

# Changing the Intensity of Interaction Based on Individual Behavior in the Iterated Prisoner's Dilemma Game

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**Abstract**—We present a model of changing the intensity of interaction based on the individual behavior to study the iterated prisoner's dilemma game in social networks. In this model, each individual has an assessed score of reputation which is obtained by considering the evaluation level of interactive partners for its present behavior. We focus on the effect of evaluation level on the changing intensity of interaction between individuals. For an individual with good behavior, the higher the evaluation level of its partners for its good behavior, the better its reputation, and the higher the probability of surrounding partners interaction with it. On the contrary, for an individual with bad behavior, the lower the evaluation level of its partners for its bad behavior, the worse its reputation, and the less the probability of surrounding neighbors interaction with it. Simulation results show that this effective mechanism can drastically facilitate the emergence and maintenance of cooperation in the population under a treacherous chip. Interestingly, for a small or moderate treacherous chip, the cooperation level monotonously ascends as the evaluation level increases; however, for a higher treacherous chip, existing an optimal evaluation level, which can result in the best promotion of cooperation. Furthermore, we find better agreement between simulation results and theoretical predictions obtained from an extended pair-approximation method, although there are some tiny deviations. We also show some typical snapshots of the system and investigate the reason for appearance and persistence of cooperation. The results further show the importance of evaluation level of individual behavior in coevolutionary relationships.

**Index Terms**—Evolutionary game theory, prisoner's dilemma, intensity of interaction, cooperation, spatial structure.

## I. INTRODUCTION

MUTUAL cooperation is ubiquitously observed in human society, as well as other animal species. How to understand and analyze the widespread cooperative behaviors among selfish individuals has become an active topic in recent decades [1], [2], [3]. Fortunately, evolutionary game theory, by using social dilemmas models, has been confirmed to be the most effective approaches to investigate this problem [4], [5], [6], [7], [8], [9]. In particular, the iterated prisoner's dilemma game (PDG) is often selected to explore the evolution of cooperation between pairwise individuals since it represents the

most adverse situation to cooperation [10], and for this reason, it also become the typical representative of describing the conflict of interest between the individual and the collective.

To understand the evolution of cooperation in social and biological systems, a large number of scholars pay the price for it. Noteworthy, Nowak and May, who are the foregoers among numerous scholars in resolving social dilemmas, showed that spatial structure may spring up and maintain cooperative behavior in the iterated PDG [11]. Subsequently, Nowak summarized five rules for the evolution of cooperation including kin selection, direct and indirect reciprocity, group interaction, and networking reciprocity [2], [12], [13], [14]. Very importantly, the combination of original evolutionary games and graph theory provides an extended framework to investigate the cooperation behavior in social systems. For example, many real systems display the characteristic small-world effect and scale-free properties which are found to highly promote the cooperation among individuals on networking communities. Besides, a variety of mechanisms have also been put forward to promote the cooperative behaviors between individuals, such as time scale [15], [16], [17], individual selection [18], [19], teaching activity [20], individual rationality [21], individual aspiration [22], environmental noise [23], asymmetric payoff [24], multilevel selection [25] and multiplayer groups [26], etc. In recent years, the coevolution of cooperative dynamics and interaction patterns has been aroused great interest of the scholars [9], [27], especially involving the transference of interactive relationship, birth and death process and migration [28], [29], [30], [31], [32], [33], [34]. It is also worth mentioning that interdependent or multiplexing network becomes a novel platform for us to account for the emergency of cooperation through the network reciprocity [35], [36]. These interesting works provide various valuable clues to understand the emergence and maintenance of cooperation within the structured populations.

However, we would like to point out that, to our knowledge, all these works of evolutionary games on networks mostly focus on full interaction with their nearest neighbors. In other words, most previous studies of games on graphs based on a common simplifying assumption that players always interact with all of their neighbors with sufficient interaction strength during the evolutionary stage.

Actually, in the real society, not all individuals always interact with all of their neighbors, but under certain conditions this kind of interaction relations are efficient. Remarkably, in

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Ref. [37] Traulsen et al. modified the setting that individuals absolutely interact with their neighbors, and first studied the case of finite well-mixed populations in which each pair of individuals interact with a probability, leading to different numbers of interactions each individual. Moreover, in Ref. [38] authors also studied the effects of random interaction through a fixed intensity of interaction on the evolution of cooperation in spatial repeated PDG, and found there was an optimal region of interaction strength resulting in maximum cooperation level.

Notably, in real lives the intensity of interaction between individuals is also not set in stone, which will be likely changed as time goes on. It is worth noting that the reputation can be used to help individuals recognize *good and bad guys* and has been used as an important factor to help individuals adjust their partnerships or carry out the selective interactions [19], [31], [39]. It is well-known that an individual's good behavior can increase its reputation. Nevertheless, an individual's bad behavior can also reduce its reputation, not just the good behavior can increase the reputation, partially inspired by the work of Nowak and Sigmund [12], [40]. More importantly, the reputation of an individual should also consider the evaluation level of its actual interaction partners to its behavior. For example, one day a friend of mine did a good thing for public, we probably showed such as good, very good, wonderful et al. this kind of different levels of terms to praise his (her) good behavior.

In view of the above-mentioned facts, presently we also abandon the assumption of deterministic interactions and investigate the effects of changing the intensity of interaction by considering evaluation level of individual behavior in spatial iterated PDG. We assume that each player engages in pairwise interaction according to an intensity of interaction  $W_{x,y}(t)$ .  $W_{x,y}(t)$  denotes the possibility that player  $x$  and one of its neighbors  $y$  participate in game at time  $t$  step. Namely, at time  $t$  step, whether  $x$  and  $y$  will attend interaction depends on the value of  $W_{x,y}(t)$ . The bigger the value of  $W_{x,y}(t)$ , the higher the likelihood of interaction and vice versa. In our model, the values of  $W_{x,y}(t)$  likely are gradually changed with evolution time owing to the dynamically changing of individual reputation. Interestingly, by using Monte Carlo simulations we demonstrate that this mechanism can promote cooperation in the population.

The rest of this paper is organized as follows. In Section II, we briefly introduce some background knowledge including prisoner's dilemma game, reputation by considering the evaluation level of individual behavior, and the formation of the intensity of interaction. In Section III, we describe our model, next present the results and analysis. Finally, conclusions are drawn in Section V.

## II. BACKGROUND KNOWLEDGE

### A. Prisoner's dilemma game

The prisoner's dilemma game presents a story of two prisoners being interrogated in isolation. A confession from one and silence from the other would result in a heavy sentence for the latter while the former walks free. However, if both prisoners remain silent, each would get a lighter sentence.

Under the circumstance whereby both confess, they would receive a sentence slightly less severe than the heavy one.

In this primitive PDG version, two players simultaneously decide whether to cooperate (remain silent) or to defect (confess). Mutual cooperation leads to the reward ( $R$ ), and mutual defection results in the punishment ( $P$ ). While a cooperator receives the sucker's payoff ( $S$ ) when confronted to a defector, which in turn receives the temptation to defect ( $T$ ). These payoffs obey the rankings:  $T > R > P > S$ , and  $2R > T + S$ . Evidently, for one-shot PDG defection is unbeatable and thus preferred by rational players, although they can realize that mutual cooperation obtains higher payoff than mutual defection [41], [42].

At the most elementary level, the above scenario is one-off. However, in real lives individuals often repeatedly interact with the surrounding neighbors. Therefore, by extending the one-off game into a repeated form, the iterated PDG can be formalized. In general, we denote by  $s_x$  the strategy of player  $x$  and it can follow two simple strategies,  $s_x(t) = [1, 0]^T$  corresponds to play cooperation ( $C$ ), and  $s_x(t) = [0, 1]^T$  corresponds to play defection ( $D$ ) at time  $t$  step.

