Fuzzy Logic System-Based Adaptive Fault-Tolerant Control for Near-Space Vehicle Attitude Dynamics With Actuator Faults

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***Abstract*—This paper addresses the problem of fault-tolerant control (FTC) for near-space vehicle (NSV) attitude dynamics with actuator faults, which is described by a Takagi–Sugeno (T–S) fuzzy model. First, a general actuator fault model that integrated varying bias and gain faults, which are assumed to be dependent on the system state, is proposed. Then, sliding mode observers (SMOs) are designed to provide a bank of residuals for fault detection and isolation. Based on Lyapunov stability theory, a novel fault diagnostic algorithm is proposed, which removes the classical assumption that the time derivative of the output error should be known. Further, for the two cases where the state is available or not, two accommodation schemes are proposed to compensate for the effect of the faults. These schemes do not need the condition that the bounds of the time derivative of the faults should be known. In addition, a sufficient condition for the existence of SMOs is derived according to Lyapunov stability theory. Finally, simulation results of NSV are presented to demonstrate the efficiency of the proposed FTC approach.**

***Index Terms*—Fault diagnosis and accommodation, fuzzy logic system, near-space vehicle (NSV), Takagi–Sugeno (T–S) fuzzy model.**

I. Introduction

OVER THE PAST FEW decades, fault-tolerant control (FTC) has received considerable attention and obtained significant results; see [1]–[20] and references therein. In [3], unknown input observers were designed for a class of nonlinear systems. The estimation scheme was included in the closed-loop system to compensate for the actuator faults. Adaptive actuator failure compensation for parametric strict feedback systems in the presence of actuator failures was studied, and adaptive backstepping schemes were proposed in [5]–[7]. However, the results in [3]–[8] only work under the condition that all states are measurable. This is a very restrictive assumption for practical systems, and relaxing this hypothesis is one of the motivations of our study. In [10], an actuator fault diagnosis scheme was proposed for a class of affine nonlinear systems with both known and unknown inputs, which was designed by making use of the derived input/output relation and the recently developed high-order sliding-mode robust differentiators. However, this scheme works under the restrictive condition that only one actuator fault occurs at one time. Considering multiple actuator faults simultaneously is another motivation of this paper. In [19], a general active FTC framework was proposed for nonlinear systems with actuator faults. However, this control framework works under the assumption that system states are bounded in all cases. Furthermore, the fault estimation algorithm was not given. It is valuable to point out that most results reported in the literature and concerning actuator faults only considered bias faults. Gain faults did not attract enough attention, which motivates this paper. Another motivation of this study is, thus, to provide a fault indicator with an associated decision algorithm which is efficient in practical application.

The concept of near-space hypersonic vehicle was first proposed by the American Air Force in a military exercise called "Schrieffer" in 2005. Near-space vehicle (NSV) is a class of vehicle flying in near space which offers a promising and new, lower cost technology for future spacecraft. It can advance space transportation as well as prompt global strike capabilities. Such complex technological system attracts considerable interests from the control research community and aeronautical engineering in the past couple of decades and significant results were reported [21]–[24]. For such high technological system, with great economical and societal issues, it is of course essential to maintain high reliability against possible faults [25]–[27].

During the past two decades, the stability analysis for Takagi–Sugeno (T–S) fuzzy systems has attracted increasing attention [28]–[33], [41]. Recently, the T–S fuzzy system was used to describe the NSV attitude dynamics. In [24], the problem of fault-tolerant tracking control for near-space-vehicle attitude dynamics with bias actuator fault is studied, where the bias fault was assumed to be unknown constant. However, in practical application, the fault may be time varying. In this paper, we propose a more general FTC scheme that handles such non-constant faults. On the other hand, as a universal approximation, fuzzy logic system (FLS) played an important role in controlling

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uncertain systems (see [6], [34]–[38], and references therein). In this paper, we use the aforementioned FLSs to approximate the unknown gain and bias functions.

In this paper, we investigate the problem of FTC for T–S fuzzy systems with actuator varying faults, with the objective to provide an efficient solution to control NSV in faulty situations. Compared with existing literature, the following contributions are worth to be emphasized. 1) The actuator fault model that is presented in this paper integrates not only unknown gain faults, but also unknown bias faults, where both faults are dependent on the system state and will be approximated by FLS; 2) differing from some design scheme in the literature, the fault-tolerant control scheme does not need the condition that the bounds of the time derivatives of the varying faults should be known constants, which thus enlarges the practical application scope; 3) compared with some results [9]–[11], [17], a decision threshold for fault detection and isolation (FDI) is defined and applied on an online computable fault indicator and not on an asymptotic value of a criterion which is not easily available to use. The decision algorithm is, thus, more practical than in other work such as [9]–[11] and [17].

The rest of this paper is organized as follows. In Section II, the T–S fuzzy model for NSV attitude dynamics is first briefly recalled. Actuator faults are integrated in such a model, and the FTC objective is formulated. In addition, mathematical description of FLS is given. In Section III, the main technical results of this paper are given, which include fault detection, isolation, and fuzzy logic system-based fault accommodation in the two cases where the state is available or not. The NSV application is presented in Section IV. Simulation results of NSV are presented to demonstrate the effectiveness of the proposed technique. Finally, Section V draws the conclusion.

Notations: In this paper, R, R^n, and $R^{n\times m}$ denote, respectively, the real numbers, the real n-vectors, and the real $n \times m$ matrices. We define the norm of a vector $x \in R^n$ as $\|x\|_2 = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$ and the norm of a matrix $A \in R^{n\times m}$ as $\|A\|_2 = \sqrt{\lambda_{\max}(A^TA)} = \sigma_{\max}(A)$; $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are the smallest and largest eigenvalues of a matrix A, respectively, and $\sigma_{\max}(A)$ is the maximum singular value. The absolute value is denoted by $|\cdot|$.

II. Takagi–Sugeno Fuzzy Model for Near-Space Vehicle Dynamics and Mathematical Description of a Fuzzy Logic System

A. Takagi–Sugeno Fuzzy Model for Near-Space Vehicle Dynamics

In this paper, a NSV attitude dynamics in a re-entry phase is given as [41]

$$\begin{cases} J\dot{\omega} = -\Omega J\omega + \delta \\ \dot{\gamma} = R(\cdot)\omega \end{cases} \tag{1}$$

where $J \in R^{3\times 3}$ is the symmetric positive-definite moment of inertia tensor; $\omega = [p, q, r]^T = [\omega_1, \omega_2, \omega_3]^T$ is the angular rate vector composed of roll p, pitch q, and yaw rate r; $\delta = [\delta_e, \delta_\alpha, \delta_r]^T \in R^{3\times 1}$ is the control surface deflection; δ_e, δ_α, and δ_r are the elevator deflection, the aileron deflection, and the rudder deflection, respectively. The skew symmetric matrix Ω is given by

$$\Omega = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ \omega_2 & \omega_1 & 0 \end{bmatrix}. \tag{2}$$

In the re-entry phase, $R(\cdot)$ is defined as follows:

$$R(\cdot) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ \sin\alpha & 0 & -\cos\alpha \\ 0 & 1 & 0 \end{bmatrix} \tag{3}$$

where $\gamma = [\phi, \beta, \alpha]^T$, and ϕ, β, α are the bank, sideslip, and the attack angles, respectively. According to the singular perturbation theory, the aforementioned six equations can be expressed in the form of inner loop ω and outer loop γ; ω and γ are also, respectively, called fast loop and slow loop.

From the motion law of NSV, it is easy to find that the response of the angular rate ω is faster than the one of the attitude angle γ. Based on the time-scale principle, we define ω as a fast state and γ as a slow state; thus, system (1) can be divided into the following two subsystems: fast subsystem (4a) related to the fast state ω and slow subsystem (4b) related to the slow state γ:

$$\begin{cases} \dot{x}_\omega = f(x_\omega) + g(x_\omega)u(t) \\ y_\omega = x_\omega \end{cases} \tag{4a}$$

$$\begin{cases} \dot{x}_\gamma = f(x_\gamma, t)y_\omega \\ y_\gamma = x_\gamma \end{cases} \tag{4b}$$

where $f(x_\omega) = J^{-1}\Omega(\omega)Jx_\omega, g(x_\omega) = J^{-1}, f(x_\gamma) = R(\cdot)$, and $x_\omega = \omega, x_\gamma = \gamma$.

The control objectives are 1) for the slow subsystem (the outer loop), to design the ideal angular rate $y_\omega(=\omega_d)$ such that the subsystem output y_γ follows the desired reference signal y_d whose first derivative is available and bounded; 2) for the fast subsystem (the inner loop), to design the control $u(t)$ such that the angular rate x_ω follows the ideal angular rate $y_\omega(=\omega_d)$. That is to say, the main task is to design proper control input u such that $\lim_{t\to\infty}(x_\omega - \omega_d) = 0 \Rightarrow \lim_{t\to\infty}(\gamma_\gamma - \gamma_d) = 0$.

A fuzzy linear dynamic model has been proposed by Takagi and Sugeno in 1985 to represent a nonlinear system as an aggregation of local linear input/output relations [28]. The fuzzy linear model is described by fuzzy IF–THEN rules and will be employed to deal with the fuzzy control problem for inner loop dynamics described by (4a) in this paper.

Consider the following T–S fuzzy model which is composed of a set of fuzzy implications, where each implication is expressed by a linear state-space model. The ith rule of this T–S fuzzy model is of the following form.

