Developing Graph-Based Co-Scheduling Algorithms on Multicore Computers

Ligang He, *Member, IEEE*, Huanzhou Zhu, and Stephen A. Jarvis, *Member, IEEE*

Abstract—It is common that multiple cores reside on the same chip and share the on-chip cache. As a result, resource sharing can cause performance degradation of co-running jobs. Job co-scheduling is a technique that can effectively alleviate this contention and many co-schedulers have been reported in related literature. Most solutions however do not aim to find the optimal co-scheduling solution. Being able to determine the optimal solution is critical for evaluating co-scheduling systems. Moreover, most co-schedulers only consider serial jobs, and there often exist both parallel and serial jobs in real-world systems. In this paper a graph-based method is developed to find the optimal co-scheduling solution for serial jobs; the method is then extended to incorporate parallel jobs, including multi-process, and multi-threaded parallel jobs. A number of optimization measures are also developed to accelerate the solving process. Moreover, a flexible approximation technique is proposed to strike a balance between the solving speed and the solution quality. Extensive experiments are conducted to evaluate the effectiveness of the proposed co-scheduling algorithms. The results show that the proposed algorithms can find the optimal co-scheduling solution for both serial and parallel jobs. The proposed approximation technique is also shown to be flexible in the sense that we can control the solving speed by setting the requirement for the solution quality.

Index Terms—Multicore, scheduling, graph, parallel computing

1 Introduction

Multicore processors have become mainstream products in the CPU industry. In a multicore processor, multiple cores reside and share resources on the same chip. However, with such a design, running multiple applications on different cores can cause resource contention, which in turn leads to performance degradation [1]. Many researchers have shown that it is possible to isolate some resources, such as disk [2] and network [3] bandwidth for co-running jobs. However, it is very difficult to isolate the on-chip last level cache (LLC). This is known as the *shared cache contention problem* and has been studied extensively, including [1], [4], [5]. Existing approaches to addressing on-chip shared cache contention fall into three categories: 1) Architectural-level solutions to improve hardware and provide isolation among threads [6], [7], 2) System-level solutions to partition the cache for each application [8], [9], and 3) Software-level solutions to develop schedulers to reduce contention [10], [11]. Within these three categories, architectural-level solutions remain under development by processor vendors, and cache partitioning solutions require many changes to existing system-level software (such as the operating system) and therefore incur high implementation cost. The third approach, developing contention-aware schedulers, is regarded as a lightweight approach, and has therefore attracted much attention by researchers—it is this last category that is the focus of this research.

A number of contention-aware co-schedulers have been developed [12], [13], [14]. These studies demonstrate that contention-aware schedulers can deliver better performance than conventional schedulers. However, they do not aim to determine or deliver optimal co-scheduling performance. Such a benchmark is useful to understand, even if it is obtained offline, as this allows system designers to assess potential for improvement. In addition, knowing the gap between current and optimal performance can also help scheduler designers make tradeoffs between scheduling efficiency (i.e., the time that the algorithm takes to compute the scheduling solution) and scheduling quality (i.e., how good the obtained scheduling solution is).

Co-schedulers in the literature that proport to be optimal only consider serial jobs (each of which runs on a single core). For example, the work in [4] modelled the optimal co-scheduling problem for serial jobs as an integer programming (IP) problem. However, in modern multi-core systems, especially those in cluster and cloud platforms, both parallel and serial jobs co-exist [15], [16], [17]. In order to address this problem, this paper proposes a new method to determine the optimal co-scheduling solution for a mix of serial and parallel jobs. Two types of parallel jobs are considered in this paper: Multi-Process Parallel (MPP) jobs, such as MPI jobs, and Multi-Thread Parallel (MTP) jobs, such as OpenMP jobs. In this paper, we first propose a method to co-schedule MPP and serial jobs, and then extend this method to handle MTP jobs.

Resource contention presents different features in single processor and multi-processor machines. In this paper, a layered graph is first constructed to model the co-scheduling problem on single processor machines. The problem of finding the optimal co-scheduling solutions is then modelled as finding the shortest *valid* path (SVP) in the graph. Further, this paper develops a set of algorithms to find the

• *The authors are with the Department of Computer Science, University of Warwick, Coventry CV4 7AL, United Kingdom.*
E-mail: {liganghe, zhz44, saj}@dcs.warwick.ac.uk.

Manuscript received 20 June 2014; revised 28 Jan. 2015; accepted 11 May 2015. Date of publication 12 Aug. 2015; date of current version 18 May 2016. Recommended for acceptance by H. Shen.
For information on obtaining reprints of this article, please send e-mail to: reprints@ieee.org, and reference the Digital Object Identifier below.
Digital Object Identifier no. 10.1109/TPDS.2015.2468223

shortest valid path for both serial and parallel jobs. A number of optimization measures are also developed to increase the scheduling efficiency of these proposed algorithms (i.e., to accelerate the solving process of finding the optimal co-scheduling solution). After these, the graph model and proposed algorithms are extended to co-scheduling parallel jobs on multi-processor machines.

It has been shown that the A*-search algorithm is able to effectively avoid unnecessary searches when finding optimal solutions. In this paper, an A*-search-based algorithm is also developed to combine the ability of the A*-search algorithm and the proposed optimization measures in terms of accelerating the solving process. Finally, a flexible approximation technique is proposed so that we can control the scheduling efficiency by setting the requirement for the solution quality.

We conduct experiments with real jobs to evaluate the effectiveness of the proposed co-scheduling algorithms. The results show that i) the proposed algorithms can find the optimal co-scheduling solution for both serial and parallel jobs, ii) the proposed optimization measures can significantly increase the scheduling efficiency, and iii) the proposed approximation technique is effective in the sense that it is able to balance the scheduling efficiency and the solution quality.

The remainder of this paper is organized as follows. Section 2 discusses related work. Section 3 formalizes the co-scheduling problem for both serial and MPP jobs, and presents a graph-based model for the problem. Section 4.1 presents methods and optimization measures to determine the optimal co-scheduling solution for serial jobs. Section 5 extends the methods proposed in Section 4.1 to incorporate MPP jobs and presents an optimization technique for the extended algorithm. In Section 6, we extend the graph-based model and proposed algorithms for multi-processor machines; Section 7 then adjusts the graph model and the algorithms for MTP jobs. Section 8 presents the A*-search-based algorithm; a clustering approximation technique is proposed in Section 9 to control the scheduling efficiency according to the required solution quality. Experimental results are presented in Section 10 and finally, Section 11 concludes the paper and presents avenues for future work.

2 Related Work

This section first discusses co-scheduling strategies proposed in the literature. Similar to the work presented in [4], our method needs to know the performance degradation of the jobs when they co-run on a multi-core machine. Therefore, this section also presents methods proposed for acquiring such performance degradation information.

2.1 Co-Scheduling Strategies

Many co-scheduling schemes have been proposed to reduce shared cache contention in multi-core processors. Different metrics can be used to capture resource contention, including cache miss rate (CMR), overuse of memory bandwidth, and performance degradation of co-running jobs. These schemes fall into the following two classes:

The first is co-scheduling schemes that aim to improve runtime scheduling by providing online scheduling solutions. The work in [5], [18], [19] develops co-schedulers that reduce the cache miss rate of co-running jobs; the fundamental idea is to uniformly distribute the jobs with high cache requirements across the processors. Wang et al. [20] demonstrate that cache contention can be reduced by rearranging the scheduling order of tasks.

The research cited above only considers the co-scheduling of serial jobs. In some cluster systems managed by conventional cluster management software such as PBS, the systems are configured in such a way that parallel and serial jobs cannot share different cores on the same chip. Such configurations are also seen in data centers, where a user can specify through job configuration files that co-scheduling with other jobs on different cores on the same chip is not permitted [21]. There are several reasons for doing this, including data security and improved performance, however, disallowing co-scheduling of parallel and serial jobs causes very poor resource utilization, especially as the number of cores in multicore machines increases.

As a result, considerable recent research [16], [14] has been dedicated to developing accurate and reliable prediction methodologies for performance interference. Coupled with these techniques for accurately predicting performance interference, popular cluster management systems have been developed to co-schedule parallel and serial jobs, aiming to improve resource utilization [16], [17], [21]. The work in [21] presents a characterization methodology called Bubble-Up to enable the accurate prediction of performance degradation (with an accuracy of 98-99 percent) due to interference in data centers. The research found in [16] applies classification techniques to accurately determine the impact of interference on performance for each job. A cluster management system called Quasar is then developed to increase resource utilization in data centers through co-scheduling. Quasar co-schedules parallel and serial jobs, using serial jobs to fill any cluster capacity unused by parallel jobs. Another platform called Mesos [17] has been designed for sharing commodity clusters between multiple diverse cluster management frameworks, such as Hadoop, Torque and Spark etc., aiming to improve cluster utilization. In Mesos, the tasks from different cluster management frameworks (e.g., MPI or serial jobs submitted to Torque, and MapReduce jobs submitted to Hadoop) can be co-located in the same multicore server.

A second class of co-scheduling scheme focuses on providing a basis for conducting performance analysis. It primarily aims to determine optimal co-scheduling performance in an offline manner, in order to provide a performance target for other co-scheduling systems. Extensive research is conducted in [4] to find co-scheduling solutions; the work models the co-scheduling problem for serial jobs as an integer programming problem, and then uses an existing IP solver to find the optimal co-scheduling solution. This research also provides a set of heuristic algorithms to determine near optimal co-scheduling. Co-scheduling studies in this category of solution only consider serial jobs, and primarily apply a heuristic approach to finding solutions.

The research presented in this paper falls into the second class of solution. Here a new offline method is developed to find the optimal co-scheduling solution for both serial and parallel jobs.

2.2 Acquiring Information on Performance Degradation

When a job co-runs with a set of other jobs, its performance degradation can be obtained either through prediction [22], [23], [24], [25] or through offline profiling [26].

Predicting performance degradation has been well studied in the literature [24], [25], [27], [28]. One of the best-known methods is Stack Distance Competition (SDC) [25], a method which uses a Stack Distance Profile (SDP) to record the hits and misses of each cache line when each process is running alone. The SDC model tries to construct a new SDP that merges the separate SDPs of individual processes that are to be co-run together. This model relies on the intuition that a process that reuses its cache lines more frequently will occupy more cache space than other processes. Based on this, the SDC model examines the cache hit count of each process's stack distance position. For each position, the process with the highest cache hit count is selected and copied into the merged profile. After the algorithm reaches the last position, the effective cache space for each process is computed based on the number of stack distance counters in the merged profile.

Offline profiling can obtain more accurate degradation information, although the process itself is more time consuming. Since the goal of our research is to find the optimal co-scheduling solutions offline, this method is also applicable in our work.

3 Formalizing the Co-Scheduling Problem

First, in Section 3.1, we briefly summarize the approach in [4] to formalizing the co-scheduling of serial jobs. Section 3.2 then formalizes the objective function for co-scheduling a mix of serial and MPP jobs, and Section 3.3 presents a graph model for the co-scheduling problem. This section focusses on single processor machines, i.e., all CPU cores reside on the same chip.