### B. Formation of the intensity of interaction

In the theory of probability, for two independent events  $A$  and  $B$ , if the probability of events  $A$  and  $B$  occurrence are  $P(A)$  and  $P(B)$ , respectively, then the probability of two events simultaneous occurrence is equal to the product of their probabilities, that is,  $P(AB) = P(A)P(B)$ . Based on the basic principle, for paired player  $x$  and  $y$ , we denote by  $w_{x \rightarrow y}(t)$  [ $w_{y \rightarrow x}(t)$ ] the probability of interaction from player  $x$  ( $y$ ) to player  $y$  ( $x$ ) at time  $t$  step. Initially, the  $w_{x \rightarrow y}(t)$  [ $w_{y \rightarrow x}(t)$ ] is assigned a score randomly chosen in the interval  $(0, 1)$ , that is,  $w_{x \rightarrow y}(0) \in (0, 1)$ ,  $w_{y \rightarrow x}(0) \in (0, 1)$ , owing to the independence of unilateral intention interaction, the initial intensity of interaction between  $x$  and  $y$  is

$$W_{x,y}(0) = w_{x \rightarrow y}(0) \cdot w_{y \rightarrow x}(0). \quad (1)$$

Once the system is running, the intensity of interaction between them probably gradually updates. We assume the intensity of interaction at time  $t$  step ( $t \geq 1$ ) as follows,

$$W_{x,y}(t) = w_{x \rightarrow y}(t) \cdot w_{y \rightarrow x}(t). \quad (2)$$

After interaction, player  $x$  ( $y$ ) probably will unilaterally revise the intensity of interaction from  $x$  ( $y$ ) to  $y$  ( $x$ ), the variable quantity of intensity of interaction is denoted by  $\Delta w_x$  ( $\Delta w_y$ ). Accordingly, at time  $(t+1)$  step the intensity of interaction between them is as follows (here assumption  $\Delta w_x$  is increment quantity and  $\Delta w_y$  is decrease one),

$$\begin{aligned} W_{x,y}(t+1) &= w_{x \rightarrow y}(t+1) \cdot w_{y \rightarrow x}(t+1) \\ &= [w_{x \rightarrow y}(t) + \Delta w_x] \cdot [w_{y \rightarrow x}(t) - \Delta w_y]. \end{aligned} \quad (3)$$

### C. Reputation of individual

We first introduce the concept of effective neighbor.

**Definition 1. Effective Neighbor** In the process of random interaction, an individual (say  $x$ ) interact with all its neighbors according to the corresponding intensity of interaction between

it and its partners. Thus, individual  $x$  possibly can successfully interact with some neighbors (if any), but the remainder interactions are failing. In this case, we regard the neighbors who actually participate in game with  $x$  among player  $x$ 's partners as the effective neighbors of  $x$ .

Then we present the scrollable assessed value of player  $x$ 's reputation at time  $t$  step, which is determined by the following utility function,

$$R_x(t) = 0.5R_x(t-1) + 0.5sgn[s_x(t)]\frac{N_x(t)}{k_x}\phi, \quad (4)$$

in the above equation,

$$sgn[s_x(t)] = \begin{cases} 1 & s_x(t) = C \\ -1 & s_x(t) = D \end{cases}, \quad 0 \leq \phi \leq 1$$

Just as discussed in the Introduction, where  $\phi$  characterises the evaluation level of individual behavior. For  $\phi=0$ , the assessment method of reputation has completely lost efficacy, and the value of player  $x$ 's reputation stepwise varies from the initial value and gradually approach zero as time evolution. According to the rules (6) (in what follows) we can know this kind of change of reputation simply do not work anymore for the adjustment of the intensity of interaction between individuals. In other words, it will degenerate to the classical random interaction. For  $0 < \phi < 1$ , the bigger the value of  $\phi$ , the higher the distinguishing extent for player  $x$ 's behavior. For  $\phi=1$ ,  $x$ 's interaction neighbors fully broadcast the long and short of  $x$ 's behavior, i.e., the interaction partners of player  $x$  unexpectedly objectively evaluate its behavior, whether it is good or bad.  $N_x(t)$  means the number of player  $x$ 's effective neighbors at time  $t$  step. And  $k_x$  denotes the number of all  $x$ 's neighbors (i.e., the degree of node  $x$ ; here the aim of divide by  $k_x$  is the normalization processing).

The implication of equation (4) is twofold: first, an individual's reputation could be a continuous variable, which should depend on the previous and current performances; second, there should be a distinction of good or bad for an individual's reputation, and the reputation score is closely related to the evaluation level of the effective neighbor for the individual's behavior. In real lives, most of us have been struck with the thought that if the stand or fall of an enterprise's product is determined by consumers' word of mouth, the good or bad of an individual's reputation should be evaluated by his partners. For example, a man who has a good behavior (e.g., from a selfish standpoint, the cooperation is a good behavior because the surrounding neighbor can obtain interest by interaction with cooperator.) in his friends circle, the higher the evaluation of his friends for its good behavior, the better his reputation. Conversely, for a man who has bad behavior (e.g., the defection is a bad behavior because the surrounding partner probably cannot obtain even lose benefit by interaction with defector.) in his friends, the worse the evaluation of his partners for its bad behavior, the worse his reputation.

### III. MODEL

We consider the iterated PDG on a  $L \times L$  square lattice with periodic boundary conditions, each player engages in

pairwise interactions within its von Neumann neighborhood, the vertices of dynamical graph represent players, the edges and their weights denote the pairwise partnership and the intensity of interaction between players and their neighbors respectively. Following common practice [11], we adopt the rescaled payoff matrix  $M$  depending on one single parameter  $b$ :  $T=b>1$ ,  $R=1$ , and  $P=S=0$ .

#### A. Framework of model

In this context, the game occurs at discrete time  $t$  and each step is divided into three stages.

##### Stage 1 stochastic interaction

Each player  $x$  engages in pairwise interactions with the corresponding intensity of interaction  $W_{x,y}(t)$  and using the same strategy  $s_x(t)$  within its neighbors, and collects an aggregate payoff,

$$U_x(t) = \sum_{y \in \Omega_x(t)} s_x^T(t) M s_y(t), \quad (5)$$

where  $\Omega_x(t)$  represents the set of effective neighbors of player  $x$ . Notably,  $\Omega_x(t)$  is likely to dynamically changed as time goes by. And  $M$  is the payoff matrix.

##### Stage 2 revise the intensity of interaction

After playing the game, each individual's reputation score needs to be updated according to the equation (4). Besides, the intensity of interaction between player  $x$  and its partner  $y$  is also likely to be adjusted.

Whether the intensity of interaction from player  $x$  ( $y$ ) to its partner  $y$  ( $x$ ) will be unilaterally varied is determined by player  $y$ 's ( $x$ 's) reputation information. At time  $t$  step, if player  $y$  has good reputation [ $R_y(t) > 0$ ], and its present reputation score higher than the previous one [ $R_y(t) > R_y(t-1)$ ],  $x$  will unilaterally increase the intensity of interaction from it to  $y$ , and the value of increment depends on the variable quantity of  $y$ 's reputation [ $\Delta R_y(t-1, t) = R_y(t) - R_y(t-1)$ ] for the reason that: the better the reputation of player  $y$ , the higher the possibility of surrounding neighbors interaction with it. Nevertheless, player  $x$  will unilaterally reduce the intensity of interaction from it to  $y$  because player  $y$  has bad reputation ( $R_y(t) < 0$ ) at present, and its reputation score less than previous one [ $R_y(t) < R_y(t-1)$ ]. More explicitly, at time  $(t+1)$  step, the intensity of interaction [ $w_{x \rightarrow y}(t+1)$ ] from player  $x$  to its partner  $y$  is as follows,

$$\begin{cases} w_{x \rightarrow y}(t) + \Delta w_x & \text{if } R_y(t) > 0 \text{ and } R_y(t) > R_y(t-1) \\ w_{x \rightarrow y}(t) - \Delta w_x & \text{if } R_y(t) < 0 \text{ and } R_y(t) < R_y(t-1) \end{cases} \quad (6)$$

In other cases, the intensity of interaction remain unchanged.