Plant Rule i: IF $z_1(t)$ is M_{i1} and $z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{x}(t) = A_ix(t) + B_iu(t) \\ y(t) = C_ix(t) \end{cases} \tag{5}$$

where $i = 1, \ldots, r, r$ is the number of the IF–THEN rules, $M_{ij}, j = 1, \ldots, q$, is the fuzzy set, $z(t) = [z_1(t), \ldots, z_q(t)]^T$ are the premise variables which are supposed to be known,

$x(t) = [x_1(t), \ldots, x_n(t)]^T \in R^n$, $u(t) \in R^m$, $A_i \in R^{n\times n}$, and $B_i \in R^{n\times m}$.

The overall fuzzy system is inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} h_i(z(t)) C_i x(t) \end{cases} \tag{6}$$

where $h_i(z(t))$ is defined as

$$h_i(z(t)) = \prod_{j=1}^{n} M_{ij}[z(t)] \Big/ \sum_{i=1}^{r}\prod_{j=1}^{n} M_{ij}[z(t)] \tag{7}$$

where $M_{ij}[z(t)]$ is the grade of membership of $z_j(t)$ in M_{ij}. It is assumed in this paper that $\prod_{j=1}^{n} M_{ij}[z(t)] \geq 0$ for all t. Hence, we have $\sum_{i=1}^{r} h_i(z(t)) = 1$, $0 \leq h_i(z(t)) \leq 1$, for all t. In the following, for describing simply, $h_i(z)$ denotes $h_i(z(t))$.

In this paper, the state feedback control strategy is chosen as a parallel distributed compensation (PDC), which can be described as follows.

Control Rule i: IF $z_1(t)$ is M_{i1} and $z_q(t)$ is M_{iq}, THEN

$$u_i(t) = K_i x(t) \tag{8}$$

where K_i is the controller gain matrix which is to be determined later.

The overall fuzzy controller is given as follows:

$$u(t) = \sum_{i=1}^{r} h_i(z) K_i x(t). \tag{9}$$

The control objective under normal conditions is to design a proper state feedback control input u such that

$$\lim_{t\to\infty}(x_\omega - \omega_d) = 0 \Rightarrow \lim_{t\to\infty}(\gamma_\gamma - \gamma_d) = 0.$$

In this paper, two kinds of actuator faults are considered: loss of effectiveness of the actuators and actuator bias faults. The first kind of fault is modeled as follows:

$$u_i^f(t) = (1 - \rho_i(x))u_i(t), \quad i = 1, \ldots, m, \, t \geq t_j \tag{10}$$

where $\rho_i(x)(0 \leq \rho_i(x) < 1)$, which is supposed to be unknown, denotes the remaining control rate, and t_j is unknown fault occurrence time. The second kind of fault, namely, actuator bias fault, can be described as

$$u_i^f(t) = u_i(t) + d_i(x), \quad i = 1, \ldots, m, \, t \geq t_j \tag{11}$$

where $d_i(x)$ denotes a bounded signal. Therefore, the aforementioned two kinds of actuator faults can be uniformly described as

$$u_i^f(t) = (1 - \rho_i(x))u_i(t) + d_i(x), \;\; t \geq t_j. \tag{12}$$

Furthermore, a more general fault model can be given as

$$u_i^f(t) = (1 - \rho_i(x))u_i(t) + \sum_{j=1}^{p_i} g_{i,j} d_{i,j}(x), \quad t \geq t_j \tag{13}$$

where $d_{i,j}(x), i = 1, \ldots, m, j = 1, \ldots, p_i$ denotes a bounded signal, and p_i is a known positive constant. $g_{i,j}$ denotes an unknown constant. With no restriction, let us suppose that $p_1 = \cdots = p_m = p$, with p being a known positive constant. Consider the following notation $a_{i,j}(x) = g_{i,j} d_{i,j}(x)$, (13) can be rewritten as follows:

$$u_i^f(t) = (1 - \rho_i(x))u_i(t) + \sum_{j=1}^{p} a_{i,j}(x), \quad t \geq t_j \tag{14}$$

where the nonlinear functions $\rho_i(x)$, $a_{i,j}(x)$ and the failure time instant t_j are unknown. In this paper, both bias and gain faults are handled by considering the general fault model (14). For the fault model (14), the following assumption is made.

Assumption 1: There exist real constants $\bar{\rho}_{s_k}$, $\bar{a}_{s_k,j}$ such that $|\rho_{s_k}(x)| \leq \bar{\rho}_{s_k}$, $|a_{s_k,j}(x)| \leq \bar{a}_{s_k,j}$.

Now, the control objective is redefined as follows. An active FTC approach is proposed to obtain the aforementioned tracking objective under normal and faulty conditions, namely $\lim_{t\to\infty}(x_\omega - \omega_d) = 0$ and $\lim_{t\to\infty}(\gamma_\gamma - \gamma_d) = 0$. Under the normal condition (no fault), a state feedback control input u is designed, such that $\lim_{t\to\infty}(x_\omega - \omega_d) = 0$. Meanwhile, the FDI algorithm is working. As soon as actuator faults are detected and isolated, the fault accommodation algorithm is activated and a proper fault-tolerant control input $u(t)$ is used such that the tracking performance ($\lim_{t\to\infty}(x_\omega - \omega_d) = 0$) is still maintained stable under a faulty case.

B. Mathematical Description of a Fuzzy Logic System [40]

An FLS consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The knowledge base for FLS comprises a collection of fuzzy IF–THEN rules of the following form:

R^l: IF x_1 is A_1^l and $\cdots$ and x_n is A_n^l, THEN y is B^l

where $l = 1, 2, \ldots, M$, $\underline{x} = [x_1, x_2, \ldots, x_n]^T \in U \subset R^n$, and y are the FLS input and output, respectively. Fuzzy sets A_i^l and B^l are associated with the fuzzy functions $\mu_{A_i^l}(x_i) = \exp(-((x_i - a_i^l)/b_i^l)^2)$ and $\mu_{B^l}(y^l) = 1$, respectively. M is the rules number. Through singleton function, center average defuzzification, and product inference, the FLS can be expressed as

$$y(x) = \left\{\sum_{l=1}^{M} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)\right\} \Big/ \left\{\sum_{l=1}^{M}\left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)\right\} \tag{15}$$

where $\bar{y}^l = \max_{y\in R} \mu_{B^l}$. Define the fuzzy basis functions as

$$\xi^l(x) = \left[\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right] \Big/ \left[\sum_{l=1}^{M}\left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)\right] \tag{16}$$

and define $\theta^T = [\bar{y}^1, \bar{y}^2, \ldots, \bar{y}^M] = [\theta_1, \theta_2, \ldots, \theta_M]$ and $\xi = [\xi^1, \xi^2, \ldots, \xi^M]^T$; then FLS (5) can be rewritten as

$$y(x) = \theta^T \xi(x). \tag{17}$$

Lemma 1 [39]: Let $f(x)$ be a continuous function which is defined on a compact set Ω. Then, for any constant $\varepsilon > 0$, there exists an FLS (17) such as

$$\sup_{x\in\Omega} |f(x) - \theta^T \xi(x)| \le \varepsilon. \tag{18}$$

By Lemma 1, FLSs are universal approximations, i.e., they can approximate any smooth function on a compact space. Due to this approximation capability, we can assume that the nonlinear term $f(x)$ can be approximated as

$$f(x, \theta) = \theta^T \xi(x). \tag{19}$$

Define the optimal parameter vector θ^* as

$$\theta^* = \arg\min_{\theta\in\Omega}[\sup_{x\in U} |f(x) - f(x,\theta^*)|]$$

where Ω and U are compact regions for θ and x, respectively. In addition, the FLS minimum approximation error is defined as

$$\varepsilon = f(x) - \theta^{*T}\xi(x). \tag{20}$$

The aforementioned fuzzy logic system is used to approximate the unknown functions $\rho_i, a_{i,j}$, namely there exist $\theta^*_{\rho,i}$, $\theta^*_{\alpha,i,j}, \varepsilon_{\rho,i}, \varepsilon_{\alpha,i,j}$ such that $\rho_i^u(x) = \theta^{*T}_{\rho,i}\xi_{\rho,i}(x) + \varepsilon_{\rho,i} a^u_{i,j}(x) = \theta^{*T}_{\alpha,i,j}\,\xi_{\alpha,i,j}(x) + \varepsilon_{\alpha,i,j}$. Now, the following assumption is made.

Assumption 2: There exist unknown positive constants $\varepsilon^*_{\rho,i}, \varepsilon^*_{\alpha,i,j}$ and known constants $\bar{M}_{\rho,s_k}, \bar{M}_{\alpha,s_k,j}, M_{\rho,s_k}, M_{\alpha,s_k,j}$ such that $|\varepsilon_{\rho,i}| \le \varepsilon^*_{\rho,i}, |\varepsilon_{\alpha,s_k,j}| \le \varepsilon^*_{\alpha,i,j}, \varepsilon^*_{\rho,i} \le \bar{M}_{\rho,s_k}, \varepsilon^*_{\alpha,i,j} \le \bar{M}_{\alpha,s_k,j}, \|\theta^*_{\rho,s_k}\| \le M_{\rho,s_k}, \|\theta^*_{\alpha,s_k,j}\| \le M_{\alpha,s_k,j}$.

III. Fault Diagnosis and Fuzzy Logic System-Based Accommodation

Consider the T–S fuzzy faulty system which is described in (6). We assume that only actuator faults occur, and no sensor fault is involved. The following assumptions are considered.

Assumption 3: Matrix B_i is of full column rank, and the pair (A_i, C_i) is observable.

A. Fault Detection

In order to detect the actuator faults, we design a fuzzy state-space observer for the system (6), which is described as follows.

Observer Rule i: IF $z_1(t)$ is M_{i1} and $z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{\hat{x}}(t) = A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C_i\hat{x}(t) \end{cases} \tag{21}$$

where $L_i, i = 1, ...r$, is the observer gain for the ith observer rule.

