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Improving Diversity in Evolutionary Algorithms: New Best Solutions for Frequency Assignment

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***Abstract*—Metaheuristics have yielded very promising results for the Frequency Assignment Problem. However, the results obtainable using currently published methods are far from ideal in complex, large-scale instances. This paper applies and extends some of the most recent advances in Evolutionary Algorithms to two common variants of the Frequency Assignment Problem, and shows how, in traditional techniques, two common issues affect their performance: premature convergence and the way in which neutral networks are handled. A recent replacement-based diversity management strategy is successfully applied to alleviate the premature convergence drawback. Additionally, by properly defining a distance metric, the performance in the presence of neutrality can also be greatly improved. The replacement strategy combines the principle of transforming a single-objective problem into a multi-objective one by considering diversity as an additional objective, with the idea of adapting the balance induced between exploration and exploitation to the requirements of the different optimization stages. Tests with 44 publicly available instances yield very competitive results. New best-known frequency plans were generated for 11 instances, whereas in the remaining ones the best-known solutions were replicated. Comparisons with a large number of strategies designed to delay convergence of the population clearly show the advantages of our novel proposals.**

***Index Terms*—Frequency Assignment Problem, Diversity Preservation, Premature Convergence, Neutral Networks, Plateaus.**

I. INTRODUCTION

THE *Frequency Assignment Problem* (FAP) is one of the most popular optimization problems that arises in the field of telecommunications. The FAP can actually be regarded as a family of optimization problems because there is quite a large number of different FAP formulations [1]. The common feature of these problems is the existence of a set of communication devices — the transmitters or transceivers — that require the assignment of a frequency or channel to operate. The set of assignments is usually referred to as the *frequency plan*. In its most basic formulation, the FAP can be converted to the graph coloring problem [2]. Thus, it can be shown to be an NP-Hard optimization problem, meaning that the FAP is quite interesting not only because of its practical relevance, but also because of its mathematical properties [3]. As a result, the FAP has extensively been used as a benchmark problem to analyze the benefits provided by different optimization methods [4]. In fact, in this paper our main interest is the use of the FAP as a benchmark problem, although some practical instances that were formerly in place in large cities are also considered [5][1].

A taxonomy of the most popular FAP models is proposed in [6]. In the first level, the Fixed, Dynamic and Hybrid Channel Assignment schemes are identified. In the *Fixed Channel Assignment*, the channels are assigned statically to each transmitter, whereas in the dynamic case, the channels are assigned online. Hybrid methods combine the advantages of previous schemes and usually require solving a restricted version of a fixed channel assignment formulation [7]. In this paper, we focus on formulations based on the Fixed Channel Assignment formulation, which are further classified into four groups: the minimum order FAP, the minimum span FAP, the minimum blocking FAP and the minimum total interference FAP (MI-FAP). In the MI-FAP, which is the variant considered in this paper, the set of channels that can be used in each transmitter is restricted and the quality of a frequency plan is estimated by calculating the set of interferences involved. The aim is to find a valid frequency plan that minimizes the total interference.

Many different methods have been devised to solve the different variants of the FAP [4], [8]. Among them, metaheuristics are probably the most widely used alternatives. The analyses of the best results attained for different publicly available instances show that currently there is no clear winning strategy. Both trajectory-based and population-based metaheuristics have yielded quite promising results. In fact, in some of the instances used in this paper, the currently best-known solutions have been obtained with population-based metaheuristics, whereas in other ones, trajectory-based methods have offered the best results. Specifically, Evolutionary Algorithms [9] (EAs) and a population-based strategy based on path relinking [10] are population-based schemes that have yielded the best results for some instances, whereas a variant of a trajectory-based Tabu-search [11] is the winning strategy in other cases.

This paper focuses on the application of EAs to the FAP. In particular, we identified that in most current approaches, no special attention has been given to two common failure modes of EAs [12]: premature convergence and the way of dealing with neutral networks or plateaus. Thus, we hypothesized that by properly taking these issues into account, the state of the

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[1]A new website where the evaluator and instances can be downloaded has been created (http://oplink.lcc.uma.es/problems/afp.html)

art in the application of EAs to the FAP might be advanced. Note that, while there are several proposals that deal with each of these drawbacks independently when considering problems different from the FAP [13], [14], in this paper we expand a recently proposed replacement strategy [15] to deal with both kinds of drawbacks simultaneously for the first time. Additionally, we would like to remark that while in this paper we focus on the solution of the FAP, the core of our proposals is general, meaning that other problems can profit from the knowledge gained in this research. In order to adapt our proposals to other problems, in addition to the common decisions required in the design of EAs, a proper distance metric between individuals is required.

Our proposals are based on the idea of applying a replacement-based diversity management strategy to explicitly control the convergence of the population. Note that several ways of delaying convergence have been proposed in the literature [14]. The main feature that distinguishes the technique put forth in this paper [15] from other more mature, and recent proposals — which are used to validate our techniques — is that it applies, in the replacement phase, the principle of adapting the degree of exploration and exploitation to the requirements of the different optimization stages by taking into account the stopping criterion set by the user. Thus, our proposal induces a faster convergence of the population for more restricted stopping criteria. This is accomplished by treating the single-objective problem as a multi-objective one by considering diversity as an additional objective and by incorporating a dynamic penalty approach to penalize individuals that contribute too little to the diversity. Additionally, we show that the way in which the distance among individuals is measured to estimate the contribution to diversity is crucial to success, and can be used to modify the way of dealing with neutrality. Specifically, in some of the instances where the fitness landscape presents larger neutral regions, avoiding premature convergence is not enough to further improve the best-known frequency plans. However, by properly defining the distance metric, the number of individuals covering a plateau is diminished, which serves to considerably improve the results. Experimental validation with a large set of techniques and FAP instances clearly shows the advantages of our proposal. In fact, the best-known frequency plans of 11 instances could be improved further.

The rest of the paper is organized as follows. Section II presents a summary of the methods designed to deal with premature convergence and neutrality, and considers the state of the art in the application of EAs to the FAP. The specific formulations of the FAP used in this paper are defined in Section III. The features of neutrality that arise in the FAP are discussed in Section IV. Our novel proposals are presented in Section V. Section VI is devoted to our experimental validation. Finally, our conclusions and future work are presented in Section VII.

II. Literature Review

A. Failure Modes of EAs

This section reviews some of the most relevant studies that deal with two common failure modes of EAs [12]: premature convergence and the way to address neutrality. Note that while both issues have a significant effect on the performance of EAs, there is much more research on premature convergence, probably because this drawback arises more frequently. In fact, premature convergence can appear in practically any problem. However, neutral networks are not an issue in several cases, such as in many of the benchmark functions defined in popular continuous optimization contests. In any case, many problems feature plateau regions [16], so special attention must be paid to the internal operation of EAs in these cases.

Many techniques for dealing with premature convergence have been devised [17]. The methods are classified, depending on the component of the EA that is modified, into the following groups [14]: *selection-based*, *population-based*, *crossover/mutation-based*, *fitness-based*, and *replacement-based*. Additionally, depending on the number of altered components, they are referred to as *uniprocess-driven* or *multiprocess-driven*. In this section we present a brief explanation of the most popular proposals. In the cases used to show the benefits of our techniques, some additional details, such as their internal parameters, are specified.

Most of the initial approaches were selection-based methods that explicitly controlled the selection pressure in the parent selection phase [18]. However, it was found both theoretically and experimentally that these kinds of proposals were not able to preserve diversity in the long term [19].

The population-based schemes alter the panmictic population model of typical EAs. In methods with structured populations, the number of interactions between individuals is reduced, meaning that convergence is usually delayed. In the infusion techniques some new individuals are inserted into the population when certain conditions appear. The saw-tooth genetic algorithm (saw-tooth GA) [20] is a popular method that applies a variant of this principle. Specifically, this method uses a variable population size and periodic partial reinitialization of the population in the shape of a saw-tooth function. In order to adapt the technique to the particular needs of the problem, two different parameters are used: the period (P) and the amplitude (D) of a saw-tooth function.

In the group of the crossover/mutation-based schemes, the approaches that impose mating restrictions are one of the most popular [21]. For instance, in the cross-generational elitist selection, Heterogeneous recombination, Cataclysmic mutation scheme, individuals that are too close are not recombined. In other cases the variation stage is modified with the aim of better controlling the balance between exploration and exploitation in a problem-specific way [22]. Finally, methods with *population management* [23] apply mutation iteratively until the distance from the offspring to its closest individual in the population exceeds a specified threshold.