Similarly, player  $y$  will unilaterally change the intensity of interaction from it to its partner  $x$  according to the foregoing ways. Notably, during the evolution process in order to prevent the intensity of interaction from less than or equal to zero between players, we set the smallest intensity of interaction equal to 0.0001, and the maximum value equal to 1. Thus the continuous score of  $W_{x,y}(t)$  varies between 0 and 1.

##### Stage 3 Update strategy

In the evolutionary games all individuals are allowed to adopt the strategies of their neighbors after each round. We adopt the Fermi rule [43], that is, the individual  $x$  randomly

selects a neighbor  $y$  and adopts  $y$ 's strategy with the probability determined by the total payoff difference between them,

$$f(U_y - U_x) = \frac{1}{1 + \exp[-(U_y - U_x)/K]}, \quad (7)$$

where  $K$  quantifies the uncertainty related to the strategy adoption process.

For simplicity, we assume that each individual has the same evaluation level in this work, and mainly focus on how the evaluation level ( $\phi$ ) of individual behavior affects the evolution of cooperation in spatial PDG.

### B. Method of adjusting $\Delta w_x$

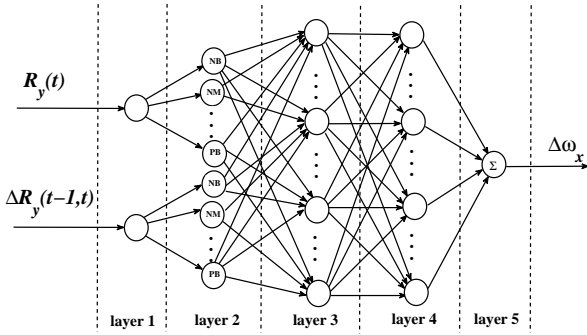


Fig. 1. Schematic diagram of a fuzzy neural network. The player  $x$  adaptively adjusts the variable quantity of the intensity of interaction from it to its neighbor  $y$  according to  $y$ 's reputation information. From left to right, layer 1 is input layer, followed by the membership layer, the fuzzy rules layer, the normalized processing layer and the output layer. The neuron number of each layer is 2, 12, 36, 36, 1, respectively.

In order to reflect the ability of individuals' adaptive feature, we use the control method of fuzzy neural network (abbreviated as FNN) [44], [45] to adaptively adjust the variable quantity of intensity of interaction. Because an individual's reputation and the variation of reputation they both belong to fuzzy language (Fuzzy language is a language which has a certain vagueness. For example, an individual's reputation perhaps is one of these terms such as worst, worse, bad, good, better and best, and they all have the character of fuzziness). Therefore, we regard the reputation score  $[R_y(t)]$  of a player  $y$  and the variable quantity  $[\Delta R_y(t-1, t)]$  of its reputation as the input vector of FNN, the variable quantity ( $\Delta w_x$ ) of intensity of interaction as the output variable of FNN, and combine with the rules (6) to compute the value of  $\Delta w_x$ . In addition, in FNN, we set the range of variable quantity of intensity of interaction  $\Delta w_x \in [-0.05, 0.05]$  because the change of intensity of interaction is gradually and faintly varied. The fuzzy neural network consists of five layers (see Fig.1). First, we introduce the function of each layer.

#### Layer 1: input layer

Each node of input layer respectively connects with each component of input vector  $R_i$ , and the total number of nodes (denote by  $N_1$ ) of this layer equal to the dimension of input vector. Notably, the nodes in this layer just transmit input signals to the next layer directly, that is,

$$I_i^{(1)} = R_i, \quad O_i^1 = I_i^{(1)}. \quad (i = 1, 2, \dots, N_1) \quad (8)$$

Obviously, in our model,  $N_1 = 2$ ,

$$\begin{bmatrix} R_y(t) \\ \Delta R_y(t-1, t) \end{bmatrix} = \begin{bmatrix} R_y(t) \\ R_y(t) - R_y(t-1) \end{bmatrix}. \quad (9)$$

#### Layer 2: membership layer

In this layer, we regard a single node as a simple membership function, thus the output of the node is the degree of membership which means a signal component is subject to the extent of corresponding fuzzy language set. Here, we choose a bell-shaped function,

$$I_{ij}^{(2)} = -\frac{(x_i - a_{ij})^2}{b_{ij}^2}, \quad O_{ij}^{(2)} = A_{ij} = \exp(I_{ij}^{(2)}), \quad (10)$$

where  $i = 1, 2, \dots, N_1$ ;  $j = 1, 2, \dots, m_i$ ;  $a_{ij}$  and  $b_{ij}$  is respectively the center and the width of the bell-shaped function of the  $j$ th term of the  $i$ th input variable  $R_i$ . The number of all the nodes of this layer  $N_2 = \sum_{i=1}^n m_i$ , where  $m_i$  denotes the number of fuzzy subsets of signal component. By the previously mentioned, we can know the reputation of the player  $y$  at time  $t$  step  $R_y(t) = R_1 \in [-1, 1]$ , and the variable quantity of its reputation between the  $t$ th step and the  $(t-1)$ th step  $\Delta R_y(t-1, t) = R_2 \in [\min\{R_y(t)\} - \max\{R_y(t-1)\}, \max\{R_y(t)\} - \min\{R_y(t-1)\}] = [-2, 2]$ . We equally divide the interval of each input component  $R_i$  ( $i=1, 2$ ) into six fuzzy classes. Accordingly, there are six fuzzy language variables from NB to PB, as shown in Fig.2. Here, it is necessary for us to explain the fuzzy variables. For the plus-minus sign of  $R_y(t)$ , we use the letter N to denote negative and P to represent positive; For the degree of membership of  $R_y(t)$ 's amplitude, we use the letters S, M and B to denote small, middle and big, respectively. In this case, there are six fuzzy sets totally consist of NB, NM, NS, PS, PM and PB. For example, NB denotes the fuzzy set of negative big. The same expressive methods for  $\Delta R_y(t-1, t)$ . Therefore, the number of membership functions  $m_1 = m_2 = C_2^1 C_3^1 = 6$ , and there are twelve nodes in this layer ( $N_2 = m_1 + m_2 = 12$ ).

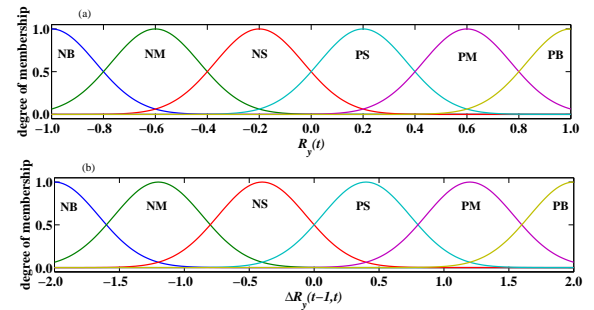


Fig. 2. The distribution of fuzzy subsets' membership degree for each input component. (a) Top panel: the reputation of player  $y$  (denoted by  $R_y(t)$ , and  $R_y(t) \in [-1, 1]$ ); (b) Bottom panel: the reputation's the variable quantity of player  $y$  (denoted by  $\Delta R_y(t-1, t)$ , and  $\Delta R_y(t-1, t) \in [-2, 2]$ ). The interval of each input component is equally divided into six fuzzy sets which denoted by six fuzzy language variables. From left to right within each panels, these sets are NB, NM, NS, PS, PM and PB, respectively.

#### Layer 3: fuzzy rules layer

Every node represents a fuzzy rule, and the rule node perform the fuzzy "and operation" because the links in this

layer are used to perform precondition matching of fuzzy logic rules. Therefore, the number of nodes of this layer  $N_3 = m = \prod_{i=1}^{N_1} m_{N_1}$ , the fitness of fuzzy rule

$$\alpha_l = \min\{A_{1j_1}, A_{2j_2}, \dots, A_{N_1j_{N_1}}\}, \quad (11)$$

where  $j_{N_1} \in \{1, 2, \dots, m_{N_1}\}$ , in our system,  $j_1 \in \{1, 2, \dots, 6\}$ ,  $j_2 \in \{1, 2, \dots, 6\}$ . Correspondingly,

$$I_l^{(3)} = \alpha_l, \quad O_l^{(3)} = I_l^{(3)}. \quad (12)$$

Layer 4: normalized processing layer

The number of nodes of this layer equal to the one of layer 3, that is,  $N_4 = N_3$ . The function of this layer is the normalized calculation,

$$\bar{\alpha}_l = \frac{\alpha_l}{\sum_{i=1}^m \alpha_i}. \quad (l = 1, 2, \dots, m) \quad (13)$$

Layer 5: output layer

The function of this layer is the defuzzification, and there is just one node, that is,  $N_5=1$ .

$$\Delta w_x = \sum_{j=1}^m v_{ij} \bar{\alpha}_j, \quad (i = 1, 2, \dots, r) \quad (14)$$

where  $r$  denotes the number of output nodes,  $v_{ij}$  represents the weight values between layer 4 and layer 5.