3.1 Formalizing the Co-Scheduling of Serial Jobs

The work in [4] shows that due to resource contention, co-running jobs generally run slower on a multi-core processor than if they run alone. This performance degradation is called the *co-run degradation*. When a job i co-runs with the jobs in a job set S, the co-run degradation of job i can be formally defined as in Eq. (1), where CT_i is the computation time when job i runs alone, S is a set of jobs and $CT_{i,S}$ is the computation time when job i co-runs with the set of jobs in S. Typically, the value of $d_{i,S}$ is a non-negative value:

$$d_{i,S} = \frac{CT_{i,S} - CT_i}{CT_i}. \tag{1}$$

In the co-scheduling problem considered in [4], n serial jobs are allocated to multiple u-core processors so that each core is allocated one job. m denotes the number of u-core processors needed, which can be calculated as $\frac{n}{u}$ (if n cannot be divided by u, we can simply add $(u - n \bmod u)$ imaginary jobs which have no performance degradation with any other jobs). The objective of the co-scheduling problem is to find the optimal way to partition n jobs into m u-cardinality sets, so that the sum of $d_{i,S}$ in Eq. (1) over all n jobs is minimized, which is expressed in Eq. (2):

$$min \sum_{i=1}^{n} d_{i,S}. \tag{2}$$

3.2 Formalizing the Co-Scheduling of Serial and Parallel Jobs

We first model the co-scheduling of embarrassingly parallel (*PE*) jobs (i.e., those with no dependency and communications between parallel processes), and then extend the model to co-schedule parallel jobs with inter-process communications (denoted by the term *PC*). An example of a PE job is parallel Monte Carlo simulation [29]. In such an application, multiple slave processes are running simultaneously to perform the Monte Carlo simulations. After a slave process completes its portion of work, it sends the result back to the master process. After the master process receives the results from all slaves, it reduces the final result (i.e., by calculating the average). An example of a PC job is an MPI application for matrix multiplication. In both types of parallel job, the completion time of a job is determined by the slowest process in the job.

Eq. (2) cannot be used to satisfy our objective of finding the optimal co-scheduling of parallel jobs. This is because Eq. (2) will sum the degradation experienced by each process of a parallel job. However, as explained above, the completion time of a parallel job is determined by its slowest process. In the case of PE jobs, a larger degradation of a process indicates a longer execution time for that process. Therefore, no matter how small the degradation other processes have, the execution flow in the parallel job must wait until the process with the largest degradation completes. Thus, the completion time of a parallel job is determined by the largest degradation experienced by all its processes; this is denoted by Eq. (3), where $d_{ij,S}$ is the degradation (measured in time) of the jth process, p_{ij}, in parallel job p_i, when p_{ij} co-runs with the jobs in the job set S. Therefore, if the set of jobs to be co-scheduled includes both serial jobs and PE jobs, the total degradation should be calculated using Eq. (4), where n is the number of all serial jobs and parallel processes, P is the number of parallel jobs, S_i and S_{ij} are the set of co-running jobs that includes job p_i and parallel process p_{ij}; $S_i - \{p_i\}$ and $S_{ij} - \{p_{ij}\}$ are then the set of jobs excluding p_i and p_{ij}, respectively. Now the objective is to find such a partition of n jobs/processes into m u-cardinality sets such that Eq. (4) is minimized

$$max_{p_{ij} \in p_i}(d_{ij,S}) \tag{3}$$

$$\sum_{i=1}^{P}(max_{p_{ij} \in p_i}(d_{ij,S_{ij}-\{p_{ij}\}})) + \sum_{i=1}^{n-P} d_{i,S_i-\{p_i\}}. \tag{4}$$

In the case of PC jobs, the slowest process in a parallel job is determined by both performance degradation and inter-process dependencies (i.e., communication time). Therefore, we define the *communication-combined degradation (CCD)*, which is expressed using Eq. (5), where $c_{ij,S}$ is the communication time taken by parallel process p_{ij} when p_{ij} co-runs with the processes in S. As with $d_{ij,S}$, $c_{ij,S}$ also varies with the co-scheduling solutions. We can see from Eq. (5) that for all process in a

parallel job, those with the largest sum of performance degradation (in terms of computation time) and communication has the greatest value of $d_{ij,S}$, since the computation time of all processes (i.e., CT_{ij}) in a parallel job is the same when a parallel job is evenly balanced. Therefore, the greatest $d_{ij,S}$ of all processes in a parallel job should be used as the communication-combined degradation for that parallel job.

When the set of jobs to be co-scheduled includes both serial jobs and PC jobs, we use Eq. (5) to calculate $d_{ij,S}$ for each parallel process p_{ij}, and then we replace $d_{ij,S}$ in Eq. (4) with that calculated by Eq. (5) to satisfy the objective of co-scheduling a mix of serial and PC jobs

$$d_{ij,S} = \frac{CT_{ij,S} - CT_{ij} + c_{ij,S}}{CT_{ij}}. \quad (5)$$

3.3 A Graph Model for Co-Scheduling

In this research we propose a graph-based approach to finding the optimal co-scheduling solution for both serial and parallel jobs. In this section, the graph model is first presented and, following this, intuitive strategies to solve the graph model are then discussed.

3.3.1 The Graph Model

As formalized in Section 3.1, the objective of solving the co-scheduling problem for serial jobs is to find a way to partition n jobs, $j_1, j_2, \ldots, j_n$, into m u-cardinality sets, so that the total degradation of all jobs is minimized. The number of all possible u-cardinality sets is $\binom{n}{u}$. In this paper, a graph is constructed, called the co-scheduling graph, to model the co-scheduling problem for serial jobs (we will discuss in Section 5 how to use this graph model to handle parallel jobs). There are $\binom{n}{u}$ nodes in the graph and a node corresponds to a u-cardinality set. Each node represents a u-core processor with u jobs assigned to it. The ID of a node consists of a list of the IDs of the jobs in the node. In the list, the job IDs are always placed in an ascending order. The weight of a node is defined as the total performance degradation of the u jobs in the node. The nodes are organized into multiple levels in the graph. The ith level contains all nodes in which the ID of the first job is i. At each level, the nodes are placed in ascending order of their ID's. A *start* node and an *end* node are added as the first (level 0) and the last level of the graph, respectively. The weights of the start and the end nodes are both 0. The edges between the nodes are dynamically established as the algorithm of finding the optimal solution progresses. Such organization of the graph nodes will be used to help reduce the time complexity of the co-scheduling algorithms proposed in this paper. Fig. 1 illustrates the case where six jobs are co-scheduled to dual-core processors. The figure also shows how to code the node IDs in the graph and how to organize the nodes into different levels. Note that for clarity we do not draw all edges.

In the constructed co-scheduling graph, a path from the start to the end node forms a co-scheduling solution if the path does not contain duplicated jobs; this we call a *valid path*. The distance of a path is defined as the sum of the weights of all nodes on the path. Finding the optimal co-scheduling solution is equivalent to finding the shortest valid path from the start to the end node. It is straightforward to know that a valid path contains at most one node from each level in the graph.

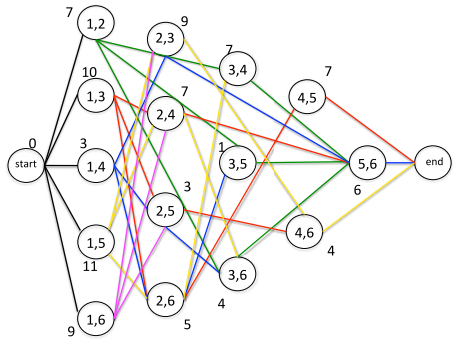


Fig. 1. The exemplar co-scheduling graph for co-scheduling six jobs on dual-core machines; the list of numbers in each node is the node ID; A number in a node ID is a job ID; The edges of the same color form the possible co-scheduling solutions; The number next to the node is the node weight, i.e., total degradation of the jobs in the node.

3.3.2 Intuitive Strategies to Solve the Graph Model

We first try to solve the graph model using Dijkstra's shortest path algorithm [30]. However, we find that Dijkstra's algorithm can not be directly applied to find the correct solution. This can be illustrated using the example in Fig. 1. In order to quickly reveal the problem, let us consider only five nodes in Fig. 1, $\langle 1,5\rangle, \langle 1,6\rangle, \langle 2,3\rangle, \langle 4,5\rangle, \langle 4,6\rangle$. Assume the weights of these nodes are 11, 9, 9, 7 and 4, respectively. Out of all these five nodes, there are two valid paths reaching node $\langle 2,3\rangle$: $< \langle 1,5\rangle, \langle 2,3\rangle >$ and $\langle\langle 1,6\rangle, \langle 2,3\rangle\rangle$. Since the distance of $\langle\langle 1,6\rangle, \langle 2,3\rangle\rangle$, which is 18, is shorter than that of $\langle\langle 1,5\rangle, \langle 2,3\rangle\rangle$, which is 20, the path $\langle\langle 1,6\rangle, \langle 2,3\rangle\rangle$ will not been examined again according to Dijkstra's algorithm. In order to form a valid schedule, the path $\langle\langle 1,6\rangle, \langle 2,3\rangle\rangle$ has to connect to node $\langle 4,5\rangle$ to form a final valid path $\langle\langle 1,6\rangle, \langle 2,3\rangle, \langle 4,5\rangle\rangle$ with the distance of 25. However, we can see that $\langle\langle 1,5\rangle, \langle 2,3\rangle, \langle 4,6\rangle\rangle$ is also a valid schedule and its distance is less than that of $\langle\langle 1,6\rangle, \langle 2,3\rangle, \langle 4,5\rangle\rangle$. However, the schedule $\langle\langle 1,5\rangle, \langle 2,3\rangle, \langle 4,6\rangle\rangle$ is dismissed by Dijkstra's algorithm during the search for the shortest path.

The main reason for this problem is that Dijkstra's algorithm only records the shortest subpaths reaching a certain node and dismisses other optional subpaths. This is fine for searching for the shortest path, but in our problem we have to search for the shortest *valid* path. Dijkstra's algorithm searches up to a certain node in the graph, recording only the shortest subpath up to that node. As such, not all nodes among the unsearched nodes can form a valid schedule with the current shortest subpath, which may cause the shortest subpath to connect to nodes with bigger weights. As illustrated, a subpath that has been dismissed by Dijkstra's algorithm may be able to connect to the unsearched nodes with smaller weights and therefore generate a shorter final valid path.

In order to address the above problem, an intuitive strategy is to revise Dijkstra's algorithm so that it will not dismiss any subpath, i.e., to allow the algorithm to record every visited subpath. Then, the path with the smallest distance among all examined and complete paths is the optimal co-scheduling result. This strategy is equivalent to enumerating all possible subpaths in the graph. The time complexity of

such a strategy is very high, which will be discussed when we compare it with the SVP algorithm presented in Section 4.1. This time complexity motivates us to design more efficient algorithms to find the shortest valid path. In the next section, we propose a more efficient algorithm to find the shortest valid path; this we call the SVP algorithm.

4 Shortest Valid Path for Serial Jobs

4.1 The SVP Algorithm

In order to tackle the problem highlighted in the application of Dijkstra's algorithm, the following *dismiss strategy* is adopted by the SVP algorithm:

SVP records all jobs that an examined sub-path contains. Assume a set of sub-paths, S, each of which contains the same set of jobs (the set of graph nodes that these paths traverse are different). SVP only keeps the path with the smallest distance and other paths are dismissed in further searches for the shortest path.

This strategy will clearly demonstrate improved efficiency compared with the intuitive, enumerative strategy, i.e., the SVP algorithm examines far fewer subpaths than the enumerative strategy. This is because, for all different subpaths that contain the same set of jobs, only one subpath (the shortest) will spawn further subpaths and all other subpaths will be discarded.

The SVP algorithm is outlined in Algorithm 1. The main differences between SVP and Dijkstra's algorithm lie in three aspects: 1) The invalid paths, which contain the duplicated jobs, are disregarded by SVP during the searching; 2) The dismiss strategy is implemented; 3) No edges are generated between nodes before SVP starts and the node connections are established as SVP progresses. In this way, only the node connections spawned by the recorded subpaths will be generated and this will therefore further improve performance.

