Learning Objectives

- Interpret the eigenvectors geometrically.
- Verify if a vector or a value is an eigenvector or eigenvalue

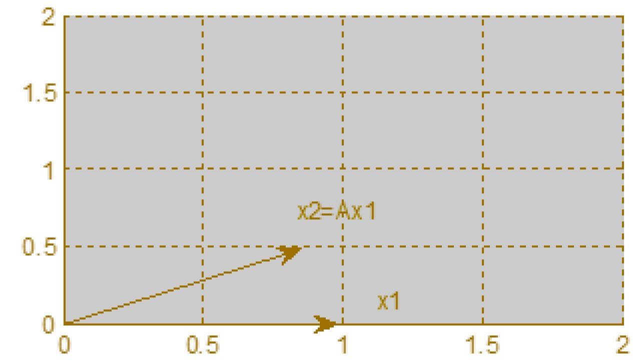
Matrix Transformation

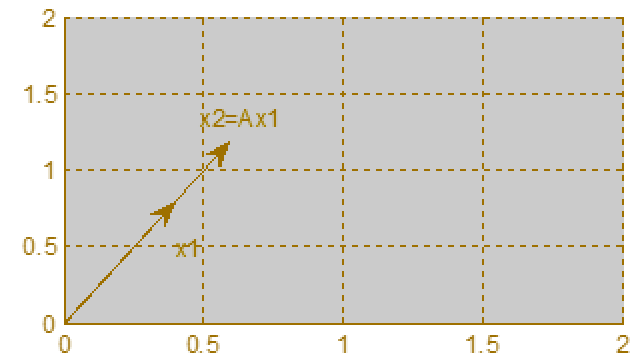
- Anytime a matrix is multiplied by a vector, that vector gets transformed to a new vector. This transformation is denoted by $x \text{ a } Ax$.
- For example matrix $A = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$

transforms vector $x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to a new vector

on the plane $x_2 = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix}$.

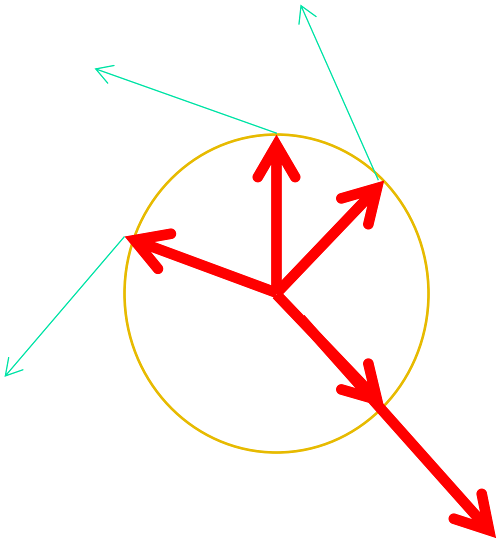
- As can be seen in the figure the new vector is a rotated version of the old vector.
- If A transforms a vector x into a parallel (same direction) vector, then x is called an eigenvector of A.





Geometrical Interpretation of Eigenvectors

- These 4 red vectors are unit vectors (inside a unit circle) each multiplied by a matrix A.
- Only 1 out of these 4 vectors is transformed to a new vector that is aligned (parallel) with the original.
- This vector is an eigenvector of matrix A.



Example

- Let $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$, $\vec{u} = \begin{bmatrix} -1 \\ +1 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} +2 \\ +1 \end{bmatrix}$.
- Are either of the two vectors an eigenvector of matrix A?

```
A = np.array([[3,-2],[1,0]])
u = np.array([[-1],[1]])
v = np.array([[2],[1]])
Au = A@u
Av = A@v
print('Au=\n',Au, '\nAv=\n',Av)
```

```
Au=
 [[-5]
 [-1]]
Av=
 [[4]
 [2]]
```

- A **v** is just 2**v**. So A only stretches v, and hence v is an eigenvector of A.
- The images of **u** and **v** under multiplication by A are shown in Fig. 1.

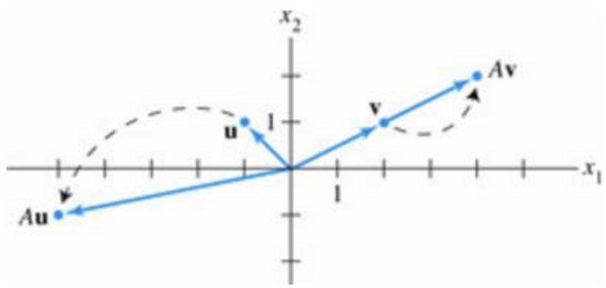


FIGURE 1 Effects of multiplication by A.