Next, we describe the learning algorithm of fuzzy neural network and the methods for obtaining the necessary spatial sample.

According to the above description, the main parameters are respectively  $a_{ij}$ ,  $b_{ij}$  and  $v_{ij}$ . By the error cost function

$$E = \frac{1}{2} \sum_{i=1}^r (t_x - \Delta w_x)^2, \quad (15)$$

where  $t_x$  is the value of expectation, and based on erroneous reversed dissemination method, we can easily obtain

$$\begin{cases} \frac{\partial E}{\partial v_{ij}} = -(t_x - \Delta w_x) \bar{\alpha}_i \\ \frac{\partial E}{\partial a_{ij}} = \frac{\partial E}{\partial I_{ij}^{(2)}} \frac{\partial I_{ij}^{(2)}}{\partial a_{ij}} \\ \frac{\partial E}{\partial b_{ij}} = \frac{\partial E}{\partial I_{ij}^{(2)}} \frac{\partial I_{ij}^{(2)}}{\partial b_{ij}} \end{cases} \quad (16)$$

So, the learning algorithm of the parameter adjustment is as follows

$$v_{ij}(k+1) = v_{ij}(k) - \beta \frac{\partial E}{\partial v_{ij}}, \quad (17)$$

$$a_{ij}(k+1) = a_{ij}(k) - \beta \frac{\partial E}{\partial a_{ij}}, \quad (18)$$

$$b_{ij}(k+1) = b_{ij}(k) - \beta \frac{\partial E}{\partial b_{ij}}, \quad (19)$$

where  $\beta$  denotes the learning efficiency. We equably divide the interval of output variable ( $\Delta w_x \in [-0.05, 0.05]$ ) into six fuzzy classes before defuzzification, accordingly, there are six fuzzy

		, and $\Delta R_y(t-1, t)$ is					
		NB	NM	NS	PS	PM	PB
if $R_y(t)$ is	NB	NB	NM	NS	$\emptyset$	$\emptyset$	$\emptyset$
	NM	NB	NM	NS	$\emptyset$	$\emptyset$	$\emptyset$
	NS	NB	NM	NS	$\emptyset$	$\emptyset$	$\emptyset$
	PS	$\emptyset$	$\emptyset$	$\emptyset$	PS	PM	PB
	PM	$\emptyset$	$\emptyset$	$\emptyset$	PS	PM	PB
	PB	$\emptyset$	$\emptyset$	$\emptyset$	PS	PM	PB

Fig. 3. The fuzzy control rules. According to an individual reputation and the variable quantity of its reputation adaptively adjust the variable quantity of intensity of interaction. For  $R_y(t) > 0$ , the higher the value of  $\Delta R_y(t-1, t)$ , the higher the value of  $\Delta w_x$ ; For  $R_y(t) < 0$ , the higher the value of  $|\Delta R_y(t-1, t)|$ , the higher the value of  $|\Delta w_x|$ . Here,  $|\Delta R_y(t-1, t)|$  denotes the absolute value of  $\Delta R_y(t-1, t)$ , and the same for  $|\Delta w_x|$ . The symbol  $\emptyset$  denotes the counterpart rules lose efficacy, once this happens,  $\Delta R_x$  identically equal to zero.

language variables from NB to PB. According to the unintended adjustment principle about interpersonal relationship in the real lives and combine with our model, we provide the fuzzy control rules, as shown in Fig.3. Experiments have shown that some training (test) samples (so long as basically meet the requirements of these rules) which can commendably train the fuzzy neural network. In other words, the main parameters of membership functions ( $a_{ij}$ ,  $b_{ij}$ ) and the weight values ( $v_{ij}$ ) can be automatically adjusted. Once the process of training network is finished, we can test network by some testing samples, if the testing results meet the demands of our model, we can utilize it as the instrument of automatically changing intensity of interaction.

#### IV. RESULTS AND ANALYSIS

Simulations are carried out on a square lattice of size  $100 \times 100$ . Initially, each player  $x$  is designated to play either  $C$  or  $D$  with equal probability, and the player  $x$ 's reputation score is randomly distributed within the interval  $[-1, 1]$ , that is,  $R_x(0) \in [-1, 1]$ . For  $R_x(0) < 0$ , it represents the individual  $x$  with bad reputation at time  $t=0$  step; for  $R_x(0) > 0$ , it denotes  $x$  with good reputation; specifically, for  $R_x(0)=0$ , it means  $x$ 's reputation is neutral, neither better nor worse. In our study, we implement this computational model with synchronous update, and compute the fraction of cooperators by averaging over the last  $2 \times 10^3$  generations of the entire  $2 \times 10^4$  generations. Moreover, the final results are averaged over up to 100 independent runs for each set of parameters to eliminate the influence of some uncertainties.

##### A. Influence of the temptation to defect $b$

In keeping with tradition, we first study the fraction of cooperators  $f_c$  as a function of  $b$  for different values of  $\phi$ , as shown in Fig.4. We can see that, compared to the traditional version of the iterated PDG ( $\phi=0$ , that is, the intensity of interaction is unchanging values which are the initial ones before system runs), the fraction of cooperators  $f_c$  is largely enhanced even can reach an absorbing state when  $\phi > 0$ . For



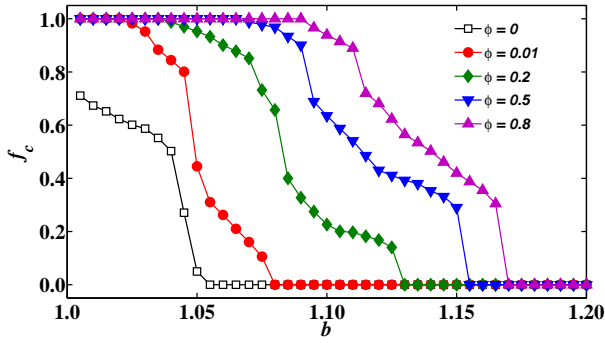


Fig. 4. The fraction of cooperators  $f_c$  as a function of the temptation to defect  $b$  for different values of  $\phi$ . From left to right, the iterated prisoner's dilemma game is carried out on a square lattice with periodic boundary conditions for the traditional version,  $\phi=0$  is the traditional version,  $\phi>0$  is our mode). When the evaluation level  $\phi$  is slightly increased, the fraction of cooperators  $f_c$  will increase to a higher value, and the extinction threshold of cooperation is extended correspondingly. With relevant parameters  $L=100$ ,  $K=0.1$ .

example, when the evaluation level  $\phi$  is slightly increased to 0.01, the fraction of cooperators  $f_c$  will increase to a higher value. Furthermore, the extinction threshold of cooperation is extended to 1.078 (for  $\phi=0.01$ ), whereas the threshold is only about 1.052 in the traditional case. Subsequently, as the  $\phi$  gradually increases, it is shown that the extinction threshold is also largely augmented.

