Recombination of Similar Parents in SMS-EMOA on Many-Objective 0/1 Knapsack Problems

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Abstract. In the evolutionary multiobjective optimization (EMO) community, indicator-based evolutionary algorithms (IBEAs) have rapidly increased their popularity in the last few years thanks to their theoretical background and high search ability. Hypervolume has often been used as an indicator to measure the quality of solution sets in IBEAs. It has been reported in the literature that IBEAs work well on a wide range of multiobjective problems including many-objective problems on which traditional Pareto dominance-based EMO algorithms such as NSGA-II and SPEA2 do not always work well. In this paper, we examine the behavior of SMS-EMOA, which is a frequently-used representative IBEA with a hypervolume indicator function, through computational experiments on many-objective 0/1 knapsack problems. We focus on the effect of two mating strategies on the performance of SMS-EMOA: One is to select extreme parents far from other solutions in the objective space, and the other is to recombine similar parents. Experimental results show that the recombination of similar parents improves the performance of SMS-EMOA on many-objective problems whereas the selection of extreme parents is effective only for a two-objective problem. For comparison, we also examine the effect of these mating strategies on the performance of NSGA-II.

Keywords: Evolutionary multiobjective optimization, evolutionary many-objective optimization, SMS-EMOA, mating schemes, knapsack problems.

1 Introduction

Evolutionary multiobjective optimization (EMO) has been one of the most active research areas in the field of evolutionary computation in the last two decades. Since Goldberg's suggestion in 1989 [9], Pareto dominance-based fitness evaluation has been the mainstream in the EMO community. Almost all of well-known traditional EMO algorithms such as NSGA-II [7], PAES [21], SPEA [35] and SPEA2 [34] are categorized as Pareto dominance-based EMO algorithms. Whereas Pareto dominance-based EMO algorithms have been successfully applied to multiobjective problems in various application fields [5], [6], [28], they do not always work well on many-objective problems with four or more objectives as repeatedly pointed out in the literature [10], [20], [23], [37]. This is because almost all solutions in the current population become non-dominated with each other in early generations in the

C.A. Coello Coello et al. (Eds.): PPSN 2012, Part II, LNCS 7492, pp. 132–142, 2012.

application of EMO algorithms to many-objective problems [16], [17], [24]. When all solutions in the current population are non-dominated, Pareto dominance-based fitness evaluation cannot generate any selection pressure towards the Pareto front. As a result, the convergence property of Pareto dominance-based EMO algorithms is deteriorated in their application to many-objective problems. Motivated by strong intentions to overcome such an undesirable behavior, evolutionary many-objective optimization has become a hot issue in the EMO community in the last few years [1], [26], [27].

Recently, two classes of nontraditional EMO algorithms have attracted a lot of attention as promising approaches to many-objective optimization. One is indicator-based EMO algorithms where an indicator function is used to measure the quality of solution sets [3], [4], [29], [33], [36]. EMO algorithms in this class are referred to as indicator-based evolutionary algorithms (IBEAs). Hypervolume has been frequently used as an indicator function in EMO algorithms in this class because it has a good theoretical background such as Pareto compliance [2], [32]. By using a fast calculation method of the exact hypervolume [30] or an efficient approximation method [3], the applicability of indicator-based EMO algorithms to many-objective problems has been improved. SMS-EMOA [4] in this class has often been used in the literature. Its high search ability on many-objective problems has been clearly demonstrated [29].

The other class is scalarizing function-based EMO algorithms where a number of scalarizing functions with different weight vectors are used to search for a wide variety of Pareto optimal solutions [11], [13], [19], [31]. One advantage of this class is the computational efficiency of scalarizing function calculation. MOEA/D [31] in this class has been frequently used as a high-performance EMO algorithm [12], [22].

Through the use of an indicator or scalarizing functions, these two classes of EMO algorithms overcome the main difficulty in the handling of many-objective problems by traditional EMO algorithms (i.e., the deterioration in their convergence property).

