# MATRIX EQUATION - Example 2

- Also note, that we can write $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

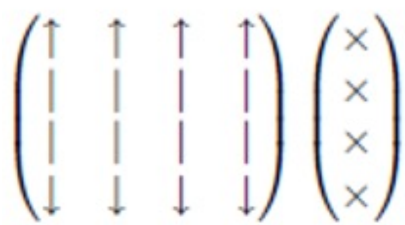
- as this: $x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ -5 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

- The above expression is called **Linear Combination** of columns of A.

# Linear Combination

- **Definition:** If $A$ is an m by n matrix, with columns $\mathbf{a}_1, \ldots, \mathbf{a}_n$, and if $\mathbf{x}$ is in $\mathbb{R}^n$, then the **product of $A$ and $\mathbf{x}$**, denoted by $A\mathbf{x}$, **is the linear combination of the columns of $A$ using the corresponding entries in $\mathbf{x}$ as weights**; that is,

$$A\mathrm{x} = \begin{bmatrix} \mathrm{a}_1 & \mathrm{a}_2 & \cdots & \mathrm{a}_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x_1\mathrm{a}_1 + x_2\mathrm{a}_2 + \ldots + x_n\mathrm{a}_n$$



- Note that $A\mathbf{x}$ is defined only if the number of columns of $A$ equals the number of entries in $\mathbf{x}$.

# Example

**EXAMPLE 4** Figure 8 identifies selected linear combinations of $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. (Note that sets of parallel grid lines are drawn through integer multiples of $\mathbf{v}_1$ and $\mathbf{v}_2$.) Estimate the linear combinations of $\mathbf{v}_1$ and $\mathbf{v}_2$ that generate the vectors $\mathbf{u}$ and $\mathbf{w}$.

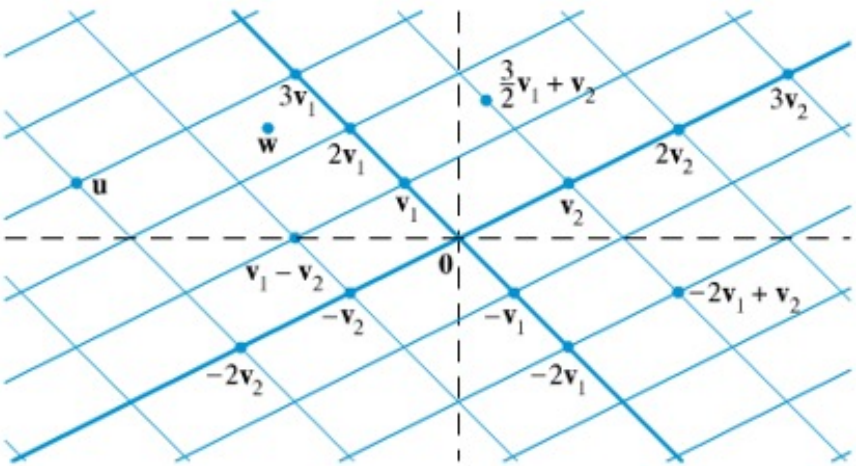
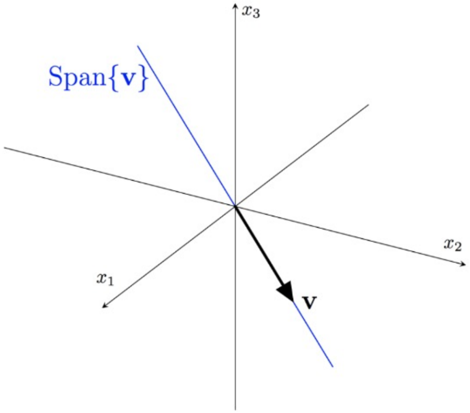



**FIGURE 8** Linear combinations of $\mathbf{v}_1$ and $\mathbf{v}_2$.

# Span

- The span of one vector forms a line that passes through the origin.




- The span of non-colinear vector forms a plane that passes through the origin.