On the Inverse Matrix of Near-Space Elevation Multi-Aperture SAR Imaging

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***Abstract*-This paper investigates the inverse matrix of near-space elevation multi-aperture synthetic aperture radar(SAR) wide-swath remote sensing. Note that near-space is defined as the altitude region between 20 and 100 km, which is much higher than air-borne height, and much lower than space-borne height. Besides, the flight speed of near-space aircraft is between air-borne aircraft and space-borne aircraft. This provides advantages for realizing high-resolution and wide-swath imaging, compared to air-borne and space-borne SAR. In this paper, the relationship between conditional number and system design specifications are analyzed. Simulation examples and simulation results are also provided.**

***Key words*-Synthetic aperture radar (SAR); range multi-beam; inverse matrix; condition number.**

I. INTRODUCTION

Future Synthetic aperture radar (SAR) will be required to produce high-resolution imagery over a wide area of surveillance [1]. However, minimum antenna area constraint makes it a contradiction to simultaneously obtain both unambiguous high azimuth resolution and wide-swath. The high orbit makes it possible for space-borne SAR to achieve wide-swath, but high-speed flight position makes high-resolution difficult to realize. Airborne SAR has an advantage to realize high-resolution, but its wide-swath is limited. Near-space provides a potential solution. Note that near-space is defined as the altitude region between 20 and 100 km [2] [3], which neither belongs to aerospace nor aviation. This region is much higher than air-borne height, and much lower than space-borne height. Besides, the flight speed of near-space aircraft is between air-borne aircraft and space-borne aircraft, this provides advantages for realizing high-resolution and wide-swath imaging, compared to air-borne and space-borne synthetic aperture radar[4][5][6][7].

In this paper, we investigated near-space elevation multi-aperture SAR for high-resolution and wide-swath imaging [8][9]. The key technology is to use multi-aperture to receive the echo data and then process through the inverse matrix to separate the different echoes of sub-swath. In this approach, the condition number of the inverse matrix is a key factor. If inverse matrix is ill-conditioned, an un-ignored, performance degradation will be caused due to even small errors.

The remaining sections are organized as follows. Section Ⅱ describe the imaging algorithm of the elevation multi-aperture SAR system, Section Ⅲ analyzes the ill-condition of the inverse matrix, followed by simulation and analysis in Section Ⅳ.Finally, Section V concludes the whole paper.

II. BASIC SCHEME OF RANGE MULTI-BEAM SAR

In a general SAR, the slant range is limited by[10]

$$\left(\frac{n}{\mathrm{PRF}}+\Delta T+T_p\right)\frac{c}{2}<R<\left(\frac{n+N}{\mathrm{PRF}}-T_p\right)\frac{c}{2} \tag{1}$$

Where n is an integer, N is the number of sub-swath, c is speed of light, T_P is the pulse repetition cycle, PRF is the pulse repetition frequency. Suppose the entire swath contains N sub-swaths. The corresponding slant range is

$$\left(\frac{n+i}{\mathrm{PRF}}+\Delta T+T_p\right)\frac{c}{2}<R_i<\left(\frac{n+i+1}{\mathrm{PRF}}-T_p\right)\frac{c}{2},\ 0\le i\le N-1 \tag{2}$$

Obviously, aliasing echoes of the N sub-swath signals will arrive at the antenna simultaneously. For example, at the time of τ (calculated from the starting time of transmit pulse), the collected signal is the sum of the returns from the resolution cells with slant ranges $\frac{c}{2}\left(\tau+\frac{n}{PRF}\right)$, $\frac{c}{2}\left(\tau+\frac{n+1}{PRF}\right)\ldots$, $\frac{c}{2}\left(\tau+\frac{n+N-1}{PRF}\right)$。If the return of each sub-swath can be separated, wide-swath imaging can then be obtained by subsequent signal processing algorithm.