Algorithm 1: The SVP Algorithm

```
1: SVP(Graph)
2:   v.jobset = {Graph.start}; v.path = Graph.start;
     v.distance = 0; v.level = 0;
3:   add v into Q;
4:   Obtain v from Q;
5:   while Graph.end is not in v.jobset
6:       for every level l from v.level + 1 to
     Graph.end.level do
7:           if job l is not in v.jobset
8:               valid_l = l;
9:               break;
10:      k = 1;
11:      while k ≤ C(n − valid_l, u − 1)
12:          if node_k.jobset ∩ v.jobset = φ
13:              distance = v.distance + node_k.weight;
14:              J = v.jobset ∪ node_k.jobset;
15:              if J is not in Q
16:                  Create an object u for J;
17:                  u.jobset = J;
18:                  u.distance = distance;
19:                  u.path = v.path + node_k;
20:                  u.level = node_k.level
21:                  Add u into Q;
22:              else
23:                  Obtain u' whose u'.jobset is J;
24:                  if distance < u'.distance
25:                      u'.distance = distance;
26:                      u'.path = v.path + node_k;
27:                      u'.level = node_k.level
28:          k+ = 1;
29:      Remove v from Q;
30:      Obtain the v with smallest v.distance from Q;
31: return v.path as the shortest valid path;
```

The time complexity of Algorithm 1 is $O(\sum_{i=1}^{m} \binom{n-i}{i\cdot(u-1)} \cdot ((n-u+1) + \frac{\binom{n}{u}}{n-u+1} + log\binom{n}{u}))$, where m is the number of u-core machines required to run n jobs. The detailed analysis of the time complexity is presented in the supplementary notes, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TPDS.2015.2468223.

4.2 Further Optimization of SVP

One of the most time-consuming steps in Algorithm 1 is to scan every node in a valid level to find a valid node for a given subpath $v.path$ (Line 11 and 28). Theorem 1 is introduced to reduce the time spent in finding a valid node in a valid level. The rational behind Theorem 1 is that once the algorithm locates a node that contains a job appearing in $v.path$, the number of nodes that follow that node and also contains that job can be calculated, since the nodes are arranged in ascending order of node ID. These nodes are all invalid and can therefore be ignored by the algorithm.

Theorem 1. *Given a subpath $v.path$, assume that level l is a valid level and node* k *(assume node* k *contains the jobs, $j_1, \ldots, j_i, \ldots, j_u$) is the first node that is found to contain a job (assume the job is j_i) appearing in $v.path$. Then, job j_i must also appear in the next $\binom{n-j_i}{u-i}$ nodes at that level.*

Proof. Since the graph nodes at a level are arranged in ascending order of node ID, the number of nodes whose ith job is j_i equals the number of possibilities of mapping the jobs whose IDs are greater than j_i to $(u-i)$ positions, which can be calculated by $\binom{n-j_i}{u-i}$. □

Based on Theorem 1, the Optimal SVP (O-SVP) algorithm is proposed to further optimize SVP. The only difference between O-SVP and SVP is that in the O-SVP algorithm, when the algorithm gets to an invalid node, instead of moving to the next node, it calculates the number of nodes that can be skipped and jumps to a valid node. Effectively, O-SVP can find a valid node in the time O(1). Therefore, the time complexity of O-SVP is $O(\sum_{i=1}^{m}\binom{n-i}{i\cdot(u-1)}\cdot((n-u+1)+log\binom{n}{u}))$. The outline algorithm for O-SVP is omitted in this paper.

In summary, SVP accelerates the solving process over the enumerative method by reducing the length of Q in the algorithm, while O-SVP further accelerates SVP by reducing the time spent in finding a valid node in a level.

5 Shortest Valid Path for Parallel Jobs

The SVP algorithm presented in the last section considers only serial jobs. This section addresses the co-scheduling of both serial and parallel jobs. Section 5.1 presents how to handle embarrassingly parallel (PE) jobs, while Section 5.2 further extends the work in Section 5.1 to handle parallel jobs with inter-process communications.

5.1 Co-scheduling PE Jobs

In Section 5.1.1, the SVPPE (SVP for PE) algorithm is proposed, extending SVP to incorporate PE jobs. Section 5.1.2 presents the optimization techniques used to accelerate the solving of SVPPE.

5.1.1 The SVPPE Algorithm

When Algorithm 1 finds a valid node, it calculates the new distance after the current path extends to that node (Line 13). This calculation is adequate for serial jobs, but cannot be applied to parallel jobs. As discussed in Section 3.2, the completion time of a parallel job is determined by Eq. (5). In order to incorporate parallel jobs, we can treat each process of a parallel job as a serial job (therefore the graph model remains the same) and extend the SVP algorithm simply by changing the means by which we calculate the path distance.

In order to calculate the performance degradation for PE jobs, a number of new attributes are introduced. First, two new attributes are added to an object v in Q. One attribute stores the total degradation of all serial jobs on $v.path$ (denoted by $v.dg\_serial$). The other attribute is an array, in which each entry stores the largest degradation of all processes of a parallel job p_i on $v.path$ (denoted by $v.dg\_p_i$). Second, two similar new attributes are also added to a graph node $node_k$. One stores the total degradation of all serials jobs in $node_k$ (denoted by $node_k.dg\_serial$). The other is also an array, in which each entry stores the degradation of a parallel job p_i in $node_k$ (denoted by $node_k.dg\_p_i$).

SVPPE is outlined in Algorithm 2. The only differences between SVPPE and SVP are: 1) Changing the means by which we calculate the subpath distance (Line 13-19 in Algorithm 2), and 2) Updating the newly introduced attributes for the case where J is not in Q (Line 28-30) and the case otherwise (Line 38-40).

Algorithm 2: The SVPPE algorithm

```
1:     SVPPE(Graph, start, end):
2-12:    ... //same as Line 2-12 in Algorithm 1;
13:        total_dg_serial = v.dg_serial + node_k.dg_serial
14:        for every parallel job, p_i, in node_k:
15:          if p_i in v.jobset:
16:            dg_p_i=max(v.dg_p_i, node_k.dg_p_i);
17:          else
18:            dg_p_i = node_k.dg_p_i;
19:        distance = ∑ dg_p_i + total_dg_serial;
20-26:       ... //same as Line14-20 in Algorithm 1
27:        u.dg_serial = total_dg_serial;
28:        for every parallel job, p_i, in node_k do
29:          u.dg_p_i = dg_p_i;
30-36:       ... //same as Line21-27 in Algorithm 1
37:          u'.dg_serial = total_dg_serial;
38:        for every parallel job, p_i, in node_k do
39:          u'.dg_p_i = dg_p_i;
40-43:     ... //same as Line28-31 in Algorithm 1
```

The maximum number of iterations of all for-loops (Line 14, 28 and 38) is u, because there are at most u jobs in a node. Each iteration takes constant time. Therefore, the worst-case complexity of computing the degradation (the first for-loop) and updating the attributes (the two other for-loops) are $O(u)$. Therefore, combined with the time complexity of Algorithm 1, the worst-case complexity of Algorithm 2 is $O(\sum_{i=1}^{m}\binom{n-i}{i\cdot(u-1)}\cdot((n-u+1)+u\cdot\frac{\binom{n}{u}}{n-u+1}+log\binom{n}{u}))$.

5.1.2 Process Condensation for Optimizing SVPPE

An obvious optimization measure for SVPPE is to skip the invalid nodes in a similar way to that given in Theorem 1, which is not repeated in this section. This section focuses on proposing another important optimization technique that is only applicable to PE jobs. The optimization technique is based on the following observation: different processes of a parallel job should have the same mutual effect with other jobs. So it is unnecessary to differentiate different processes of a parallel job, treating them as individual serial jobs.

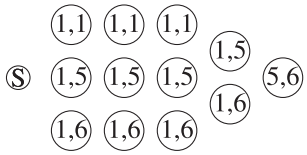


Fig. 2. The graph model for a mix of serial and parallel jobs.

Therefore, the optimization technique, which is called the *process condensation technique* in this paper, labels a process of a parallel job using its job ID, that is, it treats different processes of a parallel job as the same serial job. We illustrate this below using Fig. 1. Now assume the jobs labelled 1, 2, 3 and 4 are four processes of a parallel job, whose ID is set to be 1. Fig. 1 can be transformed to Fig. 2 after deleting the same graph nodes in each level (the edges are omitted). Compared with Fig. 1, it can be seen that the number of graph nodes in Fig. 2 is reduced. Therefore, the number of subpaths that need to be examined and consequently the time spent in finding the optimal solution is significantly reduced.

We now present the O-SVPPE (Optimal SVPPE) algorithm, which adjusts SVPPE so that it can find the shortest valid path in the optimized co-scheduling graph. The only difference between O-SVPPE and SVPPE is that a different mechanism is used to find 1) the next valid level and 2) a valid node in a valid level for parallel jobs.

Lines 6-9 in Algorithm 1 capture the mechanism used by SVPPE to find the next valid level. In O-SVPPE, for a given level l, if job l is a serial job, the condition of determining whether level l is valid is the same as that in SVPPE. However, since the same job ID is now used to label all processes of a parallel job, the condition of whether a job ID appears on the given subpath can no longer be used to determine a valid level for parallel jobs. The revised method is discussed next.

Several new attributes are added for the optimized graph model. $proc_i$ denotes the number of processes that parallel job p_i has. For a given subpath $v.path$, $v.proc_i$ is the number of times a process of parallel job p_i appears on $v.path$. $v.jobset$ is now a bag (not a set) of job IDs that appear on $v.path$, that is, there are $v.proc_i$ instances of that parallel job in $v.jobset$. As in the case of serial jobs, the adjusted $v.jobset$ is used to determine whether two subpaths consist of the same set of jobs (and parallel processes). A new attribute, $node_k.jobset$, is also added to a graph node $node_k$, where $node_k.jobset$ is also a bag of job IDs that are in $node_k$. $node_k.proc_i$ is the number of processes of parallel job p_i that are in $node_k$. $node_k.serialjobset$ is a set of all serial jobs in $node_k$.

Theorem 2 gives the condition of determining whether a level is a valid level for a given path.

Theorem 2. *Assume job l is a parallel job. For a given subpath $v.path$, level l (l starts from $v.level+1$) is a valid level if $v.proc_l < proc_l$. Otherwise, level l is not a valid level.*

Proof. Assume the jobs are co-scheduled on u-core machines. Let U be the bag of jobs that includes all serial

jobs and parallel jobs (the number of instances of a parallel job in U equals the number of processes that that a job has). Let $D = U - v.jobset$. X denotes all possible combinations of selecting $u-1$ jobs from D. Because of the way that the nodes are organized in the graph, the last $u-1$ jobs of the nodes in level l must include all possible combinations of selecting $u-1$ jobs from a set of jobs whose ID are in the range of l to n (n is the number of jobs to be co-scheduled), which is denoted by Y. Then we must have $X \cap Y \neq \phi$. This means that as long as the ID of the first job in the nodes in level l is not making the nodes invalid, which can be determined by the condition $v.proc_l < proc_l$, we must be able to find a node in level l that can append to $v.path$ and form a new valid subpath. □

After a valid level is found, O-SVPPE needs to find a valid node in that level. When there are both parallel and serial jobs, O-SVPPE uses two conditions to determine a valid node: 1) the serial jobs in the node do not appear in $v.jobset$, and 2) $\forall$ parallel job p_i in the node, $v.proc_i + node_k.proc_i \leq proc_i$.

O-SVPPE is outlined in Algorithm 3, in which Lines 7-13 implement the way of finding a valid level and Line 16 checks whether a node is valid, as discussed above.