### B. Role of evaluation level $\phi$

1) *Effect of evaluation level and verification:* In order to quantify the role of changing the intensity of interaction on the basis of individual behavior in promoting cooperation more precisely, we then study the dependence of the cooperation level  $f_c$  on  $\phi$  for different values of  $b$  (see Fig.5). On the

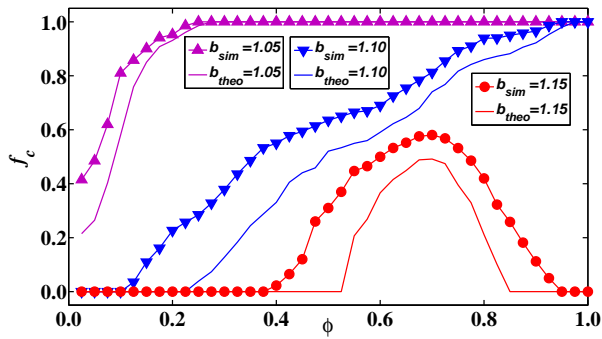


Fig. 5. The fraction of cooperators  $f_c$  as a function of the evaluation level  $\phi$  for different values of  $b$ . With the marked lines are the simulation results, and the lines of corresponding color are the theoretical analysis by pair approximation. For a small or moderate temptation to defect  $b$ , the cooperation level monotonously ascends as the evaluation level increases; however, for a higher temptation to defect  $b$ , existing an optimal evaluation level, which can result in the best promotion of cooperation. Besides, there is a good agreement between simulation results and theoretical predictions obtained from an extended pair-approximation method, although there are some tiny deviations. From left to right, the temptation to defect  $b = 1.05, 1.10, 1.15$ , respectively. With relevant parameters  $L=100$ ,  $K=0.1$ .

whole, we can see that the cooperation level exhibits diverse tendencies as the value of  $\phi$  increases, for different values

of  $b$ . For example, when the value of  $b$  is small ( $b=1.05$ ) or moderate ( $b=1.10$ ), with the value of  $\phi$  increases, the cooperation level monotonously ascends, and even the cooperators can dominate the whole population after the value of  $\phi$  is beyond a certain critical value. That is, there exist the smallest  $\phi$ , resulting in a plateau of high cooperation level for each fixed  $b$  ( $1.0 < b \leq 1.142$ ). Furthermore, with increasing  $b$ , the length of the high cooperation plateau decreases. Interestingly, when the value of  $b$  is beyond 1.142 (e.g.  $b=1.15$ ), the monotonous variation trend of cooperation level has been turned into non-monotonously one with the value of  $\phi$  increases, in other words, exists an optimal value of  $\phi$  (approximately  $\phi \approx 0.68$ ) which can lead to the most flourishing cooperation. It is shown that, for the higher value of  $b$ , the assessment level of individual's behavior too low or too high is not conducive to cooperation, appropriate assessment level is the best solution to obtain the optimal cooperation. With the continuous augmentation of  $b$  ( $b > 1.15$ ), non-monotonous trend is gradually vanishing and finally transfers to the constant one, namely, the cooperation is significantly weakened, and the cooperation level  $f_c$  approaches zero for large  $b \rightarrow 1.175$  (for clarity, the case of  $f_c=0$  for the whole interval of  $\phi$  is not included).

Altogether, these results indicate that the adjustment of the intensity of interaction between individuals, by considering the evaluation level of an individual's effective neighbors for its present behavior, can drastically facilitate the cooperators among the population and lead to the emergence of cooperation between players.

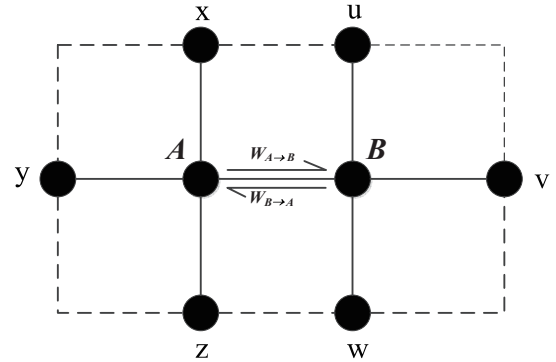


Fig. 6. A small part of the square lattice indicating the relevant configuration for the pair approximation with site  $A$  and  $B$ . This scheme is used to determine changes in the pair configuration probabilities  $p_{A,B \rightarrow B,B}$ . For clarity, we just draw the intensity of interaction between  $A$  and  $B$ , that is,  $W_{A \rightarrow B}$  and  $W_{B \rightarrow A}$ .

As is known to all, spatial structure may spring up and maintain cooperative behavior in the PDG. However, the prominent behaviors are found in the SDG where the spatial structure doesn't favor cooperation among individuals, therefore, verify the simulation results are especially important.

In what follows, we provide details of the calculations regarding the extended pair-approximation method to verify the above simulation results [46]. In spatial structures with von Neumann neighborhood, a randomly chosen site with strategy  $A$  gets updated by comparing its performance to a randomly chosen neighbor with strategy  $B$  (see Fig.6). The

payoffs  $U_A$ ,  $U_B$  of  $A$  and  $B$  are determined by interaction with their neighbors  $x, y, z$ ,  $B$  and  $u, v, w$ ,  $A$ , respectively.

In order to obtain the aggregate payoff of individual, we first need to calculate the average value of the intensity of interaction not only between cooperators but also between defectors and cooperators (We ignore the calculation which is the average value of the intensity of interaction between defectors because the payoff equals to zeros between defectors). In view of the above-mentioned idea, the computing method is as follows.

During the system evolutionary process, in the last  $m$  generations of the entire  $T_0$  generations, we regard the proportion which is the  $c, c$  links of actual interaction [denote by  $n'_{c,c}(t)$ ] account for the overall  $c, c$  links [denote by  $n_{c,c}(t)$ ] on the network, as the average intensity of interaction between cooperators (denote by  $\bar{W}_{c,c}$ ). In this way, we can also obtain the average intensity of interaction between defectors and cooperators (denote by  $\bar{W}_{d,c}$ ) on the network. More explicitly as shown below,

$$\bar{W}_{c,c} = \frac{\sum_{t=T_0-m}^{T_0} n'_{c,c}(t)}{\sum_{t=T_0-m}^{T_0} n_{c,c}(t)}, \bar{W}_{d,c} = \frac{\sum_{t=T_0-m}^{T_0} n'_{d,c}(t)}{\sum_{t=T_0-m}^{T_0} n_{d,c}(t)}. \quad (20)$$

Now we assume the site  $B$  is occupied by a cooperator and the site  $A$  is occupied by a defector, so we can compute the aggregate payoff of the cooperator for different configurations,

$$U_c(u, v, w, \bar{W}_{c,c}) = \bar{W}_{c,c} \times n_c(u, v, w) \times 1, \quad (21)$$

where  $n_c(u, v, w)$  is the number of cooperators among the neighbors  $u, v, w$ . Similarly, we can obtain the accumulation income of a defector occupying the site  $B$  plus a cooperator occupying the site  $A$  for different configurations,

$$U_d(u, v, w, \bar{W}_{d,c}) = \bar{W}_{d,c} \times [n_c(u, v, w) + 1] \times b. \quad (22)$$

Whenever  $B$  succeeds in populating site  $A$ , the pair configuration probabilities will change, thus leads to a set of ordinary differential equations which determine the time evolution of the system:

$$\begin{aligned} \dot{p}_{c,c} = & \sum_{x,y,z} [n_c(x, y, z) + 1] p_{d,x} p_{d,y} p_{d,z} \times \\ & \sum_{u,v,w} p_{c,u} p_{c,v} p_{c,w} f[\hat{U}_c(u, v, w) - \hat{U}_d(x, y, z)] + \\ & \sum_{x,y,z} [-n_c(x, y, z)] p_{c,x} p_{c,y} p_{c,z} \times \\ & \sum_{u,v,w} p_{d,u} p_{d,v} p_{d,w} f[\hat{U}_d(u, v, w) - \hat{U}_c(x, y, z)], \end{aligned} \quad (23)$$

$$\begin{aligned} \dot{p}_{c,d} = & \sum_{x,y,z} [1 - n_c(x, y, z)] p_{d,x} p_{d,y} p_{d,z} \times \\ & \sum_{u,v,w} p_{c,u} p_{c,v} p_{c,w} f[\hat{U}_c(u, v, w) - \hat{U}_d(x, y, z)] + \\ & \sum_{x,y,z} [n_c(x, y, z) - 2] p_{c,x} p_{c,y} p_{c,z} \times \\ & \sum_{u,v,w} p_{d,u} p_{d,v} p_{d,w} f[\hat{U}_d(u, v, w) - \hat{U}_c(x, y, z)]. \end{aligned} \quad (24)$$

where  $\hat{U}_c(u, v, w)$ ,  $\hat{U}_d(u, v, w)$ ,  $\hat{U}_c(x, y, z)$  and  $\hat{U}_d(x, y, z)$ , denote as follows

$$\begin{cases} \hat{U}_c(u, v, w) = U_c(u, v, w, \bar{W}_{c,c}) \\ \hat{U}_d(u, v, w) = U_d(u, v, w, \bar{W}_{d,c}) \\ \hat{U}_c(x, y, z) = U_c(x, y, z, \bar{W}_{c,c}) \\ \hat{U}_d(x, y, z) = U_d(x, y, z, \bar{W}_{d,c}). \end{cases} \quad (25)$$