In the fitness-based schemes, the way of calculating the fitness of each individual is altered. The *fitness sharing* scheme is perhaps the most popular in this group. Fitness sharing reduces the fitness of individuals placed in densely populated regions. Specifically, each individual's fitness is normalized depending on the number of individuals in its region, which is defined via the parameter σ. The *clearing* strategy [24] (CLR) is an extension of fitness sharing and alters both the fitness

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assignment and the replacement selection phase. In clearing, the individuals are grouped into niches and the resources of a niche are attributed to the best W elements in each niche. Moreover, the winners of each niche are copied to the next population. Finally, methods based on multiobjectivization [25], [26] use several fitness functions simultaneously.

Since the appearance of elitism, several replacement-based methods have also been devised. The basic principle of these methods is to induce higher levels of exploration in successive generations by diversifying the survivor of the population [15]. For instance, in *crowding* the basic principle is that offspring should replace similar individuals from the previous generation. This can be accomplished in many different ways. Some of the most popular variants are the following: *Mahfoud's deterministic crowding* [27] (DETCR), *Probabilistic crowding* [28] (POBCR), scaled probabilistic crowding (SPOBCR) [29], *adaptive generalized crowding* based on entropy [30] (AGCR) and *Restricted Tournament Selection* [31] (RTS). Of these proposals, only AGCR and RTS require additional parameters. In AGCR a scale factor (ϕ) is required, whereas in RTS the length (CF) of a set that must be sampled from the population has to be specified. The Evo-Div algorithm [32] might be seen as a special case of crowding. In Evo-Div, when the distance between an offspring and a member of the population is lower than a given threshold, the better of the two is selected to survive. When this does not happen, however, a different replacement is triggered that focuses more on the quality.

Several replacement strategies that do not rely on crowding have also been devised. In some methods, diversity is considered as an objective. For instance, in the *Hybrid Genetic Search with Adaptive Diversity Control* (HGSADC) [33] the individuals are sorted by their contribution to diversity and by their original cost. Then, the rankings of the individuals are used to calculate a score using two parameters (N_{Close} and N_{Elite}) that is used to determine the survivors. In the *contribution-diversity replace-worst* method [34] (CD/RW) new individuals enter the population by replacing another one that is worse both in diversity contribution and quality. If such an individual does not exist, the *replace-worst* strategy (RW) is applied. Finally, using concepts that arise in the field of multi-objective optimization to consider both the original objective and the diversity contribution of each individual is also a plausible choice [35]. In these kinds of schemes, the diversity contribution is referred to as the auxiliary objective [26]. Several different ways of calculating the auxiliary objective have been proposed [26]. Among them, one of the most popular is probably the *distance to the closest neighbor* (DCN) metric [35], which is used in our proposals. In DCN, the auxiliary objective of a given individual is calculated as its distance to the closest member in a reference set. A uniprocess-driven replacement-based approach (MULTI) based on these principles has also been proposed [36].

The presence of plateaus or neutral networks affects both the trajectory-based and population-based metaheuristics [16]. In the presence of plateaus there might be not enough local information in the objective function to guide the search towards better regions. As a result, one of the most typical ways of dealing with plateaus is by altering how the fitness of individuals is established [16]. One alternative based on this principle is the use of multi-objectivization [37]. The inclusion of a helper objective can transform a plateau of incomparable solutions into a region where the Pareto dominance relation provides a direction of improvement. The use of augmented cost functions is another plausible approach [16]. For instance, in guided local search, the objective function of candidate solutions that share features with previously visited solutions is penalized. As a result, solutions that are initially incomparable are turned into comparable solutions by using dynamic penalties.

Several studies have been conducted to better understand the effects of neutral networks on the dynamics of the population of EAs [38], [39]. In many cases, the effects of neutral networks were considered to be less harmful than local optima [40] because while the presence of local optima is closely linked to premature convergence, neutral networks allow maintaining some degree of diversity. In fact, the most typical behavior is that, once that a high-quality large neutral network — in comparison to the remaining individuals in the population — is found, most individuals tend to be placed quickly in such a neutral network [38], [39]. However, as the generations progress, the individuals in the neutral network become more diverse, meaning that finding a portal to escape the neutral network is usually more likely. Based on this principle, some recent methods explicitly increase diversity in neutral networks [41]. Finally, some theoretical analyses have also been carried out. Some authors have identified that, for certain neutral networks, using a population to explore the plateau is not helpful [13]. Instead, one individual can be used to explore the entire plateau [42]. However, if a population is involved and only one solution is present in a plateau, many evaluations will be done in other regions; as a result, some recent methods tend to locate a large portion of the population in the best known plateaus [41].

B. *Metaheuristics for the* FAP

A large number of metaheuristics have been devised to deal with different variants of the FAP [4], [8]. In this section, we review some of the studies that are relevant to our proposals.

First, the amount of effort devoted to designing ad-hoc genetic operators is worth noting. Initially, some simple and readily available crossover operators, such as the uniform crossover, were tested extensively [43]. However, since they were quite disruptive, more advanced operators, such as the geographic crossovers, were proposed [44]. In these cases, the positions of the transceivers are used to define regions that are exchanged between the parents. Note that in some cases, the positions of the transceivers are not available. In said cases, the underlying constraint graph can be used instead [45]. Selecting connected components to establish the regions to exchange has yielded quite promising results [45], [46]. Significant benefits have also been obtained by mutating a set of transceivers that form a connected component [47].

Another obvious conclusion that can be drawn from previous research is that when dealing with complex instances,

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using some procedures to promote intensification is mandatory to obtain high-quality solutions [4]. For instance, over half of the proposals surveyed in [8] incorporated a local search, and methods not incorporating any intensification mechanisms are clearly inferior. It is also worth noting that even some of the first methods provided in the literature included some special kinds of intensification [48].

To our knowledge, there are no extensive studies for the FAP on ways of dealing with premature convergence and neutrality that include some of the most recent advances. However, there is some clear evidence that both issues might be appearing and affecting the performance of population-based approaches. First, it was recently shown that in the graph coloring problem, which is closely related to the FAP, large neutral networks appear, clearly impacting the search process [49]. Iterated local search could be improved further by taking into account the existence of such neutral networks. However, studies with more complex strategies were left for future work. Additionally, to our knowledge, this research has not been adapted to analyze the FAP. Regarding premature convergence, while there are no studies that include comparisons with many of the methods that have been proposed recently to delay convergence [14], some of the most mature techniques have been used, and an inspection of the best reported methods shows that premature convergence is an issue that must be addressed. In fact, very early on it was discovered that stagnation was an important issue in the FAP, so some of the initial approaches included stagnation detection and restarting approaches [50]. Adapting the behavior of the genetic operators during the run is one mechanism that is typically used to avoid premature convergence [51]. For instance, in [51], the mutation probability depends on the entropy of the population. A more recent proposal, also based on identifying the diversity of the population to adapt the mutation probability, is proposed in [52]. This principle was also employed in [9] with some of the instances used in this paper. Specifically, several internal parameters of the genetic operators are altered dynamically with a fuzzy-based controller that takes diversity into account. Controlling the selection pressure is another typical approach [53]. While these schemes delay convergence, in the long term, convergence is not avoided, meaning that very long executions require the inclusion of additional mechanisms. Another clear example of the benefits provided by mechanisms that delay convergence is presented in [54], where fitness sharing and multiobjectivization are successfully applied.

Finally, based on the best results obtained with the instances used in this paper, the benefits of controlling diversity are also clear. Specifically, depending on the instance, three different methods that rely on population-based metaheuristics have yielded the best known frequency plans. They are a variant of path-relinking [10], an EA [9] and a parallel population-based scheme that includes several metaheuristics [55]. In the case of path-relinking [10], the candidate solutions are updated using a mechanism that follows the principles of crowding. In the parallel population-based model [55], the most effective single metaheuristic was a variant of the scatter search in which a large number of candidate solutions is selected by considering diversity instead of quality. Finally, the EA variant [9] applies a parameter-control strategy that takes diversity into account.

III. MATHEMATICAL FORMULATION OF THE FAP

In this paper, we consider two different formulations of the FAP. The reason for using two sets of instances is that, on the one hand, reporting results for real-world complex instances is interesting. However, the most realistic formulation that we consider [5] relies on thousands of Mobile Measurement Reports (MMRs) [56] to generate an interference matrix, and only data for two cities (Denver and Seattle) are available. For this reason, and with the aim of better showing the generality and robustness of our proposals, a more academic — but also quite complex — variant [57] is also included.