Algorithm 3: The O-SVPPE algorithm

```
1:  O-SVPPE(Graph)
2-6:  ... //same as Line 2-6 in Algorithm 1;
7:      if job l is a serial job
8-10:     ...// same as Line 7-9 in Algorithm 1;
11:     else if v.proc_l < proc_l
12:       valid_l = l;
13:       break;
14-15:  ... //same as Line 10-11 in Algorithm 1
16:     if node_k.serialjobset ∩ v.jobset = φ & ∀p_i,
    v.proc_i + node_k.proc_i ≤ proc_i
17-48:      ... //same as Line13-44 in Algorithm 2
```

5.2 Co-Scheduling PC Jobs

We now extend the SVPPE algorithm to handle PC jobs, which is called SVPPC (SVP for PC jobs). We first model the communication time, $c_{ij,S}$, in Eq. (5) and then adjust SVPPE to handle PC jobs. Moreover, since the further optimization technique developed for PE jobs, i.e., the O-SVPPE algorithm, presented in Section 5.1.2 cannot be directly applied to PC jobs, the O-SVPPE algorithm is extended to handle PC jobs in Section 5.2.2, and termed O-SVPPC.

5.2.1 Modelling Communications in PC Jobs

$c_{ij,S}$ can be modelled using Eq. (6), where γ_{ij} is the number of neighbouring processes that process p_{ij} has, corresponding to the decomposition performed on the data set to be calculated by the parallel job; $\alpha_{ij}(k)$ is the amount of data that p_{ij} needs to communicate with its kth neighbouring process; B is the bandwidth for inter-processor communications; $b_{ij}(k)$ is p_{ij}'s kth neighbouring process, and $\beta_{ij}(k,S)$ is 0 or 1 as defined in Eq. (6b). $\beta_{ij}(k,S)$ is 0 if $b_{ij}(k)$ is in the job set S co-running with p_{ij}. Otherwise, $\beta_{ij}(k,S)$ is 1. Essentially, Eq. (6) calculates the total amount of data that p_{ij} needs to communicate, which is then divided by the bandwidth B to obtain the communication time. Note that p_{ij}'s communication time can be determined by only examining which neighbouring processes are not in the job set S co-running with p_{ij}, no matter which machines that these neighbouring processes are scheduled to. In the supplementary file, available online, to this paper, an example is given to illustrate the calculation of $c_{ij,S}$:

$$c_{ij,S} = \frac{1}{B}\sum_{k=1}^{\gamma_{ij}}(\alpha_{ij}(k) \cdot \beta_{ij}(k,S)) \tag{6a}$$

$$\beta_{ij}(k,S) = \begin{cases} 0 & \text{if } b_{ij}(k) \in S \\ 1 & \text{if } b_{ij}(k) \notin S. \end{cases} \tag{6b}$$

We now adjust SVPPE to incorporate the PC jobs. In the graph model for serial and PE jobs, the weight of a graph node is calculate by summing up the weights of the individual jobs/processes, which is the performance degradation. When there are PC jobs, a process belongs to a PC job, the weight of a process p_{ij} in a PC job should be calculated by Eq. (5) instead of Eq. (1). The remainder of the SVPPC algorithm is exactly the same as SVPPE.

5.2.2 Communication-Aware Process Condensation for Optimizing SVPPC

The reason why the process condensation technique developed for PE jobs cannot be directly applied to PC jobs is because different processes in a PC job may have different communication patterns and therefore cannot be treated as identical processes. After carefully examining the characteristics of the typical inter-process communication patterns, a communication-aware process condensation technique is developed to accelerate the solving process of SVPPC, which is called Optimized SVPPC (O-SVPPC) in this paper.

We can construct the co-scheduling graph model as we did in Fig. 1 for finding the optimal solution of co-scheduling PC and serial jobs. We then define the *communication property* of a parallel job in a graph node as the number of communications that the processes of the parallel job in the graph node has to perform in each decomposition direction with other nodes. In the communication-aware process condensation, multiple graph nodes in the same level of the graph model can be condensed to one node if the following two conditions are met: 1) these nodes contain the same set of serial jobs and parallel jobs, and 2) the communication properties of all PCs in these nodes are the same. A concrete example is presented in the supplementary file, available online, to illustrate the condensation process.

6 Co-Scheduling Jobs on Multi-Processor Computers

In order to add more cores to a multicore computer, there are two general approaches: 1) increase the number of cores on a processor chip and 2) install more processors, with the number of cores in each processor remaining unchanged; both approaches are often simultaneously applied.

The co-scheduling graph previously presented is for multicore machines each of which contains a single multi-core processor, which we term a single processor multi-core machine (or a single processor for short). If there are multiple multi-core processors in a machine (which we

term a multi-processor machine), the resource contention, such as cache contention, is different. For example, only the cores on the same processor share the last-level cache on the chip, while the cores on different processors do not compete for cache. In a single processor machine, the job-to-core mapping does not affect the tasks' performance degradation. This is not the case in a multi-processor machine, as illustrated in the following example.

Consider a machine with two dual-core processors (processors p_1 and p_2) and a co-run group with four jobs $(j_1, \ldots, j_4)$. Now consider two job-to-core mappings. In the first mapping, jobs j_1 and j_2 are scheduled on processor p_1, while j_3 and j_4 are scheduled on p_2. In the second mapping, jobs j_1 and j_3 are scheduled on processor p_1, while j_2 and j_4 are scheduled on p_2. The two mappings may generate different total performance degradations for this co-run group. In the co-scheduling graph in previous sections, a graph node corresponds to a possible co-run group in a machine, which is associated with a single performance degradation value. This holds for a single processor machine. As shown in the above discussions, however, a co-run group may generate different performance degradations in a multi-processor machine, depending on the job-to-core mapping within the machine. This section presents how to adjust the methods presented in previous sections to find the optimal co-scheduling solution in multi-processor machines.

A straightforward method is to generate multiple nodes in the co-scheduling graph for a possible co-run group, with each node having a different weight that equals a different performance degradation value (which is determined by the specific job-to-core mappings). We call this method MNG (Multi-Node for a co-run Group) method. For a machine with p processors with each processor having u cores, it can be calculated that there are $\frac{\prod_{i=0}^{p-1}\binom{(p-i)\cdot u}{u}}{p!}$ different job-to-core mappings that may produce different performance degradations. The algorithms presented in previous sections can be used to find the shortest path in this co-scheduling graph, where the shortest path must correspond to the optimal co-scheduling solution on the multi-processor machines. In this straightforward solution, however, the scale of the co-scheduling graph (i.e., the number of graph nodes) increases $\frac{\prod_{i=0}^{p-1}(p-i)\cdot uu}{p!}$ fold, and consequently the solving time increases significantly compared with that for the case of single processor machines.

We now propose a method, called the Least Performance Degradation (LPD) method, to construct the new co-scheduling graph. Using this method, the optimal co-scheduling solution for multi-processor machines can be computed without increasing the scale of the co-scheduling graph. The LPD method is explained below.

As discussed above, in the case of multi-processor machines, a co-run group may produce different performance degradation in a multi-processor machine. Instead of generating multiple nodes (each being associated with a different weight, i.e., a different performance degradation value) in the co-scheduling graph for a co-run group, the LPD method constructs the co-scheduling graph for multi-processor machines in the following way: *A node is generated for a co-run group and the weight of the node is set to be the smallest performance degradation among all possible performance degradations generated by the co-run group. The remainder of the construction process is exactly the same as that for the case of single processor machines.*

Theorem 3 proves that from the co-scheduling graph constructed by the LPD method, the algorithms proposed in previous sections for the case of single processor machines still obtain the optimal co-scheduling solution on multi-processor machines.

Theorem 3. *Assume the jobs are to be co-scheduled on multi-processor machines. Using the LPD method defined above to construct the co-scheduling graph, the algorithms that have been proposed to find the optimal co-scheduling solutions on single processor machines will still find the optimal co-scheduling solutions on multi-processor machines.*

Proof. We can use the MNG method or the LPD method to construct the co-scheduling graph for the case of multi-processor machines. It has been discussed above that when using the MNG method to construct the graph, the algorithms proposed for single processor machines can still find the optimal co-scheduling solution on multi-processor machines. In the co-scheduling graph constructed by the MNG method, multiple nodes are created for a possible co-run group, each with a different weight. If a co-run group appears in the final shortest path obtained by the algorithms, the path must only contain the node with the least weight for the co-run group. Other nodes with higher weights would have been dismissed in the process of searching for the shortest path. Therefore, the shortest path obtained from the co-scheduling graph constructed by the LPD method must be the same as that from the graph by the LPD method. Consequently, the theorem holds. □

7 Co-Scheduling Multi-Thread Jobs

A parallel job considered so far in this paper is one consisting of multiple processes, such as an MPI job. In this section, we adapt the proposed graph model and algorithms so that they can handle parallel jobs consisting of multiple threads, such as OpenMP jobs. We call the former parallel jobs Multi-Process Parallel jobs and the latter Multi-Thread Parallel jobs.

In the co-scheduling graph, a thread in an MTP job is treated in the same way as a parallel process in an MPP job. Compared with MPP jobs, however, MTP jobs have the following different characteristics: 1) multiple threads of a MTP job must reside in the same node, and 2) the communication time between threads can be largely ignored. Accordingly, the co-scheduling graph model is adjusted as follows to handle the MTP jobs. For each node (i.e., every possible co-run group) in the co-scheduling graph, we check whether all threads belonging to the MTP are on the node. If not, the node is deleted from the graph since it does not satisfy the condition that all threads of a MTP job must reside in the same node. We call the above process the validity check for MTP jobs.

Since the communication time between the threads in MTP jobs can be ignored, the performance degradation of a MTP job can be calculated using Eq. (3) that is used to

compute the performance degradation of a PE job. Also, since the communication time of an MTP job is not considered, an intuitive method to find the optimal co-scheduling solution in the presence of MTP jobs is to use the algorithm for handling PE jobs, i.e., Algorithm 3. However, after closer inspection into the features of MTP jobs, it is apparent that Algorithm 3 can be adjusted to improve the performance of managing MTP jobs, a feature which is explained next.

After the validity check for MTP jobs, all threads belonging to a MTP job must only appear in the same graph node. Therefore, there is no need to perform the process condensation as we do in the presence of PE jobs. Consequently, the SVPPE algorithm (i.e., Algorithm 2) can be used to handle MTP jobs. Next, when the current path expands to a new node in the SVPPE Algorithm, for each parallel job p_i in the new node, SVPPE needs to check whether p_i appears in the current path. However, all threads in a MTP job only reside in the same node. Therefore, if a new node that the current path tries to expand to contains an MTP job, it is unnecessary to check whether threads of the MTP job appear in the current path.

In order to differentiate this approach from SVPPE, the algorithm for finding the optimal co-scheduling solution for the mix of serial and MTP jobs is denoted as SVPPT (where T stands for thread). The only difference between SVPPT and SVPPE is that Lines 15-17 in SVPPE (i.e., Algorithm 2) are removed from SVPPT.

From the above discussions, we can conclude that it is more efficient to find the optimal co-scheduling solution for MTP jobs than for PE jobs. This is because 1) the number of nodes in the co-scheduling graph for SVPPT is much less than that for PE jobs (because of the validity check for MTP jobs) and 2) SVPPT does not execute Lines 15-17 in SVPPE.

Note that the method discussed above for handling MTP jobs is applicable to both single processor machines and multi-processor machines, as defined previously.