The overall fuzzy system is inferred as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z)(A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = \sum_{i=1}^{r} h_i(z) C_i \hat{x}(t). \end{cases} \tag{22}$$

Denote

$$e_x = x(t) - \hat{x}(t), \quad e_y = y(t) - \hat{y}(t). \tag{23}$$

Then the error dynamics is described by

$$\dot{e}_x = \sum_{i=1}^{r} h_i(z)(A_i - L_iC_i)e_x(t)), \quad e_y = \sum_{i=1}^{r} h_i(z)C_ie_x(t). \tag{24}$$

Lemma 2: The estimation error e_x converges asymptotically to zero if there exist common matrices $P = P^T > 0$ and $Q > 0$ with appropriate dimensions such that the following linear matrix inequality is satisfied:

$$P(A_i - L_iC_i) + (A_i - L_iC_i)^T P \le -Q, \quad i = 1, 2, \ldots, r. \tag{25}$$

Proof: Consider the following Lyapunov function:

$$V_D(t) = e_x^T(t) P e_x(t).$$

Differentiation of V_1 with respect to time t, one has

$$\dot{V}_D(t) \le -\sum_{i=1}^{r} h_i(z(t))[e_x^T(t) Q e_x(t)] \le 0. \tag{26}$$

Since $e_x(t), \dot{e}_x(t) \in L_\infty$ and $e_x(t) \in L_2$, using the Lyapunov stability theory, we obtain $\lim_{t\to\infty} e_x(t) = 0$; furthermore, $\lim_{t\to\infty} e_y(t) = 0$. The proof is completed. □

From Lemma 2, we have

$$\dot{V}_D(t) \le -(\lambda_{\min}(Q)/\lambda_{\max}(P))V_D(t) = -\kappa V_D(t) \tag{27}$$

where $\kappa = \lambda_{\min}(Q)/\lambda_{\max}(P) \in R$. Hence

$$V_D(t) \le e^{-\kappa t} V(0). \tag{28}$$

Furthermore, we have

$$\lambda_{\min}(P)\|e_x(t)\|^2 \le e^{-\kappa t}\lambda_{\max}(P)\|e_x(0)\|^2. \tag{29}$$

Therefore, the norm of the error vector satisfies

$$\|e_x(t)\| \le \sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}\|e_x(0)\|e^{-\kappa t/2}. \tag{30}$$

Furthermore, the detection residual can be defined as

$$J(t) = \|y(t) - \hat{y}(t)\|. \tag{31}$$

From (30), the following inequality holds in the healthy case:

$$J(t) \le \sum_{i=1}^{r} h_i(z)\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}\|C_i\|\|e_x(0)\|e^{-\kappa t/2}.$$

Then, the fault detection can be performed using the mechanism

$$\begin{cases} J(t) \le T_d, & \text{no fault occurred} \\ J(t) > T_d, & \text{fault has occurred} \end{cases} \tag{32}$$

where threshold T_d is defined as follows:

$$T_d = \sum_{i=1}^{r} h_i(z)\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}\|C_i\|\|e_x(0)\|e^{-\kappa t/2}. \tag{33}$$

B. Fault Isolation

Since the system has m actuators, which maybe become faulty, we have $C_m^1 + C_m^2 + \cdots + C_m^m$ possible faulty cases, where C_m^i denotes the number of faulty cases, where there are

i faulty actuators within m actuators. Let us define the symbol $j_i^k, i = 1, 2, \ldots, m, k = 1, 2, \ldots, i$, which denotes the situation where the ith actuator fails when there are k possible faulty actuators among the m actuators. Fault patterns can be described in detail as follows.

Case 1: Only an actuator is faulty

$$\aleph_1 : \{\aleph_1^1, \aleph_1^2, \ldots, \aleph_1^{C_m^1}\} = \{\{j_1^1\}, \{j_2^1\}, \ldots, \{j_m^1\}\}.$$

In this case, there are C_m^1 fault patterns.

Case i ($2 \le i \le m-1$): Only i actuators are faulty

$$\aleph_i : \{\aleph_i^1, \aleph_i^2, \ldots, \aleph_i^{C_m^i}\} = \{\{j_1^i, j_2^i, \ldots, j_i^i\}, \ldots, \{j_{m-i+1}^2, \ldots, j_m^2\}\}$$

where the total fault pattern is $C_m^i, \quad i = 1, 2, \ldots, m$.

Case m: All m actuators are faulty

$$\aleph_m : \{\aleph_m^1, \ldots, \aleph_m^{C_m^m}\} = \{\{j_1^i, j_2^i, \ldots, j_m^i\}\}.$$

Here, there is only one fault pattern ($C_m^m = 1$). Now, let

$$\aleph_m = \{\aleph_1^1, \ldots, \aleph_1^{C_m^1}, \ldots, \aleph_i^1, \ldots, \aleph_i^{C_m^i}, \ldots, \aleph_m^1, \ldots, \aleph_m^{C_m^m}\}.$$

Obviously, there are $C_m^1 + C_m^2 + \cdots + C_m^m$ possible fault patterns that are numbered as the *1*st$, \ldots, N$th fault pattern, where $N = C_m^1 + C_m^2 + \cdots + C_m^m$.

In this paper, it is assumed that these d actuators became faulty whose pattern s is $\aleph_d^q$, namely $s = \aleph_d^q$. We also assume that the d actuators are the s_1th, s_2th$, \ldots, s_d$th actuators, where $1 \le s_1 < s_2 < \cdots < s_d \le m$. Then, the faulty model can be described as

$$\begin{cases} \dot{x}_s(t) = \sum_{i=1}^{r} h_i(z)\Bigg(A_i x_s(t) + B_i u(t) \\ \qquad - \sum_{k=1}^{d} b_{i,s_k}\left(\rho_{s_k}(x) u_{s_k}^s - \sum_{j=1}^{p} a_{s_k,j}(x)\right)\Bigg) \\ y(t) = \sum_{i=1}^{r} h_i(z) C_i x(t) \end{cases} \tag{34}$$

where $B_i = [b_{i,1}, b_{i,2}, \ldots, b_{i,m}], b_{i,l} \in R^{n\times 1}, 1 \le l \le m, \rho_{s_k}(x)$, and $a_{s_k,j}(x)$ denote the time profiles of the s_kth actuator fault, which are described by (10), and $u_{s_k}^s(t)$ is the desired controller when the s_kth actuator is healthy.

Inspired by the sliding mode observers (SMOs) in [26], we are ready to present one of the results of this paper. It is assumed that fuzzy observer and fuzzy control systems have the same premise variables $z(t)$; then the following fuzzy observers are proposed to isolate the actuator fault.

Isolation Observer Rule i: IF $z_1(t)$ is M_{i1} and $z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{\hat{x}}_{is}(t) = A_i \hat{x}_{is}(t) + L_i(y(t) - \hat{y}_{is}(t)) + B_i u(t) \\ \qquad + \sum_{k=1}^{d}\left(b_{i,s_k}\mu_{s_k}\left(\bar{\rho}_{s_k} u_{s_k}^s + \sum_{j=1}^{p} \bar{a}_{s_k,j}\right)\right) \\ \hat{y}_{is}(t) = C_{is}\hat{x}_{is}(t) \end{cases} \tag{35}$$

where $\hat{x}_{is}(t)$ and $\hat{y}_{is}(t)$ are the sth fuzzy observer's state and output, respectively. L_i is the observer's gain matrix for the ith observer. The global fuzzy observer is represented as

$$\begin{cases} \dot{\hat{x}}_s(t) = \sum_{i=1}^{r} h_i(z)\Bigg(A_i\hat{x}_{is}(t) + L_i(y(t) - \hat{y}_{is}(t)) + B_i u(t) \\ \qquad + \sum_{k=1}^{d} b_{i,s_k}\mu_{s_k}\left(\bar{\rho}_{s_k} u_{s_k}^s(t) + \sum_{j=1}^{p}\bar{a}_{s_k,j}\right)\Bigg) \\ \hat{y}_s(t) = \sum_{i=1}^{r} h_i(z) C_i \hat{x}_s(t) \\ \mu_{s_k} = -\sum_{i=1}^{r} h_i(z) F_{is_k} e_{ys}(t) \Big/ \left\| \sum_{i=1}^{r} h_i(z) F_{is_k} e_{ys}(t) \right\| \end{cases} \tag{36}$$

where $F_{is_k} \in R^{1\times n}$ is the s_kth row of $F_i \in R^{m\times n}$, which will be defined later, $L_i \in R^{n\times n}$ is chosen such that $A_i - L_iC_i$ is Hurwitz, and $e_{xs}(t) = x_s(t) - \hat{x}_s(t)$ and $e_{ys}(t) = y(t) - \hat{y}_s(t)$ are, respectively, the state error and output error between the plant and the sth SMO observer. Let l denote the practical fault pattern where the faulty actuators are the l_1th$, l_2$th$, \ldots, l_{d^*}$th actuators, where $1 \le l_1 < l_2 < \cdots < l_{d^*} \le m$.