Due to the complexity of the formulation used for the Denver and Seattle instances, this section only includes an informal description. Readers interested in a formal definition are referred to the supplementary material and to [5]. Additional information of the instances is found in [8]. Basically, the formulation operates as follows. First, a set that includes thousands of MMRs is processed to estimate the carrier-to-interference ratio probability distributions that arise among the transceivers. Then, these data are used to estimate the level of interference — or cost function — that might appear between each transceiver pair when operating on the same or adjacent channels. This is done by considering the error probability that appears in digital communications. Specifically, the cost values are an estimate of the Bit Error Rate that results when applying Gaussian Minimum Shift Keying (GMSK) modulation, which is the scheme used in the Global System for Mobile Communication (GSM). Then, the structure of the networks and the above costs are used to generate a pair-wise cost matrix that is used to calculate the total interference cost of the frequency plans. In this first formulation, data for two instances are available. The Seattle instance has 970 transceivers and 15 different channels to be assigned, whereas the Denver instance has $2,612$ transceivers with 18 channels available.

The other formulation [57] is much simpler, but in our opinion, it is quite useful to include it for several reasons. First, a large set of instances using this formulation are available. Specifically, the 42 instances provided in [57] were taken into account. Second, the simplicity of the formulation facilitates the adaptation of any optimizer to these instances, making it easier to compare against our results. Note that in some of the instances, data were adapted from other more complex and realistic formulations, whereas others are based on artificial data. In this definition, an instance is given by a set of transmitters (TRXs), a set of constraints, and the number of contiguous admissible channels (C). Each constraint defines the minimum distance allowed between a specific pair of transmitters and a value that is used as a penalty term when the constraint is not fulfilled. The cost function that must be minimized is the sum of the penalty terms associated with unfulfilled constraints. The objective of the problem is to find a complete valid assignment, i.e. an assignment where each transmitter uses a channel between 1 and C, that at the same

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time minimizes this cost function. The features of this set of instances have been analyzed in several papers. Readers are referred to [10] for additional details.

IV. NEUTRALITY IN THE FAP

In order to analyze the neutrality, the fitness landscape has to be taken into account [49]. Thus, it is not enough to specify the search space and fitness function of a problem; the neighborhood of a candidate solution also has to be defined. Moreover, in the case of MAs, the genetic operators and local search must also be considered [40] because depending on the transformation induced by them, the probabilities of facing problems with neutral networks vary.

Several metrics for studying neutrality have been proposed [58]. In our analyses, two different metrics have been considered. The first one is the percentage of neutral degree. This percentage is computed, for a given frequency plan, as the neutral degree — number of neighbors with unaltered fitness — multiplied by 100 over the size of the neighborhood. This metric represents an estimate of the sizes of the neutral networks, and thus provides an indication of the likelihood of encountering difficulties with neutral networks. Additionally, since a MA is involved, it is also important to analyze the ability to jump out of neutral networks once the genetic operators and local search are applied. Preliminar experiments showed that the probability of returning to the same neutral network was higher when the crossover was not applied. Thus, an estimate of the probability of generating solutions in the same neutral network after the application of mutation and local search is given by the second metric that is considered. Since in our MA the population is rapidly filled with individuals that have undergone several steps of the local search, these metrics are also analyzed in solutions where several steps of mutation and local search have been applied.

In our first analysis, a random sampling of $10,000$ candidate solutions is used. This allows estimating some general measures of the fitness landscape not limited to promising regions. In order to measure the neutral degree, a neighborhood must be defined. In our analysis an extension of the double move strategy (N2) [59] is considered. In N2, neighbors are generated by modifying the channels of two transceivers simultaneously. However, since it makes no sense to alter two transceivers that are unrelated, i.e. that present no conflict regardless of the channels used, N2 is extended by only considering pairs of related transceivers.

For the set of instances provided in [57], Table I shows, in the first three columns, the mean and standard deviation of the percentage of the neutral degree. In the first 22 instances the percentage of neutral degree is much higher and the standard deviations are low in every case. Thus, the differences between the first and last groups of instances are clear. The first 22 instances are unweighted, i.e. the cost associated with any constraint is 1. However, in the remaining instances, different costs are considered. In light of these results, we hypothesized that neutrality might have a more pronounced effect on the unweighted instances. Thus, it might happen — and in fact it does, as we will see experimentally — that incorporating mechanisms for dealing with neutral networks might be beneficial for the unweighted instances.

In order to complement the previous analysis, the following study that specifically considers the local search and mutation operators of our MA was performed. First, a random solution was generated and our local search was applied. Then, an iterated local search was executed for 200 iterations. In each iteration, the current solution (S) is perturbed with our mutation strategy, and local search is applied to generate S'. In each iteration, the best individual between S and S' is preserved. In each iteration, if S and S' exhibit the same cost value, it might be because they are in the same neutral network. We designed a fast heuristic procedure to check if two solutions belong to the same neutral network. Specifically, 100 randomly ordered sequences of the TRXs that have different channels in S and S' are generated, and we check if we can sequentially recover the original channel associated with each TRX following the given sequence without altering the cost value in any of the changes. Note that if we find an order where this process is successful, it implies that both solutions are in the same neutral network. In other cases, the transformation of S into S' is marked as suspicious, because not finding the path does not imply that they are not in the same neutral network. This process was applied to all the instances starting from 1,000 different solutions.

Table I shows, in its fourth and fifth column, the percentage of the suspicious and confirmed cases. Note that suspicious cases, are those where S and S' presented the same cost value, but the belonging to the same neutral network could not be confirmed. The percentage of confirmed cases is much larger in the unweighted instances. Moreover, in the unweighted instances where the percentage of confirmed cases is not too large — only in four instances it is lower than 20% —, the percentage associated with suspicious cases is quite large. Finally, we also calculated the percentage of the neutral degree of the solutions generated after the 200 perturbations, which is shown in the sixth column. The mean of the percentage of neutral degree for most of the unweighted instances is also higher than in the weighted instances. In fact, 16 out of the 22 unweighted instances had a percentage of neutral degree higher than 0.20, while this only happened in 4 out of the 20 weighted instances. However, in comparison to the analyses that considered random solutions, the differences are not that large. In any case, it is clear that the tendency to create solutions in the same neutral network when applying mutation and local search to those high-quality solutions remains large even in the unweighted cases that exhibit a low percentage of neutral degree. It is an open topic whether this event is related in some way to the fact that the neutral degree in randomly generated solutions is larger in the unweighted instances, or whether this is due to the different properties of the operators when applied to weighted and unweighted instances.

These analyses are complemented with some additional studies to check the dynamics of the population in our proposals and they are included in our experimental validation. They show that even when diversity is explicitly taken into account, the neutral networks might be an issue that affect the performance of MAs. As expected, this is specially the case

TABLE I
METRICS OF NEUTRALITY THAT SHOW THE DIFFERENT FEATURES OF WEIGHTED AND UNWEIGHTED INSTANCES

Instance	RAND. SOL.		MUT. + LS		
	Mean Neut.	σ Neut.	Neut. Move	Susp. Move	Neut. Degree
AC-45-17-7	11.17	1.94	26.98	10.02	0.22
AC-45-17-9	12.20	1.85	31.67	11.09	0.23
AC-45-25-11	9.94	1.84	31.36	14.82	0.12
AC-95-9-6	13.70	1.56	35.13	14.92	0.56
AC-95-17-15	10.50	1.21	36.77	16.32	0.11
AC-95-17-21	12.18	1.23	38.36	15.52	0.18
GSM-93-9	11.08	1.28	51.44	12.10	0.48
GSM-93-13	12.81	1.26	48.87	17.31	0.60
GSM-246-21	9.27	0.70	38.31	41.42	0.40
GSM-246-31	11.04	0.72	35.27	37.83	0.47
Test95-36	13.68	1.27	25.89	45.91	0.40
Test282-61	9.84	0.62	30.64	29.82	0.02
Test282-71	10.54	0.64	30.83	28.30	0.03
Test282-81	11.22	0.66	29.76	26.37	0.04
P06-5-11	6.67	1.12	30.28	59.06	0.53
P06-3-31	7.49	0.77	10.99	79.27	0.25
P06b-5-21	9.08	1.17	18.53	68.04	0.39
P06b-5-31	10.77	1.20	17.07	63.24	0.32
P06b-3-31	8.03	0.85	14.68	73.19	0.31
P06b-3-71	11.86	0.93	23.96	47.93	0.42
1-4-50-75-30-2-1-6	10.70	1.59	32.06	13.80	0.37
1-4-50-75-30-2-1-10	12.69	1.44	27.39	23.10	0.23
GSM2-184-39	0.13	0.02	4.65	19.95	0.06
GSM2-184-49	0.28	0.05	10.93	24.90	0.29
GSM2-184-52	0.34	0.06	12.64	27.88	0.39
GSM2-227-29	0.02	0.01	0.11	23.01	0.01
GSM2-227-39	0.06	0.01	2.23	20.04	0.02
GSM2-227-49	0.13	0.02	7.73	18.44	0.12
GSM2-272-34	0.02	0.004	0.15	18.15	0.004
GSM2-272-39	0.02	0.006	0.53	15.95	0.006
GSM2-272-49	0.06	0.01	3.95	15.70	0.04
1-1-50-75-30-2-50-5	0.69	0.20	4.53	1.25	0.03
1-1-50-75-30-2-50-10	0.85	0.11	7.55	8.03	0.09
1-1-50-75-30-2-50-11	0.91	0.11	7.38	11.26	0.12
1-1-50-75-30-2-50-12	0.97	0.11	9.33	13.81	0.19
1-2-50-75-30-4-50-9	0.78	0.12	6.81	2.22	0.02
1-2-50-75-30-4-50-11	0.81	0.11	9.11	4.16	0.04
1-3-50-75-30-0-50-7	0.97	0.17	3.94	8.30	0.07
1-5-50-75-30-2-100-10	0.49	0.08	3.84	7.93	0.04
1-5-50-75-30-2-100-12	0.59	0.09	8.84	13.61	0.13
1-6-50-75-30-0-10000-10	0.41	0.12	7.44	12.11	0.29
1-6-50-75-30-0-10000-13	0.80	0.20	14.32	12.02	0.79

