# Vectors

## Vectors

- Vectors are ordered list of numbers.

- An example of a vector with two entries is

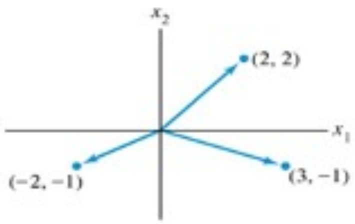
$$\mathrm{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad ,$$

where $w_1$ and $w_2$ are any real numbers.

- The set of all vectors with 2 entries is denoted by $\mathrm{R}^2$

# Geometric Descriptions of $R^2$

- Vectors are lines that have magnitude and direction.




- We may regard $R^2$ as the set of all 2D vectors in the plane.

# Vector Operations

- **Example 1:** Given $\mathrm{u} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $\mathrm{v} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, find

  $4\mathbf{u}$, $(-3)\mathrm{v}$, and $4\mathrm{u} + (-3)\mathrm{v}$.

- **Solution:**

$$4\mathrm{u} = \begin{bmatrix} 4 \\ -8 \end{bmatrix} \qquad (-3)\mathrm{v} = \begin{bmatrix} -6 \\ 15 \end{bmatrix} \qquad 4\mathrm{u} + (-3)\mathrm{v} = \begin{bmatrix} 4 \\ -8 \end{bmatrix} + \begin{bmatrix} -6 \\ 15 \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$$

# Vector Operations in Python

- **Example 1:** Given $u = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, find

  $4\mathbf{u}$, $(-3)v$, and $4u + (-3)v$.

- **Solution:**

```
import numpy as np
```

```
u = np.array([1,-2])
u.shape
```

```
(2,)
```

```
v = np.array([2,-5])
```

```
print(4*u, -3*v)
```

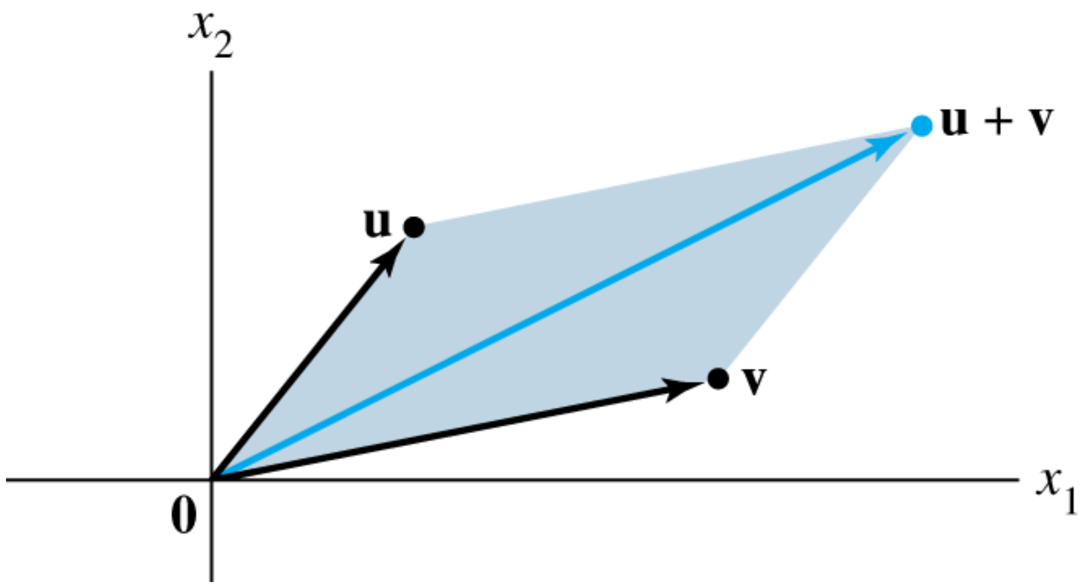
```
[ 4 -8] [-6 15]
```

```
4*u-3*v
```

```
array([-2,  7])
```

# Parallelogram Rule For Addition

- If $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^2$ are represented as points in the plane, then $\mathbf{u}+\mathbf{v}$ corresponds to the fourth vertex of the parallelogram whose other vertices are $\mathbf{u}$, $\mathbf{0}$, and $\mathbf{v}$. See Fig. 3 below.



# VECTORS IN $\mathbb{R}^3$ and $\mathbb{R}^n$

- Vectors in $\mathbb{R}^3$ are $3\times 1$ column matrices with three entries.
- They are represented geometrically by points in a three-dimensional coordinate space, with arrows from the origin sometimes included for visual clarity.
- If $n$ is a positive integer, $\mathbb{R}^n$ (read "r-n") denotes the collection of all lists (or *ordered n-tuples*) of $n$ real numbers, usually written as $n\times 1$ column matrices, such as

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}.$$