For $s = l$, namely $d = d^*$, $l_1 = s_1$, $l_2 = s_2, \ldots, l_{d^*} = s_d$, the error dynamics is obtained from (34) and (36):

$$\dot{e}_{xs} = \sum_{i=1}^{r} h_i(z)\Bigg\{(A_i - L_iC_i)e_{is} + \sum_{k=1}^{d} b_{i,s_k}\Bigg[(-\rho_{s_k} u_{s_k}^s - \mu_{s_k}\bar{\rho}_{s_k}|u_{s_k}^s|) + \sum_{j=1}^{p}(a_{s_k,j} - \mu_{s_k}\bar{a}_{s_k,j})\Bigg]\Bigg\}. \tag{37}$$

For $s \ne$ l, namely $d \ne d^*$ or $d = d^*$ and at least there exists l_i such that $l_i \ne s_i, \ i = 1, \ldots, d$, we have

$$\begin{aligned} \dot{e}_{xs} &= \sum_{i=1}^{r} h_i(z)(A_i - L_iC_i)e_{is} + \sum_{i=1}^{r} h_i(z) \\ &\times \Bigg[\Bigg(-\sum_{k=1}^{d^*} b_{i,l_k}\rho_{s_k} u_{l_k}^s - \sum_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{\rho}_{s_k}|u_{s_k}^s|\Bigg) \\ &+ \sum_{j=1}^{p}\Bigg(\sum_{k=1}^{d^*} b_{i,l_k} a_{l_k,j} - \sum_{k=1}^{d} b_{i,s_k}\mu_{s_k}\bar{a}_{s_k,j}\Bigg)\Bigg]. \end{aligned} \tag{38}$$

The stability of the state error dynamics is guaranteed by the following theorem.

Theorem 1: Under Assumptions 1–3, *if* there exist a common matrices $P = P^T > 0, Q > 0$ and matrices $L_i, F_i, i = 1, \ldots, r$, with appropriate dimensions such that the following conditions hold:

$$(A_i - L_iC_i)^T P + P(A_i - L_iC_i) \le -Q \tag{39}$$

$$PB_i = (F_iC_i)^T \tag{40}$$

then when the lth pattern is the actual fault pattern, i.e., $s = l$, we have $\lim_{t\to\infty} e_{xs}(t) = 0$, and for $s \ne l, \lim_{t\to\infty} e_{ys}(t) = 0$.

Proof:

1) For $s = l$, according to (39), we have

$$\dot{e}_{xs} = \sum_{i=1}^{r} h_i(z)\Bigg\{(A_i - L_iC_i)e_{is} + \sum_{k=1}^{d} b_{i,s_k}\Bigg[(-\rho_{s_k}u_{s_k} - \mu_{s_k}\bar{\rho}_{s_k}|u_{s_k}|) + \sum_{j=1}^{p}(a_{s_k,j} - \mu_{s_k}\bar{a}_{s_k,j})\Bigg]\Bigg\}. \quad (41)$$

Define the following Lyapunov function:

$$V_I(t) = e_{xs}^T(t)Pe_{xs}(t).$$

Differentiating V_2 with respect to time t, and using (41), one has

$$\dot{V}_I(t) \le -e_{xs}^TQe_{xs} + 2e_{xs}^TP\sum_{i=1}^{r}h_i(z)\Bigg\{\sum_{k=1}^{d}b_{i,s_k}\Bigg[(-\rho_{s_k}u_{s_k}^s - \mu_{s_k}\bar{\rho}_{s_k}|u_{s_k}^s|) + \sum_{k=1}^{d}(a_{s_k,j} - \mu_{s_k}\bar{a}_{s_k,j})\Bigg]\Bigg\}. \quad (42)$$

From the definition of μ_{s_k} in (36) and (42), one has

$$2e_{xs}^TP\sum_{i=1}^{r}h_i(z)\sum_{k=1}^{d}b_{i,s_k}[(-\rho_{s_k}u_{s_k}^s(t) - \mu_{s_k}\bar{\rho}_{s_k}|u_{s_k}^s|) \le 0$$

$$2e_{xs}^TP\sum_{i=1}^{r}h_i(z)\sum_{k=1}^{d}(a_{s_k,j} - \mu_{s_k}\bar{a}_{s_k,j}) \le 0.$$

Hence

$$\dot{V}_I(t) \le -e_{xs}^TQe_{xs} \le 0. \quad (43)$$

Similar to Lemma 2, we have $\lim_{t\to\infty} e_{xs}(t) = 0$. Furthermore, $\lim_{t\to\infty} e_{ys}(t) = 0$.

2) For $s \ne l$, it follows from (34) and (36) that

$$\dot{e}_{xs} = \sum_{i=1}^{r}h_i(z)(A_i - L_iC_i)e_{is} + \sum_{i=1}^{r}h_i(z) \cdot \Bigg[\Bigg(-\sum_{k=1}^{d^*}b_{i,l_k}\rho_{l_k}u_{l_k}^s - \sum_{k=1}^{d}b_{i,s_k}\mu_{s_k}\bar{\rho}_{s_k}|u_{s_k}^s|\Bigg) + \sum_{j=1}^{p}\Bigg(\sum_{k=1}^{d^*}b_{i,l_k}a_{l_{k1},j} - \sum_{k=1}^{d}b_{i,s_k}\mu_{s_k}\bar{a}_{s_k,j}\Bigg)\Bigg]. \quad (44)$$

Because matrix B_i is of full column rank (Assumption 1), we know that b_{is_k} and $b_{il_{k1}}$ are linearly independent. Therefore

$$\lim_{t\to\infty}\sum_{i=1}^{r}h_i(z)\Bigg[\Bigg(-\sum_{k=1}^{d^*}b_{i,l_k}\rho_{l_k}u_{l_k}^s - \sum_{k=1}^{d}b_{i,s_k}\mu_{s_k}\bar{\rho}_{s_k} \cdot |u_{s_k}^s|\Bigg) + \sum_{j=1}^{p}\Bigg(\sum_{k=1}^{d^*}b_{i,l_k}a_{l_k,j} - \sum_{k=1}^{d}b_{i,s_k}\mu_{s_k}\bar{a}_{s_k,j}\Bigg)\Bigg] \ne 0. \quad (45)$$

Thus, we have $\lim_{t\to\infty} e_{xs}(t) \ne 0$ and $\lim_{t\to\infty} e_{ys}(t) \ne 0$.

From "1" and "2," we obtain the conclusion. This ends the proof. □

Now, we denote the residuals between the real system and SMOs as follows:

$$J_s(t) = \|e_{ys}(t)\| = \|\hat{y}_s(t) - y(t)\|, \quad 1 \le \mathrm{s} \le m. \quad (46)$$

According to Theorem 1, when the actual fault pattern is $s = l$, the residual $J_s(t)$ will tend to zero, while for any $s \ne l$, $J_s(t)$ does not equal zero. Furthermore, from Lemma 1, we have if $l = s$, then

$$J_s(t) \le \sum_{i=1}^{r}h_i(z)\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}\|e_{ys}(0)\|e^{-\kappa t/2} \quad (47)$$

and if $l \ne s$, then

$$J_s(t) > \sum_{i=1}^{r}h_i(z)\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}\|e_{ys}(0)\|e^{-\kappa t/2}. \quad (48)$$

Hence, the isolation law for actuator fault can be designed as

$$\begin{cases} J_s(t) \le T_I, & l = s \Rightarrow \text{ the } l_1\text{th}, \ldots, l_d\text{th actuators are faulty} \\ J_s(t) > T_I, & l \ne s \end{cases} \quad (49)$$

where threshold T_I is defined as follows:

$$T_I = \sum_{i=1}^{r}h_i(z)\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}\|e_{ys}(0)\|e^{-\kappa t/2}.$$

Notice that the denominator of μ_{s_k} in (36) contains $e_{ys}(t)$. Just as pointed out in [26], the chattering phenomenon occurs when $e_{ys}(t) \to 0$ in practice. Inspired by [26], in order to reduce this chattering in practical applications, we modify SMOs (36) by introducing a positive constant δ as follows:

$$\begin{cases} \dot{\hat{x}}_s(t) = \sum_{i=1}^{r}h_i(z)\Bigg(A_i\hat{x}_{is}(t) + L_i(y(t) - \hat{y}_{is}(t)) + B_iu(t) + \sum_{k=1}^{d}b_{i,s_k}\mu_{s_k}\Bigg(\bar{\rho}_{s_k}u_{s_k}^s(t) + \sum_{j=1}^{p}\bar{a}_{s_k,j}\Bigg)\Bigg) \\ \hat{y}_s(t) = \sum_{i=1}^{r}h_i(z)C_i\hat{x}_s(t) \\ \mu_{s_k} = -\sum_{i=1}^{r}h_i(z)F_{is_k}e_{ys} \Bigg/ \Bigg(\Bigg\|\sum_{i=1}^{r}h_i(z)F_{is_k}e_{ys}\Bigg\| + \delta\Bigg) \end{cases} \quad (50)$$

where $\delta > 0 \in R$ is a constant.

C. Fuzzy Logic System-Based Fault Accommodation With Available System State

After fault isolation, the next task is fault accommodation. Before this task, we first investigate the following normal systems (fault-free) and drive the ideal control $u^s(t)$ when all actuators are healthy:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z)[A_i x(t) + B_i u^s(t)] \\ y(t) = \sum_{i=1}^{r} h_i(z) C_i x(t). \end{cases} \tag{51}$$

The PDC technique offers a procedure to design a fuzzy control law from a given T–S fuzzy model. In the PDC design, each control rule is designed from the corresponding rule of T–S fuzzy model. The designed fuzzy controller has the same fuzzy sets as the considered fuzzy system.

Control Rule i: IF $z_1(t)$ is M_{i1} and $z_q(t)$ is M_{iq}, THEN

$$u_i^s(t) = K_i x(t)$$

and the overall fuzzy controller is given as follows:

$$u^s(t) = \sum_{i=1}^{r} h_i(z) K_i x(t) \tag{52}$$

where the controller gain matrix K_i is determined by solving the following condition:

$$P(A_i + K_i B_i) + (A_i + K_i B_i)^T P + P S_1 P \le -Q \tag{53}$$

where $P = P^T > 0, Q > 0,\ S_1 > 0$, and $S_2 > 0$ are matrices with appropriate dimensions.

