best-known solution. The best-known solution is the best historical solution achieved during the search process, and updated only if the current solution is better than it. It is also the final output of the optimization algorithm. In general, we employ the first criterion when we compare different algorithms on efficiency. And if we need to search the optimal robust network, the second criterion is adopted.

Although this meta-heuristic algorithm does not guarantee a globally optimal solution, we will get a relatively satisfactory solution. The numerical results have been shown to confirm the availability of tabu search algorithm. The detailed description of tabu search algorithm for searching the optimal robust network is presented in Algorithm 1.

4. Numerical results analysis

Because the degree distribution of network is held constant during the optimization, the optimization results across different networks may vary as a function of their respective distributions. To comprehensively investigate the effect of degree distribution on optimization, three types of networks are considered: ER random network [26], WS small world network [27] and BA scale-free network [4]. Each of the three network has a characteristic degree distribution. Fig. 2(a) shows the fluctuation of $\bar{\lambda}$ with the iteration number n during the process of optimization based on these three types of initial networks. Both of the initial networks have the same number of nodes and links ($N = 100$ and $m = 291$). And the optimization will be terminated after 200 times of continuous iterations without improvement of the best-known solution. The numerical results are averaged over 20 independent realizations. It is clear that the value of $\bar{\lambda}$ increases progressively slower, and finally converges to a stable and desired value after a certain number of iterations. The robustness $\bar{\lambda}$ of the BA scale-free network is likely to have greater improvement than either the ER random network or the WS small world network, which indicates that the algorithm is quite effective when it is applied to those heterogeneous networks.

To identify the features of the $\bar{\lambda}$-optimized network, three properties that characterize the topology of the networks, including the degree assortativity r, the clustering coefficient C and the average shortest path length d are investigated.

The degree assortativity r is a measure of assortative mixing by degree in the networks, i.e., the tendency for high-degree nodes to associate preferentially with other high-degree nodes [28]. The degree assortativity r is defined as:

$$r = \frac{m^{-1}\sum\limits_k u_k v_k - \left(m^{-1}\sum\limits_k \frac{1}{2}(u_k + v_k)\right)^2}{m^{-1}\sum\limits_k \frac{1}{2}(u_k^2 + v_k^2) - \left(m^{-1}\sum\limits_k \frac{1}{2}(u_k + v_k)\right)^2}, \tag{2}$$

where u_k and v_k are the degrees of the nodes at the end of the kth link, and m is the total number of links in the network. In Fig. 2(b), both of the optimal networks for $\bar{\lambda}$ have a larger degree assortativity r than the initial network, which means that the structural robustness of the network is optimized along with an increase in the degree assortativity r. Additionally, the $\bar{\lambda}$-optimized network displays a larger degree assortativity than the R-optimized network (see Table 1). These results suggest that the algorithm presented by this research tends to connect the nodes of similar degree to gain a higher value of structural robustness $\bar{\lambda}$. Next we compute the average clustering coefficient C [29]. The clustering coefficient of node i is

Algorithm 1 Tabu Search Algorithm for structural robustness

Input:
G_0: Initial solution (network);
$n_{candidate}$: The size of the candidate set;
L: The length of tabu list;
$n_{iteration}$: The maximum number of iterations without improvement;
Output:
$G_{optimal}$: The solution with the highest robustness found;
Other notation:
$C(G)$: The set of candidates for the solution G;
$\tilde{C}(G)$: The "admissible" subset of $C(G)$ (i.e., non-tabu or allowed by aspiration);
$\bar{\lambda}(G)$: The structural robustness of G;
T: Tabu list;
G_{now}: The current solution;
G_{best}: The best-known solution;