in the unweighted cases. As a result, an adaptation of our proposal to better deal with neutral networks in the unweighted cases is provided.

V. PROPOSALS

In this paper we propose a novel MA that combines an extension of the genetic operators defined in [9] with the *Replacement with Multi-objective based Dynamic Diversity Control* strategy (RMDDC) [15]. A special variant of the RMDDC strategy for the FAP that yields improved performance in the presence of neutral networks is proposed for the unweighted instances. Taking into account the state of the art in the application of EAs to the FAP, the most important novelty is the special way of dealing with premature convergence and neutral networks. In the following section, the general RMDDC strategy is presented. Then, its adaptation to the FAP, as well as the general MA where RMDDC is included, is described. Note that since the most important novelty of our proposal is the application of the RMDDC strategy, we use this acronym to denote our whole proposal.

Algorithm 1 RMDDC survivor selection technique

Require: Population, Offspring
1: **for each** I $\in$ Offspring **do**
2: I.cost = Interference associated to individual I
3: CurrentIndividuals = Population $\cup$ Offspring
4: Best = Individual with lowest interference level in CurrentIndividuals
5: NewPop = { Best }
6: CurrentIndividuals = CurrentIndividuals $\setminus$ { Best }
7: Update D taking into account the elapsed time (T_e), stopping criterion (T_s) and initial value of D (D_I). For instance, in the case of a linear decrease, perform $D = D_I - D_I * \frac{T_e}{T_s}$.
8: **while** (|NewPop| $<$ N) **do**
9: **for each** I $\in$ CurrentIndividuals **do**
10: I.DCN = distance to the closest individual of I in NewPop
11: **if** I.DCN $< D$ **then**
12: I.cost = Infinity
13: ND = Non-dominated individuals of CurrentIndividuals (without repetitions) taking into account I.dcn and I.cost as the two objectives
14: Selected = Randomly select an individual from ND
15: NewPop = NewPop $\cup$ Selected
16: CurrentIndividuals = CurrentIndividuals $\setminus$ {Selected}
17: Population = NewPop

A. Survivor Selection Strategy

The key principle of our proposal is to alter the replacement strategy with the aim of inducing a proper degree of diversity by taking into account the stopping criterion set by the user. Note that this principle is radically different from others used in alternative strategies in the literature, because the stopping criterion is normally not used to bias the decisions made by the replacement strategies. However, in our opinion, relating the decisions made with the stopping criterion is a highly beneficial design decision. In order to adopt this principle, the replacement strategy operates as follows (Algorithm 1). First, the offspring are evaluated (lines 1-2). Then, the population of the previous generation and the offspring are combined in the $CurrentIndividuals$ set (line 3). In order to perform an elitist strategy, the best individual — the one with minimum interference — is selected to survive by placing it in the $NewPop$ set and it is removed from the $CurrentIndividuals$ set (line 4-6). The D value, which is used to control diversity, is updated (line 7) taking into account the elapsed time (T_e), stopping criterion (T_s) and initial value of D (D_I). The specific mechanism for updating it is described below. Then, until $NewPop$ is filled with N individuals (line 8), the following steps are executed (line 9-16). First, the contribution to diversity (DCN) of each individual in the $CurrentIndividuals$ set is calculated (line 9-10). Specifically, the DCN value of each individual is calculated as the distance to the closest individual in the $NewPop$ set. Then, the original objective of individuals that contribute too little to diversity — DCN lower than D — is set to a very low quality value, meaning that said individuals are penalized. Specifically, in our case, it is set to the maximum number that can be represented in the data type associated with the objective function (lines 11-12). Then, the non-dominated set is calculated (line 13) by taking into account the DCN and interferences of each individual as the objectives. Finally, a non-dominated individual is randomly

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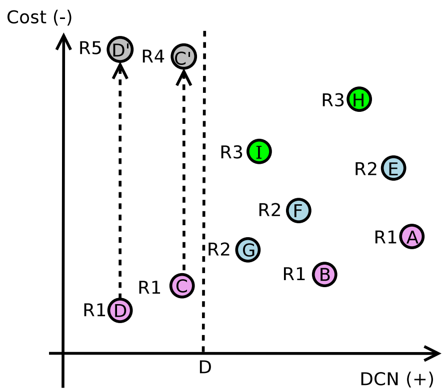

Fig. 1. Process of selecting an individual in the RMDDC strategy

Algorithm 2 Lamarckian Memetic Algorithm

Initialization: Generate an initial population P_0 with N individuals. Assign $t = 0$.
Local Search: Perform a local search for every individual in the population.
while (not stopping criterion) **do**
  Evaluation: Evaluate every individual in the population.
  Mating selection: Perform binary tournament selection on P_t in order to fill the mating pool.
  Variation: Apply genetic operators to the mating pool to create a child population CP.
  Local Search: Perform a local search for every individual in the offspring.
  Survivor selection: Apply the replacement technique to create P_{t+1}.
  $t = t + 1$

selected to survive and is added to the $NewPop$ set and removed from the $CurrentIndividuals$ set (lines 14-16). Finally, individuals in the $NewPop$ set make up the new population (line 17). The process of selecting an individual in RMDDC is illustrated in Fig. 1. In this figure, the value D represents the minimum DCN required to avoid the penalty. This figure shows a set of individuals with their corresponding DCN and cost (interference) values. Note that initially, A, B, C and D would belong to the non-dominated set. However, since C and D have a DCN value lower than D, their cost function is reset to infinity, meaning that only A and B are non-dominated individuals. Thus, in this case, RMDDC would have randomly selected A or B to survive.

In order to better understand the key principle behind RMDDC, it is useful to consider a continuous domain. In such a case, when using the Euclidean distance, RMDDC creates a set of hyper-spheres of radius D centered at every survivor that has already been selected. Any individual that is inside any hyper-sphere is penalized, meaning it will not survive (unless every pending individual is penalized). Thus, the technique avoids excessive crowding of a region. While the above approach is quite sensible, one of the key choices is how to set the value D. If we consider the benefits that can be obtained by adapting the balance between exploration and exploitation to the needs of the different optimization stages, it is clear that the value of D should vary during the optimization process. Specifically, this value should decrease as the stopping criterion is approached, the goal being to induce a higher degree of intensification in the last phases of the optimization. One alternative might be to select an initial value of D (D_I) and then use a linear reduction of D in such a way that by the end of the execution, the resulting value is 0. Thus, if T_s is the stopping criterion and T_e the elapsed time, D can be set as $D = D_I - D_I * \frac{T_e}{T_s}$. However, it is also important to consider reductions different from the linear one. For this reason, in this paper the pairwise function defined in (1) is also taken into account. In this function, C_r is the ratio of the run already executed, i.e. it is equal to $\frac{T_e}{T_s}$. The balance between exploration and exploitation can be controlled by adjusting D_I, P_1 and P_2. Basically, the initial value of D sets the initial degree of exploration, P_1 sets the fraction of the run used to linearly reduce D to 90% of the initial value, and P_2 sets the fraction of the run required to linearly reduce D to 10% of the initial value. By altering P_1 and P_2, different updating strategies can be performed. The supplementary material of this paper shows an experimental validation for several different values of P_1 and P_2. The analyses demonstrate that for long executions, the linear reduction is robust, making it unnecessary to set P_1 and P_2. Thus, the rest of the experimental validation shown in the following section relies on RMDDC with a linear reduction.