Another difficulty in the handling of many-objective problems, which has not been stressed in the literature, is negative effects of a large solution diversity on the effectiveness of recombination operators. In general, the increase in the number of objectives in a multiobjective problem leads to the increase in the number of its Pareto-optimal solutions and their diversity. As a result, the diversity of solutions in the current population becomes very large in the application of EMO algorithms to many-objective problems. That is, solutions in the current population are totally different from each other. Since good solutions are not likely to be generated from the recombination of totally different solutions, large solution diversity seems to have negative effects on the performance of EMO algorithms on many-objective problems. Actually, it was shown by Sato et al. [25] that the performance of NSGA-II on many-objective 0/1 knapsack problems was improved by local recombination. It was also shown that the performance of MOEA/D on many-objective 0/1 knapsack problems was deteriorated by increasing the size of a neighborhood structure for parent selection [15]. These reported results in the literature suggest the importance of the recombination of similar parents in EMO algorithms on many-objective problems.

The use of mating schemes has been proposed to improve the performance of traditional Pareto dominance-based EMO algorithms in the literature (e.g., [14], [25]). However, their use has not been discussed for MOEA/D or SMS-EMOA. This is because (i) these two algorithms usually show high search ability on a wide range of multiobjective problems, (ii) MOEA/D inherently has a local recombination mechanism based on a neighborhood structure of solutions, and (iii) efficient hypervolume calculation has been the main issue in hypervolume-based IBEAs. The aim of this paper is to clearly demonstrate the usefulness of mating schemes in SMS-EMOA on many-objective 0/1 knapsack problems.

This paper is organized as follows. First we briefly explain a mating scheme in our former study [14] in Section 2, which is used to implement two mating strategies: extreme parent selection and similar parent recombination. Next we explain our many-objective 0/1 knapsack problems in Section 3. Then we show the setting of our computational experiments in Section 4. In Section 5, it is demonstrated that only the similar parent recombination improves the performance of SMS-EMOA and NSGA-II on many-objective problems with four or more objectives while the extreme parent selection as well as the similar parent recombination improves their performance on two-objective problems. Finally Section 6 summarizes this paper.

2 Mating Scheme with Two Mating Strategies

In our former study [14], we proposed a mating scheme in Fig. 1 to examine the effect of the following two mating strategies on the performance of NSGA-II:

(1) Selection of extreme solutions far from other solutions in the objective space.
(2) Recombination of similar parents in the objective space.

In the left part of Fig. 1, α candidates are selected by iterating binary tournament selection with replacement α times. Their average vector is calculated in the objective space. The farthest candidate with the largest distance from the average vector is chosen as Parent A. In the right part, β candidates are selected in the same manner. The closest candidate to Parent A in the objective space is chosen as Parent B.

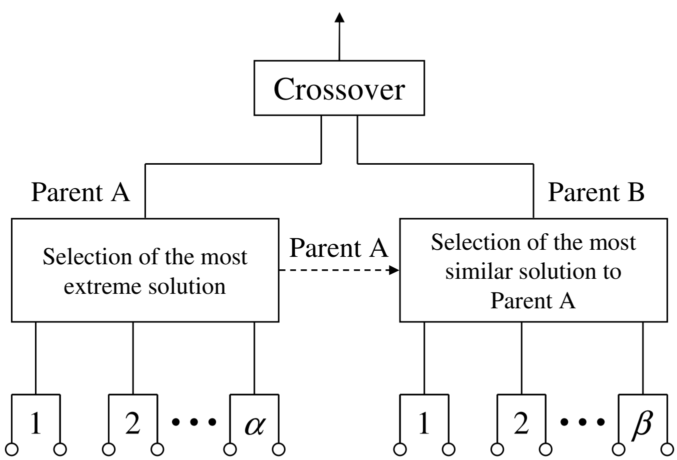


Fig. 1. Mating scheme for NSGA-II with binary tournament selection [14]. Binary tournament selection is replaced with random selection when the mating scheme is used in SMS-EMOA.

When our mating scheme is used in SMS-EMOA, α and β candidates are randomly selected from the population because SMS-EMOA randomly chooses parents from the population. The values of α and β can be viewed as showing the strength of the tendency to choose extreme candidates and to recombine similar candidates, respectively. When $\alpha = \beta = 1$, our mating scheme has no effects on EMO algorithms. In our computational experiments, we examine four values of α and β: 1, 5, 10 and 20.