8 The A*-Search-Based Algorithm

The dismiss strategy designed for the SVP algorithm in Section 4.1 and the optimization strategies developed in O-SVPPE and O-SVPPC can avoid unnecessary searches in the co-scheduling graph. It has been shown that the A*-search algorithm is also able to find the optimal solution and during the searching, effectively prune the graph branches that will not lead to the optimal solution. In order to further accelerate the solving process, an A*-search-based algorithm is developed in this section to combine the ability of avoiding the unnecessary searches in the traditional A*-search algorithm and the algorithms presented in this paper so far (SVP, O-SVP, O-SVPPE and O-SVPPC).

This section presents how to design the A*-search-based algorithm to find the optimal co-scheduling solution in the co-scheduling graph. We only consider the co-scheduling of serial and PC jobs for the sake of generality. The presented A*-search-based algorithm is called SVPPC-A*. SVP-A* (i.e., co-scheduling serial jobs), SVPPE-A* (i.e., co-scheduling both serial and PE jobs) and SVPPT-A* can be developed in a similar way.

The traditional A*-search algorithm, which is briefly overviewed in the supplementary notes, available online, cannot be directly applied to obtain the optimal co-scheduling solution for the same reasons discussed in the presentation of the SVP and the SVPPE algorithms; namely, i) the optimal co-scheduling solution in the constructed co-scheduling graph corresponds to the shortest *valid* path, not the shortest path, and ii) since the jobs to be scheduled contain parallel jobs, the distance of a path is not the total weights of the nodes on the path, as calculated by the traditional A*-search algorithm.

Three functions are defined in the traditional A*-search algorithm. Function $g(v)$ is the actual distance from the start node to node v and $h(v)$ is the estimated length from v to the end node, while $f(v)$ is the sum of $g(v)$ and $h(v)$. In SVPPC-A*, we use the exactly same methods proposed for the SVP algorithm (i.e., the dismiss strategy) to handle and expand the valid subpaths and avoid the unnecessary searches. Also, we use the method proposed for the SVPPC algorithm to calculate the distance of the subpaths (i.e., Eq. (3) and Eq. (5)) that contain the PC jobs. This technique can be used to obtain the value of $g(v)$. Note that the communication-aware process condensation technique proposed in Section 5.2.2 can also be used to accelerate SVPPC-A*.

The estimation of $h(v)$ is one of the most critical parts in designing an A*-search algorithm. The following two properties reflect the importance of $h(v)$ [4]: i) The result of an A* search is optimal if the estimation of $h(v)$ is not higher than the lowest cost to reach the end node, and ii) the closer the result of $h(v)$ is from the lowest cost, the more effective A* search is in pruning the search space.

Therefore, in order to find the optimal solution, the $h(v)$ function must satisfy the first property. In our problem, if there are q jobs on the path corresponding to $g(v)$, then the aim of setting the $h(v)$ function is to find a function of the remaining $n-q$ jobs such as the value of the function is less than the shortest distance from node v to the end node. The following two strategies are proposed to set the $h(v)$ function.

Strategy 1 for setting $h(v)$: Assume node v is in level l, we construct a set R that contains all the nodes from $l+1$ to the last level in the co-scheduling graph, and sort these nodes in ascending order of their weights. Then, regardless of the validity, the first $(n-q)/u$ (u is the number of cores) nodes are selected from R to form a new subpath; the distance of this subpath is $h(v)$.

Strategy 2 for setting $h(v)$: Assume node v is in level l. We find all valid levels from level $l+1$ to the last level in the co-scheduling graph. The total number of valid levels obtained must be $(n-q)/u$. We then obtain the node with the least weight from each valid level. $(n-q)/u$ nodes will be obtained. We use these $(n-q)/u$ nodes to form a new subpath and use its distance as $h(v)$.

It is easy to prove that $h(v)$, obtained through the above strategies, must be less than the actual shortest distance from v to the end node; this is the case because it uses the nodes with the smallest weights from all remaining nodes in Strategy 1 or from all valid levels in Strategy 2. We will show in the experiments that Strategy 2 is much more effective than Strategy 1 in terms of pruning unnecessary searches.

9 Clustering Approximation for Finding the Shortest Valid Path

We have presented methods and optimization strategies for solving the graph model for the shortest valid path. In order

to further shorten the solving time and strike a balance between solving efficiency and solution quality, this section proposes a flexible technique called the *clustering* technique, to rapidly find an approximate solution. The clustering technique is flexible because the solving efficiency can be adjusted by setting the desired solution quality. It can be applied to O-SVP, O-SVPPE and O-SVPPC.

As discussed in introduction and related work, the reason why co-scheduling causes performance degradation is because the co-running jobs compete for shared cache. Stack Distance Competition is a popular technique for calculating the impact when multiple jobs are co-running; this uses the SDPs of the multiple jobs as input. Therefore, if two jobs have similar SDPs, they will have a similar effect on other co-running jobs. The fundamental idea of the proposed clustering technique is to class the jobs with similar SDPs together and treat them as the same job. Reflected in the graph model, the jobs in the same class can be given the same job ID. In so doing, the number of different nodes in the graph model will be significantly reduced. The resulting effect is the same as when different parallel processes are given the same job ID in the O-SVPPE algorithm in Section 5.1.2.

We now introduce a method of measuring the SDP similarity between two jobs. Given a job j_i, its SDP is essentially an array, in which the kth element records the number of cache hits on the kth cache line (which is denoted by $h_i[k]$). The following formula is used to calculate the Similarity Level (SL) in terms of SDP when comparing another job j_j against j_i.

$$SL = \frac{\sqrt{\sum_{k=1}^{cl}(h_i[k] - h_j[k])^2}}{\sum_{k=1}^{cl} h_i[k]}. \quad (7)$$

When SL is larger, more jobs will be classed together. Consequently, there will be fewer nodes in the graph model and hence less scheduling time is needed to calculate an accurate solution.

The O-SVP clustering algorithm is the same as the O-SVP algorithm except in the way a valid level as well as a valid node in a valid level is found, which is the same as that for O-SVPPE (Algorithm 3). The clustering technique can also be applied to O-SVPPE and O-SVPPC in a similar way. Detailed discussion of this process is not repeated.

10 Evaluation

This section evaluates the effectiveness and the efficiency of the proposed methods: O-SVP, O-SVPPE, O-SVPPC, A*-search-based algorithms (i.e., SVPPC-A* and SVP-A*) and the clustering approximation technique. In order to carry out this evaluation, we compare the algorithms proposed in this paper with existing co-scheduling algorithms proposed in [4]: Integer Programming, Hierarchical Perfect Matching (HPM), and Greedy (GR).

We conduct the experiments with real jobs. Serial jobs are taken from the NASA benchmark suit NPB3.3-SER [31] and SPEC CPU 2000 [32]. NPB3.3-SER has 10 serial programs and each program has five different problem sizes. The problem size used in the experiments is size *C*. The PC jobs are selected from the ten MPI applications in the NPB3.3-MPI benchmark suite. As for PE jobs, five embarrassingly parallel programs are used: PI [33], Mandelbrot Set(MMS) [34], RandomAccess(RA) from the HPCC benchmark [35], EP from NPB-MPI [31] and Markov Chain Monte Carlo for Bayesian inference (MCM) [36]. In all these five embarrassingly parallel programs, multiple slave processes are used to perform calculations in parallel and a master process reduces the final result after it gathers the results from all slaves. This set of parallel programs are selected because they contains both computation-intensive (e.g, MMS and PI) and memory-intensive programs (e.g., RA).

Four types of machines, Dual core, Quad core, eight core and 16 core machines, are used to run the benchmarking programs. The dual-core machine has an Intel Core 2 Dual processor and each core has a dedicated 32 KB L1 data cache and a 4 MB 16-way L2 cache shared by the two cores. The Quad-core machine has an Intel Core i7 2600 processor and each core has a dedicated 32 KB L1 cache and a dedicated 256 KB L2 cache. A further 8 MB 16-way L3 cache is shared by the four cores. The eight core machine has two Intel Xeon L5520 processors with each processor having four cores. Each core has a dedicated 32 KB L1 cache and a dedicated 256 KB L2 cache, and an 8 MB 16-way L3 cache shared by four cores. The 16 core machine has two Intel Xeon E5-2450L processors with each processor having eight cores. Each core has a dedicated 32 KB L1 cache and a dedicated 256 KB L2 cache, and a 16-way 20 MB L3 cache shared by eight cores. The network interconnecting the dual-core and quad-core machines is a 10 Gigabit Ethernet, while the network interconnecting the eight-core and 16-core Xeon machines is QLogic TrueScale 4X QDR InfiniBand. In the remainder of this section, we label the eight core and 16 core machines as 2*4 core and 2*8 core machines, to highlight the fact that they are dual-processor machines.

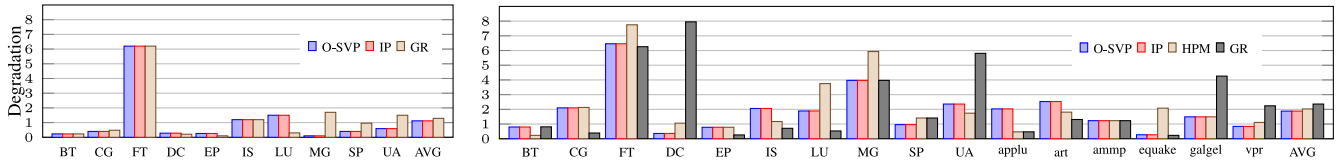
The single-run computation times of the benchmarking programs are measured. Then the method presented in [37] is used to estimate the co-running computation times of the programs, the details of which are presented in the supplementary notes, available online. With the single-run and co-run computation times, Eq. (1) is then used to compute the performance degradation.

In order to obtain the communication time of a parallel process when it is scheduled to co-run with a set of jobs/processes, i.e., $c_{ij,S}$ in Eq. (6), we examined the source codes of the benchmarking MPI programs used for the experiments and obtained the amount of data that the process needs to communicate with each of its neighbouring processes (i.e., $\alpha_{ij}(k)$ in Eq. (6)). Then Eq. (6) is used to calculate $c_{ij,S}$.

10.1 Evaluating the O-SVP Algorithm

In this section, we compare the O-SVP algorithm with the existing co-scheduling algorithms in [4].

These experiments use all 10 serial benchmark programs from the NPB-SER suite. The results are presented in Figs. 3a and 3b, which show the performance degradation of each of the 10 programs plus their average degradation under different co-scheduling strategies on Dual-core and Quad-core machines.



(a) Dual Core (b) Quad Core

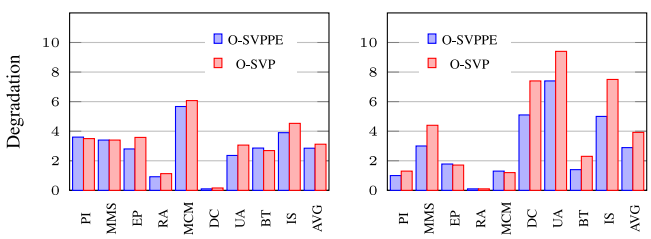
Fig. 3. Comparing the degradation of serial jobs under O-SVP, IP, HPM and GR.

The work in [4] shows that IP generates the optimal co-scheduling solutions for serial jobs. As can be seen from Fig. 3a, O-SVP achieves the same average degradation as that under IP. This suggests that O-SVP can find the optimal co-scheduling solution for serial jobs. The average degradation produced by GR is 15.2 percent worse than that of the optimal solution. It can also been seen from Fig. 3a that the degradation of FT is the largest among all 10 benchmark programs. This may be because FT is the most memory-intensive program among all those selected, and therefore endures the largest degradation when it has to share the cache with competing jobs.