$$D(C_r) = \begin{cases} D_I - \frac{C_r \times D_I}{P_1} \times 0.1 & \text{if } C_r < P_1 \\ D_I \times \frac{-0.8 \times C_r + 0.9 \times P_2 - 0.1 \times P_1}{P_2 - P_1} & \text{if } P_1 \leq C_r < P_2 \\ \frac{0.1 \times D_I \times (1 - C_r)}{1 - P_2} & \text{if } C_r \leq P_2 \end{cases} \quad (1)$$

B. Memetic Algorithm for the FAP

The above replacement strategy was incorporated into a fairly standard MA (Algorithm 2). Thus, in order to fully specify our proposal, the encoding, genetic operators and local search must be defined. The encoding is a fairly straightforward representation, probably the most popular one [1]. In this encoding, which is usually called R1, each individual is a vector $s \in \mathbb{N}^{|TRXs|}$, where $|TRXs|$ is the number of transceivers in the instance. The element s_j is simply the channel assigned to the j-th transceiver. The genetic operators applied in this paper are extensions of the ones proposed in [9]. The crossover and mutation operators are quite straightforward geographic operators. The crossover is termed *Multi-Interference-based Crossover* and it operates as follows. First, a transmitter t is selected at random. Then, every gene associated with a transmitter that interferes with t, including the gene that represents t, is tagged. This process is repeated a number of times that is selected at random between one and a number set by the user (R). However, in subsequent iterations, instead of selecting t at random from the complete set of transmitters, it is selected, without repetition, from those that were previously tagged. Finally, the parents swap the channels assigned to the transmitters that were tagged to generate the offspring. The mutation operator

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Algorithm 3 Calculation of the D_{SIM} metric

```
1: procedure PREPROCESS
2:     for each TRX1 ∈ TRXs do
3:         for each TRX2 ∈ TRXs do
4:             if (TRX1, TRX2) ∈ E //TRX1, TRX2 can interfere then
5:                 Interf[TRX1] = Interf[TRX1] ∪ TRX2
6:     for each TRX1 ∈ TRXs do
7:         for each TRX2 ∈ TRXs do
8:             InterS = |(Interf[TRX1] \ TRX2) ∩ (Interf[TRX2] \ TRX1)|
9:             UnionS = |(Interf[TRX1] \ TRX2) ∪ (Interf[TRX2] \ TRX1)|
10:            Sim[TRX1][TRX2] = InterS / UnionS
11:
12: procedure GETDISTANCE(I1, I2)
13:     S1 = 0
14:     for each TRX1 ∈ TRXs do
15:         Match = 0
16:         for each TRX2 ∈ TRXs do
17:             if I1.channel[TRX1] == I2.channel[TRX2] then
18:                 Match = max(Match, Sim[TRX1][TRX2]);
19:         S1 += Match
20:     S2 = 0
21:     for each TRX1 ∈ TRXs do
22:         Match = 0
23:         for each TRX2 ∈ TRXs do
24:             if I2.channel[TRX1] == I1.channel[TRX2] then
25:                 Match = max(Match, Sim[TRX1][TRX2]);
26:         S2 += Match
27:     Distance = 2 × |TRXs| - S1 - S2
28:     Return Distance
```

is the *Multi-Neighborhood-based Mutation*. First, a transmitter t is selected randomly and mutated with a uniform operator. Then, the transmitters that interfere with t are mutated with a probability p_m. The above step is repeated a number of times that is selected randomly between one and R, but in subsequent iterations, the transceiver is randomly selected from among those that interfere with transmitters mutated in previous iterations. Thus, similarly to the crossover, the mutation operator focuses on altering subsets of transmitters affected by common constraints.

Our local search is a simple stochastic hill climbing, i.e. the neighbors of a given candidate solution are considered in a random order and only improvement movements are accepted. In the case of the simplest formulation of *Montemanni et al.* [57], it is implemented by considering the $N2$ neighborhood previously described, and it runs until a local optimum is reached. In order to better handle large neighborhoods, some actions are usually required [16]. In our case, branch-and-bound is applied to create only promising individuals. Specifically, given a pair of transceivers to alter, if the assignment of a channel to one of the transceivers creates too much interference — higher than the interference associated with the assignment of both transceivers in the original frequency plan — there is no point in checking all the possible assignments of the second transceiver, so all these combinations are discarded. In the most complex formulation of *Luna et al.* [5], an iterated hill climber where the neighbors are generated as in [47] is applied. Specifically, in the neighbors, the transceivers located in a given sector are altered without modifying the remaining network assignments. This is thus a generalization of N1 because the same neighbors are generated when there is one transceiver serving each sector. Since the number of neighbors is quite large, branch-and-bound is also applied [47].

Note that the resources allocated to the local search might affect the degree of exploration [60]. For instance, the probability of applying a local search might be adapted with the aim of reducing the computational requirements [61]. However, since in our scheme the degree between exploration and exploitation is controlled in the replacement phase, the local search is always applied. Attempting to combine our way of controlling diversity with the adaptation of the local search is left for future work. However, note that applying a local search only to some individuals is, in some ways, contrary to the principles of our scheme. Since our proposal dynamically changes the balance between exploration and exploitation, every member of the population is considered promising. For this reason, it seems important to intensify the search in all the regions maintained in the population.

Finally, in order to apply RMDDC a very important decision involves how distances between individuals are calculated. In this paper we consider two different definitions for the distance. The first is the Hamming distance (D_{HAM}), i.e. the distance between two individuals is the number of transceivers where different channels have been assigned. Our proposal yields significant benefits in several cases when the Hamming distance is applied. However, in some of the instances with a higher percentage of neutral degree, the population tends to place too many individuals in just a few neutral networks, so a distance that avoids this behavior is also proposed. The key idea is that, in some networks, the reason for the large size of the neutral networks is that some pairs of transmitters can be considered to be almost interchangeable, i.e. they interfere with practically the same set of transceivers. Consequently, in many frequency plans their channels can be interchanged without affecting the total interference. In order to avoid filling the population with members that result from these kinds of transformations, one alternative is to define a distance in a way that these individuals are considered to be close to one another. This way, once one of them is selected to survive, RMDDC penalizes the rest, meaning they will not survive. Since the most problematic instances were the unweighted ones, a distance that ignores the weights of the constraints is defined. As a result, it makes little sense to apply this metric to weighted cases. The following steps (see Algorithm 3) are used to calculate this alternative distance (D_{SIM}). First, the set of transceivers that can potentially interfere with any other transceiver is stored (lines 2-5). Then, the similarity between each pair of transceivers is calculated as the number of transceivers that exhibit constraints with both transceivers, divided by the number of transceivers that exhibit constraints with either of the transceivers (lines 6-10). This is carried out in the PREPROCESS procedure, which is only called at the start of our method. Note that when both transceivers exhibit constraints with exactly the same set of transceivers, the similarity is one. Then, given two frequency plans *I1* and

I2, their distance is calculated in the GetDistance procedure as follows. First, for each transceiver in *I1*, the most similar transceiver in *I2* that shares the same channel is detected (lines 14-18) and the sum of the similarities is saved in *S1* (line 19). Then, *S2* is calculated using the same operation for each transceiver in *I2* but considering the similarities with respect to *I1* (lines 20-26). The similarity (*S*) is set to the sum of both values (*S* = *S1* + *S2*). Finally, given that the largest possible similarity is $2 \times |TRXs|$, where $|TRXs|$ is the number of transceivers, the distance is calculated as $2 \times |TRXs| - S$ (line 27). Note that in this case, swapping channels between similar transmitters induces lower distances than swapping channel between dissimilar transmitters.

In light of the above discussion, it is clear that in population-based schemes, placing several individuals in a plateau might reduce the time required to pass through the neutral network. However, note that in these cases, other promising regions might be quickly abandoned, which is not too reasonable, especially when considering long runs operating over large search spaces. A trivial case appears when there are sub-optimal neutral networks [42], i.e. plateaus that are not connected to regions of higher fitness. In these cases, placing the entire population in these neutral networks might have a large negative impact on the search performance. Since in this paper we adapt our distance metrics to those cases where neutral networks are larger in an effort to explicitly avoid overcrowding said regions, especially in the initial phases, additional time might be required by the MA to pass through each neutral network. However, the drawbacks associated with prematurely abandoning some regions is avoided, and in fact the experimental validation shows that the benefits do indeed outweigh the drawbacks.