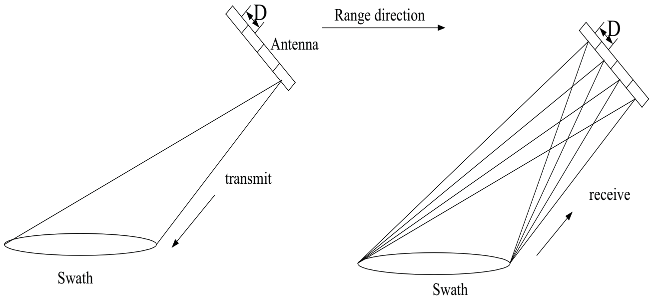

Figure1. Range multi-beam SAR systems diagram

Fig. 1 gives the imaging scheme of the elevation multi-aperture SAR system. The antenna is divided to N sub-apertures in the elevation dimension ,which are denoted by A_0, $A_1\ldots$, A_{N-1},from the top to bottom. The phase centers of the N sub-apertures are uniformly arranged by the distance D, the width of each antenna is D. The signal is transmitted from one of the sub-aperture (with N antenna can transmit), which covers the entire swath. And the returns are receives by all the sub-apertures [11].

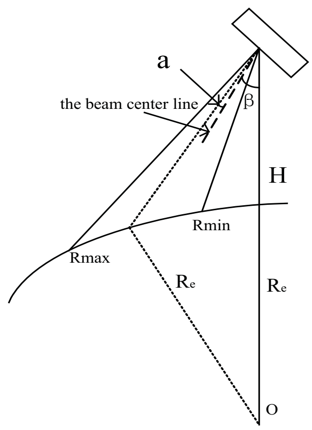

Fig. 2 the diagrammatic of the angel in elevation

Assuming that $\alpha(r)$ is the pitch angle between the resolution cell of slant range connects with the antenna and the beam center line, as shown in Fig.2.

Where the dotted line is the beam center line, O is center of the earth, H is height of the satellite, α is the angle between the slant range of the target and the beam center line, β is the angle between the beam center line and the vertical, Re is the earth radius, r is the slant range, $R\max$ is the maximum slant range, $R\min$ is the minimum slant range.

Suppose a flat earth ground from Fig.2 we can get

$$\alpha(r) = \arccos\left[\frac{r^2 + H^2 + 2HR_E}{2r\cdot(H+R_E)}\right] - \beta \tag{3}$$

Where β is the angle between the beam center line and the vertical is $\beta = (\varphi(r_{\max}) + \varphi(r_{\min}))/2$, φ is the angle between the resolution cell of slant range connects with the antenna and the vertical.

Suppose the angles between the resolution cell of N slant range connects with the antenna and antenna normal, respectively, $\alpha_0(r)$, $\alpha_1(r)$, …, $\alpha_{K-1}(r)$. The phase shifts of the first resolution cell in each sub-aperture is 0, $\frac{2\pi D\sin\alpha_0(r)}{\lambda}$, …, $\frac{2\pi(N-1)D\sin\alpha_0(r)}{\lambda}$; the phase shift of the second resolution cell in each sub-aperture is 0, $\frac{2\pi D\sin\alpha_1(r)}{\lambda}$, …, $\frac{2\pi(N-1)D\sin\alpha_1(r)}{\lambda}$; the remaining phase shifts can be obtained in a similar way[12] [13].

The inverse matrix can then be expressed as Eq.(4)

$$W(\tau) = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \exp\left(j2\pi\frac{D}{\lambda}\sin\alpha_0\right) & \exp\left(j2\pi\frac{D}{\lambda}\sin\alpha_1\right) & \cdots & \exp\left(j2\pi\frac{D}{\lambda}\sin\alpha_{N-1}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \exp\left[j2\pi\frac{(N-1)D}{\lambda}\sin\alpha_0\right] & \exp\left[j2\pi\frac{(N-1)D}{\lambda}\sin\alpha_1\right] & \cdots & \exp\left[j2\pi\frac{(N-1)D}{\lambda}\sin\alpha_{N-1}\right] \end{bmatrix} \tag{4}$$

where $\alpha_i = \alpha[(r + \frac{n+i}{PRF})\frac{c}{2}], 0 \le i \le N-1$。

The signal of each data channel after pulse compression can be represented by Eq. (5)

$$F(\tau,t) = W_1(\tau)\xi(\tau,t) \tag{5}$$

where $F(\tau,t)$ is the signal of each channel in the time τ, $\xi(\tau,t)$ is the signal of each sub-swath in the time τ, $W_1(\tau)$ is the relevant summation matrix of targets in each sub-swath.