Define tracking error $\bar{e} = y - \omega_d$. The tracking error dynamics is obtained from the aforementioned equations:

$$\dot{\bar{e}} = \dot{y} - \dot{\omega}_d = \sum_{i=1}^{r} h_i(z)(C_i A_i x + C_i B_i u^s) - \dot{\omega}_d.$$

Because all the states are supposed to be available, we have $C_i = I_{m\times m}$. The tracking error dynamics is simplified as follows:

$$\dot{\bar{e}} = \sum_{i=1}^{r} h_i(z)[(A_i + B_i K_i)\bar{e} + B_i K_i \omega_d - \dot{\omega}_d]. \tag{54}$$

Define the following Lyapunov function:

$$V_0 = \bar{e}^T P \bar{e}.$$

Differentiation of V_0 with respect to time t leads to

$$\dot{V}_0 = \sum_{i=1}^{r} h_i(z)[\bar{e}^T(P(A_i + K_i B_i) + (A_i + K_i B_i)^T P)\bar{e} + 2\bar{e}^T P(K_i B_i \omega_d - \dot{\omega}_d)]. \tag{55}$$

Since $2\bar{e}^T P(B_i K_i \omega_d - \dot{\omega}_d) \le \bar{e}^T(t) P S_1 P \bar{e} + (B_i K_i \omega_d - \dot{\omega}_d)^T \cdot S_1^{-1}(B_i K_i \omega_d - \dot{\omega}_d)$, (55) can be rewritten as follows:

$$\dot{V}_0 \le \sum_{i=1}^{r} h_i(z)[\bar{e}^T \Delta_1 \bar{e}] + \sum_{i=1}^{r} h_i(z)[(K_i B_i \omega_d - \dot{\omega}_d)^T S_1^{-1}(K_i B_i \omega_d - \dot{\omega}_d)]$$

where $\Delta_1 = P(A_i + K_i B_i) + (A_i + K_i B_i)^T P + P S_1 P$. Obviously, if $\Delta_1 \le -Q$, then

$$\dot{V}_0 \le -\sum_{i=1}^{r} h_i(z)[\bar{e}^T Q \bar{e}] + \mu_0 \le -\lambda_0 V_0 + \mu_0$$

where $\lambda_0 = \lambda_{\min}(Q)/\lambda_{\max}(P)$, and $\mu_0 = \sum_{i=1}^{r} h_i(z)[(K_i B_i \omega_d - \dot{\omega}_d)^T S_1^{-1}(K_i B_i \omega_d - \dot{\omega}_d)]$.

Then, one has $\frac{d}{dt}(V_0(t) e^{\lambda_0 t}) \le e^{\lambda_0 t}\mu_0$. Furthermore

$$0 \le V_0(t) \le \mu_0/\lambda_0 + [V_0(0) - \mu_0/\lambda_0] e^{-\lambda_0 t} \le \mu_0/\lambda_0 + V_0(0).$$

Therefore, the error system (54) is asymptotically stable. Moreover, $\bar{e}(t)$ is semiglobally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely, $|\bar{e}| \le \sqrt{\alpha/\lambda_{\min}(P)}$, where $\alpha = \mu_0/\lambda_0 + V_0(0)$.

Lemma 3: Under the normal condition (no fault), controlled by feedback control $u^s(t) = \sum_{i=1}^{r} h_i(z) K_i x(t)$, system tracking error $\bar{e} = y - \omega_d$ converges asymptotically to zero if there exist common matrices $P = P^T > 0$ and $Q > 0$ with appropriate dimensions such that the following linear matrix inequality is satisfied:

$$P(A_i + K_i B_i) + (A_i + K_i B_i)^T P + P S_1 P \le -Q \qquad i = 1, 2, \ldots, r.$$

Proof: From the aforementioned analysis, it is easy to obtain the conclusions. The detailed proof is omitted. □

Consider the following faulty system:

$$\dot{x}_s(t) = \sum_{i=1}^{r} h_i(z)\left(A_i x_s(t) + B_i u^s(t) - \sum_{k=1}^{d} b_{i,s_k}\left(\rho_{s_k}(x) u_{s_k}^s(t) - \sum_{j=1}^{p} a_{s_k,j}(x)\right)\right). \tag{56}$$

In order to estimate the faults, the following observer is constructed as follows:

$$\begin{cases} \dot{\hat{x}}_s = \sum_{i=1}^{r} h_i(z)\left(A_i \hat{x}_s + B_i u^s + L_i(y_s - \hat{y}_s) - \sum_{k=1}^{d} b_{i,s_k} \cdot \left((\hat{\rho}_{s_k} - \bar{\hat{\varepsilon}}_{\rho,s_k}) u_{s_k}^s - \sum_{j=1}^{p} (\hat{a}_{s_k,j} + \bar{\hat{\varepsilon}}_{\alpha,s_k,j})\right)\right) \\ \hat{y}_s = \sum_{i=1}^{r} h_i(z) C_i \hat{x}_s \end{cases} \tag{57}$$

where $\hat{\theta}_{\rho,s_k}, \hat{\theta}_{\alpha,s_k,j}, \hat{\rho}_{\rho,s_k}$, and $\hat{a}_{s_k,j}$ are the estimations of $\theta_{\rho,s_k}^*, \theta_{\alpha,s_k,j}^*, \rho_{s_k}(x, \theta_{\rho,s_k}^*)$, and $a_{s_k,j}(x, \theta_{\alpha,s_k,j}^*)$, which are used to compensate for the faults $\rho_{s_k}, \alpha_{s_k,j}, \rho_{s_k}(x) = \rho_{s_k}(x, \theta_{\rho,s_k}^*) + \varepsilon_{\rho,s_k}$ and $a_{s_k,j}(x) = a_{s_k,j}(x, \theta_{\alpha,s_k,j}^*) + \varepsilon_{\alpha,s_k,j}$, $\varepsilon_{\rho,s_k}, \varepsilon_{\alpha,s_k,j}$ are approximation errors, $\theta_{\rho,s_k}^*, \theta_{\alpha,s_k,j}^*$ are optimal vectors, $\bar{\hat{\varepsilon}}_{\rho,s_k} = \mathrm{sgn}(2e_s^T P b_{i,s_k} u_{s_k}^s)\hat{\varepsilon}_{\rho,s_k}$, and $\bar{\hat{\varepsilon}}_{\alpha,s_k,j} = \mathrm{sgn}(2e_s^T P b_{i,s_k})\hat{\varepsilon}_{\alpha,s_k,j}$.

Define $e_s(t) = x_s(t) - \hat{x}_s(t)$; then, the observer error dynamics can be described as follows:

$$\dot{e}_s = \sum_{i=1}^{r} h_i(z)\Bigg[(A_i - L_iC_i)e_s - \sum_{k=1}^{d} b_{i,s_k}(\rho_{s_k} - \hat{\rho}_{s_k} + \hat{\varepsilon}_{s_k})u^s_{s_k} + \sum_{k=1}^{d}\sum_{j=1}^{p} b_{i,s_k}(a_{\alpha,s_k,j} - \hat{a}_{\alpha,s_k,j} - \hat{\varepsilon}_{\alpha,s_k,j})\Bigg]. \tag{58}$$

Further, one has

$$\dot{e}_s = \sum_{i=1}^{r} h_i(z)\Bigg[(A_i - L_iC_i)e_s - \sum_{k=1}^{d} b_{i,s_k}(\tilde{\theta}^T_{s_k}\xi_{s_k} + \varepsilon_{s_k} + \hat{\varepsilon}_{s_k})u^s_{s_k} + \sum_{k=1}^{d}\sum_{j=1}^{p} b_{i,s_k}(\tilde{\theta}^T_{\alpha,s_k,j}\xi_{\alpha,s_k,j} + \varepsilon_{\alpha,s_k,j} - \hat{\varepsilon}_{\alpha,s_k,j})\Bigg] \tag{59}$$

where $\tilde{\theta}_{\rho,s_k} = \hat{\theta}_{\rho,s_k} - \theta^*_{\rho,s_k}$, and $\tilde{\theta}_{\alpha,s_k,j} = \hat{\theta}_{\alpha,s_k,j} - \theta^*_{\alpha,s_k,j}$.

Define the function

$$V_1 = e_s^T P e_s.$$

Differentiation of V_1 with respect to time t leads to

$$\begin{aligned}
\dot{V}_1 = &\sum_{i=1}^{r} h_i(z)\Bigg[e_s^T(P(A_i - L_iC_i) + (A_i - L_iC_i)^T P)e_s \\
&- \sum_{k=1}^{d} 2e_s^T P b_{i,s_k}(\tilde{\theta}^T_{s_k}\xi_{s_k} + \varepsilon_{s_k} + \bar{\hat{\varepsilon}}_{s_k})u^s_{s_k} \\
&+ \sum_{k=1}^{d}\sum_{j=1}^{p} 2e_s^T P b_{i,s_k}(\tilde{\theta}^T_{\alpha,s_k,j}\xi_{\alpha,s_k,j} + \varepsilon_{\alpha,s_k,j} - \bar{\hat{\varepsilon}}_{\alpha,s_k,j})\Bigg] \\
\le &\sum_{i=1}^{r} h_i(z)\Bigg[e_s^T(P(A_i - L_iC_i) + (A_i - L_iC_i)^T P)e_s \\
&- \sum_{k=1}^{d} 2e_s^T P b_{i,s_k}\left(\tilde{\theta}^T_{s_k}\xi_{s_k} + \sum_{j=1}^{p}\tilde{\theta}^T_{\alpha,s_k,j}\xi_{\alpha,s_k,j}\right) \\
&+ \sum_{k=1}^{d} |2e_s^T P b_{i,s_k}|(\varepsilon^*_{s_k} - \hat{\varepsilon}_{s_k})u^s_{s_k} \\
&+ \sum_{k=1}^{d}\sum_{j=1}^{p} |2e_s^T P b_{i,s_k}|(\varepsilon^*_{\alpha,s_k,j} - \hat{\varepsilon}_{\alpha,s_k,j})\Bigg] \\
\le &\sum_{i=1}^{r} h_i(z)\Bigg[e_s^T(P(A_i - L_iC_i) + (A_i - L_iC_i)^T P)e_s \\
&- \sum_{k=1}^{d} 2e_s^T P b_{i,s_k}\left(\tilde{\theta}^T_{\rho,s_k}\xi_{\rho,s_k} + \sum_{j=1}^{p}\tilde{\theta}^T_{\alpha,s_k,j}\xi_{\alpha,s_k,j}\right) \\
&+ \sum_{k=1}^{d} |\left(2e_s^T P b_{i,s_k} u^s_{s_k}|\tilde{\varepsilon}_{\rho,s_k} + \sum_{j=1}^{p}|2e_s^T P b_{i,s_k}|\tilde{\varepsilon}_{\alpha,s_k,j}\right)\Bigg].
\end{aligned}$$

