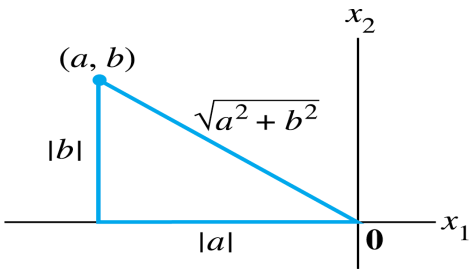
THE LENGTH OF A VECTOR

- If we identify $\mathrm{v} = \begin{bmatrix} a \\ b \end{bmatrix}$ with a geometric point in the plane, then $\|\mathrm{v}\|$ coincides with the standard notion of the length of the line segment from the origin to **v**.
- This follows from the Pythagorean Theorem applied to a triangle such as the one shown in the following figure.



Interpretation of $\|\mathbf{v}\|$ as length.

THE LENGTH OF A VECTOR

- **Definition:** The **length** (or **L2 norm**) of **v** is the nonnegative scalar $\|v\|$ defined by

$$||v||=\sqrt{v_1^2+v_2^2+\cdots+v_n^2}$$

- Can you express norm of v in terms of a dot product?
- $||v|| = \sqrt{v \cdot v}$

THE LENGTH OF A VECTOR in Python

- Find length of $u = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$ in Python.

```
import math
math.sqrt(np.dot(u,u))
```

5.099019513592785

```
from numpy.linalg import norm
norm(u)
```

5.099019513592785

Unit Vector

- A vector whose length is 1 is called a **unit vector**.

- If we *divide* a nonzero vector **v** by its length—that is, multiply by $1/\|\mathrm{v}\|$—we obtain a unit vector **u** because the length of **u** is.$(1/\|\mathrm{v}\|)\|\mathrm{v}\|$

- The process of creating **u** from **v** is sometimes called **normalizing v**, and we say that **u** is *in the same direction* as **v**.

Finding Unit Vector – Example Python

- **Example 2:** Let $\mathrm{v} = (1, -2, 2, 0)$. Find a unit vector **u** in the same direction as **v**.

```
a = np.array([1,-2,2,0])
unitA = a/norm(a)
print(unitA)
```

```
[ 0.33333333 -0.66666667  0.66666667  0.        ]
```

Verify your answer

- To check that $\|u\| = 1$, it suffices to show that $\|u\|^2 = 1$

.

$$\|u\|^2 = u \cdot u = \left(\frac{1}{3}\right)^2 + \left(-\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + (0)^2$$

$$= \frac{1}{9} + \frac{4}{9} + \frac{4}{9} + 0 = 1$$

```
In [44]: norm(unitA)

Out[44]: 1.0
```