The key technology of elevation multi-aperture SAR is the use of the inverse matrix ($W(\tau)$) to separate the aliasing echo of the N sub-swaths. If the inverse matrix $W(\tau)$ is full rank, from Eq.(6), we can obtain the signal in the time τ of each sub-swath.

$$W^{-1}(\tau)F(\tau,t) = W^{-1}(\tau)W_1(\tau)\xi(\tau,t)$$

then

$$\xi(\tau,t) = W^{-1}(\tau)F(\tau,t) \tag{6}$$

When interference signals are not existed, the summation $W_1(\tau)$ in $F(\tau,t)$ is the same as the real phase matrix $W(\tau)$ which corresponds the target of each sub-aperture. However, if the phase shift of the N groups in $W(\tau)$ is uncorrelated, we can reconstruct the N resolution cells from the aliasing returns. In this way, we can divide the signal of each sub-swath and achieve wide-swath imaging.

When the matrix $W(\tau)$ is ill-conditioned, the aliasing point can not be completely removed. Consequently it will produce interference to the target. As an example, we consider the following inverse matrix

$$\Delta\mathrm{w}(\tau) = \begin{bmatrix} A & C \\ B & D \end{bmatrix}, \quad W(\tau) = \begin{bmatrix} 1 & 1 \\ a & b \end{bmatrix}$$

then

$$W^{-1}(\tau)\cdot W_1(\tau) = \begin{bmatrix} 1 & 1 \\ a & b \end{bmatrix}^{-1} \cdot \begin{bmatrix} A & C \\ B\cdot a & D\cdot b \end{bmatrix} = \frac{1}{b-a}\begin{bmatrix} A\cdot b - B\cdot a & (C-D)\cdot b \\ (B-A)\cdot a & D\cdot b - C\cdot a \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{7}$$

In Eq.(7) $A\cdot b - B\cdot a$ and $D\cdot b - C\cdot a$ are amplification coefficient in the inverse processing operation. $(B-A)\cdot a$ and $(C-D)\cdot b$ are the interfere noise[12].

III. The Ill-condition Of The Inverse Matrix

To analyze the ill-condition of the inverse matrix, we consider the condition number is defined as Eq.(8)

$$cond[W(\tau)] = \|W(\tau)\|_2 \|W^{-1}(\tau)\|_2 = \sqrt{\frac{\lambda_{\max}}{\lambda_{\min}}} \tag{8}$$

where $\|\bullet\|_2$ is the matrix 2-norm, $\lambda_{\max}$ is the maximum eigenvalue of $W(\tau)W^H(\tau)$, $\lambda_{\min}$ is the minimum eigenvalue of $W(\tau)W^H(\tau)$. If the condition number is small, the matrix is well-conditioned. The minimum value of condition number is 1[14].

(1)When there are errors in the receiving signal $F(\tau,t)$,

$$\xi(\tau,t) + \delta\xi = W^{-1}(\tau)[F(\tau,t) + \delta F] \tag{9}$$

where δF is the error in the receiving channel and $\delta\xi$ is the signal after reserving.

