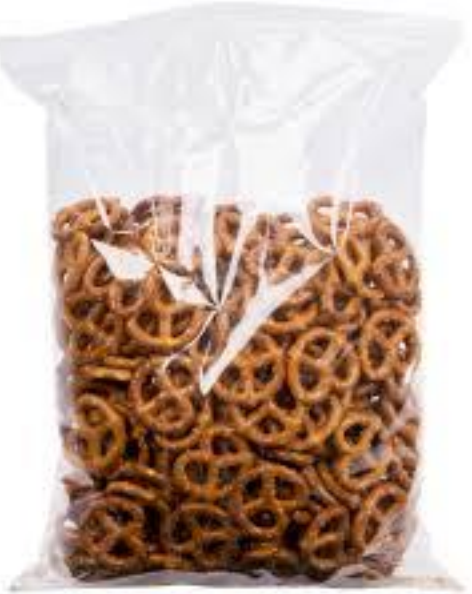
Hypothesis Testing

- Learning objectives:
  - Define a hypothesis.
  - Identify the null and alternative hypotheses.

The Nature of Hypothesis Testing

- We often use inferential statistics to make decisions about the value of a parameter, such as a population mean.
- For example, we might need to decide whether the mean weight, μ of all bags of pretzels packaged by a particular company differs from the advertised weight of 454 grams.
- As another example we might want to determine whether the mean age, μ of all cars in use has increased from the year 2020.
- One of the most commonly used methods for making such is to perform a hypothesis test.
- Typically, a hypothesis test involves two hypotheses: the null hypothesis and the alternative hypothesis (or research hypothesis).



Null and Alternative Hypotheses

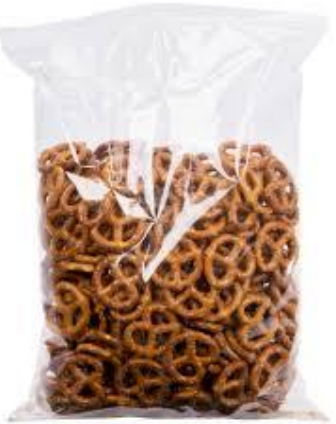
Null hypothesis: A hypothesis to be tested. We use the symbol H_0 to represent the null hypothesis.

Alternative hypothesis: A hypothesis to be considered as an alternative to the null hypothesis. We use the symbol H_a to represent the alternative hypothesis.

Hypothesis test: The problem in a hypothesis test is to decide whether the null hypothesis should be rejected in favor of the alternative hypothesis.

Null Hypothesis (H_0)

- The statement being tested in a test of significance is called the *null hypothesis*.
- Usually the null hypothesis:
  - is a statement of "no effect" or "no difference" (i.e. the difference is null),
  - is a statement about a population,
  - is expressed in terms of a (some) parameter(s).
- Example: What is the null hypotheses in the pretzel packaging example?
- Null hypothesis: the mean weight of all bags of pretzels packaged equals the advertised weight of 454 g. H_0: μ=45



Alternative Hypothesis (H_a)

The name we give to **the statement we suspect is true.**

The choice of the alternative hypothesis depends on and should reflect the purpose of the hypothesis test. Three choices are possible for the alternative hypothesis.

- If the primary concern is deciding whether a population mean, μ, is *different from* a specified value μ_0, we express the alternative hypothesis as

  $$H_a: \mu \neq \mu_0.$$

  A hypothesis test whose alternative hypothesis has this form is called a **two-tailed test.**
- If the primary concern is deciding whether a population mean, μ, is *less than* a specified value μ_0, we express the alternative hypothesis as

  $$H_a: \mu < \mu_0.$$

  A hypothesis test whose alternative hypothesis has this form is called a **left-tailed test.**
- If the primary concern is deciding whether a population mean, μ, is *greater than* a specified value μ_0, we express the alternative hypothesis as

  $$H_a: \mu > \mu_0.$$

  A hypothesis test whose alternative hypothesis has this form is called a **right-tailed test.**

A hypothesis test is called a **one-tailed test** if it is either left tailed or right tailed.

- Example: What is the alternative hypotheses in the pretzel packaging example.
- Alternative hypothesis: the mean weight of all bags of pretzels packaged differs from the advertised weight of 454 g.
- H_a: $\mu \neq 454$
- This is a two-sided hypothesis.