Notably, the above equations omit the common factor  $2p_{c,d}/(p_c^3 \cdot p_d^3)$ , which corresponds to a nonlinear transformation of the time scale but leaves the equilibrium unaffected. Thus according to the symmetry condition  $p_{c,d} = p_{d,c}$  and the obvious constraint  $p_{c,c} + p_{c,d} + p_{d,c} + p_{d,d} = 1$ , we can acquire fraction of cooperators  $f_c = p_{c,c} + p_{c,d}$ . However, if the intensity of interaction completely equals 1 between players and their neighbors (means full interaction), it will degenerate to the classical pair-approximation method. In other words, we further extend the previous method.

In Fig.5, by and large, basically coincident except some tiny deviations between simulation results ( $f_c^{sim}$ ) and theoretical predictions ( $f_c^{theo}$ ). However, we also believe, the main reason of the deviations between the two approaches is that our extended pair approximation does not fully take into account the effects of spatial clusters. But for all this, it verifies the numerical simulations and qualitatively reflects the role of changing intensity of interaction based on the individual behavior in the evolution of cooperation.

2) *Dynamic change of individual behavior*: To intuitively understand the effect of changing the intensity of interaction based on the evaluation level of individual behavior, we investigate the dynamic change of individual behavior in the population by plotting some typical snapshots of the system at different time step  $t$  for  $b=1.10$  and  $\phi=0.8$  (see Fig. 7).

For description convenience, we divide the evolutionary path starting from an initial random state and proceeding to a final equilibrium into two periods. One is the initial period of the evolutionary path which we regard it as the *enduring period* because cooperators try to endure defectors' invasion, the other is the period following the enduring period in which  $f_c$  increases, and we regard it as the *recovering and expanding period*.

Initially, each player is designated to play either  $C$  or  $D$  with equal probability, so all individuals in the population are evenly divided into cooperators and defectors ( $t=0$ ). However, for paired two individuals, by reason of the intensity of interaction from an individual to the other one is initially assigned a score randomly chosen in the interval  $(0,1)$ , therefore, once the system start running [see Fig.7(a)], some individuals as the status of loners exist in the population owing to the feeble intensity of interaction between them and their respective partners. Thus in the system there are three types of individual configuration: cooperators, defectors and loners.

During the enduring period [see Fig.7(b) and Fig.7(c)], some short-sighted individuals who prefer to adopt treacherous strategy under the temptation of betrayal obtain higher benefits, despite the fact that they know it is at the expense of the damage their reputation. In this situation, some cooperators probably also imitate the behavior of high-earning defectors

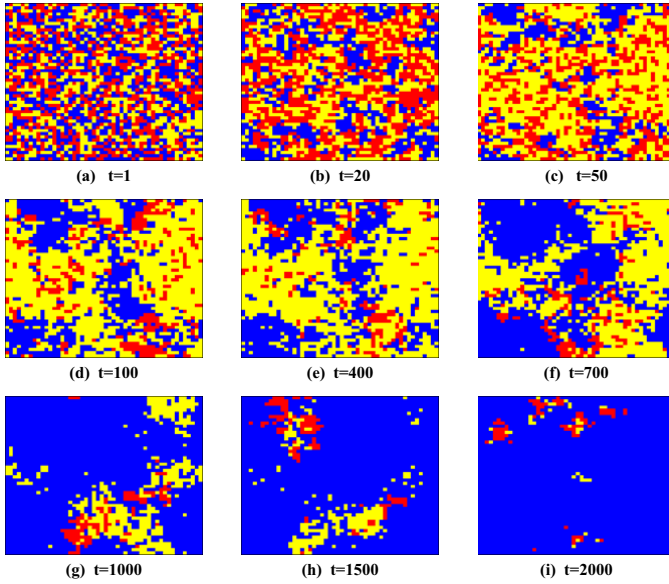


Fig. 7. Snapshots of typical distributions of cooperators (blue), defectors (red) and loners (yellow) on a square lattice obtained for  $b=1.10$  and  $\phi=0.8$  at different time step  $t$ . As time goes on, cooperators not only can survive but also gradually augment their territory by means of converting defectors into cooperators and rescuing some surrounding isolates. (a)  $t=1$  [ $f_c(1)=0.3652$ ], (b)  $t=20$  [ $f_c(20)=0.2768$ ], (c)  $t=50$  [ $f_c(50)=0.1996$ ], (d)  $t=100$  [ $f_c(100)=0.3072$ ], (e)  $t=400$  [ $f_c(400)=0.5052$ ], (f)  $t=700$  [ $f_c(700)=0.7924$ ], (g)  $t=1000$  [ $f_c(1000)=0.8724$ ], (h)  $t=1500$  [ $f_c(1500)=0.9064$ ] and (i)  $t=2000$  [ $f_c(2000)=0.9432$ ].

and turn into defectors. Thus, defectors are quickly clustered with themselves (abbreviated as D-clusters). Unfortunately, those individuals with poor reputation (defectors) likely have no choice but to also become the loners because their neighbors are more and more indifferent to them, and the *isolated* loners negatively form loners clusters (abbreviated as L-clusters). However, in spite of this, there are still some stalwart cooperators who successfully survive in the enduring period by forming cooperative clusters (abbreviated as C-clusters) for long-term benefit. Therefore, although  $f_c$  gradually decline during this period, the decrease of  $f_c$  is piece by piece restrained because C-clusters are piecemeal formed, and the end of this stage cooperators can survive and maintain.

Although each C-cluster is relatively small in scale after a suitable transient time of recovering and expanding period [Fig.7(d) and Fig.7(e)], it is the bloc of crack troops because individuals interact with their respective partners by higher intensity of interaction. Namely, for each C-cluster, there is close cooperation relationship between individuals and their own partners. Meanwhile, for each D-cluster, the interaction relationship is rather feeble between individuals and their own partners. Subsequently, with the further evolution of the system [from Fig.7(f) to Fig.7(i)], the intensity of interaction between defectors becomes increasingly weaker, but contrary phenomenon is proceeding between cooperators. As a consequence, the fragmented D-clusters is gradually disintegrated by the compact C-clusters, and the scale of C-clusters are gradually augmented by means of converting defectors into cooperators through strategy imitation. Besides, cooperators are accompanied by the rescuing work to some *isolates* which

further expands C-clusters' territory. Therefore, in this stage  $f_c$  not only can be recovered but also can be improved.

3) *Formation process of cooperator clusters and their roles:* Generally, the formation of cooperator clusters arise from the spatial effect on cooperation in the iterated PDG [11], nevertheless, through above mentioned analysis, we can know that the introduction of changing the intensity of interaction on the basis of reputation significantly influences the cluster formation process as well.

Next, in order to more clearly understand the process of C-clusters' formation, we introduce the basic concept about the node  $i$ 's in-strength and out-strength [47].