VI. Experimental Validation

This section presents an experimental validation that includes our novel proposal and the adaptation of a large number of different methods to the FAP. The optimization techniques were implemented using METCO (*Metaheuristic-based Extensible Tool for Cooperative Optimization*) [62]. The analyses were performed with two instances involving a highly realistic formulation [5], and 42 instances that consider a more academic formulation [57]. Detailed information on all the instances can be found in the original papers.

Tests have been run on the cluster "El Insurgente", using bi-processor machines with 32 Gb RAM. Each processor is an Intel(R) Xeon(TM) CPU E5-2620 at 2.1 GHz. Since stochastic algorithms were considered, each execution was repeated 30 times and comparisons were carried out by applying a set of statistical tests. Specifically, the following tests were applied, assuming a significance level of 5%. First, a *Shapiro-Wilk test* was performed to check whether or not the values of the results followed a Gaussian distribution. If they did, the *Levene test* was used to check for the homogeneity of the variances. If samples had equal variances, an ANOVA *test* was done; if not, a *Welch test* was performed. For non-Gaussian distributions, the non-parametric *Kruskal-Wallis* test was used to determine whether samples were drawn from the same distribution.

TABLE II
Parameterization of the methods applied in the first experiment

Method	Parameterization
RMDDC	$D_I = 0.75 \times T$ (number of transceivers)
GEN_ELIT	No parameterization required
CD/RW	No parameterization required
CLR	$\sigma = 0.20 \times T, W = \{1, 2, 5\}$
DETCR	No parameterization required
SPOBCR	No parameterization required
AGCR	$\phi = \{0.25, 0.75\}$
RTS	$CF = \{2, 5, 10, 25, 50\}$
HGSADC	$N_{Close} = 3, N_{Elit} = 8$
RW	No parameterization required
SAW-TOOTH-GA	$D = 99, P = 50$

In order to show the benefits of the new model and to better understand its internal operation, several sets of experiments were performed, as shown in the sections that follow.

A. Comparison with Other Schemes

In order to show the validity of our proposal, it is very important to do an extensive comparison with other methods developed in the specialized literature. Since the main distinguishing feature of our method is how it manages diversity, a large set of techniques involving a special management of diversity were selected, including several recent, as well as more mature, methods. All these methods were applied to the more realistic FAP formulation considered in this paper. In addition, the results are compared with the best-known frequency plans obtained so far by any method. The following techniques were used in our comparisons: CD/RW, CLR, DETCR, SPOBCR, AGCR, RTS, HGSADC, RW, and Saw-Tooth GA. Note that, while all these acronyms have already been defined, a table with these acronyms has also been added in the supplementary material. We also used a generational EA with elitism (GEN_ELIT) and one that relies on the replace-worst strategy (RW). All of the above proposals incorporated the genetic operators and local search previously described. Table II shows the parameterization used for each method. The symbols used for their parameters were previously described and they are the same as in the papers where they were proposed. In the methods where several values are used for a specific parameter, these are expressed as comma-separated values, and in order to denote the method with its specific parameterization, the name of the method is followed by the value of the parameter. For instance, CLR_5 means that the parameter W is set to 5. In the case of the RMDDC strategy, a linear model to decrease the value of D is employed. The reason is that we determined that for long executions, inducing a slow but continuous decrease in D provides very robust results. In fact, 66 different ways of updating D were considered by applying different parameter values in a parameterized function, but none of them yielded better results — lower cost function and statistically significant differences — than those obtained with the linear reduction. Thus, given the simplicity and robustness of the linear reduction, it is used

TABLE III
COMPARISON OF EVERY CONSIDERED SCHEME FOR THE SEATTLE INSTANCE IN 48 HOURS

Method	Statistics				Frequency Cost	
	↑	↓	↔	Score	Best	Mean
RMDDC	**17**	**0**	**0**	**17**	**349.3**	**408.4**
SPOBCR	16	1	0	15	413.8	483.4
Saw-tooth GA	13	2	2	11	462.2	553.2
AGCR_0.25	13	2	2	11	469.5	569.7
GEN_ELIT	13	2	2	11	450.2	575.5
CD/RW	10	5	2	5	503.3	619.2
CLR_1	9	5	3	4	511.7	632.6
HGSADC	9	5	3	4	537.9	638.9
CLR_2	3	6	8	-3	550.5	664.5
AGCR_0.75	2	8	7	-6	550.5	676.0
RTS_10	2	8	7	-6	547.8	682.1
RW	2	8	7	-6	513.0	688.1
RTS_25	2	8	7	-6	512.6	696.3
RTS_5	2	8	7	-6	573.0	698.1
RTS_2	2	8	7	-6	518.6	698.2
CLR_5	1	9	7	-8	592.8	711.3
RTS_50	1	15	1	-14	606.2	732.6
DETCR	0	17	0	-17	733.3	823.4

TABLE IV
COMPARISON OF EVERY CONSIDERED SCHEME FOR THE DENVER INSTANCE IN 48 HOURS

Method	Statistics				Frequency Cost	
	↑	↓	↔	Score	Best	Mean
RMDDC	**17**	**0**	**0**	**17**	**82994.9**	**83361.8**
DETCR	15	1	1	14	83270.8	83760.0
RTS_50	14	1	2	13	83207.8	83770.9
SPOBCR	13	2	2	11	83354.1	83913.2
HGSADC	12	3	2	9	83427.2	84043.4
RTS_25	11	4	2	7	83559.7	84115.5
Saw-tooth GA	7	5	5	2	83551.3	84255.5
CD/RW	6	6	5	0	83630.0	84348.8
CLR_1	6	6	5	0	83565.4	84350.8
RW	2	6	9	-4	83709.8	84407.4
GEN_ELIT	2	6	9	-4	83557.0	84415.2
RTS_10	2	7	8	-5	83558.2	84550.0
CLR_2	2	9	6	-7	83871.2	84554.1
AGCR_0.25	2	9	6	-7	84034.5	84557.2
CLR_5	2	9	6	-7	83917.7	84571.7
RTS_5	2	9	6	-7	83677.5	84591.0
RTS_2	1	16	0	-15	83969.9	84893.4
AGCR_0.75	0	17	0	-17	84936.1	85762.8

in this experiment. Readers are referred to the supplementary materials for additional details.

Considering the different parameterizations, a total set of 18 different proposals were taken into account. Common values were used for parameters that appear in every method. In order to set these parameters some preliminary experiments with four instances were taken into account in order to identify promising parameters. Both RMDDC and GEN_ELIT were taken into account in these experiments. In the population size, the values 25, 50, 100, and 150 were tested. The mutation probability was set to 0.01, 0.02, and 0.04. The crossover probability was set to 0.6, 0.8, and 1. Finally, the R parameter, which is used in the mutation and crossover operators, was set to 6, 8, and 10. Note that these preliminary experiments were performed by setting the stopping criterion to two hours. The parameterization that obtained the lowest mean considering the four instances was selected. Specifically, the population size, crossover probability, mutation probability and R were set to 50, 0.8, 0.02, and 8, respectively. Additionally, a more complete sensitivity analysis was performed subsequently. More information on this sensitivity analysis is found in the supplementary material. An analysis of the implications of using different D values is also attached as supplementary material. This analysis shows the robustness of RMDDC to small variations in D_I.

Table III shows, for the Seattle instance, a summary of the results for the different methods, when taking into account a stopping criterion of 48 hours. In the left part of the table, the results of the statistical tests are shown. In order to obtain an overall ranking of the different approaches tested, we carried out pairwise statistical comparisons between the 18 configurations. The column with the symbol ↑ shows the number of pair-wise comparisons where the model listed in each row is statistically better. The number of cases where it is worse is shown in the column with the symbol ↓. Finally, the number of cases where the differences are not statistically significant are shown in the column with the symbol ↔. In addition, a score is assigned to each model. This score is equal to the number of cases where the model was superior minus

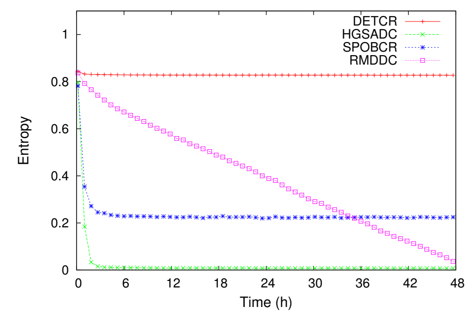

Fig. 2. Change in entropy over time for some selected techniques (Seattle instance), showing the gradual decrease of diversity in RMDDC

the number of cases where the model was inferior. The table shows the models sorted based on this score. The right part of the table shows the best and mean interference obtained by each model. The advantage of RMDDC is clear. In fact, it was statistically superior to every other technique tested in this paper, and its mean was superior to the best frequency plans output by the rest of the techniques. Additionally, it is worth noting that the previous best-known frequency plan for this instance incurred an interference level equal to 456 units [55], whereas our proposal was able to generate a frequency plan with an interference equal to 349.3 units, which is quite a remarkable improvement. Table IV shows the same information for the Denver instance. The superiority of RMDDC is also evident. Moreover, as in the Seattle instance, the best-known solution was further improved, in this case from $83,280$ units [9] to $82,994.9$.