From Eqs. (6) and (9) ,we can get

$$\delta\xi = W^{-1}(\tau)\delta F \tag{10}$$

So,

$$\|\delta\xi\|_2 = \|W^{-1}(\tau)\|_2 \|\delta F\|_2 \tag{11}$$

From the mathematics relation of the matrix 2-norm,we can obtain

$$\|\xi(\tau,t)\|_2 = \frac{\|W(\tau)\|_2\|\xi(\tau,t)\|_2}{\|W(\tau)\|_2} \ge \frac{\|W(\tau)\xi(\tau,t)\|_2}{\|W(\tau)\|_2} = \frac{\|F(\tau,t)\|_2}{\|W(\tau)\|_2} \tag{12}$$

Then from Eqs.(11) and (12) we can get

$$\frac{\|\delta\xi\|_2}{\|\xi(\tau,t)\|_2} \le \|W(\tau)\|_2\|W^{-1}(\tau)\|_2\frac{\|\delta F\|_2}{\|F(\tau,t)\|_2} = cond[W(\tau)]\frac{\|\delta F\|_2}{\|F(\tau,t)\|_2} \tag{13}$$

From Eq. (13) we can find that the signal error after the inverse matrix processing is related to the condition number $cond[W(\tau)]$.

(2)When there are errors in $W(\tau)$,

$$\xi(\tau,t) + \delta\xi = [W(\tau) + \delta W]^{-1}F(\tau,t) \tag{14}$$

where δW is the error of $W(\tau)$.

From Eqs. (6) and (14), we can obtain

$$W(\tau)\delta\xi + \delta W(\xi(\tau,t) + \delta\xi) = 0 \tag{15}$$

So,

$$\|\delta\xi\|_2 = \|-W^{-1}(\tau)\delta W[\xi(\tau,t) + \delta\xi]\|_2 \le \|W^{-1}(\tau)\|_2\|\delta W\|_2(\|\xi(\tau,t)\|_2 + \|\delta\xi\|_2) \tag{16}$$

Since δW is small we have

$$\|W^{-1}\|\|\delta W\| < 1 \tag{17}$$

From Eqs. (12) and (16) can then be further simplified into

$$\frac{\|\delta\xi\|_2}{\|\xi(\tau,t)\|_2} \le \frac{\|W^{-1}(\tau,t)\|\|\delta W\|}{1-\|W^{-1}(\tau,t)\|\|\delta W\|} = \frac{cond[W(\tau)]\frac{\|\delta W\|}{W(\tau)}}{1-cond[W(\tau)]\frac{\|\delta W\|}{W(\tau)}} \tag{18}$$

From Eq. (18), we notice that the signal error after reserving is related to the inverse matrix error and the condition number $cond[W(\tau)]$.

Therefore, we conclude that the signal error after reserving is related to the receiving signal error, the inverse signal error and the condition numbers of the inverse matrix. Because the errors of the receiving signals and the reserving signal can not be eliminated completely, so if the condition number can be designed as small as possible, then we can get smaller influences though there are still errors from received signals or reserving matrix.

The inverse matrix $W(\tau)$ is:

$$W(\tau) = \begin{bmatrix} 1 & 1 \\ \exp(j2\pi\frac{D}{\lambda}\sin\alpha_0) & \exp(j2\pi\frac{D}{\lambda}\sin\alpha_1) \end{bmatrix} \tag{19}$$

The conjugate transpose matrix $W^H(\tau)$ of the inverse matrix $W(\tau)$ is:

$$W^H(\tau) = \begin{bmatrix} 1 & \exp(-j2\pi\frac{D}{\lambda}\sin\alpha_0) \\ 1 & \exp(-j2\pi\frac{D}{\lambda}\sin\alpha_1) \end{bmatrix} \tag{20}$$

When there is no error, the aliasing matrix is the same as the inverse matrix.