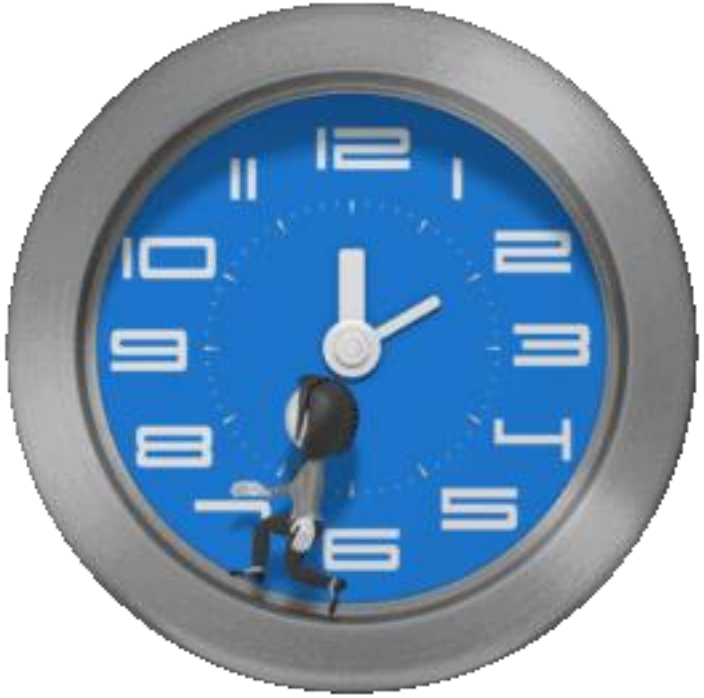
Example: Choose one-sided or two-sided

- **Example:** Your company hopes to reduce the mean time (μ) required to process customer orders. At present, this mean is 3.8 days. You study the process and eliminate some unnecessary steps.
- Q: Did you succeed in decreasing the average process time?

Target: to show that the mean is now less than 3.8 days.

- So alternative hypothesis is one-sided
- The null hypothesis is “no change” value

$$H_o : \mu = 3.8 \qquad H_a : \mu < 3.8$$



Example: Choose one-sided or two-sided

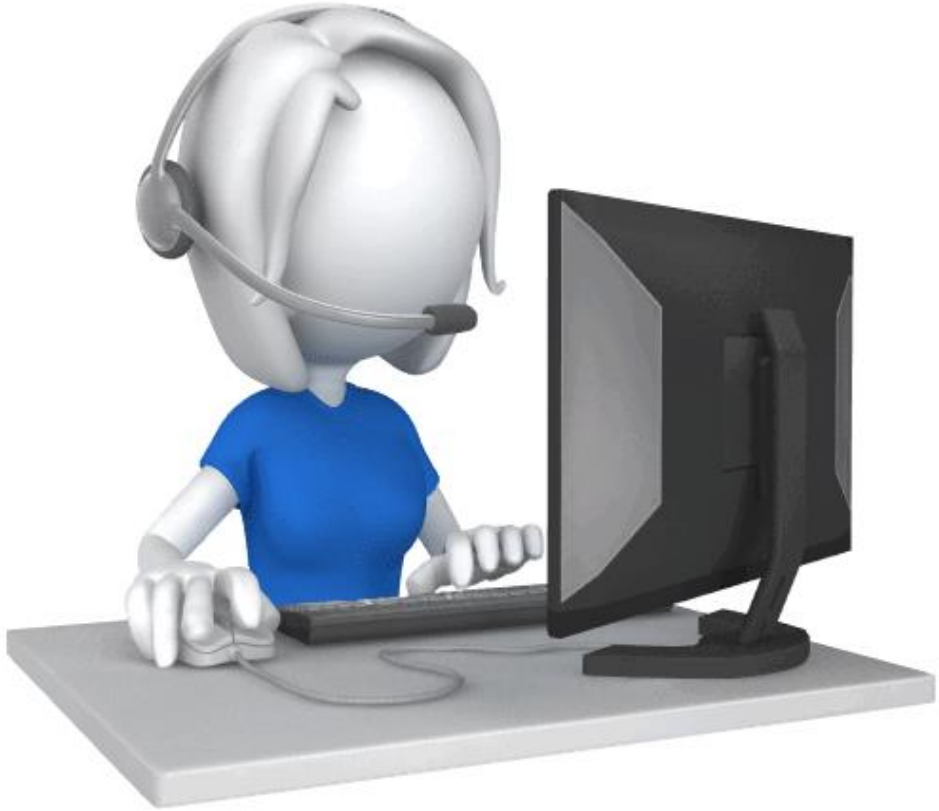
The mean area of several thousand apartments in a new development is advertised to be 1250 sqft. A tenant group thinks that the apartments are smaller than advertised. They hire an engineer to measure a sample of apartments to test their suspicion.

- H_0: $\mu=1250$ vs. H_a: $\mu<1250$

Example: Choose one-sided or two-sided

Last year, your company's service technicians took an average of 2.6 hours to respond to trouble calls from business customers who purchased service contracts. Do this year's data show a different average response time?

- H_0: $\mu = 2.6$ vs. H_a: $\mu \neq 2.6$



Example: Choose one-sided or two-sided

Experiments concerning learning in animals sometimes measure how long it takes a mouse to find its way through a maze. The mean time is 18 seconds for one particular maze. A researcher thinks that a loud noise will cause the mice to complete the maze faster. She measures how long each of 10 mice takes with a noise as stimulus.

- H_0: $\mu=18$ vs. H_a: $\mu<18$