In our former study [15], we obtained good results by dynamically changing the values of α and β during the execution of NSGA-II. However, we handle α and β as pre-specified constants in this paper to clearly examine the effect of each mating strategy. Experimental results in this paper can be improved by dynamically changing the values of α and β during the execution of NSGA-II and SMS-EMOA.

3 Multiobjective 0/1 Knapsack Problems

As test problems, we use multiobjective 0/1 knapsack problems with two, four, six and eight objectives. Our two-objective test problem is the same as the two-objective 500-item 0/1 knapsack problem with two constraint conditions in Zitzler and Thiele [35]. This two-objective test problem can be written as follows:

$$\text{Maximize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x})), \tag{1}$$

$$\text{subject to } \sum_{j=1}^{n} w_{ij} x_j \le c_i\ , \quad i = 1, 2, \tag{2}$$

$$x_j = 0 \text{ or } 1, \quad j = 1, 2, \ldots, n, \tag{3}$$

$$\text{where } f_i(\mathbf{x}) = \sum_{j=1}^{n} p_{ij} x_j\ , \quad i = 1, 2. \tag{4}$$

In (1)-(4), n = 500 (i.e., 500 items), $\mathbf{x}$ is a 500-bit binary string, p_{ij} is the profit of item j according to knapsack i, w_{ij} is the weight of item j according to knapsack i, and c_i is the capacity of knapsack i. This problem is referred to as the 2-500 problem.

As in Zitzler and Thiele [35], we can easily generate other objective functions $f_i(\mathbf{x})$ in the form of (4) for $i = 3, 4, \ldots, 8$ by randomly specifying each value of p_{ij} as an integer in [10, 100]. In this manner, we have generated 500-item 0/1 knapsack problems with four or more objectives (i.e., 4-500, 6-500 and 8-500 problems). The constraint conditions in (2) of the 2-500 problem are always used in our test problems independent of the number of objectives. This means that all of our test problems with a different number of objectives have exactly the same set of feasible solutions.

For the 2-500 problem, we use the same greedy repair method as in Zitzler and Thiele [35] for handing infeasible solutions. Infeasible solutions are repaired by removing items one by one until all the constraint conditions are satisfied. The order of the items to be removed is specified based on the maximum profit/weight ratio (see [35] for details). The same greedy repair method is used for all of our test problems because they have the same constraint conditions.

4 Setting of Computational Experiments

SMS-EMOA [4] and NSGA-II [7] are used under the following setting:

Population size: 100,
Termination condition: Evaluation of 400,000 solutions,
Crossover probability: 0.8 (Uniform crossover),
Mutation probability: 1/500 (Bit-flip mutation),
Reference point for hypervolume calculation: Origin of the objective space.

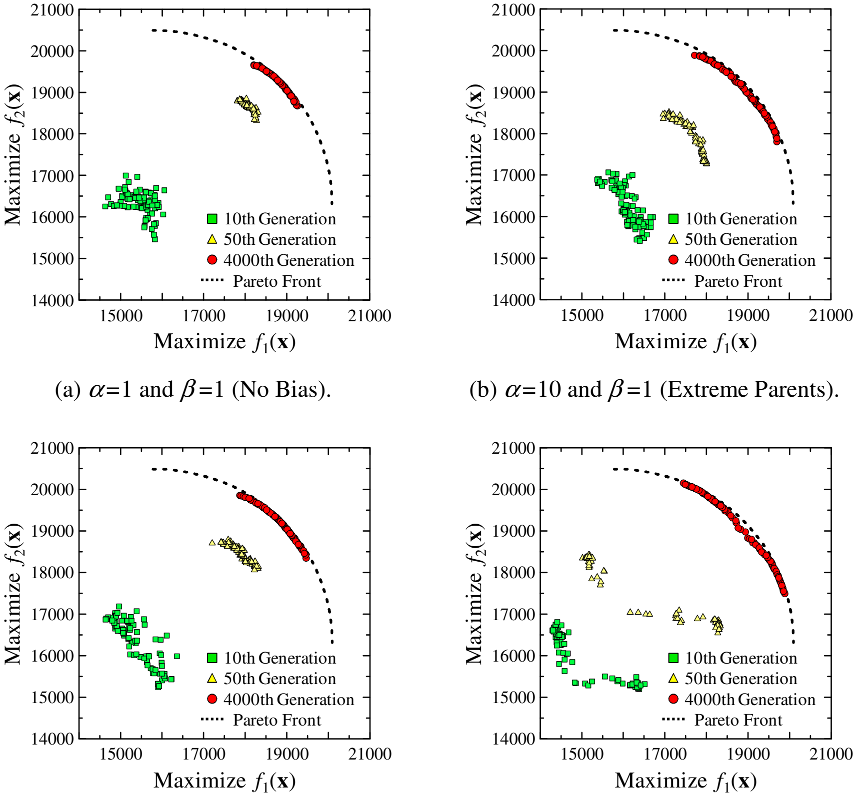
SMS-EMOA and NSGA-II are based on a (μ+λ)-ES generation update mechanism. Random selection and binary tournament selection are used for parent selection in SMS-EMOA and NSGA-II, respectively. In our computational experiments, μ and λ are specified as μ=100 and λ=1 in SMS-EMOA and μ=λ=100 in NSGA-II. An initial population is randomly generated in each algorithm. Since only a single solution is newly generated for generation update in SMS-EMOA with λ=1, 100 generations of SMS-EMOA are counted as one generation of NSGA-II when experimental results of SMS-EMOA and NSGA-II at some generations are shown. This is to show their experimental results after the same computation load. Average results are calculated over 100 runs of each algorithm on each test problem except for SMS-EMOA on the 6-500 problems (20 runs) and the 8-500 problems (10 runs).