In order to better understand the reasons for the superiority of RMDDC, it is important to analyze how diversity is managed in the different methods. Entropy [30] is a popular diversity metric that can be used for this purpose. Fig. 2 shows the evolution of the mean entropy in the Seattle instance for four sample cases. Note that the only model where there is a slow but continuous decrease in entropy is RMDDC, meaning that it is the only technique that induces a gradual switch between exploration and exploitation. The remaining models, which do

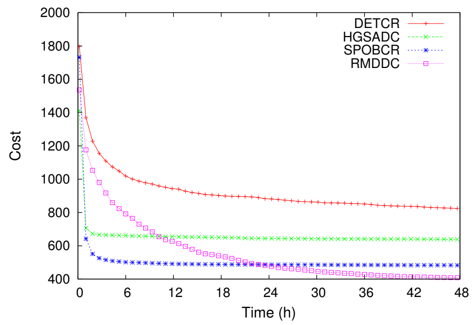

Fig. 3. Change in the mean cost over time for some selected techniques (Seattle instance), showing the good performance of RMDDC

not consider the stopping criterion internally, usually result in a reduction of diversity that is too slow or too fast. Moreover, none of them attain a reduction that can be considered to be similar to a linear reduction. Finally, Fig. 3 shows the evolution of the mean of the cost function for the selected techniques in the Seattle instance. The convergence in RMDDC is much slower than in the methods that lose diversity quickly. This means that in the first stages, the amount of intensification is much lower in the case of RMDDC, so it does not produce very promising results when compared to some other methods. However, since regions are not abandoned prematurely, the regions where intensification is promoted at the end of the runs are selected more intelligently in RMDDC, meaning that in the long term, much better results are obtained. The analyses for the Denver instance shows exactly the same pattern.

B. Robustness of Our Proposal

In order to show the robustness of our proposal, RMDDC was also used with the set of 42 instances provided in [57]. While in this case the formulation is more academic, some of these instances are highly complex. Thus, we also considered the same stopping criterion as in the previous case. Due to the large amount of instances provided in this set, we only considered the execution of RMDDC and GEN_ELIT, which is probably the most popular replacement phase. Table V shows the best and mean interference obtained by each model, as well as the best frequency plan found so far for each instance. In addition, the results of the statistical comparisons between RMDDC and GEN_ELIT are also provided. Specifically, the data from RMDDC for instances in which the statistical tests confirm its superiority against GEN_ELIT are shown in **bold**. The superiority of RMDDC is evident. In fact, it was superior with statistically significant differences in 27 cases and inferior in none. Most of the cases in which the differences were not significant correspond to the simplest instances, in which both proposals obtain the same cost in every execution. The previous best-known solutions are also shown in the table. Note that RMDDC found a new best-known frequency plan in 8 instances. These cases are marked with an '*' in the column corresponding to RMDDC. Only in two cases were the best currently known solutions not found. These last cases are marked with an '*' in the "Best Known" column. In both cases the instances are unweighted. The reasons behind the

TABLE V
SUMMARY OF RESULTS OBTAINED WITH RMDDC AND GEN_ELIT, HIGHLIGHTING THE STATISTICALLY SIGNIFICANT DIFFERENCES

Instance	GEN_ELIT		RMDDC		Best
	Mean	Best	Mean	Best	Known
AC-45-17-7	32	32	32	32	32
AC-45-17-9	15.6	15	**15**	**15**	15
AC-45-25-11	33.0	33	33	33	33
AC-95-9-6	31.0	31	31	31	31
AC-95-17-15	34.9	34	**33**	**33**	33
AC-95-17-21	10	10	10	10	10
GSM-93-9	34.7	32	**32**	**32**	32
GSM-93-13	8.2	7	**7**	**7**	7
GSM-246-21	84.7	81	**79.2**	**77***	78
GSM-246-31	28.6	27	**26.2**	**25**	24*
Test95-36	8	8	8	8	8
Test282-61	62.3	60	**54.7**	**53**	51*
Test282-71	33.5	31	**28.7**	**27**	27
Test282-81	12.8	11	**9.9**	**8***	9
P06-5-11	134.4	133	**133**	**133**	133
P06-3-31	118.3	115	**115.1**	**115**	115
P06b-5-21	52	52	52	52	52
P06b-5-31	25	25	25	25	25
P06b-3-31	112.4	112	**112**	**112**	112
P06b-3-71	26	26	26	26	26
GSM2-184-39	5252.1	5250	5251.6	5250	5250
GSM2-184-49	874	874	874	874	874
GSM2-184-52	162	162	162	162	162
GSM2-227-29	58370.9	56397	**56349**	**55339***	57790
GSM2-227-39	8948.2	8568	**8567.8**	**8283***	8656
GSM2-227-49	1998	1998	1998	1998	1998
GSM2-272-34	54390.8	52152	**51757**	**50940***	53254
GSM2-272-39	27758.1	25852	**26099.6**	**25542***	27416
GSM2-272-49	7211.5	7036	**7096.6**	**6957***	7107
1-1-...-2-50-5	1242.4	1242	**1242**	**1242**	1242
1-1-...-2-50-10	100.6	96	**96**	**96**	96
1-1-...-2-50-11	60.5	55	**55**	**55**	55
1-1-...-2-50-12	34.4	32	**32**	**32**	32
1-2-...-4-50-9	665	665	665	665	665
1-2-...-4-50-11	316.2	313	**313**	**313**	313
1-3-...-0-50-7	194.7	194	**194**	**194**	194
1-4-...-2-1-6	71.7	70	**70**	**70**	70
1-4-...-2-1-10	20.0	19	**19**	**19**	19
1-5-...-2-100-10	178.4	168	**168**	**168**	168
1-5-...-2-100-12	59.5	53	**53**	**53***	57
1-6-...-0-10000-10	6777	6777	6777	6777	6777
1-6-...-0-10000-13	1190	1190	1190	1190	1190

relatively poor behavior of the EAs lie with how plateaus are managed. This issue is explored further in the next section.

C. Management of Neutral Networks

The above analyses show that there are two instances where the best-known solutions are not found by our initial proposal. By inspecting the content of the population at different optimization stages, we ruled out premature convergence as a problem with the RMDDC strategy. However, the number of individuals in the population with the best fitness level found during the execution grew quite quickly in these instances. In fact, this issue appeared not only in the problematic instances, but in every unweighted instance. Fig. 4 shows, for each optimization stage, the maximum number of individuals that shared the best fitness level found in six test cases when the Hamming distance (D_{HAM}) was used with RMDDC. The three test cases listed at the top are unweighted, whereas the remaining ones are weighted. The differences between the weighted and unweighted cases are clear. Specifically, the number of repetitions tends to be smaller for weighted instances. Note that the appearance of several individuals with the same fitness level does not necessarily imply that they are in the same neutral network. However, the fact that this

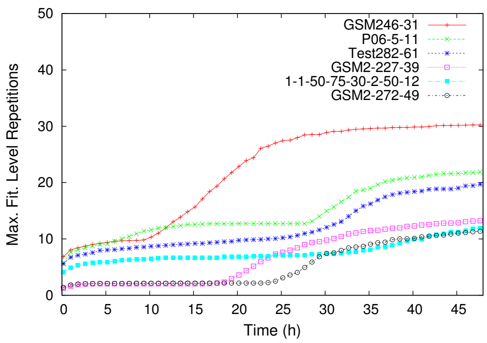

Fig. 4. Maximum number of individuals sharing the best found fitness level with RMDDC and D_{HAM} metric

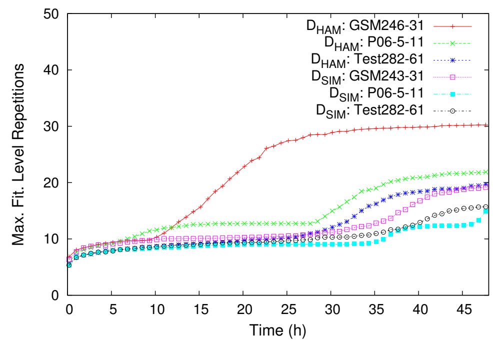

Fig. 5. Maximum amount of individuals sharing the best found fitness level with RMDDC using the D_{HAM} and D_{SIM} metric

