Learning Representations with Physics Constraints

Master's Thesis

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Acknowledgments

Thanks to my cat.

To my dog.

Committee

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Abstract

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Chapter 1

Package Demonstration

This chapter demonstrates the mathematical and typographical capabilities built into the thesis class.

1.1 Mathematical Notation

1.1.1 Basic Mathematics

The fundamental theorem of calculus states that if f is continuous on $[a, b]$ and F is an antiderivative of f, then:

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a) \tag{1.1}$$

1.1.2 Vectors and Bold Mathematics

Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^T$ be a vector in $\mathbb{R}^n$. The gradient of a function $f : \mathbb{R}^n \to \mathbb{R}$ is denoted as $\vec{\nabla} f(\boldsymbol{x})$ or $\nabla f(\boldsymbol{x})$.

For machine learning, we often work with:

$$\begin{aligned} \boldsymbol{W} &\in \mathbb{R}^{m\times n} \quad \text{(weight matrix)} && (1.2)\\ \boldsymbol{b} &\in \mathbb{R}^{m} \quad \text{(bias vector)} && (1.3)\\ \boldsymbol{y} &= \boldsymbol{W}\boldsymbol{x} + \boldsymbol{b} \quad \text{(linear transformation)} && (1.4) \end{aligned}$$

1.1.3 Calculus and Differentials

The total differential of a function $f(x, y)$ is:

$$\mathrm{d}f = \frac{\partial f}{\partial x}\,\mathrm{d}x + \frac{\partial f}{\partial y}\,\mathrm{d}y \tag{1.5}$$

For partial derivatives, we use:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \nabla^2 f \tag{1.6}$$

1.1.4 Optimization Operators

The thesis class includes predefined mathematical operators for optimization:

$$\boldsymbol{x}^* = \arg\max_{\boldsymbol{x}\in\mathcal{X}} f(\boldsymbol{x}) \tag{1.7}$$

$$\boldsymbol{w}^* = \arg\min_{\boldsymbol{w}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|^2 + \lambda\|\boldsymbol{w}\|^2 \tag{1.8}$$

These operators automatically handle proper spacing and formatting in optimization problems.

1.1.5 Probability and Statistics

The thesis class includes flexible commands for probability and statistics notation:

Basic Probability Notation

Working with probability spaces and distributions:

$$X \sim \mathcal{N}(\mu, \sigma^2) \quad \text{(normal distribution)} \tag{1.9}$$

$$Y \sim \mathcal{U}(a, b) \quad \text{(uniform distribution)} \tag{1.10}$$

$$Z \in \mathbb{R} \quad \text{(real-valued random variable)} \tag{1.11}$$

Expectation Operator

The `\Mean` command adapts to different contexts:

$$\mathbb{E} \quad \text{(plain expectation symbol)} \tag{1.12}$$

$$\mathbb{E}_X \quad \text{(expectation w.r.t. X)} \tag{1.13}$$

$$\mathbb{E}_X\left[Y\right] = \mathbb{E}_X\left[Y\right] \quad \text{(expectation of Y w.r.t. X)} \tag{1.14}$$

$$\mathbb{E}_\mu\left[X^2\right] = \mathbb{E}_\mu\left[X^2\right] \quad \text{(second moment)} \tag{1.15}$$

Probability Operator

The `\Prob` command works similarly:

$$\mathbb{P} \quad \text{(plain probability symbol)} \tag{1.16}$$

$$\mathbb{P}_X \quad \text{(probability w.r.t. X)} \tag{1.17}$$

$$\mathbb{P}_X\left(A\right) = \mathbb{P}_X\left(A\right) \quad \text{(probability of event A)} \tag{1.18}$$

$$\mathbb{P}_\mu\left(X > 0\right) = \mathbb{P}_\mu\left(X > 0\right) \quad \text{(conditional probability)} \tag{1.19}$$