As is known to all, for directed network, the degree of node including out-degree and in-degree. The out-degree of node  $i$  (denote by  $k_i^{out}$ ) means the number of links from  $i$  to its neighbor nodes; corresponding, in-degree (denote by  $k_i^{in}$ ) means the number of links from  $i$ 's neighbor nodes to node  $i$  itself. However, for weighted network, we set the weight matrix  $\Theta = [w_{ij}]$ , thus, the strength of node  $i$

$$\theta_i = \sum_{j=1}^{k_i} w_{ij}. \quad (26)$$

Taken together, for directed-weighted network, the in-strength and out-strength respectively is

$$\theta_i^{in} = \sum_{j=1}^{k_i} w_{ji}, \quad \theta_i^{out} = \sum_{j=1}^{k_i} w_{ij}. \quad (27)$$

Thus, we can gain the average value of in-strength ( $\overline{\theta_i^{in}}$ ) and out-strength ( $\overline{\theta_i^{out}}$ ) as follow,

$$\overline{\theta_i^{in}} = \frac{1}{k_i} \sum_{j=1}^{k_i} w_{ji}, \quad \overline{\theta_i^{out}} = \frac{1}{k_i} \sum_{j=1}^{k_i} w_{ij}. \quad (28)$$

**Definition 2. Attraction Degree** In the evolutionary system, the average value of a node's in-strength ( $\overline{\theta_i^{in}}$ ) characterises a player's attraction degree for surrounding partners. The higher the value of  $\overline{\theta_i^{in}}$ , the stronger the attraction degree.

Thus, in this context we can easily explain the formation process of cooperator clusters.

For a player  $x$  who has good reputation [ $R_x(t) > 0$ ] at time  $t$  step, if its present reputation higher than the previous one, its attraction degree  $\overline{\theta_x^{in}}$  will be increased, thus the surrounding friends will strengthen the communication with him, which probably results in the enhancement of its effective neighbor's quantity. Importantly, the enhancement of effective neighbor's quantity further improves its reputation on condition that it has good behaviour in the next interaction. Obviously, the positive feedback effect is formed between the individual's reputation and the quantity of effective neighbor for an individual with good reputation. In addition, the more the times of player  $x$  shows up the good behaviour, the more obvious the positive feedback effect. Remarkably, the positive feedback effect can drastically facilitate the formation of C-clusters as time evolution. The main reason is that the positive feedback effect makes the values of the intensity of interaction become increasingly higher between cooperators, i.e., the cohesion between them is gradually increased owing to the role of positive feedback.



However, for a player  $y$  who has bad reputation [ $R_y(t) < 0$ ] at time  $t$  step, if its present reputation lower than the previous one, its attraction degree  $\theta_y^{in}$  will be decreased, thus the surrounding friends will weaken the communication with him, which probably leads to the decrease of effective neighbor's quantity. Even worse, if player  $y$  habitually shows bad behaviour in the subsequent interactions, its interaction partners will become less and less. Under the circumstances, it is very likely spring up two different scenes. One scene is the player  $y$  probably mimic the behavior of cooperator of C-cluster's boundary because the cooperative neighbors whose excellent performance in collecting payoffs highly possible exceed the accumulated income of player  $y$ . The other scene is the inflexible player  $y$  is gradually discarded by its friends owing to the worsening reputation and finally become the loner. Nevertheless, although the player  $y$  has been turned into the loner from the teeth outwards, in essence, the actual interactive relation has not been *frozen*. Therefore, those loners who are analogous to the player  $y$  still have the opportunity to be rescued by collaborators so long as there are some capable cooperators (means they have collecting payoffs) around them, and these rebirth individuals are naturally subservient to the field of collaborators. Obviously, for the two cases, regardless of which one contributes to the expansion of C-clusters in scale.

In fact, during the formation process of C-clusters, whether the enhancement of internal cohesion or the augment of scale, the relationship between them is mutually complementary and inseparable.

Altogether, we can know that C-clusters play the different roles in different period according to the formation process of C-clusters. First, they are the revolutionary fighters in order to defend their respective homeland. The piecemeal formation of C-clusters is a critical aspect of preventing cooperation level  $f_c$  from decreasing during the whole evolution stage, especially the enduring period of cooperators. Second, they are the main force of cooperators. C-clusters not only gradually strengthen team cohesion by automatically changing the intensity of interaction between them but also gradually expand their field by converting defectors into cooperators. Finally, they are the savior of healing the wounded and rescuing the dying. C-clusters constantly save those individuals who have been abandoned due to their previous wretched reputation.

After analyzing the formation of cooperator clusters, we further study the effect of reputation-based changing the intensity of interaction on cooperation by plot some typical snapshots of the system at equilibrium for fixed  $b=1.10$  with respect to different  $\phi$  values, as shown in Fig.8.

In Fig.8(a), one can see that cooperators are completely died out on condition that facing the moderate temptation to defect ( $b=1.10$ ) and the very low evaluation level ( $\phi=0.1$ ) of individual behavior. Of course, in this case, there is still very faintly positive feedback effect in system and the positive feedback effect can also lead to the formation of C-clusters during the enduring period of cooperators, but the internal cohesion and resistance of C-clusters are both very unsubstantial, so they are easily destroyed and all cooperators are annihilated by defectors. With increasing  $\phi$  [from Fig.8(b)

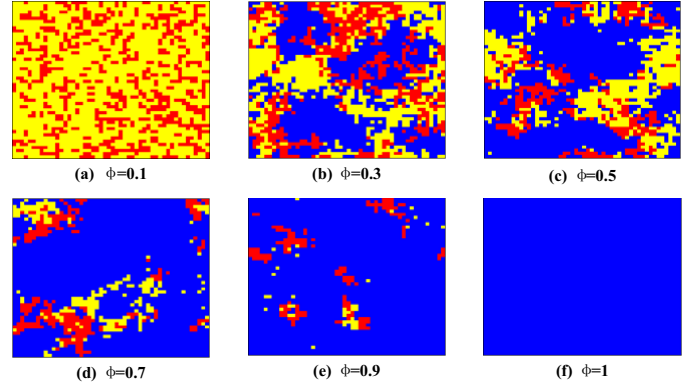


Fig. 8. Snapshots of typical distributions of cooperators (blue), defectors (red) and loners (yellow) on a square lattice obtained for  $b=1.10$  and different values of  $\phi$ . For the moderate temptation to defect, with increasing  $\phi$ , the positive feedback effect is becoming more and more prominent, the internal cohesion and resistance of cooperative clusters are also gradually increased. Furthermore, the positive feedback effect can directly determine the cooperation level in a population. The higher the value of  $\phi$ , the more obvious the positive feedback effect, and the higher the cooperation level. (a)  $\phi=0.1$  [ $f_c=0$ ], (b)  $\phi=0.3$  [ $f_c=0.4302$ ], (c)  $\phi=0.5$  [ $f_c=0.6052$ ], (d)  $\phi=0.7$  [ $f_c=0.8126$ ], (e)  $\phi=0.9$  [ $f_c=0.9425$ ] and (f)  $\phi=1.0$  [ $f_c=1$ ].

to Fig.8(f)], the positive feedback effect is becoming more and more prominent, the internal cohesion and resistance of C-clusters are also gradually increased. Once the boundary cooperators of C-clusters reach and exceed the defectors' self-defensive ability, their previous defending role immediately turn into assault force and march towards the wider field. As a consequence, the higher the value of  $\phi$ , the more powerful the legions of C-clusters, and the higher the cooperation level.

Hereto, we have investigated the effect of reputation-based changing the intensity of interaction on cooperation by some typical snapshots for a moderate value of  $b$  ( $b=1.10$ ). However, we don't present some snapshots for the higher value of  $b$  to study the effect. In fact, combine the above analysis ideas with the research results of  $f_c$  as a function of  $\phi$  for the different values of  $b$ , we can easily draw the conclusions. In Fig.2, we found that the curve of  $f_c$  is bell-shaped symmetrical for  $b=1.15$ . Namely, the value of  $\phi$  either too low or too high both restrain the cooperation, just some relatively moderate values can promote the cooperation, for the higher values of  $b$ . The optimal  $\phi^{opt}$  satisfies

$$\phi^{opt} = \arg \max_{\phi \in [0,1]} f_c. \quad (29)$$

For  $\phi < \phi^{opt}$ , with increasing  $\phi$ , the positive feedback effect is becoming more and more prominent, the internal cohesion and resistance (or aggressivity) of C-clusters is also gradually increased, accordingly, the cooperation level  $f_c$  is gradually enhanced, similar effect with  $b=1.10$ . Differently, for  $\phi > \phi^{opt}$ , with increasing  $\phi$ , a number of defectors whose reputation become worse and worse due to the higher values of  $b$ , which probably leads to they will be rapidly turned into loners. Seen another way, the number of cooperators' effective neighbors is likely to be gradually decreased, followed by the positive feedback effect weakening and the scale of C-clusters decrease, as a result, the cooperation level  $f_c$  will be reduced accordingly.