$W_1(\tau)W^H(\tau)$ is：

$$W(\tau)W^H(\tau) = \begin{bmatrix} 2 & \exp\left(j2\pi\frac{D}{\lambda}\sin\alpha_1\right)+\exp\left(-j2\pi\frac{D}{\lambda}\sin\alpha_0\right) \\ \exp\left(-j2\pi\frac{D}{\lambda}\sin\alpha_0\right)+\exp\left(j2\pi\frac{D}{\lambda}\sin\alpha_1\right) & 2 \end{bmatrix} \tag{21}$$

The eigenvalue of Eq. (21) is

$$\lambda\lambda = 2 \pm \sqrt{2}\cdot\sqrt{1+\cos(\frac{2\pi D(\sin\alpha_0-\sin\alpha_1)}{\lambda})} \tag{22}$$

The condition number is：

$$cond[W(\tau)] = \sqrt{\frac{2+\sqrt{2}\cdot\sqrt{1+\cos(\frac{2\pi D(\sin\alpha_0-\sin\alpha_1)}{\lambda})}}{2-\sqrt{2}\cdot\sqrt{1+\cos(\frac{2\pi D(\sin\alpha_0-\sin\alpha_1)}{\lambda})}}} \tag{23}$$

where $D = \frac{0.886\cdot\lambda}{2\cdot\sin(\alpha(r_{\max})-\alpha(r_{\min}))}$, $\alpha(r) = \arccos(\frac{R^2+(H+\mathrm{Re})^2-\mathrm{Re}^2}{2\cdot R\cdot(H+\mathrm{Re})})$。

The Eq.(23) can then be further simplified into:

$$cond[W(\tau)] = \sqrt{\frac{2+\sqrt{2}\cdot\sqrt{1+\cos(\frac{0.886\pi(\sin\alpha_0-\sin\alpha_1)}{2\cdot\sin(\alpha(r_{\max})-\alpha(r_{\min}))})}}{2-\sqrt{2}\cdot\sqrt{1+\cos(\frac{0.886\pi(\sin\alpha_0-\sin\alpha_1)}{2\cdot\sin(\alpha(r_{\max})-\alpha(r_{\min}))})}}} \tag{24}$$

As the minimum value of condition number is 1 with consideration of the difficulty of design and processing, if we limit the scope of the condition number between (1~3), we can obtain Eq.(25)

$$\cos(\frac{0.886\pi(\sin\alpha_0-\sin\alpha_1)}{2\cdot\sin(\alpha(r_{\max})-\alpha(r_{\min}))}) \le 0.28 \tag{25}$$

where $\alpha(r_0) = \arccos(\frac{R^2+(H+\mathrm{Re})^2-\mathrm{Re}^2}{2\cdot R\cdot(H+\mathrm{Re})})$, $\alpha(r_1) = \arccos(\frac{(R+\Delta w)^2+(H+\mathrm{Re})^2-\mathrm{Re}^2}{2\cdot(R+\Delta w)\cdot(H+\mathrm{Re})})$, $\Delta W = \frac{c}{2\cdot PRF}$ (the width of the sub-swath).

As $\sin\alpha_0-\sin\alpha_1$ increases as R, the maximum value of Eq.(25) is $\cos(\frac{0.886\pi(\sin\alpha(r_{\max}-\Delta w)-\sin\alpha(r_{\max}))}{2\cdot\sin(\alpha(r_{\max})-\alpha(r_{\min}))})$. So, if $\cos(\frac{0.886\pi(\sin\alpha(r_{\max}-\Delta w)-\sin\alpha(r_{\max}))}{2\cdot\sin(\alpha(r_{\max})-\alpha(r_{\min}))}) \le 0.28$ is satisfied the condition numbers of swath are in the range (1~3).

IV. SIMULATION RESULTS

Figure 3 shows the condition number in different platform height and the same swath bandwidth with the following parameters: carrier frequency is f=5.3GHZ, PRF=2560HZ, N=2, sub-swath width ΔW =58.594km. Note that the x-axis $x = \mathrm{PRF}\,\tau$ is the normalized receive time:

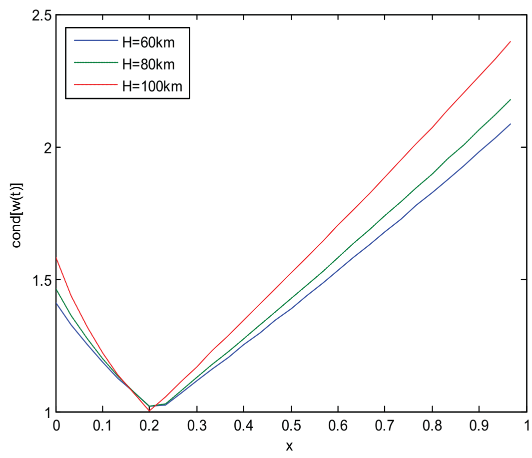

Fig.3 The condition number with the same swath bandwidth at different platform height