Variance Operator

The `\Var` command for variance calculations:

$$\mathbb{V} \quad \text{(plain variance symbol)} \tag{1.20}$$

$$\mathbb{V}_X \quad \text{(variance w.r.t. X)} \tag{1.21}$$

$$\mathbb{V}_X\left[Y\right] = \mathbb{V}_X\left[Y\right] \quad \text{(variance of Y w.r.t. X)} \tag{1.22}$$

$$\mathbb{V}_\theta\left[X\right] = \mathbb{V}_\theta\left[X\right] \quad \text{(parametric variance)} \tag{1.23}$$

Combined Example

A typical machine learning probability statement:

$$\mathbb{E}_\theta\left[\ell(X,Y)\right] = \mathbb{E}_\theta\left[\mathbb{P}_{X,Y}\left(Y \neq f_\theta(X)\right)\right] \tag{1.24}$$

where ℓ is the loss function and f_θ is our model.

1.1.6 Special Sets and Greek Letters

We work with various mathematical sets:

- Natural numbers: $\mathbb{N} = \{1, 2, 3, \ldots\}$
- Real numbers: $\mathbb{R}$
- Complex numbers: $\mathbb{C}$
- Probability space: $(\Omega, \mathcal{F}, \mathbb{P})$

Greek letters in upright form: $\pi \approx 3.14159$, $e = 2.71828$.

1.1.7 Theorems and Proofs

Theorem 1.1 (Pythagorean Theorem)**.** *In a right triangle with legs of length a and b, and hypotenuse of length c, we have:*

$$a^2 + b^2 = c^2 \tag{1.25}$$

Proof. The proof is left as an exercise for the reader. □

1.2 Cross-References

This section demonstrates smart referencing with cleveref. We can reference Eq. (1.1), Theorem 1.1, and Chapter 1 automatically.

Multiple references work too: Eq. (1.1) and Theorem 1.1 show different mathematical concepts. Use `cref` in the middle of the sentence and `Cref` at the beginning.

1.3 Citations

Mathematical typesetting follows the principles established by [1], while modern computational approaches build on [2].

1.4 Tables and Figures

1.4.1 Professional Tables with Booktabs

Here's a professional table using the booktabs package:

Table 1.1: Comparison of Machine Learning Models

Model	Accuracy (%)	Training Time (min)
Linear Regression	78.5	2.3
Random Forest	85.2	15.7
Neural Network	92.1	120.4

1.4.2 Colored Tables

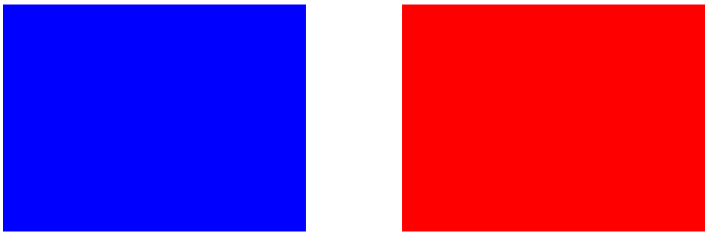
Using `xcolor` and `colortbl` packages for enhanced presentations:

1.4.3 Subfigures and Enhanced Captions

The subcaption package allows for complex figure arrangements:

Table 1.2: Performance Metrics with Color Coding

Method	Precision	Recall
Baseline	0.75	0.68
Our Method	**0.89**	**0.91**
State-of-art	0.82	0.85



(a) Training Loss (b) Validation Accuracy

Figure 1.1: Training Progress: a shows decreasing loss while b shows improving accuracy

1.4.4 Colors and Typography

The `xcolor` package enables rich text formatting:

- Important warnings can be highlighted
- Key concepts stand out
- Success indicators provide feedback
- Highlighted text draws attention

Mathematical expressions can also use colors:

$$\boldsymbol{W}\boldsymbol{x} + \boldsymbol{b} = \boldsymbol{y} \tag{1.26}$$

Note: The `parskip` package automatically improves paragraph spacing throughout the document.

Bibliography

[1] Donald E. Knuth. *The TeXbook*. Addison-Wesley, 1984.

[2] Yann LeCun, Yoshua Bengio, and Geoffrey Hinton. "Deep learning". In: *Nature* 521.7553 (2015), pp. 436–444.