Fig. 3b shows the results on Quad-core machines. In this experiment, in addition to the 10 programs from NPB-SER, six serial programs (`applu`, `art`, `ammp`, `equake`, `galgel` and `vpr`) are selected from SPEC CPU 2000. In Fig. 3b, O-SVP produces the same solution as IP, which shows the optimality of O-SVP. O-SVP also finds a better co-scheduling solution than HPM and GR. The degradation under HPM is 7.7 percent worse than that under O-SVP, while GR is 25.5 percent worse. It is worth noting that O-SVP does not produce the least degradation for all programs. The aim of O-SVP is to produce minimal total degradation. Hence, O-SVP produced a larger degradation than GR and HPM in some cases.

10.2 The O-SVPPE Algorithm

We propose O-SVPPE because 1) none of the existing co-scheduling methods are designed for parallel jobs; 2) we argue that applying existing co-scheduling methods designed for serial jobs to scheduling parallel jobs will not produce optimal solutions. In order to investigate the performance discrepancy between the methods for serial jobs and PE jobs, we apply O-SVP to solve the co-scheduling for a mix of serial and parallel jobs and compare the results with those obtained by O-SVPPE. The mix of serial and parallel jobs consist of five embarrassively parallel programs (each with 12 processes) and serial jobs from NPB-SER plus `art` from SPEC CPU 2000. Experimental results are shown in Figs. 4a and 4b.



(a) Dual Core (b) Quad Core

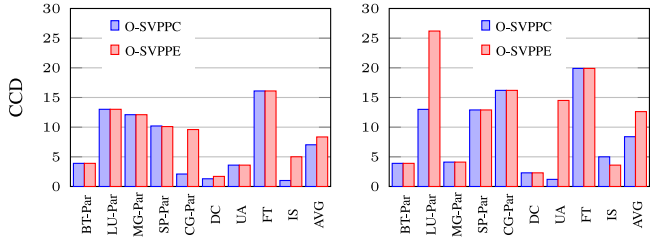
Fig. 4. Comparing the degradation under SVPPE and O-SVP for a mix of PE and serial benchmark programs.

As can be seen from the figures, SVPPE produces smaller average degradation than O-SVP in both Dual-core and Quad-core cases. In the Dual-core case, the degradation under O-SVP is worse than that under SVPPE by 9.4 percent, while in the Quad-core case, O-SVP is worse by 35.6 percent. These results suggest that it is necessary to design co-scheduling methods for parallel jobs.

10.3 The O-SVPPC Algorithm

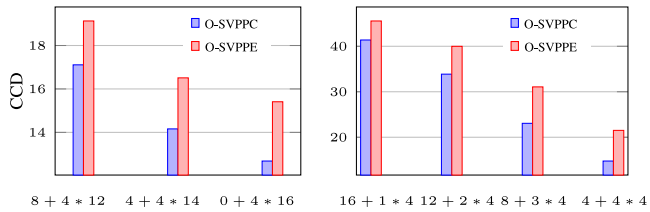
Figs. 5a and 5b show the communication-combined degradation (i.e., the value of Eq. (5)) of the co-scheduling solution obtained by the SVPPC algorithm when the applications are run on Dual-core and Quad-core machines, respectively. In this set of experiments, 5 MPI applications (i.e., `BT-Par`, `LU-Par`, `MG-Par`, `SP-Par` and `CG-Par`) are selected from the NPB3.3-MPI suite and each parallel application is run using 10 processes; the serial jobs remain the same as those used in Fig. 4. In order to demonstrate the effectiveness of SVPPC, SVPPE is also used find the co-scheduling solution for the mix of MPI jobs and serial jobs, by ignoring the inter-process communications in the MPI jobs. We then use Eq. (5) to calculate the CCD of the co-scheduling solution obtained by SVPPE. The resulting CCD is also plotted in Figs. 5a and 5b. As can be seen from these figures, the CCD under SVPPE is worse than that under SVPPC by 18.7 percent in Dual-core machines, while in Quad-core machines, the CCD obtained by SVPPE is worse than that by SVPPC by 50.4 percent. These results justify the need to develop algorithms to find effective co-scheduling solutions for PC jobs.

We further investigate the impact on CCD as the number of parallel jobs or parallel processes increases. Experimental results are shown in Figs. 6a and 6b (on Quad-core machines). In Fig. 6a, the number of total jobs/processes is 64. The number of parallel jobs is 4 (`LU-Par`, `MG-Par`, `SP-Par` and `CG-Par`) and the number of processes per job increases from 12 to 16; all other jobs are serial. For example, 8+4*12 represents a job mix with eight serial and four parallel jobs, each with 12 processes.



(a) Dual Core (b) Quad Core

Fig. 5. Comparing the communication-combined degradation obtained by SVPPC and SVPPE.



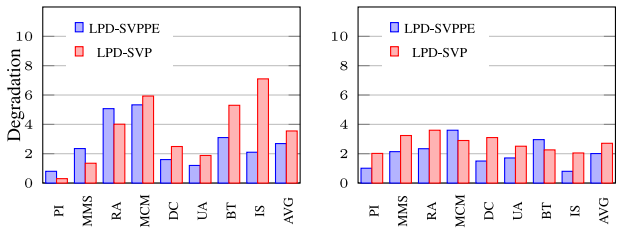
(a) Increasing the number of processes (b) Increasing the number of jobs

Fig. 6. Impact of the number of parallel jobs and parallel processes.

In Fig. 6a, the difference in CCD between SVPPC and SVPPE becomes larger as the number of parallel processes increases. This result suggests that SVPPE performs increasingly worse than SVPPC (increasing from 11.8 to 21.5 percent) as the proportion of PC jobs increases in the job mix. Another observation from this figure is that the CCD decreases as the proportion of parallel jobs increases. This is simply because the degradation experienced by multiple processes of a parallel job is only counted once. If those processes are serial jobs, their degradations will be summed and is therefore larger. In Fig. 6b, the number of processes per parallel job remains unchanged and the number of parallel jobs increases. For example, 12+2*4 represents a job mix with 12 serial jobs and two parallel jobs, each with four processes. The detailed combinations of serial and parallel jobs are: i) In the case of 16+1*4, `MG-Par` is used as the parallel job and all 16 serial programs are used as the serial jobs; ii) In the case of 12+2*4, `LU-Par` and `MG-Par` represent parallel jobs and the serial jobs are `SP`, `BT`, `FT`, `CG`, `IS`, `UA`, `applu`, `art`, `ammp`, `equake`, `galgel` and `vpr`; iii) In the case of 8 +3*4, `BT-Par`, `LU-Par`, `MG-Par` represent the parallel jobs and the serial jobs are `SP`, `BT`, `FT`, `DC`, `IS`, `UA`, `equake`, `galgel`; iv) In the case of 4+4*4, `BT-Par`, `LU-Par`, `SP-Par`, `MG-Par` represent the parallel jobs and the serial jobs are `IS`, `UA`, `equake` and `galgel`. The results in Fig. 6b show a similar pattern to those in Fig. 6a; similar reasoning holds.

10.4 Scheduling in Multi-Processor Computers

In this section, we investigate the effectiveness of the LPD method proposed to handle co-scheduling in multi-processor machines. In the experiments, we first use the MNG method discussed in Section 6 (i.e., generating multiple graph nodes for a co-run group, with each node having a different weight) to construct the co-scheduling graph. As we have discussed, from the co-scheduling graph constructed by the MNG method, the algorithm must be able to find the optimal co-scheduling solution for multi-processor machines. We then use the LPD method to construct the graph and find the shortest path in the graph. The experimental results are presented in Figs. 7a and 7b, in which a mix of four PE jobs (`PI`, `MMS`, `RA` and `MCM`, each with 31 processes) and four serial jobs (`DC`, `UA`, `BT` and `IS`) are used. It can be seen that the performance degradations obtained by the two methods are the same. This result verifies that the algorithms can produce optimal co-scheduling solutions using the LPD method.

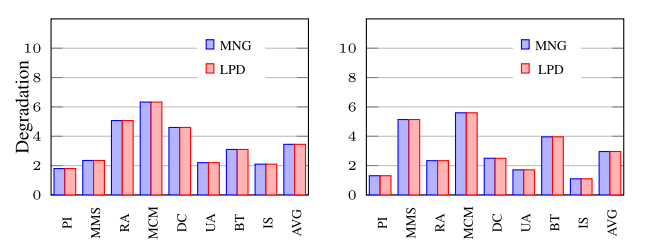


(a) 2*4 Core (b) 2*8 Core

Fig. 8. Comparing the degradation under LPD-SVP and LPD-SVPPE for a mix of PE and serial benchmark programs.

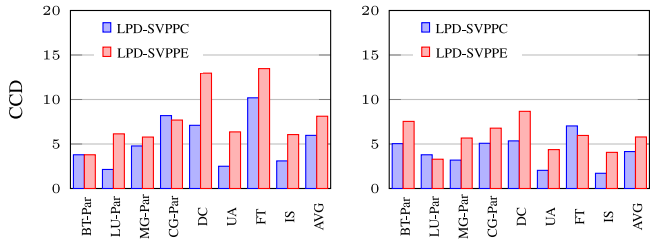
Following the same logic as in Fig. 4, we conduct experiments to investigate the performance discrepancy between the method for serial jobs and that for PE jobs on multi-processor machines. The LPD method is used to generate the co-scheduling graphs (therefore, the "LPD" prefix is added to the algorithms in the legends of the figures). In these experiments, we use the same experimental settings as in Figs. 7a and 7b. The results are shown in Figs. 8a and 8b. As can be seen from the figures, LPD-SVPPE produces smaller average degradation than LPD-SVP in both eight-core and 16-core cases. In the eight-core case, the degradation under LPD-SVP is worse than that under LPD-SVPPE by 31.9 percent, while in the 16-core case, LPD-SVP is worse by 34.8 percent. These results verify the effectiveness of the LPD method for co-scheduling PE jobs.

Similarly, following the same logic as in Fig. 5, we conduct experiments to run PC jobs using SVPPC and SVPPE on multi-processor machines and compare the performance discrepancy in terms of CCD. The same experimental settings as in Fig. 5 are used and the results are presented in Figs. 9a and 9b. In this set of experiments, 4 MPI applications (`BT-Par`, `LU-Par`, `MG-Par` and `CG-Par`) are selected from the NPB3.3-MPI suite and each parallel application is run using 31 processes, while the same serial jobs as in Fig. 7a are used. As can be seen from these figures, the CCD under LPD-SVPPE is worse than that under LPD-SVPPC by 36.1 and 39.5 percent on eight-core and 16-core machines, respectively. These results justify using SVPPC to handle



(a) 2*4 Core (b) 2*8 Core

Fig. 7. Comparing the degradation caused by the straightforward method and the LPD method.



(a) 2*4 Core (b) 2*8 Core

Fig. 9. Comparing the communication-combined degradation obtained by LPD-SVPPC and LPD-SVPPE.

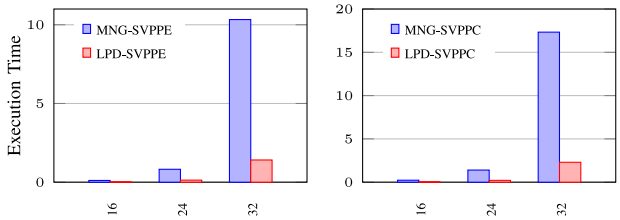


Fig. 10. Comparing the solve times of the LPD and the MNG method, coupled with SVPPE and SVPPC.

TABLE 1
Schedule Result for Multi-Threading Program

Processor	Jobs on each node		
4 Core	bt,bt, ep,ep	mg,mg, lu, sp	ft,ft,dc, ua
8 Core	bt,bt,bt, ep,ep,ep, dc, sp		mg, mg,mg, ft,ft,ft,ua, lu

PC jobs and show that the LPD method works well with the SVPPC algorithm.