$n \Leftarrow 0$; $i \Leftarrow 0$; $G_{now} \Leftarrow G_0$; $G_{best} \Leftarrow G_0$; $T \Leftarrow \phi$;
while $n < n_{iteration}$ **do**
  while $i < n_{candidate}$ **do**
    $G_{temp} \Leftarrow G_{now}$;
    Randomly rewire two edges in G_{temp} which have no common nodes;
    if G_{temp} is connected **then**
      $C(G_{now}, i) \Leftarrow G_{temp}$;
      Calculate the structural robustness of each candidate $\bar{\lambda}(C(G_{now}, i))$;
      Update $\tilde{C}(G_{now}, i)$ according to tabu list T and $\bar{\lambda}(C(G_{now}, i))$;
      $i \Leftarrow i + 1$;
    end if
  end while
  $G \Leftarrow argmax[\bar{\lambda}(G')]$, where $G' \in \tilde{C}(G_{now})$;
  if $\bar{\lambda}(G) > \bar{\lambda}(G_{best})$ **then**
    then set $G_{best} \Leftarrow G$, $G_{now} \Leftarrow G$, $n \Leftarrow 0$;
  else
    $G_{now} \Leftarrow G$;
  end if
  Update T;
  $n \Leftarrow n + 1$;
end while
$G_{optimal} \Leftarrow G_{best}$;

Table 1
Properties of the different types of networks: robustness measure R, natural connectivity $\bar{\lambda}$, the degree assortativity r, the cluster coefficient C and the average shortest path length d.

	Type of optimization	R	$\bar{\lambda}$	r	C	d
ER random network	Initial	0.310	2.711	−0.099	0.057	2.783
	R-optimized	0.363	2.789	0.079	0.077	2.814
	$\bar{\lambda}$-optimized	0.327	4.627	0.615	0.427	4.699
WS small world network	Initial	0.328	2.658	−0.031	0.113	2.844
	R-optimized	0.389	2.629	0.053	0.088	2.814
	$\bar{\lambda}$-optimized	0.342	3.644	0.426	0.324	3.879
BA scale-free network	Initial	0.192	4.820	−0.193	0.148	2.565
	R-optimized	0.322	6.207	0.040	0.150	2.744
	$\bar{\lambda}$-optimized	0.244	9.237	0.257	0.382	5.567

defined as:

$$C_i = \frac{E_i}{(k_i(k_i - 1))/2}, \tag{3}$$

where E_i is the sum of the number of links between the neighbors of node i, and k_i is the number of neighbor nodes of node i. The average clustering coefficient C is the mean of clustering coefficients for all of the nodes. The result shown in Fig. 2(c) indicates that the average clustering coefficient C in the network after the optimization for $\bar{\lambda}$ is much larger than before, which is similar to the results obtained for the degree assortativity r.

