Diagonalization

Why Diagonalization? Example

- Given $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$ calculate D^3.
- $D^2 = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 25 & 0 \\ 0 & 9 \end{bmatrix}$
- $D^3 = \begin{bmatrix} 25 & 0 \\ 0 & 9 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 125 & 0 \\ 0 & 27 \end{bmatrix}$
- In general $D^k = \begin{bmatrix} 5^k & 0 \\ 0 & 3^k \end{bmatrix}$

Why Diagonalization?

- If A is ***similar*** to D, then by definition **A=PDP⁻¹**

- $\boldsymbol{A^k} = \underbrace{(PDP^{-1})(PDP^{-1})\ldots(PDP^{-1})}_{k\ times} =$

$$\left(PD\underbrace{P^{-1}P}_{I}DP^{-1}\right)\ldots(PDP^{-1}) =$$

- $(PD^2P^{-1})\ldots(PDP^{-1}) = \boldsymbol{PD^kP^{-1}}$

Example

- Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula for A^k, given that $A = PDP^{-1}$,

where $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$

Solution

$$P^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$$

$$A^k = PD^kP^{-1} = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5^k & 0 \\ 0 & 3^k \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2\cdot 5^k - 3^k & 5^k - 3^k \\ 2\cdot 3^k - 2\cdot 5^k & 2\cdot 3^k - 5^k \end{bmatrix}$$