5 Experimental Results

First we report experimental results on the 2-500 problem. In Fig. 2, we show experimental results of a single run of SMS-EMOA for each of the four settings of our mating scheme: (α, β) = (1, 1), (10, 1), (1, 10), (10, 10). Since our mating scheme with α=1 and β=1 does not change SMS-EMOA, Fig. 2 (a) can be viewed as experimental results by SMS-EMOA without the mating scheme. The diversity of solutions is improved by the extreme parent selection with α=10 in Fig. 2 (b) and the similar parent recombination with β=10 in Fig. 2 (c). In Fig. 2 (d), the diversity is further improved by the simultaneous use of these two mating strategies. Experimental results in Fig. 2 are consistent with reported results on the 2-500 problem in the literature [14], [18], [23] where the importance of diversity maintenance was demonstrated.

For comparing the average experimental results over 100 runs between SMS-EMOA and NSGA-II, we show the 50% attainment surface [8] in Fig. 3 for each algorithm with (α, β) = (1, 1), (10, 10). In Fig. 3, our mating scheme with α=10 and β=10 has similar effects on SMS-EMOA and NSGA-II. That is, the diversity of solutions is clearly improved while the convergence is slightly degraded.

We further examine the effect of our mating scheme with various settings of α and β on the two algorithms. Experimental results are summarized in Fig. 4 where the average hypervolume over 100 runs of each algorithm is shown for the 4×4 combinations of the four values of α and β: α=1, 5, 10, 20 and β=1, 5, 10, 20. As in Fig. 2, we can see that the performance of SMS-EMOA and NSGA-II is improved by the extreme solution selection (α>1) and the similar parent recombination (β>1).



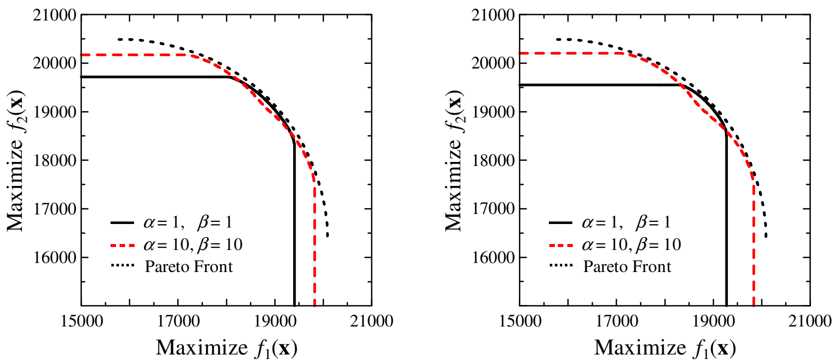
(a) α=1 and β=1 (No Bias).

