Dot Product - Example

- Example: $u = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$ $v = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$
- u.v = (1)(2)+(-3)(1)+(4)(-2) = 2-3-8=-9
- Find the dot product of u and v in Python.

```
import numpy as np
```

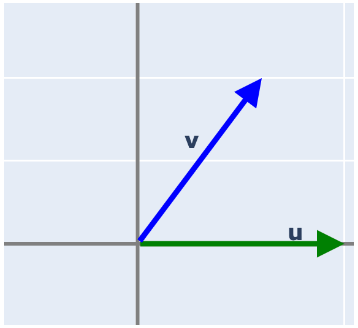
```
u = np.array([1,-3,4])
```

```
v = np.array([2,1,-2])
np.dot(u,v)
```

```
-9
```

Alternative method for finding the dot product

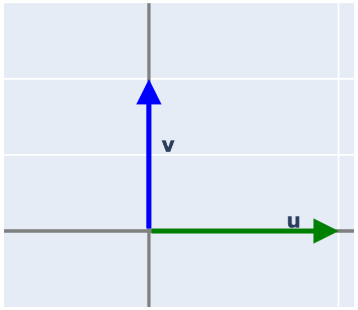
- There is a nice connection between their inner product and the **angle** between the two line segments from the origin to the points identified with **u** and **v**.



- The formula is $u \cdot v = \|\mathrm{u}\| \|\mathrm{v}\| \cos \vartheta$

(proof on the last slide)

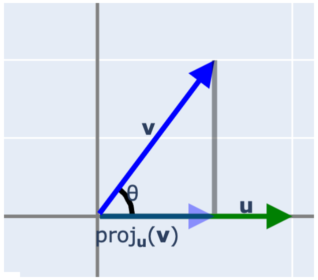
- Two vectors **u** and **v** are **orthogonal** (to each other) if $u \cdot v = 0$.



Geometry of the Dot Product

- Dot product gives information about the geometrical relationship of the two vectors.
- What is the length of projection of vector v

on to vector u in this diagram? $|Proj_u^v| = |v|\cos(\theta)$



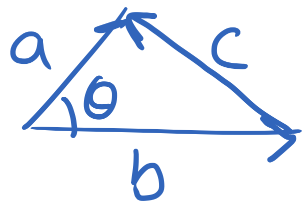
- Multiply both sides by |u|: $|u||Proj_u^v| = |u||v|\cos(\theta)$
- The right hand side of the above expression is the definition of the dot product, therefore:

$$|u||Proj_u^v| = \mathbf{v} \cdot \mathbf{u}$$

- The dot product gives you the product of the length of the projection of one, times the length of the other vector, regardless of the dimension of the original vectors.

Proof

From trig:



$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$$\|c\|^2 = \|a\|^2 + \|b\|^2 - 2\|a\|\,\|b\|\cos\theta$$

$$\|a-b\|^2 = \|a\|^2 + \|b\|^2 - 2\|a\|\,\|b\|\cos\theta \implies$$

$$\boxed{\|a\|\|b\|\cos\theta} = \frac{\|a\|^2 + \|b\|^2 - \|a-b\|^2}{2}$$

$$= \frac{1}{2}\left[a_1^2 + a_2^2 + b_1^2 + b_2^2 - (a_1-b_1)^2 - (a_2-b_2)^2\right]$$

$$= a_1 b_1 + a_2 b_2 = \boxed{a \cdot b}$$