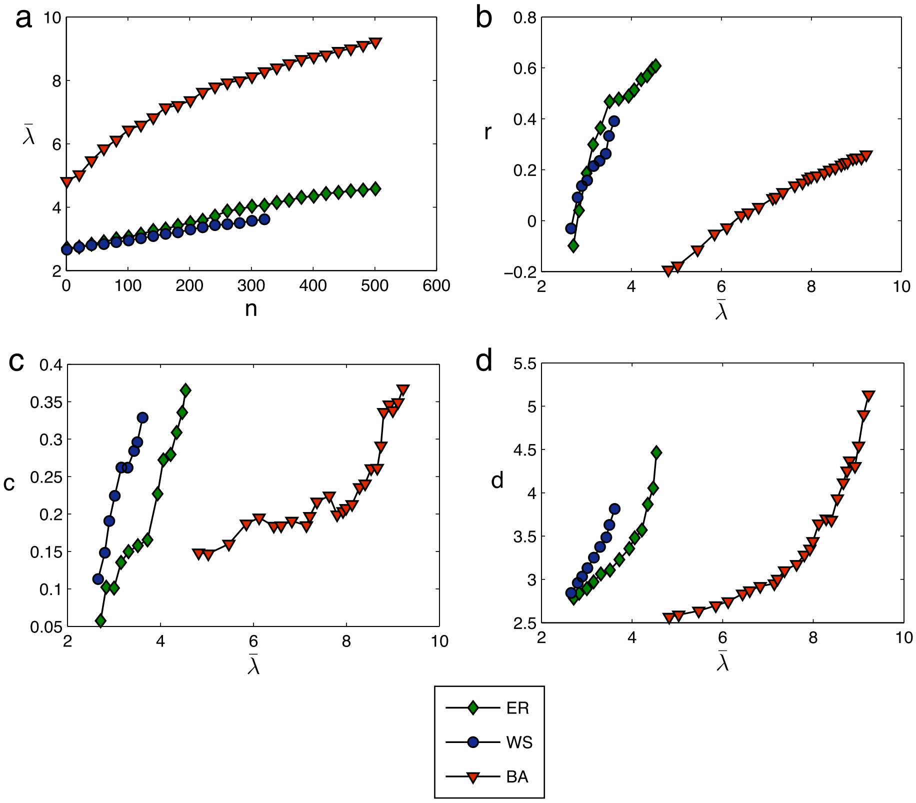

Fig. 2. (a) The fluctuation of $\bar{\lambda}$ with the iteration number n during the process of optimization for $\bar{\lambda}$ based on different types of initial network (ER random network, WS small world network and BA scale-free network); (b) The change of degree assortativity r, (c) the cluster coefficient C and (d) the average shortest path length d with the natural connectivity $\bar{\lambda}$ in the optimization, which is based on the above-mentioned networks. Both of the initial networks have the same number of nodes and links ($N = 100$ and $m = 291$).

The final topological property investigated is the average shortest path length d between each pair of nodes. The average shortest path length d generally increases during the optimization process (see Fig. 2(d)), which indicates that the increment of robustness $\bar{\lambda}$ may lead to the cost of a network efficiency.

Next, we compare the $\bar{\lambda}$-optimized network with the R-optimized network to find their similarities and differences. Table 1 shows the related numerical results. These results prove that the features of the $\bar{\lambda}$-optimized network are highly similar to the R-optimized network [11], which includes a large assortativity, a large average shortest path length and a high clustering coefficient. However, the data in Table 1 also show that optimizing R does not increase $\bar{\lambda}$ and optimizing $\bar{\lambda}$ does not always increase R either. As is known to all, the $\bar{\lambda}$-optimized network based on the BA model is confirmed to be an "onion-like" structure consisting of a core of highly connected nodes hierarchically surrounded by rings of nodes with decreasing degree [11]. We show the topologies of the $\bar{\lambda}$-optimized network and the R-optimized network which are based on different types of initial networks in Fig. 3. Both of the $\bar{\lambda}$-optimized networks have an extremely large assortativity. To obtain the maximum scaled number of closed walks of all lengths, which is considered as the redundancy of alternative routes, nodes with high degree are inclined to connect to each other to form a near "clique" structure. Other low-degree nodes prefer to attach to those nodes with higher degree, instead of nodes with similar degree. This is the primary difference between the $\bar{\lambda}$-optimized network and the R-optimized network in terms of their topological structures.

Because of the difference between the degree distributions of the ER network and the WS network, the optimizations for $\bar{\lambda}$ will lead to multiple "cliques" in their optimal networks, and these "cliques" connect to each other by passing through bits of small degree nodes. The $\bar{\lambda}$-optimized network based on the BA model exhibits a roughly "eggplant-like" topology with a cluster of high-degree nodes at its head and other low-degree nodes that are scattered across the body of the eggplant. This differs significantly from the topologies of the initial network and the onion-like network. Because of the presence of the high-degree tiny nodes in the BA scale-free network, there is only one "clique" within this "eggplant" network, and the remaining low-degree nodes prefer to attach to the nodes with high degree to form a long-strip topology. The large assortativity, the large average shortest path length and the high clustering coefficient of the $\bar{\lambda}$-optimized network has confirmed the explanations on optimal topology. It is noteworthy that the "eggplant-like" topology is something like the "extreme onion-like" topology [30], which is proposed as the optimal network topology against both failure and attack based on R. However, the "extreme onion-like" topology is constructed under a strict assumption that the number of nodes for each degree is large enough to construct a random regular graphs. The "eggplant-like" topology is obtained for general power-