(b) α=10 and β=1 (Extreme Parents).

(c) α=1 and β=10 (Similar Parents).

(d) α=10 and β=10 (Extreme and Similar).

Fig. 2. Experimental results of a single run of SMS-EMOA on the 2-500 problem



(a) SMS-EMOA.

(b) NSGA-II.

Fig. 3. 50% attainment surface over 100 runs of each EMO algorithm on the 2-500 problem

Experimental results on other test problems are shown in Figs. 5-7. From these figures, we can see that the increase in the number of objectives leads to (i) better results of SMS-EMOA over NSGA-II, (ii) positive effects of the similar parent recombination with $\beta>1$, and the negative effects of the extreme parent selection with $\alpha>1$. Fig. 8 illustrates the effects of the two mating strategies on the behavior of SMS-EMOA on the 4-500 problem by projecting the final population in the four-dimensional objective space onto the f_1-f_2 plane. Fig. 8 (c) suggests the improvement in the diversity and the convergence by the similar parent recombination.

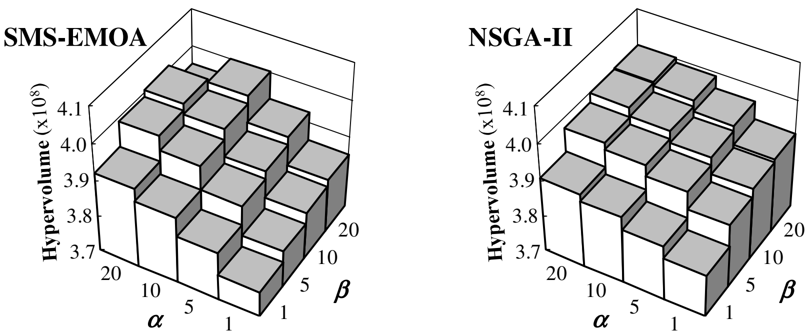


Fig. 4. Average results of SMS-EMOA (left) and NSGA-II (right) on the 2-500 problem

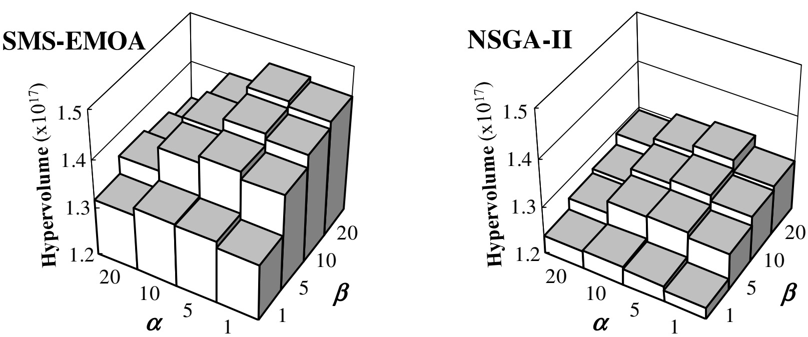


Fig. 5. Average results of SMS-EMOA (left) and NSGA-II (right) on the 4-500 problem

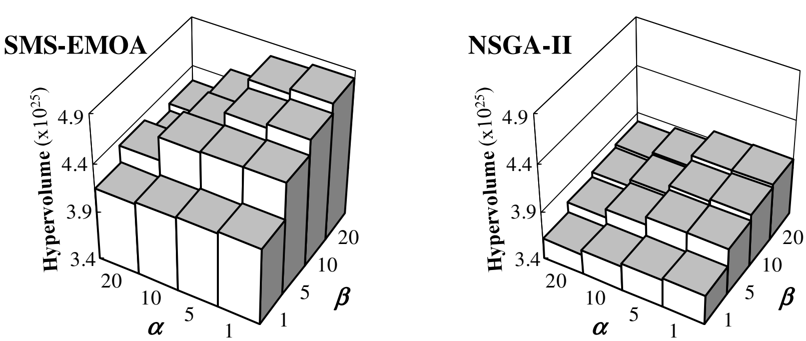


Fig. 6. Average results of SMS-EMOA (left) and NSGA-II (right) on the 6-500 problem