As discussed in Section 6, the reason why we propose the LPD method is because using the MNG method, the scale of the co-scheduling graph increased significantly in multi-processor systems. The LPD method can reduce the scale of the co-scheduling graph and consequently reduce the solving time. Therefore, we also conduct experiments to compare the solving time obtained by LPD and the MNG method. The experimental results are presented in Fig. 10, in which Figs. 10a and 10b are for PE and PC jobs, respectively. It can be seen from the figure that the solving time of LPD is significantly less than that of the straightforward method and the discrepancy increases dramatically as the number of jobs increases. These results suggest that LPD is effective in reducing the solve time compared with the MNG method.

10.5 Scheduling Multi-Threaded Jobs

In Section 7, in order to schedule the MTP jobs correctly, we need to guarantee that the threads from the same MTP job are scheduled on the same node. In order to address this, the SVPPT algorithm is proposed to construct the co-scheduling graph and find the shortest path. In this section, we first conduct experiments to examine the co-scheduling solution obtained by SVPPT. In the experiments, we chose four MTP programs (each with two threads on four Core and three threads on eight Core) from NPB3.3-OMP (`BT`, `MG`, `EP` and `FT`) and four serial jobs from NPB-SER (`DC`, `UA`, `LU` and `SP`). The experiments are conducted on two type of processors, Xeon L5520 (four cores) and Xeon E5-2450L (eight cores). The results are presented in Table 1. It can be seen that all threads from the same MTP program are mapped to the same node, which verifies SVPPT can find correct co-scheduling solutions for MTP jobs.

As discussed in Section 7, SVPPT should be more efficient than SVPPE in finding the shortest path. Therefore, we also conduct experiments to compare the solve time of SVPPT and SVPPE. The results are presented in Table 2. The experiments are conducted on four-core and eight-core machines. It can be seen that SVPPT spends much less time than SVPPE and this gap increases as the number of jobsbthreads increases. These results verify the efficiency of SVPPT.

TABLE 2
Comparing the Solve Time between MTP and SVPPE

Number of Jobs	Solving Time 4 Core	
	MTP	SVPPE
24	0.0011	0.0025
36	0.013	0.034
48	0.13	0.38
64	1.11	3.89

Number of Jobs	Solving Time 8 Core	
	MTP	SVPPE
24	0.0013	0.0022
32	0.004	0.011
48	0.078	0.15
64	0.26	1.35

TABLE 3
The Optimality of SVP-A*

Number of Jobs	Average Degradation			
	Dual Core		Quad Core	
	O-SVP	SVP-A*	O-SVP	SVP-A*
8	0.12	0.12	0.34	0.34
12	0.22	0.22	0.36	0.36
16	0.13	0.13	0.27	0.27

TABLE 4
The Optimality of SVPPC-A*

Number of Processes	Average Degradation			
	Dual Core		Quad Core	
	O-SVPPC	SVPPC-A*	O-SVPPC	SVPPC-A*
8	0.07	0.07	0.098	0.098
12	0.05	0.05	0.074	0.74
16	0.12	0.12	0.15	0.15

10.6 The A*-Search-Based Algorithms

In this section, we report results for validating the optimality of the proposed A*-search-based algorithms. We first compare the SVP-A* algorithm with the O-SVP algorithm in terms of the optimality in co-scheduling serial jobs. The experiments use all 10 serial benchmark programs from the NPB-SER suite and six serial programs (`applu`, `art`, `ammp`, `equake`, `galgel` and `vpr`) selected from SPEC CPU 2000. The experimental results are presented in Table 3. We also compare the SVPPC-A* algorithm with the O-SVPPC algorithm in terms of optimality in co-scheduling a mix of serial and parallel programs. The experiments are conducted on Quad-core machines. The results are listed in Table 4. In these experiments, two MPI applications (`MG-Par` and `LU-Par`) are selected from NPB3.3-MPI and mixed with serial programs chosen from NPE-SER and SPEC CPU 2000. The processes of each parallel application vary from 2 to 4. The detailed combinations of serial and parallel programs are: i) In the case of eight processes, `MG-Par` and `LU-Par` are combined with `applu`, `art`, `equake` and `vpr`; ii) In the case of 12 processes, `MG-Par` and `LU-Par` are combined with `applu`, `art`, `ammp`, `equake`, `galgel` and `vpr`; iii) In the case of 16 processes, `MG-Par` and `LU-Par` are combined with `BT`, `IS`, `applu`, `art`, `ammp`, `equake`, `galgel` and `vpr`.

As can be seen from Tables 3 and 4, SVP-A* and SVPPC-A* achieve the same performance degradation as those of O-SVP

TABLE 5
Comparison of the Strategies for Setting $h(v)$ with Serial Jobs

Number of Jobs	Solve time (seconds)		
	SVP-A*-1	SVP-A*-2	O-SVP
16	0.72	0.014	1.01
20	12.88	0.047	17.52
24	190.79	0.14	234.5
Number of Jobs	The number of visited paths		
	SVP-A*-1	SVP-A*-1	O-SVP
16	31,868	122	49,559
20	546,603	436	830,853
24	6,726,131	1,300	9,601,465

TABLE 6
Comparison of the Strategies for Setting $h(v)$ with Parallel Jobs

Number of Processes	Solve time (seconds)		
	SVPPC-A*-1	SVPPC-A*-2	O-SVPPC
40	0.43	0.037	0.61
80	2.44	0.17	3.38
120	10.93	0.33	17.93
160	40.05	0.64	66.85
200	99.13	0.88	212.79
Number of Processes	The number of visited paths		
	SVPPC-A*-1	SVPPC-A*-2	O-SVPPC
40	18,481	414	27,349
80	261,329	1,952	422,025
120	1,275,799	4,452	2,105,706
160	3,990,996	7,050	6,585,938
200	8,663,580	16,290	15,991,561

and O-SVPPC, respectively. These results verify the optimality of the A*-search-based algorithms. Indeed, SVPPC-A* combines the functionalities of SVPPC and the A*-search algorithm and is expected to generate the optimal solution.

Tables 5 and 6 show the scheduling efficiency of our A*-search-based algorithms under the two different strategies of setting the $h(v)$ function proposed in Section 8. SVP-A*-1 (or SVPPC-A*-1) and SVP-A*-2 (or SVPPC-A*-2) are the SVP-A* (or SVPPC-A*) algorithm that uses Strategy 1 and 2, respectively, to set $h(v)$. Table 5 shows the results for synthetic serial jobs, while Table 6 shows the results for parallel jobs. In Table 6, four synthetic parallel jobs are used and the number of processes of each parallel job increases from 10 to 50. Recall that the O-SVP algorithm is equivalent to SVP-A* with the $h(v)$ function set to 0, while O-SVPPC is equivalent to SVPPC-A* with $h(v)$ set to 0. Therefore, we also conduct experiments to show the scheduling efficiency of O-SVP and O-SVPPC, which can be used to demonstrate the effectiveness of the strategies of setting $h(v)$. The underlying reason why SVPPC-A* and SVP-A* could be effective is because they can further avoid unnecessary searches in the constructed co-scheduling graph. Therefore, we also record the number of paths visited by each algorithm and present these in Tables 5 and 6.

It can be seen from both tables that the strategies used to set $h(v)$ play a critical role in our A*-search-based algorithms. Both Strategy 1 and 2 proposed in Section 8 can reduce the number of visited paths dramatically and therefore reduce the solve time compared with the corresponding O-SVP and O-SVPPC algorithms. These results suggest that the strategies proposed in this paper can significantly reduce unnecessary searches.

Further, as observed from Tables 5 and 6, the algorithms under Strategy 2 visited fewer paths by orders of magnitude than their counterparts under Strategy 1. Therefore, SVP-A*-2 and SVPPC-A*-2 are more efficient by orders of magnitude than SVP-A*-1 and SVPPC-A*-1, respectively, in finding the optimal co-scheduling solution. This is because the estimation of $h(v)$ provided by Strategy 2 is much closer to the actual shortest path of the remaining nodes than that determined by Strategy 1, and consequently Strategy 2 is much more effective than Strategy 1 in avoiding unnecessary searches.

The scalability of the proposed algorithms can also be observed from Tables 5 and 6. It can be seen that SVPPC-A*-2 (or SVP-A*-2) shows the best scalability against SVPPC-A*-1 and O-SVPPC (or SVP-A*-1 and O-SVP). This can be explained as follows: Although the scale of the constructed co-scheduling graph and the possible search paths increase rapidly as the number of jobs/processes increases, SVPPC-A*-2 can effectively prune the graph's branches that will not lead to the optimal solution. Therefore, the increase in the scale of the graph will not increase the solve time of SVPPC-A*-2 as much as for other two algorithms.

10.7 The Optimization Techniques

We test the efficiency of the communication-aware process condensation techniques and the clustering approximation proposed in this paper. Experiments are conducted on Quad-core machines.

We first test the effectiveness of the communication-aware process condensation technique. Experiments are conducted on Quad-core machines with synthetic jobs. In this set of experiments, the number of total jobs/processes is 72, where the number of parallel jobs is 6 with the number of processes per job increasing from 1 to 12 and the remaining jobs are serial jobs. These jobs are scheduled using SVPPC-A* with and without applying the process condensation. The solve times are plotted in Fig. 11.

It can be seen from Fig. 11 that after applying the process condensation technique, the solve time decreases dramatically as the number of processes increase. This is because the number of nodes with the same communication pattern in the graph increases as the number of processes increases. Therefore, the condensation technique can eliminate more nodes from the co-scheduling graph and consequently reduce the solve time.

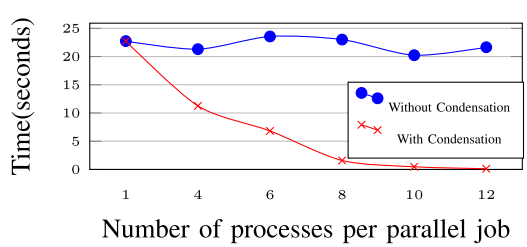


Fig. 11. Solve time with and without process condensation as the number of processes per parallel job increases. The total number of parallel processes and serial jobs is 72.

TABLE 7
Comparing the Clustering Method with O-SVP

Algorithm	visited path	Degradation	time (seconds)
O-SVP	181,267,889	19.97	708
8 class	2,115,716	21.23	14.25
4 class	141,508	23.75	1.18
2 class	17,691	25.96	0.31

The clustering approximation algorithm are tested with 32 synthetic serial jobs. These jobs are first scheduled using O-SVP. Then these jobs are grouped into eight, four and two classes by setting the Similarity Level. The experimental results are presented in Table 7. It can be observed from Table 7 that when the jobs are grouped into eight classes, the degradation increases slightly, compared with that achieved by O-SVP. However, the scheduling time under the clustering technique is reduced significantly. Moreover, as the number of classes decreases, the degradation increases further and the scheduling time continues to decrease. These results show that our clustering technique is effective. This table also lists the number of subpaths visited by the co-scheduling algorithms, which decreases by orders of magnitude as the number of classes decreases. This is the underlying reason why the scheduling time decreases after applying the clustering approximation technique.