Define the following smooth function:

$$V = V_1 + V_2 + V_3 + V_4 + V_5 \tag{60}$$

with

$$V_2 = \sum_{i=1}^{r}\sum_{k=1}^{d} h_i(z)(\tilde{\theta}^T_{\rho,s_k}\tilde{\theta}_{\rho,s_k})/2\eta_1$$

$$V_3 = \sum_{i=1}^{r}\sum_{k=1}^{d}\sum_{j=1}^{p} h_i(z)(\tilde{\theta}^T_{\alpha,s_k,j}\tilde{\theta}_{\alpha,s_k,j})/2\eta_2$$

$$V_4 = \sum_{i=1}^{r}\sum_{k=1}^{d} h_i(z)(\tilde{\varepsilon}^2_{\rho,s_k})/2\eta_3$$

$$V_5 = \sum_{i=1}^{r}\sum_{k=1}^{d}\sum_{j=1}^{p} h_i(z)(\varepsilon^2_{\alpha,s_k,j})/2\eta_4$$

where $\eta_i > 0$, $i = 1, \ldots, 4$ denote the adaptive rates.

Differentiation of V with respect to time t leads to

$$\begin{aligned}
\dot{V} \le &\sum_{i=1}^{r} h_i(z)\Bigg\{e_s^T(P(A_i - L_iC_i) + (A_i - L_iC_i)^T P)e_s \\
&- \sum_{k=1}^{d}\Bigg[\tilde{\theta}^T_{\rho,s_k}(2e_s^T P b_{i,s_k}\xi_{\rho,s_k} - \dot{\hat{\theta}}_{\rho,s_k}/\eta_1) \\
&+ \sum_{j=1}^{p}\tilde{\theta}^T_{\alpha,s_k,j}(2e_s^T P b_{i,s_k}\xi_{\alpha,s_k,j} + \dot{\hat{\theta}}_{\alpha,s_k,j}/\eta_2)\Bigg] \\
&+ \sum_{k=1}^{d}\Bigg[\tilde{\varepsilon}_{\rho,s_k}(|2e_s^T P b_{i,s_k} u^s_{s_k}| + \dot{\hat{\varepsilon}}_{\rho,s_k}) \\
&+ \sum_{j=1}^{p}\tilde{\varepsilon}_{\alpha,s_k,j}(|2e_s^T P b_{i,s_k}| + \dot{\hat{\varepsilon}}_{\rho,s_k}\Bigg]\Bigg\}.
\end{aligned} \tag{61}$$

Now, the adaptive laws are designed as follows:

$$\dot{\hat{\theta}}_{\rho,s_k} = \begin{cases} \eta_1 e_s^T P b_{i,s_k}\xi_{\rho,s_k}u^s_{s_k}, \text{ if } \|\hat{\theta}_{\rho,s_k}\| < M_{\rho,s_k} \text{ or} \\ \quad \|\hat{\theta}_{\rho,s_k}\| = M_{\rho,s_k} \text{ and } -\eta_1 e_s^T P b_{i,s_k}\xi_{\rho,s_k}u^s_{s_k} \ge 0 \\ \eta_1 e_s^T P b_{i,s_k}\xi_{\rho,s_k}u^s_{s_k} - \eta_1 e_s^T P b_{i,s_k}u^s_{s_k}\dfrac{\theta_{\rho,s_k}\theta^T_{\rho,s_k}}{\|\hat{\theta}_{if}\|^2}\xi_{\rho,s_k} \\ \quad \text{if } \|\hat{\theta}_{\rho,s_k}\| = M_{\rho,s_k} \text{ and } -\eta_1 e_s^T P b_{i,s_k}\xi_{\rho,s_k}u^s_{s_k} < 0 \end{cases} \tag{62}$$

$$\dot{\hat{\theta}}_{\alpha,s_k,j} = \begin{cases} -\eta_2 e_s^T P b_{i,s_k}\xi_{\alpha,s_k,j}, \text{ if } \|\hat{\theta}_{\alpha,s_k,j}\| < M_{\alpha,s_k,j} \text{ or} \\ \quad \|\hat{\theta}_{\alpha,s_k,j}\| = M_{\alpha,s_k,j} \text{ and } \eta_2 e_s^T P b_{i,s_k}\xi_{\alpha,s_k,j} \ge 0 \\ -\eta_2 e_s^T P b_{i,s_k}\xi_{\alpha,s_k,j} - \eta_2 e_s^T P b_{i,s_k}\dfrac{\hat{\theta}_{\alpha,s_k,j}\hat{\theta}^T_{\alpha,s_k,j}}{\|\hat{\theta}_{\alpha,s_k,j}\|^2}\xi_{\alpha,s_k,j} \\ \quad \text{if } \|\hat{\theta}_{\alpha,s_k,j}\| = M_{\alpha,s_k,j} \text{ and } e_s^T P b_{i,s_k}\xi_{\alpha,s_k,j} < 0 \end{cases} \tag{63}$$

$$\dot{\hat{\varepsilon}}_{\rho,s_k} = \begin{cases} 0, \text{ if } \hat{\varepsilon}_{\rho,s_k} = \bar{M}_{\rho,s_k} \text{ and } -\eta_3|e_s^T P b_{i,s_k}u^s_{s_k}| > 0 \\ \quad \text{or } \hat{\varepsilon}_{\rho,s_k} = -\bar{M}_{\rho,s_k} \text{ and } -\eta_3|e_s^T P b_{i,s_k}u^s_{s_k}| < 0 \\ -\eta_3|e_s^T P b_{i,s_k}u^s_{s_k}|, \text{ otherwise} \end{cases} \tag{64}$$

$$\dot{\hat{\varepsilon}}_{\alpha,s_k,j} = \begin{cases} 0, \text{if} \hat{\varepsilon}_{\alpha,s_k,j} = \bar{M}_{\alpha,s_k,j} \text{ and } -\eta_4 |e_s^T P b_{i,s_k}| > 0 \\ \text{ or } \hat{\varepsilon}_{\alpha,s_k,j} = -\bar{M}_{\alpha,s_k,j} \text{ and } -\eta_4 |e_s^T P b_{i,s_k}| < 0 \\ -\eta_4 |e_s^T P b_{i,s_k}|, \text{otherwise.} \end{cases} \tag{65}$$

Substitution of (62)–(65) into (61) yields

$$\dot{V}_1 \leq \sum_{i=1}^{r} h_i(z) \{ e_s^T (P(A_i - L_i C_i) + (A_i - L_i C_i)^T P) e_s.$$

If $P(A_i - L_i C_i) + (A_i - L_i C_i)^T P \leq -Q$, then $\dot{V}_1 \leq -e_s^T Q e_s \leq 0$. From the Lyapunov stability theory, we have $\lim_{t\to\infty} e_s(t) = 0$. Further, we have $\lim_{t\to\infty} \tilde{\theta}_{\rho,s_k}(t) = 0$, $\lim_{t\to\infty} \tilde{\varepsilon}_{\rho,s_k}(t) = 0$, $\lim_{t\to\infty} \tilde{\theta}_{\alpha,s_k,j}(t) = 0$, and $\lim_{t\to\infty} \tilde{\varepsilon}_{\alpha,s_k,j}(t) = 0$.

The stability of the fault estimation observer error dynamics is guaranteed by the following theorem.

Theorem 2: Under Assumptions 1–3, *if* there exist common matrices $P = P^T > 0, Q > 0$ and matrices $L_i, K_i, i = 1, \ldots, r$, with appropriate dimensions such that the following condition holds:

$$(A_i - L_i C_i)^T P + P(A_i - L_i C_i) \leq -Q \tag{66}$$

then $\lim_{t\to\infty} e_s(t) = 0$, $\lim_{t\to\infty} \tilde{\theta}_{\rho,s_k}(t) = 0$, $\lim_{t\to\infty} \tilde{\varepsilon}_{\rho,s_k}(t) = 0$, $\lim_{t\to\infty} \tilde{\theta}_{\alpha,s_k,j}(t) = 0$, and $\lim_{t\to\infty} \tilde{\varepsilon}_{\alpha,s_k,j}(t) = 0$.