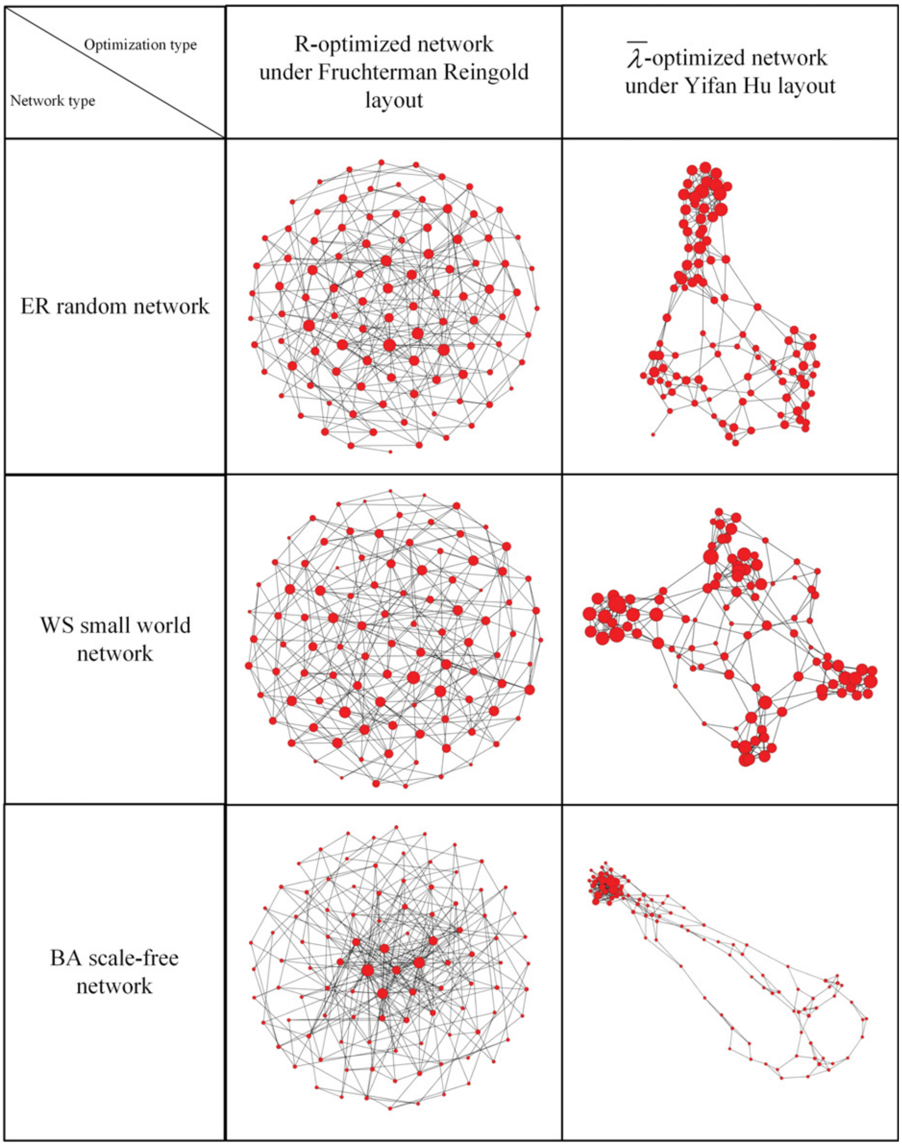

Fig. 3. The exhibition of the $\overline{\lambda}$-optimized network and the R-optimized network based on different types of initial networks. The initial networks include an ER random network, a WS small world network and a BA scale-free network. Each of the networks has the same number of nodes and links ($N = 100$, $m = 291$). Different layouts are used to show the topology of the networks clearly and without changing their connections.

law degree distribution. Thus we can consider the "extreme onion-like" topology as the extreme case of the"eggplant-like" topology.

5. Improved rewiring strategy

Often, link changes may increase costs, thus requiring a reduction in the number of changes. Considering this economical constraint, the goal of this research is to obtain a high structural robustness at a minimum number of iterations. Based on our experimental results, we find that the degree assortativity r increases monotonically with natural connectivity during the optimization process. Hence, it is inferred that if the two links are rewired to increase the assortativity of the network, then it is more likely to accelerate the constringency to achieve an acceptable optimization result. To verify this assumption, we consider three different strategies to select links for rewiring:

Assortative rewiring strategy (ARS): Preferentially select a pair of links for probability which all connect a low-degree node with a high-degree node, and rewire them to be a link with two high-degree nodes and a link with two low-degree nodes.