TABLE VI
RESULTS OBTAINED WITH RMDDC USING TWO DIFFERENT DISTANCE METRICS, HIGHLIGHTING THE STATISTICALLY SIGNIFICANT DIFFERENCES

Instance	RMDDC D_{HAM}		RMDDC D_{SIM}	
	Mean	Best	Mean	Best
GSM-246-21	79.2	77	**78.3**	**77**
GSM-246-31	26.2	25	**25.2**	**24**
Test282-61	54.7	53	**53.6**	**51**
Test282-71	28.73333	27	**27.9**	**26***
Test282-81	9.966667	8	**9.4**	**8**
P06-3-31	115.1	115	115	115

situation arises precisely in the cases with larger percentage of neutral degree is quite suspicious. Showing that in fact, the individuals belong to the same neutral network is quite difficult due to the vast size of the neutral networks. In any case, we implemented a backtracking method to test whether one individual could be converted into another by following a path of neutral neighbors. This scheme was able to determine that some of the individuals present in the population after 24 hours of execution were indeed in the same neutral network. Since the executions of backtracking were limited to 10 minutes, however, and due to the huge sizes of the neutral networks, backtracking could not be used in other pairs of selected individuals to confirm whether individuals belonged to the same neutral network or not.

Due to the appearance of the aforementioned issues involving the neutral networks, we defined the distance D_{Sim} in an effort to avoid overcrowding in neutral networks. This metric replaced the Hamming distance in RMDDC and was tested in all the unweighted instances using the same parameterization as in previous experiments. Note that due to the way it is defined, this metric is specific to unweighted instances. While there might be some advantages in defining a metric with similar principles that is applicable to the weighted cases, the lower percentage of neutrality degree of these instances indicates that any gains might be not so remarkable. As a result, extending the definition of our novel distance metric to weighted instances is left for future work.

Table VI shows the results obtained with D_{HAM} and D_{SIM} for several instances. Specifically, these are the instances where RMDDC with D_{HAM} did not find the best-known result in every execution. Note that in the six instances selected, D_{SIM} provides important benefits. In every case, the mean of the results decreases and the statistical tests show that in five of the instances (shown in bold), the differences are statistically significant. RMDDC with D_{SIM} was also tested with the rest of the unweighted instances. In these cases, the best-known solution was found in every run both with D_{HAM} and D_{SIM}, meaning that for the easiest instances, RMDDC with D_{SIM} provides similar results to D_{HAM}, and for the most complex cases, it provides better results. Moreover, the best currently known frequency plan was obtained in GSM-246-31 and Test-282-61 when D_{SIM} was applied, whereas this was not the case with D_{HAM}. Additionally, a new best-known solution was found for the Test-282-71 instance. Thus, these results show that in the long term, avoiding overcrowding in neutral networks, contrary to the principles applied in some methods [41], results in higher-quality solutions.

Note that a way to underscore the effects of the D_{SIM} metric is to show the maximum number of individuals that shared the best fitness level during the executions. Fig. 5 presents this information for RMDDC with D_{HAM} and D_{SIM} in three selected unweighted instances. It is worth noting that the use of D_{SIM} causes the number of individuals populating a fitness level to decrease, meaning that, as suspected, the reason for this increase was precisely the overcrowding of neutral networks.

Finally, we would like to note that, while our proposal was designed to deal with long-term executions, we also conducted some experiments that considered shorter stopping criteria. In the case of the formulation by Luna et al. [5], as in other papers [8], a stopping criterion of 1 hour was considered. To our knowledge, the results further improved on those obtained by any other sequential scheme published so far. In the case of the instances provided by Montemanni [57], the stopping criterion was set to 40 minutes, similar to [10]. In these short-term executions, competitive results were obtained in most cases. However, the instances with the largest number of channels were problematic because this time was not enough to cause the population to converge, meaning high-quality results could not be obtained. The main reason is that the complexity of the local search used is quadratic with respect to the number of channels, so the application of this local search is not affordable when short executions are taken into account. The incorporation of the Tabu Search scheme used in [10] solved these issues. However, in the long term we could not obtain results as good as those in our current proposal, which is why the application of stochastic hill climbing was considered in this paper.

VII. Conclusions and Future Work

The Frequency Assignment Problem (FAP) is a very important NP-hard optimization problem whose study is interesting both because of its practical and theoretical implications. The FAP is a well studied problem to which several metaheuristics have been applied. In particular, several variants of EAs have shown great promise in dealing with the FAP. However, while EAs have performed remarkably well in some cases, in others, different optimizers such as trajectory-based techniques have further improved their results.

In this paper, the weaknesses of EAs are studied using two different formulations of the FAP. Two typical failure modes of EAs are identified as important sources behind the poor performance of EAs in some cases. Specifically, the well-known problem of premature convergence and the way of dealing with neutral networks are identified as important drawbacks involving the application of EAs to the FAP. In order to advance the state of the art in the application of EAs to the FAP, some of the most promising proposals are expanded upon. Particularly, some well-known local search schemes and genetic operators are extended and incorporated into our techniques. However, the key to the good performance of our proposal is the incorporation and extension of a recently proposed method for handling premature convergence (RMDDC). One of the main new features of the method applied is that it relates the decisions made in the survivor strategy to the elapsed time and to the stopping criterion. As a result, the replacement strategy is automatically adapted to the needs of the different optimization stages. This is implemented by incorporating a penalty approach that employs a distance metric as part of a replacement strategy that converts a single-objective problem into a multi-objective one by considering the contribution to diversity as an additional objective. Specifically, the technique penalizes those individuals that contribute too little to diversity. In order to distinguish between promising individuals and those that contribute too little to diversity, the required minimum contribution is adjusted to the requirements of the different optimization stages. Several updating mechanisms are tested, concluding that updating methods that gradually switch from exploration to exploitation, such as the linear updating, are the most promising ones. Additionally, we show that in the case of the FAP, the population dynamics behaves as in other problems studied in the presence of neutral networks. Specifically, in the instances with a higher degree of neutrality, several members of the population tend to occupy the same neutral networks. This is especially true in the case of unweighted instances. Given that, in the long term this behavior has a negative impact on performance, a new distance metric is defined and incorporated into RMDDC. The principle of this metric is to induce low distances for individuals occupying the same neutral network. By doing so, this incorporation avoids the overcrowding of neutral networks, resulting in higher-quality solutions in the long term.

Experimental validation with two different formulations of the FAP and a set of 44 instances shows the remarkable performance of these new proposals. First, comparisons between our proposals and a large set of different mature, as well as more recent approaches that modify the way of managing the diversity, shows the superior performance of RMDDC. Additionally, the frequency plans obtained are compared against the best plans detailed in related papers. Our proposal generated new best-known frequency plans in 11 cases, and replicated the best currently known plans in the remaining cases. This represents a remarkable improvement in a field where many different proposals have been devised. Moreover, a sensitivity analysis shows that, in the long term, the behavior is quite robust and several different parameterizations could have been used to obtain such high-quality results. Additionally, it is important to note that while some of the proposals reported in this paper are specific to the FAP, the core of the proposals are generally applicable, so many of the advances can be directly adapted to other optimization problems. Moreover, in light of the results, we believe that more research on how to manage neutral networks in EAs is warranted.

Several lines of future work might be explored. First, given that most of the proposals are general, we would like to apply them to different combinatorial and continuous problems with the aim of better demonstrating their generality and robustness. Moreover, since there are several different formulations of the FAP, it would be interesting to study them for the purpose of analyzing whether the drawbacks arising in the two formulations studied in this paper also affect other cases. Additionally, it would be interesting to design some special distance metrics based on the principles that governed the design of D_{Sim} but applicable to the weighted instances. One of our current hypotheses is that the main reason for the superior performance of RMDDC when compared to other ways of dealing with diversity is the incorporation of the elapsed time and stopping criterion into the decisions made by the EAs. Thus, adapting some of the most widely applied techniques, such as crowding or clearing, by incorporating this principle seems to be a very promising area of research. Finally, given the long executions required to generate the new best-known frequency plans attained in this paper, parallelizing our proposals would seem to be a natural extension of our work. In addition to parallelizing our proposal using standard parallel models, we would like to propose new centralized and distributed parallel methods that explicitly consider the management of diversity.

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