Proof: From the aforementioned analysis, it is easy to obtain the conclusions. The detailed proof is omitted. □

After obtaining the desired control u^s and the fault information, we will design FTC u such that the same control objective can be achieved in spite of actuator faults. On the basis of the desired control $u^s(t)$, the fault-tolerant controller is constructed as

$$u_{s_k} = \left(u_{s_k}^s - \sum_{j=1}^{p} (\hat{a}_{s_k,j} + \hat{\varepsilon}_{\alpha,s_k,j}) \right) \Big/ (1 - \hat{\rho}_{s_k} - \hat{\varepsilon}_{\rho,s_k}). \tag{67}$$

Theorem 3: Under Assumptions 1–3, there exist a common symmetric positive-definite matrix P, and real matrices K_i and $Q > 0, i = 1, 2, \ldots, r$, with appropriate dimensions such that the following conditions hold:

$$(A_i - L_i C_i)^T P + P(A_i - L_i C_i) \leq -Q \tag{68}$$

$$P(A_i + K_i B_i) + (A_i + K_i B_i)^T P + P S_1 P \leq -Q. \tag{69}$$

Consider the control law (67) and the adaptive laws (62)–(65); then, the system output y can track asymptotically the reference signal ω_d. Moreover, $\bar{e}(t), \tilde{\theta}_{\rho,s_k}$, and $\tilde{\theta}_{\alpha,s_k,j}$ are semiglobally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely, $\|\bar{e}\| \leq \sqrt{\alpha/\lambda_{\min}(P)}$, $\|\tilde{\theta}_{\rho,s_k}\| \leq \sqrt{2\eta_1\alpha}$, $\|\tilde{\theta}_{\alpha,s_k,j}\| \leq \sqrt{2\eta_2\alpha}, \alpha = \mu/\lambda + V(0), \lambda = \min\{\lambda_{\min}(Q)/\lambda_{\max}(P),\ 1/2\eta_1,\ 1/2\eta_2\}$, $\mu = 4\sum_{i=1}^{r} h_i(z) [M_{\rho,s_k}^2/\eta_1 + \bar{M}_{\rho,s_k}^2/\eta_3 + \sum_{j=1}^{p} (M_{\alpha,s_k,j}^2/\eta_2 + \bar{M}_{\alpha,s_k,j}^2/\eta_4)] + \mu_0$, and $\mu_0 = \sum_{i=1}^{r} h_i(z) [(K_i B_i \omega_d - \dot{\omega}_d)^T S_1^{-1} (K_i B_i \omega_d - \dot{\omega}_d)]$.

Proof: From the aforementioned analysis, it is easy to obtain the conclusions. The detailed proof is omitted. □

Remark 1: In the previous section, the fault-tolerant controller was constructed as

$$u_{s_k} = \left(u_{s_k}^s - \sum_{j=1}^{p} (\hat{a}_{s_k,j} + \hat{\varepsilon}_{\alpha,s_k,j}) \right) \Big/ (1 - \hat{\rho}_{s_k} - \hat{\varepsilon}_{\rho,s_k}).$$

Unfortunately, there may exist controller singularity when $1 - \hat{\rho}_{s_k}(x, \hat{\theta}_{\rho,s_k}) - \hat{\varepsilon}_{\rho,s_k} = 0$. In order to avoid such singularity, the fault-tolerant controller is modified as follows:

$$u_{s_k} = \frac{(1 - \hat{\rho}_{s_k} - \hat{\varepsilon}_{\rho,s_k})(u_{s_k}^s - \sum_{j=1}^{p} (\hat{a}_{s_k,j} + \hat{\varepsilon}_{\alpha,s_k,j}))}{(1 - \hat{\rho}_{s_k} - \hat{\varepsilon}_{\rho,s_k})^2 + \varepsilon} \tag{70}$$

where $\varepsilon > 0 \in R$ is a design constant. Correspondingly, the adaptive laws in Theorem 3 should be revised.

D. Fuzzy Logic System-Based Fault Accommodation With Unavailable System State

Notice that the FTC (67) and the modified FTC (70) are designed under the condition that system states are measurable. In fact, in some situations, system state may be unavailable, and FTC (67) and (70) do not work. In this case, observer (57) may be used to obtain the estimation $\hat{x}$ of system state x, and design the following observer-based FTC:

$$u_{s_k} = \left[u_{s_k}^s - \sum_{j=1}^{p} (\hat{\hat{a}}_{s_k,j} + \hat{\hat{\varepsilon}}_{\alpha,s_k,j}) \right] \Big/ (1 - \hat{\hat{\rho}}_{s_k} - \hat{\hat{\varepsilon}}_{\rho,s_k}) \tag{71}$$

where $\hat{\hat{\rho}}_{s_k} = \hat{\hat{\theta}}_{\rho,s_k}^T \xi_{s_k}(\hat{x})$ and $\hat{\hat{a}}_{s_k,j} = \hat{\hat{\theta}}_{\alpha,s_k,j}^T \xi_{\alpha,s_k,j}(\hat{x})$ denote the estimates of $\hat{\theta}_{\rho,s_k}^{*T} \xi_{s_k}(\hat{x})$ and $\hat{\theta}_{\alpha,s_k,j}^{*T} \xi_{\alpha,s_k,j}(\hat{x})$, where the faults $\rho_{s_k} = \hat{\theta}_{\rho,s_k}^{*T} \xi_{s_k}(\hat{x}) + \hat{\hat{\varepsilon}}_{\rho,s_k}$, and $a_{s_k,j} = \hat{\theta}_{\alpha,s_k,j}^{*T} \xi_{\alpha,s_k,j}(\hat{x}) + \hat{\hat{\varepsilon}}_{\alpha,s_k,j}$.

Correspondingly, the adaptive laws in Theorem 3 are redesigned as follows:

$$\dot{\hat{\hat{\theta}}}_{\rho,s_k} = \begin{cases} \eta_1 e_s^T P b_{i,s_k} \xi_{\rho,s_k} u_{s_k}^s, \text{ if } \|\hat{\hat{\theta}}_{\rho,s_k}\| < M_{\rho,s_k} \text{ or} \\ \|\hat{\hat{\theta}}_{\rho,s_k}\| = M_{\rho,s_k} \text{ and } -\eta_1 e_s^T P b_{i,s_k} \xi_{\rho,s_k} u_{s_k}^s \geq 0 \\ \eta_1 e_s^T P b_{i,s_k} \xi_{\rho,s_k} u_{s_k}^s - \eta_1 e_s^T P b_{i,s_k} u_{s_k}^s \frac{\hat{\theta}_{\rho,s_k} \hat{\hat{\theta}}_{\rho,s_k}^T}{\|\hat{\theta}_{if}\|^2} \xi_{\rho,s_k} \\ \text{if } \|\hat{\hat{\theta}}_{\rho,s_k}\| = M_{\rho,s_k} \text{ and } -\eta_1 e_s^T P b_{i,s_k} \xi_{\rho,s_k} u_{s_k}^s < 0 \end{cases} \tag{72}$$

$$\dot{\hat{\hat{\theta}}}_{\alpha,s_k,j} = \begin{cases} -\eta_2 e_s^T P b_{i,s_k} \xi_{\alpha,s_k,j}, \text{ if } \|\hat{\hat{\theta}}_{\alpha,s_k,j}\| < M_{\alpha,s_k,j} \text{ or} \\ \|\hat{\hat{\theta}}_{\alpha,s_k,j}\| = M_{\alpha,s_k,j} \text{ and } \eta_2 e_s^T P b_{i,s_k} \xi_{\alpha,s_k,j} \geq 0 \\ -\eta_2 e_s^T P b_{i,s_k} \xi_{\alpha,s_k,j} - \eta_2 e_s^T P b_{i,s_k} \frac{\hat{\hat{\theta}}_{\alpha,s_k,j} \hat{\hat{\theta}}_{\alpha,s_k,j}^T}{\|\hat{\hat{\theta}}_{\alpha,s_k,j}\|^2} \xi_{\alpha,s_k,j} \\ \text{if } \|\hat{\hat{\theta}}_{\alpha,s_k,j}\| = M_{\alpha,s_k,j} \text{ and } e_s^T P b_{i,s_k} \xi_{\alpha,s_k,j} < 0 \end{cases} \tag{73}$$

$$\dot{\hat{\bar{\varepsilon}}}_{\rho,s_k} = \begin{cases} 0, \text{ if } \hat{\bar{\varepsilon}}_{\rho,s_k} = \bar{M}_{\rho,s_k} \text{ and } -\eta_3|e_s^T P b_{i,s_k} u_{s_k}^s| > 0 \\ \text{or } \hat{\bar{\varepsilon}}_{\rho,s_k} = -\bar{M}_{\rho,s_k} \text{ and } -\eta_3 |e_s^T P b_{i,s_k} u^s_{s_k}| < 0 \\ -\eta_3|e_s^T P b_{i,s_k} u^s_{s_k}|, \text{ otherwise}\end{cases} \tag{74}$$

$$\dot{\hat{\bar{\varepsilon}}}_{\alpha,s_k,j} = \begin{cases} 0, \text{ if } \hat{\bar{\varepsilon}}_{\rho,s_k} = \bar{M}_{\alpha,s_k,j} \text{ and } -\eta_4 \bar{e}^T P b_{i,s_k} > 0 \\ \text{or } \hat{\bar{\varepsilon}}_{\alpha,s_k,j} = -\bar{M}_{\alpha,s_k,j} \text{ and } -\eta_3 \bar{e}^T P b_{i,s_k} < 0 \\ \eta_4 \bar{e}^T P b_{i,s_k}, \text{ otherwise}\end{cases} \tag{75}$$

where $\eta_l > 0,\ l = 1, \ldots, 4$ denote the adaptive rates.

Now, an observer-based adaptive fault accommodation algorithm is proposed to control the faulty system. The stability of the error dynamics is guaranteed by the following theorem.