In Fig.3, the curves are corresponding to 60km, 80km and 100km respectively from bottom to top, the swath area is between 117.2km to 234.49km, the intervals between the sub-aperture are 0.8265, 0.5679 and 0.3941 respectively. From Fig.3, we can obtain that if the platform height is higher, the condition number will be bigger observed from the same swath area.

Figure 4 shows the condition number with different slant ranges of swath center at the same swath bandwidth with the parameters; platform height H=50km, wavelength is 0.057, PRF=3000HZ, sub-swath width ΔW =50km, N=2.

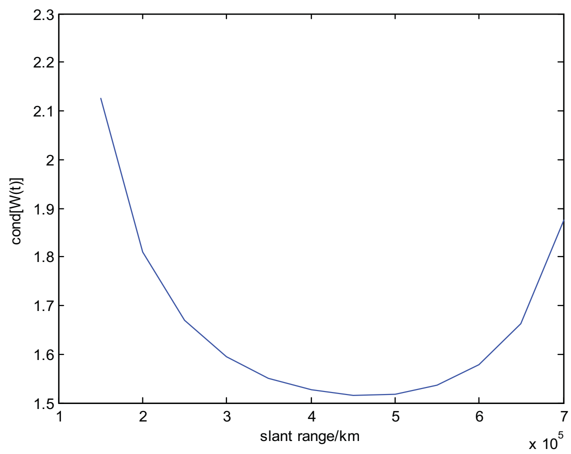

Fig.4 The condition number with the slant range of swath center increases

From Fig.4, we can conclude that, with the slant range of swath center increases, the condition number is a parabola which is opening up, so in the observation with the same size of swath, we can adjust the distance between swath and platform to make the condition number be better.

Figure 5 shows the condition number with the different number of sub-swath with the following parameters; H=20km, wavelength is 0.057, PRF=3000HZ, ΔW =50km. The number of sub-aperture and sub-swath are 2, 3, 4 respectively from bottom to top. The swath areas are respectively (50km ~150km), (50km ~200km), (50km~250km).

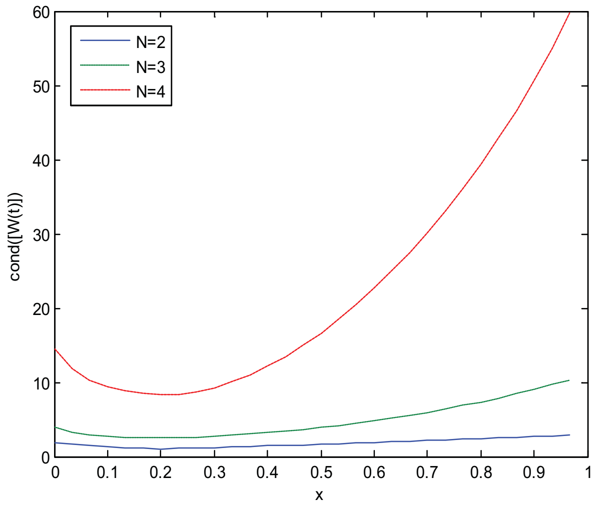

Fig.5 The condition number with the same platform height and different number of sub-swath

From Fig.5, we can conclude that the more sub- swath is the bigger condition number at the same height platform.

V. CONCLUSION

This paper investigated the inverse matrix of the elevation multi-aperture SAR for wide-swath imaging. As the important factor of the inverse matrix is its condition number, a detailed analysis and simulation have performed to show the relationship among condition number and platform height, center distance of swath, number of sub-swath. This work can provide a reference to make a better performance on anti-interference for system design.

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