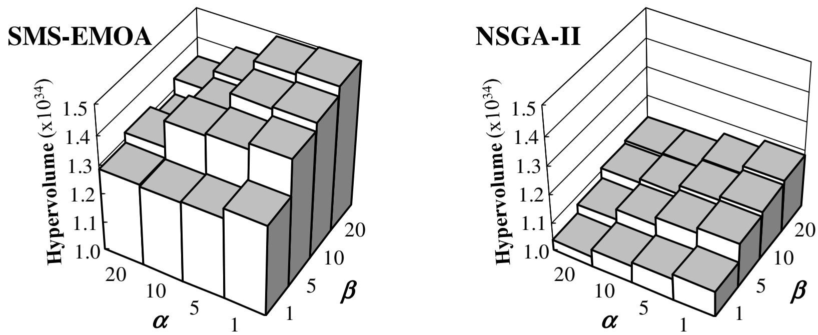
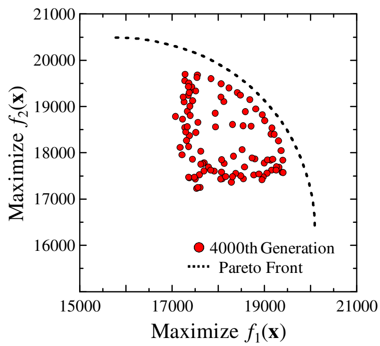
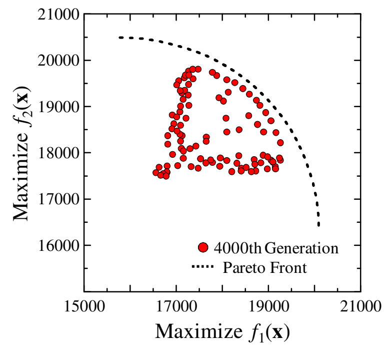


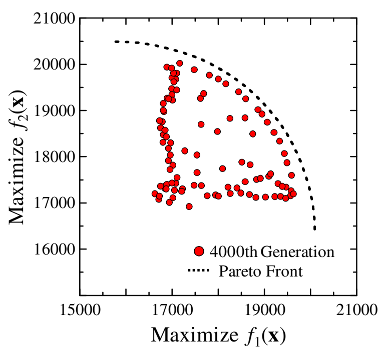
Fig. 7. Average results of SMS-EMOA (left) and NSGA-II (right) on the 8-500 problem



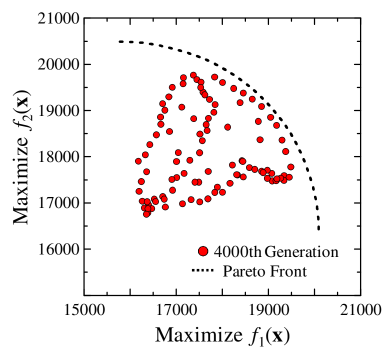
(a) α=1 and β=1 (No Bias).



(b) α=10 and β=1 (Extreme Parents).



(c) α=1 and β=10 (Similar Parents).



(d) α=10 and β=10 (Extreme and Similar).

Fig. 8. Experimental results of a single run of SMS-EMOA on the 4-500 problem

6 Conclusions

We examined the effect of the two mating strategies (i.e., extreme parent selection and similar parent recombination) on the performance of SMS-EMOA through computational experiments on multiobjective 500-item 0/1 knapsack problems with two, four, six and eight objectives. These two mating strategies improved the performance of SMS-EMOA on the 2-500 problem with two objectives. The best results on the 2-500 problem were obtained when the two mating strategies were simultaneously used. The similar parent recombination improved the performance of SMS-EMOA on our test problems independent of the number of objectives. However, the extreme parent selection improved the performance of SMS-EMOA only on the 2-500 problem. Its negative effects were observed on the performance of SMS-EMOA on our many-objective test problems. The performance of SMS-EMOA on the 2-500 problem was similar to that of NSGA-II. By increasing the number of objectives, the advantage of SMS-EMOA over NSGA-II became clear. Moreover, much larger improvements in the average hypervolume measure by the similar parent recombination were obtained in Figs. 5-8 by SMS-EMOA than NSGA-II. In Fig. 2 (c) and Fig. 8 (c), the similar parent recombination increased the diversity without deteriorating the convergence.

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