Theorem 4: Under Assumptions 1–3, if there exist a common matrix $P = P^T$ and real matrices K_i and $Q > 0, i = 1, 2, \ldots, r$, with appropriate dimensions such that the following condition holds:

$$(A_i - L_i C_i)^T P + P(A_i - L_i C_i) \leq -Q \ P(A_i + K_i B_i) + (A_i + K_i B_i)^T P + P S_1 P \leq -Q$$

when the FTC law (71) and adaptive laws (72)–(75) are applied, the error system (59) is asymptotically stable. Moreover, $\bar{e}(t), \tilde{\theta}_{\rho,s_k}$, and $\tilde{\theta}_{\alpha,s_k,j}$ are semiglobally uniformly ultimately bounded, converging asymptotically to a small neighborhood of zero, namely $\|\bar{e}\| \leq \sqrt{\alpha/\lambda_{\min}(P)}, \|\tilde{\theta}_{\rho,s_k}\| \leq \sqrt{2\eta_1\alpha}, \|\tilde{\theta}_{\alpha,s_k,j}\| \leq \sqrt{2\eta_2\alpha}, \alpha = \mu/\lambda + V(0), \lambda = \min\{\lambda_{\min}(Q)/\lambda_{\max}(P), 1/2\eta_1, 1/2\eta_2\}, \mu = 4\sum_{i=1}^r h_i(z(t))[M^2_{\rho,s_k}/\eta_1 + \bar{M}^2_{\rho,s_k}/\eta_3 + \sum_{j=1}^p (M^2_{\alpha,s_k,j}/\eta_2 + \bar{M}^2_{\alpha,s_k,j}/\eta_4)] + \mu_0$, and $\mu_0 = \sum_{i=1}^r h_i(z)[(K_i B_i \omega_d - \dot{\omega}_d)^T S_1^{-1}(K_i B_i \omega_d - \dot{\omega}_d)], \tilde{\theta}_{\rho,s_k} = \hat{\theta}^*_{\rho,s_k} - \hat{\hat{\theta}}_{\rho,s_k}$, and $\tilde{\theta}_{\alpha,s_k,j} = \hat{\theta}^*_{\alpha,s_k,j} - \hat{\hat{\theta}}_{\alpha,s_k,j}$.

Proof: Similar to the proof of Theorem 2, it is easy to obtain the conclusion. The detailed proof is omitted. □

IV. Simulation Results

To verify the effectiveness of the proposed method, we consider the re-entry phase of a NSV with the altitude $H = 40$ km and speed $V = 2500$ m/s as the initial states. The symmetric, positive-definite moment of inertia tensor is given as follows:

$$J = \begin{bmatrix} 554486 & 0 & -23002 \\ 0 & 1136949 & 0 \\ -23002 & 0 & 1376852 \end{bmatrix}.$$

Consider that the nonlinearity of NSV re-entry attitude dynamics mainly comes from the attack angle α and the attitude angular velocity ω. In the NSV re-entry phase $\alpha \in [0, \pi/4]$, we assume that α has two related fuzzy sets $\{\alpha = 0\}$and $\{\alpha = \pi/4\}$, and ω has three related fuzzy sets $\{\omega = -0.5\}, \{\omega = 0\}$, and $\{\omega = 0.5\}$; the corresponding membership functions are obtained as [41]. We choose six operating points: $[\alpha, \omega] \in \{[0, -0.5],\ [0, 0], [0, 0.5],\ [\pi/4, -0.5],\ [\pi/4, 0], [\pi/4, 0.5]\}$. Under the membership functions and the six operating points, six plant rules and six control rules can be defined. Here, the first rule is given as an example:

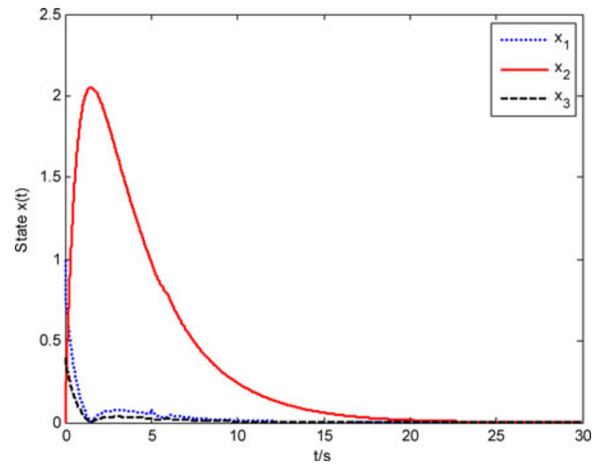

Fig. 1. State responses of NSV attitude dynamics under normal conditions.

$$\text{Rule 1: IF } \omega \text{ is about } -0.5 \text{ rad/s and } \alpha \text{ is about } 0 \text{ rad, THEN}$$
$$\dot{x}(t) = A_1 x(t) + B_1 u, \quad y(t) = C_1 x(t).$$

All A_i and B_i can be obtained by substituting the six operating points to $f(x_\omega), g(x_\omega)$. The detailed matrix parameters are given in [41]. The initial conditions are taken as follows: $\omega(0) = [0, 0, 0]^T$ and $\gamma(0) = [0, 0, 0]^T$, and the tracking command is chosen as $\omega_d = [0, 0, 0]^T$ and $\gamma_d = [1, 0, 2]^T$ during the re-entry phase. The parameters are taken as in [41] and will not be described in detail here. We consider the case where only two actuators fail at one time:

$$u_1^f(t) = \begin{cases} u_1(t), & t < 5 \\ (1 - \rho_1(x))(u_1(t) + \sum_{j=1}^p g_{1,j} f_{1,j}(x)), & t \geq 5 \end{cases}$$

$$u_2^f(t) = \begin{cases} u_2(t), & t < 5 \\ (1 - \rho_2(x))(u_2(t) + \sum_{j=1}^p g_{2,j} f_{2,j}(x)), & t \geq 5 \end{cases}$$

$$u_3^f(t) = u_3(t)$$

where $\rho_1(x) = 0.4\cos(x_1),\ p = 1,\ g_{1,1} = 0.4,\ f_{1,1}(x) = \cos(x_3),\ \ \rho_2(x) = 0.4\sin(x_2),\ \ g_{2,1} = 0.4$, and $f_{2,1}(x) = \cos(x_3)$. By using the MATLAB toolbox to solve the matrices inequalities (25), one can obtain the fault diagnostic observer gains L_i. By solving (53), one can obtain the positive-definite symmetric matrix P and the nominal controller gains K_i. Therefore, one can design the ideal control (52). Using the ideal control input (52), we can design the fault-tolerant controller (56), the modified fault-tolerant (65), and the observer-based fault-tolerant control (66). In this example, we assume that the system state is not fully measured, and thus, the observer (22) is used to estimate the system state. Consequently, the observer-based fault-tolerant control input (66) is used to control the faulty system. The simulation results are presented in Figs. 1–3. From Fig. 1, it is seen that, under the normal operating

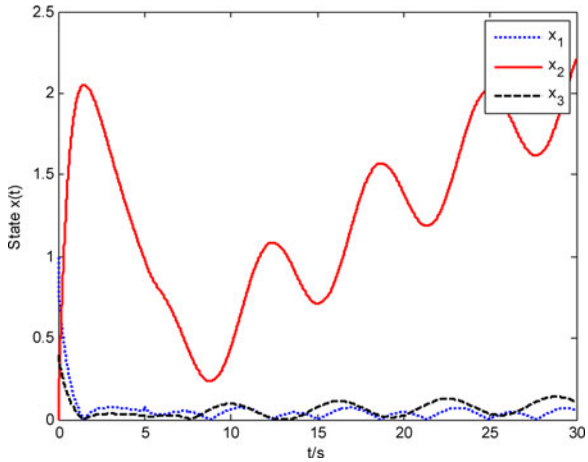

Fig. 2. State responses of NSV attitude dynamics without FTC.

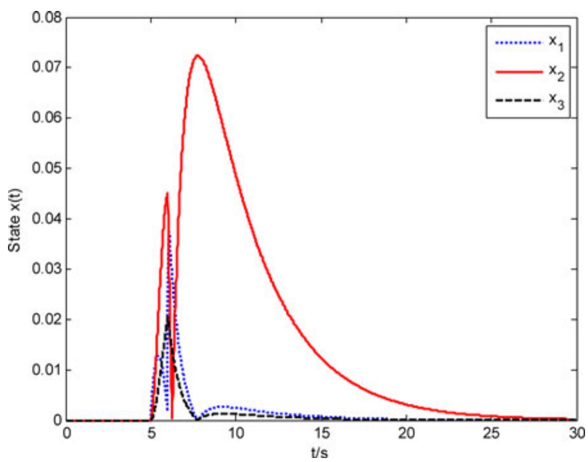

Fig. 3. State responses of NSV attitude dynamics with actuator faults and observer-based FTC (86).

condition, observation errors globally asymptotically converge to zero. If no actuator fails, the system states globally asymptotically converge to zero, as shown in Fig. 2. Fig. 3 shows that when an actuator fault occurs, keeping the normal controller, the system states deviate significantly from zero. However, as shown in Fig. 3, using the proposed FTC (66), the system states globally asymptotically converge to zero.

V. Conclusion

In this paper, the problem of FTC for T–S fuzzy systems with actuator faults has been studied. We first designed a bank of SMOs to detect and estimate the fault, and a sufficient condition for the existence of SMOs was derived. Compared with some results in the literature, the proposed fault accommodation scheme is designed to online approximate not only bias faults but gain faults as well. Moreover, it can accommodate multiple actuator faults simultaneously. In addition, the adaptive fault accommodation algorithm removes the classical assumption that the time derivative of the output errors should be known. Simulation results of NSV show that the designed fault detection, isolation, and estimation algorithms, as well as the fault-tolerant control scheme have good dynamic performances in the presence of actuator faults. However, the proposed FTC scheme does not take into account the effect of time delays which inevitably result from the FDI and FTC procedures. These time delays may have a great impact on the stability of the controlled system. This problem will be considered in our future work.

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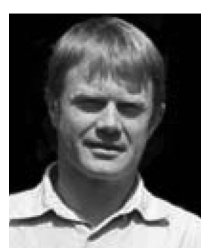

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