### C. Effect of the extended payoff matrix EM.

Finally, it is very necessary for us to mention that the payoff matrix  $M$ , we adopt, is a classic Nowak version. Although it is a *weak* PDG, scholars still widely used because the weak satisfying rule doesn't alter the qualitative results. Noticeably, in Ref. [40], [48], [49], [50] authors refined the more common philosophy in studying evolutionary game theory and collective cooperation by reviewed some novel universal scaling parameters for the dilemma strength which have the potential to unify different game scenarios. As suggested by them, pairwise games can also be classified based on gamble-intending (GID) and risk-averting (RAD) dilemmas, defined by  $D_g = T - R$  and  $D_r = P - S$ , respectively. If  $D_g$  is positive, both players should be inclined to exploit each other. By contrast, if  $D_r$  is positive, players should refrain from the exploitation. When both  $D_g$  and  $D_r$  are positive, the game is a prisoner's dilemma, whereby defector dominates cooperator. Without losing mathematical generality, we can define a PDG by presuming  $R=1$  and  $P=0$  as follows:

$$EM = \begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & -D_r \\ 1 + D_g & 0 \end{pmatrix}, \quad (30)$$

here, we limit the PDG class by assuming  $0 \leq D_g \leq 1$  and  $0 \leq D_r \leq 1$ , obviously, it covers the mode of classical spatial game.

Thus, we further study the model of changing the intensity of interaction based on the individual behavior in the extended PDG by adjusting the parameters of dilemma strength ( $D_g$  and  $D_r$ ), as shown in Fig.9.

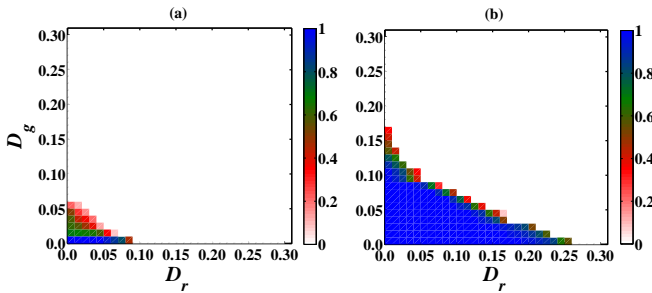


Fig. 9. The fraction of cooperators of PDG with  $0 \leq D_g \leq 1$  and  $0 \leq D_r \leq 1$ . After improving the strength of social dilemma, compared with the weak PDG version, the cooperation level is still largely enhanced. (a) Conventional interaction mode ( $\phi=0$ ); (b) By considering the evaluation level of individual behavior changing the intensity of interaction mode for a fixed  $\phi=0.8$ . Other relevant parameters  $L=100$ ,  $K=0.1$ .

Satisfactorily, compared with the traditional version ( $\phi=0$ ) of the iterated PDG, the cooperation level is still largely enhanced as the dilemma strength (consist of  $D_g$  and  $D_r$ ) is considered, for  $\phi>0$ . And for a fixed value of  $D_g$ , with the value of  $D_r$  gradually augments, the cooperation level of traditional mode rapidly declines or vanishes, nevertheless the cooperation level slowly descends for our model.

Although our simulation data is obtained on a smaller square lattice size ( $100 \times 100$ ) with von Neumann neighborhood, we also verify the simulations, regardless of whether network size

is  $200 \times 200$  or  $2 \times 200 \times 200$ , even the larger lattice size (eg.  $300 \times 300$ ), and find that the results are qualitatively not to change under the same conditions. Furthermore, we test our model on the square lattice with Moore neighborhood, the results are mainly qualitatively identical with those presented in foregoing statistics diagrams. Especially, the dependence of  $f_c$  on  $b$  is somewhat smoother. Namely, for the larger neighborhood size, the fluctuations become smaller in statistical curves including some data points which are near the transition points. Here, we simply explain the reason. For full interaction between players and their nearest neighbors (the intensity of interaction identically equals to 1), a defector has very good access to the field of triangle cooperative cluster because existing shared edge between two triangles in the network where each player has eight neighbors, which causes defectors to easily invade. However, when we introduce the mechanism, that is, automatically adjusting the intensity of interaction between individuals based on individual behavior, the traitors have fewer chance to invade. Meanwhile, the role, which is the higher clustering feature of the network (because existing the triangle structure in the lattice), can improve the clustering ability of the cooperative cluster. So, when faced the higher temptation to defect  $b$ , the small-scale cooperative cluster can exist and maintain near the transition point. With the value of  $b$  increasing, the cooperative cluster in scale gradually decreases. Under the circumstance, the fluctuations of the data points will be reduced. In addition, other strategy update mechanism (eg. the replicator dynamics rule), the updating fashion (eg. asynchronous), and the different initial frequencies of cooperators after sufficient evolution. We also confirm that the qualitative results do not change if we make the above-mentioned variations to our model. Therefore, we would like to point out that the results reported here are robust.

However, the effect of time scale ratio between strategy updating and changing the intensity of interaction for cooperation level, and the roles of diversity of individual rationality, these valuable works should be worthy of further investigation and explore the evolution of a variety of coupled dynamic each other. Besides, the next step would be to extend our model to accommodate situations where the mobility of agents is allowed in the population [33], [51]. With the continuing development of network science, the structure or evolving pattern of real-world systems has often been modeled as the interdependency and multiplexing of two or more submodules, on which the structural properties and dynamical behaviors exhibit some distinct phenomena from single, isolated networks [35], [36]. Therefore, the dynamical behaviors on the multilayer networks are also one of the focuses of investigation. Work along these lines is in progress.

## V. CONCLUSION

In the real society, an individual neither always interacts with its all partners (such as full interaction) nor interacts with them with the same intensity of interaction (such as stochastic interaction with the constant intensity of interaction), but presents a variety of interactive dynamics relation network. For example, in the circle of friends, for two partners  $x$  and

$y$ , their friendship relation perhaps just as the old saying goes, *share the joys and sorrows*, maybe just is a common relation. What's more, the relationship strength between them will be likely changed as time goes on. Here, we have proposed a reputation-based changing intensity of interaction mechanism, by considering the evaluation level of individual behavior, to investigate the evolutionary prisoner's dilemma game in social networks. The results have indicated that the effective mechanism can largely promote the cooperation level, and the enhancement of the cooperation level is highly influenced by the tunable parameter  $\phi$  (the evaluation level of individual behavior). As  $\phi > 0$ , the positive feedback effect can be formed between the individual's reputation and the quantity of effective neighbor for an individual with good reputation. It is particularly worth mentioning that the positive feedback effect can drastically facilitate the formation of cooperator clusters as time evolution. For the temptation to defect  $1.0 < b \leq 1.142$ , with increasing  $\phi$ , the positive feedback effect is increasingly outstanding, accordingly the cooperator cluster is gradually expanded. The more powerful the army (the internal cohesion and fighting capacity) of cooperator clusters, the more obvious the final dominance of cooperation is. Interestingly, for  $1.142 < b < 1.175$ , the value of  $\phi$  either too low or too high both restrain the cooperation, just some relatively moderate values can promote the cooperation. In order to confirm simulation conclusions, we adopted the extended pair-approximation method to predict the simulation results and found better agreement between the two approaches, although there is some tiny deviations. Besides, we further studied the evolutionary dynamics in the PDG by considering the wider dilemma strength and found the cooperation level is still largely enhanced than the traditional fashion and the qualitative results do not change.

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