11 Conclusions and Future Work

In this research we propose a graph-based method to co-schedule jobs in multi-core computers. A graph is constructed for the co-scheduling problem, then finding the optimal co-scheduling solution is modelled as finding the shortest valid path in the graph. An algorithm for finding the shortest valid path for serial jobs is first developed and then an optimization strategy is proposed to reduce the solve time. Further, the algorithm for serial jobs is extended to incorporate parallel jobs; the optimization strategies are also developed to accelerate the solving process for finding the optimal solution for parallel jobs. Moreover, a flexible approximation technique is proposed to balance solving efficiency and solution quality. Experiments have been conducted to verify the effectiveness of the proposed algorithms. Future work is two-fold: 1) It is possible to parallelize the proposed co-scheduling algorithms to further speedup the process of finding the optimal solution. We plan to investigate a parallel paradigm suitable for this problem and design suitable parallelization strategies; 2) We plan to extend our co-scheduling methods to solve the optimal mapping of virtual machines (VM) on physical machines. The main extension necessary to the proposed methods is to allow VM migration between physical machines.

Acknowledgments

The authors acknowledge a number of different funding sources that support this work: The Royal Society Industry Fellowship Scheme (IF090020/AM); Bull Information Systems and the Bull/Warwick Premier Partnership and the PhD funding that this provides; EPSRC and Intel Corporation Industrial CASE Account (awards 38405-2013 and 15220082-2015). Ligang He is the corresponding author.

References

[1] Y. Koh, R. Knauerhase, P. Brett, M. Bowman, Z. Wen, and C. Pu, "An analysis of performance interference effects in virtual environments," in *Proc. IEEE Int. Symp. Perform. Anal. Syst. Softw.*, 2007, pp. 200–209.

[2] J. Zhang, A. Sivasubramaniam, Q. Wang, A. Riska, and E. Riedel, "Storage performance virtualization via throughput and latency control," *ACM Trans. Storage*, vol. 2, no. 3, pp. 283–308, 2006.

[3] D. Gupta, L. Cherkasova, R. Gardner, and A. Vahdat, "Enforcing performance isolation across virtual machines in Xen," in *Proc. Int. Conf. Middleware* 2006, pp. 342–362.

[4] Y. Jiang, K. Tian, X. Shen, J. Zhang, J. Chen, and R. Tripathi, "The complexity of optimal job co-scheduling on chip multiprocessors and heuristics-based solutions," *IEEE Trans. Parallel Distrib. Syst.*, vol. 22, no. 7, pp. 1192–1205, Jul. 2011.

[5] S. Zhuravlev, S. Blagodurov, and A. Fedorova, "Addressing shared resource contention in multicore processors via scheduling," *ACM SIGARCH Comput. Archit. News.*, vol. 38, no. 1, pp. 129–142, 2010.

[6] K. Nesbit, J. Laudon, and J. Smith, "Virtual private caches," *ACM SIGARCH Comput. Archit. News*, vol. 35, no. 2, pp. 57–68, 2007.

[7] S. Srikantaiah, M. Kandemir, and M. Irwin, "Adaptive set pinning: Managing shared caches in chip multiprocessors," *ACM SIGPLAN Notices*, vol. 43, pp. 135–144, 2008.

[8] X. Zhang, S. Dwarkadas, and K. Shen, "Towards practical page coloring-based multicore cache management," in *Proc. 4th ACM Eur. Conf. Comput. Syst.*, 2009, pp. 89–102.

[9] M. Lee and K. Schwan, "Region scheduling: efficiently using the cache architectures via page-level affinity," in *Proc. 17th Int. Conf. Architectural Support Program. Languages Operating Syst.*, 2012, pp. 451–462.

[10] A. Fedorova, M. Seltzer, and M. Smith, "Cache-fair thread scheduling for multicore processors," Division Eng. Appl. Sci., Harvard Univ., Cambridge, MA, USA, Tech. Rep. TR-17-06, 2006.

[11] J. Feliu, S. Petit, J. Sahuquillo, and J. Duato, "Cache-hierarchy contention aware scheduling in CMPs," *IEEE Trans. Parallel Distrib. Syst.*, vol. 25, no. 3, pp. 581–590, Mar. 2013.

[12] S. Zhuravlev, J. C. Saez, S. Blagodurov, A. Fedorova, and M. Prieto, "Survey of scheduling techniques for addressing shared resources in multicore processors," *ACM Comput. Surveys*, vol. 45, no. 1, pp. 4:1–4:28, Dec. 2012.

[13] S. Blagodurov, S. Zhuravlev, A. Fedorova, and M. Dashti, "A case for NUMA-aware contention management on multicore systems," in *Proc. 19th Int. Conf. Parallel Archit. Compilation Techn.*, 2010, p. 1.

[14] A.-H. Haritatos, G. Goumas, N. Anastopoulos, K. Nikas, K. Kourtis, and N. Koziris, "LCA: A memory link and cache-aware co-scheduling approach for CMPs," in *Proc. 23rd Int. Conf. Parallel Archit. Compilation*, 2014, pp. 469–470.

[15] H. Yang, A. Breslow, J. Mars, and L. Tang, "Bubble-flux: Precise online QoS management for increased utilization in warehouse scale computers," in *Proc. 40th Annu. Int. Symp. Comput. Archit.*, 2013, pp. 607–618.

[16] C. Delimitrou and C. Kozyrakis, "Quasar: Resource-efficient and QoS-aware cluster management," in *Proc. 19th Int. Conf. Architectural Support Program. Languages Operating Syst.*, 2014, pp. 127–144.

[17] B. Hindman, A. Konwinski, M. Zaharia, A. Ghodsi, A. D. Joseph, R. Katz, S. Shenker, and I. Stoica, "Mesos: A platform for fine-grained resource sharing in the data center," in *Proc. 8th USENIX Conf. Netw. Syst. Des. Implementation*, 2011, pp. 295–308.

[18] S. Blagodurov, S. Zhuravlev, and A. Fedorova, "Contention-aware scheduling on multicore systems," *ACM Trans. Comput. Syst.*, vol. 28, no. 4, pp. 8:1–8:45, Dec. 2010.

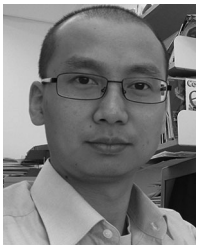
[19] A. Fedorova, S. Blagodurov, and S. Zhuravlev, "Managing contention for shared resources on multicore processors," *Commun. ACM*, vol. 53, pp. 49–57, 2010.

[20] Y. Wang, Y. Cui, P. Tao, H. Fan, Y. Chen, and Y. Shi, "Reducing shared cache contention by scheduling order adjustment on commodity multi-cores," in *Proc. IEEE Int. Symp. Parallel Distrib. Process.*, 2011, pp. 984–992.

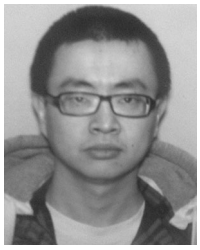
[21] J. Mars, L. Tang, R. Hundt, K. Skadron, and M. L. Soffa, "Bubble-Up: Increasing utilization in modern warehouse scale computers via sensible co-locations," in *Proc. 44th Annu. IEEE/ACM Int. Symp. Microarchit.*, 2011, pp. 248–259.

[22] S. Kim, D. Chandra, and Y. Solihin, "Fair cache sharing and partitioning in a chip multiprocessor architecture," in *Proc. 13th Int. Conf. Parallel Archit. Compilation Techn.*, 2004, pp. 111–122.

[23] A. Fedorova, M. Seltzer, and M. Smith, "Improving performance isolation on chip multiprocessors via an operating system scheduler," in *Proc. 16th Int. Conf. Parallel Archit. Compilation Techn.*, 2007, pp. 25–38.
[24] Q. Zhao, D. Koh, S. Raza, D. Bruening, W. Wong, and S. Amarasinghe, "Dynamic cache contention detection in multi-threaded applications," in *Proc. 7th ACM SIGPLAN/SIGOPS Int. Conf. Virtual Execution Environ.*, 2011, pp. 27–38.
[25] D. Chandra, F. Guo, S. Kim, and Y. Solihin, "Predicting inter-thread cache contention on a chip multi-processor architecture," in *Proc. 11th Int. Symp. High-Perform. Comput Archit.*, 2005, pp. 340–351.
[26] N. Tuck and D. Tullsen, "Initial observations of the simultaneous multithreading pentium 4 processor," in *Proc. 12th Int. Conf. Parallel Archit. Compilation Techn.*, 2003, p. 26.
[27] T. Dwyer, A. Fedorova, S. Blagodurov, M. Roth, F. Gaud, and J. Pei, "A practical method for estimating performance degradation on multicore processors, and its application to HPC workloads," in *Proc. Int. Conf. High Perform. Comput. Netw. Storage Anal.*, 2012, pp. 1–11.
[28] R. Hemani, S. Banerjee, and A. Guha "ACCORD: An analytical cache contention model using reuse distances for modern multiprocessors," Department of Computer Science and Engineering, Indraprastha Institute of Information Technology, Tech. Rep. IIITD-TR-2014-004, 2014.
[29] J. Rosenthal, "Parallel computing and monte carlo algorithms," *Far East J. Theoretical Statist.*, vol. 4, pp. 207–236, 2000.
[30] T. H. Cormen, C. Leiserson, R. Rivest, and C. Stein, *Introduction to Algorithms*. Cambridge, MA, USA: MIT Press, 2001.
[31] [Online]: Available: http://www.nas.nasa.gov/publications/npb.html, 2012.
[32] [Online]: Available: http://www.spec.org, 2015.
[33] [Online]: Available: https://computing.llnl.gov/tutorials/mpi/samples/c/mpi_pi_reduce.c, 2013.
[34] [Online]: Available: http://people.ds.cam.ac.uk/nmm1/mpi/programs/mandelbrot.c, 2011.
[35] [Online]: Available: http://icl.cs.utk.edu/hpcc/, 2015.
[36] E. Kontoghiorghes, *Handbook of Parallel Computing and Statistics*. London, U.K.: CRC Press, 2005.
[37] D. A. Patterson and J. L. Hennessy, *Computer Organization and Design Fourth Edition, Fourth Edition: The Hardware/Software. Interface (The Morgan Kaufmann Series in Computer Architecture and Design)*, 4th ed. San Mateo, CA, USA: Morgan Kaufmann, 2008.



Ligang He received the bachelor's and master's degrees from Huazhong University of Science and Technology, China, and the PhD degree in computer science from the University of Warwick, United Kingdom. He spent his early postdoctoral years as a researcher at the University of Cambridge, United Kingdom. In 2006, he joined the Department of Computer Science, University of Warwick as an assistant professor and was subsequently promoted to an associate professor. His interests include parallel and distributed computing, high-performance computing and cloud computing. He has published more than 70 papers in international conferences and journals, such as *TPDS*, *JPDC*, IPDPS, ICPP, ICWS, MASCOTS, etc. He has served as a member of the program committee or conference chair for a number of international conferences. He is a member of the IEEE.



Huanzhou Zhu received the bachelor's degree in information system engineering from Imperial College London, United Kingdom, and received the master's degree in computer science from the University of Warwick, United Kingdom. He is currently working toward the PhD student in computer science at the University of Warwick. His areas of interest include parallel and distributed computing, high-performance computing, and cloud computing.

Stephen A. Jarvis is a professor of high-performance computing at the University of Warwick; he currently also serves as the chair in the Department of Computer Science, University of Warwick, United Kingdom. While previously at the Oxford University Computing Laboratory, he worked on the development of performance tools with Oxford Parallel, Sychron Ltd., and Microsoft Research, Cambridge. He has close research ties with industry, including projects with IBM, BT, Hewlett-Packard Research Laboratories, Intel, Bull/ATOS, Rolls-Royce and AWE. He has authored more than 150 refereed publications (including three books) in the area of software and performance evaluation. He is a co-organizer for one of the United Kingdom high-end scientific computing training centers and currently holds a William Penney Fellowship, recognizing his advisory role to the United Kingdom MOD. He is a member of the IEEE.