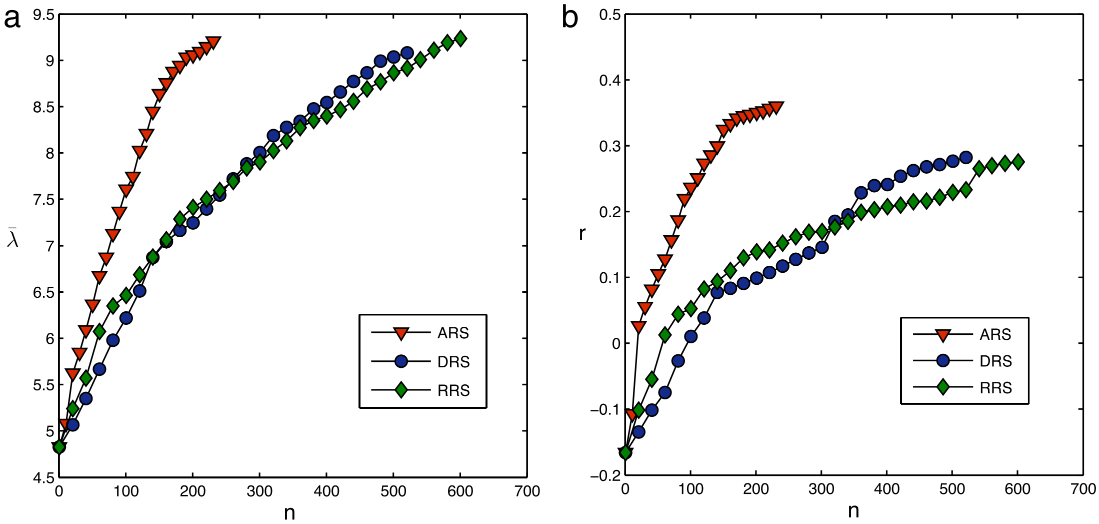

Fig. 4. (a) The robustness $\bar{\lambda}$ and (b) the degree assortativity r versus the iteration number n for BA network with 100 nodes and 291 links for different rewiring strategies, including the assortative rewiring strategy (ARS), the disassortative rewiring strategy (DRS) and the random rewiring Strategy (RRS).

Disassortative rewiring strategy (DRS): Preferentially select a link with two high-degree nodes and a link with two low-degree nodes for probability, and rewire them to be two links that all connect a low-degree node to a high-degree node.

Random rewiring strategy (RRS): Randomly select a pair of links in the network and rewire them.

When utilized, each of the rewiring strategies holds the node degrees constant during the rewiring process. By adopting the assortative rewiring strategy, the optimized network tends to be assortative. Fig. 4 shows that applying the assortative rewiring strategy can improve the natural connectivity $\bar{\lambda}$ more rapidly than applying the other two strategies. Hence, the assortative rewiring strategy can significantly reduce the number of required swaps while still obtaining the same value of structural robustness $\bar{\lambda}$.

6. Conclusions and discussions

In this paper, a new measure for the structural robustness of a networks, natural connectivity $\bar{\lambda}$, is introduced. The measure is based on the fact that the structural robustness of a network comes from the redundancy of alternate paths. Additionally, we have proposed a method, based on tabu search, to optimize the structural robustness of networks by rewiring links while keeping the node degrees constant. We have applied our method to different types of initial networks, compared the $\bar{\lambda}$-optimized network with the R-optimized network and determined their differences based on our explanations on optimization results. We found that nodes with high degree are more likely to connect to each other to form a near "clique" structure, and other low-degree nodes mainly attach to the nodes with a higher degree. In addition, the $\bar{\lambda}$-optimized network based on the BA model exhibits a roughly eggplant-like topology with a cluster of high-degree nodes at its head and other low-degree nodes that are scattered across the body of the structure (see Fig. 3). Finally, considering the economic factors that limit the number of times for rewiring links in practical applications, we have improved the algorithm presented in our research to reduce the required amount of rewiring by employing the assortative rewiring strategy.

Our results show that the optimal network has the maximum redundancy of alternative routes between each pair of nodes, which is regarded as the maximum structural robustness of the network. From a practical point of view, these results can be used to design an optimal robust topology for real-network systems or improve existing networks by rewiring a few links. Moreover, because of the ease with which $\bar{\lambda}$ can be computed, the method presented here can also be applied to large sized real-life networks, such as the Internet, the power grid and others.

Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grant Numbers 71031007 and 71371185, and the Program for New Century Excellent Talents in University under Grant Number NCET-12-0141. We thank Suo-yi Tan and Zang Yuan for useful discussions and comments.

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